Lovely Professional University

Master of Science

PERFORMANCE ANALYSIS OF SOME INDISTRIAL BASED SYSTEM THROUGH RELIABILITY APPROACH

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Transforming Education Transforming India

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Declaration of Authorship

I, Arun Kumar Garov, declare that this thesis titled, “Performance analysis of some industrial based system through reliability approach”and the works presented in it are my own. I confirm that:

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- I have acknowledged all main sources of help.
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Performance Analysis of a Paper mill Plant through Reliability Approach

Abstract

In this project we have investigated the paper manufacturing process of a paper mill plant through mathematical modeling and Markov process for finding its various preferences characteristic's. Throughout the paper making process, a paper mill plant may work in three different states according to the failure of its components (subsystem) namely good state, degraded state and failed state. The performance of a paper mill depends on the collective functioning of its subsystems namely Digester, Head box, Dandy Roll, Press section, Dryer etc. By analyzing interconnection of these components in the paper mill plant, a mathematical model is developed for finding the reliability, availability, MTTF, Sensitivity analysis and estimated profit from the paper mill plant. For better understanding, the results are shown with the help of graphs.

Key terms: Paper mill plant; Markov Process; Mathematical Modeling; Multi-state system; Performance Measures;

1. Origin of reliability theory

Reliability is a most popular technique that has been a famous for many years as a creditable quality of a person or a product. Its modest creation was in 1816, far sooner than most would guess. The word "reliability" was first started by poet Samuel Taylor Coleridge. At the beginning of the 1930s, Walter Shewhart, Harold F Dodge, and Harry G Romig laid down the theoretical basis for utilizing statistical methods in quality control of industrial products. This type's concept was not brought into use a most great level beginning of World War II. Products that were composed of a large number of parts often did not function, despite the fact that they were made up of individual high-quality components. In the 1970s interest increased, in the United States as well as in other parts of the world, in risk and safety aspects connected to the building and operation of nuclear power plants. In the United States, a large research commission, led by Professor Norman Rasmussen was set up to analyze the problem. The multimillion dollar project resulted in the so-called Rasmussen report, WASH-1400 (NUREG-75/014). Despite its weaknesses, this report represents the first serious safety analysis of so complicated a system as a nuclear power plant.

Reliability is a human quality, has been applaud for a very long time. For occupatinal systems, however, the reliability method has not been applied for more than some 60 years. It emerged with a technological meaning just beginning of World War II and was then used in relation with comparing operational safety of one-, two-, and four-engine airplanes. The reliability was measured as the number of accidents per hour of flight time.

Reliability theory allocate with the interdisciplinary use of probability, statistics and stochastic modeling, combined with engineering discernment into the design and technical understanding of the failure apparatus to study the various point of reliability. It encompasses issues such as:

- Reliability modeling
- Reliability analysis and optimization
- Reliability engineering
- Reliability science
- Reliability technology
- Reliability management `

The concept of reliability is related to one or more product functions that are required. Some function is very significant, while others may be of the category “nice to have”. When we use the term reliability, we should always define the function which is required.

Reliability, mean the probability that a failure may not occur in a given time interval. Amore rigorous definition of reliability is as follows:” Reliability of a unit or product is the probability that the unit performs its intended function adequately for a given period of time under the stated operating condition or environment”.

1.1 **Definition of Reliability**

Reliability is the probability of a product performing its purpose adequately for the period intended under the given operating conditions. This definition bring into focus four important factors, namely,

- **Reliability expressed as a probability**

This is the ratio of the number of times we can expect an event to the total number of trials undertaken. The maximum value of fraction is one and the minimum value is zero. A probability value of one means a certainly, i.e., the expected event occurs in almost every case.

- **Adequate performance**

This is the second element in the definition of reliability. It describes, in unambiguous terms, what is expected of a device or system.

- **Duration of adequate**

This is one of the most important elements in the definition since it represents a measure of the Period for which the performance is satisfactory.

- **Environmental or Operating conditions**

The environmental or operating conditions in which we expect a device to function adequately could be with regard to temperature, humidity, shocks, vibration and so on.

1.2 Availability

Availability is another measure of performance of maintained equipments. It integrates both reliability and maintainability parameters and depends on the number of failures that occur and on how quickly any faults are rectified. The long-run or steady- state availability is defined as the proportion of the time during which equipment is available for use. It can be expressed as,

$$Availability = \frac{\text{Up time}}{\text{Up time} + \text{Down time}} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

Where MTBF = Mean time between failures; MTTR=Mean time to repair

1.3 MTTF

MTTF is a basic measure of reliability of reparable systems. It is extremely similar to mean time between failures (MTBF). The difference between these terms is that while MTBF is expected time to failure after a failure and repair of the components or system, MTTF is the expected time to failure of a component or system i.e. mean time to failure of the components or systems. As a metric, MTTF represents how long a product can reasonably be expected to perform in the field based on specific testing. It's important to note, however, that the mean time to failure metrics provided by companies regarding specific products or components may not have been collected by running one unit continuously until failure. Instead, MTTF data is often collected by running many units, even many thousands of units, for a specific number of hours. It is defined as:

$$MTTF = \lim_{n\to\infty} \bar{R}(s) \text{Or} MTTF = \int_0^\infty R(t)dt$$

Where R (t) is the system's reliability and it is defined as and R(t)=P(T > t)=$\int_t^\infty f(x)dx$,and $\bar{R}(s)$ is Laplace transform Laplace of R(t).

1.4 MTTR

Mean time to repair of a system is the mean time required to repair the system. It is a much needed parameter for the management of the system. The goal of a system designer is to maintain MTTR as low as possible.

It can be defined as, $\text{MTTR} = \frac{\sum \text{Time taken in repair}}{\text{Total number of repairs}}$

It also can be defined as, $\mathrm{MTTR} = \lim_{t\to\infty}[P_{\mathrm{Down}}(t)]_{\text{with all repairs equal to zero}}$

Where, P_{down} (t) is the probability that the system is in the failed state.

1.5 MTBF

As the term indicates that MTBF is the mean time among failures of a structure. Mean time between failures is a reliability characteristic used to find out the number of failures occurred per unit time. It is the predicted elapsed time between inherent failures of a system during its operation. It can be calculated as the arithmetic mean time between failures of a system. It is calculated only for repairable system. In the case of non-reparable system MTBF is meaningless. It is one of the most common queries about a system's life cycle and it is very helpful to the management for decision making regarding system.

It can be defined as:$\mathrm{MTBF} = \frac{\sum \text{Time between failures}}{\text{Total number of failures}}$

It also can be defined in terms of density function f (t) as,

$$\mathrm{MTBF} = \int_0^\infty t\, f(t)\, dt$$

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1.6Sensitivity Analysis

Sensitivity analysis helps us to identify critical components of a system or portions of the system that are particularly sensitive to error. By this analysis one can identify that on which failure the management of the system focused most, this helps to improve the performance of a system. The sensitivity of a factor is most regularly defined as the partial derivative of that factor. This measure is then used to estimate the outcome of factor changes on the system's resultwithout requiring a full system solution for each factor change. These input factors are mostly failure rates.

1.7 Redundancy

Redundancy technique is a solution of alternative means or parallel paths in the system that all means must fail before causing a system failure. It has been noted earlier that any such additional means will increase the system reliability and mean life. In a system of complex nature, redundancy can be applied at various levels. The various approaches for introducing redundancy in the system are:

- The simplest and most straight forward approach is to provide a duplicate path for the entire system itself. This is known as system or unit redundancy.
- Another approach is to provide redundant paths for each component individually. This is called component redundancy.
- The third method suggests that the weak components should be identified and strengthened for reliability. This approach is useful when we consider reliability and coast- optimization problems.
- The last approach is to appropriately mix the above techniques depending upon the system configuration and reliability requirements. This approach is known as mixed redundancy.

The use of a particular approach depends upon many factors, such as the operating characteristics of components or system weight, size and initial cost.

It can also be defined as:

"Redundancy is the technique which is used to increase the reliability of a system"

Redundancies can be characterized in the following types:

1.7.1 Standby Redundancy

Standby redundancy is a technique which plays a crucial role for non-interruption in the functioning of a system. When a system fails, a standby unit is called for uninterrupted operation of the system. It is also defined as a failover technique to make the system more reliable. It is frequently referred as an immediate backup for an essential component without which the complete system fails. Standby Redundancy is characterized into three different sections according to their change over time from main unit to standby unit. These are:

- **Cold Standby Redundancy**

A cold standby unit is a unit which takes load when main unit of the system has failed. It is completely inactive until main unit fails, due to this it is known as a cold standby unit. It cannot be failed when it is inactive and its reliability is unchanged until it is active. The cold standby unit takes some change over time when main unit fails.

- **Warm Standby Redundancy**

A warm standby unit is a unit which takes load when main unit of the system has failed in the same manner as cold standby units. It is completely inactive though out the task of the system.

The warm standby unit is activated when main unit of the system has failed. The change over time of the warm standby unit is much lesser than a cold standby unit.

- **Hot Standby Redundancy**

A hot standby unit is a unit which takes load when main unit of the system has failed in the same manner as warm and cold standby units. It is completely active throughout the task of the system and takes immediate charge when main unit of the system has failed without any change over time, i.e. in hot standby redundancy the changeover time from main unit to standby unit is negligible.

1.7.2 *k*-out-of-*n*: G/F Redundancy

The most common mode of redundancy is *k*-out-of-*n* redundancy. This type of redundancy is further characterized into two categories, these are *k*-out-of-*n*: G and *k*-out-of-*n*: F.

A *k*-out-of-*n*: G redundancy implies that for successful operation of the system at least *k* components out of *n* components are required to be good (i.e. required to work properly). If less than *k* components are good then the system fails. A *k*-out-of-*n*: F redundancy implies that if *k* components out of *n* components have failed then the system has failed.

So, we can conclude that a *k*-out-of-*n*: G system can be written as (*n*-*k*+1)-out-of-*n*: F system. These types of systems have a wide range of application in both industry and defense. Some examples of *k*-out-of-*n* structure are given below.

- An eight cylinder automobile engine, for successful operation requires at least six cylinders is an example of 6-out-of-8: G systems or we can say 3-out-of-8: F systems.
- A shaft lift operated by four cables in which at least two are necessary for safe operation is an example of 2-out-of-4: G systems.
- In data processing with five video displays at least three displays are sufficient for full data display. In this case the systems behave like 3-out-of-5: G system.

Reliability of *k*-out-of-*n*: G system (with i. i. d. components) is given as

$$R(k,n) = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

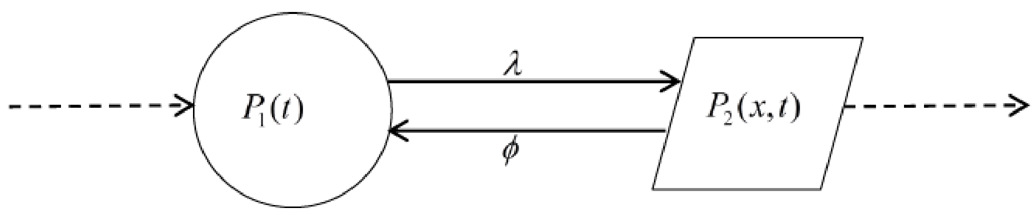
1.8 Method and Techniques for Reliability Evaluation

Here, we are discussing about the techniques which are used for reliability evaluation.

1.8.1 Markov Process

Markov process is very useful tool for analyzing random events which are dependent on each other. It is the most powerful technique in the field of reliability, which helps us to evaluate the system's various performance measures. It is named after the Russian mathematician "Andrei Andreyevich Markov" (1856-1922).

It is a process in which transition from one state to another state (future state) depends only on the present state of the system and does not depend on the past state or one can say that the transition does not depend on what happened in the past with the system or we can say that the future stage is dependent only on the present state. That's why Markov process is sometimes known as memory less process.



In the system process, initially on the basis of system configuration a state transition diagram is created, then by the Markov process a number of differential equations are generated (on the basis of input and output or repair and failure) and then by solving these equation with the help of Laplace transformation, we get the required system's transition state probabilities. Now with the help of these transition state probabilities, the various reliability measures are calculated. It is applicable to repairable and non-repairable system.

1.8.2 Stochastic Process

This process is introduced in some way of 19th century by the mathematician "Thorvald N. Thiele". "A stochastic process is a sequence of events in which the outcome at any stage depends on some probability" or we can say that it is the collection of random variables which are function of real variable t. That's why sometimes this process is known as a random process. The set of possible values of an individual member of the random process is called state space.

According to the time variable and state space, stochastic process can be classified into four sections. These sections are:

- **Discrete Time and Continuous State Space**

In this type of stochastic process, time is the discrete variable and systems state space is continuous variable For example in measuring the temperature of a day after every hour gives discrete time and continuous state space. Stock market, a time sharing computer system are the other examples.

- **Continuous Time and Continuous State Space**

In this type of stochastic process, the time and system state space are continuous variable e.g. measuring the maximum temperature of a city, a time sharing computer system with waiting time.

- **Discrete Time and Discrete State Space**

In this type of stochastic process, time and system state space are discrete variables e. g. in tossing a fair dice, time and state space both are discrete variable i.e. the set of time variable is $\{1, 2, 3, 4, 5,\ldots\ldots\}$ and the set of state space is $\{1, 2, 3, 4, 5, 6\}$.

- **Continuous Time and Discrete State Space**

In this type of stochastic process, time is continuous variable and system states are discrete variable. Under some elite conditions, a stochastic process becomes Markov process. These are listed below:

➢ The numbers of possible outcomes or states is finite.

➢ The outcome at any stage depends only on the outcome of the previous stage.

➢ The probabilities are constant over the time.

1.9 Supplementary Variable Technique

In the history of reliability theory, a lot of complex industrial systems are solved to find their various reliability measures by using different techniques. Among them, the supplementary variable technique plays a very important role. It was first time used by Cox in 1955 to solve the M/G/1 queening model. It was firstly used in the field of reliability in 1963 by Gaver.

In order to discuss about reliability measures of a system with the help of Markov process, our first and foremost concern is the system failure and repair rates, but when these rates are time dependent, the system losses its Markovian properties i.e. in this condition the transition from one state to other state is not only depend on the present state but also depends on past state. For such condition, one cannot allow to use Markov process because system loses its Markov character. In order to overcome such condition, we introduce one or more new variables to convert the non Markovian nature of the system to Markovian. Such a variable which changes non-Markovian nature of the system to Markovian nature is known as supplementary variable and this technique is known as "supplementary variable technique"

1.10 Literature review

In a field of reliability lot of research work done by researchers with the help of different processes as a fuzzy and markov processes etc. Knezevic and Odoom [1], determine reliability with the help of Petri nets and fuzzy Lambda – Tau methodology. (Pirkanniemi 2007, Hanoi 2011, Rani et al. 2011, Thanthathep et al. (2009) are calculated reliability in the field of paper plant waste water. Gupta el al. (1993) developed various models and analysis the failure computing the reliability measures such as availability, cost-estimation, mean-time to failure (MTTF). Kopra (2010) discussed refract meter computation lends itself very well to the resolution of break down basic solids from a single washer's filtrates and pulp filtrate fractions and also to the general control of washing loss levels in brown stock washing. Garg et al. (2012) discuss hybridized technique namely ABCBLT for determining the membership function of the reliability indices of complex repairable industrial system. These techniques optimize the spread of the reliability indices indicating higher sensitivity zone and thus may be useful for the reliability engineers/experts to make more sound decisions. The computed behavior analysis results of the system will help the concerned managers to plan and change suitable maintenance practices/strategies for improving system's performance and thereby reduce operational and maintenance costs. Freire et al. (2002) describes the development of a novel wal-jet flow system and also obtained a development of a laccase-based flow injection electrochemical biosensor for the determination of phenolic compounds and its application for monitoring remediation of Kraft E1 paper mill effluent. And describe a development of a novel wall-jet flow system, incorporating a small dialysis sampler and an ampere-metric laccase-based biosensor as the detector for phenolic compounds. Kopra et al. (2009) discuss the benefit of refractometer measurement compared to conductivity measurement is that the refractometer device can be installed directly in the pulp pipeline and in view of this the success or failure of washing in a previous washing stage can be detected earlier. Azaron et al. (2006) discussed a system with L-dissimilar unit's non repairable cold standby redundant system in which each unit is composed of a number of independent components with generalized Erlang distribution of life times, arranged in any general configuration and also proposed a model to the general types of non-constant hazard function. Kumar and Ram (2013) evaluated reliability measures of a coal handling unit of a thermal power plant and draw some important measures they also found expected profit for the same. Gupta and Tiwari (2011) [16] discussed the development of a performance modal of power plant using Markov technique and a probabilistic approach. This study covers two areas which are the development of a predictive modal and evaluation of performance of developed models. Wang et al. (2009) [17] modify the reliability optimization of a series-parallel system with the fuzzy path and calculation is solved by Binary Particle Swarm Optimization (BPSO) algorithm. Wei-Chang et al. (2011) [20] calculated reliability design problems using mathematical programming or heuristic optimization approach.

2 Introduction of The Problem

The first machine to make paper was developed in France around 1798. Although papermaking machines have been greatly improved and enlarged since then the basic processes remain the same. A modern papermaking machine may be several hundred feet long and may turn out hundreds of tons of paper in a single day. In the world Paper is most required in many fields, which is used as a field of study way is books and notebooks, government field where also used a paper. Many other ways for use a paper in daily use, here napkin is a simple example of daily use of paper for all person. Today use a many products where paper is most important role. Trees used for papermaking are specifically grown and harvested like a crop for that purpose. To meet tomorrow's demand, forest products companies and private landowners in Wisconsin plant millions of new seedlings every year.

To begin the process, logs are passed through a debarker, where the bark is removed, and through chippers, where spinning blades cut the wood into 1" pieces. Those wood chips are then pressure-cooked with a mixture of water and chemicals in a digester.

Digester: - “The wood chips are mixed into a large cylinder called a digester", In which they are soaked in a bath of chemicals - mainly bisulphate of lime - and cooked under pressure for about eight hours. All wood contains, along with the cellulose that make a paper, a great deal of other material that will slowly decay; the digesting process removes this other material, leaving just the cellulose.

Head box: - After the passes the digester products receive by head box. There is a wire belt and wire section the sheet pass under this section.

Dandy Roll: - pulp comes to down the screen, water is comes to outside and recycled. The resulting untreated paper sheet, or web, is crush between large rollers to remove presented water and fortify flatness and fixed width.

Pressing: - After the rolling, product will be come in the Pressing section, where will the press a seat of paper for according to size of the paper.

Dryer: - After the pressing, product will be come in the dryer section here completely remove the water which is remaining the rolling and pressing section.

Output: - After this processes comes a paper in a large roll. Which is 30 feet wide and weight around to 25 tons. A slitter cuts the paper into smaller, more manageable rolls and the paper is ready for use.

2.1 Symbolizations

t/s	Time/Inverse Laplace variable.
$P_0(t)$	The probability that at time *t* the system is working in perfect state.
$P_1(x,t)$	The probability that at time *t* the system is failed due to failure of digester.
$P_2(x,t)$	The probability that at time *t* the system is failed due to failure of head box.
$P_3(x,t)$	The probability that at time *t* the system is failed due to failure of dandy roll.
$P_4(t)$	The probability that at time *t* the system is working in degraded state due to the frailer of one unit of press section.
$P_5(t)$	The probability that at time *t* the system is working in degraded state due to the frailer of second unit press section.
$P_6(x,t)$	The probability that at time *t* the system is failed due to complete failure of Press section.
$P_7(x,t)$	The probability that at time *t* the system is failed due to failure of Dandy roll.
$P_8(x,t)$	The probability that at time *t* the system is failed due to failure of human Operator.
λ_{DG}	Failure rates of Digester.
λ_{HB}	Failure rates of Head Box.
λ_{DR}	Failure rates of Dryer.
$\lambda_{3PS}, \lambda_{2PS}, \lambda_{PS}$	Failure rates of one Press section, second Press section and Press section.
λ_{DRO}	Failure rates of Dandy Roll.
λ_{HO}	Failure rates of Human Operator.
μ_{DG}	Repair rate of digester.
μ_{HB}	Repair rate of head box.

μ_{DR}	Repair rate of Dryer.
μ_{PS}, μ	Repair rate of press section
μ_{DRO}	Repair rate of dandy roll
μ_{HO}	Repair rate of Human Operator.

2.2 Flow Diagram and Transition State Diagram

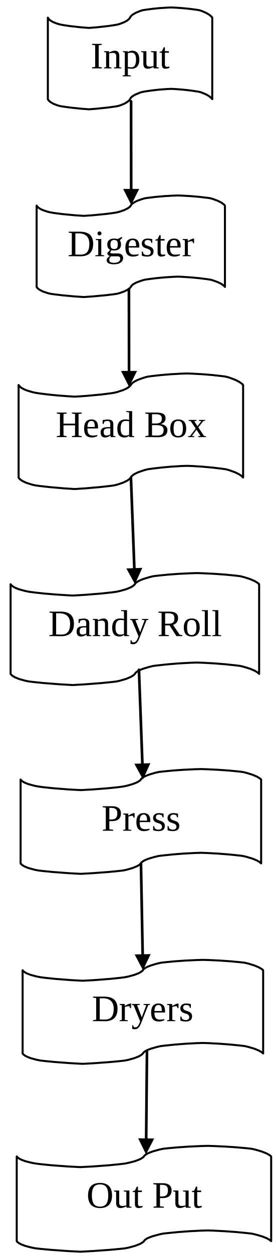


Figure 1(a). State Transition Diagram

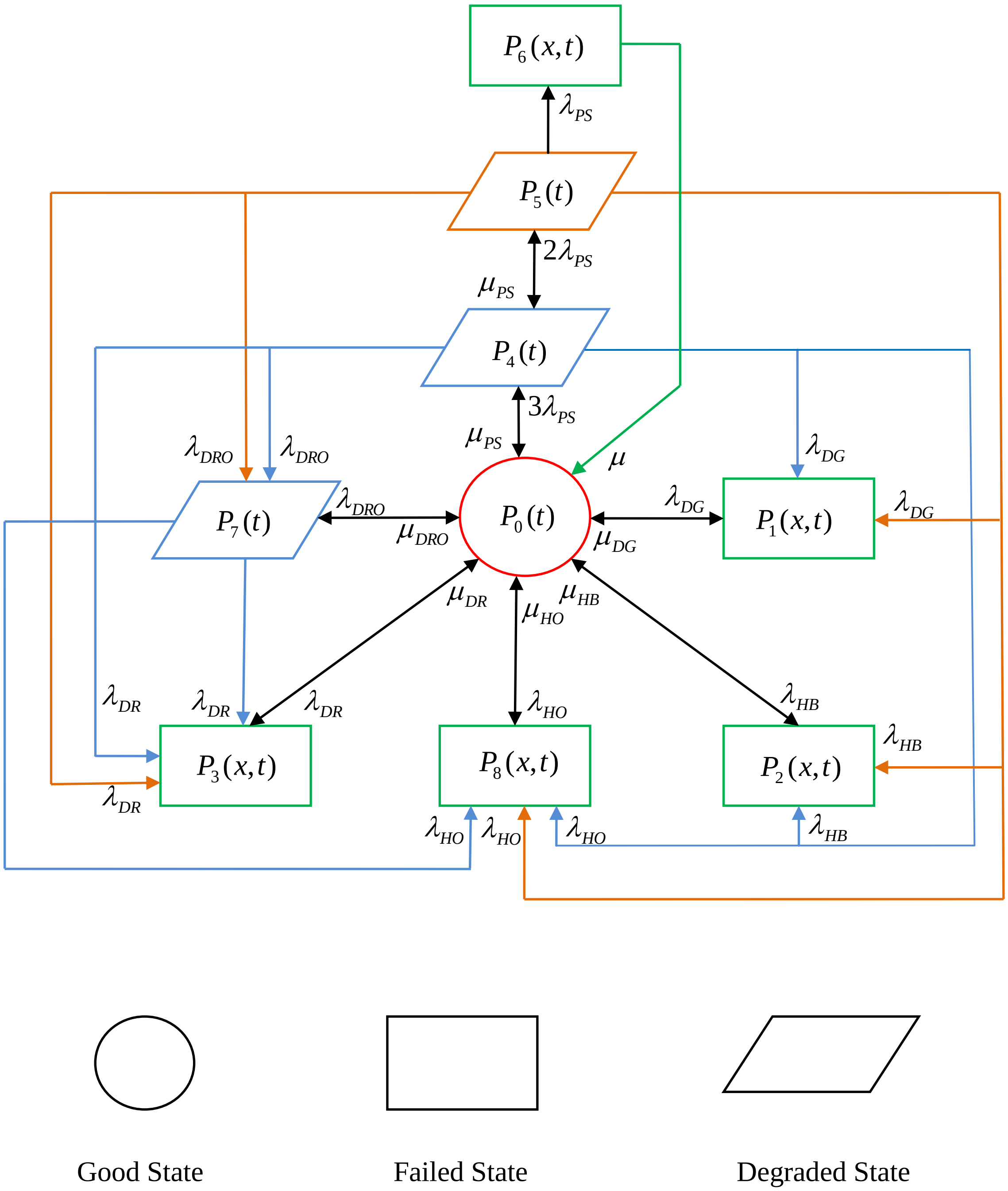


Figure 1(b). State Transition Diagram

2.3 Mathematical formulation and solution of the problem

In the transition state diagram gives the following differential equation with the help Markov process. These following differential equation are given below in the state wise

For State $P_0(t)$

$$\left(\frac{\partial}{\partial t}+\lambda_{DG}+\lambda_{HB}+\lambda_{DR}+\lambda_{3PS}+\lambda_{DRO}+\lambda_{HO}\right)P_0(t)=\mu_{DRO}P_7(t)+\mu_{PS}P_4(t)+\int_0^\infty \mu_{DG}P_1(x,t)dx+$$
$$\int_0^\infty \mu_{HB}P_2(x,t)dx+\int_0^\infty \mu_{DR}P_3(x,t)dx+\int_0^\infty \mu_{HO}P_8(x,t)dx+\int_0^\infty \mu\ P_6(x,t)dx \quad (1)$$

For State $P_1(x,t)$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{DG}\right)P_1(x,t)=0 \quad (2)$$

For State $P_2(x,t)$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{HB}\right)P_2(x,t)=0 \quad (3)$$

For State $P_3(x,t)$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{DR}\right)P_3(x,t)=0 \quad (4)$$

For State $P_4(x,t)$

$$\left(\frac{\partial}{\partial t}+\lambda_{DG}+\lambda_{HB}+\lambda_{HO}+\lambda_{DRO}+\lambda_{DR}+\lambda_{2PS}+\mu_{PS}\right)P_4(t)=\lambda_{3PS}P_0(t)+\mu_{PS}P_5(t) \quad (5)$$

For State $P_5(x,t)$

$$\left(\frac{\partial}{\partial t}+\lambda_{PS}+\lambda_{DG}+\lambda_{HB}+\lambda_{HO}+\lambda_{DR}+\lambda_{DRO}+\mu_{PS}\right)P_5(t)=\lambda_{2PS}P_4(t) \quad (6)$$

For State $P_6(x,t)$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu\right)P_6(x,t)=0 \quad (7)$$

For State $P_7(x,t)$

$$\left(\frac{\partial}{\partial t}+\lambda_{DR}+\lambda_{HO}+\lambda_{HO}+\mu_{DRO}\right)P_7(t)=\lambda_{DRO}P_0(t)+\lambda_{DRO}P_4(t)+\lambda_{DRO}P_5(t)\,(8)$$

For State $P_8(x,t)$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{HO}\right)P_8(x,t)=0\,(9)$$

Boundary condition,

$$P_1(0,t)=\lambda_{DG}P_0(t)+\lambda_{DG}P_4(t)+\lambda_{DG}P_5(t)\quad(10)$$

$$P_2(0,t)=\lambda_{HB}P_0(t)+\lambda_{HB}P_4(t)+\lambda_{HB}P_5(t)\quad(11)$$

$$P_3(0,t)=\lambda_{DR}P_0(t)+\lambda_{DR}P_4(t)+\lambda_{DR}P_5(t)+\lambda_{DR}P_7(t)\quad(12)$$

$$P_6(0,t)=\lambda_{PS}P_5(t)\qquad(13)$$

$$P_8(0,t)=\lambda_{HO}P_0(t)+\lambda_{HO}P_4(t)+\lambda_{HO}P_5(t)+\lambda_{HO}P_7(t)\qquad(14)$$

Initial condition,

$P_0(0)=1$ And all other states are zero at t=0 (15)

Taking Laplace transformation from (1) to (14); we get

$$(s+\lambda_{DG}+\lambda_{HB}+\lambda_{DR}+3\lambda_{PS}+\lambda_{DRO}+\lambda_{HO})\overline{P_0}(s)=1+\mu_{DRO}\,\overline{P_7}(s)+\mu_{PS}\,\overline{P_4}(s)+\int_0^\infty\mu_{DG}\,\overline{P_1}(x,s)dx$$
$$+\int_0^\infty\mu_{HB}\,\overline{P_2}(x,s)dx+\int_0^\infty\mu_{DR}\,\overline{P_3}(x,s)dx+\int_0^\infty\mu_{HO}\,\overline{P_8}(x,s)dx+\int_0^\infty\mu\;\overline{P_6}(x,s)dx\qquad(16)$$

$$\left(\frac{\partial}{\partial x}+s+\mu_{DG}\right)\overline{P_1}(x,s)=0\quad(17)$$

$$\left(\frac{\partial}{\partial x}+s+\mu_{HB}\right)\overline{P_2}(x,s)=0\quad(18)$$

$$\left(\frac{\partial}{\partial x}+s+\mu_{DR}\right)\overline{P_3}(x,s)=0\quad(19)$$

$$\left(s+\lambda_{DG}+\lambda_{HB}+\lambda_{HO}+\lambda_{DRO}+\lambda_{DR}+\lambda_{2PS}+\mu_{PS}\right)\overline{P_4}(s)=\lambda_{3PS}\overline{P_0}(s)+\mu_{PS}\overline{P_5}(s) \quad (20)$$

$$\left(s+\lambda_{PS}+\lambda_{DG}+\lambda_{HB}+\lambda_{HO}+\lambda_{DR}+\lambda_{DRO}+\mu_{PS}\right)\overline{P_5}(s)=\lambda_{2PS}\overline{P_4}(s) \quad (21)$$

$$\left(\frac{\partial}{\partial x}+s+\mu\right)\overline{P_6}(x,s)=0 \quad (22)$$

$$\left(s+\lambda_{DR}+\lambda_{HO}+\mu_{DRO}\right)\overline{P_7}(s)=\lambda_{DRO}\overline{P_0}(s)+\lambda_{DRO}\overline{P_4}(s)+\lambda_{DRO}\overline{P_5}(s) \quad (23)$$

$$\left(\frac{\partial}{\partial x}+s+\mu_{HO}\right)\overline{P_8}(x,s)=0 \quad (24)$$

Boundary conditions,

$$\overline{P_1}(0,s)=\lambda_{DG}\overline{P_0}(s)+\lambda_{DG}\overline{P_4}(s)+\lambda_{DG}\overline{P_5}(s) \quad (25)$$

$$\overline{P_1}(0,s)=\lambda_{HB}\overline{P_0}(s)+\lambda_{HB}\overline{P_4}(s)+\lambda_{HB}\overline{P_5}(s) \quad (26)$$

$$\overline{P_3}(0,s)=\lambda_{DR}\overline{P_0}(s)+\lambda_{DR}\overline{P_4}(s)+\lambda_{DR}\overline{P_5}(s)+\lambda_{DR}\overline{P_7}(s) \quad (27)$$

$$\overline{P_6}(0,s)=\lambda_{PS}\overline{P_5}(s) \quad (28)$$

$$\overline{P_8}(0,s)=\lambda_{HO}\overline{P_0}(s)+\lambda_{HO}\overline{P_4}(s)+\lambda_{HO}\overline{P_5}(s)+\lambda_{HO}\overline{P_7}(s) \quad (29)$$

Solving equations from (16) to (24) with the help of (25) to (29) and use a initial condition then, we get

$$\overline{P_0}(s)=\frac{1}{(H_1-H_5-H_6-H_7-H_8)}$$

$$\overline{P_4}(s)=\left(\frac{\lambda_{3PS}H_3}{(H_2H_3-\lambda_{2PS}\mu_{PS})}\right)\overline{P_0}(s)$$

$$\overline{P_5}(s)=\left(\frac{\lambda_{2PS}\lambda_{3PS}}{(H_2H_3-\lambda_{2PS}\mu_{PS})}\right)\overline{P_0}(s)$$

$$\overline{P_7}(s)=\left(\frac{\lambda_{DRO}}{H_4}+\frac{\lambda_{DRO}\lambda_{3PS}H_3}{H_4(H_2H_3-\lambda_{2PS}\mu_{PS})}+\frac{\lambda_{DRO}\lambda_{2PS}\lambda_{3PS}}{H_4(H_2H_3-\lambda_{2PS}\mu_{PS})}\right)\overline{P_0}(s)$$

Where,

$$H_1 = s + \lambda_{DG} + \lambda_{HB} + \lambda_{DR} + \lambda_{3PS} + \lambda_{DRO} + \lambda_{HO}$$

$$H_2 = s + \lambda_{DG} + \lambda_{HB} + \lambda_{HO} + \lambda_{DRO} + \lambda_{DR} + \lambda_{2PS} + \mu_{PS}$$

$$H_3 = s + \lambda_{PS} + \lambda_{DG} + \lambda_{HB} + \lambda_{HO} + \lambda_{DR} + \lambda_{DRO} + \mu_{PS}$$

$$H_4 = s + \lambda_{DR} + \lambda_{HO} + \mu_{DRO}$$

$$H_5 = \left(\frac{\lambda_{DRO}}{H_4} + \frac{\lambda_{3PS} H_3}{H_4 (H_2 H_3 - \mu_{PS} \lambda_{2PS})} + \frac{\lambda_{DRO} \lambda_{2PS} \lambda_{3PS}}{H_4 (H_2 H_3 - \mu_{PS} \lambda_{2PS})} \right) \left(\mu_{DRO} + \lambda_{DR} \left(\frac{\mu_{DR}}{s + \mu_{DR}} \right) + \lambda_{HO} \left(\frac{\mu_{HO}}{s + \mu_{HO}} \right) \right)$$

$$H_6 = \frac{\lambda_{3PS} H_3}{(H_2 H_3 - \mu_{PS} \lambda_{2PS})} \left(\mu_{PS} + \lambda_{DG} \left(\frac{\mu_{DG}}{s + \mu_{DG}} \right) + \lambda_{HO} \left(\frac{\mu_{HO}}{s + \mu_{HO}} \right) + \lambda_{HB} \left(\frac{\mu_{HB}}{s + \mu_{HB}} \right) + \lambda_{DR} \left(\frac{\mu_{DR}}{s + \mu_{DR}} \right) \right)$$

$$H_7 = \frac{\lambda_{2PS} \lambda_{3PS}}{(H_2 H_3 - \mu_{PS} \lambda_{2PS})} \left(\lambda_{DG} \left(\frac{\mu_{DG}}{s + \mu_{DG}} \right) + \lambda_{HO} \left(\frac{\mu_{HO}}{s + \mu_{HO}} \right) + \lambda_{HB} \left(\frac{\mu_{HB}}{s + \mu_{HB}} \right) + \lambda_{DR} \left(\frac{\mu_{DR}}{s + \mu_{DR}} \right) + \lambda_{PS} \left(\frac{\mu_{PS}}{s + \mu_{PS}} \right) \right)$$

$$H_8 = \lambda_{DG} \left(\frac{\mu_{DG}}{s + \mu_{DG}} \right) + \lambda_{HO} \left(\frac{\mu_{HO}}{s + \mu_{HO}} \right) + \lambda_{HB} \left(\frac{\mu_{HB}}{s + \mu_{HB}} \right) + \lambda_{DR} \left(\frac{\mu_{DR}}{s + \mu_{DR}} \right)$$

The probability of the system is in the form the up (good and degraded state) and down (failed state) state are given as;

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_4(s) + \overline{P}_5(s) + \overline{P}_7(s) \qquad (30)$$

$$\overline{P}_{down}(x,s) = \overline{P}_1(x,s) + \overline{P}_2(x,s) + \overline{P}_3(x,s) + \overline{P}_6(x,s) + \overline{P}_8(x,s) \qquad (31)$$

2.4 Computation of Various performance measures of Paper Plant

2.4.1 Availability Analysis

Taking the value of different parameters as, $\lambda_{DG} = 0.04, \lambda_{HB} = 0.01, \lambda_{DR} = 0.04, \lambda_{3PS} = 0.03, \lambda_{2PS} = 0.03, \lambda_{DRO} = 0.02, \lambda_{HO} = 0.02$, and repair rates $\mu_{DG} = 1, \mu_{HB} = 1, \mu_{DR} = 1, \mu_{PS} = 1, \mu = 1, \mu_{DRO} = 1, \mu_{HO} = 1$, in the Eq. (30) then taking inverse Laplace transformation, we get the availability of the system,

$$P_{up}(t) := \left\{ \begin{array}{l} .01575856444\, e^{-1.060000000\, t} - .01606940080\, e^{-1.331930933\, t} \\ + .08159120238\, e^{-1.127484712\, t} - .0004037295295\, e^{-.9954426696\, t} \\ + .9191233635\, e^{-.02514168492\, t} \end{array} \right\} \qquad (32)$$

Now fluctuate time t from 0 to 10 in (32), we get the following Table.1 and figure.2 for availability of the considered structure.

Time unit (t)	Availability (P_{up})
0	1.000000
1	0.923794
2	0.883323
3	0.855458
4	0.832224
5	0.810895
6	0.790539
7	0.770837
8	0.751674
9	0.733003
10	0.714801

Table 1. Availability as function of time

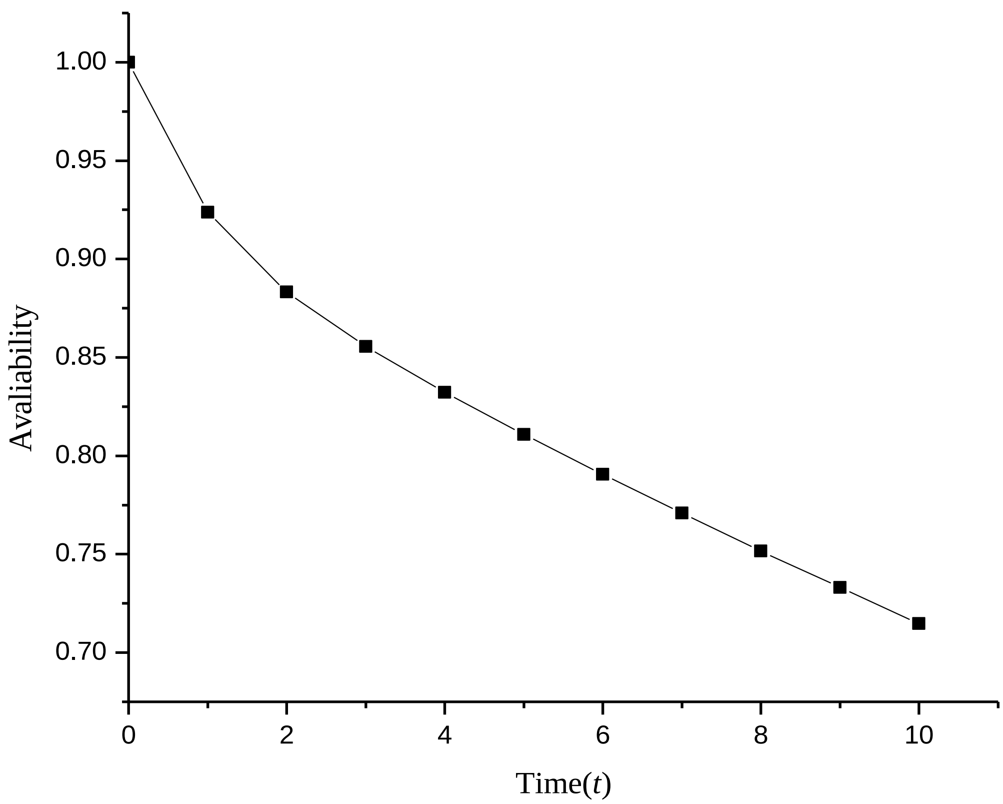


Figure 2. Availability as function of time

2.4.2 Reliability

Taking all repairs rates equal to zero and failure rates as, $\lambda_{DG} = 0.04, \lambda_{HB} = 0.01, \lambda_{DR} = 0.04, \lambda_{3PS} = 0.03, \lambda_{2PS} = 0.03, \lambda_{PS} = 0.03, \lambda_{DRO} = 0.02, \lambda_{HO} = 0.02$ in the Eq. (30) and taking inverse Laplace transformation, we get the reliability of the system,

$$R(t) = \begin{Bmatrix} (.7220000000 + .02220000000\,t + .0003600000000\,t^2)e^{-.1600000000\,t} \\ + .27800000000\,e^{-.06000000000\,t} \end{Bmatrix} \tag{33}$$

Now fluctuate time t from 0 to 10 in the Eq. (33), we get a table.2 and figure.3

Time unit (t)	**Reliability (R)**
0	1.000000
1	0.896282
2	0.804130
3	0.722182
4	0.649248
5	0.584282
6	0.526366
7	0.474692
8	0.428550
9	0.387312
10	0.350428

Table 2. Reliability vs function of time

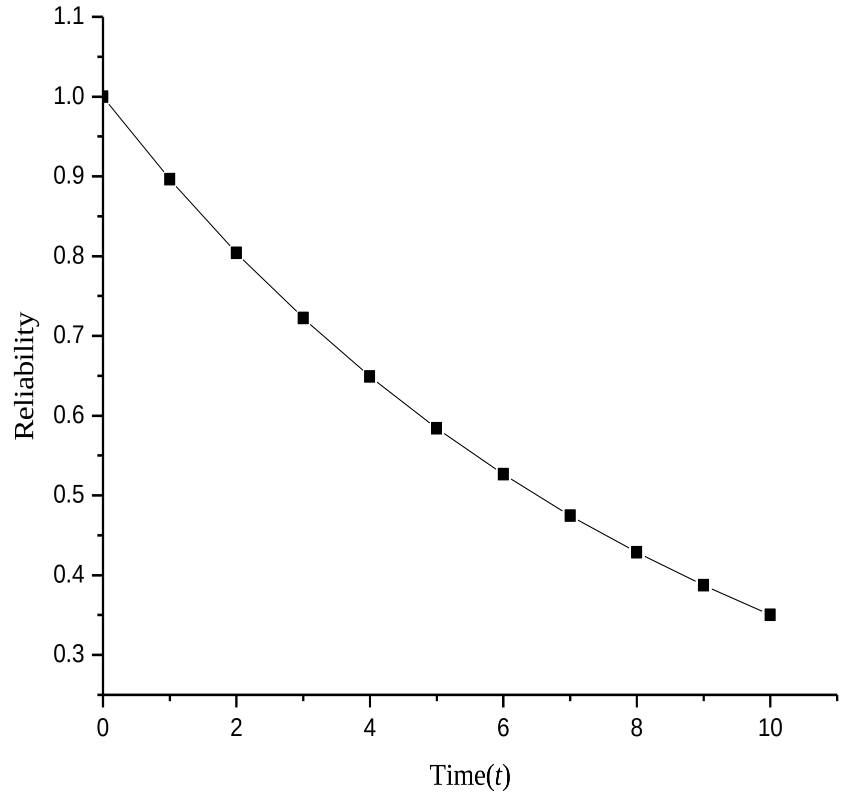


Fig.3. Reliability vs function of time

2.4.3 Mean time to failure (MTTF) of Paper Plant

Taking all repairs rates equal to zero and failure rates as, $\lambda_{DG} = 0.04, \lambda_{HB} = 0.01, \lambda_{DR} = 0.04, \lambda_{3PS} = 0.03, \lambda_{2PS} = 0.03, \lambda_{PS} = 0.03, \lambda_{DRO} = 0.02, \lambda_{HO} = 0.02$, in the Eq. (30) and taking s tends to zero, we can obtain the MTTF of the system,

Failure Rates	**MTTF with respect to failure rates**					
	λ_{DG}	λ_{HB}	λ_{DR}	λ_{PS}	λ_{DRO}	λ_{HO}
0.01	13.16994	10.18880	16.46184	10.25267	9.644444	11.57333
0.02	12.00194	9.471470	13.50218	10.23209	10.18880	10.18880
0.03	11.02222	8.847736	11.57333	10.18880	10.65540	9.133203
0.04	10.18880	8.300529	10.18880	10.12280	11.05967	8.294753
0.05	9.471470	7.816666	9.133203	10.03657	11.41322	7.608818

0.06	8.847736	7.385811	8.294753	9.933420	11.72500	7.035000
0.07	8.300529	6.999749	7.608818	9.816666	12.00194	6.546514
0.08	7.816666	6.651872	7.035000	9.689378	12.24956	6.124780
0.09	7.385811	6.336805	6.546514	9.554219	12.47226	5.756428
0.10	6.999749	6.050133	6.124780	9.41344	12.67361	5.431547

Table 3. MTTF as function of failure rates

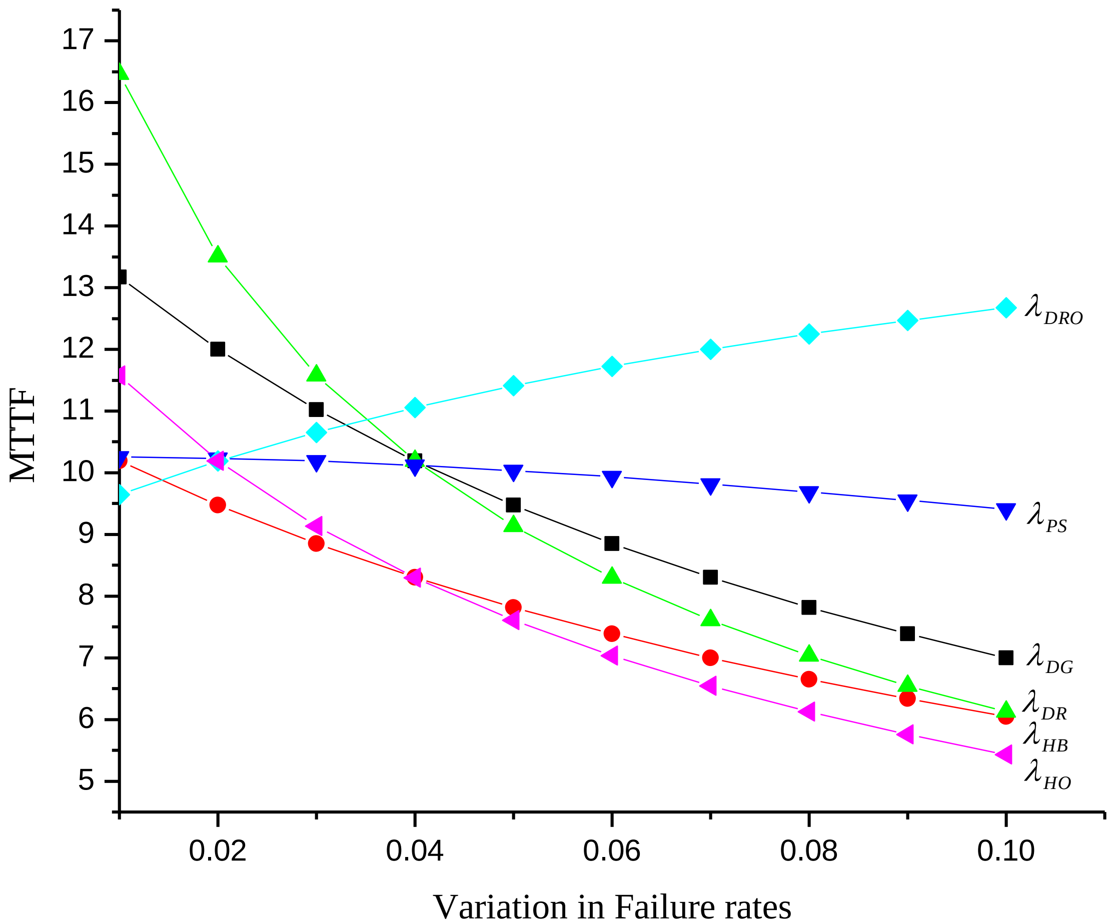


Figure4. Behavior of MTTF with respect to failure rates

2.4.4 Estimated Profit Analysis

The expected profit from the Paper Plant in the interval [0, t) is calculated by,

$$E_P(t) = C_1 \int_0^t P_{up}(t)dt - C_2(t) \qquad (34)$$

Using Equation (30) in (34), Expected profit function for the same set of failure and repair rates is given by,

$$E_P(t) := \left\{ \begin{array}{l} C_1\,[-.01486657023\,e^{-1.060000000\,t} + .01206474030\,e^{-1.331930933\,t} \\ -.07236568400\,e^{-1.127484712\,t} + .0004055778819\,e^{-.9954426696\,t} \\ -3.655774728\,e^{-.2514168492\,t} + 3.730536664] - C_2\ t \end{array} \right\} \quad (35)$$

Now $C_1 = 1$ and $C_2 = 0.1, 0.2, 0.35, 0.5, 0.65$ respectively, and using t from 0 to 10 in (35) and we get the table.6 and figure.7 for the expected profit of the considered system.

Time (t)	**Estimate Profit** $E_P(t)$				
	$C_2 = 0.1$	$C_2 = 0.2$	$C_2 = 0.35$	$C_2 = 0.5$	$C_2 = 0.65$
0	0	0	0	0	0
0.01	0.762196	0.662196	0.512196	0.362196	0.212196
0.02	1.310993	1.110993	0.810993	0.510993	0.210993
0.03	1.708161	1.408161	0.958161	0.508161	0.058161
0.04	1.992308	1.592308	0.992308	0.392308	-0.207691
0.05	2.196021	1.690219	0.940219	0.190219	-0.559780
0.06	2.321624	1.721624	0.821624	-0.078375	-0.978375
0.07	2.401493	1.701493	0.651493	-0.398506	-1.448506
0.08	2.441346	1.641346	0.441346	-0.758653	-1.958653
0.09	2.450099	1.550099	0.200099	-1.149900	-2.499900
0.10	2.434672	1.434672	-0.065327	-1.565327	-3.065327

Table 4. Expected profit vs. Service cost and time (t)

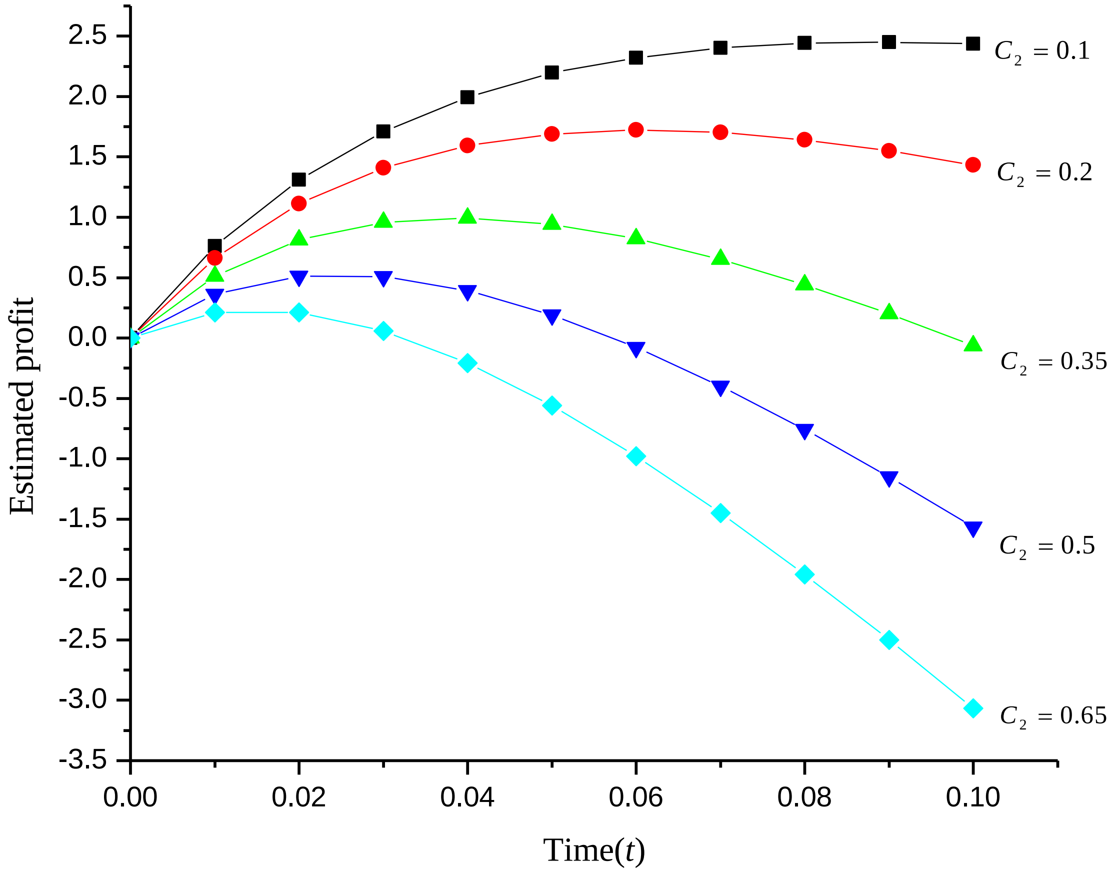


Fig.5 Expected profit vs. Service cost and time (t)

2.5 Result and discussion

In this paper, we analyzed and try to find various performance majors of a paper mill plant by the ad of mathematical modeling and supplementary variable technique. By critically analyzing the various graphs of system performance it is observed from figure 2 that the availability of the system decreases smoothly with respect to time. It shows that after a specific time interval the availability of the paper mill plant seems to be constant. The behavior of system reliability with respect to time unit is shown in Figure 3. It reflects reliability of the system depicting in a quick manner then the system availability. The difference between the graph of availability and reliability shows the importance of a good maintenance policy. The behavior of MTTF of the system with respect to various failure rates is shown in Figure 4. It reflects that system MTTF increases with respect to failure rates of Dandy roll of paper mill system. So, the MTTF is the highest with respect to Dryer failure and least with respect to Dandy roll failure.Keeping the revenue cost per unit time fixed as 1 and varying service cost as 0.1, 0.2, 0.35, 0.5, and 0.65 one can obtain Figure7. It is clear that when increases a service cost then profit will be decreases.

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