

LOVELY PROFESSIONAL UNIVERSITY

INTEGRATED B.Sc. M.Sc.(H) MATHEMATICS

An Investigation of an Incompressible Carreau Fluid between Parallel Plates

*A synopsis submitted in fulfilment of the requirements
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Abstract

In the present article, the fundamental flows of incompressible Carreau fluid with slip boundary conditions have been considered. The flow geometry is assumed in cartesian co-ordinate system. The effects of radiation parameter Joule heating are also considered the temperature distribution. The non dimensional problem is solved by analytical method. It is observed from our investigation that, the velocity decreasing with increasing of magnetic parameter, the tends is reserved in the case of applied constant pressure gradient. The temperature is increasing function of brinkman number and radiation parameter.

Contents

1	Introduction	3
2	Literature review	4
3	Formulation of the problem	5
3.1	Plates moving in the opposite directions	6
3.2	Upper plate is moving with constant velocity	7
3.3	Both plates are moving	7
4	Velocity graph	9
5	Temperature graph	13
6	Future Work	16

1 Introduction

A fluid is a substance which deforms continuously under the application of a shear stress. The fluids are divided into two groups as a Newtonian fluid and non-Newtonian fluid. Those fluids which follow the Newton law of viscosity are called Newtonian fluids. These fluids do not have a constant viscosity with changing shear stress. In these fluids the viscosity of fluids change with applied pressure. Examples of Newtonian fluids are Water, milk etc. Those fluids which do not follow Newton law of viscosity are called non-Newtonian fluids. These fluids have a constant viscosity with changing shear stress. In these fluids the viscosity of fluids change with applied pressure. Examples of non-Newtonian fluids are ketchup, toothpaste, custard, blood etc. The non-Newtonian fluids are classified in various types due to the behaviour of the fluid. Such as Carreau fluid, Couple stress fluid, Bingham fluid, Casson fluid etc. In our topic, we have concentrated the Carreau fluid model between parallel plates in various directions under the effects of heat transfer and magnetic field.

2 Literature review

The study of non-Newtonian has increased a great attention of numerous researchers. It is due to wide range of applications in science and engineering. According to the behaviour of fluids, many researchers have given different constitute relationship between stress and rate of strain for various fluid model. Recently the type of non-Newtonian fluid (named Carreau fluid) has received much attention to the researchers in which viscosity depends the shear rate i.e. the viscosity of the fluid shows a dependence on the shear rate. At low shear rate Carreau fluid acts as a Newtonian fluid and at high shear rate as a power law fluid. Khellaf and Lauriat [1] have presented the discussion on the non-Newtonian Carreau fluid between revolving concentric vertical cylinders. Ellahi et al. [2] have discussed the peristaltic flow of Carreau Fluid in a rectangular tube through a porous medium. Hayat et al. [3] have studied the effect of magnetic field on peristaltic transport of a Carreau fluid. Akbar et al. [4] have presented the numerical result on peristaltic flow of a Carreau nano-fluid in an asymmetric channel. Riaz et al. [5] have studied the peristaltic transport of a Carreau fluid in a compliant rectangular tube. Uddin et al. [6] have discussed the squeeze flow of a Carreau fluid through sphere impact. Phan-thien et al. [7] have presented the squeeze flow of a viscoelastic fluid. Khan et al. [8] have studied the heat transfer squeezed flow of Carreau fluid over a sensor surface with variable thermal conductivity. Tshela [9] have discussed the flow of a Carreau fluid down a disposed with a free surface.

Recent days, the study of magnetohydrodynamic flows has received much interest due to its wide number of applications in science and engineering. Sobamowo and Akinshilo [10] have presented the numerical study on squeezing flow of nano-fluid between two parallel plates under the effect of magnetic field. Kudenatti et al. [11] have studied the effects of presented on the study of surface roughness and magnetic field between rube and permeable rectangular plates. Rajeev and Srivastava [12] have discussed the flow of an Unsteady couple stress fluid under the effects of magnetic field between parallel plates having permeable medium. Rashidi et al. [13] have given a model on the magnetohydrodynamic squeezing flow of nano-fluid between parallel plates in the existence of aligned magnetic field. Khuswaha and sahu [14] have studied the effect of heat transfer in the slip move area between parallel plates. Duwairi et al. [15] have discussed on the heat transfer effects of a viscous fluid squeezed and extruded between parallel plates. Mahmood et al. [16] have presented on a squeezed flow and heat transfer over a permeable surface for viscous fluid. Mohyud-Din et al. [17] have studied on heat and mass transfer result for the move of a nano-fluid between rotating parallel plates.

Heat is the flow of energy from body at high temperature to body at low temperature. These bodies could be both are solid form, a solid and a liquid or a gas. There are three different ways the heat can transfer. 1. Conduction (The transfer of heat between two solid bodies) 2. Convection (The transfer of heat between the solid surface and the liquids) 3. Radiation (When two bodies are at different temperatures and separated by distance, the heat transfer between them is called radiation). Examples are Greenhouse effect, Heat transfer in Human body, Thermal energy storage, Laser cooling. Schenk and Beckers [18] have presented the heat transfer in laminar move between parallel plates. Hsia and Love [19] presented a radiative heat transfer between parallel plates divided by a non-isothermal medium with anisotropic strewing. Murty and Prakash [20] have discussed magnetohydrodynamic two fluid flow and heat transfer between parallel plates in a revolving system. Welling and Wooldridge [21] have studied a free convection heat transfer coefficients from rectangular vertical fins. Viskanta and Grosh [22] have presented heat transfer by concurrent conduction and radiation in an digesting medium. Oleslami and Ganji [23] have discussed a nano-fluid and heat transfer between parallel plates examine Brownian movement using Differential Transformation Method. Oleslami and Ganji [24] have studied heat transfer of Cu-water nano fluid move between parallel plates. Alpher [25] have presented the heat transfer in magneto hydro energetic move between parallel plates. Dropkin and Somerscales [26] have discussed the heat transfer by real

convection in liquids compact by two parallel plates which are disposed at different angles with respect to the horizontal. Hollands et al.[27] have studied free convective heat transfer over disposed air sheets.

3 Formulation of the problem

Consider an incompressible Carreau fluid is placed between horizontal, infinite, parallel plates. The flow is due to constant pressure gradient and the moving plates in various directions. With these assumptions the governing equation for the flow of an incompressible Carreau fluid are

$$\nabla \cdot \bar{q} = 0, \quad (1)$$

$$\rho \left(\frac{\partial \bar{q}}{\partial t} + (\bar{q} \cdot \nabla) \bar{q} \right) = -\nabla \cdot p + \nabla \cdot S - \sigma B_0^2 \bar{q}, \quad (2)$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + \bar{q} \cdot \nabla T \right) = \nabla \cdot (k^* \nabla T) - \frac{\partial q_r}{\partial y} + \frac{\bar{J}^2}{\sigma} + Q_0, \quad (3)$$

where

$$S = (\eta_\infty + (\eta_0 + \eta_\infty)(1 - \Gamma \bar{\gamma})^{-1} \bar{\gamma}), \quad (4)$$

$$\bar{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \bar{\gamma}_{ij} \bar{\gamma}_{ji}} = \sqrt{\frac{1}{2} \mathbb{II}}, \quad (5)$$

Here \mathbb{II} is the second invariant strain tensor, \bar{q} is the velocity vector, ρ is the density, t is the time, S is the extra stress tensor, \bar{J} is the current vector due to ohm's law, T is the temperature, C_p is the specific heat at constant pressure, k^* is the thermal conductivity, Q_0 is the heat generation parameter. The steady flow between parallel plates, we consider the velocity field $\bar{q} = (u_i(y), 0, 0)$. The velocity automatically satisfied the continuity equation (1) and reduce the equation (2) is

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(\eta_0 \frac{\partial u_i}{\partial y} \left(1 + \frac{(p-1)}{2} \Gamma^2 \left(\frac{\partial u_i}{\partial y} \right)^2 \right) \right) - \sigma B^2 u_i, \quad (6)$$

$$k^* \frac{d^2 T}{dy^2} + \sigma B_0^2 u^2 - 4\alpha^2 (T_0 - T) + Q = 0, \quad (7)$$

non-dimensional quantities are

$$\bar{u}_i = \frac{u_i}{U_i}; \bar{y} = \frac{y}{h}; \gamma = \frac{\Gamma U}{h}; \mu = \sqrt{\frac{\sigma}{\eta_0}} B_0 h; \bar{p} = \frac{h}{\eta_0 U}; \bar{x} = \frac{x}{h}; B_* = \frac{qh^2}{k^*(T_1 - T_0)}; N = \frac{2\alpha h}{\sqrt{k^*}}$$

$$Br = \frac{\mu U^2}{k^*(T_1 - T_0)}; \theta = \frac{T - T_0}{T_1 - T_0}$$

using the above dimensionless parameter, the non-dimensional problem for momentum and temperature equations are

$$\left(\frac{d^2 u}{dy^2} + \frac{3}{2} (n-1) \gamma^2 \left(\frac{du}{dy} \right)^2 \left(\frac{d^2 u}{dy^2} \right) \right) - \mu^2 u = -G \quad (8)$$

$$\frac{d^2\theta}{dy^2} + N^2\theta + Br\mu^2u^2 + B_* = 0 \quad (9)$$

with dimensionless conditions are

$$u - \beta \left(\frac{du}{dy} + \frac{1}{2}(n-1)\gamma^2 \left(\frac{du}{dy} \right)^3 \right) = 1; \theta_1 - \xi \frac{d\theta_1}{dy} = 0 \text{ at } y = -h \quad (10)$$

$$u + \beta \left(\frac{du}{dy} + \frac{1}{2}(n-1)\gamma^2 \left(\frac{du}{dy} \right)^3 \right) = -1; \theta_1 + \xi \frac{d\theta_1}{dy} = 0 \text{ at } y = h \quad (11)$$

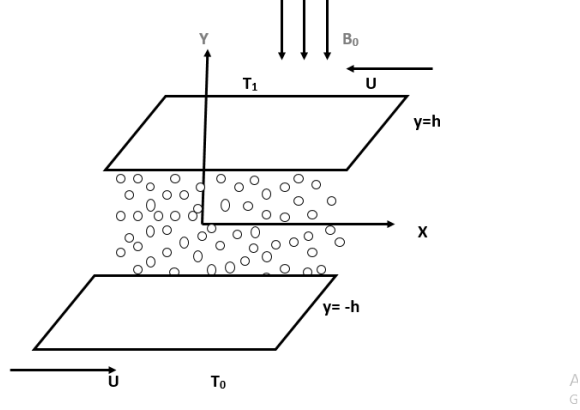


Figure 1: Geometry of the problem

3.1 Plates moving in the opposite directions

In this case, we consider the flow due to a constant pressure gradient and the plates are moving in opposite directions (see figure 1). The upper plate $y = h$ is suddenly jerked with constant velocity u in the left side while the lower plate $y = -h$ is moving in the right side of the geometry. Using the equations (8) and (9) corresponding non-dimensional problem can be written as

$$\left(\frac{d^2u_1}{dy^2} + \frac{3}{2}(p-1)\gamma^2 \left(\frac{du_1}{dy} \right)^2 \left(\frac{d^2u_1}{dy^2} \right) \right) - \mu^2u_1 = -G \quad (12)$$

$$\frac{d^2\theta_1}{dy^2} + N^2\theta_1 + Br\mu^2u_1^2 + B_* = 0 \quad (13)$$

With dimensionless boundary conditions

$$u_1 - \beta \left(\frac{du_1}{dy} + \frac{1}{2}(p-1)\gamma^2 \left(\frac{du_1}{dy} \right)^3 \right) = 1; \theta_1 - \xi \frac{d\theta_1}{dy} = 0 \text{ at } y = -1 \quad (14)$$

$$u_1 + \beta \left(\frac{du_1}{dy} + \frac{1}{2}(p-1)\gamma^2 \left(\frac{du_1}{dy} \right)^3 \right) = -1; \theta_1 + \xi \frac{d\theta_1}{dy} = 0 \text{ at } y = 1 \quad (15)$$

the solutions of equations (12) and (13) with the help of conditions (14) and (15), we get

$$u_1 = C_1e^{\mu y} + C_2e^{-\mu y} + \frac{G}{\mu^2} + \gamma^2 \left(C_3e^{\mu y} + C_4e^{-\mu y} - \frac{3}{16}(n-1)\mu^2 (C_1^3e^{3\mu y} + C_2^3e^{-3\mu y} + 4\mu y C_1 C_2 (-C_1e^{\mu y} + C_2e^{-\mu y})) \right) \quad (16)$$

$$\theta_1 = C_5 \cos(Ny) + C_6 \sin(Ny) - \frac{B_*}{N^2} - Br\mu^2 (A_{11}e^{\mu y} + A_{12}e^{-\mu y} + A_{13}e^{2\mu y} + A_{14}e^{-2\mu y} + A_{15}e^{3\mu y} + A_{16}e^{-3\mu y} + A_{17}e^{4\mu y} + A_{18}e^{-4\mu y} + A_{19}e^{6\mu y} + A_{20}e^{-6\mu y} + A_{21}) \quad (17)$$

3.2 Upper plate is moving with constant velocity

In this case, we consider the flow due to a constant pressure gradient and the plates are moving (see figure 1). The upper plate $y = h$ is suddenly jerked with constant velocity u while the lower plate $y = -h$ is at rest in the geometry. Using the equations (8) and (9) corresponding non-dimensional problem can be written as

$$\left(\frac{d^2 u_2}{dy^2} + \frac{3}{2}(p-1)\gamma^2 \left(\frac{du_2}{dy} \right)^2 \left(\frac{d^2 u_2}{dy^2} \right) \right) - \mu^2 u_2 = -G \quad (18)$$

$$\frac{d^2 \theta_2}{dy^2} + N^2 \theta_2 + Br\mu^2 u_2^2 + B_* = 0 \quad (19)$$

With dimensionless boundary conditions

$$u_2 - \beta \left(\frac{du_2}{dy} + \frac{1}{2}(p-1)\gamma^2 \left(\frac{du_2}{dy} \right)^3 \right) = 1; \theta_2 - \xi \frac{d\theta_2}{dy} = 0 \text{ at } y = -1 \quad (20)$$

$$u_2 + \beta \left(\frac{du_2}{dy} + \frac{1}{2}(p-1)\gamma^2 \left(\frac{du_2}{dy} \right)^3 \right) = 0; \theta_2 + \xi \frac{d\theta_2}{dy} = 0 \text{ at } y = 1 \quad (21)$$

the solutions of equation (18) and (19) with the help of conditions (20) and (21), we get

$$u_2 = C_7 e^{\mu y} + C_8 e^{-\mu y} + \frac{G}{\mu^2} + \gamma^2 (C_9 e^{\mu y} + C_{10} e^{-\mu y} - \frac{3}{16}(n-1)\mu^2 (C_7^3 e^{3\mu y} + C_8^3 e^{-3\mu y} + 4\mu y C_7 C_8 (-C_7 e^{\mu y} + C_8 e^{-\mu y}))) \quad (22)$$

$$\theta_2 = C_{11} \cos(Ny) + C_{12} \sin(Ny) - \frac{B_*}{N^2} - Br\mu^2 (B_{11}e^{\mu y} + B_{12}e^{-\mu y} + B_{13}e^{2\mu y} + B_{14}e^{-2\mu y} + B_{15}e^{3\mu y} + B_{16}e^{-3\mu y} + B_{17}e^{4\mu y} + B_{18}e^{-4\mu y} + B_{19}e^{6\mu y} + B_{20}e^{-6\mu y} + B_{21}) \quad (23)$$

3.3 Both plates are moving

In this case, we consider the flow due to a constant pressure gradient and both the plates are moving (see figure 1). The upper plate $y = h$ is suddenly jerked with constant velocity u while the lower plate $y = -h$ is moving with constant velocity u of the geometry. Using the equations (8) and (9) corresponding non-dimensional problem can be written as

$$\left(\frac{d^2 u_3}{dy^2} + \frac{3}{2}(p-1)\gamma^2 \left(\frac{du_3}{dy} \right)^2 \left(\frac{d^2 u_3}{dy^2} \right) \right) - \mu^2 u_3 = -G \quad (24)$$

$$\frac{d^2 \theta_3}{dy^2} + N^2 \theta_3 + Br\mu^2 (u_3)^2 + B_* = 0 \quad (25)$$

With dimensionless boundary conditions

$$u_3 - \beta \left(\frac{du_3}{dy} + \frac{1}{2}(p-1)\gamma^2 \left(\frac{du_3}{dy} \right)^3 \right) = 1; \theta_3 - \xi \frac{d\theta_3}{dy} = 0 \text{ at } y = -1 \quad (26)$$

$$u_3 + \beta \left(\frac{du_3}{dy} + \frac{1}{2}(p-1)\gamma^2 \left(\frac{du_3}{dy} \right)^3 \right) = 1; \theta_3 + \xi \frac{d\theta_3}{dy} = 0 \text{ at } y = 1 \quad (27)$$

the solutions of equations (24) and (25) with the help of conditions (26) and (27), we get

$$u_3 = C_{13}e^{\mu y} + C_{14}e^{-\mu y} + \frac{G}{\mu^2} + \gamma^2 \left(C_{15}e^{\mu y} + C_{16}e^{-\mu y} - \frac{3}{16}(n-1)\mu^2 \right. \\ \left. (C_{13}^3 e^{3\mu y} + C_{14}^3 e^{-3\mu y} + 4\mu y C_{13} C_{14} (-C_{13}e^{\mu y} + C_{14}e^{-\mu y})) \right) \quad (28)$$

$$\theta_3 = C_{17}\cos(Ny) + C_{18}\sin(Ny) - \frac{B_*}{N^2} - Br\mu^2 (D_{11}e^{\mu y} + D_{12}e^{-\mu y} + D_{13}e^{2\mu y} + D_{14}e^{-2\mu y} + \\ D_{15}e^{3\mu y} + D_{16}e^{-3\mu y} + D_{17}e^{4\mu y} + D_{18}e^{-4\mu y} + D_{19}e^{6\mu y} + D_{20}e^{-6\mu y} + D_{21}) \quad (29)$$

where $A_{11}, A_{12} \dots A_{57}, B_{11}, B_{12} \dots B_{57}, C_1, C_2 \dots C_{18}, D_{11}, D_{12} \dots D_{57}$ are constants.

4 Velocity graph

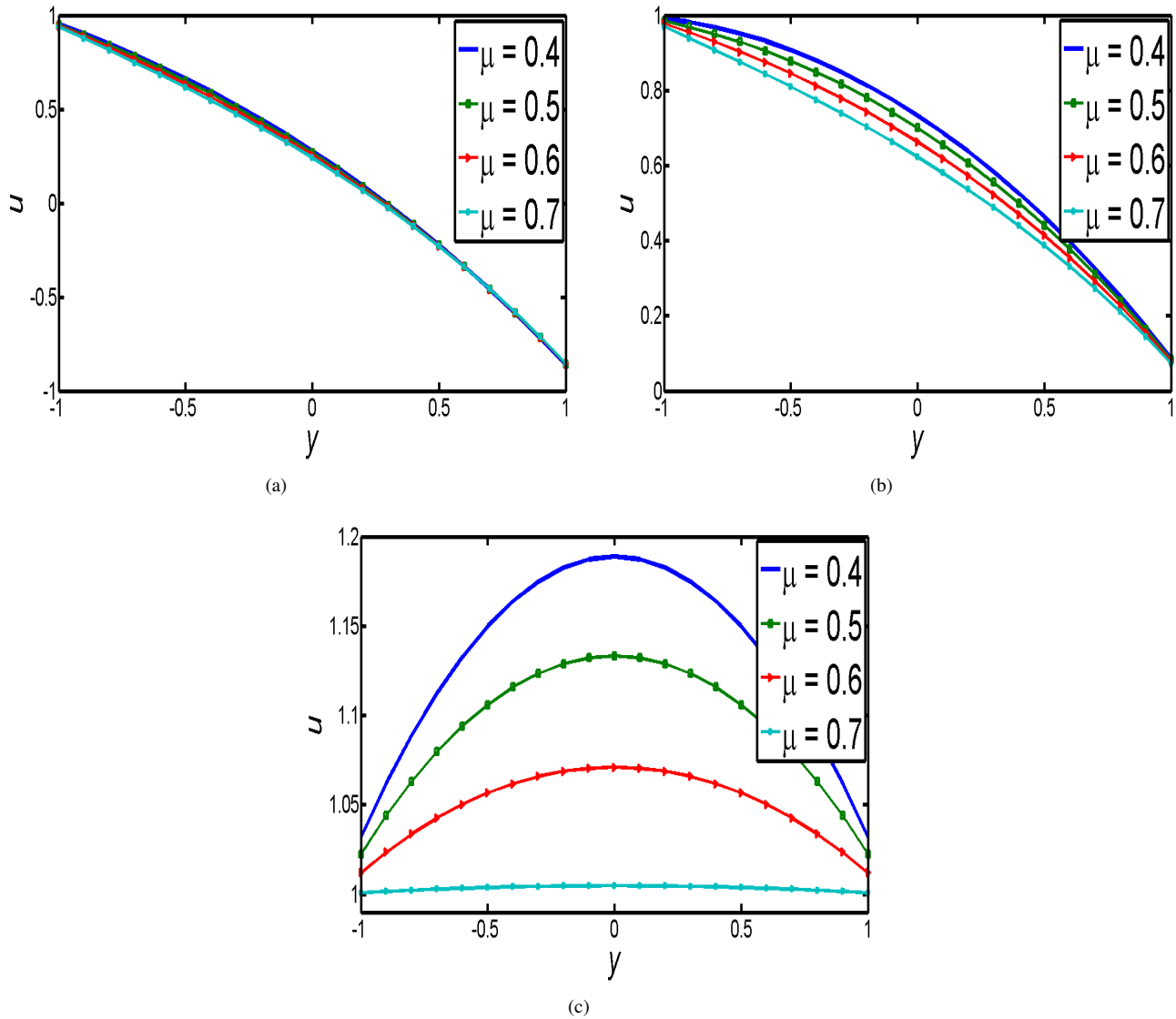


Figure 2: Velocity graph for various value of Hartmann number(μ)

Figure 2 is prepared to see the behaviour of Hartmann number μ on the velocity in different flow situation. It is noted from this graph that the velocity of fluid decrease with increase the value of Hartmann number. It is due to the fact that Hartmann number depends on the Lorentz force and Lorentz force is an agent which resists the flow. Increase of the Hartmann number tends to that the Lorentz force increase which leads to a decrease in velocity of fluid.

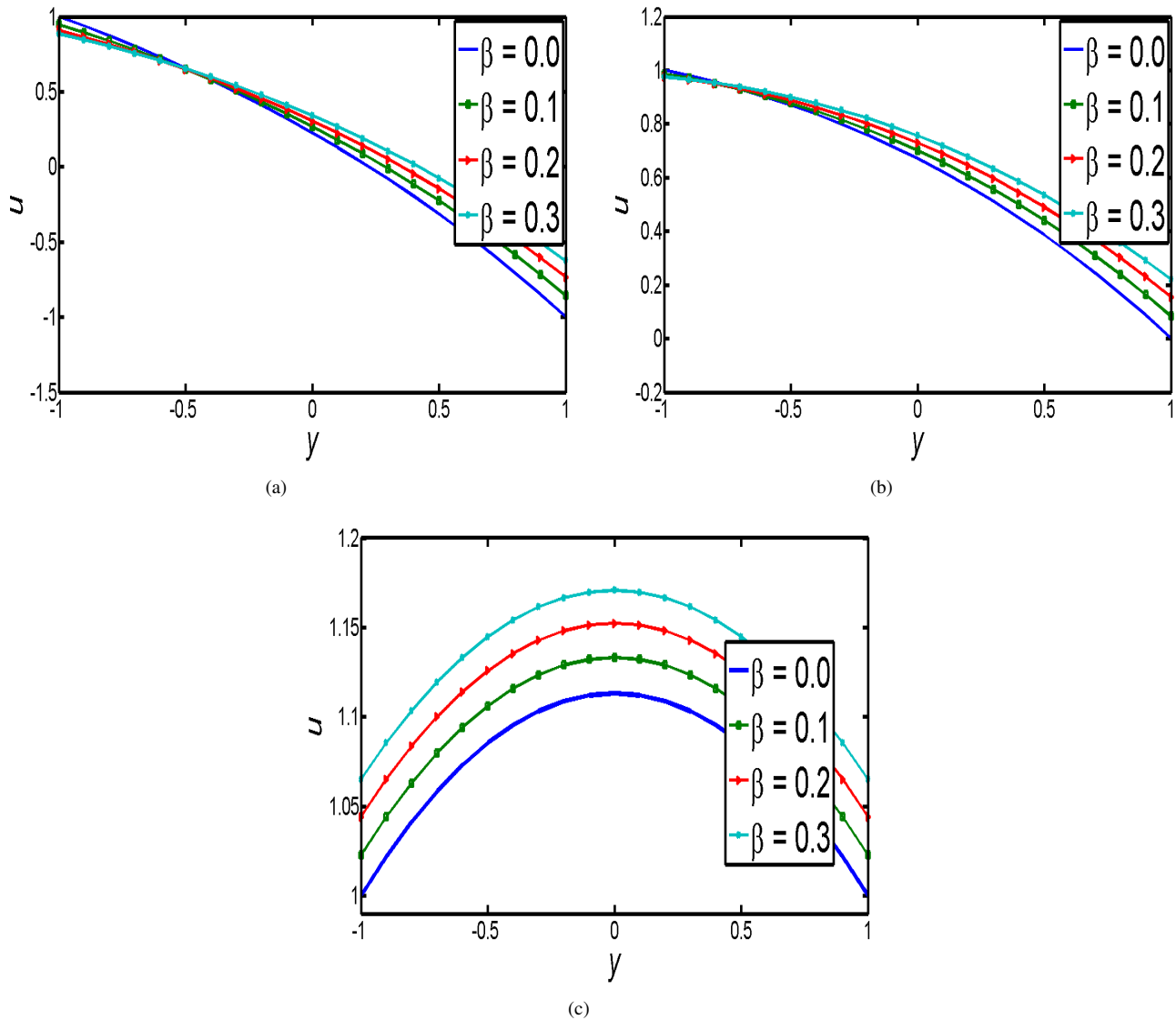


Figure 3: Velocity graph for various value of slip parameter(β)

The behaviour of slip parameter β on the velocity in different flow situation is plotted in Figure 3. In graph (a) and (b) to see the velocity of upper plate is increases with increases of the slip parameter while lower plate is decreases with increases of the slip parameter. In graph (c) to observed the velocity of fluid increase with increase of the slip parameter. It is due to the fact that increase of heat generation causes reduction of the fluid viscosity which will increase the fluid velocity.

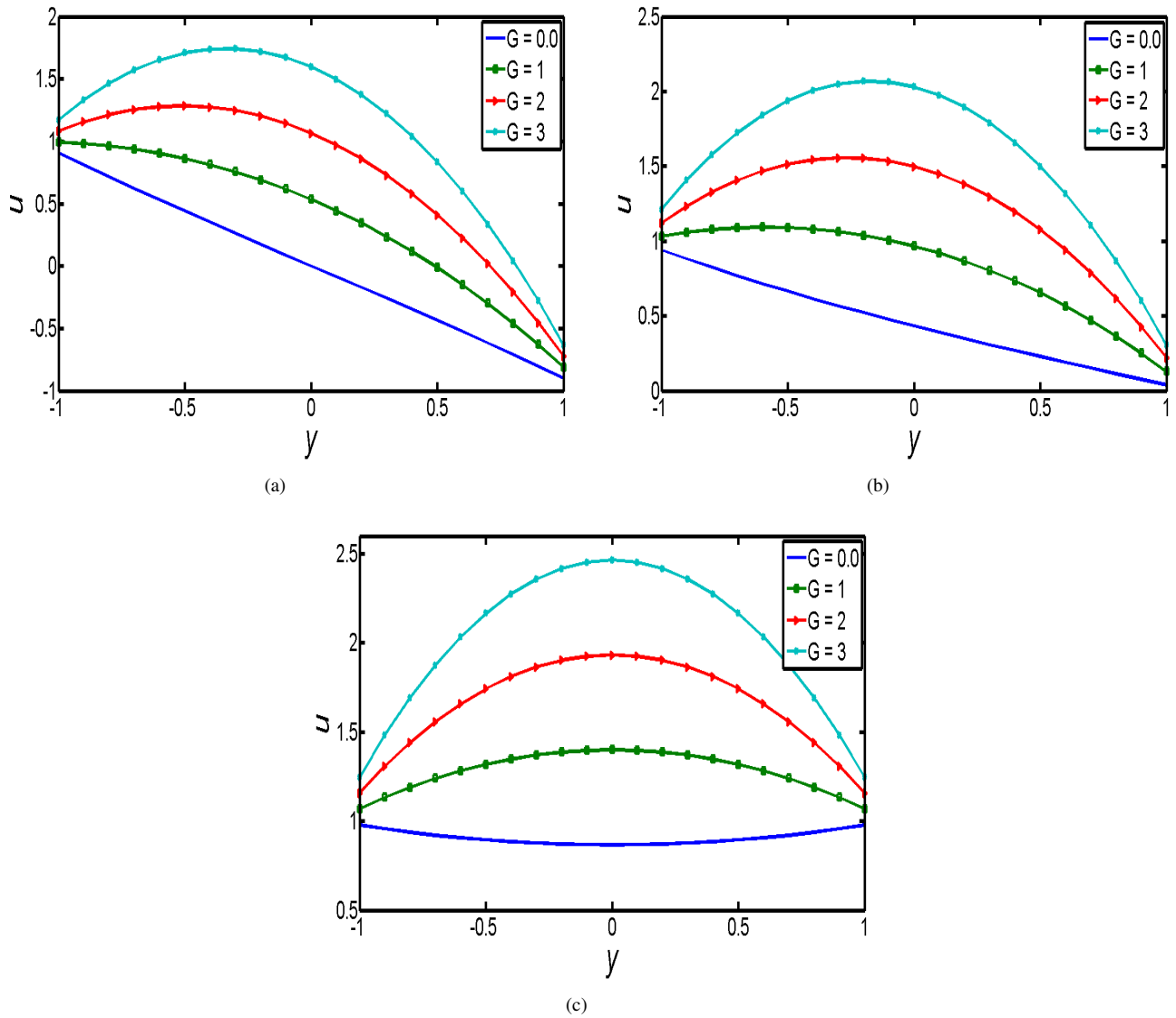


Figure 4: Velocity graph for various value of pressure gradient(G)

The influence of pressure gradient (G) on the velocity of hydromagnetic Carreau fluid in different flow situation is framed in Figure 4. It is noted from this graph that the velocity of fluid increases with increases of pressure gradient. Since as more pressure increases to the fluid, more fluid take place with the high velocity in the plate.

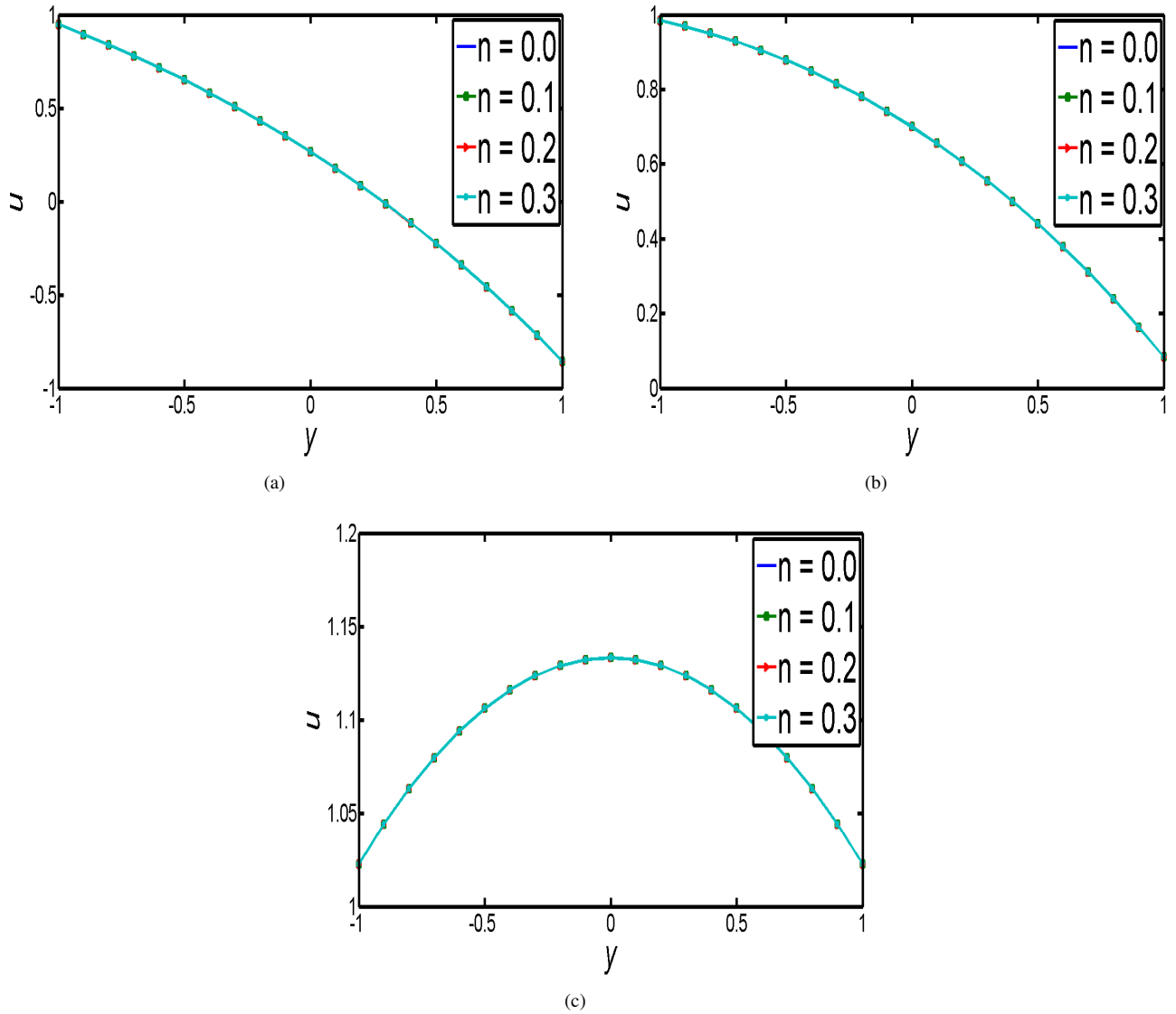


Figure 5: Velocity graph for various value of Carreau fluid parameter(n)

The effect of Carreau fluid parameter (n) on the velocity of fluid in different flow condition is constructed in Figure 5. It is observed from this graph that the velocity of fluid increases with increases of Carreau fluid parameter (n).

5 Temperature graph

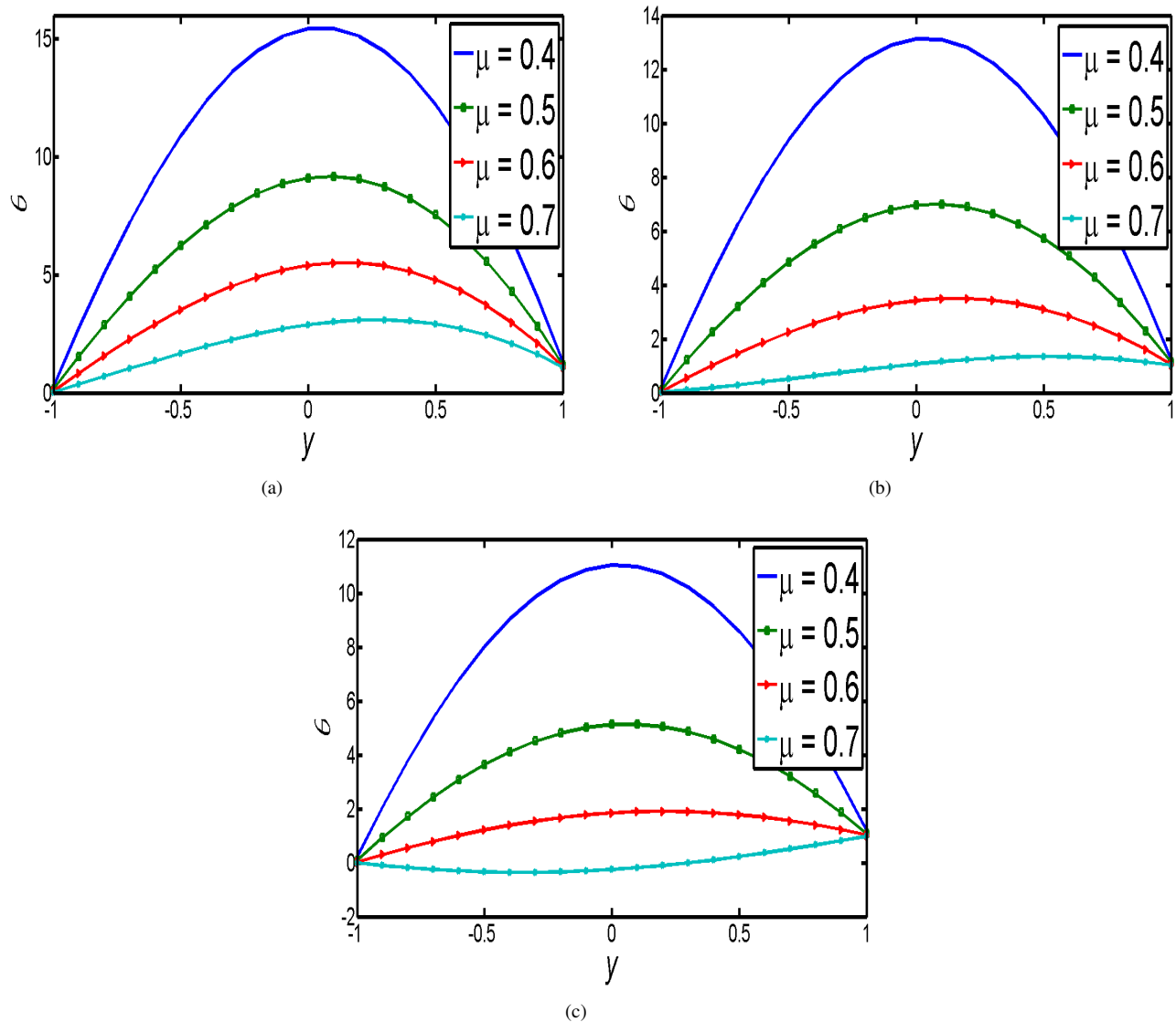


Figure 6: Temperature graph for various value of hartmann number(μ)

Figure 6 is prepared to see the behaviour of Hartmann number (μ) on the temperature in different flow situation. It is noted from this graph that, the temperature of fluid decreases with increases value of Hartmann number (μ). It is due to the fact that, the transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and thus decreases its temperature.

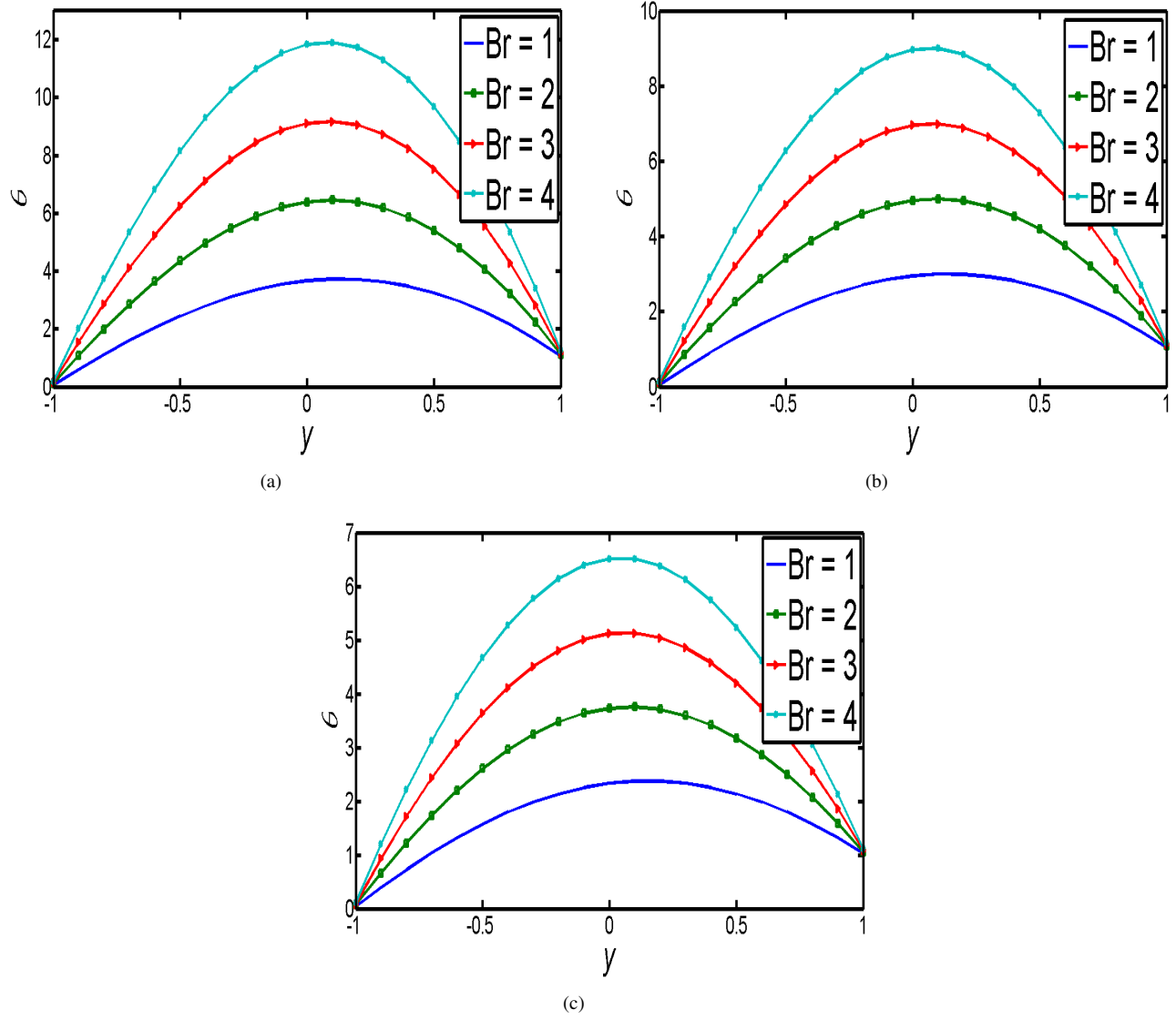


Figure 7: Temperature graph for various value of brinkman number(Br)

The behaviour of Brinkman number(Br) on the temperature of fluid in different flow situation is plotted in Figure 7. In this graph to observed the temperature of fluid increases with increases of Brinkman number(Br).

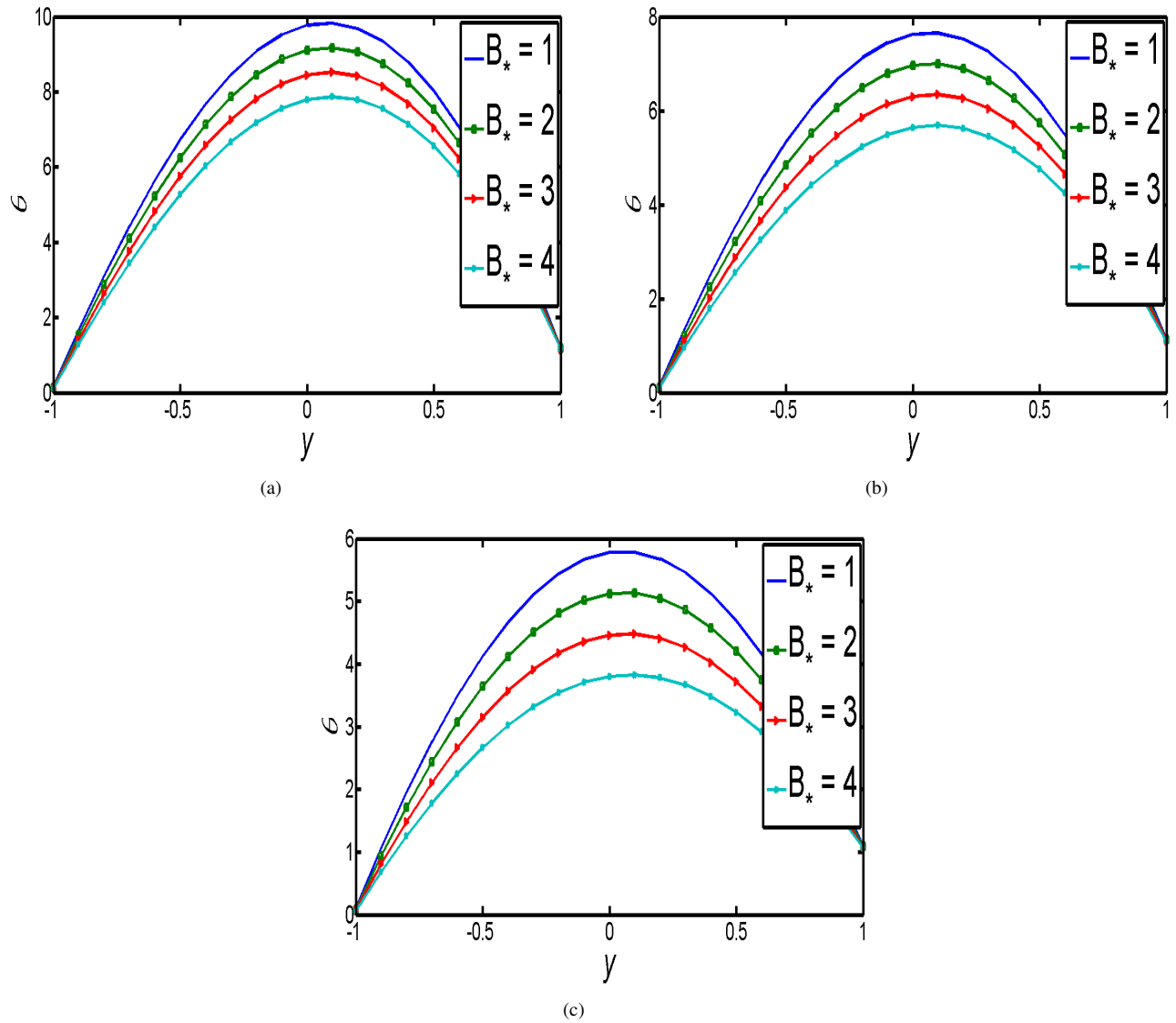


Figure 8: Temperature graph for various value of heat generation parameter(B_*)

The influence of heat generation parameter (B_*) on the temperature of hydromagnetic Carreau fluid in different flow situation is framed in Figure 8. It is noted from this graph that the temperature of fluid decreases with increases of heat generation parameter (B_*).

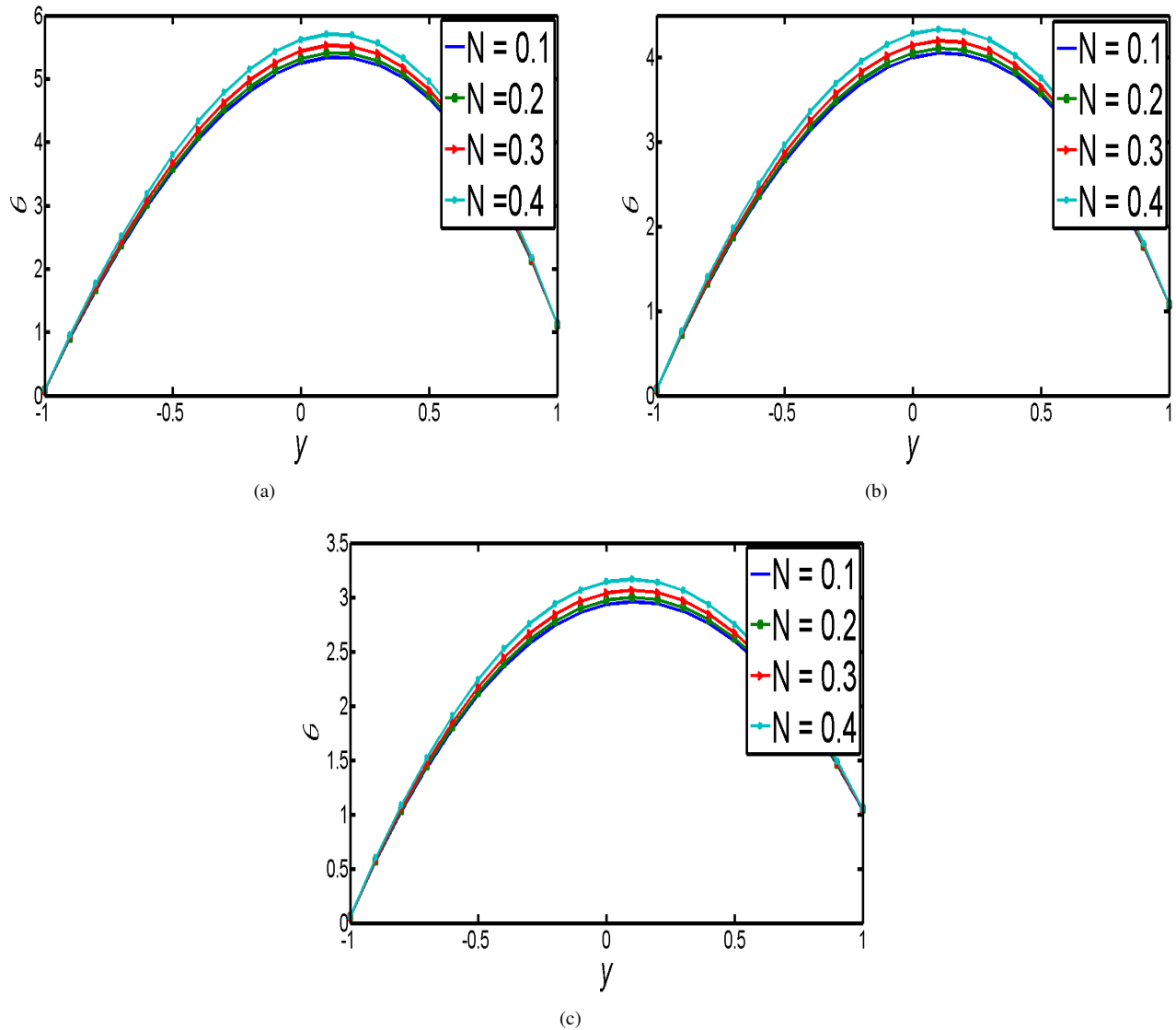


Figure 9: Temperature graph for various value of radiation parameter(N)

The effect of radiation parameter(N) on the temperature of fluid in different flow condition is constructed in Figure 9. It is observed from this graph that the temperature of fluid increases with increases of radiation parameter(N).

6 Future Work

We have discussed, in the synopsis, the Carreau fluid between parallel plates, the flow is arrived in the presence of constant pressure gradient and the plates moving in some directions. We are constructing this material in a paper format, if possible, we will communicate this article in next term.

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