LOVELY PROFESSIONAL UNIVERSITY

MASTER OF SCIENCE(H)

MHD BOUNDARY LAYER FLOW AND HEAT TRNSFER OVER A STRETCHING SHEET

A Project submitted in partial fulfillment of the requirements

for the Degree of Master of Science(H)

in the

Department of Mathematics

School of Chemical Engineering and Physical Sciences

Lovely Faculty of Technology and Sciences

Author: Charanjit Kaur

Supervisor: Dr. Narayan Prasad

Registration No.: 11510152

CERTIFICATE

This is certify that **Charanjit Kaur** has completed project title, **“MHD Boundary Layer Flow and Heat Transfer Over a Stretching Sheet ”** under my guidance and supervision. This project is fit for the submission and the partial fulfillment of the conditions for the award of master, in Mathematics.

Signed:

____

Supervisor: Dr. Narayan Prasad

Date:

DECLARATION

I Charanjit Kaur declare that this project titled “MHD Boundary Layer Flow and Heat Transfer Over as Stretching Sheet” and the work presented in it are my own. I confirm that

- This work done wholly or mainly whiles the candidature foe a masters degree at this university.
- Where any part of this project has previously been submitted foe a degree and any other qualification at this university or any other institution, this has been clearly stated.
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Signed:

Author: Charanjit Kaur

Registration No. 11510152

Date: April 2017

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Abstract

Master of Science

MHD BOUNDARY LAYER FLOW AND HEAT TRNSFER OVER A STRETCHING SHEET

By Charanjit Kaur

The main aim of this work is to study MHD boundary layers flow and heat transfer of a fluids over a stretching sheet in the presence of heat transfer and thermal radiation The similarity solution of the governing equation have been obtained and the reduced equation have been solved by using Numerical method .Numerical solutions of these equations are obtained by Runge- Kutta fourth order with shooting method .Numerical results obtained for different parameters such as magnetic parameter M ,Prandtl number(Pr),Radiation parameter (R)on velocity and temperature have been analyzed and discussed.

ACKNOWLEDGEMENT

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Words at my command are inadequate to convey my profound to my parents whose love, affection and blessings has inspired me the most. I also want to thank my friends for their constant motivation and support.

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Chapter 1

1.1 INTRODUCTION

MAGNETOHYDRODYNAMICS (MHD) & ITS APPPLICATIONS

Magneto hydrodynamics (magneto fluid dynamics or hydro magnetic) is the study of the magnetic properties of electrically conducting fluids. Examples of such magneto fluids include plasmas, liquid metals, and salt water or electrolytes. MHD is the physical-mathematical framework that concerns the dynamics of magnetic field in electrically conducting fluid, e.g. in plasma and liquid metals. The word magneto hydrodynamics is comprised of the words magneto-meaning magnetic, hydro-meaning water (or liquid) and dynamics referring to the movement of an object by forces. Synonyms of MHD that are less frequently used are the terms magnetofluid dynamics and hydro magnetic. The set of equations that describe MHD are a combination of the Navier-Stokes equation of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations must be solve simultaneously, either analytically or numerically. For fundamental work and discovers in magneto hydrodynamics with fruitful application in different parts of plasma physics. The central point of MHD theory is that conductive fluid can support magnetic fields. The presence of magnetic field leads to forces that in turn act on the fluid(typically a plasma),thereby potentially altering the geometry(or topology) and strength of magnetic fields themselves for a particular conducting fluid is the relative strength of the advancing motions in the fluid, compared to the diffusive effects caused by the electrical resistivity. These are two basic effects of MHD. The essential features of the physical situation is the common relation between velocity field and electromagnetic field.*MHD covers phenomena in electrically conducting fluids, where the velocity field V, and the magnetic field B are couple*

Hannes Alfven (1908-1995)

1.2 MHD APPLICATIONS

- Astrophysics(planetary magnetic field)
- MHD pumps (1907)
- MHD generators (1923)
- MHD flow meters (1935)
- Dispersion of metals
- Ship propulsion
- Crystal growth
- Magnetic filtration and separation
- Jet printers
- Fusion reactors
- Accelerator
- Plasma get engines
- Controlled thermonuclear reactor
- In some MHD applications, the electric current is applied to create MHD propulsion force.

1.3 BOUNDARY VALUE PROBLEM

In mathematics, something that indicates bound or limit or a boundary line is known as boundary. A Boundary value problem is a system of ordinary differential equations with solution and derivative values specified at more than one point. Most commonly, the solution and derivative are specified at just two points (the boundaries) defining a two-point boundary value problem.

A Boundary value problem is a problem, typically an ordinary differential equation or a partial differential equation, which has values assigned on the physical boundary of the domain in which the problem is specified. In differential equations, the differential equation that contains a set of restrains or limitations is known as boundary conditions.

These are following types of boundary conditions

1.3.1 DIRICHLET BOUNDARY CONDITION:

In mathematics, the dirichlet boundary condition is a type of boundary condition, named after Peter Gustav Lejeune Dirichlet(1805-1859) when imposed on an ordinary or a partially differential equation, it specifies the values that a solution needs to take on along the boundary of the domain. It specify the value of function on the surface $T=f(r, t)$

1.3.2 NEUMANN BOUNDARY CONDITIONS

In mathematics, the Neumann boundary condition is a type of boundary condition, named after Carl Neumann when imposed on an ordinary or a partial differential equation, it specifies the values that the derivative of a solution is to take on the boundary of the domain .Neumann Boundary condition is also known as second type boundary condition

1.3.3 CAUCHY BOUNDARY CONDITION

In mathematics a Cauchy boundary condition an ordinary differential equation or a partial differential equation with conditions that the solution must satisfy on the boundary, ideally so to ensure that a unique solution exits A Cauchy boundary condition specifies both the function values and normal derivative on the boundary of the domain .This corresponds to imposing both a Dirichlet and Neumann boundary condition

1.3.4 ROBIN BOUNDARY CONDITION

Robin boundary conditions are weighted combination of Dirichlet boundary conditions and Neumann boundary conditions. Robin boundary conditions are also called impedance boundary condition, from their application in electromagnetic problems, or in convective boundary conditions

1.4 HEAT TRANSFER

- Heat always moves from a warmer place to a cooler place Hot objects in a cooler room will cool to room temperature.
- Cold objects in a warmer room will heat up to room temperature.

1.4.1 HEAT TRANSFER METHODS

-Conduction
-Convection

CONDUCTION:

When we heat a metal strip at one end, the heat travels to the other end
As we heat the metals, the particles vibrate, and so on ,the vibrations are passed along the metal and so is the heat. We call this Conduction

Convective Heat Transfer:

- Heat transfer between a solid and a moving fluid is called Convection.
- Heat energy transferred between a surface and a moving fluid at different temperature is known as Convection.

In reality this is a combination of diffusion and bulk motion of molecules. Near the surface the fluid velocity is low, and diffusion dominates. Away from the surface, bulk motion increases the influence and dominates. The particles in a liquid or a gas when we heat them the particles spread out and become less dense.

Convective heat transfer may take the form of either

- Forced or assisted convection
- Natural or free convection

Forced or Assisted convection

Forced convection occurs when a fluid flow is induced by an external force, such as a pump, fan, or mixer

Natural or Free convection

- Natural convection is caused by buoyancy forces due to density differences caused by temperature variations in the fluid. At heating the density change in the boundary layer will cause the fluid to rise and be replaced by cooler fluid that also will heat and rise. This continues phenomena is called free or natural convection.

Boiling or condensing processes are also referred as a convective heat transfer processes.

The heat transfer per unit surface through convection was first described by Newton and the relation is known as the Newton's Law of cooling.

The particles in a liquid or a gas when we heat them the particles spread out asnd become less dense.

The study of heat transfer problems is of general interest due to its varied and wide applications in the problems of natural events and technology such as designing of power stations, chemical and food plants, aerodynamic heating, cooling of high power motors, extraction of energy from atomic piles, high speeds aircraft, atmospheric re-entry of vehicles, utilization of heat stored in subterranean layer of the Earth, heat exchangers utilizing liquid metal coolant etc.

The mode of heat transfer in fluids, wherein the moving fluid particles carry heat in the form of internal energy is called convection. Heat transfer by convection is of two types, namely,

(i) forced convection and (ii) free convection. A convection process when the motion are created by external influences such as pressure drop or an agitator is known as forced convection. In incompressible fluids such flows are characterized by the fact that the distribution of velocity is not affected by temperature field but the [converse is not true. In such flows, heat diffuses and at the same time is swept by the fluid motion without in any way affecting the local density of the fluid. Hence in forced convection flow velocities are exactly as they would be if there were no temperature variations so that the parts of motion arising from the differences caused by thermal expansion can be ignored. On the other hand the essential feature of a free convection flow is that the distributions of velocity and temperature field are coupled. The motion here is caused entirely by the buoyancy forces arising from the density variations in a field of gravity and this same distribution of density changes as soon as motion starts. Thus we find that in such flows the distribution of velocity and temperature are interconnected and must be considered together. If the fluid is incompressible then the density variation due to change in pressure are negligible. However, density changes due to non-uniform heating of the fluid cannot be neglected since such changes are responsible for imitating free convection. It is widely accepted that the free convection takes place in field of gravity. In a rotating fluid it can be also set up by the action of centrifugal force which is proportional to the density of the fluid. Flow and heat transfer in gas turbines

is an example of such situation. The subject designated as MHD heat transfer can be roughly divided into two parts viz. one in which heating is an incidental by-product of the electromagnetic fields. This part includes devices like MHD generators or accelerators and to a lesser degree pumps and flow meters. These are broadly characterized as channel and duct flows. The other one in which the primary use of electromagnetic fields is to control heat transfer. This part includes free or natural convection flow and aerodynamic heating where geometric configurations are varied. A comprehensive review of these basic areas are well documented by Romig and Moffatt. It is interesting to mention that in the MHD heat transfer problems the usual Reynolds analogy between skin friction and heat transfer, as in non-conducting fluids, does not hold in general. This is because, in addition to viscous dissipation, there is a Joule dissipation of heat due to the flow of electric current in the field.

1.5 FLUID DYNAMICS

- Fluid is a substance that deforms continuously under the application of a shear stress.
- Shear stress is a stress that is applied parallel or tangential to the face of material.
- Fluid comprise the liquid or gas.
- Fluid mechanics assumes every fluid obeys conservation of energy.

  Conservation of energy

  For incompressible, non-viscous fluid, the sum of the pressure, potential and kinetic energies per unit volume is constant.

  It is the branch of Physics that studies fluid (liquids, gases and plasmas) and the forces on them. It can be divided into three following parts

i) Fluid statics that is the study of fluids at rest
ii) Fluid kinematics that is the study of fluids in motion and
iii) Fluid dynamics that is the study of the effect of forces on fluid motion. It is also a branch of Continuum Mechanics

1.5.1 Classification of Fluid Flows

- **Viscous vs. Inviscid Region of flow**
  Viscous flow region-flows in which the frictional effect is significant
  Inviscid flow region-viscous forces are negligibly small compared to inertial or pressure forces
- **Internal vs. external flow**
  Internal flow-flows in which the fluid is completely bounded by solid surface
  E.g. flow in a pipe or duct
  External flow-flows in which the fluid is unbounded over solid surface
  E.g. flow over a plate, wire, sphere object
- **Compressible Vs. Incompressible Flow**
  Compressible Flow-density changes of fluid is significant
  Gases at high speeds
  Incompressible Flow-density of fluid remains nearly constant throughout
  Liquid, gases at low speeds
- **Natural Vs. Forced Flow**
  Forced Flow-fluid is forced to flow over a surface or in a pipe by external means such as pump or a fan
  Natural Flow-any fluid motion is due to natural means such as buoyancy effect, where warmer (and thus lighter) fluid rises and cooler (and thus denser) fluid falls
- **Steady Vs. Unsteady Flow**
  Steady Flow-no change of fluid properties at a point with time
  Devices that are intended for continuous operation
  E.g. turbines, pumps, boilers, condenser
  Unsteady Flow-fluid properties change at a point with time
- **Steady uniform flow**
  Condition do not change with position and with time e.g flow of water in a pipe of constant diameter at constant velocity
- **Steady non-uniform flow**
  Conditions change from point to point in the stream but do not change with time e.g. flow in tapering pipe with constant velocity at inlet, but velocity change along the length of the pipe toward the exit
- **Unsteady uniform flow**

At a given instant of time ,the conditions at every point are the same, but will change with time e.g. pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off

- **Unsteady non-uniform flow**
  Every condition of the flow may change from point to point and with time at every point e.g. waves in channel

It is the branch of Physics that studies fluid (liquids, gases and plasmas) and the forces on them. It is a Fluid Mechanics which deals with the fluid flow — the ordinary science of fluids (liquids and gases) in movement. It has some sub-disciplines itself, including Aerodynamics (the study of air and other gases in movement) and Hydrodynamics (the study of liquids in proposition). Fluid dynamics has a wide range of applications, including manipulative forces and moments on aircraft, influential the mass flow rate of petroleum through pipelines, predicting weather conditions, thoughtful nebulae in interstellar space and modeling fission weapon detonation. Some of its values are even used in interchange engineering, where interchange is treated as a continuous fluid, and crowd dynamics

Fluid dynamics offers a regular structure— which underlies these useful disciplines— that embraces empirical and semi-empirical laws resulting from flow quantity and used to solve practical problems. The solution to a fluid dynamics problem naturally involves calculating various properties of the fluid, for example velocity, pressure, density, and temperature, as functions of space and time.

1.5.2 APPLICATIONS OF FLUID DYNAMICS

- Building
  Water supply system
  Sewerage System
- Aircraft
  Aero foil design
  Gas turbine
- Industry
  Cooling of electronics
  Automation system
- Ship , submarines ,hovercraft
  Hydrodynamics design
  Buoyancy and stability

1.6 BASIC EQUATIONS

The Navier–Stokes equations (named after Claude-Louis Navier and George Gabriel Stokes) are the set of equations that express the motion of fluid substances such as liquids and gases. These equations state that changes in momentum (force) of flowing particles depend only on the external pressure and internal viscous forces (similar to friction) acting on the fluid. The Navier-Stokes equations describe the balance of forces acting at any given region of the solution. The Navier–Stokes equastions are differential equations which describe the motion of a fluid. Such equations establish relation among the rate of change of the variable of interest. For example, the Navier–Stokes equation for an ideal fluid with zero thickness states that acceleration (the rate of change of velocity) is proportional to the derivative of internal pressure. Solutions of the Navier–Stokes equations for a given physical problem must be sought with the help of calculus. In convenient terms only the simplest cases can be solved exactly in this way. These cases generally involve non-turbulent, steady flow (flow that does not change with time) in which the Reynolds number is small. For more composite situation, involving turbulence, such as global weather systems, aerodynamics, hydrodynamics

1.6.1 CONTINUITY EQUATION

"In fluid dynamics, the continuity equation states that, in any stable state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system." The equation of continuity is developed simply by applying the law of conservation of mass to a small volume element within a flowing fluid.

Equation of continuity (conservation of mass)

Mass flowing in =mass flowing out

$$massflowingin = massflowingout$$

$$m_1 \;=\; m_2 \; \rho V_1 \;=\; \rho V_2$$

$$\rho A_1 V_1 \Delta t \;=\; \rho A_2 V_2 \Delta t$$

$$A_1 V_1 = A_2 V_2 = Q = V/t \;= constant$$

the continuity equation for compressible fluid is given by

$$\frac{\partial p}{\partial t} + \nabla.(p.\vec{q}) = 0$$

Where, ρ is fluid density, t is time, q is the flow velocity vector field.

The continuity equation for incompressible fluid is given by

$$\nabla.\vec{q} = 0$$

Physically, the local volume dilation rate is zero.

1.6.2 MOMENTUM EQUATION

The rate of momentum accumulation is equivalent to the difference of the rate of momentum in and the rate of momentum out along with the sum of forces acting on the system. It is a declaration of Newton's Second Law and relates the sum of the forces acting on an element of fluid to its rate of change of momentum. Newton's Second law when applied to the moving fluid element says that the net force of the fluid is equals to its mass then the acceleration of the element. This is a vector relation and it can be split into three scalar parts along the x, y and z-axes. The moving fluid elements experience a force in the x-direction.

What is the cause of this force?

There are two cases:

1) Body forces, which act straight on the volumetric mass of the fluid element. These forces 'act at a detachment'; examples are gravitational, electric and attractive forces.

2) Surface forces, which take steps directly on the surface of the fluid element. They are due to only two sources:

(a) The pressure distribution acting on the surface, forced by the outside fluid neighboring the fluid element

(b) The shear and normal stress distributions substitute the surface also forced by the outside fluid 'tugging' or 'pushing' on the surface by means of friction. The shear and normal stresses in a fluid are related to the time-rate-of-change of the deformation of the fluid element.

The basic equations of Hydrodynamics for the path of a homogenous, isotropic, thick incompressible fluid of constant density r and constant coefficient of viscosity u is given by

$$\frac{\partial q}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho}\nabla p - \nabla \varphi + \upsilon \nabla^2 \vec{q}$$

1.6.3 MAXWELL EQUATION

It is a set of four complicated equations that describe the world of electromagnetic. These equations describe how electric and magnetic fields propagate act together and how they are influenced by objects and all four of Maxwell's equation are as follow

1. Gauss's Law

$$\nabla . E = \frac{\rho}{\varepsilon_0}$$

2. No Magnetic Monopole Law

$$\nabla . B = 0$$

3. Faraday's Law

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

4. Ampere's Law with displacement current

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

These four equations is called Maxwell's Equation.

Where E = Electric field, B = Magnetic field, ε_0 = permittivity,

μ_0 = permeability, J= current density respectively.

1.7 STRETCHING SHEETS

Stretching flow:

The flow, produced due to stretching of elastic sheet which moves in its plane with velocity varying with the distance from fixed point due to application of a stress, is known as stretching flow

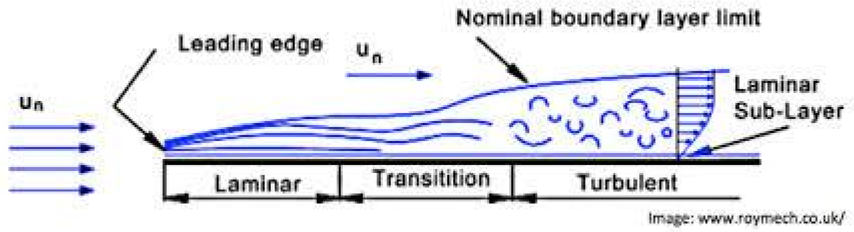
Stretch forming:

- Forming by using tensile forces to stretch the material over a tool or a block.
- Used most extensively in the aircraft industry

  Manufacturing of both metals and polymer sheets in industrial manufacturing process, the material is in molten phase when thrust through an extrusion die and then the material cools and solidified and some distance away from the die before arriving at the cooling stage. Tangential velocity imported by the sheets induces motion in the surrounding fluid, which alters the convection of the sheets.

Similar situation during the manufacture of plastic and rubber sheet where it is often to blow a gaseous medium through material yet to be solidified where the stretching force depends upon time .due to high viscosity of the fluid close to the sheet only. Thus the fluid problems can be idealized to the case of fluid distributed by a tangential moving boundary.

1.8 BOUNDARY LAYER



The concept of boundary layer was first introduced by L. Prandtl in 1904 and since then it has been applied to several fluid flow problems.

The layer adjacent to the boundary is known as boundary layer When a real fluid (viscous fluid) flows a stationary solid boundary ,a layer of fluid which comes in contact with the boundary surface, adheres to it and condition of no-slip occurs(the no-slip condition)implies that the velocity of fluid at a solid boundary must be same as that of boundary itself).Thus layer of fluid which cannot slip away from the boundary surface undergoes retardation, this retarded layer further causes retardation for the adjacent layer of the fluid, thereby developing a small region in the immediate vicinity of boundary surface in which the velocity of flowing fluid increases rapidly from zero at the boundary surface and approaches the velocity of stream.

Boundary layer is formed whenever there is a relative motion between the boundary and the fluid.

Shear resistance: the fluid exerts a shear stress on the boundary and boundary exerts an equal and opposite force on the fluid **known as the shear resistance.**

According to boundary layer theory, the extensive fluid medium around bodies moving in fluids can be divided into following two regions:

- A thin layer adjoining to the boundary called as the boundary layer where the viscous shear takes place
- A region outside the boundary layer where the flow behavior is quite like that of an ideal fluid and the potential flow theory is applicable**.**

- **Laminar flow:** The highly ordered fluid motion characterized by smooth layer of fluid the flow of high viscosity fluids such as oils at low velocities is typically Laminar
- **Turbulent flow:** The highly disordered fluid motion characterized by velocity fluctuations the flow of low viscosity fluids such as air at high velocities is typically turbulent.
- **Transitional flow:** A flow that alternates between being Laminar and Turbulent.

CHAPTER 2

LITERATURE REVIEW

This section is devoted to the review of the earlier investigations made on the flow and the heat transfer over the stretching sheet. During the last decades the problem of flow of incompressible viscous fluid and heat transfer phenomena over stretching sheets gets the great attention. This problem owns plenty of practical applications in chemical and manufacturing processes like Aerodynamics, continuous casting of metals, glass fibers and paper production, extrusion of plastic.

Study of Hydro magnetic flow of an electrically conducting fluid, due to its extensive industrial applications has attracted the interest of many researchers. The cause of the study of hydrodynamic flow of an electrically conducting fluid is the deformation of the wall of a vessel containing a fluid which is of considerable interest in a modern metal-working process and modern metallurgical. The boundary layer flow which is passing a Stretching Plane Surface in the presence of a uniform magnetic field has practical relevance in Polymer Processes.

The study of boundary layer flow over a continuous solid surface moving with constant speed is initiated by Sakelaris (1961). The steady two-dimensional boundary layer flow caused by the stretching of an elastic flat surface which moves in its plane with velocity varying linearly with distance from a fixed point was extended to analyze by Crane (1970).

Carrageen and Crane (1982) investigated the heat transfer aspect of this problem, under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. The steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution were investigated by Magyari and Keller (1999). The unsteady magneto hydrodynamic flow due to the impulsive motion of a stretching sheet was investigated by Takers et al. (2001) and reported that the surface heat transfer increase up to a certain portion of time, beyond that it decreases. In fluid flow process porous medium play an important role. The problem of viscoelastic fluid flow and warmth transfer in a porous medium over a stretching sheet has solved by sub has and Veena (1998). Vajravelu (1994) has obtained the solution for the flow problem and heat transfer in a saturated porous medium. Eldabe and Mohamed (2002) have studied both heat and mass transfer in hydro magnetic flow of a non-Newtonian fluid with a warmth source over an accelerating surface through porous medium. Recently Venkateswalu et al. (2011) have discussed finite difference analysis on convective heat transfer flow through a porous medium in a vertical channel with magnetic field.

OBJECTIVE OF THE PROPOSED WORK

The objectives of the proposed work are following:

1) To solve real world problem by using analytical and computational method.

2) To develop a model that reflects the real world problem.

3) To determine what insight the mathematical model has provided to the original problem.

CHAPTER 3

FORMULATION OF THE PROBLEM

Consider the steady two dimensional flow of viscous incompressible electrically conducting fluid over a stretching sheet in a presence of uniform heat transfer in the region y>0. Keep the origin is fixed two equal and opposite forces are applied along the x-axis which result in sheet is stretched with a speed is proportional to the distance from the origin .x-axis is taken along the stretching sheet and y axis is taken perpendicular to x-axis. A magnetic field of strength B_0 is applied normally to the stretching sheet which produces a magnetic field It is also assumed that the fluid is weakly electrically conducting so that the induced magnetic field is negligible, which is justified for MHD The continuity, momentum and energy equations are written as follows .The steady two dimensional boundary layer equations for this flow in usual notations are

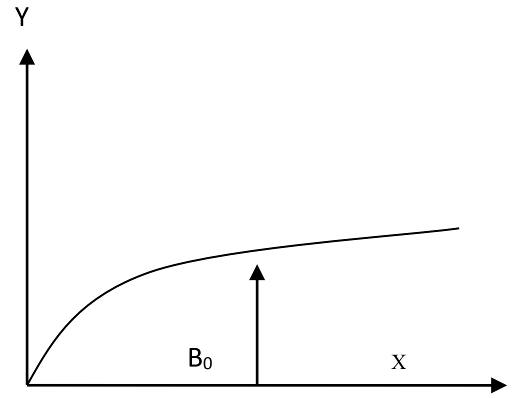


Fig 3.1

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0{}^2 u}{\rho} \tag{3.2}$$

$$u\frac{\partial T}{\partial u} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y^2} \tag{3.3}$$

Where u and v are velocity components in x and y directions respectively, ρ is the density of fluid, ν is the kinematic viscosity of the fluid , $U(x) = ax$ is the straining velocity of the stagnation – point flow , a is the straining rate, T is the temperature , κ is the thermal conductivity , σ_0 is the electric conductivity, c is stretching rate, c_p is specific heat, q_r is radiative heat flux, B_0 is the uniform magnetic field along the y axis.

The boundary conditions for the velocity field are given by:

BOUNDARY CONDITIONS:

$$u = cx,\ v = 0,\ T = T_w \quad at \quad y = 0 \tag{3.4}$$

$$T = T_\infty,\ u \to 0, \quad \frac{\partial u}{\partial y} \to 0 \quad \text{as } y \to \infty. \tag{3.5}$$

Employing the Rosseland approximation for radiation [Chen. et al (2008)] expressed as,

$$q_r = -\frac{4\sigma}{3\alpha}\frac{\partial T^4}{\partial y}$$

where a is mean absorption co-efficient and σ is the Stefen Boltzman constant. It is assumed that the temperature difference within the flow sufficiently small such that T^4 can be expressed as a linear function of temperature which after expanding using Taylor's series about T_∞ and neglecting the higher order term, reduces to

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

Hence the change in radiative flux with respect to y has been obtained as,

$$\frac{\partial q_r}{\partial y} = -\frac{16}{3\alpha}\frac{T_\infty^3 \sigma}{\rho c_p}\frac{\partial^2 T}{\partial y^2}$$

and equation (3.3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16}{3\alpha}\frac{T_\infty^3}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3.5 a}$$

METHOD OF SOLUTION

Let u and v be defined as the new variables. Then, similar transformations and dimensionless variables are used to transform equation (3.2) into a set of ordinary differential equations. The new variables introduced are as follows:

$$u = cxf'(\eta), \qquad \upsilon = -(c.v)^{1/2} f(\eta) \tag{3.6}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \eta = \left(\frac{c}{\nu}\right)^{1/2} y \tag{3.7}$$

Where θ is dimensionless temperature, T is temperature of the fluid, T_w is temperature of the sheet, T_∞ is free stream temperature, f is dimensionless stream function , η is similarity variable.

By using these new variables, i.e. equation (3.6) and equation (3.7) in equation (3.2), then these variables are satisfied in equation (3.2) by substitution, thereby giving the following result

$$f''' - f'^2 + ff'' + Mf' = 0 \tag{3.8}$$

$$\left(1 + \frac{4}{3R}\right)\theta'' + \Pr(f'\theta - f\theta') = 0 \tag{3.9}$$

The corresponding boundary conditions are:

$$f = 0,\ f' = 1,\ \theta = 1 \text{ at } \eta = 0 \tag{3.10}$$

$$\theta \to 0\ f' \to 0, \quad f'' \to 0 \text{ as } \eta \to \infty \tag{3.11}$$

Here f' denotes first order derivative with respect to η and f'' denotes second order derivative with respect to η, $M = \frac{\sigma_0 B_0^2}{\rho c}$ is the magnetic parameter and $\Pr =$ Prandtl number and $R =$ radiation parameter.

NUMERICAL SIMULATION

Runge – Kutta fourth order technique is used to solve the non-linear boundary layer equation (3.8) and (3.9) together with the boundary conditions (3.10) and (3.11), along with the shooting method. Firstly, the higher order non- differential equation (3.8) and (3.9) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem (IVP) by applying the shooting technique (Jain et al). Thereafter, by applying Runge- Kutta fourth order technique, the resultant initial value is solved. The semi- infinite integration domain $0 \le \eta < \infty$ is replaced by a finite domain $0 \le \eta < \eta_\infty$ where η_∞ is the numerically infinity, which is sufficient large and hence the numerical solution closely approximates the boundary conditions. A finite value is taken for to ensure that the solution is not affected by imposing the asymptotic condition at a finite distance. The computation in this study is carried out by taking $\eta_\infty (= 5)$ and the numerical procedure is started by guessing the value of $f''(0)$ This is done in order to initiate the shooting technique and improve the guess until the end boundary conditions is satisfied.

0.001 is the step size that is employed and the solution is assumed to converge when the difference reaches 10^{-6} between the current and previous iterations that are used.

$$f = w$$

$$f' = p$$

$$p' = q$$

$$q' = p^2 - wq - Mp$$

$$\theta' = h$$

$$h' = -\Pr[p\theta - fh] \tag{3.12}$$

$$\theta'' = -\left(\frac{3R}{3R+4}\right)\Pr\left(f'\theta - f\theta'\right)$$

Where p, q are variables.

With the transform boundary conditions

$$f = 0,\ f' = 1,\ \theta = 1 \text{ at } \quad \eta = 0$$

$$\theta \to 0\ f' \to 0, \quad f'' \to 0 \quad \text{as} \quad \eta \to \infty \tag{3.13}$$

In order to solve (3.12) as an initial value problem we must need values for $q(0)$ i.e. $\theta'(0)$ but no such values are given in the boundary. The initial guess values for $f''(0)$ and $\theta'(0)$ are chosen and fourth order Runge-Kutta method is applied to obtain the solution. we compare the computed values of $f'(\eta)$ and $\theta(\eta)$ at $\eta_\infty(=5)$ with the given boundary conditions $f'(\eta_\infty) = 0$ and $\theta(\eta_\infty) = 0$ then we adjust the values of $f'(0)$ and $\theta'(0)$ using Secant method to get better approximation for the solution. The step- size is taken as $h = 0.01.$ The process is repeated until we get the results correct up to the desired accuracy of 10^{-6} level.

4. RESULTS AND DISCUSSION

The effect of Magnetic parameter (M), Prandtl number$(\Pr)$, and Radiation parameter R on velocity and temperature are investigated and analyzed with the help of their graphical representation as below

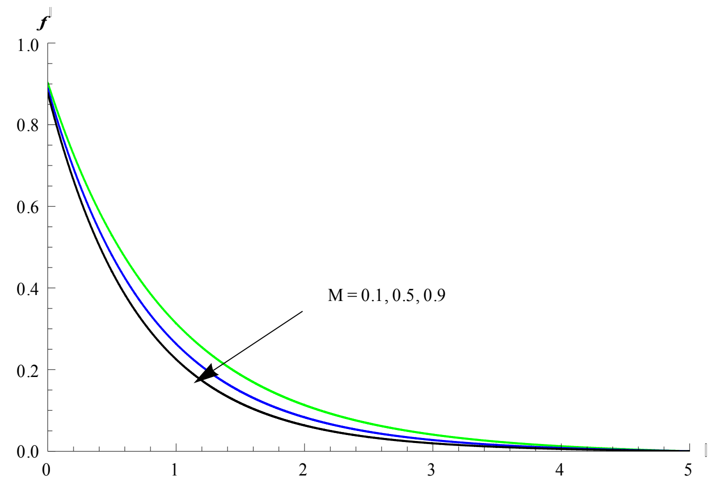


Fig.4.1. velocity profile $f'(\eta)$ for different values of M for fixed value of $R = 0.1$ *and* $\Pr = 0.72$.

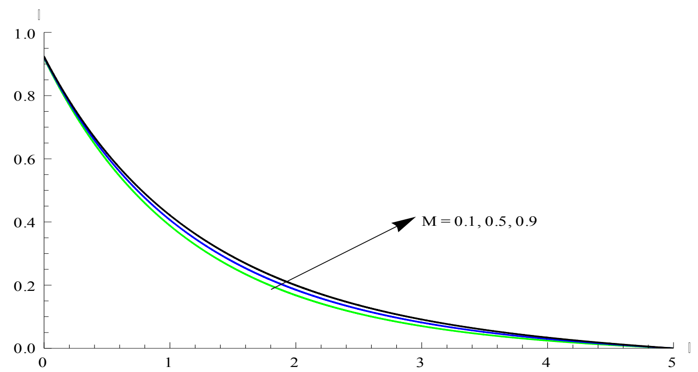


Fig.4.2. temperature profile $\theta(\eta)$ for different values of M for fixed value of $R = 0.1$ *and* $\Pr = 0.72$.

Fig. shows that the nature of velocity field for the variation of magnetic parameter M .When the value of M increases , then value of velocity is decrease.

From Fig. It clears that as the value of M increases then temperature is also increases and the motion of the fluid is opposed by the transverse magnetic field. Considerably the rate of transport is reduced. Lorentz force increases and it produces more resistance to flow due to the increasing in *M*. thickness of thermal boundary layer increases due to increase in *M*.

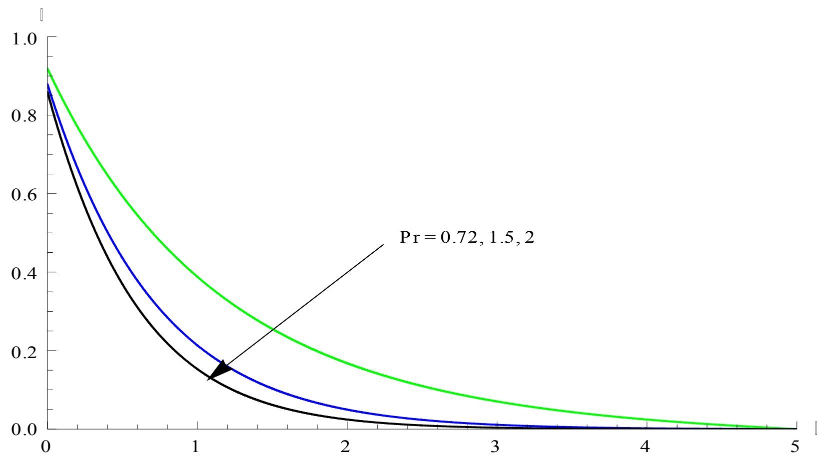


Fig.4.3. velocity profile $f'(\eta)$ for different values of Pr for fixed value of $R = 0.1$ *and* $M = 0.1$.

Fig. shows the effect of Prandtl number Pr on temperature profile. Prandtl number Pr defines the ratio of momentum diffusivity to thermal diffusivity. The temperature decreases due to increase in Pr. Hence, The thermal diffusivity decreases when we increase the Pr. So this phenomenon elaborates that the energy ability is decreasing which reduce the thermal boundary layer thickness.

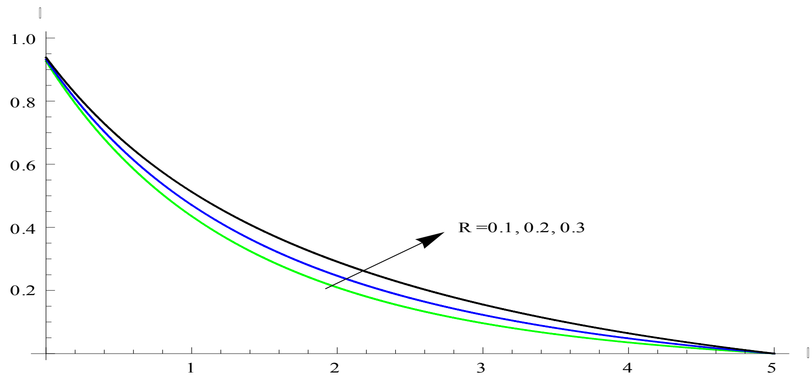


Fig.4.4. velocity profile $f'(\eta)$ for different values of Pr for fixed value of

$R = 0.1$ *and* $PM = 0.1$.

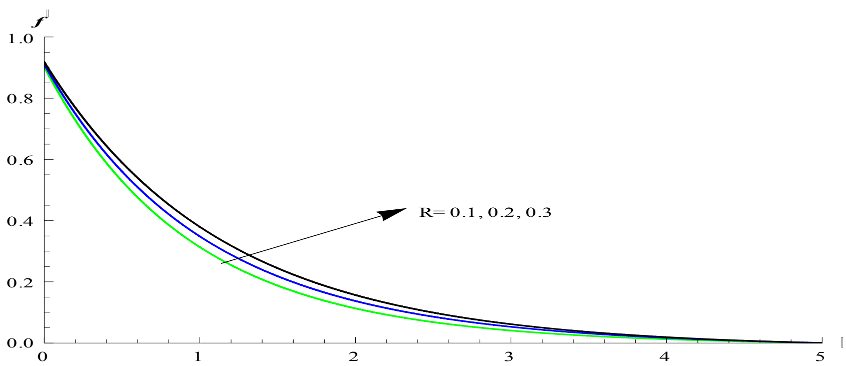


Fig.4.5. velocity profile $f'(\eta)$ for different values of Pr for fixed value of $R = 0.1$ *and* $PM = 0.1$.

Figure 4.4 shows the effect of the thermal radiation , we shows that there is a tendency to increase the velocity and temperature boundary layers as velocity of R increases .Figure 4.5 gives the effects of parameter R increased due to increase in temperature of boundary layer From here ,we observe that mean absorption coefficient k decreases and divergence of radioactive heat flux increases with the increase in thermal radiation parameter therefore , we can say that rate of irradiative heat transferred to fluid will be increased .so that the fluid temperature will be increased.

CONCLUSION

The present study gives the similarity solution of unsteady two dimensional boundary layer flows and heat transfer of viscous , incompressible , electrically conducting fluid along a stretching sheet in presence of transfer magnetic field .The results pertaining to the present study indicate that the flow and temperature field are significantly influenced by the unsteadiness parameter , Magnetic parameter , Radiation parameter .Based on the present investigation ,the following observation are made

i) With increasing *M,* velocity is decreases but the temperature increases.
ii) As Pr increase , temperature is decreases .the thermal diffusivity decreases due to Pr increases So this phenomena elaborates that the energy ability is decreasing which reduce the thermal boundary layer thickness.
iii) Velocity $f'(\eta)$ and temperature $\theta(\eta)$ along the sheet increases with increase of Radiation parameter.

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