SOME DYNAMICAL PROBLEMS IN MAGNETO MICROPOLAR THERMO ELASTICITY

A Thesis submitted to the Lovely Professional University

For the award of

Doctor of Philosophy

in
Department of Mathematics

BY

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Supervised by

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June 2015

Declaration

I declare that the thesis entitled **Some dynamical problems in magneto micropolar thermo elasticity** has been prepared by me under the guidance of Dr. Ranjit Singh, Associate Professor, Department of Mathematics, S.G.A.D. Govt. College, Punjab. No part of thesis has formed the basis for the award of any degree or fellowship previously.

Varun Kumar

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Certificate

I certify that Varun Kumar has prepared his thesis entitled **SOME DYNAMICAL PROBLEMS IN MAGNETO MICROPOLAR THERMO ELASTICITY** for the award of Ph.D. Degree of the Lovely Professional University under my guidance. He has carried out the work at the Department of Mathematics, Lovely Professional University.

Advisor

Dr.Ranjit Singh

Department of Mathematics,

S.G.A.D. Govt. College,

Tarn Taran, Punjab.

DATE:

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I certify that work presented in thesis **SOME DYNAMICAL PROBLEMS IN MAGNETO MICROPOLAR THERMO ELASTICITY** for the award of Ph.D. Degree of Lovely Professional University by Mr. Varun Kumar under my guidance. He has carried out the work at the Department of Mathematics, Lovely Professional University.

Co-Advisor

Dr.Sanjay Mishra

Department of Mathematics,

Lovely Professional University,

Punjab.

DATE:

Abstract

The theory of the elasticity undoubtedly can be regarded as one of the most important branch of solid mechanics which deals with the stresses and deformations produced in elastic media under the action of external forces or due to temperature gradients. Hooke's law forms the core of entire theory of elasticity. But its failure to explain response of materials like polymers, fibrous materials, coarse grains or any material in which its microstructure plays important role, lead many researchers to focus on new theory which can remain consistent with experimental observations. Breakthrough achievement in this regard can be considered as development of micropolar theory of elasticity. Micropolar theory assumes materials to be made up of small dumb-well like interconnected molecules, which can undergo rotational motion independently in addition to translational motion. Later on, this theory was extended to include thermal and electromagnetic effects to explain the elastic response of material subjected to thermal or magnetic source. This thesis comprises of five chapters containing the detailed analysis of elastic media subjected to different sources. Response of homogenous isotropic media placed in magnetic field and subjected to thermal and mechanical sources has been investigated in this study. Integral transformations have been applied to simplify system of partial differential equations. Use of numerical inversion technique has been done to obtain the solution in physical domain from frequency domain. Graphical analysis has been done at the end of each chapter to explain the outcome of study. First chapter contains brief developments in the theory of elasticity. Starting from Galileo's study to two major breakthroughs in the history of elasticity namely; Hooke's law and Navier's general equations and then recent developments in this field. It acknowledges the contribution of modern elasticians like Eringen and Nowacki. Second chapter contains solution of an axi-symmetric problem in infinite space using Laplace and Hankel transforms. Mechanical source was applied in the presence of transverse magnetic field. Next chapter highlights the application of a two dimensional plane strain generalized magneto micropolar thermoelastic model to analyse the effect of concentrated force on perfectly conducting media. A combination of Laplace and Fourier transform has been exercised to solve system of resulting partial differential equations. Fourth chapter includes the impact of rotation on generalized magneto micropolar thermoelastic medium. An eigen value

approach instead of usual treatment for solving system of differential equations has been applied. Such elastic models can be very useful in analysing the planetary motion. Final chapter investigates the effect of interaction of electromagnetic, elastic and thermal field by using modified Ohm's and Fourier's law. Both thermal and mechanical sources have been applied to magneto micropolar thermoelastic medium. Special cases of interest for continuous and concentrated source have been discussed to explain the utility of the approach.

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Foremost, I express my deepest gratitude to my advisor Dr. Ranjit Singh, Associate Professor, Department of Mathematics, S.G.A.D. Govt. College, Tarn Taran, for his continuous motivation and support throughout my research work. Words fail to acknowledge the kind of support he has been in the process. Indeed he has been a source of great inspiration and motivational mentor.

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It will not be justified if I forget to acknowledge the support of friends and colleagues encouraging me during difficult times.

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Last but not the least, I express heartfelt thanks to my parents, wife and son for their unconditional support and sacrifices, without which, it could never have been possible to accomplish this task.

Date: Varun Kumar

Place:

Table of Contents

Table of Figures

Nomenclature

$\boldsymbol{\lambda}, \boldsymbol{\mu}$	Lame's constants
$\alpha, \beta, \gamma, \kappa$	Micropolar Elastic constants
ρ	Density
j	Micro inertia
$\vec{\phi}$	Microrotation vector
$\vec{u}$	Displacement vector
σ_{ij}	Stress tensor
μ_{ij}	Couple stress tensor
$\vec{H}_0$	External applied magnetic field
$\vec{h}$	Induced magnetic field
$\vec{E}$	Induced electric field
μ_0	Magnetic permeability
ϵ_0	Electric permeability
δ_{ij}	Kronecker delta
I	Identity matrix
O	Null matrix
$\vec{B}$	Magnetic induction vector
c^*	Specific heat at constant strain
$\vec{J}$	Current density vector
$\vec{D}$	Electric displacement vector
e	Cubic dilation
α_T	Coefficient of linear thermal expansion
ϵ_{ijk}	Alternating tensor
ρ_e	Volume charge density
τ_0, τ_1	Relaxation times

Chapter 1

Introduction

1.1 Classical theory of Elasticity

The Mathematical theory of elasticity is an endeavour to lessen the work involved in determining stress-strain, or relative movement of constituents of a solid body, which is subjected to an equilibrating system of forces, or might be in state of slight internal relative motion. It aims to derive results which shall be essentially vital in the fields of structural design and all other beneficial fields in which the material of construction is used. Classical theory of elasticity is one of the most important branches of continuum mechanics, which deals with the stresses and deformations in elastic materials generated due to action of external forces or change in temperature. The classical theory of elasticity serves as an excellent model for studying the mechanical behaviour of a wide variety of solid materials and is used extensively in civil, mechanical and aeronautical engineering design. This is the oldest established theory governing the behaviour of deformable solid materials, which was founded in the early nineteenth century. Under this theory it is assumed that an elastic, continuous medium in which loadings are transmitted through an area element $\boldsymbol{dA}$ in the body by means of the stress vector only. The results obtained with the application of the classical theory of elasticity are in harmony with experiments carried out of many construction materials (steel, aluminium and concrete) provided the stresses are within the limits of elasticity of the material. However in many cases discrepancies between the experiments and the classical theory of elasticity is exceptionally prominent in dynamical problems which can be understood clearly in the case of elastic vibrations characterising combination of short wave length and high frequency.

Basis of this theory is formed by Hooke's Law which was discovered in 1660 and published in 1676. In general form Hooke's law (Constitutive relations) can be expressed as

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

And equation of motion as

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i,$$

Where c_{ijkl} is a fourth order tensor having 81 components which depend upon the nature of medium.

Actually Galileo was the first mathematician who studied the resistance of solids to rupture by taking solids as an inelastic object. His investigations laid the foundation of a field which was later investigated by many researchers. Two major breakthroughs in the history of the elasticity initiated by the Galileo's observations, are the discovery of Hooke's Law in 1660 by British mathematician Robert Hooke and the formulation of the general equations by French engineer Navier in 1821. Hooke's law has influenced the scientific thoughts for a considerably long period and its results agreed with experiments quite well.

Any solid is called elastic when it can undergo deformation under an applied load and can return to its original configuration after release of the deforming loads. An elastic solid that undergoes only an infinitesimal deformation and for which the governing material law is linear is called linear elastic solid. If the elastic properties of a body are same in all directions about any given point, then the body is said to be *isotropic*. If it happens that the elastic properties of the body are independent of the positions of the point, then the body is said to be *homogeneous*. For such materials Hooke's law reduces to

$$\sigma_{ij} = \lambda\delta_{ij}\mathrm{v} + 2\mu e_{ij}$$

Where λ and μ are material constants known as *Lame's* constants; $\nu = e_{11} + e_{22} + e_{33}$ and δ_{ij} is the kronecker delta.

1.2 Thermoelasticity

The theory of thermoelasticity which is generalization of concepts of classical theory of elasticity and theory of thermal conductivity, deals with the effect of thermomechanical disturbances on an elastic body. The heating of a body leads to temperature change and deformation in structure which causes thermal deformation. Interaction between elastic and temperature fields leads to coupling between deformation and temperature distribution, so Hooke's law gets replaced with Duhamel-Neumann equation

$$\sigma_{ij} = \lambda\delta_{ij}\mathrm{v} + 2\mu e_{ij} + \beta_{ij}T,$$

here β_{ij} are thermal moduli and T is the temperature change. For homogeneous isotropic material

$$\beta_{ij} = \beta = -(3\lambda + 2\mu)\alpha$$

Where α is the coefficient of thermal expansion.

Lack of thermodynamical justification of Duhemal's equation was later provided by Biot [1]. Biot also introduced the concept of thermal force. Year 1967 saw the introduction of generalized thermoelasticity (L-S theory) with single relaxation time for the special case of an isotropic body to overcome one of the two limitations of theory of thermoelasticity proposed by Lord and Shulman [2]. Actually till then it had two drawbacks which were in contradiction to the physical experiments, first it suggested for an elastic body, its mechanical state has nothing to do with the temperature distribution and second, the parabolic nature of heat equation resulting in infinite speed of heat propagation. In L-S theory, Fourier law was replaced with a modified law of heat conduction which included both the heat flux and its time derivative. The second shortcoming was addressed by introduction of two relaxation times by Green and Lindsay [3] which is known as the theory of temperature-rate-dependent thermoelasticity (G-L theory). When the body under consideration has a centre of symmetry, G-L theory is in harmony with Fourier's law of heat conduction, and it is valid for both kind of bodies isotropic as well as anisotropic. A detailed survey of thermoelasticity is available in [4].

1.3 Micropolar Elasticity

So far classical theory, which is based on Hooke's law was being used to analyse the behaviour of commonly used materials in engineering. This theory was formed with the assumption that each point of the continuum possesses three degrees of freedom or displacements in three mutually orthogonal directions. But certain other materials like wood, bones and liquid crystal elastomers having no symmetry in their microstructure and which cannot be properly modelled by using the classic theory. All this happens due to presence of additional mechanisms which resists deformation. Micropolar theory of elasticity which is part of Solid Mechanics came into existence aftermath of failure of classical theory to explain the behaviour of material possessing internal structure and all such materials with fibrous, polycrystalline materials, coarse grain, polymeric materials, microcracks, microfractures and fiberglass. Developed by Eringen [5], Micropolar theory successfully deals with deformation of these materials or any material whose microstructure plays crucial part in their macroscopic reactions. Basically a Micropolar continuum can be treated as a collection of small rigid bodies which are interconnected and can undergo both rotational and translational motions. These elements are allowed to rotate independently. So, both deformation and microrotation describe the state of motion in case of micropolar bodies, giving rise to six degrees of freedom. In this theory it is assumed that the action across an infinitesimal surface element inside a material is equivalent to a force and couple as compared to earlier understanding that the action across a hypothetical plane within material should be statically equivalent to a force. Usually in micropolar three displacement components (u_1, u_2, u_3) are used to represent the macroscopic motion of the material point, and three additional microrotational angles (ϕ_1, ϕ_2, ϕ_3) are used to characterize the rotation of microstructure within the material point. The interaction taking place between two parts of a body is transmitted by a torque vector along with force vector which results in asymmetric force stresses and couple stresses. This media represents the entire class of materials which are formed by dipole atoms or dumb-bell like molecules and are subjected to surface and body couples. Micropolar materials are assigned an additional object called "director" to represent their orientation in comparison to classical continuum mechanics where its constituents are assigned a fixed position regardless of their orientation, inside the body of material at any instant of time. So a micropolar continuum can be regarded as a collection of interconnected small elements which can undergo both translational as well as rotational motion. Following Eringen [5] mathematical model for representing stress-strain relation in homogeneous isotropic micropolar elastic media is taken as:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \varepsilon_{ijr}\phi_r\right),$$

$$\mu_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{ij} + \gamma\phi_j.$$

First attempt to remove discrepancies of the classical theory was made by Voigt [6] by who while investigating the interaction between two particles of a body through an elementary area found the presence of moment vector in addition to a force vector. This observation gave way to the origin of an additional parameter in the theory of elasticity namely couple stress. Subsequently Cosserat and Cosserat [7] showed that the deformation of the material can be explained by a displacement vector and an independent rotation vector and laid foundation of a unified theory. Further it said that during the deformation process every material particle goes through both linear displacement and rotation. Another advancement in the field was by introduction of micromorphic continuum theory which suggested that each particle within the body possesses twelve degrees of freedom; three are contributed by macromotion and remaining by micromotion. Thus all material which are capable for undergoing classical motion and microrotation can be regarded as micromorphic continuum and in these cases deformation of material can assumed to be affine. Hence Cosserat continuum and the indeterminate couple stress theory can be considered as a particular case of this theory.

Micropolar theory of elasticity had the potential of wide-ranging engineering applications in acoustics, optics and geophysics or wherever the contemporary engineering materials area being used. As these can be regarded as materials made up of constituents possessing internal structures and any small-scale effect plays a vital role in the prediction of the overall mechanical behaviour of these materials. Recently lot of work is going on in this field for isotropic as well as anisotropic media and literature now contains several hundred papers in this and in related field. Disturbance caused due to a thermomechanical source in homogeneous isotropic heat flux which was dependent upon micropolar thermelastic medium was investigated by Kumar et al. [8] by using eigen value approach. An axi-symmetric problem corresponding to time harmonic vertical and horizontal loads in micropolar medium was investigated by Kumar and Choudhary [9] with the help of integral transforms. Hu et al. [10] proposed a variational method approach for evaluating the behaviour of non-linear micropolar composites and found that the calculated values were in agreement with the available experimental data. Kumar and Ailawalia [11] studied the micropolar cubic crystal subjected to mechanical source by deploying eigen value approach. Propagation of plane waves in micropolar elastic space was investigated by Tomar and Singh [12] and proved the existence of three longitudinal waves and two sets of coupled waves. Concept of energy pairs was introduced by Ramezani and Naghdabadi [13] in micropolar continuum. Bauer et al. [14] proposed a three dimensional finite element technique which can be used to obtain large displacements and small strains. Also transition between micropolar and classical continua was reproduced by using this model. A numerical manifold (NMM) approach for obtaining solution of plane problems in micropolar elasticity has been studied by Zhao et al. [15] and proved that results are in agreement with the analytical approach. Some existence and uniqueness results were obtained for micropolar solid-solid mixture by Ghiba and Gales [16]. Also analysis of longitudinal and displacement waves for displacement and microrotation has been presented.

1.4 Micropolar thermoelasticity

The theory which is formed by including the thermal effects in micropolar theory known as micropolar coupled theory of thermoelasticity, was developed by Nowacki [17]. Temperature dependence of displacements is required to be considered in all elasto-dynamic problems giving rise to coupled thermoelastic equations. This theory consists of heat conduction equation and stress strain that are produced due to the flow of heat. It makes it possible to calculate the stresses produced by the temperature field and to determine the temperature distribution due to the action of time dependent forces and heat sources. Following Lord and Shulman [2], Green and Lindsay [3] and Eringen [18], the mathematical model for calculating stress-strain together with modified Fourier law of heat conduction for homogeneous isotropic micropolar generalized thermoelastic solid is:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \varepsilon_{\mathrm{ijr}}\phi_r\right) - \nu\left(T + \tau_1\frac{\partial T}{\partial t}\right)\delta_{ij},$$

$$\mu_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i},$$

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 \mathrm{T}}{\partial t^2}\right) + \nu T_0\left(\frac{\partial}{\partial t} + \Xi\,\tau_0\frac{\partial^2}{\partial t^2}\right)u_{i,i}.$$

For a linearly coupled thermoelastic solid, the governing equations consists of diffusion type (parabolic) equation of heat conduction and wave type (hyperbolic) equation of motion. As discussed earlier due to deficiencies in this equation it was observed that its solution was in violation of physical laws. A part of the solution of this heat equation was tending to infinity which means if any isotropic and homogeneous elastic material is subjected to mechanical or thermal disturbances; the effects in the temperature and displacement field can be immediately felt at an infinity. In simple words it means that some part of the disturbance is travelling with velocity even greater than velocity of light which is practically not possible. To overcome this shortcomings, the need was felt to develop theories of generalized thermoelasticity. Soon these were addressed with L-S and G-L theories. Lord and Shulman [2], introduced the first generalization to this theory by obtaining a head equation of wave nature and also proposed a modified law of heat conduction. Wave type heat equation of this theory ensured that thermal and elastic disturbances had finite speed of propagation. Other than this both equations of motion and constitutive relations remained same for this theory. Later the second generalization to the coupled theory of elasticity was made by introduction of theory of thermoelasticity with two relaxation times known as temperature–rate-dependent thermoelasticity. Constitutive relations in explicit form for this theory were obtained by Green and Lindsay [3]. Green and Naghdi [19] proposed the theory (G-N theory) of thermoelasticity without energy dissipation and obtained the derivation of a complete set of governing equations in the linearized form for isotropic and homogeneous materials in terms of temperature and displacement. Uniqueness of the solution for the corresponding initial mixed boundary value problem was also established by them.

An intensive study is going on in recent times in the field of micropolar thermoelasticity. A detailed review of works on this subject was done by Eringen [20] and Nowacki [21]. Lately number of authors have contributed to the field. Kumar et al. [22] investigated source problem in micropolar theory of thermoelasticity and obtained the general solution for a half-space subjected to arbitrary heat source. For particulate composites, impact of reinforcement size on the yielding and strain hardening was also investigated by them. A problem of one relaxation time due to time harmonic series was studied by Kumar and Ailawalia [23] to find the response of a micropolar thermoelastic medium possessing cubic symmetry. They also studied [24] themomechanical interactions in a micropolar thermoelastic medium possessing cubic symmetry. A comprehensive study was made by Othman and Singh [25] in micropolar thermoelastic medium under five theories of generalized thermoelasticity namely Lord-Shulman (L-S) with one relaxation time, Green-Lindsay (G-L) with two relaxation times, Green and Naghdi (G-N) theory without energy dissipation and Chandrasekharaiah-Tzou theory with dual phase lag and coupled theory. Propagation of waves using L-S theory in stress free homogeneous isotropic thermoelastic plate possessing cubic symmetry was studied by Kumar and Partap [26]. Passarella and Zampoli [27] investigated a problem on thermoelasticity without energy dissipation allowing propagation of thermal waves at finite speed. Othman et al. [28] presented a paper on disturbances in a homogeneous, isotropic elastic medium using G-N theory with generalized thermoelastic diffusion. Othman *et al.* [29] with the help of a general model of equations of generalized thermo-microstretch studied three theories; L-S theory, G-L theory and classical dynamical coupled theory for a homogeneous isotropic elastic half-space. They used normal mode analyses to find the solution and showed the impact of reference temperature on modulus of elasticity and also proved that it has significant effect on the thermomechanical interactions.

Features of thermo elastic waves with thermal relaxation using generalized theory of Lord-Shulman in isotropic micropolar plate was investigated by Shaw and Mukhopadhyay [30] and result found in this study were in agreement in the context of various theories of classical as well as micropolar thermoelasticity with those predicted earlier by Sharma and Eringen. Kumar and Kansal [31] constructed a fundamental solution of differential equations in the theory of micropolar thermoelastic diffusion in case of steady oscillations in terms of elementary functions. The uniqueness and reciprocal theorems were proved by El-Karamany and Ezzat [32] for three-phase-lag micropolar thermoelastic solid. Variational principle was established for a linear anisotropic and inhomogeneous solid. Variational principal was obtained for two temperature homogeneous istropic thermoelastic media by Youssef [33]. Sherief and Latief [34] studied a problem in half space in the context of fractional order theory of thermoelasticity by taking thermal conductivity as a variable rather than a constant.

Othman et al. [35] introduced dual-phase lag theory to study the effect of rotation on a two-dimensional problem of micropolar thermoelastic isotropic medium with two temperatures. Response of moving heat source in homogeneous, isotropic, micropolar medium was analysed by Shaw and Mukhopadhyay [36] subjected to finite rotation about its axis. Two temperature generalized thermoelasticity theory was deployed for obtain the results. Kumar et al. [37] studied a two temperature problem consisting of propagation of waves in homogenous micropolar thermoelastic solid whose boundary is stress free, thermally insulated and isothermal. Sherief and El-Latief [38] applied the fractional order theory of thermoelasticity to a two dimensional problem in half-space in the absence of heat sources and body forces. Kumar et al. [39] investigated a two temperature problem to study the reflection of plane waves at the free surface of thermally conducting micropolar conducting thermoelastic medium with and without energy dissipation.

1.5 Magneto Micropolar Elasticity

In the recent years the interaction of electromagnetic fields with elastic media is an area interest for many researchers working in the field of continuum mechanics and geophysics for both theoretical and experimental investigations. So magneto micropolar theory of elasticity is study of thermos-elastic deformations of a solid body subjected to an externally applied magnetic field. Both magnetic as well as elastic fields contribute to the total deformation of the body. Interaction of both fields causes changes in the governing laws of both fields. Elastic field enters into governing equations of electromagnetism i.e. Maxwell's equations by modifying the Ohm's law and in turn electro-magnetic field effects the elastic field by inclusion of Lorentz's ponderomotive force in Hooke's law.

So modified model [40] for this type of media by considering the Lorentz force, is taken as

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \epsilon_{ijk}\,\phi_k\right),$$

$$\mu_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i},$$

$$(\mu + \kappa)u_{i,jj} + (\lambda + \mu)u_{j,ji} + \kappa\epsilon_{ijk}\phi_{k,j} + \epsilon_{ijk}\,J_j B_k = \rho\ddot{u}_i, ,$$

$$\kappa\epsilon_{ijk}u_{k,j} - 2\kappa\phi_i + (\alpha+\beta)\phi_{j,ji} + \gamma\phi_{i,jj} = \rho j\ddot{\phi}_i \,.$$

1.6 Magneto Micropolar thermoelasticity

This theory deals with the effects of magnetic field on the elastic deformation produced by uneven heating throughout the body which may or may not be subjected to mechanical forces. In this case, besides the elastic and electro-magnetic fields, thermal field is also present. Each of these fields interact with each other and contribute to the total deformation of the body. The electro-magnetic field is still governed by Maxwell's equations with, of course, a modified Ohm's law, while the elastic field is determined by the modified Hooke's law and the thermal field by Fourier's law of heat conduction in its modified form. Current area of study magneto micropolar thermoelasticity is an extension of this theory. This theory deals the effects of magnetic field on the elastic deformation produced by uneven heating throughout the body which may or may not be subjected to mechanical forces. In this case, in addition to elastic and electro-magnetic fields, thermal field is also present. Maxwell's equations still govern the electro-magnetic field while the elastic field is determined by the modified Hooke's law and the thermal field by Fourier's law of heat conduction in its modified form. Due to superposition of electromagnetic field on elastic field, elastic-stress relation gets modified by addition of a new body force namely Lorentz's force and in turn elastic field causes changes in the the electro-magnetic field by modifying ohm's law.

The theories of magneto elasticity and thermos magneto elasticity have been developed to study the thermoelastic deformation when medium is under an externally applied magnetic field. In recent years, because of the possibilities of their extensive practical applications in diverse fields such as acoustics, geophysics, optics, damping of acoustic waves in the magnetic field and so on, these theories are being rapidly. It can also find application in analysis of propagation of seismic waves from the earth's mantle to its core. For explaining certain phenomena concerning these waves, Cagniard [41] suggested that the existence of the earth's magnetic field may be taken into consideration. Basic equations of magneto micro thermoelasticity were derived by Kaliski [42]. Later on, Knopoff [43] attempted to determine the effects of the magnetic field on the propagation of elastic waves on a geophysical scale.

In recent years number of authors have contributed to the development of this field. Baksi, Bera and Debnath [44] studied magneto–thermal elastic problems with thermal relaxations and heat sources in a three dimensional infinite rotating elastic media. Youssef [45] studied a generalized magneto thermoelastic problem with variable material properties in conducting medium. An axi-symmetric problem subjected to thermomechanical source in electromagnetic micropolar thermoelastic medium was analysed by Kumar and Rupender [46], by using a two dimensional model in cylindrical polar coordinates. A comparative study between one-temperature theory and two temperature theory in generalized magneto thermoelastic medium in perfectly conducting medium was made by Ezzat and Bary [47] by using state space approach and found that two-temperature generalized theory more accurately describes the behaviour of the particles of an elastic body than the one-temperature theory. Ezzat and Awad [48] studied a problem in micropolar generalized magneto thermoelasticity and introduced the modified Ohm's law which contained the charge density effects and also temperature gradient by using the generalized Fourier's law containing current density term. Normal mode analysis is used to obtain the solution. He and Cao [49] used generalized thermoelastic theory in context of L-S theory with thermal relaxation and investigated the response of thin film strip placed in magnetic field and moving heat source and found that displacement and stress were significantly influenced by magnetic field but temperature remains unaffected. Effect of rotation was analysed by Kumar and Rupender [40] in an electromagnetic micropolar generalized thermoelastic medium, which is subjected to mechanical force or thermal source. By means of a two dimensional model they also proved that application of thermal source plays dominant role in the stress-strain distribution as compared to mechanical force in the presence of a transverse magnetic field. Singh and Kumar [50] studied the interaction of electromagnetic field with elastic field in the presence of temperature by applying Mechanical force and thermal source by using modified Fourier and Ohm's law.

A mathematical model [48] for isotropic micropolar thermoelastic homogenous solid placed in the externally applied magnetic field is usually taken as,

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \epsilon_{ijk}\,\phi_k\right) - \nu\,T\delta_{ij},$$

Chapter 1

Introduction

1.1 Classical theory of Elasticity

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$$\sigma_{ij} = \lambda\delta_{ij}\mathrm{v} + 2\mu e_{ij} + \beta_{ij}T,$$

here β_{ij} are thermal moduli and T is the temperature change. For homogeneous isotropic material

$$\beta_{ij} = \beta = -(3\lambda + 2\mu)\alpha$$

Where α is the coefficient of thermal expansion.

Lack of thermodynamical justification of Duhemal's equation was later provided by Biot [1]. Biot also introduced the concept of thermal force. Year 1967 saw the introduction of generalized thermoelasticity (L-S theory) with single relaxation time for the special case of an isotropic body to overcome one of the two limitations of theory of thermoelasticity proposed by Lord and Shulman [2]. Actually till then it had two drawbacks which were in contradiction to the physical experiments, first it suggested for an elastic body, its mechanical state has nothing to do with the temperature distribution and second, the parabolic nature of heat equation resulting in infinite speed of heat propagation. In L-S theory, Fourier law was replaced with a modified law of heat conduction which included both the heat flux and its time derivative. The second shortcoming was addressed by introduction of two relaxation times by Green and Lindsay [3] which is known as the theory of temperature-rate-dependent thermoelasticity (G-L theory). When the body under consideration has a centre of symmetry, G-L theory is in harmony with Fourier's law of heat conduction, and it is valid for both kind of bodies isotropic as well as anisotropic. A detailed survey of thermoelasticity is available in [4].

1.3 Micropolar Elasticity

So far classical theory, which is based on Hooke's law was being used to analyse the behaviour of commonly used materials in engineering. This theory was formed with the assumption that each point of the continuum possesses three degrees of freedom or displacements in three mutually orthogonal directions. But certain other materials like wood, bones and liquid crystal elastomers having no symmetry in their microstructure and which cannot be properly modelled by using the classic theory. All this happens due to presence of additional mechanisms which resists deformation. Micropolar theory of elasticity which is part of Solid Mechanics came into existence aftermath of failure of classical theory to explain the behaviour of material possessing internal structure and all such materials with fibrous, polycrystalline materials, coarse grain, polymeric materials, microcracks, microfractures and fiberglass. Developed by Eringen [5], Micropolar theory successfully deals with deformation of these materials or any material whose microstructure plays crucial part in their macroscopic reactions. Basically a Micropolar continuum can be treated as a collection of small rigid bodies which are interconnected and can undergo both rotational and translational motions. These elements are allowed to rotate independently. So, both deformation and microrotation describe the state of motion in case of micropolar bodies, giving rise to six degrees of freedom. In this theory it is assumed that the action across an infinitesimal surface element inside a material is equivalent to a force and couple as compared to earlier understanding that the action across a hypothetical plane within material should be statically equivalent to a force. Usually in micropolar three displacement components (u_1, u_2, u_3) are used to represent the macroscopic motion of the material point, and three additional microrotational angles (ϕ_1, ϕ_2, ϕ_3) are used to characterize the rotation of microstructure within the material point. The interaction taking place between two parts of a body is transmitted by a torque vector along with force vector which results in asymmetric force stresses and couple stresses. This media represents the entire class of materials which are formed by dipole atoms or dumb-bell like molecules and are subjected to surface and body couples. Micropolar materials are assigned an additional object called "director" to represent their orientation in comparison to classical continuum mechanics where its constituents are assigned a fixed position regardless of their orientation, inside the body of material at any instant of time. So a micropolar continuum can be regarded as a collection of interconnected small elements which can undergo both translational as well as rotational motion. Following Eringen [5] mathematical model for representing stress-strain relation in homogeneous isotropic micropolar elastic media is taken as:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \varepsilon_{ijr}\phi_r\right),$$

$$\mu_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{ij} + \gamma\phi_j.$$

First attempt to remove discrepancies of the classical theory was made by Voigt [6] by who while investigating the interaction between two particles of a body through an elementary area found the presence of moment vector in addition to a force vector. This observation gave way to the origin of an additional parameter in the theory of elasticity namely couple stress. Subsequently Cosserat and Cosserat [7] showed that the deformation of the material can be explained by a displacement vector and an independent rotation vector and laid foundation of a unified theory. Further it said that during the deformation process every material particle goes through both linear displacement and rotation. Another advancement in the field was by introduction of micromorphic continuum theory which suggested that each particle within the body possesses twelve degrees of freedom; three are contributed by macromotion and remaining by micromotion. Thus all material which are capable for undergoing classical motion and microrotation can be regarded as micromorphic continuum and in these cases deformation of material can assumed to be affine. Hence Cosserat continuum and the indeterminate couple stress theory can be considered as a particular case of this theory.

Micropolar theory of elasticity had the potential of wide-ranging engineering applications in acoustics, optics and geophysics or wherever the contemporary engineering materials area being used. As these can be regarded as materials made up of constituents possessing internal structures and any small-scale effect plays a vital role in the prediction of the overall mechanical behaviour of these materials. Recently lot of work is going on in this field for isotropic as well as anisotropic media and literature now contains several hundred papers in this and in related field. Disturbance caused due to a thermomechanical source in homogeneous isotropic heat flux which was dependent upon micropolar thermelastic medium was investigated by Kumar et al. [8] by using eigen value approach. An axi-symmetric problem corresponding to time harmonic vertical and horizontal loads in micropolar medium was investigated by Kumar and Choudhary [9] with the help of integral transforms. Hu et al. [10] proposed a variational method approach for evaluating the behaviour of non-linear micropolar composites and found that the calculated values were in agreement with the available experimental data. Kumar and Ailawalia [11] studied the micropolar cubic crystal subjected to mechanical source by deploying eigen value approach. Propagation of plane waves in micropolar elastic space was investigated by Tomar and Singh [12] and proved the existence of three longitudinal waves and two sets of coupled waves. Concept of energy pairs was introduced by Ramezani and Naghdabadi [13] in micropolar continuum. Bauer et al. [14] proposed a three dimensional finite element technique which can be used to obtain large displacements and small strains. Also transition between micropolar and classical continua was reproduced by using this model. A numerical manifold (NMM) approach for obtaining solution of plane problems in micropolar elasticity has been studied by Zhao et al. [15] and proved that results are in agreement with the analytical approach. Some existence and uniqueness results were obtained for micropolar solid-solid mixture by Ghiba and Gales [16]. Also analysis of longitudinal and displacement waves for displacement and microrotation has been presented.

1.4 Micropolar thermoelasticity

The theory which is formed by including the thermal effects in micropolar theory known as micropolar coupled theory of thermoelasticity, was developed by Nowacki [17]. Temperature dependence of displacements is required to be considered in all elasto-dynamic problems giving rise to coupled thermoelastic equations. This theory consists of heat conduction equation and stress strain that are produced due to the flow of heat. It makes it possible to calculate the stresses produced by the temperature field and to determine the temperature distribution due to the action of time dependent forces and heat sources. Following Lord and Shulman [2], Green and Lindsay [3] and Eringen [18], the mathematical model for calculating stress-strain together with modified Fourier law of heat conduction for homogeneous isotropic micropolar generalized thermoelastic solid is:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa(u_{j,i} - \varepsilon_{ijr}\phi_r) - \nu\left(T + \tau_1 \frac{\partial T}{\partial t}\right)\delta_{ij},$$

$$\mu_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i},$$

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 \mathrm{T}}{\partial t^2}\right) + \nu T_0\left(\frac{\partial}{\partial t} + \Xi\,\tau_0\frac{\partial^2}{\partial t^2}\right)u_{i,i}.$$

For a linearly coupled thermoelastic solid, the governing equations consists of diffusion type (parabolic) equation of heat conduction and wave type (hyperbolic) equation of motion. As discussed earlier due to deficiencies in this equation it was observed that its solution was in violation of physical laws. A part of the solution of this heat equation was tending to infinity which means if any isotropic and homogeneous elastic material is subjected to mechanical or thermal disturbances; the effects in the temperature and displacement field can be immediately felt at an infinity. In simple words it means that some part of the disturbance is travelling with velocity even greater than velocity of light which is practically not possible. To overcome this shortcomings, the need was felt to develop theories of generalized thermoelasticity. Soon these were addressed with L-S and G-L theories. Lord and Shulman [2], introduced the first generalization to this theory by obtaining a head equation of wave nature and also proposed a modified law of heat conduction. Wave type heat equation of this theory ensured that thermal and elastic disturbances had finite speed of propagation. Other than this both equations of motion and constitutive relations remained same for this theory. Later the second generalization to the coupled theory of elasticity was made by introduction of theory of thermoelasticity with two relaxation times known as temperature–rate-dependent thermoelasticity. Constitutive relations in explicit form for this theory were obtained by Green and Lindsay [3]. Green and Naghdi [19] proposed the theory (G-N theory) of thermoelasticity without energy dissipation and obtained the derivation of a complete set of governing equations in the linearized form for isotropic and homogeneous materials in terms of temperature and displacement. Uniqueness of the solution for the corresponding initial mixed boundary value problem was also established by them.

An intensive study is going on in recent times in the field of micropolar thermoelasticity. A detailed review of works on this subject was done by Eringen [20] and Nowacki [21]. Lately number of authors have contributed to the field. Kumar et al. [22] investigated source problem in micropolar theory of thermoelasticity and obtained the general solution for a half-space subjected to arbitrary heat source. For particulate composites, impact of reinforcement size on the yielding and strain hardening was also investigated by them. A problem of one relaxation time due to time harmonic series was studied by Kumar and Ailawalia [23] to find the response of a micropolar thermoelastic medium possessing cubic symmetry. They also studied [24] themomechanical interactions in a micropolar thermoelastic medium possessing cubic symmetry. A comprehensive study was made by Othman and Singh [25] in micropolar thermoelastic medium under five theories of generalized thermoelasticity namely Lord-Shulman (L-S) with one relaxation time, Green-Lindsay (G-L) with two relaxation times, Green and Naghdi (G-N) theory without energy dissipation and Chandrasekharaiah-Tzou theory with dual phase lag and coupled theory. Propagation of waves using L-S theory in stress free homogeneous isotropic thermoelastic plate possessing cubic symmetry was studied by Kumar and Partap [26]. Passarella and Zampoli [27] investigated a problem on thermoelasticity without energy dissipation allowing propagation of thermal waves at finite speed. Othman et al. [28] presented a paper on disturbances in a homogeneous, isotropic elastic medium using G-N theory with generalized thermoelastic diffusion. Othman *et al.* [29] with the help of a general model of equations of generalized thermo-microstretch studied three theories; L-S theory, G-L theory and classical dynamical coupled theory for a homogeneous isotropic elastic half-space. They used normal mode analyses to find the solution and showed the impact of reference temperature on modulus of elasticity and also proved that it has significant effect on the thermomechanical interactions.

Features of thermo elastic waves with thermal relaxation using generalized theory of Lord-Shulman in isotropic micropolar plate was investigated by Shaw and Mukhopadhyay [30] and result found in this study were in agreement in the context of various theories of classical as well as micropolar thermoelasticity with those predicted earlier by Sharma and Eringen. Kumar and Kansal [31] constructed a fundamental solution of differential equations in the theory of micropolar thermoelastic diffusion in case of steady oscillations in terms of elementary functions. The uniqueness and reciprocal theorems were proved by El-Karamany and Ezzat [32] for three-phase-lag micropolar thermoelastic solid. Variational principle was established for a linear anisotropic and inhomogeneous solid. Variational principal was obtained for two temperature homogeneous istropic thermoelastic media by Youssef [33]. Sherief and Latief [34] studied a problem in half space in the context of fractional order theory of thermoelasticity by taking thermal conductivity as a variable rather than a constant.

Othman et al. [35] introduced dual-phase lag theory to study the effect of rotation on a two-dimensional problem of micropolar thermoelastic isotropic medium with two temperatures. Response of moving heat source in homogeneous, isotropic, micropolar medium was analysed by Shaw and Mukhopadhyay [36] subjected to finite rotation about its axis. Two temperature generalized thermoelasticity theory was deployed for obtain the results. Kumar et al. [37] studied a two temperature problem consisting of propagation of waves in homogenous micropolar thermoelastic solid whose boundary is stress free, thermally insulated and isothermal. Sherief and El-Latief [38] applied the fractional order theory of thermoelasticity to a two dimensional problem in half-space in the absence of heat sources and body forces. Kumar et al. [39] investigated a two temperature problem to study the reflection of plane waves at the free surface of thermally conducting micropolar conducting thermoelastic medium with and without energy dissipation.

1.5 Magneto Micropolar Elasticity

In the recent years the interaction of electromagnetic fields with elastic media is an area interest for many researchers working in the field of continuum mechanics and geophysics for both theoretical and experimental investigations. So magneto micropolar theory of elasticity is study of thermos-elastic deformations of a solid body subjected to an externally applied magnetic field. Both magnetic as well as elastic fields contribute to the total deformation of the body. Interaction of both fields causes changes in the governing laws of both fields. Elastic field enters into governing equations of electromagnetism i.e. Maxwell's equations by modifying the Ohm's law and in turn electro-magnetic field effects the elastic field by inclusion of Lorentz's ponderomotive force in Hooke's law.

So modified model [40] for this type of media by considering the Lorentz force, is taken as

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \epsilon_{ijk}\,\phi_k\right),$$

$$\mu_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i},$$

$$(\mu+\kappa)u_{i,jj} + (\lambda+\mu)u_{j,ji} + \kappa\epsilon_{ijk}\phi_{k,j} + \epsilon_{ijk}\,J_j B_k = \rho\ddot{u}_i, ,$$

$$\kappa\epsilon_{ijk}u_{k,j} - 2\kappa\phi_i + (\alpha+\beta)\phi_{j,ji} + \gamma\phi_{i,jj} = \rho j\ddot{\phi}_i \,.$$

1.6 Magneto Micropolar thermoelasticity

This theory deals with the effects of magnetic field on the elastic deformation produced by uneven heating throughout the body which may or may not be subjected to mechanical forces. In this case, besides the elastic and electro-magnetic fields, thermal field is also present. Each of these fields interact with each other and contribute to the total deformation of the body. The electro-magnetic field is still governed by Maxwell's equations with, of course, a modified Ohm's law, while the elastic field is determined by the modified Hooke's law and the thermal field by Fourier's law of heat conduction in its modified form. Current area of study magneto micropolar thermoelasticity is an extension of this theory. This theory deals the effects of magnetic field on the elastic deformation produced by uneven heating throughout the body which may or may not be subjected to mechanical forces. In this case, in addition to elastic and electro-magnetic fields, thermal field is also present. Maxwell's equations still govern the electro-magnetic field while the elastic field is determined by the modified Hooke's law and the thermal field by Fourier's law of heat conduction in its modified form. Due to superposition of electromagnetic field on elastic field, elastic-stress relation gets modified by addition of a new body force namely Lorentz's force and in turn elastic field causes changes in the the electro-magnetic field by modifying ohm's law.

The theories of magneto elasticity and thermos magneto elasticity have been developed to study the thermoelastic deformation when medium is under an externally applied magnetic field. In recent years, because of the possibilities of their extensive practical applications in diverse fields such as acoustics, geophysics, optics, damping of acoustic waves in the magnetic field and so on, these theories are being rapidly. It can also find application in analysis of propagation of seismic waves from the earth's mantle to its core. For explaining certain phenomena concerning these waves, Cagniard [41] suggested that the existence of the earth's magnetic field may be taken into consideration. Basic equations of magneto micro thermoelasticity were derived by Kaliski [42]. Later on, Knopoff [43] attempted to determine the effects of the magnetic field on the propagation of elastic waves on a geophysical scale.

In recent years number of authors have contributed to the development of this field. Baksi, Bera and Debnath [44] studied magneto–thermal elastic problems with thermal relaxations and heat sources in a three dimensional infinite rotating elastic media. Youssef [45] studied a generalized magneto thermoelastic problem with variable material properties in conducting medium. An axi-symmetric problem subjected to thermomechanical source in electromagnetic micropolar thermoelastic medium was analysed by Kumar and Rupender [46], by using a two dimensional model in cylindrical polar coordinates. A comparative study between one-temperature theory and two temperature theory in generalized magneto thermoelastic medium in perfectly conducting medium was made by Ezzat and Bary [47] by using state space approach and found that two-temperature generalized theory more accurately describes the behaviour of the particles of an elastic body than the one-temperature theory. Ezzat and Awad [48] studied a problem in micropolar generalized magneto thermoelasticity and introduced the modified Ohm's law which contained the charge density effects and also temperature gradient by using the generalized Fourier's law containing current density term. Normal mode analysis is used to obtain the solution. He and Cao [49] used generalized thermoelastic theory in context of L-S theory with thermal relaxation and investigated the response of thin film strip placed in magnetic field and moving heat source and found that displacement and stress were significantly influenced by magnetic field but temperature remains unaffected. Effect of rotation was analysed by Kumar and Rupender [40] in an electromagnetic micropolar generalized thermoelastic medium, which is subjected to mechanical force or thermal source. By means of a two dimensional model they also proved that application of thermal source plays dominant role in the stress-strain distribution as compared to mechanical force in the presence of a transverse magnetic field. Singh and Kumar [50] studied the interaction of electromagnetic field with elastic field in the presence of temperature by applying Mechanical force and thermal source by using modified Fourier and Ohm's law.

A mathematical model [48] for isotropic micropolar thermoelastic homogenous solid placed in the externally applied magnetic field is usually taken as,

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \epsilon_{ijk}\,\phi_k\right) - \nu\,T\delta_{ij},$$

$$\mu_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i},$$

$$K^*\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)(\rho c^* T + T_o \nu e) + \pi_o J_{i,i},$$

$$(\mu + \kappa)u_{i,jj} + (\lambda + \mu)u_{j,ji} + \kappa\epsilon_{ijk}\phi_{k,j} + \in_{ijk} J_j B_k - \nu T_{,i} = \rho\ddot{u}_i,$$

$$\kappa\epsilon_{ijk}u_{k,j} - 2\kappa\phi_i + (\alpha + \beta)\phi_{j,ji} + \gamma\phi_{i,jj} = \rho j\ddot{\phi}_i .$$

Ezzat et al. [51], by taking into consideration the heat effects and permitting the magnetic field effects, investigated a problem in an isotropic homogeneous micropolar medium. They also introduced a model of equations for generalized magneto micropolar thermoelasticity in the context of theory of two temperatures. Zakaria and Zakaria [52] discussed a two dimensional problem in generalized magneto micropolar thermoelastic medium rotating with uniform angular velocity in the presence of a transverse magnetic field by taking into consideration the effect of Hall current- subjected to ramp-type heating. Othman and Lotfy [53] studied the effects of gravity, magnetic field and rotation with the help of a two dimensional fiber-reinforced thermoelastic problem. For analysis they made use of three different theories; coupled theory, L–S theory with one relaxation time and G–L theory with two relaxation times. Sarkar and Lahiri [54] used model proposed by Sherief et al. for generalized thermoelasticity for fractional order time derivatives for analysing the propagation of electro-magneto-thermoelastic disturbances in a perfectly conducting elastic half-space. Effects of magnetic field on the generalized thermoelastic diffusion using L-S theory were investigated by Othman and Elmaklizi [55] by considering modulus of elasticity linearly dependent on reference temperature. Deswal and Kalaka [56] investigated magneto-thermoelastic interactions in perfectly conducting unbounded half-space surface which is subjected to a time harmonic thermal source with fractional order heat transfer allowing second sound effect. Ezzat et al. [57] proposed a new mathematical model for perfectly conducting solid for two-temperature. Model was applied to analyse one dimensional problem subjected to heat source in the presence of constant magnetic field. Magneto-thermoelastic interactions in an isotropic homogeneous elastic half-space with two temperatures were discussed by Lotfy [58] using mathematical methods under the purview of the Lord–Şhulman (LS) and Green–Lindsay (GL) theories, as well as the classical dynamical coupled theory (CD). Sarkar [59] studied a thermal shock problem by deploying three different theories of generalized electromagnetothermo-elasticity namely coupled (CD) theory, Lord-Shulman (LS) theory and Green-Lindsay (GL) theory for an isotropic and homogeneous hals space solid which was thermally and electrically conducting.

Due to its applications in the other branches of science like plasma physics, geophysics, crystal physics to name a few, interaction between magnetic fields and strain in a micropolar thermoelastic solid is receiving lot of attention. To study deformations at any point of the medium around mining tremors and drilling into the crust of the earth, this theory can be deployed. In fourth chapter an elastic model is presented for studying the earth's planetary motion as it involves rotational velocity in addition to its thermal and electromagnetic field. Due to such applications, this field is witnessing active research in recent times.

Chapter 2

Elastodynamics of two dimensional plane problem in magneto micropolar elastic solid

This chapter contains study of response of solid in magneto micropolar medium when it is subjected to mechanical source in an infinite space. Due to superposition of electromagnetic field on elastic field, elastic-stress relation gets modified with introduction of Lorentz's force as body force and in turn elastic field influences the electro-magnetic field by modifying Ohm's law. At present, number of researchers in the field of continuum mechanics and geophysics are keenly working on elastic media to analyse outcome of its interaction with electromagnetic fields. Significant contributions in the field of Magneto elasticity have been made by Paria [60], Knopoff [43], Chadwick [61] and Purushothama [62]. Effect of magnetic field, rotation and initial stress in a circular cylindrical flexible tube with viscoelastic or elastic wall properties on peristaltic motion of micropolar fluid were investigated by Abd-All et al. [63]. Runge-Kutta method was used to solve the governing equations of motion. Jamia et al. [64] extended Eringen's non-local theory to investigate the mixed-mode crack problem in a functionally graded magneto-electro-elastic medium.

In current study, we have used system of cylindrical polar coordinates to analyse axi-symmetric problem with the help of potential method. Integral transformations along with potential method have been applied to simplify the system of partial differential equations. Normal force at the boundary has been applied to explain the utility of the entire approach.

2.1. Basic Equations

Following Nowacki [65], the field equations, constitutive relations in micropolar elastic solid by taking the Lorentz force into consideration along with equations of electromagnetism are given by

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+(\mu+\kappa)\nabla^2\vec{u}+\kappa\nabla\times\vec{\phi}+\vec{J}\times\vec{B}=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{2.1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi})-\gamma\nabla\times\nabla\times\vec{\phi}+\kappa\nabla\times\vec{u}-2\kappa\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{2.2}$$

$$\sigma_{ij}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+\kappa(u_{j,i}-\epsilon_{ijk}\phi_k), \tag{2.3}$$

$$\mu_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}. \tag{2.4}$$

$$curl\,\vec{h}=\vec{J}+\frac{\partial\vec{D}}{\partial t},\qquad curl\,\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \tag{2.5}$$

$$\vec{E}=-\mu_0\left(\frac{\partial\vec{u}}{\partial t}\times\vec{H}_0\right), \tag{2.6}$$

$$div\,\vec{H}=0,\;\vec{B}=\mu_0\vec{H}\text{ where }\vec{D}=\epsilon_0\vec{E},\quad\vec{H}=\vec{H}_0+\vec{h}. \tag{2.7}$$

Also Ohm' law in this case is

$$\vec{J}=\sigma_0\vec{E}. \tag{2.8}$$

For the purpose of linearizing the problem we take $\vec{H}=\vec{h}+\vec{H}_0$, where $\vec{H}_0=(0,H_0,0)$.

Using equations (2.5)-(2.8), it can be shown that

$$\vec{J}\times\vec{B}=-\mu_0^2\epsilon_0H_0^2\frac{\partial\,\vec{u}}{\partial t}, \tag{2.9}$$

The equations of motion (2.1)-(2.2) with the aid of equation (2.9) in cylindrical polar co-ordinate system (r,θ,z) in component form become

$$\begin{aligned}(\lambda+\mu)\left(\frac{\partial^2u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}-\frac{1}{r^2}\frac{\partial u_\theta}{\partial\theta}+\frac{1}{r}\frac{\partial^2u_\theta}{\partial\theta\partial r}+\frac{\partial^2u_z}{\partial r\partial z}\right)+\frac{\kappa}{r}\left(\frac{\partial\phi_z}{\partial\theta}-r\frac{\partial\phi_\theta}{\partial z}\right)\\+(\mu+\kappa)\left(\frac{\partial^2u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2u_r}{\partial\theta^2}+\frac{\partial^2u_r}{\partial z^2}\right)\\-\mu_0^2\epsilon_0H_0^2\frac{\partial u_r}{\partial t}=\rho\frac{\partial^2u_r}{\partial t^2},\end{aligned} \tag{2.10}$$

$$(\lambda+\mu)\left(\frac{1}{r}\frac{\partial^2 u_r}{\partial\theta\,\partial r}+\frac{1}{r^2}\frac{\partial u_r}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{1}{r}\frac{\partial^2 u_z}{\partial\theta\partial z}\right)+\kappa\left(\frac{\partial\phi_r}{\partial z}-\frac{\partial\phi_\theta}{\partial z}\right)$$
$$+(\mu+\kappa)\left(\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial\theta^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}+\frac{\partial^2 u_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta}{r^2}\right) \quad (2.11)$$
$$-\mu_0^2\epsilon_0 H_0^2\frac{\partial u_\theta}{\partial t}=\rho\frac{\partial^2 u_\theta}{\partial t^2},$$

$$(\lambda+\mu)\left(\frac{\partial^2 u_r}{\partial r\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}+\frac{1}{r}\frac{\partial^2 u_\theta}{\partial\theta\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right)+\frac{\kappa}{r}\left(\frac{\partial(r\phi_\theta)}{\partial r}-\frac{\partial\phi_r}{\partial\theta}\right)$$
$$+(\mu+\kappa)\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial\theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right)-\mu_0^2\epsilon_0 H_0^2\frac{\partial u_z}{\partial t} \quad (2.12)$$
$$=\rho\frac{\partial^2 u_z}{\partial t^2},$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_r}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_r}{\partial r}-\frac{\phi_r}{r^2}-\frac{1}{r^2}\frac{\partial\phi_\theta}{\partial\theta}+\frac{1}{r}\frac{\partial^2\phi_\theta}{\partial\theta\partial r}+\frac{\partial^2\phi_z}{\partial r\partial z}\right)+\frac{\kappa}{r}\left(\frac{\partial u_z}{\partial\theta}-r\frac{\partial u_\theta}{\partial z}\right)$$
$$+\gamma\left(\frac{\partial^2\phi_r}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_r}{\partial r}-\frac{\phi_r}{r^2}-\frac{2}{r^2}\frac{\partial\phi_\theta}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2\phi_r}{\partial\theta^2}+\frac{\partial^2\phi_r}{\partial z^2}\right)-2\kappa\phi_r \quad (2.13)$$
$$=\rho j\frac{\partial^2\phi_r}{\partial t^2},$$

$$(\alpha+\beta)\left(\frac{1}{r}\frac{\partial^2\phi_r}{\partial\theta\,\partial r}+\frac{1}{r^2}\frac{\partial\phi_r}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2\phi_\theta}{\partial\theta^2}+\frac{1}{r}\frac{\partial^2\phi_z}{\partial\theta\partial z}\right)+\kappa\left(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right)$$
$$+\gamma\left(\frac{\partial^2\phi_\theta}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\phi_\theta}{\partial\theta^2}+\frac{1}{r}\frac{\partial\phi_\theta}{\partial r}+\frac{\partial^2\phi_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial\phi_r}{\partial\theta}-\frac{\phi_\theta}{r^2}\right) \quad (2.14)$$
$$-2\kappa\phi_\theta=\rho j\frac{\partial^2\phi_\theta}{\partial t^2},$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_r}{\partial r\partial z}+\frac{1}{r}\frac{\partial\phi_r}{\partial z}+\frac{1}{r}\frac{\partial^2\phi_\theta}{\partial\theta\partial z}+\frac{\partial^2\phi_z}{\partial z^2}\right)+\frac{\kappa}{r}\left(\frac{\partial(ru_\theta)}{\partial r}-\frac{\partial u_r}{\partial\theta}\right)$$
$$+\gamma\left(\frac{\partial^2\phi_z}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi_z}{\partial\theta^2}+\frac{\partial^2\phi_z}{\partial z^2}\right)-2\kappa\phi_z=\rho j\frac{\partial^2\phi_z}{\partial t^2}, \quad (2.15)$$

where (u_r, u_θ, u_z) and $(\phi_r, \phi_\theta, \phi_z)$ are the cylindrical polar components of displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ respectively.

2.2. Formulation and Solution of the problem

We consider homogeneous, isotropic micropolar elastic media which is moving slowly. Cylindrical polar coordinate system *(r,θ,z)* with *z*-axis pointing into the medium have been employed to study the problem.

As we are considering two-dimensional axisymmetric problem, so we take the components of displacement vector $\vec{u}$, microrotation vector $\vec{\phi}$ and magnetic field intensity vector $\vec{h}$ of the form

$$\vec{u} = (u_r, 0, u_z),\ \vec{\phi} = (0, \phi_\theta, 0),\ \vec{h} = (0, h, 0). \tag{2.16}$$

Equations (2.5)-(2.8) after simplification give

$$\left(\nabla^2 - \mu_0\sigma_0\frac{\partial}{\partial t} - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)\vec{h} = 0. \tag{2.17}$$

Due to axial symmetry about z-axis, the quantities are independent of θ. With these considerations, the equations (2.11),(2.13) and (2.15) become identically zero and the equations (2.10), (2.12) and (2.14) along with constitutive relations (2.3)-(2.4) takes the form

$$(\lambda+\mu)\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial r\partial z}\right) - \kappa\frac{\partial \phi_\theta}{\partial z} + (\mu+\kappa)\left(\nabla^2 - \frac{1}{r^2}\right)u_r - \mu_0^2\epsilon_0 H_0^2\frac{\partial u_r}{\partial t} = \rho\frac{\partial^2 u_r}{\partial t^2}, \tag{2.18}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u_r}{\partial r\partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z^2}\right) + \frac{\kappa}{r}\frac{\partial (r\phi_\theta)}{\partial r} + (\mu+\kappa)\nabla^2 u_z - \mu_0^2\epsilon_0 H_0^2\frac{\partial u_z}{\partial t} = \rho\frac{\partial^2 u_z}{\partial t^2}, \tag{2.19}$$

$$\kappa\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) + \gamma\left(\nabla^2 - \frac{1}{r^2}\right)\phi_\theta - 2\kappa\phi_\theta = \rho j\frac{\partial^2 \phi_\theta}{\partial t^2}, \tag{2.20}$$

$$\sigma_{zr} = (\mu+\kappa)\frac{\partial u_r}{\partial z} + \mu\frac{\partial u_z}{\partial r} - \kappa\phi_\theta, \tag{2.21}$$

$$\sigma_{zz} = \lambda\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) + (\lambda+2\mu+\kappa)\frac{\partial u_z}{\partial z}, \tag{2.22}$$

$$\mu_{z\theta} = \gamma\frac{\partial \phi_\theta}{\partial z}, \tag{2.23}$$

Introducing the scalar potential ψ and vector potential $\vec{\eta}$ as

$$\vec{u} = \nabla\psi + \nabla \times \vec{\eta}, \quad \text{where } \nabla.\vec{\eta} = 0 \text{ and } \vec{\eta} = (o, \eta_\theta, 0) \tag{2.24}$$

Using equations (2.24) in equations (2.18)-(2.20), we obtain

$$(\lambda + 2\mu + \kappa)\left(\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2}\right) - \epsilon_0\mu_0^2 H_0^2\frac{\partial\psi}{\partial t} - \rho\frac{\partial^2\psi}{\partial t^2} = 0, \tag{2.25}$$

$$(\mu + \kappa)\left(\frac{\partial^2\eta_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\eta_\theta}{\partial r} - \frac{\eta_\theta}{r^2} + \frac{\partial^2\eta_\theta}{\partial z^2}\right) + \kappa\mu\phi_\theta - \epsilon_0\mu_0^2 H_0^2\frac{\partial\eta_\theta}{\partial t} - \rho\frac{\partial^2\eta_\theta}{\partial t^2} = 0, \tag{2.26}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2}\right)h - \sigma_0\mu_0\frac{\partial h}{\partial t} - \epsilon_0\mu_0\frac{\partial^2 h}{\partial t^2} = 0, \tag{2.27}$$

As the electric field intensity vector and current density vector are normal to the magnetic field intensity vector, so its components are obtained as

$$E_r = \mu_0 H_0\frac{\partial u_z}{\partial t}, \tag{2.28}$$

$$E_z = -\mu_0 H_0\frac{\partial u_r}{\partial t}, \tag{2.29}$$

$$J_r = -\frac{\partial h}{\partial z} - \epsilon_0\frac{\partial E_r}{\partial t}, \tag{2.30}$$

$$J_z = \frac{\partial h}{\partial r} + \frac{h}{r} - \epsilon_0\frac{\partial E_z}{\partial t}. \tag{2.31}$$

Now we define the dimensionless quantities as

$$r' = \frac{r}{h_1}, \quad z' = \frac{z}{h_1}, \quad u_r' = \frac{u_r}{h_1}, \quad u_z' = \frac{u_z}{h_1}, \quad \phi_\theta' = \frac{\kappa}{\mu}\phi_\theta,$$
$$t' = \omega^* t, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\mu}, \quad \mu_{ij}' = \frac{h_1}{\gamma}\mu_{ij}, \quad J_r' = \frac{h_1}{H_0}J_r, \quad J_z' = \frac{h_1}{H_0}J_z, \tag{2.32}$$
$$E_r' = \frac{1}{\omega^* H_0\mu_0 h_1}E_r, \quad E_z' = \frac{1}{\omega^* H_0\mu_0 h_1}E_z, \quad h' = \frac{h}{H_0}.$$

where h_1 is the parameter having the dimensions of length.

After using these dimensionless quantities as defined in equation (2.32) in the equations (2.21)-(2.23) and (2.25)-(2.31), we obtain (after dropping dashes for convenience)

$$\left(\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2}\right) - a_1\frac{\partial\psi}{\partial t} - a_2\frac{\partial^2\psi}{\partial t^2} = 0, \tag{2.33}$$

$$\left(\frac{\partial^2\eta_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\eta_\theta}{\partial r} - \frac{\eta_\theta}{r^2} + \frac{\partial^2\eta_\theta}{\partial z^2}\right) + a_3\phi_\theta - a_4\frac{\partial\eta_\theta}{\partial t} - a_5\frac{\partial^2\eta_\theta}{\partial t^2} = 0, \tag{2.34}$$

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}+\frac{\partial^2}{\partial z^2}\right)h-a_6\frac{\partial h}{\partial \mathrm{t}}-a_7\frac{\partial^2 h}{\partial t^2}=0, \tag{2.35}$$

$$\left(\frac{\partial^2\phi_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_\theta}{\partial r}-\frac{\phi_\theta}{r^2}+\frac{\partial^2\phi_\theta}{\partial z^2}\right)-a_8\left(\frac{\partial^2\eta_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\eta_\theta}{\partial r}-\frac{\eta_\theta}{r^2}+\frac{\partial^2\eta_\theta}{\partial z^2}\right)-a_9\phi_\theta - a_{10}\frac{\partial^2\phi_\theta}{\partial t^2}=0, \tag{2.36}$$

$$\sigma_{zr}=a_{11}\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}-\phi_\theta, \tag{2.37}$$

$$\sigma_{zz}=a_{12}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}\right)+a_{13}\frac{\partial u_z}{\partial z}, \tag{2.38}$$

$$\mu_{z\theta}=a_{14}\frac{\partial\phi_\theta}{\partial z}, \tag{2.39}$$

$$E_r=\frac{\partial u_z}{\partial t}, \tag{2.40}$$

$$E_z=-\frac{\partial u_r}{\partial t}, \tag{2.41}$$

$$J_r=-\frac{\partial h}{\partial z}-a_{15}\frac{\partial E_r}{\partial t}, \tag{2.42}$$

$$J_z=\frac{\partial h}{\partial r}+\frac{h}{r}-a_{15}\frac{\partial E_z}{\partial t}. \tag{2.43}$$

where

$$a_1=\frac{\epsilon_0 H_0^2\mu_0^2\omega^* h_1^2}{\lambda+2\mu+\kappa},\quad a_2=\frac{\rho\omega^{*2}h_1^2}{\lambda+2\mu+\kappa},\quad a_3=\frac{\mu h_1^2}{\mu+\kappa},$$
$$a_4=\frac{\epsilon_0 H_0^2\mu_0^2\omega^* h_1^2}{\mu+\kappa},\quad a_5=\frac{\rho\omega^{*2}h_1^2}{\mu+\kappa}, a_6=\sigma_0\mu_0\omega^* h_1^2,\quad a_7=\epsilon_0\mu_0\omega^{*2}h_1^2, \tag{2.44}$$
$$a_8=\frac{\kappa}{\gamma\mu},\quad a_9=\frac{2h_1^2\kappa}{\gamma},\quad a_{10}=\frac{\rho j\omega^{*2}h_1^2}{\gamma},\quad a_{11}=\frac{\mu+\kappa}{\mu},$$
$$a_{12}=\frac{\lambda}{\mu},\quad a_{13}=\frac{\lambda+2\mu+\kappa}{\mu},\quad a_{14}=\frac{\kappa}{\mu},\quad a_{15}=\epsilon_0\omega^*\mu_0 h_1^2.$$

Now we apply the Laplace transform with respect to variable t and s as Laplace transform variable on equations (2.33)-(2.43) as

$$L\{f(r,z,t)\}=\int_0^\infty e^{-st}f(r,z,t)dt=\bar{f}(r,z,s), \tag{2.45}$$

With following conditions

$$L\left\{\frac{\partial f}{\partial t}\right\}=s\bar{f}(r,z,s)-f(r,z,0), \tag{2.46}$$

$$L\left\{\frac{\partial^2 f}{\partial t^2}\right\} = s^2\bar{f}(r,z,s) - sf(r,z,0) - \left(\frac{\partial f}{\partial t}\right)_{t=0}. \tag{2.47}$$

We obtain

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - a_1 s - a_2 s^2\right)\bar{\psi} = 0, \tag{2.48}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2} - -a_4 s - a_5 s^2\right)\bar{\eta}_\theta + a_3\bar{\phi}_\theta = 0, \tag{2.49}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2} - a_6 s - a_7 s^2\right)\bar{h} = 0, \tag{2.50}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2} - a_9 - a_{10}s^2\right)\bar{\phi}_\theta - a_8\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2}\right)\bar{\eta}_\theta = 0, \tag{2.51}$$

$$\bar{\sigma}_{zr} = a_{11}\frac{\partial \bar{u}_r}{\partial z} + \frac{\partial \bar{u}_z}{\partial r} - \bar{\phi}_\theta, \tag{2.52}$$

$$\bar{\sigma}_{zz} = a_{12}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\bar{u}_r + a_{13}\frac{\partial \bar{u}_z}{\partial z}, \tag{2.53}$$

$$\bar{\mu}_{z\theta} = a_{14}\frac{\partial \bar{\phi}_\theta}{\partial z}, \tag{2.54}$$

$$\bar{E}_r = s\bar{u}_z, \tag{2.55}$$

$$\bar{E}_z = -s\bar{u}_r, \tag{2.56}$$

$$\bar{J}_r = -\frac{\partial \bar{h}}{\partial z} - a_{15}s\bar{E}_r, \tag{2.57}$$

$$\bar{J}_z = \left(\frac{\partial}{\partial r} + \frac{\bar{1}}{r}\right)\bar{h} - a_{15}\bar{E}_z. \tag{2.58}$$

where the initial values of $u_r, u_z, \phi_\theta, h, \psi$ and η_θ and corresponding velocities are assumed as zero throughout the medium.

The Hankel transform [66] of order n of $\bar{f}(r,z,s)$ with respect to variable r is defined as

$$H_n\{\bar{f}(r,z,s)\} = \int_0^\infty r\,\bar{f}(r,z,s)J_n(\xi r)dr = \tilde{f}(\xi,z,s). \tag{2.59}$$

where ξ is the Hankel transform variable and $J_n(\xi r)$ is the Bessel function of first kind of order n having following properties,

$$H_0\left\{\frac{\partial \bar{f}}{\partial r}+\frac{1}{r}\bar{f}\right\}=\xi H_1\{\bar{f}\},\qquad H_0\left\{\frac{\partial^2 \bar{f}}{\partial r^2}+\frac{1}{r}\frac{\partial \bar{f}}{\partial r}\right\}=-\xi^2 H_0\{\bar{f}\}, \tag{2.60}$$

$$H_0\left\{\frac{\partial \bar{f}}{\partial r}\right\}=-\xi H_0\{\bar{f}\},\qquad H_1\left\{\frac{\partial^2 \bar{f}}{\partial r^2}+\frac{1}{r}\frac{\partial \bar{f}}{\partial r}-\frac{1}{r}\bar{f}\right\}=-\xi^2 H_1\{\bar{f}\}, \tag{2.61}$$

Applying the Hankel transform defined by (2.59) on equations (2.48)-(2.57) and with the help of results (2.60)-(2.61), we obtain

$$(D^2-\alpha_1)\tilde{\psi}=0, \tag{2.62}$$

$$(D^2-\alpha_2)\tilde{\eta}_\theta=-a_3\tilde{\phi}_\theta, \tag{2.63}$$

$$(D^2-\alpha_3)\tilde{h}=0, \tag{2.64}$$

$$(D^2-\alpha_4)\tilde{\phi}_\theta=a_7(D^2-\xi^2)\tilde{\eta}_\theta, \tag{2.65}$$

$$\tilde{\sigma}_{zr}=a_{10}D\tilde{u}_r-\xi\tilde{u}_z-\tilde{\phi}_\theta, \tag{2.66}$$

$$\tilde{\sigma}_{zz}=a_{11}\xi\tilde{u}_r+a_{12}D\tilde{u}_z, \tag{2.67}$$

$$\tilde{\mu}_{z\theta}=a_{13}D\tilde{\phi}_\theta, \tag{2.68}$$

$$\tilde{E}_r=s\tilde{u}_z, \tag{2.69}$$

$$\tilde{E}_z=-s\tilde{u}_r, \tag{2.70}$$

$$\tilde{J}_r=-D\tilde{h}-a_{14}s\tilde{E}_r, \tag{2.71}$$

$$\tilde{J}_z=\xi\tilde{h}-a_{14}s\tilde{E}_z. \tag{2.72}$$

where

$$\begin{aligned}&\tilde{\psi}=H_0\{\bar{\psi}\},\quad \tilde{\eta}_\theta=H_0\{\bar{\eta}_\theta\},\quad \tilde{\phi}_\theta=H_1\{\bar{\phi}_\theta\},\quad \tilde{h}=H_1\{\bar{h}\},\\ &\tilde{u}_z=H_0\{\bar{u}_z\},\quad \tilde{\sigma}_{zr}=H_1\{\bar{\sigma}_{zr}\},\quad \tilde{\sigma}_{zz}=H_0\{\bar{\sigma}_{zz}\},\quad \tilde{\mu}_{z\theta}=H_1\{\bar{\mu}_{z\theta}\},\\ &\tilde{E}_r=H_0\{\bar{E}_r\},\quad \tilde{E}_z=H_1\{\bar{E}_z\},\quad \tilde{J}_r=H_1\{\bar{J}_r\},\quad \tilde{J}_z=H_0\{\bar{J}_z\},\\ &\tilde{u}_r=H_1\{\bar{u}_r\}.\end{aligned} \tag{2.73}$$

Simplifying equations (2.63)-(2.65), we obtain

$$(D^4-\alpha_5 D^2+\alpha_6)\tilde{\eta}_\theta=0. \tag{2.74}$$

Solutions of equations (2.62),(2.64) and (2.74) are given by

$$\tilde{\psi}(\xi,z,s) = A_1 e^{\lambda_1 z} + A_2 e^{-\lambda_1 z}, \tag{2.75}$$

$$\tilde{h}(\xi,z,s) = A_3 e^{\lambda_2 z} + A_4 e^{-\lambda_2 z}, \tag{2.76}$$

$$\tilde{\eta}_\theta(\xi,z,s) = A_5 e^{\lambda_3 z} + A_6 e^{-\lambda_3 z} + A_7 e^{\lambda_4 z} + A_8 e^{-\lambda_4 z}. \tag{2.77}$$

where $A_i's$ some functions of parameter ξ and s.

$$\lambda_1 = \sqrt{\alpha_1},\ \lambda_2 = \sqrt{\alpha_3},\ \lambda_3 = \frac{1}{2}\left(\alpha_5 + \sqrt{\alpha_5^2 - 4\alpha_6}\right), \lambda_4 = \frac{1}{2}\left(\alpha_5 - \sqrt{\alpha_5^2 - 4\alpha_6}\right),$$
$$\alpha_1 = \xi^2 + a_1 s + a_2 s^2,\ \alpha_2 = \xi^2 + a_4 s + a_5 s^2,\ \alpha_3 = \xi^2 + a_6 s + a_7 s^2,\quad \alpha_4 = \xi^2 + a_8 + a_9 s^2, D = \frac{d}{dz}. \tag{2.78}$$

The remaining components of microrotation, displacement, stress, couple stress, electric field and current density in transformed domain are obtained as,

$$\tilde{\phi}_\theta(\xi,z,s) = \frac{1}{a_3}\left[A_5(1-\lambda_3^2)e^{\lambda_3 z} + A_6(1-\lambda_3^2)e^{-\lambda_3 z} + A_7(1-\lambda_4^2)e^{\lambda_4 z} + A_8(1-\lambda_4^2)e^{-\lambda_4 z}\right], \tag{2.79}$$

$$\tilde{u}_r(\xi,z,s) = -A_1\xi e^{\lambda_1 z} - A_2\xi e^{-\lambda_1 z} - A_5\lambda_3 e^{\lambda_3 z} + A_6\lambda_3 e^{-\lambda_3 z} - A_7\lambda_4 e^{\lambda_4 z} + A_8\lambda_4 e^{-\lambda_4 z}, \tag{2.80}$$

$$\tilde{u}_z(\xi,z,s) = A_1\lambda_1 e^{\lambda_1 z} - A_2\lambda_1 e^{-\lambda_1 z} + A_5 e^{\lambda_3 z} + A_6 e^{-\lambda_3 z} + A_7 e^{\lambda_4 z} + A_8 e^{-\lambda_4 z}, \tag{2.81}$$

$$\tilde{\sigma}_{zr}(\xi,z,s) = \Bigg[A_1(-a_{10}-1)\xi\lambda_1 e^{\lambda_1 z} + A_2(a_{10}+1)\xi\lambda_1 e^{-\lambda_1 z} - A_5\left(\lambda_3^2 + \xi + \frac{1-\lambda_3^2}{a_3}\right)e^{\lambda_3 z} - A_6\left(\lambda_3^2 + \xi + \frac{1-\lambda_3^2}{a_3}\right)e^{-\lambda_3 z} - A_7\left(\lambda_4^2 + \xi + \frac{1-\lambda_4^2}{a_3}\right)e^{\lambda_4 z} - A_8\left(\lambda_4^2 + \xi + \frac{1-\lambda_4^2}{a_3}\right)e^{-\lambda_4 z}\Bigg], \tag{2.82}$$

$$\tilde{\sigma}_{zz}(\xi,z,s) = A_1(-a_{11}\xi^2 + a_{12}\lambda_1^2)e^{\lambda_1 z} + A_2(-a_{11}\xi^2 + a_{12}\lambda_1^2)e^{-\lambda_1 z} + A_5(-a_{11}\xi + a_{12})\lambda_3 e^{\lambda_3 z} + A_6(a_{11}\xi - a_{12})\lambda_3 e^{-\lambda_3 z} + A_7(-a_{11}\xi - a_{12})\lambda_4 e^{\lambda_4 z} + A_8(a_{11}\xi - a_{12})\lambda_4 e^{-\lambda_4 z}, \tag{2.83}$$

$$\tilde{\mu}_{z\theta}(\xi,z,s) = \frac{a_{13}}{a_3}\left[A_5(1-\lambda_3^2)\lambda_3 e^{\lambda_3 z} - A_6(1-\lambda_3^2)\lambda_3 e^{-\lambda_3 z} + A_7(1-\lambda_4^2)\lambda_4 e^{\lambda_4 z} - A_8(1-\lambda_4^2)\lambda_4 e^{-\lambda_4 z}\right] \tag{2.84}$$

$$\tilde{E}_r(\xi,z,s) = s\left[A_1\lambda_1 e^{\lambda_1 z} - A_2\lambda_1 e^{-\lambda_1 z} + A_5 e^{\lambda_3 z} + A_6 e^{-\lambda_3 z} + A_7 e^{\lambda_4 z} + A_8 e^{-\lambda_4 z}\right], \tag{2.85}$$

$$\tilde{E}_z(\xi,z,s) = -s\big[-A_1\xi e^{\lambda_1 z} - A_2\xi e^{-\lambda_1 z} - A_5\lambda_3 e^{\lambda_3 z} + A_6\lambda_3 e^{-\lambda_3 z} - A_7\lambda_4 e^{\lambda_4 z} + A_8\lambda_4 e^{-\lambda_4 z}\big], \tag{2.86}$$

$$J_r(\xi,z,s) = -A_1 a_{14}s^2\lambda_1 e^{\lambda_1 z} + A_2 a_{14}s^2\lambda_1 e^{-\lambda_1 z} + A_3\lambda_2 e^{\lambda_2 z} - A_4\lambda_2 e^{-\lambda_2 z} - A_5 a_{14}s^2 e^{\lambda_3 z} - A_6 a_{14}s^2 e^{-\lambda_3 z} + A_7 a_{14}s^2 e^{\lambda_4 z} + A_8 a_{14}s^2 e^{-\lambda_4 z}, \tag{2.87}$$

$$J_z(\xi,z,s) = -A_1 a_{14}s^2\xi e^{\lambda_1 z} - A_2 a_{14}s^2\xi e^{-\lambda_1 z} + A_3\xi e^{\lambda_2 z} + A_4\xi e^{-\lambda_2 z} - A_5 a_{14}s^2\lambda_3 e^{\lambda_3 z} + A_6 a_{14}s^2\lambda_3 e^{-\lambda_3 z} - A_7 a_{14}s^2\lambda_4 e^{\lambda_4 z} + A_8 a_{14}s^2\lambda_4 e^{-\lambda_4 z}. \tag{2.88}$$

2.3. Boundary Conditions

We undertake an infinite elastic medium in cylindrical polar coordinate system, with a concentrated mechanical force $F = -\frac{F_0\delta(r)\delta(t)}{2\pi r}$ with magnitude F_0, is applied in the direction of z-axis at the origin, where $\delta()$ is the Dirac-delta function. We call the medium for the region in which $z > 0$ as medium-I and $z < 0$ as medium-II. Appropriate boundary conditions at $z = 0$ are

$$u_r(r,0^+,t) - u_r(r,0^-,t) = 0, \quad u_z(r,0^+,t) - u_z(r,0^-,t) = 0, \tag{2.89}$$

$$\phi_\theta(r,0^+,t) - \phi_\theta(r,0^-,t) = 0, \quad \sigma_{zr}(r,0^+,t) - \sigma_{zr}(r,0^-,t) = 0, \tag{2.90}$$

$$\sigma_{zz}(r,0^+,t) - \sigma_{zz}(r,0^-,t) = -\frac{F_0\delta(r)\delta(t)}{2\pi r}, \quad \mu_{z\theta}(r,0^+,t) - \mu_{z\theta}(r,0^-,t) = 0, \tag{2.91}$$

$$h(r,0^+,t) - h(r,0^-,t) = 0, \quad J_r(r,0^+,t) - J_r(r,0^-,t) = 0. \tag{2.92}$$

After using the dimensionless quantities as defined in equation (2.32) along with $F_0' = \frac{F_0}{\mu}$ and applying Laplace and Hankel transforms as defined in (2.45) and (2.59) on equations (2.89)-(2.92), we get

$$\tilde{u}_r(\xi,0^+,s) - \tilde{u}_r(\xi,0^-,s) = 0, \quad \tilde{u}_z(\xi,0^+,s) - \tilde{u}_z(\xi,0^-,s) = 0, \tag{2.93}$$

$$\tilde{\phi}_\theta(\xi,0^+,s) - \tilde{\phi}_\theta(\xi,0^-,s) = 0, \quad \tilde{\sigma}_{zr}(\xi,0^+,s) - \tilde{\sigma}_{zr}(\xi,0^-,s) = 0, \tag{2.94}$$

$$\tilde{\sigma}_{zz}(\xi,0^+,s)-\tilde{\sigma}_{zz}(\xi,0^-,s)=-\frac{F_0}{2\pi},\ \ \tilde{\mu}_{z\theta}(\xi,0^+,s)-\tilde{\mu}_{z\theta}(\xi,0^-,s)=0, \tag{2.95}$$

$$\tilde{h}(\xi,0^+,s)-\tilde{h}(\xi,0^-,s)=0,\ \ \tilde{J}_r(\xi,0^+,s)-\tilde{J}_r(\xi,0^-,s)=0. \tag{2.96}$$

Medium-I: Since $Z>0$ for the medium-I, coefficients A_1, A_3, A_5 and A_7 in the expressions (2.76),(2.79)-(2.84) and (2.87) of the transformed displacement, microrotation, stress components, current density and magnetic field intensity must be zero. Hence these transformed components for medium-I are given by

$$\tilde{u}_r(\xi,z,s)=-A_2\xi e^{-\lambda_1 z}+A_6\lambda_3 e^{-\lambda_3 z}+A_8\lambda_4 e^{-\lambda_4 z}, \tag{2.97}$$

$$\tilde{u}_z(\xi,z,s)=-A_2\lambda_1 e^{-\lambda_1 z}+A_6 e^{-\lambda_3 z}+A_8 e^{-\lambda_4 z}, \tag{2.98}$$

$$\tilde{\phi}_\theta(\xi,z,s)=\frac{1}{a_3}\left[A_6(1-\lambda_3^2)e^{-\lambda_3 z}+A_8(1-\lambda_4^2)e^{-\lambda_4 z}\right], \tag{2.99}$$

$$\begin{aligned}\tilde{\sigma}_{zr}(\xi,z,s)=\Bigg[&A_2(a_{10}+1)\xi\lambda_1 e^{-\lambda_1 z}-A_6\left(\lambda_3^2+\xi+\frac{1-\lambda_3^2}{a_3}\right)e^{-\lambda_3 z}\\&-A_8\left(\lambda_4^2+\xi+\frac{1-\lambda_4^2}{a_3}\right)e^{-\lambda_4 z}\Bigg],\end{aligned} \tag{2.100}$$

$$\begin{aligned}\tilde{\sigma}_{zz}(\xi,z,s)=&A_2(-a_{11}\xi^2+a_{12}\lambda_1^2)e^{-\lambda_1 z}+A_6(a_{11}\xi-a_{12})\lambda_3 e^{-\lambda_3 z}\\&+A_8(a_{11}\xi-a_{12})\lambda_4 e^{-\lambda_4 z},\end{aligned} \tag{2.101}$$

$$\tilde{\mu}_{z\theta}(\xi,z,s)=\frac{a_{13}}{a_3}\left[-A_6(1-\lambda_3^2)\lambda_3 e^{-\lambda_3 z}-A_8(1-\lambda_4^2)\lambda_4 e^{-\lambda_4 z}\right] \tag{2.102}$$

$$J_r(\xi,z,s)=A_2 a_{14}s^2\lambda_1 e^{-\lambda_1 z}-A_4\lambda_2 e^{-\lambda_2 z}-A_6 a_{14}s^2 e^{-\lambda_3 z}+A_8 a_{14}s^2 e^{-\lambda_4 z}, \tag{2.103}$$

$$\tilde{h}(\xi,z,s)=A_4 e^{-\lambda_2 z}, \tag{2.104}$$

Medium-II: Since $Z<0$ for this medium, the coefficients A_2, A_4, A_6 and A_8 in expressions (2.76),(2.79)-(2.84) and (2.87) of the transformed displacement, microrotation, stress components, current density and magnetic field intensity must be zero. Hence these transformed components for medium-II are given by

$$\tilde{u}_r(\xi,z,s)=-A_1\xi e^{\lambda_1 z}-A_5\lambda_3 e^{\lambda_3 z}-A_7\lambda_4 e^{\lambda_4 z}, \tag{2.105}$$

$$\tilde{u}_z(\xi,z,s)=A_1\lambda_1 e^{\lambda_1 z}+A_5 e^{\lambda_3 z}+A_7 e^{\lambda_4 z}, \tag{2.106}$$

$$\tilde{\phi}_\theta(\xi,z,s)=\frac{1}{a_3}\left[A_5(1-\lambda_3^2)e^{\lambda_3 z}+A_7(1-\lambda_4^2)e^{\lambda_4 z}\right], \tag{2.107}$$

$$\tilde{\sigma}_{zr}(\xi,z,s) = \left[A_1(-a_{10}-1)\xi\lambda_1 e^{\lambda_1 z} + A_5\left(\lambda_3^2+\xi+\frac{1-\lambda_3^2}{a_3}\right)e^{\lambda_3 z} - A_7\left(\lambda_4^2+\xi+\frac{1-\lambda_4^2}{a_3}\right)e^{\lambda_4 z}\right], \tag{2.108}$$

$$\tilde{\sigma}_{zz}(\xi,z,s) = A_1(-a_{11}\xi^2+a_{12}\lambda_1^2)e^{\lambda_1 z} + A_5(-a_{11}\xi+a_{12})\lambda_3 e^{\lambda_3 z} + A_7(-a_{11}\xi-a_{12})\lambda_4 e^{\lambda_4 z}, \tag{2.109}$$

$$\tilde{\mu}_{z\theta}(\xi,z,s) = \frac{a_{13}}{a_3}\left[A_5(1-\lambda_3^2)\lambda_3 e^{\lambda_3 z} + A_7(1-\lambda_4^2)\lambda_4 e^{\lambda_4 z}\right] \tag{2.110}$$

$$\tilde{J}_r(\xi,z,s) = -A_1 a_{14}s^2\lambda_1 e^{\lambda_1 z} + A_3\lambda_2 e^{\lambda_2 z} - A_5 a_{14}s^2 e^{\lambda_3 z} + A_7 a_{14}s^2 e^{\lambda_4 z}, \tag{2.111}$$

$$\tilde{h}(\xi,z,s) = A_3 e^{\lambda_2 z}, \tag{2.112}$$

After making use of the transformed components as defined in equations (2.97)-(2.104) for medium-I and equations (2.105)-(2.112) for medium-II in transformed boundary conditions (2.93)-(2.96), we obtain a system of eight equations in eight unknowns $A_i(i = 1,2,\ldots,8)$ as

$$-\xi(A_2-A_1)+\lambda_3(A_6+A_5)+\lambda_4(A_8+A_7)=0, \tag{2.113}$$

$$\lambda_1(A_2+A_1)+(A_6-A_5)+(A_8+A_5)=0, \tag{2.114}$$

$$\frac{1}{a_3}[(1-\lambda_3^2)(A_6-A_5)+(1-\lambda_4^2)(A_8-A7)]=0, \tag{2.115}$$

$$\xi\lambda_1(a_6+1)(A_2+A_1)-\left[\lambda_3^2+\xi+\frac{1-\lambda_3^2}{a_3}\right](A_6+A_5)-(A_8-A_7)=0, \tag{2.116}$$

$$(-a_7\xi^2+a_8\lambda_1^2)(A_2-A_1)+(a_7\xi\lambda_3-a_8\lambda_3)(A_6+A_5) + (\lambda_4 a_7\xi-a_8\lambda_4)(A_8+A_7) = -\frac{\tilde{F}_0}{2\pi}, \tag{2.117}$$

$$\frac{a_9}{a_3}[(1-\lambda_3^2)\lambda_3(-A_6-A_5)-\lambda_4(1-\lambda_4^2)(A_8+A_7)]=0, \tag{2.118}$$

$$A_4-A_3=0, \tag{2.119}$$

$$-a_{10}s^2\lambda_1(A_1+A_2)+\lambda_2(A_3+A_4)+a_{10}s^2(A_6-A_5)+a_{10}s^2(A_8-A_7) = 0. \tag{2.120}$$

Solving equations (2.113)-(2.120) for unknowns $A_i(i = 1,2,\ldots,8)$, we obtain

$$A_2 = -A_1 = -\frac{l_{12}}{l_{13}}\tilde{F}_0, \tag{2.121}$$

$$A_4 = A_3 = 0, \tag{2.122}$$

$$A_6 = A_5 = \frac{l_{11}}{l_{13}}\xi\tilde{F}_0, \tag{2.123}$$

$$A_8 = A_7 = -\frac{1}{l_{13}}\xi\Delta\tilde{F}_0. \tag{2.124}$$

where

$$\Delta = \lambda_3(1-\lambda_3^2),\quad l_{11} = \lambda_4(1-\lambda_4^2),\quad l_{12} = \lambda_4\Delta - \lambda_3 l_{11},\quad l_{13} = 4\pi[(a_8\lambda_1^2 - a_7\xi^2) - (a_7\xi\lambda_3 - a_8\lambda_3)l_{11}\xi + (\lambda_4 a_7\xi - a_8\lambda_4)\xi\Delta]. \tag{2.125}$$

2.4. Inversion of the transform

The transformed solutions obtained above are functions of the form $\tilde{f}(\xi,z,s)$, so to get these functions back in the form $f(r,z,t)$ i.e. in the physical domain, we first remove the Hankel transform by using the following procedure.

$$\bar{f}(r,z,s) = \int_0^\infty \xi\tilde{f}(\xi,z,s)J_n(\xi r)d\xi. \tag{2.126}$$

The expression (2.126) produces the Laplace transform $\bar{f}(r,z,s)$ of the function $f(r,z,t)$. Further, for the fixed values of r and z, the function $\bar{f}(r,z,s)$ can be considered as the Laplace transform $\bar{h}(s)$ of some function $h(t)$. Following Honig and Hirdes [67], the Laplace transformed function $\bar{h}(s)$can be inverted to obtain function $h(t)$ in the following manner

$$\mathrm{h(t)} = \frac{1}{2\pi\iota}\int_{Y-\iota\infty}^{Y+\iota\infty} e^{st}\bar{h}(s)ds, \tag{2.127}$$

where Y is any real number which is greater than all the real parts of singularities of $\bar{h}(s)$. Taking $\mathrm{s = Y + \iota z}$, we get

$$h(t) = \frac{e^{Yt}}{2\pi\iota}\int_{-\infty}^{\infty} e^{\iota tz}\,\bar{h}(Y+\iota z)dz, \tag{2.128}$$

Considering $e^{-Yt}h(t)$ as $p(t)$, we expand it in a Fourier series in interval [0,2L]. It approximately leads to the formula,

$$h(t) = h_\infty(t) + E_D,$$

where

$$h_\infty(t) = \frac{Y_0}{2} + \sum_{m=1}^{\infty} Y_m, \quad 0 \le t \le 2L, \tag{2.129}$$

and

$$Y_k = \left(\frac{e^{Yt}}{L}\right) Re\left[e^{\frac{k\pi t}{L}}\right] \bar{h}\left(Y + \left(\frac{\iota k\pi}{L}\right)\right). \tag{2.130}$$

Here E_D represents the discretization error and can be controlled by choosing appropriate values of Y. Also we have used the criteria laid by Honig and Hirdes [67] for choosing the values of Y and L. Since the infinite series in equation (2.129) can be summed up only to a finite number N terms, the approximate value of $h(t)$ becomes

$$h_N(t) = \frac{Y_0}{2} + \sum_{m=1}^{N} Y_m, \quad .0 \le t \le 2L. \tag{2.131}$$

A truncation error E_T is now introduced and is added to the discretization error to generate the total approximate error in calculating $h(t)$ using the above mentioned technique. Further we deploy technique of Honig and Hirdes [67] in order to increase the rate of convergence and reducing the amount of truncation error.

To evaluate the function $h(t)$, we use Korrecktur method,

$$h(t) = h_\infty(t) - e^{-2YL} h_\infty(2L + t) + E'_D,$$

where $|E'_D| \ll |E_D|$.

So, the estimated value of $h(t)$ becomes

$$h_{N_K}(t) = h_N(t) - e^{-2YL} h_M(2L + t), \tag{2.132}$$

where M is an integer such that $M < N$.

Now to accelerate the convergence of the series mentioned in the equation (2.131), we have used $\varepsilon - algorithm$. For defining the '$\varepsilon - sequence$', we take N be an odd natural number and $S_m = \sum_{k=1}^{m} Y_k$ be the sequence of partial sums of equation (2.131), which leads to,

$$\varepsilon_{0,m} = 0,\ \varepsilon_{1,m} = S_m,\ \varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}} \text{ for } m, n = 1,2,3 \ldots$$

The sequence $\varepsilon_{1,1}, \varepsilon_{3,1} \ldots \varepsilon_{N,1}$ converges faster than the sequence generated by partial sums S_m, $(m = 1,2,3 \ldots)$ and converges to $h(t) + E_D - \frac{X_0}{2}$.

2.5. Numerical results and discussion

Following Eringen [18], relevant parameters for Magnesium crystal can be taken as $\lambda = 9.4 \times 10^{10} N.m^{-2}$, $\mu = 4 \times 10^{10} N.m^{-2}$, $K = 1 \times 10^{10} N.m^{-2}$, $\rho = 1.74 \times 10^3\ Kg.m^{-3}$, $J = 0.2 \times 10^{-19}\ m^2$, $\epsilon_0 = 10^{-9}/36\pi\ F.m^{-1}$, $\beta = 0.98 \times 10^{-5} N.m^{-2}$, $\sigma_0 = 2.2356 \times 10^7 S.m^{-1}$, $\mu_0 = 4\pi \times 10^{-7} H.m^{-1}$, $H_o = 1\ A.m^{-1}$.

The computations are discussed for three values of non-dimensional time namely $t = 0.100, 0.125, 0.150$ at $z = 1.0$ in the range $0 \leq r \leq 10$ by assigning value $h_1 = 0.01m$. The distribution of non-dimensional normal displacement $U_z = \frac{2}{F_0} u_z$, non-dimensional normal force stress $\sigma_{zz}^* = \frac{2}{F_0} \sigma_{zz}$ and non-dimensional tangential force stress $\mu_{z\theta}^* = \frac{2}{F_0} \mu_{z\theta}$ with non-dimensional distance r have been shown in Fig. 2.1-2.3.

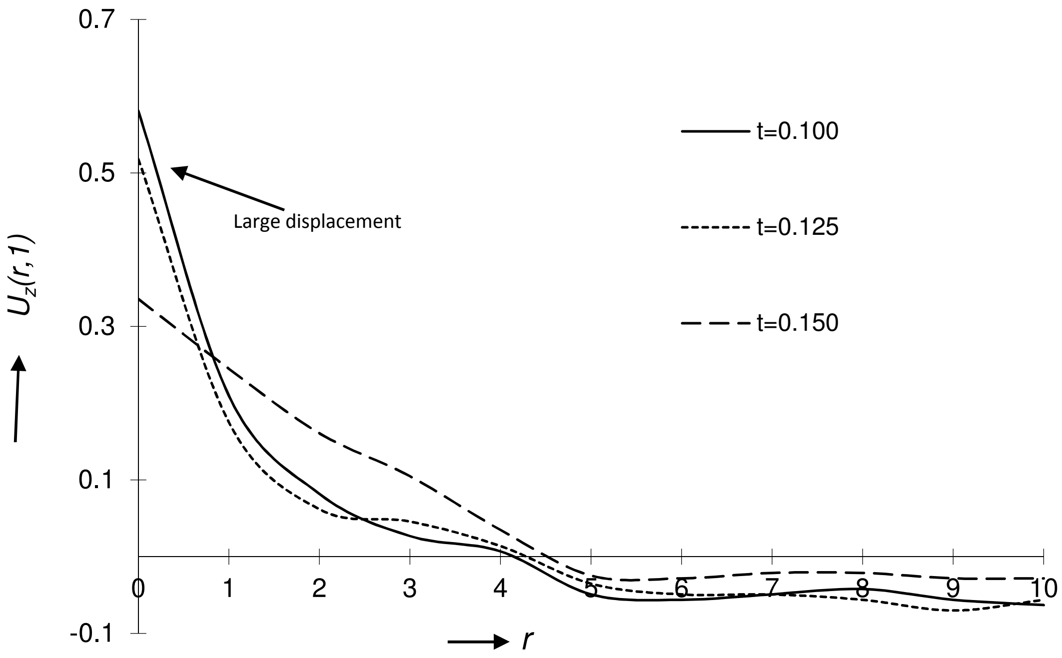

Fig. 2.5.1 Variation of normal displacement $U_z(r, 1)$

Fig. 2.5.1 shows that value of normal displacement U_z has large values near the point of application of source for large values of time and starts decreasing initially in the range $0 \leq r \leq 5$ for all three values of t but starts oscillating in the range $5 < r \leq 10$.

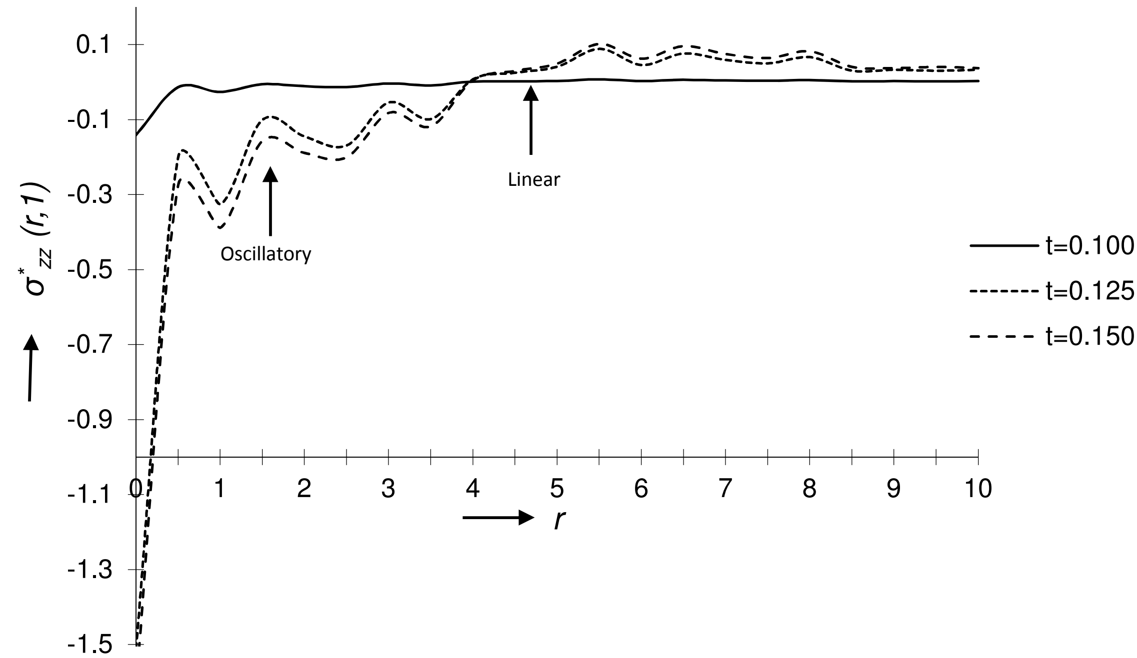

Fig. 2.5.2 Variation of normal force stress σ_{zz}^{*}

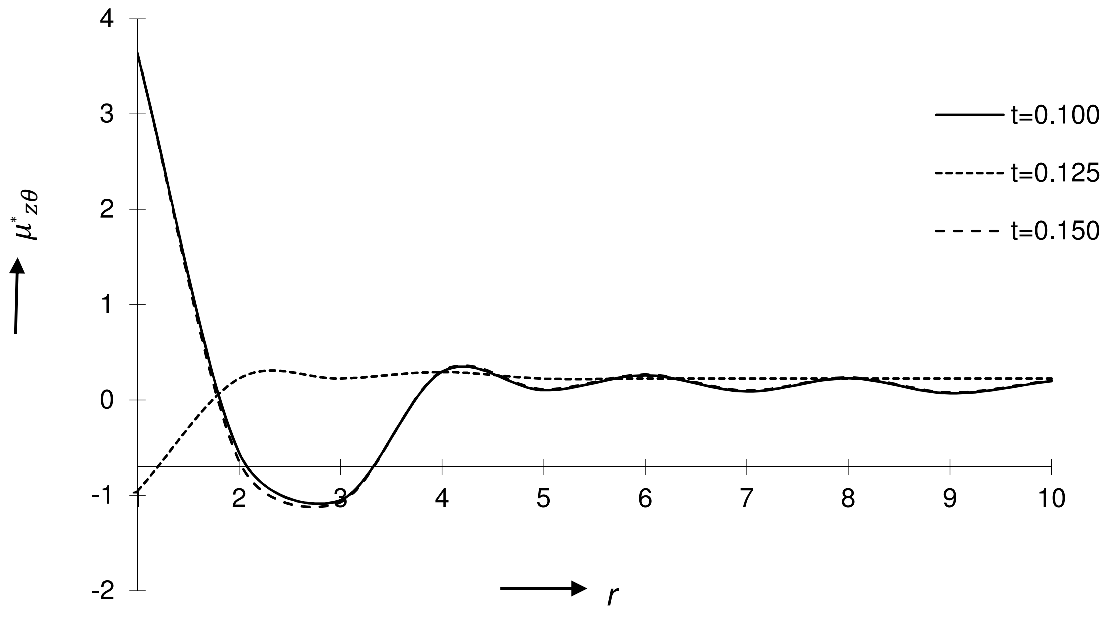

Fig. 2.5.3 Variation of tangential couple stress $\mu^*_{z\theta}$

Fig. 2.5.2 shows that for lower value of t, normal displacement σ^*_{zz} increases initially in the range $0 \leq r \leq 1$ but later becomes stationary for t=0.100 and starts oscillating in the range $1 < r \leq 4$ for t=0.125 and t=0.150. Fig. 2.5.3 shows that tangential couple stress $\mu^*_{z\theta}$ initially decreases for t=0.100 and t=0.150 in the range $0 \leq r < 3$ but then increases in the range $3 \leq r < 4$ and oscillates in the range $4 \leq r \leq 10$. But for t=0.125, tangential couple stress increases for $0 \leq r \leq 2$ and then becomes steady.

2.6. Conclusion

This study reveals a simple technique of obtaining stress-strain components for magneto micropolar elastic solid subjected to concentrated force. Considerable magnetic effect is observed on normal stress σ_{zz} and tangential couple stress $\mu_{z\theta}$ on the application of mechanical force. It is seen that near the point of application of source, normal displacement has higher values for larger values of parameter ‘t’. Same is the case with normal stress component. One can observe that deformation of a body depends on the nature of forced applied in addition to the applied boundary conditions. Further it can be concluded that decrease or increase in normal displacement and components of stress is not uniform with respect to parameter ‘t’.

Chapter 3

Dynamical problem of generalized magneto micropolar thermoelastic medium in half space

The theories developed by including the magnetic effect namely magneto-elasticity and both magnetic and thermal effect namely magneto-thermo-elasticity are being deployed to study the elastic and thermo-elastic deformations when body is under externally applied magnetic field. Due to their extensive engineering applications in the fields of geophysics, optics, acoustics, damping of acoustic waves in the magnetic field etc., these theories are being rapidly developed in recent years. A two-dimensional problem in magneto thermo elastic half-space subjected to a non-uniform thermal shock was studied by Sherief and Helmy [68] in the presence of a transverse magnetic field. They made use of theory of generalized thermoelasticity with one relaxation time for this study. Influence of rotation and the magnetic field using G-N theory of a rotating semi-infinite magneto thermoelastic medium on the plane harmonic waves was presented by Othman and Song [69]. A two-dimensional coupled problem in magneto thermoelastic half space solid which is thermally as well as electrically conducting and subjected to a time-dependent heat was studied by He and Li [70]. Othman and Song [71] studied the models of generalized thermo magneto elasticity using three different theories namely L-S theory (with one relaxation time), G-L theory (with two relaxation times) and classical dynamic coupled theory. Problem was investigated in a perfectly conducting medium which is rotating with uniform angular velocity. Lotfy et al. [72] investigated a two dimensional problem in half space possessing cubic symmetry with mode-I crack. All three models (L-S theory with one relaxation time, G-L theory with two relaxation times and coupled theory) were compared using three dimensional graphs.

In the present study a two dimensional model has been used to analyse the magneto micropolar thermoelastic problem with two temperature parameter subjected to concentrated force. Solution has been obtained in frequency domain by employing Laplace and Fourier transform and inversion has been done numerically. As an application of the approach an instantaneous thermal point source has been applied at the boundary.

3.1. Basic Equations

For a perfectly conducting micropolar elastic medium, the constitutive relations, field equations in micropolar elastic solid along with the equations of electromagnetism are given by (Following Nowacki [65]),

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \epsilon_{ijr}\phi_r\right) - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{3.1}$$

$$\mu_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}. \tag{3.2}$$

$$(\lambda+\mu)\nabla(\nabla.\vec{u}) + (\mu+\kappa)\nabla^2\vec{u} + \kappa\nabla\times\vec{\phi} - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{3.3}$$

$$(\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{\phi}\right) - \gamma\nabla\times\nabla\times\vec{\phi} + \kappa\nabla\times\vec{u} - 2\kappa\vec{\phi} = \rho j\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{3.4}$$

$$curl\vec{h} = \vec{J} + \dot{\vec{D}}, \qquad curl\vec{E} = -\dot{\vec{B}}, \tag{3.5}$$

$$\vec{E} = -\mu_0\left(\dot{\vec{u}}\times\vec{H}_0\right), \tag{3.6}$$

$$div\vec{H} = 0, \quad \vec{B} = \mu_0\vec{H}, \tag{3.7}$$

$$\vec{D} = \epsilon_0\vec{E}, \;\; \vec{H} = \vec{H}_0 + \vec{h}. \tag{3.8}$$

Following Youssef [73] relation between the heat conduction and the dynamical heat with $a > 0$ as two-temperature can be written as

$$K^*\nabla^2\varphi = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E T + \nu T_0\nabla.\vec{u}\right), \tag{3.9}$$

$$\varphi - T = a\nabla^2\varphi, \tag{3.10}$$

The equations (3.3)-(3.4) and (3.9)-(3.10) in the Cartesian coordinate system (x_1, x_2, x_3) in component take the form

$$(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_1\partial x_2} + \frac{\partial^2 u_3}{\partial x_1\partial x_3}\right) + (\mu+\kappa)\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} + \frac{\partial^2 u_1}{\partial x_3^2}\right)$$
$$+ \kappa\left(\frac{\partial\phi_3}{\partial x_2} - \frac{\partial\phi_2}{\partial x_3}\right) - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{3.11}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_2\partial x_1}+\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_3}{\partial x_2\partial x_3}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_2}{\partial x_1^2}+\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_2}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial\phi_1}{\partial x_3}-\frac{\partial\phi_3}{\partial x_1}\right)-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial \mathrm{T}}{\partial x_2}=\rho\frac{\partial^2 u_2}{\partial t^2}, \tag{3.12}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_3\partial x_1}+\frac{\partial^2 u_2}{\partial x_2\partial x_3}+\frac{\partial^2 u_3}{\partial x_3^2}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_3}{\partial x_1^2}+\frac{\partial^2 u_3}{\partial x_2^2}+\frac{\partial^2 u_3}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial\phi_2}{\partial x_1}-\frac{\partial\phi_1}{\partial x_2}\right)-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial \mathrm{T}}{\partial x_3}=\rho\frac{\partial^2 u_3}{\partial t^2}, \tag{3.13}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_1\partial x_2}+\frac{\partial^2\phi_3}{\partial x_1\partial x_3}\right)+\gamma\left(\frac{\partial^2\phi_1}{\partial x_1^2}+\frac{\partial^2\phi_1}{\partial x_2^2}+\frac{\partial^2\phi_1}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3}\right)-2\kappa\phi_1=\rho j\frac{\partial^2\phi_1}{\partial t^2}, \tag{3.14}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_2\partial x_1}+\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_3}{\partial x_2\partial x_3}\right)+\gamma\left(\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_2^2}+\frac{\partial^2\phi_2}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-2\kappa\phi_2=\rho j\frac{\partial^2\phi_2}{\partial t^2}, \tag{3.15}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_3\partial x_1}+\frac{\partial^2\phi_2}{\partial x_3\partial x_2}+\frac{\partial^2\phi_3}{\partial x_3^2}\right)+\gamma\left(\frac{\partial^2\phi_3}{\partial x_1^2}+\frac{\partial^2\phi_3}{\partial x_2^2}+\frac{\partial^2\phi_3}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right)-2\kappa\phi_3=\rho j\frac{\partial^2\phi_3}{\partial t^2}, \tag{3.16}$$

$$K^*\left(\frac{\partial^2\varphi}{\partial x_1^2}+\frac{\partial^2\varphi}{\partial x_2^2}+\frac{\partial^2\varphi}{\partial x_3^2}\right)$$
$$=\rho C_E\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right) \tag{3.17}$$
$$+\rho\nu C_E T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)$$

$$\varphi-T=a\left(\frac{\partial^2\varphi}{\partial x_1^2}+\frac{\partial^2\varphi}{\partial x_2^2}+\frac{\partial^2\varphi}{\partial x_3^2}\right), \tag{3.18}$$

3.2. Formulation and solution of the problem

Considering the region $x_3 \geq 0$ to be occupied with linear homogenous isotropic micropolar thermoelastic medium with two temperatures which is perfectly conducting.

As we are considering a two dimensional plane strain problem in which with x_3-axis is pointing vertically into the medium, we take

$$\vec{u} = (u_1, 0, u_3), \qquad \vec{\phi} = (0, \phi_2, 0), \qquad \vec{h} = (0, h, 0). \tag{3.19}$$

Using equation (3.19) in equations (3.1)-(3.18),(3.2), we obtain

$$(\lambda+\mu)\frac{\partial^2 u_1}{\partial x_1^2} + (\mu+\kappa)\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_3^2}\right) - \kappa\frac{\partial \phi_2}{\partial x_3} - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial \mathrm{T}}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{3.20}$$

$$(\lambda+\mu)\frac{\partial^2 u_3}{\partial x_3^2} + (\mu+\kappa)\left(\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_3^2}\right) + \kappa\frac{\partial \phi_2}{\partial x_1} - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial \mathrm{T}}{\partial x_3} = \rho\frac{\partial^2 u_3}{\partial t^2}, \tag{3.21}$$

$$(\alpha+\beta)\frac{\partial^2 \phi_2}{\partial x_1^2} + \gamma\left(\frac{\partial^2 \phi_2}{\partial x_1^2} + \frac{\partial^2 \phi_2}{\partial x_3^2}\right) + \kappa\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - 2\kappa\phi_2 = \rho j\frac{\partial^2 \phi_2}{\partial t^2}, \tag{3.22}$$

$$K^*\left(\frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \varphi}{\partial x_3^2}\right) = \rho C_E\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \rho\nu C_E T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) \tag{3.23}$$

$$\sigma_{33} = \lambda\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + (2\mu+\kappa)\frac{\partial u_3}{\partial x_3} - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta, \tag{3.24}$$

$$\sigma_{11} = \lambda\frac{\partial u_3}{\partial x_3} + (\lambda+2\mu+\kappa)\frac{\partial u_1}{\partial x_1} - \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta, \tag{3.25}$$

$$\sigma_{31} = \mu\frac{\partial u_3}{\partial x_1} + (\mu+\kappa)\frac{\partial u_1}{\partial x_3} + \kappa\phi_2 \tag{3.26}$$

$$\mu_{32} = \gamma\frac{\partial \phi_2}{\partial x_3}. \tag{3.27}$$

Introducing the displacement potential functions $\psi(x_1, x_3, t)$ and $\vec{\zeta}(x_1, x_3, t)$ by taking

$$\vec{u} = \nabla\psi + \nabla\times\vec{\zeta}, \qquad \nabla.\vec{\zeta} = 0, \qquad \vec{\zeta} = (0, \zeta, 0). \tag{3.28}$$

And using following dimensionless quantities in equations (3.20)-(3.27),

$$(x_1', x_3', u_1', u_3') = c_0\eta(x_1, x_3, u_1, u_3), \qquad (t', \tau_0', \tau_1') = c_0^2\eta(t, \tau_0, \tau_1),$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\lambda + 2\mu + \kappa}, \quad \mu'_{ij} = \frac{\kappa}{c_0\eta(\mu + \kappa)}\mu_{ij}, \quad J'_i = \frac{\eta}{\sigma_0^2\mu_0^2 c_0 H_0}J_i,$$
$$\phi'_2 = \frac{\kappa}{\mu + \kappa}\phi_2, \tag{3.29}$$

$$h' = \frac{\eta}{\sigma_0\mu_0 H_0}h, \qquad (\varphi', \theta') = \frac{(\varphi, T) - T_0}{T_0}, \qquad E'_i = \frac{\eta}{\sigma_0\mu_0^2 c_0 H_0}E_i$$

Where

$$\eta = \frac{\rho C_E}{K^*}, \qquad c_0^2 = \frac{\lambda + 2\mu + \kappa}{\rho}.$$

We obtain (after dropping dashes for convenience)

$$a_1\nabla^2\psi - a_2\ddot{\psi} - a_3\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta - a_4\ddot{\psi} = 0, \tag{3.30}$$

$$a_5\nabla^2\zeta - a_5\phi_2 - a_2\ddot{\zeta} - a_4\ddot{\zeta} = 0, \tag{3.31}$$

$$\nabla^2\phi_2 - a_6\nabla^2\zeta - a_7\phi_2 - a_8\ddot{\phi}_2 = 0, \tag{3.32}$$

$$\nabla^2\varphi - a_9\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\theta - a_{10}\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\psi = 0, \tag{3.33}$$

$$\varphi - \theta = a_{11}\nabla^2\varphi, \tag{3.34}$$

$$\sigma_{33} = a_{12}\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta, \tag{3.35}$$

$$\sigma_{11} = \frac{\partial u_1}{\partial x_1} + a_{12}\frac{\partial u_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta, \tag{3.36}$$

$$\sigma_{31} = a_{13}\frac{\partial u_3}{\partial x_1} + a_{14}\frac{\partial u_1}{\partial x_3} + a_{14}\phi_2, \tag{3.37}$$

$$\mu_{32} = \gamma\frac{\partial\phi_2}{\partial x_3}, \tag{3.38}$$

$$h = -a_{15}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right), \tag{3.39}$$

$$E_1 = a_{16}\frac{\partial u_3}{\partial t}, \tag{3.40}$$

$$E_3 = -a_{16}\frac{\partial u_1}{\partial t}, \tag{3.41}$$

$$J_1 = -a_{15}\frac{\partial h}{\partial x_3} + a_{17}\frac{\partial^2 u_3}{\partial t^2}, \tag{3.42}$$

$$J_3 = a_{15}\frac{\partial h}{\partial x_1} - a_{17}\frac{\partial^2 u_1}{\partial t^2}, \tag{3.43}$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, \quad a_1 = \lambda + 2\mu + \kappa, \quad a_2 = c_0^2 \epsilon_0 \mu_0 H_0^2, \quad a_3 = \nu T_0, \quad a_4 = \rho c_0^2, \quad a_5 = \mu + \kappa,$$

$$a_6 = \frac{\kappa^2}{\gamma \eta^2 c_0^2 (\mu + \kappa)}, \quad a_7 = \frac{2\kappa}{c_0^2 \eta^2 \gamma}, \quad a_8 = \frac{\rho j c_0^2 \eta}{\gamma}, \quad a_9 = \frac{\rho C_E}{K^* \eta}, \quad a_{10} = \frac{\nu}{K^* \eta}, \quad a_{11} = a c_0^2 \eta^2,$$

$$a_{12} = \frac{\lambda}{\lambda + 2\mu + \kappa}, \quad a_{13} = \frac{\mu}{\lambda + 2\mu + \kappa}, \quad a_{14} = \frac{\mu + \kappa}{\lambda + 2\mu + \kappa}, \quad a_{15} = \frac{\eta}{\sigma_0 \mu_0}, \quad a_{16} = \frac{\eta}{\sigma_0 \mu_0 c_0},$$

$$a_{17} = \frac{c_0^2 \epsilon_0 \eta^2}{\sigma_0^2 \mu_0}.$$

Now using the Laplace transform and its properties as defined in equations (2.45)-(2.47) on equations (3.30)-(3.43), we obtain

$$a_1 \nabla^2 \bar{\psi} - a_2 s^2 \bar{\psi} - a_3 (1 + \tau_1 s) \bar{\theta} - a_4 s^2 \bar{\psi} = 0, \tag{3.44}$$

$$a_5 \nabla^2 \bar{\zeta} - a_5 \bar{\phi}_2 - a_2 s^2 \bar{\zeta} - a_4 \bar{\zeta} = 0, \tag{3.45}$$

$$\nabla^2 \bar{\phi}_2 - a_6 \nabla^2 \bar{\zeta} - a_7 \bar{\phi}_2 - a_8 s^2 \bar{\phi}_2 = 0, \tag{3.46}$$

$$\nabla^2 \bar{\varphi} - a_9 (s + \tau_0 s^2) \bar{\theta} - a_{10} (s + \tau_0 s^2) \bar{\psi} = 0, \tag{3.47}$$

$$\bar{\varphi} - \bar{\theta} = a_{11} \nabla^2 \bar{\varphi}, \tag{3.48}$$

$$\bar{\sigma}_{33} = a_{12} \frac{\partial \bar{u}_1}{\partial x_1} + \frac{\partial \bar{u}_3}{\partial x_3} - \frac{a_3}{a_1} (1 + \tau_1 s) \bar{\theta}, \tag{3.49}$$

$$\bar{\sigma}_{11} = \frac{\partial \bar{u}_1}{\partial x_1} + a_{12} \frac{\partial \bar{u}_3}{\partial x_3} - \frac{a_3}{a_1} (1 + \tau_1 s) \bar{\theta}, \tag{3.50}$$

$$\bar{\sigma}_{31} = a_{13} \frac{\partial \bar{u}_3}{\partial x_1} + a_{14} \frac{\partial \bar{u}_1}{\partial x_3} + a_{14} \bar{\phi}_2, \tag{3.51}$$

$$\bar{\mu}_{32} = \gamma \frac{\partial \bar{\phi}_2}{\partial x_3}, \tag{3.52}$$

$$\bar{h} = -a_{15} \left(\frac{\partial \bar{u}_1}{\partial x_1} + \frac{\partial \bar{u}_3}{\partial x_3} \right), \tag{3.53}$$

$$\bar{E}_1 = a_{16} s \bar{u}_3, \tag{3.54}$$

$$\bar{E}_3 = -a_{16}s\bar{u}_1, \tag{3.55}$$

$$\bar{J}_1 = -a_{15}\frac{\partial \bar{h}}{\partial x_3} + a_{17}s^2\bar{u}_3, \tag{3.56}$$

$$\bar{J}_3 = a_{15}\frac{\partial \bar{h}}{\partial x_1} - a_{17}s^2\bar{u}_1, \tag{3.57}$$

The Fourier transform [66] of $\bar{f}(x_1, x_3, s)$ with respect to variable x_1 is defined through the relation

$$F\{\bar{f}(x_1, x_3, s)\} = \int_{-\infty}^{\infty} e^{-\iota\xi x_1}\bar{f}(x_1, x_3, s)dx_1 = \tilde{f}(\xi, x_3, s), \tag{3.58}$$

where ξ is the Fourier transform variable.

Applying this transform on equations (3.44)-(3.57), we get

$$(D^2 - \alpha_1)\tilde{\psi} - \alpha_2\tilde{\theta} = 0, \tag{3.59}$$

$$(D^2 - \alpha_3)\tilde{\zeta} - \tilde{\phi}_2 = 0, \tag{3.60}$$

$$(D^2 - \alpha_4)\tilde{\psi} - a_6(D^2 - \xi^2)\tilde{\zeta} = 0, \tag{3.61}$$

$$(D^2 - \xi^2)\tilde{\varphi} - \alpha_5\tilde{\theta} - \alpha_6(D^2 - \xi^2)\tilde{\psi} = 0, \tag{3.62}$$

$$a_{11}(D^2 - \alpha_7)\tilde{\varphi} + \tilde{\theta} = 0, \tag{3.63}$$

$$\tilde{\sigma}_{33} = a_{12}\frac{\partial \tilde{u}_1}{\partial x_1} + \frac{\partial \tilde{u}_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\tilde{\theta}, \tag{3.64}$$

$$\tilde{\sigma}_{11} = \frac{\partial \tilde{u}_1}{\partial x_1} + a_{12}\frac{\partial \tilde{u}_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\tilde{\theta}, \tag{3.65}$$

$$\tilde{\sigma}_{31} = a_{13}\frac{\partial \tilde{u}_3}{\partial x_1} + a_{14}\frac{\partial \tilde{u}_1}{\partial x_3} + a_{14}\tilde{\phi}_2, \tag{3.66}$$

$$\tilde{\mu}_{32} = \gamma\frac{\partial \tilde{\phi}_2}{\partial x_3}, \tag{3.67}$$

$$\tilde{h} = -a_{15}(\iota\xi\tilde{u}_1 + D\tilde{u}_3), \tag{3.68}$$

$$\tilde{E}_1 = a_{16}s\tilde{u}_3, \tag{3.69}$$

$$\tilde{E}_3 = -a_{16}s\tilde{u}_1, \tag{3.70}$$

$$\tilde{J}_1 = -a_{15}D\tilde{h} + a_{17}s^2\tilde{u}_3 \tag{3.71}$$

$$\tilde{J}_3 = a_{15}\iota\xi\tilde{h} - a_{17}s^2\tilde{u}_1, \tag{3.72}$$

Equations (3.59)-(3.63) can be re-written as

$$(D^4 - \alpha_{13}D^2 + \alpha_{14})(\tilde{\zeta}, \tilde{\phi}_2) = 0, \tag{3.73}$$

$$(D^4 - \alpha_{15}D^2 + \alpha_{16})(\tilde{\psi}, \tilde{\theta}, \tilde{\varphi}) = 0, \tag{3.74}$$

where

$$D = \frac{d}{dx_3}, \qquad \alpha_1 = \xi^2 + \frac{a_2 + a_4}{a_1}s^2, \qquad \alpha_2 = \frac{a_3}{a_1}(1 + \tau_1 s),$$
$$\alpha_3 = \xi^2 + \frac{a_2 + a_4}{a_5}s^2,$$

$$\alpha_4 = \xi^2 + a_7 + a_8 s^2, \qquad \alpha_5 = a_9(s + \tau_0 s^2), \qquad \alpha_6 = a_{10}(s + \tau_0 s^2),$$
$$\alpha_7 = \xi^2 + \frac{1}{a_{11}},$$

$$\alpha_8 = 1 + a_{11}\alpha_5, \qquad \alpha_9 = \xi^2 + a_{11}\alpha_5\alpha_7, \qquad \alpha_{10} = a_{11}\alpha_2\alpha_6 + \alpha_2,$$

$$\alpha_{11} = a_{11}\alpha_2\alpha_6(\xi^2 + \alpha_7) + \alpha_1\alpha_2 + \alpha_9, \qquad \alpha_{12} = a_{11}\alpha_2\alpha_6\alpha_7\xi^2 + \alpha_9\alpha_{11},$$

$$\alpha_{13} = \alpha_3 + \alpha_4 + a_6, \qquad \alpha_{14} = \alpha_3\alpha_4 + a_6\xi^2, \qquad \alpha_{15} = \frac{\alpha_{11}}{\alpha_{10}}, \qquad \alpha_{16} = \frac{\alpha_{12}}{\alpha_{10}}.$$

Solutions of equation (3.73)-(3.74) satisfying the radiation condition $Re(m_j) \geq 0$ are of the form,

$$\tilde{\zeta}(\xi, x_3, s) = A_1(\xi, s)e^{-m_1x_3} + A_2(\xi, s)e^{-m_2x_3}, \tag{3.75}$$

$$\tilde{\psi}(\xi, x_3, s) = A_3(\xi, s)e^{-m_3x_3} + A_4(\xi, s)e^{-m_4x_3}, \tag{3.76}$$

where

$$m_1^2 = \frac{\alpha_{13} + \sqrt{\alpha_{13}^2 - 4\alpha_{14}}}{2}, m_2^2 = \frac{\alpha_{13} - \sqrt{\alpha_{13}^2 - 4\alpha_{14}}}{2},$$

$$m_3^2 = \frac{\alpha_{15} + \sqrt{\alpha_{15}^2 - 4\alpha_{16}}}{2}, m_4^2 = \frac{\alpha_{15} - \sqrt{\alpha_{15}^2 - 4\alpha_{16}}}{2},$$

Using (3.75)-(3.76) in equations (3.59)-(3.72), we obtain

$$\tilde{\phi}_2(\xi, x_3, s) = (m_1^2 - \alpha_3)A_1e^{-m_1x_3} + (m_2^2 - \alpha_3)A_2e^{-m_2x_3}, \tag{3.77}$$

$$\tilde{\theta}(\xi, x_3, s) = \frac{1}{\alpha_2}[(m_3^2 - \alpha_1)A_3 e^{-m_3 x_3} + (m_4^2 - \alpha_1)A_4 e^{-m_4 x_3}], \tag{3.78}$$

$$\tilde{\varphi}(\xi, x_3, s) = -\frac{1}{a_{11}\alpha_2}\left[\frac{(m_3^2 - \alpha_1)}{(m_3^2 - \alpha_7)}A_3 e^{-m_3 x_3} + \frac{(m_4^2 - \alpha_1)}{(m_4^2 - \alpha_7)}A_4 e^{-m_4 x_3}\right], \tag{3.79}$$

$$\tilde{u}_1 = (m_1 A_1 e^{-m_1 x_3} + m_2 A_2 e^{-m_2 x_3}) + \iota\xi(A_3 e^{-m_3 x_3} + A_4 e^{-m_4 x_3}), \tag{3.80}$$

$$\tilde{u}_3 = \iota\xi(A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3}) - (m_3 A_3 e^{-m_3 x_3} + m_4 A_4 e^{-m_4 x_3}), \tag{3.81}$$

$$\begin{aligned}\tilde{\sigma}_{33} &= -\iota\xi(1 - a_{12})(m_1 A_1 e^{-m_1 x_3} + m_2 A_2 e^{-m_2 x_3}) \\ &\quad + (\alpha_1 - \xi^2 a_{12})(A_3 e^{-m_3 x_3} + A_4 e^{-m_4 x_3}),\end{aligned} \tag{3.82}$$

$$\begin{aligned}\tilde{\sigma}_{11} &= \iota\xi(1 - a_{12})(m_1 A_1 e^{-m_1 x_3} + m_2 A_2 e^{-m_2 x_3}) \\ &\quad + (\alpha_1 - \xi^2 - m_3^2 + a_{12} m_3^2)A_3 e^{-m_3 x_3} \\ &\quad + (\alpha_1 - \xi^2 - m_4^2 + a_{12} m_4^2)A_4 e^{-m_4 x_3},\end{aligned} \tag{3.83}$$

$$\begin{aligned}\tilde{\sigma}_{31} &= -(\xi^2 a_{13} + a_{14}\alpha_3)(A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3}) \\ &\quad - \iota\xi(a_{13} + a_{14})(m_3 A_3 e^{-m_3 x_3} + m_4 A_4 e^{-m_4 x_3}),\end{aligned} \tag{3.84}$$

$$\tilde{\mu}_{32} = -\gamma[m_1(m_1^2 - \alpha_3)A_1 e^{-m_1 x_3} + m_2(m_2^2 - \alpha_3)A_2 e^{-m_2 x_3}], \tag{3.85}$$

$$\tilde{h} = -a_{15}[(m_3^2 - \xi^2)A_3 e^{-m_3 x_3} + (m_4^2 - \xi^2)A_4 e^{-m_4 x_3}], \tag{3.86}$$

$$\begin{aligned}\tilde{J}_1 &= a_{17}s^2[\iota\xi(A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3}) - (m_3 A_3 e^{-m_3 x_3} + m_4 A_4 e^{-m_4 x_3})] \\ &\quad - a_{15}^2[m_3(m_3^2 - \xi^2)A_3 e^{-m_3 x_3} + m_4(m_4^2 - \xi^2)A_4 e^{-m_4 x_3}],\end{aligned} \tag{3.87}$$

3.3. Boundary Conditions

Boundary conditions at the plane $x_3 = 0$ which is stress free and subjected to an instantaneous thermal point source, are:

$$\theta(x_1, 0, t) = \theta_0\delta(x_1)\delta(t), \tag{3.88}$$

$$\sigma_{33}(x_1, 0, t) = 0, \tag{3.89}$$

$$\sigma_{31}(x_1, 0, t) = 0, \tag{3.90}$$

$$\mu_{32}(x_1, 0, t) = 0. \tag{3.91}$$

Where θ_0 is the maximum constant temperature applied on the boundary.

After making use of Laplace and Fourier transforms as defined in (2.45) and (3.58) on equations (3.88)-(3.91), we get

$$\tilde{\theta}(\xi,0,s)=\tilde{\theta}_0, \tag{3.92}$$

$$\tilde{\sigma}_{33}(\xi,0,s)=0, \tag{3.93}$$

$$\tilde{\sigma}_{31}(\xi,0,s)=0, \tag{3.94}$$

$$\mu_{32}(\xi,0,s)=0. \tag{3.95}$$

Using equations (3.78), (3.82), (3.84) and (3.85) in (3.92)-(3.95), we get

$$\frac{1}{\alpha_2}[(m_3^2-\alpha_1)A_3+(m_4^2-\alpha_1)A_4]=\tilde{\theta}_0, \tag{3.96}$$

$$-\iota\xi(1-a_{12})[m_1A_1+m_2A_2]+(\alpha_1-\xi^2a_{12})[A_3+A_4]=0, \tag{3.97}$$

$$-(\xi^2a_{13}+\alpha_3a_{14})[A_1+A_2]-\iota\xi(a_{14}+a_{13})[m_3A_3+m_4A_4]=0, \tag{3.98}$$

$$m_1(m_1^2-\alpha_3)A_1+m_2(m_2^2-\alpha_3)A_2=0. \tag{3.99}$$

Equations (3.96)-(3.99), after simplification give,

$$A_1=\frac{\iota\alpha_2(\alpha_1-\xi^2a_{12})(m_2^2-\alpha_3)(\Delta_1+\Delta_2)}{\xi m_1(1-a_{12})(m_1^2-m_2^2)[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0, \tag{3.100}$$

$$A_2=-\frac{\iota\alpha_2(\alpha_1-\xi^2a_{12})(m_1^2-\alpha_3)(\Delta_1+\Delta_2)}{\xi m_2(1-a_{12})(m_1^2-m_2^2)[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0, \tag{3.101}$$

$$A_3=\frac{\alpha_2\Delta_2}{[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0, \tag{3.102}$$

$$A_4=\frac{\alpha_2\Delta_1}{[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0. \tag{3.103}$$

where

$$\Delta_1=[\alpha_{17}(m_2^2-\alpha_3)-\alpha_{18}m_1m_3(m_1^2-m_2^2)],$$

$$\Delta_2=[\alpha_{17}(m_1^2-\alpha_3)+\alpha_{18}m_2m_4(m_1^2-m_2^2)],$$

$$\alpha_{17}=-\frac{\iota(\xi^2a_{13}+\alpha_3a_{14})(\alpha_1-\xi^2a_{12})}{\xi(1-a_{12})},\qquad \alpha_{18}=\iota\xi(a_{14}+a_{13}).$$

3.4. Inversion of the transforms

The transformed solutions are functions of the form $\tilde{f}(\xi, x_3, s)$, so to get these functions back in the physical domain in the form $f(x_1, x_3, t)$, we first invert the Fourier transform by using

$$\bar{f}(x_1, x_3, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(\xi, x_3, s) e^{\iota \xi x_1} d\xi. \tag{3.104}$$

Since every function can be expressed uniquely as sum of an even and odd function. Hence we write

$$\tilde{f}(\xi, x_3, s) = \tilde{f}_e(\xi, x_3, s) + \tilde{f}_o(\xi, x_3, s), \tag{3.105}$$

where

$$\tilde{f}_e(\xi, x_3, s) = \frac{1}{2}\{\tilde{f}(\xi, x_3, s) + \tilde{f}(-\xi, x_3, s)\}, \tag{3.106}$$

$$\tilde{f}_o(\xi, x_3, s) = \frac{1}{2}\{\tilde{f}(\xi, x_3, s) - \tilde{f}(-\xi, x_3, s)\}, \tag{3.107}$$

are even and odd parts of the function $\tilde{f}(\xi, x_3, s)$ with parameter ξ respectively. Thus equation (3.104) can be re-written as

$$\bar{f}(x_1, x_3, s) = \frac{1}{\pi}\int_{0}^{\infty} \left[\cos(\xi x_1)\tilde{f}_e + \sin(\xi x_1)\tilde{f}_o\right] d\xi. \tag{3.108}$$

The evaluation of integral in equation (3.108) leads to the transformed function $\bar{f}(x_1, x_3, s)$ of the function $f(x_1, x_3, t)$. Afterwards, for the fixed values of x_1 and x_3, the function $\bar{f}(x_1, x_3, s)$ can be considered as the Laplace transform $\bar{g}(s)$ of some function $g(t)$. The technique for numerical inversion of $\bar{g}(s)$ has already been explained in section 2.5 of chapter 2. Also the integral in equation (3.104) is to be calculated numerically with the help of technique mentioned in the above mentioned section.

3.5. Numerical discussion and Analysis

The analysis is conducted for a magnesium crystal. Using reference [18], the physical parameters are taken as

$\lambda = 9.4 \times 10^{10} N.m^{-2}$, $\mu = 4.0 \times 10^{10} N.m^{-2}$, $\kappa = 1.0 \times 10^{10} N.m^{-2}$, $\rho = 1.74 \times 10^{3}\ Kg.m^{-3}$, $J = 0.2 \times 10^{-19}\ m^{2}$, $\epsilon_0 = 10^{-9}/36\pi\ F.m^{-1}$, $\beta = 0.98 \times 10^{-5} N.m^{-2}$, $\sigma_0 = 2.2356 \times 10^{7} S.m^{-1}$, $\mu_0 = 4\pi \times 10^{-7} H.m^{-1}$,, $H_o = 1\ A.m^{-1}$, $\eta = 0.0168$, $K^* = 386 W.m^{-1}.K^{-1}$, $C_E = 383.1 J.Kg^{-1}.K^{-1}$, $T_0 = 293K$, $\theta_0 = 1$.

Graphical analysis of variation in temperature distribution, displacement, normal stress and tangential couple stress has been done. Results have been compared in the absence and presence of magnetic field for two different t values. In the following figures 1-4, the solid line represents the magneto micropolar thermoelastic medium (MMT1) at t=0.1; small dashed line magneto micropolar thermoelastic medium (MMT2) at t=0.5; solid line with circles signifies micropolar thermoelastic medium (MT1) at t=0.1 and small dashed line with circles represents micropolar thermoelastic medium (MT2) at t=0.5 under the application of thermal source.

Fig. 3.5.1 shows the variation in normal displacement component (u_3)with changes in value of x_1.It is observed that the presence of magnetic field leads to higher values of u_3 as compared to its values in the absence of magnetic field. Also its nature is oscillatory for MT1 and MT2 and amplitude keeps on decreasing with increase in x_1.

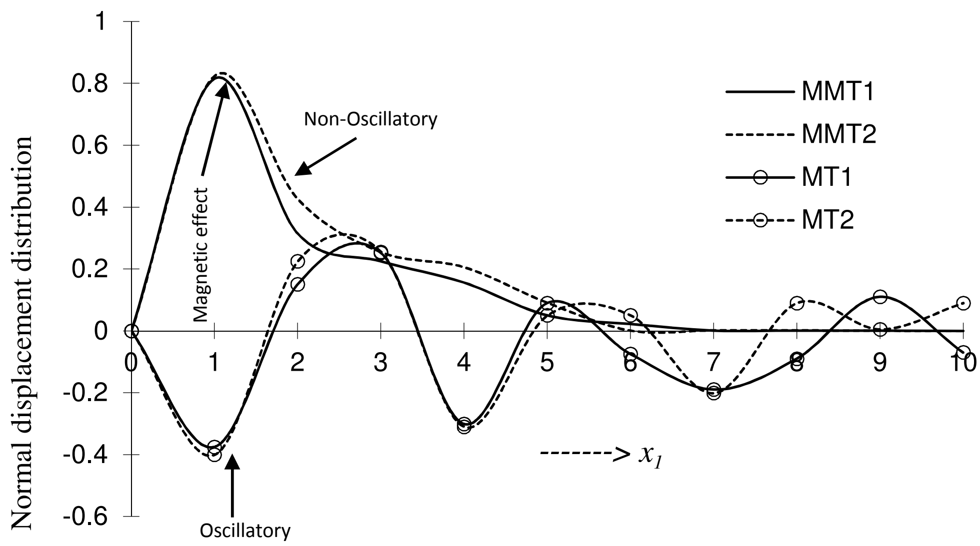

Fig. 3.5.1 Variation of displacement

Fig. 3.5.2 shows vartions in temperature distribution (θ) with x_1. Initially starting with same values for both mediums MMT and MT, value of θ has higher amplitude in the the absence of magnetic effect. For large values of x_1 $(4 \leq x_1 \leq 5)$, its values start coinciding for both theories and tend to zero.

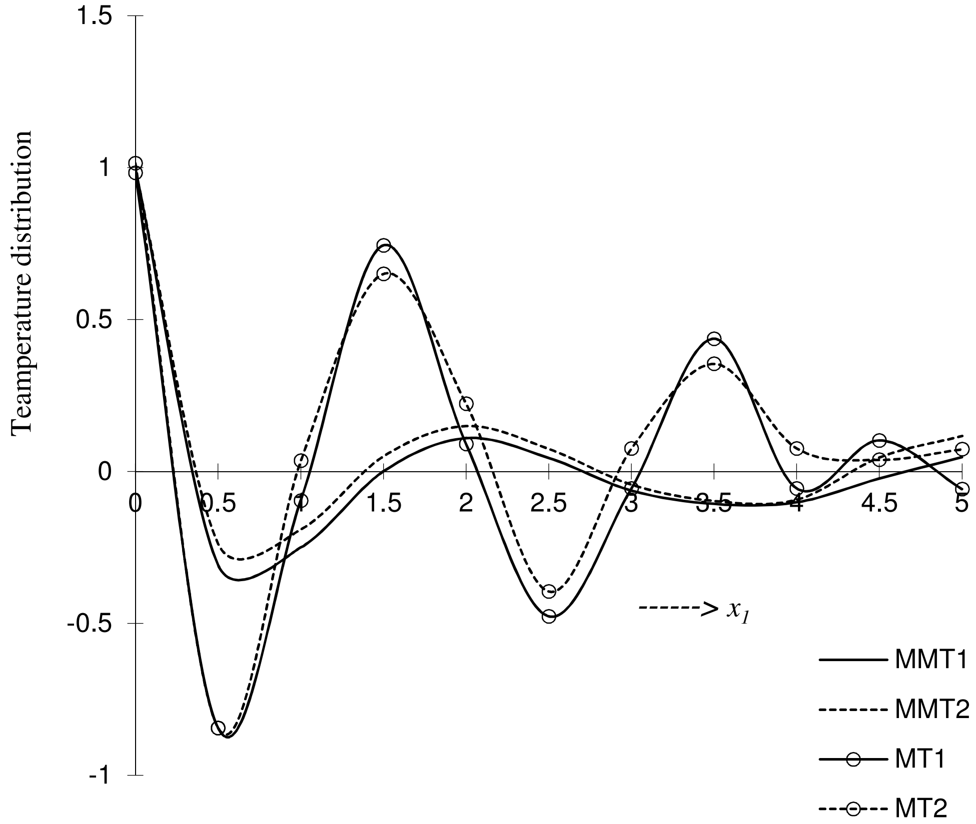

Fig. 3.5.2 Variation of temperature distribution

Variation in Normal force stress (σ_{33}) is being shown in Fig. 3.5.3. Here it is observed that σ_{33} behaves in opposite manner under both theories for range $0 \leq x_1 < 9$ but this difference tends to cease in the range $x_1 \geq 9$.

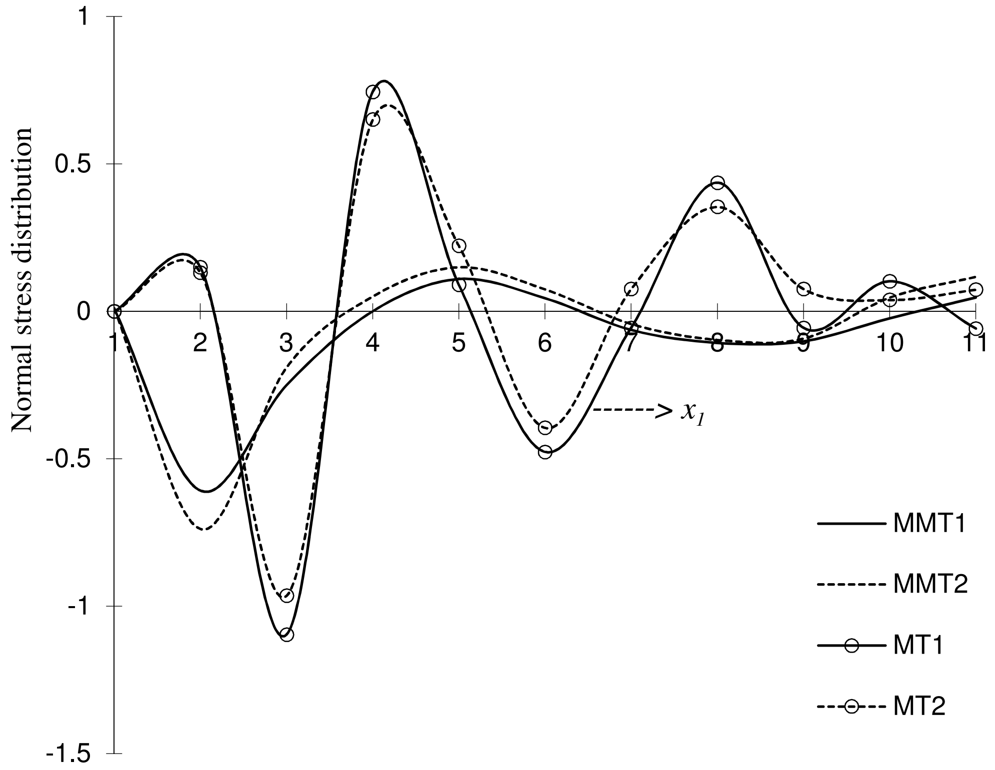

Fig. 3.5.3 Variation of Normal stress distribution

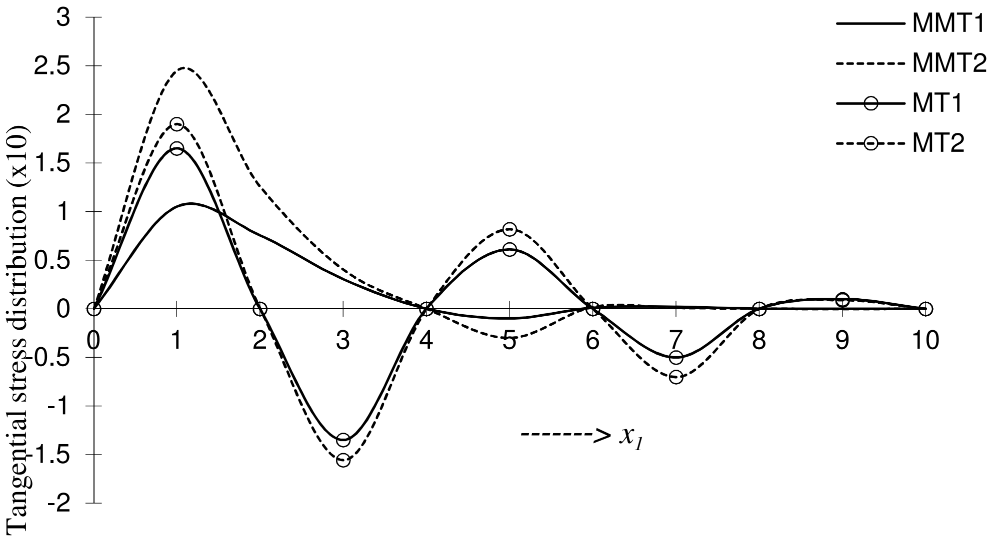

Fig. 3.5.4 behaviour of tangential couple stress (σ_{31}) is depicted. Values have been plotted after multiplying it with 10. σ_{31} again shows oscillatory nature in the absence of magnetic field (MT1 and MT2) but for large values of x_1, this behaviour tends to diminish and shows linear nature.

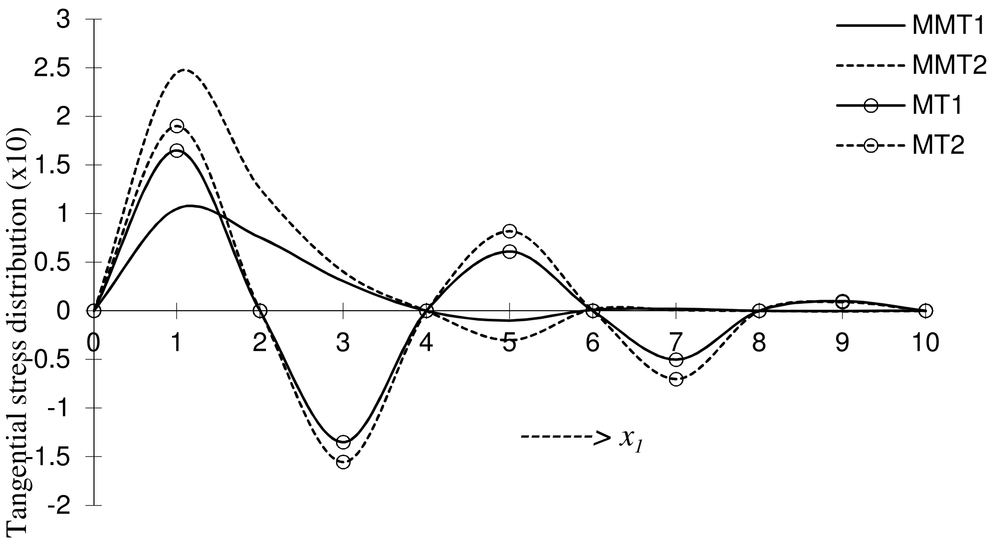

Fig. 3.5.4 Variation of tangential stress distribution

3.6. Conclusion

This study highlights a simplified technique of obtaining the stress and strain components in the case of micropolar isotropic solid subjected to thermal field in the presence of magnetic field. The trend of variations of the considered components are different in the presence and absence of magnetic field which confirms that magnetic field has significant impact on the normal displacement component u_3, temperature distribution θ, normal force stress σ_{33} and tangential couple stress σ_{31} along with application of thermal source. This study can be useful in analysing stress-strain behaviour of earth like model which is subjected to both thermal and magnetic fields.

Chapter 4

Eigen value approach to two dimensional problem in generalized magneto Micropolar thermoelastic medium with rotation effect

Last chapter represented a model for analysing the stress-strain relationship of a body under thermal and magnetic effect. In this chapter a problem with an eigen value approach has been studied to examine the effects of rotation when mechanical force is applied in the presence of transverse magnetic field in two dimensional generalized magneto micropolar thermoelastic infinite space. Present study can be regarded as a better representation of elastic model for studying the earth's planetary motion as it involves rotational velocity in addition to its thermal and electromagnetic field. Due to its many applications in the field of engineering, plasma physics, crystal physics, solid-earth geophysics and related areas, increasing attention is being given to this area. Literature contains numerous studies in this field. Singh [74] studied reflection and refraction at an interface between liquid half space and micropolar generalized thermoelastic solid half space using plane sound wave. Othman [75] investigated a problem on generalized thermoelastic plane waves under the effect of rotation by using the G-L theory. Results were also computed in the absence of rotation effect. Kong et al. [76] studied thermo magneto elastic stresses and perturbation of the magnetic field vector using an analytic method in a conducting non-homogenous hollow cylinder subjected to thermal shock. A one-dimensional problem in an infinite rotating medium for a generalized magneto-thermoelastic diffusive solid possessing a spherical cavity which is subjected to a time dependent thermal shock at its internal boundary was examined by Abd-All and Abo-Dahab [77] by considering it to be traction free. Also results were studied by including and excluding the effect of diffusion, rotation and magnetic field. Influence of number of effects in an elastic half space granular medium, like rotation, relaxation times, magnetic field, initial stress and gravity field on attenuation coefficient (Imaginary part of frequency equation root) were analysed by Mahmoud [78]. Lame's potential method was applied for obtaining the solution. Thermoelastic interactions in a homogeneous, thermally conducting cubic crystal, elastic half-plane had been studied by Abbas et al. [79] by using linear temperature ramping function.

In the current chapter results have been obtained by treating rotational velocity to be invariant. Integral transforms have been applied to solve the system of partial differential equations. Components of displacement, normal stress, tangential couple stress, electric field, temperature distribution and magnetic field have been obtained in transformed domain. Finally numerical inversion technique has been used to invert the result in physical domain. Graphical analysis has been done to describe the study.

4.1. Basic Equations

Following Baksi et al. [44] and Nowacki [65], the field equations in linearized form and constitutive relations for a slowly moving medium which is homogenous and perfectly conducting elastic solid in the simplified form taking in to account the Lorentz force are given by (4.1)-(4.9)

$$Curl\,\vec{h} = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}, \tag{4.1}$$

$$Curl\,\vec{E} = -\frac{\partial \vec{h}}{\partial t}, \tag{4.2}$$

$$\vec{E} = -\mu_0 \left(\frac{\partial \vec{u}}{\partial t} \times \vec{H}_o \right), \tag{4.3}$$

$$div\vec{h} = 0, \tag{4.4}$$

$$\begin{aligned}(\lambda + 2\mu + \kappa)\nabla(\nabla.\vec{u}) - (\mu + \kappa)\nabla \times (\nabla \times \vec{u}) + \kappa(\nabla \times \vec{\phi}) + \vec{F} \\ - v\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\nabla T = \rho\left[\frac{\partial^2 \vec{u}}{\partial t^2} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2\vec{\Omega} \times \frac{\partial \vec{u}}{\partial t}\right],\end{aligned} \tag{4.5}$$

$$\begin{aligned}(\alpha + \beta + \gamma)\nabla(\nabla.\vec{\phi}) - \gamma\nabla \times (\nabla \times \vec{\phi}) + \kappa(\nabla \times \vec{u}) - 2\kappa\vec{\phi} \\ = \rho j\left[\frac{\partial^2 \vec{\phi}}{\partial t^2} + \vec{\Omega} \times \frac{\partial \vec{\phi}}{\partial t}\right],\end{aligned} \tag{4.6}$$

$$K^*\nabla^2 T = \rho c^*\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)T + \upsilon T_o\left(\frac{\partial}{\partial t}+\tau_o\eta_o\frac{\partial^2}{\partial t^2}\right)(\nabla.\vec{u}), \tag{4.7}$$

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j}+u_{j,i}\right) + \kappa\left(u_{j,i} - \epsilon_{ijk}\,\phi_k\right) - \upsilon\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{4.8}$$

$$\mu_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \tag{4.9}$$

Where $\upsilon = (3\lambda+2\mu+\kappa)\alpha_t$ and $\vec{F} = \mu_0(\vec{J}\times\vec{H}_o)$.

The equations of motion (4.5)-(4.6) along with heat equation (4.7) in Cartesian coordinates (x_1, x_2, x_3) in component form can be written as

$$(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_1^2}+\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{\partial^2 u_3}{\partial x_1\partial x_3}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_1}{\partial x_1^2}+\frac{\partial^2 u_1}{\partial x_2^2}+\frac{\partial^2 u_1}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial\phi_3}{\partial x_2}-\frac{\partial\phi_2}{\partial x_3}\right)-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial \mathrm{T}}{\partial x_1}+\mu(J_2H_3-J_3H_2) \tag{4.10}$$
$$=\rho\frac{\partial^2 u_1}{\partial t^2}-3\Omega^2 u_1,$$

$$(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_2\partial x_1}+\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_3}{\partial x_2\partial x_3}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_2}{\partial x_1^2}+\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_2}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial\phi_1}{\partial x_3}-\frac{\partial\phi_3}{\partial x_1}\right)-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial \mathrm{T}}{\partial x_2}+\mu(J_3H_2-J_2H_3) \tag{4.11}$$
$$=\rho\frac{\partial^2 u_2}{\partial t^2}-3\Omega^2 u_2,$$

$$(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_3\partial x_1}+\frac{\partial^2 u_2}{\partial x_2\partial x_3}+\frac{\partial^2 u_3}{\partial x_3^2}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_3}{\partial x_1^2}+\frac{\partial^2 u_3}{\partial x_2^2}+\frac{\partial^2 u_3}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial\phi_2}{\partial x_1}-\frac{\partial\phi_1}{\partial x_2}\right)-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial \mathrm{T}}{\partial x_3}+\mu(J_1H_2-J_2H_1) \tag{4.12}$$
$$=\rho\frac{\partial^2 u_3}{\partial t^2}-3\Omega^2 u_3,$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_1\partial x_2}+\frac{\partial^2\phi_3}{\partial x_1\partial x_3}\right)+\gamma\left(\frac{\partial^2\phi_1}{\partial x_1^2}+\frac{\partial^2\phi_1}{\partial x_2^2}+\frac{\partial^2\phi_1}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3}\right)-2\kappa\phi_1=\rho j\frac{\partial^2\phi_1}{\partial t^2}, \tag{4.13}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_2\partial x_1}+\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_3}{\partial x_2\partial x_3}\right)+\gamma\left(\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_2^2}+\frac{\partial^2\phi_2}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-2\kappa\phi_2=\rho j\frac{\partial^2\phi_2}{\partial t^2}, \tag{4.14}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_3\partial x_1}+\frac{\partial^2\phi_2}{\partial x_3\partial x_2}+\frac{\partial^2\phi_3}{\partial x_3^2}\right)+\gamma\left(\frac{\partial^2\phi_3}{\partial x_1^2}+\frac{\partial^2\phi_3}{\partial x_2^2}+\frac{\partial^2\phi_3}{\partial x_3^2}\right)$$
$$+\kappa\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right)-2\kappa\phi_3=\rho j\frac{\partial^2\phi_3}{\partial t^2}, \tag{4.15}$$

$$K^*\left(\frac{\partial^2\varphi}{\partial x_1^2}+\frac{\partial^2\varphi}{\partial x_2^2}+\frac{\partial^2\varphi}{\partial x_3^2}\right)$$
$$=\rho C_E\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right) \tag{4.16}$$
$$+\rho\nu C_E T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)$$

Where (u_1,u_2,u_3), (ϕ_1,ϕ_2,ϕ_3), (J_1,J_2,J_3) and (H_1,H_2,H_3) are the components of displacement vector $\vec{u}$, microrotation vector $\vec{\phi}$, current density vector $\vec{J}$ and magnetic field vector $\vec{H}$ respectively.

4.2. Formulation and solution of the problem

We consider a generalized micropolar thermoelastic medium which is perfectly conducting, homogenous, isotropic and permeated by an initial magnetic field $\vec{H}_0$ which is acting along the x_2-axis. For two dimensional problem we take the displacement vector $\vec{u}$, rotation vector $\vec{\Omega}$ and microrotation vector $\vec{\phi}$ (by assuming $\vec{\Omega}$ to be invariant) as

$$\vec{u}=(u_1,0,u_3),\qquad \vec{\phi}=(0,\phi_2,0),\qquad \vec{\Omega}=(0,\Omega_2,0),\qquad \vec{E}=(E_1,0,E_3),$$
$$\vec{h}=(0,h,0),\qquad \vec{H}_0=(0,H_{02},0) \tag{4.17}$$

Using expressions mentioned in equation (4.17) in equations (4.1)-(4.4) and (4.8)-(4.9), we get

$$(\lambda+2\mu+\kappa)\frac{\partial^2 u_1}{\partial x_1^2}+(\lambda+\mu)\frac{\partial^2 u_3}{\partial x_1\partial x_3}+(\mu+\kappa)\frac{\partial^2 u_1}{\partial x_3^2}-\kappa\frac{\partial\phi_2}{\partial x_3}-\mu_0 H_{02}J_3$$
$$-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}=\rho\left[\frac{\partial^2 u_1}{\partial t^2}-3\Omega_2^2 u_1\right], \tag{4.18}$$

$$(\lambda+\mu)\frac{\partial^2 u_3}{\partial x_1\partial x_3}+(\lambda+2\mu+\kappa)\frac{\partial^2 u_3}{\partial x_3^2}+(\mu+\kappa)\frac{\partial^2 u_3}{\partial x_1^2}+K\frac{\partial\phi_2}{\partial x_3}+\mu_0 H_0 J_1$$
$$-\nu\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}=\rho\left[\frac{\partial^2 u_3}{\partial t^2}-3\Omega_2^2\, u_3\right], \tag{4.19}$$

$$-\gamma\left(\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_3^2}\right)+\kappa\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-2\kappa\phi_2 = \rho j\frac{\partial^2\phi_2}{\partial t^2}, \tag{4.20}$$

$$K^*\nabla^2 T = \rho c^*\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)T+\upsilon T_o\left(\frac{\partial}{\partial t}+\tau_o\eta_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right), \tag{4.21}$$

$$\sigma_{31} = \mu\frac{\partial u_3}{\partial x_1}+(\mu+\kappa)\frac{\partial u_1}{\partial x_3}+\kappa\phi_2, \tag{4.22}$$

$$\sigma_{33} = \lambda\frac{\partial u_1}{\partial x_1}+(\lambda+2\mu+\kappa)\frac{\partial u_3}{\partial x_3}- \nu\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{4.23}$$

$$\mu_{32} = \gamma\frac{\partial\phi_2}{\partial x_3}, \tag{4.24}$$

$$E_1 = \mu_0 H_{02}\frac{\partial u_3}{\partial t}, \tag{4.25}$$

$$E_3 = -\mu_0 H_{02}\frac{\partial u_1}{\partial t}, \tag{4.26}$$

$$h = -H_{02}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right). \tag{4.27}$$

We define the dimensionless quantities as

$$\begin{gathered} x_i{}^* = \frac{\bar{\omega}}{c_1}x_i, \quad u_i{}^* = \frac{\rho c_1\bar{\omega}}{\nu T_0}u_i, \quad t^* = \bar{\omega}t, \quad \tau_0{}^* = \bar{\omega}\tau_0, \quad \tau_1{}^* = \bar{\omega}\tau_1, \\ J_i{}^* = \frac{\eta_0}{\sigma_0{}^2\mu_0{}^2H_{02}c_0}J_i, \quad h^* = \frac{\eta_0}{\sigma_0\mu_0H_{02}}h, \quad \sigma_{ij}{}^* = \frac{1}{\nu T_0}\sigma_{ij}, \\ \mu_{ij}{}^* = \frac{\bar{\omega}}{c_1\nu T_0}\mu_{ij}, \quad E_i{}^* = \frac{E_i}{\mu_0 H_{02}c_1}, \quad \Omega_2^* = \frac{\Omega_2}{\bar{\omega}}, \\ T^* = \frac{\nu T}{\rho c_0{}^2}, \quad \phi_2^* = \frac{\rho c_1^2\bar{\omega}}{\nu T_0}\phi_2, i = 1,3 \end{gathered} \tag{4.28}$$

Where

$c_1^2 = \frac{\lambda+2\mu+\kappa}{\rho}$ and $\bar{\omega} = \frac{\rho c^* c_1^2}{\kappa}$.

Using the dimensionless quantities as defined in equation (4.28), the system of equations (4.18)-(4.27) can be rewritten as after suppressing the asterisks

$$\begin{aligned}(\alpha_1+\alpha_5)\frac{\partial^2 u_1}{\partial x_1^2}+(\alpha_2+\alpha_5)\frac{\partial^2 u_3}{\partial x_1\partial x_3}+\alpha_3\frac{\partial^2 u_1}{\partial x_3^2}-\alpha_4\frac{\partial\phi_2}{\partial x_3} \\ -\alpha_7\left(1+\tau_1\bar{\omega}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = (\alpha_6+\alpha_7)\frac{\partial^2 u_1}{\partial t^2}-\alpha_9 u_1,\end{aligned} \tag{4.29}$$

$$\alpha_3 \frac{\partial^2 u_3}{\partial x_1^2} + (\alpha_2 + \alpha_5)\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + (\alpha_1 + \alpha_5)\frac{\partial^2 u_3}{\partial x_3^2} + \alpha_4 \frac{\partial \phi_2}{\partial x_1}$$
$$- \alpha_7 \left(1 + \tau_1 \varpi \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = (\alpha_6 + \alpha_7)\frac{\partial^2 u_3}{\partial t^2} - \alpha_9 u_3, \tag{4.30}$$

$$-\alpha_{11}\left(\frac{\partial^2 \phi_2}{\partial x_1^2} + \frac{\partial^2 \phi_2}{\partial x_3^2}\right) + \alpha_{12}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - \alpha_{13}\phi_2 = \alpha_{14}\frac{\partial^2 \phi_2}{\partial t^2}, \tag{4.31}$$

$$\alpha_{15}\nabla^2 T = \alpha_{16}\left(\frac{\partial}{\partial t} + \tau_o \varpi \frac{\partial^2}{\partial t^2}\right)T + \alpha_{17}\left(\frac{\partial}{\partial t} + \tau_o \eta_o \varpi \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right), \tag{4.32}$$

$$\sigma_{31} = \alpha_{20}\frac{\partial u_3}{\partial x_1} + \alpha_{21}\frac{\partial u_1}{\partial x_3} + \alpha_{22}\phi_2, \tag{4.33}$$

$$\sigma_{33} = \alpha_{18}\frac{\partial u_1}{\partial x_1} + \alpha_{19}\frac{\partial u_3}{\partial x_3} - \alpha_{20}T, \tag{4.34}$$

$$\mu_{32} = \alpha_{23}\frac{\partial \phi_2}{\partial x_3}, \tag{4.35}$$

$$E_1 = \alpha_{24}\frac{\partial u_3}{\partial t}, \tag{4.36}$$

$$E_3 = -\alpha_{24}\frac{\partial u_1}{\partial t}, \tag{4.37}$$

$$h = -H_{02}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right), \tag{4.38}$$

Where

$$\alpha_1 = \frac{(\lambda + 2\mu + \kappa)}{\rho c_1^3}\nu T_0 \varpi, \quad \alpha_2 = \frac{(\lambda + \mu)}{\rho c_1^3}\nu T_0 \varpi, \quad \alpha_3 = \frac{(\mu + \kappa)}{\rho c_1^3}\nu T_0 \varpi,$$
$$\alpha_4 = \frac{\kappa \nu T_0 \varpi}{\rho c_1^3}, \quad \alpha_5 = \frac{\mu_0 H_{02}^2 \nu T_0 \varpi}{\rho c_1^3}, \quad \alpha_6 = \frac{\varepsilon_0 \mu_0^2 H_{02}^2 \nu T_0 \varpi}{\rho c_1},$$
$$\alpha_7 = \frac{\nu T_0 \varpi}{c_1}, \quad \alpha_8 = \frac{\nu T_0 \varpi}{c_1}, \quad \alpha_9 = \frac{3\Omega_2^2 \kappa \nu T_0 \varpi}{\rho c_1^3},$$

$$\alpha_{11}=\frac{\gamma\nu T_0\overline{\omega}^2}{\rho c_1^4},\qquad \alpha_{12}=\frac{\kappa\nu T_0}{\rho c_1^2},\qquad \alpha_{13}=\frac{2\kappa\nu T_0}{\rho c_1^2},\qquad \alpha_{14}=\frac{j\nu T_0\overline{\omega}^2}{c_1^2},$$
$$\alpha_{15}=\frac{K^*T_0\overline{\omega}^2}{c_1^2},\qquad \alpha_{16}=\rho\overline{\omega}T_0,\qquad \alpha_{17}=\frac{\nu^2T_0^2\overline{\omega}}{\rho c_1^2},$$
$$\alpha_{18}=\frac{\lambda}{\rho c_1^2},\qquad \alpha_{19}=\frac{(\lambda+2\mu+\kappa)}{\rho c_1^2},\qquad \alpha_{20}=\frac{\mu}{\rho c_1^2},\tag{4.39}$$
$$\alpha_{21}=\frac{\mu+\kappa}{\rho c_1^2},\qquad \alpha_{22}=\frac{\kappa}{\rho c_1^2},\qquad \alpha_{23}=\frac{\gamma\overline{\omega}^2}{\rho c_1^4},\qquad \alpha_{24}=\frac{\nu T_0}{\rho c_1^2},$$
$$\alpha_{25}=\frac{\nu T_0}{\rho c_1^2\overline{\omega}}$$

After applying Laplace transform and its properties as defined in equations (2.45)-(2.47) on equations (4.29)-(4.38), we get

$$(\alpha_1+\alpha_5)\frac{\partial^2\bar{u}_1}{\partial x_1^2}+(\alpha_2+\alpha_5)\frac{\partial^2\bar{u}_3}{\partial x_1\partial x_3}+\alpha_3\frac{\partial^2\bar{u}_1}{\partial x_3^2}-\alpha_4\frac{\partial\bar{\phi}_2}{\partial x_3}-\alpha_7(1+\tau_1\overline{\omega}s)\frac{\partial\bar{T}}{\partial x_1}=[(\alpha_6+\alpha_7)s^2-\alpha_9]\bar{u}_1,\tag{4.40}$$

$$\alpha_3\frac{\partial^2\bar{u}_3}{\partial x_1^2}+(\alpha_2+\alpha_5)\frac{\partial^2\bar{u}_1}{\partial x_1\partial x_3}+(\alpha_1+\alpha_5)\frac{\partial^2\bar{u}_3}{\partial x_3^2}+\alpha_4\frac{\partial\bar{\phi}_2}{\partial x_1}-\alpha_7(1+\tau_1\overline{\omega}s)\frac{\partial\bar{T}}{\partial x_3}=[(\alpha_6+\alpha_7)s^2-\alpha_9]\bar{u}_3,\tag{4.41}$$

$$-\alpha_{11}\left(\frac{\partial^2\bar{\phi}_2}{\partial x_1^2}+\frac{\partial^2\bar{\phi}_2}{\partial x_3^2}\right)+\alpha_{12}\left(\frac{\partial\bar{u}_1}{\partial x_3}-\frac{\partial\bar{u}_3}{\partial x_1}\right)-\alpha_{13}\bar{\phi}_2=\alpha_{14}s^2\bar{\phi}_2,\tag{4.42}$$

$$\alpha_{15}\nabla^2\bar{T}=\alpha_{16}(s+\tau_o\overline{\omega}s^2)\bar{T}+\alpha_{17}(s+\tau_o\eta_o\overline{\omega}s^2)\left(\frac{\partial\bar{u}_1}{\partial x_1}+\frac{\partial\bar{u}_3}{\partial x_3}\right),\tag{4.43}$$

$$\bar{\sigma}_{31}=\alpha_{20}\frac{\partial\bar{u}_3}{\partial x_1}+\alpha_{21}\frac{\partial\bar{u}_1}{\partial x_3}+\alpha_{22}\bar{\phi}_2,\tag{4.44}$$

$$\bar{\sigma}_{33}=\alpha_{18}\frac{\partial\bar{u}_1}{\partial x_1}+\alpha_{19}\frac{\partial\bar{u}_3}{\partial x_3}-\alpha_{20}\bar{T},\tag{4.45}$$

$$\bar{\mu}_{32}=\alpha_{23}\frac{\partial\bar{\phi}_2}{\partial x_3},\tag{4.46}$$

$$\bar{E}_1=\alpha_{24}s\bar{u}_3,\tag{4.47}$$

$$\bar{E}_3=-\alpha_{24}s\bar{u}_1,\tag{4.48}$$

$$\bar{h}=-H_{02}\left(\frac{\partial\bar{u}_1}{\partial x_1}+\frac{\partial\bar{u}_3}{\partial x_3}\right),\tag{4.49}$$

Also applying Fourier transform as defined in (3.58) on equations (4.40)-(4.49), we obtain

$$D^2\tilde{u}_1 = \frac{1}{\alpha_3}\big[\{(\alpha_1+\alpha_5)\xi^2 + (\alpha_6+\alpha_8)s^2 - \alpha_9\}\tilde{u}_1 - \iota\xi(\alpha_2+\alpha_5)D\tilde{u}_3 + \alpha_4 D\tilde{\phi}_2 - \iota\xi\alpha_7(1+\varpi\,\tau_1)\tilde{T}\big], \tag{4.50}$$

$$D^2\tilde{u}_3 = \frac{1}{\alpha_1+\alpha_5}\big[-\iota\xi(\alpha_2+\alpha_5)D\tilde{u}_1 + \{\alpha_3\xi^2 + (\alpha_6+\alpha_8)s^2 - \alpha_9\}\tilde{u}_3 - \iota\xi\alpha_4\tilde{\phi}_2 + \alpha_7(1+\varpi\,\tau_1 s)D\tilde{T}\big], \tag{4.51}$$

$$D^2\tilde{\phi}_2 = \frac{1}{\alpha_{11}}\big[\alpha_{12}(D\tilde{u}_1 - \iota\xi\tilde{u}_3) + (\alpha_{11}\xi^2 - \alpha_{13} - \alpha_{14}s^2)\tilde{\phi}_2\big], \tag{4.52}$$

$$D^2\tilde{T} = \frac{1}{\alpha_{15}}[\iota\xi\alpha_{17}(s+\tau_0\eta_0\varpi s^2)\tilde{u}_1 + \alpha_{17}(s+\tau_0\eta_0\varpi s^2)D\tilde{u}_3 + (\alpha_{15}\xi^2 + s + \varpi\tau_0 s^2)\tilde{T}], \tag{4.53}$$

$$\tilde{\sigma}_{31} = \iota f_{52}\tilde{u}_3 + \alpha_{21}D\tilde{u}_1 + \alpha_{22}\tilde{\phi}_2, \tag{4.54}$$

$$\tilde{\sigma}_{33} = \iota f_{51}\tilde{u}_1 + \alpha_{19}D\tilde{u}_3 - \alpha_{20}\tilde{T}, \tag{4.55}$$

$$\tilde{\mu}_{32} = \alpha_{23}D\tilde{\phi}_2, \tag{4.56}$$

$$\tilde{E}_1 = f_{53}\tilde{u}_3, \tag{4.57}$$

$$\tilde{E}_3 = -f_{53}\tilde{u}_1, \tag{4.58}$$

$$\tilde{h} = -\iota f_{54}\tilde{u}_1 - \alpha_{25}D\tilde{u}_3, \tag{4.59}$$

where $D = \frac{d}{dx_3}$.

Equations (4.50)-(4.53) can be written in matrix form as

$$DW(\xi, x_3, s) = AW(\xi, x_3, s), \tag{4.60}$$

where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix},\ U = [\tilde{u}_1 \quad \tilde{u}_3 \quad \tilde{\phi}_2 \quad \tilde{T}]', \tag{4.61}$$

$$A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & -\iota g_{12} & g_{13} & 0 \\ \iota g_{21} & 0 & 0 & g_{24} \\ g31 & 0 & 0 & 0 \\ 0 & g_{42} & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} f_{11} & 0 & 0 & -\iota f_{14} \\ 0 & f_{22} & if_{23} & 0 \\ 0 & -\iota f_{32} & f_{33} & 0 \\ \iota f_{41} & 0 & 0 & f_{44} \end{bmatrix}, \tag{4.62}$$

Here I is identity matrix of order 4, O is Null matrix of order of 4 and $[\quad]'$ is transpose of matrix.

$$f_{11} = \frac{1}{\alpha_3}\,[(\alpha_1+\alpha_5)\xi^2 + (\alpha_6+\alpha_8)s^2 - \alpha_9], \qquad f_{14} = \frac{\xi\alpha_7}{\alpha_3}(1+\bar{\omega}\tau_1 s),$$

$$f_{22} = \frac{\alpha_3\xi^2 + \alpha_6 s^2 + \alpha_8 s^2 - \alpha_9}{\alpha_1+\alpha_5}, \qquad f_{23} = -\frac{\alpha_4\xi}{\alpha_1+\alpha_5},$$

$$f_{32} = \frac{\xi\alpha_{12}}{\alpha_{11}}, \qquad f_{33} = \frac{\alpha_{11}\xi^2 - \alpha_{14}s^2 - \alpha_{13}}{\alpha_{11}},$$

$$f_{41} = -\frac{\xi\alpha_{17}}{\alpha_{15}}(s+\tau_0\eta_0\bar{\omega}s^2), \tag{4.63}$$

$$f_{44} = \frac{1}{\alpha_{15}}(\alpha_{15}\xi^2 + s + \tau_0\bar{\omega}s^2), \qquad g_{12} = -\frac{\xi(\alpha_2+\alpha_5)}{\alpha_3},$$

$$g_{13} = \frac{\alpha_4}{\alpha_3}, \qquad g_{21} = -\frac{\xi(\alpha_5+\alpha_2)}{\alpha_1+\alpha_5},$$

$$g_{24} = \frac{\alpha_7}{\alpha_1+\alpha_5}(1+\tau_1\bar{\omega}s), \qquad g_{31} = \frac{\alpha_{12}}{\alpha_{11}},$$

$$g_{42} = \frac{\alpha_{17}}{\alpha_{15}}(s+\tau_0\eta_0\bar{\omega}s^2).$$

Solution of equation (4.60) is of the form

$W(\xi,x_3,s) = X(\xi,s)e^{qx_3}$, for some parameter q. (4.64)

Using this value in equation (4.60), we get

$$AW(\xi,x_3,s) = qW(\xi,x_3,s), \tag{4.65}$$

which gives rise to an eigen value problem.

Now corresponding to the matrix A, characteristic equation can be written as

$$|A - qI| = 0, \tag{4.66}$$

On expansion it can be written as

$$q^8 - \lambda_1 q^6 + \lambda_2 q^4 - \lambda_3 q^2 + \lambda_4 = 0, \tag{4.67}$$

where

$$\lambda_1 = g_{24}g_4 + f_{11} + f_{44} + f_{22} + f_{33} - g_{12}g_{21} + g_{13}g_{31},$$

$$\begin{aligned}\lambda_2 = {} & -f_{41}g_{12}g_{24} + f_{14}f_{41} + g_{13}g_{24}g_{31}g_{42} + f_{33}g_{24}g_{42} + f_{11}g_{42} - f_{14}f_{21}g_{42} \\ & + f_{11}f_{44} + f_{11_{f_{22}}} + f_{11}f_{33} + f_{22}f_{44} + f_{33}f_{44} + f_{22}f_{33} + f_{23}f_{32} \\ & - f_{44}g_{12}g_{21} - f_{33}g_{12}g_{21} + f_{23}g_{12}g_{31} + f_{44}g_{13}g_{31} + f_{32}g_{13}g_{21} \\ & + f_{22}g_{13}g_{31},\end{aligned} \tag{4.68}$$

$$\begin{aligned}\lambda_3 = {} & -f_{32}f_{41}g_{13}g_{24} - f_{33}f_{41}g_{12}g_{24} + f_{14}f_{33}f_{41} + f_{14}f_{22}f_{41} + f_{14}f_{23}g_{31}g_{42} \\ & + f_{11}f_{33}g_{42} - f_{14}f_{21}f_{33}g_{42} + f_{22}f_{11}f_{44} + f_{11}f_{33}f_{44} + f_{11}f_{22}f_{33} \\ & + f_{22}f_{33}f_{44} + f_{23}f_{32}f_{44} - f_{33}f_{44}g_{12}g_{21} + f_{23}f_{44}g_{12}g_{31} \\ & + f_{32}f_{44}g_{13}g_{21} + f_{22}f_{44}g_{13}g_{31},\end{aligned}$$

$$\lambda_4 = f_{14}f_{23}f_{32}f_{41} + f_{14}f_{33}f_{41}f_{22} + f_{11}f_{22}f_{33}f_{44} + f_{11}f_{23}f_{32}f_{44}.$$

The eigen values of matrix A are the characteristic roots of the equation (4.67). The eigen vectors $X(\xi, s)$ corresponding to eigen value q_p can be determined by solving the homogenous equations

$$[A - qI]X(\xi, s) = 0, \tag{4.69}$$

Which gives

$$X_p(\xi, s) = \begin{bmatrix} X_{p1} \\ X_{p2} \end{bmatrix}, X_{p1} = \begin{bmatrix} a_p q_p \\ b_p \\ c_p q_p \\ d_p \end{bmatrix}, X_{p2} = q_p X_{p1} \text{for } q = q_p, p = 1,2,3,4 \text{ and}$$

$$X_j(\xi, s) = \begin{bmatrix} X_{j1} \\ X_{j2} \end{bmatrix},\ X_{j1} = \begin{bmatrix} -a_p q_p \\ b_p \\ -c_p q_p \\ d_p \end{bmatrix},\ X_{p2} = q_p X_{p1},\ \text{for}\ \ j = p + 4, q = -q_p, p = \tag{4.70}$$

1,2,3,4

where

$$a_p = [\{-g_{12}(f_{44} - q_p^2) + \iota f_{44}g_{42}\}(f_{33} - q_p^2) - g_{13}g_{31}q_p^2(f_{44} - q_p^2)], \tag{4.71}$$

$$b_p = [\{(f_{11} - q_s^2)(f_{44} - q_p^2) + \iota f_{14}f_{41}\}(f_{23} - q_p^2) - g_{13}g_{31}q_p^2(f_{44} - q_p^2)], \tag{4.72}$$

$$\begin{aligned}c_p = {} & [\{(f_{11} - q_s^2)(f_{44} - q_p^2) + f_{14}f_{41}\}f_{32} \\ & + g_{31}q_p^2\{-g_{12}(f_{44} - q_p^2) + \iota f_{14}g_{42}\}],\end{aligned} \tag{4.73}$$

$$d_p = -\frac{\iota(f_{41}\, a_p + g_{42}b_p)}{f_{44} - q_p^2}, \tag{4.74}$$

Thus a solution of equation (4.65) becomes

$$W(\xi,s)=\sum_{p=1}^{4}\left[B_pX_p(\xi,s)e^{q_px_3}+B_{p+4}X_{p+4}(\xi,s)e^{-q_px_3}\right], \tag{4.75}$$

Where $B_i's$ are eight arbitrary constants.

Now after using equations (4.50)-(4.59), (4.61) and (4.75), we obtain values of $\tilde{u}_1,\tilde{u}_3,\tilde{\phi}_2,\tilde{T},\tilde{\sigma}_{31},\tilde{\sigma}_{33},\ \tilde{\mu}_{32},\tilde{E}_1,\tilde{E}_3$ and $\tilde{h}$ as

$$\tilde{u}_1=\sum_{p=1}^{4}\left[a_pq_pB_pe^{q_px_3}-a_pq_pB_{p+4}e^{-q_px_3}\right], \tag{4.76}$$

$$\tilde{u}_3=\sum_{p=1}^{4}\left[b_pB_pe^{q_px_3}+b_pB_{p+4}e^{-q_px_3}\right], \tag{4.77}$$

$$\tilde{\phi}_2=-\sum_{p=1}^{4}\left[c_pB_pe^{q_px_3}+c_pB_{p+4}e^{-q_px_3}\right], \tag{4.78}$$

$$\tilde{T}=\sum_{p=1}^{4}\left[d_pB_pe^{q_px_3}+d_pB_{p+4}e^{-q_px_3}\right], \tag{4.79}$$

$$\tilde{\sigma}_{33}=\sum_{p=1}^{4}\left[(\iota a_pq_pf_{51}+\alpha_{19}b_pq_p-\alpha_{20}d_p)B_pe^{q_px_3}+(-\iota f_{51}a_pq_p-\alpha_{19}b_pq_p-\alpha_{20}d_p)B_{p+4}e^{-q_px_3}\right], \tag{4.80}$$

$$\tilde{\sigma}_{31}=\sum_{p=1}^{4}\left[(\iota f_{52}b_p+\alpha_{21}a_pq_p^2-\alpha_{22}c_p)B_pe^{q_px_3}+(\iota f_{52}b_p+\alpha_{21}a_pq_p^2-\alpha_{22}c_p)B_{p+4}e^{-q_px_3}\right], \tag{4.81}$$

$$\tilde{\mu}_{32}=-\alpha_{23}\sum_{p=1}^{4}\left[b_pc_pq_pB_pe^{q_px_3}-c_pq_pB_{p+4}e^{-q_px_3}\right], \tag{4.82}$$

$$\tilde{E}_1=f_{53}\sum_{p=1}^{4}\left[b_pB_pe^{q_px_3}+b_pB_{p+4}e^{-q_px_3}\right], \tag{4.83}$$

$$\tilde{E}_3=-f_{53}\sum_{p=1}^{4}\left[a_pq_pB_pe^{q_px_3}-a_pq_pB_{p+4}e^{-q_px_3}\right], \tag{4.84}$$

$$\tilde{h} = -if_{54}\sum_{p=1}^{4}\left[\left(-\iota a_p q_p f_{54} + \alpha_{25} b_p q_p\right)B_p e^{q_p x_3} + \left(\iota f_{54} a_p q_p + \alpha_{25} b_p - q_p\right)B_{p+4} e^{-q_p x_3}\right]. \tag{4.85}$$

4.3. Boundary Conditions

To obtain $B_i{}'s$, we consider an infinite micropolar elastic space in which at origin a concentrated force $F = -P_0\delta(x_1)\delta(t)$,where P_0 is the magnitude of the force, has been applied in the direction of the x_3-axis. We call the region $x_3 > 0$ as Medium-I and $x_3 < 0$ as Medium-II. The boundary condition for present problem on the plane $x_3 = 0$ are

$$u_1(x_1, 0^+, t) - u_1(x_1, 0^-, t) = 0, \tag{4.86}$$

$$u_3(x_1, 0^+, t) - u_3(x_1, 0^-, t) = 0, \tag{4.87}$$

$$\phi_2(x_1, 0^+, t) - \phi_2(x_1, 0^-, t) = 0, \tag{4.88}$$

$$T(x_1, 0^+, t) - T(x_1, 0^-, t) = 0, \tag{4.89}$$

$$\frac{\partial T}{\partial x_3}(x_1, 0^+, t) - \frac{\partial T}{\partial x_3}(x_1, 0^-, t) = 0, \tag{4.90}$$

$$\sigma_{31}(x_1, 0^+, t) - \sigma_{31}(x_1, 0^-, t) = 0, \tag{4.91}$$

$$\sigma_{33}(x_1, 0^+, t) - \sigma_{33}(x_1, 0^-, t) = -P_0\delta(x_1)\delta(t), \tag{4.92}$$

$$\mu_{32}(x_1, 0^+, t) - \mu_{32}(x_1, 0^-, t) = 0, \tag{4.93}$$

After using the dimensionless quantities as defined in equation (4.28) and after applying the transforms defined in equation (2.45) and (3.58) along with $L\{\delta(t)\} = 1$ on equations (4.86)-(4.93), we obtain

$$\tilde{u}_1(\xi, 0^+, s) - \tilde{u}_1(\xi, 0^-, s) = 0, \tag{4.94}$$

$$\tilde{u}_3(\xi, 0^+, s) - \tilde{u}_3(\xi, 0^-, s) = 0, \tag{4.95}$$

$$\tilde{\phi}_2(\xi, 0^+, s) - \tilde{\phi}_2(\xi, 0^-, s) = 0, \tag{4.96}$$

$$\tilde{T}(\xi, 0^+, s) - \tilde{T}(\xi, 0^-, s) = 0, \tag{4.97}$$

$$D\tilde{T}(\xi,0^+,s) - D\tilde{T}(\xi,0^-,s) = 0, \tag{4.98}$$

$$\tilde{\sigma}_{31}(\xi,0^+,s) - \tilde{\sigma}_{31}(\xi,0^-,s) = 0, \tag{4.99}$$

$$\tilde{\sigma}_{33}(\xi,0^+,s) - \tilde{\sigma}_{33}(\xi,0^-,s) = -\tilde{P}_0, \tag{4.100}$$

$$\tilde{\mu}_{32}(\xi,0^+,s) - \tilde{\mu}_{32}(\xi,0^-,s) = 0, \tag{4.101}$$

Medium-I: As $x_3 > 0$ for this region, coefficients B_1, B_2, B_3 and B_4 in the expressions (4.69)-(4.75) of the transformed displacement, microrotation, stress components, temperature distribution and couple stress components must be zero. Hence these transformed components for medium-I are given by

$$\tilde{u}_1 = -\sum_{p=1}^{4} a_p q_p B_{p+4} e^{-q_p x_3}, \tag{4.102}$$

$$\tilde{u}_3 = \sum_{p=1}^{4} b_p B_{p+4} e^{-q_p x_3}, \tag{4.103}$$

$$\tilde{\phi}_2 = -\sum_{p=1}^{4} c_p B_{p+4} e^{-q_p x_3}, \tag{4.104}$$

$$\tilde{T} = \sum_{p=1}^{4} d_p B_{p+4} e^{-q_p x_3}, \tag{4.105}$$

$$\tilde{\sigma}_{33} = -\sum_{p=1}^{4} (\iota f_{51} a_p q_p + \alpha_{19} b_p q_p + \alpha_{20} d_p) B_{p+4} e^{-q_p x_3}, \tag{4.106}$$

$$\tilde{\sigma}_{31} = \sum_{p=1}^{4} (\iota f_{52} b_p + \alpha_{21} a_p q_p^2 - \alpha_{22} c_p) B_{p+4} e^{-q_p x_3}, \tag{4.107}$$

$$\tilde{\mu}_{32} = \alpha_{23} \sum_{p=1}^{4} c_p q_p B_{p+4} e^{-q_p x_3}, \tag{4.108}$$

Medium-II: As $x_3 < 0$ for this region, coefficients B_5, B_6, B_7 and B_8 in the expressions (4.69)-(4.75) of the transformed displacement, microrotation, stress components, temperature distribution and couple stress components must be zero. Hence these transformed components for medium-II are given by

$$\tilde{u}_1 = \sum_{p=1}^{4} a_p q_p B_p e^{q_p x_3}, \tag{4.109}$$

$$\tilde{u}_3 = \sum_{p=1}^{4} b_p B_p e^{q_p x_3}, \tag{4.110}$$

$$\tilde{\phi}_2 = -\sum_{p=1}^{4} c_p B_p e^{q_p x_3}, \tag{4.111}$$

$$\tilde{T} = \sum_{p=1}^{4} d_p B_p e^{q_p x_3}, \tag{4.112}$$

$$\tilde{\sigma}_{33} = \sum_{p=1}^{4} (\iota a_p q_p f_{51} + \alpha_{19} b_p q_p - \alpha_{20} d_p) B_p e^{q_p x_3}, \tag{4.113}$$

$$\tilde{\sigma}_{31} = \sum_{p=1}^{4} (\iota f_{52} b_p + \alpha_{21} a_p q_p^2 - \alpha_{22} c_p) B_p e^{q_p x_3}, \tag{4.114}$$

$$\tilde{\mu}_{32} = -\alpha_{23} \sum_{p=1}^{4} b_p c_p q_p B_p e^{q_p x_3}, \tag{4.115}$$

After making use of the transformed components as defined in equations (4.102)-(4.108) for medium-I and equations (4.109)-(4.115) for medium-II in transformed boundary conditions (4.94)-(4.101), we obtain a system of eight equations in eight unknowns $B_i (i = 1,2, \dots ,8)$ as

$$-\sum_{p=1}^{4} (B_{p+4} + B_p) a_p q_p = 0, \tag{4.116}$$

$$\sum_{p=1}^{4} (B_{p+4} - B_p) b_p = 0, \tag{4.117}$$

$$-\sum_{p=1}^{4} (B_{p+4} - B_p) c_p = 0, \tag{4.118}$$

$$\sum_{p=1}^{4} (B_{p+4} - B_p) d_p = 0, \tag{4.119}$$

$$\sum_{p=1}^{4} (B_{p+4} + B_p) d_p q_p = 0, \tag{4.120}$$

$$\sum_{p=1}^{4}[B_{p+4}(\iota f_{52}b_p+\alpha_{21}a_pq_p^2-\alpha_{22}c_p)-B_p(\iota f_{52}b_p+\alpha_{21}a_pq_p^2-\alpha_{22}c_p)] = 0, \tag{4.121}$$

$$\sum_{p=1}^{4}[B_{p+4}(-\iota f_{51}a_pq_p-\alpha_{19}b_pq_p-\alpha_{20}d_p) - B_p(\iota f_{51}a_pq_p+\alpha_{19}b_pq_p-\alpha_{20}d_p)] = -\tilde{P}_0, \tag{4.122}$$

$$\sum_{p=1}^{4}(B_{p+4}+B_p)c_pq_p = 0, \tag{4.123}$$

After solving equations (4.116)-(4.123) for unknown $B_i's$, we get

$$B_1 = B_5 = \frac{P_0c_4}{l_{11}q_1}\left[\frac{l_{12}l_{23}-l_{13}l_{22}}{l_{22}l_{33}-l_{32}l_{23}}\right], \tag{4.124}$$

$$B_2 = B_6 = -\frac{P_0l_{23}c_4}{q_2(l_{22}l_{33}-l_{32}l_{23})}, \tag{4.125}$$

$$B_3 = B_7 = \frac{P_0l_{22}c_4}{q_3(l_{22}l_{33}-l_{32}l_{23})}, \tag{4.126}$$

$$B_4 = B_8 = -\frac{P_0c_4[a_1(l_{12}l_{23}-l_{13}l_{22})-a_2l_{11}l_{23}+a_3l_{22}l_{11}]}{a_4q_4(l_{22}l_{33}-l_{32}l_{23})}, \tag{4.127}$$

where

$$l_{11} = a_2c_4-c_1a_4, \quad l_{12} = a_2c_4-a_4c_2, \quad l_{13} = a_3c_4-a_4c_3,$$

$$\begin{aligned} l_{22} &= (d_2c_4-c_2d_4)-\frac{(d_1c_4-c_1d_4)}{(a_1c_4-c_1a_4)}(a_2c_4-c_2a_4), \\ l_{23} &= (d_3c_4-c_3d_4)-\frac{(d_1c_4-c_1d_4)}{(a_1c_4-c_1a_4)}(a_3c_4-c_3a_4), \\ l_{32} &= (b_2c_4-c_2b_4)-\frac{(b_1c_4-c_1b_4)}{(a_1c_4-c_1a_4)}(a_2c_4-c_2a_4), \\ l_{33} &= (b_3c_4-c_3b_4)-\frac{(b_1c_4-c_1b_4)}{(a_1c_4-c_1a_4)}(a_3c_4-c_3a_4). \end{aligned} \tag{4.128}$$

Using these values of $B_i's$ in equations (4.76)-(4.85), we obtain transformed components of displacement, microrotation, temperature distribution, tangential and normal stress, couple stress, induced electric field and magnetic field.

4.4. Inversion of the transforms

The transformed components of displacement, tangential and normal stress, couple stress, microrotation, temperature distribution, magnetic field and induced electric field are dependent on parameters x_3, s and ξ. To obtain them in the physical domain in the form of $f(x_1, x_3, t)$, we invert integral transforms by using the inversion technique as mentioned in section 3.5 of chapter 3.

4.5. Numerical discussion and Analysis

Following values for the case of magnesium crystal are taken for analysis by making use of Eringen [18],

$$\lambda = 9.4 \times 10^{10} N.m^{-2}, \quad \mu = 4 \times 10^{10} N.m^{-2}, \quad \kappa = 1 \times 10^{10} N.m^{-2},$$
$$\rho = 1.74 \times 10^{3} Kg.m^{-3}, \quad x_3 = 1, \quad j = 0.2 \times 10^{-19} m^2,$$
$$K^* = 1.1753 \times 10^{-19} m^2, \quad \omega^* = 0.0787 \times 10^{-1} N.m^{-2}.s,$$
$$\tau_0 = 6.131 \times 10^{-13} s, \quad \tau_1 = 8.765 \times 10^{-13} s, \quad \varepsilon = 0.073,$$
$$T_0 = 296K, \quad \varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} F.m^{-1}, \quad \mu_0 = 4\pi \times 10^{-7} H.m^{-1},$$
$$\Omega = 1, \quad \alpha_0 = 0.779 \times 10^{-9} N.$$

The computations are carried out for the non-dimensional time $t = \frac{1}{2}$ and range $0 \le x_1 \le 9$. The distribution of non-dimensional normal displacement u_3, non-dimensional normal stress σ_{33}, non-dimensional tangential couple stress μ_{32} and non-dimensional temperature distribution T with non-dimensional distance x_1 have been shown in Figs. 4.6.1–4.6.4.

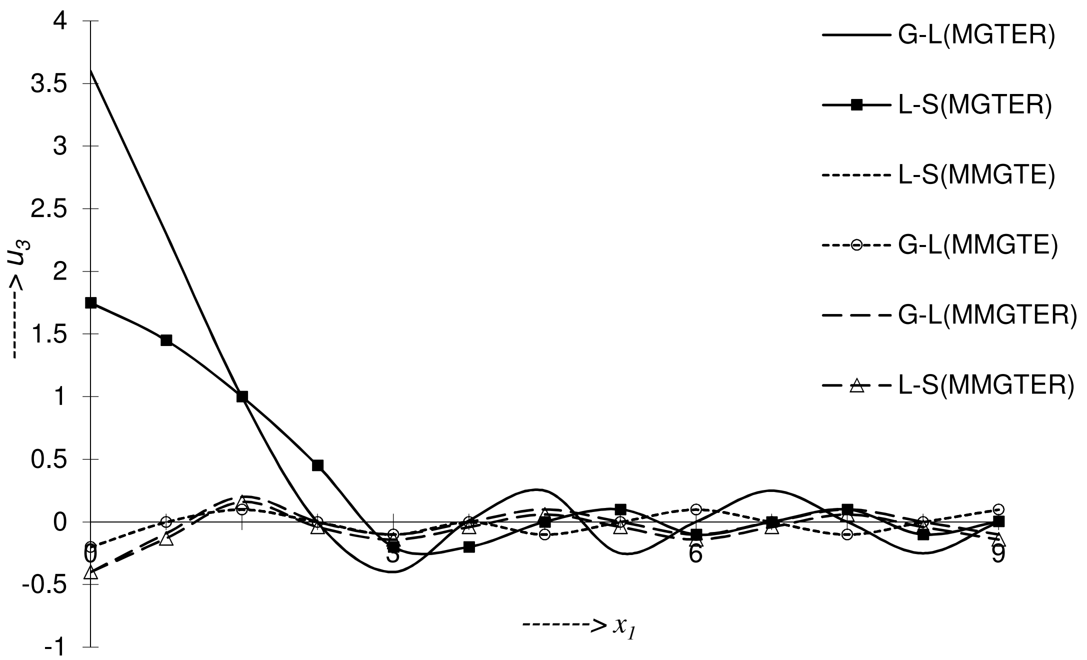

Fig. 4.5.1 Variation in normal displacement u_3

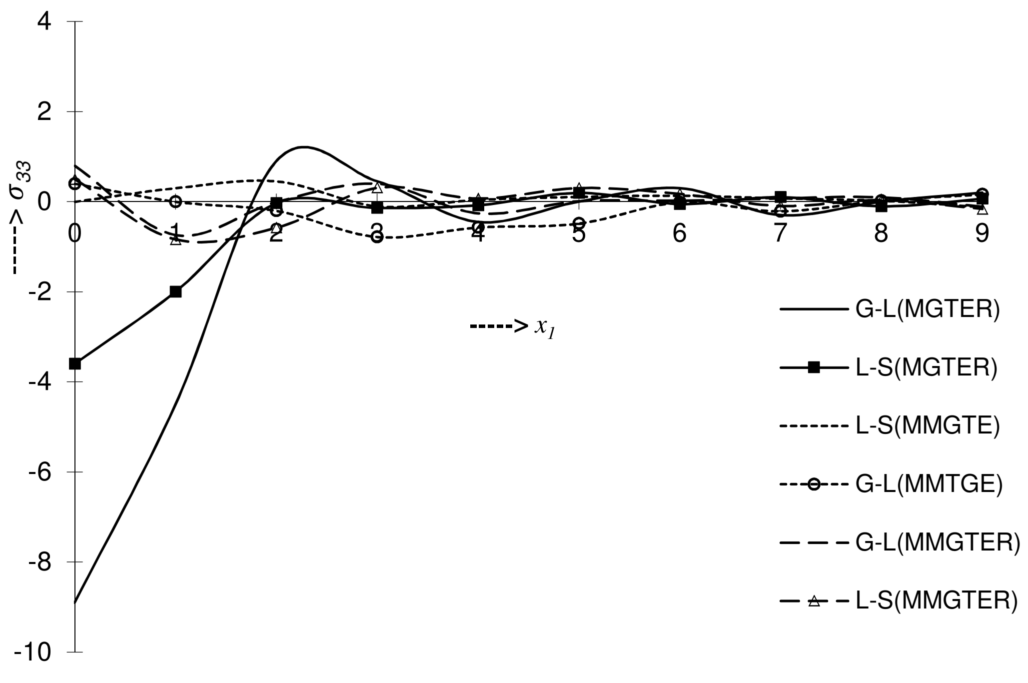

Fig. 4.5.2 Variation in normal force stress σ_{33}

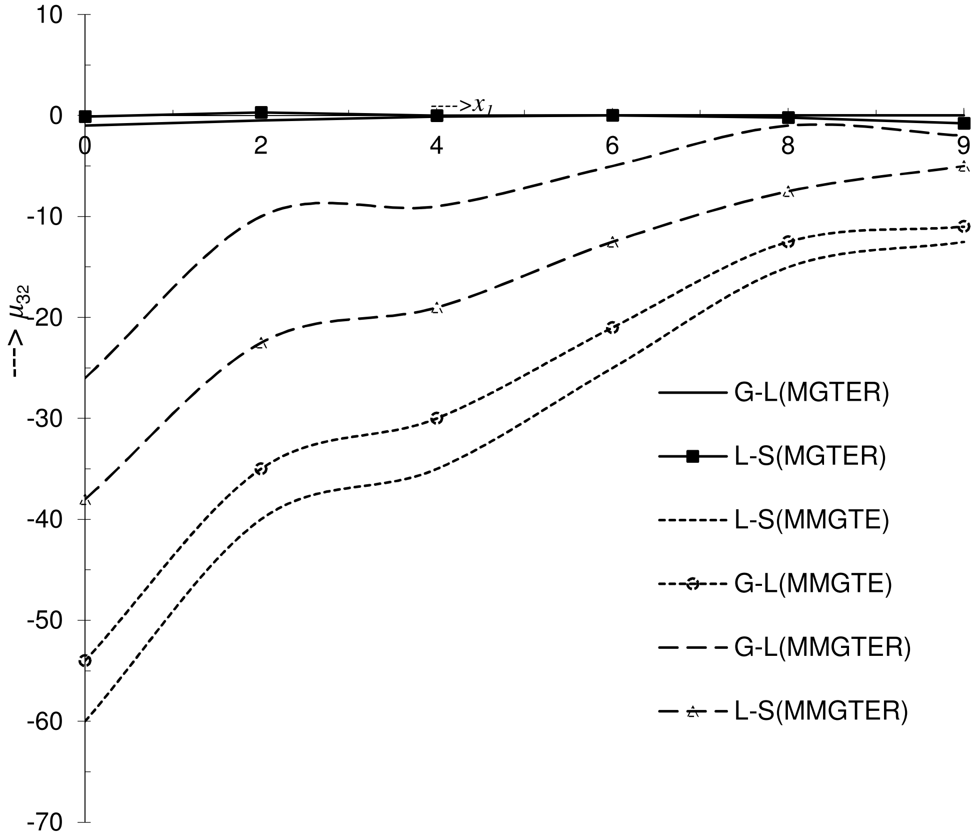

Fig. 4.5.3 Variation in tangential couple stress μ_{32}

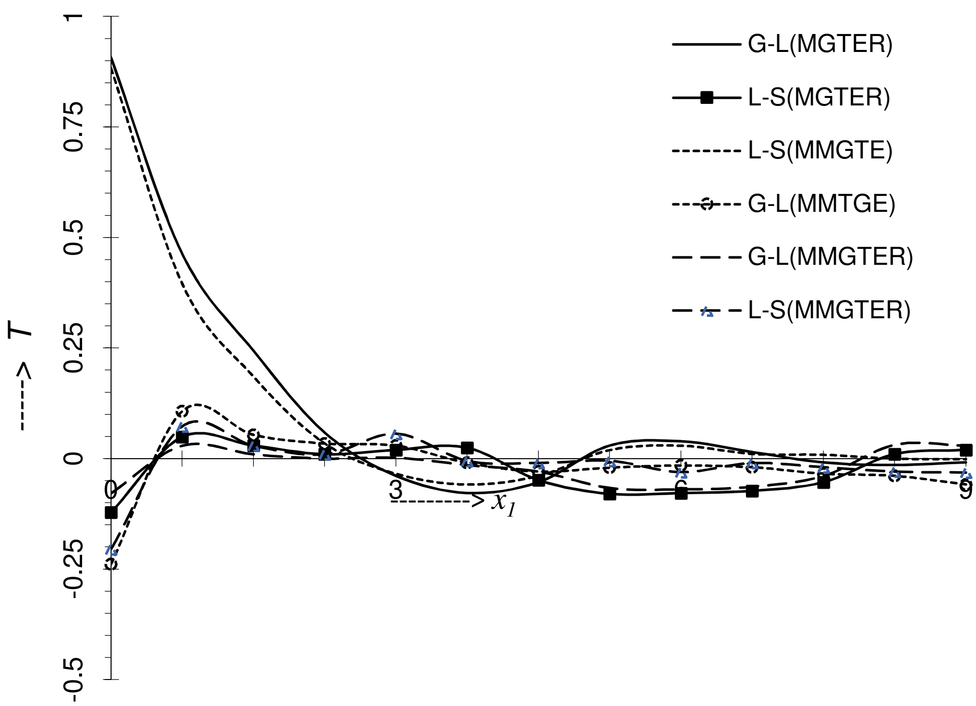

Fig. 4.5.4 Variation in temperature field T

The smooth-solid lines with and without solid squares have been used to signify generalized micropolar thermoelastic medium, for Green and Lindsay [3] theory as G-L (MGTER) and for Lord and Shulman [2] theory as L-S (MGTER), respectively with rotation effect. Again lines made up of small dashes with and without circles signify generalized magneto micropolar thermoelastic, for L–S theory as L-S (MMGTE) and for G–L theory as G-L (MMGTE), respectively. Finally lines with large dashes with and without triangles denotes generalized magneto micropolar thermoelastic medium with rotation effect, for G-L theory as G-L (MMGTER) and for L-S theory as L-S (MMGTER), respectively. Variations with distance x_1 in normal displacement u_3, normal stress σ_{33}, tangential couple stress μ_{32} and temperature distribution T have been shown for mechanical force in figs. 4.6.1–4.6.4. It is clear from Fig. 4.5.1 that near the source u_3 has higher values for G-L (MGTER) and L-S (MGTER) theories as compared to its values for all other theories. Also as x_1 increases, electromagnetic and rotation effect tend to diminish. Fig. 4.5.2 again shows that electromagnetism and rotation effect have very less impact in the range $3 \leq x_1 \leq 9$ for normal stress σ_{33}. Fig. 4.5.3 shows that tangential couple stress keeps on increasing as we move away from the point of application of source for all theories. Finally Fig. 4.5.4 depicts that variation in the temperature distribution T with rotation effect, near the source has higher values and then keeps on decreasing with x_1 whereas without rotation effect has lower values near the source and then keeps on increasing with x_1.

4.6. Conclusion

Eigen value approach has been applied to obtain solution of two dimensional plane strain problem in generalized magneto micropolar thermoelastic infinite space by including the rotation effect. Graphical analysis suggests that normal displacement, normal stress, tangential couple stress and temperature distribution T are affected significantly by application of rotation and magnetic field. Significant difference can be obtained in the temperature distribution by including the rotation effect. Also in case rotation effect is considered, normal stress shows opposite behaviour for L-S and G-L theories.

Chapter 5

Thermo-Mechanical deformation in Magneto Micropolar thermoelastic medium with modified Fourier and Ohm's law

Contemporary engineering material are usually made up of particles possessing internal structures. Classical elasticity lacks justification to explain the elastic behaviour of such materials. To analyse such materials, theory which incorporates the orientation of particles is required. The present investigation is concerned with a two dimensional problem in a magnetic micropolar thermoelastic half space, whose surface in the presence of transverse magnetic field is subjected to thermo-mechanical sources with modified Ohm's and Fourier's law. Integral transforms, Laplace and Fourier transforms have been used to solve the problem. In the past Ezzat and Youssef [80] investigated three different theories; L-S theory with one relaxation time, G-L theory with two relaxation time and coupled theory in generalized magneto thermoelastic perfectly conducting medium. An exact solution based upon Fourier Hankel series and Laplace transform was obtained by Jabbari and Dehbani [81] for classical coupled magneto thermo elasticity in cylindrical coordinates. Thermo magneto elastic interaction due to thermal shock of a stress free boundary of perfectly conducting medium were analysed by [82] in the context of two temperature generalized thermoelasticity with energy dissipation. Lotfy and Hassan [83] investigated a problem on two temperature generalized theory using normal mode analysis. Abo-Dahab and Elsagheer [84] studied the effects of relaxation times, magnetic field and rotation on the reflection of P-wave and SV-wave on the boundary of thermoelastic stress free and insulated medium which is homogeneous and isotropic.

In this chapter an application of concentrated normal force and thermal source have been taken to illustrate the utility of the approach. The transformed components of displacement, stress, couple stress, electric field and current density vector have been derived. Some special cases of interest have also been deduced from the present investigation.

5.1. Basic Equations

Following Ezzat and Emad [48], the basic field equations and constitutive relation in magneto Micropolar thermoelasticity are given by (5.1)-(5.4)

$$Curl\,\vec{h} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \tag{5.1}$$

$$Curl\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{5.2}$$

$$div\,\vec{B} = 0\ ,\ div\vec{D} = \rho_e, \tag{5.3}$$

$$\vec{B} = \mu_o \vec{H}\ \ ,\ \vec{D} = \epsilon_o\,\vec{E},\ \text{where}\ \vec{H} = \vec{H}_o + \vec{h}. \tag{5.4}$$

Here $\vec{h}$ is the perturbation caused by induction in the total magnetic field and dot represents the derivative w.r.t time.

$$K^* T_{,ii} = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)(\rho c^* T + T_o v e) + \pi_o J_{i,i}, \tag{5.5}$$

Maxwell components of stress are given by

$$T_{ij} = \mu_0\left(H_i h_j + H_j h_i - H_k h_k \delta_{ij}\right). \tag{5.6}$$

Due to interaction of electromagnetic and elastic fields, Ohm's law with finite conductivity gets modified and can be written as

$$J_i = \sigma_o\left(E_i + \dot{u}_j B_k\right) + \rho_e \dot{u}_i - k_o\,T_{,i}. \tag{5.7}$$

where k_0 is the coefficient connecting the electric current density and temperature gradient.

After considering the effect of Lorentz force in the absence of body couples and taking in to account electromagnetic couples, the equation of balance of linear momentum and angular momentum can be written as follows.

$$\sigma_{ji,j} + \in_{ijk} J_j B_k + \rho_e E_i = \rho \ddot{u}_i, \tag{5.8}$$

$$\in_{ijk} \sigma_{jk} + u_{ji,j} + \in_{ijk} x_j (\in_{klm} J_l B_m) = \rho j \ddot{\phi}_i, \tag{5.9}$$

Constitutive relations are

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + \kappa\left(u_{j,i} - \in_{ijk} \phi_k\right) - \nu T\delta_{ij}, \tag{5.10}$$

$$\mu_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}. \tag{5.11}$$

Now we consider a micropolar thermoelastic half space defined as

R=$\{(x_1, x_2, x_3) : x_1 \geq 0, -\infty < x_2, x_3 < \infty\}$ and R*=$\{(x_1, x_2, x_3) : x_1 \leq 0, -\infty < x_2, x_3 < \infty\}$ is vacuum. (5.12)

Let $\vec{h}_0 = (0, h_0, 0)$ and $\vec{E}_0 = (E_{10}, 0, E_{30})$ denote the induced magnetic field and electric field in R*.

In the vacuum, constructing the micropolar elastic half space, the system of equations of electrodynamics is

$$Curl\, \vec{h}_0 = \in_0 \frac{\partial \vec{E}_0}{\partial t}, \tag{5.13}$$

$$Curl\, \vec{E}_0 = -\mu_0 \frac{\partial \vec{h}_0}{\partial t}, \tag{5.14}$$

$$div\, \vec{h}_0 = 0. \tag{5.15}$$

In R* components of Maxwell stress are

$$T_{ij}{}^0 = \mu_0\left(H_i h_j{}^0 + H_j h_i{}^0 - H_k h_k{}^0 \delta_{ij}\right). \tag{5.16}$$

Thus equations (5.1)-(5.5), (5.7)-(5.9) and (5.10)-(5.11) form the field equations and constitutive relations in the linearized form for a generalized magneto micropolar thermoelastic medium with modified Ohm's law and modified generalized Fourier's law and equations (5.12)-(5.14) represent the equations of vacuum.

With the aid of (5.10)-(5.11) equations (5.8)-(5.9) after linearizing take the form

$$(\lambda + 2\mu + \kappa)\nabla(\nabla.\vec{u}) - (\mu + \kappa)\nabla \times \nabla \times \vec{u} + \kappa\nabla \times \vec{\phi} + \mu_0(\vec{J} \times H_0) - \nu\nabla T = \rho\frac{\partial^2 \vec{u}}{\partial t^2}, \tag{5.17}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi})-\nabla\times\nabla\times\vec{\phi}+\kappa\nabla\times\vec{u}-2\kappa\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2}. \tag{5.18}$$

Re-writing the equations (5.17)-(5.18) in component form in Cartesian coordinate system (x_1, x_2, x_3) as

$$\begin{aligned}(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_1^2}+\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{\partial^2 u_3}{\partial x_1\partial x_3}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_1}{\partial x_1^2}+\frac{\partial^2 u_1}{\partial x_2^2}+\frac{\partial^2 u_1}{\partial x_3^2}\right)\\ +\kappa\left(\frac{\partial\phi_3}{\partial x_2}-\frac{\partial\phi_2}{\partial x_3}\right)-\nu\frac{\partial \mathrm{T}}{\partial x_1}+\mu(J_2H_3-J_3H_2)=\rho\frac{\partial^2 u_1}{\partial t^2},\end{aligned} \tag{5.19}$$

$$\begin{aligned}(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_2\partial x_1}+\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_3}{\partial x_2\partial x_3}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_2}{\partial x_1^2}+\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_2}{\partial x_3^2}\right)\\ +\kappa\left(\frac{\partial\phi_1}{\partial x_3}-\frac{\partial\phi_3}{\partial x_1}\right)-\nu\frac{\partial \mathrm{T}}{\partial x_2}+\mu(J_3H_2-J_2H_3)=\rho\frac{\partial^2 u_2}{\partial t^2},\end{aligned} \tag{5.20}$$

$$\begin{aligned}(\lambda+\mu)\left(\frac{\partial^2 u_1}{\partial x_3\partial x_1}+\frac{\partial^2 u_2}{\partial x_2\partial x_3}+\frac{\partial^2 u_3}{\partial x_3^2}\right)+(\mu+\kappa)\left(\frac{\partial^2 u_3}{\partial x_1^2}+\frac{\partial^2 u_3}{\partial x_2^2}+\frac{\partial^2 u_3}{\partial x_3^2}\right)\\ +\kappa\left(\frac{\partial\phi_2}{\partial x_1}-\frac{\partial\phi_1}{\partial x_2}\right)-\nu\frac{\partial \mathrm{T}}{\partial x_3}+\mu(J_1H_2-J_2H_1)=\rho\frac{\partial^2 u_3}{\partial t^2},\end{aligned} \tag{5.21}$$

$$\begin{aligned}(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_1\partial x_2}+\frac{\partial^2\phi_3}{\partial x_1\partial x_3}\right)+\gamma\left(\frac{\partial^2\phi_1}{\partial x_1^2}+\frac{\partial^2\phi_1}{\partial x_2^2}+\frac{\partial^2\phi_1}{\partial x_3^2}\right)\\ +\kappa\left(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3}\right)-2\kappa\phi_1=\rho j\frac{\partial^2\phi_1}{\partial t^2},\end{aligned} \tag{5.22}$$

$$\begin{aligned}(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_2\partial x_1}+\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_3}{\partial x_2\partial x_3}\right)+\gamma\left(\frac{\partial^2\phi_2}{\partial x_1^2}+\frac{\partial^2\phi_2}{\partial x_2^2}+\frac{\partial^2\phi_2}{\partial x_3^2}\right)\\ +\kappa\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-2\kappa\phi_2=\rho j\frac{\partial^2\phi_2}{\partial t^2},\end{aligned} \tag{5.23}$$

$$\begin{aligned}(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x_3\partial x_1}+\frac{\partial^2\phi_2}{\partial x_3\partial x_2}+\frac{\partial^2\phi_3}{\partial x_3^2}\right)+\gamma\left(\frac{\partial^2\phi_3}{\partial x_1^2}+\frac{\partial^2\phi_3}{\partial x_2^2}+\frac{\partial^2\phi_3}{\partial x_3^2}\right)\\ +\kappa\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right)-2\kappa\phi_3=\rho j\frac{\partial^2\phi_3}{\partial t^2}.\end{aligned} \tag{5.24}$$

5.2. Formulation and solution of the problem

We consider a homogeneous generalized micropolar thermoelastic medium which is isotropic in nature and undergoing a constant applied magnetic field in the direction of x_2-

axis, $\vec{H}_0 = (0, H_0, 0)$ and induced magnetic field can also be considered in the same direction as of the applied magnetic field, $\vec{h} = (0, h, 0)$.

A Cartesian coordinate system $OX_1X_2X_3$ is taken with x_3 pointing normally into the medium. For the two dimensional problem, we assume that all quantities are dependent on x_1 and x_3 co-ordinates. For this we take the vector $\vec{u}$ and microrotation vector $\vec{\phi}$ *as*

$\vec{u} = (u_{1,} 0, u_{3,}), \vec{\phi} = (0, \phi, 0)$.

Also, the electric field is taken normal to the undertaken magnetic field $\vec{E} = (E_{1,} 0, E_{3,})$ and the electric current density is parallel to the electric field i.e. $\vec{J} = (J_{1,} 0, J_{3,})$. Using the above assumptions in the equations (5.1)-(5.4), (5.7) and (5.19)-(5.24), we obtain

$$\frac{\partial h}{\partial x_3} = -\sigma_o(E_1 - \mu_o \dot{u}_3 H_o) + k_o \frac{\partial T}{\partial x_1} - \epsilon_o \dot{E}_1, \tag{5.25}$$

$$\frac{\partial h}{\partial x_1} = \sigma_o(E_3 + \mu_o \dot{u}_1 H_o) - k_o \frac{\partial T}{\partial x_3} + \epsilon_o \dot{E}_3, \tag{5.26}$$

$$\frac{\partial E_3}{\partial x_1} - \frac{\partial E_1}{\partial x_3} = \mu_o \dot{H}, \tag{5.27}$$

$$J_1 = \sigma_o(E_1 - \mu_o \dot{u}_3 H_o) + k_o \frac{\partial T}{\partial x_1}, \tag{5.28}$$

$$J_3 = \sigma_o(E_3 + \mu_o \dot{u}_1 H_o) + k_o \frac{\partial T}{\partial x_3}. \tag{5.29}$$

$$(\lambda + \mu)\frac{\partial^2 u_1}{\partial x_1^2} + (\mu + \kappa)\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_3^2}\right) - \kappa \frac{\partial \phi}{\partial x_3} - \nu \frac{\partial T}{\partial x_1} - \mu J_3 H_0 = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{5.30}$$

$$\begin{aligned}(\lambda + \mu)\left(\frac{\partial^2 u_1}{\partial x_3 \partial x_1} + \frac{\partial^2 u_3}{\partial x_3^2}\right) + (\mu + \kappa)\left(\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_3^2}\right) + \kappa \frac{\partial \phi}{\partial x_1} - \nu \frac{\partial T}{\partial x_3} \\ + \mu J_1 H_0 = \rho \frac{\partial^2 u_3}{\partial t^2},\end{aligned} \tag{5.31}$$

$$(\alpha + \beta)\frac{\partial^2 \phi}{\partial x_1^2} + \gamma\left(\frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_3^2}\right) + \kappa\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - 2\kappa\phi = \rho j \frac{\partial^2 \phi}{\partial t^2}, \tag{5.32}$$

Where dot denotes derivative w.r.t. time t.

Introducing the following non-dimensional variables in equations (5.5) and (5.25)-(5.29),

$$x_i^* = c_0 \eta_0 x_i, \qquad u_i^* = c_0 \eta_0 u_i, \qquad t^* = {c_0}^2 \eta_0 t, \qquad \tau_0^* = {c_0}^2 \eta_0 \tau_0, \tag{5.33}$$

$$\phi^* = \frac{\mu}{(\mu+\kappa)}\phi, \qquad {\sigma_{ij}}^* = \frac{1}{(\mu+\kappa)}\sigma_{ij}, \qquad {\mu_{ij}}^* = \frac{\kappa}{c_0\eta_0(\mu+\kappa)(\beta+\gamma)}\mu_{ij},$$

$$h^* = \frac{\eta_0}{\sigma_0\mu_0 H_0}h, \qquad {E_i}^* = \frac{\eta_0}{\sigma_0{\mu_0}^2 H_0 c_0}E_i\,, \qquad T^* = \frac{\nu T}{\rho {c_0}^2},$$

$$ {J_i}^* = \frac{\eta_0}{{\sigma_0}^2{\mu_0}^2 H_0 c_0}J_i \;\; for \;\; i = 1,3$$

where

$$c_0^2 = \frac{\lambda+2\mu+\kappa}{\rho}, \;\; \eta_0 = \frac{\rho c^*}{K^*}.$$

we obtain (asterisks dropped for convenience)

$$a_3\frac{\partial e}{\partial x_1} + \frac{\partial \Omega}{\partial x_3} - \frac{\partial \phi}{\partial x_3} - a_{12}E_3 - a_3\left(\frac{\partial T}{\partial x_1} - a_{13}\frac{\partial T}{\partial x_3}\right) = a_3\left(\frac{\partial^2 u_1}{\partial t^2} + a_{19}\frac{\partial u_1}{\partial t}\right), \tag{5.34}$$

$$a_3\frac{\partial e}{\partial x_3} - \frac{\partial \Omega}{\partial x_1} + \frac{\partial \phi}{\partial x_1} + a_{12}E_1 - a_3\left(\frac{\partial T}{\partial x_3} + a_{13}\frac{\partial T}{\partial x_1}\right) = a_3\left(\frac{\partial^2 u_3}{\partial t^2} + a_{19}\frac{\partial u_3}{\partial t}\right), \tag{5.35}$$

$$\nabla^2\phi + a_{15}\Omega - a_{16}\phi = a_{17}\frac{\partial^2\phi}{\partial t^2}, \tag{5.36}$$

$$\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)(\,T + a_1 e\,) - a_2\frac{\partial}{\partial t}div\,\vec{E}, \tag{5.37}$$

$$\frac{\partial h}{\partial x_3} = -a_8E_1 + \frac{\partial u_3}{\partial t} + a_9\frac{\partial T}{\partial x_1} - a_{10}\frac{\partial E_1}{\partial t}, \tag{5.38}$$

$$\frac{\partial h}{\partial x_1} = a_8E_3 + \frac{\partial u_1}{\partial t} - a_9\frac{\partial T}{\partial x_3} + a_{10}\frac{\partial E_3}{\partial t}, \tag{5.39}$$

$$\frac{\partial E_3}{\partial x_1} - \frac{\partial E_1}{\partial x_3} = \frac{\partial h}{\partial t}, \tag{5.40}$$

$$J_1 = E_1 - \frac{1}{a_8}\dot{u}_3 - a_{11}\frac{\partial T}{\partial x_1}, \tag{5.41}$$

$$J_3 = E_3 + \frac{1}{a_8}\dot{u}_1 - a_{11}\frac{\partial T}{\partial x_3}, \tag{5.42}$$

where

$$\Omega = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\,, \qquad e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}, \tag{5.43}$$

$$a_1 = \frac{\nu^2 T_0}{\rho^2 {c_0}^2 c^*}, \qquad a_2 = \frac{\pi_0 \in_0 \sigma_0 {\mu_0}^2 H_0 \nu}{\rho^2 c^*}, \qquad a_3 = \frac{\rho {c_0}^2}{\mu + \kappa}, \qquad a_4 = \frac{\lambda}{\mu + \kappa},$$
$$a_5 = \frac{\mu}{\mu + \kappa}, \qquad a_6 = \frac{\gamma}{\beta + \gamma}, \qquad a_7 = \frac{\beta}{\beta + \gamma}, \qquad a_8 = \frac{\sigma_0 \mu_0}{\eta_0},$$
$$a_9 = \frac{\rho {c_0}^2 \eta_0}{\nu \sigma_0 \mu_0 H_0}, \qquad a_{10} = \in_0 \mu_0 {c_0}^2, \qquad a_{11} = \frac{k_0 \rho {\eta_0}^2 {c_0}^2}{\nu {\sigma_0}^2 {\mu_0}^2 H_0}, \tag{5.44}$$
$$a_{12} = \frac{{\mu_0}^3 {\sigma_0}^2 {H_0}^2}{{\eta_0}^2 (\mu + \kappa)}, \qquad a_{13} = \frac{k_0 \mu_0 H_0}{\nu}, \qquad a_{18} = a_3 a_{13},$$
$$a_{19} = \frac{{\mu_0}^2 {H_0}^2 \sigma_0}{{c_0}^2 \rho \eta_0}, \qquad a_{14} = a_3 a_{19}, \qquad a_{15} = \frac{\kappa^2}{\gamma(\mu + \kappa){c_0}^2 {\eta_0}^2},$$
$$a_{16} = \frac{2\kappa}{\gamma {c_0}^2 {\eta_0}^2}, \qquad a_{17} = \rho j {c_0}^2 .$$

Using Laplace transforms as defined in equation (2.45) and its properties (2.46)-(2.47) on equations (5.34)-(5.42), we obtain

$$a_3 \frac{\partial \bar{e}}{\partial x_1} + \frac{\partial \bar{\Omega}}{\partial x_3} - \frac{\partial \bar{\phi}}{\partial x_3} - a_{12} \bar{E}_3 - a_3 \left(\frac{\partial \bar{T}}{\partial x_1} - a_{13} \frac{\partial \bar{T}}{\partial x_3} \right) = a_3 (s^2 + a_{19} s) \bar{u}_1, \tag{5.45}$$

$$a_3 \frac{\partial \bar{e}}{\partial x_3} - \frac{\partial \bar{\Omega}}{\partial x_1} + \frac{\partial \bar{\phi}}{\partial x_1} + a_{12} \bar{E}_1 - a_3 \left(\frac{\partial \bar{T}}{\partial x_3} + a_{13} \frac{\partial \bar{T}}{\partial x_1} \right) = a_3 (s^2 + a_{19} s) \bar{u}_3, \tag{5.46}$$

$$\nabla^2 \bar{\phi} + a_{15} \bar{\Omega} - a_{16} \bar{\phi} = a_{17} s^2 \bar{\phi}, \tag{5.47}$$

$$\nabla^2 \bar{T} = (s + \tau_o s^2)(\bar{T} + a_1 \bar{e}) - a_2 s \left(\frac{\partial \bar{E}_1}{\partial x_1} + \frac{\partial \bar{E}_3}{\partial x_3} \right), \tag{5.48}$$

$$\frac{\partial \bar{h}}{\partial x_3} = -a_8 \bar{E}_1 + s \bar{u}_3 + a_9 \frac{\partial \bar{T}}{\partial x_1} - a_{10} s \bar{E}_1, \tag{5.49}$$

$$\frac{\partial \bar{h}}{\partial x_1} = a_8 \bar{E}_3 + s \bar{u}_1 - a_9 \frac{\partial \bar{T}}{\partial x_3} + a_{10} s \bar{E}_3, \tag{5.50}$$

$$\frac{\partial \bar{E}_3}{\partial x_1} - \frac{\partial \bar{E}_1}{\partial x_3} = s \bar{h}, \tag{5.51}$$

$$\bar{J}_1 = \bar{E}_1 - \frac{1}{a_8} s \bar{u}_3 - a_{11} \frac{\partial \bar{T}}{\partial x_1}, \tag{5.52}$$

$$\bar{J}_3 = \bar{E}_3 + \frac{1}{a_8} s \bar{u}_1 - a_{11} \frac{\partial \bar{T}}{\partial x_3}, \tag{5.53}$$

Now applying the Fourier transforms as defined in equation (3.58) on equations (5.45)-(5.62), after some simplification, we obtain

$$-a_3(\xi^2\tilde{u}_1+\iota\xi\tilde{u}_3)+(D^2\tilde{u}_1+\xi^2\tilde{u}_3)-D\tilde{\phi}-a_{12}\tilde{E}_3-a_3\iota\xi\tilde{T}-a_{13}\,D\bar{T} \\ =a_3(s^2\;+a_{19}s)\tilde{u}_1, \tag{5.54}$$

$$a_3(\iota\xi D\tilde{u}_1+D^2\tilde{u}_3)-(\iota\xi D\tilde{u}_1+D^2\tilde{u}_3)+D\tilde{\phi}+a_{12}\tilde{E}_1-a_3\left(D\tilde{T}+a_{13}\iota\xi\tilde{T}\right) \\ =a_3(s^2\;+a_{19}s)\tilde{u}_3, \tag{5.55}$$

$$(\mathrm{D}^2-\xi^2)\tilde{\phi}+a_{15}(D\tilde{u}_1-\iota\xi\tilde{u}_3)-a_{16}\tilde{\phi}=a_{17}\,s^2\tilde{\phi}, \tag{5.56}$$

$$(\mathrm{D}^2-\xi^2)\tilde{T}=(s+\tau_o s^2)\,\tilde{T} \\ +a_1(s+\tau_o s^2)(\iota\xi\tilde{u}_1+D\tilde{u}_3)-a_2 s\left(-\iota\xi\tilde{E}_1+D\tilde{E}_3\right), \tag{5.57}$$

$$D\tilde{h}=-a_8\tilde{E}_1+s\tilde{u}_3+a_9\iota\xi\tilde{T}-a_{10}\,s\tilde{E}_1, \tag{5.58}$$

$$\iota\tilde{h}=a_8\tilde{E}_3+s\tilde{u}_1-a_9 D\tilde{T}+a_{10}s\tilde{E}_3, \tag{5.59}$$

$$\iota\tilde{E}_3-D\tilde{E}_1=s\tilde{h}, \tag{5.60}$$

$$\tilde{J}_1=\tilde{E}_1-\frac{1}{a_8}s\tilde{u}_3-a_{11}\,\tilde{T}, \tag{5.61}$$

$$\tilde{J}_3=\tilde{E}_3+\frac{1}{a_8}s\tilde{u}_1-a_{11}\,D\tilde{T}, \tag{5.62}$$

where D=$\frac{\partial}{\partial x_3}$.

After some simplification, we obtain

$$(D^{10}+\alpha_{28}D^8+\alpha_{29}D^6+\alpha_{30}D^4+\alpha_{31}D^2+\alpha_{32})\left\{\tilde{\phi},\tilde{u}_1,\tilde{u}_3,\tilde{h},\tilde{T}\,\right\}=0\;. \tag{5.63}$$

where

$\alpha_1=\xi^2+a_{16}+a_{17}s^2,\qquad \alpha_2=\xi^2+s+\tau_0 s^2,\qquad \alpha_3=a_1(s+\tau_0 s^2),$

$\alpha_4=a_2 s,$

$\alpha_5=\xi^2+s^2+a_{19}s,\;\; \alpha_6=\frac{a_{12}}{a_3}s,\;\; \alpha_7=\xi^2+a_3(s^2+a_{19}s),$

$\alpha_8=a_3a_{13},\;\; \alpha_9=\xi^2+a_8 s+a_{10}s^2,\;\; \alpha_{10}=\frac{a_8\;+s\,a_{10}}{a_9},\;\; \alpha_{11}$
$=-\frac{s}{a_9},\;\; \alpha_{12}=\alpha_9+\alpha_5,\;\; \alpha_{34}=a_{12}-\alpha_8\alpha_{10},\qquad \alpha_{13}=\alpha_9\alpha_5-\alpha_6 s,$

$\alpha_{14}=\xi^2+\alpha_9,$

$\alpha_{15}=\alpha_9\xi^2,\;\; \alpha_{16}=\alpha_{12}+\alpha_2+\alpha_3,\;\; \alpha_{17}=\alpha_{13}+\alpha_{12}\alpha_2+\alpha_3\alpha_{14},\;\; \alpha_{18}$
$=\alpha_3\alpha_{15}-\alpha_2\alpha_{13},\;\; \alpha_{19}=\alpha_4+\alpha_{10},\qquad \alpha_{20}=\alpha_4(\alpha_{12}+\xi^2)+\alpha_{10}\alpha_6,$

$$\alpha_{21} = \alpha_4(\alpha_{13} + \alpha_{12}\,\xi^2) + \alpha_{10}\alpha_7, \qquad \alpha_{22} = \alpha_4\alpha_{13}\xi^2 + \alpha_{10}\alpha_{18},$$

$$\alpha_{23} = -[\alpha_1\alpha_{19} + \{\alpha_{20} + \alpha_7\alpha_{19} - (\alpha_{11}\alpha_{12} + \alpha_4\alpha_8\alpha_{11})\} - a_{15}\alpha_{19}],\ \ \alpha_{24} = \alpha_1(\alpha_{11}\alpha_{12} + \alpha_4\alpha_8\alpha_{11}) + \alpha_{21} + \alpha_7\alpha_{20} - \left(\alpha_{11}\alpha_{12}\alpha_{16} + \alpha_4\alpha_8\alpha_{11}(\xi^2 + \alpha_{12})\right) - a_{15}(\xi^2\alpha_{19} + \alpha_{20}),\ \ \alpha_{25} = -(\alpha_{22} + \alpha_7\alpha_{21}) + \alpha_{11}\alpha_{12}\alpha_{17} + \alpha_4\alpha_8\alpha_{11}(\alpha_{13} + \xi^2\alpha_{12}) - \alpha_1\{\alpha_{21} + \alpha_7\alpha_{20} - (\alpha_{11}\alpha_{12}\alpha_{16} + \alpha_4\alpha_8\alpha_{11}(\xi^2 + \alpha_{12})\} + a_{15}\alpha_{21} + a_{15}\alpha_{20}\xi^2,\ \ \alpha_{26} = \alpha_7\alpha_{22} - (\alpha_{11}\alpha_{12}\alpha_{18} + \alpha_4\alpha_8\alpha_{11}\alpha_{13}\xi^2) - \alpha_1\{-(\alpha_{22} + \alpha_7\alpha_{21}) + \alpha_{11}\alpha_{12}\alpha_{17} + \alpha_4\alpha_8\alpha_{11}(\alpha_{13} + \alpha_{12}\xi^2)\} - a_{15}\alpha_{22} - a_{15}\alpha_{21}\xi^2,$$

$$\alpha_{27} = -\alpha_1\{\alpha_7\alpha_{22} - (\alpha_{11}\alpha_{12}\alpha_{18} + \alpha_4\alpha_8\alpha_{11}\alpha_{13}\xi^2)\} + a_{15}\alpha_{22}\xi^2,$$

$$\alpha_{28} = \frac{\alpha_{23}}{\alpha_{19}},\ \ \alpha_{29} = \frac{\alpha_{24}}{\alpha_{19}},\ \ \alpha_{30} = \frac{\alpha_{25}}{\alpha_{19}},\qquad \alpha_{31} = \frac{\alpha_{26}}{\alpha_{19}},\ \ \alpha_{32} = \frac{\alpha_{27}}{\alpha_{19}}.$$

The characteristic equation of differential equation *(5.63)* is given by

$$m^{10} + \alpha_{28}m^8 + \alpha_{29}m^6 + \alpha_{30}m^4 + \alpha_{31}m^2 + \alpha_{32} = 0. \tag{5.64}$$

Solution of equation (5.63) satisfying the radiation condition $Re(m_j) \geq 0$ is given by

$$\tilde{\phi} = \sum_{j=1}^{5} A_j(s,\xi)e^{-m_j x_3}, \tag{5.65}$$

where $m_j{}'s$ are the roots of equation (5.64), for j = 1,2,3,4,5

Similarly we can obtain

$$(\tilde{e}, \tilde{h}, \tilde{T}, \tilde{\Omega}) = \sum_{j=1}^{5}(A_j{}^{(1)}, A_j{}^{(2)}, A_j{}^{(3)}, A_j{}^{(4)})e^{-m_j x_3}, \tag{5.66}$$

where

$A_j, A_j{}^{(1)}, A_j{}^{(2)}, A_j{}^{(3)}$ *and* $A_j{}^{(4)}$ are related as

$$A_j{}^{(1)} = \left[\frac{(m_j{}^2 - \alpha_9)(m_j{}^2 - \xi^2)}{l_j}\right]A_j, \tag{5.67}$$

$$A_j{}^{(2)} = \left[\frac{s(m_j{}^2 - \xi^2)}{l_j}\right]A_j, \tag{5.68}$$

$$A_j^{(3)} = -\left[\frac{(m_j^2-\alpha_5)(m_j^2-\alpha_9)-\alpha_6 s}{l_j}\right]A_j, \tag{5.69}$$

$$A_j^{(4)} = -\left[\frac{(m_j^2-\alpha_1)}{\alpha_{15}}\right]A_j \text{ for j} = 1,2,3,4,5. \tag{5.70}$$

where

$$l_j = -\frac{\alpha_{15}}{\alpha_4\alpha_{11}(m_j^2-\alpha_1)}\left[\{\alpha_{10}(m_j^2-\alpha_2)+\alpha_4(m_j^2-\xi^2)\}\{(m_j^2-\alpha_5)(m_j^2-\alpha_9)-\alpha_6 s\}-\alpha_3\alpha_{10}(m_j^2-\xi^2)(m_j^2-\alpha_9)\right],$$

Using (5.71)-(5.74) in equation (5.63), we obtain

$$\tilde{T} = \sum_{j=1}^{5}\left[\frac{(m_j^2-\alpha_5)(m_j^2-\alpha_9)-\alpha_6 s}{l_j}\right]A_j e^{-m_j x_3}, \tag{5.71}$$

$$\tilde{\Omega} = \sum_{j=1}^{5}\left[-\frac{(m_j^2-\alpha_1)}{\alpha_{15}}\right]A_j e^{-m_j x_3}, \tag{5.72}$$

$$\tilde{e} = \sum_{j=1}^{5}\left[\frac{(m_j^2-\alpha_9)(m_j^2-\xi^2)}{l_j}\right]A_j e^{-m_j x_3}, \tag{5.73}$$

$$\tilde{h} = \sum_{j=1}^{5}\left[\frac{s(m_j^2-\xi^2)}{l_j}\right]A_j e^{-m_j x_3}. \tag{5.74}$$

Now with the help of equations (2.47), (3.58), (5.10), (5.11), (5.43) and (5.58)-(5.62) we obtain the transformed components of displacement, normal stress, tangential stress, tangential couple stress, electric field and current density as

$$\tilde{u}_1 = \sum_{j=1}^{5}\left[\frac{m_j(m_j^2-\alpha_1)}{\alpha_{15}(m_j^2-\xi^2)}+\iota\frac{\xi(m_j^2-\alpha_9)}{l_j}\right]A_j e^{-m_j x_3}, \tag{5.75}$$

$$\tilde{u}_3 = \sum_{j=1}^{5}\left[\frac{m_j(m_j^2-\alpha_9)}{l_j}+\iota\frac{m_j\xi(m_j^2-\alpha_1)}{\alpha_{15}(m_j^2-\xi^2)}\right]A_j e^{-m_j x_3}, \tag{5.76}$$

$$\tilde{E}_1 = \frac{1}{a_8 + a_{10}s}\sum_{j=1}^{5}\left[\frac{sm_j\left(2m_j^2 - \alpha_9 - \xi^2\right)}{l_j} + \iota\frac{\xi a_9 m_j\left\{\left(m_j{}^2 - \alpha_5\right)\left(m_j{}^2 - \alpha_9\right) - \alpha_6 s\right\}}{l_j} + \frac{\xi s\left(m_j{}^2 - \alpha_1\right)}{a_{15}\left(m_j{}^2 - \xi^2\right)}\right]A_j e^{-m_j x_3}, \tag{5.77}$$

$$\tilde{E}_3 = \frac{1}{a_8 + a_{10}s}\sum_{j=1}^{5}\left[\frac{-a_9 m_j\left\{\left(m_j{}^2 - \alpha_5\right)\left(m_j{}^2 - \alpha_9\right) - \alpha_6 s\right\}}{l_j} + \frac{sm_j\left(m_j{}^2 - \alpha_9\right)}{a_{15}\left(m_j{}^2 - \xi^2\right)} - \frac{\iota\xi s(\xi^2 - \alpha_9)}{l_j}\right]A_j e^{-m_j x_3}, \tag{5.78}$$

$$\tilde{J}_1 = \sum_{j=1}^{5}\left[\frac{sm_j}{l_j}\left(\frac{2m_j^2 - \alpha_9 - \xi^2}{a_8 + a_{10}s} - \frac{m_j{}^2 - \alpha_9}{a_8}\right) + \iota\frac{\xi\left\{\left(m_j{}^2 - \alpha_5\right)\left(m_j{}^2 - \alpha_9\right) - \alpha_6 s\right\}}{l_j}\left(\frac{a_9}{a_8 + a_{10}s} - a_{11}\right) + \iota\frac{\xi s}{a_{15}\left(m_j{}^2 - \xi^2\right)}\left\{\frac{m_j{}^2 - \alpha_9}{a_8 + a_{10}s} - \frac{m_j{}^2 - \alpha_1}{a_8}\right\}\right]A_j e^{-m_j x_3}, \tag{5.79}$$

$$\tilde{J}_3 = \sum_{j=1}^{5}\left[\frac{m_j\left\{\left(m_j{}^2 - \alpha_5\right)\left(m_j{}^2 - \alpha_9\right) - \alpha_6 s\right\}}{l_j}\left(a_{11} - \frac{a_9}{a_8 + a_{10}s}\right) + \frac{sm_j\left(m_j{}^2 - \alpha_1\right)}{a_{15}\left(m_j{}^2 - \xi^2\right)}\left\{\frac{1}{a_8} + \frac{1}{a_8 + a_{10}s}\right\} + \frac{\iota\xi s}{l_j}\left(\frac{\alpha_9 - \xi^2}{a_8 + a_{10}s} + \frac{m_j{}^2 - \alpha_9}{a_8}\right)\right]A_j e^{-m_j x_3}, \tag{5.80}$$

$$\tilde{\sigma}_{33} = \sum_{j=1}^{5}\left[\frac{-\left(m_j{}^2 - \alpha_9\right)\left(m_j{}^2 a_3 + a_4\xi^2\right) - \left(m_j{}^2 - \alpha_5\right)\left(m_j{}^2 - \alpha_9\right) + \alpha_6 s}{l_j} + \iota\frac{\xi m_j\left(m_j{}^2 - \alpha_1\right)(a_4 - a_3)}{a_{15}\left(m_j{}^2 - \xi^2\right)}\right]A_j e^{-m_j x_3}, \tag{5.81}$$

$$\tilde{\sigma}_{31} = \sum_{j=1}^{5}\left[-1 - \frac{\left(m_j{}^2 - \alpha_1\right)\left(m_j{}^2 + a_5\xi^2\right)}{a_{15}\left(m_j{}^2 - \xi^2\right)} + \iota\frac{\xi m_j\left(m_j{}^2 - \alpha_9\right)(a_5 - 1)}{l_j}\right]A_j e^{-m_j x_3}, \tag{5.82}$$

$$\tilde{\mu}_{32} = -a_6 \sum_{j=1}^{5} m_j A_j e^{-m_j x_3}\,. \tag{5.83}$$

Using the dimensionless quantities given by equation (5.33) in equations (5.13)-(5.16) and adopting the same procedure as above, we obtain the values of $\tilde{h}_0, \tilde{E}_{10}, \tilde{E}_{30}$ satisfying the radiation condition as

$$\tilde{h}_0 = A_6(s,\xi)e^{nx_3}, \tag{5.84}$$

$$\tilde{E}_{10} = -\frac{1}{sa_{10}} nA_6(s,\xi)e^{nx_3}, \tag{5.85}$$

$$\tilde{E}_{30} = -\frac{i\xi}{sa_{10}} nA_6(s,\xi)e^{nx_3}, \tag{5.86}$$

where

$$n^2 = \xi^2 + \frac{s^2}{c^2}.$$

5.3. Boundary Conditions

The dimensionless boundary conditions are

$$\sigma_{33} + T_{33} - T_{33}{}^{0} = -F_1(x_1, 0, t), \tag{5.87}$$

$$\sigma_{31} = 0, \tag{5.88}$$

$$\mu_{32} = 0, \tag{5.89}$$

$$\frac{\partial T}{\partial x_3} = F_2(x_1, 0, t), \tag{5.90}$$

$$h(x_1, 0, t) = h_0(x_1, 0, t), \tag{5.91}$$

$$E_3(x_1, 0, t) = E_{30}(x_1, 0, t). \tag{5.92}$$

Because the relative permeabilities are nearly one, it leads to $T_{33} = T_{33}{}^{0}$. So equation (4.1) becomes

$$\sigma_{33} = -F_1(x_1, 0, t). \tag{5.93}$$

Applying Laplace and Fourier transforms defined by (2.47) and (3.58) on equations (5.88)-(5.93), we obtain

$$\tilde{\sigma}_{31} = 0, \tag{5.94}$$

$$\tilde{\mu}_{32} = 0, \tag{5.95}$$

$$D\tilde{T} = \tilde{F}_2(\xi, 0, s), \tag{5.96}$$

$$\tilde{h}(\xi, 0, s) = \tilde{h}_0(\xi, 0, s), \tag{5.97}$$

$$\tilde{E}_3(\xi, 0, s) = \tilde{E}_{30}(\xi, 0, s). \tag{5.98}$$

$$\tilde{\sigma}_{33} = -\tilde{F}_1(\xi, 0, s). \tag{5.99}$$

Now using equations (5.71), (5.74), (5.78) and (5.81)-(5.83) in equations (5.88)-(5.93), we obtain six equations in six unknowns A_j $(j = 1,2,\ldots,6)$ as

$$\sum_{j=1}^{5}\left[\frac{(m_j{}^2-\alpha_9)(m_j{}^2a_3+a_4\xi^2)+(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s}{l_j} - \iota\frac{\xi m_j(m_j{}^2-\alpha_1)(a_3-a_4)}{a_{15}(m_j{}^2-\xi^2)}\right]A_j = \tilde{F}_1(\xi, 0, s), \tag{5.100}$$

$$\sum_{j=1}^{5}\left[-1-\frac{(m_j{}^2-\alpha_1)(m_j{}^2+a_5\xi^2)}{a_{15}(m_j{}^2-\xi^2)}+\iota\frac{\xi m_j(m_j{}^2-\alpha_9)(a_5-1)}{l_j}\right]A_j = 0, \tag{5.101}$$

$$\sum_{j=1}^{5} m_j A_j = 0, \tag{5.102}$$

$$\sum_{j=1}^{5}\left[-\frac{(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s}{l_j}\right]m_j A_j = \tilde{F}_2(\xi, 0, s), \tag{5.103}$$

$$\sum_{j=1}^{5}\left[\frac{s(m_j{}^2-\xi^2)}{l_j}\right]A_j - A_6 = 0, \tag{5.104}$$

$$\sum_{j=1}^{5}\left[\frac{s m_j(2m_j^2-\alpha_9-\xi^2)}{l_j}+\iota\frac{\xi a_9 m_j\{(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s\}}{l_j} + \frac{\xi s(m_j{}^2-\alpha_9)}{a_{15}(m_j{}^2-\xi^2)}\right]A_j + \alpha_{33}A_6 = 0\,. \tag{5.105}$$

Solution of system of equations (5.100)-(5.105) is given by

$A_j(s,\xi) = \frac{\Delta_j}{\Delta}$ for j=1,2,3,4,5,6 (5.106)

where $\Delta = \left|n_l^{(k)}\right|_{6x6}$,

$\Delta_j = \tilde{F}_1\left|n_l^{(k)}\right|_{\substack{k\neq 1\\ l\neq j}} - \tilde{F}_2\left|n_l^{(k)}\right|_{\substack{k\neq 4\\ l\neq j}}$ for j,k, l =1,2,3,4,5,6.

$$n_j^{(1)} = \frac{(m_j{}^2-\alpha_9)(m_j{}^2a_3+a_4\xi^2)+(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s}{l_j} - \iota\frac{\xi m_j(m_j{}^2-\alpha_1)(a_3-a_4)}{a_{15}(m_j{}^2-\xi^2)},$$

$$n_j^{(2)} = -1-\frac{(m_j{}^2-\alpha_1)(m_j{}^2+a_5\xi^2)}{a_{15}(m_j{}^2-\xi^2)}+\iota\frac{\xi m_j(m_j{}^2-\alpha_9)(a_5-1)}{l_j},$$

$$n_j^{(3)} = m_j,$$

$$n_j^{(4)} = -\frac{(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s}{l_j},$$

$$n_j^{(5)} = \frac{s(m_j{}^2-\xi^2)}{l_j},$$

$$n_j^{(6)} = \frac{sm_j(2m_j^2-\alpha_9-\xi^2)}{l_j}+\iota\frac{\xi a_9 m_j\{(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s\}}{l_j} + \frac{\xi s(m_j{}^2-\alpha_9)}{a_{15}(m_j{}^2-\xi^2)}, \quad for\ j=1,2,3,4,5.$$

$$n_6^{(1)}=0,\quad n_6^{(2)}=0,\quad n_6^{(3)}=0,\quad n_6^{(4)}=0,\quad n_6^{(5)}=-1,\quad n_6^{(6)}=\alpha_{33},$$

$$\alpha_{33} = \frac{\iota\xi(a_8+a_{10}s)}{sa_{10}}.$$

Using equation (5.106) in equations (5.75)-(5.83), we obtain

$$\tilde{u}_1 = \sum_{j=1}^{5}\left[\frac{m_j(m_j{}^2-\alpha_1)}{a_{15}(m_j{}^2-\xi^2)}+\iota\frac{\xi(m_j{}^2-\alpha_9)}{l_j}\right]\frac{\Delta_j}{\Delta}e^{-m_j x_3}, \tag{5.107}$$

$$\tilde{u}_3 = \sum_{j=1}^{5}\left[\frac{m_j(m_j{}^2-\alpha_9)}{l_j}+\iota\frac{m_j\xi(m_j{}^2-\alpha_1)}{a_{15}(m_j{}^2-\xi^2)}\right]\frac{\Delta_j}{\Delta}e^{-m_j x_3}, \tag{5.108}$$

$$\tilde{E}_1 = \frac{1}{a_8 + a_{10}s}\sum_{j=1}^{5}\left[\frac{-a_9 m_j\{(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s\}}{l_j} + \frac{s m_j(m_j{}^2-\alpha_1)}{a_{15}(m_j{}^2-\xi^2)} + \frac{\iota\xi s(\xi^2-\alpha_9)}{l_j}\right]\frac{\Delta_j}{\Delta}e^{-m_j x_3}, \tag{5.109}$$

$$\tilde{E}_3 = \frac{1}{a_8 + a_{10}s}\sum_{j=1}^{5}\left[\frac{s m_j(\alpha_9-\xi^2)}{l_j} - \iota\frac{\xi a_9 m_j\{(m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)-\alpha_6 s\}}{l_j} + \frac{s m_j(m_j{}^2-\alpha_1)}{a_{15}(m_j{}^2-\xi^2)}\right]\frac{\Delta_j}{\Delta}e^{-m_j x_3}, \tag{5.110}$$

$$\tilde{\sigma}_{33} = \sum_{j=1}^{5}\left[\frac{(m_j{}^2-\alpha_9)(m_j{}^2 a_3 + a_4\xi^2) - (m_j{}^2-\alpha_5)(m_j{}^2-\alpha_9)}{l_j} + \iota\frac{\xi m_j(m_j{}^2-\alpha_1)(a_3-a_4)}{a_{15}(m_j{}^2-\xi^2)}\right]\frac{\Delta_j}{\Delta}e^{-m_j x_3}, \tag{5.111}$$

$$\tilde{\sigma}_{31} = \sum_{j=1}^{5}\left[-1 - \frac{(m_j{}^2-\alpha_1)(m_j{}^2 + a_5\xi^2)}{a_{15}(m_j{}^2-\xi^2)} + \iota\frac{\xi m_j(m_j{}^2-\alpha_9)(a_5+1)}{l_j}\right]\frac{\Delta_j}{\Delta}e^{-m_j x_3}, \tag{5.112}$$

$$\tilde{\mu}_{32} = -a_6\sum_{j=1}^{5} m_j\frac{\Delta_j}{\Delta}e^{-m_j x_3}. \tag{5.113}$$

Case I: Normal concentrated force or continuous force

In this $F_1(x_1, 0, t) = \begin{cases} \delta(x_1)\delta(t) & for\ normal\ concentrated\ force \\ \delta(x_1)H(t) & for\ continuous\ force \end{cases}$

$F_2(x_1, 0, t) = 0.$

Applying Laplace and Fourier transforms defined by (2.45) and (3.58) we obtain

$$\tilde{F}_1(\xi, 0, s) = \begin{cases} 1 & for\ normal\ concentrated\ force \\ \dfrac{1}{s} & for\ continuous\ force \end{cases}$$

$$\tilde{F}_2(\xi, 0, s) = 0.$$

Case II: Concentrated thermal source or continuous thermal source

In this case

$$F_1(x_1, 0, t) = 0,$$

$$F_2(x_1, 0, t) = \begin{cases} \delta(x_1)\delta(t) & for\ concentrated\ thermal\ source \\ \delta(x_1)H(t) & for\ continuous\ thermal\ source \end{cases}$$

Applying Laplace and Fourier transforms defined by (2.45) and (3.58), we obtain

$$\tilde{F}_1(x_1, 0, t) = 0,$$

$$\tilde{F}_2(\xi, 0, s) = \begin{cases} 1 & for\ concentrated\ thermal\ source \\ \frac{1}{s} & for\ continuous\ thermal\ source \end{cases}$$

Replacing values of $\tilde{F}_1$ and $\tilde{F}_2$ in equations (5.107)-(5.113), we obtain expressions for transformed components of displacement, Electric field, current density, tangential stress and tangential couple stress for Normal concentrated force, normal continuous force, concentrated thermal source and continuous thermal source.

5.4. Inversion of the transforms

To obtain solution of the problem in physical domain, the transformed components given by equations (5.107)-(5.113) must be inverted. These components are functions of x_3, parameter of Laplace transform (s) and Fourier transform (ξ). To obtain them in the physical domain in the form of $f(x_1, x_3, t)$, we invert integral transforms by using the inversion technique as mentioned in section 3.5 of chapter 3.

5.5. Numerical discussion and Analysis

Magnesium crystal material is used for analysis. Following reference [18] the values of physical constants are

$\lambda = 9.4 \times 10^{10} N.m^{-2},\ \mu = 4 \times 10^{10} N.m^{-2},\ \kappa = 1 \times 10^{10} N.m^{-2},\ \rho = 1.74 \times 10^3\ Kg\,.m^{-3},\ \ \gamma = 0.779 \times 10^{-5} N.m^{-2},\ \beta = 0.98 \times 10^{-5} N.m^{-2}, \sigma_0 = 2.2356 \times$

$10^7 S.m^{-1}$, $\mu_0 = 4\pi \times 10^{-7} H.m^{-1}$. $\epsilon_0 = 10^{-9}/36\pi\, F.m^{-1}$,$\vartheta = 0.0268 \times 10^8 N.m^{-2}.K^{-1}$, $j = 0.2 \times 10^{-19} m^2$, $c^* = 1.04$ J.$Kg^{-1}.K^{-1}$ $H_o = 1\, A.m^{-1}$, $\tau_0 = 0.01$.

The computations are carried out for the dimensionless time t=0.5 in the range$0 \leq x_1 \leq 6$. Effect of modification in Ohm's and Fourier's law in magneto micropolar thermoelastic medium (MMT) have been shown by assigning different values to a_2 and a_{13}. In graphs 5.6.1-5.6.6 variations have been represented by solid line for MMT1 ($\tau_0 = 0$, $a_2 = 0$, $a_{13} = 0$); dashed line with centered symbol for MMT2 ($\tau_0 = 0$, $a_2 = 0.5$, $a_{13} = 0.25$); small dashed line for MMT3 ($\tau_0 = 0.01$, $a_2 = 0$, $a_{13} = 0$) and dotted line for MMT4 ($\tau_0 = 0.1$, $a_2 = 0.5$, $a_{13} = 0.25$). The variations in the dimensionless normal stress σ_{33}, dimensionless couple stress μ_{32}, and dimensionless temperature distribution T with dimensionless distance x_1 for concentrated source have been shown in figures 5.6.1-5.6.3 whereas for continuous source the same have been shown in figures 5.6.4-5.6.6.

5.6.1 Concentrated Source

Fig. 5.5.1 shows the variations in dimensionless normal stress σ_{33} with the dimensionless distance x_1. Initially it decreases in the range $0 \leq x_1 \leq 2$ and is larger for MMT4 as compared to MMT1, MMT2 and MMT3. Further as x_1 increases in the range $2 < x_1 \leq 6$, it increases. Fig. 5.5.2 shows that tangential couple stress μ_{32} increases in the range $0 \leq x_1 \leq 2$ and decreases in the range $2 < x_1 \leq 6$. Fig. 5.5.3 shows the variation in temperature distribution T. It keeps on decreasing with x_1

5.6.2 Continuous Source

Fig. 5.5.4 shows that for $0 < x_1 < 2$ value of normal stress force σ_{33} decreases for MMT1, MMT3, MMT4 but its behaviour is of opposite nature for MMT2 i.e. it increases for MMT2. In the range $2 < x_1 < 6$, σ_{33} becomes oscillatory. Fig. 5.5.5 depicts that the variations in μ_{32} are very small for MMT1 whereas for MMT2, MMT3 and MMT4 it becomes oscillatory in the range $1 < x_1 < 6$ with decreasing amplitude. Fig. 5.5.6 shows that the temperature distribution T keeps on decreasing as x_1 increases for MMT2 and MMT3. For MMT4 it is oscillatory however the variations are very small for MMT1.

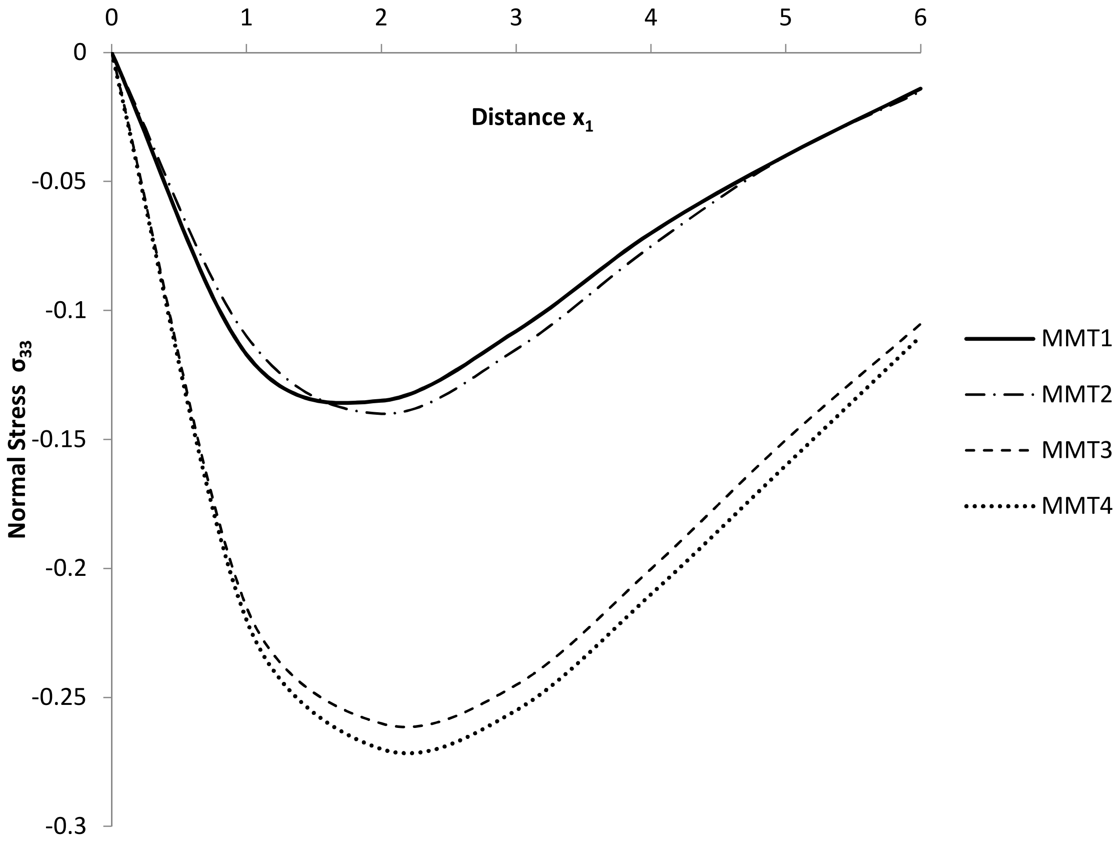

Fig. 5.5.1 Variation of normal stress σ_{33} with x_1

(Concentrated source)

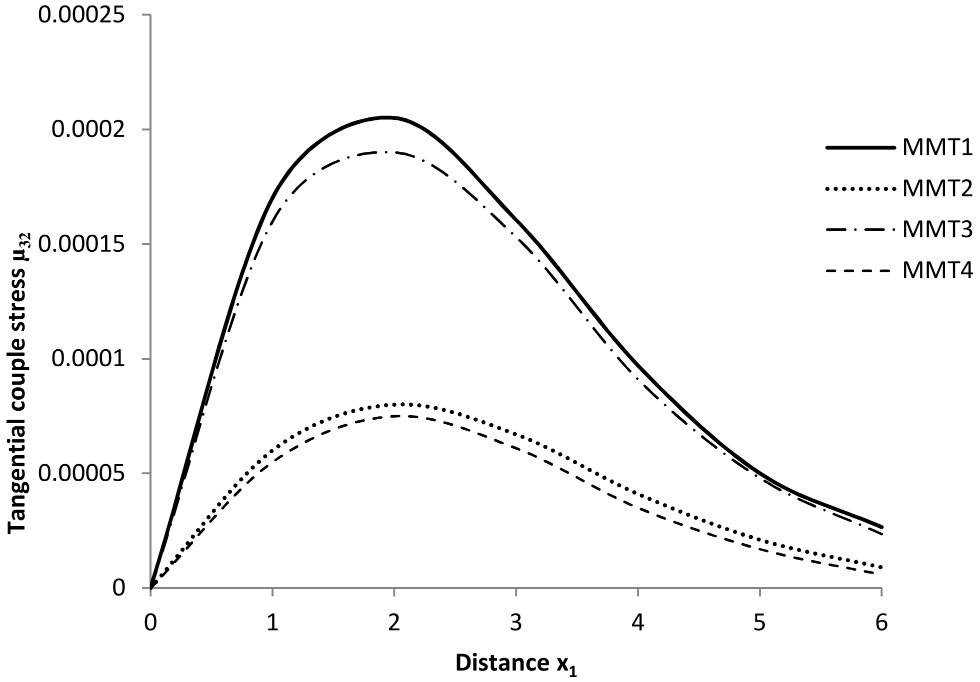

Fig. 5.5.2 Variation of tangential couple stress μ_{32} with x_1

(Concentrated source)

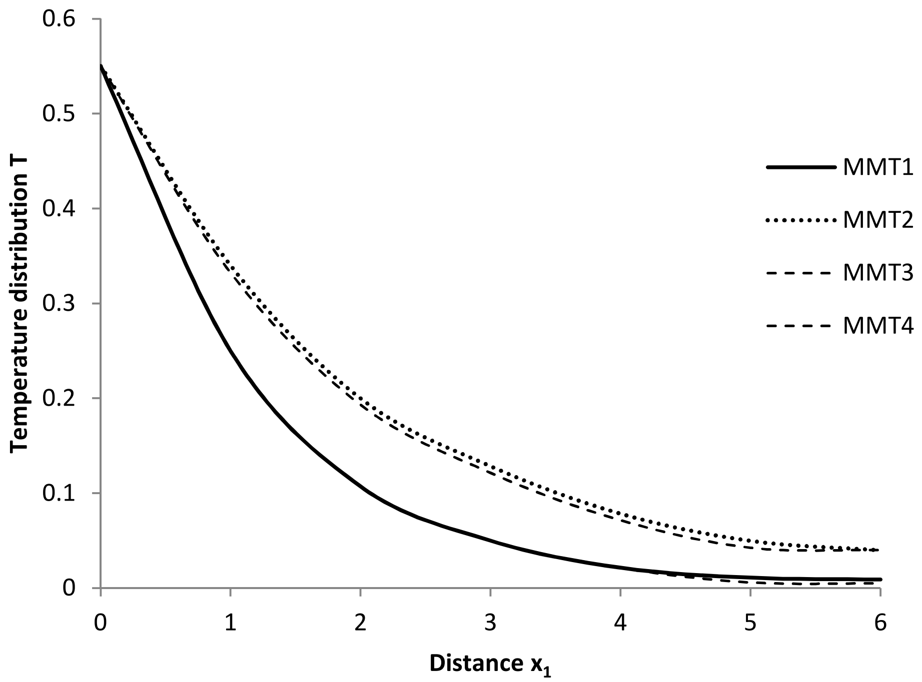

Fig. 5.5.3 Variation of temperature distribution T with x_1

(Concentrated source)

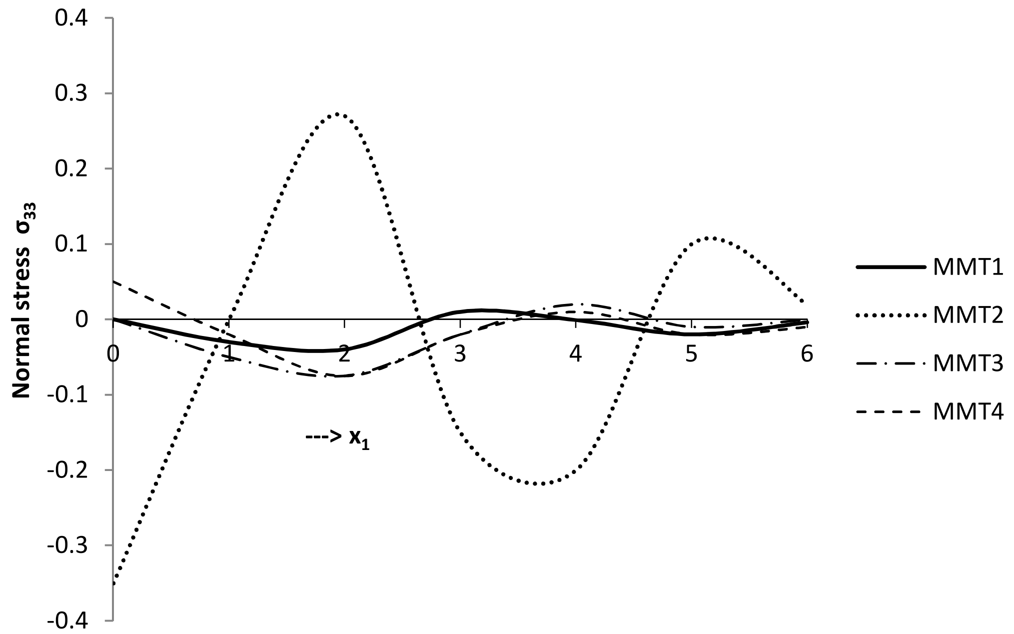

Fig. 5.5.4 Variation of normal stress σ_{33} with x_1

(Continuous source)

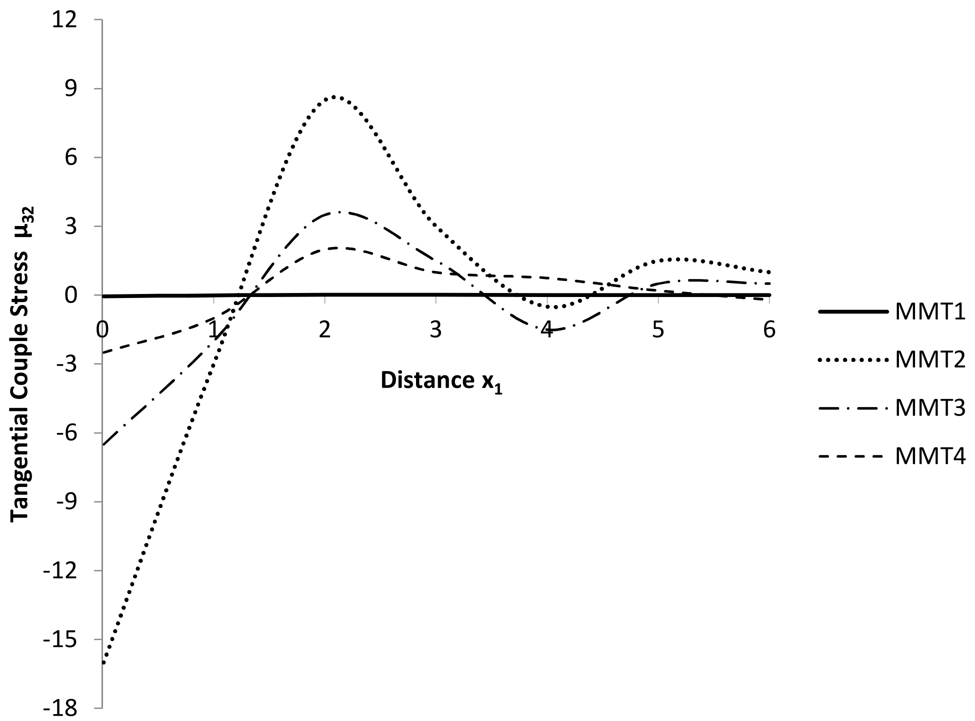

Fig. 5.5.5 Variation of tangential couple stress μ_{32} with x_1

(Continuous source)

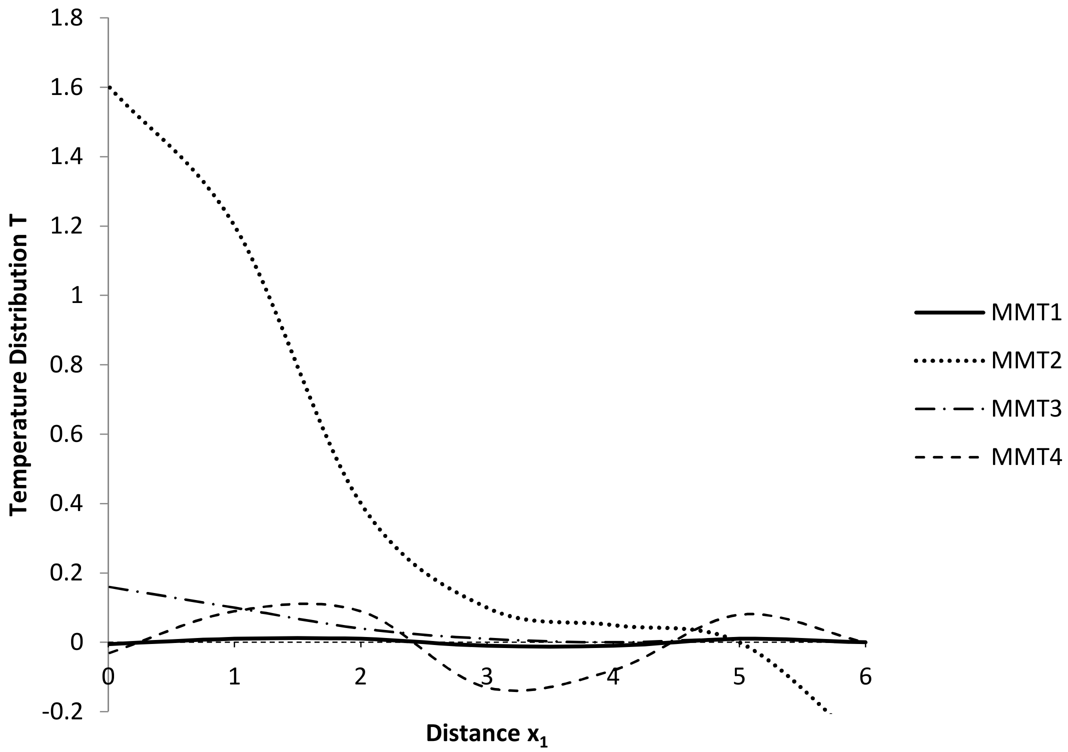

Fig. 5.5.6 Variation of temperature distribution T with x_1

(Continuous source)

5.6. Conclusion

Modified Ohm's and Fourier's law have been used to investigate the problem in magneto micropolar thermoelastic half-space. Both concentrated and continuous sources have been applied on the boundary. Laplace and Fourier transform techniques were applied to obtain the values of stresses, strains, displacements and temperature distribution in transformed domain. This study can be useful in analysing the stress-strain of modern engineering materials subjected to thermal and mechanical disturbances. From the above discussion it is observed that the effect of modification in Ohm's and Fourier's law is evident as stresses, couple stresses and temperature distribution are showing different trend for new model.

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