LOVELY PROFESSIONAL UNIVERSITY

MASTER OF SCIENCE

MHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A STRETCHING SHEET IN POROUS MEDIUM

A project submitted in fulfilment of the requirements

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Declaration of Authorship

I, Mwiya Mpishi, declare that this thesis titled, "MHD Boundary Layer Flow and Heat Transfer over a Stretching Sheet in Porous Medium" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in the candidature for a research degree at this university.
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This is to certify that Mwiya Mpishi has completed project titled "MHD Boundary Layer Flow and Heat Transfer over a Stretching Sheet in Porous Medium under my guidance and supervision. To the best of my knowledge, the present work is the result of his original investigation and study. No part of the project has been submitted for any other degree at any university.

The project is fit for the submission and the partial fulfilment of the conditions for the award of Master of Science (Hons) in Mathematics.

Signed:

Supervisor: Dr. Narayan Prasad

Date:

"Keep your dreams alive. Understand to achieve anything requires faith and belief in yourself, vision, hard work, determination, and dedication. Remember all things are possible for those who believe."

Gail Devers

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Department of Mathematics

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Abstract

Master of Science (Hons) in Mathematics

MHD Boundary Layer Flow and Heat Transfer over a Stretching Sheet in a Porous Medium

By Mwiya Mpishi.

The main aim of this work is to study MHD boundary layer flow and heat transfer of a fluids over a stretching sheet in a porous medium in presence of heat transfer. The similarity solutions of the governing equation have been obtained and the reduced equation have been solved by using numerical method. Numerical solutions of these equations are obtained by Runge-Kutta fourth order with shooting method. Numerical results obtained for different parameters such as magnetic parameter M , porosity parameter N and Prandtl number ($\Pr$) on velocity and temperature have been analyzed and discussed.

Contents

Symbols

β	Velocity ratio parameter
υ	Kinematic viscosity
k	Permeability of the porous medium
Pr	Prandtl number
M	Magnetic parameter
N	Permeability parameter
p,q	Variables
κ	Thermal conductivity
T	Temperature of the fluid
T_∞	Free stream temperature
T_w	Temperature of the wall of the surface
u,v	Components of velocity in x and y directions
z	Variable
η	Simiarity variable
K	The coefficient of thermal diffusivity
ρ	Density of the fluid
θ	Non-dimensional temperature

CHAPTER 1

Introduction

1.1 Magneto–hydrodynamics (MHD)

They science of dynamics of matter moving in an electromagnetic field is known as MHD. In MHD or Magneto–hydrodynamics, the currents in matter are established by induction, and modified in the field so that the field and dynamics equations are coupled. The set of equations that which describe MHD are a combination of Navier−Stoke equations of fluid dynamics and Maxwell's equations of electromagnetism. The interactions of moving conducting fluids with electric and magnetic fields provides a rich variety of phenomena associated with electric−fluid−mechanical energy conversion which can be observed in liquids, gases, two−phase mixtures or plasma. Originally, MHD was applied to astrophysical and geophysical problems and is still very important particularly to the problem of fusion power, where the application is the creation and containment of hot plasma by electromagnetic forces and further prevents the material walls from being destroyed. Astrophysical problems include solar wind bathing the earth and other planets, and interstellar magnetic fields. The primary geophysical problem is planetary magnetism, produced by currents deep in the planet, a problem that has not been solved to any degree of satisfaction.

However, the boundary layer flow on a stretching sheet has gained significant attention due to many practical applications in the industrial manufacturing process. These include the polymer industry where the production of plastic sheets are dealt with, the aerodynamics extrusion of plastic sheet, glass blowing and metal spinning. The sheeting materials production arises in a number of industrial manufacturing processes and it includes both metal and polymer sheets. The quality of the production depends on the rate of heat transfer at the stretching surface. Therefore, the heat transfer in porous fibrous medium is very complex and a thorough understanding is essential of such materials. The heat transfer and transportation phenomena in porous media are also important process in many engineering application. Examples include heat exchange, pack sphere−bed, electronic cooling, chemical catalytic reactors and heat pipe technology. The objective of thermal management is to ensure that the temperature of each component in an electronic system remains within specified operating limits, or to ensure the enhancement of forced convection heat transfer in many engineering applications such as nuclear cooling, heat exchangers and solar collectors.

1.2 Boundary Value Problem

In relation to mathematics, a boundary value problem is basically a differential equation together with a set of additional constraints. These constrains are known as the boundary conditions of the equation. A solution to a boundary value problem is a solution to the differential equation and satisfies the boundary conditions.

The boundary conditions that govern mathematics are as follows:

1.2.1 Dirichlet Boundary Condition

Dirichlet boundary condition in the Laplace equation imposes the restriction on the potential in some value at some location. For example, a common case of Dirichelt boundary conditions are surfaces of perfectly conductive electrodes. Free charges in such a conduction will rearrange themselves over the conductive surfaces so that the potential will be uniform over the entire conductor. The condition is known but the conducting surfaces may alternately be floating.

1.2.2 Neumann Boundary Condition

These boundary conditions specify the value of a normal derivative, or some combination of derivatives, along a boundary surface. These arise when a flux has been specified on the boundary, for instance, a heat transfer, or a surface traction in solid mechanics. In homogenous boundary condition, the boundary flux is zero like in insulating surfaces in heat transfer and free surfaces in solid mechanics. Hence the Neumann boundary conditions are referred to as natural boundary conditions in finite elements.

Neumann boundary condition in Laplace or poison equation imposes the constraint that the directional derivative of ϕ is some value at some location. The directional derivative normal to some boundary surface known as normal derivative is zero. These boundary conditions occur in 2D cylindrically symmetric systems. The axis of rotation has infinitely many mirror planes confident with the axis, so the cylindrical axis is also a Neumann boundary condition. They also occur in a repeating element such as modeling a small section of a large grid wire mesh in which case all sides of that element to the right have a Neumann boundary condition.

1.2.3 Robin Boundary Condition

This is a linear combination of a field value and its normal derivative. It occurs on a surface from which heat is carried by convection. Robin boundary conditions are handled similarly to Neumann's boundary conditions.

1.2.4 Cauchy Boundary Condition

When the boundary condition is applied to either an ordinary differential equation or a partial differential equation, a complete solution is determined where both function value and normal derivatives are specified on the boundary of the domain.

1.3 Fluid Dynamics

This is the study of fluids in motion. The term fluid is a substance that deforms continuously when subjected to shear stress no matter how that shear stress may be. Fluids are classified as ideal and real fluids. Ideal fluids are incapable of sustaining any tangential force or shearing stress but the normal force acts between the adjoining layers of the fluid and offers no internal resistance to change in its shape. These have low viscosity such as air and water. On the other hand, real fluids are also known as viscous fluids. A fluid is viscous when the normal as well as shearing stress exist. Due to shearing stress, viscous fluid offers resistance to the body moving through it as well as between its particles of the fluid itself. An example of a real ideal is heavy oils and syrup which are termed as viscous fluids.

Water and most liquids most liquids are assumed to be incompressible. Incompressible means that the density is independent of pressure but can vary with temperature. Viscosity is a measure of a fluid resistance to relative motion within the fluid. It influences energy, drag force or flow separation and further cause the velocity of a flowing fluid to vary with the distance. Normal stress produces deformation associated with volume change and shear stress is just the ratio of tangential force to area. Therefore, Newtonian fluids are fluids for which shear stress is directly proportional to the rate of strain. If the viscosity is a constant, independent of flow speed, then the fluid is called Newtonian fluid and water is consider to be an example of Newtonian fluid. If the fluid viscosity varies with the rate of deformation, then it is said to be non−Newtonian fluid. The viscosity of polymeric liquids with shear rate is known as non−Newtonian fluids.

1.4 Convective Heat Transfer

Heat transfer has a variety of applications in the problems of natural events and technology. These heat transfer problems include designing of power stations, chemical and food plants, aerodynamic heating, cooling of high powered motors, extraction of energy form atomic piles and heat exchanges utilizing liquid metal coolant. The heat transfer in fluids in which moving fluid particles carry heat in the form of energy is called convection. This is classified as force convection and free or natural convection, and depends on how the fluid motion is initiated. In forced convection, incompressible fluids are characterized by the distribution of velocity which is not affected by temperature field. Heat diffusion in such flows occurs, and is simultaneously swept by the fluid motions without any way of affecting the local density of the fluid. The velocities in the forced convection are exact such that there is no temperature variations in the motion arising from the differences caused by thermal expansion, which is ignored. The motion on the other hand in which heat caused by natural means in which the distribution of velocity and temperature field coupled together is referred to as free or natural convection. Taking a fluid in consideration, the buoyancy effect of free convection causes the rise of warm fluid and fall of the cooler fluid. In such flows the distribution of velocity and temperature are interconnected and can be considered together. If the fluid is incompressible, then the density variations due to changes in pressure are negligible. The changes are responsible for imitating free convection because of density changes due to non−uniform heating of the fluid which cannot be neglected. Hence free convection occurs in the field of gravity and in the rotating fluid. It can be set up by the action of centrifugal force which is proportional to the density of the fluid.This is evident in the flow and heat transfer in gas turbines.

Free convection and forced convection occur interchangeably and this is understood further if a common practical example is taken into account such as convection in ovens. Here, convectional ovens use natural convection to heat food while baking. Ovens typically contain two heating elements, that is, on top and bottom of the oven. During baking, the bottom heats up which heats the air inside the oven. The hot air rises and creates a current which helps distribute throughout the oven. Natural convection currents are easily blocked by large pans, creating non−uniform temperatures within the oven. Again the convection oven improve the temperature distribution by using a fan which is located within the oven and thereby creating forced convection. The forced convection currents efficiently run the air inside the oven and creating uniform temperatures, even in the presence of large fans. The practical example generally demonstrates convective heat transfer of both free and forced convection.

1.5 Basic Equation of MHD

MHD is described by a set of equations of Navier−Stoke equations of the fluid dynamics and Maxwell's equation of electromagnetism. The Navierr−Stoke equations are equations which describe the motion of fluid substances which can flow. The equations arise from applying Newton Second Law to fluids, together with the assumption that the fluid stress is the sum of a diffusing viscous term proportional to the gradient of velocity and pressure. Navier−Stoke have applications in the modeling of the weather, ocean currents, water flow in pipes, the air flow around a wing and motion of stars inside a galaxy. They also help with the design of cars and aircraft, the study of blood flow, the design of power stations and analysis of pollution. When Navier−Stoke equations are coupled with Maxwell equations, they can be used to model and study MHD.

1.5.1 Continuity Equation

The continuity equation states that in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. The differential form $\frac{\partial p}{\partial t}+\nabla.\left(\rho \vec{q}\right)=0$

where ρ is the fluid density, t is time and $\vec{q}$ is the flow velocity vector field. If ρ is a constant as in the case of incompressible flow where the density does not change then the mass continuity equation simplifies to a volume continuity equation of the form $\nabla.\vec{q}=0$. This means that the divergence of the velocity field is zero everywhere. The volume of any fluid cannot be changes resulting to incompressible flows. Some examples where the application of continuity is useful can be in pipes with cross section which changes along their length. A liquid flowing from left to right in a pipe which is narrowing in the direction then by continuity principle, the mass flow rate must be same at each section of which the mass going into the pipe is equal to the mass going out of the pipe. Another example of application of continuity principle is when determining the velocities in pipes coming from a junction, then the total mass flow into a junction is equal to the total mass flowing out of the junction.

1.5.2 Momentum Equation

Moving fluid exert forces. The lifting force on an aircraft is exerted by the air moving over the wing and a jet of water from a horse exerts a force on whatever it hits. The analysis of motion in fluid mechanics is performed in the same way as in solid mechanics by the use of newton's law of motion. Special properties of fluids are also taken into account when in motion. The momentum equation is a statement of newton's second law, which states that "the rate of change of momentum of a body is equal to the sum of the forces acting on an element of fluid to its acceleration or rate of change of momentum. A different form of equation is used rather than the basic $F = ma$ because the mass of the fluid is unknown. Therefore, the equation for momentum has a dimension of force. The analysis of forces on a fluid element consisting of two kinds of forces of which forces such as gravity or electromagnetic forces are called body forces because they act over the entire volume of the body and secondly, forces such as pressure and viscous stress act on the surface of a fluid element.

The body force which is mainly considered in fluid dynamics is gravity and it has three directional components of x, y, z. Surface forces consist of normal or shear stress forces on a face of the fluid. This is resolved into component in each coordinate direction of, say i and j. For example, the force denoted by σ_{yx} is a force or stress is considered positive when it is exerted by the fluid above an element on the fluid below an element. The force that results from stress is the area, by definition of stress, which is force per unit area on the same fluid or on which the force acts on. The equation $\frac{\partial q}{\partial t} + \left(\vec{q}.\nabla\right)\vec{q} = -\frac{1}{\rho}\nabla\varphi + \upsilon\nabla^2\vec{q}$ is the basic equation of hydrodynamics and also for the path of a homogeneous isotopic and incompressible fluid of constant density ρ and constant coefficient υ of viscosity.

1.5.3 Maxwell's Equation

Maxwell equations represent ways to state the fundamental for an electric field, $E(r,t)$ and a magnetic field $B(r,t)$. They both depend on the position r and t.

The following are a summary of the Maxwell's equations;

i. Gauss's Law

$$\nabla.E = \frac{\rho}{\varepsilon_0}$$

It states that the divergence of the electric filed is proportional to the density of the electric charge, that is, the electric field diverges away from the point where a source of electric charge is situated. The constant ε_0 of proportionality is called the electric permittivity of the vacuum. Its value depends on the system of units chosen.

ii. No Magnetic Monopole Law,

$$\nabla.B = 0$$

The equation above implies that there are no free magnetic charges. An isolated magnetic charge is impossible to see under laboratory conditions. The simplest source for the magnetic field is a magnetic dipole which is viewed as a pair of magnetic charges close to one another and of opposite sign, so the total magnetic charge is zero.

iii. Faraday's Law,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is the law of electromagnetic induction and it refers to the form of the electric field generated by a time−varying magnetic field. The minus sign on the right side represents Lenz's Law. It states that the electric current generated by E will always be in the direction such as to oppose the change in B.

iv. Ampere's Law with Displacement Current

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

The equation describes how an electric current density J acts as a source for the magnetic field and a time varying electric filed μ_0 is the permeability of the vacuum.

1.6 Stretching Sheets

Consider an elastic flat sheet in which the flow is produced due to stretching in it and this flow moves in its plane. Its velocity varies with the distance from a fixed point due to the application of stress. Then this is referred to as stretching flow. It has application in industrial manufacturing processes which includes both metal and polymer sheets. Motion is create by tangential velocity introduced by the sheets of the surrounding fluid, thereby altering the convection of the sheet. This is evident form the manufacturing of plastics and rubber sheets with a necessary condition of blowing a gaseous medium through a not yet solidified material where the stretching force is dependent upon time. It is also evident in the situation involving the cooling of a large metallic plate in a bath which may be an electrolyte. The fluid flow here is induced on the plate by shrinkage. The stretching surface also occurs in glass blowing, continuous casting and spinning of fibers. Hence the fluid is affected by the sheet because of high viscosity of the fluid near the sheet. Therefore, tangential moving boundary idealizes the case of fluid disturbances of fluid problem.

1.7 Boundary Layer Theory

Boundary layer is a layer which arises when a viscous incompressible electrically conducting fluid which is bounded by a rigid surface is rotated rapidly and produces a thin layer near the boundary surface based on the balance between Coriolis and viscous forces. It was first noticed by Ekman (1905) in his study of the wind stress on the surface of oceans. It plays an important role in determining the flow features of various problems of astrophysical and geophysical interest and engineering Greenspan (1963). In a non−rotating system, a thin boundary layer appears when a viscous incompressible electrically conducting fluid flows past a rigid surface in the presence of an applied magnetic field and magnetic force which is stronger than the viscous force. Hartmann (1937) observed this boundary layer while studying the flow of a uniform transverse magnetic field. A thin boundary layer is formed adjacent to the rigid boundary when combined effects of rotating and magnetic field in the flow of an electrically conducting viscous incompressible fluid bounded by a rigid surface are considered and if the Coriolis forces are stronger than the magnetic and viscous forces.

1.8 Porous Medium

A material congaing pores or voids is a porous medium and its pores are filled with a fluid. A porous medium of volume is a fixed solid matrix with a connected void space through which a fluid can flow or consist of solid particles so that fluids can flow through the voids and passages. Thus, the definition of porosity and permeability is essential. Porosity is the percentage of a volume medium that is empty space and contributes to the fluid flow while permeability measures quantitatively the ability of the porous medium to permit fluid flow. Porous medium has several applications some of which are in the flow through packed beds, extraction of energy from the geothermal regions, filtration of solids form liquids, flow of liquid ion−exchange beds, the evaluation of the capability of heat removal from articular nuclear fuel debris and in chemical reactors for economical separation of purification of mixtures.

CHAPTER 2

The literature review

2.1 Literature

Flow in porous media has been the subject of numerous investigations during the past several decades. The interest in this subject has been stimulated, to a large extent, by the fact that thermally driven flows in porous media have several applications in chemical and mechanical engineering. For example, food processing and storage, geophysical systems, electro−chemistry, fibrous insulation, metallurgy, the design of pebble bed nuclear reactors, underground disposal of nuclear or non−nuclear waste, microelectronics cooling, etc. detailed literature review can be found in the books by Pop and Ingham (2001), Ingham and Pop (2005), Nield and Bejan (2006). One of the fundamental problems in porous media is the flow and heat transfer driven by a linearly stretching surface through a porous medium. It seems that the first study of the steady flows of a viscous incompressible fluid (non−porous media) driven by a linearly stretching surface through a quiescent fluid has been reported by Crane (1970). Further, Elbashbeshy and Bazid (2004) studied flow and heat transfer in a porous medium over a stretching surface with internal heat generation and suction or blowing when the surface is held at a constant temperature. The problem of flow and heat transfer of an incompressible fluid over a stretching surface placed in a porous medium has received considerable attention in recent years because it is an important type of flow occurring in the polymer industry. A class of flow problems with obvious relevance to polymer extrusion is the flow induced by the stretching motion of a flat elastic sheet. For instance, in a melt−spinning process, the extradite from the die is generally drawn and simultaneously stretched into a filament or sheet, which is thereafter solidified through rapid quenching or gradual cooling by direct contact with water or chilled metal rolls. In fact, stretching impart a unidirectional orientation to extradite, thereby improving its mechanical properties and the quality of the product which greatly depends on the rate of cooling.

Recent books by Nield and Benjan (2006), Ingham and Pop (1998) excellently described the extent of the research information in this area. Sakiadis (1961) initiated the study of boundary layer flow over a continuous solid surface moving with constant speed. Crane (1970) extended it to analyze the steady two−dimensional boundary layer flow caused by the stretching of an elastic flat surface 90 which moves in its plane with velocity varying linearly with distance from a fixed point. Dutta et al. (1985) determined the temperature distribution in the flow over a stretching surface subject to uniform heat flux. Chen and Char (1988) investigated the heat transfer characteristics over a continuous stretching sheet with variable surface temperature. Gupta and Gupta (1977) have analyzed the stretching problem with constant surface temperature, while Soundalgekar and Ramana (1980) investigated the constant surface velocity. Grubka and Bobba (1985) have analyzed the stretching problem for a surface moving with linear velocity and with a variable surface temperature. In all the previous investigations, the effects of internal heat source or sink on heat transfer were not studied. When there is an appreciable difference between the surface and the ambient fluid, one need to consider the temperature dependent heat source or sink which may exert strong influence on the natural convection boundary layer induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation. The unsteady heat transfer problem over a stretching surface, which is stretched with a velocity that depends on time, is considered by Anderson et al. (2000), Elbashbeshy and Bazid (2004) and Ishak et al. (2008).

All the above mentioned studies continued their discussion by assuming the no−slip boundary conditions. The no−slip boundary condition is the assumption that a liquid adheres to a solid boundary, is one of the central tenets of the Navier−Stoke theory. However, there are situations where this condition does not hold. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. The non−adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances. Recently, many researchers investigated the flow problem taking slip flow condition at the boundary. The fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities. Beavers and Joseph (1967) studied the fluid flow past a permeable wall using the slip condition at the boundary. Effects of the slip boundary condition on the flow of Newtonian fluid over a stretching sheet were considered by Anderson (2002) and Wang (2009) Ariel et al. (2006) studied the flow of an elastic−viscous fluid over a stretching sheet with partial slip. Analytical solutions of the flow of a second grade fluid and the heat transfer over a stretching sheet under the slip condition were obtained by Hayat et al. (2009) using HAM. Very significant aspects of the slip flow were discussed by Bhattacharyya et al. (2011). All of the above mentioned studies were carried out under a steady−state condition.

However, in certain aspects, flow becomes time dependent and, consequently, it becomes necessary to consider unsteady flow conditions. Surma Devi et al. (1986) investigated the heat and mass transfer in an unsteady three−dimensional flow due to stretching of a flat surface. Takhar et al. (1993) and Pop and N (1996) also explored some important aspects of unsteady flow. Heat transfer past an unsteady stretching sheet under different physical conditions was analyzed by Tsai et al. (2008). Nazar et al. (2009) considered an unsteady boundary layer flow in the region of the stagnation point on a stretching sheet. Some other important properties of flow due to an unsteady stretching sheet were discussed by Ishak et al. (2009), Mukhopadhyay (2010) and Zheng et al. (2011). Suction or injection (blowing) of a fluid through the bounding surface can significantly change the flow field. The process of suction and blowing has also its importance in many engineering activities such as in design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag. The radiative effects have important applications in physics and engineering. The radiation heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industries where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, magneto−hydrodynamics (MHD) accelerators and power generation systems are some of the important applications of radiative heat transfer from a vertical wall to conductive fluids.

CHAPTER 3

Formulation of the problem

3.1 Formulation

A steady two dimensional MHD boundary layer flow of incompressible, viscous and electrical conducting fluid in porous medium with heat transfer is considered. The $x-$ axis has been taken along the stretching sheet and $y-$ axis is taken perpendicular to the surface. The flow is confined to $y>0$,

A uniform magnetic field strength B_0 is applied normal to the stretching surface which produce a magnetic effect. It is also assumed that fluid is weakly electrically conducting so that induced magnetic field is negligible, which is justified for MHD flow at a small magnetic Reynolds number.

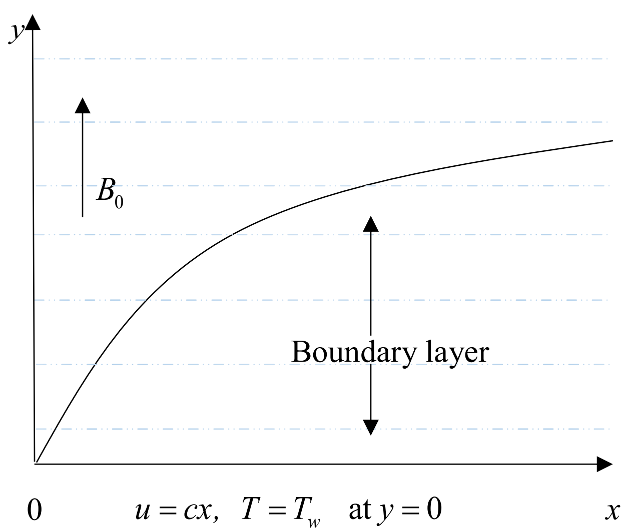

Fig. 3.1 Sketch of the physical problem.

Under the usual boundary layer approximations, the flow and heat transfer with radiation effect (Bansal, 1997; Schlichting el al. 1999) are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_0 B_0}{\rho}-\frac{\upsilon}{k}u \tag{3.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3.3}$$

where u and v are components of velocity respectively in x and y directions, B_0 is the uniform magnetic field along the $y-$axis, $\upsilon=\dfrac{\mu}{\rho}$ is the kinetic coefficient of viscosity, T is the temperature, κ is the coefficient of thermal diffusivity, c_p is specific heat capacity and k is the permeability of the porous medium.

3.2 Boundary Condition

The boundary conditions for the velocity filed are given by:

$$u=cx, \qquad v=0, \qquad T=T_w \qquad \text{at} \quad y=0, \quad c>0 \tag{3.4}$$

$$T=T_\infty, \qquad u\to 0, \qquad \frac{\partial u}{\partial y}\to 0 \quad \text{as} \quad y\to\infty \tag{3.5}$$

From the above $c(>0)$ is considered as the stretching rate, T_w is the uniform wall temperature and T_∞ is the temperature far away from the sheet.

3.3 Method of Solution

Let u and v be defined as the new variables. Then similar transformation and dimensionless variables are used to transform equation (3.2) and (3.3) into a set of ordinary differential equation. The new variables introduced are as follows:

$$u = cxf'(\eta), \qquad \upsilon = -(c.v)^{1/2} f(\eta) \tag{3.6}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \eta = \left(\frac{c}{v}\right)^{1/2} y \tag{3.7}$$

By using these new variables, that is, equation (3.6) and equation (3.7) in equation (3.2) and (3.3), then these variables are satisfied in equation (3.2) and in equation (3.3) by substitution and so giving the following result:

$$f''' - f'^2 + ff'' + Mf' + Nf' = 0 \tag{3.8}$$

$$\theta'' + \Pr\left(f'\theta - f\theta'\right) = 0 \tag{3.9}$$

The boundary conditions are:

$$f' = 1, \qquad f = R \quad \text{at} \quad \eta = 0 \tag{3.10}$$

$$\theta \to 0, \qquad f' \to 0, \qquad f'' \to 0 \quad \text{as} \quad \eta \to \infty \tag{3.11}$$

The prime in the above equations denotes differentiation with respect to η, R is the Radiation parameter, N is equal to the Porosity parameter, $M = \frac{\sigma_0 B_0{}^2}{\rho c}$ is the Magnetic parameter and $\Pr = \frac{v}{k}$ is the Prandtl number.

To assess the accuracy of the present method, comparison with previously reported data available in the literature has been made. It is clear from that the numerical values of $-\theta'(0)$ in the present paper are in agreement with results obtained by Magyari and Keller (1999).

3.4 Numerical Simulation

The Runger−Kutta fourth order technique is used to solve the non−linear boundary layer equation (3.8) and equation (3.9) together with the boundary conditions (3.10) and (3.11), along with the shooting method. Firstly, the higher order non−differential equations (3.8) and (3.9) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem (IVP) by applying the shooting method technique (Jain et al). Therefore, by applying Runge−Kutta fourth order technique, the resultant initial value is solved. The semi−infinite integration domain $0 \le \eta < \infty$ is replaced by a finite domain $0 \le \eta \le \eta_\infty$ where η_∞ is the numerical infinity which is sufficient. Thus the numerical solution closely approximates the boundary conditions. A finite value is taken for η_∞ to ensure that the solution is not affected by imposing the asymptotic condition at a finite distance. The computation in this study are carried out by taking $\eta_\infty\left(=5\right)$ and the numerical procedure is started by guessing the value of $f''\left(0\right)$. This is done in order to initiate the shooting technique and improve the guess until the end boundary condition are satisfied. The step size 0.001 is employed and the solution is assumed to converge when the difference reaches 10^{-6} between the current and previous iterations used. In terms of first order differential equations, the higher order equations mentioned above are as follows:

$$f = w \tag{3.12}$$

$$f' = p \tag{3.13}$$

$$p' = q \tag{3.14}$$

$$q' = p^2 - wq - \left(M + N\right)p \tag{3.15}$$

$$\theta' = h \tag{3.16}$$

$$h' = -\Pr\left(p\theta - fh\right) \tag{3.17}$$

The transformation boundary conditions are:

$$f\left(0\right) = 0, \qquad f'\left(0\right) = 1, \qquad f\left(0\right) = 0, \qquad \theta\left(0\right) = 1$$

The value for $f''(0)$ and $\theta'(0)$ is required to solve the equation as an initial value problem (IVP) but no such value is given in the boundary condition. The fourth order Runger−Kutta method is applied to obtain the solution form the chosen guess of $f'(\eta_\infty) = 0$ and $\theta(\eta_\infty) = 0$. The estimated value of $f''(0)$ and $\theta'(0)$ are adjusted and so this gives a better approximation for the solution. The secant method is used with step size $h = 0.001$ and the result corrects up to the desired accuracy of 10^{-6}.

3.5 Results and discussion

The effect of different parameter like magnetic parameter M, porosity parameter N and Prandtl number Pr on velocity and temperature are investigated and analyzed with the help of their graphical representation.

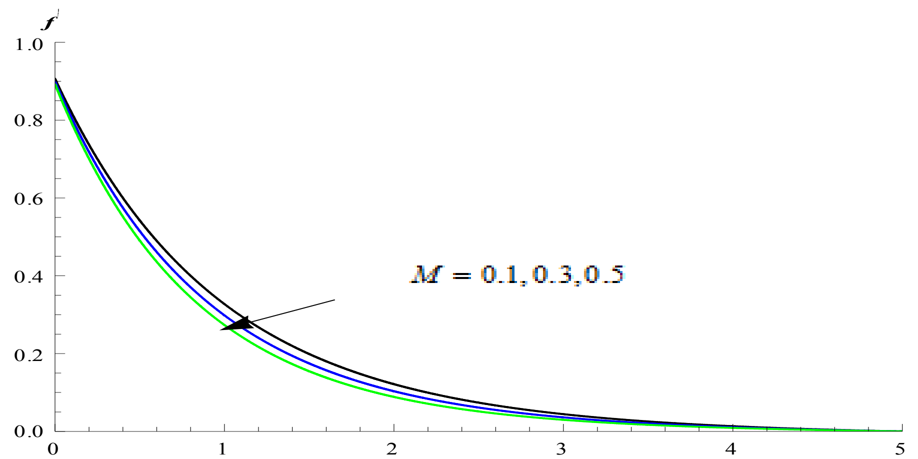

Fig. 3.5 (a) Velocity profiles for different values of magnetic parameter M

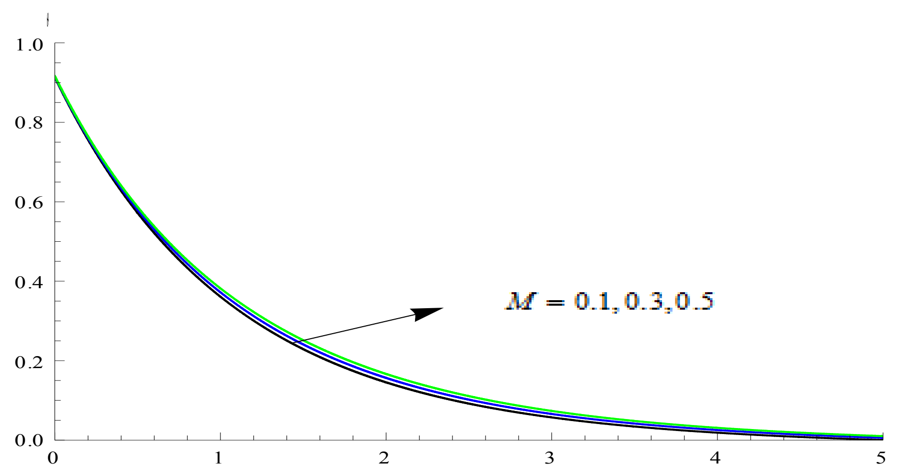

Fig. 3.5 (b) Temperature profiles for different values of magnetic parameter M

Fig. 3.5 (a) illustrates the effect of magnetic parameter M on the behavior of velocity filed. The velocity decreases with an increase in the magnetic parameter because both primary and secondary velocities u and v decrease on increasing magnetic parameter M . Furthermore, the application of transverse magnetic field that result in a resistive type of force (Lorentz force) which is similar to the drag force. This tends to resist the fluid flow and thus reducing its velocity.

Fig. 3.5 (b) illustrates the effect of magnetic parameter with temperature. Clearly, as the velocity profiles decrease, the temperature profiles increase. The increase in temperature is due to the application of transverse magnetic field in an electrically conducting fluid. This produces a resistive force similar to drag force which is the Lorentz force. As a result, the force slow down the fluid motion and results in the increase of temperature. Hence the thermal boundary layer increases in the presence of a magnetic field.

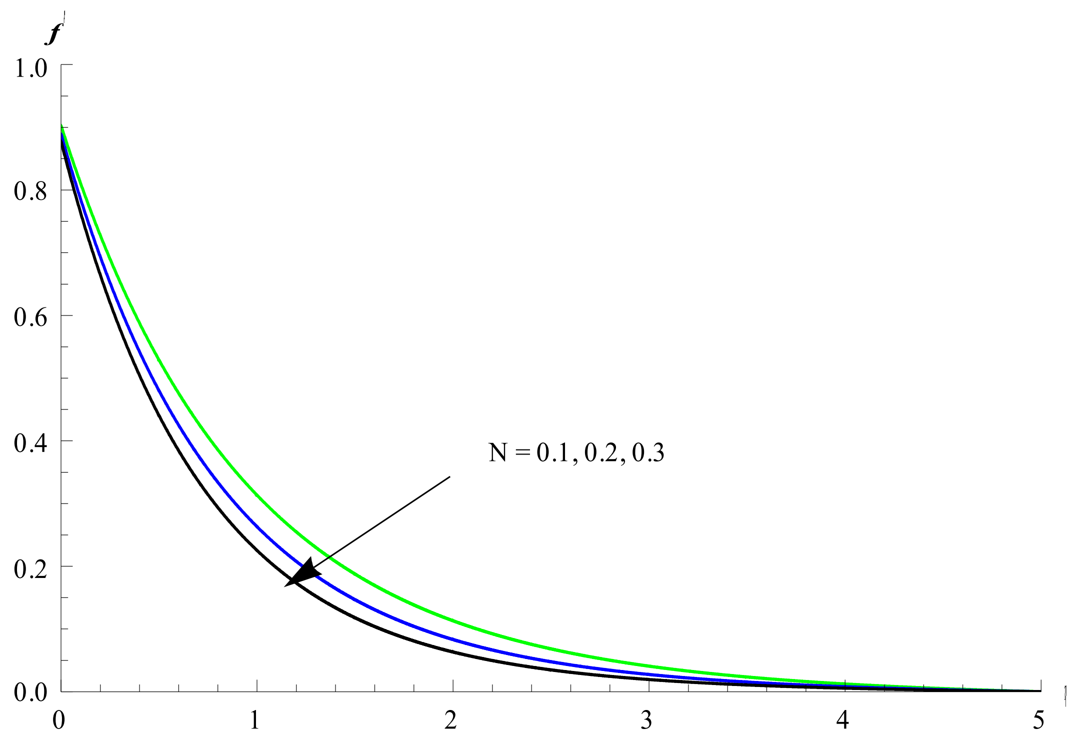

Fig. 3.6 Velocity profiles for different values of porosity parameter N

The fig. 3.6 show that the velocity $f'(\eta)$ decrease with the increase in the porosity parameter with respect to the horizontal velocity in porous medium. It is found that for different values of porosity parameter N, the velocity decreases because the porosity parameter resists the flow and thus restricts the motion of the fluid along the surface. The thickness of the velocity boundary layer increases as the value of N increases. This results in a decrease in velocity due to an increase in porosity parameter N with opposes the flow.

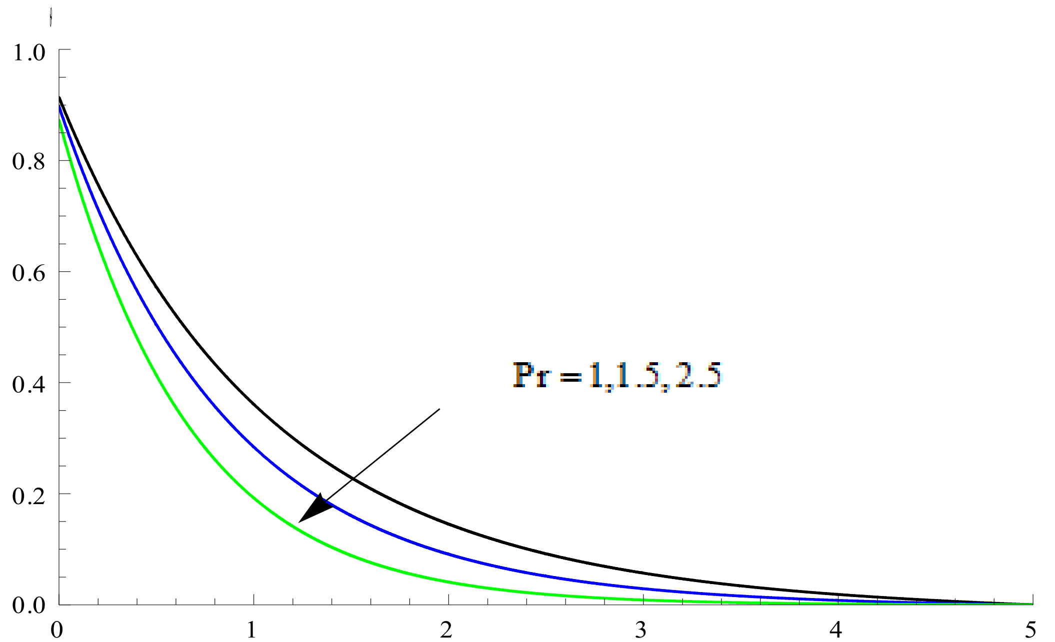

Fig 3.7 Temperature profiles for different values of Prandtl number Pr

Fig 3.7 depicts the temperature profiles to the effect of Prandtl number. An increase in the Prandtl number Pr reduces the thermal boundary layer thickness. Prandtl number Pr signifies the ratio of the momentum diffusivity to thermal diffusivity. It can be noticed that as Pr decreases, the thickness of the boundary layer becomes greater than the thickness of the velocity boundary layer according to the well−known relation $\delta T/\delta \cong 1/\Pr$ where δT the thickness of the velocity thermal boundary layer is and δ is the thickness of the velocity boundary layer. So the thickness of the thermal boundary layer increases as Prandtl number decreases. Hence the temperature decreases as the Prandtl number Pr increases.

3.6 Conclusion

The present study gives the similarity solution for MHD boundary layer flow and heat transfer over a stretching sheet in porous medium. The results obtained show that the velocity and temperature field are significantly influenced by the magnetic parameter M, porosity parameter N and Prandtl number $\Pr$.

The following findings of this investigation can be summarized as follows.

i. The effect of transverse magnetic field on a viscous incompressible conducing fluid flow is to suppress the velocity fluid which in turn causes the enhancement of the temperature field. An increase in magnetic parameter results in decrease of dimensionless velocity profiles and increase in temperature profiles.
ii. By increasing the porosity parameter N, velocity profiles decreases.
iii. Due to the thermal boundary layer thickness, it is found that as Prandtl number $\Pr$ increases, the temperature decreases.

OBJECTIVE OF THE PROPOSED WORK

The objectives of the proposed work are following:

1) To solve real world problem by using analytical and computational method.

2) To develop a model that reflects the real world problem.

3) To determine what insight the mathematical model has provided to the original problem

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