REDUCING COMPUTATIONAL COMPLEXITY FOR SPARSE FIR FILTER

DISSERTATION

Submitted in partial fulfillment of the requirement for the Award of Degree of

MASTER OF TECHNOLOGY

IN

Electronics & Communication Engineering

Submitted by

Rana Sameer Pratap Singh

Regd. No: 11510494

Under the guidance of

Mrs. Rosepreet Kaur Bhogal

Assistant Professor

Lovely Faculty of Technology and Sciences

School of Electrical and Electronics Engineering (SEEE)

Lovely Professional University

Phagwara (Punjab), India

MAY-2017

ABSTRACT

"Black and White is abstract: colour is not. Looking at a black and white photograph, you are looking at a strange world"- *Joel Sternfeld*

Sparsity has evolved as an efficient technology, to make our system economically sound, in respect to the design complexity and computational cost. In practice, one often comes across systems having sparse impulse response. One example, of such systems is the network echo channel, which has a very small active region out of the total echo response, thus removing the redundant coefficients. For most of these systems, the impulse response is not just sparse, but the degree of sparsity varies with time and context. A large number of experiments have demonstrated, that for an FIR filter the sparsity of filter coefficients is highly related to its filter order. The intent of this research is to provide an insight into the sparseness of the FIR filters, which will thereby provide cost effective solutions with reduced implementation complexity. First a linear-phase FIR filter is designed with the help of Haar wavelet method, after which the diverse attributes are calculated, to comment on the computational complexity. The second method is WLS, through which the filter coefficients are procured, and after that different algorithms are employed to make the filter quotients sparse. The various attribute values such as l_o-norm, l_1-norm, l_2-norm, and l_∞-norm are computed. The analysis is done on the basis of these attributes to comment on the computational complexity associated with the filter design. Experimental results depict which algorithm worked efficiently in the design procedure of the filter.

ACKNOWLEDGEMENT

Foremost, I would like to express my sincere gratitude towards my guide, **Mrs. Rosepreet Kaur Bhogal**, who gave her full support in the working of this dissertation work with her stimulating suggestions and encouragement all the time. She has always been a source of ideas.

I am thankful to management of Lovely Professional University for giving me an opportunity to carry out studies at the university and providing the proper infrastructure like internet facility which is pool of vast knowledge to work in.

I owe my heartiest thanks to my parents & all those guidepost who really acted as lightening pillars to enlighten my way throughout this work that has led to successful and satisfactory completion of my research.

At last but not the least I am thankful to all my friends, who have been instrumental in creating proper, healthy and conductive environment and including new and fresh innovative ideas for me during the research. Without them, it would have been extremely difficult for me to prepare this research in a time bound framework.

I would like to thank God for the strength that keeps me standing and for the hope that keeps me believing that this research would be possible.

Rana Sameer Pratap Singh

Regd. No: 11510494

School of Electrical and Electronics Engineering

Lovely Professional University

DECLARATION STATEMENT

I hereby declare that this submission is my own work and that to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which has been accepted for the award of any other degree or diploma of the university or other institute of higher learning, except where the acknowledgement has been made in the text.

Date:

Rana Sameer Pratap Singh

Regd. No: 11510494

School of Electrical and Electronics Engineering

Lovely Professional University

CERTIFICATE

This is to certify that the Research entitled "**Reducing Computational Complexity for Sparse FIR filter**" submitted by **Rana Sameer Pratap Singh** in partial fulfillment of the requirement for the award of the degree of Master of Technology (M.Tech) in Electronics and Communication engineering, is a record of bona-fide work carried out by him under my supervision.

Date:

Mrs. Rosepreet Kaur Bhogal
Supervisor
Assistant Professor
School of Electrical and Electronics Engineering
Lovely Professional University

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATION

DSP	Digital Signal Processing
CT	Continuous time
FIR	Finite Impulse Response
LPF	Low Pass Filter
HPF	High Pass Filter
BPF	Band Pass Filter
BSF	Band Stop Filter
OMP	Orthogonal Matching Pursuit
LSOMP	Least Square Orthogonal Matching Pursuit
MP	Matching Pursuit
WMP	Weak Matching Pursuit
CS	Compressive Sensing
DWT	Discrete Wavelet Transform
WLS	Weighted Least Square

1

INTRODUCTION

"The last thing that we find in making a book is to know what we must put first." -Blaise Pascal

In this era, signal processing is a qualifying technology that circumscribe the underlying theory, applications, algorithms, experimentation, formulation utilized for the processing or transmission information contained in plethora of diverse physical, figurative, or intellectual formats predominately designated as signals. It employs numerical, statistical, computational, heuristic, linguistic depiction, concoction, and methodologies for representation, modeling, analysis, synthesis, extraction, recovery, sensing, acquisition, learning, or security. Digital signal processing became a traditional method of filtering signals, and the kindred hypothesis of discrete-time systems may frequently be utilized in a series of disciplines related to science and technology. Furthermore a lot of applications associated with DSP came into picture, such as, analyzing biomedical signals, picture processing, seismic signal analysis and audio analysis.

A signal emanates information, and the grail of signal processing is to extricate useful details fetched by the signal. Signal processing involves the numerical entitlement of the signal and the algorithmic manoeuvre performed to extricate the details contained in it. The prompt progress of science and engineering relies on the eloquent breakthrough in digital system technology and integrated circuit manufacturing.

In the year 1970, eminent pace digital systems were rapidly prospered and they were procured for managing the digital description of electrical waveforms. Accordingly, it became plausible to employ the simple established theoretical ideas of Fourier analysis, waveform sampling, Z-transforms, etc., in the delineation of digital filters. Consequently, with the development of computer technology the digital filtering of signals became a pragmatic reality. In the year 1965 the algorithm written by Cooley-Tukey got published, thereby promising an essential contribution to the blooming of DSP field. Another noteworthy benefaction to the early procurement of digital filters was forged by James Kaiser, who used the concept of bilinear Z-transformation for delineating the digital filters. Nevertheless, these weren't the only quintessential contributions, and form the beginning of mid 60s many benefaction to R & D of digital filters was been published. A signal is a ramification of independent quotients such as time, position, temperature, pressure and distance.

With the evolving rate of computer technology, it is fairly possible that many traditional CT filter systems will get exchanged by identical digital filter systems. Also as the IC-technology is evolving fast, thus the computational cost to develop digital filters will fairly get reduced. This decrease in the designing cost will render more practical applications in digital signal processing. Credence is reckoned to this prophecy when one considers the innate merits of digital filters, specifically

i. can manage low frequency signals;
ii. frequency response attributes can be formulated to approximate nearly to the ideal;
iii. they can be procured with negligible insertion loss;
iv. plausible linear phase characteristics;
v. accuracy of filter can be fairly controlled;
vi. Cost and availability is generally not an issue.

Digital filters are very expensive than an equivalent analog filter because of their increased complexity, but they actually make practical many designs, that are impractical or intractable as analog filters. When employed in the context for real-time-analog systems, digital filters sometimes have complicated latency (the contrast in time of the excitation and the response) due to the associated analog-to-digital and digital-to-analog transformation and anti-aliasing filters, or may be due to delays in their implementation. A plethora of mathematical methods are employed to visualize the behavior of a digital filter. Many of these techniques are included in the filter design, which form the specification while the propounding of the filter. Typically, one characterizes filters by calculating how they will respond to a simple input such as an impulse. One can then extend this information to compute the filter's response to more complex-signals.

The digital systems and analogous digital equipment of previous decades were extensive and expensive and, as a concomitant, their employment was scarce to comprehensive disconnected scientific operation and business related applications. Accordingly innumerable signal processing errand that was traditionally accomplished through analog systems is perceived today by cheap and usually more dependable digital hardware [1].

1.1 Filters used in Signal Processing

Digital filters render a crucial mantle in DSP. Candor to be stated, their exceptional performance is one of the prime bases that DSP has transpired to be so favored. Filters possess two necessities: signal severance and signal refurbishment. Signal severance is entailed when a signal is actually adulterated with interference, Babel, or divergent signals. Such as, envisage a contrivance for gauging the electrical movement of an infant's heart (EKG) whilst still in womb. The crude signal will doubtless be debauch by the respiring and heartbeat of the mother. A filter perchance entailed to dismantle these signals with the key that they might be solely dilapidated and gauged. Signal refurbishment is entailed when a signal has been contorted somehow. Such as, an aural recording made with pauperized apparatus might be exuded to better enact the phonics as it legitimately occlude. Another illustration is the deblurring of an icon accomplished with an abhorrent concentrated lens, or a wobbly camera.

It is typical in DSP to enunciate that a filter's excitation and response signals fall under time domain. This is reckoned on the fact that the signals are usually composed by sampling at frequent span of time. Nevertheless, by all account this is not the only way sampling can occur. Another familiar process is doing sampling at equal intervals in space. This can be justified by considering an instance, such as, by taking successive readings from a muster of strain sensors clinked at one cm augments along the extent of an aircraft wing. Numerous disparate domains are plausible; nevertheless, time and space is well renowned. When you come across the phrase time domain in DSP, it actually refers to samples presumed control time. Each linear filter consists of a spectrum, i.e. an amplitude response and a frequency response. Each of these responses embraces cease information related to the filter, though in an interspersed form. For instance if one out of three is cited, the rest are reconciled and is meticulously calculated. Every representation is crucial, contrary to the fact that they portray how the filter will respond under various circumstances. The simplest form to enunciate a digital filter is by convolving the excitation signal with the digital filter's impulse response. Almost every possible linear filter can be contrived in this style.

1.2 FIR Filter

Filters can be categorized in various divergent genuses, depending on what benchmark are employed for classification. The two prime kind of digital filters are finite impulse response digital filters (FIR filters) and infinite impulse response digital filters (IIR filters). Both types possess some merits and demerits that must be diligently reviewed while delineating a filter. Apart from this, it is mandatory to consider all elementary attributes of signal to be sieved since they are prime factors while deciding which filter to use. Furthermore in most instances, there is solely one characteristic that truly matters and it is irrespective of the condition that filter possesses, it is checked whether such specifications relate to the linear phase symptoms or not. Speech signal, for instance, can be dealt in systems possessing non-linear phase attributes. The phase attributes of speech signal is not of concern and as such may be abandoned, which culminates the probability to wield much spacious ambit of systems management.

There are instances where the phase attributes are of real concern. Typical examples include signals that are acquired from diverse sensors in industry. Therefore, it is mandatory that a filter must possess linear phase characteristic preventing the loss of essential information.

When a signal that is to be filtered is scrutinized in such a way, it is facile to determine the perfect kind of digital filter that is to be employed. Consequently, if prime concern is the phase attributes, FIR filters should be employed since they possess linear phase characteristic.. FIR filters are termed as the digital filters possessing finite impulse response. Another abbreviation that is commonly used for FIR filters is non-recursive digital filters since they do not own any feedback (a recursive section of a filter), although recursive algorithms may be utilized for FIR filter accomplishment.

FIR filters are delineated using divergent methods, but mostly are established through ideal filter approximation. The grail is not to attain ideal attributes, as it is difficult anyway, but to attain adequately good attributes of a filter. The FIR filter transfer function resembles the ideal with the increase in filter order, thus increasing complexity issues and quantity of duration required for processing input specimens of a signal.

1.3 Types of Filters

1.3.1 Low-Pass

A LPF advances low frequency quotients, and attenuates components with frequencies more than filter's cutoff frequency. Furthermore, the diverse approximations to the impervious ideal LP amplitude features take divergent forms, some having a continuous negative slope i.e. being monotonic, and others possessing ruffle in the passband along with the stopband. LPF are exploited wherever the high frequency quotients must be eliminated from a signal. An instance may be a light-sensing appliance employing a photodiode.

1.3.2 High-Pass

The converse of the LPF is the HPF, which attenuates signals lying under its cutoff frequency and allowing other frequencies to pass through. A high-pass filter can be made by rearranging the components. HPF is employed as an enactment, requiring the impairment of low-frequency quotients. An instance where HPF is employed is high-fidelity loudspeaker systems. In concurrence with a LPF for the low-frequency driver (and similarly different filters for various other drivers), the HPF is utilized as a chunk of what is called "crossover network".

1.3.3 Bandpass

There are five vital filter kinds (bandpass, notch, low-pass, high-pass, and all-pass). The amount of feasible bandpass response attribute is infinite; still they all share indistinguishable rudimentary form. The meander in the diagrams of bandpass may depict an "ideal" response, with absolutely persistent gain interior of the passband, zero gain exterior of the passband, and brisk partition between the two. This response attribute is intractable to accomplish in general, though it may be estimated to proliferate degrees of exactness by factual filters. On the contrary, some bandpass responses enunciates eminent mellow, other have ruffle (gain variations) in their passband and even in stopband.

The BPF are predominantly exploited as a section of wireless communicator and recipient. The prime concern of such filters in a communicator is to bind the bandwidth of the response signal to the band allotted for transmission. This averts the communicator from impeding with variant stations. At the recipient end, a bandpass filter permits signals within a desired span of frequencies to be perceived or deciphered, while averting signals of unfavorable frequencies to get through. Apart from electronics and signal concoction, one exemplar of band-pass filters usage is dedicated to atmospheric sciences.

1.3.4 Notch or Band-Reject

A filter with efficaciously obverse purpose of the BP is the band-reject or notch filter. Notch filters are employed to eliminate undesirable frequency components from a signal, while other frequencies are left less affected. BR filters are universally exploited as a section of communication, instrumentation, control, and bio-remedial engineering, besides an emcee of divergent fields, to abolish clamor and power line intervention.

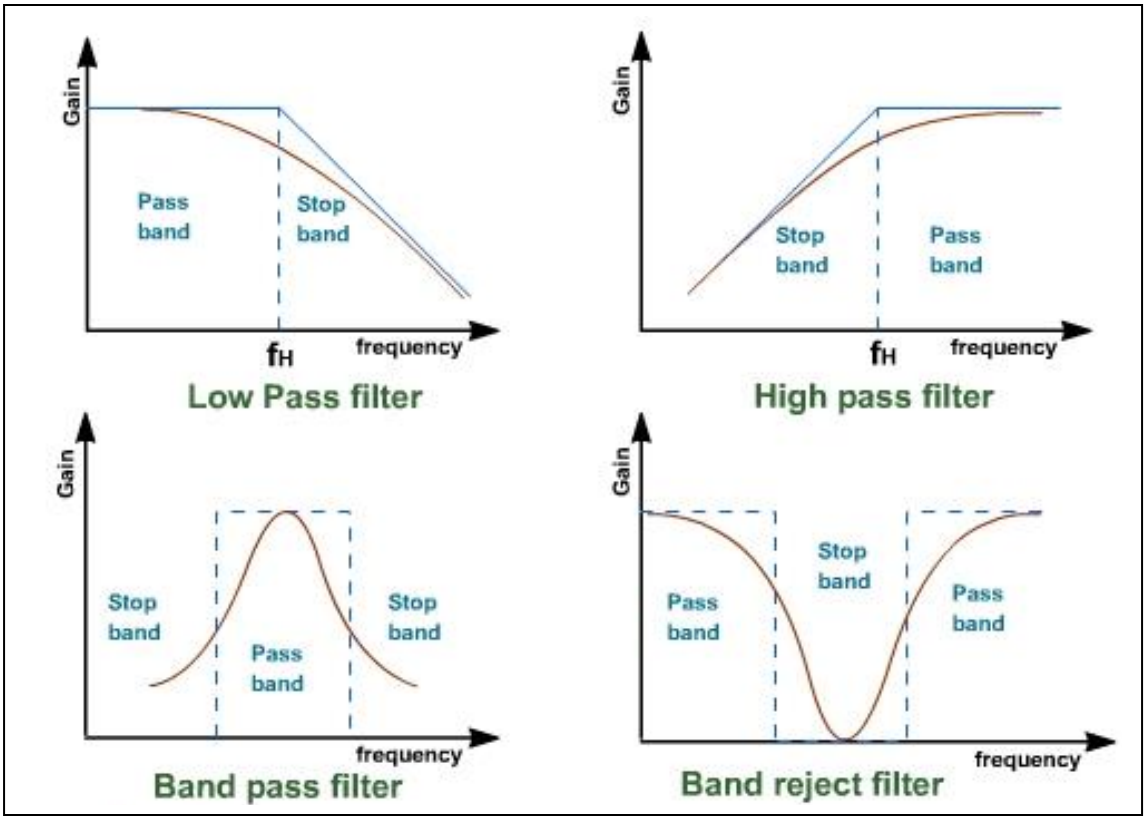

Fig.1.1 Frequency responses of the filters

1.4 Sparse FIR Filter

What is meant by the term sparse? This question is most important because of its significance in the real world. The term sparse is referred as the measurable property of a vector. It means that a particular vector is in sense small but actually the length of the vector is not our real concern. Instead in Sparsity the number of non-zero entries in the vector is most important. Here the l_0-norm is used to measure the sparsity of a vector [2]. While working with sparse vectors a lot of advantages are gained. A typical example is calculations involving multiplying a vector by a matrix will take less computational time in general if the vector is sparse. Also sparse vectors require less memory when being stored on a computer because here the space is required only to record the position and value of the entries. Over a decade now, Sparsity has evolved as one of the growing phenomenon in a wide range of signal-processing applications such as feature extraction, source separation, compressive sensing etc. Sparsity has also been a prevalent source in many theoretical and practical applications of mathematics such as statistical estimation, harmonic analysis and theoretical signal processing [2]. This area of research has grown very fast in recent years, with hundreds of interested researches, various workshops, sessions, and conferences, and an exponentially growing number of papers.

Modeling of sources is the prime concern in signal and image processing. Initiating a good and efficient model helps one to accomplish various tasks such as denoising, restoration, interpolation and extrapolation, detection, recognition etc. [9]. Here Sparsity is the model which leads to fascinating and extraordinary results because of its theoretical base, superior performance, its versatility and flexibility in serving various data sources which make all the above signal processing tasks simple and clear.

At the heart of this model lies a simple linear set of equations, which is studied in linear algebra. A full-rank matrix 'A' with number of rows less than the number of columns generates an underdetermined system of linear equations $b = Ax$ having infinite many solutions. The main objective is seeking its sparsest solution, i.e., a vector with the fewest nonzero entries [9].

Filters are employed to remove the unwanted glitches from the signal. This Unwanted constituent present in the signal is termed as noise. Thus the removal of the noise from the signal provides the accurate information of the system. FIR filters are enormously used in a wide range of applications on signal processing and communications [10]. If a set of specifications are given, traditional design method finds an FIR filter whose frequency response or magnitude response can best approximate the ideal one under some conditions. It is required to cast a convex optimization strategy and formulate efficient methods to overcome FIR filter design problems [11], [12]. Another problem faced by FIR filter is implementation complexity. With the rapid growth in sparse representation of signals, designers pay attention on designing a filter with a majority of coefficients being zero. Designing a Sparse FIR filter is our main concern since it helps in lowering the implementation complexity.

1.5 Type of Systems

1.5.1 Underdetermined Linear Systems

Many essential technological issues entail panacea to underdetermined framework of linear equations, i.e., mode of linear equations possessing fewer mathematical equations than unknowns. Instances appear in linear filtering, array signal handling, and inverse issues. For underdetermined framework of linear equations, existence of any solution, relates to infinitely many solutions. Thus, it is desired in numerous applications, “simplest” solution is greatly acceptable. This solution is grasped from the minimalist postulate of Occam’s razor. For instance, if the attributes of a system are being gauged then among all systems the one that describe the data competently, i.e. the system with the lesser amount of parameters is most prudent. The system with the minimum amount of attributes is, in reality, the sparsest solution. Thus in this exemplar, one is searching for the sparsest solution where coherence corresponds to sparseness.

Consider a matrix $A \in R^{n*m}$ with$n < m$, and define the underdetermined linear system of equations$Ax = b$. In particular this system has more number of unknowns than equations, and thus it has either no solution owing to the condition that ‘b’ is not in the span of the columns of the matrix A, or infinitely many solutions [2]. In order to overcome this anomaly of having no solutions, we assume A to be a full-rank matrix, implying that its columns span the entire space R^n.

Several applications related to science and nature is imitated through underdetermined linear system which is designated using fewer expressions than unknowns. Therefore, realizing a solution for an underdetermined linear system is a predominant subject for a plethora of areas and applications. Some of the concerned domain include: Compressive Sensing (CS) [3, 4, 5, 6, 7], error correction [8], least distance issues in coding theory, and numerous inverse problems. In many applications of signal and image processing we come across a lot of problems that are been formulated by an underdetermined linear systems of equations.

A typical example of this is from the field of image processing, where an unknown image faces a blur and scale-down situation, and the outcome obtained is lower quality and smaller image *b*. The matrix *A* is used as an asset for the degradation operations. Since the main grail is to reconstruct the original image *x* from the given measurements *b*. Although there are infinitely many possible images *x* that can possibly explain *b* but there is only little that may look better than others.

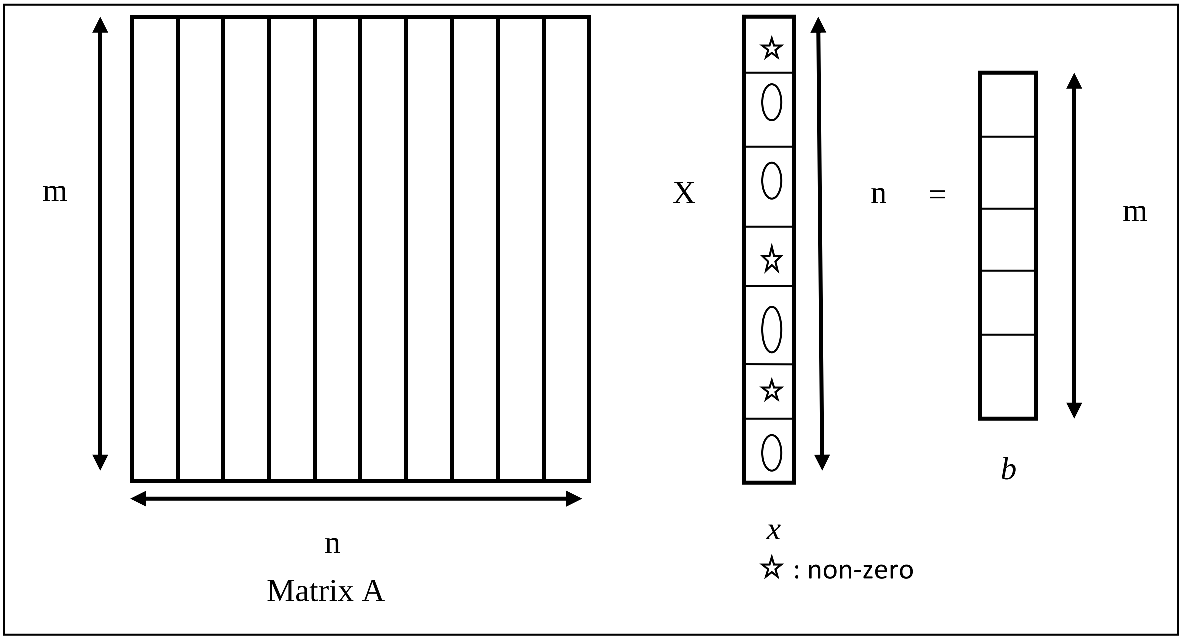

Fig.1.2 Sparse solution of Underdetermined Linear System

1.5.2 Overdetermined Linear Systems

A system, in which the amount of equations is more than the number of unknowns, is known as an over determined system. In the linear case, such a system is given by a rectangular $(m \times n)$matrix,$m > n$, where *m* is the number of equations and *n* is the number of unknowns. The main question for an over determined system is its solvability, expressed by compatibility conditions.

For instance, an over determined system of linear algebraic equations

$$\sum_{j=1}^{m} a_{ij}\, x_j \quad 1 \le i \le n \tag{1.1}$$

is solvable if and only if the rank of the matrix$A = |aij|$.

1.6 Regularization

As being discussed in the above example of image processing and a lot others that undertake the same formulation, the ultimate desire is of a single solution out of the infinitely many solutions. An additional criterion is needed for achieving the same and one of the best used ways to do so is regularization. In this a function $J(x)$ evaluates the desirability of a would-be solution x and giving preference to the smaller values. A general optimization problem (P_j) is defined as

$$(P_j): \quad \min_x J(x) \text{ Subject to } b = A \tag{1.2}$$

The squared Euclidean norm $||x||_2^2$ is the best choice of $J(x)$. Using the Lagrange multiplier we define the lagrangian as

$$£(x) = ||x||_2^2 + \lambda^T(Ax - b), \tag{1.3}$$

Where λ being the Lagrange multiplier for the constraint set. Now we take the derivative of $£(x)$ with respect to x, we obtain the requirement

$$\frac{\partial £(x)}{\partial x} = 2x + A^T\lambda \tag{1.4}$$

Thus an optimum solution is obtained as

$$x_{opt} = -\frac{1}{2}A^T\lambda \tag{1.5}$$

Putting this solution into the constraint $Ax = b$ leads to

$$Ax_{opt} = -\frac{1}{2}AA^T\lambda = b \rightarrow \lambda = -2(AA^T)^{-1}b. \tag{1.6}$$

Substituting this in Equation (1.4) gives the well-known closed-form pseudo-inverse solution

$$x_{opt} = -\frac{1}{2}A^T\lambda = A^T(AA^T)^{-1}b \tag{1.7}$$

Note that since we have assumed that A is full-rank, the matrix AA^T is positive-definite and thus invertible.

1.7 Sparse Decomposition

Consider a linear set of equations $b = Ax$, where A is an underdetermined n x m matrix with n < m and $b \in R^n, x \in R^m$. $'A'$ is called as the dictionary or the design matrix which is been provided. The problem lies in the estimation of the signal x which is subjected to a constraint that it is sparse. Sparsity implies that only a few components of b are non-zero and the rest are zero. Furthermore, b can be decomposed as a linear combination of only a few $n \times 1$ vectors in A, named atoms. $'A'$ itself is an over complete matrix since $n < m$, thus termed as the basis.

The sparse decomposition problem is represented as following,

$$\min_{x \in R^n} ||x||_0 \text{ such that } b = Ax \tag{1.7}$$

Where $||x||_0 = \#\{i: x_i \neq 0, i = 1, \ldots, m\}$ is a pseudo-norm l_0, which counts the number of non-zero components of $x = [x_1, \ldots, x_m]^T$. This is NP-Hard problem and we can obtain its convex form by taking the standard l_1 norm instead of the l_0 norm. But there are conditions set for obtaining sparsity using the l_1 norm.

1.8 Need of Sparse Approximation

Finding a sparse approximation is far more than just an abstract mathematical problem. Sparse approximations have been recognized due to its use in wide variety of practical applications. As it is known that vectors are required to represent a large amount of data which can be difficult to store or transmit? By using the method of sparse approximation the amount of space needed to store the vector would be reduced to a fraction of what was originally needed. Sparse approximations can also be used for analyzing data by the view how column vectors in a given basis come together to produce the data. Areas of science and technology have immensely benefited from the advancement involving sparse approximations. Sparse approximations are used in denoising, in painting, feature extraction and gene micro array analysis. The prominent nature of sparse approximations has prompted great interest and a lot of researches have been made out of it. The sparse representation terminology is used in representative methodology of the LRBM and is proven an extraordinary powerful solution to a plethora of application fields, especially in signal processing, image processing, machine learning, and computer vision, such as image denoising, de-blurring, in-painting, image restoration, super-resolution, visual tracking, image classification and image segmentation.

1.9 Haar Wavelet

The Haar wavelet is progression of rescaled functions which cooperatively form a wavelet basis. Wavelet analysis resembles Fourier analysis as it allows a target function over a span to be expressed in the form of an orthonormal basis. Haar wavelet is a simplest form of DWT. The Haar sequence is recognized as the first known wavelet basis and extensively used as a teaching example.

Haar wavelet generates basis vectors for wavelet and scaling at different levels. In short Haar wavelet gives sum and difference after decompositions performed. The inverse Haar operation is very easy because it is orthonormal in nature and it can be explained by

$$H^{-1} = H^{T} \tag{1.8}$$

Where H is a transformation matrix and it possess columns as basis vector.

The Haar wavelet possesses notable properties:

- Any CT real function with compact support can be coarse-grained uniformly by the linear combination $\emptyset_t$ and their scaled versions. This expands to the function spaces, where the function considered can be approximated by continuous functions.
- Any CT real function on [0, 1] range can be uniformly approximated on [0, 1] by the linear combination of wavelet function and their shifted version.
- The wavelet function is orthogonal and orthonormal in nature.

1.10 Weighted Least Square

Weighted least square method, is employed to display the characteristics of the random errors in any model. This method can be employed, with the functions having either linear or non-linear parameters. It toils by utilizing extra non-negative constants and weights. One of the main issues, related with WLS, is designing FIR filters with reduced computational complexity. By elucidating a linear operator, that is influenced, on the coefficient vector of the filter, the optimality constraint of the design issue is represented as a linear operator equation.

*****

2

LITERATURE REVIEW

"I may not agree with what you say, but I'll defend to the death your right to say it"

-Mr.Voltaire

In the introduction to sparse FIR filter, the main objective underlined is to learn the basic fundamentals behind it and what all are the benefits regarding the designing of such a filter. Researchers developed various methods and used different algorithms in achieving the sparse version of the filter with an ease. Thus this section includes the researches carried out by far in order to design an efficient Sparse FIR filter.

Aimin Jiang, Hon Keung Kwan, Yanping Zhu, Xianfeng Liu, Ning Xu and Yibin Tang (2015): In this tract, two novel algorithms are employed for optimizing the filter order and the sparsity of filter coefficients. The original sparse filter design problem is regularized in the objective function by using an extra term to penalize large filter lengths. The IRLS algorithm is employed with suitable moderation to resolve both weighted l_o-norm minimization issue. Compared to traditional sparse FIR filter, the above proposed algorithm isolate the greedy strategy which leads to better designs in terms of the number of nonzero coefficients and the effective filter order [13].

Yuhua Yang, Wei-Ping Zhu and Dalei Wu (2015): A new design technique is presented in this tract for delineating a sparse FIR filter. Since it is known that computational cost related to digital filter design greatly depends upon the amount of filter quotients, thus filters with sparse quotients are of appreciable engrossment. Here sparsity is obtained through two aspects i.e. one by utilizing l_1 minimization stratagem and the other with approximation of sparse filter quotients attained through the minimax and the LS design criteria [14].

V. Sowmya*, Neethu Mohan and K. P. Soman (2015): Image denoising based on sparse banded filter matrices is propounded in this paper. Since it is evident that noise is the quintessential element that demeans the image standards, therefore image denoising proves to be an efficient image enhancement technique. Here a LP-sparse banded filter matrices are employed for image denoising. Sparsity is considered as the prime concept in the filter delineation. Designed filter to denoise the image is formulated row-wise and column-wise. The proposed technique is examined on canonical test images liable to different noises with varying noise level. The potency of denoising acquired by our propounded technique is proved by the notable enhancement in canonical quality metric known as PSNR facilitated by the visual inspection [15].

Aimin Jiang, Hon Keung Kwan, Yibin Tang and Yanping Zhu (2014): In this tract, a novel method is developed to design sparse LP FIR filters. Conventional sparse FIR filter depiction procedure emphasize on how to increment the amount of zero-valued quotients, but overlook the influence of filter order on draft performance The design method focus on how to enhance the sparsity of filter coefficients under a set of specifications and thus describe the impact of filter orders on final designs. The design issue is then recasted as a l_0-norm optimization, elucidated through an efficient method that is built on the IRLS algorithm. Thus this proposed method can be used to determine appropriate filter order of FIR filter automatically while improving the sparsity of the filter [16].

Aiming Jiang and Hon Keung Kwan (2013): In this exegesis, a novel algorithm for the delineation of sparse FIR filter is presented. The main grail about the sparse digital filter depiction is to lessen the non-zero filter quotients that are subjected to weighted least square (WLS) estimation error restraint. The propounded design procedure is inspired by IST algorithms, worn by the sparse and unnecessary description for signals. A comparison between the results obtained from the proposed algorithm and hard thresholding algorithm show that a much better designs can be obtained by the proposed algorithm as the original WLS design problem does not take the sparsity requirement into account. In future it can be applied to design variable fractional delay (VFD) FIR filters [17].

Chien-Cheng Tseng and Su-Ling Lee (2012): In this it is delineated, the orthogonal matching pursuit (OMP) as the base to design the sparse constrained FIR filter. Further, two kinds of filters are studied. The advantages of the proposed design method are met through the tradeoff between sparsity of filter quotients and magnitude response errors. Numerical examples demonstrate the effectiveness of the propounded design method. Thus the above proffered method shows that it can be used in medical field to determine electrocardiogram (ECG) signal by neutralizing the consequence of 60-Hz power-line intervention [18].

Aimin Jiang, Hon Keung Kwan and Yanping Zhu (2012): In this tract, a novel algorithm is propounded to delineate sparse FIR filters. It is evident from the design methodology that the issue is exceptionally non-convex owing to the presence of l_o-norm in the intended function of filter quotient vector. Thus an iterative procedure to find a potential sparsity pattern for the FIR filters is developed, which is then utilized to enumerate the end solution through the resolution of a convex optimization issue. Analysis indicates that the proposed algorithm can efficiently and reliably deal with the l_0- norm design problem with a multiple set of quadratic constraints. The future scope of this proposed algorithm is that it can be used to address any optimization task related to FIR filters [19].

Aimin Jiang, Hon Keung Kwan, Yanping Zhu, and Xiaofeng Liu (2012): In this tract, it is demonstrated that a sparse FIR filter can be propounded effectively using minimax design. It is capable of handling nonconvexity problem and develops an efficient iterative procedure to find sparse pattern. A sub problem is created in every repetition in an effortless way and we won't solve sub problem directly. Rather it is expedient through their specific dual problems. The comprehensive iterative stratagem can coincide to optimal solution of the aboriginal draft problem. The actual minimax draft can then be obtained by purifying the FIR filter procured through the iterative procedure [20].

Dennis Wei and Alan V. Oppenheim (2011): In this exegesis, we propounded an algorithm established on branch-and-bound technique for delineating maximally sparse filters subject to a quadratic restraint on filter performance. Also an optimization method is utilized that either assure an optimal solution or furnish a sparse solution. Also to minimize the complexity of branch and bound, diverse techniques are initiated for limiting the filter's computational cost.

The results obtained show that the complexity is reduced substantially by employing diagonal relaxations. The techniques in this paper make optimal design more accessible not only to the filter designers but also to developers of design algorithms. Future work is directed at using more sophisticated implementations of the branch-and-bound algorithm [21].

Thomas Baran, Dennis Wei and Alan V. Oppenheim (2010): In this tract two approaches are bestowed to delineate sparse FIR filters. Conventionally the delineation of discrete filters measured the computational cost on the basis of the span of impulse response. As it is known, that the system complexity greatly depends on the arithmetic operations been carried out while designing it, thus non-zero quotients are a metric of concern, which is effectively reduced, thereby omitting the arithmetic operations carried out on zero-valued quotients. This enhances the computational requirements a lot. First approach utilizes an impulse response which is successively thinned according to predefined rules and the remaining coefficients are recomputed after each thinning to minimize the error in the frequency domain. The second approach uses a minimal 1-norm to determine which coefficients are constrained to have zero value in subsequent optimizations aimed at increasing sparsity. This design method can be used to reduce the area and power consumed while implementing an Application Specific Integrated Circuit (ASIC) [22].

Wu-Sheng Lu and Takao Hinamoto (2010): In this tract, a new visualization aroused in the authors' mind to examine half-band sparse FIR filters. Issue related to sparsity in the delineation of filter is discussed, along with its importance. Further, designing of digital filter with sparse impulse response is portrayed; owing to the fact that efficient implementation is attained with sparse version of the filter respective to its non-sparse analogue. Also there exist several techniques like frequency response masking technique that guarantees an efficient implementation of FIR filters [23].

Wu-Sheng Lu, Takao Hinamoto (2010): In this exegesis it is demonstrated that by employing a sparsity-assisting norm in a two-phase convex optimization scheme, a 2-D FIR filters with sparse coefficients can be prospered. Simulation reviews are dispensed to manifest that with suitable options of design attributes, predominantly the values of mew (μ) and epsilon (ε_t), optimal LS and minimax filters liable to a target quotient sparsity K, can be procured to surmount their analogue nonsparse counterparts. A hitch of the classification of sparse filters measured in this tract is their colossal group delay respective to their nonsparse analogue. Therefore, studies should be carried out for sparse digital filters possessing low group delay. It is also perceived through the experiments that whilst optimized sparse filters render refined performance compared to their nonsparse analogue, the quantity of enhancement proliferates from case to case [24].

Allen Y. Yang, S. Shankar Sastry, Arvind Ganesh and Yi Ma (2010): This exegesis has given an insight to the five fast l_1-minimization techniques, i.e. homotopy, GP, IST, PR, and elevated Lagrange multiplier. The algorithms are analyzed on the premise of complexity parameter pertaining to attain sparse signals and to strengthen the performance. In general the special CAB model can be used in applications like robust face recognition on real training images, using an already established sparse characterization framework that identifies or recuperate human characteristics from facial pictures that get ostentatious by occlusion, illumination alteration and facial camouflage [25].

Oscar Gustafsson, Linda S. DeBrunner, Victor Debrunner and Hakan Johansson (2007): In this exegesis, proposed design of linear-phase FIR filters is done with the use of few non-zero multiplier coefficients. Relaxed specifications are formulated to design filters close to half-band filters. It is noticed that the amount of non-zero filter quotients is diminished with the propounded design methodology. An increase in passband ripple is utilized to decrease the number of multiplications. For future work it is worth noticing that minimizing the l_1- norm of the solution to a set of underdetermined equations will result in efficient sparse solution [26].

Joel A. Tropp and Anna C. Gilbert (2007): In this exegesis, it is manifested theoretically and factually that OMP can genuinely recuperate a signal with m nonzero entries through a certain random linear quantification. It is also inferred that the OMP is an effective alternative to BP for signal recovery from random measurements. There is a significant increase in performance obtained from OMP algorithms and it provides faster and easier way to implement problems for signal recovery [27].

Joel A. Tropp (2004): This paper provides an insight to the orthogonal matching pursuit (OMP) algorithm to solve sparse problems defined over random dictionaries. It guarantees an adequate state in which both OMP and BP paradigm can recuperate the primal representation of absolutely sparse signal. This hypothesis proclaims that OMP and BP prosper for each sparse excitation signal occupied from a plethora of dictionaries. Also a progressive coherence function is established to fathom the rank of incoherence. These examinations amalgamate all recent outcomes on BP and expand them to OMP. Also the paper provides a sufficient condition under which OMP can identify atoms from an optimal approximation of a non-sparse signal. Therefore, it asserts that OMP quantifies to be an approximation algorithm under any sparse representation subject to a quasi-incoherent glossary [28].

Davide Mattera, Francesco Palmieri and Simon Haykin (2002): In this tract, a novel algorithm is propounded for sparse FIR filter design. Here the equivalence between the considered problem and the problem of determining a sparse solution of a linear system of equation is used as the base for design procedure. It is contrary to the earlier design methods that were employed that utilized brilliant forage over all plausible anatomy for sparse filter. The proposed method provides a good compromise between the computational complexity and the performance of the obtained filter [29].

Young-Seog Song and Yong Hoon Lee (1997): The branch and bound algorithm used in this exegesis depicts an efficient manner to discover the deliberately zeroed tap positions and enables us to delineate a superlative sparse FIR filter under disparate optimization criterions. Future work of branch-and-bound algorithm is that it can be used to design unequally spaced antenna arrays. Design exemplar manifest that the propounded procedure entail less computation cost than the traditional optimization method [30].

John W. Adams and Alan N. Willson, Jr. (1983): In this exegesis, it is inferred that equalization has been quintessential in specific circumstances due to discoloration in diverse segments of a system. Here it is forced to remunerate for flawed amplifiers, transmission lines and so on. Furthermore, it is propounded to intentionally employ equalization concept in order to deal with the matter of computational complexity related to digital filters. It is manifested that at the cost off a minor increment in the quantity of delays, a cardinal diminution in the amount of multipliers and adders can be procured. Only linear phase FIR digital filters are reviewed; evidently the technique could also be enacted on IIR filters which can be an area of exploration [31].

*****

3

SCOPE OF STUDY AND OBJECTIVES

"The formulation of the problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill." *-Albert Einstein*

In the last chapter, after analyzing the parameters like filter order, number of zero value coefficients etc., researchers found different methods for designing sparse FIR filters. Algorithms like matching pursuit, orthogonal matching pursuit and IRLS method emerged as a solution to design the sparse version of an FIR filter. The performance on the basis of reduced filter coefficients that were achieved after applying these algorithms enhanced it to a large extent. Also the complexity issue that is faced while designing the linear phase FIR filters got minimized, after the design of the sparse version of the filter. In this research, WLS error constraint is used to delineate the FIR filter which is then made sparse using different greedy algorithms to examine all the restraints that lead to an efficient design of sparse FIR filter.

3.1 Scope of Study

Over a decade of years, signal processing domain was implementing FIR filters i.e., finite impulse response. There are numerous techniques accessible for delineating linear phase FIR filters. Every propounded method has its own merits and demerits. In various signal processing applications like noise cancellation, equalization FIR filters are implemented at high sampling rates, but this increases the complexity of the system because of large number of non-zero coefficients. Due to this Sparse FIR filters came into existence because of less number of non-zero valued coefficients and finally reducing the system complexity. These filters are based on Linear Programming. Sparsity of the filter quotients is related to the order of the filter i.e. as the order of the filter is increased the amount of sparsity is enhanced. In sparse FIR filters arithmetic operations are less which reduces the hardware requirement and this leads to less complexity and reduced computation cost. By optimizing filter length an efficient sparse FIR filter can be designed and various algorithms like IRLS, IST etc. try to optimize the filter length to reduce complexity. With the help of above knowledge, one can delineate two dimensional filters easily by employing of sparsity.

3.2 Objective of the Study

The use of sparsity led to many reforms in the implementation of the linear-phase FIR filters. This designing of the filter was useful in many ways, but below are the underlying objectives that are to be achieved in this dissertation:

i. Is to reduce the computational cost by decreasing the implementation complexity.
ii. Generally for an FIR filter the Sparsity of filter coefficients is directly related to its filter order. While applying Sparsity to the FIR filter, if the filter order is high, less is the number of multiplication and additions to be done due to the presence of more zero valued coefficients. So a novel method is employed to achieve the lesser number of multipliers and adders in the design of the FIR filter.
iii. Improve design efficiency along with algorithm’s stability as it owns to be a critical factor while delineating a filter.
iv. Thus the underlining objective for delineation of sparse FIR filter is to provide a tradeoff between the two constraints i.e., implementation complexity and filter order.

3.3 Workplan

Different FIR filters are there in use but are implemented at a high cost. So sparsity has evolved which is an effective way to formulate results related to study FIR filters. Implementation complexity and computational cost is reduced to considerable amount by using the sparse algorithms. First, the delineation of sparse FIR filter was accomplished by employing the Haar wavelet method. Various factors are studied on the basis of which it is rendered that the sparse FIR filter does reduce the computational cost by minimizing the implementation complexity.

At present, sparse FIR filter was accomplished through WLS error constraint for procuring the filter coefficients and then subjected to different algorithms to get the sparse version of the filter. Also diverse factors are computed for each algorithm that is utilized while delineating the sparse FIR filter. The work plan that was carried out throughout this research is shown with the help of below flowcharts depicted in the fig.3.1 and fig.3.2. This was the whole approach that was carried out from the February of 2016 till the end of April 2017.

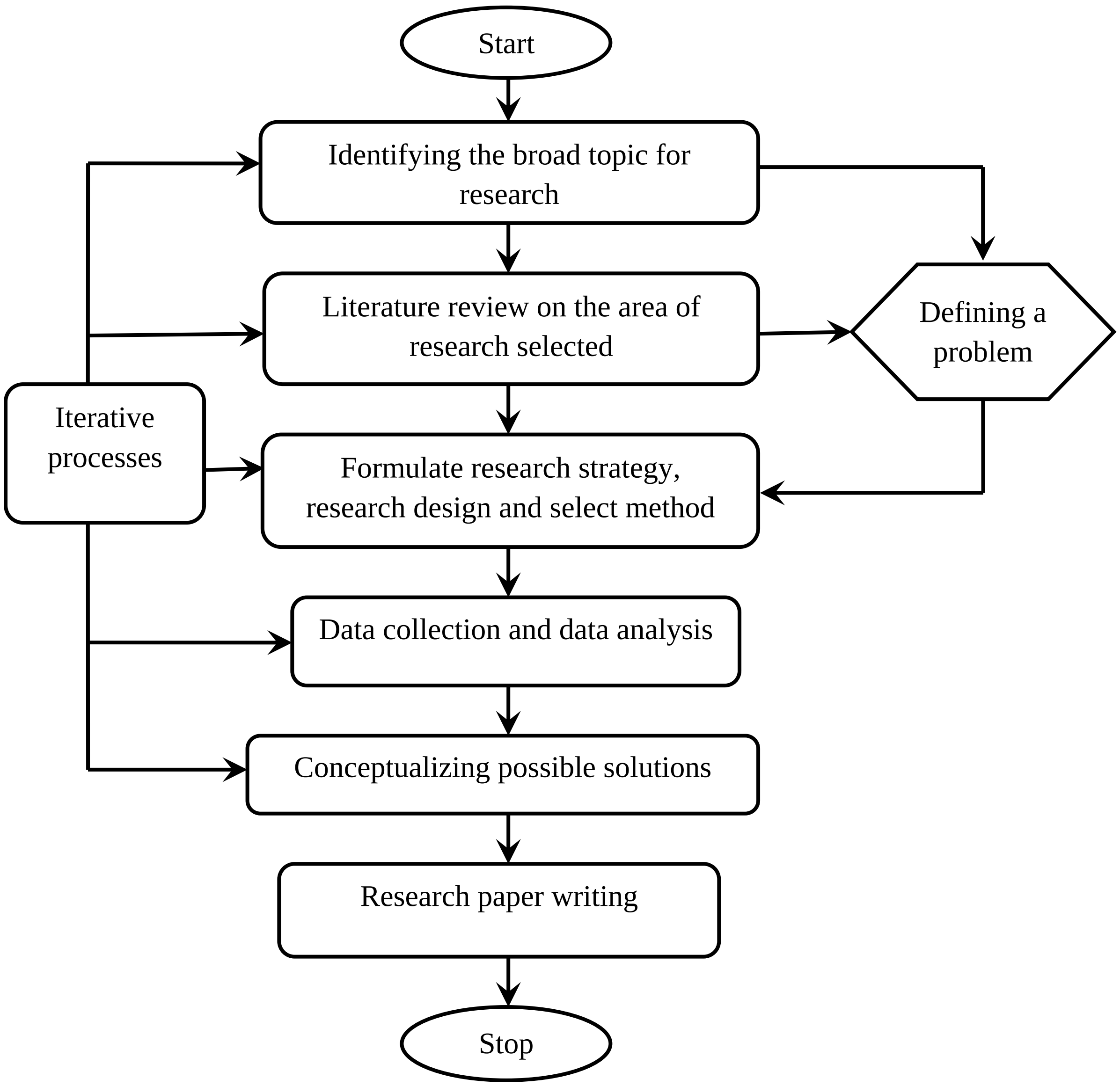

Fig. 3.1 Analysis on designing of sparse FIR filter

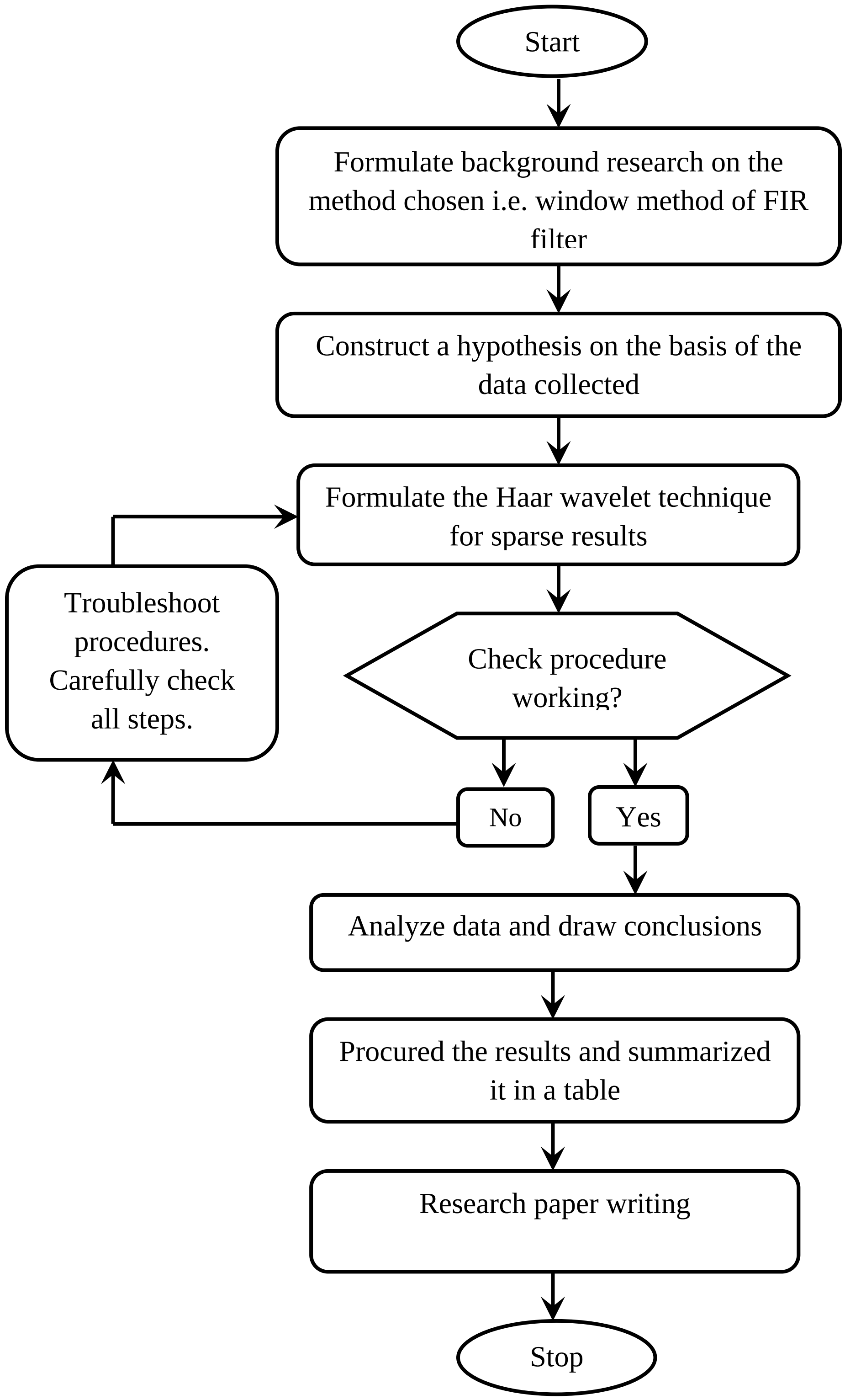

Fig.3.2 Designing of sparse FIR using Haar Wavelet

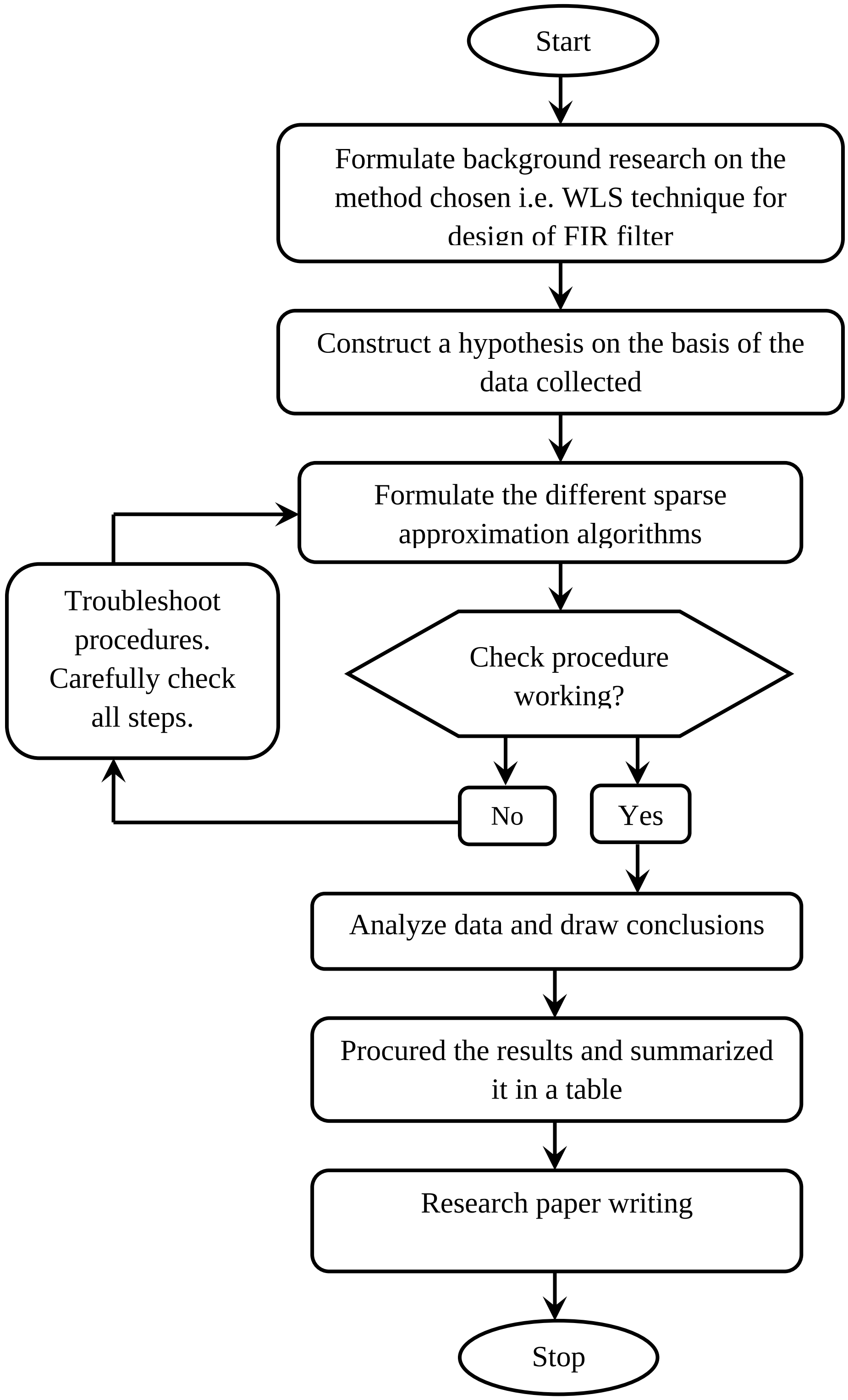

Fig.3.3 Designing of sparse FIR filter using WLS

*****

4

RESEARCH METHODOLOGY

"Get the habit of analysis; analysis well in time enables synthesis to become your habit of mind." *-Frank Lloyd Wright*

The objectives as underlined in the foregoing chapter i.e. minimization of the implementation complexity is been analyzed and fulfilled through the diverse methods employed. As it is mentioned in introduction, that while designing an FIR filter one need consider the ways by which the design procedure does not turn out to be cumbersome resulting in the increase of computational cost. Various studies are going on in the current period to overcome these problems. A perfect solution to this problem is done, by introducing the concept of sparsity, in the design procedure of the filter.

4.1 Designing of Sparse FIR using WLS

The algorithms formulated are analyzed and data is well assembled in order to meet the specifications that are standardized in the objective section. This section outlines how to delineate LP FIR filters established upon square error criterion. Recollect that the interpolation method to filter design authorizes one to identify the frequency response described only at finite point of instances. Nevertheless, square error criterion can contain the complete frequency band. Here the design methodology of FIR filter is observed through the WLS method. The FIR filters that reduce the square error are formulated through the linear set of equations.

Since it is known that a weighted integral square fallacy (i.e. l_2-error) is given by the equation,

$$\varepsilon_2 = \int_0^{\pi} W(\omega)(A(\omega) - D(w))^2 d\omega \qquad (4.1)$$

The weighting function plays important role by assigning additional significance to specific portions of the frequency spectrum. After the order $'N'$ of the filter is procured by setting the values of different attributes like passband frequency, stopband frequency etc. the grail is to realize the filter quotients *h (n)* that minimizesε_2.

Recollect that for a Type-1 FIR filter,

$$A(\omega) = \sum_{m=0}^{M} a(n)\cos(n\omega) \tag{4.2}$$

Where

$$a(0) = h(M), \qquad a(n) = 2h(M-n), \qquad 1 \le n \le M$$

In order to procure the quotients $a(n)$ to minimize errorε_2, the derivatives are set to zero,

$$\frac{d\varepsilon_2}{da(r)} = 0, \;\; 0 \le r \le M \tag{4.3}$$

After solving the above equation, the following results are obtained

$$V(r,n) = \frac{1}{\pi}\int_0^{\pi} W(\omega)\cos(n\omega)\cos(r\omega)d\omega \tag{4.4}$$

And

$$b(r) = \frac{1}{\pi}\int_0^{\pi} W(\omega)\mathrm{D}(\omega)\cos(r\omega)d\omega \tag{4.5}$$

The derivative conditions are written as

$$\sum_{n=0}^{M} V(r,n)a(n) = b(r), \qquad 0 \le r \le M \tag{4.6}$$

The above equality can be represented in the form of linear equations and it can be composed in matrix form as given below

$$\begin{bmatrix} V(0,0) & \cdots & V(0,M) \\ \vdots & \ddots & \vdots \\ V(M,0) & \cdots & V(M,M) \end{bmatrix} \times \begin{bmatrix} a(0) \\ a(1) \\ \vdots \\ a(M) \end{bmatrix} = \begin{bmatrix} b(0) \\ b(1) \\ \vdots \\ b(M) \end{bmatrix} \tag{4.7}$$

or simply

$$V \times a = b \tag{4.8}$$

the type-1 FIR filter can simply be realized by calculating this linear set of equations,

$$a = V^{-1}b \tag{4.9}$$

The matrix $V(r,n)$ is further broken into two matrices V_1 and V_2 where V_1 represents a symmetric Toeplitz matrix and V_2 represents a Hankel matrix. The Toeplitz matrix has constant values along its diagonal and the Hankel matrix has constant values along its anti-diagonals.

Consequently the matrices thus obtained require less memory storage i.e. $(M+1)$ instead of $(M+1)^2$. The Toeplitz and the Hankel matrices thus eventually decrease the computational complexity from $O(M^3)$ to $O(M^2)$.

After further manipulations with $V(r,n)$ and $b(n)$ we procure the value of $a(n)$ stated as

$$a(n) = \frac{2}{\pi}\int_0^{\pi} D(\omega) \times \cos(n\omega)\, d\omega, \qquad 1 \le n \le M \tag{4.10}$$

This in turn gives the solution for the filter coefficients of FIR filter i.e.

$$h(n) = \frac{1}{\pi}\int_0^{\pi} D(\omega) \cos\big((n-M)\omega\big)\, d\omega, \qquad 0 \le n \le N \tag{4.11}$$

The weighting function $W(\omega)$ can be used to improve the FIR low-pass filter because

i. It entitles to banish Gibbs phenomenon by erasing a neighborhood around the band edge, and
ii. It entitles to allocate different weights to PB and SB.

After formulation of the above listed equations and manipulations the value of the filter coefficients $h(n)$ are procured. Further, for making the FIR filter sparse diverse algorithms are implemented, which helps in reducing the computational complexity. Explanation of the algorithmic steps is properly stated in the near context, which will help in analyzing the problem effectively.

4.2 Algorithms

In order to locate the sparse solution different algorithms are been employed which are standardized through a set of specifications. These set of algorithms are orthogonal matching pursuit (OMP), matching pursuit (MP), least-square orthogonal matching pursuit (LS-OMP), weak matching pursuit (MP) and hard thresholding, thus providing us the best sparse solution for the given filter coefficients. The steps involved in the various algorithms are stated below.

4.2.1 Orthogonal matching pursuit

Input:

- Given a signal $'b'$ and matrix $'A'$.
- Ending criterion which symbolizes the level of accuracy.

Output:

- Approximation vector $'x'$.

Algorithm:

- Undertake the residual $R_o = b$, the time $t=0$ and the index set $V_o = \emptyset$.
- Set $s_t = i$, a_i gives the panacea of max $<R_t, a_k>$, here a_k are the row vectors of D
- Upgrade the set V_t with $s_t: V_t = V_{t-1} \cup \{s_t\}$
- Enumerate the recent residual employing x

$$R_t = R_{t-1} - \sum_{j=1}^{t} x(s_j)\, a_{v_j}$$

- Augment the value of iteration.
- Check the stopping criterion and if it is not satisfied return to step 2

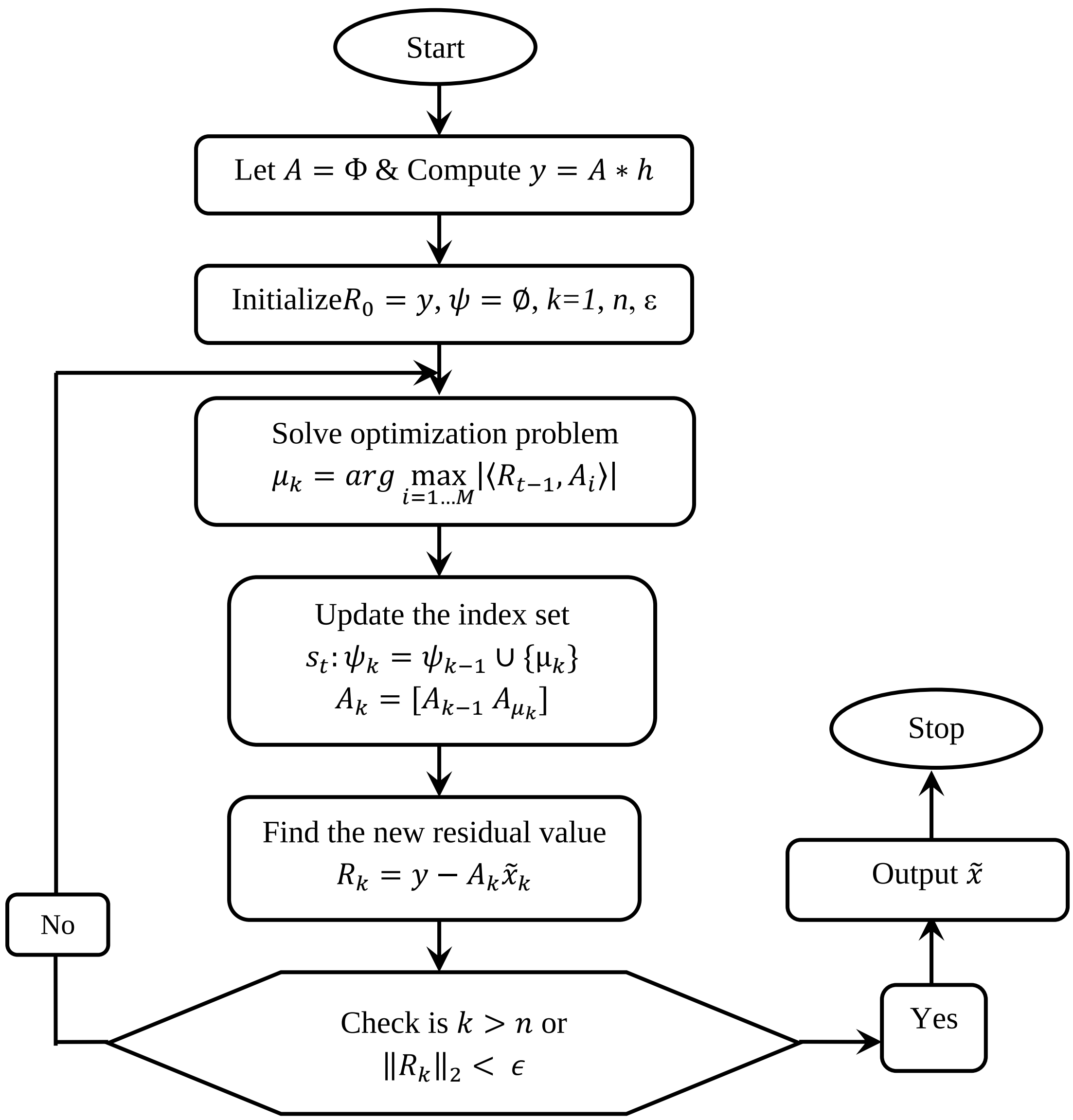

Fig.4.1 Flowchart for Orthogonal Matching Pursuit

Here the main zest is to produce the minimum error that is similar to the maximum absolute value of the inner product obtained between the residual and the normalized matrix A.

4.2.2 Least-square orthogonal matching pursuit (LS-OMP)

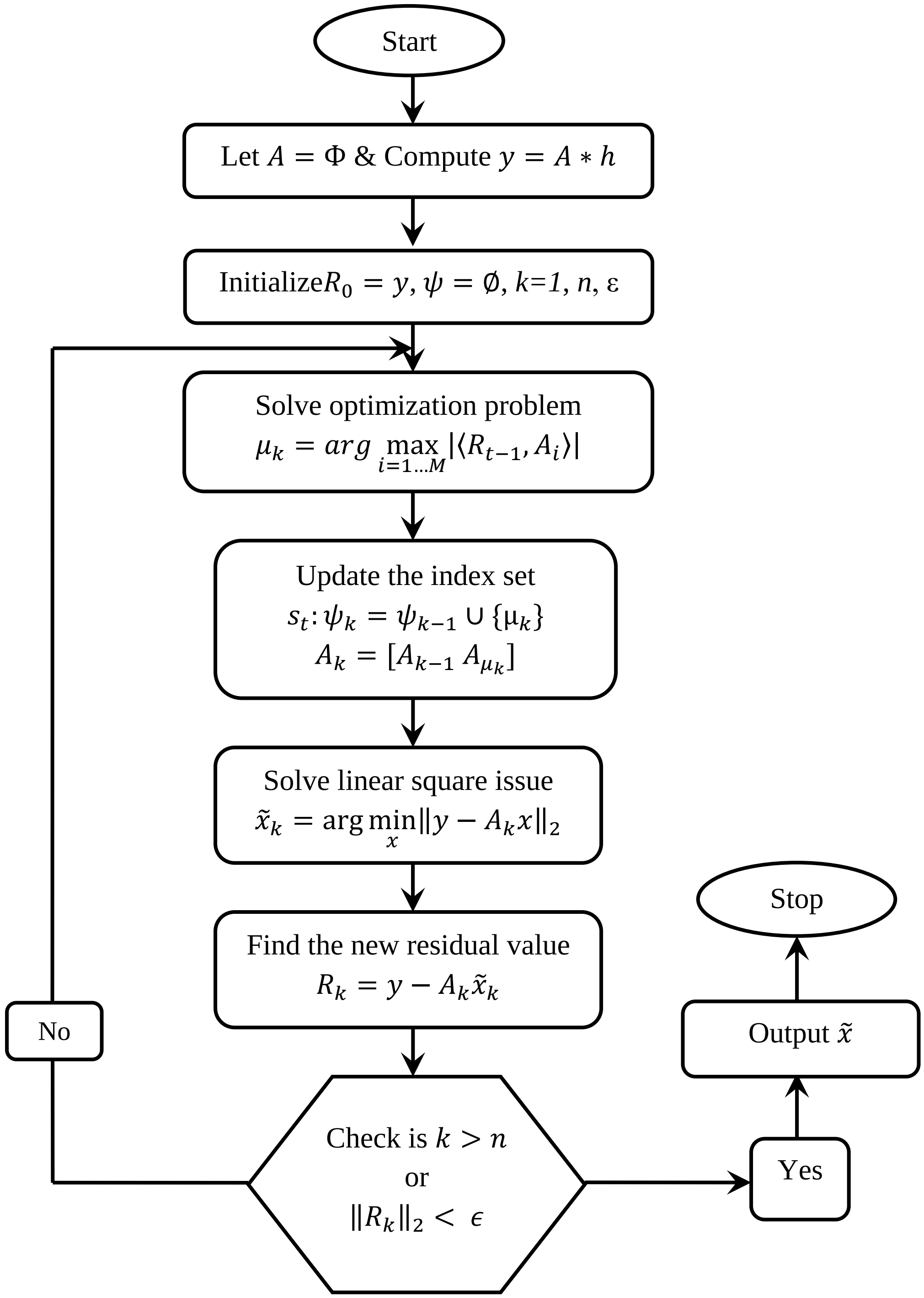

Fig.4.2 Flowchart for Least-Square Orthogonal Matching Pursuit

Input:

- Given a signal ‘$'b'$ and matrix$'A'$.
- Ending criteria which symbolize the level of accuracy.

Output:

- Approximation vector$'x'$.

Algorithm:

- Begin by setting the residual $R_o = b$, the time *t=0* and the index set $V_o = \emptyset$
- Assume $s_t = i$, $a_i$gives the solution of max <R_t, a_k> here a_k are the row vectors of D
- Upgrade the set V_t with s_t: $V_t = V_{t-1} \cup \{s_t\}$
- Resolve the least-squares issue

$$\min_{c \in C^{V_t}} ||y - \sum_{j=1}^{t} x(s_j) a_{V_j} ||_2$$

- Enumerate the recent residual using x

$$R_t = R_{t-1} - \sum_{j=1}^{t} x(s_j)\, a_{v_j}$$

- Set t←t+1
- Check the stopping criterion and if it is not satisfied return to step 2.

It is somewhat similar to the orthogonal matching pursuit (OMP), but it performs an additional step of least square that helps in trading higher accuracy level with simplified computation. Thus the error is significantly reduced in the case of least square OMP.

4.2.3 Matching pursuit (MP)

Input:

- Given a signal $'b'$ and matrix$'A'$.
- Ending criteria which symbolize the level of accuracy.

Output:

- Approximation solution x: $min_x ||x||_0$ subject to $Ax = b$

Initialization: Initialize iteration value k=0, and set

- The starting solution x^0= 0.
- The starting residual$r^0 = b - Ax^0 = b$.
- The starting solution support $S^0 = Support\{x^0\} = \emptyset$

Algorithm:

- Sweep: Calculate the errors $\in(i) = min_{z_i}||a_i z_i - r^{k-1}||_2^2$ for all i with the optimal choice $z_i^* = a_i^T r^{k-1}/||a_i||_2^2$.
- Upgrade support: Locate a minimized value i_0 of $\in(i)$: where $1 \le i \le m, \in(i_0) \le \in(i)$, and update the support $S^k = S^{k-1} \cup \{i_0\}$.
- Upgrade provisional solution: Set $x^k = x^{k-1}$, and update the entry $x^k(i_0) = x^k(i_0) + z_i^*$.
- Upgrade residual: Compute $r^k = b - Ax^k = r^{k-1} - z_{i_0}^* a_{i_0}$.
- Ending rule: If $||r^k||_2 < \in_0$, then stop the iteration. Otherwise perform iteration.

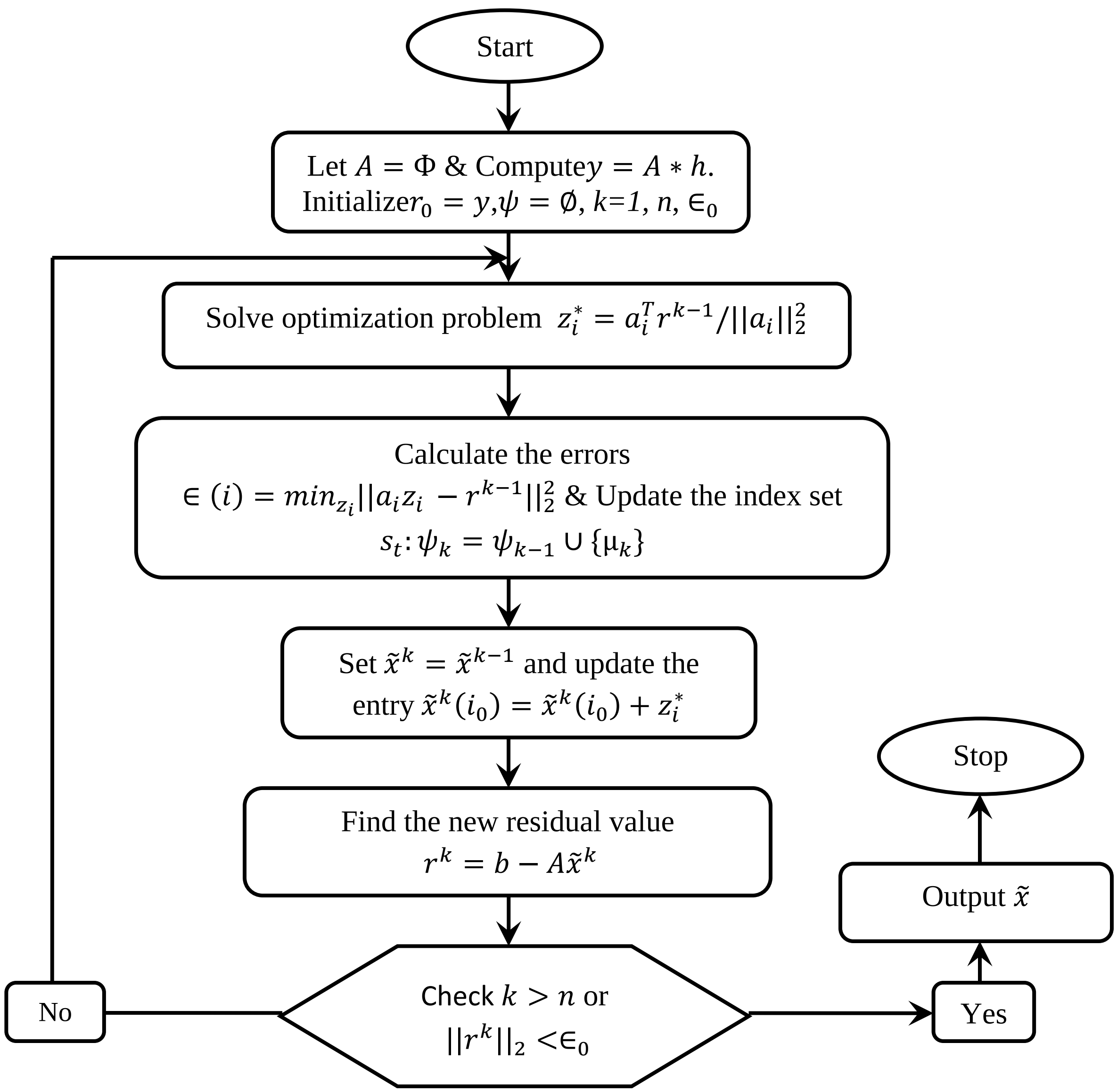

Fig.4.3 Flowchart for Matching Pursuit

In this algorithm, main motive is to find an atom in the dictionary that best resembles the given input signal. After this the weighted value obtained for this atom is removed, and we again find an atom from the dictionary that completely resembles the remaining signal. This is continued till the end rule is satisfied. Since, it is clear from the sweep step, that we had already determined quotients that minimize residual error, so again reckoning the whole quotient vector is overkill. This algorithm aims at achieving accuracy in order to have simplified computation. The main drawback with matching pursuit relates to the computational complexity imposed by the design procedure.

4.2.4 Weak matching pursuit (WMP)

Input:

- Given a signal $'b'$ and matrix$'A'$.
- Ending criteria which symbolize the level of accuracy.
- Scalar$'t'$ set within the range of $0 < t < 1$.

Output:

- Approximation solution x: $min_x ||x||_0$ subject to $Ax = b$

Initialization: Initialize iteration value k=0, and set

- The starting solution x^0= 0.
- The starting residual$r^0 = b - Ax^0 = b$.
- The starting solution support$S^0 = Support\{x^0\} = \emptyset$.

Algorithm:

- Sweep: Calculate the errors $\in (i) = min_{z_i} ||a_i z_i - r^{k-1}||_2^2$ for all i with the optimal choice $z_i^* = a_i^T r^{k-1} / ||a_i||_2^2$. Stop sweep criteria when $|a_i^T r^{k-1}| / \; ||a_i||_2 \geq t. ||r^{k-1}||_2$.
- Upgrade support: Locate a minimized value i_0 of $\in (i)$: where$1 \leq i \leq m$,$\in (i_0) \leq \in (i)$, and update the support $S^k = S^{k-1} \cup \{i_0\}$.
- Renovate provisional solution: Set $x^k = x^{k-1}$, and update the entry $x^k(i_0) = x^k(i_0) + z_i^*$.
- Renovate Residual: Compute $r^k = b - Ax^k = r^{k-1} - z_{i_0}^* a_{i_0}$.
- Ending rule: If $||r^k||_2 < \in_0$, then stop the iteration. Otherwise perform iteration.

This algorithm gives more simplifications in terms of approximation error as compared to the matching pursuit. The main advantage is that it reduces the sweep step so time consumed in formulating the error is comparatively shortened.

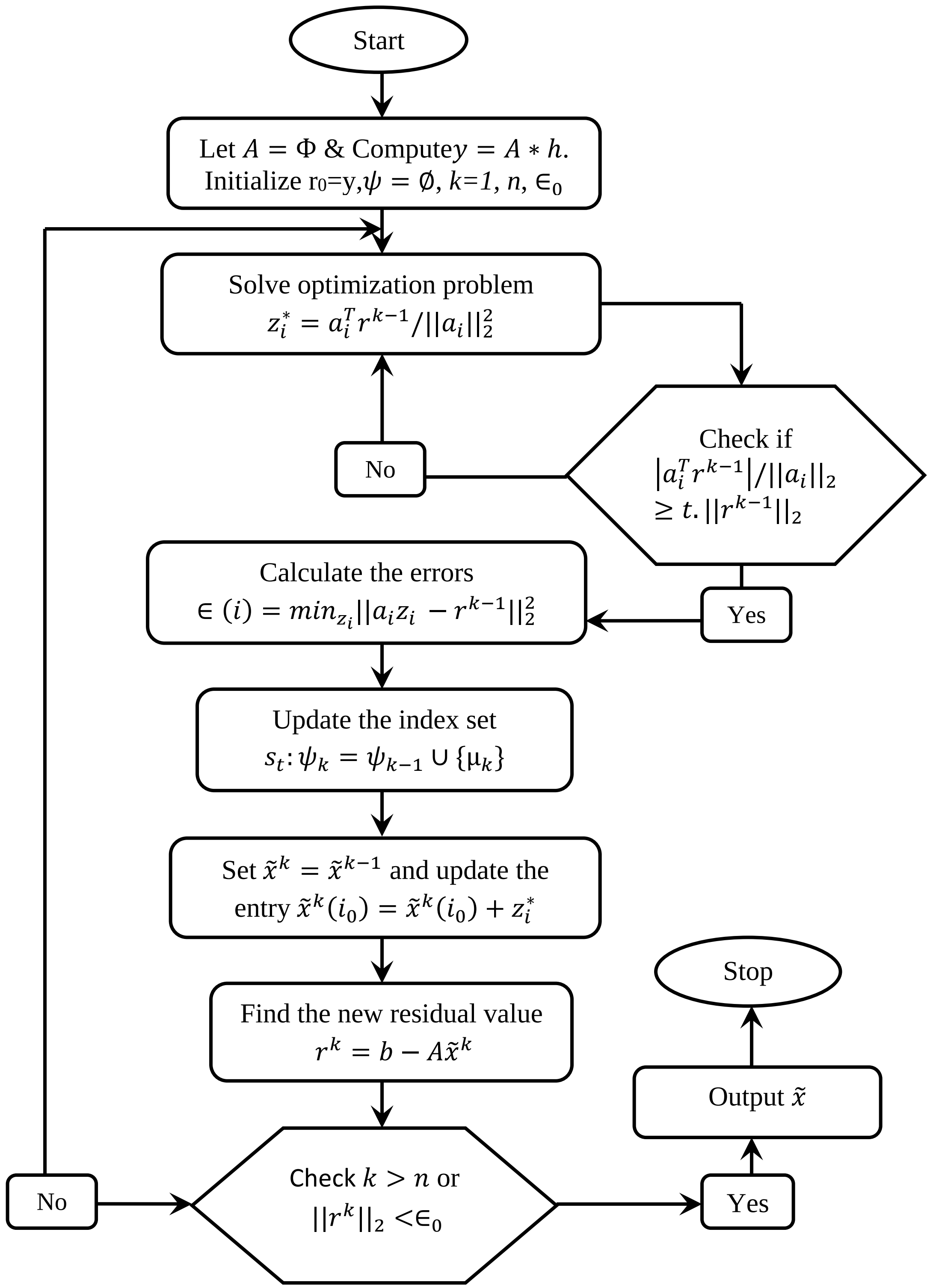

Fig.4.4 Flowchart for Weak Matching Pursuit

4.2.5 Hard thresholding

Input:

- Given a signal $'b'$ and matrix$'A'$.
- Ending criteria which symbolize the level of accuracy.
- Number of atoms that are desired i.e. $'k'$.

Output:

- Approximation solution $'x'$: $min_x||x||_0$ subject to $Ax = b$

Algorithm:

- Begin Evaluation: Calculate the errors $\in(i) = min_{z_i}||a_i z_i - b||_2^2$ for all i with the optimal choice$z_i^* = a_i^T b/||a_i||_2^2$.
- Upgrade support: Locate a minimized value of error by looking in the S indices with cardinality$k$: where$i \in S, \epsilon(j) \leq min_{j \notin S}\epsilon(i)$.
- Renovate provisional solution: Calculate x^k minimized value of $||Ax - b||_2^2$.
- End result: Thus the proposed solution obtained is$'x'$.

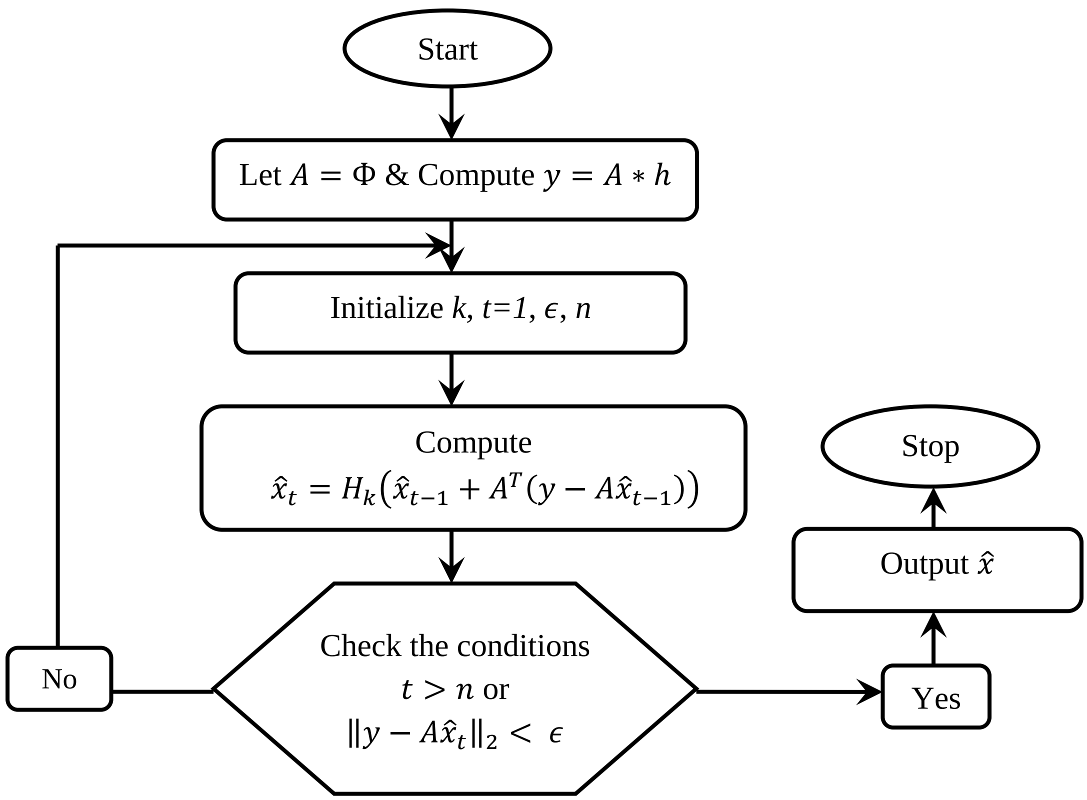

Fig.4.5 Flowchart for Hard Thresholding

The thresholding algorithm is far simpler than the above mentioned matching pursuit algorithms. Here the sweep is computed once and then finally a k largest inner product value is chosen as the support. Error is computed using the formula stated in the algorithm steps.

4.3 Designing of sparse FIR using Haar Wavelet

In general designing of the FIR filter using window technique, the filter quotients procured are made sparse using the Haar wavelet transform. While delineating the filter, the basic issue of computational complexity arises. This is because of the number of calculations performed are high. It is effectively removed, by the help of this method, as it reduces the number of non-zero coefficients, measured by the l_0 -norm. Therefore, the number of calculations for the zero-valued coefficients is omitted. Also computational complexity measured through l_2-norm is very less. The steps that are followed for obtaining this sparse version of the filter are stated below:

4.3.1 Algorithmic Steps

- Setting the value of different attributes like sampling frequency (F_s), passband ripple (δ_p), stopband ripple (δ_s), passband frequency (ω_p), stopband frequency (ω_s) and cutoff frequency (f_c).
- Calculate the order value $'N'$ by the use of above stated parameters.
- Using window methods design the FIR filter for different order values.
- Compute the filter coefficients through this method and store it in a variable i.e. *h (n).*
- Apply the Haar wavelet method of DWT transform on these coefficients and store them in a variable *x.*
- Then apply some predefined threshold to make the coefficients of *x* sparse.
- The values thus obtained give us the sparse version of the filter, meaning the more information containing quotients are kept while the others are neglected.
- Obtain the values of the attributes i.e. l_0-norm, l_1-norm, l_2-norm and l_∞-norm
- Compare the attribute values of the nonsparse FIR filter and the one obtained using the Haar wavelet technique.
- Computational complications are measured using these two attributes.

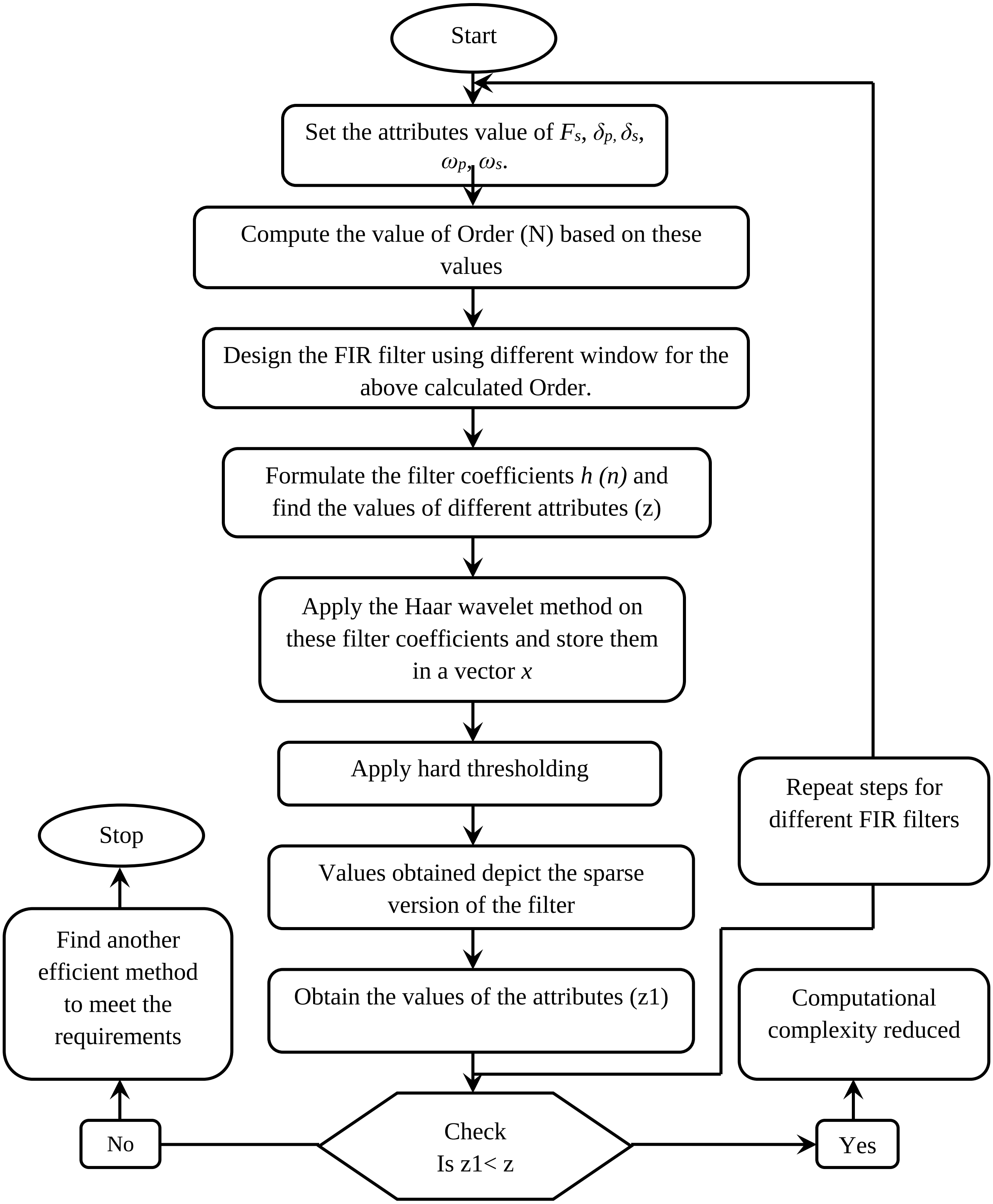

Fig.4.6 Flowchart of Haar Wavelet Method

*****

5

RESULTS AND DISCUSSIONS

"The aim of argument, or of discussion, should not be victory, but progress." -Joseph Joubert

The flowchart drawn in the foregoing chapter helps in delineating the numerous types of sparse FIR filters. The Haar transform method is discussed first, on the basis of which the procurement of sparse FIR filter is done. While delineation of the filter type various parameters are considered which help in analyzing the efficiency of the method. Further, the WLS technique for delineation of FIR filter is formulated. Consequently, a bunch of diverse algorithms are applied on the filter coefficients procured through the WLS method. Results obtained gave reduced implementation complexity, which is derived from the parameters considered while designing the filter. In order to locate the sparse solution different algorithms are been employed which are standardized through a set of specifications. These set of algorithms are orthogonal matching pursuit (OMP), matching pursuit (MP), least-square orthogonal matching pursuit (LS-OMP), weak matching pursuit (MP) and hard thresholding, thus providing us the best sparse solution for the given filter coefficients.

While formulating the above algorithms a random sensing matrix A of size $M \times N$with normalized entries is created. Here N is the order of the filter defined. For different values of $M$and N the random matrix A is created. The columns of this matrix is normalized using the l_2-norm. The generated filter coefficients using the WLS method are then made sparse with the erratic supports of cardinalities. Iteration value specified is 1000 for every algorithm. The cardinality range is from 1 to 10 and the non-zero values to be drawn as arbitrary random uniform coefficients. Once the value of this signal x is computed, where x is the value of the filter coefficients i.e. 'h' which is derived through the use of the WLS method, formulation of $b=Ax$ is done followed by the implementation of the above mentioned algorithms to seek for sparse value of x.

5.1 Attributes for analysis

For the evaluation of the above algorithms and methods, diverse attributes were employed in order to measure the efficiency and effectiveness. Following are the list of attributes exploited:

5.1.1 Number of states

The FIR number of states basically tells about the coefficient pairs involved. It also indicates the following

- Memory requirement for designing of the filter.
- Amount of calculations needed, and
- Measure of filtering the filter can provide.

5.1.2 Passband ripple (δ_p)

The filter passband and stopband may possess oscillations that are termed as ripples. δ_p specify the value of passband ripple, which is equivalent to the max divergence from unity.

$$P.B.ripple = -20\, log_{10}(1 - \delta_p) \tag{5.1}$$

5.1.3 Stopband ripple (δ_s)

δ_s stipulate the value of stopband ripple, which is equivalent to the max divergence from the base value zero.

$$S.B.ripple = -20\, log_{10}(1 - \delta_s) \tag{5.2}$$

5.1.4 Passband frequency (ω_p)

A passband frequency is the part of frequency spectrum that is allowed by the filter to pass through. It is characterized by the least relative loss or extreme relative gain.

5.1.5 Stopband frequency (ω_s)

A stopband is the range of frequencies, within specified bounds, through which a filter, terminates signals to pass, or attenuation is beyond the prescribed stopband attenuation range.

5.1.6 Sampling frequency (F_s)

The sampling frequency, F_s, is the average amount of samples procured in one second (i.e. *samples per second*).

5.1.7 Order (N)

When an excitation is given to a filter, response is computed using present inputs, previous inputs and past outputs. Past inputs and previous outputs are just the delayed excitation and delayed responses. Order of any filter is defined as the maximum measure of delay that is employed in the computation of any output. The filter order is directly proportional to the no. of calculations required or no. of elements requisite to procure the filter.

5.1.8 l_0-norm

Norms quantify the length of vectors in a particular domain - but they also tell us about the distance functions i.e. the norm is applied to the difference of coefficients of any two vectors. The l_0-norm is the measure of the amount of non-zero elements in a given vector. The l_0-norm is widespread in the compressive sensing area which endeavors to procure the sparsest solution to an underdetermined set of equations.

5.1.9 l_1-norm

l_1 –norm a.k.a Manhattan-norm alias least absolute deviations (LAD), least absolute errors (LAE) is usually utilized for miniaturizing the summation of absolute differences between the target value $(y_i)$and the approximated values $f(z_i)$

$$||x||_1 = \sum_{i=1}^{m} |y_i - f(z_i)| \tag{5.3}$$

5.1.10 l_2-norm

l_2 -norm a.k.a Euclidean-norm alias least squares is basically employed for miniaturizing the summation of square of differences between the target value (y_i) and the approximated values $f(z_i)$

$$||x||_2 = \sum_{i=1}^{m} (y_i - f(z_i))^2 \tag{5.4}$$

5.1.11 l_∞-norm

The l_∞-norm is basically used for computing the largest absolute value of the vector coefficients. Therefore the l_∞-norm is just the measure of the maximum derivative quotient.

$$||\bar{x}||_\infty = \max\_abs\ (\bar{x}) \tag{5.5}$$

The LAD or the l_1-norm is instable in nature, since for a slight adjustment in the data, the regression line has to move a large distance. There may be solutions that can give possible results for the data selected, but due to this long jump by the regression line, it passes over this region of solutions. Thus there is a considerable change in the slope obtained after this, which deviates a lot from the previous ascent. In contrast, the euclidean-norm alias l_2-norm is highly stable, since for a minor adjustment in the data point, there is only a slight movement of regression line. This states that regression attributes are continuous form of data.

The method of LAD i.e. l_1-norm is much more robust than the l_2-norm, since it does not bother about the outliers in data. Thus it turns to be effective in applications where outliers are effectively ignored. But as here the basic need is reducing computational complications thus it is desired to consider the outliers. Therefore, the l_2-norm is most preferable in such cases. But still the l_1-norm value will help us understand the robustness of the design procedure.

Another advantage of l_2-norm over l_1-norm is that it provides unique solutions. This is understood by the fact that l_2-norm uses direct path to a solution i.e. unique minuscule trail. But the l_1-norm has an in-built feature of selecting the useful coefficients from a given vector. Such as, if a filter has 100 coefficients out of which only 20 are non-zero quotients, then the l_1-norm will utilize these 20 quotients without considering the effect of the rest. The l_1-norm also bears the property of sparsity, i.e. only few quotients in a vector are non-zero, and producing a large amount of zero quotients.

Euclidean norm possess an important property of efficient computational solutions. The l_1-norm does not possess any analytical approach to a particular solution. But the l_1-norm does guarantees sparse solutions, thus allowing it to be employed along with the sparse algorithms like OMP, LS-OMP, MP, WMP, Thresholding and many more. This helps in achieving calculations which are much more computationally efficient.

5.2 Designing of sparse FIR using Haar Wavelet

The procurement of the sparse FIR filter is done with the Haar Wavelet method. The computation results are analyzed, through which the sparsity level of the FIR filter is judged. The main grail of computational complexity is measured with the help of the diverse attributes like l_0-norm, l_1-norm, l_2-norm, and l_∞-norm, which portray the effectiveness of the applied methodology.

The order of the filter is calculated by setting the values of passband ripple, stopband ripple, passband frequency, stopband frequency and sampling frequency. After this computation, the filter is designed using the three windows i.e. Kaiser, hamming and hanning. Haar wavelet is applied after procuring the filter coefficients. A predefined threshold value is set, and the coefficients obtained after implementing Haar wavelet are made sparse. Further, the value of attributes i.e. l_o-norm, l_1-norm, l_2-norm, and l_∞-norm are computed. Table.5.1 illustrates the value obtained for these parameters.

Table.5.1 Various parameter values of passband frequency, stopband frequency, l_0 (sparsity) l_1-error, l_2-error, l_∞ for Haar wavelet algorithm

Algorithm	ω_p	ω_s	Window	N	l_o	l_1 Manhattan Norm (10^-2)	l_2 Euclidean Norm(10^-4)	$l_{infinity}$
Haar Wavelet	0.0397pi	0.0596pi	Kaiser	30	9	3.19	13	0.0543
	0.0367pi	0.0489pi		50	12	2.124	6.7481	0.0397
	0.0227pi	0.0303pi		80	14	1.80	5.5818	1.4594
	0.0207pi	0.0276pi		**88**	14	1.79	5.6539	1.5944
	0.0122pi	0.0184pi		100	16	1.78	5.8281	1.7999
	0.0397pi	0.0596pi	Hamming	30	8	3.53	18	0.0668
	0.0367pi	0.0489pi		50	11	2.38	8.9497	0.0500
	0.0227pi	0.0303pi		80	13	1.77	5.8289	0.0487
	0.0207pi	0.0276pi		**88**	14	1.73	5.6511	0.0499
	0.0122pi	0.0184pi		100	15	1.63	5.3860	0.0520
	0.0397pi	0.0596pi	Hanning	30	7	3.79	22	0.0727
	0.0367pi	0.0489pi		50	10	2.38	9.5065	0.0533
	0.0227pi	0.0303pi		80	13	1.74	5.8349	0.0493
	0.0207pi	0.0276pi		**88**	14	1.69	5.5750	0.0500
	0.0122pi	0.0184pi		100	14	1.59	5.2428	0.0516

Sparsity is always defined with the help of l_o-norm. The value of sparsity shows a high increase in the count of zero-valued coefficients with the increase in order. Thus the notion that l_o-norm defined for sparse representation issues is very convenient and intuitive, is true.

The values of l_1-norm actually give sparser results, providing unique solutions for a given problem. Here it is inferred that as the order value is increased the l_1-norm value reduces greatly. Thus the vector obtained, after application of Haar wavelet algorithm, has greater number of zeros, which are further increased by imposing l_1-norm on it. Analytically the l_1-norm is not very effective for a particular solution, but still the sparse results obtained are due to this norm only. The non-zero quotients are less in case of higher order values which is due to the l_1-norm. Note that when the order is 88 the value of l_0-norm is 14, this signifies that the optimal solution of reduced computational complexity is achieved. The same value of sparsity is obtained for all the three windows.

The issue of reducing computational complication is accomplished by calculating the l_2-norm alias Euclidean-norm. There is a relation between the order and the value of l_2-norm as depicted through the values procured i.e. there is considerable decrease in l_2-norm value with the increase in the order (N). The norm is therefore defined as a global convex, smooth differentiable function. Note that the value of l_2-norm is almost same for order 88, thus this denote that, one can select any a window among the three, for designing the filter. The values of different attributes can be easily analyzed through the graphs illustrated in the fig.5.1 to fig.5.4.

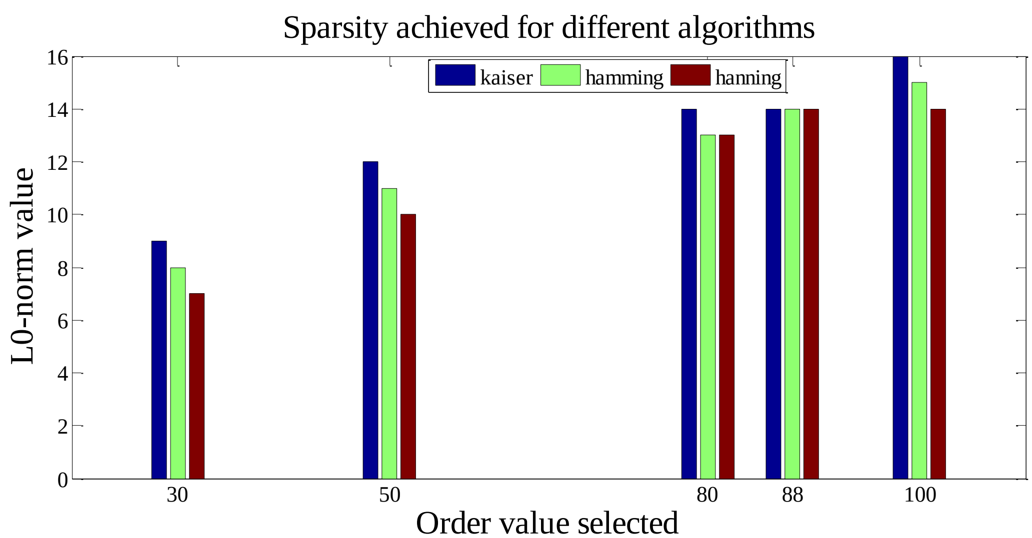

Fig.5.1 l_0-norm value for different windows using Haar wavelet method

The sparsity obtained for order 88, is almost same for all the three windows employed. This result represents that the order value does play an important role in procuring less number of non-zero coefficients. Also for an increase in order from 30 to 88, subsequently helps in reducing the computational complexity. For the Kaiser window the l_0-norm value obtained for orders 30 to 100 is very high in comparison to the other three algorithms, therefore, the hamming and hanning window stands out to be more effective. The best out of the above windows for reducing computational complexity is hanning through which the designing of the filter is easier.

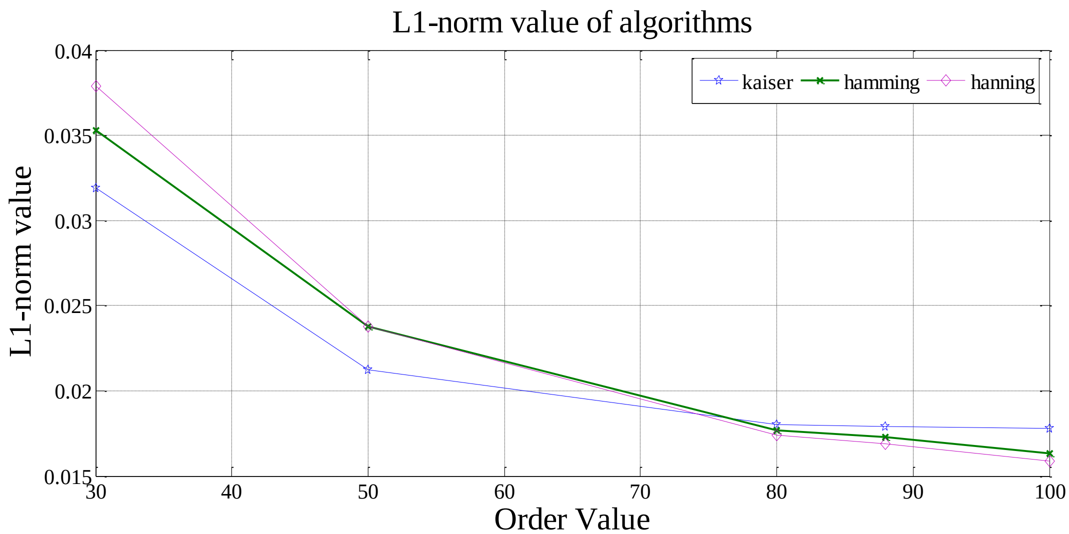

Fig.5.2 l_1-norm value for different windows using Haar wavelet method

For the different value set, the filter coefficients obtained through the Haar Wavelet Algorithm are penalized on the basis of the l_1-norm. The l_0-norm just gives us the zest of how many are the non-zero components present in the vector. From the above fig.5.2, the value of l_1-norm decreases for any further increase in the order (N). Since the l_1-norm provides solution by considering each and every quotients of the vector, therefore it ends up giving a solution with more number of zeroes. Accuracy is not the aim with l_1-norm, since we only need to find a sparser result for optimization. Therefore for analysis of filter spectrum is done on the basis of l_1-norm values, which quite possibly resembles the non-sparse filter. Note that for order ranging from 30 to 100, the l_1-norm values are smaller for hanning window in comparison to the other.

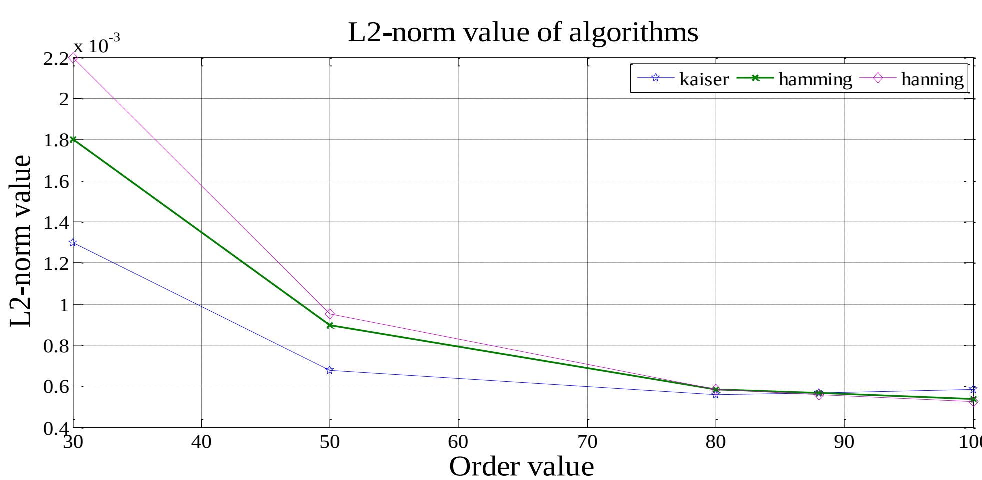

Fig.5.3 l_2-norm value for different windows using Haar wavelet method

The l_2-norm possesses the advantage of convexity, invariant rotation and analytical competency, therefore the issue of reducing computational complexity is accomplished by calculating the l_2-norm alias Euclidean-norm. There exists a correlation between the order and the value of l_2-norm as depicted through the values procured. A considerable decrease in l_2-norm value is estimated with the increase in the order (N). Note for order '88', all of the three windows have the same value for l_2-norm as shown in the fig.5.3. Therefore, choosing any one out of the three windows will definitely reduce the complexity issues related to the filter design. But, as the order value is increased to 100 the Hanning window gave the best results for l_2-norm. Therefore, for the analysis of filter both the constraints are very important, since sparseness and reduced computational complexity are to be achieved effectively.

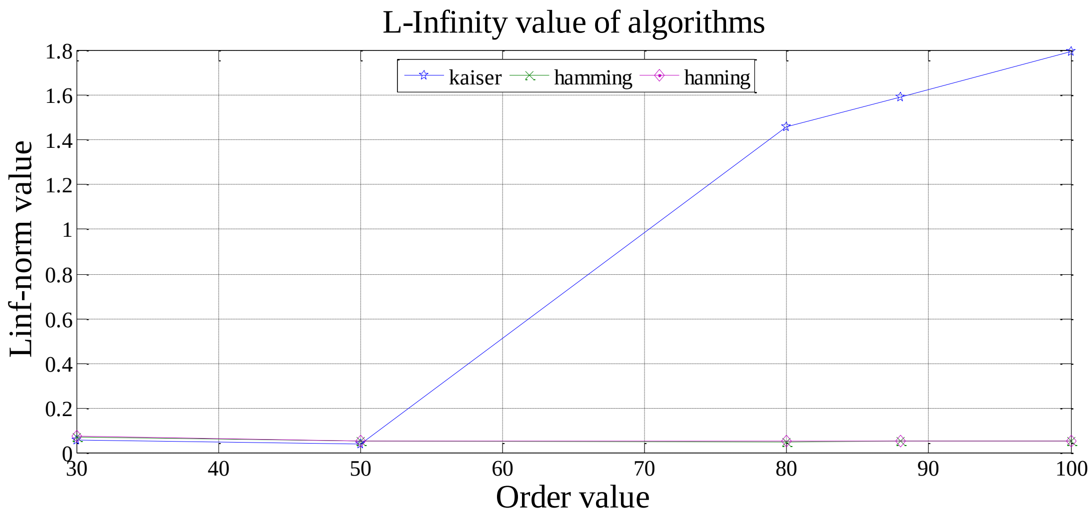

Fig.5.4 l_∞-norm value for different windows using Haar wavelet method

The maximum absolute value is obtained using the l_∞-norm. From the fig.5.4, the l_∞-norm value is almost the same for Hanning and hamming window. While for the Kaiser window, it is maximum and changes for each order value.

5.3 Designing of sparse FIR using WLS Method

The FIR filter is delineated using the WLS method. Using the weighted function helps in eliminating the Gibbs phenomenon by effectively erasing the neighbourhood around the band edge, and it also allows setting different weights for the pass-band and stop-band. Here the weight value is set at the beginning defining the structure accordingly.

By the help of l_0-norm, a sparse FIR filter designed in WLS sense can be manifested by the following expression

$$\min ||h||_0 \quad \text{Subject to (s.t)}$$

$$\sum_{\omega_i \in \Omega_I} W(\omega_i)|H(e^{j\omega_i}) - D(\omega_i)|^2 \leq \delta_d \tag{5.6}$$

Where Ω_i denotes union of the frequency bands of heed, $D(\omega)$ represents the given ideal frequency response, and d, $W(\omega)$ represents the nonnegative weighting functions. Here $h = [h_0 h_1 \dots h_n]^T$ for the obtained frequency response. The set of coefficients that minimizes the square error are obtained by solving a linear set of equations given by

$$a = Q^{-1}b \tag{5.7}$$

In order to minimize the error $\in_2$, the derivative of the error with respect to the obtained coefficients $a(n)$ is set to zero with n ranging from 0 to M. After further manipulations the $Q(n,k)$ and $b(n)$ coefficients are formed given us a set of linear system of equations which is additionally depicted in the matrix form. Thus the FIR filter coefficients are obtained by regulating the $a(n)$ coefficient vector within the range defined.

In order to locate the sparse solution different algorithms are been employed which are standardized through a set of specifications. These set of algorithms are orthogonal matching pursuit (OMP), matching pursuit (MP), least-square orthogonal matching pursuit (LS-OMP), weak matching pursuit (MP) and hard thresholding, thus providing us the best sparse solution for the given filter coefficients *h(n)*.

The filter quotients obtained through the WLS method are made sparse through the five algorithms, therefore optimizing them through the set of steps defined in the methodology. Table 5.2 depicts the values of diverse attributes employed while delineating the filter. In the literature survey, the attribute sparsity is always defined with the help of l_o-norm. As the $||x||_p$ is convex for the values of p greater than and equal to 1. Therefore it is more difficult to optimize the solution using l_o-norm. But still the l_o-norm can be considered as a very convenient and effective attribute for sparse representation issues. It signifies the standard for obtaining a filter with reduced computations.

As p value increases, the value of norm moves toward the maximum functions. This effectively penalizes the largest argument in the set. But the l_1-norm donate similar penalty to all attributes, thus enforcing sparsity. Thus the vector obtained after application of l_1-norm on the different algorithms has greater number of zeros.

Table.5.2 Various parameter values of passband frequency, stopband frequency, l_0 (sparsity) l_1-error, l_2-error, l_∞ for different algorithms

Algorithms	M	ω_p	ω_s	N	l_o	l_1 Manhattan Norm (10^-2)	l_2 Euclidean Norm(10^-4)	$l_{infinity(10^{-2})}$
OMP	20	0.0397pi	0.0596pi	30	14	4.6447	7.0148	7.0878
	30	0.0367pi	0.0489pi	50	24	1.7514	0.7144	7.8617
	70	0.0227pi	0.0303pi	80	39	0.3062	0.7728	1.6716
	60	0.0207pi	0.0276pi	**88**	14	0.5956	**0.7482**	2.7224
	70				20	0.6081	**0.7498**	3.2075
	90	0.0122pi	0.0184pi	100	45	0.2039	0.1137	0.9908
LS-OMP	20	0.0397pi	0.0596pi	30	14	2.2944	6.5988	5.6232
	30	0.0367pi	0.0489pi	50	23	1.6543	0.9115	8.2046
	70	0.0227pi	0.0303pi	80	36	0.2676	0.7734	1.5038
	60	0.0207pi	0.0276pi	**88**	14	0.2945	**0.1667**	1.0811
	70				20	0.6494	**0.8829**	3.2075
	90	0.0122pi	0.0184pi	100	44	0.1984	0.0975	0.6586
MP	20	0.0397pi	0.0596pi	30	25	4.2565	4.6266	3.9095
	30	0.0367pi	0.0489pi	50	38	1.9165	2.7833	4.9652
	70	0.0227pi	0.0303pi	80	64	0.4395	1.1859	2.9630
	60	0.0207pi	0.0276pi	**88**	14	0.7813	**0.9861**	2.4835
	70				20	0.6730	**0.9502**	3.5047
	90	0.0122pi	0.0184pi	100	70	0.4785	0.4077	2.2673
WMP	20	0.0397pi	0.0596pi	30	26	4.2565	4.6266	26.39
	30	0.0367pi	0.0489pi	50	40	1.9319	1.3508	13.70
	70	0.0227pi	0.0303pi	80	67	0.5302	1.1859	1.942
	60	0.0207pi	0.0276pi	**88**	14	0.7813	**0.9861**	2.483
	70				20	0.6468	**0.7763**	2.722
	90	0.0122pi	0.0184pi	100	71	0.4785	0.4077	1.814
Thresholding	20	0.0397pi	0.0596pi	30	19	4.2706	129.020	20.39
	30	0.0367pi	0.0489pi	50	37	2.0807	64.281	26.31
	70	0.0227pi	0.0303pi	80	65	2.1317	139.477	18.06
	60	0.0207pi	0.0276pi	**88**	14	3.6791	**26.8502**	14.67
	70				20	1.4717	**4.0035**	6.62
	90	0.0122pi	0.0184pi	100	85	2.7904	6.7347	6.15

Here, the results obtained through l_1-norm are more robust and reduces with the increase in order. Since the l_1-norm is diamond shape therefore it is not appropriately differentiable.

Therefore, analytically the l_1-norm is not very effective. The issue of reducing computational complexity is accomplished by calculating the l_2-norm alias Euclidean-norm. There is a considerable decrease in l_2-norm value with the increase in the order (N). The values of different attributes can be easily analyzed through the graphs illustrated in the fig.5.5 to fig.5.8.

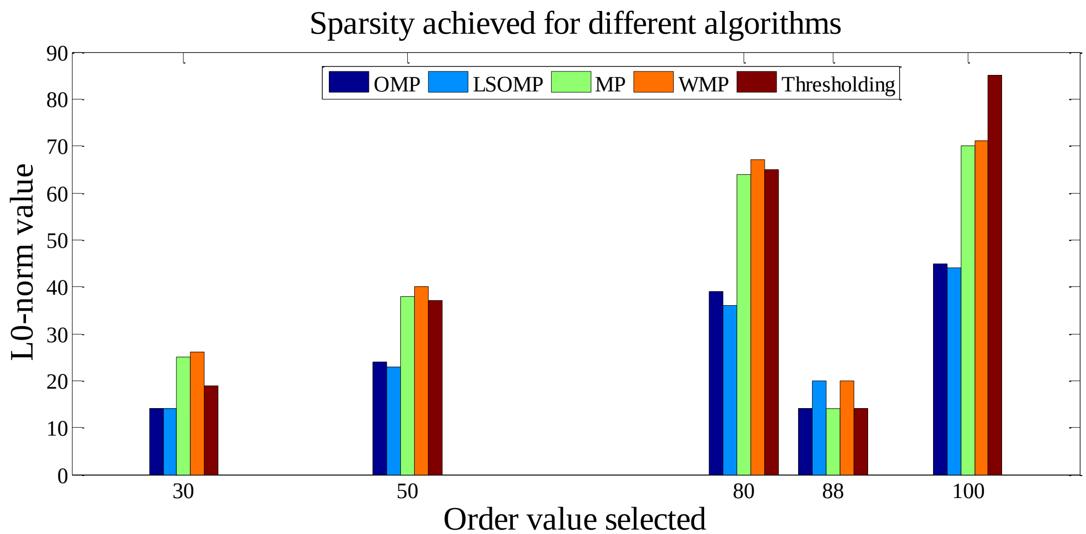

Fig.5.5 l_0-norm value for different order using different algorithms

The l_0-norm obtained for the various algorithms in fig.5.5 show that the, number of non-zero coefficients are considerably reduced for an increase in the order. The LS-OMP algorithm stands out, to be the best for each order value. Note that for order 88 the sparsity level was set at 14 and 20 for each algorithm. For all other order, the l_0-norm is computed through the steps of the different algorithms, for an iteration value of 200.

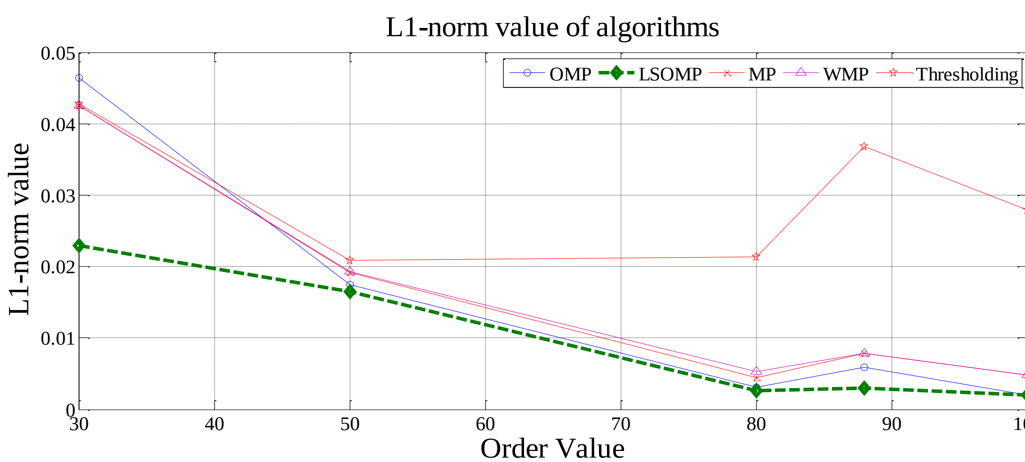

Fig.5.6 l_1-norm value for different order using different

The values of l_1-norm actually give sparser results, providing unique solutions for a given problem. As the order value is increased the l_1-norm value reduces greatly nullifying the tradeoff between the order and sparsity. Since the main issue is computational efficiency, therefore the overall process includes only scalar operations. Note that for each order value, as depicted in the fig.5.6, the l_1-norm is least for the LS-OMP followed by the OMP algorithm. The sparser results obtained for these two algorithms are far better than the other three algorithms.

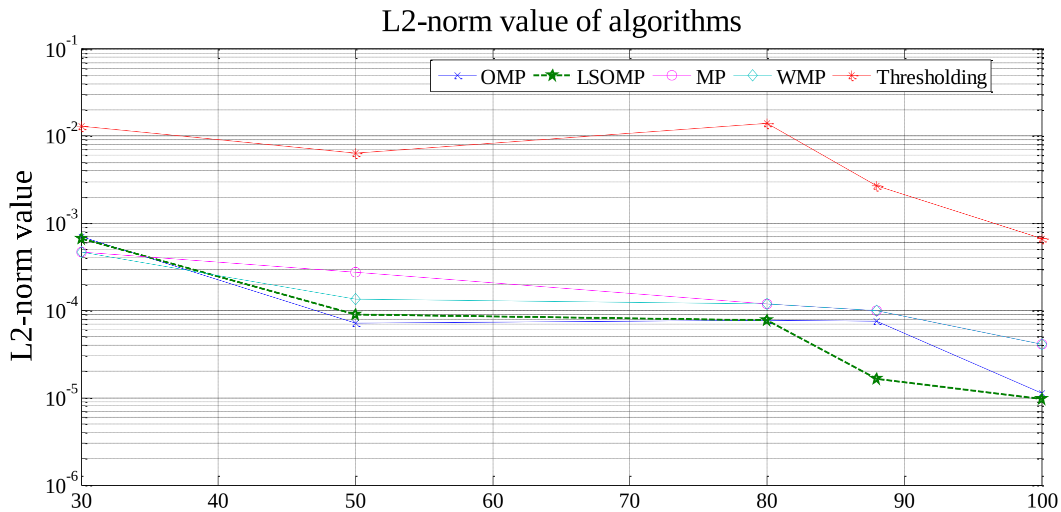

Fig.5.7 l_2-norm value for different order using different algorithms

The l_2-norm procured is considerably less for every increase in the order. The LSOMP and OMP algorithm has the least value among the other algorithms, as seen from the fig.5.7. Note that for order 88 the LSOMP procure the least value, thus the computational complexity is resolved best with this algorithm. Therefore for the analysis of filter both the constraints i.e. l_0-norm, l_2-norm are very important, since sparseness and reduced computational complexity are to be achieved effectively. For the thresholding algorithm, the l_2-norm value is large enough i.e. almost close to zero, therefore the issue of reducing computational complexity, cannot be accomplished properly and successfully. Thus it can be stated that the thresholding algorithm will work efficiently if minimum number of observations are to be considered. Its performance is nearly uniform as it depends only on the sparsity level and the sampling attributes, but since here the sparseness obtained is very less, therefore, it isn't suitable to consider this method for filter designing.

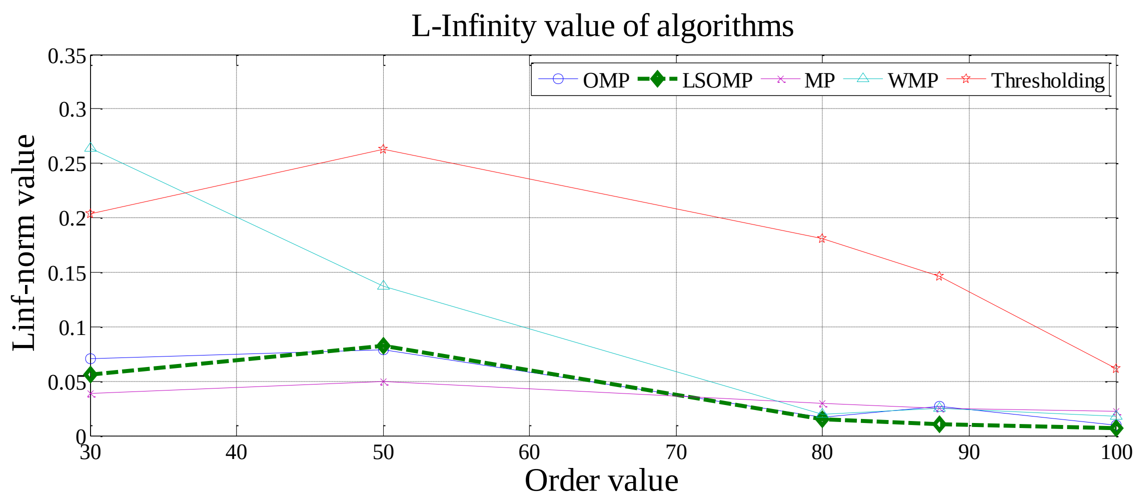

Fig.5.8 l_∞-norm value for different order using different algorithms

The l_∞-norm gives the maximum absolute value for the given vector, obtained from different algorithms. From the fig.5.8, the l_∞-norm value is almost the same and least for LSOMP and OMP algorithm. While for the thresholding algorithm, the l_∞-norm values are much higher than the other algorithms.

5.4 Comparison of WLS and Haar wavelet for designing of FIR filter

The designing of sparse FIR filter is done with the help of two methods i.e. WLS and Haar wavelet. The results obtained on the basis of different parameters such as l_0-norm, l_1-norm, l_2-norm and l_∞-norm show that the WLS method give far better result than Haar. The two algorithms are further compared on the basis of their magnitude response obtained. Here two examples are considered in order to demonstrate the effectiveness of the two approaches.

A. *Example 1*

The first example compares the magnitude responses obtained by the two methods. For the WLS method using different algorithms i.e. LSOMP, OMP, MP, WMP and thresholding, the order considered is 50 and the number of iterations is set to 10. For the same order value the magnitude response for Haar wavelet method is plotted. In each design, the LSOMP and the OMP algorithm shows far better results. The fig.5.9 shows the spectrum obtained for the various algorithms considered, while designing the sparse FIR filter using WLS.

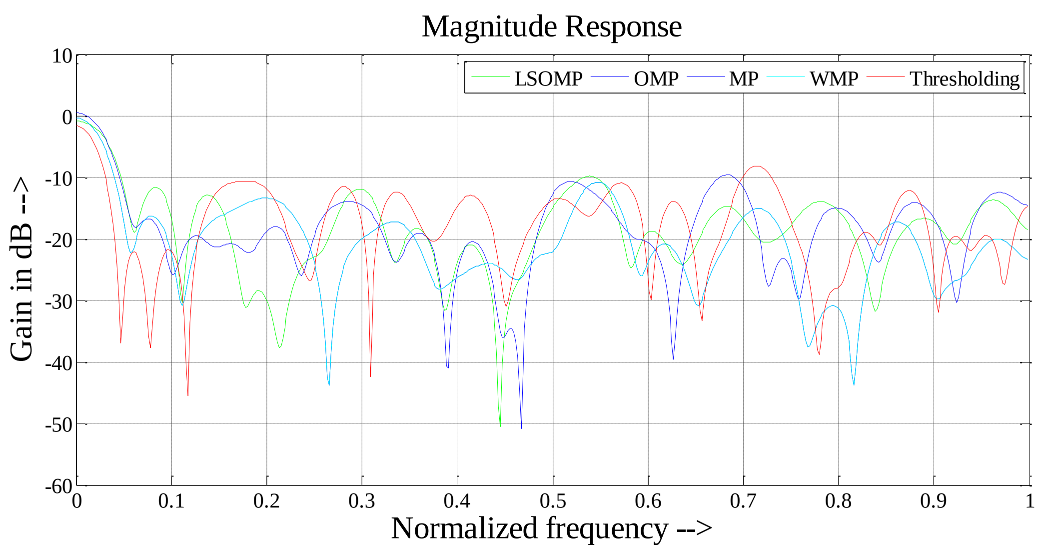

Fig.5.9 Magnitude response for various algorithms using WLS

The magnitude response obtained for the Haar wavelet method is shown in fig.5.10. The order considered is 50 same as that employed for the WLS method. The level of decomposition is set to 1, as the number of quotients would reduce considerably after imposing thresholding on it. The window considered while the design procedure is kaiser and the thresholding used is hard, with the value of 0.03.

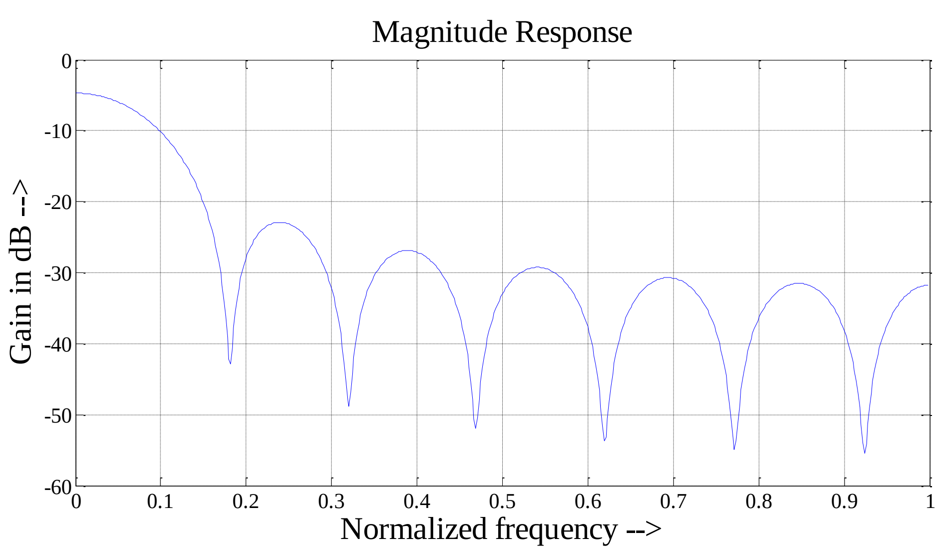

Fig.5.10 Magnitude response using Haar wavelet

The Haar wavelet method gives fewer ripples in comparison to the WLS method. Also attenuation in the ripples of the stopband are experienced while the design procedure of WLS. The gain value conveniently proclaims to start from the initial 0 dB mark in the case of WLS for different algorithms employed. The stopband ripple magnitude is also less in the WLS method. The Haar method may have fewer ripples but still their gain values are really high as compared to the filter spectrum obtained through the algorithms employed in WLS method. The cut-off frequency is at the same point in the case of WLS, but for Haar wavelet method it shifts from 0.08 to 0.18 as shown in the fig.5.10. From all the algorithms employed, during WLS sparse filter design, LSOMP gives better result in lieu to the magnitude response plotted. LSOMP also gives better result than Haar wavelet approach for the order 50.

B. Example 2

The second example illustrates, the design procedure of sparse FIR filter, using the combination of WLS and Haar wavelet. Here the order taken is 120 and the iteration value is set at 100. The l_2-norm value obtained for different algorithms i.e. LSOMP, OMP, MP, WMP and thresholding are in the range of 10^{-23}. The l_0-norm value obtained is from 18 to 20 for the five algorithms. Therefore, the reduction in computational complexity for design of the filter is very high. The LSOMP still holds the lesser value of l_0-norm and l_2-norm among the three algorithms. Hence, the efficient and effective results for computational complexity are procured, using the conjunction of the above method.

*****

6

CONCLUSION AND FUTURE SCOPE

"I think and think for months and years. Ninety-nine times, the conclusion is false. The hundredth time I am right." *-Albert Einstein*

In this research, a comprehensive study of sparse FIR filter is procured with the help of two different approaches i.e. Haar wavelet method and the WLS square criterion error. The conclusion drawn on the basis of the two methods is stated below. Further, the future scope of research is expounded.

6.1 Conclusion

The methods i.e. Haar wavelet and WLS help in finding a sparse solution for an underdetermined linear system of equations, where the sensing matrix used is random in nature. Thorough description is provided for both the methods. The Haar wavelet method used gave a reduction in the computational complexity which was based on the sparsity attributes obtained. Also, spectrum analysis was done for the original filter and its sparse version, due to which the various attribute values were examined like the cut-off frequency, ripple in stopband and passband. The WLS method used a different set of formulation while constructing the filter quotients of the filter. All the steps related to the WLS approach were clearly discussed in the foregoing chapters. The algorithms used for giving the sparse approximation solution for the filter coefficients obtained using this method were OMP, LS-OMP, MP, WMP and thresholding algorithm. The OMP, LS-OMP, MP and WMP algorithms were based on l_0-norm minimization while the thresholding algo was based on l_1-norm minimization. Even though the approaches were based on different minimizations, they all followed the underlining greedy iterative approach. They all build a sparse solution hinged on interrelation between columns of the random matrix $'A'$ and the current residual vector$'b'$. Finally they converge to a solution whenever the norm of residual vector approaches zero.

The various attribute values related to the greedy algos are examined. Through the analysis, it is proclaimed that the design of FIR filter using WLS criterion gives better results by using LS-OMP algo, as the values of l_2-norm is least in this case only. Therefore, the main grail of achieving reduced computational complexity is efficiently achieved. Also the values of l_0-norm, l_1-norm and l_∞-norm are least in the case of LS-OMP. Thus sparsity is achieved which thereby enhances the design of FIR filter. The only drawback in the above study is higher group delay w.r.t its nonsparse version. Therefore this should encourage the study of low-group delay sparse FIR filters. Although the sparse FIR filter provides efficient performance over the non-sparse version on the basis of implementation complexity, still the amount of effectiveness varies from algorithm to algorithm.

While describing the success of the above approximation algorithms, there are several ways to identify the extent between the solutions it propounds i.e. the sparse vector and the ideal coefficient matrix obtained through WLS. It is contemplated, through the measure of l_2-norm. The l_2-norm is shown in the fig.6.1, and is calculated using the formula given below

$$||x - \dot{x}||^2 / \, ||x||^2 \qquad (6.1)$$

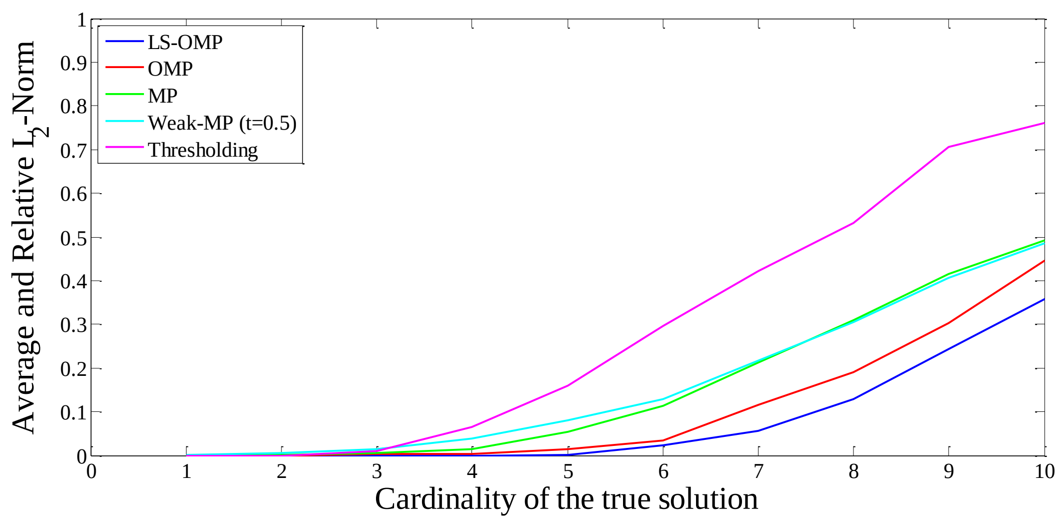

Fig.6.1 Average and Relative l_2-norm for different algorithms

As expected, the prime algorithm is the LS-OMP, which is closely followed by the OMP. This is basically demonstrated with the help of l_2-norm that is calculated for the algorithms. The MP and the weak matching pursuit (WMP) algorithms differ by a slight contrast. The algorithm that performs the worst is the thresholding algorithm, and it is the poorly rated among the other four algorithms. The error value obtained for thresholding algorithm is very high in comparison to the various other algorithms and thus the solution obtained is not perfectly reconstructed.

6.2 Future Scope

With respect to future scope, these techniques could be effectively applied to 2D-filters. Different set of criterions can be defined for developing convex optimization anatomy. With appropriate employment of parameters subjected to a specific sparsity level, can efficiently outperform its non-sparse version of the filter. For more efficient design procedure, one could use the concept of mutual coherence, R.I.P and Null space related to the sensing matrix.

*****

REFERENCES

[1] Jean-Luc Starck, Fionn Murtagh and Jalal M.Fadili, Sparse Image and Signal Processing: Wavelets, Curvelets, Morphological Diversity, NewYork: Cambridge, 2010.

[2] M. Elad, Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing, NewYork: Springer, 2010.

[3] Emmanuel J. Candès, Justin Romberg, and Terence Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," IEEE Transactions on Information Theory, vol. 52, no. 2, pp. 489 - 509, February 2006.

[4] David L. Donoho, "Compressed Sensing," IEEE Transactions on Information Theory, vol. 52, no. 4, pp. 1289–1306, April 2006.

[5] Emmanuel J. Candès and Terence Tao, "Near-Optimal Signal Recovery From Random Projections: Universal Encoding Strategies?," IEEE Transactions on Information Theory, vol. 52, no. 12, pp. 5406 - 5425, December 2006.

[6] Emmanuel J. Candès, "Compressive sampling," In Proceedings of the International Congress of Mathematicians, Madrid, Spain, August 2006.

[7] Yaakov Tsaig and David L. Donoho, "Extensions of compressed sensing," Elsevier Signal Processin*g*, vol. 86, no. 3, pp. 549-571, March 2006.

[8] Emmanuel J. Candes and Terence Tao, "Decoding by Linear Programming," IEEE Transactions on Information Theory, vol. 51, no. 12, pp. 4203-4215, December 2005.

[9] A. Antoniou, Digital Filters: Analysis, Design, and Applications, 2nd ed. NewYork: McGraw-Hill, 2000.

[10] T.N. Davidson, “Enriching the art of FIR filter design via convex optimization,” IEEE Signal Process.Mag, vol. 27, pp. 89-101, May 2010.

[11] K.M. Tsui, C.Chan, and K.S. Yeung, “Design of FIR digital filters with prescribed flatness and peak error constraints using second-order cone programming,” IEEE Trans. Circuits Syst. II, vol. 52, pp. 601-605, Sep 2005.

[12] Alfred M. Bruckstein, David L. Donoho and Michael Elad, “From Sparse Solutions of Systems of Equations to Sparse Modeling of Signals and Images,” SIAM Review, vol. 51, No. 1, pp. 34-81, Feb 2009.

[13] Aimin Jiang, Hon Kueng Kwan, Yanping Zhu, Xianfeng Liu, Ning Xu and Yibin Tang, "Design of Sparse FIR filters with Joint Optimization of Sparsity and Filter Order," IEEE Trans. Circuits Syst. I, vol. 62, No. 1, pp. 195-204, Jan. 2015.

[14] Yuhua Yang, Wei-Ping Zhu and Dalei Wu, "Design of Sparse FIR filters based on Reweighted l_1- Norm Minimization," IEEE, pp. 858-862, Feb. 2015.

[15] V. Sowmya*, Neethu Mohan and K. P. Soman, "Sparse Banded Matrix Filter for Image Denoising," IEEE, pp. 1-6, Sept. 2015.

[16] Aimin Jiang, Hon Keung Kwan, Yibin Tang and Yanping Zhu, "Efficient Design of Sparse FIR filters with Optimized Filter Length," IEEE, pp. 966-969, 2014.

[17] Aiming Jiang and Hon Keung Kwan, "WLS Design of Sparse FIR Digital Filters," IEEE Trans. Circuits Syst. I, Vol. 60, No. 1, pp. 125-135, Jan. 2013.

[18] Chien-Cheng Tseng and Su-Ling Lee, "Design of Sparse Constrained FIR Filter Using Orthogonal Matching Pursuit Method," IEEE ISPACS, pp. 308-313, Nov 2012.

[19] Aimin Jiang, Hon Keung Kwan and Yanping Zhu, "Peak-Error-Constrained Sparse FIR filter Design using Iterative SOCP," IEEE Trans. Signal Processing, Vol. 60, NO. 8, Aug. 2012

[20] Aimin Jiang, Hon Keung Kwan, Yanping Zhu, and Xiaofeng Liu, "Minimax Design of Sparse Fir Digital Filters", IEEE, pp. 3497-3500, 2012.

[21] Dennis Wei and Alan V. Oppenheim, "A Branch-and-Bound Algorithm for Quadratically-Constrained Sparse Filter Design," IEEE, pp. 1-32, 2011

[22] Thomas Baran, Dennis Wei and Alan V. Oppenheim, "Linear Programming Algorithms for Sparse Filter Design," IEEE Trans. Signal Processing, Vol. 58, No. 3, pp. 1605-1617, Mar. 2010

[23] Wu-Sheng Lu and Takao Hinamoto, "Digital Filters with Sparse Coefficients," IEEE, pp. 169-172, Jan. 2010

[24] Wu-Sheng Lu, Takao Hinamoto, "Two-dimensional digital filters with sparse coefficients." IEEE, pp. 1-16, May. 2010

[25] Allen Y. Yang, S. Shankar Sastry, Arvind Ganesh and Yi Ma, "Fast l_1- minimization algorithms and an application in Robust face recognition: A Review," ICIP, pp. 1-4, 2010.

[26] Oscar Gustafsson, Linda S. DeBrunner, Victor Debrunner and Hakan Johansson, "On the design of Sparse Half-Band like FIR filters," IEEE, pp. 1098-1102, Feb. 2007

[27] Joel A. Tropp and Anna C. Gilbert, "Signal Recovery from Random Measurements via Orthogonal Matching Pursuit," IEEE Trans. Information Theory, Vol. 53, No. 12, pp. 4655-4666, Dec. 2007

[28] Joel A. Tropp, "Greed is good: Algorithmic results for sparse approximation," IEEE Trans. Information Theory, Vol. 50, No. 10, pp. 2231-2242, Oct. 2004.

[29] Davide Mattera, Francesco Palmieri and Simon Haykin, "Efficient Sparse FIR Filter Design" IEEE, pp. 1537-1540, 2002

[30] Young-Seog Song and Yong Hoon Lee, "Design of Sparse FIR filters based on Branch-and-Bound Algorithm," IEEE, pp. 1445-1448, July 1997.

[31] John W. Adams and Alan N. Willson, Jr., "A New Approach to FIR Digital Filters with Fewer Multipliers and Reduced Sensitivity," IEEE, Vol. CAS-30, No. 5, pp. 277-283, May. 1983.

*****