Mathematical Study of Effect of Exotic Species on Native Population

A

PROJECT REPORT

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Chapter 1

Introduction

1.1 Ecology and ecosystem

Ecosystem includes all the living organisms (animals, plants etc.) in a given area or region, interacting with each other and also with non living organisms soil, air, climate etc. Ecology is the study of ecosystem or in other words it is the branch of biology deals with the relation of organisms to one another in its physical environment. Ecosystem includes the Population, which is a group of individuals, who live together in the same ecosystem. Each population has a unique physical distribution in time and space. Population is the branch of life science which deals with size and age of composition of population in a dynamical system. Example birth rate, death rate, immigration and emigration. Ecological system may be very complex and an inter related system of plants , animals, prey, flowering plants, insects etc in which new species evolve or arrive, others decline to extinction or migrate.

1.2 Population dynamics

A population describes a group of people or individuals of same species in a specific area at specific time. Population dynamics is the investigation of the adjustments in population size and structure after some time. It is the study of branch of life science that reviews the size and age sythesis of population as element framework, and the natural and ecological procedures driving them(for examples birth rates and by migration and displacement) case situations are maturing population development or population decay.

1.3 Native species and exotic species

A species which belongs to that given region by birth and live normally in particular ecosystem is called Native Species or in another word species live in the same place that they are originally from, examples Red eye frog(Agalychnis Callydrais) in forest of America and Grizzly Bear in North America, Europe and Asia. Exotic species are species which belongs to a region unintentionally through any reason like human migration or emigration. Example Africanized bees (Killer bees) in Norway, Mediterranean fruit fly (Ceratitis capitata), Purple loosestrife (Lythrum salicaria)etc. They are most destructive when they adapt readily to their new environment.

An exotic species is any species plant or animal belong to region either by accident or for a purpose other than where it evolved lived. Species which are thrive and reproduce in new environment also called alien, non native or invasive. Large variety of exotic animals and plant species have been introduces in different areas through ages. Example Coolatai grass grows, forming an almost complete monoculture and replacing native grass and wild flower species. The studies of exotic species have become a popular topic due to the potential threats they pose to other native species, environment, human being, ecosystem and economics. Many researchers try to prevent the spread of exotic species because they are a threat to their new environment. New species can crowd out native species by taking their living space and consuming their food as well as their egg and overwhelm certain ares natural resources, leaving the landscape devastated and uninhabitable for all life. The various model shows that if interaction between native plant and exotic grass follows a similar pattern in other costal grassland habitats, then the introduction of exotic grass propagules along without change in land or climate, was insufficient to the convert the region's grassland. The prey predator relationship still continues to be one of the main themes in mathematical ecology due to its complex dynamic behaviour. It is seen that the invasion or introduction of exotic species in general disrupt the trophic dynamics of native interacting prey predator species system. Introduced predators usually have dramatic effect on native prey, usually the cause of native species extinction. This harm caused by the introduced predator is broadly known and control programs are the largely identified as the best way to restore ecosystems. In view of the above, the main purposed work is to construct a general model to study the effect of exotic predator on native prey population on a system and a native predator population.

1.4 A brief review of the work already done

N.F.Britton et al.(1996) [1] explained about the population dynamics of the army ant Ecition burchelli on Barro Coloado Island in Panama is set up. It is simulated on the computer and shown to give good agreement with biological data. There are two aspects of the biological system studied in this paper that makes it of general importance. First, the population is structured, since the size of each colony of army ants is crucial. Second, the spatial behaviour of the population, as in many others, is not diffusion like although it is random. Daniel L. Kern et al.(2002) [2] explained about the competition model with spatial considerations for the spread of two exotic plant species and the corresponding replacement of single native species.The general model is a system of three Lotka-Voltera type non linear reaction- diffusion equations. The travelling wave solution is examined, giving condition for minimum wave speed for the exotic species. The work is based on Russian olive trees and tamarisks in the cottonwood woodlands of new Mexico. Michael D.Samuel et al.(2011)][3] explained about avian malaria on the islands to understand how biotic and abiotic facts affect the intensity of malaria transmission and impact on population of native Hawaiian forest bird. This paper shows that disease is a key factor in causing population and distribution of many susceptible. Hawaiian species and preventing the recovery of other vulnerable species. The model provides a framework for the evolution of factor affect disease transmission and alternative disease control program and to evaluate the impact of change on disease cycle and population of bird. O.P.Mishra et al.(2012)[4] explained the study of the effect of exotic predator population of native prey predator population on a system. The model has three state density of native prey, predator and exotic predator. All the feasible equilibria of the model are analyzed analytically and investigated all the possibility of the interior equilibrium point. Finally the results are supported by numerical simulation. O.P. Mishra (2013)[5] explained the effect of resource based exotic goose on a competitive system having native plant population and exotic grass. The model has four state variables like, biomass of native plant, exotic grass, density of resource population and exotic goose. It is assumed that the resource-based exotic goose consumes exotic grass and indirectly favouring the growth of native plant. The stability analysis of all the feasible equilibria are carried out by using stability theory and the analytical results by numerical example. Finally the criteria for the existence and extinction of native population has been discussed under the effect of exotic population.

1.5 Objective

In view of the above literature servery, the following objective for the proposed work as follow:

1. Studied some research papers and books to understand the population dynamical behaviour with mathematical modelling considering native and exotic species.

2. To define the scientific basis develop a mathematical model by using the system of non linear ordinary differential equation for native and exotic population dynamics.

3. Analysis of the mathematical model by using the stability theory like Eigen value analysis method, Routh- Hurwitz criteria, Liaponov Function and Silvester criterion.

4. Simulation and numerical results of formulated model with MATLAB.

Chapter 2

Methodology for proposed work

2.1 Autonomous and non - autonomous system

Let x(t) be vector valued function defined by

$$x(t)=\begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix}=\text{col}(x_1(t), x_2(t), ..., x_n(t))$$

and f be vector valued function given by

$$f(t,x)=\begin{pmatrix} f_1(t, x_1(t), x_2(t), ...x_n) \\ f_2(t, x_1(t), x_2(t), ...x_n) \\ . \\ . \\ . \\ f_n(t, x_1(t), x_2(t), ...x_n) \end{pmatrix}=col(f_1(t, x), f_2(t, x), ..., f_n(t, x))$$

where $f_1, f_2...f_n$ are n given functions in some domain B of n+1 dimensional Euclidean space R^{n+1} and $x_1(t), x_2(t), ...x_n$ are n unknown functions,

then the system

$$\frac{dx}{dt} = f(x, t) \tag{2.1}$$

with initial condition $x(t_0) = x_0$ is a non-autonomous system.

A differential system of the form

$$\frac{dx}{dt} = f(x) \tag{2.2}$$

with initial condition $x(t_0) = x_0$ in which right hand does not involve independent variable t, is said to be autonomous system.

2.2 Solution of system of differential equation

A set of n function $\phi_1, \phi_2, ...\phi_n$ define on I=$t : t \in$ R,$r_1 < t < r_2$, where r_1 and r_2 are any two fixed points in set of all real number R, is said to be solution of 2.1 on I if for t $\in$ I.

1. $\phi_1', \phi_2'...\phi_n'$ exists.

2. The point $(t, \phi_1(t), \phi_2(t), ...\phi_n(t))$ remain in B.

3. $\phi_i' = f_i(t, \phi_1(t), \phi_2(t), ...\phi_n(t))$, i=1,2...n.

2.3 Periodic linear system

Consider a linear homogeneous system

$$\frac{dx}{dt} = A(t)x. \tag{2.3}$$

where A is a n $\times$ n continuous matrix on the interval $-\infty < t < \infty$ and A(t+ω)= A(t) for some constant $\omega \neq 0$, then 2.3 is called a periodic system and ω is period of A.

2.4 Equilibrium point

Consider a system

$$\frac{dx_i}{dt} = f_i(x_1, x_2, ...x_n). \tag{2.4}$$

A point x^*=$(x_1^*, x_2^*, ..., x_n^*)$ is called a equilibrium point of 2.4 if

1. $x^* > 0$.

2. $f_i(x_1^*, x_2^*, ..., x_n^*)$ hold for all i=1,2,...,n.

Definition 2.4.1: The solution x(t)of 2.1 is stable if for each $\varepsilon > 0$ there exists a $\delta = \delta(\varepsilon) > 0$ such that for any solution $\bar{x}(t) = x(t, t_0, \bar{x_0})$ of 2.1 the inequality $\|\bar{x_0} - x_0\| \leq \delta$ implies $\|\bar{x}(t) - x(t)\| \leq \varepsilon$ for all $t \geq t_0$.

Definition 2.4.2: The solution x(t)of 2.1 is asymptotically stable if it is stable and if there exists a $\delta_0 > 0$ such that $\|\bar{x_0} - x_0\| \leq \delta_0$ implies $\|\bar{x}(t) - (t)\| \to 0$ as $t \to \infty$.

Definition 2.4.3: The solution x(t) of 2.1 is said to be unstable if it is not stable.

Definition 2.4.4: Let $\phi(t)$ be a fundamental matrix of 2.3 with $\phi(t_0)$= I, then 2.3 is

1. stable if and only if there exist a positive constant M such that

   $\|\phi(t)\| \leq M$ for $t \geq t_0$.

2. Asymptotically stable if and only if

   $\|\phi(t)\| \to 0$ as $t \to 0$.

Theorem 2.4.5: If all the characteristic roots of A have negative real parts, then every solution of

$$\frac{dx}{dt} = Ax \tag{2.5}$$

where A=$(\ a_{ij}\ )$ is a constant matrix, is asymptotically stable.

Theorem 2.4.6: If the characteristic roots of A having multiplicity greater than one with one negative real parts, then all its roots with multiplicity one will have non positive real parts, then all the solution of 2.5 are bounded and hence stable.

2.5 Stability by Liapunov's second method

Let

$$\frac{dx}{dt} = f(x) \tag{2.6}$$

where f $\epsilon C[R^n, R^n]$ is a autonomous differential equation system. Let $f(0) = 0$ and f(x)$\neq$ 0 for x $\neq$ 0 in some neighborhood of origin so that 2.6 has the zero solution and the origin is isolated critical point of 2.6. Let Ω be an open set in R^n containing the origin, let V(x) is a scalar continuous function which is defined on Ω. **Definition 2.5.1:** A scalar function V(x) is said to be positive definite on the set Ω if and only if $V(0) = 0$ and V(x)$>$ 0 for all x $\neq$ 0and x $\epsilon\Omega$. **Definition 2.5.2:** A scalar function V(x) is said to be negative definite on the set Ω if and only if -V(x) is positive definite on the set Ω.

2.6 Silvester criterion

Let

$$V(x) = x^T Bx = \sum b_{ij} x_i x_j \tag{2.7}$$

be a quadratic form with the system matrix B=$\left(\, b_{ij} \, \right)$, that is $b_{ij} = b_{ji}$. Necessary and sufficient condition for V(x) to be positive definite is that the determinants of all the successive principle minors of the symmetric matrix B=$\left(\, b_{ij} \, \right)$ be positive that is ,

$$b_{11} > 0, \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} > 0 \ ,... \begin{pmatrix} b_{11} & b_{12} & . & . & . & b_{1n} \\ b_{21} & b_{22} & . & . & . & b_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ b_{n1} & b_{n2} & . & . & . & b_{nn} \end{pmatrix} > 0$$

Chapter 3

North worthy contribution of work already done and proposed model

3.1 Effect of resource based exotic goose species on native plant species competing with exotic grass : a model(2013)

Let N(t), E(t) denotes the biomass of native plant and exotic grass and P(t), G(t) denotes the density of resource and goose population respectively. Let r_1, r_2, r_3 and k_1, k_2,k_3 are respectively the growth rates and carrying capacities of native plant, exotic grass and resources population. Let a_1 anda_2 are interspecific competition coefficient between exotic grass and native plant. Let e_1 is consumption rate of resources population by exotic goose and e_2 is growth rate of exotic goose due to consumption of research population. Let c_1 is consumption rate of exotic grass by exotic goose and c_2 is growth rate of exotic goose due to consumption of exotic grass. Let b is interspecific commensalism coefficient due to the presence of exotic goose, supporting the growth of native plant. Let Λ and d are birth and death rate of exotic goose respectively. Let h is intra specific competition rate of exotic goose. In view of above, the resultant system dynamics is given by the following system of differential equations:

Model 1 (With exotic species)

$$\frac{dN}{dt} = r_1 N \left(1 - \frac{N}{k_1}\right) - a_1 NE + bNG \tag{3.1}$$

$$\frac{dE}{dt} = r_2 E \left(1 - \frac{E}{k_2}\right) - a_2 NE + c_1 EG \tag{3.2}$$

$$\frac{dP}{dt} = r_3 P \left(1 - \frac{P}{k_3}\right) - e_1 PG \tag{3.3}$$

$$\frac{dG}{dt} = \Lambda G - e_2 PG + c_2 EG - dG - hG^2 \tag{3.4}$$

with initial condition as $N(0) \geq 0, E(0) \geq 0, P(0) \geq 0$ and $G(0) \geq 0$.Where a_1, a_2, c_1, c_2, e_1, e_2, r_3, k_1, k_2, k_3, b, d, h and λ are positive. In the absence of exotic goose species the above system of equations can be express as the following system od differential equations:

Model 2 (Without exotic species)

$$\frac{dN}{dt} = r_1 N \left(1 - \frac{N}{k_1}\right) - a_1 NE \tag{3.5}$$

$$\frac{dE}{dt} = r_2 E \left(1 - \frac{E}{k_2}\right) - a_2 NE \tag{3.6}$$

with non negative initial condition as $N(0) \geq 0, E(0) \geq 0$.

3.2 Proposed model

Let N(t), P(t), E(t) denotes the density of native prey population, native predator population and exotic predator population respectively. Let r and k are growth rate and carrying capacity of native prey population. Let d_1, d_2 are death rates of native and exotic predator population respectively. b is the interference due to exotic predator population. Let a_1, a_2 is the predator rate of native prey population by native predator and native predator population by exotic predator population respectively. Also, let b_1 is the growth rate of native predator population due to predator of native prey predator population and b_2 is the growth rate of exotic predator population due to predation of native predator population. Let w is the wasting time to searching native predator by exotic predator. By the above assumption, the resultant model is given by the following system of differential equation:

Model 1 (With exotic species)

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) - \frac{a_1 NP}{(1+bE)} \tag{3.7}$$

$$\frac{dP}{dt} = \frac{b_1 NP}{1+bE} - \frac{a_2 PE}{(1+\beta E)} - d_1 P \tag{3.8}$$

$$\frac{dE}{dt} = \frac{b_2 PE}{(1+\beta E)} - d_2 E \tag{3.9}$$

with initial condition $N(0) \geq 0, P(0) \geq 0, E(0) \geq 0$, where positive constant and β $=a_2$w, d_1, d_2, a_1, a_2, b_1, b_2, b are positive constant.

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