

**TO STUDY THE EFFECT OF CRACK PROPAGATION IN  
TAPER ROLLER BEARING WITH LOADED CONDITION**

Dissertation-II

Submitted in partial fulfillment of the requirement for the award of

degree

Of

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## CERTIFICATE

I hereby certify that the work being presented in the dissertation entitled “**To Study the Effect Of Crack Propagation In Taper Roller Bearing With Loaded Condition**” in partial fulfillment of the requirement of the award of the Degree of master of technology and submitted to the Department of Mechanical Engineering of Lovely Professional University, Phagwara, is an authentic record of my own work carried out under the supervision of **Dr. Manpreet Singh, Professor** Department of Mechanical Engineering, Lovely Professional University. The matter embodied in this dissertation has not been submitted in part or full to any other University or Institute for the award of any degree.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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The external viva-voce examination of the student was held on successfully \_\_\_\_\_

Signature of Examiner

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## **ABSTRACT**

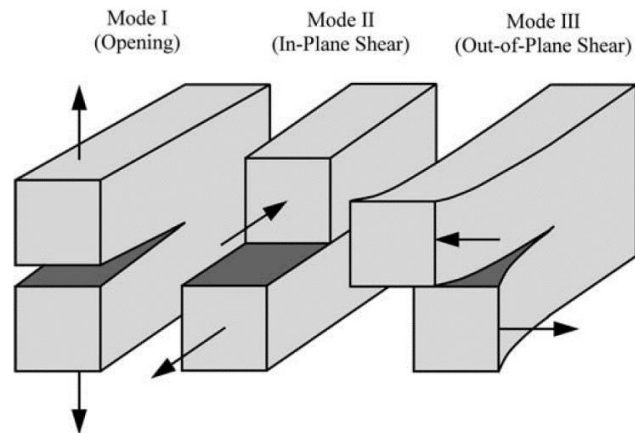
The bearing is one of the most prominent part of the rotating machinery. Prognosis and diagnosis of the bearing crack is important to averting the sudden breakdown of the machine. In this research work we will first introduced a groove defect outer race of taper roller bearing and then the bearing is made to run at constant RPM under different loading condition. We are expecting crack to be initiated near the groove defect and will grow differently at different loading condition. Bearing would be inspected visually after equal interval of time probably 4 hours. The same time if the crack would be initiated then obviously there will be change in vibration characteristics. These change in vibration characteristics will be corroborated with crack propagation by using significant parameter/different transformation of vibration signal.

## Chapter 1

### 1 INTRODUCTION

#### 1.1 Crack propagation

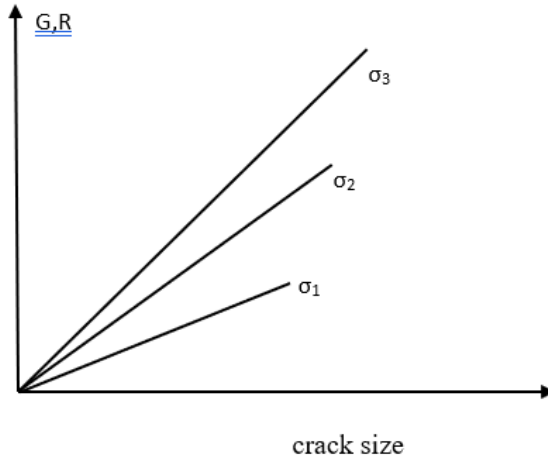
Crack means permanent detachment of atomic bonds contemporarily with formation of the new surface. A crack may arise because of mechanical, chemical, or thermal action. Crack length, crack tip radius, crack orientation is dependent on the types of loading and material properties. There are three cases of loading of cracked body can be considered, which are called as mode of deformation: Mode-1 (opening), Mode-2 (sliding or in-shear plane), Mode-3 (out-of-shear plane, Tearing) shown in Fig.1. Crack depends on the loading condition like with load and without load.



**Figure 1.1 Modes of loading that can be applied to a crack**

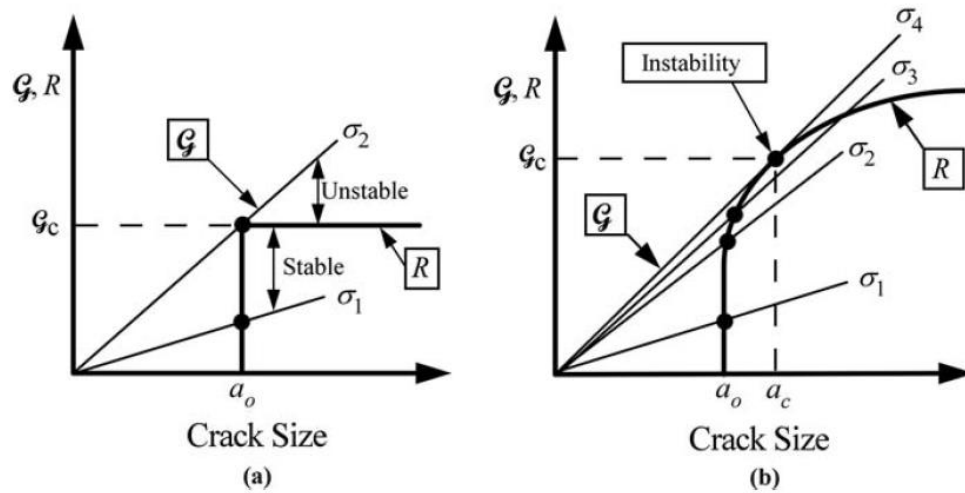
The crack will propagate if the strain energy release rate( $G$ ) during crack growth is large enough to exceed the rate of increase in surface energy( $R$ ) associated with the formation of the new crack surface.

Graph shown in figure1.2 between crack growth and various values of stress, where  $\sigma_1 < \sigma_2 < \sigma_3$  to find out the critical point or the point up to which crack is stable could be found by considering R curve in the earlier graph here R is material resistance to crack extension.



**Figure 1.2 crack growth vs stress**

Where curve of Energy release rate (crack growth) (G) and R curve meets the point is considered as critical point. The critical point is marked as  $G_c$  in the figure 1.3 [21].



**Figure 1.3 Schematic driving force vs. R curve diagrams**

There are two types of crack growth is possible, (1) stable crack growth and (2) unstable crack growth shown in figure 1.3. Stable crack growth can be expressed by the expression,

$$(1) G=R \text{ and } \frac{dG}{da} \leq \frac{dR}{da}$$

Where, G is Griffith crack growth and R is material resistance to crack extension.

$$G = - \frac{du}{da}$$



$u = \frac{a^2 \pi \sigma^2}{2E}$  is the strain energy release in the growth of crack length

a is crack length

$\sigma$  is stress

E is young's modulus

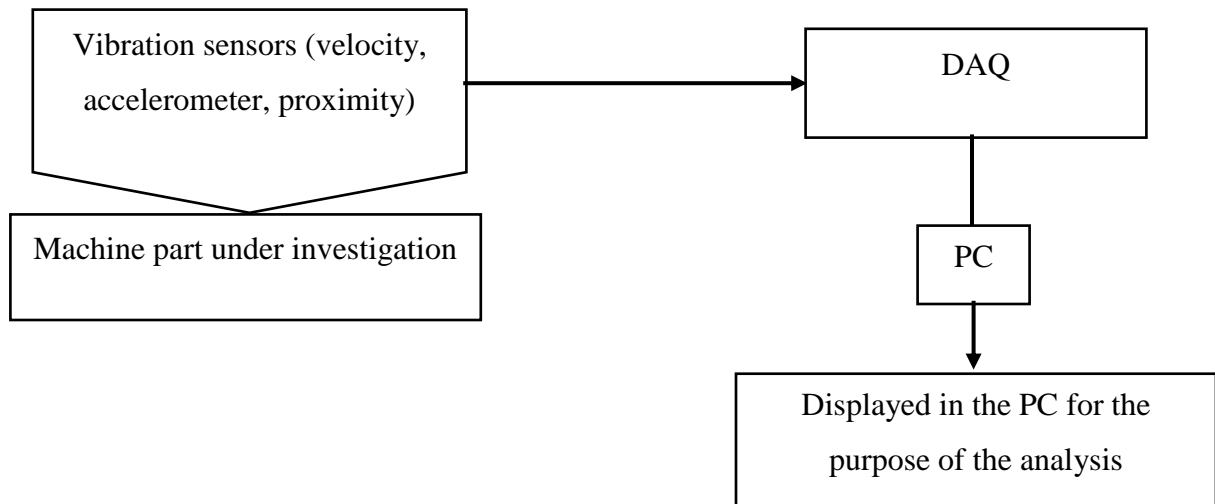
Unstable crack growth can be expressed by expression

$$(2) \frac{dG}{da} > \frac{dR}{da}$$

Where,  $R = -\frac{dW}{da}$  and  $W = 2a\gamma$  is the corresponding surface

The latest method is available for check the crack growth are XRD(X-Ray Diffraction) and SEM(Scanning Electron Microscope). To carry on these test the part needs to be disassembled for which production line has to be disrupted. The effect of crack growth could also be studied by using vibration based monitoring in which there is no need for disassembling of machine part. A similar attempt has been made in this work.

Vibration based monitoring will be done using Data Acquisition System(DAQ).



**Figure 1.4 Vibration monitoring process chart**

In this work first the Crack is initiate in outer race of Taper roller bearing and then this bearing is made to run constant rpm and with having different loading conditions. There would be a provision to record the vibration signal for this running bearing and during the experimentation

signal would be analyzed after a fix duration of time (4 hours). After this fixed duration of time bearing will be disengaged for the visual inspection. It is expected that the crack propagation will start across the seeded defect for unloaded and loaded conditions. The effect of crack propagation will be studied visually as well as with the help of vibration signal and results will be presented. This could be helpful in estimating the remaining useful life of the bearing to plan appropriate maintenance strategy.

## Chapter 2

### 2 SCOPE OF THE STUDY

The bearing is one of the most prominent components/part for the failure in the rotating machinery. For estimating the RUL of machinery, bearing needs to be investigated properly. In the present study we are trying to relate the crack propagation rate with the variations in the recorded vibration signal. To study the variations in the vibration signal different types of signal processing techniques will be used. If the relation could be developed between the crack propagation and variation in vibration signal then there will be no need to disassemble the machine component for the purpose of inspection. The remaining useful life could also be predicted through this technique and suitable maintenance strategy could be applied for the machine.

First Crack is initiate in the outer race of taper roller bearing after that crack will propagate with different load condition, also the vibration of machine is effect to propagate the crack. We will do vibration analysis of taper roller bearing 30205(TRB) and extract the signal, it can be related through mean. This data could be further helpful for estimation of bearing life.

## **Chapter 3**

### **3 OBJECTIVE OF THE STUDY**

- To analyze the crack propagation in taper roller bearing for different loading conditions by using signal processing of vibration signal.

## Chapter 4

### 4 LITERATURE REVIEW

Literature review is important to understand the work executed related this field that will provide essential data for experimental and analytical. The information of work is available in books, research paper published in National and International Journal, papers presented in conferences for Post Graduate and Doctoral research work.

**Nagi Gebraeel et al. (2004) [1]** described to predict the bearing failures using neural-network, backpropagation and vibration signal. This was done with help of 35 bearing divided into two part training set (1-25 bearing) and validation set (26-35 bearing). Bearing failure is calculated with help of weighted average of the exponential parameters and he found that 92% of the failure time prognosis figure out using validation bearings were within 20% of the actual bearing life.

**Yusuf Kayah et al. (2009) [2]** described the factor effects on the fatigue failure of taper roller bearing using the experimentally and the FEM. This bearing was used for high radial, axial and angular load. In this experimental taper roller bearing is mounted on the wheel hub. First, they inspect the bearing visually and they observed that spalling is occurred at the inner raceway of the half of the full area. This fatigue failure is occurred in two ways first is the irregularities of the contact stress among the bearing outer ring and the tapered roller surface and second is the stress concentration caused by the wear debris produced between the outer ring and roller. Then they done chemical and hardness test as per the AISI 52100 steel after that analysis is done by visual inspection, microscopic analysis (optical and SEM), at a depth of 300 micrometer. At contact surface crack initiate due to tensile stresses then extend due to the continuous loading up to at the end of the surface. It can be concluded that life of bearing can be increase by increase contact are of bearing, hardness of bearing and decrease load on lorry.

**Aditya S. et al. (2010) [3]** fatigue crack propagation in roller-rocker and rocker bearing of railways bridges. Bridge was open web of steel bridge. For crack propagation he used contact and fracture theories in combination with fatigue laws. Strain gauge was placed in the bearings to measure the surface strains on the bearing plates. Took four initial location, in these initial crack length of 0.942mm and final crack length of 9.942mm. It is seen that crack close to the

contact region propagates due to fatigue load and finally stops to propagate due to increasing in compressive strength at the end. It has not that much energy to the boundary under the crack arrest theory.

**Abd Kadir et al. (2010) [4]** described the method to predict remaining useful life of bearing like Artificial neural network(ANN). After acquiring vibration signal from experimental setup, these signals was extract by RMS and kurtosis. Extract measurement was used to train the ANN. Then with help of purposed ANN method Feed Forward Neural Network (FFNN) with Levenberg Marquardt training algorithm was prepared for RUL. Without describing threshold value ANN was best to predict RUL of bearing.

**P.K. Kankar et al. (2011) [5]** described fault diagnosis of rolling element bearing using continuous wavelet transform. For that he prepared suitable experimental setup. He took signal of different rpm and load and no-load condition. To get more precise information about fault, wavelet based feature like Energy to Shannon Entropy ratio (Mayer Wavelet) extraction was used. These extracted results were classified with Support Vector Machine (SVM), Artificial neural network(ANN), Self-organizing maps technique. After comparison he conclude that SVM and ANN was better than self-organizing map.

**T.benkedjough et al.(2013) [6]** depicted the Isometric Feature mapping Reduction Technique (ISOMAP) and Support vector Regression (SVR) for bearing defect and predict bearing RUL. They used three bearing with different life for proficiency of the method. Taking data from the experiment, feature extraction was done by using ISOMAP. RUL estimation is done with Support Vector Machine for that ho took threshold value 1.4 Root mean Square of the vibration signal.

**V.N.Patel et al.(2013) [7]** described the experimental and theoretical work the vibration signal of deep groove ball bearings in existence of local defects on inner and outer race by applying changeable frequency of applied loads. He compares either of race defect data by theoretical using Range-Kutta method and experimentally, he found that when ball pass to the either race defect, the additional displacement of the ball varies from zero to maximum at that time ball reaches from the center of the defect to the other end of the defect.

**D.P. Jena et al.(2014)[8]** depict the Signal processing technique to measure the defect width of taper roller bearing. Vibration signal also analyze with continuous wavelet transformation(CWT), Un-decimated Wavelet Transform (UWT), Ridge spectrum. By comparing both experimental and simulation work, the best result of CWT is 4.267% and ridge spectrum it is 2.385%, from above result the ridge spectrum is more convenient then CWT.

**Xia Yang et al. (2014) [9]** described how the load was distributed on the four-row taper roller bearing. In these Boundary Element Method was used because of high non-linearity, also used Hertzian contact theory for study the contact width between rollers, inner and outer races. Large pressure is induced at the end as compare to the side of the bearing. The position of axially pressurized of roller when steady on rollers and at the second roller there was more pressure compare to other raw rollers.

**Jaouher et al. (2015) [10]** described the estimation of remaining useful life based on Weibull distribution(WD) and ANN. Take seven classes of bearing: healthy bearing and six bearing for degradation. Offline and online setup proposed. For Offline full run-to-failure history was used to select optimum Simplified Fuzzy Adaptive Resonance Theory Map (SFAM) and online feature are extracted from new bearing at each ten minute for getting higher accuracy in RUL. Experimental result show the satisfactory RUL prediction result by combination of SFAM and WD.

**T. Bruce et al. (2015) [11]** explained about crack propagation when we initiate crack by smaller addition of the mean value from  $8\mu\text{m}$  to  $16\mu\text{m}$  length. When view from the side cross section area, longer WEC which means white etching crack found and to initiate this through inclusion which is closet and parallel to the raceway surface. In wind turbine gearbox bearing WTGB, failure mode was found by the white etching WEC method. White etching crack which is WEC, were attached to manganese sulphide (MnS) inclusions of span length  $3\mu\text{m}$  to  $45\mu\text{m}$  and its depth to  $630\mu\text{m}$ . Longer etching crack found the shorter inclusions crack which was closet and parallel to the raceway of bearing surface axially.

**Pavle et al. (2015) [12]** described fault detection and portending bearing fault for this he used Gaussian process model, Bayes rule with truncated prior. In statistical model in bearing vibration rolling ball and surface passing across so that surface damaged site causes a impulse

response of system. RUL is estimated by posterior distribution with Bayes rule Gaussian process model.

**Minghang Zhao et al. (2016) [13]** described that bearing RUL based on time frequency representation(TFR) and supervised dimensionality reduction. TFR was used to physique high-dimensional feature. Experiment was conducted on PRONOSTIA. First he used TFR, gaussian pyramid and Local binary pattern(LBP) assure for life. And final the RUL was predicted by using Multi Linear Regression and these method is easy to compare different feature extraction method.

**N. Harish Chandra et al. (2016) [14]** described that in rotor bearing system fault is induced due to misalignment, rotor crack, rotor to stator rub. For identifying the fault numerical simulation through FEA and experimental of rotor bearing system for that used three signal processing tools like Hilbert Huang Transform (HHT), Short Time Fourier Transform(STFT) and CWT. HHT is not suitable for low signal to noise ratio. In CWT choose high center frequency and band width parameter computation time increase and detect smaller fault in rotor system.

**Farzad Hemmati et al.(2016) [15]** described the condition monitoring of rolling element bearings for that he explains statistical analysis, defect size, fault diagnosis and prognosis, using operating speed and loading condition. For statistical analysis of the experiments he used root mean square, peak value, crest factor, skewness, kurtosis, burst duration, vibration statistical parameter for measure the acoustic emission in outer race. with acoustic emission(AE). For feature extraction of the signal he used Kurtosis to Shannon entropy ratio (KER), these methods could remove noisy signal so that weak signal also detect. He compares experimental and theoretical value which gives maximum 3% error. For finding the size of the defect Hilbert transform was used, after comparing actual value and experimental measured defect size gives maximum10% error.

**Lotfi Saidi et al.(2017) [16]** studied arrangement of bearing is mainly depending on the layout of drive train, in the wind turbine rotor blade inflict cyclic load into shaft causing bend and misalignment within the bearing. In this paper fault prognostic and life of the bearing was predicted for the wind turbine high speed shaft bearing (HSSB) using spectral kurtosis (SK) and support vector regression (SVR) model. SK used for condition based monitoring of bearing



crack, and SVR used for RUL. SK derived features can provide an early warning of bearing defect and evaluating bearing degradation. By the experiment he concludes that SVR model based on area under SK is able to estimate RUL precisely.

**Yuning Qin et al. (2017) [17]** described bearing defect and its RUL. He used two methods for predict RUL. (1) phase space warping(PSW) which track the metric and (2) auto-regression model, these methods increase defect tracking accuracy. For this he conducts experiment in this he took two case study in case study 1, he did an analysis for vibration signal in bearing 1. he used PSW and root mean square(RMS) to calculate tracking metric and degradation. Compare RUL predict with use of Paris Crack Growth(PCG) model threshold- adjusted Paris Crack Growth model, PCG method shows accuracy in crack growth. In case study 2 he did an analysis for vibration signal in bearing 3, using phase space warping PSW method. Auto-Regression construction model increase the accuracy of PSW in damage tracking. Paris crack growth model estimate RUL with reliable result.

**S. Okazaki et al. (2017) [18]** described about crack growth in a bearing steels. Torsional fatigue test has been carried out by using a hollow cylindrical test specimen. The material was used in the study is JIS-SUJ2 bearing steel. From observation, it has been seen that value below the 20MPa there was no propagating crack this value is called threshold shear stress amplitude. Above the threshold value crack does not propagate in a continuous mannered. By using Scanning Electron Microscope(SEM), Electron Backscattering Diffraction(EBSD). It has been found that shear stress amplitude was decreased (300-160MPa) with an increase of static stress (0-250Mpa),

**Pawel Rycerz et al. (2017) [19]** described, propagation of surface initiated rolling contact fatigue crack that lead to failure of the of the part. For that he used triple contact disc experimental setup and disc was attached  $120^{\circ}$  apart, test roller is placed in middle. To measure crack propagation in surface he used crack sensor which was work based on magnetic flux leakage and placed in test rings. When crack was detected the machine was stop automatic. Take picture of the surface in Scanning Electron Microscope in these SEM early crack detection, crack branching was observed. Crack growth occur in the opposite to the direction of the friction force. From the above all result he observed that crack length up to  $100\mu\text{m}$  crack

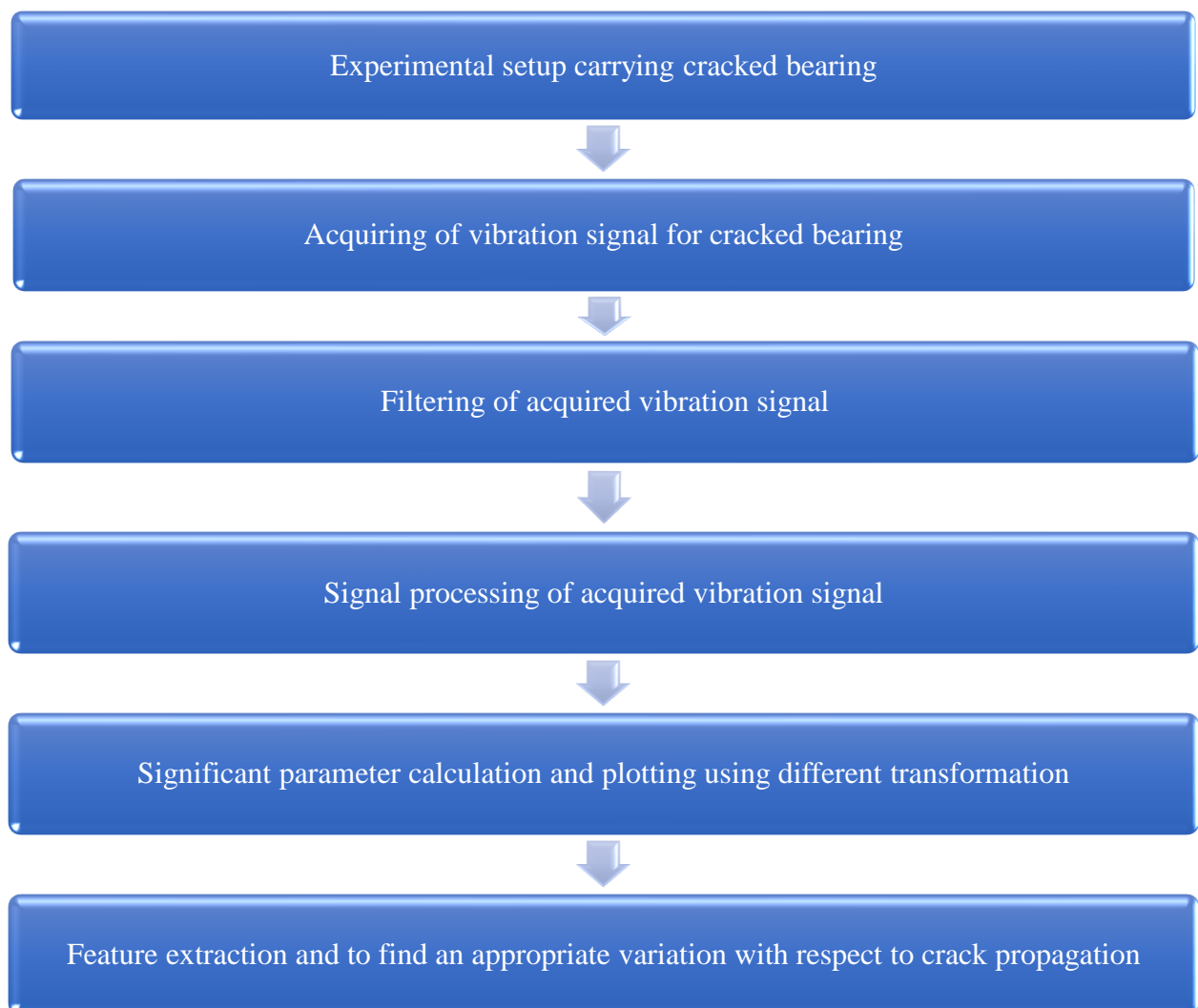
growth was low and decreasing slightly with increasing crack length. When crack length exceed 100 $\mu$ m crack propagation rate is increase with crack length.

**Akhand Rai et al. (2017)[20]** described Performance Degradation Assessment(PDA) of bearing using empirical mode decomposition(EMD) and K-medoids clustering. He prepared experimental setup to validate the result, in these two bearing was used in which run to failure test was carried out for acquired vibration signal. Feature extraction form the bearing signal was done by EMD and Singular Values(SVs) extract from the Intrinsic Mode Functions(IMFs). These feature vector was used to trained K-medoids clustering model. He compares result with fault diagnosis method kurtosis, Root Mean Square(RMS) Self-Organizing Mapping(SOM) and Fuzzy C-Means(FCM). After observation of the result K-medoids had a greater capability to sense slight variation in degradation in bearing compare to other.

## Chapter 5

### 5 RESEARCH METHODOLOGY

Research methodology is the methodical, theoretical analysis of the methods to be applied to the field of study. The experiments and testing work are to be carried out by standardized form based on calculation method and experimental.



**Figure 5.1 Flow Chart of Research Methodology**

A customized setup has to be made to be carried out for the experiment on the taper roller bearing. In the initial stage taper roller bearing is placed in the bearing casing on which provision has been provided to fix the accelerometer. The signal acquired from the accelerometer through Data Acquisition System and later on it will be processed in PC.

There is a filter provided in the DAQ system to filter the noisy signal. After acquiring signal, signal is processed in MATLAB and different plots and statistical movements will be calculated for the purpose of analysis. Finally, the change in vibration characteristic signal would be related with cracked propagation.

### **5.1 Experimental Setup**

- **Material-** Few sets of taper roller bearings with initiated groove defects.
- **Equipment-** All the equipment which is required for our experiment is available at SLIET Longowal, where we will be carrying out our experiment.

## Chapter 6

### 6 PROPOSED WORK PLAN WITH TIMELINES

- Experimental reading would be acquired before the end of January 2018
- Signal processing, plotting, feature extraction will be done by the end of February 2018
- We will startup report writing from the starting of March 2018 and probably we will finish up with the report writing before the end of April2018.

## **7 EXPECTED OUTCOMES**

We are expecting a corroborative relation between crack propagation and significant parameters/different transformation of vibration signal.

## **8 SUMMARY**

All the required information is provided in aforementioned chapters and conclusion of the work still has to be derived.

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