

Performance Evaluation of Different Wavelet Transforms for Image De-noising

A dissertation submitted by

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To

Department of CSE

In partial fulfilment of the Requirement for the

Award of the Degree of

Master of Technology in CSE

Under the guidance of

Mr. Balraj Singh

(26th Dec, 2014)

DECLARATION

I hereby declare that the dissertation entitled, **Performance Evaluation of different Wavelets Transforms for Image De-noising** submitted for the M. Tech degree is entirely my original work and all ideas and references have been duly acknowledged. It does not contain any work for the award of any other degree or diploma.

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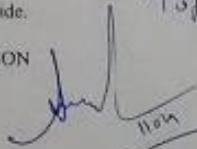

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This is to certify that Mohan Singh has completed his M. Tech. Dissertation PERFORMANCE EVALUTION OF DIFFERENT WAVELET TRANSFORM FOR IMAGE DENOISING under my guidance and supervision. To the best of my knowledge, the present work is the result of her original investigation and study. No part of the dissertation has ever been submitted for any other degree or diploma.

The dissertation is fit for the submission and the partial fulfilment of the conditions for the award of M. Tech. Computer Science & Engineering

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ABSTRACT

An image can be represented as a two-dimensional function of any pair of coordinates called the intensity or gray level of the image at that point. Images are often corrupted with noise during image acquisition and transmission. The noise degrades the quality of an image and loss the useful information details. The work in this thesis is focused on de-noising of digital image

Different techniques have been quoted in literature, which can be effectively used for image de-noising. But wavelet analysis is emerging as new method for solving difficult image processing. Wavelets are the powerful statistical tool that cut up data into different frequency components, and revise each component with a resolution coordinated to its scale. Wavelets have advantages above Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelet are functions differentiated from other transformations in that they not only cut up signals into their component frequencies, they also vary the scale at which the frequencies of components are analyzed. So, Wavelets as component pieces are used to analyze a signal are limited in space. The ability to vary the scale of the function as it addresses different frequencies also makes wavelets better suited to signals with spikes or discontinuities than traditional transformations such as the FT.

In literature different types of wavelets have been proposed. The present work is focused on the use of Haar, Bior, Symmetric, Coiflet and Daubechie wavelet transforms for image de-noising.

Thresholding is one of the efficient methods to reduce noise; it reduces the noise by killing the coefficients that are insignificantly relative to some threshold. The threshold value is calculated using hard and soft thresholding. In this thesis performance of both hard and soft thresholding has been evaluated for de-noising the image.

The performance of various wavelet transforms using hard and soft threshold has been calculated in terms of Signal to Noise Ratio (SNR) ,Peak Signal to Noise Ratio (PSNR),Mean Squared Error (MSE) ,Root Mean Square Error (RMSE) and Universal Quality index for images (UIQ).

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CHAPTER-1

INTRODUCTION

An image is a two-dimensional function $f(x, y)$ where x and y are the plane(spatial) coordinates, and the amplitude value at the pair of coordinate (x, y) is called as the intensity or gray level of an image at that point[1]. Images are often degraded with noise during image acquisition and broadcast. For example during the image possession, the performance of imaging sensors is affected by a number of factors, like environmental conditions and by the quality of the sensing elements themselves. The noise degrades the quality of an image and loss of the useful information details. The noisy image is visually unpleasant and it is difficult to perform various further studies such as segmentation, detection and compression. Therefore, it is very important to reconstruct an original image from the corrupted observations.

Noise is unwanted signal that interferes with the original signal and degrades the visual quality of digital image. Image de-noising techniques are necessary to remove such noises while retaining as much as possible the important signal features. The main objective of noise removal is to suppress the noise while preserving the original image details.

Image de-noising is required before the image data are analyzed. Conventional analyzes methods can be categorized into: time domain and frequency domain. The frequency domain analysis is more striking one because it can give more detailed information about the signal and its component frequencies whereas; the time domain analysis can give overall qualitative information. Traditionally, Fourier transform (FT) was used to perform such analysis.

It is well known from Fourier theory that a signal can be spoken as the sum of a, possibly infinite, combination of sines and cosines called as a Fourier expansion. The huge drawback of a Fourier expansion is that it has only frequency resolution and no time resolution. To overcome this problem in the past decades several solutions have been developed which are more or less able to represent signal in the time and frequency domain at the same time. The purpose of time-frequency joint representations is to cut the signal of interest into several parts and then analyze the components separately. It is clear that analyzing a signal in this way will give more accurate information.

However the Fourier analysis has some inherent limitations in the analysis of the non-linear phenomena and it is impossible to know the exact frequency and the exact time of occurrence of the frequency in a signal. Over the past 10 years, the wavelet theory has become one of the emerging and fast-evolving mathematical and signal processing tools for its many different merits. The wavelet transform can be used for multi-scale analysis of the signal through dilation and translation, so it can take out the time frequency features of the signals effectively.

In wavelet analysis the use of a fully scalable modulated window solve the signal splitting problem. The window is moved along the signal and for every position the spectrum is calculated. This process is recurring many times with a slightly shorter or longer window for each new cycle. At the end the result will be a collection of time-frequency representations of the signal, all with different resolutions. By virtue of this collection of representations we can talk about a multi-resolution analysis. [2]

Wavelets are mathematical functions that split data into dissimilar frequency components, and then study every component with a resolution related to its scale. They have advantages over conventional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelet functions are distinguished from other transformations in that they not only dissect signals into their component frequencies, they too vary the amount at which the component frequencies are analyzed. So wavelets, as component pieces used to examine a signal, are restricted in space. In other words, they have specific stopping points along the axis of a graph so they do not repeat to infinity such as sine or cosine waves do. The capability to vary the scale of the function as it addresses different frequencies also makes wavelets better suited to signals with spikes or discontinuities than traditional transformations such as the FT.

Wavelet transform, due to its brilliant localization property has quickly become an essential signal and image processing tool for a variety of applications, including compression and de-noising [3, 4, 5]. Wavelet de-noising attempts to remove the noise present in the signal while preserving the signal properties, despite of its frequency content. It concludes three steps: a

linear forward wavelet transform, non-linear thresholding step and a linear inverse wavelet transform.

Wavelet thresholding (first proposed by Donoho [3, 4, 5]) is a signal estimation technique that exploits the capabilities of wavelet transform for signal de-noising. It removes noise by killing coefficients that are not significantly relative to some threshold, and turns out to be simple and effective, depends largely on the option of a thresholding parameter and the choice of this threshold determines, to a great extent the efficacy of de-noising.

1.1 Types of de-noising techniques:

Digital images play very important role in daily life applications. The noise occurs in the digital image, during image acquisition, transmission etc. To remove the noise and to recover or to reconstruct the image various de-noising techniques are available. The de-nosing technique is selected according to the noise present in the signal. [6]

1.1.1 Spatial Domain Technique: It is the traditional way to remove the noise from images. It is further divided into linear and nonlinear filters.

Non-Linear Filters: With non-linear filters, the noise is detached or removed without any attempts to openly identify it. Spatial filters make use of a low pass filtering on groups of pixels with the supposition that the noise occupies the higher region of frequency spectrum. Generally spatial filters remove the noise to a sensible extent but at the cost of blurring the images which in turn make the edges in pictures unseen.

Linear Filters: Linear filters also tend to blur sharp edges, destroy lines and other fine image details, and perform poorly in the presence of signal-dependent noise.

1.1.2 Transform Domain Technique: Transform domain function can be subdivide according the choice of basis functions. The term “wavelets” is one of the transform domain technique used to refer to a set of orthonormal basis functions generated by dilation and translation of scaling function φ and a mother wavelet ψ .

1.2 Applications of wavelets:

Wavelet analysis is an exciting new method for solving hard problems in mathematics, physics, and engineering with current applications as diverse as wave propagation, data compression, image processing, signal processing, computer graphics, pattern recognition, the recognition of aircraft and submarines and some other medical image technology. Wavelets are the powerful statistical tool which can be used for a wide range of applications. [7]

1.1.1 Signal Processing: Wavelets process the signal by decomposing the signal using filters. One filter of the analysis (Wavelet Transform) pair is a Low Pass Filter (LPF), while the other is a high pass filter (HPF). Each filter has a down-sampler after it, to make the transform efficient. There is a synthesis (inverse wavelet transform) pair consisting of an inverse low pass filter (ILPF) and an inverse high pass filter (IHPF), each preceded by an up-sampler. A low pass filter produces the average signal, while a high pass filter produces the detail signal. While the average signal look much like the original, we need the details to make the reconstructed signal match the original.

1.1.2 Data Compression: The image is compressed by removing the redundancies e.g. if large area has uniformity It has large redundancy, and the area which having frequent changes having small redundancy and difficult to compress. Wavelet transform is used to decompose an image into approximation and detail sub signals. Small details are set to zero without notably varying the image. Larger the number of zeros larger the compression that can be achieved.

1.1.3 Image denoising: Wavelets are widely use in the application of removing noise from images. Denoising is the process of recovering the useful information of an image with the help of thresholding the values.

1.1.4 Speech recognition: Wavelets are used to analyze the speech signals, it normalize the signal and then decompose the speech signal into approximation and detail coefficients. Wavelet transform is used to extract coefficients from phonemes [8]. It helps to build a speech recognition system with improved accuracy.

1.1.5 Fingerprint verification: Fingerprints are one of the most reliable method in the identification of an individual. Wavelets are helpful in pattern recognition or texture analysis. The resolving power of wavelets extracts the texture information in Horizontal, Vertical and

Diagonal directions of the fingerprint images. The use of this local texture information can be used to increase the performance rate. [9]

Wavelets are used in many other applications such as:

- DNA analysis, protein analysis
- Blood-pressure, heart-rate and ECG analyses
- Finance (which is more surprising), for sensing the attributes of quick variation of values
- In Internet interchange explanation for building the services size
- Industrial supervision of gear-wheel
- Computer graphics and multifractal analysis etc.

CHAPTER-2

LITERATURE SURVEY

This paper is to develop a speech recognition algorithm that uses the wavelet transform to extract and represent incoming speech signals as a basis for an accurate method of identifying and matching these signals to signals in a template. The approach taken in this paper is to use the wavelet transform to extract coefficients from phonemes and to use cross correlation to classify the phoneme. Cross-correlation measures the similarities between two signals. The results show that using the wavelet transform improved the accuracy in correctly identifying the phonemes. The results also show that using the approximation coefficients to generate octaves in the wavelet transform give better accuracy than using the detail coefficients. The first three octaves give the best results, while the accuracy of using the fourth and fifth octaves declines. The results demonstrate that it is possible to build a speech recognition engine using the wavelet transform and wavelet coefficients.

S. Grace Chang et al. [10] proposed the method of wavelet thresholding for removing noise, or de-noising, has been researched extensively due to its effectiveness and simplicity. A spatially adaptive wavelet thresholding method based on context modeling, a common technique which is used in image compression to adjust the coder to change the image characteristics. Each wavelet coefficient is modeled as a random variable of a generalized Gaussian distribution with an unknown parameter. Context modeling is used to estimate the parameter for each coefficient, which is then used to adapt the thresholding strategy. This spatially adaptive thresholding is extended to the over complete wavelet expansion, which yields better results than the orthogonal transform. Results show that spatially adaptive wavelet thresholding yields significantly superior image quality and lower mean squared error than the best uniform thresholding with the original image assumed known. adapting the threshold values to local signal energy allows us to keep much of the edge and texture details, while eliminating most of the noise. The results show substantial improvement over the optimal uniform thresholding both in visual quality and mean squared error.

Grace Chang et al [11] presented an adaptive, data-driven threshold for image denoising via wavelet soft-thresholding. The threshold is derived in a Bayesian framework, and the prior used on the wavelet coefficients is the generalized Gaussian distribution (GGD) widely used in image processing applications. Threshold is simple and closed-form, and it is adaptive to each subband because it depends on data-driven estimates of the parameters. Regarding image denoising an adaptive threshold for wavelet thresholding images was proposed, based on the GGD modeling of subband coefficients, A coder was designed specifically for simultaneous compression and denoising. The proposed Bayes Shrink threshold specifies the zero-zone of the quantization step of this coder, and this zero-zone is the main agent in the coder which removes the noise.

Qu Tianshu et al [12] presented a new class of nonlinear thresholding function. Unlike the standard soft-thresholding function, the function has infinite continuous derivatives. Since the new thresholding functions perform similar operations to the standard thresholding function, similar smoothness property of the estimate using the new thresholding functions can be expected. The significance of the new thresholding functions is that they make it possible to search for optimal thresholds using gradient-based adaptive algorithms. Based on SURE risk an adaptive wavelet shrinkage method 11% been presented. The simulation result shows that the proposed method performs better than the conventional wavelet shrinkage method.

Wei Liu et al. [13] show that common denoising methods use low pass filters to get rid of noise. However, both edge and noise information is high-frequency information, so the loss of edge information is evident and inevitable in the denoising process. Edge information is the most important high-frequency information of an image, so we should try to maintain more edge information while denoising. In it, we present a new image denoising method: wavelet image threshold denoising based on edge detection. Before denoising, those wavelet coefficients of an image that correspond to an image's edges are first detected by wavelet edge detection. The detected wavelet coefficients will then be protected from denoising, and we can therefore set the denoising thresholds based solely on the noise variances, without damaging the image's edges. The theoretical analyses and experimental results presented in this paper show that, compared to commonly-used

wavelet threshold de-noising methods, our method can keep an image's edges from damage and can increase the PSNR up to 1-2dB. Finally, we can draw the conclusion that edge detection and de-noising are two important branches of image processing. If we combine edge detection with de-noising, we can overcome the shortcomings of commonly-used de-noising methods and do de-noising without notably blurring the edge.

S.Sudha et al. [14] proposed a simple and subband adaptive threshold to address the issue of image recovery from its noisy counterpart. It is based on the generalized Gaussian distribution modeling of subband coefficients. The image denoising algorithm uses soft thresholding to provide smoothness and better edge preservation at the same time. Experiments are conducted to assess the performance of the proposed Shrink in comparison with the OracleShrink, VisuShrink, BayesShrink, and Wiener. It is further suggested that the proposed threshold may be extended to the compression framework, which may further improve the denoising performance.

Ashraf Aboshosha et al. [15] presented a comparative study of image denoising techniques relying on spatial filters. Subjective and objective evaluation methods are used to judge the efficiency of different types of spatial filters applied to different types of noise. Moreover, a proposed denoising technique based on cascaded median and wavelets filter is presented. This study on denoising could be a considerable base in dealing with denoising purposes for all vital vision systems. Experimental results prove that the proposed cascaded filter is the best one for removing all types of the noise. The presented comparative study has been applied on several benchmark images to verify the attained results. A comprehensive list of key references, organized by application category, is also provided.

Lakhwinder Kaur et al. [16] In this paper, a simple and sub band semi soft threshold method is proposed to address the issue of image recovery from its noisy counter-part. It is based on the discrete wavelet transform and Gaussian distribution modeling of sub-band coefficients. The experiment shows that the traditional image de-noise methods are difficult to preserve the details of the image effectively while removing the noise.

So, compared with the above several methods, the proposed methods in this paper can preserve most satisfying image details.

Yang Qiang. [17] This paper proposes the soft thresholding method to denoise the image. The image is decomposed by using Haar wavelet transform. The threshold value is computed using Bayesian threshold. The algorithm effectively cleans the noise of image and raises the PSNR value of image.

Akhilesh Bijalwan et al. [18] This paper deals with the threshold estimation method for image denoising in the wavelet transform domain. The proposed technique is based upon the discrete wavelet transform analysis where the algorithm of wavelet threshold is used to calculate the value of threshold. The proposed method is more efficient and adaptive because the parameter required for calculating the threshold is based on sub-band data. The experiments are conducted on Lena image of size 512x512 which is corrupted by Gaussian white noise of standard deviation 0.05. Experiment 1 using wavelet hard threshold noise filter, wavelet soft threshold noise filter, wavelet BayesShrink threshold noise filter, and wavelet semisoft threshold noise filter eliminates image noise. The experiment shows that the proposed method can preserve most satisfying image details.

Jeena Joy et al. [19] This paper presents the comparative study of different wavelet denoising techniques. The denoising method removes the noise by using thresholding. It discusses the global and level-dependent thresholding technique. The performance measure chosen is the MSE between denoised and original signal. Rigsure gives the best performance and the performance of minimax and heursure is better than that of sqtwolog. Denoising performance with type of signal under considerations and wavelet chosen.

Ms. Sonam Malik et al. [20] This paper presents the comparison of the performance of Discrete cosine transform, Discrete wavelet transform and wavelets like Haar Wavelet and Daubechies Wavelet for implementation in a still image compression system and to highlight the benefit of these transforms relating to today's methods. The performance of these transforms is compared in terms of Signal to noise ratio SNR, Mean squared error (MSE) etc. The DCT shows its best results in terms of energy compaction but MSE that

is the error between original and recovered image is not acceptable. So to speed up the process and to improve the MSE, DWT based compression can be done. Among Haar and Daubechies, Daubechies shows better result.

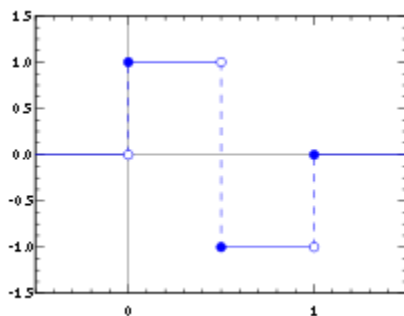
D.Gnanadurai et al. [21] In this proposed method, the choice of the threshold estimation is carried out by analyzing the statistical parameters of the wavelet subband coefficients like standard deviation, arithmetic mean and geometrical mean. The noisy image is first decomposed into many levels to obtain different frequency bands. Then soft thresholding method is used to remove the noisy coefficients, by fixing the optimum thresholding value by the proposed method. Experimental results on several test images by using this method show that this method yields significantly superior image quality and better Peak Signal to Noise Ratio (PSNR). To prove the efficiency of this method in image denoising, This method is compared with various denoising methods like Average filter, VisuShrink and BayesShrink.

WAVELET TRANSFORMS

The Haar sequence was proposed in 1909 by Alfred Haar. Haar is a sequence of rescaled "square-shaped" functions. It is a type of DWT, for an input represented by a list of 2^n numbers, It is considered to simply pair up input values, keep the dissimilarity between values and crossing the sum. This particular process is repeated recursively; combine the sums to provide the further measurement: finally resulting in $2^n - 1$ differences and one final sum.

The Haar transform is one of the simplest and basic transformations from the space/time domain to a local frequency domain, which display the space and time-variant spectrum. The magnificent features of the Haar transform, including its fast implementation and capability to study the local feature.

Conventionally, Fourier transform has been used extensively to analyze the spectral content of a signal. However, Fourier transform is not able to represent a non-stationary signal adequately; whereas time-frequency analysis function, e.g., the Haar transform, is found effective as it provides a simple approach for analyzing the local aspects of a signal.



Fig(1) The Haar Wavelet

The Haar wavelet function $\psi(t)$ can be described as

$$\psi(t) = \begin{cases} 1 & 0 \leq t < 1/2, \\ -1 & 1/2 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Its scaling function $\phi(t)$ can be explained as

$$\phi(t) = \begin{cases} 1 & 0 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases}$$

The family of N Haar functions $h_k(z)$ are defined on the interval $0 \leq z \leq 1$. [1] The shape of the Haar function, of an index k , is calculated by two parameters: p and q where

$$k = 2^p + q - 1$$

and k is in a range of $k = 0, 1, 2, \dots, N - 1$.

When $k = 0$, the Haar function is defined as a constant $h_0(z) = 1/\sqrt{N}$; when $k > 0$, the Haar function is defined as

$$h_k(z) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2} & (q-1)/2^p \leq t < (q-0.5)/2^p \\ -2^{p/2} & (q-0.5)/2^p \leq t < q/2^p \\ 0 & \text{otherwise} \end{cases}$$

From the above equation, one can analyze that p determines the amplitude and width of the non-zero part of the function, while q determines or calculate the position of the non-zero part of the Haar function.

3.1 Haar Transform

The Haar transform $HT^n(f)$ of an N -input function $X^n(f)$ is the 2^n element vector

$$HT^n(f) = \mathbf{H}^n X^n(f)$$

The Haar transform cross multiplies a function with Haar matrix that contains Haar functions with different width at different location. The Haar transform performed in levels. At each level, the Haar transform break a distinct signal into two components with half of its length: an approximation (or trend) and a detail (or fluctuation) component. The first level of

approximation $\mathbf{a}^1 = (a_1, a_2, \dots, a_{N/2})$ is defined as

$$a_m = \frac{X_{2m-1} + X_{2m}}{\sqrt{2}}$$

for $m = 1, 2, 3, \dots, N/2$, where X is the input signal. The multiplication of $\sqrt{2}$ ensures that the Haar transform preserves the energy of the signal. The values of \mathbf{a}^1 represents the average of successive pairs of X value.

The first level detail $\mathbf{d}^1 = (d_1, d_2, \dots, d_{N/2})$ is defined as

$$d_m = \frac{X_{2m-1} - X_{2m}}{\sqrt{2}}$$

for $m = 1, 2, 3, \dots, N/2$. The values of \mathbf{d}^1 represents the difference of successive pairs of X value.

The first level Haar transform is denoted as \mathbf{H}_1 . The inverse of this transformation can be achieved by

$$X = \frac{a_1 + d_1}{\sqrt{2}}, \frac{a_1 - d_1}{\sqrt{2}}, \dots, \frac{a_{N/2} + d_{N/2}}{\sqrt{2}}, \frac{a_{N/2} - d_{N/2}}{\sqrt{2}}$$

The consecutive level of Haar transform and the approximate calculation and detail component are calculate in the same way, except that these two components are calculated from the previous approximation component only.

An example: $\mathbf{X}(4, 6, 8, 10, 13, 9, 3, 3)$ the first level approximation and detail components are

$$\mathbf{a}^1 = \sqrt{2}(5, 9, 11, 3)$$

$$\mathbf{d}^1 = \sqrt{2}(-1, -1, 2, 0)$$

$$\mathbf{a}^2 = (14, 14)$$

$$\mathbf{d}^1 = (-4, 8)$$

3.2 Daubechie Wavelet:

The Daubechies wavelets are a family of orthogonal wavelet defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some provided support. With every, this type of wavelet of this class, there is a scaling function which

produces an orthogonal multi-resolution analysis. In general the Daubechies wavelets are chosen to have the highest number A of vanishing points ,(this does not apply the best quickness) for the given support width $N=2A$, and with 2^{A-1} feasible solutions the one is chosen whose scaling filter has extremal phase. Daubechies wavelets are mostly used in solving a broad range of problems, for example self-similarity properties of a signal or fractal problems and the discontinuities of the signals etc.

The Daubechies wavelet transforms are definite in the same manner as the Haar wavelet transform—by computing running averages and differences via scalar products with scaling signals and wavelets—the only difference between them consists in how these scaling signals and wavelets are defined. For the Daubechies wavelet transforms the scaling signals and wavelets have little bit more supports, i. e., they produce averages and differences using just a few more values as of the signal. This little transform, however, gives a tremendous improvement in the capabilities of these new transforms. It provides a set of powerful tools for performing basic signal processing jobs. This type of jobs includes solidity (compression) and noise removal for audio signals and for images, and include image enhancement and signal recognition.

Daubechies wavelets are very good at representing polynomial behavior within the signal. The support length of the daubechies wavelets are N_k-1 i.e D2 has support length of 1, D4 has support length of 3 and so on. The scaling function lets through the lower frequencies and hence acts as a lowpass filter and the associated wavelet lets through the higher frequencies and act as a Highpass filter. Daubechies wavelet has four scaling coefficients, the D4.The '4' the number of nonzero scaling coefficients. [23]

3.3 Thresholding

Thresholding is one of the efficient method to reduce noise, it reduce the noise by killing coefficients that are insignificantly relative to some threshold. To select the value of threshold is the critical process. If the small value of the threshold is taken, noisy coefficients will pass and signal is still noisy. If the large value of the threshold is taken, most of the coefficients will zero, image will smooth signal and destroy details.

The threshold is estimated and the coefficients are killed or remain unchanged or shrunk, depending on the type of thresholding (i.e. hard or soft). The small coefficients in the sub-bands are conquered by noise, and the coefficients with huge absolute value carry more signal information than noise. Replacing noisy coefficients (small coefficients below certain value) by zero and an inverse wavelet transform may lead to reconstruction that has lesser noise. [24]

3.1.1 HARD THRESHOLDING

Hard thresholding remove coefficients below a threshold value. If 'x' is the set of wavelet coefficients, then the threshold value 't' is specified as below

$$T(t, x) = \begin{cases} 0 & \text{if } |x| < t \\ x & \text{otherwise,} \end{cases}$$

i.e., all the values of 'x' which are less than threshold 't' are equated to zero.

3.1.2 SOFT THRESHOLDING

Soft thresholding shrinks the wavelet coefficients above and below the threshold value. It decreases the coefficients towards zero. In soft thresholding, all the coefficients x lesser than threshold t are mapped to zero. Then t is subtracted from all $x \geq t$. The condition is depicted by the following equation

$$T(t, x) = \begin{cases} 0 & \text{if } |x| < t \\ \text{sign}(x)(|x| - t) & \text{otherwise.} \end{cases}$$

4.1.1 Signal to Noise Ratio (SNR)

It gives us the relation between required signal level and surrounding noise level. It is defined as the ratio of signal power to noise power. It is measured in Decibels.

$$SNR_{db} = 10 \log_{10} \left(\frac{P_{signal}}{P_{noise}} \right) = P_{signal,db} - P_{noise,db}$$

4.1.2 Peak Signal to Noise Ratio (PSNR)

It is the fraction of the optimum power level to a desired signal and the optimized power of disturbance noise that affects the reliability of its representation expressed in logarithmic decibel scale. PSNR values can be calculated by comparing two images one is original image and other is distorted image. The PSNR has been computed using the following formula

$$PSNR = 10 * \log_{10} (255)^2 / MSE(db)$$

4.1.3 Mean Squared Error (MSE)

It deals with the values obtained by an estimator thus calculating the divergence between estimator values and optimum values of estimated quantity. Where $I(x,y)$ represents the original image and $I'(x,y)$ represents the denoised image. The pixel position of the $M \times N$ image is denoted by x and y . MSE quantifies the average of squares of the “errors”. The higher value of MSE the better.

$$MSE = \frac{1}{MN} \sum_{Y=1}^M \sum_{X=1}^N [I(x, y) - I'(x, y)]^2$$

4.1.4 Root Mean Square Error (RMSE)

It calculates the root of power two for Standard Deviation. RMSE calculates the average magnitude of the error. It is mainly valuable when large errors are specifically not desirable.

$$RMSE_{Errors} = \sqrt{\frac{\sum_{i=1}^n (\hat{y}_i - y_i)^2}{n}}$$

4.1.5 Universal Quality index for images (UIQ)

It is calculated by structuring any image abnormality as an amalgamation of parameters like loss of Correlation, contrast distortion and luminance distortion. It performs significantly better than the broadly used distortion metric mean squared error. And it consist consistency with subjective quality measurement on various models and experiments employed.

PROBLEM FORMULATION

From the literature reviewed, it has been concluded that wavelet transform can be used for vast number of applications. In the present work the use of wavelet transform will be explored for de-noising of a still image.

- Visually unpleasant.
- Difficult to analyze.
- Poor Quality.
- Loss of required details.
- Remove the noise from image. So that image can be used for further analysis.

CHAPTER- 6

OBJECTIVE

To study the need of wavelet transform in image processing by making analytical investigation on Haar, Bior, Symmetric, Coiflet and Daubechie transforms using various Quality parameters of an image with Hard and Soft thresholding at different levels for de-noising the given image.

The central idea of this thesis is to de-noise the image by using Haar and Daubechie transform. Original digital image is a grayscale image and size of the image is resized to 256.

- a) Take the boat image as Original image and add the Guassian noise at the variation (variance) of 0.01. The Wavelet toolbox is used to obtain the noisy image.
- b) Select the Haar Transform and the image will be decomposed according to the selected level. The image is analyzed at 5 levels of Haar Transform.
- c) Select the Daubechie Transform (Db4) and the image will be decomposed according to the selected level. The image is analyzed at 5 levels of Db4 Transform.
- d) The wavelet decomposition of an image is carried out as further. In the first level of breakdown, the image is dividing into 4 sub-bands namely the HH, HL, LH and LL sub-bands. The HH sub-band gives the diagonal details of the image; the HL and LH sub-bands give the horizontally and vertically appearances in that order.

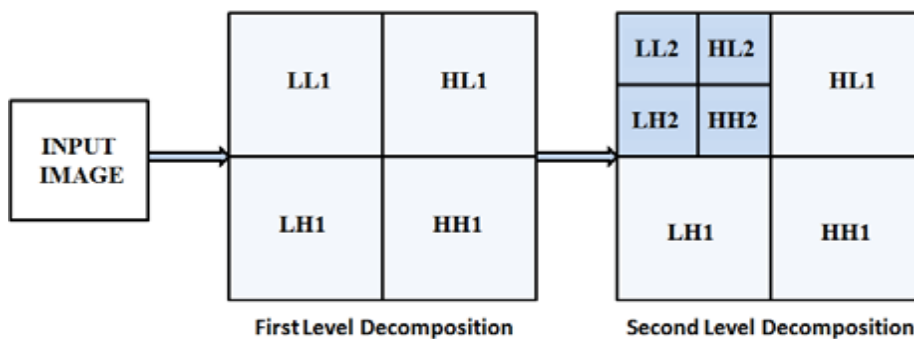


Fig 7.1: Two levels of Image decomposition

The LL sub-band is the low resolution residual consists of low frequency components and its sub-bands are further split at higher levels of decomposition. In the Wavelet sub-bands as the level number increases the coefficients of sub-band becomes smoother.HL2 is smoother than HL1 and so threshold value should be smaller than for HL1.[25]

e) At each level different threshold value is calculated with the help of Wavelet Toolbox. The threshold value can keep or kill the coefficients. We perform soft and hard thresholding at each decomposition level of an image for Haar and Daubechie Transform. At each level the value of threshold is different.

f) At the end the inverse of the Haar and Daubechie Wavelet has performed and get the de-noised image.

g) The performance of the Haar and Daubechie Wavelet has been compared with the help of various parameters.

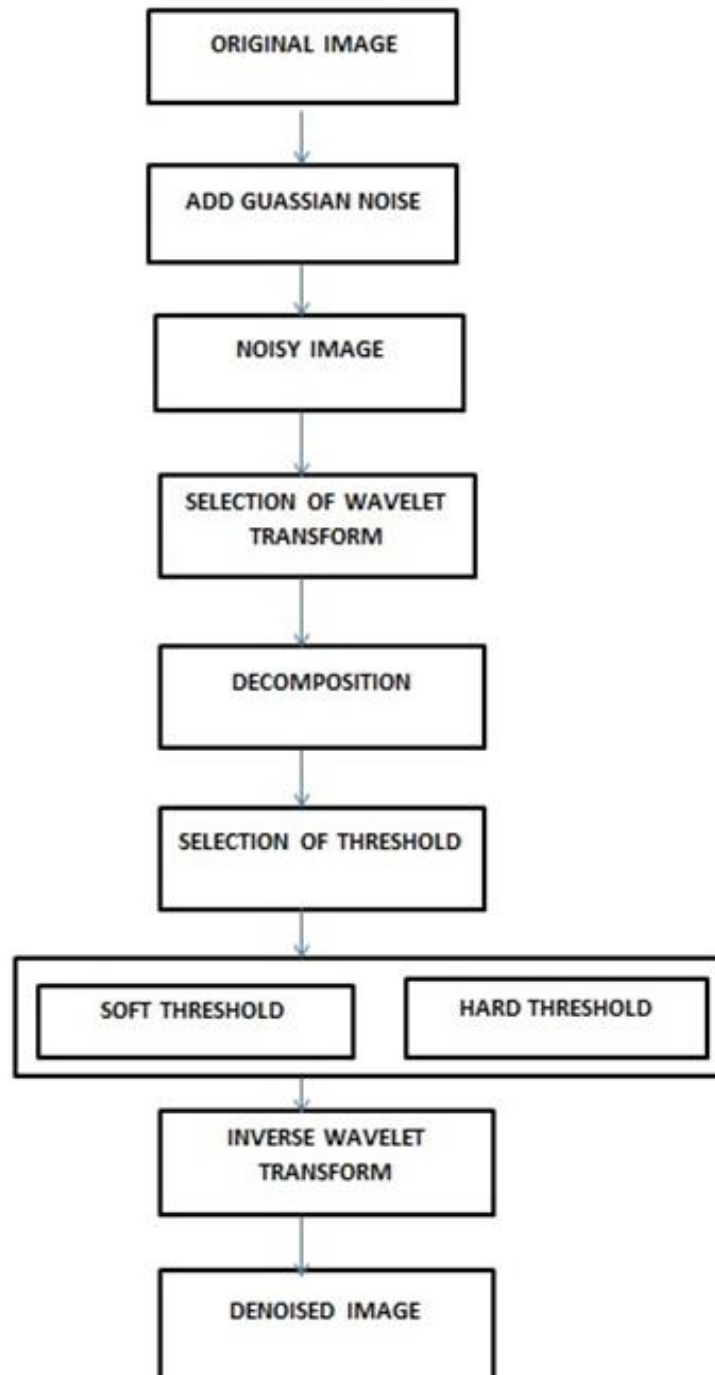


Figure 7.2 Wavelet De-noising Framework

CHAPTER- 8

EXPERIMENTAL RESULTS

The noisy image is de-noised by using Wavelet Thresholding Technique. De-noising is an important issue to be considered as the image has to be used for further analysis. To achieve the denoised image, it has to be broken into approximation and detail coefficients by using Haar and Daubechie Transform and the suitable threshold value keep or kill the coefficients and inverse transform is performed to get denoised image. Various values are generated for different parameter to check the performance. Various tables are given below.



Figure 1: Original Image



Figure 2: Noisy Image



Figure 3: Denoised image using hard threshold and using level-1 Bior Wavelet Transform



Figure 4: Denoised image using soft threshold and using level-1 Bior Wavelet Transform



Figure 5: Denoised image using hard threshold and using level-2 Bior Wavelet Transform



Figure 6: Denoised image using soft threshold and using level-2 Bior Wavelet Transform

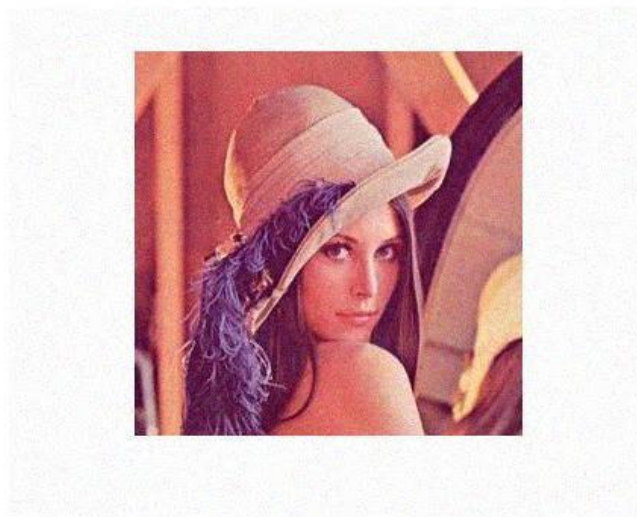


Figure 7: Denoised image using hard threshold and using level-3 Bior Wavelet Transform



Figure 8: Denoised image using soft threshold and using level-3 Bior Wavelet Transform



Figure 9: Denoised image using hard threshold and using level-4 Bior Wavelet Transform



Figure 10: Denoised image using soft threshold and using level-4 Bior Wavelet Transform



Figure 11: Denoised image using hard threshold and using level-1 Coif Wavelet Transform



Figure 12: Denoised image using soft threshold and using level-1 Coif Wavelet Transform



Figure 13: Denoised image using hard threshold and using level-2 Coif Wavelet Transform



Figure 14: Denoised image using soft threshold and using level-2 Coif Wavelet Transform



Figure 15: Denoised image using hard threshold and using level-3 Coif Wavelet Transform



Figure 16: Denoised image using soft threshold and using level-3 Coif Wavelet Transform



Figure 17: Denoised image using hard threshold and using level-4 Coif Wavelet Transform



Figure 18: Denoised image using soft threshold and using level-4 Coif Wavelet Transform



Figure 19: Denoised image using hard threshold and using level-1 dB Wavelet Transform



Figure 20: Denoised image using soft threshold and using level-1 dB Wavelet Transform



Figure 21: Denoised image using hard threshold and using level-2 dB Wavelet Transform



Figure 22: Denoised image using soft threshold and using level-2 dB Wavelet Transform

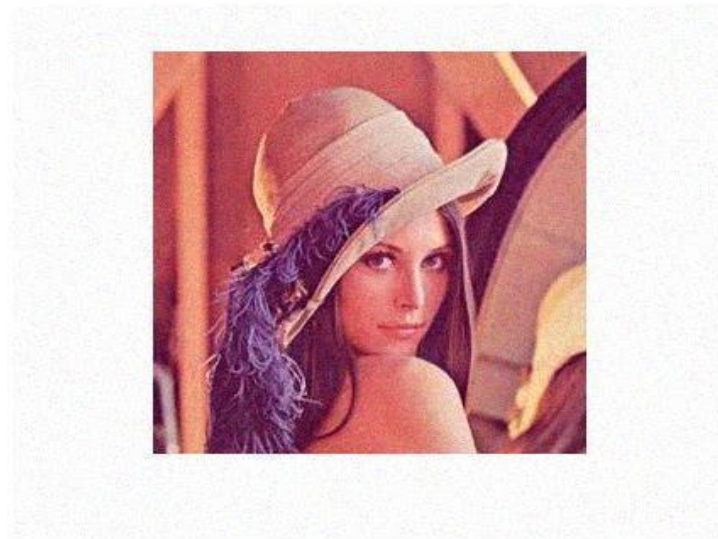


Figure 23: Denoised image using hard threshold and using level-3 dB Wavelet Transform



Figure 24: Denoised image using soft threshold and using level-3 dB Wavelet Transform



Figure 25: Denoised image using hard threshold and using level-4 dB Wavelet Transform



Figure 26: Denoised image using soft threshold and using level-4 dB Wavelet Transform



Figure 27: Denoised image using hard threshold and using level-1 Haar Wavelet Transform



Figure 28: Denoised image using soft threshold and using level-1 Haar Wavelet Transform



Figure 29: Denoised image using hard threshold and using level-2 Haar Wavelet Transform



Figure 30: Denoised image using soft threshold and using level-2 Haar Wavelet Transform



Figure 31: Denoised image using hard threshold and using level-3 Haar Wavelet Transform



Figure 32: Denoised image using soft threshold and using level-3 Haar Wavelet Transform



Figure 33: Denoised image using hard threshold and using level-4 Haar Wavelet Transform



Figure 34: Denoised image using soft threshold and using level-4 Haar Wavelet Transform



Figure 35: Denoised image using hard threshold and using level-1 sym Wavelet Transform



Figure 36: Denoised image using soft threshold and using level-1 Sym Wavelet Transform



Figure 37: Denoised image using hard threshold and using level-2 sym Wavelet Transform



Figure 38: Denoised image using soft threshold and using level-2 sym Wavelet Transform



Figure 39: Denoised image using hard threshold and using level- 3 sym Wavelet Transform



Figure 40: Denoised image using soft threshold and using level-3 sym Wavelet Transform



Figure 41: Denoised image using hard threshold and using level-4 sym Wavelet Transform



Figure 42: De-noised image using soft threshold and using level-4 sym Wavelet Transform

Table 8.1: Value of PSNR for different Levels of Wavelet Transforms and Different Thresholds

Level	Type of Wavelet Transform and Type of Threshold									
	Bior		Coif		dB		Haar		Sym	
	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
1	+50.972 57	+50.155 02	+50.988 03	+50.020 93	+50.025 52	+50.919 70	+50.919 70	+50.025 52	+50.989 81	+50.015 08
2	+48.984 12	+45.556 92	+49.904 02	+46.537 99	+49.645 30	+46.728 00	+49.645 30	+46.728 00	+49.913 71	+46.486 90
3	+48.265 03	+43.911 69	+49.528 34	+45.037 40	+49.331 72	+44.995 21	+49.331 72	+44.995 21	+49.472 25	+44.979 06
4	+48.218 49	+43.889 45	+49.545 17	+45.303 73	+49.400 80	+45.446 10	+49.400 80	+45.446 10	+49.486 36	+49.486 36

PSNR for Different Transforms and Different levels with Hard and Soft Threshold

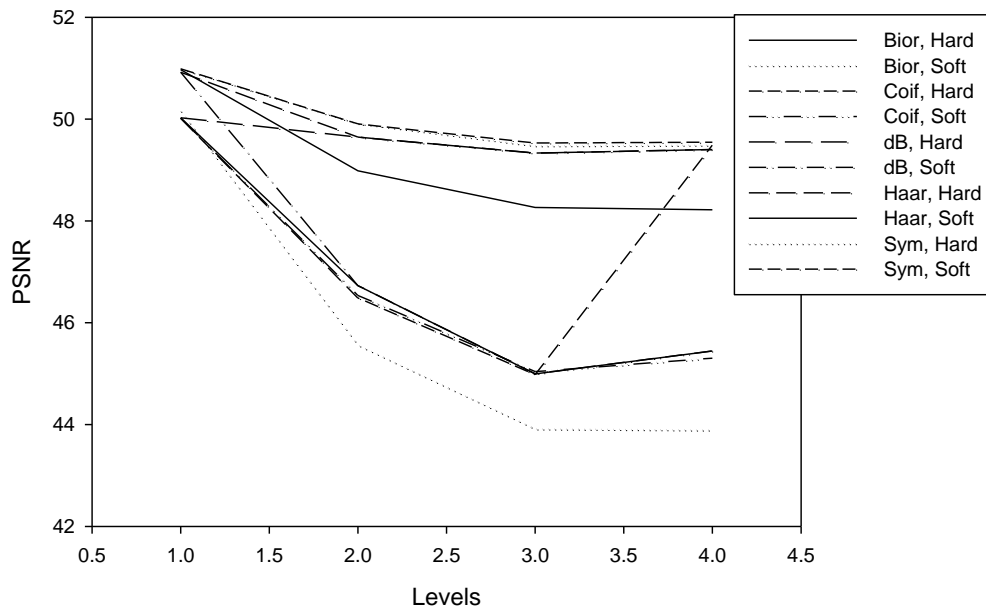


Figure 8.1 Variation of PSNR for Different Transforms and Different levels with Hard and Soft Threshold

Table 8.2: Value of MSE for different Levels of Wavelet Transforms and Different Thresholds

Level	Type of Wavelet Transform and Type of Threshold									
	Bior		Coif		dB		Haar		Sym	
	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
1	0.52387	0.63238	0.52201	0.65221	0.65152	0.53029	0.53029	0.65152	0.52179	0.65309
2	0.82807	1.82301	0.67001	1.45439	0.71113	1.39213	0.71113	1.39213	0.66851	1.47160
3	0.97719	2.66263	0.73054	2.05466	0.76438	2.07472	0.76438	2.07472	0.74004	2.08245
4	0.98771	2.67630	0.72772	1.93245	0.75231	1.87012	0.75231	1.87012	0.73764	0.73764

MSE for Different Transforms and Different levels with Hard and Soft Threshold

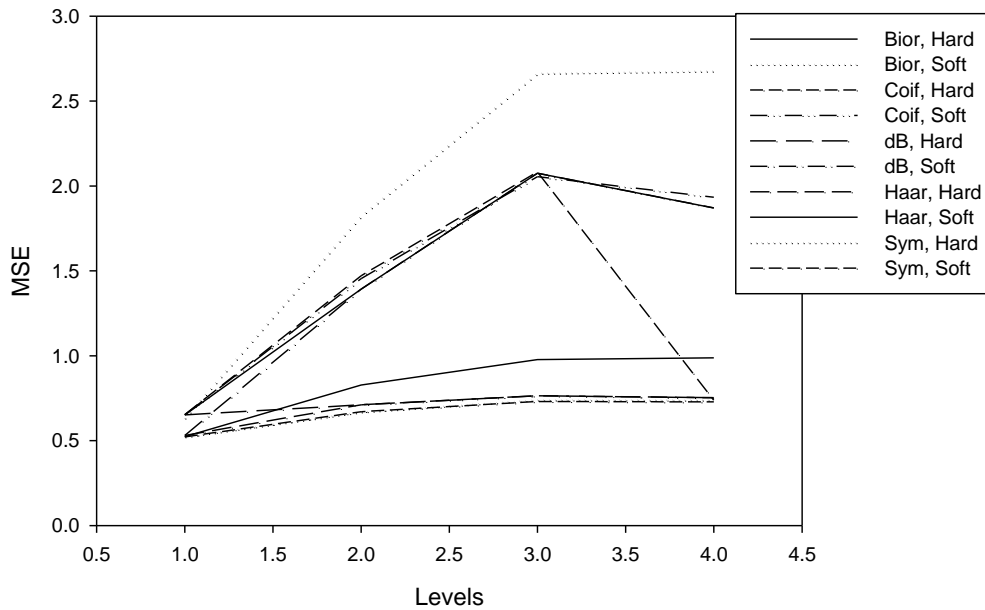


Figure 8.2 Variation of MSE for Different Transforms and Different levels with Hard and Soft Threshold

Table 8.3: Value of RMSE for different Levels of Wavelet Transforms and Different Thresholds

Level	Type of Wavelet Transform and Type of Threshold									
	Bior		Coif		dB		Haar		Sym	
	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
1	0.72379	0.79522	0.72250	0.80759	0.80717	0.72821	0.72821	0.80717	0.72235	0.80814
2	0.90998	1.35019	0.81854	1.20598	0.84329	1.17989	0.84329	1.17989	0.81763	1.21309
3	0.98853	1.63176	0.85472	1.43341	0.87429	1.44039	0.87429	1.44039	0.86026	1.44307
4	0.99384	1.63594	0.85306	1.39012	0.86736	1.36752	0.86736	1.36752	0.85886	0.85886

RMSE for Different Transforms and Different levels with Hard and Soft Threshold

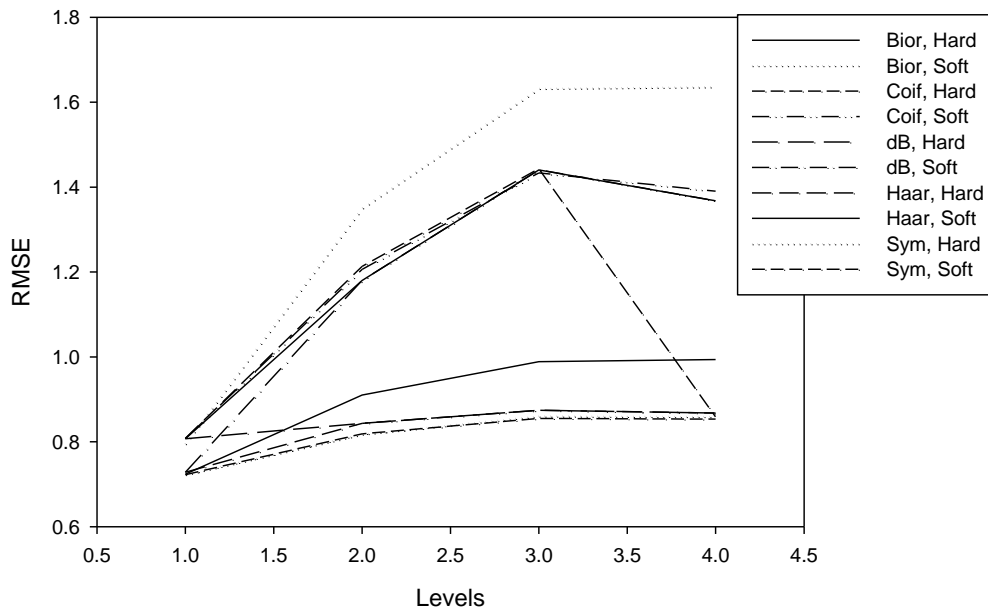


Figure 8.3 Variation of RMSE for Different Transforms and Different levels with Hard and Soft Threshold

Table 8.4: Value of UIQI for different Levels of Wavelet Transforms and Different Thresholds

Level	Type of Wavelet Transform and Type of Threshold									
	Bior		Coif		dB		Haar		Sym	
	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
1	0.97953	0.94061	0.97806	0.93804	0.94218	0.97869	0.97869	0.94218	0.97897	0.93840
2	0.93103	0.82399	0.95156	0.84152	0.95103	0.84300	0.95103	0.84300	0.94904	0.83544
3	0.98853	0.69129	0.94113	0.69907	0.94233	0.78720	0.94233	0.78720	0.93865	0.73619
4	0.89494	0.75627	0.94044	0.79205	0.94141	0.77262	0.94141	0.77262	0.93792	0.93792

UIQI for Different Transforms and Different levels with Hard and Soft Threshold

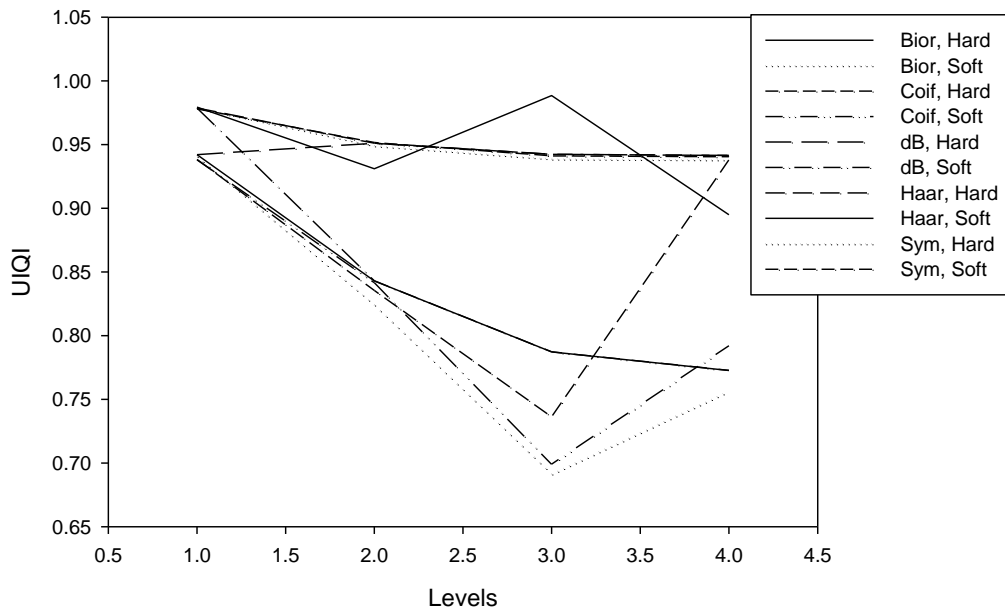


Figure 8.4 Variation of UIQI for Different Transforms and Different levels with Hard and Soft Threshold

Table 8.5: Value of SNR for different Levels of Wavelet Transforms and Different Thresholds

Leve l	Type of Wavelet Transform and Type of Threshold									
	Bior		Coif		dB		Haar		Sym	
	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
1	49.9706 0	49.1117 2	49.9860 6	48.9770 4	48.9813 7	49.9179 3	49.9179 3	48.9813 7	49.9877 8	48.9709 7
2	47.9823 6	44.5207 7	48.9021 8	45.5017 4	48.6439 5	45.6921 1	48.6439 5	45.6921 1	48.9118 6	45.4511 3
3	47.2629 6	42.8847 4	48.5265 2	44.0100 6	48.3301 7	43.9692 1	48.3301 7	43.9692 1	48.4704 1	43.9525 5
4	47.2160 6	42.8813 1	48.5430 8	44.2959 1	48.3987 6	44.4392 0	48.3987 6	44.4392 0	48.4842 9	48.4842 9

SNR for Different Transforms and Different levels with Hard and Soft Threshold

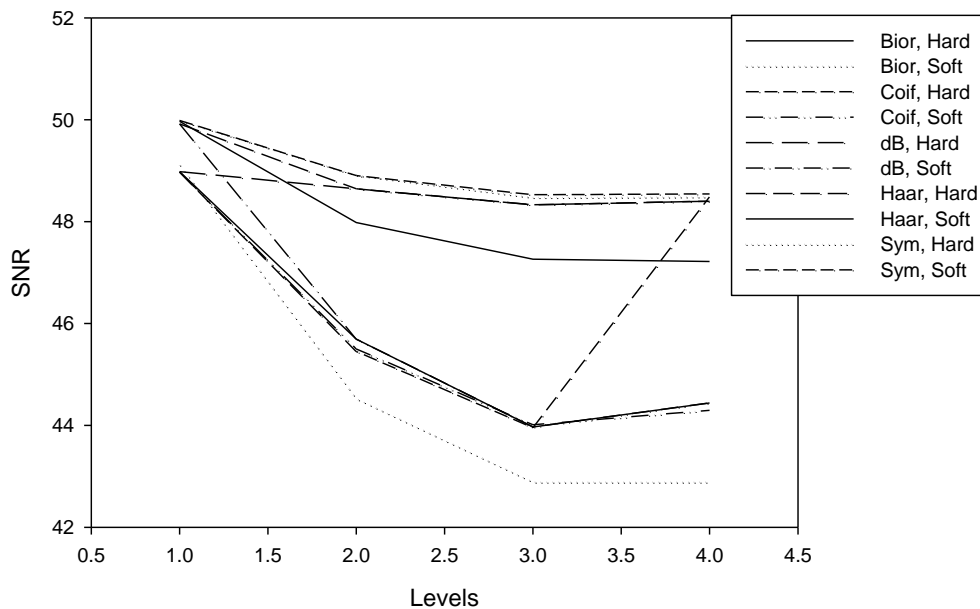


Figure 8.5 Variation of SNR for Different Transforms and Different levels with Hard and Soft Threshold

Table 8.6: Value of MAE for different Levels of Wavelet Transforms and Different Thresholds

Level	Type of Wavelet Transform and Type of Threshold									
	Bior		Coif		dB		Haar		Sym	
	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
1	0.53534	0.62948	0.53472	0.64558	0.65091	0.53916	0.53916	0.65091	0.53402	0.64902
2	0.59982	1.13032	0.56638	1.02939	0.57681	1.01149	0.57681	1.01149	0.56727	1.03676
3	0.63488	1.29195	0.58082	1.16614	0.58802	1.19636	0.58802	1.19636	0.58312	1.18093
4	0.63945	1.01040	0.57868	0.86415	0.58261	0.86475	0.58261	0.86475	0.58103	0.58103

MAE for Different Transforms and Different levels with Hard and Soft Threshold

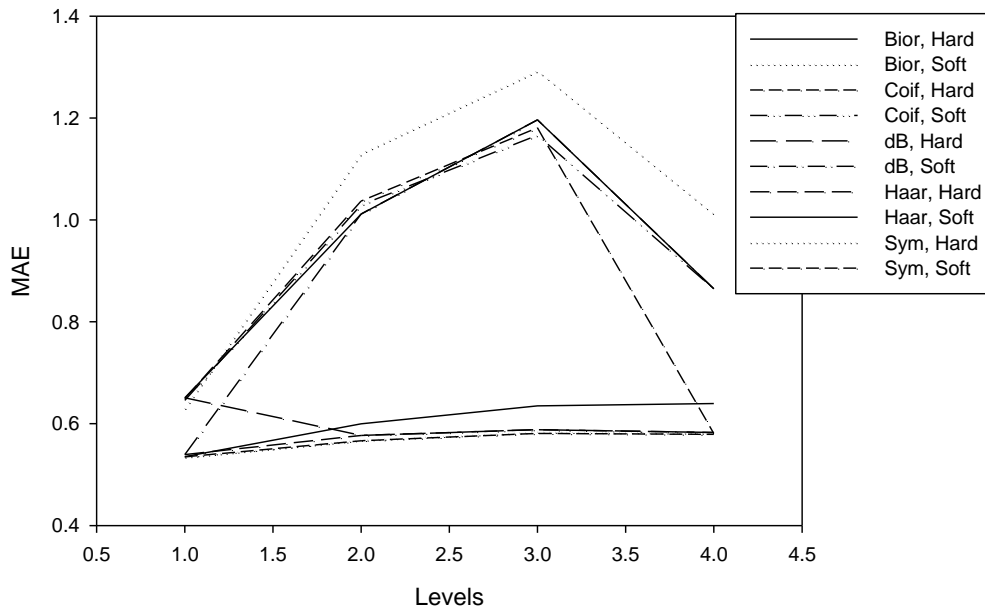


Figure 8.6 Variation of MAE for Different Transforms and Different levels with Hard and Soft Threshold

CONCLUSION AND FUTURE SCOPE**9.1 Conclusion**

Images are often corrupted with noise during image acquisition and transmission. The noise degrades the quality of an image and loss the useful information details.

Due to their capability over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes, wavelet transforms has been emerged as useful tool for image processing. Wavelets split data into dissimilar frequency elements and subsequently study each element with a resolution corresponding to its scale.

From the study it has been concluded that wavelet functions are distinguished from other transformations in that they not only dissect signals into their component frequencies, they also fluctuate the scale at which the elements frequencies are analyzed. So Wavelets, as elements (components) pieces used to analyze a signal, are limited in space. The ability to vary the scale of the function as it addresses different frequencies also makes wavelets better suited to signals with spikes or discontinuities than traditional transformations such as the Fourier transform.

In this thesis Haar, Bior, Symmetric, Coiflet and Daubechie wavelet transforms have been used for image denoising. Thresholding is one of the efficient method to reduce noise, it reduce the noise by killing coefficients that are insignificantly relative to some threshold. The threshold is estimated using hard and soft thresholding. In this thesis performance of both hard and soft thresholding has been presented in denoising the image. The work has been carried out on standard 'Lena' image. Besides using the conventional metrics like Signal to Noise Ratio, Peak Signal to Noise Ratio, we have also used important metrics like Mean Squared Error, Root Mean Square Error and Universal Quality Index for images for judging the quality of an image.

From the results it has been concluded that as the level of transform increases, the value of mean square error also increases for both types of thresholding. Results also show that in terms of Mean Square Error soft thresholding performs better as compared to hard

thresholding. Results also show that Bior transform outperforms the other transforms in terms of mean square error.

From the results it has also been concluded that as the level of transform increases, the value of root mean square error also increases for both types of thresholding. But from the results it has also been concluded that all the transforms have shown best performance at level3 and when the level has been increased beyond 3, the performance has deteriorated. Results also show that in terms of mean square error soft thresholding performs better as compared to hard thresholding. Results also show that Bior transform outperforms the other transforms in terms of mean square error.

From the results it has also been concluded that in general as the level of transform increases, the value of UIQI decreases for both types of thresholding. But from the results it has also been concluded that all the transforms except symmetric transform have shown best performance at level4 and the performance of symmetric transform is best at level 3. Results also show that in terms of mean square error soft thresholding performs better as compared to hard thresholding. Results also show that Bior transform outperforms the other transforms in terms of UIQI.

From the results it has also been concluded that in general as the level of transform increases, the value of signal to noise ratio decreases for both types of thresholding. But from the results it has also been concluded that all the transforms except Bior transform have shown best performance at level4 and the performance of other transforms is best at level 3. Results also show that in terms of signal to noise ratio soft thresholding performs better as compared to hard thresholding. Results also show that Bior transform outperforms the other transforms in terms of signal to noise ratio.

From the results it has also been concluded that in general as the level of transform increases, the value of MAE increases for both types of thresholding. By the virtue of the results it has also been completed that all the transforms have shown best performance at level4 and the performance of other transforms is best at level 4. Results also show that in terms MAE soft thresholding performs better as compared to hard thresholding. Results also show that Bior transform outperforms the other transforms in terms of signal to noise ratio.

Overall it has been concluded that selection of a particular type of transform is very important for de-noising an image, also soft thresholding outperforms hard thresholding. The selection of thresholding level is also critical and varies from parameter to parameter, which has been taken to judge the quality of an image

9.2 Scope for Future Work

As selection of threshold value is a critical issue and varies from image to image. For the suitable selection of thresholding value a hybrid techniques can be used. In hybrid technique optimization algorithms can be applied to calculate the threshold value and then wavelet transform can be applied to de-noise the image.

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ABBREVIATIONS

1. FT	Fourier Transform
2. DWT	Discrete Wavelet Transform
3. DCT	Discrete Cosine Transform
4. LPF	Low Pass Filter
5. HPF	High Pass Filter
6. IHPF	Inverse High Pass Filter
7. SNR	Signal to Noise Ratio
8. PSNR	Peak Signal to Noise Ratio
9. MSE	Mean Square Error
10. RMSE	Root Mean Square Error
11. MAE	Mean Absolute Error
12. UQI	Universal Quality Index
13. UIQI	Universal Image Quality Index