

Solution of a class of Intuitionistic Fuzzy Assignment
Problem by using Similarity measures



A

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Jalandhar

(Rajveer Kaur)

December , 2017

Declaration of Authorship

I, RAJVEER KAUR, declare that this thesis titled, Solution Of Class Of Intuitionistic Fuzzy Assignment Problem By Using Similarity Measures and the work presented in it are my own. I confirmed that:

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Certificate

This is to certify that RAJVEER KAUR has completed projected titled, Solution Of Class Of Intuitionistic Fuzzy Assignment Problem By Using Similarity Measures under my guidance and supervision. To the best of my knowledge, the present work is the result of his original investigation and study. No part of the project has ever been submitted for any other degree at any university.

The project is fits for the submission and fulfilled of the condition for award of the Masters in mathematics.

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Abstract

In this report, we form mathematical models of the assignment problem with restriction on person cost depending on efficiency/qualification and restriction on job cost where both the costs are considered as intuitionistic fuzzy numbers. For representing the costs consideration of intuitionistic Fuzzy numbers(IFNs) makes the problem more general in the sense that it considers both the degree of acceptance and degree of rejection. A well established intuitionistic fuzzy ranking method has been used for comparing the IFNs using their score functions and the accuracy degrees. Then Mathematical model of the problem has been established.

The chapter wise summary of the report is as follows:

In **Chapter 1**, some basic definitions related to Linear Programming are discussed.

In **Chapter 2**, some important definitions related to Intuitionistic Fuzzy sets are discussed. Then we define how to ranking Intuitionistic Fuzzy number by using score function and accuracy function. Then we define a matching function to find the degree of similarity between Fuzzy sets.

In **Chapter 3**, we formulate the mathematical models by considering different different conditions.

In **Chapter 4**, we developed a methodology for solving the Intuitionistic fuzzy

Assignment problem(IFAP) without and with restrictions on person and job cost by using similarity measures. The methodology for solving the problem consists of two algorithms that are discussed in this chapter.

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Chapter 1

INTRODUCTION

1.1 General Linear Programming Problem

(General Linear Programming Problem)

Let z be a linear function on R^n defined by

$$z = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n \quad (1.1)$$

where c_j are constant. Let (a_{ij}) be an $m \times n$ real matrix and $b_1, b_2, b_3, \dots, b_m$ be a set of constants such that

$$\left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \geq \text{or } \leq \text{or } = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \geq \text{or } \leq \text{or } = b_2 \\ \cdot \\ \cdot \\ \cdot \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \geq \text{or } \leq \text{or } = b_m \end{array} \right. \quad (1.2)$$

and finally let

$$x_j \geq 0, j = 1, 2, \dots, n \quad (1.3)$$

The problem of determining an n -tuple (x_1, x_2, \dots, x_n) which makes z a minimum (or maximum) and satisfies (2) and (3) is called the general linear programming problem.

1.1.1 Objective Function

The linear function

$$z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

which is to be minimized (or maximized) is called the objective function of the general L.P.P.

1.1.2 Constraints.

The inequations (2) are called the constraints of the General L.P.P.

1.1.3 Non-Negative Restrictions.

The set of inequations (3) is usually known as the set of non negative restrictions.

Solution An n-tuple (x_1, x_2, \dots, x_n) of real numbers which satisfy the constraints of a General L.P.P. is called a solution to the General L.P.P.

Feasible Solution Any solution to a General L.P.P. which also satisfy the non-negative restrictions of the problem, is called a feasible solution to the general L.P.P.

Optimum Solution

Any feasible solution which optimizes (minimizes or maximizes) the objective function of a general L.P.P. is called an optimum solution to the general L.P.P.

Example Of A General L.P.P (Diet Problem) Given the nutrient contents of a number of different foodstuffs and the daily minimum requirement of each nutrient for a diet, determine the balanced diet which satisfied the minimum daily requirements and at the same time has the minimum cost.

1.1.4 Mathematical Formulation of Diet problem

Let there be n different types of foodstuffs available and m different types of nutrients required.

Let a_{ij} denote the number of nutrient i in one unit of foodstuff j , $i=1,2,\dots,m; j=1,2,\dots,n$. Let x_j be the number of units of food j in the desired diet. Then the total number of units of nutrient i in the desired diet is

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n$$

Let b_i be the number of units of the minimum daily requirement of nutrient i . Then, we must have

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq b_i \quad i=1,2,\dots,m$$

Also, each x_j must be either positive or zero. Thus we also have $x_j \geq 0 \quad j=1,2,\dots,n$

Finally, consider the cost. Let c_j be the cost per unit of food j . Thus the total cost of the diet is given by

$$z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

Thus, the problem of selecting the best diet reduces to the following mathematical form:

Find an n -tuple (x_1, x_2, \dots, x_n) of real numbers, such that

$$(a) \quad a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq b_i \quad i=1,2,\dots,m$$

$$(b) \quad x_j \geq 0 \quad j=1,2,\dots,n$$

and for which the objective function

$$z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

may be minimum(least). This is a general L.P.P.

1.1.5 Slack And Surplus Variables

SLACK VARIABLES

Let the constraints of a general L.P.P be

$$\sum_{j=1}^n a_{ij}x_j \leq b_i \quad i = 1, 2, \dots, k$$

Then, the non-negative variables x_{n+i} which satisfy

$$\sum_{j=1}^n a_{ij}x_j + x_{n+i} = b_i \quad i = 1, 2, \dots, k$$

are called **slack variables**.

SURPLUS VARIABLES

Let the constraints of a general L.P.P. be

$$\sum_{j=1}^n a_{ij}x_j \geq b_i \quad i = k + 1, k + 2, \dots, l$$

Thus, the non-negative variables x_{n+i} which satisfy

$$\sum_{j=1}^n a_{ij}x_j - x_{n+i} = b_i \quad i = k + 1, k + 2, \dots, l$$

are called **surplus(negative slack) variables**

Chapter 2

Solution Of Class Of Intuitionistic Fuzzy Assignment Problem By Using Similarity Measures

In this chapter some preliminaries ideas on IFSs, their similarity measures and ranking method are discussed. Then we define a matching function to find the degree of similarity of A and B.

2.1 Preliminaries on Intuitionistic Fuzzy sets

2.1.1 Intuitionistic Fuzzy set

An intuitionistic Fuzzy set (IFSs) A in X is having the form:

$$A = \{ \langle x_j, \zeta_{\bar{A}}(x_j), \eta_{\bar{A}}(x_j) \rangle; x_j \in X \}$$

which is characterized by membership function $\zeta_{\bar{A}}$ and non membership function $\eta_{\bar{A}}$, where $\zeta_{\bar{A}}: X \rightarrow [0,1]$ and $\eta_{\bar{A}}: X \rightarrow [0,1]$ with the condition $\zeta_{\bar{A}}(x_j) + \eta_{\bar{A}}(x_j) \leq 1$ for all $x_j \in X$

Atansov defined $\sigma_{\bar{A}}(x_j) = 1 - \zeta_{\bar{A}}(x_j) - \eta_{\bar{A}}(x_j)$, for all $x_j \in X$ as the degree of hesitancy of x_j to A where A is an IFS in X.

If $\sigma_{\bar{A}}(x_j) = 1 - \zeta_{\bar{A}}(x_j) - \eta_{\bar{A}}(x_j) = 0$ for each $x_j \in X$ then IFS A is reduced to a Fuzzy set.

2.1.2 Intuitionistic Fuzzy Number(IFN)

An IFN \tilde{A}^i is defined as follows:

(i)an intuitionistic fuzzy sub set of the real line.

(ii)normal i.e. there is any $x_0 \in R$ such that $\zeta_{\tilde{A}^i}(x_0) = 1$ (so $\eta_{\tilde{A}^i}(x_0) = 0$)

(iii)a convex set for the membership function $\zeta_{\tilde{A}^i}(x)$ i.e.

$$\zeta_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\zeta_{\tilde{A}^i}(x_1), \zeta_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0, 1]$$

(iv)a concave set for the non membership function $\eta_{\tilde{A}^i}(x)$ i.e.

$$\eta_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \leq \max(\eta_{\tilde{A}^i}(x_1), \eta_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \in R, \lambda \in [0, 1]$$

2.1.3 Ranking of intuitionistic Fuzzy Numbers

Let $\tilde{a} = (\zeta_1, \eta_1)$ be an intuitionistic fuzzy number.

A score function S of an intuitionistic fuzzy value is given by

$$S(\tilde{a}) = \zeta_1 - \eta_1 \text{ where } S(\tilde{a}) \in [-1, 1]$$

Now an accuracy function H to evaluate degree of accuracy of the intuitionistic fuzzy

value \tilde{a} where $H(\tilde{a}) \in [0, 1]$ is defined as

$$H(\tilde{a}) = \zeta_1 + \eta_1$$

Larger value of $H(\tilde{a})$ represents more the degree of accuracy of the degree of mem-

bership of the intuitionistic fuzzy value \tilde{a} . Let $\tilde{b} = (\zeta_2, \eta_2)$ be another intuitionistic fuzzy number.

Based on score function S and accuracy function H, an order relation between two

intuitionistic fuzzy values \tilde{a} and \tilde{b} is as follows:

$$\text{If } S(\tilde{a}) < S(\tilde{b}), \text{ then } \tilde{a} < \tilde{b}$$

$$\text{If } S(\tilde{a}) > S(\tilde{b}), \text{ then } \tilde{a} > \tilde{b}$$

$$\text{If } S(\tilde{a}) = S(\tilde{b}), \text{ then } \tilde{a} = \tilde{b}$$

If $H(\tilde{a})=H(\tilde{b})$, then $\tilde{a}=\tilde{b}$

If $H(\tilde{a})<H(\tilde{b})$, then $\tilde{a}<\tilde{b}$

If $H(\tilde{a})>H(\tilde{b})$, then $\tilde{a}>\tilde{b}$

2.1.4 Similarity measures of Atanassov's intuitionistic Fuzzy Sets

Let $s:\Phi(X)^2 \rightarrow [0,1]$, then the degree of similarity between $A \in \Phi(X)$ and $B \in \Phi(X)$

is defined as $s(A,B)$, which satisfies the following properties:

(i) $0 \leq s(A,B) \leq 1$;

(ii) $s(A,B)=1$ iff $A=B$;

(iii) $s(A,B)=s(B,A)$;

(iv) $s(A,C) \leq s(A,B)$ and $s(A,C) \leq s(B,C)$, if $A \subseteq B \subseteq C, C \in \Phi(X)$.

2.1.5 Similarity measures based on matching function

Let $A \in \Phi(X)$ and $B \in \Phi(X)$, then degree of similarity of A and B has been defined

based on the matching function as:

$$s(A, B) = \frac{\sum_{j=1}^n (\zeta_A(x_j) \cdot \zeta_B(x_j) + \eta_A(x_j) \cdot \eta_B(x_j) + \sigma_A(x_j) \cdot \sigma_B(x_j))}{\max(\sum_{j=1}^n (\zeta_A^2(x_j) + \eta_A^2(x_j) + \sigma_A^2(x_j)), \sum_{j=1}^n (\zeta_B^2(x_j) + \eta_B^2(x_j) + \sigma_B^2(x_j)))} \quad (2.1)$$

Consider, the weight w_j of each element $x_j \in X$, we get

$$s(A, B) = \frac{\sum_{j=1}^n w_j (\zeta_A(x_j) \cdot \zeta_B(x_j) + \eta_A(x_j) \cdot \eta_B(x_j) + \sigma_A(x_j) \cdot \sigma_B(x_j))}{\max(\sum_{j=1}^n w_j (\zeta_A^2(x_j) + \eta_A^2(x_j) + \sigma_A^2(x_j)), \sum_{j=1}^n w_j (\zeta_B^2(x_j) + \eta_B^2(x_j) + \sigma_B^2(x_j)))} \quad (2.2)$$

If each element $x_j \in X$ has the same importance, then 2.2 is reduced to 2.1. Larger value of $s(A,B)$, represents the more similarity between A and B .

Chapter 3

Mathematical model for Intuitionistic Fuzzy assignment Problem

Let there are n persons and n jobs. Each job is to be done by exactly one person the problem is to assign the persons to the jobs so that the total cost of completing all jobs becomes minimum. The cost of person i doing the job j is considered as an Intuitionistic fuzzy number $\tilde{c}_{ij} = (\zeta_{ij}, \eta_{ij}), i, j = 1, 2, 3$. Let us now formulate models as follows:

Model 1:

$$\text{Min } z = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij} x_{ij} \quad (3.1)$$

Subject to

If each person must be assigned exactly one job.

$$\sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, \dots, n \quad (3.2)$$

If each job must be done by exactly one person.

$$\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n \quad (3.3)$$

If we have restriction on the maximum IF cost that can be spent for the job j .

$$\tilde{c}_{ij} x_{ij} \leq \tilde{c}_j, \quad i, j = 1, 2, \dots, n \quad (3.4)$$

If we have restriction on the maximum Fuzzy cost that can be offered to the i th person depending on his/her efficiency/qualification.

$$\tilde{c}_{ij}x_{ij} \leq \tilde{c}_{pi}, \quad i, j = 1, 2, \dots, n \quad (3.5)$$

$$x_{ij} = 0 \text{ or } 1, \quad i, j = 1, 2, \dots, n \quad (3.6)$$

In real situations it may not be possible to know the exact value of cost. In such an uncertain situation, instead of exact values of cost, if one knows the preference for assigning the j th job to the i th person in the form of composite relative degree (d_{ij}) of similarity to positive ideal solution, c_{ij} can be replaced by d_{ij} and can be solved by Hungarian method or by any software to get the optimal assignment.

In that case model becomes

Model 2:

$$Max \ z = \sum_{i=1}^n \sum_{j=1}^n \tilde{d}_{ij}x_{ij} \quad (3.7)$$

subject to Eqs. (3.1 – 3.6)

Model 3:

If we replace \tilde{c}_{ij} by $\tilde{c}_{ij} = (\zeta_{ij}, \eta_{ij})$ then Eq.(3.1) becomes

$$Min \ z = \sum_{i=1}^n \sum_{j=1}^n (\zeta_{ij}, \eta_{ij})x_{ij} \quad (3.8)$$

Our objective is to maximize acceptance degree ζ_{ij} and to minimize rejection degree η_{ij} . So the objective function (3.8) becomes

$$Max \ z = \sum_{i=1}^n \sum_{j=1}^n \zeta_{ij}x_{ij} \quad (3.9)$$

$$Min \ z = \sum_{i=1}^n \sum_{j=1}^n \eta_{ij}x_{ij} \quad (3.10)$$

subject to

$$(\zeta_{ij} + \eta_{ij} - 1)x_{ij} \leq 0 \quad (3.11)$$

$$\zeta_{ij}x_{ij} \geq \eta_{ij}x_{ij} \quad (3.12)$$

$$\eta_{ij}x_{ij} \geq 0 \quad (3.13)$$

along with the Eqns. (3.2)-(3.8)

Model 4:

$$Max \ z = \sum_{i=1}^n \sum_{j=1}^n (\zeta_{ij} - \eta_{ij})x_{ij} \quad (3.14)$$

subject to conditions: (3.2)-(3.6), and (3.11)-(3.13).

Chapter 4

Algorithms to find Solution of Intuitionistic Fuzzy assignment Problem

The procedure for judging the existence of the solution as well as for finding the decision matrix of the assignment problem with restriction of job cost and person cost based on their efficiency/qualification is described in Algorithm 1.

Algorithm 1

Step 1: Firstly, establish the judging matrix $A=(a_{ij})_{n \times n}$ considering the restriction on jobs such that

$$a_{ij} = \begin{cases} 1, & \text{if } \tilde{c}_{ij} \leq \tilde{c}_j \\ 0, & \tilde{c}_{ij} > \tilde{c}_j \end{cases} \text{ by using the procedure for ranking intuitionistic fuzzy numbers by using}$$

$$S(\tilde{a}) = \zeta_1 - \eta_1$$

$$H(\tilde{a}) = \zeta_1 + \eta_1$$

Then establish the judging matrix $B = (b_{ij})_{n \times n}$ considering the restriction on person-cost such that

$$b_{ij} = \begin{cases} 1, & \text{if } \tilde{c}_{ij} \leq \tilde{c}_{pi} \\ 0, & \tilde{c}_{ij} > \tilde{c}_{pi} \end{cases} \text{ by using the procedure for ranking intuitionistic fuzzy numbers by using}$$

$$S(\tilde{a}) = \zeta_1 - \eta_1$$

$$H(\tilde{a}) = \zeta_1 + \eta_1$$

Step 2: Then form the composite judging matrix $\text{Comp}(AB) = (a_{ij}b_{ij})$, where $a_{ij}b_{ij}$ is the product of the corresponding elements of the matrices A and B.

Step 3: Then calculate the number of independent 1s in judging matrix $\text{Comp}(AB)$, and denote it as k.

If $k < n$, the problem has no solution, stop;

if $k = n$, go to step 4.

Step 4: From the composite judging matrix $\text{Comp}(AB)$ and cost matrix $(\tilde{c}_{ij})_{n \times n}$, and by

$$r_{ij} = \begin{cases} \tilde{c}_{ij}, & \text{if } a_{ij} = 1 \\ -, & a_{ij} = 0 \end{cases} \quad \text{establish the decision matrix } R = (r_{ij})_{n \times n}.$$

Step 5: Find the optimal solution of assignment problem with cost/profit (acc. to the case) matrix $R = (r_{ij})_{n \times n}$ by using Algorithm 2.

Step 6: End.

After finding the decision matrix $R = (r_{ij})_{n \times n}$, it is considered to be the cost matrix for the given assignment problem. But it cannot be solved by the Hungarian method, since the elements of this matrix are in the form of IFNs. So, the concept of composite relative degree of similarity to PIIFS can be applied and this IFAP with the cost matrix as $R = (r_{ij})_{n \times n}$ can be solved by using Algorithm 2.

Algorithm 2

Input: Cost matrix with the data being IFN.

Output: Profit matrix with data being the composite relative degree of similarity to the positive ideal solution, representing the preference or suitability to offer jth job to the ith person or that the ith person is chosen for performing the jth job and hence the optimal assignment. At first the relative degree of similarity for the

jobs with respect to each person are evaluated by applying the concept of similarity measures of IFSSs for solving Intuitionistic Fuzzy Multi-Attribute Decision-Making.

The data of the first column of the assignment(cost)matrix are considered initially.

Step1: Let $\pi_{A_i}(C) = 1 - \zeta_{A_i}(C) - \eta_{A_i}(C)$, for all $i=1,2,3,\dots,m$. Determine the positive-ideal and negative-ideal solution based on intuitionistic fuzzy numbers, defined as follows, respectively:

$$A^+ = \langle \zeta_{A^+}(C), \eta_{A^+}(C) \rangle \quad (4.1)$$

and

$$A^- = \langle C, \zeta_{A^-}(C), \eta_{A^-}(C) \rangle \quad (4.2)$$

where

$$\zeta_{A^+}(C) = \max_i \zeta_{A_i}(C) \quad \eta_{A^+}(C) = \min_i \eta_{A_i}(C), \quad (4.3)$$

$$\zeta_{A^-}(C) = \max_i \zeta_{A_i}(C) \quad \eta_{A^-}(C) = \min_i \eta_{A_i}(C), \quad (4.4)$$

Step2:Based on the 2.1, the following similarity measures A_i of IFSSs have been defined. Calculate the degree of similarity of the positive ideal IFS A^+ and the alternative, and the degree of similarity of the negative ideal IFS A^- and the alternative A_i , using the following equations respectively. The degree of similarity of each alternative A_i and the positive ideal IFS A^+ is defined as:

$$s(A^+, A_i) = \frac{(\zeta_{A^+}(C) \cdot \zeta_{A_i}(C) + \eta_{A^+}(C) \cdot \eta_{A_i}(C) + \sigma_{A^+}(C) \cdot \sigma_{A_i}(C))}{\max(\zeta_{A^+}^2(C) + \eta_{A^+}^2(C) + \sigma_{A^+}^2(C), (\zeta_{A_i}^2(C) + \eta_{A_i}^2(C) + \sigma_{A_i}^2(C))} \quad (4.5)$$

$i = 1,2,\dots,n; j = 1,2,3,\dots,n$. Similarly, degree of similarity of each alternative A_i and the negative ideal IFS A^- is defined as:

$$s(A^-, A_i) = \frac{(\zeta_{A^-}(C) \cdot \zeta_{A_i}(C) + \eta_{A^-}(C) \cdot \eta_{A_i}(C) + \sigma_{A^-}(C) \cdot \sigma_{A_i}(C))}{\max(\zeta_{A^-}^2(C) + \eta_{A^-}^2(C) + \sigma_{A^-}^2(C), (\zeta_{A_i}^2(C) + \eta_{A_i}^2(C) + \sigma_{A_i}^2(C))} \quad (4.6)$$

$i = 1, 2, \dots, n; j = 1, 2, 3, \dots, n.$

Step 3: Based on (4.5) and (4.6) calculate the relative similarity measure d_i corresponding to the alternative A_i as:

$$d_i = \frac{s(A^+, A_i)}{s(A^+, A_i) + s(A^-, A_i)}, \quad i = 1, 2, 3, \dots, n. \quad (4.7)$$

Larger value of d_i , represents A_i is more similar to the positive ideal IFS A^+ and hence A_i is the better Alternative.

Step 4: Repeat Step 1 to Step 3 for the rest of the columns of the cost matrix and find the relative similarity measure d_i corresponding to the alternative A_i for these columns i.e. for the jobs with respect to the persons.

Step 5: With these relative similarity measure d_j of the jobs with respect to the persons, form the matrix R_1 where $[R_1] = [p_{ij}]_{n \times n}$, p_{ij} is the relative similarity measure representing how much the j th person prefers the i th job by considering all the intuitionistic fuzzy attributes. We put $\epsilon > 0$, a very small number (degree of similarity) in the positions of the matrix R_1 to denote the situation that the j th person cannot be assigned to the i th job for these positions, if the data in the original problem considers that option.

Step 6: Now find the relative similarity measure d_i for the persons with respect to each job. Consider the data of the first row of the cost matrix. Repeat Step 1 to Step 3 for this row and also the rest of the rows of the cost matrix and find the relative similarity measure for these rows i.e. the relative similarity measure of the persons with respect to the jobs.

Step 7: With these relative similarity measure d_i of the persons with respect to the jobs, form the matrix R_2 where $[R_2] = [q_{ij}]_{n \times n}$, q_{ij} is the relative similarity measure representing how much the i th job is suitable for the j th person considering all the

IF attributes. Put $\epsilon > 0$, a very small number (degree of similarity) in the positions of R_2 to denote the situation that the j th person cannot be assigned to the i th job for these positions, if the original problem considers this case.

Step 8: Then form the composite matrix $\text{Comp}(R_1 R_2) = (p_{ij} q_{ij})_{n \times n} = (d_{ij})_{n \times n}$ whose elements are the composite relative degree of similarity representing the preference or suitability to offer the i th job to the j th person or that the j th person is chosen for performing the i th job.

Step 9: Then considering this matrix $\text{Comp}(R_1 R_2)$ as the initial table for an assignment problem in the maximization form (Model 2), it is solved by Hungarian method or by any standard software to find the optimal assignment which maximizes the total composite relative degree of similarity.

Step 10: End.

Thus by using the above algorithm the more realistic Intuitionistic Fuzzy Assignment Problem (IFAP) can be solved.

4.1 Solution procedure by using the score function of IF costs

Algorithm 3

Input: Cost matrix with the data given in the form of IFN.

Output: Optimal assignment.

Step 1: Find the score function matrix of the given cost matrix with data in the form of IFN.

Step2: Considering this score function matrix as the profit matrix in the maximization form, solve it by Hungarian Method or by any standard software to find the optimal assignment.

References:

(1) Sathi Mukherjee, Kajla Basu, Solution of a class of Intuitionistic Fuzzy Assignment Problem by using Similarity measures, Knowledge-Based Systems (27) 2012, 170-179.