Redundancy Resolution of Kinematic Manipulator with High Degree of Freedom Using Task Priority Method

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CERTIFICATE

I hereby certify that the work being presented in the dissertation entitled "Redundancy Resolution of Kinematic Manipulator with High Degree of Freedom Using Task Priority Method" in partial fulfillment of the requirement of the award of the Degree of master of technology and submitted to the Department of Mechanical Engineering of Lovely Professional University, Phagwara, is an authentic record of my own work carried out under the supervision of Himanshu Arora, Assistant Professor, Department of Mechanical Engineering, Lovely Professional University. The matter embodied in this dissertation has not been submitted in part or full to any other University or Institute for the award of any degree.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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"Robotic and other combinations will make the world pretty fantastic compared with today."

-Bill Gates

Apart from the efforts of me, the success of this project depends largely on the encouragement and guidelines of many others. I take this opportunity to express my gratitude to all people who have extended their co-operation in various ways in doing successful completion of this project.

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ABSTRACT

Kinematics modeling of a robot manipulator is the outcome of geometry of motions of a robot arm with respect to the coordinate system which is considered as a fixed frame. There exists always a challenge while solving the problem of inverse kinematics because many solutions are available for the same problem. This work focused on obstacle and singularity avoidance of robot manipulator using task priority method.

Task priority relating to the inverse kinematic model of the redundant robot manipulator. The task is partitioned into number of subtasks as per order of priority. Various mathematical simulation is performed for various compound condition consisting of obstacles with an offered direction to demonstrate the adequacy of the redundancy control structure for obstacle and singularity avoidance. In this thesis a snake comparable behavior of redundant manipulators, which track the trajectory in narrow "L" shape tube channel. The redundant manipulators are significant in numerous challenging applications like, to check the jam of sewerage pipes, welding of pipe lines, under water welding in a narrow tubes or tanks, to perform laparoscopy operation inside a human body etc.

1. INTRODUCTION

The origin of the robot word is came from Robota, which means work in Czech. A robot is a reprogrammable multi-functional controller intended to move materials, parts, tools, or specialized devices, through variable programmed motions for execution of a tasks.

1.1 Robotics

Robotics is the interdisciplinary branch of engineering and science that incorporates mechanical designing, electrical building, software engineering, and others. Robotics manages the design, development, operation, and utilization of robots, as well as computer systems for their control, sensory feedback, and data processing. In 1942, The three laws for Robotics are given a scientist Isaac Asimov. Which are the following, 1] The human being should not be injure or allow a human being into harm through inaction by Robot. 2] The commands which are given by human being, a robot should obey them unless they are conflict with the first law .3] A robot should protect its own existence if such protection does not conflict with the First or Second Laws. A robot manipulator can be considered as an open loop articulated chain by means of numerous linkages associated in arrangement by either revolute or prismatic joints driven by various actuators. One of the end serial chain manipulator is linked to a fixed base whereas the other end is unrestricted and attached through an end-effector to operate entities or execute given operations.

1.2 Robot Kinematics

The kinematics is branch of mechanics which examines the motion of objects without considering to the forces that causes the motion. The analytical study of the geometry of movement of a robot arm is dealt by Robot arm kinematics against a fixed reference coordinate system as a function of time with no regard to the force/moment that is the reason behind the motion. Robotic manipulator is described by its kinematic equation which relates the joint configuration of the manipulator to the position and orientation of the end effector in the workspace. To solve foundational kinematics problems the robotics community has focused on proficiently applying different representation of position and direction and their derivatives with respect to time. Kinematic problems are of two different forms, they can be definite as forward kinematics and inverse kinematics.

1.2.1 Forward kinematics

By using forward kinematics for a given joint Parameters and the geometric link constraints of a manipulator, the position and alignment of the end effector of the robot arm with respect to a reference coordinate frame can be find.

1.2.2 Inverse kinematics

Inverse kinematics is for a given position and alignment of the end effector of the manipulator, finding the geometric link and joint factors with respect to a reference coordinate frame, so that the manipulator can reach the wanted position and orientation.

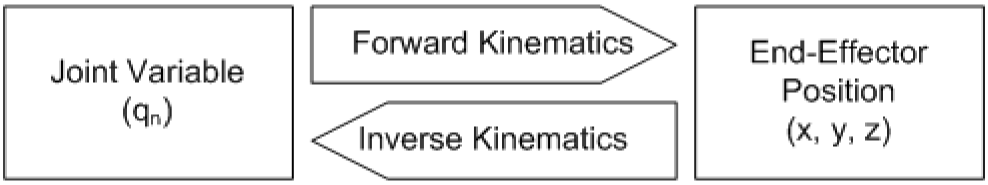

Figure 1.1 schematic representing of forward kinematics and inverse kinematics

1.3 Redundant Manipulator

Kinematic redundancy happens when a robot arm has a greater number of degrees of freedom than those entirely essential to accomplish a given task. A manipulator with at least seven joints is a distinctive example of an intrinsically redundant manipulator. Still, some robot manipulator with lesser degrees of freedom may become kinematically redundant for particular tasks, like conventional six-joint industrial robotic arms, which is simple end-effector positioning with no constraints on the direction. While avoiding obstacles in the workspace, a larger number of degrees of freedom are needed when a robotic arm is required to trace a given path of the end effector, than in a free working space. Redundancy is a positive feature particularly when trying to overcome difficulties related to an overfilled workspace or limitations due to mechanical constraints. With the help of kinematic redundancy size of the workspace could be increase and the manipulator could reach at every point without being in a singular arrangement for obtaining general motion path tracking tasks. The method of solving kinematics of non-redundant robots is by obtaining analytical solutions for a number of manipulator configurations. It is predicted that the inverse kinematics has infinite solutions in case when a manipulator is having redundant link. The kinematic redundancy resolution can be determined in two-way redundancy resolution via optimization and redundancy resolution via task-priority. In redundancy resolution via optimization there are two basic approaches one is local optimization approaches and the other is global optimization. The pseudoinverse solution is the simplest form of local optimization, which provides the joint velocity with the minimum standard among those that realize the task constraint. Global optimization has a benefit which is its simplicity because of the redundancy resolution scheme. By including a quadratic form in the joint accelerations or velocities inside the integral, problem can be solved but this can be done more easily at the second-order kinematic level. Augmenting the task vector is another approach for resolving redundancy to undertake additional objectives conveyed as constraints. A redundant robotic arm having more degrees of freedom is much appropriate to a multiple criteria problem, for example singularity avoidance and obstacle avoidance.

1.4 Importance of Redundancy Resolution

Past the manipulator are designed, which are categorized by designing the manipulator with the minimum number of joints and links required to perform the given task that give rise to a serious restriction in real worlds application. These restrictions are due to some difficulties as joint limits, singularity problem and obstacles in workspace. There restrictions lead in increasing the regions to be avoided in the joint and task workspace during the operation, therefore these manipulators requires a sensibly planned task space.

When a manipulator is designed with large degrees of freedom brings the flexibility to avoid the restrictions as specified above. This flexibility is expected to have infinite joint designs for the same manipulator posture since there are an infinite number of possible solutions for the reverse kinematics problem of the redundant manipulator. Thus, there exists joint motion which can be circulated in the null space of the Jacobian matrix of manipulator without influencing the end effector posture.

In this way, redundancy can be advantageously exploited to accomplish more dexterous robot motion. Formally, a functional complex task is obliged to be fulfilled along the end-effector task. Such typical limitations include obstacle avoidance, singularity avoidance, manipulability measures and limited joint range. In practice, if the increased dexterity of kinematically redundant manipulator may permit the manipulator to avoid singularities, obstacles and joint limits but also to minimize torque over a specified task, eventually significance that the robot arm can achieve more degree of freedom.

1.4.1 Obstacle Avoidance using Task Priority

The indirect kinematic problem is quote compelling in case of redundant manipulator since it concedes infinite solutions. The indirect kinematic procedures have been properly extended to redundant manipulators by accepting a task space augmentation technique. A task priority plan has been applied to avoid the problem of obstacle restraint task specification rendering to which the task which is having lower importance is implemented only if it is not struggling with the task which is having higher importance. This can be achieved by projecting the limitation Jacobian against the null space of the end effector Jacobian. The method has been implemented to redundancy on a free-floating spacecraft for space robotic arm.

1.4.2 Singularity Avoidance using Task Priority

Singularity regions correspond to robot configurations that are close to a singular configuration, and in which the joint velocities essential to accomplish the end effector movement in specific ways are to a great degree high. Therefore, it becomes significant for a general-purpose robot arm to have at least six DOFs. At the point when the robot is in singular configuration, there is no less than one direction in which the end effector velocity is can't move. The joint velocities essential to attain the end effector velocity component in this direction. Thus, random directional change of the end effector becomes more problematic. Numerically, this happens in case when the number of Jacobian matrix rows is more than the rank of the Jacobian matrix. There are two sorts of singularities,

1. The end effector loses all its mobility at its singular configurations these are kinematic singularities
2. Algorithmic singularities are those singularities at which the end effector task and the limitation task contrast despite the redundant DOFs.

1.4.3 Torque Minimization

Because of infinite solution of indirect kinematic problem, the solution should be picked with the goal that the torque requires to move the actuator is minimum. The following solution is chosen about the pervious one and in this way, goes for limiting the torque of the actuator.

1.4.4 Minimum Movement

To achieve the chosen position the solution is taken in such that there is minimum movement of the manipulator. It additionally points in choosing those solutions which involve minor axis movements than major axis since the mobility of a robot depends on the direction of the end effector motion desired which is signified by the manipulator velocity ratio ellipsoid. The minor axis of the manipulator velocity ratio ellipsoid signifies the minimum value of manipulator velocity ratio and the upper limit on the joint velocities necessary for the end effector to move in all direction. Therefore, for singular configuration a small minor axis indicates low flexibility and proximity.

1.4.5 Flexibility and Versatility

The redundancy of robot manipulator assumes a vital part in increasing the flexibility and versatility. Flexibility is essential with the goal that the robot can move toward any path unreservedly. It is the capacity of the manipulator to alter the direction of end effector motion. Thus, at a singular configuration or when it is close, the robot manipulator will have less flexible.

1.5 Applications of Redundant Manipulators

- Redundant Manipulators are used in inspection inside nuclear plants.
- Redundant robots are used in industries for automation.
- Used to perform complicated tasks like welding operation, handling, assembly, spray painting.
- Snake link redundant manipulator used in performing tasks in complicated areas

Figure 1.2 Robots used for welding operation

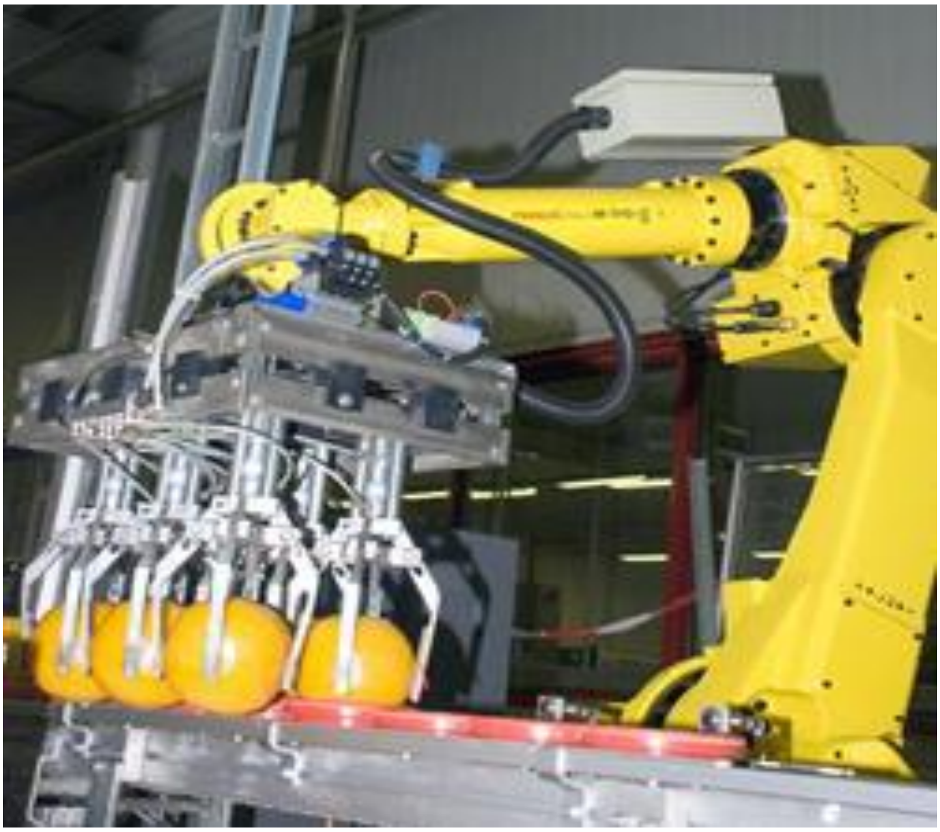
Figure 1.3 Robots used for handling operation

Figure 1.4 Robots used in assembly

Figure 1.5 Snake arm robots performing its task

1.6 Research Methodology

This thesis work has been organized as stated. In chapter 2 it has been discussed about the work/study done by researches through literature survey. Authors started using generalized inverse of Jacobian matrix to calculate the answer of joint velocity when the idea of active utilization of redundancy came in their considerations. Different algorithms have been developed to utilize the redundancy and incorporating it for task priority, obstacle avoidance, singularity avoidance, torque minimization, etc. Finally, a systematic framework of the problems of redundancy resolution is proposed.

In chapter 3, Euler-Langrange model has been used for explaining the dynamic model for the equation of motion for robot arm manipulator. Then, to find the symmetric, positive-definite mass inertia matrices, and the cariolis and centrifugal force vector for each manipulator, this approach is analytically applied to the multiple links of manipulator.

The kinematic control related to redundancy resolution as well as task priority is to be explained. To calculate the motion rate-control, the problem is to be formulated in a framework. This task will also show the value of Jacobian matrix, concept of task priority and manipulable together with the formation of equations related to it.

The efficiency of the formulation and Potential function is to be numerically simulated. Potential function is to be incorporated to develop the algorithm for obstacle avoidance and singularity avoidance using task priority. Different results for obstacle and singularity avoidance of redundant manipulator is to be coded in MATLAB using these algorithms for an 8-link manipulator which tracks 'L' shape trajectory.

1.7 Objectives of This Work

The objectives of this work are following: -

- Development of equation of motion to an 8-link manipulator which tracks 'L' shape trajectory.
- Mainly focused on obstacle avoidance and singularity avoidance, by creating a 'L' shaped narrow tube obstacles.
- This type of manipulators is used in Bio-Medical operations, underwater welding in narrow tube, welding of joint in narrow channel sections, etc.

2. LITERATURE SURVEY

2.1 Review of methods used in Kinematic Modeling

2.1.1 Forward Kinematics Problems

The forward kinematics problems are explained with the transformation matrices calculated between a coordinate frame fixed in the end effector and another coordinate frame fixed in the base (reference frame). The position vector is signified by the homogeneous transformation matrix in a 3-D space along with the rotation matrix of the body. The overall homogeneous transformation matrix is derived by simply multiplying transformations of different frames fixed in contiguous links of the chain. **Denavit and Hartenberg [1]** were the first to introduce this determination for the spatial geometric representation of a manipulator, and its benefit is in the universal algorithm to solve the kinematics of a manipulator.

2.1.2 Inverse Kinematics Problems

Indirect kinematics problems can be resolved by using two types of approaches first one is closed form solutions and second is numerical approach. Closed form solutions are robot dependent and faster than the numerical approach. This approach classified into two type of methods Analytical method **[2]** and Geometric method **[3]**. Analytical method it is also called as algebraic method, analytically invert the direct kinematics equations. The problem of inverse kinematics can be summarized by solving a system of algebraic equations. The robot kinematics leads to an algebraic system or a set of equations, when the trigonometric functions are evaded by fixed replacements. Geometric method identifies the point on the manipulator comparative to either position or both position and orientation can be defined as a function of the joint variables. This Geometric method changes spatial problem into separate planar problems. Then algebraic method is used to solve these equations that are obtained. The numerical approach method can be applied to any kinematic arrangement because it is not robot dependent. It is slower and in some case, it is not possible to compute the solution. In this approach, there are different methods like symbolic elimination method **[4]**, continuation method **[5]** and iterative method. Symbolic elimination method gives a set of nonlinear equations by eliminating variables to shorten it into a smaller set of equations. Different number of iterative methods are using now a day to resolve the inverse kinematics problems. Like Newton-Raphson method **[6]**, Optimization approach **[7,8]**, Cyclic coordinate descent method **[9]**, Pseudoinverse method **[10]**, Jacobine transpose methods **[11]**, the Levenberg-Marquardt damped least squares method **[12]**, Quasi newton and conjugate gradient methods **[13,14]** and Neural net and artificial intelligence methods **[15,16,17,18].**

2.2 Redundancy Resolution

In the addition of more degree of freedom to form a redundant arm that overcomes the restrictions founded in non-redundant robotic arm manipulator is the outcome of researchers and development since seventies in the area of robotics. By minimizing the kinetic energy of the manipulator **Whitney [19]** resolved the redundancy at the velocity level and he was one of the first researchers. The pseudo inverse control finds a minimum average solution but has the disadvantage of not being conservative for repetitive motions. **Sung-Woo Kim [20]** proposed an approach of optimal kinematic control. By using the necessary conditions of optimal control to obtain the globally optimal resolutions resolution of redundancy, this global optimal solution is obtained when the redundancy resolution problem to an optimal control problem. The main ideas are those that the kinematic resolution problem is considered as an optimal control problem, and from the necessary conditions of the optimal control the redundancy is resolved.

Joseph Wunderlich [21] presented a method of designing redundant and hyper-redundant manipulators for enclosed workspace, these manipulators dedicated to work within enclosed workspaces like welding, grinding, and spray-painting in assembly-line tasks. A variation on task-priority based redundancy control is also presented which allows many secondary-priority obstacle avoidance tasks to be satisfied simultaneously, therefore permitting easier movement through complex enclosures.

2.2.1 Task-Priority Method

Sometimes in each task for an end effector one thing, the position and orientation are more important than the other. In some operations the orientation of the end effector is less important than the position in tasks like cutting operations, welding operation, and dimension tools. But, in some operation like directing a camera to objects, spray painting, position is less important than the orientation. When a redundant manipulator is requested to avoid obstacle on its path and trace, a given trajectory for an end effector. Therefore, is trajectory tracing is the first priority and obstacle avoidance is the second priority and also for the avoiding singularity of the robot arm manipulator, trajectory tracing is given first priority and singularity is given second priority. The idea of task priority is stated by a famous robotic researcher **Yoshihiko Nakamura [22]** in relative to the indirect kinematic problem of redundant robot manipulators. In task priority the required task is split into subtasks based on their order of priority as discussed above. **Fabrizio Flacco [23]** given a new approach which is based on separating the redundancy resolution form tasks state resolution. In this the first part, all the tasks are considered with an equal priority, and the contribution of each one is computed. And used a task priority matrix for enforcing the correct task priority order.

Bruno Siciliano [24] A new method of closed loop schemes for solving the indirect kinematics of constrained redundant manipulator. The author presented that how the end effector task can be properly improved with the constraint task, and a solution based on Jacobian transpose can be developed. Priority is given to the end effector and the convergence of the constraint error have been proved via a Lya-punov argument. **Hsien-I Lin [25]** stated that by using a robot having same degree of freedom or device like haptics device to achieve the goal of an end effector to track a specified trajectory in inverse kinematic control of a redundant robot arm. To improve this kinematic control author proposed a novel method by which the control of manipulator, directly by computer simulations and human motion. A kinematic-control method is proposed to validate the redundant robot arm.

2.3 Obstacle Avoidance

An iterative solution method is presented by **Andreas Muller [26]** for the reverse kinematics of redundant serial manipulators that avoids collision of obstacles. A predictor-perturbation-corrector algorithm accomplished with inverse kinematics while the manipulator end effector is tracking a described trajectory. This predictor achieves geometric tracking of described end effector position, Perturbation adjusts the manipulator posture away from obstacles and the corrector emends the perturbed configuration in accordance with the target end effector position. Author showed results for planar 5R and spatial 10R manipulator.

In many actual situations the kinematic boundaries that lead to very low navigation performance. **Jose-Luis Blanco [27]** Author proposed an outline of controlling the manipulator kinematically and a free-flying point in new workspace is transformed for any shape robots. Most of the present available transformational methods is covered by a generalized space transformation, and a reactive navigation system to optimize the robot arm motion by multiple transformation processed simultaneously, the present available obstacle avoidance process is performed better detection by these above transformations. **Jose-Luis Blanco [27]** give an experimental result that reveals the advantages of this method.

O. Khatib [28] Used a powerful principle and simple method that has a fixed obstacle avoidance capability and the method is potential fields method. In this potential field method, a manipulator is considered to be as a particle that moves in a potential field is generated for the trajectory and for the obstacle with is present in the workspace. An attractive potential is generated for the trajectory in filed and a repulsive potential is generated for each obstacle in the work filed. Obstacles are either a priori known or an on-board sensor used for on-line detected and hence the repulsive potential generated in filed is on-line estimated. This potential field method is not only useful for the obstacle avoidance, this also defines the velocity vector for the manipulator for driving it to the trajectory while avoiding obstacles by a potential field planning method.

The dual neural network is proposed by **Yunong Zhang and Jun Wang [29]** for the online solution of kinematically redundant manipulator to impact free indirect kinematics problem. An improved problem design is proposed in such a way that the obstacle avoidance necessity is signified by dynamically updated physical constraints and inequality constraints can be formulated directly such as joint physical limits. **Yunong Zhang and Jun Wang [29]** explained this method using PA10 robot arm with a motion control simulation in the presence of point and window shaped obstacle. **Mihai Duguleana [30]** also worked in a solution that is based this process that uses Q-learning reinforcement technique this means for attaining computing trajectory planning problems or obstacle free navigation manipulator. **Yunong Zhang and Jun Wang [29]** use neural networks process for solving indirect kinematics with less obstacles like only one or two are present in the manipulator workspace, and next focused in calculating indirect kinematics for obstacle avoidance of complex manipulators in an unknown environment workspace with multiple obstacles in workspace.

2.4 Singularities Avoidance

For the avoidance of singularity in selective compliance assembly robot arm (SCARA) robot is used by **Leon Beiner [31]** to present a method to avoid singularities by using optimal work position. selective compliance assembly robot arm robot in several specified distinct locations on the manipulator workspace that performs the given task. To minimize a Jacobian related a graphical suboptimal solution, a numerical solution for workpieces of arbitrary from, an analytical solution for symmetrical workpieces, and cost function subjected to workspace boundary conditions, and are presented and explained by SCARA robots.

Samer Yahya [32] designed a singularity avoiding three-dimensional planar manipulator, without increasing the control of motors but by increasing the degrees of freedom. The PUMA arm robot manipulability ellipsoids are related with the proposed manipulator have been obtained. The manipulability measure values of both PUMA and proposed manipulators have been analyzed and calculated. Author concluded that form both of manipulator the proposed manipulator is more ability to be used for singularity avoidance as that of PUMA.

Giacomo Marani [33] Described a solution for kinematic controlled manipulator based on task priority for avoiding the presence of both kinematic singularities and algorithmic singularities. In task priority method, the Algorithmic singularities avoidance uses a successive task projection and a secondary task correction. Author concluded that in this procedure the exact pseudo-inversion is used and also stated that the measure of manipulability is never zero.

Stefano Chiaverini [34] developed a new task priority method that reduces the properties of the algorithmic singularities by reviewing the application of presently available singularity avoiding methods to the case of kinematically redundant arms. For a seven degree of freedom robotic arm this method is applied in numerical case studied to establish its usefulness.

3. DYNAMIC MODELING OF MANIPULATOR.

3.1 Introduction.

The set of numerical equations relating the dynamic behaviour are the dynamic equations of motion of a robot arm. A manipulator move at consistent speed, must accelerate, and decelerate amid the work cycle. The time varying torques are connected by the ten actuators to adjust the internal forces and external forces. The forces influenced by the environment are the external forces which include load and gravitational forces. The internal forces are caused by motion i.e. velocity and acceleration, of links e.g. Inertial, Coriolis, and Frictional forces. Links and joints have to resist stresses resulted by forces/torques balance.

In this chapter, the numerical model and properties of the dynamic equations of motion for the dynamic behaviour of robot arm is developed. Relationship between motion of links for simulation, joint actuator torques, and design of control algorithms are given by the dynamic behaviour of the manipulator. This dynamic equation of motion is considered for control purposes.

The dynamic model for a manipulator can be formulated by using known physical law such as law of Newtonian mechanics and Lagrangian mechanics. Following two approaches are used to develop a set of equation of motion for a manipulator which are Lagrange-Euler approach and Newton-Euler approach. Lagrange-Euler approach is based on the energy approach while Newton-Euler approach is based on forced balancing. Lagrange-Euler approach considers each term as scalar quantity e.g. kinetic energy, potential energy etc, while Newton-Euler approach consider each term as vector quantity e.g. displacement, velocity, acceleration etc. The Lagrange-Euler and Newton-Euler formulations of the dynamic model give a closed-form solution. These solutions are computationally intensive, making real time control based on such a dynamic model. There might be some variations in the structure of these equations as they are found for various explanations and purposes. Some are formulated to ease the control analysis, some are formulated to achieve better computation time and other are formulated to enhance computer simulation.

For the formulation of equation of motion for robot arm Lagrange-Euler approach is more simple, general and systematic as comparted to Newton-Euler approach. Thus, Lagrange-Euler approach is preferred for the formulation of dynamic model of manipulator in future work.

3.2 Lagrange-Euler Mechanics.

In physics Lagrangian mechanics is widely used to solve mechanical problems. In optimisation problems of dynamic systems Lagrange's equations are applied. The Lagrange function $'L'$ which is a scaler function is given by the difference between the total kinetic energy $'K'$ and the total potential energy $'P'$ of mechanical system.

$$L = K - P \tag{3.1}$$

To define the manipulator variables based on a set of generalized coordinates dynamic model formulation utilising Lagrange-Euler approach is used. In the generalized coordinates the joint variables are described as displacement 'q'. For prismatic joint which is defined as linear displacement $'d'$ and while for rotary joint defined as angular displacement $'\theta'$. The velocity for prismatic joint $'\dot{q}'$ describes linear velocity $'\dot{d}'$ and velocity for rotary joint $'\dot{q}'$ describes angular velocity '$\dot{\theta}'$.

To obtain the dynamic model for robot arm based on Lagrange-Euler approach is given by the Lagrangian, as a set of equations,

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_i}\right) - \frac{dL}{dq_i} = \tau_i \tag{3.2}$$

Here,

L = Lagrangian function.

q_i = Generalized coordinates of the manipulator.

$\dot{q}_i$ = Generalized coordinates of the joint velocity.

τ_i = Generalized force applied to the system at joint i.

The left-hand side of dynamic equations gives sum of the torques/forces produced because of kinetic and potential energy available in the system. While the right-hand side τ_i represent the joint torque for joint i that is produced by actuator i.

3.3 Lagrange-Euler Formulation

The Lagrange-Euler formulation is a systemic practice for deriving the dynamic model of an n degree of freedom (DOF) manipulator. The equation (3.2) creates the relation between the joint accelerations, positions, velocities and the generalized torques applied to the manipulator. The n degree of freedom open kinematic chain serial link manipulator has n joint position,

$$q = [q_1, \dots \dots q_i]^T. \tag{3.3}$$

Lagrange-Euler formulation which is discussed in equation (3.2) has the following features:

- It defines motion in real physical terms and is systematic.
- The equation of motion attained are analytical and compact.
- The matrix vector form of equations is alluring for calculations and control systems design.
- The control problem can be simplified by designing the structure of the manipulator with minimum joint connection that is coefficients and might be reduced or eliminated.
- The model is computationally intensive and is not acquiescent to online control.

The derivation of equation of motion utilizing Lagrange-Euler formulation is done in the accompanying subtopics. It makes use of link homogenous transformation matrices T, which are derived from the kinematic modeling. Initially, the link velocity is calculated and next the link inertia tensor is attained. These are employed to compute kinetic energy. Then potential energy is calculated and next the lagrangian is formed which is substituted in equation (3.2) to obtain dynamic model.

3.3.1 JOINT VELOCITY OF A POINT ON THE MANIPULATOR

For computing the kinetic energy of a link of an n- degree of freedom manipulator, link velocity is required.

In this subsection, the velocity of a point fixed in link i is derived and the effects of the motion of other joint on all the points in this link is discovered. Let's consider a point p on the link i of an n-degree of freedom manipulator as shown in fig (3.1). The coordinate frames, frame $\{0\}$, frame $\{i\text{-}1\}$, frame $\{i\}$ are chosen as per convention. The position vector r_i^i describes the point p on the link with respect to frame $\{i\}$.

$$r_i^i = [x_i \quad y_i \quad z_i \quad 1]^T \tag{3.4}$$

The position of point p with respect to frame $\{0\}$ is given by

$$r_i^0 = T_i^0 r_i^i \tag{3.5}$$

$$T_i^0 = (T_1^0 T_2^1 ....T_i^{i-1}) \tag{3.6}$$

T_i^0 is the coordinate homogeneous transformation matrix which relates the i^{th} coordinate frame to the base coordinate frame, and T_i^{i-1} is the homogeneous transformation matrix which relates the i^{th} coordinate frame with the $(i-1)^{th}$ coordinate frame.

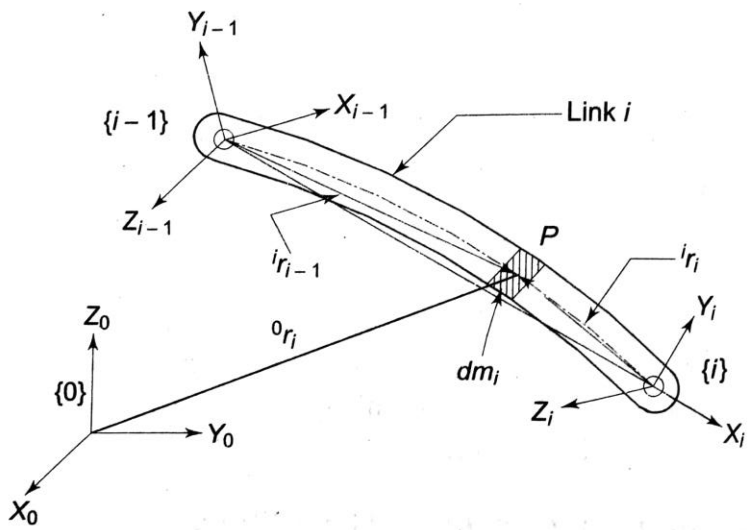

Fig 3.1 Joint velocity of a point on the manipulator

If joint i is prismatic, the homogeneous transformation matrix is given as

$$T_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\alpha_i \sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & \sin\alpha_i \cos\theta_i & 0 \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.7}$$

If joint i is rotary, the homogeneous transformation matrix is given as

$$T_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & \sin\alpha_i \cos\theta_i & a_i \cos\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.8)$$

Where $a_{i,}$ α_i are link parameters of robot arm and $d_{i,}$ θ_i are joint parameters of joint i. The partial derivate of above rotary homogeneous transformation matrix with respect to θ_i gives,

$$\frac{\partial T_i^{i-1}}{\partial \theta_i} = \begin{bmatrix} -\sin\theta_i & -\cos\theta_i cos\alpha_i & \sin\alpha_i \cos\theta_i & -a_i \sin\theta_i \\ \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.9)$$

The velocity of r_i^i expressed in the base coordinate frame can be expressed as

$$v_i^0 \equiv v_i = \frac{d}{dt}(r_i^0) = \frac{d}{dt}(\, T_i^0 \, r_i^i) \quad (3.10)$$

$$\frac{d}{dt}(r_i^0) = \left(\sum\nolimits_{j=1}^{i} \frac{\partial\, T_i^0}{\partial q_j} \dot{q}_J\right) r_i^i \quad (3.11)$$

When equation number (3.8) and (3.9) are compared they gives a pattern, that equation (3.9) can be obtained from equation (3.8) by using some matrix operations,

- Interchanging row 1 with row 2,
- Changing the sign of row 1, and
- Making row 3 and row 4 zero.

Hence by using above matrix operation can obtain the partial derivation of homogeneous transformation matrix T_i^{i-1} with respect to θ_i. Numerically these steps can be given by a 4×4 matrix Q_i. For revolute joint Q_i is defined as

$$Q_i = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.12}$$

For prismatic joint Q_i is defined as

$$Q_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.13}$$

and by premultiplying T_i^{i-1} with Q_i,

$$Q_i T_i^{i-1} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & \sin\alpha_j \cos\theta_i & a_i \cos\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.14}$$

$$Q_i T_i^{i-1} = \begin{bmatrix} -\sin\theta_i & -\cos\theta_i cos\alpha_i & \sin\alpha_i \cos\theta_i & -a_i \sin\theta_i \\ \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.15}$$

It is observed that equation (3.9) and (3.13) are same,

$$\frac{\partial T_i^{i-1}}{\partial \theta_i} = Q_i T_i^{i-1} \tag{3.16}$$

Since $T_i^0 = T_1^0 T_2^1 \ldots T_i^{i-1}$, therefore the partial derivative T_i^0 with respect to q_j,

$$\frac{\partial T_i^0}{\partial q_j} = T_1^0 T_2^1 \ldots\ldots T_j^{j-1} \frac{\partial T_j^{j-1}}{\partial q_j} T_{j+1}^j \ldots\ldots T_i^{i-1} \tag{3.17}$$

After simplifying equation (3.17), the result is valid for $j \le i$. Hence, for $i = 1,2,3 \ldots.., n$.

$$\frac{\partial T_i^0}{\partial q_j} = \begin{cases} T_{j-1}^0 Q_j T_i^{j-1} & for\ j \le i \\ 0 & for\ j \succ i \end{cases} \tag{3.18}$$

The link velocity v_i as given in equation (3.11), is simplified using equation (3.18),

$$v_i = \left(\sum_{j=1}^{i} T_{j-1}^0 Q_j T_i^{j-i} \dot{q}_J \right) r_i^i \tag{3.19}$$

3.3.2 THE INERTIA TENSOR OF THE MANIPULATOR

During the motion of links, the mass of links contributes inertia forces. All the inertial loads are reflected with respect to rotations about the origin of frame of interest, are represented by a motion of inertia tensor, which is one mass properties. It is a 4×4 symmetric matrix, which characterizes the distributions of mass of a rigid body. The moment of inertia tensor is given as

$$I_i = \begin{bmatrix} \int x_i^2 dm_i & \int x_i\, y_i\, dm_i & \int x_i\, z_i\, dm_i & \int x_i\, dm_i \\ \int x_i\, y_i\, dm_i & \int y_i^2 dm_i & \int y_i\, z_i\, dm_i & \int y_i\, dm_i \\ \int x_i\, z_i\, dm_i & \int y_i\, z_i\, dm_i & \int z_i^2 dm_i & \int z_i\, dm_i \\ \int x_i\, dm_i & \int y_i\, dm_i & \int z_i\, dm_i & \int dm_i \end{bmatrix} \tag{3.20}$$

Where dm_i is the mass of the element on link i located at $r_i^i = [x_i \quad y_i \quad z_i \quad 1]^T$.

$$I_i = \begin{bmatrix} 0.5(-I_{xx} + I_{yy} + I_{zz}) & I_{xy} & I_{xz} & m_i \overline{x_i} \\ I_{xy} & 0.5(I_{xx} - I_{yy} + I_{zz}) & I_{yz} & m_i \overline{y_i} \\ I_{xz} & I_{yz} & 0.5(I_{xx} + I_{yy} - I_{zz}) & m_i \overline{z_i} \\ m_i \overline{x_i} & m_i \overline{y_i} & m_i \overline{z_i} & m_i \end{bmatrix} \tag{3.21}$$

Where $m_i$is the mass of the i^{th} link and $\overline{r_i^i} = [\overline{x_i} \quad \overline{y_i} \quad \overline{z_i} \quad 1]$ is the center of mass. The moment of inertia tensor I_i for i^{th} link depends on the mass distribution of the link.

3.3.3 THE KINETIC ENERGY OF THE MANIPULATOR

The kinetic energy of the differential mass dm_i on link i,for $i = 1,2,3 \ldots \ldots, n$ located at r_i^0 and moving with velocity $v_i^0$with respect to the base frame $\{0\}$ is,

$$dk_i = \frac{1}{2} \, dm_i (v_i)^2 \tag{3.22}$$

$$(v_i)^2 = v_i . v_i = \dot{r_i^0} \dot{r_i^0} = Trace\left(\dot{r_i^0} \dot{r_i^0}^T\right) = Trace(v_i . {v_i}^T) \tag{3.23}$$

By substituting equation (3.19) in equation (3.22),

$$dk_i = \frac{1}{2} Trace\left[\left(\sum_{j=1}^{i} T_{j-1}^0 Q_j T_i^{j-i} \, \dot{q_J} \, r_i^i\right)\left(\sum_{k=1}^{i} T_{k-1}^0 Q_k T_i^{k-i} \, \dot{q_k} \, r_i^i\right)^T\right] dm_i \tag{3.24}$$

The total kinetic energy can be given by integration of equation (3.24),

$$k_i = \int dk_i \tag{3.25}$$

$$k_i = \frac{1}{2} Trace\left[\sum_{j=1}^{i} \sum_{k=1}^{i} \left(T_{j-1}^0 Q_j T_i^{j-i}\right) \int r_i^i {r_i^i}^T dm_i \left(T_{k-1}^0 Q_k T_i^{k-i}\right)^T \dot{q_J} \dot{q_k}\right] \tag{3.26}$$

Form equation (3.21), the term $\int r_i^i {r_i^i}^T dm_i$ of equation (3.26) is the moment of inertia tensor I_i, therefore the above equation (3.26) is ,

$$k_i = \frac{1}{2} Trace\left[\sum_{j=1}^{i} \sum_{k=1}^{i} \left(T_{j-1}^0 Q_j T_i^{j-i}\right) I_i \left(T_{k-1}^0 Q_k T_i^{k-i}\right)^T \dot{q_J} \dot{q_k}\right] \tag{3.27}$$

The total kinetic energy of manipulator for n- degree of freedom is,

$$K = \sum_{i=1}^{n} k_i$$
$$= \frac{1}{2}\sum_{i=1}^{n} Trace\left[\sum_{j=1}^{i}\sum_{k=1}^{i}\left(T_{j-1}^{0} Q_j T_i^{j-i}\right) I_i \left(T_{k-1}^{0} Q_k T_i^{k-i}\right)^T \dot{q}_j \dot{q}_k\right] \tag{3.28}$$

By simplifying equation (3.28),

$$K = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i} Trace\left[\left(T_{j-1}^{0} Q_j T_i^{j-i}\right) I_i \left(T_{k-1}^{0} Q_k T_i^{k-i}\right)^T\right] \dot{q}_j \dot{q}_k \tag{3.29}$$

$$K = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i} Trace\left[\left(U_{ij}\right) I_i \left(U_{ik}\right)^T\right] \dot{q}_j \dot{q}_k \tag{3.30}$$

3.3.4 THE POTENTIAL ENERGY OF THE MANIPULATOR

The potential energy for i^{th} link is p_i in gravity field g,

$$p_i = -m_i g\left(\overline{r_i^0}\right) = -m_i g T_i^0 (\overline{r_i^i}) \tag{3.31}$$

Here the negative sign represents the work is done on the system to raise link i against gravity. $\overline{r_i^i}$ represents the center of mass of i^{th} link with respect to the frame $\{i\}$, and $\overline{r_i^0}$ represents the center of mass of i^{th} link with respect to base frame $\{0\}$. And the acceleration due to gravity $g = \begin{bmatrix} g_x & g_y & g_z & 0 \end{bmatrix}^T$ is the gravity vector with respect to base frame $\{0\}$.

The total potential energy is given as,

$$p = \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} -m_i g T_i^0 (\overline{r_i^i}) \tag{3.32}$$

3.3.5 EQUATION OF MOTION OF THE MANIPULATOR

The equation of motion of the manipulator is formulated as following. After substituting kinetic energy equation (3.30) and potential energy equation (3.32) in Lagrange-Euler equation (3.1), $L = K - P$,

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i} Trace\left[\left(U_{ij}\right) I_i \left(U_{ik}\right)^T\right]\dot{q}_j\dot{q}_k - \left(\sum_{i=1}^{n} -m_i g T_i^0 (\overline{r_i^i})\right) \quad (3.33)$$

The generalized torque τ_i of actuator at joint i, as described in equation (3.2). by substituting above Lagrange-Euler equation (3.33) in equation (3.2), the final equation of motion is,

$$\tau_i = \sum_{j=1}^{n} M_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}\,\dot{q}_j\dot{q}_k + G_i \quad (3.34)$$

Where,

$$M_{ij} = \sum_{p=\max(i,j)}^{n} Trace\left[\left(U_{pj}\right) I_i \left(U_{pk}\right)^T\right] \quad (3.35)$$

$$h_{ijk} = \sum_{p=\max(i,j,k)}^{n} Trace\left[\frac{\partial(d_{pk})}{\partial q_p} I_p U_{pi}^T\right] \quad (3.36)$$

$$G_i = -\sum_{p=i}^{n} m_p g U_{pi}\left(\overline{r_p^p}\right) \quad (3.37)$$

$$U_{ij} = \begin{cases} T_{j-1}^0 Q_j T_i^{j-1} & for\ j \le i \\ 0 & for\ j \succ i \end{cases} \quad (3.38)$$

$$\frac{\partial U_{ij}}{\partial q_k} = \begin{cases} T_{j-1}^0 Q_j T_{k-1}^{j-1} Q_k T_i^{k-1} & for\ i \ge k \ge j \\ T_{k-1}^0 Q_k T_{j-1}^{k-1} Q_j T_i^{j-1} & for\ i \ge j \ge k \\ 0 & for\ i < j\ or\ i < k \end{cases} \quad (3.39)$$

The dynamic model for a manipulator is given in equation (3.34) which is a set of n nonlinear, coupled, second order ordinary differential equation for n degree of freedom manipulator.

3.4 DEVELOPMENT OF THE DYNAMIC MODEL FOR MULTIPLE DEGREES OF FREEDOM MANIPULATOR.

The following is the example to develop dynamic model using Lagrange-Euler equation for multiple link manipulator.

3.4.1 8-Link Manipulator

Let's consider an 8-link manipulator which is having all revolute joint. The physical dimensions of the links of the manipulator and the link parameter are given in table 3.1.

No. of links (i)	Link Masses $(m)(kg)$	Link Parameters				
		Initial Joint Angle $(\theta_i)(deg)$	Link Length $(a_i)(m)$	Joint Offset Distance $(d_i)(m)$	Twist Angle $(\alpha_i)(deg)$	Initial Joint Angle Velocity $(\dot{\theta}_i)(deg/min)$
1	20	75	0.6	0	0	1
2	17	35	0.5	0	0	1
3	20	-98	0.6	0	0	1
4	26	-26	0.5	0	0	1
5	20	13	0.5	0	0	1
6	15	-30	0.5	0	0	1
7	15	-75	0.5	0	0	1
8	15	43	0.5	0	0	1

Table no 3.1:- Physical dimensions and parameters of 8-link manipulator

To formulate dynamic model for 8 link manipulator as discussed in above section 3.3, now calculate the homogeneous transformation matrices using equation (3.8).

The homogeneous transformation matrix can be given by,

$$T_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & \sin\alpha_i \cos\theta_i & a_i \cos\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.40}$$

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \cos\alpha_1 & \sin\alpha_1 \sin\theta_1 & a_1 \cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 \cos\alpha_1 & \sin\alpha_1 \cos\theta_1 & a_1 \cos\theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.41}$$

$$T_1^0 = \begin{bmatrix} 0.9218 & 0.3878 & 0 & 0.5531 \\ -0.3878 & 0.9218 & 0 & -0.2327 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.42}$$

$$T_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \cos\alpha_2 & \sin\alpha_2 \sin\theta_2 & a_2 \cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 \cos\alpha_2 & \sin\alpha_2 \cos\theta_2 & a_2 \cos\theta_2 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.43}$$

$$T_2^1 = \begin{bmatrix} -0.9037 & 0.4282 & 0 & -0.4518 \\ -0.4282 & -0.9037 & 0 & -0.2141 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.44}$$

$$T_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 \cos\alpha_3 & \sin\alpha_3 \sin\theta_3 & a_3 \cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 \cos\alpha_3 & \sin\alpha_3 \cos\theta_3 & a_3 \cos\theta_3 \\ 0 & \sin\alpha_3 & \cos\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.45}$$

$$T_3^2 = \begin{bmatrix} -0.8193 & -0.5734 & 0 & -0.4916 \\ 0.5734 & -0.8193 & 0 & 0.3440 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.46}$$

$$T_4^3 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4\cos\alpha_4 & \sin\alpha_4\sin\theta_4 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4\cos\alpha_4 & \sin\alpha_4\cos\theta_4 & a_4\cos\theta_4 \\ 0 & \sin\alpha_4 & \cos\alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.47}$$

$$T_4^3 = \begin{bmatrix} 0.6469 & 0.7626 & 0 & 0.3235 \\ -0.7626 & 0.6469 & 0 & -0.3813 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.48}$$

$$T_5^4 = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5\cos\alpha_5 & \sin\alpha_5\sin\theta_5 & a_5\cos\theta_5 \\ \sin\theta_5 & \cos\theta_5\cos\alpha_5 & \sin\alpha_5\cos\theta_5 & a_5\cos\theta_5 \\ 0 & \sin\alpha_5 & \cos\alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.49}$$

$$T_5^4 = \begin{bmatrix} 0.9074 & -0.4202 & 0 & 0.4537 \\ 0.4202 & 0.9074 & 0 & 0.2101 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.50}$$

$$T_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6\cos\alpha_6 & \sin\alpha_6\sin\theta_6 & a_6\cos\theta_6 \\ \sin\theta_6 & \cos\theta_6\cos\alpha_6 & \sin\alpha_6\cos\theta_6 & a_6\cos\theta_6 \\ 0 & \sin\alpha_6 & \cos\alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.51}$$

$$T_6^5 = \begin{bmatrix} 0.1543 & -0.9880 & 0 & 0.0771 \\ 0.9880 & 0.1543 & 0 & 0.4940 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.52}$$

$$T_7^6 = \begin{bmatrix} \cos\theta_7 & -\sin\theta_7\cos\alpha_7 & \sin\alpha_7\sin\theta_7 & a_7\cos\theta_7 \\ \sin\theta_7 & \cos\theta_7\cos\alpha_7 & \sin\alpha_7\cos\theta_7 & a_7\cos\theta_7 \\ 0 & \sin\alpha_7 & \cos\alpha_7 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.53}$$

$$T_7^6 = \begin{bmatrix} 0.9218 & -0.3878 & 0 & 0.4609 \\ 0.3878 & 0.9218 & 0 & 0.1939 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.54}$$

$$T_8^7 = \begin{bmatrix} \cos\theta_8 & -\sin\theta_8 \cos\alpha_8 & \sin\alpha_8 \sin\theta_8 & a_8 \cos\theta_8 \\ \sin\theta_8 & \cos\theta_8 \cos\alpha_8 & \sin\alpha_8 \cos\theta_8 & a_8 \cos\theta_8 \\ 0 & \sin\alpha_8 & \cos\alpha_8 & d_8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.55}$$

$$T_8^7 = \begin{bmatrix} 0.5551 & 0.8318 & 0 & 0.1943 \\ -0.8318 & 0.5551 & 0 & -0.2911 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.56}$$

The final homogeneous transformation matrices with respect to base frame {0} is,

$$T_8^0 = \begin{bmatrix} 0.9859 & 0.1674 & 0 & 2.0441 \\ -0.1674 & 0.9859 & 0 & -0.9858 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{3.57}$$

Form equation (3.12) the Q_i matrix, for a rotary joint,

$$Q_i = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.58}$$

The first derivative of the coordinate homogeneous transformation matrices T_8^0 is given by equation (3.17). Assuming all the products of inertia are zero, the pseudo-inertia matrix $I_i$is given by equation (3.20),

$$I_i = \begin{bmatrix} 1/3(m_i l_i^2) & 0 & 0 & -1/2(m_i l_i) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_i l_i) & 0 & 0 & m_i \end{bmatrix} \tag{3.59}$$

$$I_1 = \begin{bmatrix} 1/3(m_1 l_1^2) & 0 & 0 & -1/2(m_1 l_1) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_1 l_1) & 0 & 0 & m_1 \end{bmatrix} = \begin{bmatrix} 2.4000 & 0 & 0 & -6.0000 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -6.0000 & 0 & 0 & 20.0000 \end{bmatrix} \tag{3.60}$$

$$I_2 = \begin{bmatrix} 1/3(m_2 l_2^2) & 0 & 0 & -1/2(m_2 l_2) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_2 l_2) & 0 & 0 & m_2 \end{bmatrix} = \begin{bmatrix} 1.4167 & 0 & 0 & -4.2500 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -4.2500 & 0 & 0 & 17.0000 \end{bmatrix} \tag{3.61}$$

$$I_3 = \begin{bmatrix} 1/3(m_3 l_3^2) & 0 & 0 & -1/2(m_3 l_3) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_3 l_3) & 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} 2.4000 & 0 & 0 & -6.0000 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -6.0000 & 0 & 0 & 20.0000 \end{bmatrix} \tag{3.62}$$

$$I_4 = \begin{bmatrix} 1/3(m_4 l_4^2) & 0 & 0 & -1/2(m_4 l_4) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_4 l_4) & 0 & 0 & m_4 \end{bmatrix} = \begin{bmatrix} 2.1667 & 0 & 0 & -6.5000 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -6.5000 & 0 & 0 & 26.0000 \end{bmatrix} \tag{3.63}$$

$$I_5 = \begin{bmatrix} 1/3(m_5 l_5^2) & 0 & 0 & -1/2(m_5 l_5) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_5 l_5) & 0 & 0 & m_5 \end{bmatrix} = \begin{bmatrix} 1.6667 & 0 & 0 & -5.0000 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -5.0000 & 0 & 0 & 20.0000 \end{bmatrix} \tag{3.64}$$

$$I_6 = \begin{bmatrix} 1/3(m_6 l_6^2) & 0 & 0 & -1/2(m_6 l_6) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_6 l_6) & 0 & 0 & m_6 \end{bmatrix} = \begin{bmatrix} 1.2500 & 0 & 0 & -3.7500 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -3.7500 & 0 & 0 & 15.0000 \end{bmatrix} \tag{3.65}$$

$$I_7 = \begin{bmatrix} 1/3(m_7 l_7^2) & 0 & 0 & -1/2(m_7 l_7) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_7 l_7) & 0 & 0 & m_7 \end{bmatrix} = \begin{bmatrix} 1.2500 & 0 & 0 & -3.7500 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -3.7500 & 0 & 0 & 15.0000 \end{bmatrix} \quad (3.66)$$

$$I_8 = \begin{bmatrix} 1/3(m_8 l_8^2) & 0 & 0 & -1/2(m_8 l_8) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1/2(m_8 l_8) & 0 & 0 & m_8 \end{bmatrix} = \begin{bmatrix} 0.6125 & 0 & 0 & -2.6250 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2.6250 & 0 & 0 & 15.0000 \end{bmatrix} \quad (3.67)$$

Using equation (3.35),

$$M_{11} = \sum_{p=\max(1,1)}^{8} Trace\left[(U_{11})\ I_1\ (U_{11})^T\right] = 323.4558 \quad (3.68)$$

$$M_{12} \sum_{p=\max(1,2)}^{8} Trace\left[(U_{12})\ I_1\ (U_{12})^T\right] = 235.7224 \quad (3.69)$$

$$M_{13} = \sum_{p=\max(1,3)}^{8} Trace\left[(U_{13})\ I_1\ (U_{13})^T\right] = 210.3771 \quad (3.70)$$

$$M_{14} = \sum_{p=\max(1,4)}^{8} Trace\left[(U_{14})\ I_1\ (U_{14})^T\right] = 120.3516 \quad (3.71)$$

$$M_{15} = \sum_{p=\max(1,5)}^{8} Trace\left[(U_{15})\ I_1\ (U_{15})^T\right] = 68.3713 \quad (3.72)$$

$$M_{16} = \sum_{p=\max(1,6)}^{8} Trace\left[(U_{16})\ I_1\ (U_{16})^T\right] = 19.6365 \quad (3.73)$$

$$M_{17} = \sum_{p=\max(1,7)}^{8} Trace\left[(U_{17})\ I_1\ (U_{17})^T\right] = -6.6078 \quad (3.74)$$

$$M_{18} = \sum_{p=\max(1,8)}^{8} Trace\left[(U_{18})\ I_1\ (U_{18})^T\right] = 2.9072 \quad (3.75)$$

Similarly, solving from M_{21} to M_{88} the final obtained matrix be as follows,

$$M = \begin{bmatrix} 323.4558 & 235.7224 & 210.3771 & 120.3516 & 68.3713 & 19.6365 & -6.6078 & 2.9072 \\ 235.7224 & 196.4689 & 200.4902 & 126.9718 & 75.4000 & 30.6568 & 1.3116 & 3.2676 \\ 210.3771 & 200.4902 & 233.6782 & 155.9429 & 93.4667 & 43.7960 & 7.1650 & 4.5704 \\ 120.3516 & 126.9718 & 155.9429 & 113.3675 & 71.9231 & 38.3296 & 9.9263 & 4.1627 \\ 68.3713 & 75.4000 & 93.4667 & 71.9231 & 48.8954 & 28.6995 & 9.2615 & 3.3016 \\ 19.6365 & 30.6568 & 43.7960 & 38.3296 & 28.6995 & 21.4201 & 10.1012 & 2.6855 \\ -6.6078 & 1.3116 & 7.1650 & 9.9263 & 9.2615 & 10.1012 & 7.5323 & 1.5724 \\ 2.0972 & 3.2676 & 4.5704 & 4.1627 & 3.3016 & 2.6855 & 1.5724 & 0.6125 \end{bmatrix} \quad (3.76)$$

4. MATHEMATICAL ALGORITHM USING MATLAB

```
%% Task Priority of 8 Link Robot Tracks 'L' Shape Trajectory in a 'L'
Shape Tube Using Potential Function Approach

%%settings
clc;
clear all;
close all;
format short;


% Plot Settings
% figure
grid on

% Weights
g11 =0;
g12 = 0;
g21 = 110000;
g22 = 0;

%Obstacles
X1= [0.3 2.65 2.65 0.3];
Y1= [1.79 1.79 2.04 2.04];
X2= [2.40 2.65 2.65 2.40];
Y2= [0 0 1.79 1.79];
X3= [1.35 1.60 1.60 1.35];
Y3= [0 0 0.74 0.74];
X4= [0.3 1.60 1.60 0.3];
Y4= [0.74 0.74 0.99 0.99];

% Parameters
t = 0;                                      % time
n = 8;                                      % No of Links
l = [0.6 0.5 0.6 0.5 0.5 0.5 0.5 0.35];     % Link Lenghts
m = [20 17 20 26 20 15 15 15];              % Link Masses

% Initial Values in Joint Space
th   = [75 35 -98 -26 13 -30 -75 43]'*pi/180;
thd  = [0 0 0 0 0 0 0 0]';
thdd = [0 0 0 0 0 0 0 0]';

%Position of end-effector r = [x y]
y   = inline('1.39*(2*t^3 - 3*t^2 + 1)');
yd  = inline('(t^2-t)*7.34');
ydd = inline('(2*t-1)*7.34');
x   = 2;
xd  = 0;
xdd = 0;

step = -0.005;
for t=1:step:0

    r1   = [x; y(t)];
    rd1  = [xd; yd(t)];
```

```
    rdd1 = [xdd; ydd(t)];

    [J,DJ,X,Y] = jacobian(n,th,thd,l);
    ard1 = J*thd;
    ar1 = [X(n);Y(n)];

    h1 = rdd1 - DJ*thd+g11*(rd1-ard1) + g21*(r1-ar1);

    if(t~=0)
        [M,C] = DynamicMassCoriolis(th,thd,m,n,l);
        [dp,dd] = PotenFunc2(n,l,th,thd);
        INVM=inv(M);

      INVM1=[INVM(1,1),INVM(1,2),0,0,0,0,0,0;INVM(2,1),INVM(2,2),0,0,0,0
      ,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0
      ,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0];

      thdd = (pinv(J)*h1)-((eye(n,n)-(pinv(J)*J))*(INVM1)*(dp+dd) +
      ([C,0,0,0, 0,0,0]'));

      thd = thd + thdd*step;
      th  = th + thd*step

    end

   cla
    axis square
    axis tight
    hold on
    plot([2,2],[1.39,0],'LineWidth',2,'Color',[0.5,0,0]);
    plot([0,2],[1.39,1.39],'LineWidth',2,'Color',[0.5,0,0]);
    plot([-2,3],[-2,-2],'LineWidth',0.2,'Color',[0.5,0,0]);
    plot([3,3],[-2,3],'LineWidth',0.2,'Color',[0.5,0,0]);
    plot([-2,3],[3,3],'LineWidth',0.2,'Color',[0.5,0,0]);
    plot([-2,-2],[-2,3],'LineWidth',0.2,'Color',[0.5,0,0]);
    fill(X1,Y1,[0.7969,0.7969,0.7969]);
    fill(X2,Y2,[0.7969,0.7969,0.7969]);
    fill(X3,Y3,[0.7969,0.7969,0.7969]);
    fill(X4,Y4,[0.7969,0.7969,0.7969]);

    x1=[0 X];
    y1=[0 Y];

    if rem(t,.05)==0
        plot(x1,y1,'o-','LineWidth',2,'Color','b');
        pause(1e-10);
        plot(x1,y1,'o-','LineWidth',2,'Color','b');
    end

end
plot(x1,y1,'o-','LineWidth',2,'Color','b');

%HorizontalTrackFollowing


% Weights
g11 =0;
g12 = 0;
g21 = 25000;
```

```
g22 = 0;

%Obstacles
X1=[0.3 2.65 2.65 0.3];
Y1=[1.79 1.79 2.04 2.04];
X2=[2.40 2.65 2.65 2.40];
Y2=[0 0 1.79 1.79];
X3=[1.35 1.60 1.60 1.35];
Y3=[0 0 0.74 0.74];
X4=[0.3 1.60 1.60 0.3];
Y4=[0.74 0.74 0.99 0.99];

% Parameters
t = 0;                                          % time
n = 8;                                          % No of Links
l = [0.6 0.5 0.6 0.5 0.5 0.5 0.5 0.35];         % Link Lenghts
m = [20 17 20 26 20 15 15 15];                  % Link Masses

% Initial Values in Joint Space
th   = [82.4428 99.0644 -129.2478 -37.8896 23.4740 -15.4813 -68.1877
58.4588]'*pi/180;
thd  = [0 0 0 0 0 0 0 0]';
thdd = [0 0 0 0 0 0 0 0]';

%Position of end-effector r = [x y]
y   = 1.39;
yd  = 0;
ydd = 0;
x   = inline('2*(2*t^3 - 3*t^2 + 1)');
xd  = inline('(t^2-t)*7.34');
xdd = inline('(2*t-1)*7.34');


step = 0.005;
for t=0:step:1

    r1   = [x(t); y];
    rd1  = [xd(t); yd];
    rdd1 = [xdd(t); ydd];

    [J,DJ,X,Y] = jacobian(n,th,thd,l);
    ard1 = J*thd;
    ar1 = [X(n);Y(n)];

    h1 = rdd1 - DJ*thd+g11*(rd1-ard1) + g21*(r1-ar1);

    if(t~=0)
        [M,C] = DynamicMassCoriolis(th,thd,m,n,l);
        [dp,dd] = PotenFunc2(n,l,th,thd);
        INVM=inv(M);
      INVM1=[INVM(1,1),INVM(1,2),0,0,0,0,0,0;INVM(2,1),INVM(2,2),0,0,0,0
      ,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0
      ,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0];
      thdd = (pinv(J)*h1)-((eye(n,n)-(pinv(J)*J))*(INVM1)*(dp+dd)+
      ([C,0,0,0,0,0,0]'));
      thd = thd + thdd*step;
      th  = th + thd*step;
    end
```

```
    cla
     axis square
     axis tight
     hold on
      plot([2,2],[1.39,0],'LineWidth',2,'Color',[0.5,0,0]);
      plot([0,2],[1.39,1.39],'LineWidth',2,'Color',[0.5,0,0]);
      plot([-2,3],[-2,-2],'LineWidth',0.2,'Color',[0.5,0,0]);
      plot([3,3],[-2,3],'LineWidth',0.2,'Color',[0.5,0,0]);
      plot([-2,3],[3,3],'LineWidth',0.2,'Color',[0.5,0,0]);
      plot([-2,-2],[-2,3],'LineWidth',0.2,'Color',[0.5,0,0]);
      fill(X1,Y1,[0.7969,0.7969,0.7969]);
      fill(X2,Y2,[0.7969,0.7969,0.7969]);
      fill(X3,Y3,[0.7969,0.7969,0.7969]);
      fill(X4,Y4,[0.7969,0.7969,0.7969]);

     x1=[0 X];
     y1=[0 Y];

     if rem(t,.05)==0
         plot(x1,y1,'o-','LineWidth',2,'Color','b');
         pause(1e-10);
         plot(x1,y1,'o-','LineWidth',2,'Color','b');
     end

end
plot(x1,y1,'o-','LineWidth',2,'Color','b');
```

5. CONCULSION

In this thesis, the idea of task priority technique is utilized, which splits the offered task to numerous subtasks based on order of priority. The primary two subtasks are, one is tasking the given trajectory and the second one is avoiding the obstacles in the complex workspace. Various mathematical simulation is performed for various compound environment consisting of obstacles with an offered trajectory to show the adequacy of the redundancy control structure for obstacle and singularity avoidance.

The dynamic model development for redundant manipulator is modelled. Equation of motion for a redundant manipulator which track the trajectory in narrow “L” shape tube channel by using Lagrange-Euler approach is formulated and the dynamic model is developed. MATLAB software is used for development of numerical simulation.

5.1. FUTURE WORK FOR RESEARCH

1 These methods can be applied for high DOF redundant manipulators for achieving task prioritization, obstacle avoidance, singularity avoidance, path planning etc.

2 Future research can be on high DOF redundant manipulator which are in the field of medicinal

LIST OF PUBLICATIONS

Title: - “A REVIEW OF METHODS USED FOR KINEMATIC MODELING OF A MANIPULATOR”.

Author: - Akula Umamaheswararao, Himanshu Arora, Vaibhav Nanavare and Naresh Reddy Bhumireddy.

Publication: - Scopus Index Journal titled as “International Journal of Control Theory and Application, Volume 8, Issue 7, July 2017, pp. 1854–1861, Article ID: IJMET_08_07_206.

REFERENCES

[1] J. Denavit, R.S. Hartenberg, (1995) A kinematic notation for lower-pair mechanisms based on matrices, Journal Applied Mechanics, Vol-22, Pp 215-221.

[2] Sabine Stifter, (1994) Algebraic methods for computing inverse kinematics, journal of intelligent and robotic system, Journal of Intelligent and Robotic Systems 11: Pp.79-89.

[3] Ian S.Fischer, (1990) A geometric method for determining joint rotations in the inverse kinematics of robotic manipulators, Journal of Robotic Systems 17(2), Pp. 107-117.

[4] M. Raghavan, B. Roth, (1990) Kinematics analysis of the 6R manipulator of general geometry, 5th international symposium on Robotics research, Pp. 263-269.

[5] L.W.Tsai , A.P.Morgan, (1985) Solving the kinematics of the most general six and five DOF manipulator by continuation methods, ASME Journal of mechanisms, transmissions and automation in design, Pp. 189-195.

[6] D.Pieper , (1968) The kinematics of manipulator under computer control, PH.D Thesis.

[7] J.J.Uicker Jr.,J.Denavit,R.S Hartenberg, (1964) An Interactive method for the displacement analysis of spatial mechanisms, Journal Applied Mechanics, Pp. 309-314,

[8] J.Zhao, N.Badler, (1994) Inverse kinematics positioning using nonlinear programming for highly articulated figures, Transactions on Computer Graphic, Pp.313-336.

[9] Wei Song, Guang Hu, (2011) A fast inverse kinematics Algorithm for joint animation, International conference on advances in engineering, Elsevier, 350-354.

[10] D. E. Whitney, (1969) Resolved motion rate control of manipulators and human prostheses, IEEE Transactions on Man-Machine Systems, Pp. 47-53.

[11] W. A. Wolovich and H. Elliot, (1984) A computational technique for inverse kinematics, in 23rd IEEE Conference on Decision and Control, Pp.1359-1363.

[12] C. W. Wampler, (1986) Manipulator inverse kinematic solutions based on vector formulations and damped least squares methods, IEEE Transactions on Systems, Man, and Cybernetics. Pp. 93-110.

[13] A.S.Deo,I.D.Walker, (1993) Adaptive non-linear least squares for inverse kinematics, in Proc. IEEE International Conference on Robotics and Automation, Pp.186-193.

[14] L.-C. T. Wang and C. C. Chen, (1991) A combined optimization method for solving the inverse kinematics problem of mechanical manipulators, IEEE Transactions on Robotics and Automation, Pp. 489-499.

[15] G. G. Lendaris, K. Mathia, and R. Sacks, (1999) Linear Hopfield networks and constrained optimization, IEEE Transactions on Systems, Man, and cybernetics, Pp. 114-118.

[16] E. Oyama, N. Y. Chong, A. Agah, T. Maeda, and S. Tachi, (2001) Inverse kinematics learning by modular architecture neural networks with performance prediction networks, in Proc. IEEE International Conference on Robotics and Automation, Pp.1009-1012.

[17] A. Ramdane-Cherif, B. Daachi, A. Benallegue, and N. Levy, (2002) Kinematic inversion, in Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems, 1904-1909.

[18] A. D'Souza, S. Vijayakumar, and S. Schaal, (2001) Learning inverse kinematics, in Proc. IEEE/RSJ International Conference on Intelligent Robots and Systems, Pp. 298-303.

[19] D. E. Whitney, (1969) Resolved motion rate control of manipulators and human prostheses, IEEE Transactions on Man-Machine Systems, Pp. 47-53.

[20] Sung-woo Kim, Redundancy Resolution of Robot Manipulators Using Optimal Kinematic Control. IEEE Xplore, Pp.683-688

[21] Joseph Wunderlich (1996), Local Optimization of Redundant Manipulator Kinematics within Constrained Workspaces, International Conference on Robotics and Automation, Pp.125-132.

[22] Y. Nakamura, H. Hanafusa, and T. Yoshikawa, (1987) “Task-priority based redundancy control of robot manipulators,” Int. J. Robotics Res., vol. 6, no. 2, Pp. 3-15.

[23] Fabrizio Flacco, (2018), The Tasks Priority Matrix: a new tool for hierarchical redundancy resolution, IEEE-RAS 16th International Conference on Humanoid Robots (Humanoids), Pp 1-7

[24] B. Siciliano and J. J. Slotine, (1991) “A general framework for managing multiple tasks in highly redundant robotic systems,” in Proc. 5th Int. Conf. on Advanced Robotics, Pp. 1211–1216.

[25] Hsien-I Lin, (2013), Intuitive Kinematic Control of a Robot Arm via Human Motion, 37th national conference on theoretical and applied mechanics, Pp 411-416.

[26] Andreas Muller, (2004), Collision avoiding continuation method for the inverse kinematics of redundant manipulators, IEEE International conference on ronotics and antomation,Pp 1593-1598.

[27] Jose-Luis Blanco, Javier González, Juan-Antonio Fernández-Madrigal, Extending Obstacle Avoidance Methods through Multiple Parameter-Space Transformations, Springerlink.com/Autonomousrobots,Pp 1-25.

[28] O. Khatob, (1986), Real- time obstacle avoidance for manipultor and moblie robotics, Robotics research , vol 5, Pp 90-99.

[29] Yunong Zhang and Jun Wang, (2004), Obstacle Avoidance for Kinematically Redundant Manipulators Using a Dual Neural Network, Pp- 752-759.

[30] Mihai Duguleana, (2012), Obstacle avoidance of redundant manipulators using neural networks based reinforcement learning, ScienceDirect-Robotics and Computer-Integrated Manufacturing,Pp 136-146.

[31] Leon Beiner, (1992), Singularity avoidance for SCARA robots, Robotics and Autonomous Systems, Pp 63-69.

[32] Samer Yahya, (2012), Singularity avoidance of a six degree of freedom three dimensional redundant planar manipulator, ScienceDirect Computers and Mathematics with Applications, Pp 856-868.

[33] Giacomo Marani, (2003), Algorithmic singularities avoidance in task-priority based controller for redundant manipulators, https://www.researchgate.net/publication/4046134 .

[34] Stefano Chiaverini, (1997), Singularity-Robust Task-Priority Redundancy Resolution for Real-Time Kinematic Control of Robot Manipulators, IEEE transactions on robotics and automation, vol. 13, Pp 398-410

[35] S. R. Buss and J. S. Kim, Selectively damped least squares for inverse kinematics. Typeset manuscript, April 2004. Draft available at http:/math.ucsd.edu/~sbuss /ResearchWeb. Submitted for publication.

Fig 1.1 Robots used in welding opertion Source from roboticsandautomationnews.com

Fig 1.2 Robots used in handling opertion Source from roboticsandautomationnews.com

Fig 1.3 Robotos used in assmbly opertion source from roboticsandautomationnews.com

Fig 1.4 Sanke like arm robot Screenshot for youtube video-https://www.youtube.com/ watch?v=qeXFx_npFuw.

Fig 3.1 Joint velocity of a point on the manipulator Source from text book Robotics and Control by R K Mittal.