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MASTER OF SCIENCE

Modelling Effect of Increasing Temperature on Growth Dynamics of Two Interacting Population

A project submitted in fulfilment of the requirements for the degree of MASTER OF SCIENCE

in the

Department of Mathematics
School of Chemical Engineering & Physical Sciences
Lovely Faculty of Technology and Sciences

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April 2017

Declaration of Authorship

I, Vikas Choudhary, declare that this thesis titled, "Modelling Effect of Increasing Temperature on Growth Dynamics of Two Interacting Population" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this project has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this project is entirely my own work.
- I have acknowledged all main sources of help.
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Certificate

This is to certify that VIKAS CHOUDHARY has completed Project titled "Modelling Effect of Increasing Temperature on Growth Dynamics of Two Interacting Population" under my guidance and supervision. To the best of my knowledge, the present work is the result of his/her original investigation and study. No part of the project has ever been submitted for any other degree at any University.

The project is fit for the submission and the partial fulfilment of the conditions for the award of MASTER OF SCIENCE in Mathematics.

Signed:

Supervisor: DR.PREETY KALRA

Date: April 2017

"Hard work beats talent when talent fails to work hard."

Kevin Durant

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Abstract

MASTER OF SCIENCE

Modelling Effect of Increasing Temperature on Growth Dynamics of Two Interacting Population

by VIKAS CHOUDHARY

It is well recognized that the greenhouse gas such as Chlorofluoro Carbon (CFC) is responsible directly or indirectly for the increase in the average global temperature of the Earth. The presence of CFC is responsible for the depletion of ozone concentration in the atmosphere due to which the heat accompanied with the sun rays are less absorbed causing increase in the atmosphere temperature of the Earth. The increase in the temperature level directly or indirectly affects the dynamics of interacting species system. Therefore, mathematical model are purposed and analyzed using stability theory to asses the effects of increasing temperature due to greenhouse gas CFC on the survival or extinction of interactive population.

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Dedicated To my parents

Chapter 1

Introduction

1.1 Ecology and Ecosystem

The manner in which the organisms interact with other organisms and also with the environment is being studied in ecology. Ecology is considered as a science, not any movement (like environmentalism). The Ecologist involves himself in the hypothetic method which is co-deductive so as to frame questions and generate hypotheses about ecosystems which can be tested. Usually, mathematical models which are very complex are to be generated to simulate ecosystems. The comparison of the real systems can be done with these models which represent idealized systems and the predictive value of these real systems can be found out. When a project on a huge scale is impossible sometimes to perform, then the usage of computer model is done for predicting results. An ecosystem consists of

- biotic components which are the living organisms.
- abiotic components which are the non-living factors like light, water, temperature, nutrients, topography, etc.

Ecosystems are entities made of the biological community and the abiotic environment. The state of the evironmental factors which are interrelated with each other is determined so as to know the structure and composition of ecosystem. The composition can be both biotic and abiotic. Changes in these components (for example: supplement availability, temperature, light constrain, touching force, and species masses thickness) will realize exceptional changes to the method for these structures. For example, a fire in the quiet deciduous timberland thoroughly changes the structure of that system. There are no longer any considerable trees, by far most of the herbs,greeneries and shrub beries that have the timberland floor are gone, and the supplements that were secured in the biomass are promptly released into the soil, environment and hydrologic structure. After a short time of recovery, the gathering that was once developed trees now transforms into a gathering of grasses, tree seedlings and herbaceous species.

1.2 Population and community

A population incorporates each one of the general population of a given creature bunches in a specific district or zone at a particular time. Its centrality is more than that of different individuals in light of the fact that not all individuals are indistinct. Populations contain variety inside themselves and between various populations. In fact, even major inherited characteristics, for instance, hair shading or size may fluctuate fairly from individual to individual. More basically, not all people from the masses are equal in their ability to survive and repeat.

Group alludes to every one of the population in a particular region or locale at a specific time. Its structure includes many sorts of collaborations among species. Some of these include the obtaining and utilization of sustenance, space, or other ecological assets. Others include supplement pushing through all individuals from the group and shared direction of populace sizes. In these cases, the organized cooperations of populations prompt circumstances in which people are tossed into last chance battles.

By and large, scientists trust that a group that has a high differing qualities is more intricate and stable than a group that has a low assorted qualities. This hypothesis is established on the perception that the sustenance networks of groups of high assorted qualities are more interconnected. More noteworthy interconnectivity makes these frameworks be stronger to aggravation. On the off chance that an animal types is evacuated, those species that depended on it for nourishment have the choice to change to numerous different species that involve a comparative part in that biological community. In a low differing qualities biological system, conceivable substitutes for sustenance might be non-existent.

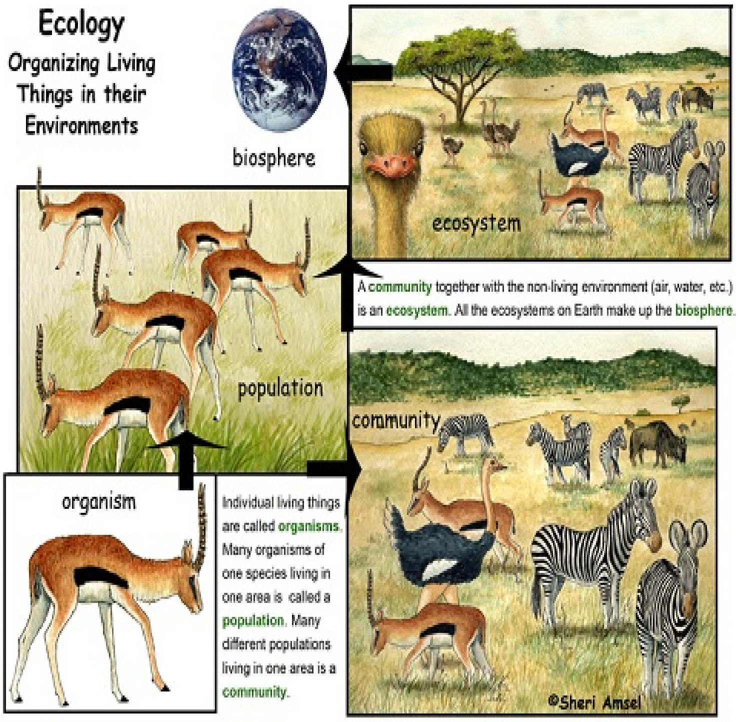

FIGURE 1.1: Ecological System

1.3 Population Dynamics

Population dynamics is the investigation of the adjustments in population size and structure after some time. Population progression is the branch of life science that reviews the size and age sythesis of population as elements frameworks, and the natural and ecological procedures driving them, (for example, birth and passing rates, and by migration and displacement), case situations are maturing populace development, or population decay.

1.4 Habitats

Variety in population thickness = (Birth+Immigration) − (Deaths + Emigration) living spaces have restricted measures of the assets required by living beings. Living beings must rival others with a specific end goal to get enough of these assets to survive. In the event that they are unsuccessful and can't move to another environment.

$$O_{t=1} = O_t + B + I - D - E$$

Where:

- O_t = the number of organisms currently
- O_{t+1} the number of organisms per year per generation in the next time step
- B = the number of births
- D = the number of deaths
- I = the number of immigrants
- E = the number of emigrants

1.5 Interaction between populations

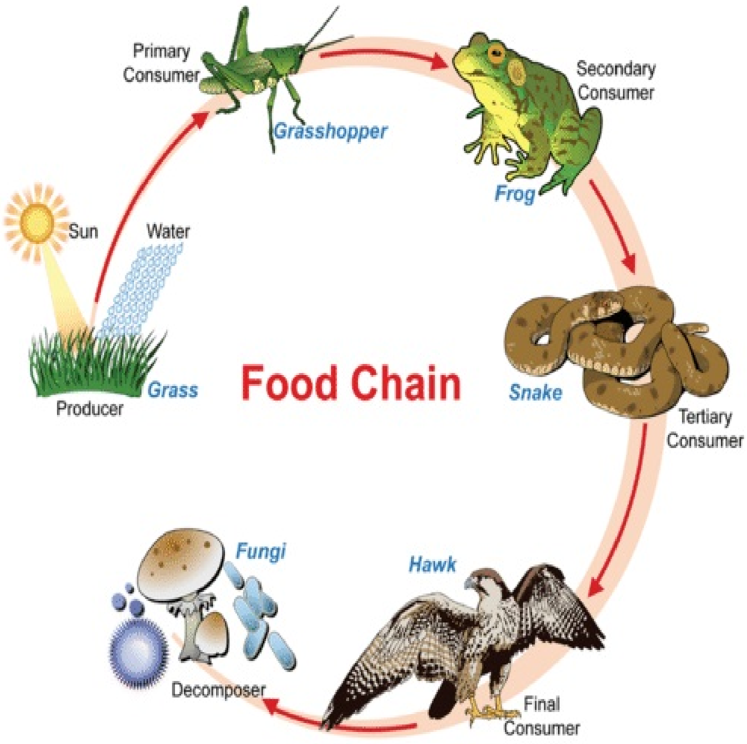

FIGURE 1.2: Interaction between populations

1.5.1 Predation

An organism which eats another living being for their sustenance is called predator while the living being that is being eaten upon is named as the prey. This sort of association between the prey and predator is known as predation. Ordinarily a predator has a tendency to be bigger than that of the prey, and thus they expend many preys amid their life cycle. Amid the demonstration of predation frequently the passing of prey will happen because of the ingestion of the prey's tissue by the predator. Common cases of predation are bats eating the bugs, snakes eating mice, and the whales eating the krill.

1.5.2 Camouflage

This is a phenomenon of a organism with the foundation shade of its living space is a typical strategy for keeping away from discovery by their predators in the biological system. An illustration is grasshoppers which can mix splendidly with the materials on which they sustain. The veins of leaves are frequently mirrored on the grasshopper's wings.

1.5.3 Mimicry

Other than camouflage, a few organisms mirror different creatures, for instance, some hoverflies emulate wasps in the biological community. The similitudes between one animal varieties to another secure one or both the animal varieties required in the mimicking procedure. The likenesses can be either in their appearance, practices, sound, development or area and which helps them shield themselves from the predators.

1.5.4 Commensalism

Commensalism is an interspecific connection between two life forms in the ecosystem where one animal groups benefits while alternate species stays unaffected. In this affiliation, for the most part a commensal can acquire supplements from the host species for their living place, development, and movement. The host stays unaffected. The host is bigger and unmodified, while the commensal is littler with some changed basic adjustments with its natural surroundings.

1.5.5 Mutualism

Not at all like commensalism, mutualism is interspecific connection between two living beings in the biological system with advantage to both the partner individuals in interaction with each other. Amid this connection, populations of each associating species develop, survive and reproduce at a larger rate within the sight of the other interfacing species. Fertilization is a decent case to clarify mutualism, where the plant gets advantage from the dispersal of dust the pollinator acquiring a food of nectar from the bloom.

1.5.6 Parasitism

A parasite bolsters on the host, yet they for the most part don't devastate it. Parasites are normally littler then the host. Parasites may have more than one host amid its life cycle. The host advanced some barrier components against the parasites; the most essential is the invulnerable reactions, for example, cell protections. Likewise parasites can generously diminish the host population sizes. The connection between the parasites and the hosts is known as Parasitism. Tapeworms and parasitic bloodsuckers are commonplace cases of parasites.

1.6 A brief review of the work already done

Petchey et al. (2010) [1] explained the effect of temperature increase on properties of food-web. It has been showed [2, 3] the effect of climatic warming on the living beings metabolic rate. In both these papers, the metabolic hypothesis of nature(Brown et al. (2004) [4] is considered to predict the changes in the rates of processes with increasing temperature.

Chaturvedi et al.(2013) [5] explained how an equilbrium state which was originally unstable becomes stable with the process of diffusion and process of advection. Depending on these results, the derivations of prey and predator populations are being done.

Srinivasan et al.(2016) [6] had proposed a method by which some unknown parameters are used to solve the problem of handling chaos of three species models of prey predator. Also the study of this three dimensional prey predator model is being proposed.

Kalra et al.(2012) [7] suggested that the ozone harming substance, for example, Chlorofluoro Carbon (CFC) is capable specifically or in a roundabout way for the expansion in the normal worldwide Earth's temperature. The existence of CFC is the reason of the exhaustion of ozone in the surroundings bringing about increment in the environmental temperature. The expansion in the level of temperature specifically or in a roundabout way influences the dynamics of systems of interacting species.

Ray et al. (2016) [8]in light of hypothetical investigations of Lotka and Volterra, Gause and his associates supplant the already utilized straight practical reaction by utilizing an immersing useful reaction with a brokenness at an edge prey thickness. In the present review, we reclassify and break down the model by utilizing Filippov regularization technique. By this continuation technique, the framework turns out to be very much acted and gives more outcomes like anticipated by Gause. Likewise predator completely relies on option eating routine to get by from eradication hazard when prey is in refuge patch and framework generally fluctuates with the accessibility of option eating routine asset however in the later case predator again changes to its essential (fundamental) sustenance. At the point when prey is in the region of the threshold density, then predator may pick its deit specially from basic or option assets as indicated by its benefit.

O.P.Mishra and Preety Kalra, et al (2012) [9]proposed the mathematical modeling of impact of expanding temperature because of the consumption of ozone layer brought about by CFC on the behaviour of two populations competing with each other. It is mathematically survey that the ozone depleting substance, for example, chlorofluoro carbon (CFC) is in charge of the expansion in the normal worldwide temperature. The CFC is in charge of the consumption of ozone layer in the earth's atmosphere because of which the warmth in the sun rays is less retained bringing about increment in the air temperature of the earth.

1.7 Objective of the Proposed Work

In this proposed work, the standard target is to learn about the connection between the creatures in the ecosystem. In particular, we would focus on communication of prey-predator where predator eats its prey. It is all around seen that the ozone hurting substance, for instance, Chlorofluoro Carbon (CFC) leads to the development in the overall temperature of the Earth. The CFC is accountable for the decrease of ozone density in the earth's atmosphere as a result of which the warmth in sun rays is less absorbed achieving increase in the temperature of atmosphere around the earth. The development in the temperature level clearly or by suggestion impacts the elements of interacting species systems. Thus, in this work a numerical model has been proposed and used the stability theory to study the effects of growing temperature due to the ozone draining substance CFC on the survival or disposal of populations in a prey-predator framework. A threshold value will be obtained which chooses the eradication or survival of populations in prey-predator system. objectives of the project are as follows:

1. To study about the relationship between the organisms in the ecosystem. Specially prey-predator where predator depends on its prey. For example, polar bear is a predator and fish is a prey.
2. To observe the basic concepts of prey-predator dynamics and to study the variation in population pattern.
3. To find the solution of model by stability theory by using different methods. To find the threshold level of growth parameters under the effect of rising temperature.

Chapter 2

Mathematical Preleminaries

2.1 Predator Functional Responses Holling type

Figure delineates the three general sorts of bends expected in different predator-prey circumstances.

Type I is a linear relationship in which the predator eats the prey as much as the abundance of prey in environment. If the predator eats 10% of the prey at low density, the predator will eat 10% of prey at high densities. The dotted line illustrates a greatest utilization rate that a few authors join to Type I scavenging.

Type II portrays a circumstance in which the quantity of prey devoured per predator at first ascents rapidly as the thickness of prey increments however then levels off with further increment in prey thickness.

Type III is similar as Type II in having a maximum farthest point to prey utilization, however varies in that the reaction of predators to prey is discouraged at low prey thickness.

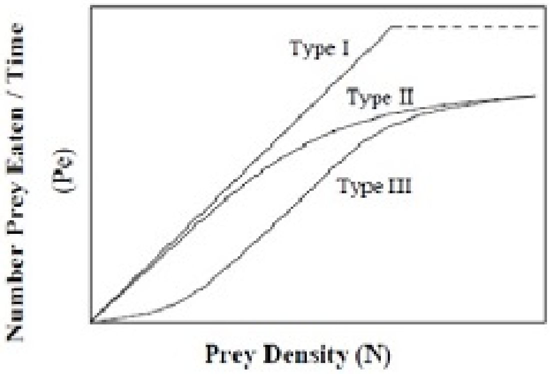

FIGURE 2.1: Three types of functional response connecting prey density (X) and the number of prey eaten by one predator (p_e).

2.2 Methodology for proposed work

2.2.1 Autonomous and non-autonomous system

Let $x(t)$ be vector valued function defined by

$$x(t) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = col(x_1(t), x_2(t) \dots x_n(t))$$

and f be vector valued function given by

$$f(t,x) = \begin{bmatrix} f_1(t, x_1, x_2, \dots x_n) \\ f_2(t, x_1, x_2, \dots x_n) \\ \vdots \\ f_n(t, x_1, x_2, \dots x_n) \end{bmatrix} = col(f_1(t,x), f_2(t,x) \dots f_n(t,x)).$$

Where $f_1, f_2 \dots f_n$ are n given functions in some domain B of $n+1$ dimensional euclidean space $R^{n+1}$and $x_1, x_2 \dots x_n$ are n unknown functions.
Then the system

$$\frac{dx}{dt} = f(x,t) \tag{2.1}$$

with initial condition $x(t_0) = x_0$ is a non-autonomous system.
A differential system of the form

$$\frac{dx}{dt} = f(x) \tag{2.2}$$

with initial condition $x(t_0) = x_0$ in which right hand does not involve independent variable t, is said to be autonomous system.

2.2.2 Solution of system of differential equations

A set of n-function $\phi_1, \phi_2 \ldots \phi_n$ define on $I = \{t : t \in R, r_1 < t < r_2\}$, where r_1 and r_2 are any two fixed points in set of all real number R, is said to be solution of (2.1) on I if for $t \in I$

1. $\phi'_1, \phi'_2 \ldots \phi'_n$ exits.
2. The point $(t, \phi_1(t), \phi_2(t) \ldots \phi_n(t))$ remain in B.
3. $\phi'_i = f_i(t, \phi_1(t), \phi_2(t) \ldots \phi_n(t))$, $i = 1, 2...n$.

2.2.3 Periodic linear system

Consider a linear homogeneous system

$$\frac{dx}{dt} = A(t)x \tag{2.3}$$

where $A(t)$ is an $n \times n$ continuous matrix on the interval $-\infty < t < \infty$ and $A(t+\omega) = A(t)$ for some constant $\omega \neq 0$, then (2.3)is called a periodic system and ω is period of A.

2.2.4 Equilibrium point

Consider a system

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots x_n) \tag{2.4}$$

A point$x^* = (x_1^*, x_2^*, \ldots x_n^*)$, is called a equilibrium point of (2.4 if

1. $x^* > 0$,
2. $f_i(x_1^*, x_2^*, \ldots x_n^*) = 0$ hold for all $i = 1, 2, \ldots n$.

2.2.5 Community matrix or Variational matrix

linearize system (2.4) about the equilibrium x_i^* to obtain.

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij}x_j, \qquad i = 1, 2, \dots n. \tag{2.5}$$

where

$$a_{ij} = (\frac{\partial f_i}{\partial x_i})(x^*), \qquad i, j = 1, 2, \dots n. \tag{2.6}$$

the matrix $A = (a_{ij})_{n\times n}$ is called the community matrix of the linearize system (2.5).

Definition 2.1. The solution $x(t)$ of (2.1) is stable if, for each $\epsilon > 0$ there exists a $\delta = \delta(\epsilon) > 0$ such that, for any solution $\bar{x}(t) = x(t, t_0, \bar{x_0})$ of (2.1), the inequality $\|\bar{x_0} - x_0\| \leq \delta$ implies $\|\bar{x}(t) - x(t)\| < \epsilon$ for all $t \geq t_0$.

Definition 2.2. The solution $x(t)$ of (2.1) is asymptotically stable if it is stable and if there exists a $\delta_0 > 0$ such that $\|\bar{x_0} - x_0\| \leq \delta_0$ implies $\|\bar{x}(t) - x(t)\| \to 0$ as $t \to \infty$.

Definition 2.3. The solution $x(t)$ of (2.1) is said to be unstable if it is not stable.

Definition 2.4. Let $\phi(t)$ be a fundamental matrix of (2.3) with $\phi(t_0) = I$. Then (2.3) is

1. stable if and only if there exist a positive constant M such that
   $\|\phi(t)\| \leq M$ for $t \geq t_0$.
2. Asymptotically stable if and only if
   $\|\phi(t)\| \to 0$ as $t \to \infty$.

Theorem 2.5. *If all the characteristic roots of A have negative real parts, then every solution of*

$$\frac{dx}{dt} = Ax \tag{2.7}$$

where, $A = [a_{ij}]$ is a constant matrix, is asymptotically stable.

Theorem 2.6. *If characteristic roots of A having multiplicity greater than one and with negative real parts, then all its roots with multiplicity one will have non positive real parts, then all the solution of (2.7) are bounded and hence stable.*

2.2.6 Hurwitzs Theorem

A necessary and sufficient condition for the negativity of real parts of all the roots of polynomial $P(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots + a_n$, with real coefficients is the positivity of all the principle diagonals of the minors of the Hurwitz matrix

$$H_n = \begin{bmatrix} a_1 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 \\ a_3 & a_2 & a_1 & 1 & 0 & 0 & \ldots & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & a_n \end{bmatrix}$$

Principal diagonals of the Hurwitzs theorem H_n, for $n = 1, 2, 3\ldots$ are given by

$$|a_1|, \begin{vmatrix} a_1 & 1 \\ a_3 & a_2 \end{vmatrix}, \ldots |H_n|$$

In the case of second, third and forth degree polynomials, the Hurwitz conditions can be written as follow:

1. For $P(\lambda) = \lambda^2 + a_1\lambda + a_2$, the hurwitz conditions are $a_1 > 0, a_2 > 0$.
2. For $P(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$, the hurwitz conditions are $a_1 > 0, a_2 > 0, a_3 > 0$ and $a_1a_2 - a_3 > 0$.
3. For $P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$, the hurwitz conditions are $a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0$, and $a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0$.

2.2.7 Stability by Liapunov'second method

Let

$$\frac{dx}{dt} = f(x) \tag{2.8}$$

where $f \in C[R^n, R^n]$ is an autonomous differential equation system. Let $f(0) = 0$ and $f(x) \neq 0$ for $x \neq 0$ in some neighborhood of origin so that (2.8) has the zero solution and the origin is isolated critical point of (2.8). Let Ω be an open set in R^n containing the origin, let $V(x)$ is a scalar continuous function which is defined on Ω.

Definition 2.7. A scalar function $V(x)$ is said to be positive definite on the set Ω if and only if $V(0) = 0$ and $V(x) > 0$ for $x \neq 0$ and $x \in \Omega$.

Definition 2.8. A scalar function $V(x)$ is negative definite on the set Ω if and only if $-V(x)$ is positive definite on the set Ω.
Let $S_\rho = x \in R^n : \|x\| < \rho$ and $J[t_0, \infty), t_0 \geq 0$. suppose $x(t) = x(t, t_0, x_0)$ is any solution of (2.8) with initial value $x(t_0) = x_0$, such that $\|x\| < \rho$ for $t \in J$.
Let

$$V^*(x) = gradV(x).f(x) = \frac{d}{dt}V(x(t))$$

Now following theorems describe the stability of zero solution of system (**??**).

Theorem 2.9. *If there exist a positive definite scalar function $V(x)$ such that $V^*(x) \leq 0$ on S_ρ then the zero solution of (2.8) is stable.*

Theorem 2.10. *If there exist a positive definite scalar function $V(x)$ such that $V^*(x)$ is negative definite on S_ρ, then the zero solution of (2.8) is asymptotically stable.*

2.2.8 Theorem on periodic solution and its stability

Theorem:(1)
Consider a non autonomous system

$$\frac{dx}{dt} = f(t, x, \mu) \tag{2.9}$$

If f is real and continuous function in (t, x, μ) when (t, x) is in some domain V of (t, x) space containing the curve $(t, p(t))$ and when $|\mu|$ is small. f has first-order partial derivative with respect to the component x_i of x which are continuous in (t, x, μ). If first variation of (2.9) for $\mu = 0$ with respect to the solition $p(t)$ has no solution of period T, then for small $|\mu|$ the equation (2.9) has a solution $q = q(t, \mu)$, periodic in t for period T, continuous in (t, μ) and with $q(t, 0) = p(t)$. there is only one such solution for each μ.
Theorem:(2)
the first variation is the linear system with periodic coefficients

$$\frac{dy}{dt} = \sum_{j=1}^{n} \frac{\partial f}{\partial x_i}(t, p(t), 0)y_i = f_x(t, p(t), 0)y \tag{2.10}$$

where the matrix $f_x(t, p(t), 0)$ has the period T. if the real part of the characteristic exponents of (2.10), the first variation of (2.9) for $\mu = 0$ with respect to p are all negative, then (2.10) can have no periodic solution, so that the conclusion of $theorem(1)$ is valid. Moreover, in this case the periodic solution $q = q(t, \mu)$ of (2.9) is asymptotically stable providing $|\mu|$ is small.

2.2.9 Floquet's theorem

If ϕ is a fundamental matrix 0f (3), then so is ψ, where

$$\psi(t)\phi(t+\omega), -\infty < t < \infty$$

, corresponding to every such ϕ, there exist a periodic nonsingular matrix P with the period ω and a constant R such that

$$\phi(t)P(t)e^{tR}$$

2.2.10 Sylvester criterion

Let

$$V(x) = x^T Bx = \sum_{i,j=1}^{n} b_{ij}x_i x_j$$

be a quadratic form with the system matrix $B = [b_{ij}]$, that is $b_{ij} = b_{ji}$. Necessary and sufficient condition for $V(x)$ in (11) to be positive definite is that the determinants of all the successive principle minors of the symmetric matrix $B = [b_{ij}]$ be positive that is,

$$b_{11} > 0, \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} > 0, \ldots \begin{vmatrix} b_{11} & b_{12} & \ldots & b_{1n} \\ b_{21} & b_{22} & \ldots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \ldots & b_{nn} \end{vmatrix} > 0$$

.

Chapter 3

Effect of Rising Temperature on Prey Predator Population:A Mathematical Model

3.1 Mathematical Model

Let X denotes logistically growing prey population density and Y is the predator population density. C is concentration of CFC. Z denotes concentration of ozone.T is normal expanded temperature of the environment where the species live.
let the searching capacity of predator per unit prey be δ_1. Consequently, searching capacity of the predator population for prey density X is $\delta_1 X$. The predator's handling capacity per unit prey is δ_2, then the handling capacity with respect to prey density X is $\delta_2 X$.
It is accepted in the model development that the searching capacity is influenced by the expanding temperature of nature and in this manner δ_1 is taken to be $\frac{\delta_1}{1+\mu(T-T_0)}$.
So also, it is additionally accepted in the model detailing that the handling capacity is likewise unfavorably influenced by the expanding temperature of nature and along these lines, δ_2 is thought to be equivalent to $\frac{\delta_2}{1+\mu(T-T_0)}$. Thus, the aggregate searching and handling capacity of a predator for prey density X is given by

$$\frac{\delta_1 X}{1+\mu(T-T_0)} + \frac{\delta_2 X}{1+\mu(T-T_0)} = \frac{(\delta_1+\delta_2)X}{1+\mu(T-T_0)} = \frac{\eta_1 X}{1+\mu(T-T_0)}, \tag{3.1}$$

where, $\eta_1 = \delta_1 + \delta_2$ From the expression (3.1), we take note of that when the environment is at the ordinary temperature; T_0, that is, at $T = T_0$, the predator acts normally and there is no adjustment in their handling and searching capacity. We additionally see from (3.1) that the predation rate may be influenced when temperature T surpasses T_0 . With the above assumptions and notations, the mathematical model of the system under thought is given by the arrangement of nonlinear differential conditions.

$$\frac{dX}{dt} = r_1(T)X - \frac{\eta_1 XY}{1 + \mu(T - T_0)} - \frac{r_{10}X^2}{K_{10}}, \tag{3.2}$$

$$\frac{dY}{dt} = -r_{20}Y + \frac{\eta_2 XY}{1 + \mu(T - T_0)}, \tag{3.3}$$

$$\frac{dC}{dt} = P - \frac{C}{\tau} - \beta CZ, \tag{3.4}$$

$$\frac{dZ}{dt} = Q_0 - \alpha_2 Z - \beta ZC, \tag{3.5}$$

$$\frac{dT}{dt} = \frac{K_1}{K_2 + Z} - \alpha_1(T - T_0), \tag{3.6}$$

with the initial condition as:
$X(0) > 0, Y(0) > 0, C(0) > 0, Z(0) > 0, T(0) > 0.$
In the present analysis we assume the following form of $r_1(T)$:

$$r_1(T) = \frac{r_{10}}{1 + r_{11}(T - T_0)}, r_1(T) > 0 \forall T, r_1(T_0) = r_{10} \tag{3.7}$$

Also, $\eta_2 = \gamma\eta_1$ where γ is transformation coefficient. It might be noted here that at the ordinary temperature T_0, the development rate of prey populace is r_{10} which is its natural development rate. The system parameters are characterized as under: $r_{20}$is natural demise rate of predator population. μ is a constant which measures the effect of temperature on both handling and searching capacity. P is inpute rate of C. τ is normal environmental residence time of CFC. $\beta$is the exhaustion rate of ozone because of CFC. Q_0 is the characteristic development rate of ozone density in the environment. T_0 is normal ordinary temperature of earth surface of the zone involved by the populations under review. $r_1(T)$ is development rate of population X. K_{10} is carrying capacity of prey population X. α_1 is coefficient of surface warmth exchange and α_2 is regular consumption rate of ozone concentration. Here, each of the parameters $K_1, K_2, r_{10}, K_{10}, r_{11}, \mu, \eta_1, \eta_2, Q_0, \beta, \alpha_1, \alpha_2$ and r_{20} are all taken to be positive constants.

3.2 Boundedness and Dynamical Behaviour

Here, we prove that the solution of the model given by the set of equation (3.2) to (3.6) with equation (3.7) are bounded in R^5_+. The boundedness of solution is given by the following lemma.

3.2.1 Lemma

Every solution of the model will be in the region

$$V_1 = \{(X, Y, C, Z, T) \in R_+^5 : 0 < X \leq K_{10}, 0 < Y \leq \frac{r_{10}K_{10}}{\eta}$$

$$= X_M, 0 < C \leq C_M, 0 < Z_m \leq Z \leq Z_M, 0 < T_m \leq T \leq T_M\},$$

as $t \to \infty$, for all positive initial values $(X(0), Y(0), C(0), Z(0), T(0)) \in R_+^5$, where,

$$C_M = P\tau, T_M = \frac{K_1}{\alpha_1(K_2 + Z_m)} + T_0, T_m = \frac{K_1}{\alpha_1(K_2 + Z_M)} + T_0, Z_M = \frac{Q_0}{\alpha_0} and Z_m = \frac{Q_0}{\alpha_2 + \beta P\tau}.$$

Proof:

From equation (3.2) and (3.3) we get,
$\frac{d(X+Y)}{dt} \leq \frac{r_{10}X}{1+r_{11}(T^*-T_0)} - r_{20}Y - \frac{r_{10}X^2}{K_{10}}$
$\leq r_{10}K_{10} - r_{20}Y - \frac{r_{10}K_{10}X}{K_{10}}$
$\leq r_{10}K_{10} - (r_{20}, r_{10})(X + Y)$
$\leq r_{10}K_{10} - \xi(X + Y)$
where,$\xi = (r_{20}, r_{10})$
Using the usual comparison theorem, we get as $t \to \infty$:
$(X + Y) \leq \frac{r_{10}K_{10}}{\xi}$.
From Equation (3.6) we get,
$\frac{dT}{dt} \leq A - \alpha_1 T$,
where
$A = \frac{K_1}{K_2+Z_m} + \alpha_1 T_0$.
Again by the usual comparison theorem, we get as $t \to \infty$:
$T \leq \frac{A}{\alpha_1}$,
i.e.,
$T \leq \frac{K_1}{\alpha_1(K_2+Z_m)} + T_0 = T_M$.
Again from Equation (3.6) we get,
$\frac{dT}{dt} \geq \frac{K_1}{(K_2+Z_m)} + \alpha_1 T_0 - \alpha_1 T$
By the usual comparison theorem, we get as $t \to \infty$:
$T \geq \frac{K_1}{\alpha_1(K_2+Z_m)} + T_0 = T_m$
Similarly from Equation. (3.4) and (3.5), we get as $t \to \infty$:
$C \leq P\tau = C_m, Z \leq \frac{Q_0}{\alpha_2} = Z_M$
again from equation (3.5), we get
$\frac{dZ}{dt} \geq Q_0 - \alpha_1 Z - \beta Z P\tau$
Then by the usual comparison theorem, we get as $t \to \infty$:
$Z \geq \frac{Q_0}{\alpha_2+\beta P\tau} = Z_m$.
This completes the proof of the Lemma (3.2.1)

All the feasible equilibria of the system (3.2)-(3.6) the system of equation (3.2)-(3.6) has three feasible equilibria $E_i (i = 1, 2, 3)$ as given below:

1. $E_1(X^*, Y^*, C^*, Z^*T^*)$, where, $X^* = 0, Y^* = 0$,

$$C^* = \frac{P\tau}{1 + \beta\tau Z^*}, \tag{3.8}$$

$$Z^{(*)} = \frac{-b_2 + \sqrt{b_2^2 - 4b_1 b_3}}{2b}, \tag{3.9}$$

$b_1 = \alpha_2\beta\tau, b_2 = \alpha_2 + \beta\tau(P - Q_0), b_3 = -Q_0,$

$$T^* = \frac{1}{\alpha_1}(\frac{K_1}{K_2 + Z^*} + \alpha_1 T_0), \tag{3.10}$$

2. $E_2(X^{(*)}, Y^*, C^*, Z^*, T^*)$, where, $X^* = \frac{r_1(T^*)K_{10}}{r_{10}}$, $Y^* = 0$, and C^*, Z^*, T^* are given by (3.8)-(3.10), respectively. The equilibrium E_2 exists if $r_1(T^*) > 0$.

3. $E_3(X^{(*)}, Y^*, C^*, Z^*, T^*)$, where,$X^* = \frac{r_{20}}{\gamma\eta_1}[1 + \mu(T^* - T_0)]$, $Y^* = \frac{(1+\mu(T^*-T_0))}{\gamma\eta_1^2 K_{10}}[\frac{r_{10}}{1+r_{11}(T^*-T_0)}\gamma\eta_1 K_{10} - r_{10}r_{20}(1+\mu(T^*-T_0))]$ and C^*, Z^*, T^* are given by (3.8)-(3.10) respectively.
   The equilibrium E_3 exists if $T^* > T_0$ and $r_1(T^*)K_{10}\gamma\eta_1 > r_{10}r_{20}(1+\mu(T^*-T_0))$ or

$$\frac{K_{10}}{r_{20}} > \frac{(1 + r_{11}(T^* - T_0))(1 + \mu(T^* - T_0))}{\gamma\eta_1}. \tag{3.11}$$

3.3 Local Stability

The local stability of equilibrium E_1 is observed by the nature of root of following characteristic equation

$$(F_1 - \lambda)(F_2 - \lambda)(F_7 - \lambda)\{(F_3 - \lambda)(F_6 - \lambda) - F_4 F_5\} = 0, \tag{3.12}$$

where,

$$F_1 = r_1(T^*); F_2 = -r_{20}; F_3 = -\frac{1}{\tau} - \beta Z^*; F_4 = -\beta C^*; F_5 = -\beta Z^*; F_6 = -\alpha_2 - \beta C^*; F_7 = -\alpha_1.$$

The equilibrium point E_1 is locally unstable because of $r_1(T^*) > 0$.

3.3.1 Remark

If $r_1(T^*) < 0$, then E_1 is locally asymptotically stable and hence predator and prey populations would die out in the long run.
The characteristic equation associated with the equilibrium point E_2 is obtained as

$$(G_1 - \lambda)(G_2 - \lambda)(G_7 - \lambda)\{(G_3 - \lambda)(G_6 - \lambda) - G_4 G_5\} = 0, \tag{3.13}$$

where,
$G_1 = -r_1(T^*); G_2 = -r_{20} + \frac{\eta_2 X^*}{1+\mu(T^*-T_0)}; G_3 = -\frac{1}{\tau} - \beta Z^*; G_4 = -\beta C^*;$
$G_5 = -\beta Z^*; G_6 = -(\alpha_2 + \beta C^*); G_7 = \alpha_1.$
The condition for the linear asymptotical stability of equilbrium point given by E_2 is as under:

$$\frac{K_{10}}{r_{20}} < \frac{(1 + \mu(T^* - T_0))(1 + r_{11}(T^* - T_0))}{\gamma\eta_1}, T^* > T_0 \tag{3.14}$$

The latent equation connected with equilibrium E_3 is as under

$$(\alpha_1 + \lambda)\{(H_1 - \lambda)(H_4 - \lambda) - H_2 H_3\}\{H_6 H_7 - (H_5 - \lambda)(H_8 - \lambda)\} \tag{3.15}$$

where,
$H_1 = -\frac{r_{10} r_{20}}{\eta_1 \gamma K_{10}}(1 + \mu(T^* - T_0)); H_2 = \frac{\eta_1 X^*}{1+\mu(T^*-T_0)}; H_3 = \frac{\eta_1 \gamma Y^*}{1+\mu(T^*-T_0)};$
$H_4 = -r_{20} + \frac{\eta_1 \gamma X^*}{1+\mu(T^*-T_0)}; H_5 = -\frac{1}{\tau} - \beta Z^*; H_6 = -\beta C^*; H_7 = -\beta Z^*; H_8 = -(\alpha_2 + \beta C^*).$
The condition for the linear asymptotical stability of the equilbrium point E_3 is $T^* > T_0$.
E_2 is linearly stable if E_3 does not exist. The unstability of E_2 leads to linear stability of E_3.
The stability conditions are dependent on the temperature level of equilbrium and also average temperature.

3.4 Global Stability

The global stability of equilibrium point E_3. is discussed below

3.4.1 Theorem

The equilibrium E_3 is non-linearly asymptotically stable in comparison to solution in the interior of V_1 for if the following inequalities are true:

$$\frac{1}{2}\left[\frac{r_{10}}{K_{10}}\right]\left[r_{20}+\frac{\gamma\eta_1 X}{1+\mu(T^*-T_0)}(\frac{\mu t_1}{1+\mu(T^*-T_0)}-1)\right] \geq \left[\frac{\eta_1}{1+\mu(T^*-T_0)}+\frac{\gamma\eta_1 Y^*}{1+\mu(T^*-T_0)}(\frac{\mu t_1}{1+\mu(T^*-T_0)}-1)\right]^2, \tag{3.16}$$

$$\frac{1}{2}\alpha_1\frac{r_{10}}{K_{10}} \geq \left[\frac{r_{10}r_{11}}{(1+\mu(T^*-T_0))^2}-\frac{\eta_1 Y^*\mu}{(1+\mu(T^*-T_0))^2}\right]^2 \tag{3.17}$$

and

$$\frac{1}{2}r_{10}\left[\frac{\gamma\eta_1 X}{1+\mu(T^*-T_0)}(\frac{\mu t_1}{1+\mu(T^*-T_0)}-1)+\frac{\alpha_1}{2}\right] \geq \left[\frac{\gamma\eta_1 X^*Y^*\mu}{(1+\mu(T^*-T_0))^2}\right]^2. \tag{3.18}$$

$$[\alpha_2+\beta C^*]\left[\frac{1}{\tau}+\beta Z_m\right] \geq [\beta Z_m]^2 \tag{3.19}$$

proof: The perturbations are taken at equilibrium value as under:
$X = X^*+u_1(t), Y = Y^*+u_2(t), C = C^*+v_1(t), Z = Z^*+x(t), T = T^*+t_1(t)$.
The non-linearised equations from (3.2) to (3.6) at equilibrium point E_3 is shown below

$$\frac{du_1}{dt} = (X^*+u_1)[\frac{-r_{10}r_{11}t_1}{(1+r_{11}(T^*-T_0))^2}+\frac{\eta_1 Y^*\mu t_1}{(1+\mu(T^*-T_0))^2}-\frac{r_{10}u_1}{K_{10}}-\frac{\eta_1 u_2}{1+\mu(T^*-T_0)}] \tag{3.20}$$

$$\frac{du_2}{dt} = [-r_{20}u_2-\frac{\gamma\eta_1 X^*Y^*\mu t_1}{(1+\mu(T^*-T_0))^2}+\frac{\gamma\eta_1(Xu_2+Y^*u_1)}{1+\mu(T^*-T_0)}-\frac{\gamma\eta_1(Xu_2+Y^*u_1)\mu t_1}{(1+\mu(T^*-T_0))^2}] \tag{3.21}$$

$$\frac{dv_1}{dt} = -\frac{v_1}{\tau}-\beta C^*x-\beta(Z^*+x)v_1 \tag{3.22}$$

$$\frac{dx}{dt} = -\alpha_2 x-\beta C^*x-\beta(Z^*+x)v_1 \tag{3.23}$$

$$\frac{dt_1}{dt} = \frac{-K_1 x}{(K_2+Z^*)^2}-\alpha_1 t_1 \tag{3.24}$$

Consider,
$G(t) = [u_1 - X^*\log(1+\frac{u_1}{X^*})]+\frac{1}{2}u_2^2+\frac{1}{2}v_1^2+\frac{1}{2}x^2+\frac{1}{2}t_1^2$,
where,$A_i(i = 1to4)$ are arbitrary positive constants.
The derivative of $G(t)$ with respect to time is as under
$\frac{dG}{dt} = \frac{u_1}{X^*+u_1}\frac{du_1}{dt}+u_2\frac{du_2}{dt}+v_1\frac{dv_1}{dt}+x\frac{dx}{dt}+t_1\frac{dt_1}{dt}$
now from the system of equations (3.20)to (3.24) in $\frac{dG}{dt}$ in the region V_1, we get
$\frac{dG}{dt} \leq -\{t_1u_1(\frac{r_{10}r_{11}}{(1+r_{11}(T^*-T_0))^2}-\frac{\eta_1 Y^*\mu}{(1+\mu(T^*-T_0))^2})+\frac{r_{10}u_1^2}{K_{10}}+u_1u_2(\frac{\eta_1}{1+\mu(T^*-T_0)}-\frac{\gamma\eta_1 Y^*}{1+\mu(T^*-T_0)}+\frac{\gamma\eta_1\mu t_1 Y^*}{(1+\mu(T^*-T_0))^2})+\frac{u_2^2}{2}(r_{20}-\frac{\gamma\eta_1 X}{1+\mu(T^*-T_0)}+\frac{\gamma\eta_1 X\mu t_1}{(1+\mu(T^*-T_0))^2})+\frac{\gamma\eta_1 X^*y^*\mu}{(1+\mu(T^*-T_0))^2}u_2t_1+v_1^2(\frac{1}{\tau}+\beta(Z_m))+\beta v_1C^*x+x^2(\alpha_2+\beta C^*)+\beta(Z_m)v_1x+\frac{K_1xt_1}{(K_2+Z^*)^2}+\alpha_1t_1^2\}$.
From Sylvester's criteria in the above expression's right hand side :
$\frac{1}{2}\left[\frac{r_{10}}{K_{10}}\right]\left[r_{20}+\frac{\gamma\eta_1 X}{1+\mu(T^*-T_0)}(\frac{\mu t_1}{1+\mu(T^*-T_0)}-1)\right]$
$\geq \left[\frac{\eta_1}{1+\mu(T^*-T_0)}+\frac{\gamma\eta_1 Y^*}{1+\mu(T^*-T_0)}(\frac{\mu t_1}{1+\mu(T^*-T_0)}-1)\right]^2$,

$\frac{1}{2}\alpha_1\frac{r_{10}}{K_{10}} \geq \left[\frac{r_{10}r_{11}}{(1+\mu(T^*-T_0))^2} - \frac{\eta_1 Y^*\mu}{(1+\mu(T^*-T_0))^2}\right]^2$
$\frac{1}{2}r_{10}\left[\frac{\gamma\eta_1 X}{1+\mu(T^*-T_0)}(\frac{\mu t_1}{1+\mu(T^*-T_0)} - 1) + \frac{\alpha_1}{2}\right] \geq \left[\frac{\gamma\eta_1 X^* Y^*\mu}{(1+\mu(T^*-T_0))^2}\right]^2.$
$[\alpha_2 + \beta C^*]\left[\frac{1}{\tau} + \beta Z_m\right] \geq [\beta Z_m]^2$
It can be shown that $\frac{dG}{dt}$ becomes negative definite if the equations (3.16) to (3.19) are being satisfied. Thus, it is proved that E_3 is globally (non-linearly) asymptotically stable in the region V_1.

3.5 Conclusion

By the linear analysis of stability of the equilbrium point E_2, it is concluded that the population with density Y would become extinct and population with density Y would survive but at lower value of the equilibrium because of the abatement in its development rate by virtue of raised temperature. The non trivial positive equilibrium point E_3 exists just when the equilibrium point E_2 is not steady. Thus, from the linear and also non-direct stability analysis of the non-trivial positive equilibrium E_3 it is shown that the prey and predator populations would exist together.

Chapter 4

Effect of Rising Temperature on Competing Populations: A Mathematical model

4.1 Introduction

Recently, [10] have anticipated that the greenhouse gases will significantly change worldwide climate design in the following century and temperature of the Earth will ascend in the years to come. A constant warming pattern, driven to a great extent by anthropogenic generation of greenhouse gases, is anticipated to make the worldwide surface temperature ascend amongst 1.4^0 and 5.8^0C before the finish of the $21st$ century (contrasted and 0.6^0C in the $20th$ century, IPCC 2001) [11]. The expansion in temperature level may straightforwardly or in a roundabout way influence the progression of interacting species systems. In this way, it is fundamental to evaluate mathematically the impacts of expanding greenhouse gases incorporating CFC on populations keeping in mind the end goal to take important measures to maintain a strategic distance from any unfavorable effect of rising temperature on a biological system. In [12][13] models depending on temperature were considered for prey-predator system yet for various targets, where as, in this paper, a numerical model is being proposed and investigated to study the impacts of temperature variation from record of greenhouse gases, for example, CFC on the survival or eradication of two contending populations. In the model it is expected that the temperature increments indirectly because of the ozone depletion in the atmosphere. In the model it is likewise expected that the ascent in temperature contrarily impacts the development rates, carrying capacities and decidedly impacts the interspecific competition of the rates of two competing populations.

4.2 Mathematical Model

Let X and Y signify the densities of contending populations, which are growing under the impact of variation in temperature. C indicates the concentration of CFC (Clorofluoro carbon) and Z signifies concentration of ozone . We consider here that T is raised temperature over the characteristic or normal increased temperature of the living space. By the above notations, the numerical model of the system under thought is given by the accompanying arrangement of nonlinear differential conditions.

$$\frac{dX}{dt} = r_1(T)X - \frac{r_{10}X^2}{K_P(T)} - \eta_1(T)XY, \tag{4.1}$$

$$\frac{dY}{dt} = r_2(T)Y - \frac{r_{20}Y^2}{K_q(T)} - \eta_2 TXY, \tag{4.2}$$

$$\frac{dC}{dt} = P - \frac{C}{\tau} - \beta CZ, \tag{4.3}$$

$$\frac{dZ}{dt} = Q_0 - \alpha_2 Z - \beta ZC, \tag{4.4}$$

$$\frac{dT}{dt} = \frac{K_1}{K_2 + Z} - \alpha_1(T - T_0), \tag{4.5}$$

with the initial condition as:
$X(0) > 0, Y(0) > 0, C(0) > 0, Z(0) > 0, T(0) = \delta > 0.$
In the present analysis we assume the following form of $r_1(T), r_2(T), \eta_1(T), \eta(2)(T), K_P(T)$, and $K_Q(T)$:

$$r_1(T) = r_{10} - r_{11}T, \qquad r_1(T) > 0 \forall T, \qquad r_1(0) = r_{10} \tag{4.6}$$

$$r_2(T) = r_{20} - r_{22}T, \qquad r_2(T) > 0 \forall T, \qquad r_2(0) = r_{20} \tag{4.7}$$

$$\eta_1(T) = \eta_{10} - \eta_{11}T, \qquad \eta_1(T) > 0 \forall T, \qquad \eta_1(0) = \eta_{10} \tag{4.8}$$

$$\eta_2(T) = \eta_{20} - \eta_{22}T, \qquad \eta_2(T) > 0 \forall T, \qquad \eta_2(0) = \eta_{20} \tag{4.9}$$

$$K_P(T) = \frac{K_{10}}{1 + K_{11}(T)}, \qquad K_P(T) > 0 \forall T, \qquad K_P(0) = K_{10} \tag{4.10}$$

$$K_Q(T) = \frac{K_{20}}{1 + K_{22}(T)}, \qquad K_Q(T) > 0 \forall T, \qquad K_Q(0) = K_{20} \tag{4.11}$$

The system parameters are defined as follows: P is input rate of C. τ is average atmospheric residence time of CFC. β is depletion rate of ozone due to CFC. Q_0 is the rate of natural formation of ozone concentration in the atmosphere. T_0 is natural temperature of the habitat. $r_1(T)$ and $r_2(T)$ are growth rates of populations X and Y respectively. K_P and $K_Q(T)$ denote the carrying capacities of populations X and Y respectively. $\eta_1(T)$ and $\eta_2(T)$ are interspecific competition coefficients for X and Y respectively. α_1 is coefficient of surface heat transfer and α_2 is natural depletion rate of ozone concentration. Here all the parameters are taken to be real and positive. $K_1, K_2, r_{10}, r_{11}, r_{20}, r_{22}, \eta_{10}, \eta_{11}, \eta_{20}, \eta 22, K_{10}, K_{11}, K_{20}$ and K_{22} are all positive constants. r_{10} and r_{20} are natural growth rates of the competing populations.

4.3 Boundedness and Dynamical Behaviour

Now we will demonstrate that the solutions of model given by (4.1) to (4.5) are bounded in a positive orthant in R_+^5. The boundedness of solutions is being given by the following lemma.

4.3.1 Lemma

All the solutions of model will lie in the region $V_1 = \{(X, Y, C, Z, T) \in R_+^5 : 0 \leq X \leq K_{10}, 0 \leq Y \leq K_{20}, 0 \leq C \leq C_M, 0 < Z_M \leq Z \leq Z_M, 0 < T_m \leq T \leq T_M\}$, as $t \to \infty$, for all positive initial values $(X(0), Y(0), C(0), Z(0), T(0)) \in R_+^5$ where, $C_M = P\tau$, $T_M = \frac{K_1}{\alpha_1(K_2+Z_m)} + T_0, T_m = T_0, Z_M = \frac{Q_0}{\alpha_2}$ and $Z_m = \frac{Q_0}{\alpha_2+\beta P\tau}$.

Proof:

From (4.5) we get,

$$\frac{dT}{dt} \leq A - \alpha_1 T$$

where $A = \frac{K_1}{K_2+Z_m} + \alpha_0$.

Then by the usual comparison theorem we get as $t \to \infty$:

$$T \leq \frac{A}{\alpha_1}$$

i.e.

$$T \leq \frac{K_1}{\alpha_1(K_2 + Z_m)} + T_0 = T_M$$

Again from (4.5) we get,

$$\frac{dT}{dt} \geq \alpha_1 T_0 - \alpha_1 T$$

Then by the usual comparison theorem we get as $t \to \infty$:

$$T \geq T_0 = T_m$$

Similarly from (4.3) and (4.4), we get as $t \to \infty$:

$$C \leq P\tau = C_M, \qquad Z \leq \frac{Q_0}{\alpha_2} = Z_M$$

Again from (4.4) we get

$$\frac{dZ}{dt} \geq Q_0 - \alpha_2 Z - \beta Z P \tau$$

By the usual comparison theorem we get as $t \to \infty$:

$$Z \geq \frac{Q_0}{\alpha_2 + \beta P \tau} = Z_m$$

This completes the proof of the lemma.

4.3.2 Equilibrium Points

The system of (4.1)-(4.5) has four feasible equilibrium as follows:

1. $E_1(X^*, Y^*, C^*, Z^*, T^*)$: where, $X^* = 0, Y^* = 0,$

$$C^* = \frac{P\tau}{1 + \beta\tau Z^*}, \tag{4.12}$$

$$Z^{(}*) = \frac{-b_2 + \sqrt{b_2^2 - 4b_1b_3}}{2b}, \tag{4.13}$$

   $b_1 = \alpha_2\beta\tau, b_2 = \alpha_2 + \beta\tau(P - Q_0), b_3 = -Q_0,$

$$T^* = \frac{1}{\alpha_1}[\frac{K_1}{K_2 + Z^*} + \alpha_1 T_0], \tag{4.14}$$

2. $E_2(X^*, Y^*, C^*, Z^*, T^*)$: where,$X^* = \frac{r_1(T^*)K_P(T^*)}{r_{10}}$; $Y^* = 0$ and C^*, Z^*, T^* are given by (4.12)-(4.14) respectively. The equilibrium E_2 exists if $r_1(T^*) > 0$ and $K_P(T^*) > 0$.

3. $E_3(X^*, Y^*, C^*, Z^*, T^*)$: where, $X^* = 0,$ $Y^* = \frac{r_2(T^*)K_Q(T^*)}{r_{20}}$ and C^*, Z^*, T^* are given by (4.12)-(4.14) respectivel. The equilibrium E_3 exists if $r_2(T^*) > 0$ and $K_Q(T^*) >)$

4. $E_4(X^*, Y^*, C^*, Z^*, T^*)$: where,

$$X^* = \frac{K_P(T^*)[\eta_1(T^*)r_2(T^*)K_Q(T^*) - r_{20}r_1(T^*)]}{[\eta_1(T^*)\eta_2(T^*)K_P(T^*)K_Q(T^*) - r_{10}r_{20}]},$$

$$Y^* = \frac{K_Q(T^*)[\eta_2(T^*)r_1(T^*)K_P(T^*) - r_{10}r_2(T^*)]}{[\eta_1(T^*)\eta_2(T^*)K_P(T^*)K_Q(T^*) - r_{10}r_{20}]}.$$

Here C^*, Z^*, T^* are same as given in (4.12)-(4.14). The equilibrium E_4 exists if either

$$\frac{\eta_1(T^*)K_Q(T^*)}{r_{20}} > \frac{r_1(T^*)}{r_2(T^*)} > \frac{r_{10}}{\eta_2(T^*)}K_P(T^*) \tag{4.15}$$

$$\frac{\eta_1(T^*)K_Q(T^*)}{r_{20}} < \frac{r_1(T^*)}{r_2(T^*)} < \frac{r_{10}}{\eta_2(T^*)}K_P(T^*) \tag{4.16}$$

is satisfied.

4.4 Local Stability

The characteristic equation associated with the variational matrix about equilibrium E_1 is given by

$$(F_1 - \lambda)(F_2 - \lambda)(F_7 - \lambda)\{(F_3 - \lambda)(F_6 - \lambda) - F_4F_5\} = 0, \tag{4.17}$$

where,

$$F_1 = r_1(T^*); F_2 = r_2(T^*); F_3 = -\frac{1}{\tau} - \beta Z^*; F_4 = -\beta C^*; F_5 = -\beta Z^*; F_6 = -\alpha_2 - \beta C^*; F_7 = -\alpha_1.$$

Form the nature of the root of the characteristic equation (4.17) we observe that the equilibrium point E_1 is locally unstable provided $r_1(T^*) > 0$.
The characteristic equation related to the equilibrium point E_2 is obtained as

$$(G_1 - \lambda)(G_2 - \lambda)(G_7 - \lambda)\{(G_3 - \lambda)(G_6 - \lambda) - G_4G_5\} = 0, \tag{4.18}$$

where,
$G_1 = -r_1(T^*); G_2 = r_2(T^*) - \eta_2(T^*)X^*; G_3 = -\frac{1}{\tau} - \beta Z^*; G_4 = -\beta C^*;$
$G_5 = -\beta Z^*; G_6 = -(\alpha_2 + \beta C^*); G_7 = -\alpha_1.$
From the characteristic equation (3.13) we find that the equilibrium point E_2 is linearly asymptotically stable under the condition given by:

$$\frac{r_1(T^*)}{r_2(T^*)} > \frac{r_{10}}{\eta_2(T^*)K_P(T^*)} \tag{4.19}$$

The characteristic equation for the variational matrix about equilibrium E_3 is given by

$$(A_1 - \lambda)(A_2 - \lambda)(A_7 - \lambda)\{(A_6 - \lambda)(A_3 - \lambda) - A_4A_5\} = 0, \tag{4.20}$$

where, $A_1 = r_1(T^*) - \eta_1(T^*)Y^*; A_2 = -r_2(T^*); A_3 = -\frac{1}{\tau} - \beta Z^*; A_4 = -\beta C^*; A_5 = -\beta Z^*; A_6 = -(\alpha_2 + \beta C^*); A_7 = -\alpha_1.$
We find from the values of the root $A_i; i = 1to7$ that all the roots A_i are negative

$$if \frac{r_1(T^*)}{r_2(T^*)} < \frac{\eta_1(T^*)K_Q(T^*)}{r_{20}} \tag{4.21}$$

Hence, from the roots of the characteristic equation (4.20) we observe that the equilibrium point E_3 is linearly asymptotically stable under the condition given by (4.21).
The characteristic equation with the variational matrix about equilibrium point E_4 is

$$(\alpha_1 + \lambda)\{(H_1 - \lambda)(H_4 - \lambda) - H_2H_3\}\{H_6H_7 - (H_5 - \lambda)(H_8 - \lambda)\} \tag{4.22}$$

where,
$H_1 = -\frac{r_{10}X^*}{K_P(T^*)}; H_2 = -\eta_1(T^*)X^*; H_3 = -\eta_2(T^*)Y^*; H_4 = -\frac{r_{20}Y^*}{K_Q(T^*)};$
$H_5 = -\frac{1}{\tau} - \beta Z^*; H_6 = -\beta C^*; H_7 = -\beta Z^*; H_8 = -(\alpha_2 + \beta C^*).$
From the nature of the roots of characteristic equation (4.22) we find that the equilibrium point E_4 is linearly asymptotically stable under the following condition:

$$\frac{\eta_1(T^*)K_Q(T^*)}{r_{20}} < \frac{r_1(T^*)}{r_2(T^*)} < \frac{r_{10}}{\eta_2(T^*)K_P(T^*)} \tag{4.23}$$

Moreover, from the above investigation it is noticed that E_2 and E_3 are linearly asymptotically stable, just when E_4 is unstable and E_4 is linearly asymptotically stable just when E_2 and E_3 are unstable. It is seen from the investigation that the stability conditions for the four equilbriums points now rely on the temperature however are like the aftereffects of two species competing systems with no temperature effects alluded to as invalid.

4.5 Conclusion

From the stability investigation of the equilibrium point E_2 it might be said that the number of inhabitants in Y species would not tend to extinction and population of X species would survive and correspondingly, from the linear stability of the equilibrium point E_3 it might be inferred that the number of inhabitants in X species would not tend towards extinction and the number of inhabitants in Y species would survive yet at lower balance an incentive because of the decline in its development rate and carrying capacity by virtue of hoisted temperature, demonstrating the competition principle. The non-inconsequential positive equilibrium point E_4 would exist just when both the equilibrium point E_2 and E_3 are not stable. Thus, from the direct and in addition non-straight dependability of non-trivial positive equilibrium point E_4 it might be inferred that both the contending populations would co-exist.

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