# Design and Fabrication of Bio-Inspired Multi-Fingered Gripper And Optimization of the Number of Links Based On Multi Body Dynamic Analysis

Dissertation-II

Submitted in partial fulfillment of the requirement for the award of degree

Of

**MASTER OF TECHNOLOGY**

**IN**

**MECHANICAL ENGINEERING**

By

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**DEPARTMENT OF MECHANICAL ENGINEERING**

**LOVELY PROFESSIONAL UNIVERSITY**

**PUNJAB**

**TOPIC APPROVAL PERFORMA**

School of Mechanical Engineering

**Program :**P17H::M.Tech. (Design) [Full Time]

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**Supervisor Name** : Nitin Chauhan **UID :** 16901 **Designation :** Assistant Professor

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**SPECIALIZATION AREA** : CAD/CAM & Mechatronics **Supervisor Signature:** _____

**PROPOSED TOPIC** : Design and fabrication of bio-inspired multi-finger gripper and optimization of the number of links based on multi body dynamic analysis.

| Qualitative Assessment of Proposed Topic by PAC | | |
|---|---|---|
| **Sr.No.** | **Parameter** | **Rating (out of 10)** |
| 1 | Project Novelty: Potential of the project to create new knowledge | 7.50 |
| 2 | Project Feasibility: Project can be timely carried out in-house with low-cost and available resources in the University by the students. | 7.00 |
| 3 | Project Academic Inputs: Project topic is relevant and makes extensive use of academic inputs in UG program and serves as a culminating effort for core study area of the degree program. | 7.00 |
| 4 | Project Supervision: Project supervisor's is technically competent to guide students, resolve any issues, and impart necessary skills. | 7.50 |
| 5 | Social Applicability: Project work intends to solve a practical problem. | 7.00 |
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| PAC committee Members | | |
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**Final Topic Approved by PAC:** **Design and fabrication of bio-inspired multi-finger gripper and optimization of the number of links based on multi body dynamic analysis.**

**Overall Remarks:** Approved

**PAC CHAIRPERSON Name:** 12174::Gurpreet Singh Phull **Approval Date:** 11 Nov 2017

## CERTIFICATE

I hereby certify that the work being presented in the dissertation entitled "**Design and fabrication of bio-inspired multi-fingered gripper and optimization of the number of links based on multi body dynamic analysis**" in partial fulfillment of the requirement of the award of the Degree of master of technology and submitted to the Department of Mechanical Engineering of Lovely Professional University, Phagwara, is an authentic record of my own work carried out under the supervision of (Mr. Nitin Chauhan) Department of Mechanical Engineering, Lovely Professional University. The matter embodied in this dissertation has not been submitted in part or full to any other University or Institute for the award of any degree.

(30/11/2017)

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

(30/11/2017) (Mr. Nitin Chauhan)

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The external viva-voce examination of the student was held on successfully

_____

Signature of Examiner

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## Acknowledgements

To my family, teachers and friends

## DECLARATION

I hereby declare that the report of Post Graduate Thesis entitled **"Design and fabrication of bio-inspired multi-fingered gripper and optimization of the number of links based on multi body dynamic analysis"** which is being submitted to Lovely Professional University, Phagwara, in partial fulfillment of requirements for the award of the Degree of **MASTER OF TECHNOLOGY**, Department of Mechanical Engineering, is a bonafide report of the work carried out by me. The material contained in this report has not been submitted to any university or institution for the award of any degree or diploma.

Date: 30/11/2017 Student signature

Elmigdad Surageldin Abdelgafar Omer

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## Abstract

Bio-inspiration and mimicry of the nature mechanism has become a key to the innovative engineering solutions spatially in the area of robotics. In present work biomechanical analysis of human hand has been taken from literature in order to extract the important date to make a mathematical model as platform of the design of multi-fingered gripper using multi-body dynamics techniques.

# Chapter 1 Introduction

## 1.1Introduction

Bio-inspiration and mimicry the behavior of nature is considered to be the key of successful technology leading to efficient and reliable engineering solution of our daily problems, due to its dynamic behavior, complex configuration and the ability of adaptation[1]. human hand plays a vital role in the development of mankind civilization from the beginning of the history, so in order to utilize its dexterity in our dull, dirty and dangerous application which are the famous roles of robotics and farther when high accuracy is needed , developing models of human become a hot topic in recent decades[2][3] .

Designing and analyzing a complex dynamic system in areas like biomechanical, robotics and mechatronics; multi-body system dynamics analysis has found to be a powerful way to mitigate all these parts and even it gives efficient computation models in order to perform the control part[4]

in present work a biomechanical analysis model have been taken [5] and approximation made to develop the mathematical model which gives the governing dynamic equation as foundation platform for designing.

# Chapter 2 Literature Review

## 2.1 Review of literature

Multi-body dynamics (MBD) has grown during the last 20 years to become an important and effective tool in the design, prototyping, optimization and simulation of complex mechanical systems, biomechanical and multi-physics problems. MBD as one of the most active branches of applied mechanics is based on classical and analytical mechanics. A clear historical development was given by Professor Schiehlen explaining roots, the state-of-the-art and perspectives of MBD[6]

In the time period between 1997 and 2006 multi-body dynamics was acknowledged as an independent branch of theoretical, computational and applied mechanics around the globe and the research trends were discussed with respect to the subjects and countries dealing with multi-body dynamics by Professor Schiehlen in[4]

In review part at the beginning of present work research trends were analyzed as a brief extension of Professor Schiehlen work [4], with respect to areas, topics and applications as contributions published in "Multibody System Dynamics" journal in last 10 years. It has been found that the number of contributions has been increasing from 302 paper in the period (1997 to 2006) by 50% in last decade (2007 to 2016), these contributions were analyzed and summarized in Table 1 showing research trends and hot active topics also challenging topics. Hot and active topics has found to be theoretical and computational methods, flexible multi-body systems, contact and impact problems, biomechanical problems and robotics and walking machines[7],from this platform we have proceed further to biomechanical and robotics as they have identified as hot topics in multi-body dynamics.

An experimental model which gives the background of human hand from biological point of view and provide standard model especially for biomechanical analysis has been proposed based on anatomical analysis of diffrent10 specimens [8]. In same context a three-dimensional scalable biomechanical model of the four fingers of the hand to evaluate power grip has been proposed [9]

Different kinematic and dynamic models of human hand have been proposed using different approximation. One was a 24 degree of freedom kinematic model to describe to

real motion and 2 simplified hand models with 9 and 6 degrees of freedom have been developed[10]. Further, another model proposed using two sets one for thumb as 4 joints 3 link and for others as 5 joints 4 links[5].

A survey paper has given a useful conclusion about the grasping and manipulation part which is the main task of the present work, it was concluded that the problem of manipulation divided into sub parts consist of: task planning, hand design, grasping issue control and sensing part[11].Further, a guidance for practically implementation of grasp model an evaluating method for the quality of grasping and its planning algorithms has been introduced by simulating 7 different robot hands with 126 daily used object models as a quality of grasp matrix[12] . Furthermore, based on dynamic simulation a generic approach has been proposed to optimize robot grippers including gripper mechanism parameters and the geometry of finger used in the gripper combined with the grasp quality matrix[13]. Furthermore, classification of grasp model based the grasped object shape has been intruded as eight types[14] and important point the mentioned has found to be that not all types has been implemented in one algorithm and that will be a challenge and research gap.

Control and stability of grasped object has considered to be a challenging area regarding the problem of designing such type of grippers. Using dynamic analysis and the theory of controllability an examination of manipulation possibilities using grasping equilibrium under gravity has been proposed[15]. A novel formulation of grasp planning analysis and control multi-fingered robot hands using the ray-shooting technique, the proposed method may lead to algorithm used in real time application[16].

For industrial application a multi-fingered gripper with 6 degrees of freedom has been proposed using the idea of generating a closed loop kinematic chain with object and gripper fingers as a key idea [17]. Further a dynamic model of underwater applications including hydrodynamic terms has been introduced[18]. Furthermore, a 3D printed myoelectric multi-fingered hand model has been proposed for prosthetic applications[19].

## 2.2 Research gabs

From design point of view some considerations have found to be taken:

- Joint friction force effect has found to be larger than the effect of inertia and gravity forces, in order to improve the grasp model and control this consideration has to be taken[20].
- Including the disturbance due friction in control model[21].
- Considering the contact movement due to the shape of fingertip[21].
- For achieving all possible orientations and position fully actuated system has to be considered.
- Based on the object shape classifications algorithm containing all types has to be developed to deal with various object shapes[14]
- Optimizing the number of fingers in proposed hand for enhancing the grasp model[14]

## 2.3 Objective of the study

The objective of this study is:

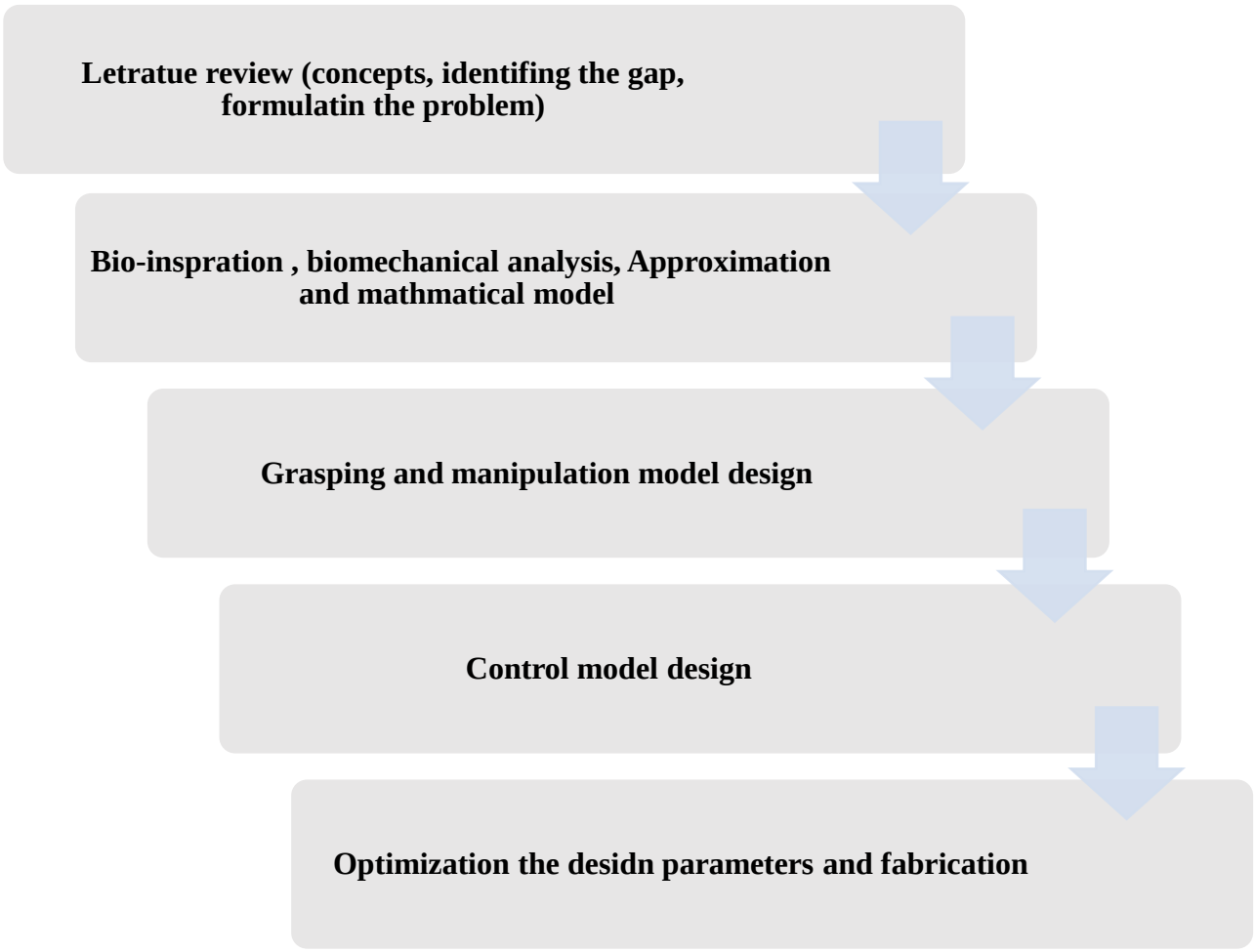
1. To develop and design multi-fingered gripper which is able to grasp and manipulate irregular shape objects.
2. To study the effect of number of links in a finger on performance of multi-fingered gripper.
3. To study the effect of number of fingers on performance of the gripper.

*Table 1 Research trends in years 2007 to 2016[7]*

| No | Topic | 2016 | 2015 | 2014 | 2013 | 2012 | 2011 | 2010 | 2009 | 2008 | 2007 | total from 2007 to 2016 | total from 1997 to 2006 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Algorithms, integration codes, software | 5 | 1 | 4 | 7 | 1 | 2 | 0 | 3 | 2 | 3 | 28 | 28 |
| 2 | Biomechanical problems | 6 | 7 | 2 | 6 | 13 | 4 | 7 | 1 | 2 | 1 | 49 | 9 |
| 3 | Contact and impact problems | 4 | 8 | 2 | 10 | 4 | 6 | 7 | 6 | 5 | 2 | 54 | 21 |
| 4 | Control and mechatronics | 2 | 4 | 3 | 3 | 1 | 1 | 1 | 2 | 0 | 1 | 18 | 17 |
| 5 | Education in multibody dynamics | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 3 |
| 6 | Experiments and numerical verification | 1 | 0 | 1 | 0 | 0 | 0 | 2 | 0 | 2 | 0 | 6 | 8 |
| 7 | Flexible multibody systems | 4 | 1 | 5 | 7 | 2 | 11 | 6 | 7 | 9 | 4 | 56 | 40 |
| 8 | Large deformation phenomena | 0 | 0 | 0 | 1 | 0 | 2 | 1 | 1 | 1 | 1 | 7 | 8 |
| 9 | Machines and mechanisms | 3 | 3 | 1 | 2 | 1 | 1 | 1 | 3 | 1 | 2 | 18 | 22 |
| 10 | Multibody dynamics as a tool for historical research | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 11 | Multiphysics problems | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 2 | 4 | 16 |
| 12 | Nonholonomic systems | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 3 | 4 |
| 13 | Optimization of mechanical systems | 0 | 2 | 1 | 5 | 2 | 1 | 1 | 0 | 2 | 1 | 15 | 14 |
| 14 | Parameter identification and data models | 0 | 2 | 1 | 5 | 2 | 1 | 0 | 1 | 0 | 2 | 14 | 5 |
| 15 | review part | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 4 | 6 | 0 |
| 16 | Robotics and walking machines | 5 | 4 | 4 | 9 | 3 | 1 | 4 | 4 | 2 | 5 | 41 | 27 |
| 17 | Simulation and virtual reality | 2 | 2 | 4 | 3 | 4 | 2 | 1 | 1 | 1 | 4 | 24 | 14 |
| 18 | Theoretical and computational methods | 1 | 4 | 6 | 19 | 6 | 8 | 8 | 8 | 8 | 11 | 79 | 41 |
| 19 | Vehicle dynamics | 2 | 1 | 0 | 3 | 4 | 5 | 1 | 5 | 4 | 3 | 28 | 25 |
| Total | | 37 | 39 | 35 | 82 | 44 | 46 | 42 | 42 | 39 | 46 | 452 | 302 |

# Chapter 3 Research Methodology

## 3.1 Research methodology flow chart




From the above flow chart, we have introduced the research methodology in which will be followed to conduct the research work.

In the literature review part, we have first analyzed the area of multi body dynamics as hot growing and active field area and it has found to be a powerful technique for design and analysis. Tends regarding multi body dynamics has been analyzed using data collected from "Multibody system dynamics" journal as highly ranked journal in this field, contributions published in this journal we have taken for the last 10 years and classified them with respect to the applications as shown in table 1.

From the above stage hot topics and research gaps have been identified and then problem and the objective of study have been formulated. Further, analysis has been done of the problem along with the approximation in order to develop the mathematical model.

Using the above outputs in the form of mathematical model as platform of the design grasp model and control model have to be developed and simulated with CAD and CAE tools to validate the obtained models results. Furthermore. Optimizing the obtained design parameters for specifying the final model specification to select the actuating system type, material of structure, manufacturing technology, control system hardware and final CAD model for fabrication.

### 3.2 Proposed work plan with timelines

Below table has introduced the summarized activities of research work through the time line as proposed work plan steps according to the research methodology.

*Table 2 Activities of research work through the time line*

| Activity | Jan-Aug | Aug | Sep | Oct | Nov | Dec | Jan | Feb | Mar |
|---|---|---|---|---|---|---|---|---|---|
| Literature review | | | | | | | | | |
| Identification of gap | | | | | | | | | |
| Problem formulation | | | | | | | | | |
| Kinematic analysis | | | | | | | | | |
| Force analysis | | | | | | | | | |
| Formulation of mathematical model for one finger | | | | | | | | | |
| Formulation of mathematical model for multiple fingers | | | | | | | | | |
| Formulation of grasp and control models, comparison of models and optimization | | | | | | | | | |
| CAD, CAE simulation of optimized model, Fabrication and testing | | | | | | | | | |

# Chapter 4 Mathematical Modeling

## 4.1 Mathematical model

Based on literature review of bio-mechanical models[5] referred to Figure 1, approximations made in order to model thumb and index fingers:

1. Thumb finger modeled as 4 joints and 4 links.
2. Index finger modeled as 3 joints and 3 links.
3. Palm approximated to be one rigid body.
4. Bone links assumed to be slender members with mass at distal end.

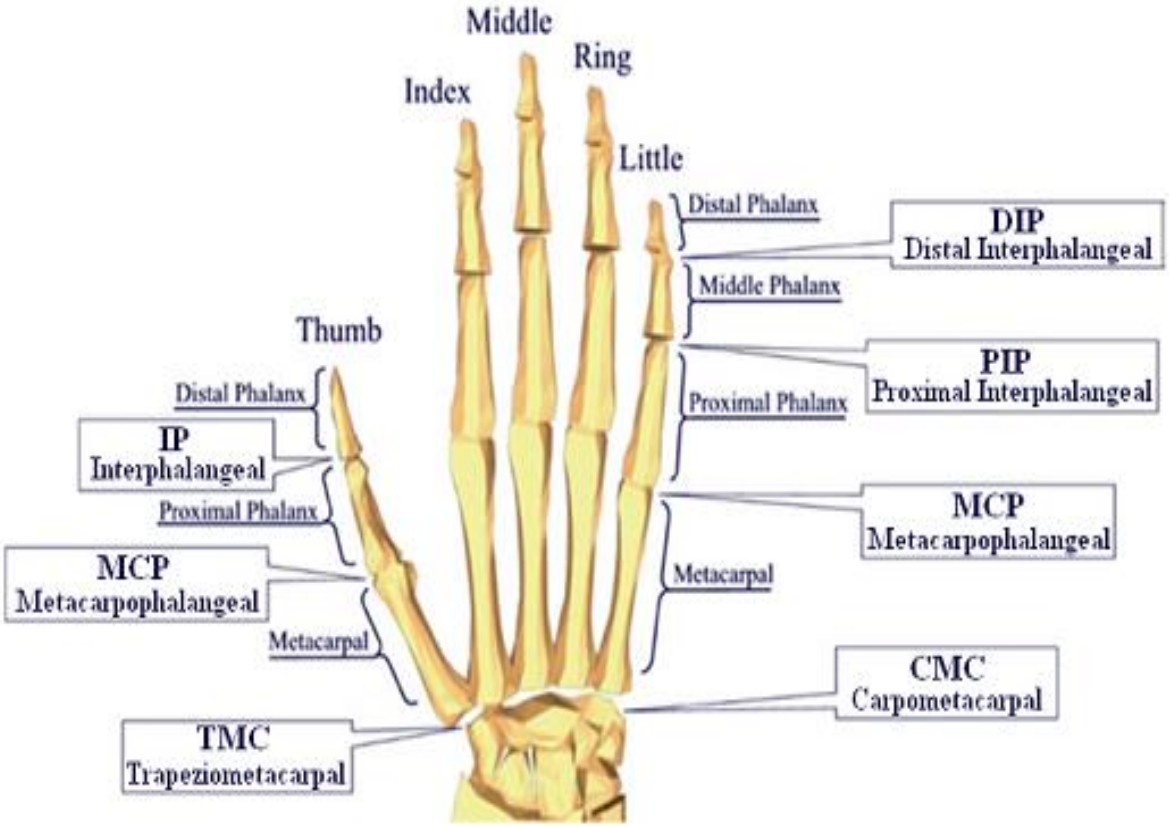



*Figure 1 Anatomical details of the hand skeleton*[5]

Using Denavit-Hartenberg formulation for obtaining kinematics models shown in figures 1, 2 below is illustrated on this Table 3:

*Table 3 DH notation of thumb and index fingers models*

| link | A(m) | Alpha(degree) | d(m) | theta(degree) |
|---|---|---|---|---|
| Thumb finger model | | | | |
| 1 | 0 | +90 | 0 | th1 |
| 2 | L1 | 0 | 0 | th2 |
| 3 | L2 | 0 | 0 | th3 |
| 4 | L3 | 0 | 0 | th4 |
| Index finger model | | | | |
| 1 | 0 | 0 | 0 | th1 |
| 2 | 0 | 0 | 0 | th2 |
| 3 | 0 | 0 | 0 | th3 |

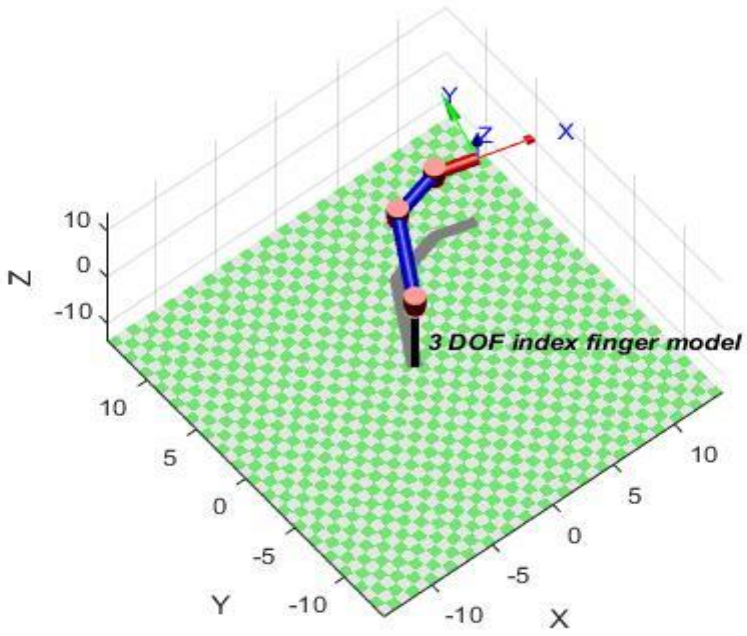



*Figure 2 3-DOF index finger model*

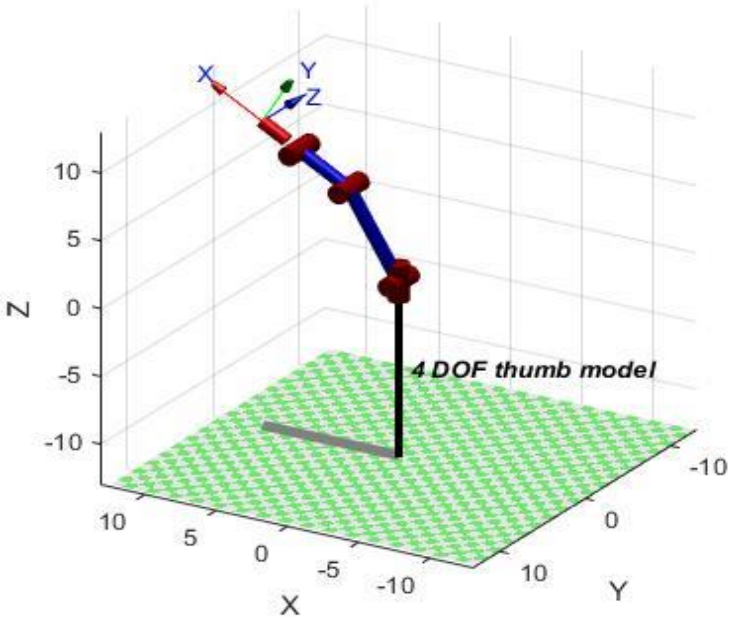



*Figure 3 4-DOF thumb finger model*

Using the above data, we obtained the forward kinematics models with the help of Matlab software:

the generalized equations are given below [22] :

$$^{i-1}\boldsymbol{T}_i = \boldsymbol{T}_z(\theta_i)\boldsymbol{T}_z(d_i)\boldsymbol{T}_x(a_i)\boldsymbol{T}_x(\alpha_i) \qquad \textit{Equation 1}$$

$$^{i-1}\boldsymbol{T}_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textit{Equation 2}$$

where $C\theta_i = \cos\ \theta_i$, $S\theta_i = \sin\ \theta_i$, $C\alpha_i = \cos\ \alpha_i$, and $S\alpha_i = \sin\ \alpha_i$.

Equation 1 and 2 called the homogenous transformation matrix used to give the information about the position and orientation of the end-effector and the origin the first 3*3 sub-matrix of the transformation matrix represents the orientation and the top right 3*1 sub-matrix represents the position and the bottom 1*3 sub-matrix represents the perspective transformation matrix and the 1*1 bottom right is the scale factor.

Then the transformation matrix of thumb:

| | | | |
|---|---|---|---|
| $C_{234}C1$ | $-S_{234}C_1$ | $S_1$ | $C_1(L2C23+L1C_2+L_3C_{234})$ |
| $C_{234}S_1$ | $-S_{234}S_1$ | $-C_1$ | $S_1(L_2C_{23}+L_1C_2+L_3C_{234})$ |
| $S_{234}$ | $C_{234}$ | 0 | $L_2S_{23}+L_1S_2+L_3S_{234}$ |
| 0 | 0 | 0 | 1 |

*Equation 3*

Where $C_{ijk}$= Cos ($th_i$ +$th_j$+ $th_k$), $S_{ijk}$= Sin ($th_i$ +$th_j$+ $th_k$).

Transformation matrix of index:

| | | | |
|---|---|---|---|
| $C_{123}$ | $-S_{123}$ | 0 | $L_2C_{12}+L_1C_1+L_3C_{123}$ |
| $S_{123}$ | $C_{123}$ | 0 | $L_2S_{12}+L_1S_1+L_3S_{123}$ |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

*Equation 4*

The differential motion model represented by Jacobin matrix is given below[22]:

$$V_e(t) = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \dot{d} \\ \dot{\theta} \end{bmatrix} = J(q)\,\dot{q}(t) \qquad \textit{Equation 5}$$

Where:

V: Cartesian velocity vector, J: Jacobin matrix, q dot: joint velocity vector

The Jacobin matrix gives the relation between the differential motion (velocity) of the joint and differential position (velocity) of the end-effector.

for Jacobin computation the below equations are used[22]:

$$J_i(q) = \begin{bmatrix} J_{vi} \\ J_{\omega i} \end{bmatrix} = \begin{cases} \begin{bmatrix} P_{i-1} \\ 0 \end{bmatrix} & \text{for a prismatic joint} \\ \begin{bmatrix} P_{i-1} \times {}^{i-1}P_n \\ P_{i-1} \end{bmatrix} & \text{for a revolute joint} \end{cases} \qquad \textit{Equation 6}$$

$$^{i-1}P_n = {}^0T_n O_n - {}^0T_{i-1}\,O_n \qquad \textbf{\textit{Equation 7}}$$

Where:

P: position vector, $O_n = [0\ 0\ 0\ 1]^T$.

Then the resulting Jacobin matrix for thumb finger is:

| | | | |
|---|---|---|---|
| $-S_1(L_2C_{23}+L_1C_2+L_3C_{234})$ | $-C_1(L_2S_{23}+L_1S_2+L_3S_{234})$ | $-C_1\ (L_2S_{23}\ L_3S_{234})$ | $-L_3S_{234}\ C_1$ |
| $C_1(L_2C_{23}+L_1C_2+L_3C_{234})$ | $-S_1(L_2S_{23}+L_1S_2+L_3S_{234})$ | $-S_1(L_2S_{23}+\ L_3S_{234}))$ | $-L_3S_{234}\ S_1$ |
| 0 | $(L_2C_{23}+L_1C_2+L_3C_{234})$ | $(L_2C_{23}+L_3C_{234})$ | $L_3C_{234}$ |
| 0 | $S_1$ | $S_1$ | $S_1$ |
| 0 | $-C_1$ | $-C_1$ | $-C_1$ |
| 1 | 0 | 0 | 0 |

*Equation 8*

Also Jacobin matrix for index finger:

| | | |
|---|---|---|
| $-\ L_2S_{12}-\ L_1S_1\ -\ L_3S_{123}$ | $-\ L_2S_{12}\ -\ L_3S_{123}$ | $-L_3S_{123}$ |
| $L_2C_{12}\ +\ L_1C_1\ +\ L_3C_{123}$ | $L_2C_{12}\ +\ L_3C_{123}$ | $L_3C_{123}$ |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 1 | 1 | 1 |

*Equation 9*

Finally, the dynamic model is obtained using Recursive Newton-Euler formulation for obtaining joint torques:

$tourque1 = \sum_{k=1}^{100} A$

*Equation 10*

$tourque2 = \sum_{k=1}^{80} B$

*Equation 11*

$tourque2 = \sum_{k=1}^{30} C$

*Equation 12*

$tourque3 = \sum_{k=1}^{4} D$

*Equation 13*

Where: the value of A, B, C and D are mentioned in appendix (1)

Note: the value of link length has taken as unity for simplification of above expressions and gravity as 10($m/sec^2$).

In section 4.2we have introduced the Matlab codes which have been used to obtain the results of mathematical models in this section

### 4.2 Matlab codes

Function for obtaining transformation matrix

```
function T=transformationL(a,alpha,d,th)
%TRANSFORMATIONL obtains transformation matrix T of link i using DH notation
%T(i) means tensformation of frame i wrt frame i-1
% if the jpint variable is th set it to 0 or if it is d set it to 1
%syms theta dis
% if q==0
%     th=th; %revolute
% elseif q==1
%     d=dis; %prismatic
% else
%     error('invalid choice of joint type')
% end
    e11=cos(th);
    e12=-sin(th)*cos(alpha);
    e13=sin(th)*sin(alpha);
    e14=a*cos(th);
    e21=sin(th);
    e22=cos(th)*cos(alpha);
    e23=-cos(th)*sin(alpha);
    e24=a*sin(th);
    e31=0;
    e32=sin(alpha);
    e33=cos(alpha);
    e34=d;
    T=[e11 e12 e13 e14
        e21 e22 e23 e24
        e31 e32 e33 e34
        0 0 0 1];
end
```

Code for obtaining forward kinematic model:

```
clc
clear
%--------forward kinematic model-----------------------------------
%---------for 4DOF thumb finger-------------------------------------
%---------using DH notation-----------------------------------------
%user must define the link and joint variables below for the specific
%problem using syms
disp('forward kinematic model for thumb finger')
syms th1 th2 th3 th4 L1 L2 L3
%q_data matrix is the inputs of joint -link parameter
%each row defines parameteres of each link
%according to this order[a,alpha,d,th]
disp('q_data matrix is joint-link parameters')
q_data=[0 pi/2 0 th1;L1 0 0 th2;L2 0 0 th3;L3 0 0 th4]
%n= represent DOF
disp('number of degree of freedom is:')
n=size(q_data,1)
disp('transformation and rotation matrix of each link:')
%transmat(:,:,i) is the transformation matrix of (i)th link
%Rmat(:,:,i) is the rotation matrix of (i)th link
 for i=1:n
transmat(:,:,i)=transformationL(q_data(i,1),q_data(i,2),q_data(i,3),q_data(i,4))
Rmat(:,:,i)=transmat(1:3,1:3,i);
 end
T=transmat(:,:,1)*transmat(:,:,2)*transmat(:,:,3)*transmat(:,:,4);
disp('the overall transformation matrix is:')
Transmat_final=simplify(T)
%---------------------------end-----------------------
```

Code for obtaining the Jacobean matrix:

```
%--------Jacobian for 4DOF thumb finger----------------
%using transformation matreces from forward kinematics model
%T0_4 means transformation of frame {4} wrt frame{0}; this will be the
%notation uesd for sud and supperscripts
%Ji(q)=[Pi-1xPi-1_n
%           Pi-1]   for revolute joints
%Pi-1_n=(T0_n*On)-(T0_i-1*On)
%On=[0 0 0 1]' , u=[0 0 1]' ,P0=[0 0 1]' ,R0_0=T0_0=[I]
%Pi-1=R0_i-1*u
%-----------------------------------------------------
T0_1=transmat(:,:,1);  T1_2=transmat(:,:,2);
T2_3=transmat(:,:,3);  T3_4=transmat(:,:,4);
a1=T0_1*T1_2;T0_2=simplify(a1); a2=T0_2*T2_3;T0_3=simplify(a2);
a3=T0_3*T3_4; T0_4=simplify(a3);
On=[0 0 0 1]';T0_0=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];
P0=[0 0 1]'; P1=T0_1(1:3,3); P2=T0_2(1:3,3); P3=T0_3(1:3,3);
a4=(T0_4*On)-(T0_0*On); a5=(T0_4*On)-(T0_1*On);
a6=(T0_4*On)-(T0_2*On); a7=(T0_4*On)-(T0_3*On);
P0_4=a4(1:3,1); P1_4=a5(1:3,1); P2_4=a6(1:3,1); P3_4=a7(1:3,1);
a8=cross(P0,P0_4); a9=cross(P1,P1_4); a10=cross(P2,P2_4); a11=cross(P3,P3_4);
a12=simplify(a8); a13=simplify(a9); a14=simplify(a10); a15=simplify(a11);
J1=[a12;P0], J2=[a13;P1], J3=[a14;P2], J4=[a15;P3]
disp('the total jacobian matrix for 4DOF thumb finger is')
J_thumb=[J1 J2 J3 J4]
%---------------------------end-----------------------
```

Code for obtaining dynamic model:

```
%--------Jacobian for 4DOF thumb finger----------------
%using transformation matreces from forward kinematics model
%T0_4 means transformation of frame {4} wrt frame{0}; this will be the
%notation uesd for sud and supperscripts
%Ji(q)=[Pi-1xPi-1_n
% Pi-1]   for revolute joints
%Pi-1_n=(T0_n*On)-(T0_i-1*On)
%On=[0 0 0 1]' , u=[0 0 1]' ,P0=[0 0 1]' ,R0_0=T0_0=[I]
%Pi-1=R0_i-1*u
%-------------------------------------------------------
T0_1=transmat(:,:,1);  T1_2=transmat(:,:,2);
T2_3=transmat(:,:,3);  T3_4=transmat(:,:,4);
a1=T0_1*T1_2;T0_2=simplify(a1); a2=T0_2*T2_3;T0_3=simplify(a2);
a3=T0_3*T3_4; T0_4=simplify(a3);
On=[0 0 0 1]';T0_0=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];
P0=[0 0 1]'; P1=T0_1(1:3,3); P2=T0_2(1:3,3); P3=T0_3(1:3,3);
a4=(T0_4*On)-(T0_0*On); a5=(T0_4*On)-(T0_1*On);
a6=(T0_4*On)-(T0_2*On); a7=(T0_4*On)-(T0_3*On);
P0_4=a4(1:3,1); P1_4=a5(1:3,1); P2_4=a6(1:3,1); P3_4=a7(1:3,1);
a8=cross(P0,P0_4); a9=cross(P1,P1_4); a10=cross(P2,P2_4); a11=cross(P3,P3_4);
a12=simplify(a8); a13=simplify(a9); a14=simplify(a10); a15=simplify(a11);
J1=[a12;P0], J2=[a13;P1], J3=[a14;P2], J4=[a15;P3]
disp('the total jacobian matrix for 4DOF thumb finger is')
J_thumb=[J1 J2 J3 J4]
%----------------------------end-----------------------
```

# Chapter 5 Expected outcome and conclusion

## 5.1 Expected outcomes

According to the objective of this study we expect, data representation about the effect of number of fingers in a gripper and the effect of number of links in a finger, using this data to fabricate a prototype of multi-fingered gripper for grasping irregular shapes.

## 5.2Conclusion

In present work we introduced the research trends in multi-body dynamics and highlighting active and hot topics in last 10 years, and they are theoretical and computational methods, flexible multi-body systems, contact and impact problems, biomechanical problems and robotics and walking machines, through this trends we took combined problem between biomechanical and robotics in order to design multi-fingered gripper, further, we have presented the mathematical model as platform of the design.

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## Appendix

The below results has been obtained using Matlab software:

```
torque1

A1 =

(5*cos(2*th1 - th2 - th3 + conj(th3)))/2 - (5*cos(2*th1 - th2 - th3 +
conj(th2)))/2 - (5*cos(th2 + th3 - 2*conj(th2) - conj(th3)))/2

A2 =

- (15*cos(th2 + th3 - conj(th2) - 2*conj(th3)))/2 - 5*cos(2*th1 + th2 -
2*conj(th2)) - 5*cos(2*th1 - th2) - 10*cos(th2 - 2*conj(th2))

A3 =

- 5*cos(th2 + th3 + th4 - conj(th2) - conj(th3)) - (5*cos(th2 + th3 + th4 -
conj(th2) - conj(th4)))/2 - (5*cos(th2 + th3 + th4 - conj(th3) - conj(th4)))/2

A4 =

(5*cos(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) - conj(th4)))/2 +
(5*cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) - conj(th3) - conj(th4)))/2 +
(5*cos(2*th1 - th2 - th3 - th4 + conj(th3) + conj(th4)))/2

A5 =

(5*cos(2*th1 - th2 - th3 - th4 + conj(th2) + conj(th4)))/2 - (5*cos(th2 + th3
+ th4 - 2*conj(th2) - conj(th3) - conj(th4)))/2

A6 =

(5*cos(2*th1 - th2 - th3 - th4 + conj(th2) + conj(th4)))/2 - (5*cos(th2 + th3
+ th4 - conj(th2) - 2*conj(th3) - conj(th4)))/2 - (5*cos(th2 + th3 + th4 -
2*conj(th2) - conj(th3) - conj(th4)))/2

A7=

- 5*cos(th2 + th4 - conj(th2) - conj(th3) - 2*conj(th4)) - 10*cos(th2)-
(5*cos(th2 + th3 - conj(th3)))/2

A8 =

(5*cos(2*th1 + th2 + th3 - 2*conj(th2) - conj(th3)))/2 - (5*cos(2*th1 + th2 +
th3 - conj(th2) - 2*conj(th3)))/2 - (15*cos(th2 + th3 - conj(th2)))/2
```

A9 =

(cos(2*th1 - 2*real(th2))*conj(thddot1))/2 + (cos(2*th1 - 2*real(th2))*conj(thddot2))/4 + (cos(th2 + th3 + th4 - conj(th2) - conj(th3) - conj(th4))*conj(thddot1))/2

A10 =

(sin(2*th1 - th2 - 2*real(th3) + conj(th2))*conj(thdot1)^2)/4 + (3*cos(th2 + th3 + th4 - conj(th2) - conj(th3) - conj(th4))*conj(thddot1))/2

A11 =

- (sin(2*th1 - th2 - 2*real(th3) + conj(th2))*conj(thdot2)^2)/4 - (sin(2*th1 - th3 - 2*real(th2) + conj(th3))*conj(thdot1)^2)/4 - (sin(2*th1 - th3 - 2*real(th2) + conj(th3))*conj(thdot2)^2)/4

A12 =

(sin(th2 - 2*th1 + 2*real(th3))*conj(thdot3)^2)/4 - (conj(thddot1)*cos(2*th1 + th2 + th3 - 3*conj(th2) - conj(th3)))/4

A13 =

(conj(thddot1)*cos(2*th1 + th2 + th3 - conj(th2) - 3*conj(th3)))/4 - (conj(thddot2)*cos(2*th1 + th2 + th3 - 3*conj(th2) - conj(th3)))/4

A14 =

-(cos(2*th1 + th2 + th3 - conj(th2) - 3*conj(th3))*(conj(thddot2) + conj(thddot3)))/4

A15 =

-(conj(thdot1)^2*(sin(2*th1 + th2 + th3 + th4 - conj(th2) - 3*conj(th3) - conj(th4)) + sin(2*th1 + th2 + th3 + th4 - 3*conj(th2) - conj(th3) - conj(th4))))/4

A16 =

-(conj(thdot2)^2*(sin(2*th1 + th2 + th3 + th4 - conj(th2) - 3*conj(th3) - conj(th4)) + sin(2*th1 + th2 + th3 + th4 - 3*conj(th2) - conj(th3) - conj(th4))))/4

```
A17 =

-(sin(2*th1 + th2 + th3 + th4 - conj(th2) - conj(th3) -
3*conj(th4))*conj(thdot2)^2)/4

A18 =

-(conj(thdot3)^2*(sin(2*th1 + th2 + th3 + th4 - conj(th2) - 3*conj(th3)) +
sin(2*th1 + th2 + th3 + th4 - conj(th2) - conj(th3) - 3*conj(th4))))/4

A19 =

- (sin(2*th1 + th2 + th3 + th4 - conj(th2) - conj(th3) -
3*conj(th4))*conj(thdot4)^2)/4 - (cos(2*th1 - th2 - th4 - 2*real(th3) +
conj(th2) + conj(th4))*conj(thddot1))/4

A20 =

(cos(2*th1 - th2 - th3 - 2*real(th4) + conj(th2) + conj(th3))*conj(thddot2))/4
+ (cos(2*th1 - th2 - th4 - 2*real(th3) + conj(th2) +
conj(th4))*conj(thddot2))/4 - (cos(2*th1 - th3 - th4 - 2*real(th2) + conj(th3)
+ conj(th4))*conj(thddot1))/4

A21 =

(cos(2*th1 - th2 - th3 - 2*real(th4) + conj(th2) + conj(th3))*conj(thddot3))/4
+ (cos(2*th1 - th2 - th4 - 2*real(th3) + conj(th2) +
conj(th4))*conj(thddot3))/4 + (cos(2*th1 - th3 - th4 - 2*real(th2) + conj(th3)
+ conj(th4))*conj(thddot2))/4

A22 =

(conj(thddot1)*cos(th2 + th3 - conj(th2))/2 + (cos(2*th1 - th2 - th3 -
2*real(th4) + conj(th2) + conj(th3))*conj(thddot4))/4

A23 =

(3*cos(th2 + th3 - conj(th2) - conj(th3))*conj(thddot1))/2 + (sin(2*th1 -
2*real(th2))*conj(thdot1)^2)/2 - (sin(2*th1 - 2*real(th2))*conj(thdot2)^2)/4

A24 =

(cos(2*th1 + th2 - 3*conj(th2))*conj(thddot1))/2 - (cos(2*th1 + th2 -
3*conj(th2))*conj(thddot2))/4 - (conj(thdot1)^2*sin(2*th1 + th2 + th3 -
3*conj(th2) - conj(th3)))/4
```

```
A25 =

(conj(thdot1)^2*sin(2*th1 + th2 + th3 - conj(th2) - 3*conj(th3)))/4 -
(conj(thdot2)^2*sin(2*th1 + th2 + th3 - conj(th2) - 3*conj(th3)))/4 -
(conj(thdot2)^2*sin(2*th1 + th2 + th3 - 3*conj(th2) - conj(th3)))/4

A26 =

(cos(2*th1 - th2 - 2*real(th3) + conj(th2))*conj(thddot1))/4 -
(conj(thdot3)^2*sin(2*th1 + th2 + th3 - conj(th2) - 3*conj(th3)))/4 -
(cos(2*th1 - th3 - 2*real(th2) + conj(th3))*conj(thddot1))/4

A26 =

(cos(2*th1 - th3 + conj(th3))*conj(thddot2))/4 + (cos(2*th1 - th2 -
2*real(th3) + conj(th2))*conj(thddot2))/4 + (cos(2*th1 - th2 - 2*real(th3) +
conj(th2))*conj(thddot3))/4

A27 =

2*cos(th2 - conj(th2))*conj(thddot1) - (sin(2*th1 - th2 - th4 - 2*real(th3) +
conj(th2) + conj(th4))*conj(thdot1)^2)/4 - (sin(2*th1 - th3 - th4 -
2*real(th2) + conj(th3) + conj(th4))*conj(thdot1)^2)/4

A28 =

-(conj(thdot2)^2*(sin(2*th1 - th2 - th3 - 2*real(th4) + conj(th2) + conj(th3))
+ sin(2*th1 - th2 - th4 - 2*real(th3) + conj(th2) + conj(th4)) + sin(2*th1 -
th3 - th4 - 2*real(th2) + conj(th3) + conj(th4))))/4

A29 =

- (sin(2*th1 - th2 - th3 - 2*real(th4) + conj(th2) +
conj(th3))*conj(thdot3)^2)/4 - (sin(2*th1 - th2 - th3 - 2*real(th4) +
conj(th2) + conj(th3))*conj(thdot4)^2)/4 - (sin(2*th1 - th2 - th4 -
2*real(th3) + conj(th2) + conj(th4))*conj(thdot3)^2)/4

A30 =

- (cos(2*th1 + th2 + th3 + th4 - conj(th2) - 3*conj(th3) -
conj(th4))*conj(thddot1))/4 - (cos(2*th1 + th2 + th3 + th4 - 3*conj(th2) -
conj(th3) - conj(th4))*conj(thddot1))/4
```

A31 =

-(conj(thddot2)*(cos(2*th1 + th2 + th3 + th4 - conj(th2) - 3*conj(th3) - conj(th4)) + cos(2*th1 + th2 + th3 + th4 - 3*conj(th2) - conj(th3) - conj(th4))))/4

A32 =

- (cos(2*th1 + th2 + th3 + th4 - conj(th2) - conj(th3) - 3*conj(th4))*conj(thddot2))/4 - (cos(2*th1 + th2 + th3 + th4 - conj(th2) - 3*conj(th3) - conj(th4))*conj(thddot3))/4

A33 =

-(cos(2*th1 + th2 + th3 + th4 - conj(th2) - conj(th3) - 3*conj(th4))*(conj(thddot3) + conj(thddot4)))/4

A34 =

(cos(th2 - conj(th3))*conj(thddot1))/2 + (sin(2*th1 + th2 - 3*conj(th2))*conj(thdot1)^2)/2 - (sin(2*th1 + th2 - 3*conj(th2))*conj(thdot2)^2)/4

A35 =

(conj(thddot1)*(cos(th2 - conj(th4)) + 2*cos(2*th1 + th2 - 2*conj(th2) - conj(th3)) + cos(2*th1 + th2 - 2*conj(th2) - conj(th4))))/4

A36 =

(cos(th2 - 2*th1 + th3)*conj(thddot2))/2 + (cos(th2 - 2*th1 + th3)*conj(thddot3))/4 - (conj(thdot2)*conj(thdot3)*sin(2*th1 + th2 + th3 - conj(th2) - 3*conj(th3)))/2

A37 =

(sin(th2 + th3 - conj(th2) - conj(th4))*conj(thdot1)^2)/4 + (cos(th2 - 2*th1 + th3 - conj(th2) + conj(th4))*conj(thddot1))/4 + (3*cos(th2 + th3 + th4 - 2*conj(th2) - conj(th3))*conj(thddot1))/4

```
A38 =

(3*cos(th2 + th3 + th4 - conj(th2) - 2*conj(th3))*conj(thddot1))/4 + (cos(th2
+ th3 + th4 - 2*conj(th2) - conj(th4))*conj(thddot1))/2 + (cos(th2 + th3 + th4
- 2*conj(th3) - conj(th4))*conj(thddot1))/2

A39 =

(3*cos(th2 + th3 + th4 - conj(th2) - 2*conj(th4)))/4 - (cos(th2 + th3 + th4 -
2*conj(th2) - conj(th3))*conj(thddot3))/4 - (cos(th2 + th3 + th4 - 2*conj(th2)
- conj(th4))*conj(thddot3))/4

A40 =

(3*cos(th2 + th3 + th4 - conj(th3) - 2*conj(th4))*conj(thddot1))/4 - (cos(th2
+ th3 + th4 - 2*conj(th2) - conj(th3))*conj(thddot4))/4

A41 =

(cos(th2 + th3 + th4 - 2*conj(th3) - conj(th4))*conj(thddot3))/4 - (cos(th2 +
th3 + th4 - conj(th2) - 2*conj(th3))*conj(thddot4))/4

 A42 =

(cos(th2 + th3 + th4 - conj(th2) - 2*conj(th4))*conj(thddot4))/4 + (cos(th2 +
th3 + th4 - conj(th3) - 2*conj(th4))*conj(thddot3))/4 + (cos(th2 + th3 + th4 -
conj(th3) - 2*conj(th4))*conj(thddot4))/4

A43 =

(sin(2*th1 + th2 + th3 - conj(th2) - 2*conj(th3) -
conj(th4))*conj(thdot1)^2)/4 - (cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
2*conj(th3) - conj(th4))*conj(thddot1))/2

A44 =

(cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) - conj(th3) -
2*conj(th4))*conj(thddot1))/(2*cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
2*conj(th3) - conj(th4))*conj(thddot2))

A45 =

- (cos(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thddot1))/4 - (cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
conj(th3) - 2*conj(th4))*conj(thddot2))/2
```

```
A46 =

- (cos(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thddot2))/2 - (cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
2*conj(th3) - conj(th4))*conj(thddot3))/4

A47 =

- (cos(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thddot3))/2 - (cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
conj(th3) - 2*conj(th4))*conj(thddot3))/4

A48 =

-(conj(thddot4)*(cos(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4)) + cos(2*th1 + th2 + th3 + th4 - 2*conj(th2) - conj(th3) -
2*conj(th4))))/4

A49 =

(conj(thdot1)^2*(sin(th2 - 2*th1 + th3 + th4) + sin(th2 - 2*th1 + th3 + th4 -
conj(th3))))/4

A50 =

(sin(th2 - 2*th1 + th3 + th4 - conj(th2))*conj(thdot2)^2)/2 + (sin(th2 - 2*th1
+ th3 + th4 - conj(th3))*conj(thdot2)^2)/2 + (sin(th2 - 2*th1 + th3 + th4 -
conj(th4))*conj(thdot1)^2)/2

A51 =

(sin(th2 - 2*th1 + th3 + th4 - conj(th2))*conj(thdot3)^2)/2 + (sin(th2 - 2*th1
+ th3 + th4 - conj(th4))*conj(thdot2)^2)/2

A52 =

(sin(th2 - 2*th1 + th3 + th4 - conj(th2))*conj(thdot4)^2)/4 + (sin(th2 - 2*th1
+ th3 + th4 - conj(th3))*conj(thdot3)^2)/4 + (sin(th2 - 2*th1 + th3 + th4 -
conj(th4))*conj(thdot3)^2)/4

A53 =

(sin(th2 - 2*th1 + th3 + th4 - conj(th3))*conj(thdot4)^2)/4 - (sin(2*th1 - th2
- th3 - 2*real(th4) + conj(th2) + conj(th3))*conj(thdot2)*conj(thdot3))/2
```

```
A54 =

- (sin(2*th1 - th2 - th3 - 2*real(th4) + conj(th2) +
conj(th3))*conj(thdot2)*conj(thdot4))/2 - (sin(2*th1 - th2 - th4 - 2*real(th3)
+ conj(th2) + conj(th4))*conj(thdot2)*conj(thdot3))/2

A55 =

- (sin(th2 - 2*conj(th2) + conj(th3))*conj(thdot1)^2)/2 - (sin(2*th1 - th2 -
th3 - 2*real(th4) + conj(th2) + conj(th3))*conj(thdot3)*conj(thdot4))/2

A56 =

(sin(th2 - conj(th3))*conj(thdot1)^2)/2 + (sin(th2 -
conj(th4))*conj(thdot1)^2)/4 + (conj(thddot1)*cos(th2 - 2*th1 + conj(th3)))/2
- (sin(th2 - 2*conj(th2) + conj(th4))*conj(thdot1)^2)/4

A57 =

(conj(thddot1)*(4*cos(th2 + th3 - 2*conj(th2)) + cos(th2 - 2*th1 + conj(th4))
+ 4*cos(th2 + th3 - 2*conj(th3))))/4

 A58 =

(conj(thddot3)*cos(th2 + th3 - 2*conj(th3)))/4 - (conj(thddot3)*cos(th2 + th3
- 2*conj(th2)))/4 + (sin(2*th1 + th2 - 2*conj(th2) -
conj(th3))*conj(thdot1)^2)/2

A59 =

(sin(2*th1 + th2 - 2*conj(th2) - conj(th4))*conj(thdot1)^2)/4 -
(conj(thddot2)*cos(2*th1 + th2 + th3 - 2*conj(th2) - 2*conj(th3)))/2

A60 =

(sin(th2 - 2*th1 + th3)*conj(thdot2)^2)/2 - (conj(thddot3)*cos(2*th1 + th2 +
th3 - 2*conj(th2) - 2*conj(th3)))/4 + (sin(th2 - 2*th1 +
th3)*conj(thdot3)^2)/4

A61 =

(conj(thddot1)*cos(th2 + th3 - conj(th2) - 2*conj(th3) + conj(th4)))/4 -
(sin(th2 - 2*th1 + th3 - conj(th2) + conj(th4))*conj(thdot1)^2)/4

 A62 =
```

```
(conj(thdot1)^2*(sin(th2 + th3 + th4 - conj(th2) - 2*conj(th3)) + sin(th2 +
th3 + th4 - 2*conj(th2) - conj(th3))))/4

 A63 =

- (sin(th2 + th3 + th4 - conj(th2) - 2*conj(th4))*conj(thdot1)^2)/4 - (sin(th2
+ th3 + th4 - 2*conj(th2) - conj(th3))*conj(thdot3)^2)/4

 A64 =

- (sin(th2 + th3 + th4 - conj(th3) - 2*conj(th4))*conj(thdot1)^2)/4 - (sin(th2
+ th3 + th4 - 2*conj(th2) - conj(th4))*conj(thdot3)^2)/4

A65 =

(sin(th2 + th3 + th4 - 2*conj(th3) - conj(th4))*conj(thdot3)^2)/4 - (sin(th2 +
th3 + th4 - 2*conj(th2) - conj(th3))*conj(thdot4)^2)/4

A66 =

(sin(th2 + th3 + th4 - conj(th3) - 2*conj(th4))*conj(thdot3)^2)/4 - (sin(th2 +
th3 + th4 - conj(th2) - 2*conj(th3))*conj(thdot4)^2)/4

A67 =

(conj(thdot4)^2*(sin(th2 + th3 + th4 - conj(th2) - 2*conj(th4)) + sin(th2 +
th3 + th4 - conj(th3) - 2*conj(th4))))/4

A68 =

- (sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) - 2*conj(th3) -
conj(th4))*conj(thdot1)^2)/2 - (sin(2*th1 - th2 - 2*real(th3) +
conj(th2))*conj(thdot2)*conj(thdot3))/2

A69 =

- (sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) - conj(th3) -
2*conj(th4))*conj(thdot1)^2)/4 - (sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
2*conj(th3) - conj(th4))*conj(thdot2)^2)/2

A70 =

- (sin(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thdot1)^2)/4 - (sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
conj(th3) - 2*conj(th4))*conj(thdot2)^2)/2
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A71 =

- (sin(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thdot2)^2)/2 - (sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
2*conj(th3) - conj(th4))*conj(thdot3)^2)/4

A72 =

- (sin(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thdot3)^2)/2 - (sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
conj(th3) - 2*conj(th4))*conj(thdot3)^2)/4

A73 =

-(conj(thdot4)^2*(sin(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4)) + sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) - conj(th3) -
2*conj(th4))))/4

A74 =

(conj(thddot1)*(cos(th2 + th3 - conj(th2) - conj(th4)) + cos(2*th1 + th2 + th3
- conj(th2) - 2*conj(th3) - conj(th4))))/4

A75 =

-(conj(thddot1)*(cos(th2 - 2*th1 + th3 + th4 - conj(th2)) + cos(th2 - 2*th1 +
th3 + th4 - conj(th3)) + 2*cos(th2 - 2*th1 + th3 + th4 - conj(th4))))/4


A76 =

(cos(th2 - 2*th1 + th3 + th4 - conj(th2))*conj(thddot2))/2 + (cos(th2 - 2*th1
+ th3 + th4 - conj(th2))*conj(thddot3))/2 + (cos(th2 - 2*th1 + th3 + th4 -
conj(th3))*conj(thddot2))/2 + (cos(th2 - 2*th1 + th3 + th4 -
conj(th4))*conj(thddot2))/2

A77 =

(cos(th2 - 2*th1 + th3 + th4 - conj(th2))*conj(thddot4))/4 + (cos(th2 - 2*th1
+ th3 + th4 - conj(th3))*conj(thddot3))/4 + (cos(th2 - 2*th1 + th3 + th4 -
conj(th4))*conj(thddot3))/4
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A78 =

(cos(th2 - 2*th1 + th3 + th4 - conj(th3))*conj(thddot4))/4 - (sin(2*th1 + th2
+ th3 + th4 - conj(th2) - 3*conj(th3) -
conj(th4))*conj(thdot2)*conj(thdot3))/2

A79 =

-(sin(2*th1 + th2 + th3 + th4 - conj(th2) - conj(th3) -
3*conj(th4))*(conj(thdot2)*conj(thdot3) + conj(thdot2)*conj(thdot4) +
conj(thdot3)*conj(thdot4)))/2

A80 =

(cos(th2 - 2*conj(th2) + conj(th3))*conj(thddot1))/2 + (cos(th2 - 2*conj(th2)
+ conj(th4))*conj(thddot1))/4 - (conj(thdot1)^2*sin(th2 - 2*th1 +
conj(th3)))/2

A81 =

(conj(thdot1)^2*(sin(th2 + th3 - 2*conj(th2)) - sin(th2 + th3 -
2*conj(th3))))/2

A82 =

(conj(thdot1)^2*sin(th2 - 2*th1 + conj(th4)))/4 - (conj(thdot3)^2*sin(th2 +
th3 - 2*conj(th2)))/4 + (conj(thdot3)^2*sin(th2 + th3 - 2*conj(th3)))/4

A83 =

-(sin(2*th1 + th2 + th3 - 2*conj(th2) - 2*conj(th3))*(2*conj(thdot2)^2 +
conj(thdot3)^2))/4

A84 =

- (sin(th2 + th3 - conj(th2) - 2*conj(th3) + conj(th4))*conj(thdot1)^2)/4 -
(sin(th2 + th3 + th4 - 2*conj(th2) - conj(th3))*conj(thdot2)*conj(thdot3))/2 -
(sin(th2 + th3 + th4 - 2*conj(th2) - conj(th4))*conj(thdot2)*conj(thdot3))/2

A85 =

(sin(th2 + th3 + th4 - 2*conj(th3) - conj(th4))*conj(thdot2)*conj(thdot3))/2 -
(sin(th2 + th3 + th4 - 2*conj(th2) - conj(th3))*conj(thdot2)*conj(thdot4))/2
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A86 =

- (sin(th2 + th3 + th4 - conj(th2) - 2*conj(th3))*conj(thdot2)*conj(thdot4))/2
- (sin(th2 + th3 + th4 - 2*conj(th2) - conj(th3))*conj(thdot3)*conj(thdot4))/2

A87 =

(sin(th2 + th3 + th4 - conj(th3) - 2*conj(th4))*conj(thdot2)*conj(thdot3))/2 -
(sin(th2 + th3 + th4 - conj(th2) - 2*conj(th3))*conj(thdot3)*conj(thdot4))/2

A88 =

(conj(thdot2)*conj(thdot4)*(sin(th2 + th3 + th4 - conj(th2) - 2*conj(th4)) +
sin(th2 + th3 + th4 - conj(th3) - 2*conj(th4))))/2

A89 =

(conj(thdot3)*conj(thdot4)*(sin(th2 + th3 + th4 - conj(th2) - 2*conj(th4)) +
sin(th2 + th3 + th4 - conj(th3) - 2*conj(th4))))/2

A90 =

-(conj(thdot2)*conj(thdot3)*(sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
conj(th3) - 2*conj(th4)) + sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
2*conj(th3) - conj(th4))))/2

A91 =

- sin(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thdot2)*conj(thdot3) - (sin(2*th1 + th2 + th3 + th4 -
2*conj(th2) - conj(th3) - 2*conj(th4))*conj(thdot2)*conj(thdot4))/2

A92 =

-(sin(2*th1 + th2 + th3 + th4 - conj(th2) - 2*conj(th3) -
2*conj(th4))*conj(thdot2)*conj(thdot4))/2

A93 =

-(conj(thdot3)*conj(thdot4)*(sin(2*th1 + th2 + th3 + th4 - conj(th2) -
2*conj(th3) - 2*conj(th4)) + sin(2*th1 + th2 + th3 + th4 - 2*conj(th2) -
conj(th3) - 2*conj(th4))))/2

A94 =

-(conj(thdot2)*conj(thdot3)*(sin(th2 + th3 - 2*conj(th2)) - sin(th2 + th3 -
2*conj(th3))))/2
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A95 =

-(conj(thdot2)*conj(thdot3)*sin(2*th1 + th2 + th3 - 2*conj(th2) -
2*conj(th3)))/2

 A96 =

sin(th2 - 2*th1 + th3 + th4 - conj(th2))*conj(thdot2)*conj(thdot3)

A97 =

(conj(thdot2)*conj(thdot3)*(sin(th2 - 2*th1 + th3 + th4 - conj(th3)) + sin(th2
- 2*th1 + th3 + th4 - conj(th4))))/2

A98 =

(conj(thdot2)*conj(thdot4)*(sin(th2 - 2*th1 + th3 + th4 - conj(th2)) + sin(th2
- 2*th1 + th3 + th4 - conj(th3))))/2


A99 =

(conj(thdot3)*conj(thdot4)*(sin(th2 - 2*th1 + th3 + th4 - conj(th2)) + sin(th2
- 2*th1 + th3 + th4 - conj(th3))))/2

A100 =

(sin(th2 - 2*th1 + th3)*conj(thdot2)*conj(thdot3))/2


torque2

B1 =

(5*cos(conj(th2) - th4 - th3 + conj(th3) + conj(th4)))/2 - (5*cos(th3 + th4 +
conj(th2) - conj(th3) - conj(th4)))/2 - (5*cos(2*th1 + th3 + th4 - conj(th2) -
conj(th3) - conj(th4)))/2

 B2 =

(5*cos(2*th1 - th3 - th4 - conj(th2) + conj(th3) + conj(th4)))/2 +
conj(thddot2) + conj(thddot3)/2
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 B3 =

conj(thddot2)/2 - (5*cos(th3 + conj(th2) - conj(th3)))/2 + conj(thddot3)/2 +
conj(thddot4)/2

B4 =

(5*cos(conj(th3))*cos(th3 - conj(th2)))/2 + (5*sin(conj(th3))*sin(th3 -
conj(th2)))/2 + 5*cos(conj(th4))*sin(conj(th3))*sin(th4) -
5*sin(conj(th3))*sin(conj(th4))*cos(th4)

B5 =

(5*cos(2*th1 - th3 - conj(th2) + conj(th3)))/2 - (5*cos(2*th1 + th3 -
conj(th2) - conj(th3)))/2 - (5*cos(2*th1 + th4 - conj(th3) - conj(th4)))/2

B6 =

(5*cos(2*th1 - th4 - conj(th3) + conj(th4)))/2 + (cos(2*th1 -
2*conj(th2))*conj(thddot2))/2 + (cos(2*th1 - 2*conj(th3))*conj(thddot2))/2

B7 =

(cos(2*th1 - 2*conj(th3))*conj(thddot3))/2 + (cos(2*th1 -
2*conj(th4))*conj(thddot2))/2 + (cos(2*th1 - 2*conj(th4))*conj(thddot3))/2

B8 =

(cos(2*th1 - 2*conj(th4))*conj(thddot4))/2 + (sin(th3 -
conj(th3))*conj(thdot1)^2)/2 + (sin(th3 - conj(th3))*conj(thdot2)^2)/2

 B9 =

(sin(th4 - conj(th4))*(conj(thdot1)^2 + conj(thdot2)^2 + conj(thdot3)^2))/2

B10 =

(sin(2*th1 + th3 - 2*conj(th2) - conj(th3))*conj(thdot1)^2)/4 - (sin(2*th1 -
th3 - 2*conj(th2) + conj(th3))*conj(thdot1)^2)/4 + (sin(2*th1 + th3 -
2*conj(th2) - conj(th3))*conj(thdot2)^2)/4

B11 =

(sin(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thdot1)^2)/4 - (sin(2*th1 -
th3 - 2*conj(th2) + conj(th3))*conj(thdot2)^2)/4 - (sin(2*th1 - th4 -
2*conj(th3) + conj(th4))*conj(thdot1)^2)/4
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B12 =

(sin(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thdot2)^2)/4 - (sin(2*th1 -
th4 - 2*conj(th3) + conj(th4))*conj(thdot2)^2)/4 + (sin(2*th1 + th4 -
2*conj(th3) - conj(th4))*conj(thdot3)^2)/4

B13 =

(cos(th3 + th4 - conj(th3) - conj(th4))*conj(thddot2))/2 - (sin(2*th1 - th4 -
2*conj(th3) + conj(th4))*conj(thdot3)^2)/4 + (cos(2*th1 + th3 + th4 -
2*conj(th2) - conj(th3) - conj(th4))*conj(thddot1))/4

B14 =

(cos(2*th1 + th3 + th4 - 2*conj(th2) - conj(th3) - conj(th4))*conj(thddot2))/4
- (cos(2*th1 - th3 - th4 - 2*conj(th2) + conj(th3) +
conj(th4))*conj(thddot1))/4

B15 =

(conj(thddot2)*(cos(2*th1 - th3 - th4 - 2*conj(th2) + conj(th3) + conj(th4)) +
2*cos(th3 - conj(th3)) + 2*cos(th4 - conj(th4))))/4

B16 =

(cos(th4 - conj(th4))*conj(thddot3))/2 + (cos(2*th1 + th3 - 2*conj(th2) -
conj(th3))*conj(thddot1))/4 - (cos(2*th1 - th3 - 2*conj(th2) +
conj(th3))*conj(thddot1))/4

B17 =

(cos(2*th1 + th3 - 2*conj(th2) - conj(th3))*conj(thddot2))/4 + (cos(2*th1 -
th3 - 2*conj(th2) + conj(th3))*conj(thddot2))/4 + (cos(2*th1 + th4 -
2*conj(th3) - conj(th4))*conj(thddot1))/4

B18 =

(cos(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thddot2))/4 - (cos(2*th1 -
th4 - 2*conj(th3) + conj(th4))*conj(thddot1))/4 + (cos(2*th1 - th4 -
2*conj(th3) + conj(th4))*conj(thddot2))/4

B19 =

(conj(thddot3)*(cos(2*th1 + th4 - 2*conj(th3) - conj(th4)) + cos(2*th1 - th4 -
2*conj(th3) + conj(th4))))/4
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B20 =

(sin(2*th1 + th3 + th4 - 2*conj(th2) - conj(th3) -
conj(th4))*conj(thdot2)^2)/4 - (sin(2*th1 - th3 - th4 - 2*conj(th2) +
conj(th3) + conj(th4))*conj(thdot1)^2)/4

B21 =

- (cos(th3 - conj(th2))*conj(thddot1))/4 - (sin(2*th1 - th3 - th4 -
2*conj(th2) + conj(th3) + conj(th4))*conj(thdot2)^2)/4


B22 =

(cos(th3 - conj(th2))*conj(thddot2))/4 + (cos(th3 -
conj(th2))*conj(thddot3))/4 - (cos(th4 - conj(th3))*conj(thddot1))/4 +
(cos(th4 - conj(th3))*conj(thddot2))/4

B23 =

(cos(th4 - conj(th3))*conj(thddot3))/4 + (cos(th4 -
conj(th3))*conj(thddot4))/4 + (cos(2*th1 + th3 - conj(th2) -
2*conj(th3))*conj(thddot1))/4

B24 =

(cos(2*th1 + th3 - conj(th2) - 2*conj(th3))*(conj(thddot2) + conj(thddot3)))/4

B25 =

(cos(2*th1 + th4 - conj(th3) - 2*conj(th4))*(conj(thddot1) + conj(thddot2) +
conj(thddot3)))/4

B26 =

(cos(2*th1 + th4 - conj(th3) - 2*conj(th4))*conj(thddot4))/4 - (cos(th4 +
conj(th2) - conj(th3) - conj(th4))*conj(thddot1))/4

B27 =

(cos(th4 - conj(th2) + conj(th3) - conj(th4))*conj(thddot1))/4 + (cos(th4 +
conj(th2) - conj(th3) - conj(th4))*conj(thddot2))/4
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B28 =

(cos(th4 - conj(th2) + conj(th3) - conj(th4))*conj(thddot2))/4 + (sin(th3 +
th4 - conj(th2) - conj(th3))*conj(thdot1)^2)/4 + (sin(th3 + th4 - conj(th2) -
conj(th4))*conj(thdot1)^2)/4

B29 =

(sin(th3 + th4 - conj(th2) - conj(th3))*conj(thdot2)^2)/4 + (sin(th3 + th4 -
conj(th2) - conj(th3))*conj(thdot3)^2)/4 + (sin(th3 + th4 - conj(th2) -
conj(th4))*conj(thdot2)^2)/4

B30 =

(sin(th3 + th4 - conj(th2) - conj(th3))*conj(thdot4)^2)/4 + (sin(th3 + th4 -
conj(th2) - conj(th4))*conj(thdot3)^2)/4 - (cos(th3 - 2*th1 + th4 + conj(th2)
- conj(th3))*conj(thddot1))/4

B31 =

(cos(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thddot2))/4 - (cos(th3 -
2*th1 + th4 + conj(th2) - conj(th4))*conj(thddot1))/4 + (cos(th3 - 2*th1 + th4
+ conj(th2) - conj(th4))*conj(thddot2))/4

B32 =

(conj(thddot3)*(cos(th3 - 2*th1 + th4 + conj(th2) - conj(th3)) + cos(th3 -
2*th1 + th4 + conj(th2) - conj(th4))))/4

B33 =

(cos(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thddot4))/4 + (sin(2*th1
+ th3 + th4 - conj(th2) - 2*conj(th3) - conj(th4))*conj(thdot1)^2)/4

B34 =

(sin(2*th1 + th3 + th4 - conj(th2) - conj(th3) -
2*conj(th4))*conj(thdot1)^2)/4 + (sin(2*th1 + th3 + th4 - conj(th2) -
2*conj(th3) - conj(th4))*conj(thdot2)^2)/4

B35 =

(sin(2*th1 + th3 + th4 - conj(th2) - conj(th3) -
2*conj(th4))*conj(thdot2)^2)/4 + (sin(2*th1 + th3 + th4 - conj(th2) -
2*conj(th3) - conj(th4))*conj(thdot3)^2)/4
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B36 =

(sin(2*th1 + th3 + th4 - conj(th2) - conj(th3) - 2*conj(th4))*(conj(thdot3)^2
+ conj(thdot4)^2))/4


B37 =

(sin(th3 + conj(th2) - 2*conj(th3))*(conj(thdot1)^2 + conj(thdot2)^2 +
conj(thdot3)^2))/4

 B38 =

(sin(th4 + conj(th3) - 2*conj(th4))*(conj(thdot1)^2 + conj(thdot2)^2 +
conj(thdot3)^2))/4

B39 =

(cos(conj(th2) - conj(th3))*conj(thddot2))/2 + (cos(conj(th2) -
conj(th4))*conj(thddot2))/2 + (sin(th4 + conj(th3) -
2*conj(th4))*conj(thdot4)^2)/4

B40 =

(cos(conj(th3) - conj(th4))*conj(thddot2))/2 + (cos(conj(th3) -
conj(th4))*conj(thddot3))/2 + (sin(th3 - conj(th2))*conj(thdot1)^2)/4

B41 =

(sin(th3 - conj(th2))*conj(thdot2)^2)/4 + (sin(th3 -
conj(th2))*conj(thdot3)^2)/4 + (sin(th4 - conj(th3))*conj(thdot1)^2)/4

B42 =

(sin(th4 - conj(th3))*(conj(thdot2)^2 + conj(thdot3)^2 + conj(thdot4)^2))/4

B43 =

(cos(th3 - 2*th1 + conj(th2))*(conj(thddot2) - conj(thddot1) +
conj(thddot3)))/4

B44 =

(cos(th4 - 2*th1 + conj(th3))*(conj(thddot2) - conj(thddot1) +
conj(thddot3)))/4
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B45 =

(conj(thddot4)*cos(th4 - 2*th1 + conj(th3)))/4 + (sin(2*th1 + th3 - conj(th2)
- 2*conj(th3))*conj(thdot1)^2)/4

B46 =

(sin(2*th1 + th3 - conj(th2) - 2*conj(th3))*conj(thdot2)^2)/4 + (sin(2*th1 +
th3 - conj(th2) - 2*conj(th3))*conj(thdot3)^2)/4 + (sin(2*th1 + th4 -
conj(th3) - 2*conj(th4))*conj(thdot1)^2)/4

B47 =

(sin(2*th1 + th4 - conj(th3) - 2*conj(th4))*(conj(thdot2)^2 + conj(thdot3)^2 +
conj(thdot4)^2))/4

B48 =

(sin(th4 + conj(th2) - conj(th3) - conj(th4))*conj(thdot1)^2)/4 + (sin(th4 -
conj(th2) + conj(th3) - conj(th4))*conj(thdot1)^2)/4 + (sin(th4 + conj(th2) -
conj(th3) - conj(th4))*conj(thdot2)^2)/4

B49 =

(sin(th4 - conj(th2) + conj(th3) - conj(th4))*conj(thdot2)^2)/4 -
(conj(thddot1)*cos(th4 - 2*th1 + conj(th2) + conj(th3) - conj(th4)))/4 +
(conj(thddot1)*cos(conj(th2) - th4 - 2*th1 + conj(th3) + conj(th4)))/4

B50 =

(conj(thddot2)*cos(th4 - 2*th1 + conj(th2) + conj(th3) - conj(th4)))/4 +
(conj(thddot2)*cos(conj(th2) - th4 - 2*th1 + conj(th3) + conj(th4)))/4 +
(conj(thddot1)*cos(th3 + th4 + conj(th2) - 2*conj(th3) - conj(th4)))/4

 B51 =

(conj(thddot1)*cos(th3 + th4 + conj(th2) - conj(th3) - 2*conj(th4)))/4 +
(conj(thddot2)*cos(th3 + th4 + conj(th2) - 2*conj(th3) - conj(th4)))/4 +
(conj(thddot3)*cos(th3 + th4 + conj(th2) - 2*conj(th3) - conj(th4)))/4

B52 =

(cos(th3 + th4 + conj(th2) - conj(th3) - 2*conj(th4))*(conj(thddot2) +
conj(thddot3) + conj(thddot4)))/4
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B53 =

(sin(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thdot1)^2)/4 + (sin(th3 -
2*th1 + th4 + conj(th2) - conj(th3))*conj(thdot2)^2)/4 + (sin(th3 - 2*th1 +
th4 + conj(th2) - conj(th4))*conj(thdot1)^2)/4

B54 =

(sin(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thdot3)^2)/4 + (sin(th3 -
2*th1 + th4 + conj(th2) - conj(th4))*conj(thdot2)^2)/4 + (sin(th3 - 2*th1 +
th4 + conj(th2) - conj(th4))*conj(thdot3)^2)/4

B55 =

(sin(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thdot4)^2)/4 + sin(th4 -
conj(th4))*conj(thdot2)*conj(thdot3) + (sin(2*th1 + th4 - 2*conj(th3) -
conj(th4))*conj(thdot2)*conj(thdot3))/2 - (sin(2*th1 - th4 - 2*conj(th3) +
conj(th4))*conj(thdot2)*conj(thdot3))/2

B56 =

(cos(th3 + th4 - conj(th2) - conj(th3))*conj(thddot2))/4 - (cos(th3 + th4 -
conj(th2) - conj(th3))*conj(thddot1))/4 - (cos(th3 + th4 - conj(th2) -
conj(th4))*conj(thddot1))/4

B57 =

(cos(th3 + th4 - conj(th2) - conj(th3))*conj(thddot3))/4 + (cos(th3 + th4 -
conj(th2) - conj(th4))*conj(thddot2))/4 + (cos(th3 + th4 - conj(th2) -
conj(th4))*conj(thddot3))/4

B58 =

(cos(th3 + th4 - conj(th2) - conj(th3))*conj(thddot4))/4 + (cos(2*th1 + th3 +
th4 - conj(th2) - conj(th3) - 2*conj(th4))*conj(thddot1))/4 + (cos(2*th1 + th3
+ th4 - conj(th2) - 2*conj(th3) - conj(th4))*conj(thddot1))/4

B59 =

(cos(2*th1 + th3 + th4 - conj(th2) - conj(th3) - 2*conj(th4))*conj(thddot2))/4
+ (cos(2*th1 + th3 + th4 - conj(th2) - 2*conj(th3) -
conj(th4))*conj(thddot2))/4 + (cos(2*th1 + th3 + th4 - conj(th2) - 2*conj(th3)
- conj(th4))*conj(thddot3))/4
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B60 =

(cos(2*th1 + th3 + th4 - conj(th2) - conj(th3) - 2*conj(th4))*(conj(thddot3) +
conj(thddot4)))/4

B61 =

(cos(conj(th2) - 2*th1 + conj(th3))*conj(thddot2))/2 + (cos(conj(th2) - 2*th1
+ conj(th4))*conj(thddot2))/2 + (cos(th3 + conj(th2) -
2*conj(th3))*conj(thddot1))/4 + (cos(th3 + conj(th2) -
2*conj(th3))*conj(thddot2))/4

B62 =

(cos(conj(th3) - 2*th1 + conj(th4))*conj(thddot2))/2 + (cos(th3 + conj(th2) -
2*conj(th3))*conj(thddot3))/4 + (cos(conj(th3) - 2*th1 +
conj(th4))*conj(thddot3))/2 + (cos(th4 + conj(th3) -
2*conj(th4))*conj(thddot1))/4

B63 =

(cos(th4 + conj(th3) - 2*conj(th4))*conj(thddot2))/4 + (cos(th4 + conj(th3) -
2*conj(th4))*conj(thddot3))/4 + (cos(th4 + conj(th3) -
2*conj(th4))*conj(thddot4))/4 + (conj(thdot1)^2*sin(th3 - 2*th1 +
conj(th2)))/4

B64 =

(conj(thdot2)^2*sin(th3 - 2*th1 + conj(th2)))/4 + (conj(thdot1)^2*sin(th4 -
2*th1 + conj(th3)))/4 + (conj(thdot3)^2*sin(th3 - 2*th1 + conj(th2)))/4 +
(conj(thdot2)^2*sin(th4 - 2*th1 + conj(th3)))/4

B65 =

(conj(thdot3)^2*sin(th4 - 2*th1 + conj(th3)))/4 + (conj(thdot4)^2*sin(th4 -
2*th1 + conj(th3)))/4 + (sin(th4 - 2*th1 + conj(th2) + conj(th3) -
conj(th4))*conj(thdot1)^2)/4

B66 =

(sin(th4 - 2*th1 + conj(th2) + conj(th3) - conj(th4))*conj(thdot2)^2)/4 -
(sin(conj(th2) - th4 - 2*th1 + conj(th3) + conj(th4))*conj(thdot1)^2)/4
```

```
B67 =

(sin(th3 + th4 + conj(th2) - conj(th3) - 2*conj(th4))*conj(thdot1)^2)/4 -
(sin(conj(th2) - th4 - 2*th1 + conj(th3) + conj(th4))*conj(thdot2)^2)/4 +
(sin(th3 + th4 + conj(th2) - 2*conj(th3) - conj(th4))*conj(thdot1)^2)/4

B68 =

((sin(th3 + th4 + conj(th2) - conj(th3) - 2*conj(th4)) + sin(th3 + th4 +
conj(th2) - 2*conj(th3) - conj(th4)))*(conj(thdot2)^2 + conj(thdot3)^2))/4

B69 =

(sin(th3 + th4 + conj(th2) - conj(th3) - 2*conj(th4))*conj(thdot4)^2)/4 +
(sin(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thdot2)*conj(thdot3))/2 +
(sin(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thdot2)*conj(thdot4))/2 +
(sin(th3 - 2*th1 + th4 + conj(th2) - conj(th4))*conj(thdot2)*conj(thdot3))/2 +
(sin(th3 - 2*th1 + th4 + conj(th2) - conj(th3))*conj(thdot3)*conj(thdot4))/2

B70 =

(conj(thdot2)*(conj(thdot3)*sin(th3 - 2*th1 + conj(th2)) +
conj(thdot3)*sin(th4 - 2*th1 + conj(th3)) + conj(thdot4)*sin(th4 - 2*th1 +
conj(th3))))/2

B71 =

(conj(thdot3)*conj(thdot4)*sin(th4 - 2*th1 + conj(th3)))/2 + (sin(th3 + th4 +
conj(th2) - 2*conj(th3) - conj(th4))*conj(thdot2)*conj(thdot3))/2

B72 =

(sin(th3 + th4 + conj(th2) - conj(th3) -
2*conj(th4))*(conj(thdot2)*conj(thdot3) + conj(thdot2)*conj(thdot4) +
conj(thdot3)*conj(thdot4)))/2

B73 =

(sin(th3 + th4 - conj(th2) - conj(th3))*conj(thdot2)*conj(thdot3))/2 +
(sin(th3 + th4 - conj(th2) - conj(th3))*conj(thdot2)*conj(thdot4))/2 +
(sin(th3 + th4 - conj(th2) - conj(th4))*conj(thdot2)*conj(thdot3))/2
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B74 =

(sin(th3 + th4 - conj(th2) - conj(th3))*conj(thdot3)*conj(thdot4))/2 +
(sin(2*th1 + th3 + th4 - conj(th2) - 2*conj(th3) -
conj(th4))*conj(thdot2)*conj(thdot3))/2

B75 =

(sin(2*th1 + th3 + th4 - conj(th2) - conj(th3) -
2*conj(th4))*conj(thdot2)*(conj(thdot3) + conj(thdot4)))/2

B76 =

(sin(2*th1 + th3 + th4 - conj(th2) - conj(th3) -
2*conj(th4))*conj(thdot3)*conj(thdot4))/2 + (sin(th3 + conj(th2) -
2*conj(th3))*conj(thdot2)*conj(thdot3))/2

B77 =

(sin(th4 + conj(th3) - 2*conj(th4))*conj(thdot2)*(conj(thdot3) +
conj(thdot4)))/2


B78 =

(conj(thdot3)*(sin(th4 + conj(th3) - 2*conj(th4))*conj(thdot4) + sin(th3 -
conj(th2))*conj(thdot2)))/2

B79 =

(sin(th4 - conj(th3))*(conj(thdot2)*conj(thdot3) + conj(thdot2)*conj(thdot4) +
conj(thdot3)*conj(thdot4)))/2

B80 =

(sin(2*th1 + th3 - conj(th2) - 2*conj(th3))*conj(thdot2)*conj(thdot3))/2 +
(sin(2*th1 + th4 - conj(th3) - 2*conj(th4))*conj(thdot2)*conj(thdot3))/2 +
(sin(2*th1 + th4 - conj(th3) - 2*conj(th4))*conj(thdot2)*conj(thdot4))/2 +
(sin(2*th1 + th4 - conj(th3) - 2*conj(th4))*conj(thdot3)*conj(thdot4))/2
```

torque3

C1 =

conj(thddot2) - (5*cos(th4 + conj(th3) - conj(th4)))/2 + conj(thddot3) + conj(thddot4)/2

C2 =

(5*cos(conj(th3) - th4 + conj(th4)))/2 - (5*cos(2*th1 + th4 - conj(th3) - conj(th4)))/2 + (5*cos(2*th1 - th4 - conj(th3) + conj(th4)))/2 + (cos(2*th1 - 2*conj(th3))*conj(thddot2))/2

C3 =

(cos(2*th1 - 2*conj(th3))*conj(thddot3))/2 + (cos(2*th1 - 2*conj(th4))*conj(thddot2))/2 + (cos(2*th1 - 2*conj(th4))*conj(thddot3))/2

C4 =

(cos(2*th1 - 2*conj(th4))*conj(thddot4))/2 + (sin(th4 - conj(th4))*conj(thdot1)^2)/2 + (sin(th4 - conj(th4))*conj(thdot2)^2)/2 + (sin(th4 - conj(th4))*conj(thdot3)^2)/2

C5 =

(sin(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thdot1)^2)/4 - (sin(2*th1 - th4 - 2*conj(th3) + conj(th4))*conj(thdot1)^2)/4 + (sin(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thdot2)^2)/4

C6 =

(sin(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thdot3)^2)/4 - (sin(2*th1 - th4 - 2*conj(th3) + conj(th4))*conj(thdot2)^2)/4 - (sin(2*th1 - th4 - 2*conj(th3) + conj(th4))*conj(thdot3)^2)/4

C7 =

(cos(th4 - conj(th4))*conj(thddot2))/2 + (cos(th4 - conj(th4))*conj(thddot3))/2 + (cos(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thddot1))/4 - (cos(2*th1 - th4 - 2*conj(th3) + conj(th4))*conj(thddot1))/4

```
C8 =

(cos(2*th1 + th4 - 2*conj(th3) - conj(th4))*conj(thddot2))/4 + (cos(2*th1 -
th4 - 2*conj(th3) + conj(th4))*conj(thddot2))/4 + (cos(2*th1 + th4 -
2*conj(th3) - conj(th4))*conj(thddot3))/4

C9 =

(cos(th4 - conj(th3))*conj(thddot2))/4 - (cos(th4 -
conj(th3))*conj(thddot1))/4 + (cos(2*th1 - th4 - 2*conj(th3) +
conj(th4))*conj(thddot3))/4

C10 =

(cos(th4 - conj(th3))*conj(thddot3))/4 + (cos(th4 -
conj(th3))*conj(thddot4))/4 + (cos(2*th1 + th4 - conj(th3) -
2*conj(th4))*conj(thddot1))/4

C11 =

(cos(2*th1 + th4 - conj(th3) - 2*conj(th4))*(conj(thddot2) + conj(thddot3) +
conj(thddot4)))/4

C12 =

(cos(th4 - conj(th2) + conj(th3) - conj(th4))*conj(thddot1))/4 - (cos(th4 +
conj(th2) - conj(th3) - conj(th4))*conj(thddot1))/4 + (cos(th4 + conj(th2) -
conj(th3) - conj(th4))*conj(thddot2))/4

C13 =

(cos(th4 - conj(th2) + conj(th3) - conj(th4))*conj(thddot2))/4 + (sin(th4 +
conj(th3) - 2*conj(th4))*conj(thdot1)^2)/4 + (sin(th4 + conj(th3) -
2*conj(th4))*conj(thdot2)^2)/4

C14 =

(cos(conj(th2) - conj(th3))*conj(thddot2))/2 + (sin(th4 + conj(th3) -
2*conj(th4))*conj(thdot3)^2)/4 + (sin(th4 + conj(th3) -
2*conj(th4))*conj(thdot4)^2)/4
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C15 =

(cos(conj(th2) - conj(th4))*conj(thddot2))/2 + (cos(conj(th3) -
conj(th4))*conj(thddot2))/2 + (cos(conj(th3) - conj(th4))*conj(thddot3))/2 +
(sin(th4 - conj(th3))*conj(thdot1)^2)/4

C16 =

(sin(th4 - conj(th3))*conj(thdot2)^2)/4 + (sin(th4 -
conj(th3))*conj(thdot3)^2)/4 + (sin(th4 - conj(th3))*conj(thdot4)^2)/4 -
(conj(thddot1)*cos(th4 - 2*th1 + conj(th3)))/4

C17 =

(cos(th4 - 2*th1 + conj(th3))*(conj(thddot2) + conj(thddot3) +
conj(thddot4)))/4

C18 =

(sin(2*th1 + th4 - conj(th3) - 2*conj(th4))*(conj(thdot1)^2 + conj(thdot2)^2 +
conj(thdot3)^2))/4

C19 =

(sin(2*th1 + th4 - conj(th3) - 2*conj(th4))*conj(thdot4)^2)/4 + (sin(th4 +
conj(th2) - conj(th3) - conj(th4))*conj(thdot1)^2)/4

C20 =

(sin(th4 - conj(th2) + conj(th3) - conj(th4))*conj(thdot1)^2)/4 + (sin(th4 +
conj(th2) - conj(th3) - conj(th4))*conj(thdot2)^2)/4 + (sin(th4 - conj(th2) +
conj(th3) - conj(th4))*conj(thdot2)^2)/4

C21 =

(conj(thddot1)*cos(conj(th2) - th4 - 2*th1 + conj(th3) + conj(th4)))/4 -
(conj(thddot1)*cos(th4 - 2*th1 + conj(th2) + conj(th3) - conj(th4)))/4 +
(conj(thddot2)*cos(th4 - 2*th1 + conj(th2) + conj(th3) - conj(th4)))/4

C22 =

(conj(thddot2)*cos(conj(th2) - th4 - 2*th1 + conj(th3) + conj(th4)))/4 +
sin(th4 - conj(th4))*conj(thdot2)*conj(thdot3) + (sin(2*th1 + th4 -
2*conj(th3) - conj(th4))*conj(thdot2)*conj(thdot3))/2
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C23 =

(cos(conj(th2) - 2*th1 + conj(th3))*conj(thddot2))/2 + (cos(conj(th2) - 2*th1
+ conj(th4))*conj(thddot2))/2 - (sin(2*th1 - th4 - 2*conj(th3) +
conj(th4))*conj(thdot2)*conj(thdot3))/2

C24 =

(cos(conj(th3) - 2*th1 + conj(th4))*conj(thddot2))/2 + (cos(conj(th3) - 2*th1
+ conj(th4))*conj(thddot3))/2 + (cos(th4 + conj(th3) -
2*conj(th4))*conj(thddot1))/4

C25 =

(cos(th4 + conj(th3) - 2*conj(th4))*(conj(thddot2) + conj(thddot3) +
conj(thddot4)))/4

C26 =

(sin(th4 - 2*th1 + conj(th3))*(conj(thdot1)^2 + conj(thdot2)^2 +
conj(thdot3)^2 + conj(thdot4)^2))/4

C27 =

(sin(th4 - 2*th1 + conj(th2) + conj(th3) - conj(th4))*conj(thdot1)^2)/4 -
(sin(conj(th2) - th4 - 2*th1 + conj(th3) + conj(th4))*conj(thdot1)^2)/4 +
(sin(th4 - 2*th1 + conj(th2) + conj(th3) - conj(th4))*conj(thdot2)^2)/4

C28 =

(conj(thdot2)*conj(thdot3)*sin(th4 - 2*th1 + conj(th3)))/2 - (sin(conj(th2) -
th4 - 2*th1 + conj(th3) + conj(th4))*conj(thdot2)^2)/4 +
(conj(thdot2)*conj(thdot4)*sin(th4 - 2*th1 + conj(th3)))/2 +
(conj(thdot3)*conj(thdot4)*sin(th4 - 2*th1 + conj(th3)))/2

C29 =

(sin(th4 + conj(th3) - 2*conj(th4))*(conj(thdot2)*conj(thdot3) +
conj(thdot2)*conj(thdot4) + conj(thdot3)*conj(thdot4)))/2

C30 =

((sin(th4 - conj(th3)) + sin(2*th1 + th4 - conj(th3) -
2*conj(th4)))*(conj(thdot2)*conj(thdot3) + conj(thdot2)*conj(thdot4) +
conj(thdot3)*conj(thdot4)))/2
```

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torque4

D1 =

conj(thddot2)/2 + conj(thddot3)/2 + conj(thddot4)/2 + (cos(2*th1 -
2*conj(th4))*conj(thddot2))/2

 D2 =

(cos(2*th1 - 2*conj(th4))*conj(thddot3))/2 + (cos(2*th1 -
2*conj(th4))*conj(thddot4))/2 + (cos(conj(th2) - conj(th4))*conj(thddot2))/2


D3 =

(cos(conj(th3) - conj(th4))*(conj(thddot2) + conj(thddot3)))/2

D4 =

(cos(conj(th2) - 2*th1 + conj(th4))*conj(thddot2))/2 + (cos(conj(th3) - 2*th1
+ conj(th4))*conj(thddot2))/2 + (cos(conj(th3) - 2*th1 +
conj(th4))*conj(thddot3))/2
```