LOVELY PROFESSIONAL UNIVERSITY

MASTER OF SCIENCE

SLIP EFFECTS ON UNSTEADY HYDRO MAGNETIC MIXED CONVECTIVE FLOW AND HEAT TRANSFER OVER A POROUS STRETCHING SURFACE

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Abstract

The aim of this project work is to study the boundary layer flow of an incompressible electrically conducting fluid over a stretching sheet under the effect of magnetic field and heat transfer. The governing boundary layer equation is converted into self-similar nonlinear ordinary differential equations, using similarity transformations and then solved numerically by using shooting method. The effect of magnetic parameter, porosity parameter and Prandtl number are studied with respect to similarity variable (η).

CONTENTS

1. INTRODUCTION

1.1 MAGNETO-HYDRODYNAMICS

Magneto-hydrodynamics (MHD) deals with the process of change of electrically conducting fluids, like plasmas (highly ionized gases), salt water, liquid metals, electrolytes etc. It is well known that magnetic field can cause current in a moving conducting fluid, which generates forces on the fluid and that can also change the magnetic field. As a result current experiences a mechanical force called Lorentz force because of the presence of magnetic field which tends to modify the original fluid motion. Therefore, the relation of electrically conducting fluids with magnetic fields is essential features of the physical situation in MHD fluid flow problems.

Magneto-hydrodynamic (MHD)is the phenomena that occurs in Earth's interior naturally, establishing the dynamo which produces the Earth's magnetic field, in magnetosphere which surrounds the earth and in the Sun and in every part of the universe. Magneto-hydrodynamics principles are used in plasma accelerator for spacecraft propulsion and for Magneto-hydrodynamic power generation.

Faraday in 1832 thought that the motion of the sea might account for the observed disturbance of motion of Earth's magnetic field. However, the use of such idea to natural events did not take place for the rest of the century and it received belated interest when astrophysicists came to realize the prevalence of conducting ionizes gases (plasmas) and significantly strong magnetic field in every part of universe. In 1889 Bigelow suggested from the resemblance of the coronal plum seen at the time of total solar eclipse that the Sun is a great magnet. Larmor in 1918 made an attractive suggestion that the Sun's magnetic field and magnetic field of other heavenly bodies is due to dynamo action, where by the conducting material of the stars act as a frame and stator of a self-exciting dynamo. Subsequently geophysicists considered the similar problem explaining the earth's magnetic field by the motion of assumed core of conducting liquid. These suggestions led to many different investigations of magneto-hydrodynamic phenomena in astrophysical and geophysical problems .The existence of the general magnetic field which is of order 1 Gauss on the Sun's surface, the yielding of high magnetic fields of order of few thousand Gauss in Sun spots. The interstellar clouds which produce polarization by orienting the charged particle during the presence of the magnetic field which is of order of 105 Gauss and the energy particles in cosmic rays are some of the phenomena which have given boost of different types of studies. Between the periods of World wars I & II, the astrophysicists, Cowling and Ferraro began to explore the theory of MHD and its possible future effects and results. However, the engineer-astrophysicist in 1942 Alfven published his classical paper which helps in emerging MHD. He discovered MHD wave which is also known as Alfven's wave and which in turn made many applications of MHD to astrophysical problems. He has received Nobel Prize in 1970 in physics for these earliest works. However, the experimental exploration of the modern MHD flow in laboratory was carried out by Hartmann and Luzarus who designed a magnetic pump to put mercury in motion where transverse magnetic field is present. After this famous work a series of experiments were performed by engineers and applied physicists to study the basic features of MHD flow and to find its applications in fluid engineering, namely, power generation, electromagnetic flow meters, electromagnetic pumps, accelerators, plasma jet engines, shock tubes and wind tunnels, controlled thermonuclear reactors and the control of hyper velocity vehicles.

1.2 BOUNDARY VALUE PROBLEM

Anything that has bound or limit or a boundary line is known as boundary. In differential equations, the differential equation that contains a set of restrains or limitations is known as boundary conditions. Let there exist a solution of boundary value problem then it should also be a solution of differential equation satisfying boundary conditions. These constraints are indeed boundary condition of the equation.

The boundary conditions given in mathematics are:

1.2.1 Dirichlet Boundary Conditions

Dirichlet boundary conditions in Laplace equation impose the restriction on the potential in some value at some location. For example, a common case of Dirichlet boundary conditions are surfaces of perfectly conductive electrodes. Free charges in such a condition will rearrange themselves over conductive surfaces so that the potential will be uniform aver entire conductive conductor. The condition is known but the conducting surfaces may alternately be floating.

1.2.2 Neumann Boundary Condition

This boundary condition specifies the value of normal derivative or some combination of derivatives along the boundary surface. This arises when a flux has been specified on the boundary for instance, a heat transfer, sugar traction in solid mechanics. In homogenous boundary condition, the boundary flux is zero like in insulating surface in heat transfer and free surfaces in solid mechanics. Hence the Neumann boundary conditions are referred to as natural boundary conditions for finite elements.

Neumann boundary condition in Laplace or poison equation imposes the constraint that the directional derivative of ϕ is some value at some location. The directional derivative normal to some boundary surface known as normal derivative is zero. These boundary conditions occur in two dimensional cylindrically symmetric systems. The axis of rotation has infinitely many mirror planes confident with the axis, so the cylindrical axis is also a Neumann boundary condition. They also occur in a repeating element such as modeling a small section of a large grid wire mesh in which case all sides of that element to the right have Neumann boundary condition.

1.2.3 Robin Boundary Condition

This is a linear combination of a field value and its normal derivative. It occurs on a surface from which heat is carried by convection. Robin boundary conditions are handled similarity to Neumann’s boundary condition.

1.2.4 Cauchy Boundary Condition

When the boundary condition is applied to either an ordinary differential equation or a partial differential equation, a complete solution is determined where both function value and normal derivatives are specified on the boundary of the domain.

1.3 FLUID DYNAMICS

Fluid dynamics is the study of fluids which are in motion. The term fluid is a substance that deforms continuously when subjected to shear stress no matter how that shear stress may be. Fluids are classified as ideal and real fluids. Ideal fluids are incapable of sustaining any tangential force or shearing stress but the normal force acts between the adjoining layers of the fluid and offers no internal resistance to change its shape. These have low viscosity such as air and water. On the other hand, real fluids are also known as viscous fluids. A fluid is viscous when normal as well as shearing stress exist. Due to shearing stress, viscous fluid offers resistance to the body moving through it as well as between its particles of fluid itself. An example of a real ideal is heavy oils and syrup which are termed as viscous fluids.

Water and most liquids are incompressible which means that the density in independent of pressure but can vary with the distance. Normal stress produces deformation associated with volume change and shear stress is just the ratio of tangential force to area. Therefore, Newtonian fluids are fluids for which shear stress is directly proportional to the rate of strain. If the viscosity is a constant, independent of flow speed, then the fluid is called Newtonian fluid and water is considered to be example of Newtonian fluid. If the fluid viscosity varies with the rate of deformation, then it is said to be non-Newtonian fluids.

1.4 BASIC EQUATION OF FLUID DYNAMICS

1.4.1 Continuity Equation

In fluid dynamics, the continuity equation states that, in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. In Fluid dynamics, the continuity equation for compressible fluid is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho.\vec{q}) = 0 \tag{1.1}$$

where, ρ is fluid density, t is time, q is the flow velocity vector field.
The continuity equation for incompressible fluid is given by

$$\nabla.\vec{q} = 0 \tag{1.2}$$

Physically, the local volume dilation rate is zero

1.4.2 Momentum Equation

The rate of momentum accumulation is equivalent to the difference of the rate of momentum in and the rate of momentum out along with the sum of forces acting on the system
The basic equations of Hydrodynamics for the flow of a homogenous, isotropic, viscous incompressible fluid is given by

$$\frac{\partial q}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho}\nabla p - \nabla\varphi + \nu\nabla^2\vec{q} \tag{1.3}$$

where, ρ is fluid density q is the flow velocity vector field.

1.4.3 Maxwell's Equation

Maxwell's equations are the set of four complicated equations that describe the world of electromagnetic. These equations describe how electric and magnetic field propagate, interact and how they are influenced by objects. Maxwell's Equations shows that separated charge (positive and negative) gives rise to an electric field and if this is varying in time will give rise to a propagating electric field, further giving rise to a propagating magnetic field.

1) Gauss's Law

$$\nabla . E = \frac{\rho}{\varepsilon_0} \tag{1.4}$$

2) No Magnetic Monopole Law

$$\nabla . B = 0 \tag{1.5}$$

3) Faraday's Law

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1.6}$$

4) Ampere's Law with displacement current

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \tag{1.7}$$

These four equations are called Maxwell's equations. Maxwell brought these four equations together along with the Lorentz force to summarize the theoretical content of electrodynamics. Maxwell's equations tells how charges produce fields and the Lorentz force how fields affect charges. In this section, E=Electric field, *B*=Magnetic field, ε_0=permittivity, μ_0=Permeability, *J*=current density respectively.

1.5 STRETCHING SHEETS

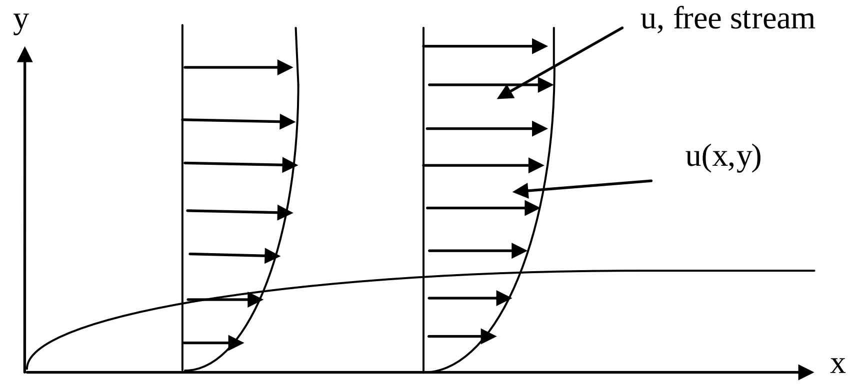
The stretching flow is the flow produced due to the stretching of an elastic sheet which moves in its plane varying with some distance from a fixed point due to stress. Manufacturing of the both metal and polymer sheets in industrial manufacturing process, the material is in molten phase when thrust through an extrusion die and then the material cools and solidified and some distance away from the die before arriving at the cooling stage. The tangential velocity imported by the sheet induces motion in the surrounding fluid, which alters the convection of the sheet.

Similar situation prevails during the manufacture of plastic and rubber sheet where it is often necessary to blow a gaseous medium through material yet to be solidified where the stretching force depends upon time. Another example that belongs to this class of problem where the fluid flow is induced due to shrinking of the plate is the cooling of a large metallic plate in a bath, known as electrolyte.

Due to very high viscosity of the fluid near the sheet, one can assume that the fluid is affected by the sheet only. Thus the fluid problem can be idealized to the case of fluid disturbed by a tangential moving boundary

1.6 BOUNDARY LAYER THEORY

When a viscous fluid (real fluid) flows on a stationary solid boundary, a layer of fluid that comes in contact with the boundary surface stick firmly to it and no slip condition occurs. No slip conditions means that the velocity of fluid at a solid boundary must be same as that of boundary itself. Hence the layer of fluid that cannot slip away from the boundary surface undergoes retardation. This retarded layer further causes retardation for the adjacent layers of the fluid. This results in the development of small regions in the immediate surroundings of the boundary surface in which the flowing fluid velocity increases rapidly from zero at the boundary surface and reaches the velocity of the main stream. The layer near to the boundary is called as boundary layer.



1.6(a) Boundary layer

1.7 CONVECTIVE HEAT TRANSFER

Heat transfer has a variety of applications in the problems of natural events and technology. These heat transfer problems include designing of power stations, chemical and food plants, aerodynamic heating, cooling of high powered motors, extraction of energy from atomic piles and heat exchanges utilizing liquid metal coolant. The heat transfer in fluids in which moving fluid particles carry heat in the form of energy is called convection and depends upon how the fluid motion is initiated. In forced convection and free or natural convection and depends on how the fluid motion is initiated. In forced convection, incompressible fluids are characterized by the distribution of velocity which is not affected by temperature field. Heat diffusion in such flows occurs and is simultaneously swept by the fluid motions without any way of affecting the local density of the fluid. The velocities in the forces convection are exact such that there is no temperature variation in the motion arising from the differences caused by natural means in which the distribution of velocity and temperature field coupled together is referred to as free or natural convection. Taking a fluid in consideration, the effect of free convection causes the rise of warm fluid and fall of the cooler fluid. In such flows the distribution of velocity and temperature are interconnected and can be considered together. If the fluid is incompressible, then the density variations due to changes in pressure are negligible. These changes are responsible for imitating free convection because of density changes due to non-uniform heating of the fluids which cannot be neglected. Hence free convection occurs in the field of gravity and in the rotating fluid. It can be set up by the action of centrifugal force which is proportional to the density of the fluid. This is evident in the flow and heat transfer in gas turbines.

Free convection and forced convection occur interchangeably and this is understood further if a common practical example is taken into account such as convection in ovens. Here, convectional ovens use natural convection to heat food while baking. Ovens typically contain two heating elements that is, on top and bottom of the oven. During baking, the bottom heats up which heats the air inside the oven. The hot air rises and creates a current which helps distribute throughout the oven. Natural convection currents are blocked by large pans and create non-uniform temperatures in oven. Again the convection oven improves the temperature distribution by using a fan which is located within the oven and thereby creating forced convection. The forced convection currents efficiently run the air inside the oven and creating uniform temperatures even in the presence of large fans. The practical example generally demonstrates convective heat transfer of both free and forced convection.

2. LITERATURE REVIEW

This section is devoted to the review of the earlier investigations made on the flow and the heat transfer over the stretching sheet. During the last decades the problem of flow of incompressible viscous fluid and heat transfer phenomena over stretching sheets gets the great attention. This problem owns plenty of practical applications in chemical and manufacturing processes like Aerodynamics, continuous casting of metals, glass fibers and paper production, extrusion of plastic.

Study of Hydro magnetic flow of an electrically conducting fluid, due to its extensive industrial applications has attracted the interest of many researchers. The cause of the study of hydrodynamic flow of an electrically conducting fluid is the deformation of the wall of a vessel containing a fluid which is of considerable interest in a modern metal-working process and modern metallurgical. The boundary layer flow which is passing a Stretching Plane Surface in the presence of a uniform magnetic field has practical relevance in Polymer Processes.

The study of boundary layer flow over a continuous solid surface moving with constant speed is initiated by Sakiadis (1961). The steady two-dimensional boundary layer flow caused by the stretching of an elastic flat surface which moves in its plane with velocity varying linearly with distance from a fixed point was extended to analyze by Crane (1970).

Carragher and Crane (1982) investigated the heat transfer aspect of this problem, under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. The steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution were investigated by Magyari and Keller (1999). The unsteady magneto hydrodynamic flow due to the impulsive motion of a stretching sheet was investigated by Takher et al. (2001) and reported that the surface heat transfer increase upto a certain portion of time, beyond that it decreases.

At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment Seddeek[2002].

The study of magneto-hydrodynamic has important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field, Ganesan & Palani [2004]. Ali et al. [2007] have studied the problem of unsteady fluid and heat induced by submerged stretching surface.

Nazar et al.(2009) considered an unsteady boundary layer flow in the region of the stagnation point on a stretching sheet. Some other important properties of flow due to an unsteady stretching sheet were discussed by Ishak et al.(2009), Mukhopadhyay (2010) and Zheng et al.(2011).

In fluid flow process porous medium play an important role. The problem of viscoelastic fluid flow and warmth transfer in a porous medium over a stretching sheet has solved by subhas and Veena (1998). Vajravelu (1994) has obtained the solution for the flow problem and heat transfer in a saturated porous medium. Eldabe and Mohamed (2002) have studied both heat and mass transfer in hydro magnetic flow of a non Newtonian fluid with a warmth source over an accelerating surface through porous medium. Recently Venkateswalu et al. (2011) have discussed finite difference analysis on convective heat transfer flow through a porous medium in a vertical channel with magnetic field.

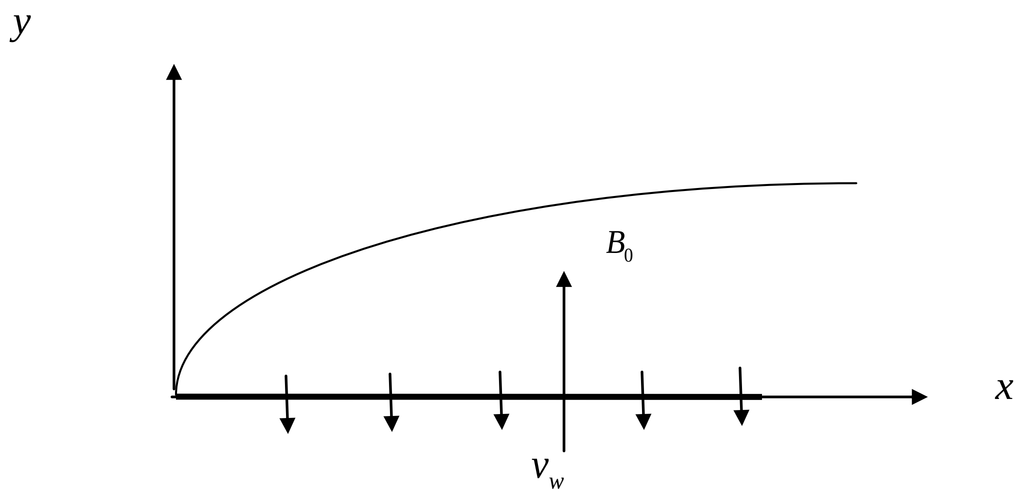
Flow in porous medium has been the subject of numerous investigations during the past several decades. The concentration in this subject has been stimulated, to a large extent, by the fact that thermally driven flows in porous media have more applications in chemical and mechanical engineering, e.g. food processing and storage space, geophysical system, electro- chemistry, fibrous filling metallurgy, the design of pebble bed nuclear reactors, underground removal of nuclear or non-nuclear waste, microelectronics cooling, etc. Detailed literature review can be found in the books by Pop and Ingham (2001), Ingham and Pop (2005), Nield and Bejan (2006), Vafai (2005) and Vadasz (2008). One of the basic problems in porous media is the flow and heat transfer determined by a linearly stretching surface through a porous material. It seems that the initial study of the steady flows of a viscous incompressible fluid (nonporous media) driven by a linearly stretching plane through a quiescent fluid has been reported by Crane (1970). Further, Elbashbeshy and Bazid (2004) considered flow in a porous medium over a stretching surface with internal heat production and suction/ blowing when the surface is held at a constant temperature.

3. FORMULATION OF THE PROBLEM

We consider the time dependent (unsteady) two dimensional mixed convective boundary layer flow and heat transfer of a viscous, incompressible, electrically conducting fluid over a porous stretching sheet in presence of transverse magnetic field. The x-axis is directed along the continuous stretching surface and points in the direction of motion while the y-axis is perpendicular to the surface. The flow is confined to y> 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The flow is assumed to be generated by stretching of the elastic boundary sheet from a slit with a large force. The stretching sheet has the time dependent surface velocity $U_w = \frac{bx}{1-\alpha t}$ and the time dependent wall temperature $T_w = T_\infty + T_0 (bx/\upsilon)(1-\alpha t)^{-2}$, where b is the initial stretching rate, $\frac{b}{1-\alpha t}$ is the effecting stretching rate which is increasing with time, $T_0 (0 \le T_0 \le T_w)$ is the reference temperature and T_∞ is the fluid temperature far away from the stretching sheet.

In order to get the effect of temperature difference between the sheet and the ambient fluid, we consider temperature dependent heat source/sink in the flow region. It is considered that the rate of heat generation is equal to $Q_0 (T - T_\infty)$ for $T \ge T_\infty$ and equal to zero for $T \le T_\infty$, where $Q_0 (> 0)$ is the heat generation and $Q_0 (< 0)$ is the heat absorption (Vajravelu and Hadjinicolaou).

A uniform magnetic field of strength B_0 is applied normal to the stretching surface. We have neglected the induced magnetic field since the magnetic Reynolds number for the flow is considered to be very small. No external electric field is applied so the effect of polarization of fluid is neglected. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects (Bansal, 1977; Schlichting *et al.*, 1999.) are written by the following continuity, momentum and energy equations governing such type of flow as



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u \tag{2.2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{2.3}$$

where u and v are velocity component in x and y direction respectively, t is the time, υ is the kinematic viscosity, T is the temperature inside the boundary layer, β is the volumetric coefficient of thermal expansion, g is the gravity field. σ is the electrical conductivity, K is the permeability of porous medium, κ is the thermal conductivity, ρ is the density, c_ρ is the specific heat at constant pressure , q_r is the radiation heat flux, Q is the heat source when $Q>0$ and/or heat sink when $Q<0$, T_w is the surface temperature, T_∞ is the temperature at infinity.

2.1 BOUNDARY CONDITIONS

The appropriate boundary conditions for the problem are

$$u=u_w(x,t)+N_1\upsilon\frac{\partial u}{\partial y},\ \ v=v_w(x,t)\quad T=T_w(x,t)+D_1\frac{\partial T}{\partial y},\quad \text{at } y=0 \tag{2.4}$$

$$u\to 0,\qquad T\to T_\infty\qquad \text{at } y\to\infty \tag{2.5}$$

Here $N_1 = N\sqrt{1-\alpha t}$ and $D_1 = D\sqrt{1-\alpha t}$ are the velocity and thermal slip factors. The slip factor N_1 and D_1 have dimension of (velocity)$^{-1}$ and length. N and D are initial values of velocity and thermal slip factors. The no slip case considered by Anderson et.al [2002] is recovered for $N = 0$ and $D = 0$. $v_w(x,t) = -v_0(1-\alpha t)^{1/2}$ where $v_0 > 0$ is the velocity of suction.

2.2 METHOD OF SOLUTION:

The continuity equation (2.1) is satisfied by the Cauchy- Riemann equation

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{2.6}$$

where $\psi(x, y)$ is the steam function.

In order to transform equation (2.2) and (2.3) into a set of ordinary differential equations, the following similarity transformation and dimensionless variables are introduced

$$\psi = \sqrt{\frac{\upsilon c}{1-\alpha t}}\, x f(\eta), \quad \eta = \sqrt{\frac{c}{\upsilon(1-\alpha t)}}\, x f(\eta), \quad \theta(\eta) T = \frac{(T - T_\infty)(1-\alpha t)^2 \theta(\eta)}{T_0\left(\frac{cx}{\upsilon}\right)} \tag{2.7}$$

$$\lambda = \frac{g\beta T_0}{\upsilon b} = \frac{Gr_x}{\mathrm{Re}^2{}_x}, \quad Gr_x = \frac{g\beta\left(T_L - T_w\right)x^3}{\upsilon^2}, \ \mathrm{Re}^x = \frac{Ux}{\upsilon}, \quad \Pr = \frac{\upsilon}{\kappa}, \tag{2.8}$$

$$f_0 = \frac{\upsilon_0}{\sqrt{\upsilon b}}(>0), \quad \gamma = \frac{D}{\sqrt{b/\upsilon}}, \quad \beta = \sqrt{b\upsilon}, \quad A = \frac{b}{\alpha}, \quad M = \frac{\sigma B_0}{\rho b} \tag{2.9}$$

where, $f(\eta)$ is the dimensionless stream function, θ-dimensionless temperature, η-similarity variables, M -magnetic parameter, α and b are constant, A is the unsteadiness parameter, λ is the mixed convection parameter, Gr_x is the Grashof number, Re_x is the local Reynolds number, $\Pr$ is the Prandlt number, f_0 is the suction parameter, S is the velocity slip parameter and γ is the dimensionless temperature slip parameter.

In view of equations (2.6) to (2.9), the above momentum and energy equations transform into

$$f''' - f'^2 + ff'' - A\left(\frac{1}{2}\eta f'' + f'\right) + \lambda\theta + Mf' + Nf' = 0 \tag{2.10}$$

$$\theta'' + \frac{\Pr}{1+R}\left[f'\theta - f\theta' + A\left(\frac{1}{2}\eta\theta' + 2\theta\right) + \delta\theta \right] = 0 \tag{2.11}$$

The corresponding boundary conditions are,

$$f = f_0, \qquad f' = 1 + Sf'', \qquad \theta = 1 + \gamma\theta' \qquad \text{at } \eta = 0 \tag{2.12}$$

$$f' \to 0, \qquad \theta \to 0 \qquad \text{at } \eta = \infty \tag{2.13}$$

where, the prime denote differentiation with respect to η.

The physical quantity of interest are the skin friction co-efficient c_f and the Nusselt number Nu_x which are defined as

$$c_f \sqrt{R_{e_x}} = -2R_{e_x}^{-1/2} f''(0) \tag{2.14}$$

$$Nu_x \frac{N}{\sqrt{R_{e_x}}} = -\theta'(0) \tag{2.15}$$

To assess the accuracy of the present method, comparison with previously reported data available in the literature has been made. It is clear from that the numerical values of $-\theta'(0)$ in the present paper are in agreement with results obtained by Magyari and Keller (1999).

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