Study on Thermal Instability of Rivlin-Ericksen Fluid with Suspended Particles

A

Thesis

Submitted to

Lovely Professional University

for the award of the degree

of

DOCTOR OF PHILOSOPHY

IN

MATHEMATICS

By

Rajesh Kumar Gupta

Guide

Dr. Mahinder Singh

Faculty of Technology and Sciences

Lovely Professional University

Phagwara

August,2014

DECLARATION

I declare that the thesis entitled "**Study on Thermal Instability of Rivlin-Ericksen Fluid with Suspended Particles"** has been prepared by me under the guidance of Dr. Mahinder Singh, Assistant Professor of Mathematics, Govt. P.G. College, Seema (Rohru), District Shimla (Himachal Pradesh) - 171207. No part of this thesis has formed the basis for the award of any degree or fellowship previously.

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CERTIFICATE

I certify that Rajesh Kumar Gupta has prepared his thesis entitled " Study on Thermal Instability of Rivlin-Ericksen Fluid with Suspended Particles ", for the award of Ph.D degree of the Lovely Professional University, under my guidance. He has carried out the work at the Department of Mathematics, Lovely Professional University ,Phagwara.

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DATE:

ABSTRACT

My research aims to analyze hall current effects and double diffusive effects in the presence of suspended particles on the thermal instability of the non-Newtonian viscoelastic fluid whose non linear relation between the stress and strain rate (which includes deformation, rotation and extension) is given by Rivlin-Ericksen in 1955.

To understand the applied problem of real life, one must know the physics of the problem and able to interpret the results obtained. In the introductory chapter, all the basics which are essential for the understanding of the problems discussed in thesis are well explained. Basic terms explained with the help of examples and real life applications. No problem can be solved without assumptions, so fundamental assumptions are also explained. In this chapter all the terms used in the thesis are explained for the understanding of general investigations in the subsequent chapters 2, 3 and 4. Flow governing equations based on the various principles of conservation like mass, momentum and energy are discussed in detail. Fluid properties, fluid types like Newtonian and non-Newtonian fluid with their specific applications and uses are explained in this introductory chapter. Concept of hydrodynamic stability of the system in terms of various parameters is also explained. Also light is thrown on the procedure of the problems formulated in the subsequent chapters.

Problem is formulated for non-Newtonian and viscoelastic fluid named Rivlin-Ericksen in porous medium in chapter 2. Fluid is permeated with suspended particles and uniform magnetic field is also considered. Governing equations for the problem were obtained and the initial state of the system described in terms of various parameters like velocity field, Pressure, magnetic field etc. is perturbed or disturbed. All the disturbances analyzed and it is found that relation between strain rate and stress become linear in case of stationary convection. Perturbations due to the magnetic field were decaying while the perturbations due to the suspended particles and medium permeability were growing. Oscillatory modes exist only due to the presence of magnetic field.

Study devoted to the effect of magnetic field which change the direction of flow of electric current when applied at right angle to electric field on the thermal instability in porous medium of dusty viscoelastic fluid in chapter 3. Problem related to the effect of

hall current on the thermal instability of viscoelastic fluid with dust in porous medium was modeled in terms of mathematical equations, initial state of the system is perturbed as in previous chapter by giving small perturbations to the physical quantities like pressure, velocity, temperature, density and magnetic field etc. Linearize the system by neglecting all the non linear terms. Dispersion relation is obtained after the normal mode analysis. It is observed that perturbations due to suspended particles and hall current were growing while the perturbations due to the magnetic field and compressibility were decaying in the system for the case of stationary convection. Magnetic field stabilize the effect of permeability on thermal instability. Oscillatory modes were introduced by viscoelastic parameter, magnetic field and hall current. Behavior of hall current, permeability, magnetic field and suspended particles on the critical thermal Rayleigh number were shown graphically.

Double diffusive or thermosolutal convection i.e. the presence of more than one component with different diffusivities like heat and salt in the fluid layer, explained in chapter 4. Now temperature and salt field are two destabilizing sources for the density difference whereas in standard Bénard problem, temperature field is the only destabilizing source. This situation is similar to ocean where both salt and heat are present simultaneously and chemical engineering with two or more components of different molecular diffusivities. Also in case of stellar helium acts like salt in raising the density and diffusing more slowly than heat. Mathematical model for the problem of double-diffusive convection in presence of compressible fluid with fine dust was designed in terms of equation. Using the same procedure and techniques or methods as in previous chapters to find the solution. It is observed that relation between strain rate and stress become linear in case of stationary convection due to vanishing of viscoelastic parameter. Presence of stable solute gradient, suspended particles and viscoelasticity introduced oscillatory modes. The stable solute gradient and compressibility has a stabilizing effect and suspended particles hasten the onset of thermosolutal instability.

Programming codes were written for the variations of Rayleigh numbers obtained in the chapters 2, 3 and 4 by assigning numerical values to all other parameters, these codes will calculate the Rayleigh number and will also plot the graph.

ACKNOWLEDGEMENT

My thesis is based on the research in applied mathematics for developing mathematical models relevant to the study of non-Newtonian Rivlin-Ericksen fluid. The research took place at Department of Mathematics, Lovely Professional University, Phagwara.

I express my deep gratitude to my supervisor **Dr. Mahinder Singh** for his support, guidance, inspiring collaboration and for providing background on which the entire work is based.

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I wish to express my gratitude to my family, relatives and friends for their sincere interest in my work. Most of all, I thank my beloved wife, **Neetu Gupta**, without her support, this work would never have been completed.

I DEDICATE MY THESIS TO MY PARENTS.

August, 2014 Rajesh Kumar Gupta

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Chapter 1

Introduction

In the introductory chapter, all the basic terms and procedures have been explained for the understanding of general investigation in the subsequent chapters.

1.1 Introduction

Fluid dynamics is subject of my research in which continuous movement of a non-Newtonian viscoelastic Rivlin-Ericksen fluid is modelled. This subject is challenging as the fluid is in motion. Fluid dynamics and Electromagnetic theory were being developed independently of each other almost upto the first half of the 20^{th} century. A systematic study of the hydrodynamics of a conducting fluid immersed in a magnetic field was started in 1942 by Alfvn. This study known as Hydromagnetics or Magnetohydrodynamics(MHD).

Magnetohydrodynamics is the science where in the presence of magnetic field, the motion of electrically conducting fluid is considered. The study of the interaction between magnetic field and electrically *conducting fluids* is currently receiving considerable interest. This interest has been spurred primarily by *astrophysical* problems and by problems associated with the fusion reactor. Thus in a very lucid manner, hydromagnetics or MHD is the union of fluid dynamics and electromagnetic theory. It is concerned with physical systems specified by the equations that result from the fusion of those of hydrodynamcis and *electromagnetic* theory. It is a well known fact that when a conductor moves in a magnetic field, *electric currents* are induced in it. These currents experience a mechanical force called the '*Lorentz force*', due to magnetic field. This force tends to modify the initial motion of the conductor. Moreover, a magnetic field which is generated by the induced currents is added on to the applied magnetic field. Thus there is a coupling between the motion of the conductor and electromagnetic field, which is exhibited in a more pronounced form in liquid and gaseous conductors. This is due to the fact that the molecules composing the liquids and gases enjoy more freedom of movement than those of solid conductors. The *Lorentz force* is usually small unless inordinately high magnetic fields are applied. Therefore, this force is too small to alter the motion as a whole considerably but if it acts for a sufficiently long period, the *molecules* of gases and liquids may get accelerated considerably to change the initial state of motion of these types of conductors. Therefore, the *coupling* between the electromagnetic fields and the motion of a conductor could only be judged appreciably by confining attention to liquid and gaseous conductors.

1.2 Fluid

Fluid is something which can flow it can be gas or liquid. The study of characteristic of fluid in motion is *hydrodynamics* and the study of characteristic of fluid at rest is hydrostatic. Pressure difference applies force, which can create motion. It is substance that flows or *deforms* continuously under the action of forces applied may pressure difference or shearing (acting tangentially). Fluid has no ability to resist the force of deformation. If there is no pressure difference or *shearing force*, it implies that fluid is at rest and all other forces are perpendicular to the plane in which these force acting.

1.2.1 Properties of Fluid

Temperature, *density* and pressure describe the thermodynamic state of the fluid along with other properties like internal energy or *entropy*. Viscosity is unique property of fluid by which we can differentiate between two fluids. Fluid has also other properties like kinematic viscosity, velocity and surface tension.

Density

Density $= \frac{\text{Mass}}{\text{Volume}} =$ Mass per unit volume.

It is the distribution of mass and denoted by ρ. Its dimension is $\frac{M}{L^3} = ML^{-3}$. Density is different for different liquids as :

- Density of water = 1000 kg/m^3.
- Density of blood = 1060 kg/m^3.
- Density of salty water = 1027 kg/m^3.
- Density of air = 1.29 kg/m^3.

Relative Density

Relative density is the *dimensionless* or unit less number which is used to compare the heaviness of different fluids. It is defined as, Relative Density $= \frac{\text{Density of Material}}{\text{Density of water}}$.

Relative density of Mercury = 13.6, which means that mass of mercury in the *volume* of 1 m^3 is 13600 kg which is 13.6 times the mass of water.

Pressure

Pressure is proportionate to force and inversely proportionate to area. It is defined as, Pressure $= \frac{\text{Force}}{\text{Area}}$ = Force per unit area. It is denoted by p and its unit is *Pascal*, dimension of pressure is given by, 1 Pascal $= \frac{\text{1 Newton}}{m^2} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

Stress has also the same units. A physical quantity with no direction is scalar and quantity having one direction is vector and quantity having more than one or many directions is tensor. So pressure is the tensor as it has direction in all the directions. For instance, fill air in the balloon, now pressure is inside the balloon, puncture the balloon at the top and flow of air will be in upward direction so pressure will also have an upward direction, similarly repeat it in any direction and you will find the pressure in all the directions.

Pascal's Law says that pressure inside a fluid is same throughout. Its direction is always normal (at right angle) to the surface in contact. This law is for the case where gravity is not included but pressure varies with depth inside fluid because of *gravity* i.e. pressure will be different at different heights inside the fluid. Above the surface of earth, atmosphere consists of molecules (having mass) so gravity will be in effect. Atmosphere presses the earth. Molecules apply force on the surface of earth. Effect of this force is F/A i.e. the pressure created by atmosphere called atmospheric pressure. It is maximum on the earth and decreases gradually as we move up. Barometer is used to measure atmospheric pressure.

Viscosity

Viscosity is the Property of fluid when it is in motion. In flowing liquid there is a force which resist or opposes the motion is called viscosity or *viscous drag*. It is assumed that fluid flow in layers and all the layers move with different velocity. The layer near the lower fixed surface has zero velocity and the layers away from it have larger velocity that is change in velocity as the height increases. When *tangential force* is applied on surface of upper layer, stress is created which is called tangential stress. As we move down from the upper layer, the velocity decreases. More the *tangential stress* will increase the velocity of all layers of fluid. It is denoted by τ and is directly proportional to *velocity gradient* because during flow of liquid when tangential stress is increased the velocity gradient also increases and $\tau = \mu \frac{du}{dy}$, where μ is constant of proportionality, known as the *coefficient of viscosity*. So, $\mu = \frac{\text{Tangential stress}}{\text{Velocity gradient}} = \frac{\text{Force/area}}{\text{Velocity/length}}$.

For fixed tangential stress, liquid with greater value of constant μ will have less velocity i.e. fluid is more viscous and vice versa. Viscous drag is opposite to the direction of the tangential stress and it is between the two consecutive layers of fluid. Viscosity is characteristic of liquid which is fixed. It will not change by increasing the stress. Variables are force and velocity gradient .
Dimension of $\mu =$ Dimension of $\left[\frac{\text{Force/area}}{\text{Velocity/length}}\right] = \frac{MLT^{-2}/L^2}{(L/T)/L} = \frac{M}{LT} = ML^{-1}T^{-1}$.

Kinematic Viscosity

The ratio of coefficient of viscosity μ to the density ρ determines the effect of viscosity on the motion of fluid is called *kinematic viscosity*. It is denoted by ν and defined as $\nu = \frac{\mu}{\rho}$. Its dimension is $\frac{ML^{-1}T^{-1}}{ML^{-3}} = \frac{L^2}{T} = L^2T^{-1}$.

Stokes' Drag

It is the quantity of force due to viscous drag.
$F = 6\pi *$ coefficient of viscosity $*$ radius r of body $*$ velocity of body $= 6\pi\mu rv$.

Surface Tension

Surface tension is characteristic of the surface of liquid due to which it tries to decrease its area and for this purpose applies a force of attraction between *molecules* in the surface. For this reason surface of liquid behaves like stretched membrane.

Surface tension is the force that acts at each point of the surface of a fluid due to interaction of the neighbouring molecules on the molecule situated at this point. It expressed as the force per unit length of the surface in the tangential direction. It is property of *static liquid* and it does not depend on the quantity. *Surface tension* is calculated as force per unit length across an imaginary line drawn on the surface. Its unit is Newton per meter which is different from the unit of force.

Blade of steel does not sink in the water whenever its density is more than water because of surface tension.

1.3 Buoyancy Force

When a solid is dip in a liquid and displaces its molecules, those *displaced molecules* apply a force on the solid and trying to eject it out. This force is called *buoyancy* force and phenomenon is buoyancy. It is the Natural force and solid dip inside the water because of gravity i.e. thrust applied by solid in downward direction. Liquid molecule apply the force in upward direction i.e. *upthurst* by the liquid.

1.4 Fluid Types

- Newtonian fluid.
- Non-Newtonian fluid.

Newtonian fluid

The fluid in which stresses are the linear composite function of the instantaneous velocity gradients are called Newtonian fluids. In other words stresses are the linear function of strain rate and strain rate are expressible in velocity gradients. Graph of this relation of stress and strain rate is a straight line. Flow governing equations for the Newtonian fluid are Navier-Stokes' equations. Moreover *Newtonian* fluid cannot explain every type of phenomenon.

Non-Newtonian fluid

The fluid in which stresses are the non linear composite function of the instantaneous velocity gradients are called *non-Newtonian* fluids. In other words stresses are the non linear function of strain rate and strain rate are expressible in velocity gradients. Graph of this relation of stress and strain rate is a curve not a straight line. Equations which govern fluid flow are obtained by using the principle of conservation of mass and momentum.

Rivlin-Ericksen Fluid

Several Models have been proposed for non-Newtonian fluids (having non linear relation between the shearing stress and strain rate) like :

- Ostwaldde Waele power law model (1925, Ball point pen ink, molten chocolate).
- Carreau Yasuda model (1972, Properties of polystyrene fluids).
- Newtonian fluid Cross model (1965, Pseudoplastic systems).
- Sisko model (1958, Lubricating greases).
- Bingham Herschel-Bulkley model (1922, Paints, toothpaste, mango jam etc.)
- Rivlin-Ericksen model (1955), Known as Rivlin-Ericksen fluids proposed by Ronald Samuel Rivlin and Jerald LaVerne Ericksen. This fluid model is known as order fluid model: 2^{nd} order, 3^{rd} order or n^{th} order. And many more models.

I focused my study on the Rivlin-Ericksen model for non-Newtonian fluid because it can be used in various shear damping fluid devices, modeling of blood and in many other safety equipments which can be helpful to the society.

In 1955, Rivlin-Ericksen proposed a theory of non-linear viscoelasticity based on the assumption that the stress can be expresses in terms of velocity gradients. The resulting constitutive equations for an isotropic incompressible viscoelastic liquid were shown in the form :

$$T_{kl} = -p\delta_{kl} + \tau_{kl}, \tag{1.1}$$

$$\tau_{kl} = \rho\left(\nu + \nu'\frac{\partial}{\partial t}\right)e_{kl}, \tag{1.2}$$

$$e_{kl} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k}\right) \tag{1.3}$$

where $T_{ji} \rightarrow$ *stress tensor*, $\tau_{ji} \rightarrow$ *shear stress tensor*, $e_{ji} \rightarrow$ *strain rate tensor*, $\delta_{ji} \rightarrow$ *Kronecker delta*, $v_i \rightarrow$ velocity vector, $x_i \rightarrow$ position vector, $p \rightarrow$ *isotropic pressure*, $\mu \rightarrow$ viscosity and $\mu' \rightarrow$ *viscoelasticity*.

The flow of a conducting viscoelastic Rivlin-Ericksen fluid through *Porous medium* in a long uniform straight tube of rectangular cross-section under the influence of transverse uniform MF (magnetic field) has been studied by Gaurav Mishra et al. [1]. The upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in the Rivlin-Ericksen fluid heated from below in the presence of uniform vertical magnetic field was studied by Ajaib S.Banyal1 et al. [2]. The problem of thermal convection of a Rivlin-Ericksen fluid permeated with suspended particles in porous medium heated from below with variable gravity is analyzed by the method of positive operator by Pushap Lata [3]. An analysis is presented with MHD free convective viscoelastic flow of a fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime in presence of heat source by R.Choudhury and B.Das [4]. Study of Instability of Streaming Rivlin-Ericksen Fluid in Porous Medium is made by B.Jana and J.Sarkar [5]. The effect of suspended particles on thermal convection of incompressible Rivlin-Ericksen elastico-viscous fluid in a porous medium is considered G.C.Rana and R.C.Thakur [6]. A theoretical study is made to investigate the influences of relaxation and retardation times of viscoelastic fluid on the onset of convection in a horizontal fluid layer heated underneath by Rajib Basu1 and G.C.Layek [7]. The thermal instability of a layer of Rivlin-Ericksen elastico-viscous rotating fluid in a porous medium in hydromagnetics is considered by S.K.Kango and Vikram Singh [8]. An investigation is made on the effect of Hall currents and suspended particles on the hydromagnetic stability of a compressible, electrically conducting Rivlin-Ericksen elastico-viscous fluid by U.Gupta et al [9]. The unsteady Hele-Shaw flow of a viscoelastic Rivlin-Ericksen conducting fluid between two parallel walls by S.Sreekanth et al. [10]. Bertrand Rollin and Malcolm J.Andrews extended the Goncharov model for nonlinear Rayleigh-Taylor instability of perfect fluids to the case of Rivlin-Ericksen viscoelastic fluids with surface tension [11]. Oscillatory onset of convection is studied numerically for Rivlin-Ericksen, Maxwell and Jeffreys liquids by considering free-free and rigid-free isothermal/adiabatic boundaries by P.G. Siddheshwar et al. [12]. An analysis for the steady two-dimensional boundary-layer stagnation-point flow of Rivlin-Ericksen fluid of second grade with a uniform suction is carried out via symmetry analysis by M.B.Abd-el-Malek and H.S.Hassan [13]. P.Riesen, K.Hutter and M.Funk present a viscoelastic constitutive relation which describes transient creep of a modified second grade fluid enhanced with elastic properties of a solid. The material law describes a Rivlin-Ericksen material and is a generalization of existing material laws applied to study the viscoelastic properties of ice [14]. The thermosolutal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium is considered to include the effect of suspended particles and rotation. The sufficient conditions for the validity of principle of exchange of stabilities are obtained by A.K.Aggarwal [15]. Hyam Abbouda and Toni Sayah propose a finite-element scheme for solving numerically the equations of a transient two-dimensional grade-two non-Newtonian Rivlin-Ericksen fluid model [16]. Motivated by the aim of modelling the behavior of swirling flow motion, F.Carapau present a 1D hierarchical model for an Rivlin-Ericksen fluid with complexity n = 2, flowing in a circular straight tube with constant and no constant radius [17]. Ronald Rivlin was an outstanding figure in the development of modern nonlinear continuum mechanics in the second half of the 20th century. Much of his research is characterized by the innovative, systematic and effective use of methods based on invariant theory. A.J.M.Spencer had summarize his work in this area, and show that it continues to be effective in applications to recent research in the mechanics of fibre-reinforced elastic materials[18]. The flow of an unsteady third-grade Rivlin-Ericksen fluid on an oscillating plate is discussed by Muhammad R.Mohyyuddin et al [19]. The stability of the plane interface separating two viscoelastic (Rivlin-Ericksen) superposed fluids in the presence of suspended particles is studied by P.Kumar and G.J.Singh [20].

Applications of non-Newtonian fluid

It is used in many *safety equipments* and some mathematical models had developed on the basis of non -Newtonian fluid. Some useful applications are :

- It is used in the formation of various materials like rope, seatbelt and safety harness.
- Some *shear damping fluid* devices are based on the *shear thickening* property of the non-Newtonian fluid which can reduce the injuries in road accidents or sports.
- Blood behaves as a non-Newtonian fluid in the core. Thus, it is modeled as a non-Newtonian fluid.
- Magma fluid is non-Newtonian fluid because it does not obey the Newton's law of viscosity. The study of these fluids is an important area of *research*.
- It can be used in military suit which would change to solid state when the bullet hits.

- Because of shear thickening characteristics of non-Newtonian, it is used in of shoes. it remain in liquid state while running, walking, standing and change to solid state while fast running. it can prevent injuries.

1.5 Basic Hydrodynamic Terms

1.5.1 Temperature and Heat

Temperature of an object is the degree of its *hotness*. It is the physical quantity which decides the direction of flow of heat energy. Heat is a type of energy contents in an object. Heat flow from an object of higher temperature to object of lower temperature. For example, if we touch an ice, the heat will flow from our body to ice because our body is at higher temperature. If we touch a hot water then heat will flow from hot water to our body because our body is at lower *temperature*.

Convection

In Convection, heat energy is transferred from higher temperature region to lower temperature region through the displacement of the particles of the medium. Thus convective heat transfer is associated with displacement of *fluid element*.

In natural convection, fluid element is displaced due to *density difference* arising out of temperature difference. In forced *convection*, fluid element is forced to change its position by applied external energy. Heat transfer takes place due to the presence of temperature difference. The driving force is the temperature difference.

Thermal Expansion

Whenever we give heat energy to molecules of an object, the activity /vibrations of molecules increases and need a larger space to exist and what we get finally expansion called *thermal expansion*.

In other words, when an object is heated, the distance between molecules increases and therefore its volume increase. If any one dimension is negligible then we say that area has increased, if depth and width both are negligible, then we consider only longitudinal expansion.

Let us consider the volume of one cubic meter and raise the temperature by one degree by giving heat, change in volume is called the volumetric thermal expansion. Coefficient of volumetric thermal expansion is denoted by α, where

$\alpha =$ Coefficient of expansion $= \frac{\text{Change in Volume}}{\text{Original volume} * \text{Change in temperature}} = \frac{V - V_0}{V_0 * (T - T_0)}$

$\Rightarrow$ New volume, $V = V_0[1 + \alpha(T - T_0)]$.

With the increase in volume the density will decrease and it will be given by the relation $\rho = \rho_0[1 + \alpha(T - T_0)]$.

Specific Heat

Specific heat of an material is the heat required to raise the temperature of 1kg of that material by 1 degree Celsius. Its unit is joule per kg pre degree Celsius and represented by c. It is a property of material.

Specific heat at the constant volume is the heat required to raise the temperature of one mole of gas by one degree Celsius by keeping volume constant. It is denoted by C_v.

Specific heat at the constant pressure is the heat required to raise the temperature of one mole of gas by one degree Celsius by keeping the pressure constant whereas Volume may change. It is denoted by C_p.

Specific heat at the constant pressure, C_p, is always greater than *specific heat* at the constant volume, C_v.

Heat Capacity

Heat capacity of an object is the heat required to raise its temperature by one degree Celsius. Its unit is joule. It is a property of an object and defined as,

Heat capacity of an object $=$ Mass of an object $*$ Specific heat of the an object .

An object with more *heat capacity* can store more heat.

1.5.2 Laminar and Turbulent Flow

Fluid flow can happen in two ways, *Laminar*/Streamline or *Turbulent*/Random. Suppose all the fluid molecules moving in row with certain velocity, if there is no change in the sequence and velocity throughout, motion is laminar/streamline. In other words, in laminar flow, fluid particles maintain its order and cross any particular point with same velocity. There will be no extra pressure on the walls of pipe in pipe flow during laminar flow.

In turbulent flow, fluid particles do not maintain their serial order and overtake each other. There will be an extra pressure on the walls of pipe in pipe flow during turbulent flow and pipe can burst out. Fluid flows in lines and different lines can have different velocity. All the particles have the same velocity with respect to line in laminar flow. These lines are called *streamline*. Numerical value of Dimensionless *Reynold's number* decides the pattern of flow which depends on the velocity of fluid. If a liquid flows in pipe then it is defined as $R_n = \frac{\rho v d}{\mu}$

where $d \rightarrow$ is the diameter,

$v \rightarrow$ is the velocity,

$\rho \rightarrow$ is the density,

$\mu \rightarrow$ is coefficient of viscosity.

For $R_n < 1000$, flow is laminar.

For $R_n > 1000$, flow is turbulent.

For $R_n \geq 1000$ and $R_n \leq 2000$, flow is mixed.

Engineers use this number to optimize the flows in pipe.

1.5.3 Compressible and Incompressible

Gases are highly *compressible* as compare to liquids. In case of gases, small change in pressure may bring large change in *specific volume* $\left(\frac{1}{\rho}\right)$ or in volume per unit mass. In case of liquids, effect of pressure on density is neglected and we assume $\rho =$ Constant.

Dimensionless, *Mach number* decides whether the fluid flow is compressible or incompressible which depends upon the velocity of fluid. It is denoted by M and defined as, $M = \frac{\text{Fluid velocity}}{\text{Speed of sound}} = \frac{v}{a} = \frac{v}{332 m/s}$. If fluid velocity, $v > 99 m/s$, then compressibility effects are to be considered.

1.5.4 Prandtl Number

During convection, *conduction* also take place in fluid. Both processes reduce the temperature difference due to heat transfer. Rates of convection and conduction are different for different fluids. The dimensionless *Parndtl number* decides the which process will dominate and defined as

$P_r = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{c_p \mu}{\kappa}$, where

$\nu = \frac{\mu}{\rho}$, kinematic viscosity

$\alpha = \frac{\kappa}{\rho c_p}$, thermal diffusivity

κ = thermal conductivity and

ρ = density, If fluid is more viscous or stickier, then P_r is greater and the heat transfer will be less convective.

1.5.5 Porous Medium

Porous media defined as solid bodies that contain pores. *Pores* are void or the empty spaces which must be distributed more or less frequently through the *porous material*. Extremely small voids in a solid are called *molecular interstices* and very large voids are called *caverns*. Pores are the void spaces intermediate in size between caverns and molecular interstices. The pores in a porous system may be interconnected or non interconnected. The interconnected part of the pore system is called the effective pore space of the *porous medium*. Pore spaces can be ordered or disordered.

Porosity

A porous media can be characterized by a variety of geometrical properties. The ratio of void to the total volume is called *porosity* and denoted by ϵ, where $\epsilon = \frac{Ratio\,of\,Void}{Total\,Volume}$.

If the calculation of porosity is based upon the interconnected pore space interval of the total pore space, the resulting quantity is termed as effective porosity. Porosity can be measured by a variety of methods:

- **Direct Method:** Porosity is determined by measuring bulk volume of a piece of porous material and then compact the body so as to destroy all the voids, and to measure the difference between the volumes. This method is applicable only if material is very soft like bread.
- **Optical Method:** In this method porosity is determined by looking at a section of the porous medium under microscope. Numerical value of porosity obtained in this manner of the random section must be the same as that of the porous material.

- **Density Method:** If the density ρ_G of the material making up the porous medium is known, then the bulk density ρ_B of the medium, which can be calculated, is related to the fractional porosity ϵ, where $\epsilon = 1 - \frac{\rho_G}{\rho_B}$.
- **Gas Expansion Method:** The basic principle of this method is the direct measurement of the volume of air or gas contained in the pore space. This can be achieved either by continuously evacuating the air out of the specimen.

Permeability

It is the measure of ease with which a fluid can move through a porous rock.

1.6 Fundamental Assumptions

We now discuss two fundamental assumptions.

- Continuum Hypothesis.
- Newtonian Mechanics.

Continuum Hypothesis

Fluid is appeared to smooth and continuous but in reality it has *discrete structure* of molecules and atoms. A detailed molecular approach for understanding fluid flow is very difficult. Concept of property at a point has no meaning if a point is located in the void between the atoms or at the center of an atom. Let density ρ at a point P is defined

as $\quad \rho_P = \lim_{\nabla V \to 0} \frac{\Delta m}{\Delta V}$

where $\nabla V \to$ volume element surrounding a point P in a fluid containing total mass Δm. If P lies at the center of the atom, then $\Delta V = 0$ and $\rho_P \to \infty$. If P lies between in the void between two atoms then Δm = 0 and $\rho_P \to 0$.

Thus at some points the density is infinite and at some points the density is zero. In order to overcome these inconsistencies we shall assume that masses are uniformly distributed over the whole volume and consider matter as continuous. By assuming continuum hypothesis, we can give meaning to pressure, momentum, density at a point and treat them as a continuous function of space and time variables.

Newtonian Mechanics

Newtonian mechanics is one which follows the three law of motion of Newton. Thus it is assumed that fluid velocity is very small as compared to the speed of light otherwise the theory of relativity has to be considered.

1.7 Basic Hydrodynamical Equations

Fundamental equations governing fluid flow are :

- Mass Conservation (Continuity) equation.
- Momentum Conservation (Fluid Motion) equations.
- Energy equation.
- Equation of state.

1.7.1 Equation of Continuity - Conservation of Mass

Mass conservation on the fluid in the control volume states that

$$
\begin{aligned}
&\text{Rate of accumulation of mass in the control volume}\\
&\qquad = \text{Rate of inflow of mass in control volume}\\
&\qquad\quad - \text{Rate of outflow of mass from control volume}\\
&\qquad\qquad + \text{Any source.}
\end{aligned}
$$

Let u, v, w be the components of the velocity $\vec{v}$, ρ be the density at the point (x, y, z) in a fluid domain, the mathematical equivalence of the verbal statement of conservation of mass for every point in the fluid domain is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\Rightarrow \frac{D\rho}{Dt} + \rho(\nabla.\vec{v}) = 0. \tag{1.4}$$

Case 1 : If ρ is homogeneous and incompressible i.e. ρ is same at all the points and constant in the fluid domain i.e density of an element does not alter as that element moves about, then equation of continuity becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{1.5}$$

Case 2 : If ρ is heterogeneous and incompressible i.e. ρ is different at different points and constant in the fluid domain then equation of continuity becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{1.6}$$

Case 3 : For compressible steady fluid $\left(\frac{\partial}{\partial t} = 0\right)$, equation of continuity becomes

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0. \tag{1.7}$$

1.7.2 Equations of Motion - Conservation of Momentum

Momentum conservation on the fluid in the control volume states that

Rate of accumulation of momentum

$=$ Rate of inflow of momentum

$-$ Rate of outflow of momentum

$+$ Net external forces acting on the control volume.

Since momentum is a vector quantity, so there are 3 equations of momentum as:

Rate of accumulation of momentum in x-direction

$=$ Rate of inflow of momentum in x-direction

$-$ Rate of outflow of momentum in x-direction

$+$ External forces acting on control volume in x-direction.

Rate of accumulation of momentum in y-direction

$=$ Rate of inflow of momentum in y-direction

$-$ Rate of outflow of momentum in y-direction

$+$ External forces acting on control volume in y-direction.

Rate of accumulation of momentum in z-direction

$$= \text{Rate of inflow of momentum in z-direction}$$
$$- \text{Rate of outflow of momentum in z-direction}$$
$$+ \text{External forces acting on the control volume in z-direction.}$$

Let u,v,w be the components of the velocity $\vec{v}$, ρ be the density and p be the pressure at the point (x, y, z) in a fluid domain or mass of fluid, and let X,Y,Z be the components of external force $\vec{F}$ per unit mass at the same point. Mathematical equivalence of the above verbal statement for every point in the fluid domain in x-direction is :

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \frac{\partial}{\partial z}(\rho uw) = \rho X.$$

Forces present everywhere in the fluid domain like pressure force, gravitational force, viscous force, magnetic force and electric force etc. Mainly two types of forces considered are body force (gravitational force) and stress (normal and shear stress). Therefore $X = \rho g_x + \Sigma$ (stress component in x-direction x area of the surface perpendicular to stress component). There will be six such *stress components* if we consider cuboid as the control volume, three in positive direction and three in negative direction. So momentum balance equation at a point (x, y, z) in x-direction is

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \frac{\partial}{\partial z}(\rho uw) = \rho g_x + \frac{\partial}{\partial x}T_{xx} + \frac{\partial}{\partial y}T_{yx} + \frac{\partial}{\partial z}T_{zx}$$

where $T_{ij} = -p\delta_{ij} + \tau_{ij}$, δ_{ij} is the kronecker delta, and τ_{ij} is shear stress component in j direction and i is the axis to which the plane face is perpendicular and above equation reduces to

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \frac{\partial}{\partial z}(\rho uw) = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}. \tag{1.8}$$

Similarly other two equations in y-direction and z-direction are

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(\rho v^2) + \frac{\partial}{\partial z}(\rho vw) = \rho g_y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}. \tag{1.9}$$

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho u w) + \frac{\partial}{\partial y}(\rho v w) + \frac{\partial}{\partial z}(\rho w^2) = \rho g_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}. \tag{1.10}$$

There are four equations (three momentum balance equations and one continuity equation) in 12 variables $(u, v, w, \tau_{xx}, \tau_{yx}, \tau_{zx}, \tau_{xy}, \tau_{yy}, \tau_{zy}, \tau_{xz}, \tau_{yz}, \tau_{zz})$. So more equations are required. Energy equation will generate the new variable, it will not resolve the problem. More information is required to resolve the situation in the formulation of model. Constitutive model/expression is required,one of such model is $\tau = \mu\frac{du}{dy}$, where μ is coefficient of viscosity that can be measured, τ is the tangential/shearing stress and velocity gradient $\frac{du}{dy}$ is not a new variable. It means stress τ can be expressed in the known variables, only need to identify the τ with one of the nine shear stress components. It is specific for one-dimensional flow and further need of constitutive law i.e the relation between stress and rate of strain which describes the stresses within fluid.

As the fluid will continue to deform when stress is applied and does not take original shape as the stress removed but deformation stops. So rate of strain is considered not strain (considered in solid mechanics). Various combinations of velocity gradients describe the strain rates as :

Rotational strain rate $= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

Shear strain rate $= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

Extensional strain rate in x-direction $= \frac{\partial u}{\partial x}$

General isotropic (invariant to the orientation of co-ordinate axes) and linear relation between stress and strain rate is

$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\left(\frac{\partial u_k}{\partial x_k}\right)\delta_{ij}$

where μ is kinematic viscosity and λ is second coefficient of viscosity which is important only in case of compressible fluids and disappears for incompressible fluid.

Thus, for compressible fluids

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\delta_{ij}$$

and for incompressible fluids

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right).$$

Similarly other stress components can be expressed in terms of velocity gradients. Substitute the values of all stress components in the equations (1.8) - (1.10) and we get all the equations in the variables u, v, w, p and material property constants ρ, μ, λ.

Momentum balance equations (1.8)-(1.10) are called Navier-Stokes' equations for motion of viscous compressible fluid. These equations are valid only for the Newtonian fluid which obeys isotropic condition and linear relation between stress and stress rate. For non-Newtonian fluids different *constitutive relations* between stress (arises out of fluid motion) and strain rate are required.

Navier-Stokes' equation for viscous incompressible fluid are, in x-direction is

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right). \tag{1.11}$$

Similarly other two equations in y-direction and z-direction are

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right). \tag{1.12}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right). \tag{1.13}$$

Now we have closed system of equations i.e four equations in four variables u, v, w, p.

1.7.3 Equation of Energy - Conservation of Energy

It is required in case of heat transfer. Principle of the *conservation of energy* in the control volume states that

Rate of change of energy in the control volume

= Rate of inflow of energy − Rate of outflow of energy

+ Rate of heat addition to the fluid contained in control volume

+ Rate of work done by the forces acting on control volume

+ Generation of energy from sources within control volume.

The Mathematical equivalence of the above verbal statement for viscous compressible fluids is

$$\frac{\partial}{\partial t}(\rho C_v T) + \frac{\partial}{\partial x_j}(\rho C_v T u_j) = \frac{\partial}{\partial x_i}\left(q\frac{\partial T}{\partial x_j}\right) - p\frac{\partial u_j}{\partial x_j} + \Phi \tag{1.14}$$

where $\Phi = 2\mu e_{ij}^2 - \frac{2}{3}\mu\left(e_{ij}\right)^2$

is the 'rate of *viscous dissipation*' and gives the heat generated because of frictional forces and

$$e_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$$

is the 'rate-of-strain tensor', C_v is the *specific heat* when volume is constant and q is the coefficient of heat conduction. For an incompressible fluid, $e_{ij} = 0$ and the corresponding expression for Φ is given by $\Phi = 2\mu e_{ij}^2$. Thus, for an incompressible fluid, the equation of energy (1.14) takes the form

$$\rho\frac{\partial}{\partial t}(C_v T) + \rho u_j\frac{\partial}{\partial x_j}(C_v T) = \frac{\partial}{\partial x_j}\left(q\frac{\partial T}{\partial x_j}\right) + 2\mu e_{ij}^2. \tag{1.15}$$

1.7.4 Equation of State

When the motion of compressible fluid is considered, a relation is required between the *state variable*, pressure $\rightarrow p$, density $\rightarrow \rho$ or volume $\rightarrow V$ and temperature $\rightarrow T$, in order to obtain sufficient number of equations to determine the physical and dynamical variables involved in the problem. Such a relation is called the equation of state and has the general form $g(p, V, T) = 0 \quad$ or $\quad g(p, \rho, T) = 0$.

If we neglect the compressibility of a fluid, its density remains constant. Thus for an incompressible fluid the equation of state is $\rho =$ Constant.

In viscous compressible flow: Equations of energy and motions are coupled. Energy equation involves viscous dissipation function and temperature which are functions of velocity. Thus temperature and velocity are coupled. Similarly equations of motion involves velocity components u,v,w and pressure p which are function of temperature. So velocity and temperature are coupled.

In viscous incompressible flow: Density, thermal conductivity and coefficient of viscosity are fluid properties constants and equations of energy and motions are uncoupled. Therefore four equations of continuity and motion can be solved for four variables u, v, w and p. Using the values of velocity components we can solve the energy equation and find temperature. When initial and boundary conditions are specified, we can find the solution of above equations physically.

1.8 Initial and Boundary Conditions

Equations describing the motion are partial differential (not algebraic) equations which are valid at all the points in *flow domain* so it is necessary to have boundary and initial conditions in order to have a solution. *Initial conditions* may be of the form

$u(x, y, z, t_0) = f_1\,(x, y, z)$,

$v(x, y, z, t_0) = f_2\,(x, y, z)$,

$w(x, y, z, t_0) = f_3\,(x, y, z)$

and $\quad p(x, y, z, t_0) = f_4\,(x, y, z)$.

There are three types of *boundary conditions* :

- Dirichlet's Boundary Conditions of the type $u =$ Constant.
- Neumann's Boundary conditions of the type $\frac{\partial u}{\partial x} =$ Constant.
- Robin's Boundary Condition of the mixed type $au + \frac{\partial u}{\partial x} =$ Constant.

For fluid flow situation we need more realistic or physical boundary conditions. At inlet, flow entering the boundary so apply dirichlet's boundary conditions. At outlet, flow is leaving the boundary , so apply the Robin's boundary conditions and for free shear boundary apply Neumann's boundary conditions. For fully *developed flow* apply same boundary conditions as at the outlet.

1.8.1 Implication of Boundary conditions:

Any kind of boundary condition for any problem is not justified. For unique solution, solution continuously depend on initial and boundary conditions, with the change of these conditions solution will change. This type of sensitivity is exhibited by the boundary conditions. Type of conditions depend upon the physics of the problem. So the mathematical problem must be well posed for the solution.

1.9 Hydrodynamic Stability - Basic Concepts

Let the system be defined by parameters as
$Y_1 \rightarrow$ dimensions of the system ,
$Y_2 \rightarrow$ velocity field,
$Y_3 \rightarrow$ temperature gradients ,
$Y_4 \rightarrow$ pressure gradients ,
$Y_5 \rightarrow$ magnetic fields ,
$Y_6 \rightarrow$ magnitude of forces ,
$Y_7 \rightarrow$ density ,
and $Y_8 ... Y_n \rightarrow$ denotes other parameters.

The above system is stable with respect to any *disturbance*, if the initial state of *parameter* is disturbed/perturbed and disturbance gradually *decay* in amplitude. Thus system considered as stable, If it is stable with respect to all disturbances in all the parameters. Otherwise the system is unstable. In other words, *stability* means there exist no disturbance by which system is unstable and no disturbance *grow* in *amplitude*.

1.10 Flow Instabilities

Flow instability occur everywhere and effect every fluid phenomenon, there are several examples of fluid instability like smoke rises because it is lighter than surrounding air. Instability is the first step in events which generate turbulence. Some flow instabilities are:

- KH (Kelvin- Helmholtz) Instability/Double-Diffusive Convection.
- RT (Rayleigh-Taylor) Instability.
- Thermal (Bénard) Instability.
- Shock Wave Instability.

1.10.1 Thermal Instability - Bénard Convection

A layer of fluid heated from underside or below may becomes unstable because of heavier fluid at the top and lighter one at the bottom. The heating element is at the bottom. As heat is turn on, fluid become *unstable* and hot buoyant fluid get away before it loses heat and *buoyancy* to its surroundings.

The critical parameter is the *Rayleigh number* which involves gravity(g), thermal expansion coefficient(α), the vertical *temperature gradient*$\left(\frac{dT}{dz}\right)$, the effects which tends to prevent instability i.e. kinematic viscosity(ν) and thermometric conductivity(κ) and finally a length parameter or thickness of fluid layer (h) and it is given by $R_a = \frac{g\alpha h^4}{\kappa\nu}|\frac{dT}{dz}|$.

Rayleigh number can be increased by heating the bottom, As this dimensionless number goes beyond *critical value*, *instability* sets in the form of *Bénard cells*. Below the critical value the flow is stable. The earliest experiments to demonstrate the onset of *thermal instability* are those of Bénard in 1900, though the phenomenon of thermal convection itself had been recognized earlier by James Thomson(1882) and Count Rumfort (1797).

Bénard's Experiment

He carried out experiments on a very thin layers of fluid, about one mm in depth, or less, standing on a leveled *metallic plate* maintained at constant temperature. He did experiments on many liquids with different physical constants. He was particularly interested in the role of viscosity. He observed when the temperature of the lower surface was gradually increased, at a certain instant, the layer became *reticulated* and revealed its dissection into cells. There were motions inside the cells and two phases in the succeeding development of the cellular pattern in which the cells are hexagonal, equal and properly aligned. R.K.Zeytounian [21] had used the results of this experiment during his research on convection in fluids.

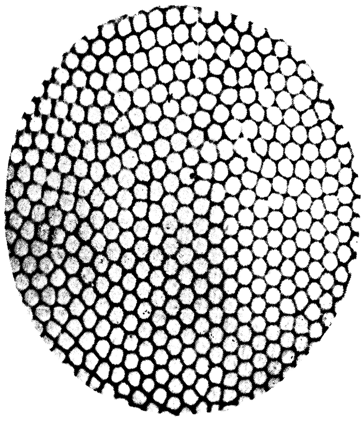

Figure 1.1: Bénard cells

Schmidt-Milverton Principle for detecting the onset of thermal instability

Schmidt and Milverton incorporated a principle for the detection of the onset of thermal instability which is so direct and simple that it served as the basis for all later experiments in this area. They applied their principle to determine the critical Rayleigh number for the onset of thermal instability in horizontal layers of water confined between two rigid planes. The critical value $R_C = 1770 \pm 140$, they derived from their experimental results and is satisfactory agreement with the theoretical value 1708.

The Precision experiments of Silveston

The experiments of Schmidt and Milverton have been repeated by Saunders, Malku, Silveston and others to achieve greater range and precision. Siveston used water, Heptane, Silicon AK-3, ethylene glycol and silicon oil AK-350 in his experiments. From an experimentation of results obtained for the Rayleigh numbers in the range 1000-10,000. Silveston derives for the critical Rayleigh number for the onset of instability the value $R_C = 1700 \pm 51$. It is very good accord with the theoretical value 1708.

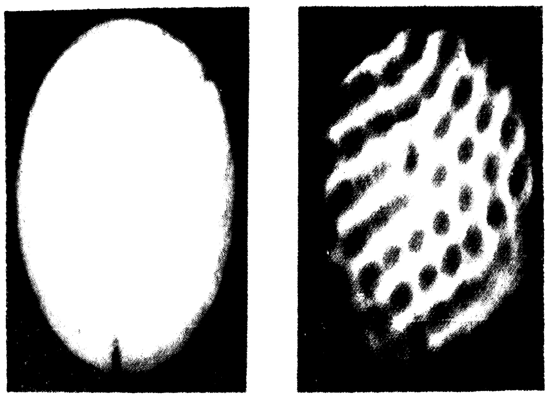

Figure 1.2: Visualization of onset of thermal convection by Silveston. The photograph on the left was obtained for the Rayleigh number 1,500 while the photograph on the right was obtained for a Rayleigh number 1,800. The depth of the layer in these experiments was 7 mm.

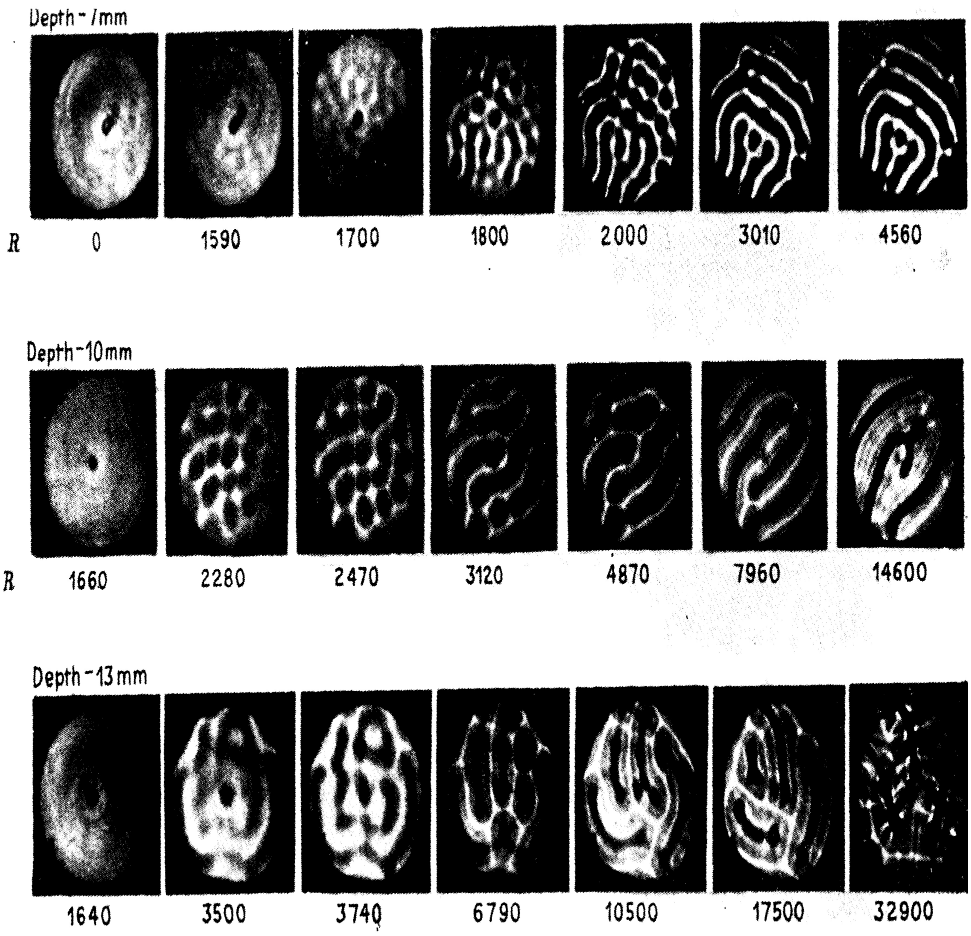

Figure 1.3: Visualization of onset of thermal convection by Silveston. Photographs for different depths and increasing Rayleigh numbers.

1.10.2 Double Diffusive Convection(or Thermosolutal Instability)

In the standard Bénard problem, the instability is driven by the density difference which is caused by a temperature difference between the upper and the lower planes bounding the fluid. If the fluid layer additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference i.e. the temperature field and the salt field. When the simultaneous presence of two or more components with different diffusivities is considered, the phenomenon of convection which arises is called thermosolutal or double diffusive convection. For the specific case involving a temperature field and sodium chloride it is frequently referred as thermohaline convection. Double-diffusive convection has been proved, when we think about ocean where both heat and salt (or some dissolved substances) are important. In thermosolutal convection, when the thermal and solutal effect are aiding each other, the convective flow behaviour remains qualitatively similar to that of pure thermal convection. In these problems, the solute is commonly, but not necessarily, as salt. Related effects have now been observed in other contexts and the name double-diffusive convection has been used to cover this wide range of phenomena.

1.11 Suspended Particles

The effect of suspended particles on the stability of superposed fluids might be of industrial and chemical engineering importance. Further motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. Also we are motivated to the study because of the decades old contradiction between the theory for onset of convection and experiement. A contradiction between the theory and his experiments for the onset of convection in fluids heated from below was observed by Chandra [22]. He performed the experiments in an air layer and found that the instability depended on the depth of the layer. A Bénard-type cellular convection with fluid descending at the cell centre was observed when the predicted gradients were imposed, for layers deeper than 10 mm. However, if the layer depth was less than 7 mm, convection, which was different in character from that in deeper layers, occurred at much lower gradients than predicted.

Chandra called this motion “Columnar instability.” A complete survey of subsequent experimental studies, which confirm Chandra’s result, can be found in report by Jones (1962) on the effect of different aerosols on stability. According to him the effects, which may be important, are thermal forces, electrostatic charges, evaporation, condensation and buoyancy forces. Jones concluded that columnar instability is not an example of single-phase natural convection and that it is most likely due to the unique properties of aerosol suspensions. Theoretical discussions of columnar instability, have been given by Sutton (1950) and Segel and Stuart (1962).

Motivated by interest in fluid-particle mixtures generally and columnar instability in particular, Scanlon and Segel [23] investigated the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles.

The effect of suspended particles was found to destabilize the layer i.e. to lower the critical temperature gradient. Sharma, Prakash and Dube (1976) have studied the effect of suspended particles on the onset of Bénard convection in the presence of magnetic field and rotation separately. They have found that the magnetic field and rotation have stabilizing effects whereas the effect of suspended particles is to destabilize the layer.

1.12 Effect of Magnetic Field

Consider a fluid to be electrically conducting and be under the influence of a magnetic field. The electrical conductivity of the fluid and the prevalence of magnetic field contribute to effects of two kinds. First, by the motion of the electrically conducting fluid across the magnetic lines of force, electric currents are generated and the associated magnetic fields contribute to changes in the existing fields, and second, the fact that the fluid elements carrying currents transverse magnetic lines of forces contributes to additional forces acting on the fluid elements. It is this two-fold interaction between the motions and the fields that is responsible for patterns of behaviour which are often striking and unexpected. The interaction between the fluid motions and magnetic fields are contained in Maxwell’s equations. As a consequence of Maxwell’s equations, equations of hdyrodynamics are modified suitably.

In the outer layers of stars like the Sun, thermal convection is affected by the presence of magnetic fields. In stellar interiors and atmospheres, the magnetic field may be variable and may altogether alter the nature of the instability. For example, Kent (1966) studied the effect of a horizontal magnetic field, which varies in the vertical direction, on the stability of parallel flows and showed that the system is unstable under certain conditions, while in the absence of magnetic field, the system is known to be stable.

1.13 Perturbation Method

Most of the physical problems facing engineers,applied mathematicians and physicists today exhibit certain essential features which preclude exact *analytical* solutions. Some of these features are *nonlinearities*, variable coefficients, complex boundary shapes, and nonlinear boundary conditions at known or unknown boundaries. Thus in order to obtain information about solutions of equations, we forced to resort to approximations, numerical solutions or both. *Perturbation method* is one of those approximation techniques. According to this technique some parameters of the initial state of the system are perturbed, and by substituting all these *perturbed variables* in the flow governing equations to obtain the perturbed or *linearized equations*. Perturbation methods are also used in the study of dynamic stability of aircraft.

Let the equilibrium conditions of flight are given by $u = u_0, v = v_0$ and $w = w_0$, non-zero velocity in forward direction only and other conditions are $p_0, q_0, r_0 = 0$, $\phi_0 = 0$ and $\theta = \theta_0$. Non linear equation of motion of aircraft is

$$m\left(\frac{\partial u}{\partial t} + qw - rv\right) = X - mg\sin\theta.$$

Now, perturb the equilibrium condition as

$u = u_0 + \delta u,$

$v = v_0 + \delta v,$

$w = w_0 + \delta w,$

$p = p_0 + \delta p,$

$q = q_0 + \delta q,$

$\phi = \phi_0 + \delta\phi$

and $\quad \theta = \theta_0 + \delta\theta$

where $\delta u, \delta v, \delta w, \delta p, \delta q, \delta\phi, \delta\theta$ are all perturbation in variables.

The above equations of motion are true for both the states, equilibrium and perturbed state. Thus linearize the equations of motion of aircraft and then study the dynamics of perturbed variables if all perturbed variables decaying in time then aircrafts stable in that particular equilibrium condition.

Chapter 2

Thermal Instability of Rivlin-Ericksen Elastico-Viscous Fluid Permeated with Suspended Particles in Hydrodynamics in a Porous Medium

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2.1 Introduction

The formulation and derivation of the basic equations of a layer of a *fluid* heated from below in a *porous medium*, using the Boussinesq approximation, has been given in treatise by Joseph [24]. When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by *Darcy's law*. The study of a layer of a fluid heated from below in a porous medium is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous media, Singh and Gupta [25].

A comprehensive account of the effect of a uniform *magnetic field* on the layer of a Newtonian fluid heated from below was given by Chandrasekhar [26]. The effect of a magnetic field on the stability of the fluid flow is of interest in geophysics, particularly in the study of earth core where the earth's mantle, which consists of a conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential *diffusion*. The results of flow through a porous medium in the presence of a magnetic field are applied in the study of the *stability* of a *convective flow* in the *geothermal* region. Lapwood [27] studied the stability of a convective flow in hydrodynamics using Rayleigh's procedure. Wooding [28] considered the Rayleigh instability of a thermal boundary layer in the flow through a porous medium.

The fluid may not be absolutely *pure* but may, instead, be *permeated* with suspended (or dust) particles. The effect of particle mass and *heat capacity* on the onset of Bénard convection was considered by Scanlon and Segel [23]. The effect of suspended particles was found to *destabilize* the layer. In another context, Palaniswamy and Purushotham [29] studied the *stability* of a *shear flow* of stratified fluids with *fine dust* and found the effect of fine dust to increase the region of instability. The *thermal instability* of fluids in a porous medium in the presence of suspended particles was studied by Sharma and Sharma [30]. The suspended particles and the permeability of the medium were found to destabilize the layer. Sharma and Kumar [31] studied the *Rayleigh-Taylor instability* of fluids in porous media in the presence of suspended particles and *variable magnetic field*. In all the above studies, the fluid has been considered to be *Newtonian*. One such class of elastico- *viscous* fluids is the *Rivlin-Ericksen* fluid [32]. Srivastava and Singh [33] studied the unsteady flow of the dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross sections in the presence of a time-dependent pressure gradient. In other study, Garg et al. [34] studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivilin-Ericksen fluid in the presence of a uniform magnetic field. Sharma and Kumar [35] studied the thermal instability of a layer of a Rivlin-Ericksen elastico-viscous fluid in the presence of suspended particles. In another study, Kumar [36] considered the stability of suspended Rivlin-Ericksen elastico-viscous fluids permeated with suspended particles in a porous medium. It is this class of elastico-viscous fluids we are particularly interested in studying the effect of suspended or dust particles on the Rivlin-Ericksen elastico-viscous fluid heated from below in a porous medium in the presence of a uniform horizontal *magnetic field*.

2.2 Formulation of the Problem

Let us consider the following *physical quantities* for the formulation of the problem. Tensor quantities like stress, rate of strain, shear stress , Kronecker delta be represented by $T_{kl}, e_{kl}, \tau_{kl}$ and δ_{kl} respectively. Vector quantities like velocity and position vector be represented by $\vec{v}$ and $\vec{x}$ respectively. p denotes the isotropic pressure and material properties viscosity and viscoelasticity be denoted by μ and μ'. Constitutive relations between the stress and rate of strain for the Rivlin-Ericksen fluid are

$$T_{kl} = -p\delta_{kl} + \tau_{kl},$$

$$\tau_{kl} = \rho\left(\nu + \nu'\frac{\partial}{\partial t}\right)e_{kl},$$

$$e_{kl} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k}\right). \tag{2.1}$$

In porous medium, an infinite *horizontal layer* of depth d of an electrically conducting viscoelastic Rivlin-Ericksen fluid which is acted on by gravity force

$\vec{g}(0,0,-g)$ and a uniform horizontal magnetic field $\vec{H}(0,0,H)$ is considered. For the study of thermal instability, layer is heated from underside and steady adverse temperature gradient β is maintained , where $\beta = |\frac{dT}{dz}|$.

Let the fluid properties like pressure, temperature, density , velocity of pure fluid, kinematic viscosity and kinematic viscoelasticity be denoted by $p, T, \rho, \vec{v}(u,v,w), \nu$ and ν' respectively. Properties of suspended particle like velocity and number density be represented by $u(\overline{x},t)$ and $N(\overline{x},t)$. $\vec{g}$ is the gravitational acceleration, $epsilon$ represents the medium porosity and k_1 represents the medium permeability. $K = 6\pi\mu\eta'$ is the Stokes' drag coefficient for the particle having the radius η'.

Then the *flow governing equations* of conservation of mass and momentum in a porous medium for the considered fluid in the presence of magnetic field and suspended particles are

$$\frac{1}{\epsilon}\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\epsilon}(\vec{v}.\nabla)\vec{v}\right] = -\frac{1}{\rho_0}\nabla p - g\left(1+\frac{\delta\rho}{\rho_0}\right)\vec{\lambda} - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN}{\rho_0\epsilon}(\vec{u}-\vec{v}) + \frac{\mu_e}{4\pi\rho_0}\left[\left(\nabla\times\vec{H}\right)\times\vec{H}\right] \quad (2.2)$$

and $\quad \nabla.\vec{v} = 0. \quad (2.3)$

In the above equations of conservation of momentum (2.2), some assumptions regarding the shape and velocity of the suspended particles are taken as

- Shape of the *suspended particles* in the fluid is uniform spherical.
- Relative velocities between the fluid and particles is small.
- Large distance between the particles as compare to their diameter. So Interparticle reactions are ignored.
- Gravity, pressure, Darcian force and magnetic field effect on the suspended particles are negligibly small, so ignored.
- Extra force due to the presence of particles is proportional to *velocity difference* between the particles and the fluid.
- Force exserted by fluid on particles and force exerted by particles on fluid balance each other.

So there must be an extra force equal in magnitude but opposite in sign in the equations of conservation of momentum or motion for the particles. If mN is the mass of particles per unit volume, then under the above assumptions, equations of conservation of momentum and mass for the particles are

$$mN\left[\frac{\partial\vec{u}}{\partial t}+\frac{1}{\epsilon}(\vec{u}.\nabla)\vec{u}\right]=KN(\vec{v}-\vec{u})\quad\text{and} \tag{2.4}$$

$$\epsilon\frac{\partial N}{\partial t}+\nabla.(N\vec{u})=0. \tag{2.5}$$

Let at *constant volume*, C_v is the *heat capacity* of the fluid, C_{pt} denote the heat capacity of the particles, T is the temperature and q is effective thermal conductivity of the pure fluid. If the fluid and the particles are in thermal equilibrium, then equation of heat conduction is

$$[\rho_0 C_v\epsilon+\rho_s C_s(1-\epsilon)]\frac{\partial T}{\partial t}+\rho_0 C_v(\vec{v}.\nabla)T+mNC_{pt}\left(\epsilon\frac{\partial}{\partial t}+\vec{u}.\nabla\right)T=q\nabla^2 T \tag{2.6}$$

where ρ_s is the density and C_s is the heat capacity of the solid matrix. Maxwell's equations yield:

$$\epsilon\frac{\partial\vec{H}}{\partial t}=(\vec{H}.\nabla)\vec{v}+\epsilon\eta\nabla^2\vec{H}\quad\text{and} \tag{2.7}$$

$$\nabla.\vec{H}=0 \tag{2.8}$$

where $\eta\rightarrow$ The electrical resistivity. Equation of state for fluid is

$$\rho=\rho_0\left(1-\alpha\delta T\right)=\rho_0\left[1-\alpha(T-T_0)\right] \tag{2.9}$$

where $\alpha\rightarrow$ co-efficient of thermal expansion, $\rho_0\rightarrow$ density of the fluid at the *bottom surface* $z=0$ and $T_0\rightarrow$ at temperature of the fluid at $z=0$. Initially the system is taken as quiescent layer (no settling) with a uniform particle distribution N_0. Initial values of the variables are

$\vec{u}=(0,0,0),\quad \vec{v}=(0,0,0),\quad N_0=Constant,\quad T=-\beta z.$

which is an exact solution to the governing equations.

2.2.1 Perturbation of Equations

Let δp denote the *perturbation* in pressure p, $\delta\rho$ denote the perturbation in density ρ, θ denote the perturbation in temperature T, $\vec{v}(u,v,w)$ denote the perturbation in fluid velocity (zero initially), $\vec{u}(l,r,s)$ denote the perturbation in particle velocity (zero initially), N denote perturbations in suspended particles number density N_0 and $\vec{h}(h_x,h_y,h_z)$ denote perturbations in magnetic field $\vec{H}(0,0,H)$. Since density is depends upon the temperature, so perturbation in temperature will bring change is density defined by the relation $\delta\rho = -\alpha\rho_0\theta$.

Governing equations of flow hold true for both the initial and perturbed state. Therefore, linearized perturbed equations of the problem are

$$\frac{1}{\epsilon}\frac{\partial\vec{v}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - g\alpha\theta\lambda - \frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN}{\rho_0\epsilon}(\vec{u}-\vec{v}) + \frac{\mu_e}{4\pi\rho_0}\left[\left(\nabla\times\vec{h}\right)\times\vec{H}\right], \tag{2.10}$$

$$\nabla.\vec{v} = 0, \tag{2.11}$$

$$mN_0\frac{\partial\vec{u}}{\partial t} = KN_0(\vec{v}-\vec{u}), \tag{2.12}$$

$$(E+h\epsilon)\frac{\partial\theta}{\partial t} = \beta(w+hs) + \kappa\nabla^2\theta, \tag{2.13}$$

$$\epsilon\frac{\partial\vec{h}}{\partial t} = (\vec{H}.\nabla)\vec{v} + \epsilon\eta\nabla^2\vec{h} \tag{2.14}$$

and $\quad \nabla.\vec{h} = 0 \qquad (2.15)$

where $\quad E = \epsilon + (1-\epsilon)\frac{\rho_s C_s}{\rho_0 C_v}, \quad h = \frac{mN_0C_{pt}}{\rho_0 C_v} \quad and \quad \kappa = \frac{q}{\rho_0 C_v}.$

Eliminating u in equation (2.10) by using equation (2.12), write the resulting equation in scalar components eliminate $u, v, \delta p, h_x, h_y$ between them, with the help of equations (2.11) and (2.15), we obtain

$$n'\nabla^2 w + \frac{\epsilon}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 w - \epsilon g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right) - \frac{\mu_e H}{4\pi\rho_0}\frac{\partial}{\partial x}\nabla^2 h_z = 0, \tag{2.16}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[(E+h\epsilon)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta=\beta\left[\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)+h\right]w \quad \text{and} \tag{2.17}$$

$$\epsilon\left[\frac{\partial}{\partial t}-\eta\nabla^2\right]h_z=H\frac{\partial w}{\partial x} \tag{2.18}$$

$$\text{where} \quad n'=\frac{\partial}{\partial t}\left[1+\frac{\frac{mN_0K}{\rho_0}}{m\frac{\partial}{\partial t}+K}\right]$$

2.3 Dispersion relation

Perturbed quantities are assumed to be of the following form for the analysis of disturbances into normal modes

$$[w,\theta,h_z]=[W(z),\Theta(z),X(z)]\,exp\,(ik_xx+ik_yy+nt) \tag{2.19}$$

where k_x is the wave number along x-direction, and k_y is wave number along y-direction. $k=\sqrt{k_x^2+k_y^2}$ = resultant wave number and n = growth rate = complex constant in general. Using expression (2.19), equations (2.16)-(2.18) in a non dimensional form become

$$\left[\frac{\sigma'}{\epsilon}+\frac{1}{p_1}(1+F\sigma)\right]\left(D^2-a^2\right)W+\frac{g\alpha d^2a^2\Theta}{\nu}-\frac{ik_x\mu_eHd^2}{4\pi\rho_0\nu}\left(D^2-a^2\right)X=0, \tag{2.20}$$

$$\left[\frac{\tau\nu\sigma}{d^2}+1\right]\left[\left(D^2-a^2\right)-(E+h\epsilon)p_3\sigma\right]\Theta=-\frac{\beta d^2}{\kappa}\left[H'+\frac{\tau\nu\sigma}{d^2}\right]W \tag{2.21}$$

$$\text{and} \quad \left[\left(D^2-a^2\right)-p_2\sigma\right]\chi=-\frac{ik_xHd^2}{\epsilon\eta}W \tag{2.22}$$

where the co-ordinates x,y,z have expressed in the new unit of length d, time t in the new unit of length $\frac{d^2}{\kappa}$ and put $a=kd,\sigma=\frac{nd^2}{\nu},p_3=\frac{\nu}{\kappa}\rightarrow$ Prandtl number, $p_2=\frac{\nu}{\eta}\rightarrow$ magnetic Prandtl number, $p_1=\frac{k_1}{d^2}\rightarrow$ dimensionless medium permeability, $F=\frac{\nu}{d^2}\rightarrow$ dimensionless kinematic viscoelasticity, $\sigma'=\frac{n'd^2}{\nu},H'=h+1,\tau=\frac{m\kappa}{Kd^2}$ and $D=\frac{d}{dz}$.

By eliminating X and Θ between equations (2.20)-(2.22), we obtain

$$\left[1+\frac{\tau\nu\sigma}{d^2}\right]\left[\left(D^2-a^2\right)-(E+h\epsilon)p_3\sigma\right]\left[\left\{\frac{\sigma'}{\epsilon}+\frac{1}{p_1}(1+F\sigma)\right\}\left[\left(D^2-a^2\right)-p_2\sigma\right]\right.$$
$$\left.-\frac{k_x^2Q}{\epsilon}\right]\left(D^2-a^2\right)W = Ra^2\left[H'+\frac{\tau\nu\sigma}{d^2}\right]\left[\left(D^2-a^2\right)-p_2\sigma\right]W \quad (2.23)$$

where $R=\frac{g\alpha\beta d^4}{\nu\kappa}$ = Rayleigh number and $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ = Chandrasekhar number.

The boundary conditions, suitable for the problem, are Chandrasekhar [26]. For the solution to the problem, free boundaries are considered which is little artificial in nature. Also Temperatures at the boundaries are kept fixed and the medium adjoining the fluid is perfectly conducting.

$$W=0,\quad D^2W=0,\quad \Theta=0,\quad X=0\quad at\quad z=0\quad and\quad z=1. \quad (2.24)$$

Obviously that the even order derivatives of W vanish on the boundaries and hence the proper solution of W characterizing the lowest mode is

$$W=W_0 sin(\pi z) \quad (2.25)$$

where $W_0=Constant$. By putting the solution (2.25) in equation (2.23), the dispersion relation can be written as

$$R_1=\frac{\begin{array}{c}(1+x)\left[(1+x)+(E+h\epsilon)i\sigma_1p_1\right]\left(1+\frac{i\nu\tau\pi^2\sigma_1}{d^2}\right)*\\ \left[\left\{\frac{i\sigma_1'}{\epsilon}+\frac{1}{P}\left(1+i\pi^2F\sigma_1\right)\right\}\left\{(1+x)+i\sigma_1p_2\right\}+\frac{Q_1xcos^2\theta}{\epsilon}\right]\end{array}}{x\left[H'+\frac{i\nu\pi^2\tau\sigma_1}{d^2}\right]\left\{(1+x)+i\sigma_1p_2\right\}} \quad (2.26)$$

where $\quad x=\frac{a^2}{\pi^2},\quad i\sigma_1=\frac{\sigma}{\pi^2},\quad P=\pi^2p_1\quad and\quad R_1=\frac{R}{\pi^4},$

$$i\sigma_1'=\frac{\sigma'}{\pi^2},\quad Q_1=\frac{Q}{\pi^2},\quad and\quad k_x=kcos\theta.$$

2.4 Stationary convection

Put $\sigma=0$, for stationary convection, and the dispersion relation (2.26) becomes

$$R_1=\frac{(1+x)\left[\frac{1+x}{P}+\frac{Q_1xcos^2\theta}{\epsilon}\right]}{xH'}. \quad (2.27)$$

Thus, it is found that for stationary convection the viscoelastic parameter F vanishes with σ and the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid. To study the effects of the magnetic field, suspended particles and medium permeability, we examine the nature of $\frac{dR_1}{dQ_1}, \frac{dR_1}{dH'}$, and $\frac{dR_1}{dP}$, Equation (2.27) yields:

$$\frac{dR_1}{dQ_1} = \frac{(1+x)cos^2\theta}{H'\epsilon}, \tag{2.28}$$

$$\frac{dR_1}{dH'} = -\frac{(1+x)\left[\frac{1+x}{P} + \frac{Q_1 x cos^2\theta}{\epsilon}\right]}{xH'^2} \tag{2.29}$$

and $$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xH'P^2}. \tag{2.30}$$

Which shows that the magnetic field has a stabilizing effect whereas the suspended particles and medium permeability have a destabilizing effect on thermal convection in the Rivlin-Ericksen fluid permeated with suspended particles in a porous medium in hydrodynamics for stationary convection. Graphically, we analyse the magnetic field, suspended particles and medium permeability as follows:

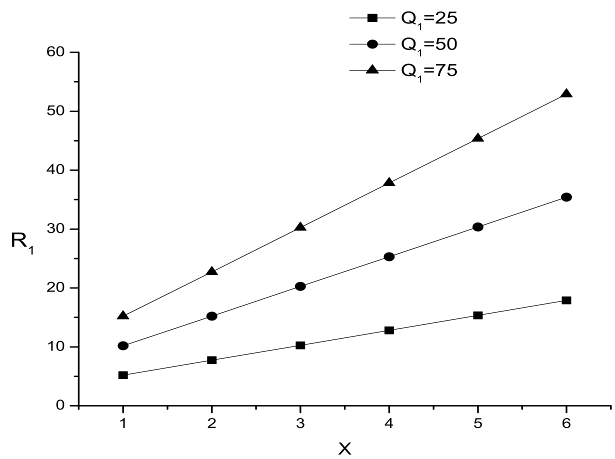

Figure 2.1: The variation of dimensionless Rayleigh number (R_1) with wave number (x), for $H' = 10, P = 2, \theta = 45^0, \epsilon = 0.5$ and $Q_1 = 25, 50, 75$.

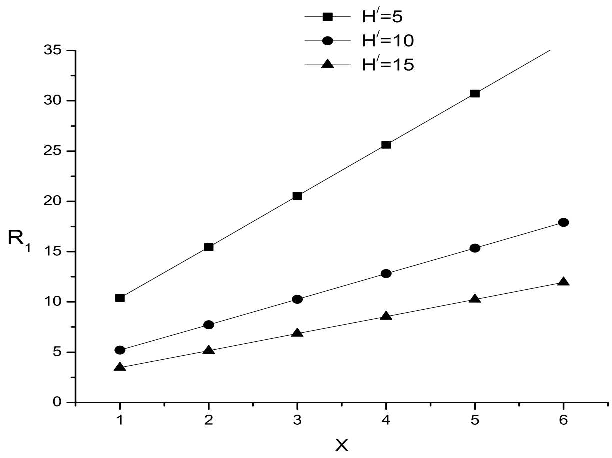

Figure 2.2: The variation of dimensionless Rayleigh number (R_1) with wave number (x), for $P = 2, Q_1 = 25, \theta = 45^0, \epsilon = 0.5$ and $H' = 5, 10, 15$.

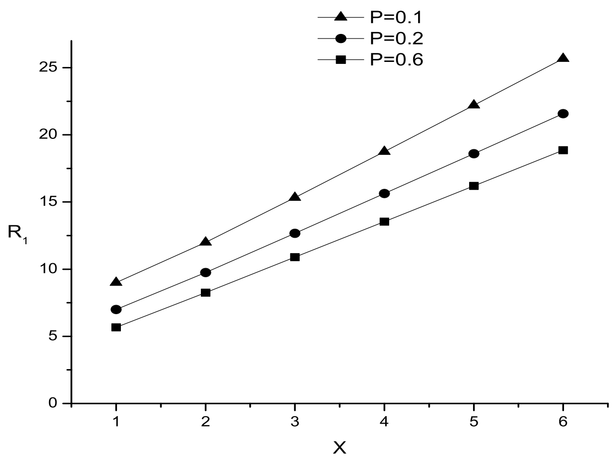

Figure 2.3: The variation of dimensionless Rayleigh number (R_1) with wave number (x), for $Q_1 = 25, H' = 10, \theta = 45^0, \epsilon = 0.5$ and $P = 0.1, 0.2, 0.6$.

In **Figure2.1** (refer table 1), we have $x = 1, 2, 3, 4, 5, 6$ and $H' = 10, P = 2, \epsilon = 0.5$, $\theta = 45^0$ and $Q_1 = 25, 50, 75$, found that, if magnetic field is increased growth rate is also increased, shows the effect of stabilization on the system.

Whereas in **Figure2.2** (refer table 2), $x = 1, 2, 3, 4, 5, 6$, $H' = 5, 10, 15, P = 2$ and $\epsilon = 0.5, \theta = 45^0, Q_1 = 25$, shows that, if suspended particles are increased growth rate is decreased, gives the effect of destabilizing effect on the system.

In **Figure2.3** (refer table 3), by using values of $H' = 10, \epsilon = 0.5, \theta = 45^0$, $Q_1 = 25$, $P = 0.1, 0.2, 0.6$ and $x = 1, 2, 3, 4, 5, 6$, found that, when medium permeability is increased, growth rate is decreased, gives the destabilizing effect on the system.

2.5 Stability of the system of oscillatory modes

Multiplying equation (2.20) by the complex conjugate of W i.e W^*, integrating over the range of z from $z = 0$ to $z = 1$ and making use of equations (2.21) and (2.22) together with the given physical boundary conditions (2.24), we obtain

$$\left[\frac{\sigma'}{\epsilon} + \frac{1}{p_1}(1+F\sigma)\right] I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}\left[\frac{d^2+\nu\tau\sigma^*}{H'd^2+\nu\tau\sigma^*}\right][I_2 + (E+h\epsilon)p_3\sigma^* I_3] + \frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}[I_4 + p_2\sigma^* I_5] = 0 \tag{2.31}$$

$$\begin{aligned} where \quad I_1 &= \int_0^1 \left(|DW|^2 + a^2|w|^2\right) dz \quad , \quad I_2 == \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz, \\ I_3 &= \int_0^1 |\Theta| dz, \qquad I_4 = \int_0^1 \left(|D^2X|^2 + 2a^2|DX|^2 + a^4|X|^2\right) dz, \\ I_5 &= \int_0^1 \left(|DX|^2 + a^2|X|^2\right) dz \end{aligned}$$

and $\sigma^* \to$ complex conjugate of σ. The integrals I_1, I_2, I_3, I_4, I_5 are all positive definite. Putting $\sigma = i\sigma, f = \frac{mN_0}{\rho_0}$, and by equating the imaginary parts of equation (2.31), we obtain

$$\sigma_i\left[\left\{\frac{1}{\epsilon}\left(1+\frac{f}{1+p_3^2\tau^2\sigma_i^2}\right)+\frac{F}{p_1}\right\} I_1 + \frac{g\alpha\kappa a^2}{\nu\beta\left(H'^2d^4+\nu^2\tau^2\sigma_i^2\right)} \left\{d^2\nu\tau h I_2 + p_3(E+h\epsilon)(H'd^4+\nu^2\tau^2\sigma_i^2)I_3\right\} + \frac{\mu_e\eta\epsilon p_2}{4\pi\rho_0\nu} I_5\right] = 0. \tag{2.32}$$

Equation (2.32) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of the magnetic field, equation (2.32) is reduced to

$$\sigma_i\left[\left\{\frac{1}{\epsilon}\left(1+\frac{f}{1+p_1^2\tau^2\sigma_i^2}\right)+\frac{F}{p_1}\right\} I_1 + \frac{g\alpha\kappa a^2}{\nu\beta\left(H'^2d^4+\nu^2\tau^2\sigma_i^2\right)} \left\{d^2\nu\tau h I_2 + p_1(E+h\epsilon)(H'd^4+\nu^2\tau^2\sigma_i^2)I_3\right\}\right] = 0. \tag{2.33}$$

Thus, $\sigma_i = 0 \Rightarrow$ the principle of exchange of stabilities is valid but oscillatory modes are not allowed. Whereas the quantity inside the brackets is positive definite. The presence of the magnetic field introduces oscillatory modes.

2.6 Conclusion

Presence of Magnetic field showed the stabilizing effect whereas presence of suspended particles and medium permeability showed the destabilizing effect in the study of Rivlin-Ericksen fluid.

Chapter 3

Hall Effect on Thermal Instability of Viscoelastic Dusty Fluid in Porous Medium

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3.1 Introduction

The theoretical and experimental results of the onset of thermal instability (Bénard convection), under varying assumptions of hydrodynamics and hydromagnetics, have been discussed by Chandrasekhar [26] in his celebrated monograph. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current is called the Hall current effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasma, Singh and Gupta [37]. Sherman and Sutton [38] considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta [39] studied the problem of thermal instability in the presence of Hall currents and found that Hall currents have a destabilizing effect on the thermal instability of a horizontal layer of a conducting fluid in the presence of a uniform vertical magnetic field. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in association with the external force. The approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density variations arise principally from thermal effects. Spiegel and Veronis [40] simplified the set of equations governing the flow of compressible fluids under the following assumptions:

- The depth of the fluid layer is much less than the scale height, as defined by them.
- The fluctuations in temperature, density and pressure, introduced due to motion, do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the *static temperature gradient* is replaced by its excess over the adiabatic one and C_v is replaced by C_p. In geophysical situations, the fluid is often not pure but contains suspended particles. Scanlon and Segel [23] considered the effects of suspended particles on the onset of Bénard convection and found that the *critical Rayleigh number* is reduced because of the heat capacity of the particles. The suspended particles were thus found to destabilize the layer.

Palaniswamy and Purushotham [29] studied the stability of *shear flow* of stratified fluids with *fine dust* and found the fine dust to increase the region of instability. The fluids were considered to be Newtonian and the medium was considered to be *non-porous* in all the above studies.

There is growing importance of *non-Newtonian* fluids in geophysical fluid dynamics, chemical technology and *petroleum industry*. Bhatia and Steiner [41] studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation was studied by Sharma [42]. There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's *constitutive relations*. The Rivlin-Ericksen elastico-viscous fluid is one such fluid. Rivlin and Ericksen [32] studied the stress, deformation, relaxations for isotropic materials. Thermal instability in viscoelastic Rivlin-Ericksen fluids in the presence of rotation and magnetic field, separately, was investigated by Sharma and Kumar [43] and [44]. Sharma and Kumar [45] studied the hydromagnetic stability of two Rivlin-Ericksen elasticoviscous superposed conducting fluids. Kumar and Singh [46] studied the stability of two superposed Rivlin-Ericksen viscoelastic fluids in the presence of suspended particles. In another study, Kumar et al. [47] studied the hydrodynamic and hydromagnetic stability of two *stratified* Rivlin-Ericksen elasticoviscous *superposed fluids*.

The flow through porous media is of considerable interest for petroleum engineers and geophysical fluid dynamicists. A great number of applications in geophysics may be found in the books by Phillips [48], Ingham and Pop [49], and Nield and Bejan [50]. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\frac{1}{k_1}\left(\mu + \mu' \frac{\partial}{\partial t}\right) q$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. Lapwood [27] studied the stability of a convective flow in hydromagnetics in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium was considered by Wooding [28]. The stability of superposed Rivlin-Ericksen elastico-viscous fluids permeated with suspended particles in a porous medium was considered by Kumar [36]. Kumar et al. [51] studied the instability of two rotating viscoelastic (Rivlin-Ericksen) superposed fluids with suspended particles in a porous medium. In another study, Kumar et al. [52] considered the MHD instability of rotating superposed Rivlin-Ericksen viscoelastic fluids through a porous medium.

Here our interest is to bring out the suspended particles effect on thermal instability of a compressible viscoelastic (Rivlin-Ericksen) fluid in a porous medium including the effect of *Hall currents*.

3.2 Formulation of the Problem

In porous medium, an infinite *horizontal layer* of thickness d confined between two planes $z = 0$ and $z = d$ of an compressible viscoelastic Rivlin-Ericksen fluid in the presence of uniform horizontal magnetic field $\vec{H}(0, 0, H)$ is considered. For the study thermal instability, layer is heated from underside and steady adverse temperature gradient β is maintained, where $\beta = |\frac{dT}{dz}|$. The equations of motion and continuity for the fluid are:

$$\frac{\rho}{\epsilon}\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\epsilon}\left(\vec{v}.\nabla\right)\vec{v}\right] = -\nabla p - \rho g\vec{\lambda} - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN}{\epsilon}\left(\vec{u} - \vec{v}\right) + \frac{\mu_e}{4\pi}\left(\nabla \times \vec{H}\right) \times \vec{H} \tag{3.1}$$

and $$\epsilon\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{v}) = 0 \tag{3.2}$$

where $\rho \rightarrow$ density, $\mu \rightarrow$ viscosity, $\mu' \rightarrow$ viscoelasticity, $p \rightarrow$ pressure and $\vec{v}(u, v, w) \rightarrow$ velocity of the pure fluid. Here $\vec{u}(l, r, s) \rightarrow$ velocity of the suspended particles, $N(\overline{x}, t) \rightarrow$ number density of the suspended particles, $\epsilon \rightarrow$ medium porosity, $k_1 \rightarrow$ medium permeability, $\mu_e \rightarrow$ magnetic permeability, $g \rightarrow$ acceleration due to gravity, $\overline{x} = (x, y, z)$, $\vec{\lambda}(0, 0, 1)$ and $K = 6\pi\mu\eta'$, η' being the particle radius, is the Stokes' drag coefficient.

In the above equations of conservation of momentum (3.1), Some assumptions regarding the shape and velocity of the suspended particles are taken as

- Shape of the suspended particles in the fluid is uniform spherical.
- Relative velocities between the fluid and particles is small.
- Large distance between the particles as compare to their diameter. So Interparticle reactions are ignored.
- Gravity, pressure, Darcian force and magnetic field effect on the suspended particles are negligibly small, so ignored.
- Extra force due to the presence of particles is proportional to velocity difference between the particles and the fluid.
- Force exserted by fluid on particles and force exerted by particles on fluid balance each other.

So there must be an extra force equal in magnitude but opposite in sign in the equations of conservation of momentum or motion for the particles. If mN is the mass of particles per unit volume, then under the above assumptions, equations of conservation of momentum and mass for the particles are

$$mN\left[\frac{\partial \vec{u}}{\partial t}+\frac{1}{\epsilon}\left(\vec{u}.\nabla\right)\vec{u}\right]=KN(\vec{v}-\vec{u}) \tag{3.3}$$

and $$\epsilon\frac{\partial N}{\partial t}+\nabla.(N\vec{u})=0. \tag{3.4}$$

Let at *constant volume*, C_v is the *heat capacity* of the fluid, at constant pressure, C_p is the *heat capacity* of the fluid, C_{pt} denote the heat capacity of the particles T is the temperature and q is *effective thermal conductivity* of the pure fluid. Assuming, fluid particles are in *thermal equilibrium*, then equation of *heat conduction* is given by

$$[\rho C_v\epsilon+\rho_s C_s(1-\epsilon)]\frac{\partial T}{\partial t}+\rho C_v(\vec{v}.\nabla)T+mNC_{pt}\left(\epsilon\frac{\partial}{\partial t}+\vec{u}.\nabla\right)T=q\nabla^2 T \tag{3.5}$$

where ρ_s is the density and C_s is the heat capacity of the solid matrix, R.C.Sharma and U.Gupta [53] had used the same parameters for their study.

Maxwell's equations in the presence of hall currents give

$$\nabla.\vec{H} = 0 \tag{3.6}$$

and $$\epsilon\frac{\partial H}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \epsilon\eta\nabla^2\vec{H} - \frac{c\epsilon}{4\pi N'e}\nabla \times \left[(\nabla \times \vec{H}) \times \vec{H}\right] \tag{3.7}$$

where $\eta \rightarrow$ resistivity, $c \rightarrow$ speed of light, $N' \rightarrow$ electron number density and e is charge of an electron. The initial state of the system is taken to be a quiescent layer (no settling) with a uniform particle distribution N_0 and is given by

$$\vec{u} = (0,0,0), \quad \vec{v} = (0,0,0), \quad \vec{H} = (0,0,H),$$

$$T = T(z), \quad p = p(z) \quad \rho = \rho(z) \quad \text{and} \quad N = N_0 = constant. \tag{3.8}$$

Following the Spiegel and Veronis' [40] we have

$$T(z) = -\beta z + T_0, \quad p(z) = p_m - g\int_0^z (\rho_m + \rho_0)\, dz,$$

$$\rho(z) = \rho_m \left[1 - \alpha_m(T - T_m) + K_m(p - p_m)\right],$$

$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right)_m \quad \text{and} \quad K_m = \left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)_m. \tag{3.9}$$

Spiegel and Veronis' [40] expressed any state variable say X, in the form

$$X = X_m + X_0(z) + X'(x,y,z,t) \tag{3.10}$$

where $X_m \rightarrow$ constant space distribution of $X, X_0 \rightarrow$ variation of X in the absence of motion and $X'(x,y,z,t) \rightarrow$ fluctuations in X due to motion of the fluid. Also, ρ_m is constant space distribution of ρ and $p_m \rightarrow$ constant space distribution of p and ρ_0 is density at the lower boundary $z = 0$ and $T_0 \rightarrow$ temperature of the fluid at $z = 0$. Again following Spiegel and Veronis[40] assumptions and results for compressible fluids, the flow equations are found to be the same as those of incompressible fluids except that the static temperature gradient β is replaced by its excess over the adiabatic $(\beta - g/C_p)$.

3.2.1 Perturbation of Equations

Let δp denote the *perturbation* in pressure p, $\delta\rho$ denote the perturbation in density ρ, θ denote the perturbation in temperature T, $\vec{v}(u, v, w)$ denote the perturbation in fluid velocity (zero initially), $\vec{u}(l, r, s)$ denote the perturbation in particle velocity (zero initially), N denote perturbations in suspended particles number density N_0 and $\vec{h}(h_x, h_y, h_z)$ denote perturbations in magnetic field $\vec{H}(0, 0, H)$. Linearized perturbed equations of the viscoelastic fluid-particle layer are:

$$\frac{1}{\epsilon}\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p - g\left(\frac{\delta\rho}{\rho_m}\right)\vec{\lambda} - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN_0}{\epsilon\rho_m}(\vec{u} - \vec{v}) + \frac{\mu_e}{4\pi\rho_m}(\nabla \times \vec{h}) \times \vec{H}, \tag{3.11}$$

$$\nabla.\vec{v} = 0, \tag{3.12}$$

$$mN_0\frac{\partial u}{\partial t} = KN_0(\vec{v} - \vec{u}), \tag{3.13}$$

$$\epsilon\frac{\partial N}{\partial t} + \nabla.(N_0\vec{u}) = 0, \tag{3.14}$$

$$(E + h\epsilon)\frac{\partial \theta}{\partial t} = (\beta - g/C_p)(w + hs) + \kappa\nabla^2\theta, \tag{3.15}$$

$$\nabla.\vec{h} = 0 \quad \text{and} \tag{3.16}$$

$$\epsilon\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \epsilon\eta\nabla^2\vec{h} - \frac{c\epsilon}{4\pi N'e}\nabla \times \left[(\nabla \times \vec{h}) \times \vec{H}\right] \tag{3.17}$$

where $\alpha_m = \frac{1}{T_m} = \alpha$ (say), $\nu = \frac{\mu}{\rho_m}, \kappa = \frac{q}{\rho_m C_v}$ and $\frac{g}{C_p} \rightarrow$ adiabatic gradient, ν is kinematic *viscosity* and κ is thermal diffusivity. Also,

$$h = \frac{fC_{pt}}{C_v}, \quad f = \frac{mN_0}{\rho_m} \quad \textit{and} \quad E = \epsilon + \frac{(1-\epsilon)\rho_s C_s}{\rho_m C_v}.$$

The linearized dimensionless perturbation equations relevant to the problem are

$$N_{p_1}^{-1}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - \frac{1}{p}\left(1 + A\frac{\partial}{\partial t}\right)u + \omega\,(l - u) + N_Q\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right), \tag{3.18}$$

$$N_{p_1}^{-1}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - \frac{1}{p}\left(1 + A\frac{\partial}{\partial t}\right)v + \omega\left(r - v\right) + N_Q\left(\frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y}\right), \tag{3.19}$$

$$N_{p_1}^{-1}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - \frac{1}{p}\left(1 + A\frac{\partial}{\partial t}\right)w + \omega\left(s - w\right) + N_R\theta, \tag{3.20}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{3.21}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)l = u, \quad \left(\tau\frac{\partial}{\partial t} + 1\right)r = v, \quad \left(\tau\frac{\partial}{\partial t} + 1\right)s = w, \tag{3.22}$$

$$\frac{\partial M}{\partial t} + \frac{\partial l}{\partial x} + \frac{\partial r}{\partial y} + \frac{\partial s}{\partial z} = 0, \tag{3.23}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0, \tag{3.24}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_x}{\partial t} = \epsilon^{-1}\frac{\partial u}{\partial z} + \nabla^2 h_x - M_1\frac{\partial}{\partial z}\left(\frac{\partial h_z}{\partial y} - \frac{\partial h_y}{\partial z}\right), \tag{3.25}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_y}{\partial t} = \epsilon^{-1}\frac{\partial v}{\partial z} + \nabla^2 h_y - M_1\frac{\partial}{\partial z}\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right), \tag{3.26}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_z}{\partial t} = \epsilon^{-1}\frac{\partial w}{\partial z} + \nabla^2 h_z - M_1\frac{\partial}{\partial z}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right) \quad \text{and} \tag{3.27}$$

$$\left(E + h\epsilon\right)\frac{\partial \theta}{\partial t} = \left(\frac{G-1}{G}\right)\left(w + hs\right) + \nabla^2\theta \tag{3.28}$$

where

$N_{p_1} = \frac{\epsilon\nu}{\kappa}$ is modified Prandtl number, $N_{p_2} = \frac{\epsilon\nu}{\eta}$ is modified magnetic Prandtl number, $N_R = \frac{g\alpha\beta d^4}{\nu\kappa}$ is *Rayleigh number*, $N_Q = \frac{\mu_e H^2 d^2}{4\pi\rho_m\nu\eta}$ is Chandrasekhar number, $M = \frac{\epsilon N}{N_0}$, $M_1 = \frac{cH}{4\pi N' e\eta}$ is Hall parameter, $\omega = \frac{KN_0 d^2}{\rho_m\nu\epsilon}$, $\tau = \frac{m\kappa}{Kd^2}$, $A = \left(\frac{\nu}{\nu}{}'\right)\frac{\kappa}{d^2}$, $f = \frac{mN_0}{\rho_m} = \tau\omega$, N_{p_1} is mass fraction, $G = \frac{C_p\beta}{g}$ and $P = \frac{k_1}{d^2}$.

Here physical variables have been scaled using $d, \frac{d^2}{\kappa}, \frac{\kappa}{d}, \frac{\rho\nu\kappa}{d^2}, \beta d$ and $\frac{H\kappa}{\eta}$ as the length, time, velocity, pressure, temperature and magnetic field scale factors, respectively. The boundary conditions suitable to the problem, two *free boundaries* and the medium adjoining the fluid as non conducting, are considered as

$$w = \frac{\partial^2 w}{\partial z^2} = \theta = 0, \quad \xi = \frac{\partial \zeta}{\partial z} = 0, \quad at \quad z = 0 \quad and \quad z = 1. \tag{3.29}$$

and $h_x, h_y, h_z \quad are \quad continuous \quad with \quad an \quad external \quad vacuum \quad field.$ (3.30)

Here $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ are the z-components of vorticity and current density, respectively. Equations (3.18)-(3.28), after eliminating u, v and δp can be expressed as

$$\left[L_1 + \frac{L_2}{P}\left(1 + A\frac{\partial}{\partial t}\right)\right]\nabla^2 w = L_2 N_Q \nabla^2 \frac{\partial h_z}{\partial z} + L_2 N_R \nabla_1^2 \theta, \tag{3.31}$$

$$\left[L_1 + \frac{L_2}{P}\left(1 + A\frac{\partial}{\partial t}\right)\right]\zeta = L_2 N_Q \frac{\partial \xi}{\partial z}, \tag{3.32}$$

$$\left[N_{p2} N_{p1}^{-1} \frac{\partial}{\partial t} - \nabla^2\right]\xi = \epsilon^{-1}\frac{\partial \zeta}{\partial z} + M_1 \frac{\partial}{\partial z}\left(\nabla^2 h_z\right), \tag{3.33}$$

$$\left[N_{p2} N_{p1}^{-1} \frac{\partial}{\partial t} - \nabla^2\right] h_z = \epsilon^{-1}\frac{\partial w}{\partial z} - M_1 \frac{\partial \xi}{\partial z} \quad \text{and} \tag{3.34}$$

$$L_2\left[(E + h\epsilon)\frac{\partial}{\partial t} - \nabla^2\right]\theta = \left(\frac{G-1}{G}\right)\left(\tau\frac{\partial}{\partial t} + \overline{H}\right) w \tag{3.35}$$

where

$$L_1 = N_{p_1}^{-1}\left(\tau\frac{\partial^2}{\partial t^2} + F\frac{\partial}{\partial t}\right), \quad F = f + 1, \quad L_2 = \tau\frac{\partial}{\partial t} + 1, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \overline{H} = h + 1.$$

3.3 The Dispersion Relation

Perturbed quantities are assumed to be of the following form and for the analysis of disturbances into normal modes by seeking solutions whose dependence on x, y and t is given by

$$[w, \theta, h_z, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)]\, exp(ik_x x + ik_y y + nt) \tag{3.36}$$

where k_x is the wave number along x-direction, and k_y is wave number along y-direction. $k = \sqrt{k_x^2 + k_y^2}$ = resultant wave number and n = growth rate. Equations (3.31)-(3.35), with the help of expression (3.36), become

$$\left[L_1 + \frac{L_2}{P}(1 + An)\right]\left(D^2 - \alpha^2\right) W = L_2 N_Q \left(D^2 - \alpha^2\right) DK - L_2 N_R \alpha^2 \Theta, \tag{3.37}$$

$$\left[L_1 + \frac{L_2}{P}(1 + An)\right] Z = L_2 N_Q DX, \tag{3.38}$$

$$\left[N_{p2} N_{p1}^{-1} n - \left(D^2 - \alpha^2\right)\right] X = \epsilon^{-1} DZ + M_1 \left(D^2 - \alpha^2\right) DK, \tag{3.39}$$

$$\left[N_{p2} N_{p1}^{-1} n - \left(D^2 - \alpha^2\right)\right] K = \epsilon^{-1} DW - M_1 DX \quad \text{and} \tag{3.40}$$

$$L_2 \left[(E + h\epsilon)\, n - \left(D^2 - \alpha^2\right)\right] \Theta = \left(\frac{G - 1}{G}\right)\left(\tau n + \overline{H}\right) W \tag{3.41}$$

where $D = \dfrac{d}{dz}$, $\quad L_1 = N_{p_1}^{-1}(\tau n^2 + Fn) \quad and \quad L_2 = \tau n + 1.$

By eliminating X, Z, K, and Θ from the equations (3.37)-(3.41), we obtain

$$\begin{aligned}
&\left[L_1 + \frac{L_2}{P}(1 + An)\right]\left[(D^2 - \alpha^2) - (E + h\epsilon)n\right]\left(D^2 - \alpha^2\right) W \\
&+ \left[\frac{L_2 N_Q \left[(D^2 - \alpha^2) - (E + h\epsilon)n\right]\left[\frac{(D^2 - \alpha^2) - N_{p_2} N_{p_1}^{-1} n}{M_1 \epsilon} + \frac{L_2 N_Q D^2}{\left[L_1 + \frac{L_2}{P}(1 + An)\right] M_1 \epsilon^2}\right] D^2}{M_1 (D^2 - \alpha^2) D^2 + \frac{\left\{(D^2 - \alpha^2) - N_{p_2} N_{p_1}^{-1} n\right\}^2}{M_1} + L_2 N_Q \frac{(D^2 - \alpha^2) - N_{p_2} N_{p_1}^{-1} n}{\left\{L_1 + \frac{L_2}{P}(1 + An)\right\} M_1 \epsilon} D^2}\right]\left(D^2 - \alpha^2\right) W \\
&= \left(\frac{G - 1}{G}\right) N_R \alpha^2)(\tau n + \overline{H}) W.
\end{aligned} \tag{3.42}$$

Using the boundary conditions and equations (3.29) and (3.30), obviously that the even order derivatives of W vanish on the boundaries and hence the proper solution of equation (3.42) characterizing the lowest mode is

$$W = W_0 sin(\pi z), \quad where \quad W_0 = Constant. \tag{3.43}$$

On substituting the solution (3.43) in equation (3.42), we get the dispersion relation as

$$N_R = \left(\frac{G}{G-1}\right)\frac{\left(\pi^2+\alpha^2\right)\left[\left(\pi^2+\alpha^2\right)+\left(E+h\epsilon\right)n\right]}{\alpha^2\left(\tau n+\overline{H}\right)}\left[\left\{L_1+\frac{L_2}{P}\left(1+An\right)\right\}+\right.$$
$$\left.\frac{L_2 N_Q \pi^2\left[\frac{\left(\pi^2+\alpha^2\right)+N_{p2}N_{p1}^{-1}n}{M_1\epsilon}+\frac{L_2 N_Q \pi^2}{\left\{L_1+\frac{L_2}{P}(1+An)\right\}M_1\epsilon^2}\right]}{\left[M_1\pi^2\left(\pi^2+\alpha^2\right)+\frac{\left\{(\pi^2+\alpha^2)+N_{p2}N_{p1}^{-1}n\right\}^2}{M_1}+\frac{L_2 N_Q\left\{(\pi^2+\alpha^2)+N_{p2}N_{p1}^{-1}n\right\}\pi^2}{\left\{L_1+\frac{L_2}{P}(1+An)\right\}M_1\epsilon}\right]}\right]. \tag{3.44}$$

3.4 Stationary Convection

When the instability sets in as *stationary convection*, the *marginal state* will be characterized by $n = 0$ and the *dispersion relation* equation (3.44) reduces to

$$N_R = \left(\frac{G}{G-1}\right)\frac{\left(\pi^2+\alpha^2\right)^2}{\alpha^2\overline{H}}\left[\frac{1}{P}+\frac{N_Q\epsilon^{-1}\pi^2\left\{\left(\pi^2+\alpha^2\right)+N_Q P\epsilon^{-1}\pi^2\right\}}{\left(\pi^2+\alpha^2\right)\left\{M_1^2\pi^2+\left(\pi^2+\alpha^2\right)+N_Q P\epsilon^{-1}\pi^2\right\}}\right]. \tag{3.45}$$

Thus for stationary convection, the viscoelastic parameter vanishes with n, and stress and strain rate showed linear realtion for Rivlin- Ericksen viscoelastic fluid. Also, for fixed values of P, N_Q, M_1 and $\overline{H}$, let the *non-dimensional* number G accounting for the compressibility effects be also kept as fixed, then we have

$$\overline{N_R^C} = \left(\frac{G}{G-1}\right)N_R^C \tag{3.46}$$

where N_R^C is critical Rayleigh number in the absence compressibility and $\overline{N_R^C}$ is critical Rayleigh number in the presence of compressibility. Since the critical Rayleigh number > 0 and finite which implies $G > 1$, which means stabilizing effect due to compressibility.

Now we study, the effect of suspended particles which depends upon the nature of $\frac{dN_R}{d\overline{H}}$, the effect of medium permeability which depends upon the nature of $\frac{dN_R}{dP}$, the effect of magnetic field which depends upon the nature of $\frac{dN_R}{dN_Q}$, the effect of hall current which depends upon the nature of $\frac{dN_R}{dM_1}$. From equation (3.45) we have

$$\frac{dN_R}{d\overline{H}} = -\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)^2}{\alpha^2\overline{H}^2}\left[\frac{1}{P}+\frac{N_Q\epsilon^{-1}\pi^2\left\{(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}{(\pi^2+\alpha^2)\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}\right]. \tag{3.47}$$

which is $< 0 \Rightarrow$ destabilizing effect of suspended particles on the thermal instability of the compressible fluid-particle layer in the presence of and hall currents through a porous medium. It is obvious from equation (3.45) that

$$\frac{dN_R}{dP} = \left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)^2}{\alpha^2\overline{H}}\left[-\frac{1}{P^2}+\frac{(N_Q\pi^2\epsilon^{-1})M_1^2\pi^2}{(\pi^2+\alpha^2)\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}^2}\right]. \tag{3.48}$$

which is > 0 if $P\left[M_1\pi-\sqrt{\pi^2+\alpha^2}\right] > \frac{\sqrt{\pi^2+\alpha^2}\left[M_1^2\pi^2+\left(\pi^2+\alpha^2\right)\right]}{N_Q\epsilon^{-1}\pi^2}$.

which is < 0 if $P\left[M_1\pi-\sqrt{\pi^2+\alpha^2}\right] < \frac{\sqrt{\pi^2+\alpha^2}\left[M_1^2\pi^2+\left(\pi^2+\alpha^2\right)\right]}{N_Q\epsilon^{-1}\pi^2}$.

Thus, for the different values of parameter, medium permeability has both destabilizing and stabilizing effect. Presence and absence of magnetic field plays an important role in stabilizing effect of permeability. Its absence destabilize the effect. Since for the case

$$\frac{dN_R}{dP} = -\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)^2}{\alpha^2\overline{H}P^2}. \tag{3.49}$$

which is always < 0. Thus, in the presence of magnetic field, medium permeability succeeds in stabilizing the thermal instability of the compressible fluid-particle layer for certain wave numbers. Now from equation (3.45), we get

$$\frac{dN_R}{dN_Q} = \left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)\,\pi^2\epsilon^{-1}}{\alpha^2\overline{H}\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}\left[\left\{\left(\pi^2+\alpha^2\right)+N_QP\epsilon^{-1}\pi^2\right\}\right.$$
$$\left.+\frac{M_1^2\pi^4N_QP\epsilon^{-1}}{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2}\right]. \tag{3.50}$$

which is always > 0 which implies that magnetic field has a stabilizing effect. To find the effect of hall currents, from equation (3.45), we have

$$\frac{dN_R}{dM_1} = -2\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)}{\alpha^2\overline{H}}\left[\frac{N_Q\epsilon^{-1}M_1\pi^4\left\{(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}{\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}^2}\right]. \tag{3.51}$$

which is always < 0 which means that, in porous medium, hall current destabilize the thermal convection in the compressible fluid-particle layer. We analyze graphically all the four effects as

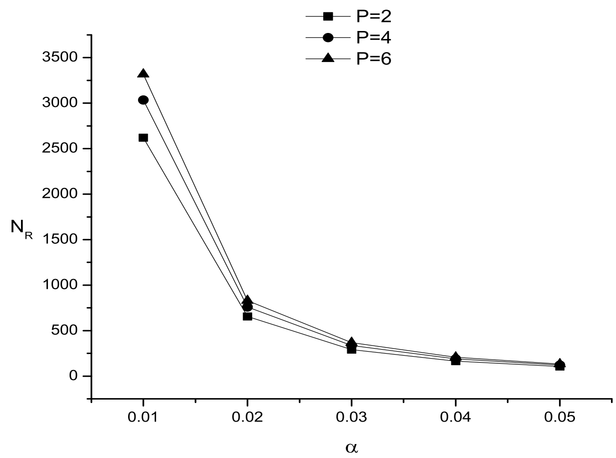

Figure 3.1: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14, N_Q = 20, M_1 = 10, \epsilon = 0.5$ and for different values of $P(2, 4, 6)$.

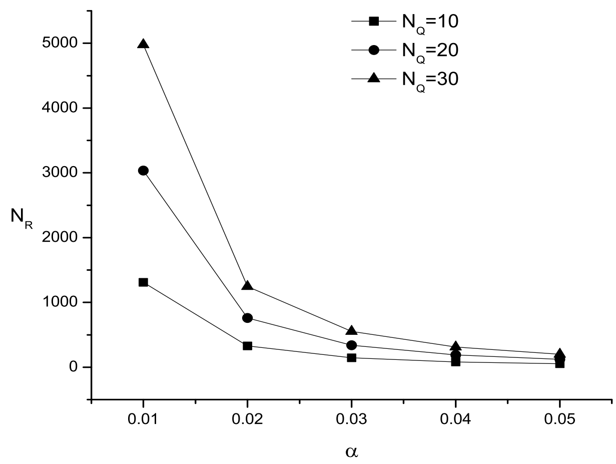

Figure 3.2: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14, P = 4, M_1 = 10, \epsilon = 0.5$ and for different values of $N_Q = (10, 20, 30)$.

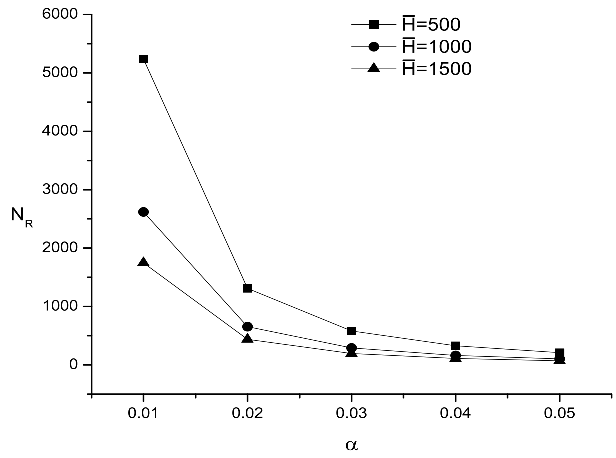

Figure 3.3: Variation of N_R with α for a different value of $\overline{H} = (500, 1000, 1500)$ for fixed values of $G = 9.8, \pi = 3.14, P = 2, M_1 = 10, \epsilon = 0.5$.

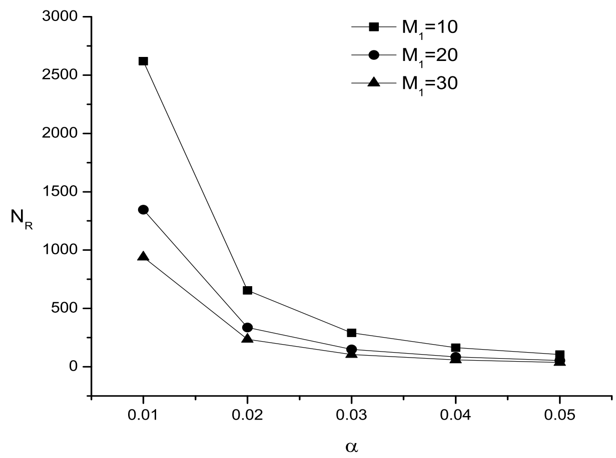

Figure 3.4: Variation of N_R with α for a fixed values $\overline{H} = 1000, G = 9.8, \pi = 3.14, P = 2, N_Q = 20, \epsilon = 0.5$ for different values of $M_1 = (10, 20, 30)$.

We find from **Figure3.1** (refer table 4), when the value of the medium permeability(P), increased then the value of N_R is increased which shows the stabilizing effect. Similarly from **Figure3.2** (refer table 5), when the value of magnetic field N_Q is increased, and the value of N_R is increased which again shows the case of stabilizing effect. **In Figure3.3** (refer table 6), as the value of suspended particle $\overline{H}$ increased, the value of N_R decreased, which shows the destabilizing effect. Also **Figure3.4** (refer table 7) shows as the value of hall currents M_1 through the porous medium increased, the value of N_R decreased, which is again the case of destabilizing effect on the system.

3.5 Oscillatory Modes

Multiplying equation (3.37) by he complex conjugate of W i.e. W^*, integrating over the range of z from $z = 0$ to $z = d$ and using equations (3.38)-(3.47) together with the boundary conditions (3.29) and (3.30)

$$\left[L_1 + \frac{L_2}{P}(1 + An)\right] I_1 + A_1 (nI_2 + n^* I_5) + L_2 N_Q \epsilon (I_3 + I_6) + \frac{L_2}{L_2^*}\left[L_1^* + \frac{L_2^*}{P}(1 + An^*)\right] I_4$$
$$= L_2 L_2^* N_R \alpha^2 \left(\frac{G-1}{G}\right)\left(\frac{1}{\tau n^* + \overline{H}}\right)[I_7 + (E + h\epsilon) n^* I_8]. \quad (3.52)$$

where $A_1 = L_2 N_Q N_{P2} N_{P1}^{-1}$ and

$$\begin{aligned}
I_1 &= \int_0^1 \left(|DW|^2 + a^2|w|^2\right) dz \quad , \quad I_2 = \int_0^1 |X|^2 dz,\\
I_3 &= \int_0^1 \left(|DX|^2 + a^2|X|^2\right) dz \quad , \quad I_4 = \int_0^1 |Z|^2 dz\\
I_5 &= \int_0^1 \left(|DK|^2 + a^2|K|^2\right) dz \quad , \quad I_6 = \int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right) dz,\\
I_7 &= \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz \quad , \quad I_8 = \int_0^1 |\Theta|^2 dz. \qquad (3.53)
\end{aligned}$$

all $I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8$ all are positive definite, take $n = in_0$ in equation (3.52), where n_0 is real, and equate imaginary parts on both sides, we get

$$n_0 = 0 \quad \text{or} \quad n_0^2 = -\tau^{-2}\frac{A - B}{C - D + E} \quad (3.54)$$

where

$$A = \left(N_{p_1}^{-1}\overline{H}F - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_Q \epsilon N_{P_2} N_{P_1}^{-1}\overline{H}(I_2 - I_5) - N_Q \epsilon \tau (I_3 + I_6),$$

$$B = \left(N_{p_1}^{-1}\overline{H}F + \frac{\tau}{P} + \frac{\tau}{P}A\right) I_4 - N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\},$$

$$C = \left(N_{p_1}^{-1}\left(\overline{H} + 1 - F\right) - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_Q \epsilon N_{P_2} N_{P_1}^{-1}\overline{H}(I_2 - I_5),$$

$$D = N_Q \epsilon \tau (I_3 + I_6) - \left(N_p^{-1}\left(1 - \overline{H} - F\right) - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_4 \quad \text{and}$$

$E = N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\}$. Whereas in the absence of magnetic field,

$$n_0^2 = \frac{-\tau^{-2}\left[\left(N_{p_1}^{-1}\overline{H}F - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\}\right]}{\left(N_{p_1}^{-1}\left(\overline{H} + 1 - F\right) - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\}}. \quad (3.55)$$

3.6 Conclusion

Problem was formulated to discuss the combined effect of compressibility, hall current, magnetic field, medium permeability and suspended particles on thermal instability of a Rivlin-Ericksen fluid and the results obtained as :

(I) Constitutive relation of Rivlin-Ericksen fluid becomes linear i.e. the relation between stress and strain becomes linear for stationary convection due to the vanishing of the *viscoelastic parameter.*

(II) Magnetic field, suspended particles and *medium permeability* introduce oscillatory modes in the system otherwise effects the principle of exchange of stabilities is hold good.

(III) When magnetic field is not present, $n_0^2 < 0$ if

$$C_{pt} > C_v \left[1 + \frac{\epsilon m}{f k_1 K d^2} \left\{ \nu d^2 + \nu' \right\} \right] \tag{3.56}$$

For all $N_R > 0$, since n_0 is real and $n_0^2 < 0$ which implies $n_0 = 0$. This shows that n is real when $N_R > 0$ in the absence of the magnetic field. If equation (3.55) holds true and that the principle of exchange of stabilities is valid for this case, however, if equation (3.55) is violated, then the oscillatory modes may come into play even in the absence of the magnetic field, Singh and Gupta [37].

(IV) Equation (3.46) indicates compressibility effect is to postpone the onset of instability.

(V) To study the various effects of suspended particles, medium permeability, magnetic filed and Hall currents in a compressible Rivlin-Ericksen viscoelastic fluid, we examined the expressions $\frac{dN_R}{dH}, \frac{dN_R}{dP}, \frac{dN_R}{dN_Q}$ and $\frac{dN_R}{dM_1}$ analytically. The magnetic field postpones the onset of instability, suspended particles and Hall currents both hasten the onset of convection, which is in contrast with the result of Gupta et al. [54].

Chapter 4

Double - Diffusive Convection in Presence of Compressible Rivlin-Ericksen Fluid with Fine Dust

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4.1 Introduction

A layer of *Newtonian fluid* heated from below, under varying assumptions of hydrodynamics, has been treated in detail by Chandrasekhar [26]. Chandra [22] performed careful experiments in an air layer and found contradiction between the theory and the experiment. He found that the *instability* depended on the depth of the layer. A Bénard type cellular convection with fluid descending at the cell centre was observed when predicted gradients were imposed, if the layer depth was more than 10 mm. But if the layer of depth was less than 7 mm, convection occurred at much lower gradients than predicted and appeared as irregular strips of elongated cells with fluid rising at the centre. Chandra called this motion *columnar instability*. The effect of particle mass and *heat capacity* on the onset of Bénard convection has been considered by Scanlon and Segel [23]. They found that the *critical Rayleigh number* was reduced solely because the heat capacity of the clean gas was supplemented by that of the particles. The effect of *suspended particles* was found to *destabilize* the layer. Palniswamy and Purushotham [29] have considered the stability of shear flow of stratified fluids with *fine dust* and have found the effect of fine dust to increase the region of instability. A study of *double-diffusive convection* with fine dust has been made by Sharma and Rani [55]. Kumar et al. [56] have studied effect of magnetic field on thermal instability of rotating Rivlin-Ericksen *viscoelastic fluid*, in which effect of magnetic field has stabilizing as well as destabilizing effect on the system. Also, Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluid through porous medium has been considered by Sharma et al. [57]. They have studied the stability aspects of the system. The effects of a uniform horizontal magnetic field and a uniform rotation on the problem have also been considered separately. Kumar [58] has also studied the stability of superposed viscoelastic Rivlin-Ericksen fluids in presence of suspended particles through a porous medium. In one other study, Kumar and Singh [59] have studied the stability of superposed viscoelastic fluids through porous medium, in which effects of uniform horizontal magnetic field and a uniform rotation are considered. Kumar et al.[47] have also studied hyderodynamic and hyderomagnetic stability of Rivlin-Ericksen fluid and found that the growth rates decrease as well as increase with the increase in *kinematic viscosity* and kinematic viscoelasticity in absence and presence of magnetic field. Singh et. al. [25] has studied thermal instability of Rivlin-Ericksen elastico viscous fluid permeated with suspended particles in hydrodynamics in a porous medium and found that magnetic field have only stabilizing effect whereas medium *permeability* have a destabilizing effect on the system. M.F.EI-Sayed et. al [60], have studied non-linear Kelvin-Helmholtz instability of Rivlin-Ericksen viscoelastic electrified fluid particle mixtures saturating porous medium and in one another study Kumar et al. [61], have also studied double-diffusive convection in compressible viscoelastic fluid through Brinkman porous media.

Presently, the study of stability of double-diffusive convection of Rivlin-Ericksen elastico-viscous fluids *permeated* with suspended particles is considered. Viscosity is a function of space and time in a large variety of fluid flows and its variation can have a dramatic effect on flow stability. Here instability due to double-diffusive effects in viscosity, permeated with suspended particles flow have been discussed. Double-diffusive systems are known to display a rich variety of instability behavior in density permeated with suspended particles fluid flow system. In viscosity permeated systems, it was found that stable flow in the context of single component systems become unstable due to double-diffusive effect. Many interesting flow patterns arise due to this instability, these aspects form the motivation for the present study, Singh and Gupta[62].

4.2 Formulation of The Problem

Infinite and *horizontal layer* of Rivlin-Ericksen fluid of depth d i.e. from $z = 0$ to $z = d$ is considered for an compressible electrically conducting viscoelastic Rivlin-Ericksen with suspended particles. This layer is given the heat from below, let the temperature at $z = 0$ is T_0 and at the upper layer, $z = d$, is T_d, and that a steady adverse temperature gradient $|\frac{dT}{dz}| = \beta$ and *solute gradient* $|\frac{dC}{dz}| = \beta'$ are maintained. Here, $\vec{g}\,(0, 0, -g)$ is acceleration due to gravity. The effect of fluid compressibility, even small in magnitude, is also considered.

Let the fluid properties like pressure, density, velocity of pure fluid, kinematic viscosity and kinematic viscoelasticity be denoted by p, ρ, $\vec{u}(u, v, w)$, ν and ν' respectively. Properties of suspended particle like velocity and number density be represented by $v(\overline{x}, t)$ and $N(\overline{x}, t)$. $\vec{x}(x, y, z)$, $\vec{\lambda}(0, 0, 1)$ and $K = 6\pi\mu\eta'$ is the Stokes' drag coefficient for the particle having the radius η'.

Then the *flow governing* equations i.e. equations of motion and continuity are

$$\rho\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u})\right] = -\nabla p + \rho\vec{g} + KN\left(\vec{v} - \vec{u}\right) + \left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2\vec{u} \tag{4.1}$$

$$\text{and} \quad \nabla.\vec{u} = 0. \tag{4.2}$$

Some assumptions regarding the shape and velocity of the suspended particles are taken as

- Shape of the suspended particles in the fluid is uniform spherical.
- The buoyancy forces on the particle are neglected.
- Large distance between the particles as compare to their diameter. So Interparticle reactions are ignored.
- Extra force due to the presence of particles is proportional to velocity difference between the particles and the fluid.
- Force exserted by fluid on particles and force exerted by particles on fluid balance each other.

So there must be an extra force equal in magnitude but opposite in sign in the equations of conservation of momentum or motion for the particles. If mN is the mass of particles per unit volume, then under the above assumptions, equations of conservation of momentum and mass for the particles are

$$mN\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v})\right] = KN\left(\vec{u} - \vec{v}\right) \quad \text{and} \tag{4.3}$$

$$\frac{\partial N}{\partial t} + \nabla.\left(N\vec{v}\right) = 0. \tag{4.4}$$

Let C_v is heat capacity of fluid at constant volume, C_{pt} is heat capacity of particles, C_p is heat capacity of fluid at constant pressure, T is temperature and q is effective thermal conductivity of the pure fluid. Volume fractions of the particles are assumed to be small; the effective properties of the suspension are considered as same as of clean fluid.

If we assume that the fluid and particles are in the *thermal equilibrium*, the equation of *heat conduction* is

$$\rho C_v\left[\frac{\partial}{\partial t}+\vec{u}.\nabla\right]T+mNC_{pt}\left[\frac{\partial}{\partial t}+\vec{v}.\nabla\right]T=q\nabla^2 T, \tag{4.5}$$

If C denotes the *solute concentration*, then equation of solute conduction gives

$$\rho C_v'\left[\frac{\partial}{\partial t}+\vec{u}.\nabla\right]C+mNC_{pt}'\left[\frac{\partial}{\partial t}+\vec{v}.\nabla\right]C=q'\nabla^2 C \tag{4.6}$$

where C_v', C_{pt}' and q' denote the analogous solute quantities. Spiegel and Veronis [40] defined f as any one of the state variables (p, ρ, or T) and expressed in the form

$$f(x,y,z,t)=f_m+f_0\left(z\right)+f'(x,y,z,t) \tag{4.7}$$

where $f_m \rightarrow$ constant space average of f, $f_0 \rightarrow$ variation in the absence of motion and $f' \rightarrow$ fluctuation resulting from motion. The initial state of the system is taken to be quiescent layer with a uniform particle distribution N_0, therefore initial state in which velocity, temperature T, solute concentration C is given by $\vec{v}=(0,0,0),\vec{u}=(0,0,0)$

and $T=T(z)=T_0-\beta z, C=C(z)=C_0+\beta' z$,

$$p=p(z)=p_m-g\int_0^z\left(\rho_m-\rho_0\right)\,dz\,,$$

$$\rho=\rho(z)=\rho_m\left[1-\alpha_m\left(T-T_m\right)+\alpha_m'\left(C-C_m\right)+K_m\left(p-p_m\right)\right],$$

$$\alpha_m=-\left[\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right]_m\left(=\alpha(say)\right),$$

$$\alpha_m'=-\left[\frac{1}{\rho}\frac{\partial\rho}{\partial C}\right]_m\left(=\alpha'(say)\right),$$

$$K_m=-\left[\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right]_m \tag{4.8}$$

$N_0=$ Constant.

Perturb the initial state of the system. Let δp denote the *perturbation* in pressure p, $\delta\rho$ denote the perturbation in density ρ, θ denote the perturbation in temperature T, γ denote the perturbation in solute concentration C, $\vec{v}(u,v,w)$ denote the perturbation in fluid velocity, $\vec{u}(l,r,s)$ denote the perturbation in particle velocity, N denote perturbations in suspended particles number density N_0. The quantity $\delta\rho$, depend on θ and γ and is given by $\quad \delta\rho = -\rho_m \left(\alpha\theta - \alpha'\gamma\right).$ (4.9)

Then the linearized perturbation equations of the problem, Spiegel and Veronis [40], Scanlon and Segel [23], and Rivlin -Ericksen [32], become

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p + g\left(\alpha\theta - \alpha'\gamma\right)\lambda + \frac{KN}{\rho_m}\left(\vec{v} - \vec{u}\right) + \left(\nu + \nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{u}, \tag{4.10}$$

$$\nabla.\vec{u} = 0, \tag{4.11}$$

$$\left[\frac{m}{K}\frac{\partial}{\partial t} + 1\right]\vec{v} = \vec{u}, \tag{4.12}$$

$$\frac{\partial N}{\partial t} + \nabla.\left(N_0\vec{v}\right) = 0, \tag{4.13}$$

$$(1+h)\frac{\partial\theta}{\partial t} = \beta\left(\frac{G-1}{G}\right)(w + hs) + \kappa\nabla^2\theta \quad \textit{and} \tag{4.14}$$

$$(1+h')\frac{\partial\theta}{\partial t} = \beta'\left(\frac{G-1}{G}\right)(w + h's) + \kappa'\nabla^2\gamma \tag{4.15}$$

where $\mu, \mu', \nu = \frac{\mu}{\rho_m}, \nu' = \frac{\mu'}{\rho_m}, \kappa = \frac{q}{\rho_m C_v}$ and $\kappa' = \frac{q'}{\rho_m C_v'}$ stand for viscosity, viscoelasticity, kinematic viscosity, kinematic viscoelasticity, thermal diffusivity and analogous solute diffusivity, respectively. Also, $h = f(C_{pt}/C_v), h' = f(C'_{pt}/C'_v)$, $f = mN_0/\rho_m$, and $G = \frac{C_p\beta}{g}$. Initially, $\vec{v} = (0,0,0), \vec{u} = (0,0,0), T = T(z)$, and $N = N_0$ which implies (4.5) yields $0 = 0$, identically. After perturbation, (4.5) becomes

$$\left(\rho_m + \delta\rho\right)C_v\left[\frac{\partial}{\partial t} + \vec{u}.\nabla\right](T+\theta) + \left(mN_0 + mN\right)C_{pt}\left[\frac{\partial}{\partial t} + \vec{v}.\nabla\right](T+\theta)$$
$$= q\nabla^2\left(T+\theta\right). \tag{4.16}$$

Follow Speigal and Veronis [40] where the flow equations are found to be the same as those for incompressible fluids except β is replaced by $\left(\beta - \frac{g}{C_p}\right)$ i.e. the *static temperature gradient* is replaced by its excess over the adiabatic and C_v is replaced by C_p.

So linearization of (4.5) gives

$$\frac{\partial\theta}{\partial t}+\frac{mN_0}{\rho_m}\frac{C_{pt}}{C_v}\frac{\partial\theta}{\partial t}=\left(\beta-\frac{g}{C_p}\right)(w+hs)+\frac{q}{\rho_m C_v}\nabla^2\theta. \tag{4.17}$$

that is, (4.14). However, β' remains unaltered and, as above, (4.6) yields (4.15).

4.3 The Dispersion Relation

Perturbed quantities are assumed to be of the following form for the analysis of disturbances into normal modes

$$[w,\theta,\gamma]=[W(z),\Theta(z),\Gamma(z)]\,exp\,(ik_x x+ik_y y+nt) \tag{4.18}$$

where k_x is the wave number along x-direction and k_y is wave number along y-direction. $k=\sqrt{k_x^2+k_y^2}=$ resultant wave number and $n=$ growth rate = complex constant in general. Non dimensional form of equations (4.16), (4.10) - (4.15) become

$$\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)-(1+F\sigma)\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W+\frac{ga^2d^2}{\nu}\left(\alpha\Theta-\alpha'\Gamma\right)=0, \tag{4.19}$$

$$\left(D^2-a^2-Hp_1\sigma\right)\Theta=-\beta\left(\frac{G-1}{G}\right)\frac{d^2}{\kappa}\frac{(H+\tau_1\sigma)}{(1+\tau_1\sigma)}W \tag{4.20}$$

$$\textit{and}\quad \left(D^2-a^2-H'q\sigma\right)\Gamma=-\beta'\frac{d^2}{\kappa'}\frac{(H'+\tau_1\sigma)}{(1+\tau_1\sigma)}W \tag{4.21}$$

where we have put $a=kd$, $\sigma=\frac{nd^2}{\nu}$, $\tau=\frac{m}{\kappa}$, $\tau_1=\frac{\tau\nu}{d^2}$, $M=\frac{mN}{\rho_m}$, $p_1=\frac{\nu}{\kappa}$, $q=\frac{\nu}{\kappa'}$, $H=1+h, H'=1+h'$, $F=\frac{\nu'}{d^2}$ and $D=\frac{d}{dz}$.

Eliminate Γ and Θ from equations (4.19) and (4.21), then

$$\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)-(1+F\sigma)\left(D^2-a^2\right)\right]\left(D^2-a^2-Hp_1\sigma\right)\left(D^2-a^2-H'q\sigma\right)\left(D^2-a^2\right)W$$
$$-R\left(\frac{G-1}{G}\right)a^2\frac{(H+\tau_1\sigma)}{(1+\tau_1\sigma)}\left(D^2-a^2-H'q\sigma\right)W+Sa^2\frac{(H'+\tau_1\sigma)}{(1+\tau_1\sigma)}\left(D^2-a^2-Hp_1\sigma\right)W=0. \tag{4.22}$$

$where \quad R = \dfrac{g\alpha\beta d^4}{\nu\kappa} \rightarrow$ thermal Rayleigh number

$S = \dfrac{g\alpha'\beta' d^4}{\nu\kappa'} \rightarrow$ analogous solute Rayleigh number

$p_1 = \dfrac{\nu}{\kappa} \rightarrow$ thermal Prandtl number

$q = \dfrac{\nu'}{\kappa} \rightarrow$ analogous Schmidt number.

For the solution of the problem boundaries considered are perfect conductors of heat and solute and free. Surrounding medium is assumed to be electrically nonconducting. So boundary conditions taken as

$$\Theta = 0, W = 0, \Gamma = 0, \quad D^2W = 0, \quad DZ = 0 \quad at \quad z = 0 \quad and \quad z = 1. \tag{4.23}$$

For the solution to the problem, free boundaries are considered which is little artificial in nature but most suitable for stellar atmospheres. Using (4.23), even order derivatives of W vanish on the boundaries and so the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{4.24}$$

where $W_0 =$ Constant. Substituting (4.24) in (4.22), the relation reduces to

$$R_1 x = \left(\frac{G}{G-1}\right)\left[\left\{i\sigma_1\left(1 + \frac{M}{1 + i\tau_1\sigma\pi^2}\right) + \left(1 + iF\sigma\pi^2\right)(1+x)\right\}\right.$$
$$\left.\left\{\frac{\left(1 + i\tau_1\sigma\pi^2\right)(1+x)(1 + x + iHp_1\sigma)}{(H + i\tau_1\sigma\pi^2)}\right\} + S_1 x \frac{\left(H' + i\tau_1\sigma\pi^2\right)(1 + x + iHp_1\sigma)}{(H + i\tau_1\sigma\pi^2)(1 + x + iH'q\sigma)}\right]. \tag{4.25}$$

where $\quad R_1 = \dfrac{R}{\pi^4}, \quad x = \dfrac{a^2}{\pi^2}, \quad i\sigma_1 = \dfrac{\sigma}{\pi^2} \quad$ and $\quad S = \dfrac{a^2}{\pi^4}.$

Dispersion relation (4.25) studying the effects of suspended particles and compressibility on the double-diffusive convection in Rivlin-Ericksen elastico- viscous fluid.

4.4 The Stability and Oscillatory Modes

Here, we examine instability, if any, which can occur as oscillatory modes in the system defined. Multiplying (4.19) by the complex conjugate of W i.e W^*, integrating over $z = 0$ to $z = 1$ and making use of (4.20) and (4.21) with the help of boundary conditions (4.23), we obtain

$$\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)I_1+(1+F\sigma)\,I_2-\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{G}{G-1}\right)\left(\frac{1+\tau_1\sigma^*}{H+\tau_1\sigma^*}\right)(I_3+Hp_1\sigma^* I_4) + \frac{g\alpha' a^2\kappa'}{\nu\beta'}\left(\frac{1+\tau_1\sigma^*}{H'+\tau_1\sigma^*}\right)(I_5+H'q\sigma^* I_6)=0 \qquad (4.26)$$

$$\begin{aligned} where \quad I_1 &= \int_0^1 \left(|DW|^2+a^2|w|^2\right)dz \quad , \quad I_2=\int_0^1\left(|D^2W|^2+2a^2|Dw|^2+a^4|W|^2\right)dz \\ I_3 &= \int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz \quad , \quad I_4=\int_0^1|\Theta|^2dz \\ I_5 &= \int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz \quad , \quad I_6=\int_0^1|\Gamma|^2dz \end{aligned}$$

All the integrals $I_1, I_2, I_3, I_4, I_5, I_6$ are positive definite. Substituting $\sigma = i\sigma_i$ and equate the imaginary parts,where σ_i is real, we get small

$$\sigma_i\left[\left(1+\frac{M}{1+\tau_1\sigma_i}\right)I_1+FI_2+\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{G}{G-1}\right)\left(\frac{\tau_1(H-1)}{H^2+\tau_1^2\sigma_i^2}I_3+\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}Hp_1\sigma^*I_4\right) - \frac{g\alpha' a^2\kappa'}{\nu\beta'}\left(\frac{\tau_1(H'-1)}{H'^2+\tau_1^2\sigma_i^2}I_5+\frac{H'+\tau_1^2\sigma_i^2}{H'^2+\tau_1^2\sigma_i^2}H'q\sigma^*I_6\right)\right]=0 \qquad (4.27)$$

Here $\sigma_i = 0$ implies that modes may be non-oscillatory or $\sigma_i \neq 0$ implies that modes may be oscillatory. Presence of stable solute gradient introduces oscillatory modes.

4.5 Stationary Convection

$\sigma = 0$ characterized the marginal state When instability sets in as stationary convection, Put $\sigma = 0$ in dispersion relation (4.25) which reduces to

$$R_1=\left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{xH}+S_1\frac{H'}{H}\right]. \qquad (4.28)$$

and constitutive relation becomes linear for Rivlin-Ericksen elastico-viscous fluid. Behavior of $\frac{dR_1}{dS_1}$ explains the effect of stable solute gradient and behavior of $\frac{dR_1}{dH}$ explains the effect of suspended particles analytically. Equation (4.28) yields

$$\frac{dR_1}{dS_1} = \left(\frac{G}{G-1}\right)\frac{H'}{H}. \tag{4.29}$$

which is positive, thereby Rayleigh number and solute parameter increases simultaneously. So, stable solute gradient shows stabilizing effect.

$$\frac{dR_1}{dH} = -\left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{x} + S_1H'\right]\frac{1}{H^2}. \tag{4.30}$$

which is negative, which means suspended particles destabilize the system as the dimensionless Rayleigh number decreases with increase in the suspended particles number density. Therefore, We studied here, these effects graphically as below

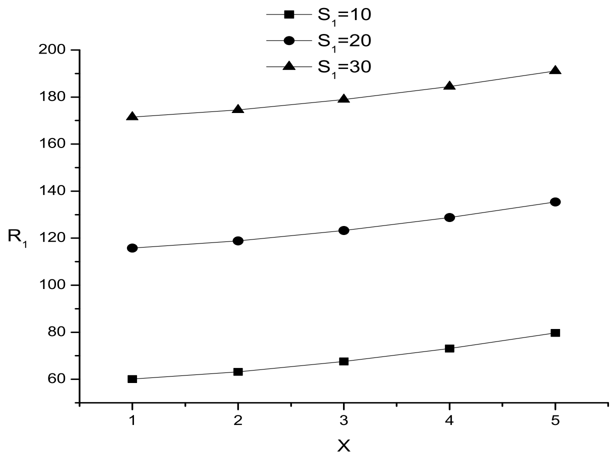

Figure 4.1: The variation of dimensionless Rayleigh number (R_1) with wave number $x(= 1, 2, 3, 4, 5)$, for $G = 9.8, H = 2, H' = 10$ and $S_1(= 10, 20, 30)$.

In **Figure4.1** (refer table 8), as the value of stable solute gradient parameter increased, so the value of Rayleigh number is increased for fixed values $G = 9.8, H = 2, H' = 10$ and $S_1(= 10, 20, 30)$ when taking values of wave number x(= 1,2,3,4,5) respectively. Therefore as value of Rayleigh number increased with increase in wave number showing the stabilizing effect.

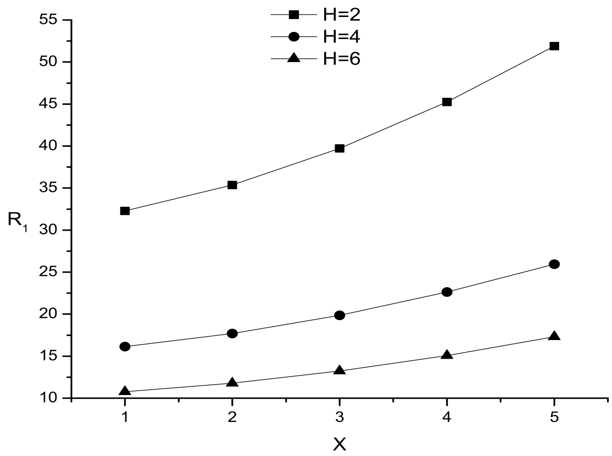

Figure 4.2: The variation of dimensionless Rayleigh number (R_1) with wave number $x(=1,2,3,4,5)$, for $G=9.8, S_1=10, H'=5$ and $H(=2,4,6)$.

In **Figure4.2** (refer table 9), Rayleigh number decreased with increase in the suspended particles by taking values of wave number x(= 1,2 3,4,5), for fixed values $G = 9.8, S_1 = 10, H' = 5$ and $H(= 2, 4, 6)$, respectively. Therefore as values of Rayleigh number has increased with decrease suspended particles parameter, showing the *destabilizing effect*.

Let G (accounting for the compressibility effects) be kept fixed for fixed S_1, H and H'. Then we have

$$\overline{R_c} = \left(\frac{G}{G-1}\right) R_c \tag{4.31}$$

where $\overline{R_c}$ is Critical Rayleigh number in the presence compressibility and R_c is Critical Rayleigh number in the absence of compressibility.In the presence of compressibility, $G < 1$ and $G = 1 \Rightarrow$ negative and infinite values of the critical Rayleigh number, which is irrelevant to the given system. $G > 1$ is relevant to the given system, thus compressibility postpone the onset of double-diffusive convection.

4.6 Conclusion

Effect of compressibility, stable solute gradient and suspended particles and has been investigated on thermosolutal convection of a Rivlin-Ericksen fluid. The study may be relevant to the stability of some polymer solutions and the problem finds its applications in chemical technology and in Geophysical situations . Hence a study has been made on thermosolutal convection in presence of compressible fluid with fine dust. Due to the vanishing of the viscoelastic parameter the constitutive relation for Rivlin-Ericksen fluid become linear for the case of stationary convection. It is obvious from the equation (4.31)that compressibility had postponed the onset of instability. The expressions $\frac{dR_1}{dS_1}$ explains the effects of stable solute gradient and $\frac{dR_1}{dH}$ explains the effect of suspended particles analytically. Stable solute gradient delay the onset of instability whereas suspended particles are found to hasten the onset of instability. **Figure1** and **Figure2**, shows the same results as obtained. The presence of viscoelasticity, suspended particles and stable solute gradient introduce the oscillatory modes. In the absence of viscoelasticity, suspended particles and stable solute gradient, the principle of exchange of stabilities holds good.

Chapter 5

Programming Codes

Programming codes to find Rayleigh number obtained in the chapters 2 ,3 and 4 by assigning numerical values to all other parameters .

5.1 Chapter 2 : Variations of Rayleigh number

Consider the equation (2.27) to find the variations in Rayleigh number R_1

5.1.1 When Q1 = 25,50,75

```
clear all

% Values assigned to various parameters

x=[1 2 3 4 5 6];
P=2;
H_dash=10;
Thetha=pi/4;
Epsilon=0.5;
Q1=[25 50 75];

% Intermediate calculations

A1=(1+x);
A2=A1/P;

A31=Q1(1,1)*power(cos(Thetha),2)*power(Epsilon,-1)*x;
A32=Q1(1,2)*power(cos(Thetha),2)*power(Epsilon,-1)*x;
A33=Q1(1,3)*power(cos(Thetha),2)*power(Epsilon,-1)*x;

A41=A2+A31;
A42=A2+A32;
A43=A2+A33;

A51=A1.*A41
```

```
A52=A1.*A42
A53=A1.*A43

B1=x*H_dash;

% Variation of Rayleigh number

Rayleigh_Number1=A51./B1     % when Q1 = 25
Rayleigh_Number2=A52./B1     % when Q1 = 50
Rayleigh_Number3=A53./B1     % when Q1 = 75

% Plot of  all Rayleigh numbers Vs. Wave number x

plot(x,Rayleigh_Number1)
hold on
plot(x,Rayleigh_Number2)
hold on
plot(x,Rayleigh_Number3)
```

5.1.2 When H-dash = 5,10,15

```
clear all

% Values assigned to various parameters

x=[1 2 3 4 5 6];
P=2;
H_dash=[5 10 15 ];
Thetha=pi/4;
Epsilon=0.5;
Q1=25;

% Intermediate calculations

A1=(1+x);
A2=A1/P;
A3=Q1*power(cos(Thetha),2)*power(Epsilon,-1)*x;
A4=A2+A3;
A5=A1.*A4

B11=x*H_dash(1,1);
B12=x*H_dash(1,2);
B13=x*H_dash(1,3);

% Variation of Rayleigh number

Rayleigh_Number1=A5./B11
Rayleigh_Number2=A5./B12
Rayleigh_Number3=A5./B13

% Plot of  all Rayleigh numbers Vs. Wave number x
```

```
plot(x,Rayleigh_Number1)
hold on
plot(x,Rayleigh_Number2)
hold on
plot(x,Rayleigh_Number3)
```

5.1.3 When P = 0.1 ,0.2,0.6

```
clear all

% Values assigned to various parameters

x=[1 2 3 4 5 6];
P=[0.1 0.2 0.6];
H_dash=10;
Thetha=pi/4;
Epsilon=0.5;
Q1=25 ;

% Intermediate calculations

A1=(1+x);

A21=A1/P(1,1);
A22=A1/P(1,2);
A23=A1/P(1,3);

A3=Q1*power(cos(Thetha),2)*power(Epsilon,-1)*x;

A41=A21+A3;
A42=A22+A3;
A43=A23+A3;


A51=A1.*A41;
A52=A1.*A42;
A53=A1.*A43;

```

```
B1=x*H_dash;

% Variation of Rayleigh number

Rayleigh_Number1=A51./B1     % when P=0.1
Rayleigh_Number2=A52./B1     % when P=0.2
Rayleigh_Number3=A53./B1     % when P=0.6

% Plot of  all Rayleigh numbers Vs. Wave number x

plot(x,Rayleigh_Number1)
hold on
plot(x,Rayleigh_Number2)
hold on
plot(x,Rayleigh_Number3)
```

5.2 Chapter 3 : Variations of Rayleigh number

Consider the equation (3.45) to find the variations in Rayleigh number N_R

5.2.1 When P = 2 , 4 , 6

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=1000;
P=[2 4 6 ];
NQ=20;
M1=10;

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G -1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);
A4=power(Alpha,2)*H_Bar;
A5=A3./A4;
A6=A1*A5;

B11=1/P(1,1);
B12=1/P(1,2);
B13=1/P(1,3);
```

```
C1=NQ*power(Pi,2)*power(Epsilon,-1);

C21=P(1,1)*C1;
C22=P(1,2)*C1  ;
C23=P(1,3)*C1;

C31=C21+A3;
C32=C22+A3;
C33=C23+A3;

C41=C1*C31;
C42=C1*C32;
C43=C1*C33;

D1=power(M1,2)*power(Pi,2);
D21=C31+D1;
D22=C32+D1;
D23=C33+D1;

D31=A3.*D21;
D32=A3.*D22;
D33=A3.*D23;

D41=C41/D31;
D42=C42/D32;
D43=C43/D33;

E11=B11+D41;
E12=B12+D42;
E13=B13+D43;
```

```
% Values of Rayleigh number NR

NR1=A6*E11          %  When P=2
NR2=A6*E12          %  When P=4
NR3=A6*E13          %  When P=6

% Graphs of NR Vs Alpha

plot(Alpha,NR1)
hold on
plot(Alpha,NR2)
hold on
plot(Alpha,NR3)
```

5.2.2 When NQ = 10,20,30

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=1000;
P=4 ;
NQ=[10 20 30];
M1=10;

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G -1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);
A4=power(Alpha,2)*H_Bar;
A5=A3./A4;
A6=A1*A5;

B1=1/P;

C11=NQ(1,1)*power(Pi,2)*power(Epsilon,-1);
C12=NQ(1,2)*power(Pi,2)*power(Epsilon,-1);
C13=NQ(1,3)*power(Pi,2)*power(Epsilon,-1);

C21=P*C11;
C22=P*C12 ;
```

```
C23=P*C13;

C31=C21+A3;
C32=C22+A3;
C33=C23+A3;

C41=C11*C31;
C42=C12*C32;
C43=C13*C33;

D1=power(M1,2)*power(Pi,2);
D21=C31+D1;
D22=C32+D1;
D23=C33+D1;

D31=A3.*D21;
D32=A3.*D22;
D33=A3.*D23;

D41=C41/D31;
D42=C42/D32;
D43=C43/D33;

E11=B1+D41;
E12=B1+D42;
E13=B1+D43;

% Values of Rayleigh number NR

NR1=A6*E11          %  When P=2
NR2=A6*E12          %  When P=4
```

```
NR3=A6*E13          %  When  P=6

% Graphs  of  NR  Vs  Alpha

plot(Alpha,NR1)
hold on
plot(Alpha,NR2)
hold on
plot(Alpha,NR3)
```

5.2.3 When H-Bar = 500,1000,1500

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=[500 1000 1500 ];
P=2 ;
NQ=20;
M1=10;

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G-1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);

A41=power(Alpha,2)*H_Bar(1,1);
A42=power(Alpha,2)*H_Bar(1,2);
A43=power(Alpha,2)*H_Bar(1,3);

A51=A3./A41;
A52=A3./A42;
A53=A3./A43;

A61=A1*A51;
A62=A1*A52;
A63=A1*A53;
```

```

B1=1/P;

C1=NQ*power(Pi,2)*power(Epsilon,-1);
C2=P*C1;
C3=C2+A3;
C4=C1*C3;


D1=power(M1,2)*power(Pi,2);
D2=C3+D1;
D3=A3.*D2;
D4=C4/D3;

E1=B1+D4;


% Values of Rayleigh number NR

NR1=A61*E1        % When P=2
NR2=A62*E1        % When P=4
NR3=A63*E1        % When P=6

% Graphs of NR Vs Alpha

plot(Alpha,NR1)
hold on
plot(Alpha,NR2)
hold on
plot(Alpha,NR3)
```

5.2.4 When M1= 10,20,30

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=1000 ;
P=2 ;
NQ=20;
M1=[10 20 30];

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G -1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);
A4=power(Alpha,2)*H_Bar;
A5=A3./A4;
A6=A1*A5;


B1=1/P;

C1=NQ*power(Pi,2)*power(Epsilon,-1);
C2=P*C1 ;
C3=C2+A3;
C4=C1*C3;

```

```
D11=power(M1(1,1),2)*power(Pi,2);
D12=power(M1(1,2),2)*power(Pi,2);
D13=power(M1(1,3),2)*power(Pi,2);

D21=C3+D11;
D22=C3+D12;
D23=C3+D13;

D31=A3.*D21;
D32=A3.*D22;
D33=A3.*D23;

D41=C4/D31;
D42=C4/D32;
D43=C4/D33;

E11=B1+D41;
E12=B1+D42;
E13=B1+D43;



% Values of Rayleigh number NR

NR1=A6*E11          %  When P=2
NR2=A6*E12          %  When P=4
NR3=A6*E13          %  When P=6

% Graphs of NR Vs Alpha

```

```
plot(Alpha,NR1)
hold on
plot(Alpha,NR2)
hold on
plot(Alpha,NR3)
```

5.3 Chapter 4 : Variations of Rayleigh number

Consider the equation (4.28) to find the variations in Rayleigh number R_1

5.3.1 When S1 = 10 , 20 , 30

```
clear all

% Assigning numertical values to various parameters

Wave_Number=[1 2 3 4 5 ];
Gravitational_Acc_G=9.8;
H_dash=10;
H=2;
S_1=[10 20 30 ];

% Intermediate calculations

A2=(Gravitational_Acc_G)/(Gravitational_Acc_G -1)

F2=power(1+Wave_Number,3)./(Wave_Number*H)

C21=S_1(1,1)*(H_dash/H)
C22=S_1(1,2)*(H_dash/H)
C23=S_1(1,3)*(H_dash/H)

% Values of Rayleigh number

Rayleigh_Number1=A2*(F2+C21)
Rayleigh_Number2=A2*(F2+C22)
Rayleigh_Number3=A2*(F2+C23)
```

```
% Graphs of Rayleigh number Vs wave number

plot(Wave_Number, Rayleigh_Number1)
hold on
plot(Wave_Number, Rayleigh_Number2)
hold on
plot(Wave_Number, Rayleigh_Number3)
```

5.3.2 When H = 2 , 4 , 6

```
clear all

% Assigning numertical values to various parameters

Wave_Number=[1 2 3 4 5 ];
Gravitational_Acc_G=9.8;
H_dash=5;
H=[2 4 6 ];
S_1=10 ;

% Intermediate calculations

A2=(Gravitational_Acc_G)/(Gravitational_Acc_G -1)

F21=power(1+Wave_Number,3) ./(Wave_Number*H(1,1))
F22=power(1+Wave_Number,3) ./(Wave_Number*H(1,2))
F23=power(1+Wave_Number,3) ./(Wave_Number*H(1,3))

C21=S_1*(H_dash/H(1,1))
C22=S_1*(H_dash/H(1,2))
C23=S_1*(H_dash/H(1,3))

% Values of Rayleigh number

Rayleigh_Number1=A2*(F21+C21)
Rayleigh_Number2=A2*(F22+C22)
Rayleigh_Number3=A2*(F23+C23)

% Graphs of Rayleigh number Vs wave number

```

```
plot(Wave_Number, Rayleigh_Number1)
hold on
plot(Wave_Number, Rayleigh_Number2)
hold on
plot(Wave_Number, Rayleigh_Number3)
```

Chapter 6

Conclusion and Future Scope

Concluding Remarks of the Thesis and Future Scope of the research work.

6.1 Concluding Remarks

The whole thesis is devided into four chapters. Chapter 1, is introductory/review of literature. In chapter 2, we have studied “Thermal instability of Rivlin-Ericksen elastico-viscous fluid with suspended particles through porous medium”, in chapter 3 we have studied, “Hall effect on thermal instability of visco-elastic dusty fluid through porous medium” and in chapter 4, “Double-diffusive convection in presence of compressible Rivlin-Ericksen fluid with fine dust”. In chapter 2, we found that magnetic field has stabilizing effect whereas suspended particles and medium permeability have destabilizing effect on the system. In chapter 3, we found that medium permeability have stabilizing as well as destabilizing effect only in presence of magnetic field, but in absence of magnetic field it holds the same result as in presence of suspended particles in chapter 2. As in the absence of suspended particles and presence of compressibility in chapter 3, magnetic field has stabilizing effect on the system. Also hall current is studied at here and found that hall current have destabilizing effect on the system. In chapter 4, we found that stable solute gradient have stabilizing effect, where has suspended particles have destabilizing effect on the system in the presence of compressibility.

From the observation of all these three chapters, we found that magnetic field has stabilizing effect, in presence of compressibility as well as incompressibility. Medium permeability have stabilizing as well as destabilizing effects on the system, Hall current have destabilizing effect whereas stable solute gradient have stabilizing effect on the system.

All these results are verified graphically and by computer programming, self created programming codes is the beauty of the thesis.

6.2 Future Scope

Fluid dynamics has many applications in all the branches of engineering like mechanical, aeronautical and chemical etc. In medical discipline it plays an important role. Observed problems of nature can be medelled by using fluid dynamics and can be solved by using appropriate analytical method or numerical method which gives approximate solution. Presently technology is driven by physics, one must know the physics of the problem only then solutions can be interpreted and useful in real life. Mathematical equations tells a lot about the problem and physics behind it. The biggest challenge is always to convert the real life fluid flow problem into mathematical equations . The questions that always arise :

a) What is appropriate element (1 D, 2 D or 3D).

b) What are appropriate initial or boundary conditions.

c) Which technique or method is well suited for the problem.

Sometime experiments can not be performed because it is time consuming and expensive, moreover resources are limited. Also it is not possible to done on all the scales. All numerical methods convert continuum problem into discrete problem and give the solution at nodal points not at all the points of domain. So simulations techniques can be useful.

Simulation : Here, Firstly the problem is observed from the real life situation, then problem is defined and converted into mathematical model which is the set of differential equations (ordinary or partial). Afterwards problem is solved by using the mathematical techniques or tools and results obtained.

Numerical Simulation : If solution to the problem is approximated by using one of the numerical methods like finite difference method, Finite element method, finite volume method, runge-Kutta method, Galerkian method or any other method which is well suited to given problem is called numerical simulation. Finite element method is numerical tool for simulation. It can be used upto micro and continuum scale but cannot be used for nano scale. Numerical techniques are those which can be programmed.

Simulation through software: For this purpose various softwares of computational fluid dynamics like ANSYS (FLUENT), COMSOL Multiphysics, ABAQUS, MARK, PAFEC, ADINA are available in the market. The processing of these softwares is based on the numerical methods.

One can pursue the research in the area of fluid structure interaction (FSI) and further simulate the results by using computational software.

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Appendices

Appendix A : Table for Graphs

x	$Q_1 = 25$	$Q_1 = 50$	$Q_1 = 75$
1	5.2	10.2	15.2
2	7.725	15.225	22.725
3	10.267	20.267	30.267
4	12.813	25.313	37.813
5	15.36	30.36	45.36
6	17.908	35.408	52.908

Table 1: The variation of Rayleigh number R_1 with wave number x, for $H' = 10, P = 2, \theta = 45^0, \epsilon = 0.5$ and $Q_1 = 25, 50, 75$.

x	$H = 5$	$H = 10$	$H = 15$
1	10.4	5.2	3.467
2	15.45	7.725	5.15
3	20.533	10.267	6.844
4	25.625	12.813	8.542
5	30.72	15.36	10.24
6	35.817	17.908	11.939

Table 2: The variation of Rayleigh number R_1 with wave number x, for $P = 2, Q_1 = 25, \theta = 45^0, \epsilon = 0.5$ and $H' = 5, 10, 15$.

x	$P = 0.1$	$P = 0.2$	$P = 0.6$
1	9	7	5.667
2	12	9.75	8.25
3	15.333	12.667	10.889
4	18.75	15.625	13.542
5	22.2	18.6	16.2
6	25.667	21.583	18.861

Table 3: The variation of Rayleigh number R_1 with wave number x, for $Q_1 = 25, H' = 10, \theta = 45^0, \epsilon = 0.5$ and $P = 0.1, 0.2, 0.6$.

α	$P = 2$	$P = 4$	$P = 6$
0.01	2620.019	3035.142	3317.077
0.02	655.015	758.791	829.273
0.03	291.125	337.244	368.568
0.04	163.764	189.703	207.321
0.05	104.814	121.412	132.687

Table 4: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14$, $N_Q = 20, M_1 = 10, \epsilon = 0.5$ and for different values of $P = 2, 4, 6$.

α	$N_Q = 10$	$N_Q = 20$	$N_Q = 30$
0.01	1310.01	3035.142	4975.616
0.02	327.507	758.791	1243.909
0.03	145.563	337.244	552.852
0.04	81.882	189.703	310.982
0.05	52.407	121.412	199.031

Table 5: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14, P = 4,$ $M_1 = 10, \epsilon = 0.5$ and for different values of $N_Q = 10, 20, 30.$

α	$\overline{H} = 500$	$\overline{H} = 1000$	$\overline{H} = 1500$
0.01	5240.039	2620.019	1746.68
0.02	1310.03	655.015	436.677
0.03	582.25	291.125	194.083
0.04	327.528	163.764	109.176
0.05	209.627	104.814	69.876

Table 6: Variation of N_R with α for a different value of $\overline{H} = (500, 1000, 1500)$ for fixed values of $G = 9.8, \pi = 3.14, P = 2, M_1 = 10, \epsilon = 0.5.$

α	$M_1 = 10$	$M_1 = 20$	$M_1 = 30$
0.01	2620.019	1346.972	940.011
0.02	655.015	336.752	235.012
0.03	291.125	149.675	104.456
0.04	163.764	84.197	58.762
0.05	104.814	53.891	37.612

Table 7: Variation of N_R with α for a fixed values $\overline{H} = 1000, G = 9.8, \pi = 3.14,$ $P = 2, N_Q = 20, \epsilon = 0.5$ for different values of $M_1 = 10, 20, 30.$

α	$S_1 = 10$	$S_1 = 20$	$S_1 = 30$
1	60.136	115.818	171.5
2	63.199	118.881	174.563
3	67.561	123.242	178.924
4	73.082	128.764	184.446
5	79.736	135.418	191.1

Table 8: The variation of Rayleigh number R_1 with wave number x, for $G = 9.8, H = 2, H' = 10$ and $S_1 = 10, 20, 30$.

α	$H = 2$	$H = 4$	$H = 6$
1	32.295	16.148	10.765
2	35.358	17.679	11.786
3	39.72	19.86	13.24
4	45.241	22.621	15.08
5	51.895	25.948	17.298

Table 9: The variation of Rayleigh number R_1 with wave number x, for $G = 9.8, S_1 = 10, H' = 5$ and $H = 2, 4, 6$.

Appendix B : List of Symbols

$6\pi\mu\eta'$	Stoke's drag coefficient
β	Temperature Gradient
β'	Solute Gradient
$\delta\rho$	Perturbations in Density
δp	Perturbations in Pressure
δ_{ij}	Kronecker Delta
ϵ	Medium Porosity
$\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$	Chandrasekhar Number
$\frac{\nu}{\eta}$	Magnetic Prandtl Number
$\frac{\nu}{\kappa}$	Prandtl Number
$\frac{cH}{4\pi N' e\eta}$	Hall parameter
$\frac{g\alpha\beta d^4}{\nu\kappa}$	Rayleigh Number
κ	Thermal Diffusivity
μ	Viscosity
μ'	Viscoelasticity
μ_e	Magnetic Permeability
ν	Kinematic Viscosity

ν'	Kinematic Viscoelasticity
ρ	Density
τ_{ij}	Shear Stress Tensor
θ	Perturbations in Temperature
$\vec{g}$	Gravity Force
$\vec{H}$	Magnetic Field
$\vec{h}$	Perturbations in Magnetic Field
e_{ij}	Rate-of-Strain Tensor
k	Resultant Wave Number
k_1	Medium Permeability
k_x	Wave Number along x- direction
k_y	Wave Number along y- direction
n	Growth Rate
N'	Electron Number Density
p	Pressure
T	Temperature
T_0	Temperature at Bottom layer
T_d	Temperature at Upper layer
T_{ij}	Stress Tensor

Appendix C : Title Pages of Published Research Papers

Int. J. of Applied Mechanics and Engineering, 2011, vol.16, No.4, pp.1169-1179

THERMAL INSTABILITY OF RIVLIN-ERICKSEN ELASTICO-VISCOUS FLUID PERMEATED WITH SUSPENDED PARTICLES IN HYDRODYNAMICS IN A POROUS MEDIUM

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The thermal instability of a layer of a Rivlin-Ericksen elastico-viscous fluid permeated with suspended particles in a porous medium acted on by a uniform magnetic field is considered. For stationary convection, the Rivlin-Ericksen elastico-viscous field behaves like a Newtonian fluid. The magnetic field is found to have a stabilizing effect, whereas suspended particles and medium permeability have a destabilizing effect for the case of stationary convection. The magnetic field introduces oscillatory modes in the systems, which were non-existent in its absence.

Key words: Rivlin-Ericksen elastico-viscous fluid, thermal instability, suspended particles, magnetic field, porous medium.

1. Introduction

The formulation and derivation of the basic equations of a layer of a fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in treatise by Joseph (1976). When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of a layer of a fluid heated from below in a porous medium is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous media.

A comprehensive account of the effect of a uniform magnetic field on the layer of a Newtonian fluid heated from below was given by Chandrasekhar (1981). The effect of a magnetic field on the stability of the fluid flow is of interest in geophysics, particularly in the study of Earth core where the Earth's mantle, which consists of a conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The results of flow through a porous medium in the presence of a magnetic field are applied in the study of the stability of a convective flow in the geothermal region. Lapwood (1948) studied the stability of a convective flow in hydrodynamics using Rayleigh's procedure. Wooding (1960) considered the Rayleigh instability of a thermal boundary layer in the flow through a porous medium.

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DOI: 10.2478/ijame-2013-0052

HALL EFFECT ON THERMAL INSTABILITY OF VISCOELASTIC DUSTY FLUID IN POROUS MEDIUM

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The effect of Hall currents and suspended dusty particles on the hydromagnetic stability of a compressible, electrically conducting Rivlin-Ericksen elastico viscous fluid in a porous medium is considered. Following the linearized stability theory and normal mode analysis the dispersion relation is obtained. For the case of stationary convection, Hall currents and suspended particles are found to have destabilizing effects whereas compressibility and magnetic field have stabilizing effects on the system. The medium permeability, however, has stabilizing and destabilizing effects on thermal instability in contrast to its destabilizing effect in the absence of the magnetic field. The critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behavior of various parameters on critical thermal Rayleigh numbers are depicted graphically. The magnetic field, Hall currents and viscoelasticity parameter are found to introduce oscillatory modes in the systems, which did not exist in the absence of these parameters.

Key words: thermal instability, Rivlin-Ericksen viscoelastic fluid, suspended particles, Hall current effect, porous medium.

AMS Classification (2010): 76D50, 76A05, 76A10, 76S05.

1. Introduction

The theoretical and experimental results of the onset of thermal instability (Benard convection), under varying assumptions of hydrodynamics and hydromagnetics, have been discussed by Chandrasekhar (1981) in his celebrated monograph. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current is called the Hall current effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasma. Sherman and Sutton (1962) considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta (1967) studied the problem of thermal instability in the presence of Hall currents and found that Hall currents have a destabilizing effect on the thermal instability of a horizontal layer of a conducting fluid in the presence of a uniform vertical magnetic field. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in association with the external force. The approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density

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Research Article

Double-Diffusive Convection in Presence of Compressible Rivlin-Ericksen Fluid with Fine Dust

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An investigation is made on the effect of suspended particles (fine dust) on double-diffusive convection of a compressible Rivlin-Ericksen elastico-viscous fluid. The perturbation equations are analyzed in terms of normal modes after linearizing the relevant set of equations. A dispersion relation governing the effects of viscoelasticity, compressibility, stable solute gradient, and suspended particles is derived. For stationary convection, Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter. The stable solute gradient compressibility has a stabilizing effect on the system whereas suspended particles hasten the onset of thermosolutal instability. The Rayleigh numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behaviour of various parameters on Rayleigh numbers has been depicted graphically. It has been observed that oscillatory modes are introduced due to the presence of viscoelasticity, suspended particles, and stable solute gradient which were not existing in the absence of these parameters.

1. Introduction

A layer of Newtonian fluid heated from below, under varying assumptions of hydrodynamics, has been treated in detail by Chandrasekhar [1]. Chandra [2] performed careful experiments in an air layer and found contradiction between the theory and the experiment. He found that the instability depended on the depth of the layer. A Bénard-type cellular convection with fluid descending at the cell centre was observed when predicted gradients were imposed, if the layer depth was more than 10 mm. But if the layer of depth was less than 7 mm, convection occurred at much lower gradients than predicted and appeared as irregular strips of elongated cells with fluid rising at the centre. Chandra called this motion columnar instability. The effect of particle mass and heat capacity on the onset of Bénard convection has been considered by Scanlon and Segel [3]. They found that the critical Rayleigh number was reduced solely because the heat capacity of the clean gas was supplemented by that of the particles. The effect of suspended particles was found to destabilize the layer. Palniswamy and purushotham [4] have considered the stability of shear flow of stratified fluids with fine dust and have found the effect of fine dust to increase the region of instability. A study of double-diffusive convection with fine dust has been made by Sharma and Rani [5]. Kumar et al. [6] have studied effect of magnetic field on thermal instability of rotating Rivlin-Ericksen viscoelastic fluid, in which effect of magnetic field has stabilizing as well as destabilizing effect on the system. Also, Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluid through porous medium has been considered by Sharma et al. [7]. They have studied the stability aspects of the system. The effects of a uniform horizontal magnetic field and a uniform rotation on the problem have also been considered separately. Kumar [8] has also studied the stability of superposed viscous-viscoelastic Rivlin-Ericksen fluids in presence of suspended particles through a porous medium. In one other study, Kumar and Singh [9] have studied the stability of superposed viscous-viscoelastic fluids through porous medium, in which effects of uniform horizontal magnetic field and a uniform rotation are considered. Kumar et al. [10] have also studied hyderodynamic and hyderomagnetic stability of Rivlin-Ericksen

ABSTRACT

My research aims to analyze hall current effects and double diffusive effects in the presence of suspended particles on the thermal instability of the non-Newtonian viscoelastic fluid whose non linear relation between the stress and strain rate (which includes deformation, rotation and extension) is given by Rivlin-Ericksen in 1955.

To understand the applied problem of real life, one must know the physics of the problem and able to interpret the results obtained. In the introductory chapter, all the basics which are essential for the understanding of the problems discussed in thesis are well explained. Basic terms explained with the help of examples and real life applications. No problem can be solved without assumptions, so fundamental assumptions are also explained. In this chapter all the terms used in the thesis are explained for the understanding of general investigations in the subsequent chapters 2, 3 and 4. Flow governing equations based on the various principles of conservation like mass, momentum and energy are discussed in detail. Fluid properties, fluid types like Newtonian and non-Newtonian fluid with their specific applications and uses are explained in this introductory chapter. Concept of hydrodynamic stability of the system in terms of various parameters is also explained. Also light is thrown on the procedure of the problems formulated in the subsequent chapters.

Problem is formulated for non-Newtonian and viscoelastic fluid named Rivlin-Ericksen in porous medium in chapter 2. Fluid is permeated with suspended particles and uniform magnetic field is also considered. Governing equations for the problem were obtained and the initial state of the system described in terms of various parameters like velocity field, Pressure, magnetic field etc. is perturbed or disturbed. All the disturbances analyzed and it is found that relation between strain rate and stress become linear in case of stationary convection. Perturbations due to the magnetic field were decaying while the perturbations due to the suspended particles and medium permeability were growing. Oscillatory modes exist only due to the presence of magnetic field.

Study devoted to the effect of magnetic field which change the direction of flow of electric current when applied at right angle to electric field on the thermal instability in porous medium of dusty viscoelastic fluid in chapter 3. Problem related to the effect of

hall current on the thermal instability of viscoelastic fluid with dust in porous medium was modeled in terms of mathematical equations, initial state of the system is perturbed as in previous chapter by giving small perturbations to the physical quantities like pressure, velocity, temperature, density and magnetic field etc. Linearize the system by neglecting all the non linear terms. Dispersion relation is obtained after the normal mode analysis. It is observed that perturbations due to suspended particles and hall current were growing while the perturbations due to the magnetic field and compressibility were decaying in the system for the case of stationary convection. Magnetic field stabilize the effect of permeability on thermal instability. Oscillatory modes were introduced by viscoelastic parameter, magnetic field and hall current. Behavior of hall current, permeability, magnetic field and suspended particles on the critical thermal Rayleigh number were shown graphically.

Double diffusive or thermosolutal convection i.e. the presence of more than one component with different diffusivities like heat and salt in the fluid layer, explained in chapter 4. Now temperature and salt field are two destabilizing sources for the density difference whereas in standard Bénard problem, temperature field is the only destabilizing source. This situation is similar to ocean where both salt and heat are present simultaneously and chemical engineering with two or more components of different molecular diffusivities. Also in case of stellar helium acts like salt in raising the density and diffusing more slowly than heat. Mathematical model for the problem of double-diffusive convection in presence of compressible fluid with fine dust was designed in terms of equation. Using the same procedure and techniques or methods as in previous chapters to find the solution. It is observed that relation between strain rate and stress become linear in case of stationary convection due to vanishing of viscoelastic parameter. Presence of stable solute gradient, suspended particles and viscoelasticity introduced oscillatory modes. The stable solute gradient and compressibility has a stabilizing effect and suspended particles hasten the onset of thermosolutal instability.

Programming codes were written for the variations of Rayleigh numbers obtained in the chapters 2, 3 and 4 by assigning numerical values to all other parameters, these codes will calculate the Rayleigh number and will also plot the graph.

ACKNOWLEDGEMENT

My thesis is based on the research in applied mathematics for developing mathematical models relevant to the study of non-Newtonian Rivlin-Ericksen fluid. The research took place at Department of Mathematics, Lovely Professional University, Phagwara.

I express my deep gratitude to my supervisor **Dr. Mahinder Singh** for his support, guidance, inspiring collaboration and for providing background on which the entire work is based.

I am grateful to the management of **Lovely Group**, for allowing me to the work and for providing me with the working facilities. I thanks **Dr. Lovi Raj Gupta** for his expert assistance in many practical concerns.

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I wish to express my gratitude to my family, relatives and friends for their sincere interest in my work. Most of all, I thank my beloved wife, **Neetu Gupta**, without her support, this work would never have been completed.

I DEDICATE MY THESIS TO MY PARENTS.

August, 2014 Rajesh Kumar Gupta

Appendices

Appendix A : Table for Graphs

x	$Q_1 = 25$	$Q_1 = 50$	$Q_1 = 75$
1	5.2	10.2	15.2
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Table 1: The variation of Rayleigh number R_1 with wave number x, for $H' = 10, P = 2, \theta = 45^0, \epsilon = 0.5$ and $Q_1 = 25, 50, 75$.

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Table 2: The variation of Rayleigh number R_1 with wave number x, for $P = 2, Q_1 = 25, \theta = 45^0, \epsilon = 0.5$ and $H' = 5, 10, 15$.

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α	$P = 2$	$P = 4$	$P = 6$
0.01	2620.019	3035.142	3317.077
0.02	655.015	758.791	829.273
0.03	291.125	337.244	368.568
0.04	163.764	189.703	207.321
0.05	104.814	121.412	132.687

Table 4: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14$, $N_Q = 20, M_1 = 10, \epsilon = 0.5$ and for different values of $P = 2, 4, 6$.

α	$N_Q = 10$	$N_Q = 20$	$N_Q = 30$
0.01	1310.01	3035.142	4975.616
0.02	327.507	758.791	1243.909
0.03	145.563	337.244	552.852
0.04	81.882	189.703	310.982
0.05	52.407	121.412	199.031

Table 5: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14, P = 4,$ $M_1 = 10, \epsilon = 0.5$ and for different values of $N_Q = 10, 20, 30$.

α	$\overline{H} = 500$	$\overline{H} = 1000$	$\overline{H} = 1500$
0.01	5240.039	2620.019	1746.68
0.02	1310.03	655.015	436.677
0.03	582.25	291.125	194.083
0.04	327.528	163.764	109.176
0.05	209.627	104.814	69.876

Table 6: Variation of N_R with α for a different value of $\overline{H} = (500, 1000, 1500)$ for fixed values of $G = 9.8, \pi = 3.14, P = 2, M_1 = 10, \epsilon = 0.5$.

α	$M_1 = 10$	$M_1 = 20$	$M_1 = 30$
0.01	2620.019	1346.972	940.011
0.02	655.015	336.752	235.012
0.03	291.125	149.675	104.456
0.04	163.764	84.197	58.762
0.05	104.814	53.891	37.612

Table 7: Variation of N_R with α for a fixed values $\overline{H} = 1000, G = 9.8, \pi = 3.14,$ $P = 2, N_Q = 20, \epsilon = 0.5$ for different values of $M_1 = 10, 20, 30$.

α	$S_1 = 10$	$S_1 = 20$	$S_1 = 30$
1	60.136	115.818	171.5
2	63.199	118.881	174.563
3	67.561	123.242	178.924
4	73.082	128.764	184.446
5	79.736	135.418	191.1

Table 8: The variation of Rayleigh number R_1 with wave number x, for $G = 9.8, H = 2, H' = 10$ and $S_1 = 10, 20, 30$.

α	$H=2$	$H=4$	$H=6$
1	32.295	16.148	10.765
2	35.358	17.679	11.786
3	39.72	19.86	13.24
4	45.241	22.621	15.08
5	51.895	25.948	17.298

Table 9: The variation of Rayleigh number R_1 with wave number x, for $G = 9.8, S_1 = 10, H' = 5$ and $H = 2, 4, 6$.

Appendix B : List of Symbols

$6\pi\mu\eta'$	Stoke's drag coefficient
β	Temperature Gradient
β'	Solute Gradient
$\delta\rho$	Perturbations in Density
δp	Perturbations in Pressure
δ_{ij}	Kronecker Delta
ϵ	Medium Porosity
$\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$	Chandrasekhar Number
$\frac{\nu}{\eta}$	Magnetic Prandtl Number
$\frac{\nu}{\kappa}$	Prandtl Number
$\frac{cH}{4\pi N' e\eta}$	Hall parameter
$\frac{g\alpha\beta d^4}{\nu\kappa}$	Rayleigh Number
κ	Thermal Diffusivity
μ	Viscosity
μ'	Viscoelasticity
μ_e	Magnetic Permeability
ν	Kinematic Viscosity

ν'	Kinematic Viscoelasticity
ρ	Density
τ_{ij}	Shear Stress Tensor
θ	Perturbations in Temperature
$\vec{g}$	Gravity Force
$\vec{H}$	Magnetic Field
$\vec{h}$	Perturbations in Magnetic Field
e_{ij}	Rate-of-Strain Tensor
k	Resultant Wave Number
k_1	Medium Permeability
k_x	Wave Number along x- direction
k_y	Wave Number along y- direction
n	Growth Rate
N'	Electron Number Density
p	Pressure
T	Temperature
T_0	Temperature at Bottom layer
T_d	Temperature at Upper layer
T_{ij}	Stress Tensor

Appendix C : Title Pages of Published Research Papers

Int. J. of Applied Mechanics and Engineering, 2011, vol.16, No.4, pp.1169-1179

THERMAL INSTABILITY OF RIVLIN-ERICKSEN ELASTICO-VISCOUS FLUID PERMEATED WITH SUSPENDED PARTICLES IN HYDRODYNAMICS IN A POROUS MEDIUM

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The thermal instability of a layer of a Rivlin-Ericksen elastico-viscous fluid permeated with suspended particles in a porous medium acted on by a uniform magnetic field is considered. For stationary convection, the Rivlin-Ericksen elastico-viscous field behaves like a Newtonian fluid. The magnetic field is found to have a stabilizing effect, whereas suspended particles and medium permeability have a destabilizing effect for the case of stationary convection. The magnetic field introduces oscillatory modes in the systems, which were non-existent in its absence.

Key words: Rivlin-Ericksen elastico-viscous fluid, thermal instability, suspended particles, magnetic field, porous medium.

1. Introduction

The formulation and derivation of the basic equations of a layer of a fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in treatise by Joseph (1976). When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of a layer of a fluid heated from below in a porous medium is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous media.

A comprehensive account of the effect of a uniform magnetic field on the layer of a Newtonian fluid heated from below was given by Chandrasekhar (1981). The effect of a magnetic field on the stability of the fluid flow is of interest in geophysics, particularly in the study of Earth core where the Earth's mantle, which consists of a conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The results of flow through a porous medium in the presence of a magnetic field are applied in the study of the stability of a convective flow in the geothermal region. Lapwood (1948) studied the stability of a convective flow in hydrodynamics using Rayleigh's procedure. Wooding (1960) considered the Rayleigh instability of a thermal boundary layer in the flow through a porous medium.

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HALL EFFECT ON THERMAL INSTABILITY OF VISCOELASTIC DUSTY FLUID IN POROUS MEDIUM

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The effect of Hall currents and suspended dusty particles on the hydromagnetic stability of a compressible, electrically conducting Rivlin-Ericksen elastico viscous fluid in a porous medium is considered. Following the linearized stability theory and normal mode analysis the dispersion relation is obtained. For the case of stationary convection, Hall currents and suspended particles are found to have destabilizing effects whereas compressibility and magnetic field have stabilizing effects on the system. The medium permeability, however, has stabilizing and destabilizing effects on thermal instability in contrast to its destabilizing effect in the absence of the magnetic field. The critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behavior of various parameters on critical thermal Rayleigh numbers are depicted graphically. The magnetic field, Hall currents and viscoelasticity parameter are found to introduce oscillatory modes in the systems, which did not exist in the absence of these parameters.

Key words: thermal instability, Rivlin-Ericksen viscoelastic fluid, suspended particles, Hall current effect, porous medium.

AMS Classification (2010): 76D50, 76A05, 76A10, 76S05.

1. Introduction

The theoretical and experimental results of the onset of thermal instability (Benard convection), under varying assumptions of hydrodynamics and hydromagnetics, have been discussed by Chandrasekhar (1981) in his celebrated monograph. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current is called the Hall current effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasma. Sherman and Sutton (1962) considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta (1967) studied the problem of thermal instability in the presence of Hall currents and found that Hall currents have a destabilizing effect on the thermal instability of a horizontal layer of a conducting fluid in the presence of a uniform vertical magnetic field. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in association with the external force. The approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density

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Research Article

Double-Diffusive Convection in Presence of Compressible Rivlin-Ericksen Fluid with Fine Dust

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An investigation is made on the effect of suspended particles (fine dust) on double-diffusive convection of a compressible Rivlin-Ericksen elastico-viscous fluid. The perturbation equations are analyzed in terms of normal modes after linearizing the relevant set of equations. A dispersion relation governing the effects of viscoelasticity, compressibility, stable solute gradient, and suspended particles is derived. For stationary convection, Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter. The stable solute gradient compressibility has a stabilizing effect on the system whereas suspended particles hasten the onset of thermosolutal instability. The Rayleigh numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behaviour of various parameters on Rayleigh numbers has been depicted graphically. It has been observed that oscillatory modes are introduced due to the presence of viscoelasticity, suspended particles, and stable solute gradient which were not existing in the absence of these parameters.

1. Introduction

A layer of Newtonian fluid heated from below, under varying assumptions of hydrodynamics, has been treated in detail by Chandrasekhar [1]. Chandra [2] performed careful experiments in an air layer and found contradiction between the theory and the experiment. He found that the instability depended on the depth of the layer. A Bénard-type cellular convection with fluid descending at the cell centre was observed when predicted gradients were imposed, if the layer depth was more than 10 mm. But if the layer of depth was less than 7 mm, convection occurred at much lower gradients than predicted and appeared as irregular strips of elongated cells with fluid rising at the centre. Chandra called this motion columnar instability. The effect of particle mass and heat capacity on the onset of Bénard convection has been considered by Scanlon and Segel [3]. They found that the critical Rayleigh number was reduced solely because the heat capacity of the clean gas was supplemented by that of the particles. The effect of suspended particles was found to destabilize the layer. Palniswamy and purushotham [4] have considered the stability of shear flow of stratified fluids with fine dust and have found the effect of fine dust to increase the region of instability. A study of double-diffusive convection with fine dust has been made by Sharma and Rani [5]. Kumar et al. [6] have studied effect of magnetic field on thermal instability of rotating Rivlin-Ericksen viscoelastic fluid, in which effect of magnetic field has stabilizing as well as destabilizing effect on the system. Also, Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluid through porous medium has been considered by Sharma et al. [7]. They have studied the stability aspects of the system. The effects of a uniform horizontal magnetic field and a uniform rotation on the problem have also been considered separately. Kumar [8] has also studied the stability of superposed viscous-viscoelastic Rivlin-Ericksen fluids in presence of suspended particles through a porous medium. In one other study, Kumar and Singh [9] have studied the stability of superposed viscous-viscoelastic fluids through porous medium, in which effects of uniform horizontal magnetic field and a uniform rotation are considered. Kumar et al. [10] have also studied hyderodynamic and hyderomagnetic stability of Rivlin-Ericksen

CERTIFICATE

I certify that Rajesh Kumar Gupta has prepared his thesis entitled " Study on Thermal Instability of Rivlin-Ericksen Fluid with Suspended Particles ", for the award of Ph.D degree of the Lovely Professional University, under my guidance. He has carried out the work at the Department of Mathematics, Lovely Professional University ,Phagwara.

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Chapter 1

Introduction

In the introductory chapter, all the basic terms and procedures have been explained for the understanding of general investigation in the subsequent chapters.

1.1 Introduction

Fluid dynamics is subject of my research in which continuous movement of a non-Newtonian viscoelastic Rivlin-Ericksen fluid is modelled. This subject is challenging as the fluid is in motion. Fluid dynamics and Electromagnetic theory were being developed independently of each other almost upto the first half of the 20^{th} century. A systematic study of the hydrodynamics of a conducting fluid immersed in a magnetic field was started in 1942 by Alfvn. This study known as Hydromagnetics or Magnetohydrodynamics(MHD).

Magnetohydrodynamics is the science where in the presence of magnetic field, the motion of electrically conducting fluid is considered. The study of the interaction between magnetic field and electrically *conducting fluids* is currently receiving considerable interest. This interest has been spurred primarily by *astrophysical* problems and by problems associated with the fusion reactor. Thus in a very lucid manner, hydromagnetics or MHD is the union of fluid dynamics and electromagnetic theory. It is concerned with physical systems specified by the equations that result from the fusion of those of hydrodynamcis and *electromagnetic* theory. It is a well known fact that when a conductor moves in a magnetic field, *electric currents* are induced in it. These currents experience a mechanical force called the '*Lorentz force*', due to magnetic field. This force tends to modify the initial motion of the conductor. Moreover, a magnetic field which is generated by the induced currents is added on to the applied magnetic field. Thus there is a coupling between the motion of the conductor and electromagnetic field, which is exhibited in a more pronounced form in liquid and gaseous conductors. This is due to the fact that the molecules composing the liquids and gases enjoy more freedom of movement than those of solid conductors. The *Lorentz force* is usually small unless inordinately high magnetic fields are applied. Therefore, this force is too small to alter the motion as a whole considerably but if it acts for a sufficiently long period, the *molecules* of gases and liquids may get accelerated considerably to change the initial state of motion of these types of conductors. Therefore, the *coupling* between the electromagnetic fields and the motion of a conductor could only be judged appreciably by confining attention to liquid and gaseous conductors.

1.2 Fluid

Fluid is something which can flow it can be gas or liquid. The study of characteristic of fluid in motion is *hydrodynamics* and the study of characteristic of fluid at rest is hydrostatic. Pressure difference applies force, which can create motion. It is substance that flows or *deforms* continuously under the action of forces applied may pressure difference or shearing (acting tangentially). Fluid has no ability to resist the force of deformation. If there is no pressure difference or *shearing force*, it implies that fluid is at rest and all other forces are perpendicular to the plane in which these force acting.

1.2.1 Properties of Fluid

Temperature, *density* and pressure describe the thermodynamic state of the fluid along with other properties like internal energy or *entropy*. Viscosity is unique property of fluid by which we can differentiate between two fluids. Fluid has also other properties like kinematic viscosity, velocity and surface tension.

Density

Density $= \frac{\text{Mass}}{\text{Volume}} =$ Mass per unit volume.

It is the distribution of mass and denoted by ρ. Its dimension is $\frac{M}{L^3} = ML^{-3}$. Density is different for different liquids as :

- Density of water = 1000 kg/m^3.
- Density of blood = 1060 kg/m^3.
- Density of salty water = 1027 kg/m^3.
- Density of air = 1.29 kg/m^3.

Relative Density

Relative density is the *dimensionless* or unit less number which is used to compare the heaviness of different fluids. It is defined as, Relative Density $= \frac{\text{Density of Material}}{\text{Density of water}}$.

Relative density of Mercury = 13.6, which means that mass of mercury in the *volume* of 1 m^3 is 13600 kg which is 13.6 times the mass of water.

Pressure

Pressure is proportionate to force and inversely proportionate to area. It is defined as, Pressure $= \frac{\text{Force}}{\text{Area}} =$ Force per unit area. It is denoted by p and its unit is *Pascal*, dimension of pressure is given by, 1 Pascal $= \frac{\text{1 Newton}}{m^2} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$.

Stress has also the same units. A physical quantity with no direction is scalar and quantity having one direction is vector and quantity having more than one or many directions is tensor. So pressure is the tensor as it has direction in all the directions. For instance, fill air in the balloon, now pressure is inside the balloon, puncture the balloon at the top and flow of air will be in upward direction so pressure will also have an upward direction, similarly repeat it in any direction and you will find the pressure in all the directions.

Pascal's Law says that pressure inside a fluid is same throughout. Its direction is always normal (at right angle) to the surface in contact. This law is for the case where gravity is not included but pressure varies with depth inside fluid because of *gravity* i.e. pressure will be different at different heights inside the fluid. Above the surface of earth, atmosphere consists of molecules (having mass) so gravity will be in effect. Atmosphere presses the earth. Molecules apply force on the surface of earth. Effect of this force is F/A i.e. the pressure created by atmosphere called atmospheric pressure. It is maximum on the earth and decreases gradually as we move up. Barometer is used to measure atmospheric pressure.

Viscosity

Viscosity is the Property of fluid when it is in motion. In flowing liquid there is a force which resist or opposes the motion is called viscosity or *viscous drag*. It is assumed that fluid flow in layers and all the layers move with different velocity. The layer near the lower fixed surface has zero velocity and the layers away from it have larger velocity that is change in velocity as the height increases. When *tangential force* is applied on surface of upper layer, stress is created which is called tangential stress. As we move down from the upper layer, the velocity decreases. More the *tangential stress* will increase the velocity of all layers of fluid. It is denoted by τ and is directly proportional to *velocity gradient* because during flow of liquid when tangential stress is increased the velocity gradient also increases and $\tau = \mu\frac{du}{dy}$, where μ is constant of proportionality, known as the *coefficient of viscosity*. So, $\mu = \frac{\text{Tangential stress}}{\text{Velocity gradient}} = \frac{\text{Force/area}}{\text{Velocity/length}}$.

For fixed tangential stress, liquid with greater value of constant μ will have less velocity i.e. fluid is more viscous and vice versa. Viscous drag is opposite to the direction of the tangential stress and it is between the two consecutive layers of fluid. Viscosity is characteristic of liquid which is fixed. It will not change by increasing the stress. Variables are force and velocity gradient .
Dimension of $\mu =$ Dimension of $\left[\frac{\text{Force/area}}{\text{Velocity/length}}\right] = \frac{MLT^{-2}/L^2}{(L/T)/L} = \frac{M}{LT} = ML^{-1}T^{-1}$.

Kinematic Viscosity

The ratio of coefficient of viscosity μ to the density ρ determines the effect of viscosity on the motion of fluid is called *kinematic viscosity*. It is denoted by ν and defined as $\nu = \frac{\mu}{\rho}$. Its dimension is $\frac{ML^{-1}T^{-1}}{ML^{-3}} = \frac{L^2}{T} = L^2T^{-1}$.

Stokes' Drag

It is the quantity of force due to viscous drag.
$F = 6\pi *$ coefficient of viscosity $*$ radius r of body $*$ velocity of body $= 6\pi\mu rv$.

Surface Tension

Surface tension is characteristic of the surface of liquid due to which it tries to decrease its area and for this purpose applies a force of attraction between *molecules* in the surface. For this reason surface of liquid behaves like stretched membrane.

Surface tension is the force that acts at each point of the surface of a fluid due to interaction of the neighbouring molecules on the molecule situated at this point. It expressed as the force per unit length of the surface in the tangential direction. It is property of *static liquid* and it does not depend on the quantity. *Surface tension* is calculated as force per unit length across an imaginary line drawn on the surface. Its unit is Newton per meter which is different from the unit of force.

Blade of steel does not sink in the water whenever its density is more than water because of surface tension.

1.3 Buoyancy Force

When a solid is dip in a liquid and displaces its molecules, those *displaced molecules* apply a force on the solid and trying to eject it out. This force is called *buoyancy* force and phenomenon is buoyancy. It is the Natural force and solid dip inside the water because of gravity i.e. thrust applied by solid in downward direction. Liquid molecule apply the force in upward direction i.e. *upthurst* by the liquid.

1.4 Fluid Types

- Newtonian fluid.
- Non-Newtonian fluid.

Newtonian fluid

The fluid in which stresses are the linear composite function of the instantaneous velocity gradients are called Newtonian fluids. In other words stresses are the linear function of strain rate and strain rate are expressible in velocity gradients. Graph of this relation of stress and strain rate is a straight line. Flow governing equations for the Newtonian fluid are Navier-Stokes' equations. Moreover *Newtonian* fluid cannot explain every type of phenomenon.

Non-Newtonian fluid

The fluid in which stresses are the non linear composite function of the instantaneous velocity gradients are called *non-Newtonian* fluids. In other words stresses are the non linear function of strain rate and strain rate are expressible in velocity gradients. Graph of this relation of stress and strain rate is a curve not a straight line. Equations which govern fluid flow are obtained by using the principle of conservation of mass and momentum.

Rivlin-Ericksen Fluid

Several Models have been proposed for non-Newtonian fluids (having non linear relation between the shearing stress and strain rate) like :

- Ostwaldde Waele power law model (1925, Ball point pen ink, molten chocolate).
- Carreau Yasuda model (1972, Properties of polystyrene fluids).
- Newtonian fluid Cross model (1965, Pseudoplastic systems).
- Sisko model (1958, Lubricating greases).
- Bingham Herschel-Bulkley model (1922, Paints, toothpaste, mango jam etc.)
- Rivlin-Ericksen model (1955), Known as Rivlin-Ericksen fluids proposed by Ronald Samuel Rivlin and Jerald LaVerne Ericksen. This fluid model is known as order fluid model: 2^{nd} order, 3^{rd} order or n^{th} order. And many more models.

I focused my study on the Rivlin-Ericksen model for non-Newtonian fluid because it can be used in various shear damping fluid devices, modeling of blood and in many other safety equipments which can be helpful to the society.

In 1955, Rivlin-Ericksen proposed a theory of non-linear viscoelasticity based on the assumption that the stress can be expresses in terms of velocity gradients. The resulting constitutive equations for an isotropic incompressible viscoelastic liquid were shown in the form :

$$T_{kl} = -p\delta_{kl} + \tau_{kl}, \tag{1.1}$$

$$\tau_{kl} = \rho \left(\nu + \nu' \frac{\partial}{\partial t} \right) e_{kl}, \tag{1.2}$$

$$e_{kl} = \frac{1}{2} \left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right) \tag{1.3}$$

where $T_{ji} \rightarrow$ *stress tensor*, $\tau_{ji} \rightarrow$ *shear stress tensor*, $e_{ji} \rightarrow$ *strain rate tensor*, $\delta_{ji} \rightarrow$ *Kronecker delta*, $v_i \rightarrow$ velocity vector, $x_i \rightarrow$ position vector, $p \rightarrow$ *isotropic pressure*, $\mu \rightarrow$ viscosity and $\mu' \rightarrow$ *viscoelasticity.*

The flow of a conducting viscoelastic Rivlin-Ericksen fluid through *Porous medium* in a long uniform straight tube of rectangular cross-section under the influence of transverse uniform MF (magnetic field) has been studied by Gaurav Mishra et al. [1]. The upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in the Rivlin-Ericksen fluid heated from below in the presence of uniform vertical magnetic field was studied by Ajaib S.Banyal1 et al. [2]. The problem of thermal convection of a Rivlin-Ericksen fluid permeated with suspended particles in porous medium heated from below with variable gravity is analyzed by the method of positive operator by Pushap Lata [3]. An analysis is presented with MHD free convective viscoelastic flow of a fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime in presence of heat source by R.Choudhury and B.Das [4]. Study of Instability of Streaming Rivlin-Ericksen Fluid in Porous Medium is made by B.Jana and J.Sarkar [5]. The effect of suspended particles on thermal convection of incompressible Rivlin-Ericksen elastico-viscous fluid in a porous medium is considered G.C.Rana and R.C.Thakur [6]. A theoretical study is made to investigate the influences of relaxation and retardation times of viscoelastic fluid on the onset of convection in a horizontal fluid layer heated underneath by Rajib Basu1 and G.C.Layek [7]. The thermal instability of a layer of Rivlin-Ericksen elastico-viscous rotating fluid in a porous medium in hydromagnetics is considered by S.K.Kango and Vikram Singh [8]. An investigation is made on the effect of Hall currents and suspended particles on the hydromagnetic stability of a compressible, electrically conducting Rivlin-Ericksen elastico-viscous fluid by U.Gupta et al [9]. The unsteady Hele-Shaw flow of a viscoelastic Rivlin-Ericksen conducting fluid between two parallel walls by S.Sreekanth et al. [10]. Bertrand Rollin and Malcolm J.Andrews extended the Goncharov model for nonlinear Rayleigh-Taylor instability of perfect fluids to the case of Rivlin-Ericksen viscoelastic fluids with surface tension [11]. Oscillatory onset of convection is studied numerically for Rivlin-Ericksen, Maxwell and Jeffreys liquids by considering free-free and rigid-free isothermal/adiabatic boundaries by P.G. Siddheshwar et al. [12]. An analysis for the steady two-dimensional boundary-layer stagnation-point flow of Rivlin-Ericksen fluid of second grade with a uniform suction is carried out via symmetry analysis by M.B.Abd-el-Malek and H.S.Hassan [13]. P.Riesen, K.Hutter and M.Funk present a viscoelastic constitutive relation which describes transient creep of a modified second grade fluid enhanced with elastic properties of a solid. The material law describes a Rivlin-Ericksen material and is a generalization of existing material laws applied to study the viscoelastic properties of ice [14]. The thermosolutal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium is considered to include the effect of suspended particles and rotation. The sufficient conditions for the validity of principle of exchange of stabilities are obtained by A.K.Aggarwal [15]. Hyam Abbouda and Toni Sayah propose a finite-element scheme for solving numerically the equations of a transient two-dimensional grade-two non-Newtonian Rivlin-Ericksen fluid model [16]. Motivated by the aim of modelling the behavior of swirling flow motion, F.Carapau present a 1D hierarchical model for an Rivlin-Ericksen fluid with complexity n = 2, flowing in a circular straight tube with constant and no constant radius [17]. Ronald Rivlin was an outstanding figure in the development of modern nonlinear continuum mechanics in the second half of the 20th century. Much of his research is characterized by the innovative, systematic and effective use of methods based on invariant theory. A.J.M.Spencer had summarize his work in this area, and show that it continues to be effective in applications to recent research in the mechanics of fibre-reinforced elastic materials[18]. The flow of an unsteady third-grade Rivlin-Ericksen fluid on an oscillating plate is discussed by Muhammad R.Mohyyuddin et al [19]. The stability of the plane interface separating two viscoelastic (Rivlin-Ericksen) superposed fluids in the presence of suspended particles is studied by P.Kumar and G.J.Singh [20].

Applications of non-Newtonian fluid

It is used in many *safety equipments* and some mathematical models had developed on the basis of non -Newtonian fluid. Some useful applications are :

- It is used in the formation of various materials like rope, seatbelt and safety harness.
- Some *shear damping fluid* devices are based on the *shear thickening* property of the non-Newtonian fluid which can reduce the injuries in road accidents or sports.
- Blood behaves as a non-Newtonian fluid in the core. Thus, it is modeled as a non-Newtonian fluid.
- Magma fluid is non-Newtonian fluid because it does not obey the Newton's law of viscosity. The study of these fluids is an important area of *research*.
- It can be used in military suit which would change to solid state when the bullet hits.

- Because of shear thickening characteristics of non-Newtonian, it is used in of shoes. it remain in liquid state while running, walking, standing and change to solid state while fast running. it can prevent injuries.

1.5 Basic Hydrodynamic Terms

1.5.1 Temperature and Heat

Temperature of an object is the degree of its *hotness*. It is the physical quantity which decides the direction of flow of heat energy. Heat is a type of energy contents in an object. Heat flow from an object of higher temperature to object of lower temperature. For example, if we touch an ice, the heat will flow from our body to ice because our body is at higher temperature. If we touch a hot water then heat will flow from hot water to our body because our body is at lower *temperature*.

Convection

In Convection, heat energy is transferred from higher temperature region to lower temperature region through the displacement of the particles of the medium. Thus convective heat transfer is associated with displacement of *fluid element*.

In natural convection, fluid element is displaced due to *density difference* arising out of temperature difference. In forced *convection*, fluid element is forced to change its position by applied external energy. Heat transfer takes place due to the presence of temperature difference. The driving force is the temperature difference.

Thermal Expansion

Whenever we give heat energy to molecules of an object, the activity /vibrations of molecules increases and need a larger space to exist and what we get finally expansion called *thermal expansion*.

In other words, when an object is heated, the distance between molecules increases and therefore its volume increase. If any one dimension is negligible then we say that area has increased, if depth and width both are negligible, then we consider only longitudinal expansion.

Let us consider the volume of one cubic meter and raise the temperature by one degree by giving heat, change in volume is called the volumetric thermal expansion. Coefficient of volumetric thermal expansion is denoted by α, where

$\alpha =$ Coefficient of expansion $= \frac{\text{Change in Volume}}{\text{Original volume} * \text{Change in temperature}} = \frac{V - V_0}{V_0 * (T - T_0)}$

$\Rightarrow$ New volume, $V = V_0[1 + \alpha(T - T_0)]$.

With the increase in volume the density will decrease and it will be given by the relation $\rho = \rho_0[1 + \alpha(T - T_0)]$.

Specific Heat

Specific heat of an material is the heat required to raise the temperature of 1kg of that material by 1 degree Celsius. Its unit is joule per kg pre degree Celsius and represented by c. It is a property of material.

Specific heat at the constant volume is the heat required to raise the temperature of one mole of gas by one degree Celsius by keeping volume constant. It is denoted by C_v.

Specific heat at the constant pressure is the heat required to raise the temperature of one mole of gas by one degree Celsius by keeping the pressure constant whereas Volume may change. It is denoted by C_p.

Specific heat at the constant pressure, C_p, is always greater than *specific heat* at the constant volume, C_v.

Heat Capacity

Heat capacity of an object is the heat required to raise its temperature by one degree Celsius. Its unit is joule. It is a property of an object and defined as,

Heat capacity of an object $=$ Mass of an object $*$ Specific heat of the an object .

An object with more *heat capacity* can store more heat.

1.5.2 Laminar and Turbulent Flow

Fluid flow can happen in two ways, *Laminar*/Streamline or *Turbulent*/Random. Suppose all the fluid molecules moving in row with certain velocity, if there is no change in the sequence and velocity throughout, motion is laminar/streamline. In other words, in laminar flow, fluid particles maintain its order and cross any particular point with same velocity. There will be no extra pressure on the walls of pipe in pipe flow during laminar flow.

In turbulent flow, fluid particles do not maintain their serial order and overtake each other. There will be an extra pressure on the walls of pipe in pipe flow during turbulent flow and pipe can burst out. Fluid flows in lines and different lines can have different velocity. All the particles have the same velocity with respect to line in laminar flow. These lines are called *streamline*. Numerical value of Dimensionless *Reynold's number* decides the pattern of flow which depends on the velocity of fluid. If a liquid flows in pipe then it is defined as $R_n = \frac{\rho v d}{\mu}$
where $d \to$ is the diameter,
$v \to$ is the velocity,
$\rho \to$ is the density,
$\mu \to$ is coefficient of viscosity.
For $R_n < 1000$, flow is laminar.
For $R_n > 1000$, flow is turbulent.
For $R_n \geq 1000$ and $R_n \leq 2000$, flow is mixed.
Engineers use this number to optimize the flows in pipe.

1.5.3 Compressible and Incompressible

Gases are highly *compressible* as compare to liquids. In case of gases, small change in pressure may bring large change in *specific volume* $\left(\frac{1}{\rho}\right)$ or in volume per unit mass. In case of liquids, effect of pressure on density is neglected and we assume $\rho =$ Constant.

Dimensionless, *Mach number* decides whether the fluid flow is compressible or incompressible which depends upon the velocity of fluid. It is denoted by M and defined as, $M = \frac{\text{Fluid velocity}}{\text{Speed of sound}} = \frac{v}{a} = \frac{v}{332m/s}$. If fluid velocity, $v > 99m/s$, then compressibility effects are to be considered.

1.5.4 Prandtl Number

During convection, *conduction* also take place in fluid. Both processes reduce the temperature difference due to heat transfer. Rates of convection and conduction are different for different fluids. The dimensionless *Parndtl number* decides the which process will dominate and defined as

$P_r = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{c_p \mu}{\kappa}$, where

$\nu = \frac{\mu}{\rho}$, kinematic viscosity

$\alpha = \frac{\kappa}{\rho c_p}$, thermal diffusivity

κ = thermal conductivity and

ρ = density, If fluid is more viscous or stickier, then P_r is greater and the heat transfer will be less convective.

1.5.5 Porous Medium

Porous media defined as solid bodies that contain pores. *Pores* are void or the empty spaces which must be distributed more or less frequently through the *porous material*. Extremely small voids in a solid are called *molecular interstices* and very large voids are called *caverns*. Pores are the void spaces intermediate in size between caverns and molecular interstices. The pores in a porous system may be interconnected or non interconnected. The interconnected part of the pore system is called the effective pore space of the *porous medium*. Pore spaces can be ordered or disordered.

Porosity

A porous media can be characterized by a variety of geometrical properties. The ratio of void to the total volume is called *porosity* and denoted by ϵ, where $\epsilon = \frac{Ratio\, of\, Void}{Total\, Volume}$.

If the calculation of porosity is based upon the interconnected pore space interval of the total pore space, the resulting quantity is termed as effective porosity. Porosity can be measured by a variety of methods:

- **Direct Method:** Porosity is determined by measuring bulk volume of a piece of porous material and then compact the body so as to destroy all the voids, and to measure the difference between the volumes. This method is applicable only if material is very soft like bread.
- **Optical Method:** In this method porosity is determined by looking at a section of the porous medium under microscope. Numerical value of porosity obtained in this manner of the random section must be the same as that of the porous material.

- **Density Method:** If the density ρ_G of the material making up the porous medium is known, then the bulk density ρ_B of the medium, which can be calculated, is related to the fractional porosity ϵ, where $\epsilon = 1 - \frac{\rho_G}{\rho_B}$.
- **Gas Expansion Method:** The basic principle of this method is the direct measurement of the volume of air or gas contained in the pore space. This can be achieved either by continuously evacuating the air out of the specimen.

Permeability

It is the measure of ease with which a fluid can move through a porous rock.

1.6 Fundamental Assumptions

We now discuss two fundamental assumptions.

- Continuum Hypothesis.
- Newtonian Mechanics.

Continuum Hypothesis

Fluid is appeared to smooth and continuous but in reality it has *discrete structure* of molecules and atoms. A detailed molecular approach for understanding fluid flow is very difficult. Concept of property at a point has no meaning if a point is located in the void between the atoms or at the center of an atom. Let density ρ at a point P is defined

as $\quad \rho_P = \lim_{\nabla V \to 0} \frac{\Delta m}{\Delta V}$

where $\nabla V \to$ volume element surrounding a point P in a fluid containing total mass Δm. If P lies at the center of the atom, then $\Delta V = 0$ and $\rho_P \to \infty$. If P lies between in the void between two atoms then Δm = 0 and $\rho_P \to 0$.

Thus at some points the density is infinite and at some points the density is zero. In order to overcome these inconsistencies we shall assume that masses are uniformly distributed over the whole volume and consider matter as continuous. By assuming continuum hypothesis, we can give meaning to pressure, momentum, density at a point and treat them as a continuous function of space and time variables.

Newtonian Mechanics

Newtonian mechanics is one which follows the three law of motion of Newton. Thus it is assumed that fluid velocity is very small as compared to the speed of light otherwise the theory of relativity has to be considered.

1.7 Basic Hydrodynamical Equations

Fundamental equations governing fluid flow are :

- Mass Conservation (Continuity) equation.
- Momentum Conservation (Fluid Motion) equations.
- Energy equation.
- Equation of state.

1.7.1 Equation of Continuity - Conservation of Mass

Mass conservation on the fluid in the control volume states that

Rate of accumulation of mass in the control volume

$$
\begin{aligned}
&= \text{Rate of inflow of mass in control volume}\\
&\quad - \text{Rate of outflow of mass from control volume}\\
&\qquad + \text{Any source.}
\end{aligned}
$$

Let u, v, w be the components of the velocity $\vec{v}$, ρ be the density at the point (x, y, z) in a fluid domain, the mathematical equivalence of the verbal statement of conservation of mass for every point in the fluid domain is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\Rightarrow \frac{D\rho}{Dt} + \rho(\nabla.\vec{v}) = 0. \tag{1.4}$$

Case 1 : If ρ is homogeneous and incompressible i.e. ρ is same at all the points and constant in the fluid domain i.e density of an element does not alter as that element moves about, then equation of continuity becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{1.5}$$

Case 2 : If ρ is heterogeneous and incompressible i.e. ρ is different at different points and constant in the fluid domain then equation of continuity becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{1.6}$$

Case 3 : For compressible steady fluid $\left(\frac{\partial}{\partial t} = 0\right)$, equation of continuity becomes

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0. \tag{1.7}$$

1.7.2 Equations of Motion - Conservation of Momentum

Momentum conservation on the fluid in the control volume states that

Rate of accumulation of momentum

$=$ Rate of inflow of momentum

$-$ Rate of outflow of momentum

$+$ Net external forces acting on the control volume.

Since momentum is a vector quantity, so there are 3 equations of momentum as:

Rate of accumulation of momentum in x-direction

$=$ Rate of inflow of momentum in x-direction

$-$ Rate of outflow of momentum in x-direction

$+$ External forces acting on control volume in x-direction.

Rate of accumulation of momentum in y-direction

$=$ Rate of inflow of momentum in y-direction

$-$ Rate of outflow of momentum in y-direction

$+$ External forces acting on control volume in y-direction.

Rate of accumulation of momentum in z-direction

$$\begin{aligned}&= \text{Rate of inflow of momentum in z-direction}\\ &\quad - \text{Rate of outflow of momentum in z-direction}\\ &\quad\quad + \text{External forces acting on the control volume in z-direction.}\end{aligned}$$

Let u,v,w be the components of the velocity $\vec{v}$, ρ be the density and p be the pressure at the point (x, y, z) in a fluid domain or mass of fluid, and let X,Y,Z be the components of external force $\vec{F}$ per unit mass at the same point. Mathematical equivalence of the above verbal statement for every point in the fluid domain in x-direction is :

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \frac{\partial}{\partial z}(\rho uw) = \rho X.$$

Forces present everywhere in the fluid domain like pressure force, gravitational force, viscous force, magnetic force and electric force etc. Mainly two types of forces considered are body force (gravitational force) and stress (normal and shear stress). Therefore $X = \rho g_x + \Sigma$ (stress component in x-direction x area of the surface perpendicular to stress component). There will be six such *stress components* if we consider cuboid as the control volume, three in positive direction and three in negative direction. So momentum balance equation at a point (x, y, z) in x-direction is

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \frac{\partial}{\partial z}(\rho uw) = \rho g_x + \frac{\partial}{\partial x}T_{xx} + \frac{\partial}{\partial y}T_{yx} + \frac{\partial}{\partial z}T_{zx}$$

where $T_{ij} = -p\delta_{ij} + \tau_{ij}$, δ_{ij} is the kronecker delta, and τ_{ij} is shear stress component in j direction and i is the axis to which the plane face is perpendicular and above equation reduces to

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \frac{\partial}{\partial z}(\rho uw) = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}. \tag{1.8}$$

Similarly other two equations in y-direction and z-direction are

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(\rho v^2) + \frac{\partial}{\partial z}(\rho vw) = \rho g_y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}. \tag{1.9}$$

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho u w) + \frac{\partial}{\partial y}(\rho v w) + \frac{\partial}{\partial z}(\rho w^2) = \rho g_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}. \tag{1.10}$$

There are four equations (three momentum balance equations and one continuity equation) in 12 variables $(u, v, w, \tau_{xx}, \tau_{yx}, \tau_{zx}, \tau_{xy}, \tau_{yy}, \tau_{zy}, \tau_{xz}, \tau_{yz}, \tau_{zz})$. So more equations are required. Energy equation will generate the new variable, it will not resolve the problem. More information is required to resolve the situation in the formulation of model. Constitutive model/expression is required,one of such model is $\tau = \mu\frac{du}{dy}$, where μ is coefficient of viscosity that can be measured, τ is the tangential/shearing stress and velocity gradient $\frac{du}{dy}$ is not a new variable. It means stress τ can be expressed in the known variables, only need to identify the τ with one of the nine shear stress components. It is specific for one-dimensional flow and further need of constitutive law i.e the relation between stress and rate of strain which describes the stresses within fluid.

As the fluid will continue to deform when stress is applied and does not take original shape as the stress removed but deformation stops. So rate of strain is considered not strain (considered in solid mechanics). Various combinations of velocity gradients describe the strain rates as :

Rotational strain rate $= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

Shear strain rate $= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

Extensional strain rate in x-direction $= \frac{\partial u}{\partial x}$

General isotropic (invariant to the orientation of co-ordinate axes) and linear relation between stress and strain rate is

$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\left(\frac{\partial u_k}{\partial x_k}\right)\delta_{ij}$

where μ is kinematic viscosity and λ is second coefficient of viscosity which is important only in case of compressible fluids and disappears for incompressible fluid.

Thus, for compressible fluids

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \delta_{ij}$$

and for incompressible fluids

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right).$$

Similarly other stress components can be expressed in terms of velocity gradients. Substitute the values of all stress components in the equations (1.8) - (1.10) and we get all the equations in the variables u, v, w, p and material property constants ρ, μ, λ.

Momentum balance equations (1.8)-(1.10) are called Navier-Stokes' equations for motion of viscous compressible fluid. These equations are valid only for the Newtonian fluid which obeys isotropic condition and linear relation between stress and stress rate. For non-Newtonian fluids different *constitutive relations* between stress (arises out of fluid motion) and strain rate are required.

Navier-Stokes' equation for viscous incompressible fluid are, in x-direction is

$$\rho \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right). \tag{1.11}$$

Similarly other two equations in y-direction and z-direction are

$$\rho \left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right) = \rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right). \tag{1.12}$$

$$\rho \left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right). \tag{1.13}$$

Now we have closed system of equations i.e four equations in four variables u, v, w, p.

1.7.3 Equation of Energy - Conservation of Energy

It is required in case of heat transfer. Principle of the *conservation of energy* in the control volume states that

Rate of change of energy in the control volume

= Rate of inflow of energy − Rate of outflow of energy

+ Rate of heat addition to the fluid contained in control volume

+ Rate of work done by the forces acting on control volume

+ Generation of energy from sources within control volume.

The Mathematical equivalence of the above verbal statement for viscous compressible fluids is

$$\frac{\partial}{\partial t}\left(\rho C_v T\right) + \frac{\partial}{\partial x_j}\left(\rho C_v T u_j\right) = \frac{\partial}{\partial x_i}\left(q\frac{\partial T}{\partial x_j}\right) - p\frac{\partial u_j}{\partial x_j} + \Phi \tag{1.14}$$

where $\quad \Phi = 2\mu e_{ij}^2 - \frac{2}{3}\mu\left(e_{ij}\right)^2$

is the 'rate of *viscous dissipation*' and gives the heat generated because of frictional forces and

$$e_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$$

is the 'rate-of-strain tensor', C_v is the *specific heat* when volume is constant and q is the coefficient of heat conduction. For an incompressible fluid, $e_{ij} = 0$ and the corresponding expression for Φ is given by $\Phi = 2\mu e_{ij}^2$. Thus, for an incompressible fluid, the equation of energy (1.14) takes the form

$$\rho\frac{\partial}{\partial t}\left(C_v T\right) + \rho u_j\frac{\partial}{\partial x_j}\left(C_v T\right) = \frac{\partial}{\partial x_j}\left(q\frac{\partial T}{\partial x_j}\right) + 2\mu e_{ij}^2. \tag{1.15}$$

1.7.4 Equation of State

When the motion of compressible fluid is considered, a relation is required between the *state variable*, pressure $\rightarrow p$, density $\rightarrow \rho$ or volume $\rightarrow V$ and temperature $\rightarrow T$, in order to obtain sufficient number of equations to determine the physical and dynamical variables involved in the problem. Such a relation is called the equation of state and has the general form $g(p, V, T) = 0 \quad$ or $\quad g(p, \rho, T) = 0$.

If we neglect the compressibility of a fluid, its density remains constant. Thus for an incompressible fluid the equation of state is $\rho =$ Constant.

In viscous compressible flow: Equations of energy and motions are coupled. Energy equation involves viscous dissipation function and temperature which are functions of velocity. Thus temperature and velocity are coupled. Similarly equations of motion involves velocity components u,v,w and pressure p which are function of temperature. So velocity and temperature are coupled.

In viscous incompressible flow: Density, thermal conductivity and coefficient of viscosity are fluid properties constants and equations of energy and motions are uncoupled. Therefore four equations of continuity and motion can be solved for four variables u, v, w and p. Using the values of velocity components we can solve the energy equation and find temperature. When initial and boundary conditions are specified, we can find the solution of above equations physically.

1.8 Initial and Boundary Conditions

Equations describing the motion are partial differential (not algebraic) equations which are valid at all the points in *flow domain* so it is necessary to have boundary and initial conditions in order to have a solution. *Initial conditions* may be of the form

$u(x, y, z, t_0) = f_1\,(x, y, z)$,

$v(x, y, z, t_0) = f_2\,(x, y, z)$,

$w(x, y, z, t_0) = f_3\,(x, y, z)$

and $\quad p(x, y, z, t_0) = f_4\,(x, y, z)$.

There are three types of *boundary conditions* :

- Dirichlet's Boundary Conditions of the type $u =$ Constant.
- Neumann's Boundary conditions of the type $\frac{\partial u}{\partial x} =$ Constant.
- Robin's Boundary Condition of the mixed type $au + \frac{\partial u}{\partial x} =$ Constant.

For fluid flow situation we need more realistic or physical boundary conditions. At inlet, flow entering the boundary so apply dirichlet's boundary conditions. At outlet, flow is leaving the boundary , so apply the Robin's boundary conditions and for free shear boundary apply Neumann's boundary conditions. For fully *developed flow* apply same boundary conditions as at the outlet.

1.8.1 Implication of Boundary conditions:

Any kind of boundary condition for any problem is not justified. For unique solution, solution continuously depend on initial and boundary conditions, with the change of these conditions solution will change. This type of sensitivity is exhibited by the boundary conditions. Type of conditions depend upon the physics of the problem. So the mathematical problem must be well posed for the solution.

1.9 Hydrodynamic Stability - Basic Concepts

Let the system be defined by parameters as
$Y_1 \to$ dimensions of the system ,
$Y_2 \to$ velocity field,
$Y_3 \to$ temperature gradients ,
$Y_4 \to$ pressure gradients ,
$Y_5 \to$ magnetic fields ,
$Y_6 \to$ magnitude of forces ,
$Y_7 \to$ density ,
and $Y_8 ... Y_n \to$ denotes other parameters.

The above system is stable with respect to any *disturbance*, if the initial state of *parameter* is disturbed/perturbed and disturbance gradually *decay* in amplitude. Thus system considered as stable, If it is stable with respect to all disturbances in all the parameters. Otherwise the system is unstable. In other words, *stability* means there exist no disturbance by which system is unstable and no disturbance *grow* in *amplitude*.

1.10 Flow Instabilities

Flow instability occur everywhere and effect every fluid phenomenon, there are several examples of fluid instability like smoke rises because it is lighter than surrounding air. Instability is the first step in events which generate turbulence. Some flow instabilities are:

- KH (Kelvin- Helmholtz) Instability/Double-Diffusive Convection.
- RT (Rayleigh-Taylor) Instability.
- Thermal (Bénard) Instability.
- Shock Wave Instability.

1.10.1 Thermal Instability - Bénard Convection

A layer of fluid heated from underside or below may becomes unstable because of heavier fluid at the top and lighter one at the bottom. The heating element is at the bottom. As heat is turn on, fluid become *unstable* and hot buoyant fluid get away before it loses heat and *buoyancy* to its surroundings.

The critical parameter is the *Rayleigh number* which involves gravity(g), thermal expansion coefficient(α), the vertical *temperature gradient*$\left(\frac{dT}{dz}\right)$, the effects which tends to prevent instability i.e. kinematic viscosity(ν) and thermometric conductivity(κ) and finally a length parameter or thickness of fluid layer (h) and it is given by $R_a = \frac{g\alpha h^4}{\kappa\nu}|\frac{dT}{dz}|$.

Rayleigh number can be increased by heating the bottom, As this dimensionless number goes beyond *critical value*, *instability* sets in the form of *Bénard cells*. Below the critical value the flow is stable. The earliest experiments to demonstrate the onset of *thermal instability* are those of Bénard in 1900, though the phenomenon of thermal convection itself had been recognized earlier by James Thomson(1882) and Count Rumfort (1797).

Bénard's Experiment

He carried out experiments on a very thin layers of fluid, about one mm in depth, or less, standing on a leveled *metallic plate* maintained at constant temperature. He did experiments on many liquids with different physical constants. He was particularly interested in the role of viscosity. He observed when the temperature of the lower surface was gradually increased, at a certain instant, the layer became *reticulated* and revealed its dissection into cells. There were motions inside the cells and two phases in the succeeding development of the cellular pattern in which the cells are hexagonal, equal and properly aligned. R.K.Zeytounian [21] had used the results of this experiment during his research on convection in fluids.

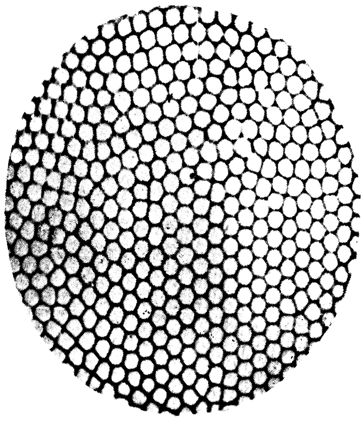

Figure 1.1: Bénard cells

Schmidt-Milverton Principle for detecting the onset of thermal instability

Schmidt and Milverton incorporated a principle for the detection of the onset of thermal instability which is so direct and simple that it served as the basis for all later experiments in this area. They applied their principle to determine the critical Rayleigh number for the onset of thermal instability in horizontal layers of water confined between two rigid planes. The critical value $R_C = 1770 \pm 140$, they derived from their experimental results and is satisfactory agreement with the theoretical value 1708.

The Precision experiments of Silveston

The experiments of Schmidt and Milverton have been repeated by Saunders, Malku, Silveston and others to achieve greater range and precision. Siveston used water, Heptane, Silicon AK-3, ethylene glycol and silicon oil AK-350 in his experiments. From an experimentation of results obtained for the Rayleigh numbers in the range 1000-10,000. Silveston derives for the critical Rayleigh number for the onset of instability the value $R_C = 1700 \pm 51$. It is very good accord with the theoretical value 1708.

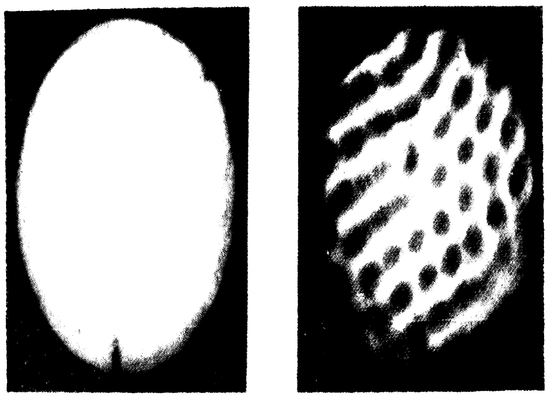

Figure 1.2: Visualization of onset of thermal convection by Silveston. The photograph on the left was obtained for the Rayleigh number 1,500 while the photograph on the right was obtained for a Rayleigh number 1,800. The depth of the layer in these experiments was 7 mm.

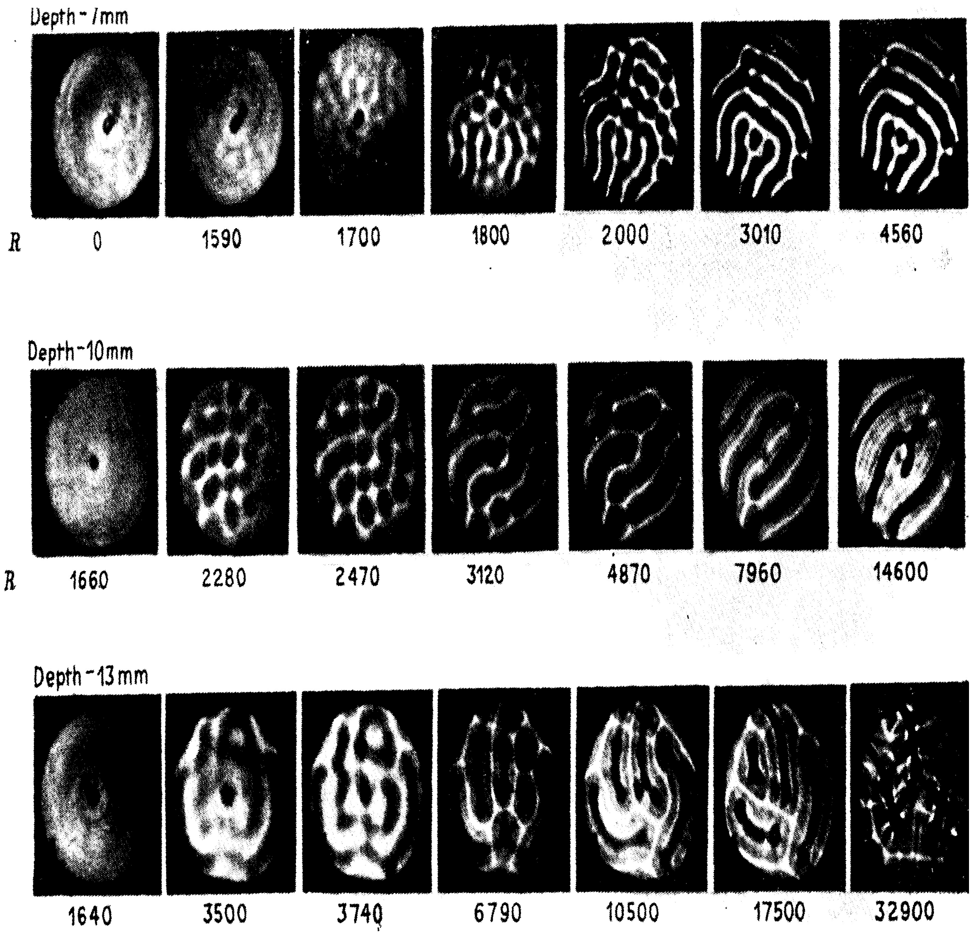

Figure 1.3: Visualization of onset of thermal convection by Silveston. Photographs for different depths and increasing Rayleigh numbers.

1.10.2 Double Diffusive Convection(or Thermosolutal Instability)

In the standard Bénard problem, the instability is driven by the density difference which is caused by a temperature difference between the upper and the lower planes bounding the fluid. If the fluid layer additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference i.e. the temperature field and the salt field. When the simultaneous presence of two or more components with different diffusivities is considered, the phenomenon of convection which arises is called thermosolutal or double diffusive convection. For the specific case involving a temperature field and sodium chloride it is frequently referred as thermohaline convection. Double-diffusive convection has been proved, when we think about ocean where both heat and salt (or some dissolved substances) are important. In thermosolutal convection, when the thermal and solutal effect are aiding each other, the convective flow behaviour remains qualitatively similar to that of pure thermal convection. In these problems, the solute is commonly, but not necessarily, as salt. Related effects have now been observed in other contexts and the name double-diffusive convection has been used to cover this wide range of phenomena.

1.11 Suspended Particles

The effect of suspended particles on the stability of superposed fluids might be of industrial and chemical engineering importance. Further motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. Also we are motivated to the study because of the decades old contradiction between the theory for onset of convection and experiement. A contradiction between the theory and his experiments for the onset of convection in fluids heated from below was observed by Chandra [22]. He performed the experiments in an air layer and found that the instability depended on the depth of the layer. A Bénard-type cellular convection with fluid descending at the cell centre was observed when the predicted gradients were imposed, for layers deeper than 10 mm. However, if the layer depth was less than 7 mm, convection, which was different in character from that in deeper layers, occurred at much lower gradients than predicted.

Chandra called this motion “Columnar instability.” A complete survey of subsequent experimental studies, which confirm Chandra’s result, can be found in report by Jones (1962) on the effect of different aerosols on stability. According to him the effects, which may be important, are thermal forces, electrostatic charges, evaporation, condensation and buoyancy forces. Jones concluded that columnar instability is not an example of single-phase natural convection and that it is most likely due to the unique properties of aerosol suspensions. Theoretical discussions of columnar instability, have been given by Sutton (1950) and Segel and Stuart (1962).

Motivated by interest in fluid-particle mixtures generally and columnar instability in particular, Scanlon and Segel [23] investigated the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles.

The effect of suspended particles was found to destabilize the layer i.e. to lower the critical temperature gradient. Sharma, Prakash and Dube (1976) have studied the effect of suspended particles on the onset of Bénard convection in the presence of magnetic field and rotation separately. They have found that the magnetic field and rotation have stabilizing effects whereas the effect of suspended particles is to destabilize the layer.

1.12 Effect of Magnetic Field

Consider a fluid to be electrically conducting and be under the influence of a magnetic field. The electrical conductivity of the fluid and the prevalence of magnetic field contribute to effects of two kinds. First, by the motion of the electrically conducting fluid across the magnetic lines of force, electric currents are generated and the associated magnetic fields contribute to changes in the existing fields, and second, the fact that the fluid elements carrying currents transverse magnetic lines of forces contributes to additional forces acting on the fluid elements. It is this two-fold interaction between the motions and the fields that is responsible for patterns of behaviour which are often striking and unexpected. The interaction between the fluid motions and magnetic fields are contained in Maxwell’s equations. As a consequence of Maxwell’s equations, equations of hdyrodynamics are modified suitably.

In the outer layers of stars like the Sun, thermal convection is affected by the presence of magnetic fields. In stellar interiors and atmospheres, the magnetic field may be variable and may altogether alter the nature of the instability. For example, Kent (1966) studied the effect of a horizontal magnetic field, which varies in the vertical direction, on the stability of parallel flows and showed that the system is unstable under certain conditions, while in the absence of magnetic field, the system is known to be stable.

1.13 Perturbation Method

Most of the physical problems facing engineers,applied mathematicians and physicists today exhibit certain essential features which preclude exact *analytical* solutions. Some of these features are *nonlinearities*, variable coefficients, complex boundary shapes, and nonlinear boundary conditions at known or unknown boundaries. Thus in order to obtain information about solutions of equations, we forced to resort to approximations, numerical solutions or both. *Perturbation method* is one of those approximation techniques. According to this technique some parameters of the initial state of the system are perturbed, and by substituting all these *perturbed variables* in the flow governing equations to obtain the perturbed or *linearized equations*. Perturbation methods are also used in the study of dynamic stability of aircraft.

Let the equilibrium conditions of flight are given by $u = u_0, v = v_0$ and $w = w_0$, non-zero velocity in forward direction only and other conditions are $p_0, q_0, r_0 = 0$, $\phi_0 = 0$ and $\theta = \theta_0$. Non linear equation of motion of aircraft is

$$m\left(\frac{\partial u}{\partial t} + qw - rv\right) = X - mg\sin\theta.$$

Now, perturb the equilibrium condition as

$u = u_0 + \delta u,$

$v = v_0 + \delta v,$

$w = w_0 + \delta w,$

$p = p_0 + \delta p,$

$q = q_0 + \delta q,$

$\phi = \phi_0 + \delta\phi$

and $\quad \theta = \theta_0 + \delta\theta$

where $\delta u, \delta v, \delta w, \delta p, \delta q, \delta\phi, \delta\theta$ are all perturbation in variables.

The above equations of motion are true for both the states, equilibrium and perturbed state. Thus linearize the equations of motion of aircraft and then study the dynamics of perturbed variables if all perturbed variables decaying in time then aircrafts stable in that particular equilibrium condition.

Chapter 2

Thermal Instability of Rivlin-Ericksen Elastico-Viscous Fluid Permeated with Suspended Particles in Hydrodynamics in a Porous Medium

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2.1 Introduction

The formulation and derivation of the basic equations of a layer of a *fluid* heated from below in a *porous medium*, using the Boussinesq approximation, has been given in treatise by Joseph [24]. When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by *Darcy's law*. The study of a layer of a fluid heated from below in a porous medium is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous media, Singh and Gupta [25].

A comprehensive account of the effect of a uniform *magnetic field* on the layer of a Newtonian fluid heated from below was given by Chandrasekhar [26]. The effect of a magnetic field on the stability of the fluid flow is of interest in geophysics, particularly in the study of earth core where the earth's mantle, which consists of a conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential *diffusion*. The results of flow through a porous medium in the presence of a magnetic field are applied in the study of the *stability* of a *convective flow* in the *geothermal* region. Lapwood [27] studied the stability of a convective flow in hydrodynamics using Rayleigh's procedure. Wooding [28] considered the Rayleigh instability of a thermal boundary layer in the flow through a porous medium.

The fluid may not be absolutely *pure* but may, instead, be *permeated* with suspended (or dust) particles. The effect of particle mass and *heat capacity* on the onset of Bénard convection was considered by Scanlon and Segel [23]. The effect of suspended particles was found to *destabilize* the layer. In another context, Palaniswamy and Purushotham [29] studied the *stability* of a *shear flow* of stratified fluids with *fine dust* and found the effect of fine dust to increase the region of instability. The *thermal instability* of fluids in a porous medium in the presence of suspended particles was studied by Sharma and Sharma [30]. The suspended particles and the permeability of the medium were found to destabilize the layer. Sharma and Kumar [31] studied the *Rayleigh-Taylor instability* of fluids in porous media in the presence of suspended particles and *variable magnetic field*. In all the above studies, the fluid has been considered to be *Newtonian*. One such class of elastico- *viscous* fluids is the *Rivlin-Ericksen* fluid [32]. Srivastava and Singh [33] studied the unsteady flow of the dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross sections in the presence of a time-dependent pressure gradient. In other study, Garg et al. [34] studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivilin-Ericksen fluid in the presence of a uniform magnetic field. Sharma and Kumar [35] studied the thermal instability of a layer of a Rivlin-Ericksen elastico-viscous fluid in the presence of suspended particles. In another study, Kumar [36] considered the stability of suspended Rivlin-Ericksen elastico-viscous fluids permeated with suspended particles in a porous medium. It is this class of elastico-viscous fluids we are particularly interested in studying the effect of suspended or dust particles on the Rivlin-Ericksen elastico-viscous fluid heated from below in a porous medium in the presence of a uniform horizontal *magnetic field*.

2.2 Formulation of the Problem

Let us consider the following *physical quantities* for the formulation of the problem. Tensor quantities like stress, rate of strain, shear stress , Kronecker delta be represented by $T_{kl}, e_{kl}, \tau_{kl}$ and δ_{kl} respectively. Vector quantities like velocity and position vector be represented by $\vec{v}$ and $\vec{x}$ respectively. p denotes the isotropic pressure and material properties viscosity and viscoelasticity be denoted by μ and μ'. Constitutive relations between the stress and rate of strain for the Rivlin-Ericksen fluid are

$$T_{kl} = -p\delta_{kl} + \tau_{kl},$$

$$\tau_{kl} = \rho\left(\nu + \nu'\frac{\partial}{\partial t}\right)e_{kl},$$

$$e_{kl} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k}\right). \tag{2.1}$$

In porous medium, an infinite *horizontal layer* of depth d of an electrically conducting viscoelastic Rivlin-Ericksen fluid which is acted on by gravity force

$\vec{g}(0,0,-g)$ and a uniform horizontal magnetic field $\vec{H}(0,0,H)$ is considered. For the study of thermal instability, layer is heated from underside and steady adverse temperature gradient β is maintained , where $\beta = |\frac{dT}{dz}|$.

Let the fluid properties like pressure, temperature, density , velocity of pure fluid, kinematic viscosity and kinematic viscoelasticity be denoted by $p, T, \rho, \vec{v}(u, v, w), \nu$ and ν' respectively. Properties of suspended particle like velocity and number density be represented by $u(\overline{x}, t)$ and $N(\overline{x}, t)$. $\vec{g}$ is the gravitational acceleration, $epsilon$ represents the medium porosity and k_1 represents the medium permeability. $K = 6\pi\mu\eta'$ is the Stokes' drag coefficient for the particle having the radius η'.

Then the *flow governing equations* of conservation of mass and momentum in a porous medium for the considered fluid in the presence of magnetic field and suspended particles are

$$\frac{1}{\epsilon}\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\epsilon}(\vec{v}.\nabla)\vec{v}\right] = -\frac{1}{\rho_0}\nabla p - g\left(1 + \frac{\delta\rho}{\rho_0}\right)\vec{\lambda}$$
$$- \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN}{\rho_0\epsilon}(\vec{u} - \vec{v}) + \frac{\mu_e}{4\pi\rho_0}\left[\left(\nabla \times \vec{H}\right) \times \vec{H}\right] \quad (2.2)$$

and $\quad \nabla.\vec{v} = 0. \quad (2.3)$

In the above equations of conservation of momentum (2.2), some assumptions regarding the shape and velocity of the suspended particles are taken as

- Shape of the *suspended particles* in the fluid is uniform spherical.
- Relative velocities between the fluid and particles is small.
- Large distance between the particles as compare to their diameter. So Interparticle reactions are ignored.
- Gravity, pressure, Darcian force and magnetic field effect on the suspended particles are negligibly small, so ignored.
- Extra force due to the presence of particles is proportional to *velocity difference* between the particles and the fluid.
- Force exserted by fluid on particles and force exerted by particles on fluid balance each other.

So there must be an extra force equal in magnitude but opposite in sign in the equations of conservation of momentum or motion for the particles. If mN is the mass of particles per unit volume, then under the above assumptions, equations of conservation of momentum and mass for the particles are

$$mN\left[\frac{\partial \vec{u}}{\partial t}+\frac{1}{\epsilon}(\vec{u}.\nabla)\vec{u}\right]=KN(\vec{v}-\vec{u}) \quad \text{and} \tag{2.4}$$

$$\epsilon\frac{\partial N}{\partial t}+\nabla.(N\vec{u})=0. \tag{2.5}$$

Let at *constant volume*, C_v is the *heat capacity* of the fluid, C_{pt} denote the heat capacity of the particles, T is the temperature and q is effective thermal conductivity of the pure fluid. If the fluid and the particles are in thermal equilibrium, then equation of heat conduction is

$$\left[\rho_0 C_v\epsilon+\rho_s C_s(1-\epsilon)\right]\frac{\partial T}{\partial t}+\rho_0 C_v(\vec{v}.\nabla)T+mNC_{pt}\left(\epsilon\frac{\partial}{\partial t}+\vec{u}.\nabla\right)T=q\nabla^2 T \tag{2.6}$$

where ρ_s is the density and C_s is the heat capacity of the solid matrix. Maxwell's equations yield:

$$\epsilon\frac{\partial \vec{H}}{\partial t}=(\vec{H}.\nabla)\vec{v}+\epsilon\eta\nabla^2\vec{H} \quad \text{and} \tag{2.7}$$

$$\nabla.\vec{H}=0 \tag{2.8}$$

where $\eta \to$ The electrical resistivity. Equation of state for fluid is

$$\rho=\rho_0\left(1-\alpha\delta T\right)=\rho_0\left[1-\alpha(T-T_0)\right] \tag{2.9}$$

where $\alpha \to$ co-efficient of thermal expansion, $\rho_0 \to$ density of the fluid at the *bottom surface* $z=0$ and $T_0 \to$ at temperature of the fluid at $z=0$. Initially the system is taken as quiescent layer (no settling) with a uniform particle distribution N_0. Initial values of the variables are

$\vec{u}=(0,0,0), \quad \vec{v}=(0,0,0), \quad N_0=Constant, \quad T=-\beta z.$

which is an exact solution to the governing equations.

2.2.1 Perturbation of Equations

Let δp denote the *perturbation* in pressure p, $\delta\rho$ denote the perturbation in density ρ, θ denote the perturbation in temperature T, $\vec{v}(u,v,w)$ denote the perturbation in fluid velocity (zero initially), $\vec{u}(l,r,s)$ denote the perturbation in particle velocity (zero initially), N denote perturbations in suspended particles number density N_0 and $\vec{h}(h_x,h_y,h_z)$ denote perturbations in magnetic field $\vec{H}(0,0,H)$. Since density is depends upon the temperature, so perturbation in temperature will bring change is density defined by the relation $\delta\rho = -\alpha\rho_0\theta$.

Governing equations of flow hold true for both the initial and perturbed state. Therefore, linearized perturbed equations of the problem are

$$\frac{1}{\epsilon}\frac{\partial\vec{v}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - g\alpha\theta\lambda - \frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN}{\rho_0\epsilon}(\vec{u}-\vec{v}) + \frac{\mu_e}{4\pi\rho_0}\left[\left(\nabla\times\vec{h}\right)\times\vec{H}\right], \tag{2.10}$$

$$\nabla.\vec{v} = 0, \tag{2.11}$$

$$mN_0\frac{\partial\vec{u}}{\partial t} = KN_0(\vec{v}-\vec{u}), \tag{2.12}$$

$$(E+h\epsilon)\frac{\partial\theta}{\partial t} = \beta(w+hs) + \kappa\nabla^2\theta, \tag{2.13}$$

$$\epsilon\frac{\partial\vec{h}}{\partial t} = (\vec{H}.\nabla)\vec{v} + \epsilon\eta\nabla^2\vec{h} \tag{2.14}$$

$$\text{and}\quad \nabla.\vec{h} = 0 \tag{2.15}$$

where $\quad E = \epsilon + (1-\epsilon)\frac{\rho_s C_s}{\rho_0 C_v},\quad h = \frac{mN_0C_{pt}}{\rho_0 C_v}\quad \textit{and}\quad \kappa = \frac{q}{\rho_0 C_v}.$

Eliminating u in equation (2.10) by using equation (2.12), write the resulting equation in scalar components eliminate $u, v, \delta p, h_x, h_y$ between them, with the help of equations (2.11) and (2.15), we obtain

$$n'\nabla^2 w + \frac{\epsilon}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 w - \epsilon g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right) - \frac{\mu_e H}{4\pi\rho_0}\frac{\partial}{\partial x}\nabla^2 h_z = 0, \tag{2.16}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[(E+h\epsilon)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta=\beta\left[\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)+h\right]w \quad \text{and} \tag{2.17}$$

$$\epsilon\left[\frac{\partial}{\partial t}-\eta\nabla^2\right]h_z=H\frac{\partial w}{\partial x} \tag{2.18}$$

$$\text{where} \quad n'=\frac{\partial}{\partial t}\left[1+\frac{\frac{mN_0K}{\rho_0}}{m\frac{\partial}{\partial t}+K}\right]$$

2.3 Dispersion relation

Perturbed quantities are assumed to be of the following form for the analysis of disturbances into normal modes

$$[w,\theta,h_z]=[W(z),\Theta(z),X(z)]\,exp\,(ik_xx+ik_yy+nt) \tag{2.19}$$

where k_x is the wave number along x-direction, and k_y is wave number along y-direction. $k=\sqrt{k_x^2+k_y^2}$ = resultant wave number and n = growth rate = complex constant in general. Using expression (2.19), equations (2.16)-(2.18) in a non dimensional form become

$$\left[\frac{\sigma'}{\epsilon}+\frac{1}{p_1}(1+F\sigma)\right]\left(D^2-a^2\right)W+\frac{g\alpha d^2a^2\Theta}{\nu}-\frac{ik_x\mu_eHd^2}{4\pi\rho_0\nu}\left(D^2-a^2\right)X=0, \tag{2.20}$$

$$\left[\frac{\tau\nu\sigma}{d^2}+1\right]\left[\left(D^2-a^2\right)-(E+h\epsilon)p_3\sigma\right]\Theta=-\frac{\beta d^2}{\kappa}\left[H'+\frac{\tau\nu\sigma}{d^2}\right]W \tag{2.21}$$

$$\text{and} \quad \left[\left(D^2-a^2\right)-p_2\sigma\right]\chi=-\frac{ik_xHd^2}{\epsilon\eta}W \tag{2.22}$$

where the co-ordinates x,y,z have expressed in the new unit of length d, time t in the new unit of length $\frac{d^2}{\kappa}$ and put $a=kd,\sigma=\frac{nd^2}{\nu},p_3=\frac{\nu}{\kappa}\rightarrow$ Prandtl number, $p_2=\frac{\nu}{\eta}\rightarrow$ magnetic Prandtl number, $p_1=\frac{k_1}{d^2}\rightarrow$ dimensionless medium permeability, $F=\frac{\nu}{d^2}\rightarrow$ dimensionless kinematic viscoelasticity, $\sigma'=\frac{n'd^2}{\nu},H'=h+1,\tau=\frac{m\kappa}{Kd^2}$ and $D=\frac{d}{dz}$.

By eliminating X and Θ between equations (2.20)-(2.22), we obtain

$$\left[1+\frac{\tau\nu\sigma}{d^2}\right]\left[\left(D^2-a^2\right)-(E+h\epsilon)p_3\sigma\right]\left[\left\{\frac{\sigma'}{\epsilon}+\frac{1}{p_1}(1+F\sigma)\right\}\left[\left(D^2-a^2\right)-p_2\sigma\right]\right.$$
$$\left.-\frac{k_x^2Q}{\epsilon}\right]\left(D^2-a^2\right)W=Ra^2\left[H'+\frac{\tau\nu\sigma}{d^2}\right]\left[\left(D^2-a^2\right)-p_2\sigma\right]W \quad (2.23)$$

where $R=\frac{g\alpha\beta d^4}{\nu\kappa}$ = Rayleigh number and $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ = Chandrasekhar number.

The boundary conditions, suitable for the problem, are Chandrasekhar [26]. For the solution to the problem, free boundaries are considered which is little artificial in nature. Also Temperatures at the boundaries are kept fixed and the medium adjoining the fluid is perfectly conducting.

$$W=0, \quad D^2W=0, \quad \Theta=0, \quad X=0 \quad at \quad z=0 \quad and \quad z=1. \quad (2.24)$$

Obviously that the even order derivatives of W vanish on the boundaries and hence the proper solution of W characterizing the lowest mode is

$$W=W_0 sin(\pi z) \quad (2.25)$$

where $W_0=Constant$. By putting the solution (2.25) in equation (2.23), the dispersion relation can be written as

$$R_1=\frac{\begin{array}{c}(1+x)\left[(1+x)+(E+h\epsilon)i\sigma_1p_1\right]\left(1+\frac{i\nu\tau\pi^2\sigma_1}{d^2}\right)*\\ \left[\left\{\frac{i\sigma_1'}{\epsilon}+\frac{1}{P}\left(1+i\pi^2F\sigma_1\right)\right\}\left\{(1+x)+i\sigma_1p_2\right\}+\frac{Q_1xcos^2\theta}{\epsilon}\right]\end{array}}{x\left[H'+\frac{i\nu\pi^2\tau\sigma_1}{d^2}\right]\left\{(1+x)+i\sigma_1p_2\right\}} \quad (2.26)$$

where $\quad x=\frac{a^2}{\pi^2}, \quad i\sigma_1=\frac{\sigma}{\pi^2}, \quad P=\pi^2p_1 \quad and \quad R_1=\frac{R}{\pi^4},$

$$i\sigma_1'=\frac{\sigma'}{\pi^2}, \quad Q_1=\frac{Q}{\pi^2}, \quad and \quad k_x=kcos\theta.$$

2.4 Stationary convection

Put $\sigma=0$, for stationary convection, and the dispersion relation (2.26) becomes

$$R_1=\frac{(1+x)\left[\frac{1+x}{P}+\frac{Q_1xcos^2\theta}{\epsilon}\right]}{xH'}. \quad (2.27)$$

Thus, it is found that for stationary convection the viscoelastic parameter F vanishes with σ and the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid. To study the effects of the magnetic field, suspended particles and medium permeability, we examine the nature of $\frac{dR_1}{dQ_1}, \frac{dR_1}{dH'}$, and $\frac{dR_1}{dP}$, Equation (2.27) yields:

$$\frac{dR_1}{dQ_1} = \frac{(1+x)cos^2\theta}{H'\epsilon}, \tag{2.28}$$

$$\frac{dR_1}{dH'} = -\frac{(1+x)\left[\frac{1+x}{P} + \frac{Q_1 x cos^2\theta}{\epsilon}\right]}{xH'^2} \tag{2.29}$$

and $$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xH'P^2}. \tag{2.30}$$

Which shows that the magnetic field has a stabilizing effect whereas the suspended particles and medium permeability have a destabilizing effect on thermal convection in the Rivlin-Ericksen fluid permeated with suspended particles in a porous medium in hydrodynamics for stationary convection. Graphically, we analyse the magnetic field, suspended particles and medium permeability as follows:

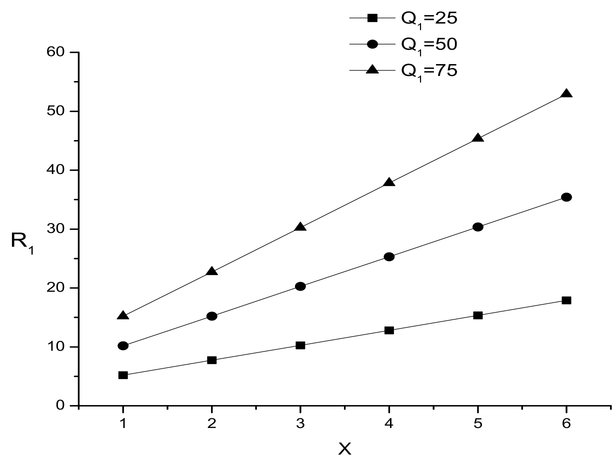

Figure 2.1: The variation of dimensionless Rayleigh number (R_1) with wave number (x), for $H' = 10, P = 2, \theta = 45^0, \epsilon = 0.5$ and $Q_1 = 25, 50, 75$.

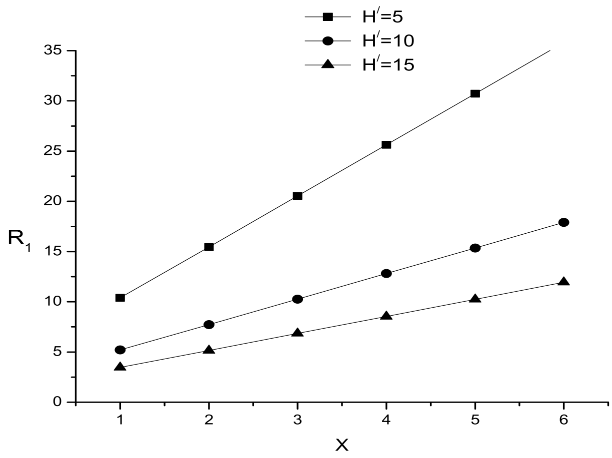

Figure 2.2: The variation of dimensionless Rayleigh number (R_1) with wave number (x), for $P = 2, Q_1 = 25, \theta = 45^0, \epsilon = 0.5$ and $H' = 5, 10, 15$.

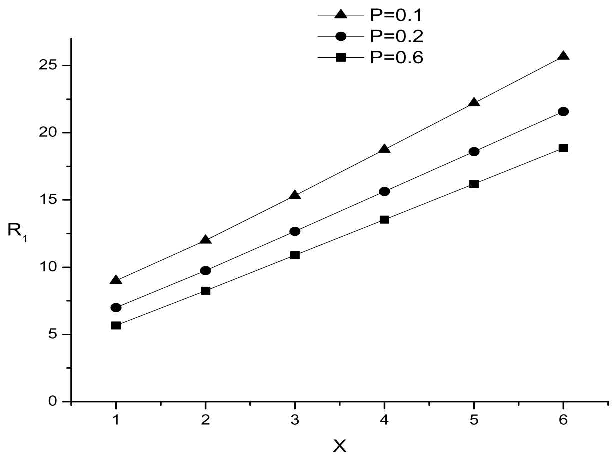

Figure 2.3: The variation of dimensionless Rayleigh number (R_1) with wave number (x), for $Q_1 = 25, H' = 10, \theta = 45^0, \epsilon = 0.5$ and $P = 0.1, 0.2, 0.6$.

In **Figure2.1** (refer table 1), we have $x = 1, 2, 3, 4, 5, 6$ and $H' = 10, P = 2, \epsilon = 0.5$, $\theta = 45^0$ and $Q_1 = 25, 50, 75$, found that, if magnetic field is increased growth rate is also increased, shows the effect of stabilization on the system.

Whereas in **Figure2.2** (refer table 2), $x = 1, 2, 3, 4, 5, 6$, $H' = 5, 10, 15, P = 2$ and $\epsilon = 0.5, \theta = 45^0, Q_1 = 25$, shows that, if suspended particles are increased growth rate is decreased, gives the effect of destabilizing effect on the system.

In **Figure2.3** (refer table 3), by using values of $H' = 10, \epsilon = 0.5, \theta = 45^0$, $Q_1 = 25$, $P = 0.1, 0.2, 0.6$ and $x = 1, 2, 3, 4, 5, 6$, found that, when medium permeability is increased, growth rate is decreased, gives the destabilizing effect on the system.

2.5 Stability of the system of oscillatory modes

Multiplying equation (2.20) by the complex conjugate of W i.e W^*, integrating over the range of z from $z = 0$ to $z = 1$ and making use of equations (2.21) and (2.22) together with the given physical boundary conditions (2.24), we obtain

$$\left[\frac{\sigma'}{\epsilon} + \frac{1}{p_1}(1 + F\sigma)\right] I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}\left[\frac{d^2 + \nu\tau\sigma^*}{H'd^2 + \nu\tau\sigma^*}\right][I_2 + (E + h\epsilon)p_3\sigma^* I_3] + \frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}[I_4 + p_2\sigma^* I_5] = 0 \tag{2.31}$$

$$where \quad I_1 = \int_0^1 \left(|DW|^2 + a^2|w|^2\right) dz \quad , \quad I_2 == \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz,$$
$$I_3 = \int_0^1 |\Theta| dz, \qquad I_4 = \int_0^1 \left(|D^2X|^2 + 2a^2|DX|^2 + a^4|X|^2\right) dz,$$
$$I_5 = \int_0^1 \left(|DX|^2 + a^2|X|^2\right) dz$$

and $\sigma^* \to$ complex conjugate of σ. The integrals I_1, I_2, I_3, I_4, I_5 are all positive definite. Putting $\sigma = i\sigma, f = \frac{mN_0}{\rho_0}$, and by equating the imaginary parts of equation (2.31), we obtain

$$\sigma_i \left[\left\{\frac{1}{\epsilon}\left(1 + \frac{f}{1 + p_3^2\tau^2\sigma_i^2}\right) + \frac{F}{p_1}\right\} I_1 + \frac{g\alpha\kappa a^2}{\nu\beta\left(H'^2d^4 + \nu^2\tau^2\sigma_i^2\right)} \left\{d^2\nu\tau h I_2 + p_3(E + h\epsilon)(H'd^4 + \nu^2\tau^2\sigma_i^2)I_3\right\} + \frac{\mu_e\eta\epsilon p_2}{4\pi\rho_0\nu} I_5\right] = 0. \tag{2.32}$$

Equation (2.32) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of the magnetic field, equation (2.32) is reduced to

$$\sigma_i \left[\left\{\frac{1}{\epsilon}\left(1 + \frac{f}{1 + p_1^2\tau^2\sigma_i^2}\right) + \frac{F}{p_1}\right\} I_1 + \frac{g\alpha\kappa a^2}{\nu\beta\left(H'^2d^4 + \nu^2\tau^2\sigma_i^2\right)} \left\{d^2\nu\tau h I_2 + p_1(E + h\epsilon)(H'd^4 + \nu^2\tau^2\sigma_i^2)I_3\right\}\right] = 0. \tag{2.33}$$

Thus, $\sigma_i = 0 \Rightarrow$ the principle of exchange of stabilities is valid but oscillatory modes are not allowed. Whereas the quantity inside the brackets is positive definite. The presence of the magnetic field introduces oscillatory modes.

2.6 Conclusion

Presence of Magnetic field showed the stabilizing effect whereas presence of suspended particles and medium permeability showed the destabilizing effect in the study of Rivlin-Ericksen fluid.

Chapter 3

Hall Effect on Thermal Instability of Viscoelastic Dusty Fluid in Porous Medium

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3.1 Introduction

The theoretical and experimental results of the onset of thermal instability (Bénard convection), under varying assumptions of hydrodynamics and hydromagnetics, have been discussed by Chandrasekhar [26] in his celebrated monograph. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current is called the Hall current effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasma, Singh and Gupta [37]. Sherman and Sutton [38] considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta [39] studied the problem of thermal instability in the presence of Hall currents and found that Hall currents have a destabilizing effect on the thermal instability of a horizontal layer of a conducting fluid in the presence of a uniform vertical magnetic field. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in association with the external force. The approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density variations arise principally from thermal effects. Spiegel and Veronis [40] simplified the set of equations governing the flow of compressible fluids under the following assumptions:

- The depth of the fluid layer is much less than the scale height, as defined by them.
- The fluctuations in temperature, density and pressure, introduced due to motion, do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the *static temperature gradient* is replaced by its excess over the adiabatic one and C_v is replaced by C_p. In geophysical situations, the fluid is often not pure but contains suspended particles. Scanlon and Segel [23] considered the effects of suspended particles on the onset of Bénard convection and found that the *critical Rayleigh number* is reduced because of the heat capacity of the particles. The suspended particles were thus found to destabilize the layer.

Palaniswamy and Purushotham [29] studied the stability of *shear flow* of stratified fluids with *fine dust* and found the fine dust to increase the region of instability. The fluids were considered to be Newtonian and the medium was considered to be *non-porous* in all the above studies.

There is growing importance of *non-Newtonian* fluids in geophysical fluid dynamics, chemical technology and *petroleum industry*. Bhatia and Steiner [41] studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation was studied by Sharma [42]. There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's *constitutive relations*. The Rivlin-Ericksen elastico-viscous fluid is one such fluid. Rivlin and Ericksen [32] studied the stress, deformation, relaxations for isotropic materials. Thermal instability in viscoelastic Rivlin-Ericksen fluids in the presence of rotation and magnetic field, separately, was investigated by Sharma and Kumar [43] and [44]. Sharma and Kumar [45] studied the hydromagnetic stability of two Rivlin-Ericksen elasticoviscous superposed conducting fluids. Kumar and Singh [46] studied the stability of two superposed Rivlin-Ericksen viscoelastic fluids in the presence of suspended particles. In another study, Kumar et al. [47] studied the hydrodynamic and hydromagnetic stability of two *stratified* Rivlin-Ericksen elasticoviscous *superposed fluids*.

The flow through porous media is of considerable interest for petroleum engineers and geophysical fluid dynamicists. A great number of applications in geophysics may be found in the books by Phillips [48], Ingham and Pop [49], and Nield and Bejan [50]. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\frac{1}{k_1}\left(\mu + \mu' \frac{\partial}{\partial t}\right) q$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. Lapwood [27] studied the stability of a convective flow in hydromagnetics in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium was considered by Wooding [28]. The stability of superposed Rivlin-Ericksen elastico-viscous fluids permeated with suspended particles in a porous medium was considered by Kumar [36]. Kumar et al. [51] studied the instability of two rotating viscoelastic (Rivlin-Ericksen) superposed fluids with suspended particles in a porous medium. In another study, Kumar et al. [52] considered the MHD instability of rotating superposed Rivlin-Ericksen viscoelastic fluids through a porous medium.

Here our interest is to bring out the suspended particles effect on thermal instability of a compressible viscoelastic (Rivlin-Ericksen) fluid in a porous medium including the effect of *Hall currents*.

3.2 Formulation of the Problem

In porous medium, an infinite *horizontal layer* of thickness d confined between two planes $z = 0$ and $z = d$ of an compressible viscoelastic Rivlin-Ericksen fluid in the presence of uniform horizontal magnetic field $\vec{H}(0, 0, H)$ is considered. For the study thermal instability, layer is heated from underside and steady adverse temperature gradient β is maintained, where $\beta = |\frac{dT}{dz}|$. The equations of motion and continuity for the fluid are:

$$\frac{\rho}{\epsilon}\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\epsilon}(\vec{v}.\nabla)\vec{v}\right] = -\nabla p - \rho g\vec{\lambda} - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN}{\epsilon}(\vec{u} - \vec{v}) + \frac{\mu_e}{4\pi}\left(\nabla \times \vec{H}\right) \times \vec{H} \tag{3.1}$$

and $$\epsilon\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{v}) = 0 \tag{3.2}$$

where $\rho \rightarrow$ density, $\mu \rightarrow$ viscosity, $\mu' \rightarrow$ viscoelasticity, $p \rightarrow$ pressure and $\vec{v}(u, v, w) \rightarrow$ velocity of the pure fluid. Here $\vec{u}(l, r, s) \rightarrow$ velocity of the suspended particles, $N(\overline{x}, t) \rightarrow$ number density of the suspended particles, $\epsilon \rightarrow$ medium porosity, $k_1 \rightarrow$ medium permeability, $\mu_e \rightarrow$ magnetic permeability, $g \rightarrow$ acceleration due to gravity, $\overline{x} = (x, y, z)$, $\vec{\lambda}(0, 0, 1)$ and $K = 6\pi\mu\eta'$, η' being the particle radius, is the Stokes' drag coefficient.

In the above equations of conservation of momentum (3.1), Some assumptions regarding the shape and velocity of the suspended particles are taken as

- Shape of the suspended particles in the fluid is uniform spherical.
- Relative velocities between the fluid and particles is small.
- Large distance between the particles as compare to their diameter. So Interparticle reactions are ignored.
- Gravity, pressure, Darcian force and magnetic field effect on the suspended particles are negligibly small, so ignored.
- Extra force due to the presence of particles is proportional to velocity difference between the particles and the fluid.
- Force exserted by fluid on particles and force exerted by particles on fluid balance each other.

So there must be an extra force equal in magnitude but opposite in sign in the equations of conservation of momentum or motion for the particles. If mN is the mass of particles per unit volume, then under the above assumptions, equations of conservation of momentum and mass for the particles are

$$mN\left[\frac{\partial\vec{u}}{\partial t}+\frac{1}{\epsilon}\left(\vec{u}.\nabla\right)\vec{u}\right]=KN(\vec{v}-\vec{u}) \tag{3.3}$$

and $$\epsilon\frac{\partial N}{\partial t}+\nabla.(N\vec{u})=0. \tag{3.4}$$

Let at *constant volume*, C_v is the *heat capacity* of the fluid, at constant pressure, C_p is the *heat capacity* of the fluid, C_{pt} denote the heat capacity of the particles T is the temperature and q is *effective thermal conductivity* of the pure fluid. Assuming, fluid particles are in *thermal equilibrium*, then equation of *heat conduction* is given by

$$\left[\rho C_v\epsilon+\rho_s C_s(1-\epsilon)\right]\frac{\partial T}{\partial t}+\rho C_v(\vec{v}.\nabla)T+mNC_{pt}\left(\epsilon\frac{\partial}{\partial t}+\vec{u}.\nabla\right)T=q\nabla^2 T \tag{3.5}$$

where ρ_s is the density and C_s is the heat capacity of the solid matrix, R.C.Sharma and U.Gupta [53] had used the same parameters for their study.

Maxwell's equations in the presence of hall currents give

$$\nabla.\vec{H} = 0 \tag{3.6}$$

and $$\epsilon\frac{\partial H}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \epsilon\eta\nabla^2\vec{H} - \frac{c\epsilon}{4\pi N'e}\nabla \times \left[(\nabla \times \vec{H}) \times \vec{H}\right] \tag{3.7}$$

where $\eta \rightarrow$ resistivity, $c \rightarrow$ speed of light, $N' \rightarrow$ electron number density and e is charge of an electron. The initial state of the system is taken to be a quiescent layer (no settling) with a uniform particle distribution N_0 and is given by

$$\vec{u} = (0,0,0), \quad \vec{v} = (0,0,0), \quad \vec{H} = (0,0,H),$$

$$T = T(z), \quad p = p(z) \quad \rho = \rho(z) \quad \text{and} \quad N = N_0 = constant. \tag{3.8}$$

Following the Spiegel and Veronis' [40] we have

$$T(z) = -\beta z + T_0, \quad p(z) = p_m - g\int_0^z (\rho_m + \rho_0)\, dz,$$

$$\rho(z) = \rho_m\left[1 - \alpha_m(T - T_m) + K_m(p - p_m)\right],$$

$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m \quad \text{and} \quad K_m = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m. \tag{3.9}$$

Spiegel and Veronis' [40] expressed any state variable say X, in the form

$$X = X_m + X_0(z) + X'(x,y,z,t) \tag{3.10}$$

where $X_m \rightarrow$ constant space distribution of $X, X_0 \rightarrow$ variation of X in the absence of motion and $X'(x,y,z,t) \rightarrow$ fluctuations in X due to motion of the fluid. Also, ρ_m is constant space distribution of ρ and $p_m \rightarrow$ constant space distribution of p and ρ_0 is density at the lower boundary $z = 0$ and $T_0 \rightarrow$ temperature of the fluid at $z = 0$. Again following Spiegel and Veronis[40] assumptions and results for compressible fluids, the flow equations are found to be the same as those of incompressible fluids except that the static temperature gradient β is replaced by its excess over the adiabatic $(\beta - g/C_p)$.

3.2.1 Perturbation of Equations

Let δp denote the *perturbation* in pressure p, $\delta\rho$ denote the perturbation in density ρ, θ denote the perturbation in temperature T, $\vec{v}(u, v, w)$ denote the perturbation in fluid velocity (zero initially), $\vec{u}(l, r, s)$ denote the perturbation in particle velocity (zero initially), N denote perturbations in suspended particles number density N_0 and $\vec{h}(h_x, h_y, h_z)$ denote perturbations in magnetic field $\vec{H}(0, 0, H)$. Linearized perturbed equations of the viscoelastic fluid-particle layer are:

$$\frac{1}{\epsilon}\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p - g\left(\frac{\delta\rho}{\rho_m}\right)\vec{\lambda} - \frac{1}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN_0}{\epsilon\rho_m}(\vec{u} - \vec{v}) + \frac{\mu_e}{4\pi\rho_m}(\nabla \times \vec{h}) \times \vec{H}, \tag{3.11}$$

$$\nabla.\vec{v} = 0, \tag{3.12}$$

$$mN_0\frac{\partial u}{\partial t} = KN_0(\vec{v} - \vec{u}), \tag{3.13}$$

$$\epsilon\frac{\partial N}{\partial t} + \nabla.(N_0\vec{u}) = 0, \tag{3.14}$$

$$(E + h\epsilon)\frac{\partial \theta}{\partial t} = (\beta - g/C_p)(w + hs) + \kappa\nabla^2\theta, \tag{3.15}$$

$$\nabla.\vec{h} = 0 \quad \text{and} \tag{3.16}$$

$$\epsilon\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \epsilon\eta\nabla^2\vec{h} - \frac{c\epsilon}{4\pi N'e}\nabla \times \left[(\nabla \times \vec{h}) \times \vec{H}\right] \tag{3.17}$$

where $\alpha_m = \frac{1}{T_m} = \alpha$ (say), $\nu = \frac{\mu}{\rho_m}, \kappa = \frac{q}{\rho_m C_v}$ and $\frac{g}{C_p} \to$ adiabatic gradient, ν is kinematic *viscosity* and κ is thermal diffusivity. Also,

$$h = \frac{fC_{pt}}{C_v}, \quad f = \frac{mN_0}{\rho_m} \quad \textit{and} \quad E = \epsilon + \frac{(1-\epsilon)\rho_s C_s}{\rho_m C_v}.$$

The linearized dimensionless perturbation equations relevant to the problem are

$$N_{p_1}^{-1}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - \frac{1}{p}\left(1 + A\frac{\partial}{\partial t}\right)u + \omega\,(l - u) + N_Q\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right), \tag{3.18}$$

$$N_{p_1}^{-1}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - \frac{1}{p}\left(1 + A\frac{\partial}{\partial t}\right)v + \omega\left(r - v\right) + N_Q\left(\frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y}\right), \tag{3.19}$$

$$N_{p_1}^{-1}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - \frac{1}{p}\left(1 + A\frac{\partial}{\partial t}\right)w + \omega\left(s - w\right) + N_R\theta, \tag{3.20}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{3.21}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)l = u, \quad \left(\tau\frac{\partial}{\partial t} + 1\right)r = v, \quad \left(\tau\frac{\partial}{\partial t} + 1\right)s = w, \tag{3.22}$$

$$\frac{\partial M}{\partial t} + \frac{\partial l}{\partial x} + \frac{\partial r}{\partial y} + \frac{\partial s}{\partial z} = 0, \tag{3.23}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0, \tag{3.24}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_x}{\partial t} = \epsilon^{-1}\frac{\partial u}{\partial z} + \nabla^2 h_x - M_1\frac{\partial}{\partial z}\left(\frac{\partial h_z}{\partial y} - \frac{\partial h_y}{\partial z}\right), \tag{3.25}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_y}{\partial t} = \epsilon^{-1}\frac{\partial v}{\partial z} + \nabla^2 h_y - M_1\frac{\partial}{\partial z}\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right), \tag{3.26}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_z}{\partial t} = \epsilon^{-1}\frac{\partial w}{\partial z} + \nabla^2 h_z - M_1\frac{\partial}{\partial z}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right) \quad \text{and} \tag{3.27}$$

$$\left(E + h\epsilon\right)\frac{\partial \theta}{\partial t} = \left(\frac{G-1}{G}\right)\left(w + hs\right) + \nabla^2\theta \tag{3.28}$$

where

$N_{p_1} = \frac{\epsilon\nu}{\kappa}$ is modified Prandtl number, $N_{p_2} = \frac{\epsilon\nu}{\eta}$ is modified magnetic Prandtl number, $N_R = \frac{g\alpha\beta d^4}{\nu\kappa}$ is *Rayleigh number*, $N_Q = \frac{\mu_e H^2 d^2}{4\pi\rho_m\nu\eta}$ is Chandrasekhar number, $M = \frac{\epsilon N}{N_0}$, $M_1 = \frac{cH}{4\pi N' e\eta}$ is Hall parameter, $\omega = \frac{KN_0 d^2}{\rho_m \nu\epsilon}$, $\tau = \frac{m\kappa}{Kd^2}$, $A = \left(\frac{\nu'}{\nu}\right)\frac{\kappa}{d^2}$, $f = \frac{mN_0}{\rho_m} = \tau\omega$, N_{p_1} is mass fraction, $G = \frac{C_p\beta}{g}$ and $P = \frac{k_1}{d^2}$.

Here physical variables have been scaled using $d, \frac{d^2}{\kappa}, \frac{\kappa}{d}, \frac{\rho\nu\kappa}{d^2}, \beta d$ and $\frac{H\kappa}{\eta}$ as the length, time, velocity, pressure, temperature and magnetic field scale factors, respectively. The boundary conditions suitable to the problem, two *free boundaries* and the medium adjoining the fluid as non conducting, are considered as

$$w = \frac{\partial^2 w}{\partial z^2} = \theta = 0, \quad \xi = \frac{\partial \zeta}{\partial z} = 0, \quad at \quad z = 0 \quad and \quad z = 1. \tag{3.29}$$

and $h_x, h_y, h_z \quad are \quad continuous \quad with \quad an \quad external \quad vacuum \quad field.$ (3.30)

Here $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ are the z-components of vorticity and current density, respectively. Equations (3.18)-(3.28), after eliminating u, v and δp can be expressed as

$$\left[L_1 + \frac{L_2}{P}\left(1 + A\frac{\partial}{\partial t}\right)\right]\nabla^2 w = L_2 N_Q \nabla^2 \frac{\partial h_z}{\partial z} + L_2 N_R \nabla_1^2 \theta, \tag{3.31}$$

$$\left[L_1 + \frac{L_2}{P}\left(1 + A\frac{\partial}{\partial t}\right)\right]\zeta = L_2 N_Q \frac{\partial \xi}{\partial z}, \tag{3.32}$$

$$\left[N_{p2}N_{p1}^{-1}\frac{\partial}{\partial t} - \nabla^2\right]\xi = \epsilon^{-1}\frac{\partial \zeta}{\partial z} + M_1\frac{\partial}{\partial z}\left(\nabla^2 h_z\right), \tag{3.33}$$

$$\left[N_{p2}N_{p1}^{-1}\frac{\partial}{\partial t} - \nabla^2\right]h_z = \epsilon^{-1}\frac{\partial w}{\partial z} - M_1\frac{\partial \xi}{\partial z} \quad \text{and} \tag{3.34}$$

$$L_2\left[(E + h\epsilon)\frac{\partial}{\partial t} - \nabla^2\right]\theta = \left(\frac{G-1}{G}\right)\left(\tau\frac{\partial}{\partial t} + \overline{H}\right)w \tag{3.35}$$

where

$$L_1 = N_{p_1}^{-1}\left(\tau\frac{\partial^2}{\partial t^2} + F\frac{\partial}{\partial t}\right), \quad F = f + 1, \quad L_2 = \tau\frac{\partial}{\partial t} + 1, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \overline{H} = h + 1.$$

3.3 The Dispersion Relation

Perturbed quantities are assumed to be of the following form and for the analysis of disturbances into normal modes by seeking solutions whose dependence on x, y and t is given by

$$[w, \theta, h_z, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)]\, exp(ik_x x + ik_y y + nt) \tag{3.36}$$

where k_x is the wave number along x-direction, and k_y is wave number along y-direction. $k = \sqrt{k_x^2 + k_y^2}$ = resultant wave number and n = growth rate. Equations (3.31)-(3.35), with the help of expression (3.36), become

$$\left[L_1 + \frac{L_2}{P}(1 + An)\right]\left(D^2 - \alpha^2\right) W = L_2 N_Q \left(D^2 - \alpha^2\right) DK - L_2 N_R \alpha^2 \Theta, \tag{3.37}$$

$$\left[L_1 + \frac{L_2}{P}(1 + An)\right] Z = L_2 N_Q DX, \tag{3.38}$$

$$\left[N_{p2} N_{p1}^{-1} n - \left(D^2 - \alpha^2\right)\right] X = \epsilon^{-1} DZ + M_1 \left(D^2 - \alpha^2\right) DK, \tag{3.39}$$

$$\left[N_{p2} N_{p1}^{-1} n - \left(D^2 - \alpha^2\right)\right] K = \epsilon^{-1} DW - M_1 DX \quad \text{and} \tag{3.40}$$

$$L_2 \left[(E + h\epsilon)\, n - \left(D^2 - \alpha^2\right)\right] \Theta = \left(\frac{G-1}{G}\right)\left(\tau n + \overline{H}\right) W \tag{3.41}$$

where $D = \dfrac{d}{dz}, \quad L_1 = N_{p_1}^{-1}(\tau n^2 + Fn) \quad and \quad L_2 = \tau n + 1.$

By eliminating $X, Z, K,$ and Θ from the equations (3.37)-(3.41), we obtain

$$\left[L_1 + \frac{L_2}{P}(1 + An)\right]\left[(D^2 - \alpha^2) - (E + h\epsilon)n\right]\left(D^2 - \alpha^2\right) W$$
$$+ \left[\frac{L_2 N_Q \left[(D^2 - \alpha^2) - (E + h\epsilon)n\right]\left[\frac{(D^2-\alpha^2) - N_{p_2} N_{p_1}^{-1} n}{M_1 \epsilon} + \frac{L_2 N_Q D^2}{\left[L_1 + \frac{L_2}{P}(1+An)\right] M_1 \epsilon^2}\right] D^2}{M_1 (D^2 - \alpha^2) D^2 + \frac{\left\{(D^2-\alpha^2) - N_{p_2} N_{p_1}^{-1} n\right\}^2}{M_1} + L_2 N_Q \frac{(D^2-\alpha^2) - N_{p_2} N_{p_1}^{-1} n}{\left\{L_1 + \frac{L_2}{P}(1+An)\right\} M_1 \epsilon} D^2}\right]\left(D^2 - \alpha^2\right) W$$
$$= \left(\frac{G-1}{G}\right) N_R \alpha^2)(\tau n + \overline{H}) W. \tag{3.42}$$

Using the boundary conditions and equations (3.29) and (3.30), obviously that the even order derivatives of W vanish on the boundaries and hence the proper solution of equation (3.42) characterizing the lowest mode is

$$W = W_0 sin(\pi z), \quad where \quad W_0 = Constant. \tag{3.43}$$

On substituting the solution (3.43) in equation (3.42), we get the dispersion relation as

$$N_R = \left(\frac{G}{G-1}\right) \frac{\left(\pi^2+\alpha^2\right)\left[\left(\pi^2+\alpha^2\right)+(E+h\epsilon)\,n\right]}{\alpha^2\left(\tau n+\overline{H}\right)} \left[\left\{L_1+\frac{L_2}{P}(1+An)\right\} + \frac{L_2 N_Q \pi^2 \left[\frac{\left(\pi^2+\alpha^2\right)+N_{p2}N_{p1}^{-1}n}{M_1\epsilon} + \frac{L_2 N_Q \pi^2}{\left\{L_1+\frac{L_2}{P}(1+An)\right\}M_1\epsilon^2}\right]}{\left[M_1\pi^2\left(\pi^2+\alpha^2\right) + \frac{\left\{\left(\pi^2+\alpha^2\right)+N_{p2}N_{p1}^{-1}n\right\}^2}{M_1} + \frac{L_2 N_Q\left\{\left(\pi^2+\alpha^2\right)+N_{p2}N_{p1}^{-1}n\right\}\pi^2}{\left\{L_1+\frac{L_2}{P}(1+An)\right\}M_1\epsilon}\right]}\right]. \tag{3.44}$$

3.4 Stationary Convection

When the instability sets in as *stationary convection*, the *marginal state* will be characterized by $n = 0$ and the *dispersion relation* equation (3.44) reduces to

$$N_R = \left(\frac{G}{G-1}\right) \frac{\left(\pi^2+\alpha^2\right)^2}{\alpha^2\overline{H}} \left[\frac{1}{P} + \frac{N_Q\epsilon^{-1}\pi^2\left\{\left(\pi^2+\alpha^2\right)+N_Q P\epsilon^{-1}\pi^2\right\}}{\left(\pi^2+\alpha^2\right)\left\{M_1^2\pi^2+\left(\pi^2+\alpha^2\right)+N_Q P\epsilon^{-1}\pi^2\right\}}\right]. \tag{3.45}$$

Thus for stationary convection, the viscoelastic parameter vanishes with n, and stress and strain rate showed linear realtion for Rivlin- Ericksen viscoelastic fluid. Also, for fixed values of P, N_Q, M_1 and $\overline{H}$, let the *non-dimensional* number G accounting for the compressibility effects be also kept as fixed, then we have

$$\overline{N_R^C} = \left(\frac{G}{G-1}\right) N_R^C \tag{3.46}$$

where N_R^C is critical Rayleigh number in the absence compressibility and $\overline{N_R^C}$ is critical Rayleigh number in the presence of compressibility. Since the critical Rayleigh number > 0 and finite which implies $G > 1$, which means stabilizing effect due to compressibility.

Now we study, the effect of suspended particles which depends upon the nature of $\frac{dN_R}{d\overline{H}}$, the effect of medium permeability which depends upon the nature of $\frac{dN_R}{dP}$, the effect of magnetic field which depends upon the nature of $\frac{dN_R}{dN_Q}$, the effect of hall current which depends upon the nature of $\frac{dN_R}{dM_1}$. From equation (3.45) we have

$$\frac{dN_R}{d\overline{H}} = -\left(\frac{G}{G-1}\right)\frac{\left(\pi^2+\alpha^2\right)^2}{\alpha^2\overline{H}^2}\left[\frac{1}{P}+\frac{N_Q\epsilon^{-1}\pi^2\left\{(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}{(\pi^2+\alpha^2)\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}\right]. \tag{3.47}$$

which is $< 0 \Rightarrow$ destabilizing effect of suspended particles on the thermal instability of the compressible fluid-particle layer in the presence of and hall currents through a porous medium. It is obvious from equation (3.45) that

$$\frac{dN_R}{dP} = \left(\frac{G}{G-1}\right)\frac{\left(\pi^2+\alpha^2\right)^2}{\alpha^2\overline{H}}\left[-\frac{1}{P^2}+\frac{(N_Q\pi^2\epsilon^{-1})M_1^2\pi^2}{(\pi^2+\alpha^2)\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}^2}\right]. \tag{3.48}$$

which is > 0 if $P\left[M_1\pi-\sqrt{\pi^2+\alpha^2}\right] > \frac{\sqrt{\pi^2+\alpha^2}\left[M_1^2\pi^2+\left(\pi^2+\alpha^2\right)\right]}{N_Q\epsilon^{-1}\pi^2}$.

which is < 0 if $P\left[M_1\pi-\sqrt{\pi^2+\alpha^2}\right] < \frac{\sqrt{\pi^2+\alpha^2}\left[M_1^2\pi^2+\left(\pi^2+\alpha^2\right)\right]}{N_Q\epsilon^{-1}\pi^2}$.

Thus, for the different values of parameter, medium permeability has both destabilizing and stabilizing effect. Presence and absence of magnetic field plays an important role in stabilizing effect of permeability. Its absence destabilize the effect. Since for the case

$$\frac{dN_R}{dP} = -\left(\frac{G}{G-1}\right)\frac{\left(\pi^2+\alpha^2\right)^2}{\alpha^2\overline{H}P^2}. \tag{3.49}$$

which is always < 0. Thus, in the presence of magnetic field, medium permeability succeeds in stabilizing the thermal instability of the compressible fluid-particle layer for certain wave numbers. Now from equation (3.45), we get

$$\frac{dN_R}{dN_Q} = \left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)\,\pi^2\epsilon^{-1}}{\alpha^2\overline{H}\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}\left[\left\{\left(\pi^2+\alpha^2\right)+N_QP\epsilon^{-1}\pi^2\right\}\right.$$
$$\left.+\frac{M_1^2\pi^4N_QP\epsilon^{-1}}{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2}\right]. \tag{3.50}$$

which is always > 0 which implies that magnetic field has a stabilizing effect. To find the effect of hall currents, from equation (3.45), we have

$$\frac{dN_R}{dM_1} = -2\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)}{\alpha^2\overline{H}}\left[\frac{N_Q\epsilon^{-1}M_1\pi^4\left\{(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}}{\left\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\epsilon^{-1}\pi^2\right\}^2}\right]. \tag{3.51}$$

which is always < 0 which means that, in porous medium, hall current destabilize the thermal convection in the compressible fluid-particle layer. We analyze graphically all the four effects as

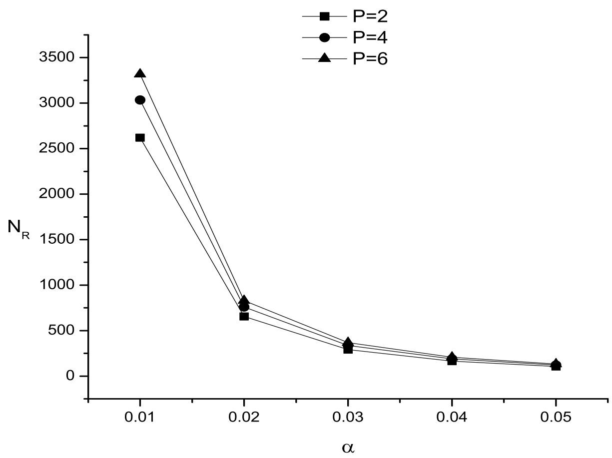

Figure 3.1: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14, N_Q = 20, M_1 = 10, \epsilon = 0.5$ and for different values of $P(2, 4, 6)$.

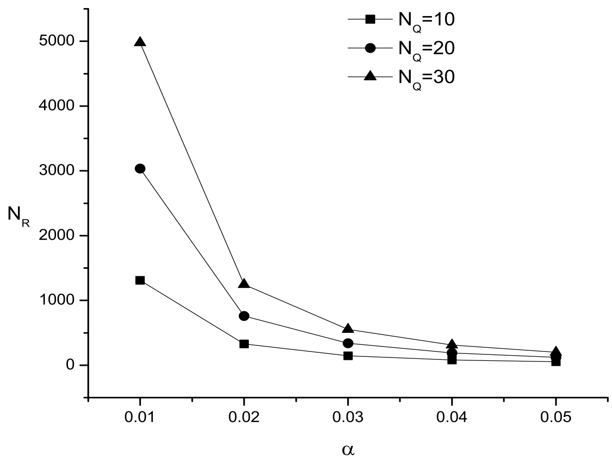

Figure 3.2: Variation of N_R with α for a fixed $\overline{H} = 1000, G = 9.8, \pi = 3.14, P = 4, M_1 = 10, \epsilon = 0.5$ and for different values of $N_Q = (10, 20, 30)$.

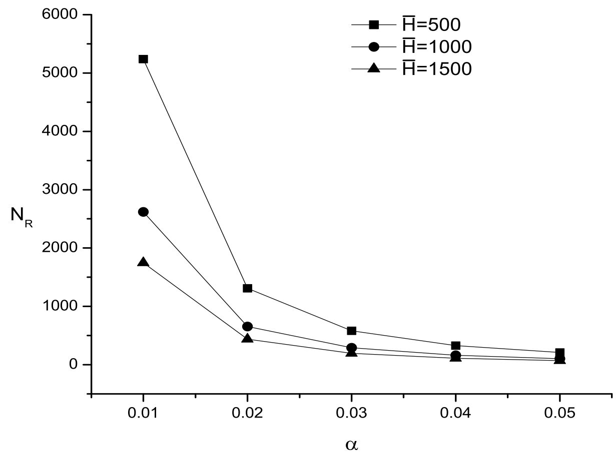

Figure 3.3: Variation of N_R with α for a different value of $\overline{H} = (500, 1000, 1500)$ for fixed values of $G = 9.8, \pi = 3.14, P = 2, M_1 = 10, \epsilon = 0.5$.

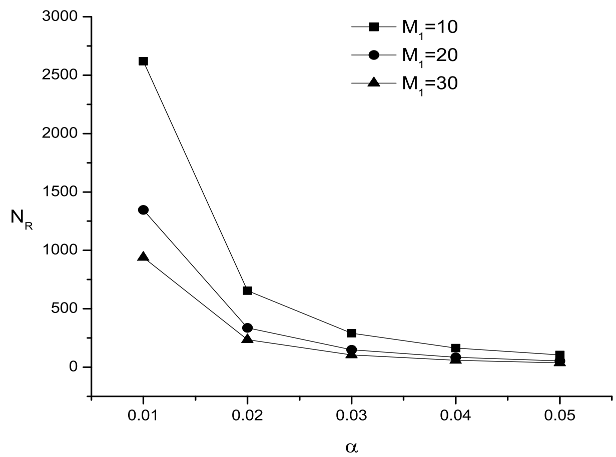

Figure 3.4: Variation of N_R with α for a fixed values $\overline{H} = 1000, G = 9.8, \pi = 3.14, P = 2, N_Q = 20, \epsilon = 0.5$ for different values of $M_1 = (10, 20, 30)$.

We find from **Figure3.1** (refer table 4), when the value of the medium permeability(P), increased then the value of N_R is increased which shows the stabilizing effect. Similarly from **Figure3.2** (refer table 5), when the value of magnetic field N_Q is increased, and the value of N_R is increased which again shows the case of stabilizing effect. **In Figure3.3** (refer table 6), as the value of suspended particle $\overline{H}$ increased, the value of N_R decreased, which shows the destabilizing effect. Also **Figure3.4** (refer table 7) shows as the value of hall currents M_1 through the porous medium increased, the value of N_R decreased, which is again the case of destabilizing effect on the system.

3.5 Oscillatory Modes

Multiplying equation (3.37) by he complex conjugate of W i.e. W^*, integrating over the range of z from $z = 0$ to $z = d$ and using equations (3.38)-(3.47) together with the boundary conditions (3.29) and (3.30)

$$\left[L_1 + \frac{L_2}{P}(1 + An)\right] I_1 + A_1 (nI_2 + n^* I_5) + L_2 N_Q \epsilon (I_3 + I_6) + \frac{L_2}{L_2^*}\left[L_1^* + \frac{L_2^*}{P}(1 + An^*)\right] I_4$$
$$= L_2 L_2^* N_R \alpha^2 \left(\frac{G-1}{G}\right)\left(\frac{1}{\tau n^* + \overline{H}}\right)[I_7 + (E + h\epsilon) n^* I_8]. \tag{3.52}$$

where $A_1 = L_2 N_Q N_{P2} N_{P1}^{-1}$ and

$$\begin{aligned}
I_1 &= \int_0^1 \left(|DW|^2 + a^2|w|^2\right) dz \quad , \quad I_2 = \int_0^1 |X|^2 dz,\\
I_3 &= \int_0^1 \left(|DX|^2 + a^2|X|^2\right) dz \quad , \quad I_4 = \int_0^1 |Z|^2 dz\\
I_5 &= \int_0^1 \left(|DK|^2 + a^2|K|^2\right) dz \quad , \quad I_6 = \int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right) dz,\\
I_7 &= \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz \quad , \quad I_8 = \int_0^1 |\Theta|^2 dz.
\end{aligned} \tag{3.53}$$

all $I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8$ all are positive definite, take $n = in_0$ in equation (3.52), where n_0 is real, and equate imaginary parts on both sides, we get

$$n_0 = 0 \quad \text{or} \quad n_0^2 = -\tau^{-2}\frac{A - B}{C - D + E} \tag{3.54}$$

where

$$A = \left(N_{p_1}^{-1}\overline{H}F - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_Q \epsilon N_{P_2} N_{P_1}^{-1}\overline{H}(I_2 - I_5) - N_Q \epsilon \tau (I_3 + I_6),$$

$$B = \left(N_{p_1}^{-1}\overline{H}F + \frac{\tau}{P} + \frac{\tau}{P}A\right) I_4 - N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\},$$

$$C = \left(N_{p_1}^{-1}\left(\overline{H} + 1 - F\right) - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_Q \epsilon N_{P_2} N_{P_1}^{-1}\overline{H}(I_2 - I_5),$$

$$D = N_Q \epsilon \tau (I_3 + I_6) - \left(N_p^{-1}\left(1 - \overline{H} - F\right) - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_4 \quad \text{and}$$

$E = N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\}$. Whereas in the absence of magnetic field,

$$n_0^2 = \frac{-\tau^{-2}\left[\left(N_{p_1}^{-1}\overline{H}F - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\}\right]}{\left(N_{p_1}^{-1}\left(\overline{H} + 1 - F\right) - \frac{\tau}{P} - \frac{\tau}{P}A\right) I_1 + N_R \alpha^2 \left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\epsilon) I_8\}}. \tag{3.55}$$

3.6 Conclusion

Problem was formulated to discuss the combined effect of compressibility, hall current, magnetic field, medium permeability and suspended particles on thermal instability of a Rivlin-Ericksen fluid and the results obtained as :

(I) Constitutive relation of Rivlin-Ericksen fluid becomes linear i.e. the relation between stress and strain becomes linear for stationary convection due to the vanishing of the *viscoelastic parameter.*

(II) Magnetic field, suspended particles and *medium permeability* introduce oscillatory modes in the system otherwise effects the principle of exchange of stabilities is hold good.

(III) When magnetic field is not present, $n_0^2 < 0$ if

$$C_{pt} > C_v \left[1 + \frac{\epsilon m}{f k_1 K d^2} \left\{\nu d^2 + \nu'\right\}\right] \tag{3.56}$$

For all $N_R > 0$, since n_0 is real and $n_0^2 < 0$ which implies $n_0 = 0$. This shows that n is real when $N_R > 0$ in the absence of the magnetic field. If equation (3.55) holds true and that the principle of exchange of stabilities is valid for this case, however, if equation (3.55) is violated, then the oscillatory modes may come into play even in the absence of the magnetic field, Singh and Gupta [37].

(IV) Equation (3.46) indicates compressibility effect is to postpone the onset of instability.

(V) To study the various effects of suspended particles, medium permeability, magnetic filed and Hall currents in a compressible Rivlin-Ericksen viscoelastic fluid, we examined the expressions $\frac{dN_R}{dH}, \frac{dN_R}{dP}, \frac{dN_R}{dN_Q}$ and $\frac{dN_R}{dM_1}$ analytically. The magnetic field postpones the onset of instability, suspended particles and Hall currents both hasten the onset of convection, which is in contrast with the result of Gupta et al. [54].

Chapter 4

Double - Diffusive Convection in Presence of Compressible Rivlin-Ericksen Fluid with Fine Dust

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4.1 Introduction

A layer of *Newtonian fluid* heated from below, under varying assumptions of hydrodynamics, has been treated in detail by Chandrasekhar [26]. Chandra [22] performed careful experiments in an air layer and found contradiction between the theory and the experiment. He found that the *instability* depended on the depth of the layer. A Bénard type cellular convection with fluid descending at the cell centre was observed when predicted gradients were imposed, if the layer depth was more than 10 mm. But if the layer of depth was less than 7 mm, convection occurred at much lower gradients than predicted and appeared as irregular strips of elongated cells with fluid rising at the centre. Chandra called this motion *columnar instability*. The effect of particle mass and *heat capacity* on the onset of Bénard convection has been considered by Scanlon and Segel [23]. They found that the *critical Rayleigh number* was reduced solely because the heat capacity of the clean gas was supplemented by that of the particles. The effect of *suspended particles* was found to *destabilize* the layer. Palniswamy and Purushotham [29] have considered the stability of shear flow of stratified fluids with *fine dust* and have found the effect of fine dust to increase the region of instability. A study of *double-diffusive convection* with fine dust has been made by Sharma and Rani [55]. Kumar et al. [56] have studied effect of magnetic field on thermal instability of rotating Rivlin-Ericksen *viscoelastic fluid*, in which effect of magnetic field has stabilizing as well as destabilizing effect on the system. Also, Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluid through porous medium has been considered by Sharma et al. [57]. They have studied the stability aspects of the system. The effects of a uniform horizontal magnetic field and a uniform rotation on the problem have also been considered separately. Kumar [58] has also studied the stability of superposed viscoelastic Rivlin-Ericksen fluids in presence of suspended particles through a porous medium. In one other study, Kumar and Singh [59] have studied the stability of superposed viscoelastic fluids through porous medium, in which effects of uniform horizontal magnetic field and a uniform rotation are considered. Kumar et al.[47] have also studied hyderodynamic and hyderomagnetic stability of Rivlin-Ericksen fluid and found that the growth rates decrease as well as increase with the increase in *kinematic viscosity* and kinematic viscoelasticity in absence and presence of magnetic field. Singh et. al. [25] has studied thermal instability of Rivlin-Ericksen elastico viscous fluid permeated with suspended particles in hydrodynamics in a porous medium and found that magnetic field have only stabilizing effect whereas medium *permeability* have a destabilizing effect on the system. M.F.EI-Sayed et. al [60], have studied non-linear Kelvin-Helmholtz instability of Rivlin-Ericksen viscoelastic electrified fluid particle mixtures saturating porous medium and in one another study Kumar et al. [61], have also studied double-diffusive convection in compressible viscoelastic fluid through Brinkman porous media.

Presently, the study of stability of double-diffusive convection of Rivlin-Ericksen elastico-viscous fluids *permeated* with suspended particles is considered. Viscosity is a function of space and time in a large variety of fluid flows and its variation can have a dramatic effect on flow stability. Here instability due to double-diffusive effects in viscosity, permeated with suspended particles flow have been discussed. Double-diffusive systems are known to display a rich variety of instability behavior in density permeated with suspended particles fluid flow system. In viscosity permeated systems, it was found that stable flow in the context of single component systems become unstable due to double-diffusive effect. Many interesting flow patterns arise due to this instability, these aspects form the motivation for the present study, Singh and Gupta[62].

4.2 Formulation of The Problem

Infinite and *horizontal layer* of Rivlin-Ericksen fluid of depth d i.e. from $z = 0$ to $z = d$ is considered for an compressible electrically conducting viscoelastic Rivlin-Ericksen with suspended particles. This layer is given the heat from below, let the temperature at $z = 0$ is T_0 and at the upper layer, $z = d$, is T_d, and that a steady adverse temperature gradient $|\frac{dT}{dz}| = \beta$ and *solute gradient* $|\frac{dC}{dz}| = \beta'$ are maintained. Here, $\vec{g}\,(0, 0, -g)$ is acceleration due to gravity. The effect of fluid compressibility, even small in magnitude, is also considered.

Let the fluid properties like pressure, density, velocity of pure fluid, kinematic viscosity and kinematic viscoelasticity be denoted by p, ρ, $\vec{u}(u, v, w)$, ν and ν' respectively. Properties of suspended particle like velocity and number density be represented by $v(\overline{x},t)$ and $N(\overline{x},t)$. $\vec{x}(x,y,z)$, $\vec{\lambda}(0,0,1)$ and $K = 6\pi\mu\eta'$ is the Stokes' drag coefficient for the particle having the radius η'.

Then the *flow governing* equations i.e. equations of motion and continuity are

$$\rho\left[\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u})\right]=-\nabla p+\rho\vec{g}+KN\left(\vec{v}-\vec{u}\right)+\left(\mu+\mu'\frac{\partial}{\partial t}\right)\nabla^2\vec{u} \tag{4.1}$$

and $\quad \nabla.\vec{u}=0.$ (4.2)

Some assumptions regarding the shape and velocity of the suspended particles are taken as

- Shape of the suspended particles in the fluid is uniform spherical.
- The buoyancy forces on the particle are neglected.
- Large distance between the particles as compare to their diameter. So Interparticle reactions are ignored.
- Extra force due to the presence of particles is proportional to velocity difference between the particles and the fluid.
- Force exserted by fluid on particles and force exerted by particles on fluid balance each other.

So there must be an extra force equal in magnitude but opposite in sign in the equations of conservation of momentum or motion for the particles. If mN is the mass of particles per unit volume, then under the above assumptions, equations of conservation of momentum and mass for the particles are

$$mN\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}.\nabla)\vec{v})\right]=KN\left(\vec{u}-\vec{v}\right)\quad \text{and} \tag{4.3}$$

$$\frac{\partial N}{\partial t}+\nabla.\left(N\vec{v}\right)=0. \tag{4.4}$$

Let C_v is heat capacity of fluid at constant volume, C_{pt} is heat capacity of particles, C_p is heat capacity of fluid at constant pressure, T is temperature and q is effective thermal conductivity of the pure fluid. Volume fractions of the particles are assumed to be small; the effective properties of the suspension are considered as same as of clean fluid.

If we assume that the fluid and particles are in the *thermal equilibrium*, the equation of *heat conduction* is

$$\rho C_v \left[\frac{\partial}{\partial t} + \vec{u}.\nabla\right] T + mNC_{pt} \left[\frac{\partial}{\partial t} + \vec{v}.\nabla\right] T = q\nabla^2 T, \tag{4.5}$$

If C denotes the *solute concentration*, then equation of solute conduction gives

$$\rho C'_v \left[\frac{\partial}{\partial t} + \vec{u}.\nabla\right] C + mNC'_{pt} \left[\frac{\partial}{\partial t} + \vec{v}.\nabla\right] C = q'\nabla^2 C \tag{4.6}$$

where C'_v, C'_{pt} and q' denote the analogous solute quantities. Spiegel and Veronis [40] defined f as any one of the state variables (p, ρ, or T) and expressed in the form

$$f(x,y,z,t) = f_m + f_0\,(z) + f'(x,y,z,t) \tag{4.7}$$

where $f_m \rightarrow$ constant space average of f, $f_0 \rightarrow$ variation in the absence of motion and $f' \rightarrow$ fluctuation resulting from motion. The initial state of the system is taken to be quiescent layer with a uniform particle distribution N_0, therefore initial state in which velocity, temperature T, solute concentration C is given by $\vec{v} = (0,0,0), \vec{u} = (0,0,0)$

and $\quad T = T(z) = T_0 - \beta z, C = C(z) = C_0 + \beta' z,$

$$p = p(z) = p_m - g\int_0^z (\rho_m - \rho_0)\ dz\,,$$

$$\rho = \rho(z) = \rho_m \left[1 - \alpha_m (T - T_m) + \alpha'_m (C - C_m) + K_m (p - p_m)\right],$$

$$\alpha_m = -\left[\frac{1}{\rho}\frac{\partial \rho}{\partial T}\right]_m (= \alpha(say))\,,$$

$$\alpha'_m = -\left[\frac{1}{\rho}\frac{\partial \rho}{\partial C}\right]_m (= \alpha'(say))\,,$$

$$K_m = -\left[\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right]_m \tag{4.8}$$

$N_0 =$ Constant.

Perturb the initial state of the system. Let δp denote the *perturbation* in pressure p, $\delta\rho$ denote the perturbation in density ρ, θ denote the perturbation in temperature T, γ denote the perturbation in solute concentration C, $\vec{v}(u,v,w)$ denote the perturbation in fluid velocity, $\vec{u}(l,r,s)$ denote the perturbation in particle velocity, N denote perturbations in suspended particles number density N_0. The quantity $\delta\rho$, depend on θ and γ and is given by $\quad \delta\rho = -\rho_m \left(\alpha\theta - \alpha'\gamma\right).$ (4.9)

Then the linearized perturbation equations of the problem, Spiegel and Veronis [40], Scanlon and Segel [23], and Rivlin -Ericksen [32], become

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p + g\left(\alpha\theta - \alpha'\gamma\right)\lambda + \frac{KN}{\rho_m}\left(\vec{v} - \vec{u}\right) + \left(\nu + \nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{u}, \tag{4.10}$$

$$\nabla.\vec{u} = 0, \tag{4.11}$$

$$\left[\frac{m}{K}\frac{\partial}{\partial t} + 1\right]\vec{v} = \vec{u}, \tag{4.12}$$

$$\frac{\partial N}{\partial t} + \nabla.\left(N_0\vec{v}\right) = 0, \tag{4.13}$$

$$(1+h)\frac{\partial \theta}{\partial t} = \beta\left(\frac{G-1}{G}\right)(w + hs) + \kappa\nabla^2\theta \quad \textit{and} \tag{4.14}$$

$$(1+h')\frac{\partial \theta}{\partial t} = \beta'\left(\frac{G-1}{G}\right)(w + h's) + \kappa'\nabla^2\gamma \tag{4.15}$$

where $\mu, \mu', \nu = \frac{\mu}{\rho_m}, \nu' = \frac{\mu'}{\rho_m}, \kappa = \frac{q}{\rho_m C_v}$ and $\kappa' = \frac{q'}{\rho_m C'_v}$ stand for viscosity, viscoelasticity, kinematic viscosity, kinematic viscoelasticity, thermal diffusivity and analogous solute diffusivity, respectively. Also, $h = f(C_{pt}/C_v), h' = f(C'_{pt}/C'_v)$, $f = mN_0/\rho_m$, and $G = \frac{C_p\beta}{g}$. Initially, $\vec{v} = (0,0,0), \vec{u} = (0,0,0), T = T(z)$, and $N = N_0$ which implies (4.5) yields $0 = 0$, identically. After perturbation, (4.5) becomes

$$\left(\rho_m + \delta\rho\right)C_v\left[\frac{\partial}{\partial t} + \vec{u}.\nabla\right](T+\theta) + \left(mN_0 + mN\right)C_{pt}\left[\frac{\partial}{\partial t} + \vec{v}.\nabla\right](T+\theta) = q\nabla^2\left(T+\theta\right). \tag{4.16}$$

Follow Speigal and Veronis [40] where the flow equations are found to be the same as those for incompressible fluids except β is replaced by $\left(\beta - \frac{g}{C_p}\right)$ i.e. the *static temperature gradient* is replaced by its excess over the adiabatic and C_v is replaced by C_p.

So linearization of (4.5) gives

$$\frac{\partial \theta}{\partial t}+\frac{mN_0}{\rho_m}\frac{C_{pt}}{C_v}\frac{\partial \theta}{\partial t}=\left(\beta-\frac{g}{C_p}\right)(w+hs)+\frac{q}{\rho_m C_v}\nabla^2\theta. \tag{4.17}$$

that is, (4.14). However, β' remains unaltered and, as above, (4.6) yields (4.15).

4.3 The Dispersion Relation

Perturbed quantities are assumed to be of the following form for the analysis of disturbances into normal modes

$$[w,\theta,\gamma]=[W(z),\Theta(z),\Gamma(z)]\,exp\,(ik_x x+ik_y y+nt) \tag{4.18}$$

where k_x is the wave number along x-direction and k_y is wave number along y-direction. $k=\sqrt{k_x^2+k_y^2}$ = resultant wave number and n = growth rate = complex constant in general. Non dimensional form of equations (4.16), (4.10) - (4.15) become

$$\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)-(1+F\sigma)\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W+\frac{ga^2d^2}{\nu}\left(\alpha\Theta-\alpha'\Gamma\right)=0, \tag{4.19}$$

$$\left(D^2-a^2-Hp_1\sigma\right)\Theta=-\beta\left(\frac{G-1}{G}\right)\frac{d^2}{\kappa}\frac{(H+\tau_1\sigma)}{(1+\tau_1\sigma)}W \tag{4.20}$$

$$\textit{and}\quad \left(D^2-a^2-H'q\sigma\right)\Gamma=-\beta'\frac{d^2}{\kappa'}\frac{(H'+\tau_1\sigma)}{(1+\tau_1\sigma)}W \tag{4.21}$$

where we have put $a=kd$, $\sigma=\frac{nd^2}{\nu}$, $\tau=\frac{m}{\kappa}$, $\tau_1=\frac{\tau\nu}{d^2}$, $M=\frac{mN}{\rho_m}$, $p_1=\frac{\nu}{\kappa}$, $q=\frac{\nu}{\kappa'}$, $H=1+h, H'=1+h'$, $F=\frac{\nu'}{d^2}$ and $D=\frac{d}{dz}$.

Eliminate Γ and Θ from equations (4.19) and (4.21), then

$$\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)-(1+F\sigma)\left(D^2-a^2\right)\right]\left(D^2-a^2-Hp_1\sigma\right)\left(D^2-a^2-H'q\sigma\right)\left(D^2-a^2\right)W$$
$$-R\left(\frac{G-1}{G}\right)a^2\frac{(H+\tau_1\sigma)}{(1+\tau_1\sigma)}\left(D^2-a^2-H'q\sigma\right)W+Sa^2\frac{(H'+\tau_1\sigma)}{(1+\tau_1\sigma)}\left(D^2-a^2-Hp_1\sigma\right)W=0. \tag{4.22}$$

$where \quad R = \dfrac{g\alpha\beta d^4}{\nu\kappa}$ $\rightarrow$ thermal Rayleigh number

$S = \dfrac{g\alpha'\beta' d^4}{\nu\kappa'}$ $\rightarrow$ analogous solute Rayleigh number

$p_1 = \dfrac{\nu}{\kappa}$ $\rightarrow$ thermal Prandtl number

$q = \dfrac{\nu'}{\kappa}$ $\rightarrow$ analogous Schmidt number.

For the solution of the problem boundaries considered are perfect conductors of heat and solute and free. Surrounding medium is assumed to be electrically nonconducting. So boundary conditions taken as

$$\Theta = 0, W = 0, \Gamma = 0, \quad D^2W = 0, \quad DZ = 0 \quad at \quad z = 0 \quad and \quad z = 1. \tag{4.23}$$

For the solution to the problem, free boundaries are considered which is little artificial in nature but most suitable for stellar atmospheres. Using (4.23), even order derivatives of W vanish on the boundaries and so the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{4.24}$$

where W_0 = Constant. Substituting (4.24) in (4.22), the relation reduces to

$$R_1 x = \left(\frac{G}{G-1}\right)\left[\left\{i\sigma_1\left(1 + \frac{M}{1 + i\tau_1\sigma\pi^2}\right) + \left(1 + iF\sigma\pi^2\right)(1+x)\right\}\right.$$
$$\left.\left\{\frac{\left(1 + i\tau_1\sigma\pi^2\right)(1+x)(1+x+iHp_1\sigma)}{(H + i\tau_1\sigma\pi^2)}\right\} + S_1 x \frac{\left(H' + i\tau_1\sigma\pi^2\right)(1+x+iHp_1\sigma)}{(H + i\tau_1\sigma\pi^2)(1+x+iH'q\sigma)}\right]. \tag{4.25}$$

where $\quad R_1 = \dfrac{R}{\pi^4}, \quad x = \dfrac{a^2}{\pi^2}, \quad i\sigma_1 = \dfrac{\sigma}{\pi^2} \quad$ and $\quad S = \dfrac{a^2}{\pi^4}.$

Dispersion relation (4.25) studying the effects of suspended particles and compressibility on the double-diffusive convection in Rivlin-Ericksen elastico- viscous fluid.

4.4 The Stability and Oscillatory Modes

Here, we examine instability, if any, which can occur as oscillatory modes in the system defined. Multiplying (4.19) by the complex conjugate of W i.e W^*, integrating over $z = 0$ to $z = 1$ and making use of (4.20) and (4.21) with the help of boundary conditions (4.23), we obtain

$$\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)I_1+(1+F\sigma)I_2-\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{G}{G-1}\right)\left(\frac{1+\tau_1\sigma^*}{H+\tau_1\sigma^*}\right)(I_3+Hp_1\sigma^*I_4) + \frac{g\alpha' a^2\kappa'}{\nu\beta'}\left(\frac{1+\tau_1\sigma^*}{H'+\tau_1\sigma^*}\right)(I_5+H'q\sigma^*I_6)=0 \qquad (4.26)$$

$$\begin{aligned} where \quad I_1 &= \int_0^1 \left(|DW|^2+a^2|w|^2\right)dz \quad , \quad I_2=\int_0^1 \left(|D^2W|^2+2a^2|Dw|^2+a^4|W|^2\right)dz \\ I_3 &= \int_0^1 \left(|D\Theta|^2+a^2|\Theta|^2\right)dz \quad , \quad I_4=\int_0^1 |\Theta|^2 dz \\ I_5 &= \int_0^1 \left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz \quad , \quad I_6=\int_0^1 |\Gamma|^2 dz \end{aligned}$$

All the integrals $I_1, I_2, I_3, I_4, I_5, I_6$ are positive definite. Substituting $\sigma = i\sigma_i$ and equate the imaginary parts,where σ_i is real, we get small

$$\sigma_i\left[\left(1+\frac{M}{1+\tau_1\sigma_i}\right)I_1+FI_2+\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{G}{G-1}\right)\left(\frac{\tau_1(H-1)}{H^2+\tau_1^2\sigma_i^2}I_3+\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}Hp_1\sigma^*I_4\right) -\frac{g\alpha' a^2\kappa'}{\nu\beta'}\left(\frac{\tau_1(H'-1)}{H'^2+\tau_1^2\sigma_i^2}I_5+\frac{H'+\tau_1^2\sigma_i^2}{H'^2+\tau_1^2\sigma_i^2}H'q\sigma^*I_6\right)\right]=0 \qquad (4.27)$$

Here $\sigma_i = 0$ implies that modes may be non-oscillatory or $\sigma_i \neq 0$ implies that modes may be oscillatory. Presence of stable solute gradient introduces oscillatory modes.

4.5 Stationary Convection

$\sigma = 0$ characterized the marginal state When instability sets in as stationary convection, Put $\sigma = 0$ in dispersion relation (4.25) which reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{xH}+S_1\frac{H'}{H}\right]. \qquad (4.28)$$

and constitutive relation becomes linear for Rivlin-Ericksen elastico-viscous fluid. Behavior of $\frac{dR_1}{dS_1}$ explains the effect of stable solute gradient and behavior of $\frac{dR_1}{dH}$ explains the effect of suspended particles analytically. Equation (4.28) yields

$$\frac{dR_1}{dS_1} = \left(\frac{G}{G-1}\right)\frac{H'}{H}. \tag{4.29}$$

which is positive, thereby Rayleigh number and solute parameter increases simultaneously. So, stable solute gradient shows stabilizing effect.

$$\frac{dR_1}{dH} = -\left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{x} + S_1H'\right]\frac{1}{H^2}. \tag{4.30}$$

which is negative, which means suspended particles destabilize the system as the dimensionless Rayleigh number decreases with increase in the suspended particles number density. Therefore, We studied here, these effects graphically as below

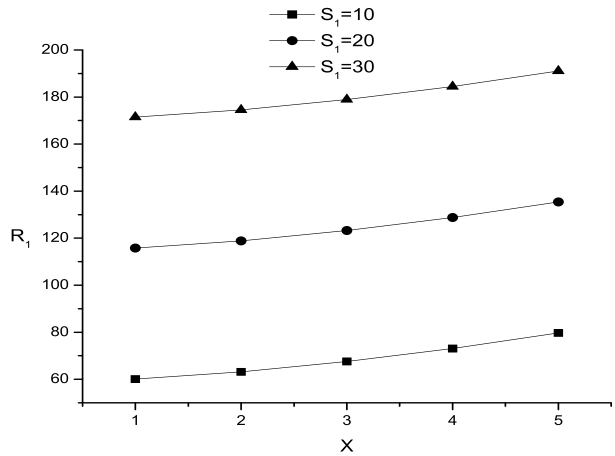

Figure 4.1: The variation of dimensionless Rayleigh number (R_1) with wave number $x(= 1, 2, 3, 4, 5)$, for $G = 9.8, H = 2, H' = 10$ and $S_1(= 10, 20, 30)$.

In **Figure4.1** (refer table 8), as the value of stable solute gradient parameter increased, so the value of Rayleigh number is increased for fixed values $G = 9.8, H = 2, H' = 10$ and $S_1(= 10, 20, 30)$ when taking values of wave number x(= 1,2,3,4,5) respectively. Therefore as value of Rayleigh number increased with increase in wave number showing the stabilizing effect.

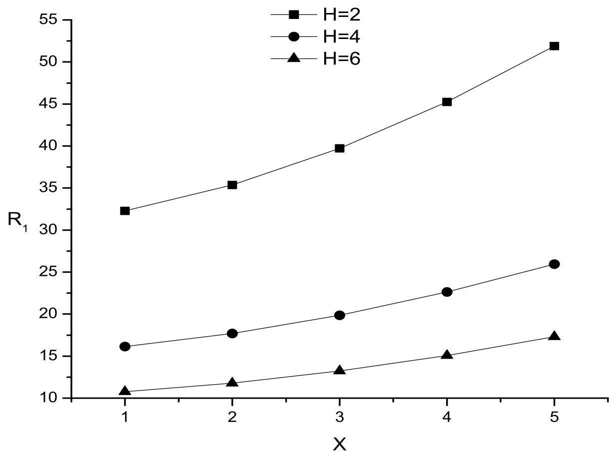

Figure 4.2: The variation of dimensionless Rayleigh number (R_1) with wave number $x(= 1, 2, 3, 4, 5)$, for $G = 9.8, S_1 = 10, H' = 5$ and $H(= 2, 4, 6)$.

In **Figure4.2** (refer table 9), Rayleigh number decreased with increase in the suspended particles by taking values of wave number x(= 1,2 3,4,5), for fixed values $G = 9.8, S_1 = 10, H' = 5$ and $H(= 2, 4, 6)$, respectively. Therefore as values of Rayleigh number has increased with decrease suspended particles parameter, showing the *destabilizing effect*.

Let G (accounting for the compressibility effects) be kept fixed for fixed S_1, H and H'. Then we have

$$\overline{R_c} = \left(\frac{G}{G-1}\right) R_c \tag{4.31}$$

where $\overline{R_c}$ is Critical Rayleigh number in the presence compressibility and R_c is Critical Rayleigh number in the absence of compressibility.In the presence of compressibility, $G < 1$ and $G = 1 \Rightarrow$ negative and infinite values of the critical Rayleigh number, which is irrelevant to the given system. $G > 1$ is relevant to the given system, thus compressibility postpone the onset of double-diffusive convection.

4.6 Conclusion

Effect of compressibility, stable solute gradient and suspended particles and has been investigated on thermosolutal convection of a Rivlin-Ericksen fluid. The study may be relevant to the stability of some polymer solutions and the problem finds its applications in chemical technology and in Geophysical situations . Hence a study has been made on thermosolutal convection in presence of compressible fluid with fine dust. Due to the vanishing of the viscoelastic parameter the constitutive relation for Rivlin-Ericksen fluid become linear for the case of stationary convection. It is obvious from the equation (4.31)that compressibility had postponed the onset of instability. The expressions $\frac{dR_1}{dS_1}$ explains the effects of stable solute gradient and $\frac{dR_1}{dH}$ explains the effect of suspended particles analytically. Stable solute gradient delay the onset of instability whereas suspended particles are found to hasten the onset of instability. **Figure1** and **Figure2**, shows the same results as obtained. The presence of viscoelasticity, suspended particles and stable solute gradient introduce the oscillatory modes. In the absence of viscoelasticity, suspended particles and stable solute gradient, the principle of exchange of stabilities holds good.

Chapter 5

Programming Codes

Programming codes to find Rayleigh number obtained in the chapters 2 ,3 and 4 by assigning numerical values to all other parameters .

5.1 Chapter 2 : Variations of Rayleigh number

Consider the equation (2.27) to find the variations in Rayleigh number R_1

5.1.1 When Q1 = 25,50,75

```
clear all

% Values assigned to various parameters

x=[1 2 3 4 5 6];
P=2;
H_dash=10;
Thetha=pi/4;
Epsilon=0.5;
Q1=[25 50 75];

% Intermediate calculations

A1=(1+x);
A2=A1/P;

A31=Q1(1,1)*power(cos(Thetha),2)*power(Epsilon,-1)*x;
A32=Q1(1,2)*power(cos(Thetha),2)*power(Epsilon,-1)*x;
A33=Q1(1,3)*power(cos(Thetha),2)*power(Epsilon,-1)*x;

A41=A2+A31;
A42=A2+A32;
A43=A2+A33;

A51=A1.*A41
```

```
A52=A1.*A42
A53=A1.*A43

B1=x*H_dash;

% Variation of Rayleigh number

Rayleigh_Number1=A51./B1     % when Q1 = 25
Rayleigh_Number2=A52./B1     % when Q1 = 50
Rayleigh_Number3=A53./B1     % when Q1 = 75

% Plot of  all Rayleigh numbers Vs. Wave number x

plot(x,Rayleigh_Number1)
hold on
plot(x,Rayleigh_Number2)
hold on
plot(x,Rayleigh_Number3)
```

5.1.2 When H-dash = 5,10,15

```
clear all

% Values assigned to various parameters

x=[1 2 3 4 5 6];
P=2;
H_dash=[5 10 15 ];
Thetha=pi/4;
Epsilon=0.5;
Q1=25;

% Intermediate calculations

A1=(1+x);
A2=A1/P;
A3=Q1*power(cos(Thetha),2)*power(Epsilon,-1)*x;
A4=A2+A3;
A5=A1.*A4

B11=x*H_dash(1,1);
B12=x*H_dash(1,2);
B13=x*H_dash(1,3);

% Variation of Rayleigh number

Rayleigh_Number1=A5./B11
Rayleigh_Number2=A5./B12
Rayleigh_Number3=A5./B13

% Plot of all Rayleigh numbers Vs. Wave number x
```

```
plot(x,Rayleigh_Number1)
hold on
plot(x,Rayleigh_Number2)
hold on
plot(x,Rayleigh_Number3)
```

5.1.3 When P = 0.1 ,0.2,0.6

```
clear all

% Values assigned to various parameters

x=[1 2 3 4 5 6];
P=[0.1 0.2 0.6];
H_dash=10;
Thetha=pi/4;
Epsilon=0.5;
Q1=25 ;

% Intermediate calculations

A1=(1+x);

A21=A1/P(1,1);
A22=A1/P(1,2);
A23=A1/P(1,3);

A3=Q1*power(cos(Thetha),2)*power(Epsilon,-1)*x;

A41=A21+A3;
A42=A22+A3;
A43=A23+A3;


A51=A1.*A41;
A52=A1.*A42;
A53=A1.*A43;

```

```
B1=x*H_dash;

% Variation of Rayleigh number

Rayleigh_Number1=A51./B1     % when P=0.1
Rayleigh_Number2=A52./B1     % when P=0.2
Rayleigh_Number3=A53./B1     % when P=0.6

% Plot of  all Rayleigh numbers Vs. Wave number x

plot(x,Rayleigh_Number1)
hold on
plot(x,Rayleigh_Number2)
hold on
plot(x,Rayleigh_Number3)
```

5.2 Chapter 3 : Variations of Rayleigh number

Consider the equation (3.45) to find the variations in Rayleigh number N_R

5.2.1 When P = 2 , 4 , 6

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=1000;
P=[2 4 6 ];
NQ=20;
M1=10;

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G -1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);
A4=power(Alpha,2)*H_Bar;
A5=A3./A4;
A6=A1*A5;

B11=1/P(1,1);
B12=1/P(1,2);
B13=1/P(1,3);
```

```
C1=NQ*power(Pi,2)*power(Epsilon,-1);

C21=P(1,1)*C1;
C22=P(1,2)*C1  ;
C23=P(1,3)*C1;

C31=C21+A3;
C32=C22+A3;
C33=C23+A3;

C41=C1*C31;
C42=C1*C32;
C43=C1*C33;

D1=power(M1,2)*power(Pi,2);
D21=C31+D1;
D22=C32+D1;
D23=C33+D1;

D31=A3.*D21;
D32=A3.*D22;
D33=A3.*D23;

D41=C41/D31;
D42=C42/D32;
D43=C43/D33;

E11=B11+D41;
E12=B12+D42;
E13=B13+D43;
```

```
% Values of Rayleigh number NR

NR1=A6*E11          %  When P=2
NR2=A6*E12          %  When P=4
NR3=A6*E13          %  When P=6

% Graphs of NR Vs Alpha

 plot(Alpha,NR1)
 hold on
 plot(Alpha,NR2)
 hold on
 plot(Alpha,NR3)
```

5.2.2 When NQ = 10,20,30

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=1000;
P=4 ;
NQ=[10 20 30];
M1=10;

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G -1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);
A4=power(Alpha,2)*H_Bar;
A5=A3./A4;
A6=A1*A5;

B1=1/P;

C11=NQ(1,1)*power(Pi,2)*power(Epsilon,-1);
C12=NQ(1,2)*power(Pi,2)*power(Epsilon,-1);
C13=NQ(1,3)*power(Pi,2)*power(Epsilon,-1);

C21=P*C11;
C22=P*C12 ;
```

```
C23=P*C13;

C31=C21+A3;
C32=C22+A3;
C33=C23+A3;

C41=C11*C31;
C42=C12*C32;
C43=C13*C33;

D1=power(M1,2)*power(Pi,2);
D21=C31+D1;
D22=C32+D1;
D23=C33+D1;

D31=A3.*D21;
D32=A3.*D22;
D33=A3.*D23;

D41=C41/D31;
D42=C42/D32;
D43=C43/D33;

E11=B1+D41;
E12=B1+D42;
E13=B1+D43;

% Values of Rayleigh number NR

NR1=A6*E11          %  When  P=2
NR2=A6*E12          %  When  P=4
```

```
NR3=A6*E13          %  When P=6

% Graphs of NR Vs Alpha

plot(Alpha,NR1)
hold on
plot(Alpha,NR2)
hold on
plot(Alpha,NR3)
```

5.2.3 When H-Bar = 500,1000,1500

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=[500 1000 1500 ];
P=2 ;
NQ=20;
M1=10;

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G -1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);

A41=power(Alpha,2)*H_Bar(1,1);
A42=power(Alpha,2)*H_Bar(1,2);
A43=power(Alpha,2)*H_Bar(1,3);

A51=A3./A41;
A52=A3./A42;
A53=A3./A43;

A61=A1*A51;
A62=A1*A52;
A63=A1*A53;
```

```
B1=1/P;

C1=NQ*power(Pi,2)*power(Epsilon,-1);
C2=P*C1;
C3=C2+A3;
C4=C1*C3;


D1=power(M1,2)*power(Pi,2);
D2=C3+D1;
D3=A3.*D2;
D4=C4/D3;

E1=B1+D4;


% Values of Rayleigh number NR

NR1=A61*E1        %  When P=2
NR2=A62*E1        %  When P=4
NR3=A63*E1        %  When P=6

% Graphs of NR Vs Alpha

plot(Alpha,NR1)
hold on
plot(Alpha,NR2)
hold on
plot(Alpha,NR3)
```

5.2.4 When M1= 10,20,30

```
clear all

% Assigning numertical values to various parameters

Alpha=[0.01 0.02 0.03 0.04 0.05 ];
Gravitational_Acc_G=9.8;
Pi=3.14;
Epsilon=0.5;
H_Bar=1000 ;
P=2 ;
NQ=20;
M1=[10 20 30];

% Intermediate calculations

A1=(Gravitational_Acc_G)/(Gravitational_Acc_G -1);
A2=power(Pi,2)+power(Alpha,2);
A3=power(A2,2);
A4=power(Alpha,2)*H_Bar;
A5=A3./A4;
A6=A1*A5;


B1=1/P;

C1=NQ*power(Pi,2)*power(Epsilon,-1);
C2=P*C1 ;
C3=C2+A3;
C4=C1*C3;

```

```
D11=power(M1(1,1),2)*power(Pi,2);
D12=power(M1(1,2),2)*power(Pi,2);
D13=power(M1(1,3),2)*power(Pi,2);

D21=C3+D11;
D22=C3+D12;
D23=C3+D13;

D31=A3.*D21;
D32=A3.*D22;
D33=A3.*D23;

D41=C4/D31;
D42=C4/D32;
D43=C4/D33;

E11=B1+D41;
E12=B1+D42;
E13=B1+D43;



% Values of Rayleigh number NR

NR1=A6*E11          % When P=2
NR2=A6*E12          % When P=4
NR3=A6*E13          % When P=6

% Graphs of NR Vs Alpha

```

```
plot(Alpha,NR1)
hold on
plot(Alpha,NR2)
hold on
plot(Alpha,NR3)
```

5.3 Chapter 4 : Variations of Rayleigh number

Consider the equation (4.28) to find the variations in Rayleigh number R_1

5.3.1 When S1 = 10 , 20 , 30

```
clear all

% Assigning numertical values to various parameters

Wave_Number=[1 2 3 4 5 ];
Gravitational_Acc_G=9.8;
H_dash=10;
H=2;
S_1=[10 20 30 ];

% Intermediate calculations

A2=(Gravitational_Acc_G)/(Gravitational_Acc_G -1)

F2=power(1+Wave_Number,3)./(Wave_Number*H)

C21=S_1(1,1)*(H_dash/H)
C22=S_1(1,2)*(H_dash/H)
C23=S_1(1,3)*(H_dash/H)

% Values of Rayleigh number

Rayleigh_Number1=A2*(F2+C21)
Rayleigh_Number2=A2*(F2+C22)
Rayleigh_Number3=A2*(F2+C23)
```

```
% Graphs of Rayleigh number Vs wave number

plot(Wave_Number, Rayleigh_Number1)
hold on
plot(Wave_Number, Rayleigh_Number2)
hold on
plot(Wave_Number, Rayleigh_Number3)
```

5.3.2 When H = 2 , 4 , 6

```
clear all

% Assigning numertical values to various parameters

Wave_Number=[1 2 3 4 5 ];
Gravitational_Acc_G=9.8;
H_dash=5;
H=[2 4 6 ];
S_1=10 ;

% Intermediate calculations

A2=(Gravitational_Acc_G)/(Gravitational_Acc_G -1)

F21=power(1+Wave_Number,3)./(Wave_Number*H(1,1))
F22=power(1+Wave_Number,3)./(Wave_Number*H(1,2))
F23=power(1+Wave_Number,3)./(Wave_Number*H(1,3))

C21=S_1*(H_dash/H(1,1))
C22=S_1*(H_dash/H(1,2))
C23=S_1*(H_dash/H(1,3))

% Values of Rayleigh number

Rayleigh_Number1=A2*(F21+C21)
Rayleigh_Number2=A2*(F22+C22)
Rayleigh_Number3=A2*(F23+C23)

% Graphs of Rayleigh number Vs wave number

```

```
plot(Wave_Number,Rayleigh_Number1)
hold on
plot(Wave_Number,Rayleigh_Number2)
hold on
plot(Wave_Number,Rayleigh_Number3)
```

Chapter 6

Conclusion and Future Scope

Concluding Remarks of the Thesis and Future Scope of the research work.

6.1 Concluding Remarks

The whole thesis is devided into four chapters. Chapter 1, is introductory/review of literature. In chapter 2, we have studied “Thermal instability of Rivlin-Ericksen elastico-viscous fluid with suspended particles through porous medium”, in chapter 3 we have studied, “Hall effect on thermal instability of visco-elastic dusty fluid through porous medium” and in chapter 4, “Double-diffusive convection in presence of compressible Rivlin-Ericksen fluid with fine dust”. In chapter 2, we found that magnetic field has stabilizing effect whereas suspended particles and medium permeability have destabilizing effect on the system. In chapter 3, we found that medium permeability have stabilizing as well as destabilizing effect only in presence of magnetic field, but in absence of magnetic field it holds the same result as in presence of suspended particles in chapter 2. As in the absence of suspended particles and presence of compressibility in chapter 3, magnetic field has stabilizing effect on the system. Also hall current is studied at here and found that hall current have destabilizing effect on the system. In chapter 4, we found that stable solute gradient have stabilizing effect, where has suspended particles have destabilizing effect on the system in the presence of compressibility.

From the observation of all these three chapters, we found that magnetic field has stabilizing effect, in presence of compressibility as well as incompressibility. Medium permeability have stabilizing as well as destabilizing effects on the system, Hall current have destabilizing effect whereas stable solute gradient have stabilizing effect on the system.

All these results are verified graphically and by computer programming, self created programming codes is the beauty of the thesis.

6.2 Future Scope

Fluid dynamics has many applications in all the branches of engineering like mechanical, aeronautical and chemical etc. In medical discipline it plays an important role. Observed problems of nature can be medelled by using fluid dynamics and can be solved by using appropriate analytical method or numerical method which gives approximate solution. Presently technology is driven by physics, one must know the physics of the problem only then solutions can be interpreted and useful in real life. Mathematical equations tells a lot about the problem and physics behind it. The biggest challenge is always to convert the real life fluid flow problem into mathematical equations . The questions that always arise :

a) What is appropriate element (1 D, 2 D or 3D).

b) What are appropriate initial or boundary conditions.

c) Which technique or method is well suited for the problem.

Sometime experiments can not be performed because it is time consuming and expensive, moreover resources are limited. Also it is not possible to done on all the scales. All numerical methods convert continuum problem into discrete problem and give the solution at nodal points not at all the points of domain. So simulations techniques can be useful.

Simulation : Here, Firstly the problem is observed from the real life situation, then problem is defined and converted into mathematical model which is the set of differential equations (ordinary or partial). Afterwards problem is solved by using the mathematical techniques or tools and results obtained.

Numerical Simulation : If solution to the problem is approximated by using one of the numerical methods like finite difference method, Finite element method, finite volume method, runge-Kutta method, Galerkian method or any other method which is well suited to given problem is called numerical simulation. Finite element method is numerical tool for simulation. It can be used upto micro and continuum scale but cannot be used for nano scale. Numerical techniques are those which can be programmed.

Simulation through software: For this purpose various softwares of computational fluid dynamics like ANSYS (FLUENT), COMSOL Multiphysics, ABAQUS, MARK, PAFEC, ADINA are available in the market. The processing of these softwares is based on the numerical methods.

One can pursue the research in the area of fluid structure interaction (FSI) and further simulate the results by using computational software.

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