A Study of Production Inventory Model with Time Dependent Quadratic Demand and Variable Holding Cost

A Synopsis submitted

To

Department of Mathematics

In partial fulfillment of the requirement for the

Award of the degree of

Masters of Mathematics

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May, 2017

DECLARATION OF AUTHORSHIP

I, Simran Preet Kaur, declare that this project titled, "A Study of Production Inventory Model with Time Dependent Quadratic Demand and Variable Holding Cost" and the work presented in it my own. I confirm that:

- This work was done wholly or mainly whiles the candidature for a bachelor degree at this university.
- Where any part of this project has previously been submitted for a degree or any other qualification at this university or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotation, this project work is entirely my own work.
- I have acknowledged all main sources of help.
- Where the project is based on work done by myself jointly with others, I have made clearly what was done by others and what I have contributed myself

Signature

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CERTIFICATE

This is to certify that Simran Preet Kaur has completed project titled, "A Study of Production Inventory Model with Time Dependent Quadratic Demand and Variable Holding Cost" under the guidance and supervision. To the best of my knowledge, the present work is the result of the original investigation and study. No part of the project has ever been submitted for any degree at any university.

The project is fit for the submission and the partial fulfillment of the conditions for the award of Master in Mathematics.

Signature

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ABSTRACT

This paper presents production lot size inventory models for deteriorating items with time dependent demand rate. It is assumed that the deterioration rate is constant and the holding cost is a linear function in time. Inventory models are developed without considering shortages. The salvage value is used while calculating the optimal policies that maximize the revenue of the system. Numerical examples and the sensitivity of these models will be discussed in the next semester.

ACKNOWLEDGEMENT

It is true that one can't express his/her feelings completely by just writing a few words on a piece of paper. But some time a few words also make a difference. I express my deep sense of gratitude to my guide Dr. Ashutosh Pandey Assistant Professor, Dept. Of Mathematics Lovely Professional University whose cooperation, enthusiastic attitude, active participation, encouragement and devotion towards work helped me in completing this work timely successfully. I thank him from the core of my heart.

My sincere thanks Mr. Kulwinder Singh (Head of dept. of Mathematics) for his continuous motivation and encouragement throughout my research work.

My thanks also go to faculty members of dept. Of Mathematics and especially to my friends.

I would be going astray from my path of duty if I do not pay my heartfelt gratitude to my entire family, my brother and my parents who are my world and whose devotion, love, affection and financial support enabled me to complete this important work successfully. I also wish to thank to my mates Krishna, Sachin and others, we have shared every moment together and made the life beautiful which would have been incomplete without them.

And Finally thanks to Almighty who made all the things possible for me. Last but not the least, I have no words to thank God for his countless blessings.

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Chapter1:

INTRODUCTION [1]

Inventory is defined as the money, capital or labor that is being used in an enterprise. It is important to maintain inventories for smooth functioning of an enterprise. The objective is to minimize total i.e. actual or expected cost.

Types of inventories [3]:

There are basically five types of inventories.

1) Fluctuation inventories: These are inventories to meet uncertainties of demand and supply.
2) Anticipation inventories: These inventories keep men and machine ready in advance for future use. Like storing crackers well before Diwali.
3) Cycle inventories: In these inventories items are purchased in large amount rather than the exact quantities required.
4) Decoupling inventories: The inventories used to minimize the interdependence of various stages of the production system are called decoupling inventories.
5) Transportation inventories: This is also known as process inventory in which the significant amount of time is used in the shipment of items from production center to various distribution centers and customers.

Inventory costs [2]:

1) Purchased cost: It is the amount of money paid to produce an item.
2) Holding cost: These include carrying or holding goods in stock. various components of holding costs are:
   a) Cost of capital tied up in inventories.

b) Cost of storage space.
c) Depreciation and deterioration costs.
d) Pilferage cost.
e) Obsolescence cost.
f) Handling cost.
g) Record-keeping and administrative cost.
h) Taxes and insurance.

3) Set-up cost: These costs include the fixed cost associated with placing of an order. They include costs of purchase, follow up, receiving the goods, cost of mailing etc.
4) Shortage costs: These costs are associated with delay in fulfilling demands.

Inventory control problem [5]:

The inventory control problem is to determine three basic factors:

1) When to order?
2) How much to order?
3) And How much safety stock should be kept?

Chapter2:

Literature review

1.) **Fuzzy inventory model for two parameter Weibull deteriorating items [1]:** This paper, present the development of a fuzzy inventory model with linear demand two parameter Weibull deterioration and shortage under fully backlogged. The deterioration cost, holding cost and shortages are taken as hexagonal fuzzy members. Signed distance method was used to defuzzify the total cost function.

2.) **Time dependent quadratic demand inventory models when delay in payments is acceptable [2]:** This paper constructs an EOQ model for deteriorating items with time dependent quadratic demand rate. An assumption is made that the deterioration rate is constant and the supplier offers his retailer the credit period to settle the account of the procurement units. It is assumed that shortages are not allowed and the replenishment rate is instantaneous, to solve the model. The objective of this paper is to minimize the retailers total inventory cost.

3.) **Considering lost sale in inventory routing problems for perishable goods [3]:** This paper presents a mathematical model for an inventory routing problem. This model is especially designed for allocating the stock of perishable goods. It is assumed that the age of perishable inventory has a negative impact on the demand of end customers and a percentage of the demand is considered as lost sale. The proposed model balances the transportation cost, the cost of inventory holding and lost sale. In this paper, the cost of lost sale is considered as linear or exponential function of inventory age.

4.) **Optimal pricing and marketing planning for deteriorating items [4]:** optimal pricing and marketing planning plays an essential role in production decisions on deteriorating items. This paper presents a mathematical model for a three-level supply chain, which includes one producer, one distributor and one retailer. The proposed study considers the production of a deteriorating item where demand is influenced by price, marketing expenditure, quality of product and after-sales service expenditures.

5.) **An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging [5]:** In this study, we consider a problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. In this model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown.

6.) **An order-level inventory model for a deteriorating item with Weibull distribution deterioration, time-quadratic demand and shortages [8]:** In this paper, an inventory model is developed for a deteriorating item which have an instantaneous supply, a quadratic time-varying demand and shortages in inventory. The time to deterioration is represented by two-parameter Weibull distribution. The optimal solution of the problem is obtained by solving the model analytically.

Chapter 3:

Assumptions and Notations [1]:

1. The demand rate is assumed to be $R(x) = a + bx + cx^2$, where a, b and c are constants.
2. $y(x)$ is the inventory level at time x.
3. The lead-time is zero and shortages are allowed.
4. The production rate $K = \gamma R(x)$, where γ>1.
5. The fraction of the on-hand inventory deterioration per unit time is fixed, $\theta(x) = \theta$.
6. There is no production during the time x_1 to T and demand dominates so the inventory level slowly decreases to zero.
7. Holding cost is linear function of time $h(x) = a_1 + a_2 x, a_1 \geq 0, a_2 \geq 0$.
8. b_1 is the deteriorating cost per unit time.
9. γ_1 is the salvage value, associated with deteriorated units during a cycle time.
10. s_r is the selling price per unit.

Formulation of Mathematical Model [1]:

The objective of the model is to determine the optimum profit for items having time dependent quadratic demand and the rate of deterioration is constant. We assumed that the production dominates demand during tome 0 to x_1 and there is no production during the time x_1 to T and demand dominates, so that the inventory level slowly decreases to zero at the end.

If $y(x)$ be the inventory level at time x, the differential equations describing the inventory level at time x are given by

$$\frac{dy}{dx} + \theta . y(x) = K - R(x) \qquad 0 \leq x \leq x_1 \qquad (1)$$

$$\frac{dy}{dx} + \theta . y(x) = -R(x) \qquad x_1 \leq x \leq T \qquad (2)$$

Where $K = \gamma R(x), R(x) = a + bx + cx^2$.

According to the assumptions

$$y(0) = 0,\ y(x_1) = S,\ y(T) = 0 \tag{3}$$

The solution of the equation (1) with condition $y(0) = 0$ is

$$y(x) = -\frac{2c}{\theta^3} + \frac{2ce^{-x\theta}}{\theta^3} + \frac{2c\gamma}{\theta^3} - \frac{2ce^{-x\theta}\gamma}{\theta^3} + \frac{b}{\theta^2} - \frac{be^{-x\theta}}{\theta^2} + \frac{2cx}{\theta^2} - \frac{b\gamma}{\theta^2} + \frac{be^{-x\theta}\gamma}{\theta^2} - \frac{2cx\gamma}{\theta^2} - \frac{a}{\theta} + \frac{ae^{-x\theta}}{\theta} - \frac{bx}{\theta} - \frac{cx^2}{\theta} + \frac{a\gamma}{\theta} - \frac{ae^{-x\theta}\gamma}{\theta} + \frac{bx\gamma}{\theta} + \frac{cx^2\gamma}{\theta} \tag{4}$$

Now, the solution of equation (2) is

$$y(x) = e^{-x_1\theta+T\theta}S - \frac{2c}{\theta^3} + \frac{2ce^{-x_1\theta+T\theta}}{\theta^3} + \frac{b}{\theta^2} - \frac{be^{-x_1\theta+T\theta}}{\theta^2} + \frac{2cx_1}{\theta^2} - \frac{2ce^{-x_1\theta+T\theta}T}{\theta^2} - \frac{a}{\theta} + \frac{ae^{-x_1\theta+T\theta}}{\theta} - \frac{bx_1}{\theta} - \frac{c{x_1}^2}{\theta} + \frac{be^{-x_1\theta+T\theta}T}{\theta} + \frac{ce^{-x_1\theta+T\theta}T^2}{\theta} \tag{5}$$

The total cost (TC) is given by

$$TC = OC + IHC + DC - SV \tag{6}$$

Where OC is the ordering cost, IHC is the holding cost, DC is the deterioration cost and SV is the salvage cost.

Now

1. The ordering cost = A (7)
2. Inventory holding cost per unit is given by

$$IHC = \int_0^T h.y(x)\,dx$$

$$= \int_0^{x_1} h.y(x)dx + \int_{x_1}^{T} h.y(x)dx \tag{8}$$

$$\int_0^{x_1} h.y(x)dx = \frac{2ca_1}{\theta^4} - \frac{2ce^{-\theta x_1}a_1}{\theta^4} - \frac{2c\gamma a_1}{\theta^4} + \frac{2ce^{-\theta x_1}\gamma a_1}{\theta^4} - \frac{ba_1}{\theta^3} + \frac{be^{-\theta x_1}a_1}{\theta^3} + \frac{b\gamma a_1}{\theta^3} - \frac{be^{-\theta x_1}\gamma a_1}{\theta^3} + \frac{aa_1}{\theta^2} - \frac{ae^{-\theta x_1}a_1}{\theta^2} - \frac{a\gamma a_1}{\theta^2} + \frac{ae^{-\theta x_1}\gamma a_1}{\theta^2} + \frac{2ca_2}{\theta^5} - \frac{2ce^{-\theta x_1}a_2}{\theta^5} - \frac{2c\gamma a_2}{\theta^5} + \frac{2ce^{-\theta x_1}\gamma a_2}{\theta^5} - \frac{ba_2}{\theta^4} + \frac{be^{-\theta x_1}a_2}{\theta^4} + \frac{b\gamma a_2}{\theta^4} - \frac{be^{-\theta x_1}\gamma a_2}{\theta^4} + \frac{aa_2}{\theta^3} - \frac{ae^{-\theta x_1}a_2}{\theta^3} - \frac{a\gamma a_2}{\theta^3} + \frac{ae^{-\theta x_1}\gamma a_2}{\theta^3} - \frac{2ca_1x_1}{\theta^3} + \frac{2c\gamma a_1x_1}{\theta^3} + \frac{ba_1x_1}{\theta^2} - \frac{b\gamma a_1x_1}{\theta^2} - \frac{aa_1x_1}{\theta} + \frac{a\gamma a_1x_1}{\theta} - \frac{2ce^{-\theta x_1}a_2x_1}{\theta^4} + \frac{2ce^{-\theta x_1}\gamma a_2x_1}{\theta^4} + \frac{be^{-\theta x_1}a_2x_1}{\theta^3} - \frac{be^{-\theta x_1}\gamma a_2x_1}{\theta^3} - \frac{ae^{-\theta x_1}a_2x_1}{\theta^2} + \frac{ae^{-\theta x_1}\gamma a_2x_1}{\theta^2} + \frac{ca_1x_1^2}{\theta^2} - \frac{c\gamma a_1x_1^2}{\theta^2} - \frac{ba_1x_1^2}{2\theta} + \frac{b\gamma a_1x_1^2}{2\theta} - \frac{ca_2x_1^2}{\theta^3} + \frac{c\gamma a_2x_1^2}{\theta^3} + \frac{ba_2x_1^2}{2\theta^2} - \frac{b\gamma a_2x_1^2}{2\theta^2} - \frac{aa_2x_1^2}{2\theta} + \frac{a\gamma a_2x_1^2}{2\theta} - \frac{ca_1x_1^3}{3\theta} + \frac{c\gamma a_1x_1^3}{3\theta} + \frac{2ca_2x_1^3}{3\theta^2} - \frac{2c\gamma a_2x_1^3}{3\theta^2} - \frac{ba_2x_1^3}{3\theta} + \frac{b\gamma a_2x_1^3}{3\theta} - \frac{ca_2x_1^4}{4\theta} + \frac{c\gamma a_2x_1^4}{4\theta} \qquad (8.a)$$

$$\int_{x_1}^{T} h.y(x)dx = -\frac{2ca_1}{\theta^4} + \frac{2ce^{\theta(T-x_1)}a_1}{\theta^4} + \frac{ba_1}{\theta^3} - \frac{be^{\theta(T-x_1)}a_1}{\theta^3} - \frac{2ce^{\theta(T-x_1)}Ta_1}{\theta^3} - \frac{aa_1}{\theta^2} + \frac{ae^{\theta(T-x_1)}a_1}{\theta^2} + \frac{be^{\theta(T-x_1)}Ta_1}{\theta^2} + \frac{ce^{\theta(T-x_1)}T^2a_1}{\theta^2} - \frac{Sa_1}{\theta} + \frac{e^{\theta(T-x_1)}Sa_1}{\theta} - \frac{aTa_1}{\theta} - \frac{bT^2a_1}{2\theta} - \frac{cT^3a_1}{3\theta} - \frac{2ca_2}{\theta^5} + \frac{2ce^{\theta(T-x_1)}a_2}{\theta^5} + \frac{ba_2}{\theta^4} - \frac{be^{\theta(T-x_1)}a_2}{\theta^4} - \frac{2ce^{\theta(T-x_1)}Ta_2}{\theta^4} - \frac{aa_2}{\theta^3} + \frac{ae^{\theta(T-x_1)}a_2}{\theta^3} + \frac{be^{\theta(T-x_1)}Ta_2}{\theta^3} + \frac{ce^{\theta(T-x_1)}T^2a_2}{\theta^3} - \frac{Sa_2}{\theta^2} + \frac{e^{\theta(T-x_1)}Sa_2}{\theta^2} - \frac{aTa_2}{\theta^2} - \frac{bT^2a_2}{2\theta^2} - \frac{cT^3a_2}{3\theta^2} - \frac{STa_2}{\theta} - \frac{aT^2a_2}{2\theta} - \frac{bT^3a_2}{3\theta} - \frac{cT^4a_2}{4\theta} + \frac{2ca_1x_1}{\theta^3} - \frac{ba_1x_1}{\theta^2} + \frac{aa_1x_1}{\theta} - \frac{2ce^{\theta(T-x_1)}a_2x_1}{\theta^4} - \frac{be^{\theta(T-x_1)}a_2x_1}{\theta^3} - \frac{2ce^{\theta(T-x_1)}Ta_2x_1}{\theta^3} + \frac{ae^{\theta(T-x_1)}a_2x_1}{\theta^2} + \frac{be^{\theta(T-x_1)}Ta_2x_1}{\theta^2} + \frac{ce^{\theta(T-x_1)}T^2a_2x_1}{\theta^2} + \frac{e^{\theta(T-x_1)}Sa_2x_1}{\theta} - \frac{ca_1x_1^2}{\theta^2} + \frac{ba_1x_1^2}{2\theta} + \frac{ca_2x_1^2}{\theta^3} - \frac{ba_2x_1^2}{2\theta^2} + \frac{aa_2x_1^2}{2\theta} + \frac{ca_1x_1^3}{3\theta} - \frac{2ca_2x_1^3}{3\theta^2} + \frac{ba_2x_1^3}{3\theta} + \frac{ca_2x_1^4}{4\theta} \qquad (8.b)$$

Using eq. (8.a) and (8.b) in eq. (8), we get

$$IHC = \frac{2ce^{\theta(T-x_1)}a_1}{\theta^4} - \frac{2ce^{-\theta x_1}a_1}{\theta^4} - \frac{2c\gamma a_1}{\theta^4} + \frac{2ce^{-\theta x_1}\gamma a_1}{\theta^4} - \frac{be^{\theta(T-x_1)}a_1}{\theta^3} + \frac{be^{-\theta x_1}a_1}{\theta^3} - \frac{2ce^{\theta(T-x_1)}Ta_1}{\theta^3} + \frac{b\gamma a_1}{\theta^3} - \frac{be^{-\theta x_1}\gamma a_1}{\theta^3} + \frac{ae^{\theta(T-x_1)}a_1}{\theta^2} - \frac{ae^{-\theta x_1}a_1}{\theta^2} + \frac{be^{\theta(T-x_1)}Ta_1}{\theta^2} + \frac{ce^{\theta(T-x_1)}T^2a_1}{\theta^2} - \frac{a\gamma a_1}{\theta^2} + \frac{ae^{-\theta x_1}\gamma a_1}{\theta^2} - \frac{Sa_1}{\theta} + \frac{e^{\theta(T-x_1)}Sa_1}{\theta} - \frac{aTa_1}{\theta} - \frac{bT^2a_1}{2\theta} - \frac{cT^3a_1}{3\theta} + \frac{2ce^{\theta(T-x_1)}a_2}{\theta^5} - \frac{2ce^{-\theta x_1}a_2}{\theta^5} - \frac{2c\gamma a_2}{\theta^5} + \frac{2ce^{-\theta x_1}\gamma a_2}{\theta^5} - \frac{be^{\theta(T-x_1)}a_2}{\theta^4} + \frac{be^{-\theta x_1}a_2}{\theta^4} - \frac{2ce^{\theta(T-x_1)}Ta_2}{\theta^4} + \frac{b\gamma a_2}{\theta^4} - \frac{be^{-\theta x_1}\gamma a_2}{\theta^4} + \frac{ae^{\theta(T-x_1)}a_2}{\theta^3} - \frac{ae^{-\theta x_1}a_2}{\theta^3} + \frac{be^{\theta(T-x_1)}Ta_2}{\theta^3} + \frac{ce^{\theta(T-x_1)}T^2a_2}{\theta^3} - \frac{a\gamma a_2}{\theta^3} + \frac{ae^{-\theta x_1}\gamma a_2}{\theta^3} - \frac{Sa_2}{\theta^2} + \frac{e^{\theta(T-x_1)}Sa_2}{\theta^2} - \frac{aTa_2}{\theta^2} - \frac{bT^2a_2}{2\theta^2} - \frac{cT^3a_2}{3\theta^2} - \frac{STa_2}{\theta} - \frac{aT^2a_2}{2\theta} - \frac{bT^3a_2}{3\theta} - \frac{cT^4a_2}{4\theta} + \frac{2c\gamma a_1x_1}{\theta^3} - \frac{b\gamma a_1x_1}{\theta^2} + \frac{a\gamma a_1x_1}{\theta} + \frac{2ce^{\theta(T-x_1)}a_2x_1}{\theta^4} - \frac{2ce^{-\theta x_1}a_2x_1}{\theta^4} + \frac{2ce^{-\theta x_1}\gamma a_2x_1}{\theta^4} - \frac{be^{\theta(T-x_1)}a_2x_1}{\theta^3} + \frac{be^{-\theta x_1}a_2x_1}{\theta^3} - \frac{2ce^{\theta(T-x_1)}Ta_2x_1}{\theta^3} - \frac{be^{-\theta x_1}\gamma a_2x_1}{\theta^3} + \frac{ae^{\theta(T-x_1)}a_2x_1}{\theta^2} - \frac{ae^{-\theta x_1}a_2x_1}{\theta^2} + \frac{be^{\theta(T-x_1)}Ta_2x_1}{\theta^2} + \frac{ce^{\theta(T-x_1)}T^2a_2x_1}{\theta^2} + \frac{ae^{-\theta x_1}\gamma a_2x_1}{\theta^2} + \frac{e^{\theta(T-x_1)}Sa_2x_1}{\theta} - \frac{c\gamma a_1x_1^2}{\theta^2} + \frac{b\gamma a_1x_1^2}{2\theta} + \frac{c\gamma a_2x_1^2}{\theta^3} - \frac{b\gamma a_2x_1^2}{2\theta^2} + \frac{a\gamma a_2x_1^2}{2\theta} + \frac{c\gamma a_1x_1^3}{3\theta} - \frac{2c\gamma a_2x_1^3}{3\theta^2} + \frac{b\gamma a_2x_1^3}{3\theta} + \frac{c\gamma a_2x_1^4}{4\theta} \quad (9)$$

3. The deterioration cost is given by

$$DC = \int_0^{x_1} (\gamma ab_1 + \gamma bb_1x + \gamma cb_1x^2 - ab_1 - bb_1x - cb_1x^2)\,dx + \int_{x_1}^{T} (-ab_1 - bb_1x_1 - cb_1{x_1}^2)\,dr$$

$$DC = -aTb_1 - \frac{1}{2}bT^2b_1 - \frac{1}{3}cT^3b_1 + a\gamma b_1x_1 + \frac{1}{2}b\gamma b_1x_1^2 + \frac{1}{3}c\gamma b_1x_1^3 \quad (10)$$

4. Salvage value is given by

$$SV = \gamma_1 \mathrm{DC}$$

$$SV = -aTb_1\gamma_1 - \frac{1}{2}bT^2b_1\gamma_1 - \frac{1}{3}cT^3b_1\gamma_1 + a\gamma b_1x_1\gamma_1 + \frac{1}{2}b\gamma b_1x_1^2\gamma_1 + \frac{1}{3}c\gamma b_1x_1^3\gamma_1 \tag{11}$$

Using eq. (7), (8), (9), (10), (11) in eq. (6), the Total Cost is

$$TC = A + \frac{2ce^{\theta(T-x_1)}a_1}{\theta^4} - \frac{2ce^{-\theta x_1}a_1}{\theta^4} - \frac{2c\gamma a_1}{\theta^4} + \frac{2ce^{-\theta x_1}\gamma a_1}{\theta^4} - \frac{be^{\theta(T-x_1)}a_1}{\theta^3} + \frac{be^{-\theta x_1}a_1}{\theta^3} - \frac{2ce^{\theta(T-x_1)}Ta_1}{\theta^3} + \frac{b\gamma a_1}{\theta^3} - \frac{be^{-\theta x_1}\gamma a_1}{\theta^3} + \frac{ae^{\theta(T-x_1)}a_1}{\theta^2} - \frac{ae^{-\theta x_1}a_1}{\theta^2} + \frac{be^{\theta(T-x_1)}Ta_1}{\theta^2} + \frac{ce^{\theta(T-x_1)}T^2a_1}{\theta^2} - \frac{a\gamma a_1}{\theta^2} + \frac{ae^{-\theta x_1}\gamma a_1}{\theta^2} - \frac{Sa_1}{\theta} + \frac{e^{\theta(T-x_1)}Sa_1}{\theta} - \frac{aTa_1}{\theta} - \frac{bT^2a_1}{2\theta} - \frac{cT^3a_1}{3\theta} + \frac{2ce^{\theta(T-x_1)}a_2}{\theta^5} - \frac{2ce^{-\theta x_1}a_2}{\theta^5} - \frac{2c\gamma a_2}{\theta^5} + \frac{2ce^{-\theta x_1}\gamma a_2}{\theta^5} - \frac{be^{\theta(T-x_1)}a_2}{\theta^4} + \frac{be^{-\theta x_1}a_2}{\theta^4} - \frac{2ce^{\theta(T-x_1)}Ta_2}{\theta^4} + \frac{b\gamma a_2}{\theta^4} - \frac{be^{-\theta x_1}\gamma a_2}{\theta^4} + \frac{ae^{\theta(T-x_1)}a_2}{\theta^3} - \frac{ae^{-\theta x_1}a_2}{\theta^3} + \frac{be^{\theta(T-x_1)}Ta_2}{\theta^3} + \frac{ce^{\theta(T-x_1)}T^2a_2}{\theta^3} - \frac{a\gamma a_2}{\theta^3} + \frac{ae^{-\theta x_1}\gamma a_2}{\theta^3} - \frac{Sa_2}{\theta^2} + \frac{e^{\theta(T-x_1)}Sa_2}{\theta^2} - \frac{aTa_2}{\theta^2} - \frac{bT^2a_2}{2\theta^2} - \frac{cT^3a_2}{3\theta^2} - \frac{STa_2}{\theta} - \frac{aT^2a_2}{2\theta} - \frac{bT^3a_2}{3\theta} - \frac{cT^4a_2}{4\theta} - aTb_1 - \frac{1}{2}bT^2b_1 - \frac{1}{3}cT^3b_1 + \frac{2c\gamma a_1x_1}{\theta^3} - \frac{b\gamma a_1x_1}{\theta^2} + \frac{a\gamma a_1x_1}{\theta} + \frac{2ce^{\theta(T-x_1)}a_2x_1}{\theta^4} - \frac{2ce^{-\theta x_1}a_2x_1}{\theta^4} + \frac{2ce^{-\theta x_1}\gamma a_2x_1}{\theta^4} - \frac{be^{\theta(T-x_1)}a_2x_1}{\theta^3} + \frac{be^{-\theta x_1}a_2x_1}{\theta^3} - \frac{2ce^{\theta(T-x_1)}Ta_2x_1}{\theta^3} - \frac{be^{-\theta x_1}\gamma a_2x_1}{\theta^3} + \frac{ae^{\theta(T-x_1)}a_2x_1}{\theta^2} - \frac{ae^{-\theta x_1}a_2x_1}{\theta^2} + \frac{be^{\theta(T-x_1)}Ta_2x_1}{\theta^2} + \frac{ce^{\theta(T-x_1)}T^2a_2x_1}{\theta^2} + \frac{ae^{-\theta x_1}\gamma a_2x_1}{\theta^2} + \frac{e^{\theta(T-x_1)}Sa_2x_1}{\theta} + a\gamma b_1x_1 - \frac{c\gamma a_1x_1^2}{\theta^2} + \frac{b\gamma a_1x_1^2}{2\theta} + \frac{c\gamma a_2x_1^2}{\theta^3} - \frac{b\gamma a_2x_1^2}{2\theta^2} + \frac{a\gamma a_2x_1^2}{2\theta} + \frac{1}{2}b\gamma b_1x_1^2 + \frac{c\gamma a_1x_1^3}{3\theta} - \frac{2c\gamma a_2x_1^3}{3\theta^2} + \frac{b\gamma a_2x_1^3}{3\theta} + \frac{1}{3}c\gamma b_1x_1^3 + \frac{c\gamma a_2x_1^4}{4\theta} + aTb_1\gamma_1 + \frac{1}{2}bT^2b_1\gamma_1 + \frac{1}{3}cT^3b_1\gamma_1 - a\gamma b_1x_1\gamma_1 - \frac{1}{2}b\gamma b_1x_1^2\gamma_1 - \frac{1}{3}c\gamma b_1x_1^3\gamma_1 \tag{12}$$

The Sales Revenue of the system is

$$SR = s_r\left(\int_0^{x_1}(\gamma a + \gamma bx + \gamma cx^2 - a - bx - cx^2)\,dx + \int_{x_1}^{T}(a + bx_1 + cx_1{}^2)\,dx_1\right)$$

`

$$SR = aTs_r + \frac{1}{2}bT^2s_r + \frac{1}{3}cT^3s_r - 2as_rx_1 + a\gamma s_rx_1 - bs_rx_1^2 + \frac{1}{2}b\gamma s_rx_1^2 - \frac{2}{3}cs_rx_1^3 + \frac{1}{3}c\gamma s_rx_1^3 \quad (13)$$

Thus, the Total Profit of the system is

$$P(x_1, T) = \frac{1}{t}(\mathrm{SR} - \mathrm{TC})$$

$$P(x_1,T) = -\frac{A}{t} - \frac{2ce^{\theta(T-x_1)}a_1}{t\theta^4} + \frac{2ce^{-\theta x_1}a_1}{t\theta^4} + \frac{2c\gamma a_1}{t\theta^4} - \frac{2ce^{-\theta x_1}\gamma a_1}{t\theta^4} + \frac{be^{\theta(T-x_1)}a_1}{t\theta^3} - \frac{be^{-\theta x_1}a_1}{t\theta^3} + \frac{2ce^{\theta(T-x_1)}Ta_1}{t\theta^3} - \frac{b\gamma a_1}{t\theta^3} + \frac{be^{-\theta x_1}\gamma a_1}{t\theta^3} - \frac{ae^{\theta(T-x_1)}a_1}{t\theta^2} + \frac{ae^{-\theta x_1}a_1}{t\theta^2} - \frac{be^{\theta(T-x_1)}Ta_1}{t\theta^2} - \frac{ce^{\theta(T-x_1)}T^2a_1}{t\theta^2} + \frac{a\gamma a_1}{t\theta^2} - \frac{ae^{-\theta x_1}\gamma a_1}{t\theta^2} + \frac{Sa_1}{t\theta} - \frac{e^{\theta(T-x_1)}Sa_1}{t\theta} + \frac{aTa_1}{t\theta} + \frac{bT^2a_1}{2t\theta} + \frac{cT^3a_1}{3t\theta} - \frac{2ce^{\theta(T-x_1)}a_2}{t\theta^5} + \frac{2ce^{-\theta x_1}a_2}{t\theta^5} + \frac{2c\gamma a_2}{t\theta^5} - \frac{2ce^{-\theta x_1}\gamma a_2}{t\theta^5} + \frac{be^{\theta(T-x_1)}a_2}{t\theta^4} - \frac{be^{-\theta x_1}a_2}{t\theta^4} + \frac{2ce^{\theta(T-x_1)}Ta_2}{t\theta^4} - \frac{b\gamma a_2}{t\theta^4} + \frac{be^{-\theta x_1}\gamma a_2}{t\theta^4} - \frac{ae^{\theta(T-x_1)}a_2}{t\theta^3} + \frac{ae^{-\theta x_1}a_2}{t\theta^3} - \frac{be^{\theta(T-x_1)}Ta_2}{t\theta^3} - \frac{ce^{\theta(T-x_1)}T^2a_2}{t\theta^3} + \frac{a\gamma a_2}{t\theta^3} - \frac{ae^{-\theta x_1}\gamma a_2}{t\theta^3} + \frac{Sa_2}{t\theta^2} - \frac{e^{\theta(T-x_1)}Sa_2}{t\theta^2} + \frac{aTa_2}{t\theta^2} + \frac{bT^2a_2}{2t\theta^2} + \frac{cT^3a_2}{3t\theta^2} + \frac{STa_2}{t\theta} + \frac{aT^2a_2}{2t\theta} + \frac{bT^3a_2}{3t\theta} + \frac{cT^4a_2}{4t\theta} + \frac{aTb_1}{t} + \frac{bT^2b_1}{2t} + \frac{cT^3b_1}{3t} + \frac{aTs_r}{t} + \frac{bT^2s_r}{2t} + \frac{cT^3s_r}{3t} - \frac{2c\gamma a_1x_1}{t\theta^3} + \frac{b\gamma a_1x_1}{t\theta^2} - \frac{a\gamma a_1x_1}{t\theta} - \frac{2ce^{\theta(T-x_1)}a_2x_1}{t\theta^4} + \frac{2ce^{-\theta x_1}a_2x_1}{t\theta^4} - \frac{2ce^{-\theta x_1}\gamma a_2x_1}{t\theta^4} + \frac{be^{\theta(T-x_1)}a_2x_1}{t\theta^3} - \frac{be^{-\theta x_1}a_2x_1}{t\theta^3} + \frac{2ce^{\theta(T-x_1)}Ta_2x_1}{t\theta^3} + \frac{be^{-\theta x_1}\gamma a_2x_1}{t\theta^3} - \frac{ae^{\theta(T-x_1)}a_2x_1}{t\theta^2} + \frac{ae^{-\theta x_1}a_2x_1}{t\theta^2} - \frac{c\gamma b_1x_1^3}{3t} - \frac{be^{\theta(T-x_1)}Ta_2x_1}{t\theta^2} - \frac{ce^{\theta(T-x_1)}T^2a_2x_1}{t\theta^2} - \frac{ae^{-\theta x_1}\gamma a_2x_1}{t\theta^2} - \frac{e^{\theta(T-x_1)}Sa_2x_1}{t\theta} - \frac{a\gamma b_1x_1}{t} - \frac{2as_rx_1}{t} + \frac{a\gamma s_rx_1}{t} + \frac{c\gamma a_1x_1^2}{t\theta^2} - \frac{b\gamma a_1x_1^2}{2t\theta} - \frac{c\gamma a_2x_1^2}{t\theta^3} + \frac{b\gamma a_2x_1^2}{2t\theta^2} - \frac{a\gamma a_2x_1^2}{2t\theta} - \frac{b\gamma b_1x_1^2}{2t} - \frac{bs_rx_1^2}{t} + \frac{b\gamma s_rx_1^2}{2t} - \frac{c\gamma a_1x_1^3}{3t\theta} + \frac{2c\gamma a_2x_1^3}{3t\theta^2} - \frac{b\gamma a_2x_1^3}{3t\theta} - \frac{2cs_rx_1^3}{3t} + \frac{c\gamma s_rx_1^3}{3t} - \frac{c\gamma a_2x_1^4}{4t\theta} - \frac{aTb_1\gamma_1}{t} - \frac{bT^2b_1\gamma_1}{2t} - \frac{cT^3b_1\gamma_1}{3t} + \frac{a\gamma b_1x_1\gamma_1}{t} + \frac{b\gamma b_1x_1^2\gamma_1}{2t} + \frac{c\gamma b_1x_1^3\gamma_1}{3t} \quad (14)$$

The optimum value of x_1 and T are obtained by solving

$\frac{\partial}{\partial x_1}P(x_1,T)=0$ and $\frac{\partial}{\partial T}P(x_1,T)=0$, we get

$$\frac{e^{\theta(T-x_1)}Sa_1}{t}+\frac{2ce^{\theta(T-x_1)}a_1}{t\theta^3}-\frac{2ce^{-\theta x_1}a_1}{t\theta^3}-\frac{2c\gamma a_1}{t\theta^3}+\frac{2ce^{-\theta x_1}\gamma a_1}{t\theta^3}-\frac{be^{\theta(T-x_1)}a_1}{t\theta^2}+$$
$$\frac{be^{-\theta x_1}a_1}{t\theta^2}-\frac{2ce^{\theta(T-x_1)}Ta_1}{t\theta^2}+\frac{b\gamma a_1}{t\theta^2}-\frac{be^{-\theta x_1}\gamma a_1}{t\theta^2}+\frac{ae^{\theta(T-x_1)}a_1}{t\theta}-\frac{ae^{-\theta x_1}a_1}{t\theta}+$$
$$\frac{be^{\theta(T-x_1)}Ta_1}{t\theta}+\frac{ce^{\theta(T-x_1)}T^2a_1}{t\theta}-\frac{a\gamma a_1}{t\theta}+\frac{ae^{-\theta x_1}\gamma a_1}{t\theta}-\frac{a\gamma b_1}{t}-\frac{2as_r}{t}+\frac{a\gamma s_r}{t}+\frac{2c\gamma a_1x_1}{t\theta^2}-$$
$$\frac{b\gamma a_1x_1}{t\theta}+\frac{e^{\theta(T-x_1)}Sa_2x_1}{t}+\frac{2ce^{\theta(T-x_1)}a_2x_1}{t\theta^3}-\frac{2ce^{-\theta x_1}a_2x_1}{t\theta^3}-\frac{2c\gamma a_2x_1}{t\theta^3}+\frac{2ce^{-\theta x_1}\gamma a_2x_1}{t\theta^3}-$$
$$\frac{be^{\theta(T-x_1)}a_2x_1}{t\theta^2}+\frac{be^{-\theta x_1}a_2x_1}{t\theta^2}-\frac{2ce^{\theta(T-x_1)}Ta_2x_1}{t\theta^2}+\frac{b\gamma a_2x_1}{t\theta^2}-\frac{be^{-\theta x_1}\gamma a_2x_1}{t\theta^2}+$$
$$\frac{ae^{\theta(T-x_1)}a_2x_1}{t\theta}-\frac{ae^{-\theta x_1}a_2x_1}{t\theta}+\frac{be^{\theta(T-x_1)}Ta_2x_1}{t\theta}+\frac{ce^{\theta(T-x_1)}T^2a_2x_1}{t\theta}-\frac{a\gamma a_2x_1}{t\theta}+$$
$$\frac{ae^{-\theta x_1}\gamma a_2x_1}{t\theta}-\frac{b\gamma b_1x_1}{t}-\frac{2bs_rx_1}{t}+\frac{b\gamma s_rx_1}{t}-\frac{c\gamma a_1x_1^2}{t\theta}+\frac{2c\gamma a_2x_1^2}{t\theta^2}-\frac{b\gamma a_2x_1^2}{t\theta}-\frac{c\gamma b_1x_1^2}{t}-$$
$$\frac{2cs_rx_1^2}{t}+\frac{c\gamma s_rx_1^2}{t}-\frac{c\gamma a_2x_1^3}{t\theta}+\frac{a\gamma b_1\gamma_1}{t}+\frac{b\gamma b_1x_1\gamma_1}{t}+\frac{c\gamma b_1x_1^2\gamma_1}{t}=0$$

and

$$-\frac{e^{\theta(T-x_1)}Sa_1}{t}+\frac{aa_1}{t\theta}-\frac{ae^{\theta(T-x_1)}a_1}{t\theta}+\frac{bTa_1}{t\theta}-\frac{be^{\theta(T-x_1)}Ta_1}{t\theta}+\frac{cT^2a_1}{t\theta}-\frac{ce^{\theta(T-x_1)}T^2a_1}{t\theta}+$$
$$\frac{aa_2}{t\theta^2}-\frac{ae^{\theta(T-x_1)}a_2}{t\theta^2}+\frac{bTa_2}{t\theta^2}-\frac{be^{\theta(T-x_1)}Ta_2}{t\theta^2}+\frac{cT^2a_2}{t\theta^2}-\frac{ce^{\theta(T-x_1)}T^2a_2}{t\theta^2}+\frac{Sa_2}{t\theta}-\frac{e^{\theta(T-x_1)}Sa_2}{t\theta}+$$
$$\frac{aTa_2}{t\theta}+\frac{bT^2a_2}{t\theta}+\frac{cT^3a_2}{t\theta}+\frac{ab_1}{t}+\frac{bTb_1}{t}+\frac{cT^2b_1}{t}+\frac{as_r}{t}+\frac{bTs_r}{t}+\frac{cT^2s_r}{t}-\frac{e^{\theta(T-x_1)}Sa_2x_1}{t}-$$
$$\frac{ae^{\theta(T-x_1)}a_2x_1}{t\theta}-\frac{be^{\theta(T-x_1)}Ta_2x_1}{t\theta}-\frac{ce^{\theta(T-x_1)}T^2a_2x_1}{t\theta}-\frac{ab_1\gamma_1}{t}-\frac{bTb_1\gamma_1}{t}-\frac{cT^2b_1\gamma_1}{t}=0$$

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