Study of Non Newtonian Fluids

A Synopsis submitted

To

Department of Mathematics

In partial fulfillment of the requirement for the

Award of the degree of

Masters of Mathematics

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DECLARATION OF AUTHORSHIP

I, Sachin Narang, declare that this project titled, “A Study of Non Newtonian Fluids” and the work presented in it my own. I confirm that:

- This work was done wholly or mainly whiles the candidature for a Master degree at this university.
- Where any part of this project has previously been submitted for a degree or any other qualification at this university or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotation, this project work is entirely my own work.
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CERTIFICATE

This is to certify that Sachin Narang has completed project titled, "A Study of Non Newtonian Fluids" under my guidance and supervision. To the best of my knowledge, the present work is the result of the original investigation and study. No part of the project has ever been submitted for any degree at any university.

The project is fit for the submission and the partial fulfillment of the conditions for the award of Master in Mathematics.

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ABSTRACT

The objective of this chapter is to introduce and to illustrate the frequent and wide occurrence of non-Newtonian fluid behaviour in a diverse range of applications, both in nature and in technology. Starting with the definition of a non- Newtonian fluid, different types of non-Newtonian characteristics are briefly described. Representative examples of materials .We will also discuss some applications of non Newtonian fluids and the various types of fluid instabilities.

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1. Introduction:

1.1 Fluid Dynamics and Basic Concepts

Fluid dynamics is the science, which deals with the properties of fluids in motion. Before dealing with fluid dynamics in a deep manner, let us first make clear the terms of fluid and dynamics. Dynamics is nothing but the science of motion and force. Now the question arises that what is meant by "Fluid"? It is a hard fact that all materials undergo deformation under the action of forces. If the deformation in the material increases continually without limit under the action of shearing forces, however small, the material is called fluid. This continuous deformation under the action of forces compels the fluid to flow and this tendency is called "**fluidity**".

As we know that the matter exists in four forms namely i) solid, ii) liquid, iii) gas, iv) plasma. Liquids and gases taken together are classified as fluids. It has been believed by the physicists for a long time that there is no clear dividing line between solids and fluids, since there are many materials which in some respect behave like a solid and in other respect like a fluid. For example, jelly, paint and pitch have dual character. However, a loose distinction can be made between solid and fluids. A solid mass has a definite shape, while a mass of fluid has no preferred shape and assumes the shape of the container more or less instantaneously. The deformation in the piece of solid is small even under the action of large external forces, whereas in the case of fluids the deformation may be large under the suitably chosen forces, however small in magnitude.

Fluids are classified as liquids and gases. As a result the distinction between liquids and gases is much less fundamental so far the dynamical studies are concerned. The most important difference between the mechanical properties of liquids and gases lies in their compressibility. Liquids have strong intermolecular forces whereas the gases experience weak intermolecular forces. As a result of these, the liquids are incompressible fluids and the gases are highly compressible fluids. It should be mentioned that for velocities which are not comparable with the velocity of sound, the effect of compressibility on atmospheric air can be neglected and it may be considered to be a liquid and in this sense it is called incompressible air.

The fourth state of matter is called plasma. Plasma is essentially a highly ionized matter. Therefore, in plasma we have to take into account the charges on its particles and associated electromagnetic phenomena. We go to plasma state when we deal with Earth's molten core, ionosphere, stellar interiors and atmospheres.

The study of water flowing in rivers, waves in ocean and the motion of aeroplane in the lower parts of Earth's atmosphere are the best examples of classical fluid dynamics. As the fluid molecules are considered electrically neutral here, the gross properties of various states of matter are directly related to the molecular structure and the nature of intermolecular forces that operate between the constituent molecules. In solids, the arrangement of molecules is virtually permanent and under normal conditions may have a simple periodic structure as in case of crystals, and molecules are acted upon by strong intermolecular forces. Our knowledge of the liquid state is incomplete, but it appears that the arrangement of molecules is partially ordered and are acted upon by medium intermolecular forces. In case of gases and plasmas the particles are acted upon by weak short-range intermolecular forces and molecular arrangements are disordered.

The history of the science of fluid dynamics is very difficult to describe because the excavation of the Indus Valley Civilization and Egyptian ruins show that even as long as four thousand years ago, the principles of flow and resistance to flow were known. The drainage and irrigation systems of Mohan-jo-daro, Egypt and China; the use of siphons and bellows and the construction of wind mills and paddle wheels dates back from ancient times. However, the systematic study of the fluid dynamics started only after Euler's discovery of the equations of motion of an inviscid fluid. Earlier attempt to describe the effect of fluid motion is due to Newton, who conceived the idea that the fluid consisted of a granulated structure of discrete particles. The range of validity of the method, as defined by the agreement of the results with experiment was limited.

Later, some other significant contributions to this subject were given by following. Lagrange gave the concept of velocity potential and stream function. The principle of resistance to flow in capillary tubes was given by Poiseuille. The credit for the equations of motion of viscous fluids goes to Navier and Stokes'. Reynolds discovered the equations of turbulent motion. Prandtl put forward the boundary layer theory. G.I. Taylor and Lord Rayleigh gave the theories of turbulence and stabilities. Later on, many more contributions were given by many famous scientists, which include Bénard, Kutta, Prandtl, Lord Kelvin, Orr, Sommerfeld, Rayleigh, Zhukovskii and Karmán etc.

Practically, fluid dynamics bears a lot of importance. Some practical situations where fluid dynamics plays a significant role are lubrication, flight of aeroplanes, ship science, meteorology, the influence of wind upon building structures, ground water seepage, the extraction of oils from underground reservoirs, the use of pipelines, pumps and turbines. Even the swinging of cricket ball is an example of the fluid forces used by the bowler, to deceive the batsman. The flow of fluids affects each one of us throughout our lives. The flow of blood in veins and pumping action of heart are familiar examples. These days fluid dynamics has became a very vast subject and has given birth to many other subjects like meteorology, gas dynamics, aerodynamics, non-Newtonian flows, megnetohydrodynamics etc.

1.2 The Basic Hydrodynamical Equations

The fundamental equations of the flow of viscous compressible fluids are:

i) Equation of state, (one)

ii) Equation of continuity, (one)

iii) Equations of motion, (three) and

iv) Equation of energy, (one).

These equations are mathematical expressions of basic physical laws. These are six in number and therefore, determine the six unknowns of the fluid motion viz., the three components of velocity u_i (u, v, w), the temperature T, the pressure p, and the density ρ, which are functions of both space coordinates and time.

Equation of State

Variables that depend only upon the state of a system are called variables of state. The variables of state are the pressure p, the density ρ and the temperature T. It is an experimental fact that a relationship between these three thermodynamic variables exist and can be written as

$$F(p, \rho, T) = 0, \tag{1.1}$$

which is commonly called the **'Equation of state'**. For substances with which we shall be principally concerned, we can write the equation of state as

$$\rho = \rho_o [1 + \alpha (T_o - T)], \tag{1.2}$$

where α is the coefficient of volume expansion and T_o is the temperature at which $\rho = \rho_o$.

Equation of Continuity – Conservation of Mass

This equation expresses that the rate of generation of mass within a given volume is entirely due to the net inflow of mass through the surface enclosing the given volume (assuming that there are no internal sources). It amounts to the basic physical law that the mass is conserved; it is neither being created nor destroyed. For viscous compressible fluids, the equation of continuity is

$$\frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = -\rho \frac{\partial u_j}{\partial x_j} \ , \qquad (1.3)$$

where u_j is the j^{th} component of velocity.

For an incompressible fluid

$$\frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = 0 \ , \qquad (1.4)$$

so that equation (1.3) reduces to

$$\frac{\partial u_j}{\partial x_j} = 0 \ . \qquad (1.5)$$

Equations of Motion (Navier-Stokes' Equations)-Conservation of Momentum

The equations of motion are derived from Newton's second law of motion, which states that

Rate of change of linear momentum = Total force.

For viscous compressible fluids, Navier-Stokes' equations can be expressed as

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = \rho X_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \delta_{ij} \right] \ , \qquad (1.6)$$

where X_i (0, 0, -g) is the external force, μ is the coefficient of viscosity and δ_{ij} is the Kronecker delta.

In case of incompressible fluid flow, the equation of continuity is

$$\frac{\partial u_j}{\partial x_j} = 0 \ ,$$

and if μ is also regarded as constant, the equations (1.6) can be simplified to

$$\rho\left[\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right]=\rho X_i-\frac{\partial p}{\partial x_i}+\mu\frac{\partial^2 u_i}{\partial x_j^2}\ , \tag{1.7}$$

keeping in view that

$$\frac{\partial}{\partial x_j}\left(\frac{\partial u_j}{\partial x_i}\right)=\frac{\partial}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j}\right)=0\ .$$

Equation of Energy – Conservation of Energy

To obtain the energy equation we have to apply the law of conservation of energy which requires that, the difference in the rate of supply of energy to a controlled surface S enclosing a volume V in the region occupied by a moving fluid and the rate at which the energy goes out through S must be equal to the net rate of increase of energy in the enclosed volume V.

For viscous compressible fluids, the equation of energy is

$$\frac{\partial}{\partial t}(\rho C_V T)+\frac{\partial}{\partial x_j}(\rho C_V T u_j)=\frac{\partial}{\partial x_j}\left(q\frac{\partial T}{\partial x_j}\right)-p\frac{\partial u_j}{\partial x_j}+\Phi\ , \tag{1.8}$$

where $\Phi = 2\mu e_{ij}^2-\frac{2}{3}\mu(e_{jj})^2$, (1.9)

is the 'rate of viscous dissipation' (which gives the rate at which energy is dissipated irreversibly by the viscosity in each element of volume of the fluid).

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\ , \tag{1.10}$$

is the 'rate-of-strain tensor', C_V is the specific heat at constant volume and q is the coefficient of heat conduction.

For an incompressible fluid, e_{jj} = o and the corresponding expression for Φ is given by

$$\Phi = 2\mu e_{ij}^2\ .$$

Thus, for an incompressible fluid, the equation of energy (1.8) takes the form

$$\rho\frac{\partial}{\partial t}(C_V T)+\rho u_j\frac{\partial}{\partial x_j}(C_V T)=\frac{\partial}{\partial x_j}\left(q\frac{\partial T}{\partial x_j}\right)+2\mu e_{ij}^2\ . \tag{1.11}$$

1.3 STABILITY CONCEPT

Due to the paramount importance in the quest for thermonuclear fusion, the subject of hydromagnetic stability is a large one and much has been written about it. Even in the case of turbulence, the importance of stability arises because a motion which is definitely unstable for small perturbations cannot remain steady for speeds higher than that at which instability sets in. On the other hand, a motion, which is definitely stable for small disturbances, may become turbulent when finite disturbances are imposed on it. Thus **'stability'** may be defined as the quality of being immune to small disturbances. Thus, by stability we mean permanent type of equilibrium state. An equilibrium state or steady flow, to be of permanent type, should not only satisfy the mechanical equation, but must be stable against arbitrary perturbations.

As every system in nature is subject to many small perturbations, the investigation of stability of a physical system is of great importance. To check the stability of a hydrodynamical system, the system is given some arbitrary perturbations. If the system is disturbed and the disturbances gradually die down or if the system never departs appreciably from this stationary state, the system is said to be stable with respect to that particular disturbance. If the disturbance that grows in amplitude in such a way that the system progressively departs from the initial state and never reverts to it, the system is called unstable with respect to that particular disturbance. A system must be considered as unstable even if there is only one special mode of disturbance with respect to which it is unstable. And a system cannot be considered as stable unless it is stable with respect to every possible disturbance to which it can be subject. The state of 'neutral stability' is called the marginal state.

Although the solutions of the equations of hydrodynamics as well as magnetohydrodynamics are somewhat complex, still they allow some simple patterns of flow (such as between parallel planes or rotating cylinders) as stationary solutions. These patterns of flow can, however, be realised only for certain ranges of the parameters characterizing them. They cannot be realised outside these ranges. The reason for this lies in their inherent instability, i.e. in their inability to sustain themselves against small perturbations to which any physical system is subjected. Problems of hydrodynamic instability thus originated from the differentiation of the unstable flow from the stable patterns of permissible flows.

Now-a-days, to investigate the stability problems deeply, the interest in hydrodynamic flow of electrically conducting fluids in the presence of magnetic fields has been considered. This is the domain of hydromagnetics, as we have discussed earlier; and there are problems of hydromagnetic stability. Let us make it clear here that if at the onset of instability stationary patterns of motion prevails, then one says that the **'Principle of exchange of stabilities'** is valid and that instability sets in as stationary cellular convection or secondary flow. On the other hand, if at the onset of instability oscillatory motions prevail, then it is called the case of **'overstability'**.

Let us consider a stationary state, in which a hydromagnetic system is in accordance with the equations governing it. Let R_1, R_2, ---, R_j be a set of parameters which define the system. While considering the stability of such a system, we seek to determine the reaction of the system to small disturbances.

According to the criteria stated above, if all the initial states are classified as stable or unstable, then in space of parameters; R_1, R_2, ---, R_j, the locus which separates the two classes of states defines the state of **'marginal stability'** of the system. The locus of the marginal states in the (R_1, R_2, ---, R_j) space will be defined by an equation of the form

$$\Sigma(R_1,\ R_2, \text{—}, Rj) = 0. \tag{1.16}$$

The determination of this locus is one of the prime objects of an investigation on hydrodynamic stability. If the amplitude of a small disturbance can grow or be damped aperiodically, the transition from stability to instability takes place via a marginal state exhibiting a stationary pattern of motions. If the amplitude of a small disturbance can grow or be damped by oscillations of increasing or decreasing amplitude, the transition takes place via a marginal state exhibiting oscillatory motions with a certain definite characteristic frequency.

2. Review of Literature

The first major contribution to the study of hydrodynamic stability can be found in theoretical papers of Helmholtz (1868). Even earlier, many scholars had certainly become aware of the question but their efforts did not progress beyond the stage of description. For example, the drawings of vorticities by the Lenardo-da-Vinci (fifteenth century) and the experimental observations of Hagan (1855) deserve mention. Lord Rayleigh (1880) developed a general linear stability theory for inviscid plane-parallel shear flows, which was mathematically tractable and had intuitively sensible results. The combined efforts of Reynolds (1883), Kelvin (1880, 1887) and Rayleigh [1879, 1880, 1892(a), 1892(b), 1913, 1914, 1916(a), 1916(b)] produced a rich harvest of knowledge. Reynolds (1883) predicted that Reynolds number was a crude measure of the relative importance of inertial (non-linear) effects relative to viscous processes in determining the evolution of a flow. He discovered the first experimental evidence of **'sinuous'** motions in water and is generally credited for a first description of random or **'turbulent'** flow. He pointed out that disorder begins when Reynolds number exceeds a critical value and that special stresses must be taken into account. The founder of hydrodynamical stability is Lord Rayleigh, who published a great number of papers (as cited above) regarding the importance of inflection points in the velocity profile and the instability of rotating flows between cylinders. Thus, hydrodynamic stability is concerned with when and how laminar flows break down, their subsequent development and their eventual transition to turbulence. It has many applications in engineering, in meteorology and oceanography and in astrophysics and geophysics.

The best-known contribution and principle was that of Taylor (1923) on vorticities between concentric rotating cylinders. Indeed this was a dual effort where theory and experiment were matched simultaneously. The analysis of Heisenberg (1924) was more abstract and points towards the possibility of resistive instability. Jeffreys demonstrated in 1928, the mathematical equivalence of the two stability problems of convection and flow between rotating cylinders. In fact, it was the application of newer mathematical techniques that brought the initial success to Tollmein. Soon, following the same track, Schlichting [1932(a), 1932(b), 1933(a), 1933(b), 1933(c), 1934, 1935] made further evaluations of the critical Reynolds number and amplification rates of disturbances.

A large body of theoretical work by Lin (1955), Joseph (1976) and Drazin and Reid (1981), has been developed in an attempt to understand and predict the phenomena of stability or instability. Early in this century, studies on hydrodynamic stability were connected with the Bénard experiments on thermal convection in thin liquid layers. Around 1907, it became apparent that the existence of a critical Reynolds number could not be explained easily and that the problem involved both the effects of the second derivative of the mean flow and of the viscous forces. The key equation was arrived at independently by Orr

(1907) and by Sommerfeld (1908). This Orr-Sommerfeld equation remained unsolved for twenty two years, until Tollmein (1929) calculated the first neutral eigen values and obtained a critical Reynolds number.

The improved mathematical procedure used by Lin (1944, 1945) not only removed the controversial issue of stability of Poiseuille flows but also laid the basis for the general expansion of the stability analysis. Any additional doubts with respect to this system were finally settled down by the first use of a digital computer in hydrodynamical stabilities. This success and the experimental results of Schubauer and Skramstad (1943) made perfectly clear that the critical Reynolds number marked only the threshold of **'sinuous'** motion and not that of turbulence. And the turbulent transition still remains an engima.

Magnetic, gravitational and convective effects were examined by Bénard (1901) and further elaborated by Chandrasekhar (1981). The monograph of Lin (1955) settled many controversial questions that had been built over the years. The study of compressible flow was started with the work of Landau (1944) and Lees (1947) and continued by Dunn and Lin (1955). Finally, the theory of non-linear processes was set up by Meksyn and Stuart (1951). Later, some simple non-linear problems have been successfully treated by Fromm and Harlow (1963). This work used a totally numerical method and demonstrated the sources of modern computers. Soon other good works in non-linear theory, which need mention, are by Coles (1965), Segel (1966), Reynolds and Potter (1967). Büsse (1969), Kirchgessner and Sorger (1969), Stewartson and Stuart (1971) and Weissman (1979) etc.

In the present work, we have worked on various hydrodynamic and hydromagnetic instability problems. Therefore, for the clear and better understanding of he work, we feel it necessary to explain and also review, in brief, some fundamental contributions relating to these instabilities. We take them one by one below:

2.1 Thermal Instability (or Bénard Problem)

Stability of a physical system is its ability to sustain itself against small perturbations to which the system can be subjected, while the instability of the system is its inability to sustain itself against the above small perturbations.

Consider a horizontal layer of fluid of uniform density in which an adverse temperature gradient is maintained by heating from below. Due to the thermal expansion, the liquid at the bottom becomes lighter than that at the top; and this is a top-heavy arrangement which is potentially unstable. Because of this unstable arrangement, there will be natural tendency on the part of the fluid to redistribute itself and make up the weakness in its arrangement. Therefore, the liquid at the bottom goes up and the cooler heavy liquid from the top layer comes down giving rise to thermal instability. But, this thermal motion in the fluid is prevented to certain extent by its own viscosity and therefore, instability can set in only when the adverse temperature gradient exceeds a certain limit. The temperature gradient, thus, maintained is qualified as adverse because on account of thermal expansion, the fluid at the bottom becomes lighter than the fluid at the top and thus, making an unstable arrangement.

2.2 Rayleigh-Taylor Instability

Rayleigh-Taylor instability arises from the character of equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid). The simplest, nevertheless important example demonstrating the Rayleigh-Taylor instability is when we consider two fluids of different densities superposed one over the other (or accelerated towards each other), the instability of the plane interface between the two fluids, if it occurs, is known as Rayleigh-Taylor instability. Rayleigh (1900) was first to investigate the character of equilibrium of an inviscid, non-heat conducting as well as incompressible heavy fluid of variable density which is continuously stratified in the vertical direction. The cases of (i) two uniform fluids of different densities superposed one over the other and (ii) an exponentially varying density distribution, were also treated by him. The main result in all such cases is that the configuration is stable or unstable with respect to infinitesimal small perturbation according as the higher density fluid underlies or overlies the lower density fluid.

3. Various Effects on Instability Problems

3.1 Effect of Uniform/Variable Magnetic Field

Consider a fluid to be electrically conducting and be under the influence of a magnetic field. The electrical conductivity of the fluid and the prevalence of magnetic field contribute to effects of two kinds. First, by the motion of the electrically conducting fluid across the magnetic lines of force, electric currents are generated and the associated magnetic fields contribute to changes in the existing fields; and second, the fact that the fluid elements carrying currents transverse magnetic lines of forces contributes to additional forces acting on the fluid elements. It is this two-fold interaction between the motions and the fields that is responsible for patterns of behaviour which are often striking and unexpected. The interaction between the fluid motions and magnetic fields are contained in Maxwell's equations. As a consequence of Maxwell's equations, equations of hdyrodynamics are modified suitably.

In the outer layers of stars like the Sun, thermal convection is affected by the presence of magnetic fields. In stellar interiors and atmospheres, the magnetic field may be variable and may altogether alter the nature of the instability. For example, Kent (1966) studied the effect of a horizontal magnetic field, which varies in the vertical direction, on the stability of parallel flows and showed that the system is unstable under certain conditions, while in the absence of magnetic field, the system is known to be stable.

3.2 Effect of Rotation

Rotation introduces a number of new elements in fluid dynamics, for example, under certain circumstances the role of viscosity is inverted. In fact, the consequences of rotation are the results of certain general theorems relating to vorticities, in the dynamics of rotating fluids.

When a fluid spreads under gravity in a rotating system, motions normal to the rotation vector induce Coriolis forces that tend to oppose the spreading. In the absence of boundaries intersecting isopotential surfaces and of instability or viscous dissipation, the flow approaches a state of geostropic equilibrium in which buoyancy and Coriolis forces are in balance. The rotation with an angular velocity introduces two new terms, in the

Navier-Stokes' equations; $(2\vec{\Omega} \times \vec{u})$ represents the Coriolis acceleration and the term $-\frac{1}{2}$ grad $|\vec{\Omega} \times \vec{r}|^2$ represents the centrifugal force. In the case of an inviscid fluid in which external forces are derived from a potential function V", the Taylor-Proudman theorem states that "All steady, slow motions in a rotating inviscid fluid are necessarily two dimensional". That is to say, the motion transverse to $\vec{\Omega}$ can not vary in the direction of $\vec{\Omega}$ means that any two fluid elements which are initially on a line parallel to $\vec{\Omega}$ will always remain on that line, and the fact that motions in the direction of $\vec{\Omega}$ cannot also vary along this direction means that two fluid elements which are initially a certain distance apart will always remain at the same distance apart.

3.3 Effect of Rotation and Magnetic Field

Generally, the effects of rotation and magnetic field, on the onset of thermal instability in layers of fluid heated from below, are remarkably alike, when acting separately, they both inhibit the onset of instability and they both elongate the cells which appear at marginal stability. On the contrary, acting together they do not reinforce each other, but tend to oppose each other. The viscosity facilitates the onset of instability when rotation is present, and a magnetic field imparts to the fluid certain aspect of viscosity. Hence, even though the two acting separately inhibit the onset of instability; they will have conflicting tendencies when acting together. Rotation induces a component of vorticity in the direction of $\vec{\Omega}$, and for large Taylor number it results in the streamlines becoming closely wound spiral with motions principally confined to planes transverse to $\vec{\Omega}$. Instead, magnetic field does not induce similar component of vorticity and there are no comparable effects: for large Chandrasekhar numbers, the motions transverse to $\vec{H}$ are much reduced and the motions along the magnetic lines of force become predominant.

Instability sets in mostly as overstability when rotation is present, in liquid metals like mercury; but in the presence of magnetic field, it sets in as stationary convection. For all these reasons, the study of thermal instability in the presence of both rotation and magnetic field is an instructive work.

3.4 Porous Medium

Media which are solid bodies containing pores are called ***"porous media"***. Extremely small void spaces in a solid are ***'molecular interstices'*** and very large ones are called ***'caverns'***. Pores are void spaces intermediate in size between caverns and molecular interstices. Flow of fluid is possible only if at least part of the pore space is interconnected. The interconnected part of the pore system is called effective pore space of the porous medium.

The oil recovery from within the Earth has made the flow of a fluid in porous medium of great interest and importance. There has been considerable interest especially among geophysical fluid dynamicists to study the breakdown of stability of a layer of fluid subject to a vertical temperature gradient in a porous medium and to study the possibility of convective flow. The effect of geomagnetic field on the stability of such flows is of interest in the physics of the Earth, particularly in the study of Earth's core where the molten fluid is electrically conducting, which can become convectively unstable as a result of differential diffusion.

4. Methods to Investigate Stability

4.1 Perturbation Method

To establish the instability of any hydrodynamic system, the system is imagined to undergo a specific, small trial displacement. If the additional forces thus produced tend to increase the displacement, thereby enhancing the deformation of the system still further, the system is unstable. This is the most suitable method for establishing instability of a system.

4.2 Energy Method

The energy principle technique, is another method for the investigation of stability which depends upon a variational formulation of the equations of motion. This method leads to a variational problem for the first critical of energy theory and to definite criterion viscosity which is sufficient for the global stability of the basic flow. It is sometimes possible to find positive definite functionals of the disturbances of basic flow, other than the energy, which decrease on solutions when the viscosity is larger than a critical value. Such functionals, which may be called generalized energy functionals of the Liapounov type, are of interest because they can lead to a larger interval of viscosities on which the global stability of the basic flow can be guaranteed.

This principle was first used by Rayleigh (1877) in calculation of the frequencies of vibrating systems. Reynolds (1895) and Orr (1907) used this method in their early works. In this method we make use of energy principle. In a mechanical system for which there exists a potential energy function V', a stationary state of system will be unstable or stable according as V' is strictly maximum or minimum. In such a system, when dissipation forces are neglected,

$T' + V' =$ constant,

where T' denotes the kinetic energy of the system.

Suppose that V' attains a strict minimum V'_0 for a stationary configuration. When the system is disturbed, $V' > V'_0$ in a neighbouring configuration if T'_0 is the initial kinetic energy of the system generated by the small disturbance, then we have

$$T' + V' = T'_0 + V'_0,$$

which gives

$$T' = T'_0 - (V' - V'_0),$$

i.e. $\quad T' < T'_0 \qquad\qquad (\because \; V' - V'_0 > 0)$

Thus, the system does not tend to deviate further from the stationary configuration, but remains in its proximity. The system is therefore stable.

If V'_0 is strict maximum, then $T' > T'_0$ and the system will tend to depart further from its initial state. The system is thus unstable. So we calculate the change $V' - V'_0$ in the potential energy of the system, when it is given a small displacement satisfying the boundary conditions. The system is stable if this change is positive for all possible infinitesimal displacements and is unstable if $V' - V'_0$ can be shown negative for any one particular trial displacement.

4.3 Normal Mode Analysis

The normal mode analysis method is quite general and has found extensive applications. It is preferred because it gives complete information about the instability including the rate of growth of any unstable perturbation.Chandrasekhar has used throughout this method in his book ***"Hydrodynamic and Hydromagnetic Stability"*** (Dover Publication, New York, 1981) while discussing the various instability problems of flow under varying assumptions of hydrodynamics and hydromagnetics.

If the value of n determined by the dispersion relation is:

1) real and negative, the system is stable;
2) real and positive, the system is unstable;
3) complex, say $n = n_r + in_i$, where n_r and n_i are real and
   a) $n_r < 0$, the system is stable;
   b) $n_r > 0$, the system is unstable;
   c) $n_r = 0$, the modes are oscillatory;
4) further, if $n_r = 0$ implies that $n_i = 0$, then the stationary (cellular) pattern of flow prevails on the onset of instability. In other words, "principle of exchange of stabilities" is valid.
5) If $n_r = 0$, does not imply that $n_i = 0$, then overstability occurs.

From this it follows that if n is real, then n = 0 will separate the stable and unstable modes and we will always have exchange of stabilities.

the magnetic field and decreases with the increase in kinematic viscoelasticity.

5. Nomenclature

T	Temperature
p	Pressure
ρ	Density
α	Coefficient of volume expansion
T_o	Temperature at $\rho = \rho_o$

μ	Coefficient of viscosity
δ_{ij}	Kronecker delta
e_{ij}	Rate-of-strain tensor
q	Coefficient of heat conduction

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