Inventory Models under Inflation in finite planning horizon

A Thesis

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Mathematics

By

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DECLARATION

I, Seema Saxena, declare that this research thesis titled, "Inventory models under inflation in finite planning horizon" is an original work done by me wholly while in candidature for the research degree at Department of Mathematics, Lovely Professional University, Phagwara, Punjab under the guidance of Dr. Rajesh Kumar Gupta, Dr. Vikramjeet Singh and Dr. Pushpinder Singh. I corroborate that this thesis or any partial work of the thesis has not been previously submitted for the award of degree at any other institution. I clearly state that wherever I have conferred any published research of any researcher or have quoted others research work, the reference is always mentioned. Besides this the research work is entirely my own research work. I have conveyed my gratitude to all sources of help.

Signed:

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Date:

CERTIFICATE

This is to certify that the work incorporated in the thesis titled "Inventory models under inflation in finite planning horizon" submitted by Ms. Seema Saxena was carried out by the candidate under my guidance. In my opinion the thesis fulfills the requirements laid down by the Lovely Professional University, Phagwara, Punjab for the award of degree of Doctor of Philosophy in Mathematics.

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ABSTRACT

Inventory plays a significant role in ascertaining a company's financial position. This resource has an economic evaluation, its maintenance is a dire need for any organization to fulfill the present and coming requirements. Inventory models are mathematically formulated, owing to which certain decisive rules/policies are framed for the effective management of the inventory. In this research work, various models on inventory are proposed for the retailer and the supplier under inflation. Moreover, the formulated models analyze inflationary environment for the parameters such as shortages, deterioration, lost sales and time quadratic demand.

The admissible delay period given by the supplier is a benefit in the form of price discount as the retailer pays later in turn reducing the cost of purchase and making an income through the interest on the money locked. The models developed incorporates this concept and the extra savings realized due to this co-ordination is divided amongst the two parties. In addition to this, the models are developed for the supply chain where the supplier makes the retailer agreed for the less number of ordering cycles with an increase in the quantity ordered. The efficiency of the integrated models depend upon the coordination between the vendor and the vendee.

Green technology is an add on to the inventory models. As every organization is moving to adopt the green concept to reduce the amount of waste which pollute the environment and also decreases the profit margin. The concept of re-manufacturing is included along with the delay time under inflation. The study shows a percentage reduction in the cost under these constraints and can be effectively followed by the organizations.

Finally to incorporate the uncertainty prevailing in the market for the parameters of the inventory system, the concept of fuzzy logic is included in the models. The inflation rate, demand, price etc.,are not fixed. So fuzzifying these parameters becomes more realistically valid to formulate the solution to the inventory problem. The defuzzification is done by the centroid and the signed distance method. Later a comparative study is made to ascertain the results.

For each inventory problem model an algorithm is prepared and the problem is elucidated by the numerical. Sensitivity analysis is performed on the major parameters and the optimizations is verified. Various managerial and logical insights are developed.

In appendix A the list of published, accepted and communicated research is provided.

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Dedicated to my ANGEL SRISHTI

Chapter 1

INTRODUCTION

1.1 Operation Research and Inventory

Operation research is a field which relates to an upgraded logical ways of investigating the real world problems and figuring the best possible way for the organization. Operation research accumulates the modern ways for picking the most beneficial choice for various difficulties coming up in any industry or business of prominent workforce, machines, money and material. As defined by churchman et al.[81] operation research is primarily the practical application of various scientific tools and techniques for the decision oriented real problems so as to yield a check on the trading operations and determine the most effective solution for the problem.
Operation research came into existence since World War II. During the time of war, as there was shortages of military equipments, England expressed a dire need for effectively utilizing the land and air armed resources. Since the research was on armed forces operations, the project was called as Operational Research. Later the experts from industry also tried to look for the solutions to the various complicated industrial problems. Presently operation research strikes every area. Apart from business and military it also includes assignments of work, transportation, planning and scheduling management of resources, etc. When it comes to the observation regarding the demand, lead interval, ordering cost, quantity ordered and of finding a solution for over-carrying and under-carrying of inventory, then the study is termed as inventory control.
Inventory is the movement of finished and unfinished products from the vendor to the vendee which is again passed on from the vendee to the customer. Inventory is defined as that portion of assets in any business that accounts for sale or is under manufacturing stage for selling. This includes raw material, work in progress, which is unfinished goods and finally the finished goods. Inventory constitutes to be an important part of any organization as the incoming and outgoing of inventory measures the optimal value of cost and the revenue generated to the owners of the organization. Although inventory cannot be held for a longer period of time as it leads to extra holding, storage, spoilage and deteriorating cost. However, the shortage of the inventory within a period is also not good for the organization. The organization incurs shortage cost due to the loss of

potential customers and the decrease in the hold over the market, by reduction in the goodwill of the product. This is the reason by “inventory is a necessary evil”. Inventory control is the most widely studied area as researchers, practitioners figure out the best possible solution to the two important decisions for any organization that is the best time period when which best fits into placing of an order and the optimal quantity that is to be ordered. The various parameters involved in framing an inventory policy as lead time gap, the demand perception, several cost values etc. must be studied before hand before formulating and inventory policy.

1.2 Need for holding Inventory

Inventory policies help in minimizing the various cost of the model and framing out the strategies such as JIT, which facilitates to receive or manufactur the goods as the need arises. Few reasons why organization whether small or large scale managers manages some kind of inventory are:

1. to deal with the alterations in the cost of raw stuff
2. for smooth manufacturing process
3. to facilitate the rate of production
4. to fulfill the variability in the demand throughout the year
5. to fulfill the customer’s demand through the stock and avoid delay
6. placing bulk orders entitles for discount
7. having economic transportation cost
8. efficiently planning the operational strategies through the chain of procedures and receiving goods, manufacturing, transporting to the warehouse and finally to the customers
9. more the inventory, more is the availability of the goods and so customers can be attracted to make the purchase by displaying the majority of the goods
10. for products which have variability in prices due to seasonal fluctuations, bulk purchase can avoid paying extra
11. also low inventories results in back orders, lost sales, manufacturing, constriction and displeased customers.

The study of inventory control is basically done to select the best strategy which achieves a balance between the holding or carrying cost of inventory and the cost of not having the inventory that is the shortage cost. Some more reasons are:

1. **Control the costs**: Keeping reports about the inventory helps to figure out which stock is doing well and which is just acquiring the shelf space. The unavailability of inventory at right time can lead to back orders, loss of goodwill which drives up the cost.

2. **Lead time:** Lead time is duration between time of inventory ordered and the time of inventory received/replenished by the system. Most of the model considers lead time as zero but it is due to the existence of this lead time having an inventory becomes essential.

3. **Reduce the time for managing inventory:** Having an appropriate solution to inventory management, reduces the time taken in keeping the track of products which are on hand and on order. In addition to this, time recounts if the records of inventory are not up to date.

4. **Transportation:** When all the transportation in Indian supply system is either by train or by trucks the uncertainty of receiving the goods in time becomes really high. Therefore rather than “Just in time”, ”Just in case” type environment is proposed to be considered which again necessitates the need of inventory. It is only in “Just in Time’ type of environment either no inventory or less inventory is required. Simply speaking more the uncertainty more the amount of inventory required.

5. **Manage planning and resources:** By analyzing the data trends of well performing stocks, pertinent demand is forecasted which in turn reduces the holding cost and handling cost of the inventory keeping the customers’ need satisfied.

6. **Demand fluctuation:** It is always not possible to predict the demand for a long period of time or for a planning horizon in years. Sometimes these days for a product like mobile phones where Samsung has taken a lead from Nokia and fast developing mobile technology even Samsung is facing the heat from several other companies it is totally not possible to predict the market of a technology even for few months. More over greater the demand more is the inventory requirement.

7. **Seasonal inventory:** For product like Cement, the demand is seasonal demand therefore the lean season is used to maintain inventory to meet the demand in peak season.

8. **Tracking inventory in multiple stage production:** In view of stock across the multiple channels, a good inventory system keeps centralized record of inventory. It allows to keep track of how much is the inventory and how much is to be allocated to specific sales channel. Such as for Car etc their spares are produced in multiple units and are then transited for assembling purposes. During this period of time where the inventory are of different raw unprepared goods are shifted from one place to another necessitates one to hold the inventory until the next consignment which is in pipeline is received. These inventories perform as essential joints in a supply chain system.

9. **Inflation:** As inventory represents a very important part of the company's financial assets, it is very much affected by the market's response to various situations, specially inflation. Inflation is the primary reason for which the companies are needed to put so much effort into valuing their inventory. The cost parameters are affected by the rate of inflation. High rate of inflation affects the organizations financial conditions. It creates a number of uncertainties because of rising prices in raw materials, semi finished and finished goods.

1.3 Costs Associated With Inventory

While maintaining a desired level of inventory stock there are various cost which either increase or decrease for the system. Such associated costs are important to study for an inventory. These costs have a prominent role in assessing the overall cost of the system. Following is a list of various relevant cost:-

1.3.1 Ordering Cost

This cost is associated with the ordering of the inventory. This cost has a fixed nature. The cost of ordering includes in it:

1. the cost of Administration

2. cost related to the staff for the clerical work

3. cost of transportation etc

1.3.2 Cost of Holding

While carrying the inventory as a stock over a period of time costs such as

1. the rent of the storage space

2. insurance
3. the amount of interest for the locked Money
4. taxes etc

are taken under the Cost of Holding.

1.3.3 Cost of Purchasing

The unit cost of procuring an item of the inventory for the manufacturing/ production from outside source is the Cost of Purchasing. It is proportional to the demand of the product. The cost is affected by the discount given in the quantity purchased as it lowers the unit price.

1.3.4 Cost of Stock-Outs/Shortages

This cost is taken as a penalty in the inventory system. This occurs when the available inventory stock is not sufficient to fulfill the demand of the customers. This leads to the loss in the profit.

1.3.5 Cost of Lost Sales

In today's scenario when the goods of the type are available from different manufacturers, it is quite often that on account of shortages, the customers may not intend to wait for the replenishment of the stock, rather switches for the different manufacturer. This results in the loss of profit and also the loss of goodwill.

1.3.6 Stock Clearance Cost

At the end of the cycle time of an inventory the leftover goods are sold at a lower price to dispose of the stock. This in turn reduces the other cost incurred in holding the inventory, but due to this, the revenue generated for these goods is less and is termed as the Stock Clearance/Disposal Cost.

1.3.7 Deterioration Cost

The goods as the enter into the inventory tends to deteriorate with the passage of time. Such goods turns into defectives or perishable and are either disposed in waste or are sold for a reasonably lower price. This is deterioration cost.

1.4 Fundamental Attributes in Inventory

The inventory coordinating structure involves various parameters like the demand, lead period, refilling or reordering, various associated cost etc. Decision making model for inventory is classified in the figure 1.1.

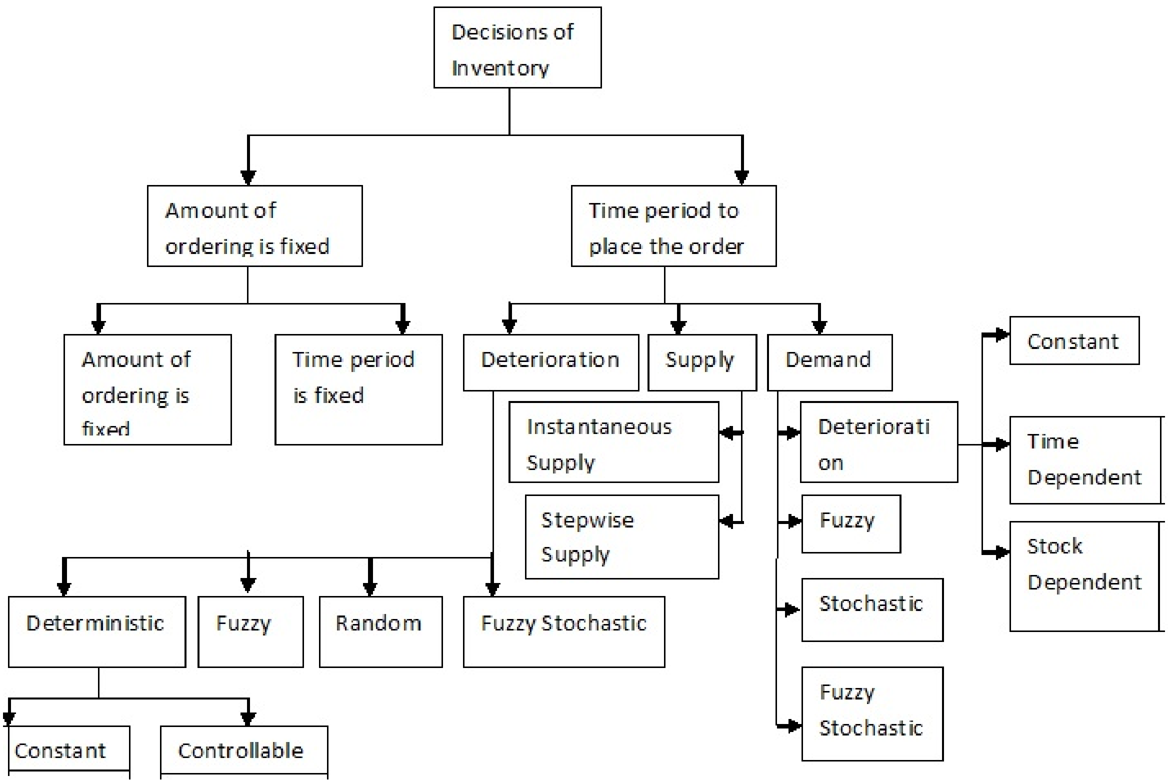

FIGURE 1.1: Inventory Attributes

1.5 Chief Components in Inventory Modelling Analysis

1. **Demand:-** is taken as a prominent factor in the study of any inventory framework. It influences the modelling of the whole inventory organization. A quadratically defined demand pattern with respect to time for any product follows the expression Demand(t) = $x + y * t + z * t^2$ where x is the constant basic demand, y is the increase in demand with respect to time and z represents the change in demand

with respect to time. When the demand for any product varies linearly with respect to time it is called linear demand expressed as Demand(t) = $x + y * t$ with x being the constant demand and y being the increase in demand with respect to time. Demand of any product also depend upon its availability. If there is a large stock of any product, it can be displayed to the customers by the sellers, which in turn motivates the purchase. This type of nature of demand is called as stock dependent demand.

2. **Deterioration:-** It is defined as the rate of spoilage of any product. It can be constant or can vary with respect to time. It is important to ascertain deterioration rate as this decreases the inventory level in the system thus effecting the system's overall cost and revenue generation.

3. **Shortage:-** These occur due to some unanticipated circumstances. Another important key parameter in the system. Shortages are completely backlogged if the customers are willing to wait for a period of time for any specific product. But if the customers move to some other product available in the market, then these customers are lost. It refers to partially backlogged shortages where lost sales occurs and the cost is charged to the seller.

4. **Trade Credit:-** It is the admissible delay time in paying back time offered by the supplier to the retailer. This policy is gaining acceptance in the market as it not only allows the supplier to take a hold over the potential retailers and capture the market but also allows the retailer to earn in the form of sales and interest while holding the money throughout the delay period.

5. **Inflation:-** It is the increase in the value of money with respect to time. It indicates a decline in the purchasing ability of the currency. With inflation every single currency loses its value and purchase fewer products and services than earlier. In inventory every associated cost should be efficiently inflated in order to determine a realistic cost value of the system.

6. **Green Inventory Model:-** With the increase in the awareness and responsibility towards the environment, organizations are also heading towards a system which does not involve any threat to the environment. The green inventory models aims at reducing the amount of waste being thrown in the surroundings. The inventory model includes reusing, recycling and remanufacturing of the defective goods. These are again sent back to the retailer for the consumption in the market.

7. **Fuzzy logic:-** Due to the uncertainties prevailing in the market for every parameter, it becomes quite unrealistic to frame a mathematical model which gives an

accurate result for the overall cost or the overall revenue generation for the system. The parameters like the demand, price, rate of inflation, discount rate, various costs are all subject to the market variations and suffer an increase or decrease depending on the market. To come across a realistic nature of the real problem, it becomes necessary to fuzzify these parameters so that an accuracy towards the optimality of the cost or profit can be effectively achieved.

8. **Planning Time Horizon:-** The inventory is controlled for a finite time interval or infinite time interval. It depends upon the demand forecasted.

9. **Inflation and Time Value of Money:-** Inflation is the rise in the amount of money to be spent for any product or service. It brings a decline in the buying power of each unit of currency. This implies that with inflation, one has to pay more for any unit of the product. For example, if the rate of inflation for any product is 5

1.6 Factors Responsible for Inflation

1.6.1 Demand pull Inflation

When the consumers willingly pay higher cost for any product, then the demand of that good surpasses its supply.

1.6.2 Cost pull Inflation

This is not frequently observed. In this type of inflation, the supply for any good becomes restricted. This commonly happens during any natural calamity, when the demand of any product increases, but due to constricted supply there, is a price rise of that product.

1.6.3 National debt

With increase in the country's debt, the governing authorities look for the two possibilities, either there is an increase in the tax or the currency is printed for paying the debt. With the former option, as the tax is increased, the corporate world will react to it by increasing the prices of their products in order to compensate the increase in the tax. So the authorities mostly prefer the latter option of currency to be printed. This leads to intensify supply of the currency leading to its devaluation and price rise causing inflation.

Buzacot, mathematically derived the concept of inflation by assuming the cost of any product at any time t as $Z(t)$ and at any time t+ δt as the change in the cost being Z(t+δt) due to a constant rate of inflation a($ per unit).

Z(t+δt) = Z(t)+ a.Z(t)δt as δt $\to 0$

$\frac{Z(t+\delta t)-Z(t)}{\delta t}$ = a.Z(t) as δt $\to 0$

$\frac{dZ(t)}{dt}$ = a.Z(t)

$\frac{dZ(t)}{Z(t)}$ = adt which is further solved

Z(t)= $Z(0).e^a t$,where Z(0) is the initial cost at t=0.

1.7 Time Value of Money:

It refers to the variations in the buying ability of a currency with a passage of time. The fundamental principle underlying the time value is that the money worth $100 today will have more value if invested for sometime. That is the earning from the money invested in the form of interest. If a sum of $100 is invested at 10$\%$ per annum, then after 1 year the money will amount to $ 110.

1.8 Future cash value of money or discounted present value of money

It is the key concept in financing any system. Due to this, organizations promote to have money soon after the supply rather than with the passage of time, as this will make a loss of the earning through the interest amount if the money is invested. The following rules define cash value of money:

1.8.1 Present Cash Value:-

This rule converts the future cash into its equivalent present cash.
Present Cash = $\frac{FutureCash}{(1+interestrate)^t}$

1.8.2 Future Cash Value:-

This rule converts the present cash into its equivalent future cash value.
Future Cash= $PresentCash * (1 + interestrate)^t$

1.8.3 Discounting:-

Refers to the process of determining the present value for money at any point of time in future. The future value is reduced by the currently prevailing discount value. The discount value at any time t is

D(t) = e^{-rt} for fixed compounded rate of discount r.

Thus every future cost value is to be multiplied by the discount value in order to arrive at its current value. The cost parameters are affected by the rate of inflation. High rate of inflation affects the organization's financial conditions. It creates a number of uncertainties because of rising prices in raw materials, semi finished and finished goods.

1.9 Effects of inflation:-

1. **EOQ :** Inventory carrying cost determines the level of inventory to be kept on-hand. Under inflation, there is an increase in the carrying costs of an EOQ model as the interest rate is pushed up by the inflation resulting in decreased economic order quantity. . Hence, to calculate the optimum order size, cost to carry should be reduced by inflation's impact on interest cost.

2. **Effect:** With a LIFO-method inventory, your cost of goods sold reflects the most current costs: Newer, higher costs are being reported as COGS. Older, lower costs remain "locked up" in inventory. Thus, during inflation, LIFO will overstate your turnover ratio. With a FIFO inventory, by contrast, older, lower costs will be reported as COGS, while your inventory includes newer, higher costs. So, during inflation FIFO will understate your turnover ratio. In both cases, the higher the inflation, the greater the distortion.

1.10 Trade Credit:

In the traditional EOQ formulation, the fundamental assumptions included no lead time, constant or fixed demand, infinite storage and immediate payment by the retailers for the items received to the supplier. Practically, these conditions are not met in real lity. In real life, the supplier offers the retailer certain time period to pay back for the goods. This relaxation in paying back is termed as permissible or admissible delay. The policy of credit time has gained attention of several researchers. Credit time is adopted by suppliers as with this they can have a better hold over the market, can retain their potential retailers. Also retailers enjoy the benefit of dual earning. Earnings which is made by selling the goods and the earning through the interest earned on the amount held. Thus the retailer delays the payment till the last admissible period offered to him. The researchers are working on the formulation of the inventory models which workout to be profitable for both the retailers as well as the suppliers.

With trade credit the primary motive of the supplier is to make the retailer agree for ordering large quantity and in optimal time interval. By increasing the ordering quantity helps supplier in a way that the setup cost is reduced for the supplier and only restricted to the optimal number of times and order is placed. This reduction in the setup cost of the supplier is an additional cost savings to him/her. On the contrary, the retailer makes an additional earning in the form of the interest accrued on the amount held for the delay time. Also he earns by selling the goods. Thus trade credit is a profitable policy for both the retailers and the suppliers and modelling a supply chain model under various constraints is presently done by several researches.

1.11 Trade period for credit includes the following benefits:

1. Credit time offered by the supplier to the retailer is an agreement based on the mutual understanding
2. Both the retailer and the supplier agree to enter into terms and conditions of the adm admissible delay period and hence no formula is used letter of agreement on admissible delay period and hence no formalized letter of agreement of debt is required
3. Credit term is a self generated resource for a business by both retailer and supplier.
4. Admissible delay in paying back reduces the basic principle requirement of investment.
5. With trade credit, organizations focus on the chief actions of looking onto the market, acquiring control over the business for future prospects
6. Permissible delay easy business activities without any formulas letter of contact .

From the previous decades, firms are paying more emphasis on the supply chain, as it is believed to be one of the driving forces for the firm to find the competition in the market. Since the year 2000, the data reveals how the companies make hefty Investments to streamline the operations in the supply chain for the betterment of the internal

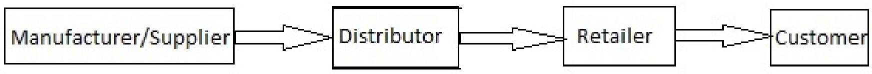

FIGURE 1.2: Supply Chain

production and to satisfy the customers. The primary focus in developing the supply chain process lies in the basic fact of maximizing the profit and minimizing the overall cost of the system. This is chiefly achieved by decreasing setup cost, by reducing the replenishment schedules or by increasing the order size. In supply chain, apart from the supplier and the retailer, warehouses, transporters and the customers are also included. Supply chain include operations and activities through joint organizational efforts and an efficient business practices to have a sustainable advantage in the competitive market.

It involves challenges of creating trust and cooperation among the associates in the supply chain. To have an effective supply chain management trade credit is an important tool.

1.12 Green Inventory System:-

The stressing condition of the increase of waste resources in the environment, a decrease in the non-renewable energy resources and consistently increase in the imperfect/defective goods occupying the land has forced the companies to adopt the strategies such as recycle, remanufacture, re-generate or repair instead of throwing these goods in the surroundings. With the objective of minimizing the environmental threats and re-utilizing the resources, the entire inventory system should be redefined. In assessing the green inventory or the reverse supplying system, the key point is to minimize the cost of the supply chain system and also to have a check on customer's satisfaction. Retrieving the defective goods for recycling and reusing repairing remanufacturing and cannibalizing are part of green inventory and rivers supplying creating new business opportunities and accentuating the sustainability of the overall system. Including the green concept in the inventory modeling is that the goods as received by the retailer undergoes screening and the defective/imperfect goods are taken back by the supplier and are not discarded elsewhere. These goods are again recycled and remanufactured and sent back to the retailer for the consumption in the market. This concept is being taken up by the researchers so as to identify the key areas where there can be a profitable situation for the allies of the supply chain thereby ensuring minimum hazard to

the environment and have sustainability in the environment management. Having incertitudeness over the measure of the goods returned, the challenge is thrown over the overall planning of the inventory system. With the inclusion of the remanufacturing, recycling or reutilizing of the returned products in the inventory system, following two complexities appear in the inventory modeling.
The quantity of items returned would vary which would initiate randomness in the system, and would have an impact on the cost and time of the inventory system. After remanufacturing, the goods are again transported back for its absorption in the market, so it becomes necessary that the coordination for the goods between the retailer and the supply is regular and efficient.

With the remanufacturing or recycling of the defective goods for the green environment, an additional difficulty of determining the production flow becomes uncertain, as it largely depends upon the measure of the goods returned and their quality. Apart from these difficulties, there are numerous untapped possibilities through green inventory system for a business, human resources, customers and our environment.

1.12.1 Following is the list of few of the beneficiaries from the properly managed and productive green manufacturing system:

1. **Business firms:** decrease in the investment of the capital. Profit is increased by remanufacturing/recycling of the defective goods which are again remarketed. A small scale enterprise with less budget can land into profitability by utilizing the remanufactured goods as they cost relatively less, approximately 40 to 60

2. **Customers:** The product is available with extra benefits and at a low cost. Large number of usable options for the product is available in the market.

3. **Lead time:** is less for the manufactured goods and so the cosumers have to wait less for the products.

4. **Human resource/power:** For the process of remanufacturing/recycling broader set of workforce is employed and a more satisfaction on the engagement in work.

5. **Environment surrounding :** While adapting to the remanufacturing/recycling or cannibalizing the goods, the consumption of energy is only 15

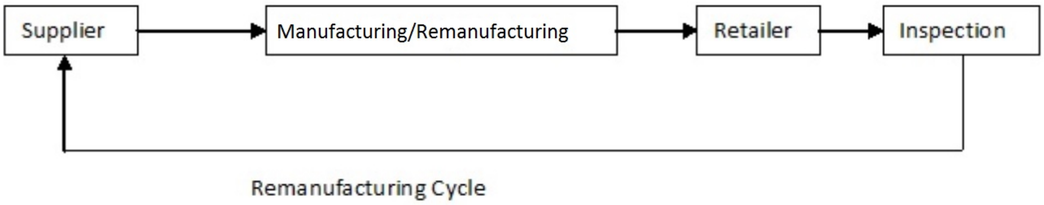

FIGURE 1.3: Green Concept Process

1.13 Need to Fuzzify the Inventory System:-

In traditional literature of inventory every parameter related to the inventory namely various cost, lead time, price, demand, accessible resources etc are taken in crisp form. But these parameters are subject to variations owing to market fluctuations. Rate of inflation and discounted rate are also subjected to rapid variations owing to the current economic conditions. The parameters have randomness in their value and so the problem modeled cannot predict a realistic solution. Thus every inventory model survives in the following two environments:

In a crisp/precise environment, every parameter involved in the system has a deterministic value, whereas, in the uncertain environment, fuzziness prevails when no information regarding the valuation of the parameters can be made. As if any product is newly launched in the market, the organizations cannot predict the demand of the product. For this, they rely on the data from the field experts. If the opinions from the experts too are imprecise, then the demand is considered as fuzzy. Lead time is one of the parameters which cannot be predicted. In basic models, though lead time is taken as a constant, but in practicality it consists of various components as vendor's supply time, preparation time, and time in transit, installing time etc. It also includes various unforeseen factors which increases the lead time like the natural calamities, strikes, jams etc. Such parameters have to be dealt with fuzzy approach. The measure of the inflation rate and the discount rate also depends upon the prevailing economic conditions and so cannot have a certain value for a longer span of time. If taken as constant, then the cost accrued with these values will not be accurate and the model will not rightly depict the problem. Fuzzy approach is essential for all these parameters in formulating the models for the real world problems. Many inventory related decisions are made under fuzzy environment as these parameters cannot be precisely valued. Fuzzy theory integrates the subjectivity and the uncertainty in modeling and solves the inventory problem. In the research work, a fuzzy inventory model is developed under admissible delay with demand, price, rate of inflation and discount being uncertain and taken as triangular fuzzy number in fuzzy logic.

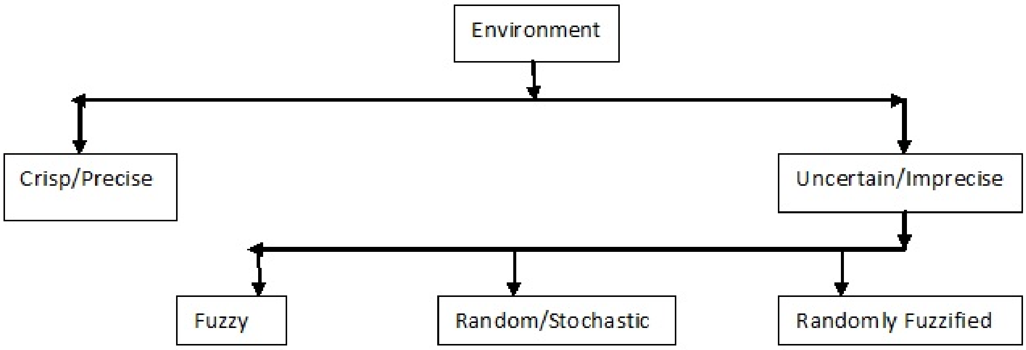

FIGURE 1.4: Inventory Environment: Crisp and Fuzzy

1.13.1 Fuzzy set:-

Fuzzy Set is defined as

$\tilde{F} = \{(X, \lambda_{\tilde{F}}(x)) : x \epsilon X, \lambda_{\tilde{F}}(x) \epsilon [0, 1]\}$

$(X, \lambda_{\tilde{F}}(x))$ signifies that X is classified in the standard set F and the element $\lambda_{\tilde{F}}(x)$ is classified in the interval [0,1]. This element is called as the membership function. This function indicates the compatibility of truth of variable X in the fuzzy set F. Lofti A. Zadeh(1965) extended the traditional theory of set to the fuzzy concept of set. Fuzzy conceptualizes the set by permitting the assessment of the elements from the set and aims at valuing the membership function into the real closed interval [0,1]. A fuzzified number can be expressed as extending a standard figure so that it is not represented by a single value, rather it is described by a set of potential values, where each value is assigned for its weight lying amongst 0 and 1.

1.13.2 Triangular Fuzzy Number

In a fuzzified set $\tilde{F}$ = (p,q,r) where p < q <r characterized in R is said to be a triangular fuzzy number, if its membership function is stated as follows:

$$\lambda_{\tilde{F}}(x) = \begin{cases} \dfrac{x - p_0}{p - q}, & p \leq x \leq q \\ \dfrac{r - x}{r - q}, & q \leq x \leq r \\ 0, & Otherwise \end{cases}$$

The graph of the triangular fuzzy number is as follows:

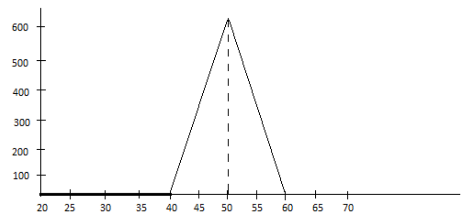

FIGURE 1.5: Triangular Fuzzy Number

1.13.3 Trapezoidal Fuzzy Number

For a trapezoidal Fuzzy Number $\tilde{F}$ = $(p_1, p_2, p_3, p_4$), its membership function is stated as follows:

$$\lambda_{\tilde{F}}(x) = \begin{cases} \dfrac{x - p_1}{p_2 - p_1}, & p_1 \leq x \leq p_2 \\ 1, & p_2 \leq x \leq p_3 \\ \dfrac{p_4 - x}{p_4 - p_3}, & p_3 \leq x \leq p_4 \end{cases}$$

The graph of the trapezoidal fuzzy number is as follows:

1.14 Prominent characteristics of the thesis

Within an inflationary environment, it becomes difficult to have inventory management and show the inventory models with inflation are gaining awareness. With an increase

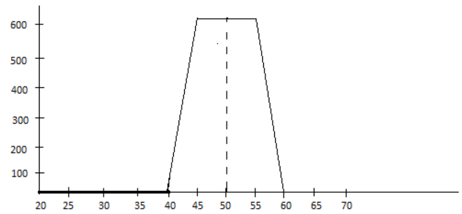

FIGURE 1.6: Trapezoidal Fuzzy Number

in inflation and a quick decline in the buying power of the money particularly in the developed countries with rising economy, it becomes practically impossible to take a note on the consequences of rising prices. Ground this work attempts to formulate the theory of optimising the inventory system under inflation. For their the supplier in order to capture the market and keep a hold over the existing scenario grantz a fixed delay period to the retailer. This delay time also encourages the Wendy to increase the order size. Earlier, the traditional models of inventory considered immediate paying back by the retailer to the supplier as soon as the goods are received. But the concept of delay in paying bank has come out to be profitable concept to both the retailer and the supplier. This delay in paying back is called as trade credit. If the retailer pays after this admissible period, then an interest is charged on him. Within this admissible period, no interest is charged on the amount held by the retailer. So this is a Win-Win strategy for both the retailer and the supplier. With the increase in the awareness towards a surroundings, the forms are initiating that steps towards green environment and so the inventory modelling is formulated in such a way that there is a reduction in the amount of goods wasted or and utilised in the system. These goods were earlier disposed in the surroundings and were a major threat to the environment. But now the concept of cannibalising or remanufacturing has played an important role in reducing this wastage. The research work progress is for the green inventory system with the admissible delay concept. Also the work includes various other parameters to minimise the cost. The demand is one such factor. Demand is a significant factor in determining the level of inventory. This work includes time quadratic and price dependent demand. Demand is inversely proportional to the price of the product. And inventory modelling including price dependent demand is done to ensure minimum cost in the sun supply chain process. These inventory models are framed under arkwrist environment. But many a times the decisions in the inventory system are done in the environment where the parameters are taken in fuzzy form. The research progress is with Fauji find the parameters like the demand, price, inflation and the discount rate. With defining the uncertainty prevailing in the inventory system a more realistic solution to the real world inventory problems is determined. The inventory problems discussed in the research work provide an elementary Framework for the various situations arising in the inventory system and its analysis the work is useful in studying the real life problems and the mathematical solution providing and edge in managing the inventory. The objective of the research work is to formulate the models for the inventory management for unit item and single retailer and single supplier in the supply chain.

1.15 Thesis Organization

Inventory policies are developed to determine the optimal refilling time period for the retailer and supplier when coordination and no coordination exist under inflation. Also optimal measure of the ordering quantity is determined with minimising the overall total cost of the system. Trade credit is considered as an important factor for the models. With this delayed period factor the savings in the cost realised are divided amongst the retailer and the supplier.

Chapter 1 is the introduction to inventory models.The different types of inventory models on the green concept, fuzzy logic study. Also the models are subject to different demand patterns and coordination between the retailer and the supplier is considered.

Chapter 2: Literature prevailing on inventory model formulation is reviewed.The study on the Greens theory for environment saving on the inventory policy formulation is done. Also the application on various parameters of an inventory model is studied for realistically framing a real problem. Demand plays an important role in minimising the objective function. Different demand patterns are analysed by the researchers. The studying on these different demand function is done in this chapter.

Chapter 3,Chapter 4 and Chapter 5 formulates the concept of coordination between the retailer and the supplier which is becoming a prominent subject for study to the researches. Since this concept helps both the members of the supply chain, this chapter studies the significant role of co ordination. A study is developed to determine an optimal time period in a supply chain model optimizing the objective function of minimizing total cost for both the retailer and the supplier under inflation.

Chapter 6: Now a days government puts extra emphasis on the organizations to reduce the waste products which are a threat to the environment. In this section, a study is conducted to develop mathematical models for green inventory. The defective present in the lord received by the retailer are taken back by the supplier. This defective goods are recycled and he manufactured and again sent back to be observed in the market. This in turn reduces the hazards. These imperfect good poses on the environment and also if unintentionally consumed in the market the can have negative impact on Goodwill of the product. Inflation is compounded on every cost and in a supply chain, the benefits realized on account of delay time are divided amongst the retailer and the supplier.

Chapter 7: Demand is a dominating parameter while formulating a mathematical model. Demand of a product depends upon the availability of the product and also on the price of the product. If product has reasonable price plus offers, it attracts more customers. Also if price is more, it has adverse effects on the demand of the product. In this chapter a mathematical solution for a real problem is examined numerically. The demand is a function of price. Also green preservation is kept in mind while developing the model.

Chapter 8:The rate of inflation, price or the discount rate are subjected to unpredictedness. As these parameters are more prone to the market fluctuations, it is very difficult to have a precise value to these parameters. In this section these parameters are fuzzified to have a more realistic approach to the mathematical solution of the problem. The defuzzification of the total cost is done using two methods centroid and the signed distance method. The defuzzified cost value from both the methods are compared and analyzed for better insights to the problem.

Chapter 9 This chapter concludes the study of the inventory model featuring inflation and credit period. Greens preservation technology and various demand patterns are included in the study. Finally a crisp nature of the inventory model is compared with the fuzzified inventory policy.

Chapter 2

Literature Review

2.1 Inventory Review

The analysis of inventory management through mathematical models is done with the upcoming of various manufacturing and industrial firms. In order to have a sustainable growth of in the market, these industrialists should have the inventory of finished goods so that these can be made available as and when the demand occurs. If the product is not available with their retailer, he has to place the back order of the goods. This is backlogging. Nowadays, there are plenty of resources available in the market for any particular good. If the customer moves to some other supplier and gets the goods from that source, then this customer is lost and it is lost sale which in turn affects the goodwill of the product and also results in the decrease in revenue. Also a manufacturing firm has to keep a watch on the inventory of raw materials and unfinished goods so that a streamline production process is always maintained. Running short of any item during manufacturing process can hampers the progress and completion of the finished products. The primary motive of holding inventory is to assure timely delivery of the proper material and in exact quantity as required at right station and time. With these constraints, inventories are essential for any organization but a right order and amount will help the organization to fulfill its needs and watch out for the excess cost. Hence every firm has to organize and control its inventory in a productive manner. Thus, a major problem which is to be taken care by every organization is the maintenance of an optimum level of stocks of inventory. A level which will not affect the production process and also will not increase the system's overall cost.

Many researchers throughout the world are engaged in formulating the mathematical possibilities for the real time problems faced by the firms in managing the inventory. The introduction to the economically ordering the quantity in the stock was made by Haris, 1915. The economic expert Arrow, Harris, and Marschak, 1951 took the initiative in analyzing the mathematical procedure for an inventory theory. He consecrated a book for the mathematical logics and properties in the inventory theory. Hadley and whitin wrote a book on the inventory analysis namely "Analysis of Inventory Systems".

Also, "Inventory System" book is penned by Waddon, 1996. Thus, the area of inventory was consistently under study and progress gradually. Many research works and books are published in the area in the previous decades considering variety in the constraints and parameters. This section of thesis focuses on the literature survey and outlines a brief knowledge on the association between the various constraints involved in inventory research.

2.2 Inventory theory with trade credit:

It is defined as the time period made admissible by the supplier to pay back for the goods received by the retailer. This admissible delay period is gaining attention by the researchers. In today's competitive market, it has become increasingly important for the supplier to keep hold of their potential retailers so that they can have a hold over the market in a passive way.

During the credit period, the retailer is allowed to have income through two ways. Firstly, by selling the goods throughout the delay time and secondly by earning through the interest on the amount of money held. The supplier also gives the privilege to the retailer by not charging interest, if the payback is made within the delay time. In this way, credit time is a profitable strategy for the retailers. Although, the supplier too earns during the delay time. This is done by increasing the ordering size and reducing the replenishing orders. In this way the supplier reduces the increase in setup cost. Therefore this delay time is a win-win strategy for both the retailers and the suppliers. The research works done in this field also incorporates various other factors and figure out the optimum solutions for both the retailer and the supplier. Goyal, 1985a initially made a study on the admissible delay period by covering this area in the basic inventory ordering theory. Incorporating the delay time and the stock varying demand, Liao, Tsai, and Su, 2000 formulated and inventory theory. Teng et al., 2002, followed a secondary approach to formulate the inventory theory along with the permissible delay period. An ordering model was studied by Musa and Sani, 2012 for the deteriorating goods with the permissible delay time. For non-instantaneous deteriorating goods Maihami and Kamalabadi, 2012a developed an inventory model for the effective price and the trade credit including shortages within the inventory system. Khanra and Chaudhuri, 2003, modeled a system for the time varying demand. For the production of the imperfect goods along with the delay time, Chen, Cárdenas-Barrón, and Teng, 2014 proposed a beneficial production model for the inventory. The credit time given by the vendors are also associated with the quantity ordered by the vendee. Such association of the delay time with order quantity was studied by Chen and Teng, 2014. With two stage trade credit paying, Wu and Chan, 2014 evaluated the best suitable duration for the credit time. They considered the product of deteriorating nature and a concerned with the expiry dates on the products. For a definite life period of products Sarkar, Saren, and Cárdenas-Barrón, 2015 suggested a theory integrating the credit policy and the randomly deteriorating nature. Considering multiple scenarios for the permissible delay and shortages, Jaggi et al., 2015 developed the ordering quantity theory. Warehouses restrict the capacity of the inventory. A study by Ouyang et al., 2015 examined the constraint of capacity and also linked the delay time with the quantity ordered. Three different approaches of trade credit based on the discount factor for the advance payment is applied in the study done by Gupta, Banerjee, and Agrawal, 2018. They formulated the model for the optimal period and paying back decisions with the price simulating demand. Wu and Chan, 2014 and Chen and Teng, 2014 studied the inventory framework considering complete or partial credit policies where they linked the delay time with the quantity to be ordered. Amongst credit period, pre-payment option was examined to determine the optimality in the amount deposited. Zhang, 1996 modelled a problem to determine the optimal figure of cash to be deposited under advance paying option. Taleizadeh, Niaki, and Nikousokhan, 2011 modelled the inventory theory for goods considering advancement in payment and the doubtfulness in the discount offered. Lashgari, Taleizadeh, and Sana, 2015 proposed a study integrating the daily time in payment for the upper and below levels in the supply chain with the prepayment.

Cárdenas-Barrón et al., 2018 formulated a theory for the optimal ordering quantity based on the nonlinearity concept of the stock supporting demand and the cost of carrying inventory. They have studied the model within a supply chain process with an admissible delay time given to the retailer by the supplier. Also they have incorporated a concept of inventory in the last cycle which can be positive, zero or negative. By negative inventory, the authors include the completely or partially backlog shortages. Tiwari et al., 2018 discussed an inventory theory for the products with the expiry dates. They also included the partial back orders along with the two chain credit policy. To determine the overall system cost for an inventory management, warehouses (owned for rented) too plays a significant role. Gaur et al., 2018 introduced the concept of the learning strategy in the inventory modeling. The authors have brought out that with experience, the cost associated to an inventory model can be reduced. The model formulated also integrates the delay policy and the determination of an optimal replenishing policy for the vendee.

2.3 Inventory models with Inflation

Inflation is one of the key components in determining the overall cost of the system. It influences every cost related to the inventory. With the increase in the rate of inflation, the currency declines in its value. It implies that the future amount of currency measures less as compared to the current value. Hence the application of inflation is a must for determining the cost value in the inventory management. Buzacott, 1975 introduced the concept of rate of inflation on the basic inventory problem. Datta and Pal, 1990 viewed the significance of the rate of inflation and the money value for an inventory theory with the demand rate being linear with time. An ordering quantity formula was developed by Bose, Goswami, and Chaudhuri, 1995 for the demand being linear and depending on time. They included shortages and inflation with the discounted approach. Chang, 2004 formulated an ordering quantity theory under inflation when the credit time given by the supplier depended upon the quantity ordered. An optimal refilling policy was determined by Uthayakumar and Geetha, 2009b. They modeled the inventory problem for single product and also integrated the effect of inflation. A discounted money flow concept was involved in determining the cost in the inventory system for the perishable products and with partially exponential backlogging. A model with demand being influenced by inflation was studied by Thangam and Uthayakumar, 2010. An optimal manufacturing quantity model for the faulty product was prepared by Sarkar and Moon, 2011. They further considered time dependent demand along with inflation. Tripathi, 2011 modeled a credit policy considering two cases in the inventory problem. They studied the cases for the credit time less than or greater than the cycle time. The model was prepared under inflation and demand being time fluctuant. They derived the optimal ordering time.

Taheri-Tolgari, Mirzazadeh, and Jolai, 2012 proposed an inventory theory for the corrupt goods by inspecting the goods for errors. They formulated the model under the effects of inflation. Incorporating the delay time and inflation Yang and Chang, 2013 considered an inventory theory for partial shortages and two warehouses. For a finite planning Horizon Gilding, 2014 formulated the refilling schedule in an inventory problem including inflation. Chen and Teng, 2015 simulated an inventory decision making model with the upstream and downstream concept of delay time permitting the discounted approach and inflation on all the inventory related cost for the perishable goods. Allowing inflation, Jaggi, Khanna, and Nidhi, 2016a formulated a profit show casing inventory model for the deteriorating goods and partially accumulated shortages. They included the benefits given to the consumers by the retailers to protect the sales from getting lost. The benefits included were as offering the current price at the time when the product will be made available to them etc. Rani, Ali, and Agarwal, 2017 studied the inventory theory involving inflation. An interesting and emerging need for the green inventory was also included in their study. They also formulated the model for the weibull deteriorated products. Puja 2017 analyzed the model for the deteriorated goods and fluctuations in the holding cost with respect to time. The research work modeled the inventory problem with the discounted technique and inflation. Alvarez, Lippi, and Robatto, 2019 adopted the beneficial approach of inflation on the variety of inventory problems. The research work followed the demand graph and came to a result that the economic ad through inflation can be approximately calculated. Kwofie and Ansah, 2018 study the consequence of inflation on the cost and stock exchange market of Ghana

2.4 Inventory models with inventory dependent time quadratic and price sensitive demand

In today's competitive scenario, price dominates demand of the product. It is commonly seen that when the price of the product increases, it leads to the decrease in the demand and vice versa attitude is observed when there is decrease in the product's price. Another important factor which influences the demand is the inventory available. It is generally observed that the stock if are displayed in large number in the market tend to attract more customers. This happens due to its popularity, diverseness or visibility. There are a variety of researches done in this field by several researchers. Inventory problems modeled depicts more realistically the real situations regarding the demand of any product. The inventory models framed aims at maximizing the sales with these constraints on the demand, with minimizing the overall cost of the system.

The connection with the increase and decrease of the sales for any product with its availability to the customers was studied by Levin, 1972. In his research work he pointed out the motivational effect which is the prominent measure of displayed goods brings on to the customers and influences them to buy more. Silver and Peterson, 1985 mentioned in their research the linking of the inventory visible on the shelf with the sales at the retailer's end. This connection of the displayed stock with the increase in the demand and sales has encouraged many researchers to examine the inventory framework for the stock stimulating consumption rate and the inventory in hand. Padmanabhan and Vrat, 1990 exhibited a model for the several perishable items. They further incorporated the nonlinear goal programming technique to solve the problem including the stock varying demand under several other constraints. Datta and Pal, 1990 analyze the problem with the rate of demand being dependent on the instant inventory measure till the desired demand level is achieved. Urban, 1992 further extended the Datta and Pal, 1990 inventory model by withdrawing the zero level of inventory at the extremes of the cycle length. Bar-Lev, Parlar, and Perry, 1994 established and inventory framework for the stock varying demand with stochastic yield. Taking demand as a function of time and price, Urban and Baker, 1997 formulated a model for determining economically ordering measure.

Giri and Chaudhuri, 1998 elaborately explained the inventory model with the two-dimensional cost of holding. Chang, 2004 considered the case of non linearity of the holding cost for the stock depending usage rate and the deteriorating items. An optimal ordering model was discussed by Sana and Chaudhuri, 2008 integrating the inventory stimulating demand and the delay time along with the price discounts linked with the quantity ordered. An inventory framework for the stock stimulating demand considering the impact of inflation and the shortages which are partially backlogged was studied by Yang, Teng, and Chern, 2010. Dye and Hsieh, 2011 formulated a model for deriving the deterministic policy for the ordering quantity with the demand being stock and price dependent. The work also incorporated the fluctuations in the cost and included the constraints of restricted capacity. Duan et al., 2012 and Lee and Dye, 2012 modeled a problem for inventory management where the usage depends on the level of inventory for the perishable products with manageable rate of perishability. Tripathi, 2013 simulated the inventory management job studying the inflationary significance on various cost with admissible delay concept. Yang, 2014 explained the model on inventory with two parameters as demand and holding cost being stock varying.

It is usually observed that the demand is generally higher for a new product launched in the market. The demand is very likely to depend on the price of the product and the various attractive offers the product carries along with it. This makes an increase in the retailers quantity to be ordered and also increase the order size of the customers. Various researchers have worked and are still working on the connection between the price stimulating demand and the optimization of the inventory models.

2.4.1 Time dependent

In the recent studies, researchers have also focused on the demand being time-dependent. For the goods like the electronic goods, fashionable products, vehicles etc, the demand of these products increases with the passage of time. Goswami and Chaudhuri, 1991 considered demand as a linear function of time. They formulated the inventory model for deteriorating goods with shortages. Hariga, 1995 studied an ordering inventory model for the perishable products with the demand depending upon time. Chang and Dye, 1999a proposed a mathematical model for the time changing demand and shortage including lost sales. Lin, Tan, and Lee, 2000, Papachristos and Skouri, 2000a presented a study for the time adapting intake of the goods. They integrated the model for the exponential shortages. The inventory problems structure discussed by Skouri and Papachristos, 2002 and Wu, 2002 integrated weibull way of deterioration with the time varying demand. Goyal and Giri, 2003 and Lee and Wu, 2004 modeled an inventory managing job for the exponential is distributed the duration with the time relying demand. Ghosh and Chaudhuri, 2006 formulated and optimal refilling schedule model for the demand being time quadratic and deterioration also recycling on time relying on time. They include shortages for all the cycle in the horizon. Mohan and Venkateswarlu, 2013 model an inventory problem for the time quadratic demand along with shortages. Their study also incorporated the deteriorated goods and the resale value for the defectives. A study including the salvage amount for the deteriorated goods in determining the overall cost of the system is presented by Venkateswarlu and Mohan, 2014. The demand pattern included in the study was nonlinear. Venkateswarlu and Reddy, 2014 studied the inventory model where every cost associated is subject to inflation and the non linearity of demand.

2.4.2 Price-Dependent

Demand is treated as an external variable, not within the governance of the organization. In fact, demand of the product is related with the price of the product. The significance of price in the food and beverage organizations is noticeable. The food products cannot be held for a longer period as stock. This will lead to its deterioration and even spoilage. Moreover, the price of these products plays an important role in ascertaining its demand. This is due to the availability of the products through several companies. If the customer is not satisfied with the price of any product, the customer can move for the alternative companies manufacturing the same product. The customer to looks for the discount option available with the product. Kim, 1995 proposed an inventory model for the price stimulating demand and stock consumption. The model so worked included the delay time in the supply chain process for the defective goods. Wee, 1997 developed the model for determining an optimal refilling policy for the perishable products subject to the price elasticity of demand. They included the random decay rate for the items in stock. Other zestful research works considering demand as a linearly increasing function of price is done by Chang et al.(2006), Dye, Hsieh, and Ouyang, 2007. Chern et al., 2008 suggested in the work done that the firms cannot impose the pricing strategy at their will. So the estimation of the demand depending on the price of the product is to be carefully done in order to avoid any shortages or backlogging. Wee, 1997 proposed an inventory model for the random rate of deterioration with price varying demand.

Mondal, Bhunia, and Maiti, 2003 studied and inventory model with demand being linearly depending on the price. The research work included time relying deterioration rate for the goods in stock. Teng, Chang, and Goyal, 2005 formulated and optimal refilling policy for the inventory problem having price varying demand along with the consideration of delay time action as one of the case. Ding, 2010 suggested an inventory management system in an infinite horizon for the price depending model presuming normal distributive characteristic curve for the demand function. Hou and Lin, 2006 investigated an inventory problem for the deteriorated goods where the demand function collectively varies according to the price and the level of stock. An inventory framework for a single manufacturer with several retailers is examined by Panda, Maiti, and Maiti,

2010. Their work included the demand which is price dependent. Khanna, Gautam, and Jaggi, 2017 studied an inventory system for the defective and deteriorating goods. The work progressed with the admissible delay time financing for the price relying demand. Chen(2010) dealt with the problem of constriction in the storage space for the goods that deterioate with the passage of time. In their study the demand was considered as price and inventory dependent. Many researchers have taken lead time as negligible, but Maiti, Maiti, and Maiti, 2009 considered random value of lead time for their model. They formulated the model for the price depending demand along with the paying the amount in advance condition.

For a single vendor and vendee and single product optimal pricing strategy and optimal refilling schedule was determined by Kim, Hong, and Kim, 2011 for a price defining demand. Yang, 2011 synchronized the price depending random demand with the supply chain concept modeling an inventory problem. They consider two cases in the supply chain namely the centralized and the decentralized case. With the supply chain management, the included sharing of the revenue generated. Annadurai, 2013 proposed an inventory model for the deteriorated goods integrating price defining demand. The work modeled the credit time financing depending upon the quantity ordered. Forghani, Mirzazadeh, and Rafiee, 2013 modeled an inventory system in the infinite planning horizon for the problem of price defining demand with price varying according to the quantity ordered. Swami et al., 2015 considered the price fixing demand for an inventory framework with the impact of inflation on every associated cost. The model also included the concept of credit financing and the random cost of holding. Alfares and Ghaithan, 2016 examine the impact of price relying demand on the optimality of the overall cost. The paper discusses the changeability or randomness in the holding cost and the purchasing cost. Maragatham and Palani, 2017 simulated a situation for the deteriorating goods considering the parameters as the cost of holding, ordering and the rate of deterioration vary with time. They included shortages along with lead time.

2.5 Inventory model with deterioration

2.5.1 Instantaneous deterioration

Deterioration has a prominent role in determining the inventory levels for a system. Deterioration can be explained as the damage, decay, spoilage, obsolescence or loss of marginal value for any product which leads to a decline in its usefulness. In general, there are goods which starts deteriorating as they enter the system and certain goods as medicines, blood banks etc, which starts deteriorating with the passage of time. The goods on behalf of the rate of deterioration are classified into three categories: (i) Instant deterioration:- The goods as the enter the system starts deteriorating. For example fresh fruits, crops, foodstuff, milk, crop etc. (ii) Substantial/essential deterioration:- This type of deterioration is seen in the chemical products as the medicine and the petrol. (iii) Non-instantaneous deterioration:- The deterioration within a passage of time, as for the medicines, electronic goods etc. Adding with the lifespan of the products, deterioration is further classified as: (i) Fixed or certain life span:- There are certain products with a certain life period. After the expiry of the duration, the product is bound to become useless. Practically, the quality of the product declines with the passage of time, example blood, milk, food and beverages etc. (ii) Variable lifespan:- Some goods do not have a fixed lifespan. They starts deteriorating after a certain time period. The rate of deterioration varies according to the time and is taken as a function of time. Normally the distribution rate of deterioration follows weibull, gamma or exponential ways. The goods that follow such time-varying deterioration are medicines, chemicals, electronic goods etc. The introduction to the deterioration rate in determining the optimal ordering Policy was done by Ghare and Schrader, 1963. Further the extension of the work by Ghare and Schrader, 1963 was done by Covert and Philip, 1973. They extended the model for the exponential decaying stock following weibull and gamma distribution. An inventory model considering deterioration for three parameters distributing in a weibull mode was done by Covert and Philip, 1973. Shah, 1977 examined the two ways of deterioration namely weibull and exponential for the time relying rate of deterioration in determining the optimal level of refilling for the inventory system. Time defining deterioration along with the partial shortages was discussed by Sana and Chaudhuri, 2008 to determine the best possible values for the selling price and the best ordering measure for the deteriorated goods. Ghosh and Chaudhuri, 2006 devised a framework determining the optimal price value for the product and the optimal level of the order placed when the goods are subjected to the time fixing rate of deterioration. They formulated the model in a finite planning horizon and also included partial shortages with lost sales. The concept of the delay in paying back integrated with non-instant goods which deteriorate exponentially was studied by Mahata and Goswami, 2007 in determining the best strategy for the ordering model. Liao, 2007 with supply chain policy developed the production model for the time limiting deterioration goods. Also with the variations in the rate of deterioration, Sarkar, 2012 presented an ordering quantity model. Sarkar, 2012 examined the probabilistic time varying deteriorating inventory model to determine the number of refilling schedules and the ordering level. Zhou et al., 2014 formulated an inventory framework for the two staged supply chain system to calculate the optimal level of the ordering measure and optimal cycle time along with the best suitable pricing policy for the vendee. Kumar and Kumar, 2016 analyzed an inventory model for the deteriorating products. They involved the preservation mode for reducing the rate of deterioration. The model also incorporates other parameters of inflation and credit time financing. Aliyu and Sani, 2018 modeled an inferred demand varying exponentially problem for determining the optimal value of the gross cost. They included linear nature of deterioration relying on time for the inventory system.

2.5.2 Non instant deterioration

The concept of the deterioration cost for the products as they enter into the system is questionable for many products. Moreover, goods and certain shelf life span and can retain their originality and quality for that period. The product starts to deteriorate after this passage of time, as the vegetables, fruits etc. Wu, Ouyang, and Yang, 2006 discussed in their research the concept of non instant deterioration. In practicality, the goods directly from the source have a certain time period in which they maintain their original taste, color and properties. Within this time period, there is no decay. If during the framing of an optimal refilling time and quantity, the above mentioned condition is not considered then the results would very likely be inappropriate. Hence in valuation of the inventory, the products which are non instantaneous in deterioration should be viewed. Ouyang, Wu, and Yang, 2006a defined an inventory model for the non-instant deteriorating products. The work done for optimizing the inventory system included supply chain concept. Liao, 2008 formulated an ordering policy for the non instantaneous deteriorated goods, taking deterioration of exponential form. They also considered for the idea of supply chain at two stage. Uthayakumar and Geetha, 2009a prepared a refilling schedule for an inventory system for the non-instantly deteriorated goods. They prepare the model integrating the concept of lost sales.

Geetha and Uthayakumar, 2010 designed a system for the crediting period with the non instantaneous perishable goods. Maihami and Kamalabadi, 2012a specified a plan for the non-instantly deteriorating goods in the inventory structure. They prepared the optimized cost project considering shortages which were background partially and demand varied according to price and time. An investigation related to decrease in the rate of deterioration for the non instant goods was done by Dye, 2013. The work included preservation and protection technology for the goods in stock. Including the effect of inflation on various cost in the inventory area, Ghoreishi, Mirzazadeh, and Weber, 2014 designed the best possible pricing and replenishing plan of action for the non-instantly deteriorated products. The model was based on the return of the goods by the customers back to the retailer or seller. Tat, Taleizadeh, and Esmaeili, 2015 developed the ordering size model for the non instant goods including the strategy of managing the inventory at the suppliers end. Tiwari 2017 incorporated the capacity constraint at two different warehouses for the non-instant deteriorated goods. They gave model based on the demand relying on the available stock and the impact of inflation on the cost. They studied the particle swarm optimization technique in the model. Benkherof 2018 developed a model in an infinite planning time zone for a single product. The model included goods which deteriorate with the passage of time and are non-repairable. The work also considered that all the backorders are met in total.

2.6 Inventory model with completely/partially backlogged shortages

From the realistic nature of inventory structure, it can be presumed that there is significant linking between the demand and the consumption of any item with the shortages occurring in the system. Shortages occur when the stock of inventory is not sufficient to fulfill the demand of the customers. Shortages can be fully or partially backlogged. In full or complete backlogged shortages, the demand of the customers waiting is fulfilled as the next order arrives. Also owing to the goodwill of the product or the retailer customers tend to wait for the arrival of the stock rather than to switch to other options available in the market. It also happens for some fashionable and trendy goods which are available at a particular shop or for some electronic equipments etc. The goods like the food and beverage products are available at different options in the market. So for these goods, the customers may move from one option to the other depending on the price offered at other outlets and the discount offered with the particular product. This type of characteristic of the customers is studied under partial backlogging. Chang and Dye, 1999a and Dye, Ouyang, and Hsieh, 2007 designed an inventory model for the deteriorating goods with shortages being time relying. Inventory policy in determining the optimal refilling time for deteriorated goods with partially backlogged shortages is formulated by Park, 1982 and Wang, 2002. Min and Zhou, 2009 developed and inventory system for the perishable products with the consumption rate being stock limiting and the orders undone are partially backordered.

For deteriorating rate being weibull, Tripathy and Pradhan, 2010 derived an optimal ordering policy model for partial shortages. Yang, 2011 formulated a production model for shortages which are partially fulfilled. The goods under consideration undergo time limiting deterioration. Roy, Sana, and Chaudhuri, 2011 modeled an ordering policy for the defective and imperfect type of products with partially fulfilled shortages. Sarkar, Ghosh, and Chaudhuri, 2012a formulated a replenishment plan of action for the time quadratic demand and partial backlogs in shortages for all cycles. An EOQ framework involving credit payment in partial mode and partial shortages was developed by Taleizadeh et al., 2013. The backlogging done partially is taken as a function of waiting time. Considering the prepayments and backlogs for the perishable goods Taleizadeh, 2014 designed and optimal policy model. Following demand as ramp type and including shortages Pal, Mahapatra, and Samanta, 2015 give a production model for the deteriorated goods. A time fixing approach for demand was introduced in the inventory model by Pervin, Roy, and Weber, 2018. The work investigated the optimal refilling decisions for the time varied demand and deterioration. The model is prepared to include shortages. Saranya and Varadarajan, 2018 investigator model under fuzzy logics and also included complete shortage in optimizing the profit for the inventory system. The research included shortages and generalized the concept of fresh products and products directly ordered from the source. The shortages included signifies that customers can wait for the fresh product and the outlet too owns its goodwill. The objective of the paper is to optimize the total cost of the system.

2.7 Inventory model with green inventory system

In the present scenario of competitiveness prevailing in the market, the manufacturers and the units are not close to the retailer or selling outlets. The products, when received at the vendee end, is bound to have certain percentage of defective goods. These imperfect goods are to an extent realized as low-cost products in the system or are treated as waste products. If these goods go waste they are thrown in the surroundings and pollute the environment in many ways. To have sustainability in preserving the environment, it is important to treat these goods in a prominent manner. Also these goods, add to the cost of the inventory system in the form of deterioration, shortages and sometimes lost sales, and if unintentionally sold to a consumer can lead to the loss of goodwill of the product and the organization. Thus to ensure effective inventory management, quality of the items and the reputation of the brand/organization, the goods can be repaired, remanufactured or even recycled for their absorption in the system. Green inventory system not only is the way to protect the environment against the hazards of these waste/imperfect goods but also adds in the revenue generation of the inventory system. Hence the theory of green system not only includes responsive supervising but also the proactive policies, implementation through diverse R's such as re-manufacture, recycle, refurbished, reclaim, reuse, reduce, reverse logistics, rework etc.

Improving the quality of environment is considered for safe and healthy food and drinking water, tidy physical systems, freedom from the toxicity prevailing in the communities, safe and proper waste disposal and management and the refurbishment of the polluted waste sites. At present, there is a rise in the public interest towards the cleanliness of the natural surroundings. This is happen due to the increase in the number of non profitable organizations working in the interest of the consumers and the environment Fiksel, 1996 1996. The various manufacturing units and the production processes are frequently observed as the biggest threat to the environment. As these operations ejects waste and the defectives in the surroundings and decrease the natural assets. This indicates that a change in the manufacturing trend is an emergent necessity. A requirement of change towards affirmability of the environment is obtained through the declining the amount of waste produced and optimally utilizing the resources. Beamon, 1999 investigated the various channels for improving the quality of the environment. The work describes the differences between the traditional model and the supply chain models incorporating them to maintain the green inventory system.

Adequate work including several facets and aspects of the green system is available. Detailed reviews on the green system design is done in the world by Zhang et al., 1997 and a comprehensive study on repairable product inventory is done by Guide Jr, Srivastava, and Spencer, 1997 and Guide, Kraus, and Srivastava, 1997, on remanufacturing control and planning of the production process is done by Bras and McIntosh, 1999, Guide Jr et al., 2000 and on the supplying network project design work published by Fleischmann et al., 2000,Fleischmann et al., 2001, Jayaraman, Patterson, and Rolland, 2003. Incorporating green system in a supply chain inventory process is advantages to both environment and the supply management. The green inventory in supply chain management regulates and maintains the relationship among the environment and the supply chain system. The green inventory supply chain model as defined in the literature included the green modeling and purchasing to the inclusion of the supply chain enterprise in two or three levels from the vendor to the vendee and then to the customer sometimes incorporating reverse Logistics Zhu and Sarkis, 2004. Integrating the concepts of return through reverse supply is an effective way towards the green inventory framework. Sheu, Chou, and Hu, 2005 integrated the return of used logistics to optimize both the operations of production and the used goods return policy for the green inventory supply framework.

A detailed and comprehensive review of the green supply chain management is done by Srivastava, 2007. He incorporated various facets of green system as the importance of green inventory, design of green system, and the operations done for green inventory system. An important role in maintaining a green environment is played by reverse logistics. The defective product is returned to the manufacturer for recycling or remanufacturing. This type of platform not only helps in gaining the customers confidence and satisfaction towards the brand but also increases the profit of the system. Huang, 2010 presented an integrated model with the admissible delay period along with the process of the reduction in the cost. Kim, Goyal, and Kim, 2013 proposed an ordering streaming line of action with the forward reverse logistics supply of the defective goods. Luthra, Garg, and Haleem, 2014 investigated a detailed state of the art review of the green supply management and discussed the usage of green inventory like the eco-friendly raw materials, recycled papers used in packaging, reducing dependency on the petroleum and its products and many others. The review discussed the research work from 1995 to 2010. The key concepts studied in the work were the green product design and development, Green product manufacturing and transportation, green raw material purchase and procurement. A study on the emission of carbon under the manufacturing and trading mechanism was done by Hua, Cheng, and Wang, 2011. The model optimizing the total cost and investigating the impact of carbon emission and price on the inventory cost.

Hammami, Nouira, and Frein, 2015 modeled a three level supply chain from the supplier to manufacturer/retailer to the customers. They used the technique of inventory control to frame the inventory model. The work included the constraints on lead time, the time period to place an order and the storage capacity. They also involved the influence of the emission of the carbon on the production process. The research study derives some significant conclusions regarding a decline in the emission of carbon due to an increase in the order size. Panda, 2014 suggested the impact of the environment friendly models on the society. The model incorporated the sharing of the profit generated on account of implementing the green inventory system. Jiang, Chen, and Zhou, 2015 presented an optimization model for several units in the inventory under preservation and maintenance. Sarkar, 2016 implemented the coordination mechanism at various levels including the inspection of the goods received. The defective goods are dealt according to the green inventory system. The model also included the policy of discount for the products having fixed long life. A mixed policy for the manufacturing and production design with the delay in payment strategy is proposed by Saxena, Singh, and Sana, 2017. The research work included the determination of optimal lot streaming period. They analyzed the effects of various parameters on the cost and profit of the inventory system. Yang, Tu, and Lai, 2017 worked on the concept of reworking on the imperfect goods which are unavoidable to any company. To sustain a profitable situation a proper screening and re-manufacturing process is to be followed so as to provide assistance in green technology. They studied on the reduction of the carbon dioxide gases and sustained the business along with saving the environment in maximizing the profit.

2.8 Inventory models with Fuzzy logics

Fuzzy logics is applied in the inventory theory to bring the model close to the reality. The exact prediction of the real scenario is not possible. The calculations regarding a real-world problem can only be made and for this the fuzzification of the parameters involved helps in approaching towards a more realistic solution. Thus fuzzifying a model integrates the uncertainty prevailing in the model thus reaching to a more practical solution. With the progress in the field of fuzzy inventory theory, several researchers have worked on problems integrating the two concepts. Since the formulation of the primary inventory ordering and production model, the research is then progressed by eliminating the various relaxed assumptions of time-varying demand, partial backlogging, inflation, supply chain and delay in paying back, quantity and price discounts etc. But keeping all these significant parameters in mind while developing the inventory models, one very unrealistic scenario that was considered was a deterministic and certain environment for all the parameters involved in the inventory model. The introduction of fuzzy concept in the inventory ordering model was done by Park, 1987. Moreover in today's changing world business world, the data for the inventory problem cannot be exactly predicted. To overcome with this indeterminable nature, Zadeh, 1965 used the concept of fuzzy in developing the inventory model. With the advancement in the literature, different real life problems as of newsvendor, trade credit, optimal re-ordering point, inventory management problems etc were included in the basic EOQ model. An effective inventory management relies on the proper and efficient prediction for the demand and all the cost related to the inventory.

Aissaoui, Haouari, and Hassini, 2007 reviewed research works with the fuzzy logics on the inventory models related in determining the optimal reordering interval for the supplier. A fuzzified inventory model is also compared under different constraints with the crispy inventory model. Lee and Yao, 1999 in their study derived that the fuzzified total cost of the fuzzified inventory model is marginally greater than the total cost of the crisp inventory model. A comparison of the defuzzification methods as the centroid and signed distance methods in obtaining the value is done by Yao and Chiang, 2003a. They also suggested certain measures and policies for choosing the defuzzifying methods. Chen, Wang, and Arthur, 1996 first attempted to formulate an ordering model considering backlogging. A fuzzification of the shortage quantity and the ordered quantity is done by Yao and Lee, 1996 and Chang, Yao, and Lee, 1998 in their study. Wu and Yao, 2003 derived through their study that when both the order quantity and the back ordered quantity are fuzzified, a more realistic model is derived as compared to a model with fuzzifying any one of the above said parameters. Björk, 2009 suggested that an increase in the number of ordering schedule by 6 percent is observed in the fuzzified model as compared with the crisp model. Kazemi, Ehsani, and Jaber, 2010 proposed the model which is fully fuzzified and derived that the model is more responsive to any change in the triangular fuzzy number as compared to a change in the trapezoidal fuzzy number.

Practically there are defective / imperfect goods in the lot received / manufactured. There are numerous reasons for defective goods as imperfect manufacturing process, outdated machines, natural calamity, breakage or spoilage on way, evaporation etc. The goods that are of deteriorating nature are milk and milk products, medicines, fruits/vegetables, oils, fertilizers, electronic goods and many more. Thus inclusion of the defective goods with fuzziness becomes more realistic. Chang, 2003, extending the work of Porteus, 1986 formulated an ordering model by using statistic fuzzy number for the rate of capital cost. Integrating the imperfect goods in each lot received, Wang, Tang, and Zhao, 2007 maximized the profit inccured by developing a Fuzzy- Stochastic model. Making demand as a fuzzy variable and depending upon the rate of advertisement, Yadav, Singh, and Kumari, 2012 developed a fuzzified ordering model. An extension to the work of Wee and Chung c, 2009 was done by Mahata and Goswami, 2013, by incorporating the complete backordering in fuzzy sense. Jana, Das, and Maiti, 2014 included fuzzy concept for the multi item inventory model. The work is explored in the random horizon and demand being inventory variant. Jamal, Sarker, and Wang, 1997 explode and inventory ordering system for perishable goods including shortages and delay time. Chen and Ouyang, 2006 further made an extension in the work done by Jamal, Sarker, and Wang, 1997 by incorporating the fuzzy concept to the model. Mahata and Goswami, 2007 discussed the fuzzy system with cost and delay time fuzzified. Their model also showed the concept of being profitable to both the retailer and the supplier in the supply chain. Ouyang, Teng, and Cheng, 2010 fuzzified the association of the delay time given by the supplier with the quantity ordered thus extending the work done by Chang, Ouyang, and Teng, 2003. Guchhait, Maiti, and Maiti, 2014 investigated a fuzzy two level trade credit inventory model from supplier to the retailer and from retailer to the customer with a discount option of the payment being done early. The defective goods can be recycled or remanufactured. Roy, Maity, Maiti, et al., 2009 diagnosed a fuzzy inventory system where percentage of the defectives / imperfect goods is reworked to become perfect goods. Mondal et al., 2013 considered in their study the re-manufacturing of defective goods in a fuzzy production process. Shekarian et al., 2014 worked on the two fuzzy numbers namely trapezoidal and triangular and concluded that the total cost of the Inventory model using trapezoidal fuzzy number is more than that using triangular fuzzy number.

Mandal et al., 2010 developed a fuzzy model for the imperfect goods where demand is taken as a fuzzy time under constant, linear and quadratic. Chen and Ouyang, 2006 fuzzified the holding cost and the amount of interest paid and earns using the triangular fuzzy number. The Fuzzy inventory system derived a convexity of the total cost. Kao and Hsu, 2002 analysed a fuzzy inventory model in infinite planning horizon with demand being fuzzified. The fuzzified value of total cost ranks the different fuzzy logics. Hsieh, 2002 introduced to fuzzy models based on fuzzy constraints with crisp/fuzzy manufacturing quantity. The final solution to the two models is derived by graded mean integration representation method for defuzzification. Liu and Zheng, 2012 considered fuzzification in the inspection method, whereby an error can be made while inspecting thus fuzzifying the fraction of imperfect goods in the lot describes the problem realistically. The cost is minimized under these constraints. Hsieh, 2004 analyzed an inventory system for fuzzified demand and time lag. The trapezoidal fuzzy number is used in decision making and deriving the optimal refilling period with minimizing the total cost. Function principle and graded mean integration representation methods are used to defuzzify the model. An ordering inventory system without backorders is formulated by Dutta and Kumar, 2012 in the fuzzified environment. Trapezoidal fuzzy number is used to model the problem and minimization of total cost with optimal refilling period is determined. Signed distance method is used to defuzzify the model and achieve the objective. Pathak and Sarkar, 2012 proposed an inventory system with weibull deterioration, time-varying demand and allowable backorders. Trapezoidal and triangular fuzzy numbers are used to model the situation in the fuzzy environment. The cost are fuzzified with an objective to minimize the total cost. Vijayan and Kumaran, 2008 modeled an inventory system with partially backordered shortages. To fuzzify the parameters, trapezoidal fuzzy number is used and signed distance method is used for defuzzification. Wang, Tang, and Zhao, 2007 explored an inventory system by integrating the fuzzification of the parameters with the particle swarm optimization method. The model is proposed to minimize the fuzzy total cost and optimal refilling period is determined. Yao and Chiang, 2003b analyzed a fuzzy inventory model without shortages, fuzzifying the cost parameters and then defuzzification is done through centroid and signed distance method. The model derives the minimum value of total cost and optimal ordering level. A comparison of result is done by the two defuzzification methods.

Chapter 3

Replenishment policy for an inventory model under Inflation

Abstract

The purpose of replenishment is to keep the flow of inventory in the system. To determine an optimal replenishment policy is a great challenge in developing an inventory model. Inflation is defined as the rate at which the prices of goods and services are rising over a time period. The cost parameters are affected by the rate of inflation. High rate of inflation affects the organization's financial conditions. Based on the above backdrop the present paper proposes the retailer's replenishment policy for deteriorating items with different cycle lengths under inflation. The shortages are partially backlogged. At last numerical examples validate the results.

3.1 Introduction

Inflation is the rate at which the prices of the goods and services are rising over a period of time and as a result, the purchasing power of money is decreasing. Most of the models are designed keeping the costs as constant, but it is not so in reality, as the costs get effected by the time value of money. The effect of inflation is important in the practical environment. Buzacott, 1975 evolved an inventory model considering inflation and modeled two cases. Afterwords (Buzacott, 1975, MISRA, 1975, Bierman and Thomas, 1977 framed inventory policies considering the different phases of inflation. Uthayakumar and Geetha, 2009b established an optimal ordering policy for inventory dependent demand. Yang, Teng, and Chern, 2010 established a model where demand is stock dependent, considering deteriorating goods and partial backlogging. Bansal proposed an inventory model allowing deterioration with inflation, considering the first case with shortages completely backlogged and the second without shortages. Bhaula and Kumar presented an optimal ordering policy for stock dependent demand with two-parameter Weibull deterioration under inflation and time value of money. Jaggi, Khanna, and

Nidhi, 2016a derived optimal replenishment cycles for deteriorating goods and partially backlogged shortages under inflation.

Also, deterioration is decay, damage, decline, loss of water, or spoilage of goods or services. It means the degradation of quality. The inventory is subjected to deterioration with time. Ghare and Schrader, 1963 first introduced an inventory model considering exponential deterioration. Covert and Philip, 1973 modified the work of Ghare and Schrader, 1963 by proposing a model where deterioration is subjected to two-parameter Weibull distribution. Jaggi, Khanna, and Nidhi, 2016a derived an EOQ model with deterioration under inflation. Hsieh, 2002 studied a lot size model incorporating deteriorating items with inflation to find the optimal selling price. Other zestful research works on deterioration is done by researchers as Ouyang, Wu, and Yang, 2006b, Bazan, Jaber, and El Saadany, 2015, and Goyal and Giri, 2001.

However, in the present scenario of frequently changing market trends, the customer is reluctant to wait for the product during the shortage period. The electronic products and fashionable goods have a short product life cycle, so the customers do not wait for the longer period. Consequently, some of the shortages are turned into lost sales.Abad, 1996 developed the first paper to present the customer's impatience function. Chang and Dye, 1999a developed a model where the backlogging rate is the reciprocal of the waiting time. Papachristos and Skouri, 2000a proposed an inventory model with partial backlogging where with the increase in waiting time, there is an exponential decrease in the backlogging rate. Teng and Yang, 2004 established a partial backlogging inventory model with time-varying purchasing cost. Dye and Hsieh, 2011 extended Abad, 1996Abad's model for lost sales and backorder cost.

The present chapter develops an inventory model for optimal replenishment cycles fro deteriorating goods with partial backlogging considering time value of money in a finite planning horizon. Numerical examples are shown to validate the analytical solution of the proposed mathematical model.

3.2 Assumptions and the nomenclature

3.2.1 Assumptions

1. Inflation and time value of money is considered..
2. Every cost related to inventory is influenced by the inflation rate r.
3. Single item inventory is used.
4. Lead time is zero.

5. Deterioration starts as the goods come in stock. In the planning period the deteriorated goods are not repaired or replaced. A part of inventory in hand deteriorates with time and deterioration rate is given by ϕ= βt,where $0 < \beta < 1$.

6. Shortages are permitted. A(t) is the part of shortages that is backordered.

7. Unfulfilled demand turns into lost sales. Lost sale costs includes the loss in the revenue generated and also the decline in the goodwill of the firm. Thus 1-A(t) is the fraction of the demand that is lost.

8. The model is developed in finite planning horizon H.

9. Demand of the product varies with time.

3.2.2 Nomenclature

1. r = inflation rate per unit time
2. d = discounting rate of time value of money
3. P_0 = Purchasing cost per unit of the product
4. D_0 = Deterioration cost per unit of the product
5. H_0 = Unit holding cost
6. S_0 = Unit shortage cost
7. L_0 = Lost sale cost per unit
8. C_0 = Cost of replenishment per order
9. $g(t)$ = The demand function of time
10. $A(t) = \dfrac{1}{1 + \delta t}$,$\delta > 0$, and t is the waiting time of the customer

3.3 Mathematical formulation and solution of the proposed model

The model is as shown in fig.1.The level of inventory is zero at the starting time. Shortages begins to accumulate at the beginning of the cycle. The inventory reaching at time t_i is used to accomplish the shortages during the last cycle and the demand of

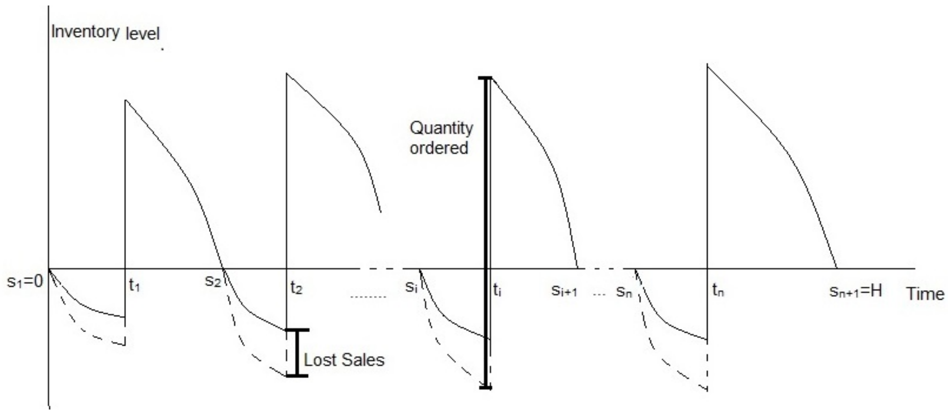

FIGURE 3.1: Pictorial representation of the discussed inventory Model

the current cycle with deterioration. There is a decline in the inventory level during the cycle due to demand and deterioration and the inventor level ultimately becomes zero at time s_{i+1} . In the next cycle $[s_{i+1}, t_{i+1}]$ the demand in this period again accounts for the shortages.

The formulation and the explanation of the inventory model aims to get the optimal refilling period n and the optimal ordering time t_i and s_i so that the total cost is minimized.

The inventory level at time t is given by $I_{1i}(t)$. The depletion in the inventory level during $[t_i, s_{i+1}]$ is due to the fulfilled demand and deterioration. This is given by the differential equation as follows:

The instantaneous level of shortages in the shortage period is $I_{2i}(t)$. The differential equation is given by

$$\frac{d\left(I_{1i}(t)\right)}{\text{dt}} + \phi(t)I_{1i}(t) = -g(t), t_i \leq t \leq s_{i+1}(i = 1, 2, ...n) \tag{3.1}$$

the boundary condition is $I_{1i}\left(s_{i+1}\right) = 0$.

$$\frac{d\left(I_{2i}(t)\right)}{\text{dt}} = g(t)A(t) = g(t).\frac{1}{\delta\left(t_i - t\right)}, s_i \leq t \leq t_i(i = 1, 2, 3, ...n) \tag{3.2}$$

the boundary condition is $I_{2i}\left(s_i\right) = 0$.

Taking $\phi = \beta t (0 < \beta < 1)$,

the solution of the equation(1) is

$$I_{1i}(t) = \int_{t_i}^{s_{i+1}} e^{(\beta/2)\left(u^2-t^2\right)} f(u)du, t_i \leq t \leq s_{+1},\ (i = 1, 2, 3, ...n) \tag{3.3}$$

During the interval $[t_i, s_{i+1}]$, the total amount of inventory is

$$I_1 = \int_{t_i}^{s_{i+1}} I_{1i}(t)dt = \int_{t_i}^{s_{i+1}} \left\{ \int_{t_i}^{s_{i+1}} e^{(\beta/2)\left(u^2-t^2\right)} f(u)du \right\} dt, (i = 1, 2, 3, ..., n) \tag{3.4}$$

β being small, β^2 and higher powers of β are neglected and applying the change in the order of integration

$$I_1 = \int_{t_i}^{s_{i+1}} \left[\left(1 + (\beta/2)t^2\right)(t - t_i) - (\beta/6)\left(t^3 - t_i^3\right)\right] g(t)dt,\ (i = 1, 2, 3, ..., n) \tag{3.5}$$

During the interval $[s_i, t_i]$, the total amount of shortages is

$$I_2 = \int_{s_i}^{t_i} I_{2i}(t)dt = \int_{s_i}^{t_i} \left\{ \int_{s_i}^{t_i} g(t).\left(1/1 + \delta\left(t_i - t\right)\right) dt \right\} dt,\ (i = 1, 2, 3, ..., n) \tag{3.6}$$

$$I_2 = \int_{s_i}^{t_i} \left(\left(\left(t_i - t\right)\right) \div \left(1 + \delta\left(t_i - t\right)\right)\right) g(t)dt,\ (i = 1, 2, 3, ..., n) \tag{3.7}$$

The total amount of deterioration for the ith cycle is

TD=

$$\int_{t_i}^{s_{i+1}} (\beta * t)\left[\left(1 + (\beta/2)t^2\right)(t - t_i) - (\beta/6)\left(t^3 - t_i^3\right)\right] g(t)dt, (i = 1, 2, 3, ..., n) \tag{3.8}$$

During the interval $[s_i, t_i]$, the total quantity of lost sales is

TL= $\int_{s_i}^{t_i} [g(t) - g(t)A(t)]\, dt =$

$$\int_{s_i}^{t_i} \left(\left(\delta\left(t_i - t\right)\right) \div \left(1 + \delta\left(t_i - t\right)\right)\right) g(t)dt, (i = 1, 2, 3, ..., n) \tag{3.9}$$

Present worth of the ordering cost C_i is

$$C_i = C_0 * e^{(d-r)*t_{i-1}} \tag{3.10}$$

Present worth of the holding cost is

$$H_i = H_0 * e^{(d-r)*t_{i-1}} * \int_{t_i}^{s_{i+1}} \left[\left(1 + (\beta/2)t^2\right)(t - t_i) - (\beta/6)\left(t^3 - t_i^3\right)\right] g(t)dt \quad (3.11)$$

Present worth of the deterioration cost D_i is

$$D_i = D_0 * e^{(d-r)*t_{i-1}} * \int_{t_i}^{s_{i+1}} (\beta * t)\left[\left(1 + (\beta/2)t^2\right)(t - t_i) - (\beta/6)\left(t^3 - t_i^3\right)\right] g(t)dt \quad (3.12)$$

Present worth of the shortage cost S_i is

$$S_i = S_0 * e^{(d-r)*t_{i-1}} * \int_{s_i}^{t_i} \left(\frac{(t_i - t)}{(1 + \delta(t_i - t))}\right) g(t)dt \quad (3.13)$$

Present worth of the cost of lost sales is

$$L_i = L_0 * e^{(d-r)*t_{i-1}} * \int_{s_i}^{t_i} \left(\frac{\delta(t_i - t)}{(1 + \delta(t_i - t))}\right) g(t)dt \quad (3.14)$$

Present worth of the purchase cost is

$$P_i = P_0 * e^{(d-r)*t_{i-1}} * \left(\int_{t_i}^{s_{i+1}} [(1 + (\beta/2)t^2)(t - t_i)\right.$$

$$\left. - (\beta/6)\left(t^3 - t_i^3\right) g(t)dt + \int_{s_i}^{t_i} \left(\frac{(t_i - t)}{(1 + \delta(t_i - t))}\right) g(t)dt \quad (3.15)$$

Present worth of the total relevant cost is

PWTC= $\sum_{i=1}^{i=n}(C_i + D_i + S_i + L_i + P_i + H_i)$

or

$$PWTC(t_i, s_i, n) = \sum_{i=1}^{i=n}\Big(C_0 * e^{(d-r)*t_{i-1}} + D_0 * e^{(d-r)*t_{i-1}} * \int_{t_i}^{s_{i+1}}(\beta * t)$$

$$[(1 + (\beta/2)t^2)(t - t_i) - (\beta/6)(t^3 - t_i^3)] g(t)dt + S_0 * e^{(d-r)*t_{i-1}} *$$

$$\int_{s_i}^{t_i}\left(\frac{(t_i - t)}{(1+\delta(t_i - t))}\right) g(t)dt + L_0 * e^{(d-r)*t_{i-1}} * \int_{s_i}^{t_i}\left(\frac{\delta(t_i - t)}{(1+\delta(t_i - t))}\right) g(t)dt$$

$$+P_0 * e^{(d-r)*t_{i-1}} * \Big(\int_{t_i}^{s_{i+1}} [(1 + (\beta/2)t^2)(t - t_i) - (\beta/6)(t^3 - t_i^3)] g(t)dt +$$

$$\int_{s_i}^{t_i} (((t_i - t)) \div (1 + \delta(t_i - t))) g(t)dt$$

$$+ H_0 * e^{(d-r)*t_{i-1}} * \int_{t_i}^{s_{i+1}} \left[\left(1 + (\beta/2)t^2\right)(t - t_i) - (\beta/6)\left(t^3 - t_i^3\right)\right] g(t)dt \quad (3.16)$$

The purpose of the inventory model is to find the values of t_i, s_i and n that optimizes the total cost

$$\frac{\partial \text{PWTC}\,(t_i, s_i, n)}{\partial t_i} = 0,\ (i = 1, 2, 3, ..., n) \tag{3.17}$$

and

$$\frac{\partial \text{PWTC}\,(t_i, s_i, n)}{\partial s_i} = 0,\ (i = 1, 2, 3, ..., n) \tag{3.18}$$

$$\frac{\partial \text{PWTC}\,(t_i, s_i, n)}{\partial t_i} = H_0 * e^{(d-r)*t_{i-1}} * \int_{t_i}^{s_{i+1}} ((\beta/2) * ((t_i^2) - t^2 - 1))\, g(t)dt -$$
$$D_0 * e^{(d-r)*t_{i-1}} * \int_{t_i}^{s_{i+1}} (\beta * t) * g(t)dt + S_0 * e^{(d-r)*t_{i-1}} * \int_{s_i}^{t_i} \frac{g(t)}{/\left((1+\delta(t_i-t))^2\right)} dt +$$
$$\left(L_0 * e^{(d-r)*t_{i-1}} * \delta\right) * \int_{s_i}^{t_i} g(t) \,/\left((1 + \delta\,(t_i - t))^2\right)\, dt + P_0 * e^{(d-r)*t_{i-1}} *$$
$$\left(\left(\int_{t_i}^{s_{i+1}} ((\beta/2) * \left((t_i^2) - t^2\right) - 1)\, g(t)dt\right) + \left(\int_{s_i}^{t_i} g(t) \,/\left((1 + \delta\,(t_i - t))^2\right)\, dt\right)\right) \tag{3.19}$$

and

$$\frac{\partial \text{PWTC}\,(t_i, s_i, n)}{\partial s_i}$$
$$= H_0 * e^{(d-r)t_{i-2}} * ((1 + (\beta/2)\,(s_i\hat{}2))\,(s_i - t_{i-1}) - (\beta/6)\,((s_i\hat{}3) - (t_{i-1}\hat{}3)))\, f\,(s_i) +$$
$$D_0 * e^{(d-r)t_{i-2}} * (\beta * s_i) \left[(1 + (\beta/2) s_i^2)\,(s_i - t_{i-1}) - (\beta/6)\left(s_i^3 - t_{i-1}^3\right)\right] f\,(s_i) -$$
$$S_0 * e^{(d-r)*t_{i-1}} * ((t_i - s_i)\,/(1 + \delta * (t_i - s_i)))\, f\,(s_i) - L_0 * e^{(d-r)*t_{i-1}} *$$
$$(\delta * (t_i - s_i)\,/(1 + \delta * (t_i - s_i)))\, f\,(s_i) + P_0 * e^{(d-r)t_{i-2}} *$$
$$((1 + (\beta/2)\,(s_i\hat{}2))\,(s_i - t_{i-1}) - (\beta/6)\,((s_i\hat{}3) - (t_{i-1}\hat{}3))) * f\,(s_i) -$$
$$P_0 * e^{(d-r)*t_{i-1}} * ((t_i - s_i)\,/(1 + \delta * (t_i - s_i)))\, f\,(s_i) \tag{3.20}$$

For total cost $PWTC$ to be minimum, the sufficient condition is that the Hessian Matrix of present worth of total cost $PWTC$ should be positive definite.

3.4 Sufficient condition for Optimality of total cost $PWTC$

The Hessian matrix of $\nabla^2 PWTC$ for a fixed n is positive definite Sarkar, Ghosh, and Chaudhuri, 2012a.

$$\nabla^2\text{PWTC} = \begin{pmatrix} \frac{\partial^2\text{PWTC}}{\partial t_1^2} & \frac{\partial^2\text{PWTC}}{\partial t_1 \partial s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2\text{PWTC}}{\partial s_1 \partial t_1} & \frac{\partial^2\text{PWTC}}{\partial s_1^2} & \frac{\partial^2\text{PWTC}}{\partial s_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2\text{PWTC}}{\partial t_2 \partial s_1} & \frac{\partial^2\text{PWTC}}{\partial t_1^2} & \frac{\partial^2\text{PWTC}}{\partial t_2 \partial s_2} & 0 & 0 & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial^2\text{PWTC}}{\partial t_{n-1} \partial s_{n-2}} & \frac{\partial^2\text{PWTC}}{\partial t_{n-1}^2} & \frac{\partial^2\text{PWTC}}{\partial t_{n-1} \partial s_{n-1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2\text{PWTC}}{\partial s_{n-1} \partial t_{n-1}} & \frac{\partial^2\text{PWTC}}{\partial s_{n-1}^2} & \frac{\partial^2\text{PWTC}}{\partial s_{n-1} \partial t_n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2\text{PWTC}}{\partial t_n \partial s_{n-1}} & \frac{\partial^2\text{PWTC}}{\partial t_n^2} \end{pmatrix}$$

Where

$\frac{\partial^2\text{PWTC}}{\partial t_i^2}$ $=H_0 * e^{(d-r)*t_{i-1}} * \left(\int_{t_i}^{s_{i+1}} \beta * t_i * g(t)dt + f\left(t_i\right)\right) + D_0 * e^{(d-r)*t_{i-1}} * \beta * t_i * f\left(t_i\right) - 2\left(S_0 + L_0 * \delta\right) * e^{(d-r)*t_{i-1}} * \int_{s_i}^{t_i} g(t) \,/ \left(\left(1+\delta\left(t_i - t\right)\right)^3\right) dt + \left(S_0 + L_0 * \delta\right) * e^{(d-r)*t_{i-1}} * f\left(t_i\right) + P_0 * e^{(d-r)*t_{i-1}} *$

$$\left(\left(\int_{t_i}^{s_{i+1}} \beta * t_i * g(t)dt + f\left(t_i\right)\right) - 2 * \delta * \int_{s_i}^{t_i} g(t) \,/ \left(\left(1+\delta\left(t_i - t\right)\right)\text{^}3\right) dt\right) \tag{3.22}$$

$\frac{\partial^2\text{PWTC}}{\partial s_i^2} =$

$$\left(H_0 * e^{(d-r)t_{i-2}} * \left(\left(1+(\beta/2)\left(s_i^2\right)\right)\left(s_i - t_{i-1}\right) - (\beta/6)\left(\left(s_i^3\right) - \left(t_{i-1}^3\right)\right)\right) + \right.$$

$$D_0 * e^{(d-r)t_{i-2}} * \left(\beta * s_i\right)\left[\left(1+(\beta/2)s_i^2\right)\left(s_i - t_{i-1}\right) - (\beta/6)\left(s_i^3 - t_{i-1}^3\right)\right] -$$

$$S_0 * e^{(d-r)*t_{i-1}} * \left(\frac{(t_i - s_i)}{(1+\delta*(t_i - s_i))}\right) - L_0 * e^{(d-r)*t_{i-1}} * \left(\frac{\delta*(t_i - s_i)}{(1+\delta*(t_i - s_i))}\right) +$$

$$P_0 * e^{(d-r)t_{i-2}} * \left(\left(1+(\beta/2)\left(s_i^2\right)\right)\left(s_i - t_{i-1}\right) - (\beta/6)\left(\left(s_i^3\right) - \left(t_{i-1}^3\right)\right)\right) -$$

$$\left. P_0 * e^{(d-r)*t_{i-1}} * \left(\frac{(t_i - s_i)}{/(1+\delta*(t_i - s_i))}\right)\right) f'\left(s_i\right) + H_0 * e^{(d-r)t_{i-2}} *$$

$$\left(\beta * s_i * \left(s_i - t_{i-1}\right) + 1\right) f\left(s_i\right) + D_0 * e^{(d-r)t_{i-2}} * \left(\beta * \left(1+(\beta/2)s_i^2\right)\right.$$

$$\left(s_i - t_{i-1}\right) + \left(\beta * s_i\right)\left(s_i - t_{i-1}\right) + \left(\beta * s_i\right)\left(1 + \left(\frac{\beta * s_i^2}{2}\right)\right) - \left(\beta * s_i\right)\left(\frac{\beta * s_i^2}{2}\right)$$

$$-\beta * \left(\frac{\beta}{6}\right)\left(s_i^3 - t_{i-1}^3\right) f\left(s_i\right) + S_0 * e^{(d-r)*t_{i-1}} * \left(\frac{1}{(1+\delta(t_i - s_i))^2}\right) f\left(s_i\right)$$

$$+L_0 * e^{(d-r)*t_{i-1}} * \left(\tfrac{\delta}{(1+\delta(t_i-s_i))^2}\right) f(s_i) + P_0 * e^{(d-r)t_{i-2}} *$$

$$(\beta * s_i * (s_i - t_{i-1}) + 1) f(s_i) + P_0 * e^{(d-r)*t_{i-1}} * \left(\frac{1}{(1+\delta(t_i - s_i))^2}\right) f(s_i) \quad (3.23)$$

$$\frac{\partial^2 \text{PWTC}}{\partial s_i \partial t_i} = \frac{\partial^2 \text{PWTC}}{\partial t_i \partial s_i}$$

$$= -e^{(d-r)*t_{i-1}} * (S_0 + L_0 * \delta + P_0) * \left(f(s_i) \div (1 + \delta * (t_i - s_i))^2\right) \quad (3.24)$$

It also follows that

$$\frac{\partial^2 \text{PWTC}}{\partial t_i \partial s_{i-1}} = \frac{\partial^2 \text{PWTC}}{\partial s_i \partial t_{i+1}} = 0 \quad (3.25)$$

From equation (22) ∇^2PWTC is a tridiagonal matrix. Therefore using equation 23, 24, 25 and 26, properties of tri-diagonal matrix and the theorem stated below it follows that ∇^2PWTC is positive definite.

Theorem 1: If t_i and s_i satisfy inequations (i) $\frac{\partial^2 \text{PWTC}}{\partial t_i^2} > 0$, (ii) $\frac{\partial^2 \text{PWTC}}{\partial s_i^2} > 0$, (iii) $\frac{\partial^2 \text{PWTC}}{\partial t_i^2} - \left|\frac{\partial^2 \text{PWTC}}{\partial t_i \partial s_i}\right| > 0$ and (iv) $\frac{\partial^2 \text{PWTC}}{\partial s_i^2} - \left|\frac{\partial^2 \text{PWTC}}{\partial s_i \partial t_i}\right| > 0$ for $i = 1, 2, ..., n$ then ∇^2PWTC is positive definite.

3.5 Algorithm

1. Allocate the values to the parameter $C_0, H_0, D_0, S_0, L_0, P_0, a, \beta, b, d, r, c, \delta$.
2. Using equation (20) calculate s_2,by initialising the value of $t_1$and taking $s_1 = 0$.
3. From equation (21),find t_2,using the calculated values of s_2 and t_1.
4. Again taking the values of s_2 and t_2, calculate s_3 from equation (20). Continuing in the same way till s_{n+1} is obtained.

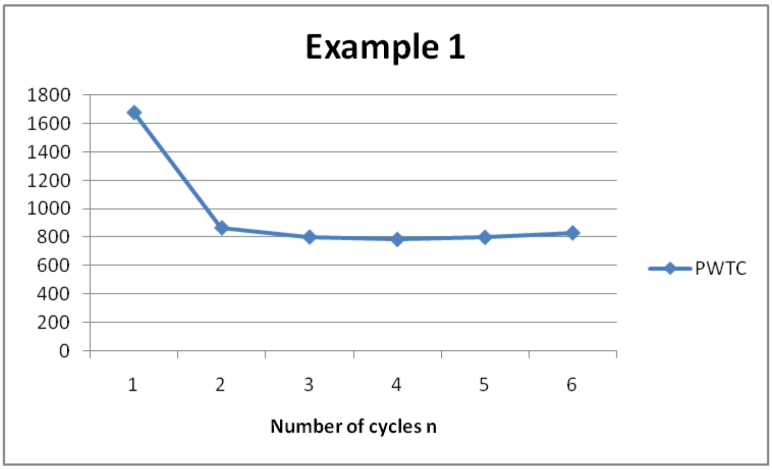

FIGURE 3.2

5. Having the values of s_{n+1} nearly equal to H,and the values of $t_i^{'}$s and $s_i^{'}$s satisfying the theorem, obtain the value of the present worth of total cost PWTC and the replenishment cycle n.

6. For each $n = 1, 2, 3, ...$PWTC is obtained to determine the optimal value of the total cost $PWTC$.

3.5.1 Numerical Examples

1. Assuming complete backlogging for the shortages, and the quadratic demand $g(t) = 25 + 10t + 5t^2$, with the other values as $H = 4$,$D_0 = 4$, $P_0 = 4$, $S_0 = 2$, $L_0 = 10$,$d = 0.12$,$r = 0.05$, α=0.001,$\delta = \theta$ and $C_0 = 60$. The optimal total cost with quadratic demand and complete backlogging for $n = 1, 2, \ldots.$.is shown in table 1 and the corresponding values of ti and si are shown in table 2.

2. When the demand is quadratic $g(t) = 25+10t+5t^2$, with shortages are partially backlogged, the values of other parameters being $H = 4, D_0 = 4, P_0 = 4, S_0 = 2, L_0 = 10, d = 0.12, r = 0.05, \alpha = 0.001, \delta = 4$ and $C_0 = 60$. The optimal solution for total cost along with the optimal values of t_i and s_i are given in table 3 and table 4 respectively.

3. When the demand is linear $g(t) = 25 + 10t$, with partially backlogged shortages, the other parameters as $H = 4, D_0 = 4, P_0 = 4, S_0 = 2, L_0 = 10, d = 0.12, r = 0.05, \alpha = 0.001, \delta = 4andC_0 = 60$. The optimal result is shown in table 5 and 6.

TABLE 3.1: Present Worth of total cost and the no of cycles for Numerical Example 1

n	PWTC
1	1681.86458
2	868.99854
3	803.18126
4	787.06828
5	801.51287
6	833.03759

TABLE 3.2: Optimal values of t_i and s_i for Example 1

i	$t_i^{'}$	$s_{i+1}^{'}$
1	0.858	1.45985
2	2.06855	2.54288
3	2.97967	3.35012
4	3.69866	4.00

TABLE 3.3: Present Worth of total cost and the no of cycles for Numerical Example 2

n	PWTC
1	1940.72
2	1633.87
3	1306.82
4	1159.75
5	1084.30
6	1054.07
7	1054.65

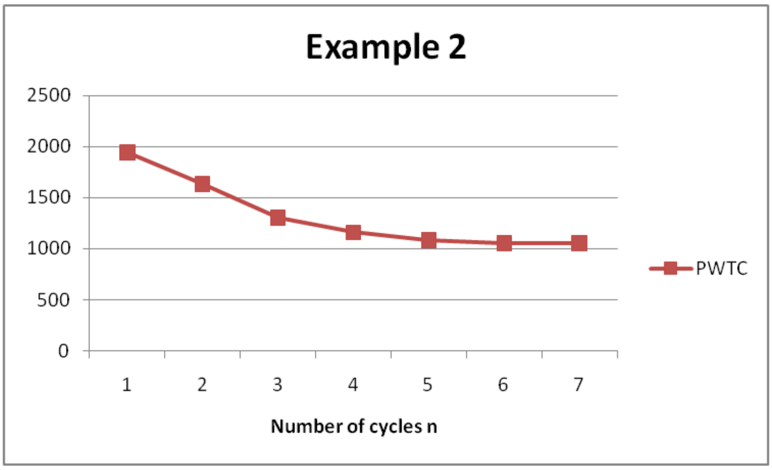

FIGURE 3.3

TABLE 3.4: Optimal values of t_i and s_i for Example 2

i	t_i^*	s_{i+1}^*
1	0.267	1.04751
2	1.23541	1.90397
3	2.02519	2.56404
4	2.65493	3.11048
5	3.18434	3.58263
6	3.64547	4.00

TABLE 3.5: Present Worth of total cost and the no of cycles for Numerical Example 3

n	PWTC
1	1343.622
2	1119.538
3	904.949
4	838.143
5	820.494
6	832.898
7	861.388

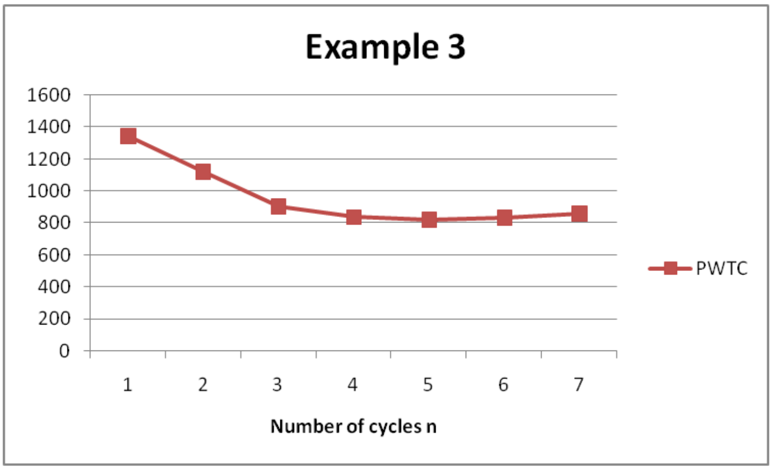

FIGURE 3.4

TABLE 3.6: Optimal values of t_i and s_i for Example 3

i	$t_i^{'}$	$s_{i+1}^{'}$
1	0.235	1.01942
2	1.20972	1.94638
3	2.08681	2.72167
4	2.83521	3.39694
5	3.49368	4.00

3.6 Conclusion

The study proposes an inventory model with time-varying deterioration, partially backlogged shortages, and time increasing demand. Here the costs are under the influence of money value concerning time. The model implements the DCF approach on all costs and its effect on the present worth of total cost. The projected model provides support to the firm/retailer in succinctly formulating the order time and the total cost. This model emphasizes the role of inflation and money value with time on the replenishment period and the total cost of the retailer. Three numerical examples discussed on the time quadratic demand with and without partial backlogging and linear demand under inflation. The numerical outcome precisely validates the theoretical aspects. Future studies can incorporate the selling price along with the profit maximization. Also, the model can be customized with the concept of permissible delay in payment.

Chapter 4

A Supply Chain Model with deteriorating items under inflation

Abstract

Inventory models are an important part of an organization. This chapter presents a model for deteriorating goods within an inflationary environment. The finite planning horizon is considered along with constant demand where the credit term is offered to the retailer by the supplier. The model considers completely back-ordered shortages. Numerical illustration at the end validates the model.

4.1 INTRODUCTION

Deterioration refers to the spoilage or damage to the computer peripherals and other electronic appliances. Deterioration cost affects the total cost of an inventory model. The constant deterioration concept was introduced by Ghare and Schrader, 1963 Ghare and Schrader, 1963. Later many zestful works like time-varying, Weibull deterioration, exponential deterioration is done by researchers Chang, 2004,Das, Roy, and Kar, 2015. Shortages tend to occur and are completely or partially back ordered. Hou, 2006 and Manna, Chaudhuri, and Chiang, 2007 developed an inventory model with completely back ordered shortages. The customer gets impatient and switches to the other options available in the market. The first model that studied the customer's impatience was developed by Dye, Hsieh, and Ouyang, 2007. The partial back ordered shortages along with deterioration is also studied by Singh et al., 2017a,Selvi et al., 2017 and many others.
Trade credit is a helpful phenomenon in the inventory system which helps both the parties enter into a profitable business. Chung and Huang, 2009 discussed the delay in payment in the inventory model. Aggarwal and Jaggi, 1995a,Goyal, 1985b discussed the permissible delay. Demand is the crucial parameter in determining the inventory cost. The concept of time-varying demand is used by many researchers as Donaldson, 1977, Jeganathan et al., 2018, Bose, Goswami, and Chaudhuri, 1995 and others. many

times organizations suffer loss due to neglecting the effects of inflation. Buzacott, 1975 was the first to discuss the effect of inflation on an inventory model. Bierman and Thomas, 1977,Chang, 2004,Jain and Aggarwal, 2012,Singh et al., 2017c studied the inventory model under inflationary environment. The fuzzy concept of the inventory parameters is discussed by Selvi et al., 2017 and Singh et al., 2018a.

The chapter progresses with the postulates and terminology followed by the mathematical solution in both independent and dependent systems. The optimization for the total cost is done next. In the latter part of the article, an algorithm is unionized with the numerical and conclusion.

4.2 Postulates and Terminology

4.2.1 Postulates

1. A Single item, a single retailer, and a single supplier was considered with no lead time.
2. Permissible delay(η) was given by the supplier to the retailer in a finite planning horizon.
3. The demand f(t) was constant and deterioration of goods starts when entering the inventory. The rate of deterioration was $\rho\ (t)\ =\beta\ t, with(\beta > 0)$ and $(t > 0)$.
4. Completely back-ordered shortages were considered.
5. The model was developed under inflationary conditions with the constant inflation rate. All cost are subjected to the same rate of inflation.

4.2.2 Terminology

For retailer

1. IC, ID, IH, IS, IP denotes the cost of ordering, deterioration cost, cost of holding, cost of lost sale, shortage cost and purchase cost respectively.
2. $I_{1i}^{I}(t)$ is the level of inventory during the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system with no co-ordination, $I_{1j}^{D}(t)$ is the inventory level during the time interval $[t_j, s_{j+1}]$, $[j = 1, 2, 3, \ldots, n_2]$ for the dependent system where credit period is offered by the supplier to the retailer.
3. $R_i^I(t)$ and $D_i^I(t)$ is the total level of inventory and the total quantity deteriorated in the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system and

$R_j^D(t)$ and $D_j^D(t)$ is the total inventory level and total quantity deteriorated during the time interval $[t_j, s_{j+1}]$, $[j = 1, 2, 3, \ldots, n_2]$ for the dependent system.

4. $S_i^I(t)$ is the total shortage level during the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system and $S_j^D(t)$ is the total shortage level during the time interval $[t_j, s_{j+1}]$, $[j = 1, 2, 3, \ldots, n_2]$ for the dependent system.

5. The retailer's capital cost is the same as the capital cost of the supplier. It is $RC(\$/unit/year)$.

6. The order quantity for the i^{th} cycle at time t_i,$\{i = 1, 2, ..., n_1\}$ in independent system is $Q_i^I = R_i^I + S_i^I$, whereas $Q_j^D = R_j^D + S_j^D$, is the order quantity for j^{th} cycle $\{j = 1, 2, ..., n_2\}$ at time t_j^C for dependent system.

7. In the centralized system, credit period W_j^D is given to the retailer by the the supplier for j^{th} cycle $\{j = 1, 2, ..., n_2\}$.

8. T_i^I is the duration of the i^{th} ordering cycle, $\{i = 1, 2, ..., n_1\}$ in the independent system and T_j^D is the duration of the j^{th} ordering cycle, $\{j = 1, 2, ..., n_2\}$ for the dependent system.

9. $PWTC_r^I$ and $PWTC_r^D$ is the total cost under inflation for the retailer in the independent and dependent system respectively for the planning horizon H.

For supplier

1. The setup cost per order was SU_s(\$/order).

2. The per unit purchasing cost was IP_s(\$/unit) and $IP_s < IP$.

3. During the planning horizon H, the present worth of the total cost of the supplier in independent and dependent systems was $PWTC_s^I$ and $PWTC_s^D$ respectively.

4.2.3 Decision variables

1. The time period for refilling t_i $\{i = 1, 2, ..., n_1\}$ in the independent system and t_j^D $\{j = 1, 2, ..., n_2\}$ in the dependent system.

2. The time at which the level of inventory becomes zero on account of demand getting fulfilled and deterioration is s_i $\{i = 1, 2, ..., n_1\}$ in the independent system and s_j^D $\{j = 1, 2, ..., n_2\}$ in the dependent system respectively with the initial and final time being $s_1 = 0$ and $s_{n_1+1} = H$.

3. During the planning horizon H, the total number of refilling cycles are n_1 and n_2 for independent and dependent system respectively.

4. Credit period rate is η.

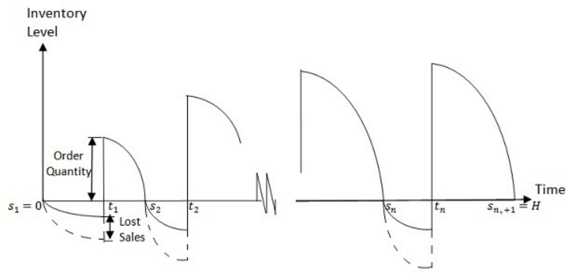

FIGURE 4.1: Graphical representation of Inventory Model

4.3 Mathematical approach and analysis of the suggested model

The model under study considers a constant demand with time proportion deterioration and completely back ordered shortages. In every cycle demand decreases to zero on account of its fulfillment and due to deterioration. Fig.1 illustrates the suggested model under study.

Case 1: Independent system

$$\frac{d\left(I_{1i}^{I}(t)\right)}{dt} + \rho(t)I_{1i}^{I}(t) = \text{-f(t)},\ t_i \leq t \leq s_{i+1}, (i = 1, 2, \ldots, n_1) \tag{4.1}$$

the boundary condition is $I_{1i}^{I}\left(s_{i+1}\right) = 0$

$$\frac{d\left(I_{2i}^{I}(t)\right)}{dt} = f(t)A(t) = \frac{f(t)}{1+\delta\left(t_i - t\right)}, s_i \leq t \leq t_i, \{i = 1, 2, \ldots, n_1\} \tag{4.2}$$

Taking $\rho = \beta t, (0 < \beta < 1)$, the solution of the differential eq (1) is

$$I_{1i}^{I}(t) = \int_{t}^{s_{i+1}} e^{(\beta/2)\left(u^2 - t^2\right)} f(u)du, t_i \leq t \leq s_{i+1}, \quad \{i = 1, 2, \ldots, n_1\}. \tag{4.3}$$

During the interval $[t_i, s_{i+1}]$, the total quantity of inventory in the non-coordinated system is

$$R_i^{I} = \int_{t_i}^{s_{i+1}} I_{1i}^{I}(t)dt = \int_{t_i}^{s_{i+1}} \left\{\int_{t}^{s_{i+1}} e^{(\beta/2)\left(u^2 - t^2\right)} f(u)du\right\} dt, \quad (i = 1, 2, 3, \ldots, n_1). \tag{4.4}$$

β being small, $\beta^2$and higher degree of β are omitted. The change of order of integration ia applied to further solve it. We get,

$$R_i^I = \int_{t_i}^{s_{i+1}} \left[(1+(\beta/2)t^2)(t-t_i) - (\beta/6)(t^3-t_i^3) \right] f(t)dt, \quad (i=1,2,3,\ldots,n_1). \tag{4.5}$$

For the time period $[s_i, t_i]$, the total amount of shortages in non-coordinated system is

$$S_i^I = \int_{s_i}^{t_i} I_{2i}(t)dt = \int_{s_i}^{t_i} \left\{ \int_{s_i}^{t_i} f(t).\left(1/1+\delta(t_i-t)\right)dt \right\} dt, \quad (i=1,2,3,\ldots,n_1)$$

$$S_i^I = \int_{s_i}^{t_i} \left(((t_i-t)) \div (1+\delta(t_i-t)) \right) f(t)dt, \quad (i=1,2,3,\ldots,n_1) \tag{4.6}$$

Total quantity during the planning horizon =

$$\begin{aligned} Q^I &= \sum_{i=1}^{n_1} Q_i^I \\ &= \sum_{i=1}^{n_1} \left(R_i^I + S_i^I \right). \end{aligned} \tag{4.7}$$

The quantity deteriorated in the i^{th} cycle is

$$D_i^I = \int_{t_i}^{s_{i+1}} (\beta * t) \left[\left(1+\frac{\beta}{2}t^2\right)(t-t_i) - \frac{\beta}{6}(t^3-t_i^3) \right] f(t)dt, \quad (i=1,2,3,\ldots,n_1)$$

$$D_i^D = \int_{t_i}^{s_{i+1}} (\beta * t) * (t-t_i) f(t)dt, \quad (i=1,2,3,;n_1) \tag{4.8}$$

neglecting β^2 and higher terms

Present Measure of the ordering cost C_i is

$$C_i = IC * e^{(d-r)*t_i} \tag{4.9}$$

Present Measure of the holding cost H_i is

$$H_i = IH * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} * \left[\left(1+\frac{\beta}{2}t^2\right)(t-t_i) - \frac{\beta}{6}\left(t^3-t_i^3\right) \right] f(t)dt \tag{4.10}$$

Present Measure of the deterioration cost D_i is

$$D_i = ID * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} * (\beta * t)(t - t_i) f(t)dt \tag{4.11}$$

Present Measure of the shortage cost S_i is

$$S_i = IS * \int_{s_i}^{t_i} e^{(d-r)*t} * (((t_i - t)) \div (1 + \delta(t_i - t))) * f(t)dt \tag{4.12}$$

Present Measure of the purchase cost P_i is

$$\begin{aligned} P_i = IP * e^{(d-r)*t_{i-1}} \\ * \left(\int_{t_i}^{s_{i+1}} \left[1 + (\beta/2)\left(t^2 - t_i^2\right)\right] f(t)dt + \int_{s_i}^{t_i} (1 \div (1 + \delta(t_i - t))) f(t)dt \right) \end{aligned} \tag{4.13}$$

Retailer's total cost is the sum of the purchasing cost, ordering cost, shortage cost, deterioration cost and inventory holding cost. Present measure of the retailer's total cost $PWTC_r^I$ is

$$\begin{aligned} & PWTC_r^I(n_1, s_1, t_1, \ldots, s_{n_1+1}) = \\ & \sum_{i=1}^{i=n_1} \Bigg(IC * e^{(d-r)*t_i} + ID * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} * (\beta * t)(t - t_i) f(t)dt + IS * \int_{s_i}^{t_i} e^{(d-r)*t} \\ & * \frac{(t_i - t) f(t)dt}{(1 + \delta(t_i - t))} + IP * e^{(d-r)*t_{i-1}} \\ & * \left(\int_{t_i}^{s_{i+1}} \left[1 + (\beta/2)\left(t^2 - t_i^2\right)\right] f(t)dt + \int_{s_i}^{t_i} \frac{f(t)dt}{(1 + \delta(t_i - t))} \right) + IH * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} \\ & * \left[\left(1 + (\beta/2)t^2\right)(t - t_i) - (\beta/6)\left(t^3 - t_i^3\right)\right] f(t)dt \Bigg), \quad \{i = 1, 2, \ldots, n_1\} \end{aligned} \tag{4.14}$$

For optimum value of the total cost, the necessary conditions for $PWTC_r^I$ to be minimum are

$\dfrac{\partial PWTC_r^I(t_i, s_i; n_1)}{\partial t_i} = 0$ and $\dfrac{\partial PWTC_r^I(t_i, s_i; n_1)}{\partial s_i} = 0$.

Where $\dfrac{\partial PWTC_r^I(t_i, s_i; n_1)}{\partial t_i} =$

$e^{((d-r)t_i)} * IC * (d - r) - ID * \int_{t_i}^{s_{i+1}} (\beta * t) * a * (1 + ((d - r) * t) +$

$$\frac{(d-r)^2 * t^2}{2} + \frac{(d-r)^3 * t^3}{6} + \frac{(d-r)^4 * t^4}{24} + \frac{(d-r)^5 * t^5}{120} +$$
$$\frac{(d-r)^6 * t^6}{720} dt + IS * \int_{s_i}^{t_i} \left(\frac{1}{1+\delta\,(t_i - t)^2} dt \right) * a * (1 + ((d-r) * t)$$
$$+ \frac{(d-r)^2 * t^2}{2} + \frac{(d-r)^3 * t^3}{6} + \frac{(d-r)^4 * t^4}{24} + \frac{(d-r)^5 * t^5}{120}$$
$$+ \frac{(d-r)^6 * t^6}{720} dt + IP * e^{((d-r)t_i)} * (d-r) *$$
$$\left(\int_{t_i}^{s_{i+1}} (1 + (\beta/2) * ((t^2) - (t_i^2))) * adt + \int_{s_i}^{t_i} \left(\frac{1}{1+\delta\,(t_i - t)} \right) * adt \right)$$
$$+ IP * e^{((d-r)t_i)} * \left(\int_{t_i}^{s_{i+1}} (-1 * \beta * t_i) * adt - \int_{s_i}^{t_i} \left(\frac{\delta}{1+\delta\,(t_i - t)^2} \right) * adt \right)$$
$$+ IH * \int_{t_i}^{s_{i+1}} (-1 - (\beta/2) * (t^2) + (\beta/2) * (t_i^2)) * a * (1 + ((d-r) * t) +$$

$$\left(\frac{(d-r)^2 * t^2}{2} + \frac{(d-r)^3 * t^3}{6} + \frac{(d-r)^4 * t^4}{24} + \frac{(d-r)^5 * t^5}{120} + \frac{(d-r)^6 * t^6}{720} \right) dt \tag{4.15}$$

as β is very small β^2 and higher terms are neglected.

$$\frac{\partial PWTC_r^D\,(t_i, s_i; n_1)}{\partial s_i} =$$
$$IH * \left((d-r) * \left((1 + (\beta/2)\,(s_i^2))\,(s_i - t_{i-1}) - (\beta/6)\left((s_i^3) - \left(t_{i-1}^3\right)\right)\right) +\right.$$
$$(\beta * s_i * (s_i - t_{i-1}) + 1) * (1 + ((d-r) * s_i) + (((d-r)^2 * s_i^2)/\,2)$$
$$+ (((d-r)^3 * s_i^3)/\,6) + (((d-r)^4 * s_i^4)/\,24) + (((d-r)^5 * s_i^5)/\,120)$$
$$+ (((d-r)^6 * s_i^6)/\,720) * a + ID * ((d-r) * (\beta * s_i)\,(s_i - t_{i-1})$$
$$* \beta * s_i + \beta\,(s_i - t_{i-1}) * (1 + ((d-r) * s_i) + (((d-r)^2 * s_i^2)/\,2)$$
$$+ (((d-r)^3 * s_i^3)/\,6) + (((d-r)^4 * s_i^4)/\,24) + (((d-r)^5 * s_i^5)/\,120) +$$
$$(((d-r)^6 * s_i^6)/\,720) * a + IS * ((1\,/(1 + \delta * (t_i - s_i))^{\,2}\,) -$$

$(d - r) * ((t_i - s_i) / (1 + \delta * (t_i - s_i))) * (1 + ((d - r) * s_i) +$

$(((d - r)^2 * s_i^2) / 2) + (((d - r)^3 * s_i^3) / 6) + (((d - r)^4 * s_i^4) / 24) +$

$(((d - r)^5 * s_i^5) / 120) + (((d - r)^6 * s_i^6) / 720) * a + IP*$

$$\left(e^{(d-r)t_{i-1}} * (\beta * s_i) - e^{(d-r)t_i} * \left(\delta * \left(1 / (1 + \delta * (t_i - s_i))^2\right)\right)\right) * a \quad (4.16)$$

as β is very small β^2 and higher terms are neglected.

After solving eq(15) and eq(16) for Min $PWTC_r^I\,(n_1, s_1, t_1, \ldots, s_{n_1+1})$, the values are n_1^{IO}, s_1, t_1^{IO}, s_2^{IO},...,$s_{n_1+1}^{IO} = H$,
Present measure of the cost for the supplier in the independent system is obtained by adding the set up cost to the purchasing cost of the supplier for the optimal refilling schedules obtained for the retailer.

$PWTC_s^I\left(n_1^{IO}, s_1, t_1^{IO}, s_2^{IO}, \ldots, s_{n_1+1}^{IO} = H\right) = n_1^{IO} * e^{(d-r)*t_i} * SU_s + n_1^{IO} * e^{(d-r)*t_i} * IP_s * Q_{OPT}^*$

$$= n_1^{IO} * e^{(d-r)*t_i} * SU_s + n_1^{IO} * e^{(d-r)*t_i} * IP_s * \left(R_i^{IO} + S_i^{IO}\right). \quad (4.17)$$

Case 2: Dependent system

In the dependent system, the supplier gives the retailer a new re-ordering time period which increases the overall cost of the retailer in the dependent system. The supplier convinces the retailer for compensating the increase in the cost.

For the new reordering cycle n_2, the present measure of the cost for supplier is the sum of the purchasing cost, ordering cost, shortage cost, deterioration cost and inventory holding cost.
$PWTC_r^C(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C) =$

$$= \sum_{j=1}^{j=n_2} \Bigg(IC * e^{(d-r)*t_j^c} + ID * \int_{t_j^c}^{s_{j+1}^c} e^{(d-r)*t} * (\beta * t) \left(t - t_j^c\right) f(t)dt + IS * \int_{s_j^c}^{t_j^c} e^{(d-r)*t} * \left(\left(\left(t_j^c - t\right)\right) \div \left(1 + \delta \left(t_j^c - t\right)\right)\right) f(t)dt + IP * e^{(d-r)*t_{j-1}^c} * \left(\int_{t_j^c}^{s_{j+1}^c} \left[1 + (\beta/2)\left(t^2 - \left(t_j^c\right)^2\right)\right] f(t)dt + \int_{s_j^c}^{t_j^c} \left(1 \div \left(1 + \delta\left(t_j^c - t\right)\right)\right) f(t)dt \right) + IH * \int_{t_j^c}^{s_{j+1}^c} e^{(d-r)*t} * \left[\left(1 + (\beta/2)t^2\right)\left(t - t_j^c\right) - (\beta/6)\left(t^3 - \left(t_j^c\right)^3\right)\right] f(t)dt \Bigg) \tag{4.18}$$

The present measure of the total cost of the supplier after including the increase in the cost of the retailer is

$PWTC_s^D\left(n_2, s_1, t_1^D, \ldots, s_{n_2+1}^D\right) = n_2 * e^{(d-r)*t_j} * SU_s + n_2 * e^{(d-r)*t_j} * IP_s * Q_j^D + PWTC_r^D\left(n_2, s_1, t_1^D, \ldots, s_{n_2+1}^D\right) - PWTC_r^I\left(n_1^{IO}, s_1, t_1^{IO}, s_2^{IO}, \ldots, s_{n_1+1}^{IO} = H\right)$

on further simplification, we get

$$\begin{aligned} =& n_2 * (SU_s + IC) * e^{(d-r)*t_j} + n_2 * (IP_s + IP) * e^{(d-r)*t_j} \\ & * Q_j^D + IH * \textstyle\int_{t_j^D}^{s_{j+1}^D} e^{(d-r)*t} * \left[\left(1 + (\beta/2)t^2\right)(t - t_j)\right. \\ & \left. - (\beta/6)\left(t^3 - t_j^3\right)\right] f(t)dt + ID * \textstyle\int_{t_j^D}^{s_{j+1}^D} e^{(d-r)*t} * (\beta * t)(t - t_j) f(t)dt \\ & + (IS) * \textstyle\int_{s_j^D}^{t_j^D} e^{(d-r)*t} * (((t_j - t)) \div (1 + \delta(t_j - t))) f(t)dt \end{aligned} \tag{4.19}$$

the optimal solution for the minimum value of the cost of supplier

$PWTC_s^D\left(n_2, s_1, t_1^D, \ldots, s_{n_2+1}^D = H\right)$ be $n_2^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_2+1}^{DO} = H$

The optimized order measure for the planning horizon H in the dependent system is

$Q^D = \sum_{j=1}^{n_2^{DO}} Q_j^{DO} = \sum_{j=1}^{n_2^{DO}} \left(R_j^{DO} + S_j^{DO}\right)$

The cost of the retailer in the dependent system is less than the cost in the independent system so the retailer agrees for the new replenishment schedule, i.e.

$PWTC_r^I\left(n_1^{IO}, s_1, t_1^{IO}, s_2^{IO}, \ldots, s_{n_1+1}^{IO} = H\right) \geq$

$$PWTC_r^D\left(n_2^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_2+1}^{DO}=H\right)-\sum_{j=1}^{n_2^{DO}} e^{(d-r)*t_j} RC.\eta\left(s_{j+1}^{DO}-s_j^{DO}\right) Q_j^{DO} \tag{4.20}$$

where the rate of credit period η is same for all ordering interval and Credit period duration W_j^D is given by the equation:
$W_j^D=\eta\left(s_{j+1}^{DO}-s_j^{DO}\right)$

The total cost of the retailer in the dependent system is calculated by adding the profit accrued through supply chain with the minimum value of credit period rate.

$$\begin{aligned} & PWTC_r^D\left(n_2^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_2+1}^{DO}=H\right) \\ & -\sum_{j=1}^{n_2^{DO}} e^{(d-r)*t_j} RC.\eta_{min}\left(s_{j+1}^{DO}-s_j^{DO}\right) Q_j^{DO}= \\ & PWTC_r^I\left(n_1^{IO}, s_1, t_1^{IO}, s_2^{IO}, \ldots, s_{n_1+1}^{IO}=H\right)\end{aligned} \tag{4.21}$$

The total cost for the supplier is calculated by adding the profit share for the maximum credit period.

$$\begin{aligned} & n_2^{DO} SU_s.e^{(d-r)*t_j}+\sum_{j=1}^{n_2^{DO}} IP_s.e^{(d-r)*t_j}.Q_j^{DO}+\sum_{j=1}^{n_2^{DO}}.e^{(d-r)*t_j}.RC.\eta_{\max}\left(s_{j+1}^{DO}-s_j^{DO}\right) Q_j^{DO} \\ & = PWTC_s^I\left(n_1^{IO}, s_1, t_1^{IO}, s_2^{IO}, \ldots, s_{n_1+1}^{IO}=H\right)\end{aligned} \tag{4.22}$$

The minimum rate of credit period and maximum rate of credit period are determined using the equations (21) and (22):
$\eta_{\min}=$

$$\frac{PWTC_r^D\left(n_2^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_2+1}^{DO}=H\right)}{\sum_{j=1}^{n_2^{DO}}.e^{(d-r)*t_j}.RC\left(s_{j+1}^{DO}-s_j^{DO}\right) Q_j^{DO}} \frac{-PWTC_r^I\left(n_1^{IO}, s_1, t_1^{IO}, s_2^{IO}, \ldots, s_{n_1+1}^{IO}=H\right)}{\sum_{j=1}^{n_2^{DO}}.e^{(d-r)*t_j}.RC\left(s_{j+1}^{DO}-s_j^{DO}\right) Q_j^{DO}} \tag{4.23}$$

$$\eta_{\max}=\frac{PWTC_s^I\left(n_1^{IO}, s_1, t_1^{IO}, s_2^{IO}, \ldots, s_{n_1+1}^{IO}=H\right)}{\sum_{j=1}^{n_2^{DO}}.e^{(d-r)*t_j}.RC\left(s_{j+1}^{DO}-s_j^{DO}\right) Q_j^{DO}} \frac{-n_2^{DO}.e^{(d-r)*t_j}.SU_s+\sum_{j=1}^{n_2^{DO}} IP_s.e^{(d-r)*t_j}.Q_j^{DO}}{\sum_{j=1}^{n_2^{DO}}.e^{(d-r)*t_j}.RC\left(s_{j+1}^{DO}-s_j^{DO}\right) Q_j^{DO}} \tag{4.24}$$

The cost savings realized in the dependent model is equally distributed by the average value of the rate of credit period $\bar{\eta}$. So the present optimal measure for the finalized cost of the retailer and the supplier is given by:

$$\begin{aligned} & PWTC_r^{DO\eta}\left(n_2^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_2+1}^{DO} = H\right) \\ & = PWTC_r^{D}\left(n_2^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_2+1}^{DO} = H\right) \\ & - \sum_{j=1}^{n_2^{DO}} .e^{(d-r)*t_j}.RC\bar{\eta}\left(s_{j+1}^{DO} - s_j^{DO}\right) Q_j^{DO} \end{aligned} \tag{4.25}$$

and

$$PWTC_s^{DO\eta}\left(n_2^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_2+1}^{DO} = H\right)$$
$$= n_2^{DO}.e^{(d-r)*t_j}.SU_s + \sum_{j=1}^{n_2^{DO}} IP_s.e^{(d-r)*t_j}.Q_j^{DO} + \sum_{j=1}^{n_2^{DO}} e^{(d-r)*t_j}.RC.\bar{\eta}\left(s_{j+1}^{DO} - s_j^{DO}\right) Q_j^{DO} \tag{4.26}$$

4.4 Optimality condition for $PWTC_r^D$ and $PWTC_s^C$

For $PWTC_r^I$ to be minimum, the following theorem is used:

Theorem 1: If t_i and s_i satisfy inequations (i) $\dfrac{\partial^2 PWTC_r^I}{\partial t_i^2}$ >0, (ii) $\dfrac{\partial^2 PWTC_r^I}{\partial s_i^2}$ >0, (iii) $\dfrac{\partial^2 PWTC_r^I}{\partial t_i^2} - \left|\dfrac{\partial^2 PWTC_r^I}{\partial t_i \partial s_i}\right|$ >0 and (iv) $\dfrac{\partial^2 PWTC_r^I}{\partial s_i^2} - \left|\dfrac{\partial^2 PWTC_r^I}{\partial s_i \partial t_i}\right|$ >0 for $i = 1, 2, \ldots, n_1$ then $\nabla^2 PWTC_r^I$ is positive definite.

The same can be used to show that $\nabla^2 PWTC_s^D$ is a positive definite and $PWTC_s^D(n_2$, s_1, t_1^c,s_2^c, ..., $s_{n+1}^c)$ reaches a minimal.

TABLE 4.1: Inflated amount for $a = \{400\}$ for retailer in an independent system

$\downarrow a$ $\rightarrow n_1$	1	2	3	4	5	6	7
400	10683.3	6254.72	4922.17	4445.82	4305.59	4410.94	4574.63

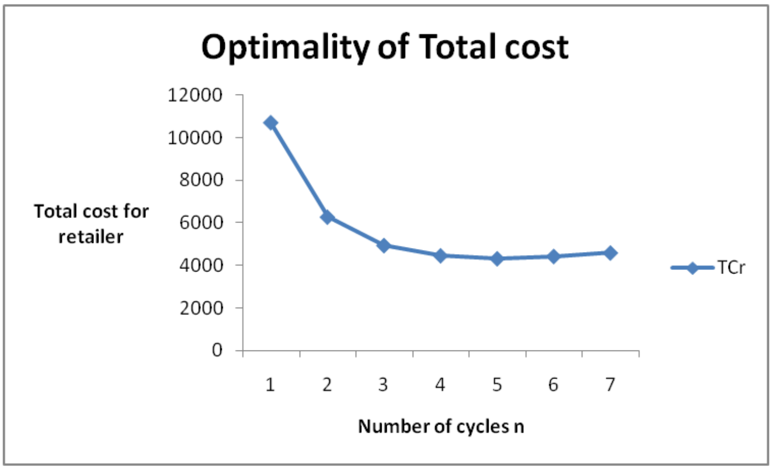

FIGURE 4.2

4.5 Numerical Example

To validate the mathematical procedure for obtaining an optimal solution to the model, following example is demonstrated:

Example1: Taking $a = 400$ items/year,$IP = 0.3$ \$/item, $IS = 18$ \$/item,$\delta = 0.8$, $\beta = 0.01$, $s_1 = 0$, $IC = 300$ \$/ purchase order, $SU_s = 320$ \$/arrangement, $H = 4$, $IP_s = 0.3$ \$/item, $RC = 2.5$ \$/item/year , $IH = 3$ \$/item/year, $d = 0.115$, $r = 0.05$, $ID = 12$ \$/item/year. Mathematica(version 8.0) is used to solve the numerical calculation of the non linear system of equations.

TABLE 4.2: Retailer's optimal cycle in an independent system

a	n_1^{IO}	s_1	s_2	s_3	s_4	s_5	s_6	s_7
400	5	0	0.82767	1.6401	2.4385	3.2238	3.9971	2.0137

a	n_1^{IO}	t_1	t_2	t_3	t_4	t_5	t_6
400	5	0.13216	0.9613	1,77531	2.5751	3.3618	1.7036

TABLE 4.3: Inflated amount for $a = \{400\}$ for supplier in a dependent system

	$\rightarrow n_2$						
a	1	2	3	4	5	6	7
400	7214.32	3220.8	2274.44	2173.38	2390.69	2767.83	3235.43

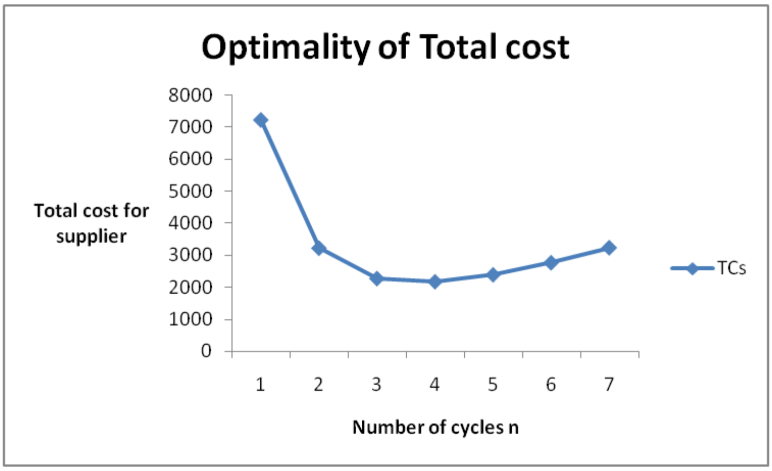

FIGURE 4.3

4.6 Conclusion

The chapter studies the impact of inflation on the various cost associated with inventory in two cases, one when the retailer and supplier follow their schedule and the second when the retailer agrees with the replenishment schedule of the supplier and gets the benefit of credit term. With the cost-saving table (Table 5) of the chapter, it is clear that in an inflationary environment even when the permissible delay is offered by the supplier to the retailer both the parties end into a profitable situation. With trade credit for the amount given to the retailer, the supplier ends up with a cost savings of 4.6 percent, and the retailer is also benefited by the cost savings margin of 2.56 percent.

TABLE 4.4: Supplier's optimal cycle in a dependent system

a	n_2^{DO}	s_1	s_2	s_3	s_4	s_5
400	4	0	1.01825	2.02299	3.01527	3.99858

a	n_2^{DO}	t_1	t_2	t_3	t_4	t_5
400	4	0.1662	1.18906	2.1982	3.1947	2.25939

TABLE 4.5: Cost saving percentage for $a = \{400\}$

Independent system

a	$PWTC_r^{IO}$	$PWTC_s^{IO}$	n_1^{IO}	Q^{IO}	$\eta_{\min}$	$\eta_{\max}$	$\bar{\eta}$
400	4293.87	2393.87	5	1593.66	0.008	0.021	0.014

Dependent system

a	$PWTC_r^{DO\eta}$	$PWTC_s^{DO\eta}$	n_2^{DO}	Q^{DO}
400	4183.63	2283.62	4	1591.27

%Cost saving

a	$\frac{\Delta PWTC}{PWTC_r^{IO}}$	$\frac{\Delta PWTC}{PWTC_s^{IO}}$
400	2.56	4.60

Chapter 5

A Supply Chain Replenishment Inflationary Inventory Model with Trade Credit

Abstract

This chapter considers the problem of obtaining the optimal replenishment schedule in a supply chain with a parameter of credit period rate. The model is designed for time-dependent quadratic demand and deterioration. The model is generalized considering partially backlogged shortages under inflation. However, as a peculiar case, an example is aimed for a model without lost sales. Ultimately sensitivity analysis is performed to analyze the mathematical formulation and numerical examples are examined to study the effect of inflation and time value of money on the economic order quantity model.

5.1 INTRODUCTION

Considering real-life scenarios some goods/commodities, for example, medicines, vegetables, volatile liquids, fruits, electronic goods etc. deteriorate constantly with respect to time due to spoilage, evaporation etc. The traditional approach in inventory model is that the inventory is depleted on account of the fulfillment of demand, but practically the depletion in the inventory level is due to the fulfillment of the demand and also on account of the loss caused by deterioration. Ghare and Schrader 1963 Ghare and Schrader, 1963 first introduced a model for an exponential deterioration rate of inventory. An extension in the inventory model of Ghare and Schrader 1963 was done by Philip, 1974. They extended the model where deterioration is a two-parameter Weibull distribution. Other related research papers on deteriorations are summed up in a review paper by Raafat 1991.Chang, 2004 presented an inventory replenishment model considering deterioration and trade credit where the delay in payment is offered for large purchase quantity under the effects of inflation.Chung and Huang 2009 demonstrated an optimal replenishment schedule for deteriorating goods incorporating delay in payment under inflation during the finite planning horizon.

Shortages occur when the inventory does not meet the demand. However, shortages are completely or partially backlogged. For fashionable goods or electronic equipment where the product has a short life cycle, the consumers do not wait for a long period and so switch to the other options available in the market, thus resulting in lost sales. Chang and Dye 1999b first modeled an inventory theory for the customer's impatience. For partial backlogging Chang and Dye 1999b formulated an EOQ model taking backlogging as the inverse of the linear waiting time.Papachristos and Skouri 2000b stated in their inventory model that there is an exponential decrease in the backlogging rate when the waiting time of customer increases in the system.Hou, 2006 framed an inventory model for deteriorating products, under the time value of money and inflation with shortages. Other works in this area are done by Yang, Teng, and Chern 2002,Teng and Yang 2004 and Teng et al. 2002.Palanivel and Uthayakumar 2015 studied a linear trend of the demand function for a deterministic EOQ model with two separate warehouses and different deterioration rate under inflation allowing partial backlogging. In the paper of Das, Roy, and Kar 2015, they developed a multi-item multi-warehouse inventory refilling model for deteriorating items for two warehouses with demand which is depending on price and stock when the delay in payment is permissible allowing shortages and partial backlogging. Also, a model is developed in the paper of Palanivel and Uthayakumar 2016 under-inflation, and the finite planning horizon allowed shortages with partially backlogging, where shortages are with a rate dependent on the duration of waiting time up to the arrival of next lot.

To demonstrate sales of the product during the product life cycle the inventory models are made for the demand varying with respect to time. Donaldson 1977 introduced the EOQ model for a linear function of demand. Thereafter much zestful research works have been done comprising time-dependent demand and deterioration with or without shortages as Hariga 1995.Sarkar, Ghosh, and Chaudhuri 2012a stated in his research work the advantage of time quadratic demand over the other demand functions. Time quadratic demand includes all the three faces of increase, decrease, and demand to be constant. The research articles which include time quadratic demand are done by Khanra and Chaudhuri 2003,Ghosh and Chaudhuri 2006,Manna, Chaudhuri, and Chiang 2007 and Singh et al. 2017a. Generally, the inventory models presume that the retailer pays for the products as they are received but realistically this does not happen. The supplier extends to the retailer a discount, either in terms of quantity or permissible delay of payment.Goyal 1985b formulated and EOQ model with trade credit. Later Aggarwal and Jaggi 1995b extended the work of Goyal 1985b to incorporate deterioration rate.Jamal, Sarker, and Wang, 1997 derived a replenishment policy under permissible delay in payment considering shortages and deterioration. Palanivel and Uthayakumar 2015 framed an inventory model for time-varying demand function with trade credit under the effect of inflation. Inflation has been a key concern for developing countries like

India and China where the rate of inflation is in double digits. The traditional inventory model does not consider the effect of inflation on the various cost associated with the model. Jain and Aggarwal 2012 has developed a model, for items that are exponentially deteriorating and are of imperfect quality, here the supplier offers trade credit by the 'β /M1 net M' trade credit policy using a discounted cash flow (DCF) approach. Palanivel and Uthayakumar 2016 have analyzed an inventory replenishment model for deteriorating items under the inflationary environment and the time value of money with seller granting trade credit to its buyer.Yang, Lee, and Zhang 2013 has studied an inventory model for perishable products with the stock-dependent demand under inflation with trade credit policy between supplier and retailer.

However, the effect cannot be ignored while framing an EOQ model. Buzacott, 1975 introduced an inventory model relaxing the assumption of no inflation on the cost with different pricing policies. Bierman and Thomas 1977 studied the inventory model under inflation and time discount. Bose, Goswami, and Chaudhuri, 1995 formulated the in general inventory model considering the effect of inflation and time value of money on the various costs associated with inventory. The two inventory models arrogate the rate of deterioration with respect to time as constant, allowing shortages. Chang, 2004 propose an inflationary model considering deterioration and where the delay in payment is linked with the quantity ordered.Yang, Teng, and Chern, 2010 extended the model of Teng et al. 2002 including the effects of inflation and calculating the economic ordering schedule for an EOQ model with time-dependent demand under shortages and deterioration. Jaggi, Khanna, and Nidhi 2016b derived an inventory model when profit is maximized over a finite planning Horizon. His model incorporated deterioration with partially backlogged shortages. The subject of Gilding, 2014 and Singh et al. 2017c was to find optimal replenishment policy to an inventory problem within a finite time planning horizon with inflation. Palanivel and Uthayakumar 2015 also has discussed their economic order quantity (EOQ) model over a finite planning horizon. Some authors who have recently studied permissible delay with deteriorating under inflation are Das, Roy, and Kar 2015, Yang and Chang, 2013 and Muniappan, Uthayakumar, and Ganesh 2015.

The chapter progresses with the postulates and terminology followed by the mathematical solution in both decentralised and centralised systems. The optimization for the total cost is done next. In the later part of the article, an algorithm is unionized with the exercise of numericals, sensitivity analysis for different parameters and conclusion.

5.2 Postulates and Terminology

5.2.1 Postulates

1. Negligible lead time is considered.

2. The supplier offers the retailer credit period ρ to settle the account.
3. The demand f(t) is time dependent and quadratic in nature.
4. The deterioration of goods starts as they arrive in the stock. The rate of deterioration is $\psi\ (t) = \gamma\ t$, with $(\gamma > 0)$ and $(t > 0)$.
5. Deteriorated goods are neither replaced nor repaired.
6. Shortages are countenanced and the unfulfilled demand is back-ordered. The notation $A(t)$ denotes the unfulfilled demand. The back ordered shortages is the fraction $1/(1 + \delta t)$ of demand, where $0 \leq \delta \leq 1$ and t is the waiting time for the next replenishment.
7. The inventory model is for the finite planning horizon H.
8. During shortage period $1 - A(t)$ fraction of demand turns into lost sales.
9. The model is developed under inflationary conditions with the constant inflation rate. All cost are subjected to the same rate of inflation.
10. The model is assumed to have zero initial inventory level. It starts with shortages in the beginning until first replenishment takes place. Also, the last cycle does not have any shortages.
11. Single item single retailer and single supplier is considered in the supply chain.

5.2.2 Terminology

For retailer

1. Co, Do, Ho, Lo, So, Po denotes the cost of ordering, deterioration cost, cost of holding, cost of lost sale, shortage cost and purchase cost respectively.
2. $I_{1i}^{D}(t)$ is the level of inventory during the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system where there is no co-ordination between the retailer and the supplier, $I_{1j}^{C}(t)$ is the inventory level during the time interval $[t_j, s_{j+1}]$, $[j = 1, 2, 3, \ldots, n_2]$ for the dependent system where credit period is given by the supplier to the retailer.
3. $R_i^D(t)$ and $D_i^D(t)$ is the total level of inventory and the total quantity deteriorated in the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system where there is no co-ordination between the retailer and the supplier and $R_j^C(t)$ and $D_j^C(t)$ is the total inventory level and total quantity deteriorated during the time interval $[t_j, s_{j+1}]$, $[j = 1, 2, 3, \ldots, n_2]$ for the dependent systemwhere credit period is given by the supplier to the retailer.

4. $S_i^D(t)$ and L_i^D is the total shortage level and total quantity of lost sales during the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system where there is no co-ordination between the retailer and the supplier and $S_j^C(t)$ and L_j^C is the total shortage level and total quantity of lost sales during the time interval $[t_j, s_{j+1}]$, $[j = 1, 2, 3, \ldots, n_2]$ for the dependent system where the supplier offers the trade credit to the retailer respectively.

5. The retailer's capital cost is the same as the capital cost of the supplier. It is $C_c(\$/unit/year)$.

6. The cost of carrying inventory is $h(\$/unit/year)$, and $h = I_{hr} + C_c$.

7. The order quantity for the i^{th} cycle at time t_i,$\{i = 1, 2, ..., n_1\}$ in decentralize system is $Q_i^D = R_i^D + S_i^D$, whereas $Q_j^C = R_j^C + S_j^C$, is the order quantity for j^{th} cycle $\{j = 1, 2, ..., n_2\}$ at time t_j^C for centralized system.

8. In the centralized system, credit period M_j^C is given to the retailer by the the supplier for j^{th} cycle $\{j = 1, 2, ..., n_2\}$.

9. T_i^D is the duration of the i^{th} ordering cycle, $\{i = 1, 2, ..., n_1\}$ in the decentralized system and T_j^C is the duration of the j^{th} ordering cycle, $\{j = 1, 2, ..., n_2\}$ for the centralized system.

10. $PWTC_r^D$ and $PWTC_r^C$ is the total cost under inflation for the retailer in the decentralized and centralized system respectively for the planning horizon H.

For supplier

1. The setup cost per order is S_s($/order).

2. The per unit purchasing cost is P_s($/unit) and $P_s < P_o$.

3. During the planning horizon H, the present worth of the total cost of the supplier in decentralized and centralized system is $PWTC_s^D$ and $PWTC_s^C$ respectively.

5.2.3 Decision variables

1. The renewal schedule is t_i $\{i = 1, 2, ..., n_1\}$ in the decentralised system and t_j^C $\{j = 1, 2, ..., n_2\}$ in the centralized system.

2. The time at which the level of inventory becomes zero on account of demand getting fulfilled and deterioration is s_i $\{i = 1, 2, ..., n_1\}$ in the decentralised system and s_j^C $\{j = 1, 2, ..., n_2\}$ in the centralized system respectively with the initial and final time being $s_1 = 0$ and $s_{n_1+1} = H$.

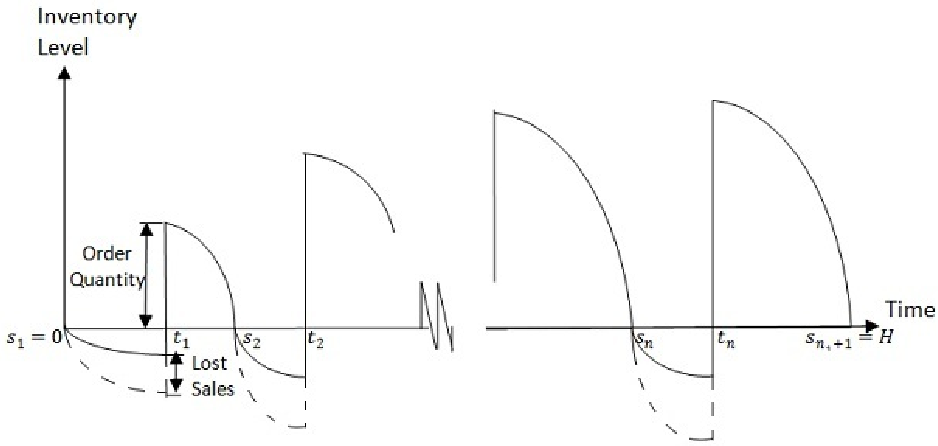

FIGURE 5.1: Graphical representation of Inventory Model

3. During the planning horizon H, the total number of refilling cycles are n_1 and n_2 for decentralized and centralized system respectively.

4. Credit period rate is ρ.

5.3 Mathematical approach and analysis of the suggested model

The study considers a time-varying deterioration and demand with shortages that are partially backlogged. Fig. 1 depicts the inventory model under study. The model starts with zero inventory level initially where shortages start piling up. The first replenishment occurs at time t_1. At this point in time, the level of inventory is maximum. The time interval from t_1 to s_2 pictures a gradual decrease in the level of inventory on account of demand consummation and product deterioration. At s_2 the inventory turns zero. After this, for the time period [s_2, t_2] the inventory accumulates in shortages which are backlogged partially.This repeats for every cycle $[s_i, s_{i+1}]$ where $\{i = 1, 2, ..., n_1\}$.

Case 1: Decentralized system

$$\frac{d\left(I_{1i}^{D}(t)\right)}{dt} + \psi(t)I_{1i}^{D}(t) = -f(t),\ t_i \leq t \leq s_{i+1},(i = 1, 2, \ldots, n_1) \tag{5.1}$$

the boundary condition is $I_{1i}^{D}\left(s_{i+1}\right)=0$

$$\frac{d\left(I_{2i}^{D}(t)\right)}{dt} = f(t)A(t) = \frac{f(t)}{1+\delta\left(t_i-t\right)}, s_i\leq t\leq t_i,\{i=1,2,\ldots,n_1\} \quad (5.2)$$

Taking $\psi=\gamma t, (0<\gamma<1)$, the solution of the differential eq (1) is

$$I_{1i}^{D}(t)=\int_{t}^{s_{i+1}} e^{(\gamma/2)\left(u^2-t^2\right)} f(u)du, t_i\leq t\leq s_{i+1}, \quad \{i=1,2,\ldots,n_1\}. \quad (5.3)$$

During the interval $[t_i, s_{i+1}]$, the total quantity of inventory in the non-coordinated system is

$$R_i^D=\int_{t_i}^{s_{i+1}} I_{1i}^{D}(t)dt = \int_{t_i}^{s_{i+1}}\left\{\int_{t}^{s_{i+1}} e^{(\gamma/2)\left(u^2-t^2\right)} f(u)du\right\}dt, \quad (i=1,2,3,\ldots,n_1). \quad (5.4)$$

γ being small, $\gamma^2$and higher degree of γ are omitted. The change of order of integration ia applied to further solve it. We get,

$$R_i^D = \int_{t_i}^{s_{i+1}}\left[\left(1+(\gamma/2)t^2\right)(t-t_i)-(\gamma/6)\left(t^3-t_i^3\right)\right]f(t)dt, \quad (i=1,2,3,\ldots,n_1). \quad (5.5)$$

For the time period $[s_i, t_i]$, the total amount of shortages in non-coordinated system is

$$S_i^D = \int_{s_i}^{t_i} I_{2i}(t)dt = \int_{s_i}^{t_i}\left\{\int_{s_i}^{t_i} f(t).\left(1/1+\delta\left(t_i-t\right)\right)dt\right\}dt, \quad (i=1,2,3,\ldots,n_1)$$

$$S_i^D = \int_{s_i}^{t_i}\left(\left(\left(t_i-t\right)\right)\div\left(1+\delta\left(t_i-t\right)\right)\right)f(t)dt, \quad (i=1,2,3,\ldots,n_1) \quad (5.6)$$

Total quantity during the planning horizon =

$$\begin{aligned} Q^D &= \sum_{i=1}^{n_1} Q_i^D \\ &= \sum_{i=1}^{n_1}\left(R_i^D+S_i^D\right). \end{aligned} \quad (5.7)$$

The quantity deteriorated in the i^{th} cycle is

$$D_i^D = \int_{t_i}^{s_{i+1}}(\gamma * t)\left[\left(1+\frac{\gamma}{2}t^2\right)(t-t_i)-\frac{\gamma}{6}\left(t^3-t_i^3\right)\right]f(t)dt, \quad (i=1,2,3,\ldots,n_1)$$

$$D_i^D = \int_{t_i}^{s_{i+1}} (\gamma * t) * (t - t_i)\, f(t) dt, \quad (i = 1, 2, 3, ; n_1) \tag{5.8}$$

neglecting γ^2 and higher terms

During the interval $[s_i, t_i]$, the total quantity of lost sales is

$$L_i^D = \int_{s_i}^{t_i} [f(t) - f(t)A(t)]\, dt = \int_{s_i}^{t_i} \frac{\delta\,(t_i - t)\, f(t)}{1 + \delta\,(t_i - t)} dt, \quad (i = 1, 2, \ldots, n_1)\,. \tag{5.9}$$

Present Measure of the ordering cost C_i is

$$C_i = C_0 * e^{(d-r)*t_i} \tag{5.10}$$

Present Measure of the holding cost H_i is

$$H_i = H_0 * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} * \left[\left(1 + \frac{\gamma}{2} t^2\right)(t - t_i) - \frac{\gamma}{6}\left(t^3 - t_i^3\right)\right] f(t) dt \tag{5.11}$$

Present Measure of the deterioration cost D_i is

$$D_i = D_0 * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} * (\gamma * t)\,(t - t_i)\, f(t) dt \tag{5.12}$$

Present Measure of the shortage cost S_i is

$$S_i = S_0 * \int_{s_i}^{t_i} e^{(d-r)*t} * (((t_i - t)) \div (1 + \delta\,(t_i - t))) * f(t) dt \tag{5.13}$$

Present Measure of the cost of lost sales L_i is

$$L_i = L_0 * \int_{s_i}^{t_i} e^{(d-r)*t} * \frac{\delta\,(t_i - t)\, f(t)}{1 + \delta\,(t_i - t)} dt \tag{5.14}$$

Present Measure of the purchase cost P_i is

$$\begin{aligned} P_i = P_0 * e^{(d-r)*t_{i-1}} \\ * \left(\int_{t_i}^{s_{i+1}} \left[1 + (\gamma/2)\left(t^2 - t_i^2\right)\right] f(t) dt + \int_{s_i}^{t_i} (1 \div (1 + \delta\,(t_i - t)))\, f(t) dt \right) \end{aligned} \tag{5.15}$$

Retailer's total cost is the sum of the purchasing cost, ordering cost, shortage cost, deterioration cost, inventory holding cost and the lost sales cost. Present measure of the retailer's total cost $PWTC_r^D$ is

$$PWTC_r^D\left(n_1, s_1, t_1, \ldots, s_{n_1+1}\right)=\sum_{i=1}^{i=n_1}\left(C_i + D_i + S_i + L_i + P_i + H_i\right)$$

$$PWTC_r^D\left(n_1, s_1, t_1, \ldots, s_{n_1+1}\right) =$$

$$\sum_{i=1}^{i=n_1}\Bigg(C_0 * e^{(d-r)*t_i} + D_0 * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} * (\gamma * t)\left(t - t_i\right) f(t)dt + S_0 * \int_{s_i}^{t_i} e^{(d-r)*t}$$
$$* \frac{\left(t_i - t\right) f(t)dt}{\left(1 + \delta\left(t_i - t\right)\right)} + L_0 * \int_{s_i}^{t_i} e^{(d-r)*t} * \frac{\delta\left(t_i - t\right) f(t)dt}{\left(1 + \delta\left(t_i - t\right)\right)} + P_0 * e^{(d-r)*t_{i-1}} *$$
$$\left(\int_{t_i}^{s_{i+1}}\left[1 + (\gamma/2)\left(t^2 - t_i^2\right)\right] f(t)dt + \int_{s_i}^{t_i}\frac{f(t)dt}{\left(1 + \delta\left(t_i - t\right)\right)}\right)$$
$$+ H_0 * \int_{t_i}^{s_{i+1}} e^{(d-r)*t} * \left[(1 + (\gamma/2)t_2\left(t - t_i\right)\right.$$
$$- (\gamma/6)\left(t^3 - t_i^3\right) f(t)dt, \quad \{i = 1, 2, \ldots, n_1\} \tag{5.16}$$

The inventory system is designed to optimize the present worth of total cost by determining the values of s_i, t_i and n_1. The necessary conditions for $PWTC_r^D$ to be minimum are

$$\frac{\partial PWTC_r^D\left(t_i, s_i; n_1\right)}{\partial t_i} = 0 \text{ and } \frac{\partial PWTC_r^D\left(t_i, s_i; n_1\right)}{\partial s_i} = 0.$$

Where $\frac{\partial PWTC_r^D\left(t_i, s_i; n_1\right)}{\partial t_i} =$

$$e^{((d-r)t_i)} * Co * (d-r) - DC * \int_{t_i}^{s_{i+1}}(\gamma * t) * (a + b * t + c * t^2) * (1 + ((d-r) * t) +$$
$$\frac{(d-r)^2 * t^2}{2} + \frac{(d-r)^3 * t^3}{6} + \frac{(d-r)^4 * t^4}{24} + \frac{(d-r)^5 * t^5}{120} + \frac{(d-r)^6 * t^6}{720} dt +$$
$$So * \int_{s_i}^{t_i}\frac{1}{\left(1 + \delta\left(t_i - t\right)\right)^2} * (a + b * t + c * t^2) * \Bigg(1 + ((d-r) * t) + \frac{(d-r)^2 * t^2}{2}$$
$$+ \frac{(d-r)^3 * t^3}{6} + \frac{(d-r)^4 * t^4}{24} + \frac{(d-r)^5 * t^5}{120} + \frac{(d-r)^6 * t^6}{720} dt +$$
$$Lo * \int_{s_i}^{t_i}\frac{1}{\left(1 + \delta\left(t_i - t\right)\right)^2} * (a + b * t + c * t^2) * \Bigg(1 + ((d-r) * t) + \frac{(d-r)^2 * t^2}{2}$$
$$+ \frac{(d-r)^3 * t^3}{6} + \frac{(d-r)^4 * t^4}{24} + \frac{(d-r)^5 * t^5}{120} + \frac{(d-r)^6 * t^6}{720} dt +$$

$$Po * e^{((d-r)t_i)} * (d-r) * \Big(\int_{t_i}^{s_{i+1}} (1 + (\gamma/2) * ((t^2) - (t_i^2))) * (a + b*t + c*t^2)\, dt$$

$$+ \int_{s_i}^{t_i} (1/(1+\delta\,(t_i - t)))*(a + b*t + c*t^2)\, dt + Po*e^{((d-r)t_i)}*\Big(\int_{t_i}^{s_{i+1}} (-1*\gamma*t_i)$$

$$*(a + b*t + c*t^2)\, dt - \int_{s_i}^{t_i} (\delta/((1+\delta\,(t_i - t))^2))*(a + b*t + c*t^2)\, dt +$$

$$Ho * \int_{t_i}^{s_{i+1}} (-1 - (\gamma/2)*(t^2) + (\gamma/2)*(t_i^2)) * (a + b*t + c*t^2) *$$

$$\Big(1 + ((d-r)*t) + \frac{(d-r)^2 * t^2}{2} + \frac{(d-r)^3 * t^3}{6} + \frac{(d-r)^4 * t^4}{24} +$$

$$\frac{(d-r)^5 * t^5}{120} + \frac{(d-r)^6 * t^6}{720} dt \tag{5.17}$$

as γ is very small γ^2 and higher terms are neglected.

$$\frac{\partial PWTC_r^D(t_i, s_i; n_1)}{\partial s_i} =$$

$$H_0 * \left((d-r) * \left((1 + (\gamma/2)(s_i^2))(s_i - t_{i-1}) - (\gamma/6)\left((s_i^3) - \left(t_{i-1}^3\right)\right)\right) +\right.$$

$$(\gamma * s_i * (s_i - t_{i-1}) + 1) * (1 + ((d-r)*s_i) +$$

$$\frac{(d-r)^2 * s_i^2}{2} + \frac{(d-r)^3 * s_i^3}{6} + \frac{(d-r)^4 * s_i^4}{24} + \frac{(d-r)^5 * s_i^5}{120} +$$

$$\frac{(d-r)^6 * s_i^6}{720} * (a + b*s_i + c*(s_i^2)) +$$

$$D_0 * ((d-r)*(\gamma * s_i)(s_i - t_{i-1}) * \gamma * s_i + \gamma(s_i - t_{i-1})) *$$

$$\Big(1 + ((d-r)*s_i) + \frac{(d-r)^2 * s_i^2}{2} + \frac{(d-r)^3 * s_i^3}{6} + \frac{(d-r)^4 * s_i^4}{24} +$$

$$\frac{(d-r)^5 * s_i^5}{120} + \frac{(d-r)^6 * s_i^6}{720} * (a + b*s_i + c*(s_i^2)) + \frac{S_0}{(1+\delta*(t_i - s_i))^2}$$

$$-(d-r) * \frac{(t_i - s_i)}{1 + \delta*(t_i - s_i)} * \Big(1 + ((d-r)*s_i) + \frac{(d-r)^2 * s_i^2}{2} +$$

$$\frac{(d-r)^3 * s_i^3}{6} + \frac{(d-r)^4 * s_i^4}{24} + \frac{(d-r)^5 * s_i^5}{120} + \frac{(d-r)^6 * s_i^6}{720} *$$

$$(a + b*s_i + c*(s_i^2)) + L_0 * \frac{\delta}{(1+\delta*(t_i - s_i))^2} + (d-r) * \frac{\delta*(t_i - s_i)}{1+\delta*(t_i - s_i)}$$

$$* \left(1 + ((d-r) * s_i) + \frac{(d-r)^2 * s_i^2}{2} + \frac{(d-r)^3 * s_i^3}{6} + \frac{(d-r)^4 * s_i^4}{24} + \frac{(d-r)^5 * s_i^5}{120} + \frac{(d-r)^6 * s_i^6}{720} * (a + b * s_i + c * (s_i^2)) + \right.$$

$$P_0 * \left(e^{(d-r)t_{i-1}} * (\gamma * s_i) - e^{(d-r)t_i} * (\delta * (1 / (1 + \delta * (t_i - s_i))^2)) \right) *$$

$$(a + b * s_i + c * (s_i^2)) + H_0 * \left((1 + (\gamma/2)(s_i^2))(s_i - t_{i-1}) - (\gamma/6)\left((s_i^3) - \left(t_{i-1}^3\right)\right)\right) *$$

$$\left(1 + ((d-r) * s_i) + \frac{(d-r)^2 * s_i^2}{2} + \frac{(d-r)^3 * s_i^3}{6} + \frac{(d-r)^4 * s_i^4}{24} + \frac{(d-r)^5 * s_i^5}{120} + \frac{(d-r)^6 * s_i^6}{720} * f'(s_i) + D_0 * (\gamma * s_i)(s_i - t_{i-1}) * \right.$$

$$\left(1 + ((d-r) * s_i) + \frac{(d-r)^2 * s_i^2}{2} + \frac{(d-r)^3 * s_i^3}{6} + \frac{(d-r)^4 * s_i^4}{24} + \frac{(d-r)^5 * s_i^5}{120} + \frac{(d-r)^6 * s_i^6}{720} * f'(s_i) - S_0 * \frac{(t_i - s_i)}{1 + \delta * (t_i - s_i)} * \right.$$

$$\begin{aligned} &\left(1 + ((d-r) * s_i) + \frac{(d-r)^2 * s_i^2}{2} + \frac{(d-r)^3 * s_i^3}{6} + \frac{(d-r)^4 * s_i^4}{24} \right. \\ &\left. + \frac{(d-r)^5 * s_i^5}{120} + \frac{(d-r)^6 * s_i^6}{720} \right) * f'(s_i) - L_0 * \frac{\delta * (t_i - s_i)}{1 + \delta * (t_i - s_i)} \\ &* \left(1 + ((d-r) * s_i) + \frac{(d-r)^2 * s_i^2}{2} + \frac{(d-r)^3 * s_i^3}{6} \right. \\ &\left. + \frac{(d-r)^4 * s_i^4}{24} + \frac{(d-r)^5 * s_i^5}{120} + \frac{(d-r)^6 * s_i^6}{720} \right) * f'(s_i) + P_0 \\ &* e^{(d-r)t_{i-1}} * \left(1 + (\gamma/2)\left(s_i^2 - t_{i-1}^2\right)\right) * f'(s_i) - P_0 * e^{(d-r)*t_i} * * f'(s_i) \end{aligned} \tag{5.18}$$

as γ is very small γ^2 and higher terms are neglected.

Let the optimal solution (see section 4 for optimality condition) obtained from equation (17) and (18) for Min $PWTC_r^D(n_1, s_1, t_1, \ldots, s_{n_1+1})$ be n_1^{DO}, s_1, t_1^{DO}, s_2^{DO},...,$s_{n_1+1}^{DO} = H$,
Now in a decentralized system, the supplier determines the present value of the total cost by summing the setup cost and production cost for the planning horizon H, following the optimal replenishment cycles of the retailer. The equation is as follows:

$PWTC_s^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) = n_1^{DO} * e^{(d-r)*t_i} * S_s + n_1^{DO} * e^{(d-r)*t_i} * P_s * Q_{OPT}^*$

$$= n_1^{DO} * e^{(d-r)*t_i} * S_s + n_1^{DO} * e^{(d-r)*t_i} * P_s * \left(R_i^{DO} + S_i^{DO}\right). \tag{5.19}$$

And the total optimal order quantity during the planning horizon H is

$$\begin{aligned} Q^*_{OPT} &= \sum_{i=1}^{n_1^{DO}} Q_i^{DO} \\ &= \sum_{i=1}^{n_1^{DO}} \left(R_i^{DO} + S_i^{DO}\right). \end{aligned} \tag{5.20}$$

Case 2: Centralized system

The initiation of coordination is made by the supplier in the centralized system. The retailer's cost of ordering is less than the supplier's set up cost. Due to this, the total cost of the system reduces significantly in the co-ordinated system as compared to the uncoordinated system. As the number of replenishment schedules reduces in the centralized system, accordingly there is a reduction in the supplier's set up cost.

The supplier expects the retailer to agree to this new replenishment schedule. The increase in the holding cost tends to increase the overall cost of the retailer in the co-ordinated system, yet the increase is not only compensated by the supplier, but also the savings realised are shared by him in the coordinated system.

For reordering cycle n_2, the present measure of the overall cost is
$PWTC_r^C(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C) =$

$$\sum_{j=1}^{j=n_2} (C_j + D_j + S_j + L_j + P_j + H_j) \tag{5.21}$$

$$= \sum_{j=1}^{j=n_2} \Bigg(C_0 * e^{(d-r)*t_j^c} + D_0 * \int_{t_j^c}^{s_{j+1}^c} e^{(d-r)*t} * (\gamma * t) \left(t - t_j^c\right) f(t)dt + S_0 * \int_{s_j^c}^{t_j^c} e^{(d-r)*t}$$
$$* \left(\left(\left(t_j^c - t\right)\right) \div \left(1 + \delta\left(t_j^c - t\right)\right)\right) f(t)dt + L_0 * \int_{s_j^c}^{t_j^c} e^{(d-r)*t}$$
$$* \left(\left(\delta\left(t_j^c - t\right)\right) \div \left(1 + \delta\left(t_j^c - t\right)\right)\right) f(t)dt + P_0 * e^{(d-r)*t_{j-1}^c}$$
$$* \left(\int_{t_j^c}^{s_{j+1}^c} \left[1 + (\gamma/2)\left(t^2 - \left(t_j^c\right)^2\right)\right] f(t)dt + \int_{s_j^c}^{t_j^c} \left(1 \div \left(1 + \delta\left(t_j^c - t\right)\right)\right) f(t)dt \right)$$
$$+ H_0 * \int_{t_j^c}^{s_{j+1}^c} e^{(d-r)*t} * \left[\left(1 + (\gamma/2)t^2\right)\left(t - t_j^c\right) - (\gamma/6)\left(t^3 - \left(t_j^c\right)^3\right)\right] f(t)dt \Bigg) \tag{5.22}$$

The increase in the cost of the retailer in centralized system is given by

$$PWTC_r^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C\right) - PWTC_r^D(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H). \tag{5.23}$$

The supplier's total cost after including the increase in the cost of the retailer in the centralised model is

$PWTC_s^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C\right) = n_2 * e^{(d-r)*t_j} * S_s + n_2 * e^{(d-r)*t_j} * P_s * Q_j^c +$
$PWTC_r^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C\right) - PWTC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right)$

$$\begin{aligned} =& n_2 * (S_s + C_0) * e^{(d-r)*t_j} + n_2 * (P_s + P_0) * e^{(d-r)*t_j} * Q_j^c + H_0 \\ & * \textstyle\int_{t_j^c}^{s_{j+1}^c} e^{(d-r)*t} * \left[\left(1 + (\gamma/2)t^2\right)(t - t_j) - (\gamma/6)\left(t^3 - t_j^3\right)\right] f(t)dt \\ & + D_0 * \textstyle\int_{t_j^c}^{s_{j+1}^c} e^{(d-r)*t} * (\gamma * t)(t - t_j) f(t)dt + (S_0 + L_0 * \delta) \\ & * \textstyle\int_{s_j^c}^{t_j^c} e^{(d-r)*t} * (((t_j - t)) \div (1 + \delta(t_j - t))) f(t)dt \end{aligned} \tag{5.24}$$

The present value of the total cost for the supplier in the centralized system $PWTC_s^C$ is determined by finding values to the time s_j^C and t_j^C by the similar method as used for determining the present worth of the total cost of retailer in the decentralized system $PWTC_r^D$.

Let the optimal solution for Min $PWTC_s^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C = H\right)$ be

$n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H$

The optimized order measure for the planning horizon H is

$Q^C = \sum_{j=1}^{n_2^{CO}} Q_j^{CO} = \sum_{j=1}^{n_2^{CO}} \left(R_j^{CO} + S_j^{CO}\right)$

Retailer agrees with the new replenishment schedule only when the present measure of the overall cost in the co-ordinated system is not greater than the overall cost in uncoordinated system , i.e.

$$PWTC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) \geq$$

$$PWTC_r^C\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H\right) - \sum_{j=1}^{n_2^{CO}} e^{(d-r)*t_j} C_c.\rho\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO} \tag{5.25}$$

where Credit period duration is M_j^C and the rate of credit period ρ, is same for all ordering interval.

$M_j^C = \rho\left(s_{j+1}^{CO} - s_j^{CO}\right)$

The present measure of the total cost of the retailer so obtained in the co-ordinated system after his due profit share is added to it equals the optimised value of his total cost in the uncoordinated system. The credit period rate is minimum ρ_{min} at this point of time.

$$PWTC_r^C\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H\right) - \sum_{j=1}^{n_2^{CO}} e^{(d-r)*t_j} C_c.\rho_{min}\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO} = PWTC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) \tag{5.26}$$

Also for the supplier, the rate of credit period is maximum, ρ_{max}. The present measure of the total cost in the uncoordinated system after the profit is shared equals the cost in the co-ordinated system.

$$n_2^{CO} S_s.e^{(d-r)*t_j} + \sum_{j=1}^{n_2^{CO}} P_s.e^{(d-r)*t_j}.Q_j^{CO} + \sum_{j=1}^{n_2^{CO}} .e^{(d-r)*t_j}.C_c.\rho_{\max}\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO} = PWTC_s^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) \tag{5.27}$$

The value for the minimum rate of credit period and maximum rate of credit period are determined using the equations (26) and (27):

$$\rho_{\min} = \frac{PWTC_r^C\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H\right) - PWTC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right)}{\sum_{j=1}^{n_2^{CO}} .e^{(d-r)*t_j}.C_c\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO}} \tag{5.28}$$

$$\rho_{\max} = \frac{PWTC_s^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) - n_2^{CO}.e^{(d-r)*t_j}.S_s + \sum_{j=1}^{n_2^{CO}} P_s.e^{(d-r)*t_j}.Q_j^{CO}}{\sum_{j=1}^{n_2^{CO}} .e^{(d-r)*t_j}.C_c\left(s_{j+1}^{CO} - s_j^{CO}\right)Q_j^{CO}} \tag{5.29}$$

The extra cost saving in centralized model is equally distributed by $\bar{\rho}$, i.e. average of $\rho_{\max}$ and $\rho_{\min}$. Using $\bar{\rho}$ the present measure for the finalized cost of the retailer and the supplier is given by:

$$PWTC_r^{CO\rho}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n2+1}^{CO} = H\right) = PWTC_r^{C}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n2+1}^{CO} = H\right) - \sum_{j=1}^{n_2^{CO}} .e^{(d-r)*t_j}.C_c\bar{\rho}\left(s_{j+1}^{CO} - s_j^{CO}\right)Q_j^{CO} \tag{5.30}$$

and

$$PWTC_s^{CO\rho}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n2+1}^{CO} = H\right) = n_2^{CO}.e^{(d-r)*t_j}.S_s + \sum_{j=1}^{n_2^{CO}} P_s.e^{(d-r)*t_j}.Q_j^{CO} + \sum_{j=1}^{n_2^{CO}} .e^{(d-r)*t_j}.C_c.\bar{\rho}\left(s_{j+1}^{CO} - s_j^{CO}\right)Q_j^{CO} \tag{5.31}$$

5.4 Optimality condition for $PWTC_r^D$ and $PWTC_s^C$

For particular n_1, the sufficient condition for the nominal value of $PWTC_r^D$ is that the Hessian matrix of the present measure of the finalized cost $\nabla^2 PWTC_r^D$ of the retailer in decentralized system is positive definite[Sarkar et al[38]].

$$\nabla^2 PWTC_r^D = \begin{pmatrix} \frac{\partial^2 PWTC_r^D}{\partial t_1^2} & \frac{\partial^2 PWTC_r^D}{\partial t_1 \partial s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 PWTC_r^D}{\partial s_1 \partial t_1} & \frac{\partial^2 PWTC_r^D}{\partial s_1^2} & \frac{\partial^2 PWTC_r^D}{\partial s_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 PWTC_r^D}{\partial t_2 \partial s_1} & \frac{\partial^2 PWTC_r^D}{\partial t_1^2} & \frac{\partial^2 PWTC_r^D}{\partial t_2 \partial s_2} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 PWTC_r^D}{\partial t_{n_1-1}^2} & \frac{\partial^2 PWTC_r^D}{\partial t_{n_1-1} \partial s_{n_1-1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 PWTC_r^D}{\partial s_{n_1-1}^2} & \frac{\partial^2 PWTC_r^D}{\partial s_{n_1-1} \partial t_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 PWTC_r^D}{\partial t_{n_1} \partial s_{n_1-1}} & \frac{\partial^2 PWTC_r^D}{\partial t_{n_1}^2} \end{pmatrix} \tag{5.32}$$

Where

$$\begin{aligned}
&\frac{\partial^2 PWTC_r^D\left(n_1, s_1, t_1, \ldots, s_{n_1+1}\right)}{\partial t_i^2}\\
&= e^{((d-r)t_j)} * Co * (d-r)^2 + H_0 * (\gamma * t_j) * (a + b*t + c*t^2)\\
&\quad * (1 + ((d-r)*t) + (((d-r)^2 * t^2)/\,2) + (((d-r)^3 * t^3)/\,6)\\
&\qquad + (((d-r)^4 * t^4)/\,24) + (((d-r)^5 * t^5)/\,120) + (((d-r)^6 * t^6)/\,720))\,dt\\
&\quad - (2*\delta)\,(S_0 + L_0) * \textstyle\int_{s_j}^{t_j} (1\,/(1 + \delta\,(t_j - t))\,\hat{}\,3) * (a + b*t + c*t^2)\\
&\quad * (1 + ((d-r)*t) + (((d-r)^2 * t^2)/\,2) + (((d-r)^3 * t^3)/\,6)\\
&\quad + (((d-r)^4 * t^4)/\,24) + (((d-r)^5 * t^5)/\,120) + (((d-r)^6 * t^6)/\,720))\,dt + P_0\\
&\quad * (d-r)^2 * e^{(d-r)*t_j} * \Big(\textstyle\int_{t_j}^{s_{j+1}} (1 + (\gamma/2) * ((t^2) - (t_j\hat{}2))) * (a + b*t + c*t^2)\,dt\\
&\quad + \textstyle\int_{s_j}^{t_j} (1\,/(1 + \delta\,(t_j - t))\,) * (a + b*t + c*t^2)\,dt\Big) + 2 * P_0 * (d-r) * e^{(d-r)*t_j}\\
&\quad * \Big(\textstyle\int_{t_j}^{s_{j+1}} (-1*\gamma*t_j) * (a + b*t + c*t^2)\,dt - \int_{s_j}^{t_j} (\delta\,/((1 + \delta\,(t_j - t))\,^2)\,)\\
&\quad * (a + b*t + c*t^2)\,dt\Big) + P_0 * (d-r) * e^{(d-r)*t_j} * \Big(\textstyle\int_{t_j}^{s_{j+1}} (-1*\gamma)\\
&\quad * (a + b*t + c*t^2)\,dt - \textstyle\int_{s_j}^{t_j} (2*\delta^2/((1 + \delta\,(t_j - t))\,^3)) * (a + b*t + c*t^2)\,dt\Big)
\end{aligned} \tag{5.33}$$

$$\frac{\partial^2 PWTC_r^D\left(n_1, s_1, t_1, \ldots, s_{n_1+1}\right)}{\partial s_i \partial t_{i+1}} = \frac{\partial^2 PWTC_r^D\left(n_1, s_1, t_1, \ldots, s_{n_1+1}\right)}{\partial t_i \partial s_{i-1}} = 0 \tag{5.34}$$

and

$$\begin{aligned}
&\frac{\partial^2 PWTC_r^D\left(n_1, s_1, t_1, \ldots, s_{n_1+1}\right)}{\partial s_i^2} =\\
&H_0 * \big((d-r) * \big(\big(1 + (\gamma/2)\,(s_j^2)\big)\,(s_j - t_{j-1}) - (\gamma/6)\,\big((s_j^3) - (t_{j-1}^3)\big)\big) +\\
&(\gamma * s_j * (s_j - t_{j-1}) + 1) * \Bigg(1 + ((d-r)*s_j) + \frac{(d-r)^2 * s_j^2}{2} +\\
&\frac{(d-r)^3 * s_j^3}{6} + \frac{(d-r)^4 * s_j^4}{24} + \frac{(d-r)^5 * s_j^5}{120} + \frac{(d-r)^6 * s_j^6}{720} *\\
&\big(a + b*s_j + c*s_j^2\big) + D_0 * ((d-r) * (\gamma * s_j)\,(s_j - t_{j-1}) + \gamma\,(s_j - t_{j-1})) *\\
&\Bigg(1 + ((d-r)*s_j) + \frac{(d-r)^2 * s_j^2}{2} + \frac{(d-r)^3 * s_j^3}{6} + \frac{(d-r)^4 * s_j^4}{24} +
\end{aligned}$$

$$\frac{(d-r)^5*s_j^5}{120}+\frac{(d-r)^6*s_j^6}{720}*\left(a+b*s_j+c*\left(s_j^2\right)\right)+S_0*\frac{1}{\left(1+\delta*(t_j-s_j)\right)^2}$$
$$-(d-r)*\left((t_j-s_j)/(1+\delta*(t_j-s_j))\right)*\left(1+((d-r)*s_j)+\frac{(d-r)^2*s_j^2}{2}+\right.$$
$$\frac{(d-r)^3*s_j^3}{6}+\frac{(d-r)^4*s_j^4}{24}+\frac{(d-r)^5*s_j^5}{120}+\frac{(d-r)^6*s_j^6}{720}*$$
$$\left(a+b*s_j+c*\left(s_j^2\right)\right)+L_0*\frac{1}{\left(1+\delta*(t_j-s_j)\right)^2}-(d-r)*\frac{\delta*(t_j-s_j)}{1+\delta*(t_j-s_j)}$$
$$*\left(1+((d-r)*s_j)+\frac{(d-r)^2*s_j^2}{2}+\frac{(d-r)^3*s_j^3}{6}+\frac{(d-r)^4*s_j^4}{24}+\right.$$
$$\frac{(d-r)^5*s_j^5}{120}+\frac{(d-r)^6*s_j^6}{720}*\left(a+b*s_j+c*\left(s_j^2\right)\right)+P_0*$$
$$\left(e^{(d-r)t_{j-1}}*(\gamma*s_j)-e^{(d-r)t_j}*\frac{\delta}{\left(1+\delta*(t_j-s_j)\right)^2}*\left(a+b*s_j+c*\left(s_j^2\right)\right)+\right.$$
$$H_0*\left(\left(1+(\gamma/2)\left(s_j^2\right)\right)(s_j-t_{j-1})-(\gamma/6)\left(s_j^3-t_{j-1}^3\right)\right)*(1+((d-r)*s_j)+$$
$$\frac{(d-r)^2*s_j^2}{2}+\frac{(d-r)^3*s_j^3}{6}+\frac{(d-r)^4*s_j^4}{24}+\frac{(d-r)^5*s_j^5}{120}+$$
$$\frac{(d-r)^6*s_j^6}{720}*f'(s_j)+D_0*(\gamma*s_j)(s_j-t_{j-1})*(1+((d-r)*s_j)+$$
$$\frac{(d-r)^2*s_j^2}{2}+\frac{(d-r)^3*s_j^3}{6}+\frac{(d-r)^4*s_j^4}{24}+\frac{(d-r)^5*s_j^5}{120}+$$
$$\frac{(d-r)^6*s_j^6}{720}*f'(s_j)-S_0*\frac{(t_j-s_j)}{(1+\delta*(t_j-s_j)}*(1+((d-r)*s_j)+$$
$$\frac{(d-r)^2*s_j^2}{2}+\frac{(d-r)^3*s_j^3}{6}+\frac{(d-r)^4*s_j^4}{24}+\frac{(d-r)^5*s_j^5}{120}+$$
$$\frac{(d-r)^6*s_j^6}{720}*f'(s_j)-L_0*\frac{\delta*(t_j-s_j)}{(1+\delta*(t_j-s_j)}*(1+((d-r)*s_j)+$$
$$\frac{(d-r)^2*s_j^2}{2}+\frac{(d-r)^3*s_j^3}{6}+\frac{(d-r)^4*s_j^4}{24}+\frac{(d-r)^5*s_j^5}{120}+$$
$$\frac{(d-r)^6*s_j^6}{720}*f'(s_j)+P_0*e^{(d-r)t_{j-1}}*\left(1+(\gamma/2)\left(s_j^2-t_{j-1}^2\right)\right)*f'(s_j)-$$
$$P_0*e^{(d-r)*t_j}*\frac{1}{(1+\delta*(t_j-s_j)}*f'(s_j) \tag{5.35}$$

From equation (32),it follows that $\nabla^2 PWTC_r^D$ is a tridiagonal matrix. Therefore using the theorem mentioned below, equation (33),(34) and (35), and also applying the concepts of the tridiagonal matrix, it is observed that $\nabla^2 PWTC_r^D$ is positive definite.

Theorem 1: If t_i and s_i satisfy inequations (i) $\frac{\partial^2 PWTC_r^D}{\partial t_i^2}$ >0, (ii) $\frac{\partial^2 PWTC_r^D}{\partial s_i^2}$ >0, (iii) $\frac{\partial^2 PWTC_r^D}{\partial t_i^2} - \left|\frac{\partial^2 PWTC_r^D}{\partial t_i \partial s_i}\right|$ >0 and (iv) $\frac{\partial^2 PWTC_r^D}{\partial s_i^2} - \left|\frac{\partial^2 PWTC_r^D}{\partial s_i \partial t_i}\right|$ >0 for $i = 1, 2, \ldots, n_1$ then $\nabla^2 PWTC_r^D$ is positive definite.

The same can be used to show that $\nabla^2 PWTC_s^C$ is a positive definite and $PWTC_s^C(n_2$, s_1, t_1^c,s_2^c, ..., $s_{n+1}^c)$ reaches a minimal.

The algorithm to determine the results of the model is as follows:

5.5 Algorithm

1. The factors a_1, b_1, c_1, H_o, C_o, S_o, L_o, D_o, γ ,δ , P_o, P_s, C_c and s_1 are assigned with the values.

2. The optimum replenishment schedule in the decentralized system for the retailer is determined by the following steps:.

   (a) Taking s_1 = 0, n_1 = 1 and s_2 = H. Initializing t_1, and calculating it from equation (17).

   (b) Taking n_1 = 2.

   (c) Initializing t_1 and then from t_1 and $\{s_1 = 0\}$, the value of s_2 is calculated from equation (17).

   (d) From the values of t_1 and s_2 the value of t_2 is calculated from equation (17).

   (e) Using t_2 and s_2, the value of s_3 is calculated fron equation (18). Continuing the process to find all optimal $t_i's$ and $s_i's$ {i=$1, 2, \ldots, n_1$} for all n_1.

   (f) The steps $2(d)$ and $2(e)$ are repeated for all n_1.

   (g) For $n_1 = 1$ and if $PWTC_r^D\,(n_1) < PWTC_r^D(n_1 + 1)$,then $PWTC_r^D\,(n_1) = PWTC_r^{DO}(n_1)$.Stop.

3. For $n_1 \geq 2$ and if $PWTC_r^D\,(n_1) < PWTC_r^D\,(n_1 - 1)$ and $PWTC_r^D\,(n_1) < PWTC_r^D\,(n_1 + 1)$, then $PWTC_r^D\,(n_1) = PWTC_r^{DO}\,(n_1)$ and stop or else let $n_1 = n_1 + 1$, and goto step 2(c).

4. $n_1^{DO} = n_1$ is the optimum replenishment schedule in the decentralised system for the retailer and the supplier.

5. $PWTC_r^{DO}\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1^{DO}+1}\right)$,
$PWTC_s^{DO}\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1^{DO}+1}\right)$ and Q^{DO} are calculated from equations (16), (19) and (20) respectively.

6. Continuing the steps 2 to 4, n_2^{CO}, s_1,t_1^{CO},s_2^{CO},... and
$PWTC_r^{CO}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2^{CO}+1}\right)$,
$PWTC_s^{CO}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2^{CO}+1}\right)$ and Q^{CO} are calculated in centralised model.

7. Calculate $\rho_{\min}$, $\rho_{\max}$,$PWTC_r^{CO\rho}$ and $PWTC_s^{CO\rho}$ from equations (28),(29),(30) and (31) respectively.

TABLE 5.1: Cost saving percentage for $S_s = 160, 320, 480$ without lost sales

	Decentralized system						
S_s	$PWTC_r^{DO}$	$PWTC_s^{DO}$	n_1^{DO}	Q^{DO}	$\rho_{\min}$	$\rho_{\max}$	$\bar{\rho}$
160	6145.29	2329.85	7	2791.43	0.0035	0.0060	0.0047
320	6145.29	3602.85	7	2791.43	0.013	0.023	0.018
480	6145.29	4875.85	7	2791.43	0.0131	0.0358	0.0244

	Centralized system				% Cost saving	
S_s	$PWTC_r^{CO\rho}$	$PWTC_s^{CO\rho}$	n_2^{CO}	Q^{CO}	$\frac{\Delta PWTC}{PWTC_r^{DO}}$	$\frac{\Delta PWTC}{PWTC_s^{DO}}$
160	6105.89	2290.45	6	2791.97	0.64	1.69
320	5975.06	3432.62	5	2792.43	2.77	4.72
480	5790.18	4520.75	5	2792.78	5.77	7.28

5.5.1 Numerical Example

To validate the mathematical procedure for obtaining an optimal solution to the model, following examples are demonstrated:
Example1: To particularize this model, we assume no lost sales. The shortages are completely backordered. The measures for other factors are as follows: $a_1 = 450$ items/year, $b_1 = 110$ items/year, $c_1 = 5$ items/year, $P_o = 0.3$ \$/item, $s = 28$ \$/item, $\delta = 0.8, \gamma = 0.01, s_1 = 0, C_o = 300$ \$/purchase order, $S_s = 320$ \$/arrangement, $H = 4$, $P_s = 0.3$ \$/item, $C_c = 2.5$ \$/item/year , $H_o = 3$ \$/item/year, d = 0.115, r = 0.05, DC = 12. The parameter S_s is made variable to study the impact of increase in set up cost. For this particular case, table 1 displays the optimal reordering schedule and the optimal quantity to be ordered in a de-centralized and centralized system. It also shows the percentage change in the overall cost for the retailer and the supplier with the increase in the set-up cost.

TABLE 5.2: Inflated amount for $a_1 = \{225, 450, 675\}$ for retailer in an uncoordinated system

$\downarrow a_1$	$\rightarrow n_1$	1	2	3	4	5	6
225		15639.3	8383.29	6267.48	5457.99	5129.8	5037.26
450		22817	11895.4	8736.77	7392.07	6750.84	6453.63
675		29770.7	15419	11181	9312.8	8363.5	7864.32

$\downarrow a_1$	$\rightarrow n_1$	7	8	9	10	11
225		5077.04	5186.41	5367.41	5582.42	5822.16
450		6349.8	6348.32	6448.43	6601.09	6791.53
675		7594.98	7512.75	7531.98	7621.65	7762.1

Example2: Taking $a_1 = \{225, 450, 675\}$ items/year, $b_1 = 110$ items/year, $c_1 = 5$ items/year, $P_o = 0.3$ \$/item, $S_o = 18$ \$/item, $L_o = 22$ \$/item, $\delta = 0.8$, $\gamma = 0.01$, $s_1 = 0$, $C_o = 300$ \$/ purchase order, $S_s = 320$ \$/arrangement, $H = 4$, $P_s = 0.3$ \$/item, $C_c = 2.5$ \$/item/year , $H_o = 3$ \$/item/year, $d = 0.115$, $r = 0.05$, $DC = 12$ \$/item/year. Mathematica(version 8.0) is used to solve the numerical calculation of the non linear system of equations in decentralized system for different replenishment cycles n_1= {1, 2, ...7}. Table 2 shows the present measure of the overall cost of the retailer for a_1 = {225, 450, 675}. The optimized total cost for retailer under inflation for the values of a_1 = {225, 450, 675} are 5037.26, 6348.32 and 7512.75 for replenishment cycles 6, 8 and 8 respectively. The decrement in the inflated cost from n_1=1, till it reaches a minimum or optimum value at n_1=6 after which the value again gradually increases for remaining cycles following its convexity for different values of a_1.
The graphical representation as in Fig.2, for the overall cost function shows the convex nature of the cost subject to inflation rate.

TABLE 5.3: Retailer's optimal cycle in an uncoordinated system

a_1	n_1^{DO}	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
225	6	0	0.662171	1.3173	1.97399	2.6376	3.31183	3.99944		
450	8	0	0.381201	0.796402	1.24557	1.72856	2.24516	2.79512	3.37812	3.99389
675	8	0	0.381201	0.796402	1.24557	1.72856	2.24516	2.79512	3.37812	3.99389

a_1	n_1^{DO}	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
225	6	0.0551381	0.717079	1.37316	2.03152	2.6973	3.37412		
450	8	0.0287974	0.423739	0.850875	1.30927	1.79819	2.31705	2.86538	3.44282
675	8	0.0278348	0.412205	0.83079	1.2835	1.7702	2.29067	2.84468	3.43192

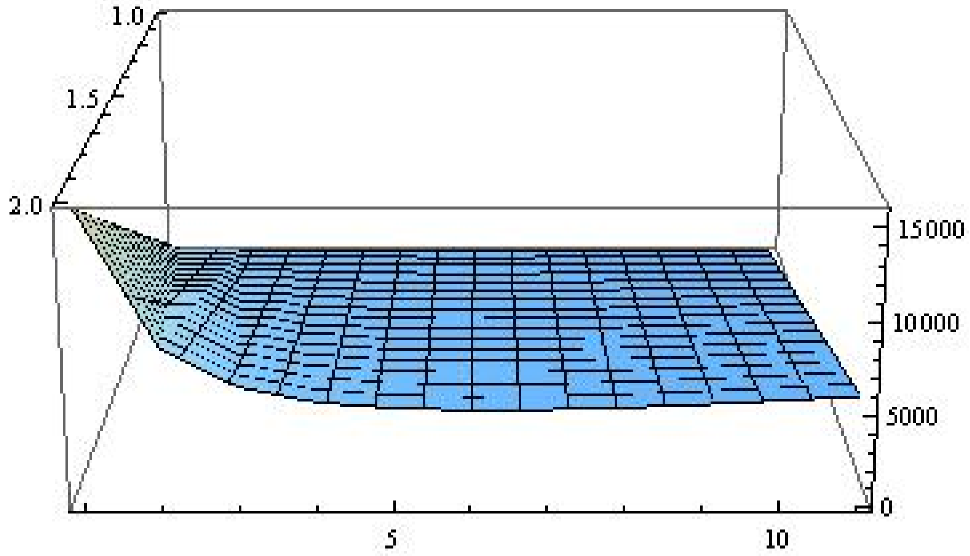

FIGURE 5.2: Convexity of total cost for $a_1 = 225$ in decentralised system

TABLE 5.4: Inflated amount for $a_1 = \{225, 450, 675\}$ for supplier in an co-ordinated system

a_1 \ $\rightarrow n_2$	1	2	3	4	5	6	7
225	11542.	4719.51	2996.54	2564.47	2608.93	2867.12	3287.23
450	17681.4	7227.68	4472.31	3511.13	3246.13	3291.92	3571.4
675	23748.6	9899.22	6074.79	4595.07	4024.9	3869.4	4006.74

The optimum values of time s_i's and t_i's alongwith the optimal cycles are presented in table 3 for distinct values of a_1.

Table 4 displays the optimized total cost for supplier under inflation in a centralized model for different values of $a_1 = \{225, 450, 675\}$ and for different replenishment schedules n_2 going from 1 to 7. The optimum value of total cost for $a_1 = \{225, 450, 675\}$ are \$2564.47, \$3246.13 and \$3869.4.37 respectively. The optimal refilling schedules for $a_1 = \{225, 450, 675\}$ are therefore 4, 5 and 6 respectively. The inflated overall cost is convex in nature. The value of the cost starts declining from the first cycle, till it reaches a minimal at 4th cycle. After that it gradually increases for remaining n_2. Table 5 comprises of the optimal replenishment cycles and the optimal time s_i's and t_i's for different a_i.

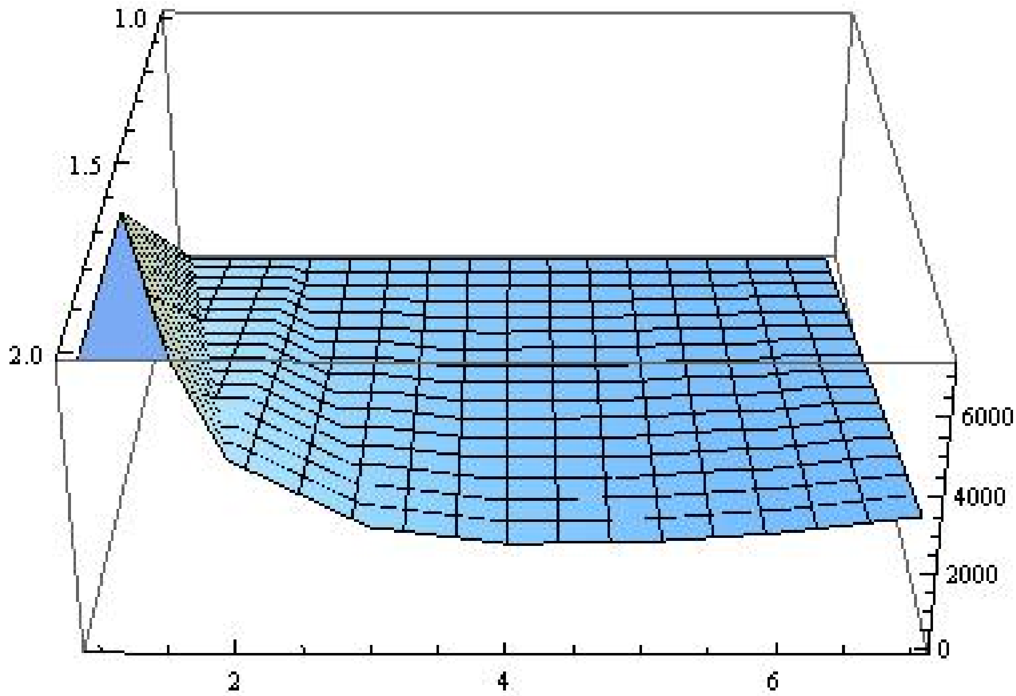

FIGURE 5.3: Convexity of total cost for $a_1 = 225$ in centralised system

The convex nature of the final cost under inflation is graphically representation for a_1 = 225 in fig.3.

A comparison of the total cost under inflation,optimum reordering schedule and economic order quantity for both decentralized and centralized models is done in table 6.The table incorporates the optimum values of the inflated overall cost for the retailer as well as for the supplier for distinct a_1 in both the decentralized and centralized system. The rate of the credit period rate(minimum, maximum and average) for each a_i is also shown in the table. The cost saving percentage resulting in centralized model due to credit period rate ρ for both retailer and supplier is shown in the last two columns of the table. There is an increase in the cost savings from $a_1 = 225$ to $a_1 = 675$.

TABLE 5.5: Cost saving percentage for $a_1 = \{225, 450, 675\}$

	Decentralized system						
a_1	$PWTC_r^{DO}$	$PWTC_s^{DO}$	n_1^{DO}	Q^{DO}	$\rho_{\min}$	$\rho_{\max}$	$\bar{\rho}$
225	5037.26	2859.29	6	1895.32	0.020	0.034	0.27
450	6348.32	3900.48	8	2789.86	0.01	0.03	0.02
675	7512.75	4235.08	8	3693.18	0.008	0.017	0.013

	Centralized system				% Cost saving	
a_1	$PWTC_r^{CO\rho}$	$PWTC_s^{CO\rho}$	n_2^{CO}	Q^{CO}	$\frac{\Delta PWTC}{PWTC_r^{DO}}$	$\frac{\Delta PWTC}{PWTC_s^{DO}}$
225	4889.85	2711.88	4	1897.92	2.92	5.15
450	6021.14	3573.31	5	2801.31	5.15	8.38
675	7329.92	4052.24	6	3694.45	2.43	4.31

TABLE 5.6: Supplier's optimal cycle in an co-ordinated system

a_1	n_2^{CO}	s_1	s_2	s_3	s_4	s_5	s_6	s_7
225	4	0	1.08096	2.08518	3.05131	3.99858		
450	5	0	0.765073	1.54729	2.34682	3.16411	3.99971	
675	6	0	0.571706	1.18357	1.83338	2.51918	3.23928	3.99227

a_1	n_2^{CO}	t_1	t_2	t_3	t_4	t_5	t_6
225	4	0.10232	1.17385	2.17539	3.14158		
450	5	0.060896	0.829556	1.6155	2.41894	3.24034	2.72997
675	6	0.0427336	0.619103	1.23571	1.89035	2.5811	3.30629

5.5.2 Sensitivity analysis

Sensitivity analysis is done for the inventory model analyzing whether the formulated model is influenced by the alterations in the input parameters.We analyze the consequence of the alterations in the factors against the changes in the total cost of the retailer and the supplier,the replenishment schedule,credit period rate and the economic order quantity.

The following table demonstrates the result when one of the parameter is changed keeping others unaltered. Every parameter is altered by -50%, -25%, 25% and 50%, of the initial cost taken in Example 2.

1. $PWTC_r^{CO\rho}$ increases for parameters a_1, b_1, l, H_o, P_o, C_o, d and H. It decreases for the parameters P_s, S_s, r and δ. Also $PWTC_r^{CO\rho}$ is highly sensitive to the parameter H, moderately sensitive to the parameter a_1, b_1, H_o, C_o and δ. Also $PWTC_r^{CO\rho}$ less sensitive to c_1, γ, s, l, P_o, P_s, S_s, DC, d and r.

2. $PWTC_s^{CO\rho}$ increases for the parameters a_1, b_1, c_1, γ, H_o, s, P_s, S_s, DC, d and H. It decreases for the parameters l, P_o, C_o, r and δ. Also $PWTC_s^{CO\rho}$ is extremely responsive to the parameters H and S_s, fairly responsive to a_1, H_o, P_s, C_o, d and δ. $PWTC_s^{CO\rho}$ is unresponsive to the parameters r, DC, P_o, l, s, b_1, c_1, γ.

3. With the increase in the parameters c_1, γ, s, P_s, S_s, DC, d and H_o, credit period rate $\overline{\rho}$ increases. It decreases with the changes in the parameters a_1, b_1, l, P_o, r δ, H and C_o. $\overline{\rho}$ is extremely responsive to the parameters H_o, S_s, C_o, δ and H, fairly responsive to a_1, b_1, γ, s, DC and d. Also that it is unresponsive to b_1, l, P_o, P_s and r.

4. n_1^{DO} is majorly responsive to the parameters C_o and H. With the increase in C_o, n_1^{DO} decreases but with increase in H, n_1^{DO} increases. n_1^{DO} is fairly responsive to the parameters a_1 and δ while it is unresponsive to every other parameters. n_2^{CO} is majorly responsive to the parameter H. With the increase in H, n_2^{CO} increases. n_2^{CO} is fairly responsive to the alterations in parameters a_1, γ and H_o and unresponsive to the alterations in every other parameters.

5. The optimal order quantity after coordination between the retailer and the supplier QJT increases with increase in every parameter except the holding cost H_o. It clearly shows that when per unit cost of holding increases then the quantity to order decreases. Also when the rate of backlogging δ diminishes then the optimal order quantity reduces. The optimal order quantity remains unaltered with the increase in the parameters P_o, P_s, d, r and DC.

	$Values$	$\frac{\Delta PWTC_r^{CO\rho}*100\%}{PWTC_{rO}^{CO\rho}}$	$\frac{\Delta PWTC_s^{CO\rho}*100\%}{PWTC_{sO}^{CO\rho}}$	$\frac{\Delta\overline{p}*100\%}{\overline{p}O}$	n_1^{DO}	n_2^{CO}	QJT
	225	−18.7886	−24.1073	15.4291	6	4	1897.72
	337.5	−9.046	−11.9912	−22.5636	7	5	2349.1
a_1	562.5	11.0582	7.01499	−45.043	8	6	3242.54
	675	21.7364	13.4031	−45.5749	8	6	3694.45
	55	−12.7777	−7.65148	−0.769666	7	5	2358.33
	82.5	−6.38878	−3.82572	−0.351696	7	5	2579.85
b_1	137.5	3.58144	9.09425	28.6754	8	5	3022.9
	165	9.67571	13.1159	−25.5657	8	6	3243.34
	2.5	1.31821	−5.61953	−22.3758	7	5	2747.73
c_1	3.75	2.2008	−5.40583	−23.5367	7	5	2774.52
	6.25	0.698756	0.516567	0.0787466	8	5	2828.1
	7.5	1.39578	1.03274	0.156234	8	5	2854.88
	0.005	0.435129	−5.5699	−25.3156	7	6	2791.64
γ	0.0075	1.60703	−5.11528	−23.7379	7	7	2796.53
	0.0125	1.10117	0.530871	1.92396	8	8	2806.01
	0.015	2.19322	1.05689	3.8348	8	8	2810.61
	1.5	−21.2359	−17.6589	−34.2654	5	4	2810.92
H_o	2.25	−9.43805	−8.03695	−37.9377	5	5	2804.61
	3.75	10.689	3.37463	−32.5716	5	6	2796.73
	4.5	20.324	5.57955	−22.3167	5	6	2793.27
	9	2.73164	−6.09767	−27.8596	7	5	2797.12
s	13.5	2.80691	−5.43692	−25.3656	7	5	2799.61
	22.5	0.0693557	0.507132	2.05165	8	5	2802.55
	27	0.158604	0.904935	3.66269	8	5	2803.47
	11	−2.92412	0.170032	−0.655025	8	5	2794.46
l	16.5	−1.316	0.0915276	−0.201671	8	5	2798.55
	27.5	1.08737	−0.0904029	0.0751969	8	5	2803.29
	33	4.73686	−4.79181	−21.8518	7	5	2804.77
	0.15	−8.66318	0.0735532	0.343108	8	5	2801.32
P_o	0.225	−4.33156	0.0367678	0.171506	8	5	2801.32
	0.375	4.3315	−0.0367505	−0.171412	8	5	2801.31
	0.45	8.66295	−0.0734837	−0.34273	8	5	2801.31
	0.15	0.0368718	−14.5863	−0.338364	8	5	2801.32
P_s	0.225	0.0184476	−7.29314	−0.169334	8	5	2801.32
	0.375	−0.0184711	7.2931	0.169641	8	5	2801.31
	0.45	−0.0369657	14.5862	0.33959	8	5	2801.31
	160	3.4603	−34.0989	−68.1289	8	6	2799.59
S_s	240	2.04299	−16.5223	−55.746	8	6	2799.73
	400	−2.18021	16.2911	18.2192	8	5	2801.44
	480	−4.3617	32.58	36.4644	8	5	2801.56
	150	−25.877	17.2343	33.0672	10	6	2799.61
C_o	225	−9.30514	3.5487	−31.6309	8	6	2799.74
	375	11.26	−7.14465	−33.861	7	5	2801.43
	450	22.2856	−13.8979	−65.8601	6	5	2801.55
	6	0.591263	−5.39095	−25.3308	7	5	2802.19
DC	9	1.68416	−5.02671	−23.7475	7	5	2801.75
	15	1.02882	0.438901	1.95475	8	5	2800.87
	18	2.05003	0.874761	3.902	8	5	2800.43
	0.0575	−10.2732	−16.4019	−25.6792	7	5	2801.43
d	0.08625	−6.70085	−6.27349	−1.41664	8	5	2801.34
	0.14375	7.29999	6.8106	1.36123	8	5	2801.3
	0.1725	15.2526	14.2153	2.69045	8	5	2801.3
	0.025	6.31203	5.88979	1.18572	8	5	2801.31
r	0.0375	3.09734	2.89179	0.59666	8	5	2801.31
	0.0625	−2.98423	−2.79019	−0.606405	8	5	2801.32
	0.075	−5.85919	−5.48356	−1.22668	8	5	2801.33
	0.4	23.6259	−15.8831	−65.535	6	5	2804.68
δ	0.6	9.40406	−7.67832	−32.0588	7	5	2805.75
	1.0	−1.99311	1.9056	5.26013	8	5	2797.6
	1.2	3.12699	−3.00943	−21.3598	7	5	2793.95
	1.995	−59.7446	−60.2372	194.596	3	2	1131.58
H	2.9925	−33.3863	−32.4546	−11.3016	5	4	1889.9
	4.9875	41.9328	37.7438	−44.334	9	7	3845.52
	5.985	96.2282	84.7661	−11.9866	11	7	5070.49

5.6 Conclusions

The financial state of an organization is substantially affected by inflation. Every cost associated with the inventory model is greatly influenced as there is an increase in the rate of inflation due to which there is a decline in the purchasing index of money. The present paper demonstrates an inferred inventory framework with the credit period being offered by the supplier to the retailer. The present measure of the total cost is determined for the retailer as well as for the supplier. Also, numerical examples validate the model where the partially backlogged shortages are turned to completely backlogged shortages. Lastly, sensitivity analysis is done for every factor to ascertain their impact on the present measure of the overall cost.

Chapter 6

Green Inventory Integrated Model with Inflation under Permissible Delay

Abstract

Trade credit is an important cost reduction tool in inventory management. The effect of trade credit is studied on the integrated system for sharing the cost benefits realized due to the permissible delay. A credit term factor is introduced to divide the cost benefits between the retailer and the supplier. The various costs in the inventory model are subjected to the same inflation rate. This chapter revisits the traditional EOQ model for the re-manufacturing process under the green supply chain with the permissible delay available to the retailer. Numerical examples prove that the optimal re-ordering schedule exists and is unique. Also, sensitivity analysis is performed on certain parameters to ascertain their logical implications.

6.1 INTRODUCTION

This paper is an extension of the work Singh et al., 2018b orginally presented in the 4th International Conference on Computing Science(ICCS),2018. This extended research work incorporates the green inventory concept. The harmful effects of the waste and outdated products are imposing a threat to the environment. In present scenario, world is facing pollution as a big hazard to mankind. Every organization is moving towards reducing and reusing the waste/imperfect goods. In this direction this EOQ model aims at a single stage remanufacturing process where the imperfect goods after screening are taken back by the supplier and are remanufactured. This is again transported to the retailer. Replenishment schedule is derived for the retailer and the supplier, considering green product life cycle and the time value of money. Various countries have adopted several measures for waste product management, reusing and remanufacturing programs. International standards as European Union's proposal for Waste Electrical and Electronic equipment (WEEE) introduces the concept of product design, to bring a decrease in the cost of disassembly and remanufacturing,Toffel, 2002. Product, if designed significantly reduces the cost of inspection, disassembly, repair, remanufacturing and recycling. Schrady, 1967 was the first to determine an optimal ordering size with recovery and remanufacturing. He studied the traditional EOQ model with continuous and deterministic demand and return. Bonney, Ratchev, and Moualek, 2003 probed the changes taking place in the market and in the manufacturing organizations. He studied the changes for the process of product design, with respect to the technology introduced for new material and new production methods, including tools and techniques altered for the manufacturing, inspection, reusing and remanufacturing. Brito, 2004 provided a comprehensive and immense knowledge for the remanufacturing process from the literature surveyed. The supplier and retailer are stationed far from each other so it is not possible for the supplier to send all perfect goods. Thus to be assure of the brand and quality, retailer screens the lot as it is received. Sarkar et al., 2018 studied the remanufacturing of the imperfect items to maximize the total profit. The effect of deterioration is dominant and its consequences cannot be ignored while framing an EOQ model. Electronic goods, blood, fruits, grain products, alcohol are some of the deteriorating products. Ghare and Schrader, 1963 were the first to study deterioration in an inventory model. Several researchers as Covert and Philip, 1973, Raafat, 1991, Wee, 1993, Wee and Chung c, 2009 studied different patterns of deterioration in inventory models. Chung, 2003 developed an inventory model for the stock dependent demand for deteriorating products under two level credit.Rameswari and Uthayakumar, 2018 studied a two-echelon supply chain model for deteriorating products under trade credit. They analyzed two models, one with demand being stock dependent and the second with demand being selling price dependent for the perishable products. Singh et al., 2017a studied an inventory model for the deteriorating products where the deterioration being time dependent. They studied permissible delay policy in their model. Partially backlogged shortages were als incorporated. Annadurai, 2013 discusses the inventory model for perishable items where deterioration follows exponential distribution under permissible delay.

In conventional EOQ model, assumes that the retailer must pay for the goods as they arrive. But as practice, sometimes the supplier allows permissible delay to the retailer for the settlement of the goods. Numbers of research papers have been published with inventory models under trade credit. The rudest approach of an inventory model under trade credit was done by Goyal, 1985b. Chung and Cárdenas-Barrón, 2013 developed an EOQ model under two level supply chain with trade credit for deteriorating products and demand being stock dependent. Sarkar, Ghosh, and Chaudhuri, 2012a,Sarkar, Ghosh, and Chaudhuri, 2012b revisited the inventory models with time and stock dependent demand for imperfect and perishable products allowing trade credit. Wang and Liu, 2018 studies the effect of sales effort on demand in a supply chain inventory model when permissible delay is allowed. He also included quantity discounts in his model with two level supply chain. Trade credit is widely used by the industries in

U.S., China and Europe as discussed in Lin and Chou, 2015,Zhou, Wen, and Wu, 2015 and Coulibaly, Sapriza, and Zlate, 2013. Suppliers give credit period to the retailer to postpone their payments, thereby increasing their participation and retaining the market. On the other hand, retailer can collect the revenue of the goods during the credit period and is encouraged to increase the order quantity.The concept of trade credit incorporated with the green inventory and weibull deterioration was studied by Singh et al., 2019. Pal, 2018 analyses an inventory system with trade credit for imperfect goods and demand being quality dependent. He has considered two different approaches of trade credit in the supply chain model.
The formal EOQ Model establishing various results does not include the inflation for all the inventory related costs, whereas its effect should not be ignored. The first initiation for an inventory model under the time value of money was considered by Buzacott, 1975. Lo, Wee, and Huang, 2007 analyzed an inventory model with the time value of money and weibull deterioration. Alizadeh et al., 2011 proposed a deteriorating EOQ model when demand is poisson and lead time is non zero. Ghoreishi, Weber, and Mirzazadeh, 2015 developed a replenishment model for perishable products with return policy under inflation. Singh et al., 2017c determine a reordering policy in an inventory model for deteriorating goods,partial backordering under inflation. Singh et al., 2018b represented supply chain inventory model with inflation for time quadratic demand and deterioration. Rani, Ali, and Agarwal, 2017 developed an inventory model for deteriorating products considering inflation in a green supply chain for remanufacturing and recycling. Also a fuzzified model was developed by Singh et al., 2018c.

6.2 Assumptions and Terminology

6.2.1 Assumptions

The following assumptions are followed:

1. All inventory related cost are subjected to the same constant rate of inflation r.
2. The supplier offers the retailer credit period ρ to settle the account.
3. Finite replenishment and non zero lead time is considered.
4. The available inventory deteriorates with a constant fraction in a finite planning horizon.
5. Single item single retailer and single supplier is considered in the supply chain.
6. A percentage of products received in the lot are of imperfect quality.

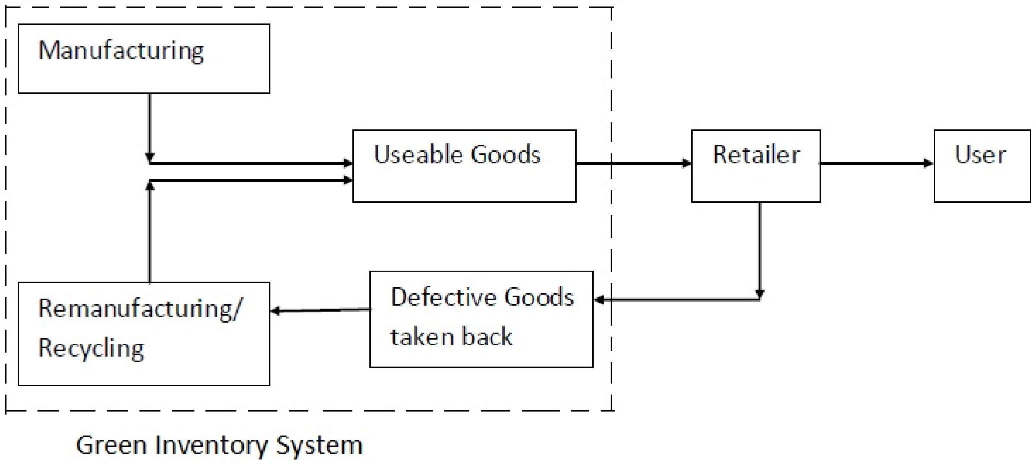

FIGURE 6.1: Flow of Inventory with Green Technology

7. Commencing with the inventory related costs under the same rate of inflation, the ordering cost, the purchasing cost, the holding costs, the deterioration cost and the screening cost are respectively. The model is proposed in a finite planning horizon H, having I_{oi} units at the start of the cycle.

6.2.2 Terminology

1. The Opportunity cost $C_c(\$/unit/year)$ is same for the retailer and the supplier.
2. M_j^S is the permissible delay given by the supplier to the retailer for the settlement of the account in the synchronized model.
3. Setup cost for the supplier is S_s($/order).
4. The purchasing price for the supplier and the retailer is P_s($/unit) and P_s($/unit) with $P_s < P_o$.
5. In the synchronised model, the rate at which the additional cost is distributed by both the divisions is ρ.

6.3 Model presentation and Analysis

The model is developed for two different scenarios:

Scenario 1 : Desynchronised Model

The retailer orders the quantity as per his convenience and the supplier follows the replenishment schedule of the retailer under inflation and green's technology of remanufacturing and recycling.

Scenario 2: Synchronised Model under permissible delay

The supplier offers credit period to the retailer to pay back the money for the lot received. Also shares the extra cost incurred by the retailer on account of following the replenishment schedule of the supplier with the order size increased. The present estimate of the total cost for the retailer and the supplier is calculated under inflation and green's technology of remanufacturing and recycling.

Model Description

The lot received at the beginning of the cycle has p percentage of imperfect quality goods. The complete lot undergoes screening process by the retailer and the defective goods are taken back by the supplier for remanufacturing and recycling. The cycle starts at t_i with I_{oi} units. The fall in the inventory is due to demand and deterioration till $t_i^{'}$.The level of inventory at this first stage of the process is $I_{1i}(t)$. At $t_i^{'}$, a decrease in demand occurs when the imperfect quality goods are taken back by the supplier. From $t_i^{'}$ to $t_i^{''}$, the fall in inventory is due to demand and deterioration. $I_{2i}(t)$ is the inventory for the second stage of the process. At $t_i^{''}$, an increase in the level of inventory is observed when the imperfect goods remanufactured are transported back to the retailer. Further at $t_i^{''}$, the inventory level $I_{3i}(t)$ declines due to demand and deterioration till it reaches zero. This behaviour is illustrated by figure 1.

Scenario 1: Desynchronised Model

$$\frac{d\left(I_{1i}(t)\right)}{dt} + \theta_1(t) I_{1i}(t) = \text{-f(t)},\ t_i \leq t \leq t_i^{'}, (i = 1, 2, \ldots, n_1) \tag{6.1}$$

the boundary condition is $I_{1i}\left(t_i\right) = I_{oi}$ and $I_{1i}\left(t_i^{'}\right) = I_{si}$

$$\frac{d\left(I_{2i}(t)\right)}{dt} + \theta_1(t) I_{2i}(t) = \text{-f(t)},\ t_i^{'} \leq t \leq t_i^{''}, (i = 1, 2, \ldots, n_1) \tag{6.2}$$

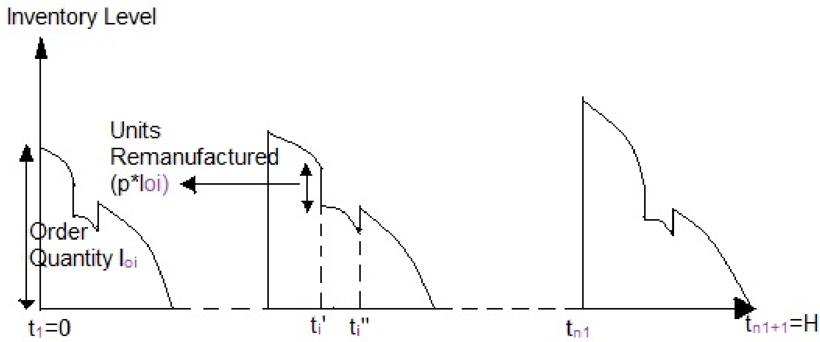

FIGURE 6.2: Graphical representation of Inventory Model

the boundary condition is $I_{2i}\left(t_i^{'}\right) = I_{si} - p.I_{oi}$ and $I_{2i}\left(t_i^{''}\right) = I_{fi}$

$$\frac{d\left(I_{3i}(t)\right)}{dt} + \theta_1(t)I_{3i}(t) = \text{-f(t)},\ t_i^{''} \le t \le t_{i+1}, (i = 1, 2, \ldots, n_1) \tag{6.3}$$

the boundary condition is $I_{3i}\left(t_i^{''}\right) = I_{fi} + p.I_{oi}$ and $I_{3i}\left(t_{i+1}\right) = 0$

Solving these equations, we get

$$\begin{aligned} I_{\text{oi}} = a\left(t_{i+1} - t_i\right) + \left((\text{a}\theta_1 + b)/\,2\right).\left(t_{i+1}^2 - t_i^2\right) + \left(\text{b}\theta_1/\,3\right)\left(t_{i+1}^3 - t_i^3\right) \\ - \left(\text{p}\theta_1\left(t_{i+1}^{"} - t_i^{'}\right) + \theta_1 t_i\right).\left(a\left(t_{i+1} - t_i\right) + (b/2).\left(t_{i+1}^2 - t_i^2\right)\right) \end{aligned} \tag{6.4}$$

$$\begin{aligned} I_{si} = e^{-\theta_1.t_i^{'}}\left(1 - p\left(-e^{-\theta_1\left(t_i - t_i^{'}\right)} + e^{-\theta_1\left(t_i - t_i^{''}\right)}\right)\right).\int_{t_i}^{t_{i+1}} e^{\theta_1.u}.f(u)du \\ - \int_{t_i}^{t_i^{'}} e^{\theta_1.\left(u - t_i^{'}\right)}.f(u)du \end{aligned} \tag{6.5}$$

$$\begin{aligned} I_{fi} = I_{oi}.e^{\theta_1.\left(t_i - t_i^{''}\right)} - e^{-\theta_1 t_i^{''}}.\int_{t_i}^{t_i^{'}} e^{\theta_1.u}.f(u)du \\ - p.I_{oi}.e^{\theta_1.\left(t_i^{'} - t_i^{''}\right)} - e^{-\theta_1 t_i^{''}}.\int_{t_i^{'}}^{t_i^{''}} e^{\theta_1.u}.f(u)du \end{aligned} \tag{6.6}$$

substituting these values, the solution of the differential eq (1) is

$$I_{1i}(t) = a\left(t_{i+1} - t\right) + \left(\left(a.\theta_1 + b\right)/\,2\right).\left(t_{i+1}^2 - t^2\right) + \left(\left(b.\theta_1\right)/\,3\right).\left(t_{i+1}^3 - t^3\right) + \Big(p.\theta_1.t_i^{'} - p.\theta_1.t_i^{"}\Big).\left(a\left(t_{i+1} - t_i\right) + (b/2).\left(t_{i+1}^2 - t_i^2\right)\right) - \theta_1.t.\left(a\left(t_{i+1} - t\right) + (b/2).\left(t_{i+1}^2 - t^2\right)\right), t_i \le t \le t_i{}', \quad \{i = 1, 2, \ldots, n_1\}. \tag{6.7}$$

the solution of the differential eq (2) is

$$I_{2i}(t) = \left(1 - \theta_1.t\right).\left(a\left(t_{i+1} - t\right) + (b/2).\left(t_{i+1}^2 - t^2\right)\right) + \left(\left(a.\theta_1\right)/\,2\right).\left(t_{i+1}^2 - t^2\right) + \left(\left(b.\theta_1\right)/\,3\right).\left(t_{i+1}^3 - t^3\right) - p.\left(\left(\left(a.\theta_1\right)/\,2\right).\left(t_{i+1}^2 - t_i^2\right) + \left(\left(b.\theta_1\right)/\,3\right).\left(t_{i+1}^3 - t_i^3\right)\right) - \left(p.\left(1 + \theta_1.t_i^{"} - \theta_1.t - \theta_1 t_i\right).\left(a\left(t_{i+1} - t_i\right) + (b/2).\left(t_{i+1}^2 - t_i^2\right)\right) + p^2.\Big(\theta_1.t_i^{"} - \theta_1.t_i^{'}\Big)\right), t_i{}' \le t \le t_i{}'', \quad \{i = 1, 2, \ldots, n_1\} \tag{6.8}$$

the solution of the differential eq (3) is

$$I_{3i}(t) = \left(1 - \theta_1.t\right).\left(a\left(t_{i+1} - t\right) + (b/2).\left(t_{i+1}^2 - t^2\right)\right) + \left(\left(\left(a.\theta_1\right)/\,2\right).\left(t_{i+1}^2 - t^2\right) + \left(\left(b.\theta_1\right)/\,3\right).\left(t_{i+1}^3 - t^3\right)\right), t_i{}'' \le t \le t_{i+1}, \quad \{i = 1, 2, \ldots, n_1\}. \tag{6.9}$$

Total inventory carried during the interval $[t_i, t_{i+1}]\, is$

$$R_i^D = \int_{t_i}^{t_i^{'}} I_{1i}(t)dt + \int_{t_i^{'}}^{t_i^{''}} I_{2i}(t)dt + \int_{t_i^{''}}^{t_{i+1}} I_{3i}(t)dt\,, \quad (i = 1, 2, 3, \ldots, n_1). \tag{6.10}$$

Total quantity during the planning horizon =

$$Q^D = \sum_{i=1}^{n_1} I_{oi}^D \tag{6.11}$$

Retailer's total cost includes present estimate of the ordering cost, holding cost, deterioration cost, purchasing cost and screening cost. Present estimate of the retailer's total cost $PETC_r^D$ is

$$PETC_r^D = \sum_{i\,=1}^{i\,=n_1}\left(PEOC + PEHC + PEDC + PEPC + PESC\right)$$

$$\sum_{i=1}^{i=n_1} e^{(d-r)t_i}.n_1.O_r$$
$$+ H_o.\left(\int_{t_i}^{t_i^{'}} e^{(d-r).t}.I_{1i}(t)dt + \int_{t_i^{'}}^{t_i^{''}} e^{(d-r).t}.I_{2i}(t)dt + \int_{t_i^{''}}^{t_{i+1}} e^{(d-r).t}.I_{3i}(t)dt\right)$$
$$+ \text{DC}.\left(\int_{t_i}^{t_i^{'}} e^{(d-r).t}.\theta_1.I_{1i}(t)dt + \int_{t_i^{'}}^{t_i^{''}} e^{(d-r).t}.\theta_1.I_{2i}(t)dt\right.$$
$$\left. + \int_{t_i^{''}}^{t_{i+1}} e^{(d-r).t}.\theta_1.I_{3i}(t)dt\right) + (P_o + SC).e^{(d-r).t_i}.I_{oi} \tag{6.12}$$

By determining the $t_i's$, the green's inventory model is optimized. Equation the first order derivative of the total cost to zero, the values are calculated.

$$\frac{\partial PWTC_r^D\left(t_i; n_1\right)}{\partial t_i} = 0 \tag{6.13}$$

The optimal solution so obtained by solving the above equation is n_1^{DO}, t_0, t_1^{DO}, t_2^{DO},...,$t_{n_1+1}^{DO} = H$,

With the existing replenishment schedule in the desynchronized model, the supplier determines his/her present estimate of the total cost by summing the setup cost,purchasing cost, holding cost, transportation cost, dissemmbly cost and the remaufacturing cost for the planning horizon H. The equation is as follows:

$$PETC_s^D =$$

$$n_1^{DO}.e^{(d-r)t_i}.Ss + P_s.e^{(d-r)t_i}.I_{oi} + H_s.\int_{t_i^{'}}^{t_i^{''}} e^{(d-r)t}.p.I_{oi}\,dt$$
$$+ (TC + DSM + REM).p.e^{(d-r)t}.I_{oi} \tag{6.14}$$

The optimized quantity to be ordered is:

$$Q^* = \sum_{i=1}^{n_1^{DO}} I_{oi}^{DO} \tag{6.15}$$

Scenario 2: Synchronized model with permissible delay

in the synchronized model supplier offers permissible delay to the retailer. On account of which the number of reordering cycle decreases,which further leads to a decrease in the setup cost of the supplier. With the new ordering plan, there is an increase in the cost for the retailer which the supplier compensates by distributing it through the parameter credit period rate.

For the new ordering plan n_2, the present estimate of the cost for the retailer is $PETC_r^S =$

$$\sum_{j=1}^{j=n_2} (PEOC + PEHC + PEDC + PEPC + PESC) \tag{6.16}$$

The addition in the retailer's cost is given by:

$$PETC_r^S - PETC_r^D \tag{6.17}$$

The supplier compensates the retailer by adding the increase to his/her total cost and so the present estimate of the cost is

$PETC_s^S =$

$$\begin{aligned} & n_2.e^{(d-r)t_j}.Ss + P_s.e^{(d-r)t_j}.I_{oj} + H_s.\int_{t_j'}^{t_j''} e^{(d-r)t}.p.I_{oj}dt \\ & + (TC + DSM + REM).p.e^{(d-r)t}.I_{oj} + PETC_r^S - PETC_r^D \end{aligned} \tag{6.18}$$

The optimized value of the cost for the supplier $PETC_s^S$ in the synchronized model is determined in the same way as done for the retailer $PETC_r^D$ in the desynchronized model.

$n_2^{SO}, t_1^{SO}, t_2^{SO}, \ldots, t_{n_2+1}^{SO} = H$ be the optimal solution for $PETC_s^S$.

The optimized ordering quantity in synchronized model is
$Q^S = \sum_{j=1}^{n_2^{SO}} I_{oj}^{SO}$

Equitable distribution of the extra cost incurred during synchronized model

For the retailer,the present estimate of the total cost in the synchronised model should be less than that in the desynchronised model, only then the new replenishment program is followed by the retailer.

$$PETC_r^D \geq PETC_r^S - \sum_{j=1}^{n_2^{SO}} e^{(d-r)*t_j} C_c.\rho \left(t_{j+1}^{SO} - t_j^{SO}\right) I_{oj}^{SO} \tag{6.19}$$

where ρ is the credit period rate which is the distributing factor for the extra cost at the retailer's end and the rate is same in all ordering intervals.
The credit period duration is given by:
$M_j^S = \rho \left(t_{j+1}^{SO} - t_j^{SO}\right)$

In the synchronized model,the supplier offers retailer credit for the minimum period.The credit period rate is kept minimum ρ_{min} to divide the benefits realized.The present estimate of his total cost in the synchronized model is then given by:

$$PETC_r^S - \sum_{j=1}^{n_2^{SO}} e^{(d-r)*t_j} C_c.\rho_{min} \left(t_{j+1}^{SO} - t_j^{SO}\right) I_{oj}^{SO} = PETC_r^D \tag{6.20}$$

which implies the value of ρ_{min} is:

$$\rho_{\min} = \frac{PETC_r^S - PETC_r^D}{\sum_{j=1}^{n_2^{SO}} .e^{(d-r)*t_j}.C_c \left(t_{j+1}^{SO} - t_j^{SO}\right) I_{oj}^{SO}} \tag{6.21}$$

As supplier bears the additional cost borne at the retailer's end, the credit period rate is maximum at his end ρ_{max}. His total cost in the synchronised model is given by the following equation:

$$n_2^{SO} S_s.e^{(d-r)*t_j} + \sum_{j=1}^{n_2^{SO}} P_s.e^{(d-r)*t_j}.I_{oj}^{SO} + \sum_{j=1}^{n_2^{SO}} .e^{(d-r)*t_j}.C_c.\rho_{\max} \left(t_{j+1}^{SO} - t_j^{SO}\right) I_{oj}^{SO} \tag{6.22}$$
$$= PETC_s^D$$

and ρ_{max} equals

$$\rho_{\max} = \frac{PETC_s^D - n_2^{SO}.e^{(d-r)*t_j}.S_s + \sum_{j=1}^{n_2^{SO}} P_s.e^{(d-r)*t_j}.I_{oj}^{SO}}{\sum_{j=1}^{n_2^{SO}} .e^{(d-r)*t_j}.C_c \left(t_{j+1}^{SO} - t_j^{SO}\right) I_{oj}^{SO}} \tag{6.23}$$

The additional expenditure due to the replenishment plan of the synchronized model is fractioned on the basis of $\bar{\rho}$, which is the average of the minimum and the maximum credit period rate. The present estimate of the net cost for both the retailer and the supplier is given by:

$$PETC_r^{SO\rho} = PETC_r^S \sum_{j=1}^{n_2^{SO}} .e^{(d-r)*t_j}.C_c\bar{\rho}\left(t_{j+1}^{SO} - t_j^{SO}\right) I_{oj}^{SO} \tag{6.24}$$

and $PETC_s^{SO\rho} =$

$$n_2^{SO}.e^{(d-r)*t_j}.S_s + \sum_{j=1}^{n_2^{SO}} P_s.e^{(d-r)*t_j}.I_{oj}^{SO} + \sum_{j=1}^{n_2^{SO}} .e^{(d-r)*t_j}.C_c.\bar{\rho}\left(t_{j+1}^{SO} - t_j^{SO}\right) I_{oj}^{SO} \tag{6.25}$$

6.4 Optimality Evaluation and Solution

The model is aimed at minimizing the total cost for the above cited two scenarios. The cycle time at which the cost is minimum is determined and the following theorem is verified with the help of the graphs.

Theorem 1: For any n_1,solution for the green supply chain EOQ model exists and is unique.

The convexity of the total cost can be seen through the following figures. Fig.2 plots the costs of retailer for the parameter SC = 4.5, and the value is minimum at the fourth cycle. The same is seen in Fig.3 for the supplier in the synchronized model where third cycle has a minimum value and the convexity of the cost is validated.
The table values for the following figures is seen in Table 1 and Table 2 of the numerical example.

6.5 Algorithm

1. All the parameters are assigned the hypothetical values.

2. In the desynchronized structure, the optimal reordering point for the retailer is determined.

   (a) Taking t_1 = 0, n_1 = 1 and t_3 = H. Initializing t_2, and calculating it from equation (13).

   (b) Taking n_1 = 2.

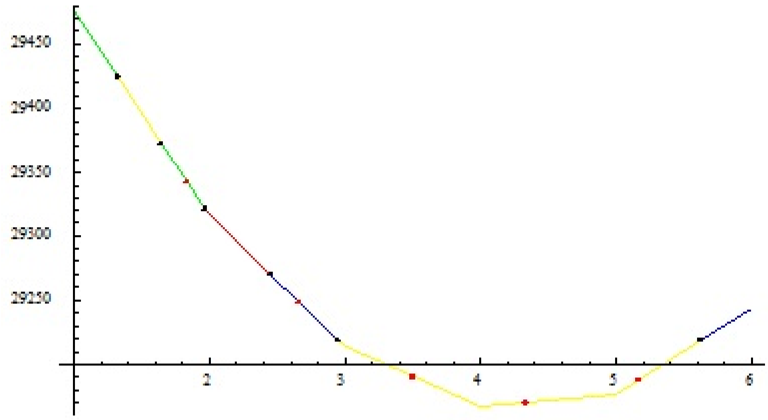

FIGURE 6.3: Convexity of total cost for retailer at SC=4.5 in desynchronized system

(c) From the values of t_1 and t_2 the value of t_3 is calculated from equation (13).

(d) The optimal values of $t_i's$ is determined for every n_1.

(e) For $n_1 = 1$ and if $PETC_r^D(n_1) < PETC_r^D(n_1+1)$,then $PETC_r^D(n_1) = PETC_r^{DO}(n_1)$.Stop.

3. For $n_1 \geq 2$ and if $PETC_r^D(n_1) < PETC_r^D(n_1-1)$ and $PETC_r^D(n_1) < PETC_r^D(n_1+1)$, then $PETC_r^D(n_1) = PETC_r^{DO}(n_1)$ and stop or else let $n_1 = n_1 + 1$, and goto step 2(c).

4. $n_1^{DO} = n_1$ is the optimised cycle for both the retailer and the supplier in the desynchronised model.

5. $PETC_r^{DO}$, $PETC_s^{DO}$ and Q^{DO} are determined from the respective equations.

6. In synchronized model,the optimal cycle time n_2^{SO}, the present estimate of the cost for the retailer and the supplier $PETC_r^{SO}$ and $PETC_s^{SO}$ are calculated as done in steps from 2 to 4.

7. The values of $\rho_{\min}$, $\rho_{\max}$,$PETC_r^{SO\rho}$ and $PETC_s^{SO\rho}$ are determined from the respective equations.

6.5.1 Numerical Example

Hypothetical statistics is taken to validate the potency of the green inventory model.One principal parameter Screening Cost,SC is undertaken to observe the effects of changes

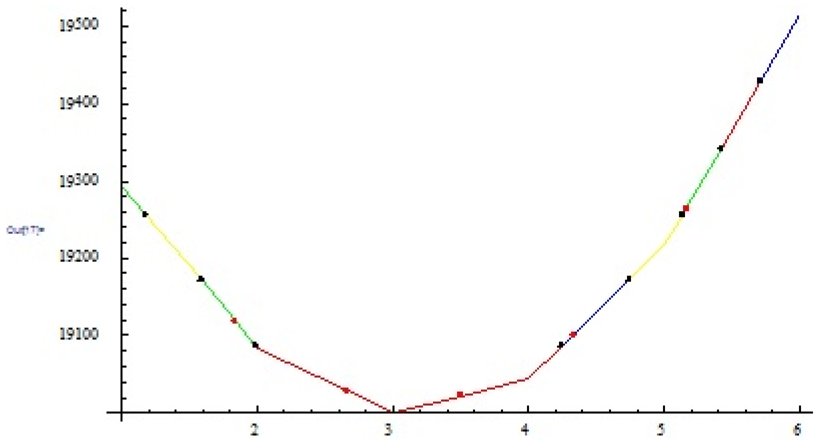

FIGURE 6.4: Convexity of total cost for supplier at SC=4.5 in synchronized system

in it on the optimal outcomes.
Example 1: $a_1 = 20$ items/year, $b_1 = 5$ items/year, $\theta_1 = 0.05$ items/year, $P_o = 4$ \$/item, $\gamma = 0.01, t_1 = 0, O_r = 25$ \$/purchase order, $S_s = 30$ \$/arrangement, $H = 4$, $P_s = 0.01$ \$/item, $C_c = 2.5$ \$/item/year , $H_o = 100$ \$/item/year, d = 0.1, r = 0.05, DC = 70, CT = 0.1, TC=0.01 \$/item, REM=0.01 \$/item, DSM=0.01 \$/item.

TABLE 6.1: Inflated cost of retailer in the desynchronized model under green's inventory system

$\downarrow SC$ $\rightarrow n_1$	1	2	3	4	5	6
3	29222.1	28946.4	28728.6	28568.6	28466.4	**28422.2**
3.75	29398.8	29131.3	28970.8	28867.3	**28821**	28831.7
4.5	29475.8	29316.6	29213.7	**29167.3**	29177.2	29243.6

The parameter of screening cost is further analyzed to study the impact of its changes on the green inventory model. Table 1 and Table 2 gives the total cost and the optimal ordering time period for both the retailer and the supplier in the desynchronized and the synchronized model respectively. Figure 2 and Figure 3 show the convexity of the cost.

Table 3 shows that the optimal number of replenishment schedules after the synchronization diminishes when the screening cost increases. This signifies that the supplier will be benefited is he/she reduces the number of ordering cycles. To add on, the

TABLE 6.2: Inflated cost of supplier in the synchronized model under green's inventory system

SC \ $\rightarrow n_2$	1	2	3	4	5	6
3	19707.5	19299.4	19020.9	18872.2	**18853.3**	18964.4
3.75	19474.8	19165.2	18984	**18931.1**	19006.8	19211
4.5	19294.9	19084.3	**19000.7**	19044.5	19215.5	19514

percentage in the cost savings of the supplier also increases when the parameter SC increases, which is agreeable as the supplier compensates for the increase in the cost of the retailer in the synchronized system.
Moreover same trend in the cost savings is seen for the retailer also. the percentage cost savings increases for the retailer when SC increases, which will assure the retailer's participation in the synchronized scheme.
Furthermore, the data also show that allowing for credit period, the optimal order quantity in the synchronized model is more than that as compared to the desynchronized model,thus raising the quantity ordered with reduced cycles.

TABLE 6.3: Percentage savings in cost for the retailer and the supplier in green's inventory system

	Desynchronized system				
SC	$PETC_r^{DO}$	$PETC_s^{DO}$	n_1^{DO}	Q^{DO}	$\bar{\rho}$
3	28422.2	1914.67	6	118.912	0.728528
3.75	28821	1518.23	5	118.078	0.479235
4.5	29167.3	1195.08	4	117.79	0.273768

	Synchronized system				% Cost saving	
SC	$PETC_r^{SO\rho}$	$PETC_s^{SO\rho}$	n_2^{SO}	Q^{SO}	$\frac{\Delta PETC}{PETC_r^{DO}}$	$\frac{\Delta PETC}{PETC_s^{DO}}$
3	27730.2	1222.63	5	119.485	2.43485	36.144
3.75	28131.3	828.518	4	119.485	2.3931	45.4288
4.5	28469.5	497.275	3	119.485	2.39243	58.3899

6.6 Theoretical aspects in green inventory model on trade credit

Hypothesis 1: Permissible delay is negatively associated with the inflation rate.

To deal with the changes in the financial costs that incur due to the changing rate of inflation, organisations frequently alter the policy of trade credit. There is a decrease in the credit period to cope with the inflation. This study has been made by several

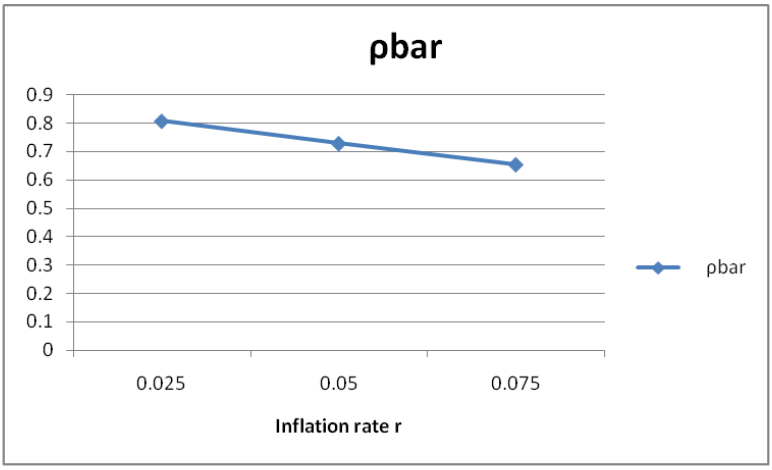

FIGURE 6.5

researchers.The following table shows that as the rate of inflation increases, there is a decrease in the credit period rate. The data also reveals that although inflation is anticipated yet there is a realization of cost savings for both the retailer and the supplier in the green inventory model.

TABLE 6.4: Effect of change in rate of inflation on credit period rate

r	$\bar{\rho}$	$\frac{\Delta PETC}{PETC_r^{DO}}$	$\frac{\Delta PETC}{PETC_s^{DO}}$
0.025	0.807762	2.30863	33.2966
0.05	0.728549	2.43486	36.1441
0.075	0.653612	2.55951	39.0887

Hypothesis 2: Permissible delay is negatively associated to the capital cost.

Supplier incurs extra cost due to permissible delay. However, both the retailer and the supplier at distinct rates can make financial investments. So with the increase in the capital cost, there is a reduction in the credit period rate. Furthermore,the percentage savings in the cost for the retailer and the supplier is not associated with the alterations in the capital cost.

Hypothesis 3: Credit period rate is negatively associated with the deterioration.

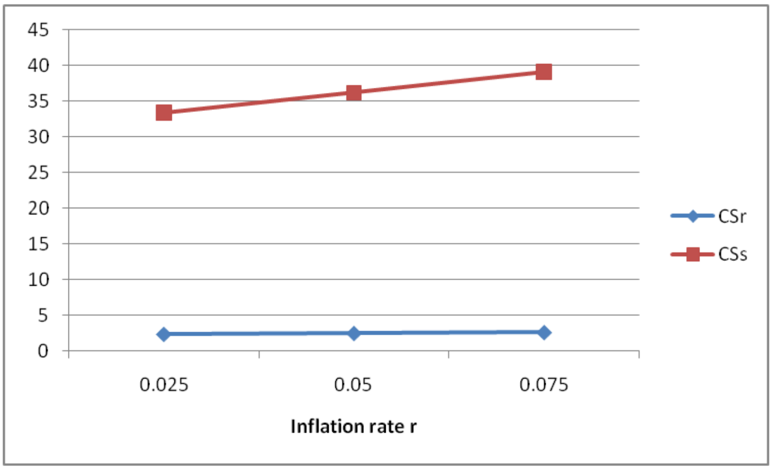

FIGURE 6.6

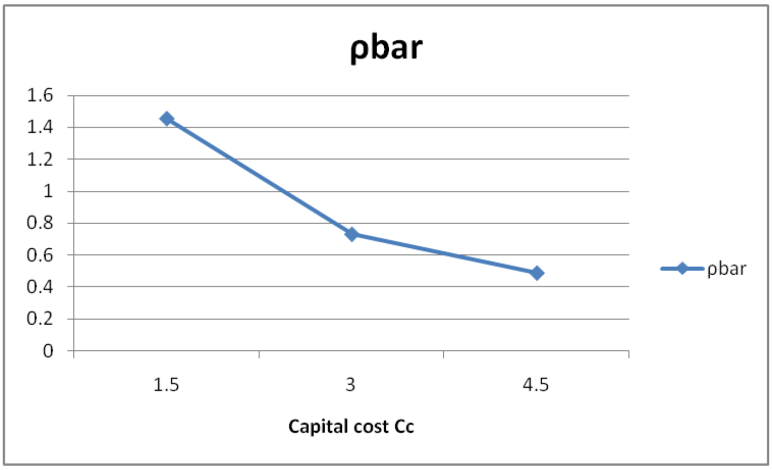

FIGURE 6.7

TABLE 6.5: Effect of change in rate of Capital cost on credit period rate

Cc	$\bar{\rho}$	$\frac{\Delta PETC}{PETC_r^{DO}}$	$\frac{\Delta PETC}{PETC_s^{DO}}$
1.5	1.45327	2.4252	36.1042
3	0.728548	2.43485	36.144
4.5	0.485699	2.43485	36.144

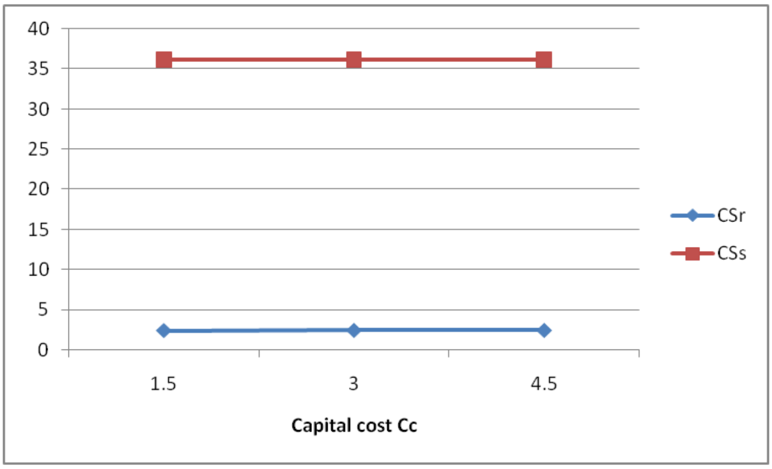

FIGURE 6.8

When the inventory of finished goods is perishable, there is a decrease in the revenue collected due to deterioration. For the deteriorated goods the time duration for permissible delay also decreases.The hypothesis is validated by the numerical example and the following table.

TABLE 6.6: Effect of change in rate of deterioration on credit period rate

θ_1	$\bar{\rho}$	$\frac{\Delta PETC}{PETC_r^{DO}}$	$\frac{\Delta PETC}{PETC_s^{DO}}$
0.0025	0.762206	2.26233	33.3653
0.005	0.728544	2.43483	36.144
0.0075	0.696866	2.70232	40.493

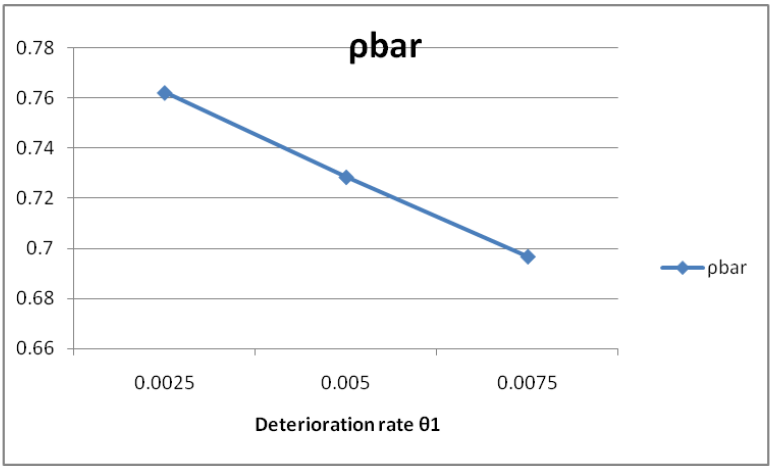

FIGURE 6.9

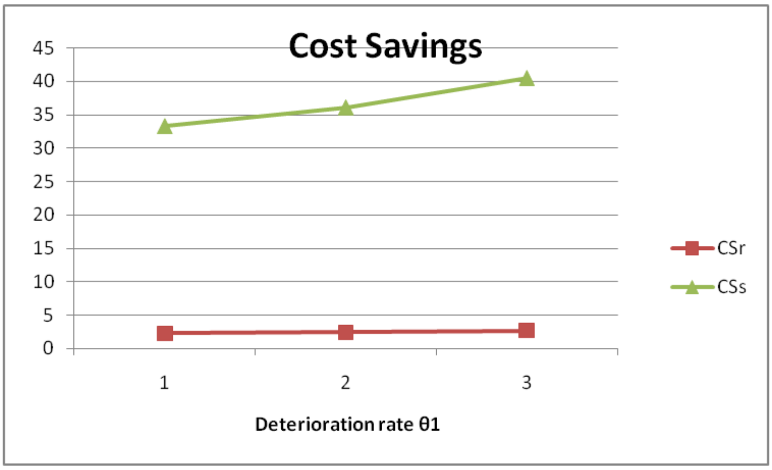

FIGURE 6.10

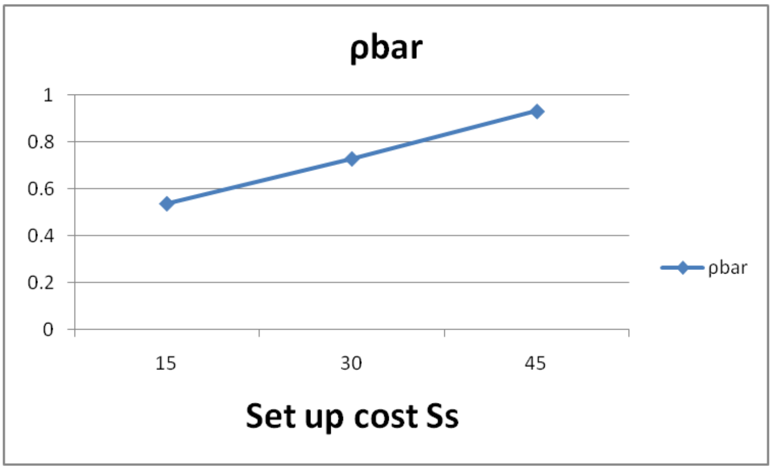

FIGURE 6.11

Hypothesis 4: Permissible delay is positively associated with the supplier's set up cost.

There is an increase in the credit period rate when the parameter Ss increases. This guarantees that the supplier prolongs the permissible delay period to attain the benefit from synchronization. There is percentage cost savings fruition for the supplier which enables him to extend credit period incentive to the retailer. The retailer too generates percentage savings in the cost,thereby motivational for him to accept the synchronization scheme.

TABLE 6.7: Effect of change in supplier's set up cost on credit period rate

Ss	$\bar{\rho}$	$\frac{\Delta PETC}{PETC_r^{DO}}$	$\frac{\Delta PETC}{PETC_s^{DO}}$
15	0.536511	1.38002	30.8952
30	0.728547	2.43485	36.144
45	0.931589	3.54378	39.4326

Hypothesis 5: The change in the optimal total cost of the retailer and the supplier is negatively associated with the parameter p, but permissible delay is positively associated with p.

With the high value of the inspection rate,the imperfect goods are quickly removed,thus reducing the cost of holding. The percentage savings is negatively associated as the

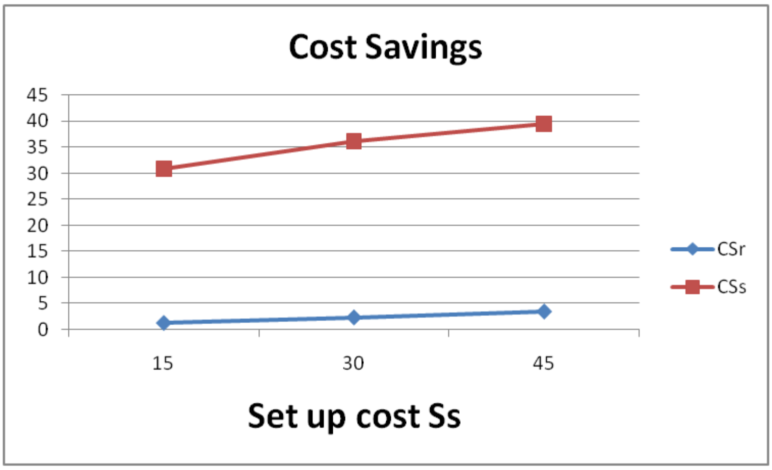

FIGURE 6.12

goods removed decreases the time period of the revenue generation. The credit period increases as it takes time for the imperfect goods to be remanufactured and get again absorbed as demand in the greens inventory model, thus validating the hypothesis.

TABLE 6.8: Effect of change in the parameter p on credit period rate

p	$\bar{\rho}$	$\frac{\Delta PETC}{PETC_r^{DO}}$	$\frac{\Delta PETC}{PETC_s^{DO}}$
0.375	0.694987	2.78661	41.9272
0.75	0.737651	2.64678	39.2901
1.125	0.778312	2.49702	36.6757

6.6.1 Sensitivity analysis

Sensitivity analysis is carried on the green inventory model analyzing whether the formulated model is influenced by the alterations in the input parameters.We analyze the consequence of the alterations in the factors against the changes in the total cost of the retailer, supplier and the credit period rate. Every parameter is altered by -50%, -25%, 25% and 50%, of the initial cost taken in Example 1.

The table[Table 9] below shows that the change in the cost of the retailer and the supplier increases as demand increases which is agreeable. Also there is a decrease in the percentage change in the credit period with the increase in the demand.

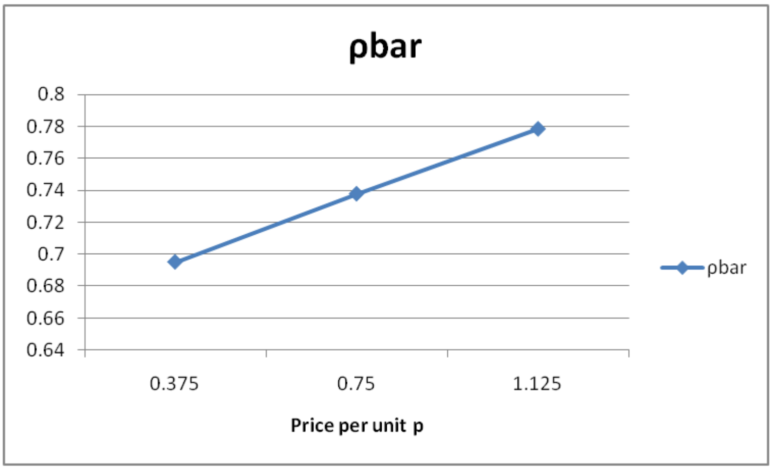

FIGURE 6.13

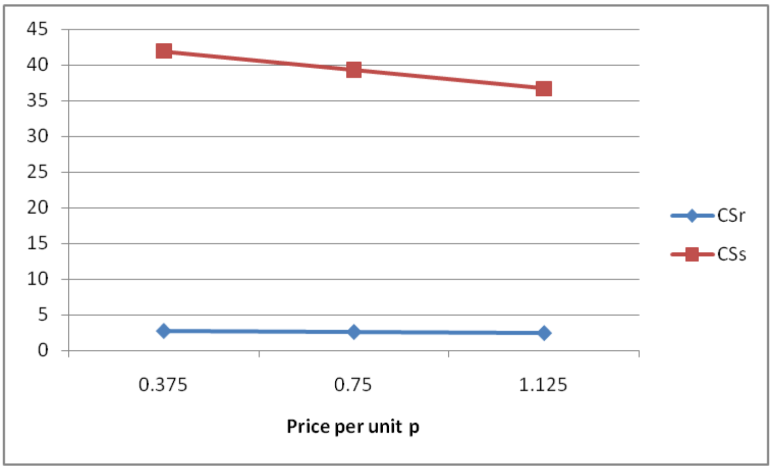

FIGURE 6.14

TABLE 6.9: Analysis on demand variable 'a'

value	$\dfrac{\Delta PETC_r^{SO\rho} * 100\%}{PETC_{rO}^{SO\rho}}$	$\dfrac{\Delta PETC_s^{SO\rho} * 100\%}{PETC_{sO}^{SO\rho}}$	$\dfrac{\Delta\overline{\rho} * 100\%}{\overline{\rho}_O}$
0.5	-30.0024	-29.83	2.8461
0.75	-15.36	-6.576	15.2355
1.25	15.3233	5.503	-8.733
1.5	30.6353	10.7837	-15.479

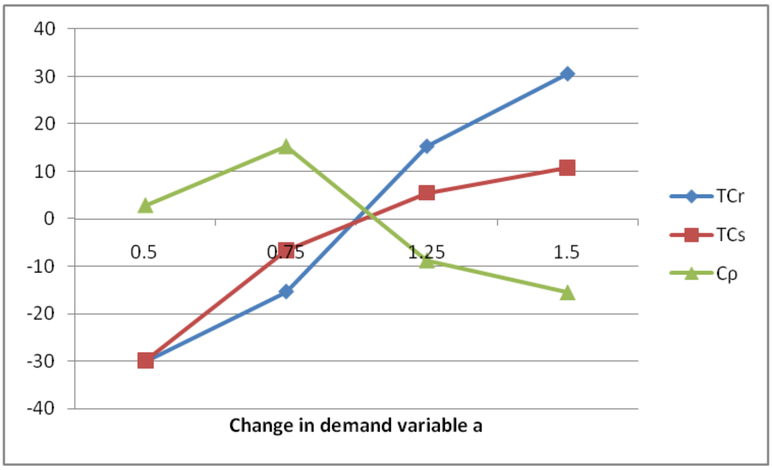

FIGURE 6.15

The analysis on the holding cost of the supplier indicates that with the increase in the cost value the change in the cost of the supplier increases and also the rate of credit period increases.This implies that the supplier's total cost is sensitive to the holding cost as shown in Table 10.

With the analysis done on the ordering cost of the retailer as in Table 11, it is evident that when the ordering size increases the total cost of the retailer increases. But there is a decrease in the cost of the supplier as the number of cycles decreases which is the synchronized model.

When the parameters P_s and REM are examined,the data is recorded in Table 12 and Table 13. It is seen that with the increase in the purchasing cost and the remanufacturing cost, percentage change in the the total cost of the supplier also increases which is apparent. The supplier's total cost is sensitive to these cost parameters. The credit

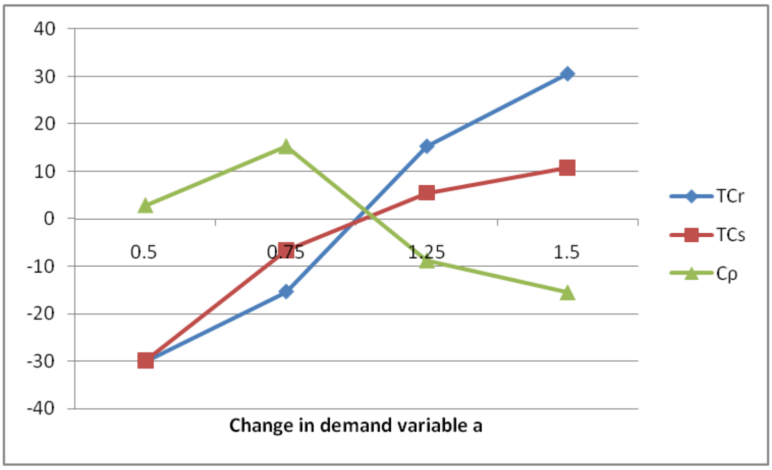

FIGURE 6.16

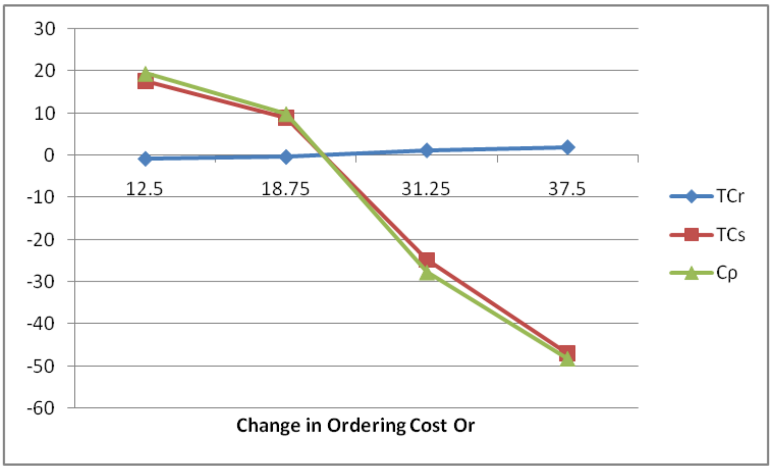

FIGURE 6.17

TABLE 6.10: Analysis on holding cost of supplier H_s

value	$\frac{\Delta PETC_r^{SO\rho} * 100\%}{PETC_{rO}^{SO\rho}}$	$\frac{\Delta PETC_s^{SO\rho} * 100\%}{PETC_{sO}^{SO\rho}}$	$\frac{\Delta\overline{\rho} * 100\%}{\overline{\rho}_O}$
35	0.5753	-13.003	-15.219
52.5	0.2850	-6.449	-7.548
87.5	-0.2848	6.452	7.551
105	-0.5697	12.905	15.105

TABLE 6.11: Analysis on the Ordering cost O_r

value	$\frac{\Delta PETC_r^{SO\rho} * 100\%}{PETC_{rO}^{SO\rho}}$	$\frac{\Delta PETC_s^{SO\rho} * 100\%}{PETC_{sO}^{SO\rho}}$	$\frac{\Delta\overline{\rho} * 100\%}{\overline{\rho}_O}$
12.5	-0.8777	17.69	19.444
18.75	-0.444	8.96	9.851
31.25	1.110	-24.81	-27.772
37.5	1.849	-47.07	-48.271

period rate also increases wth the increase in these parameters. Although the retailer's cost is less responsive to these cost parameters. With the increase in the credit period the retailer can invests the amount to generate gains.

TABLE 6.12: Analysis on the Purchasing cost of the supplier P_s

value	$\frac{\Delta PETC_r^{SO\rho} * 100\%}{PETC_{rO}^{SO\rho}}$	$\frac{\Delta PETC_s^{SO\rho} * 100\%}{PETC_{sO}^{SO\rho}}$	$\frac{\Delta\overline{\rho} * 100\%}{\overline{\rho}_O}$
0.005	0.0093	-0.3012	-0.2619
0.0075	-0.000293	-0.0328	-0.00151
0.0125	0.000299	0.0327	0.00136
0.015	0.000594	0.0655	0.00283

6.7 Conclusion

The research demonstrates a synchronized supply chain with permissible delay under inflation in a greens inventory system. The model shows that the organizations can

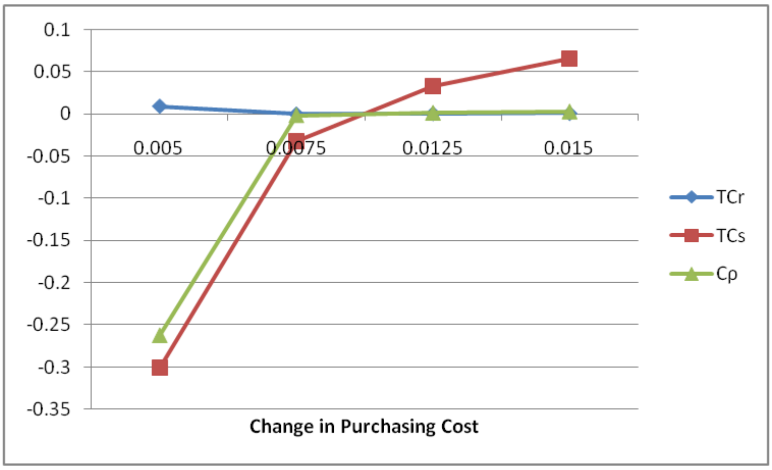

FIGURE 6.18

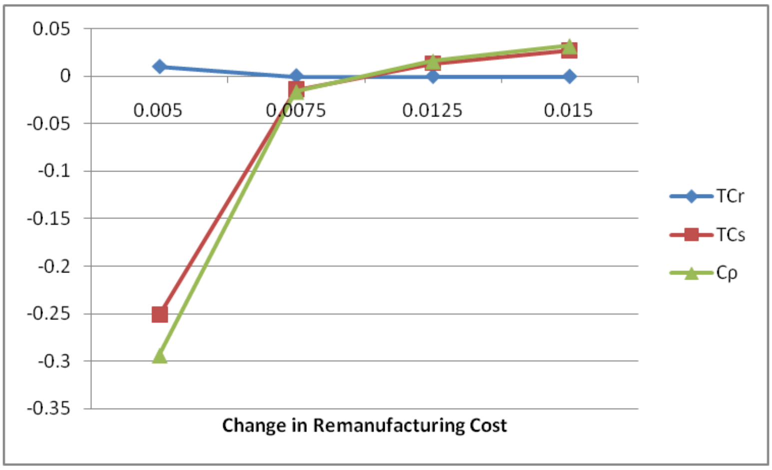

FIGURE 6.19

TABLE 6.13: Analysis on the Remanufacturing cost of the supplier REM

value	$\frac{\Delta PETC_r^{SO\rho} * 100\%}{PETC_{rO}^{SO\rho}}$	$\frac{\Delta PETC_s^{SO\rho} * 100\%}{PETC_{sO}^{SO\rho}}$	$\frac{\Delta\overline{\rho} * 100\%}{\overline{\rho}_O}$
0.005	0.0102	-0.2518	-0.2946
0.0075	0.0002	-0.0138	-0.0162
0.0125	-0.0001	0.0137	0.0160
0.015	-0.0003	0.0276	0.0322

adopt for the green technology favouring the environment and can still realize cost savings. The study shows that the cost under inflation in the synchronized model is not more than the cost in the desynchronized model. The primary characteristic of the synchronized model is the credit period rate. It divides the extra cost incurred to the retailer on account of accepting the new ordering plan by the supplier and the extra cost incurred to the supplier on account of green's technology of remanufacturing and recycling. The analysis of the model reveals that both the retailer and the supplier are able to generate financial benifits. Logical insights are proposed for the various parameters in this study.

Chapter 7

Vendor-Buyer Green's Inventory Model with Price Sensitive Demand under Inflation

Abstract

In the present scenario of globalization of products, it is nearly impossible for the organizations to rule the prices of their products. However, if the prices are lowered sales can be made to increase. Furthermore, the product's price and the ordering size dominates its demand. The present study incorporates demand which relies on the product's price. The green's inventory module in a supply chain is further considered for the study. Supply chain is increasingly gaining importance in the market as the vendor and buyer both realize cost benefits. The aim in developing this framework is to analyze the vendor-buyer association in consideration with the price dependent demand on the greens technology. The material and the related manufacturing/producing processes are aimed for the better usage of the raw materials. The product making activities requires innovation in the producing techniques and techniques to avoid wastage of the material. Green's technology is an emerging trend towards saving the environment. It includes reusing, remanufacturing and recycling of the waste and deteriorated products in the inventory system. Two different approaches for the delay in paying back time S are considered, (i) S $<$ cycle time and (ii) S $>$ cycle time. The study focuses on two major objectives: (a) reduce the wastage of the material in turn heading towards a greener manufacturing process and (b) aiming for a minimized cost value for the vendor and the buyer intending an optimal period for refilling the raw material. Numerical illustrations are done by Mathematica 8.0 to figure out the results. Sensitivity analysis is conducted on respective values with table data and graph.

7.1 INTRODUCTION

The choice of products and its reliability in the production process is examined for ensuring the reduction in the cost of manufacturing and also leads to less proportionate amount of inventory in remanufacturing. To achieve green effects of prolonged sustainability and renewable strengths at various scales, nanotechnology particularly nanophotonics is gaining paramount emphasis as discussed by Smith, 2011. It is discussed in the study that nanomaterials can be taken to a low cost production value with high benefits for protecting the environment. Trade credit is gaining popularity amongst the retailer and the supplier. It has become a prominent source for the short term financing. The benefits of permissible delay accrued to the supplier includes (a) the supplier becomes aware of the responsible or trustworthy retailers (b) the supplier can initiate for a control over the market by controlling the supplies and (c) the merchandiser can negotiate on the price. From profiting the potential buyers to the merchandiser's self operations credit time has an influential impact on increasing and capturing the market. An immense work is done with permissible delays in the theoretical context of the optimum value of the quantity ordered. The models are developed in the deterministic and stochastic frameworks. A general ordering model with predefined credit days by which the retailer can delay in the payment was established by Goyal, 1985a. A mathematical inventory ordering framework with time-variable increasing demand without deterioration was formulated by Teng, Min, and Pan, 2012. With permissible delay provided by the supplier a replenishment inventory model for environment conservation is developed by Singh et al., 2019. In this research they have considered partially backlog shortages and deterioration. A review on permissible delay and the detailed scope for the upcoming research works in this field is presented by Seifert, Seifert, and Protopappa-Sieke, 2013. Inflation is an important factor in today's economy. Inflation signifies the power of money. With the rising value of money, the buying price increases which results in reduction of the demand. Since inventory is amount to a significant figure in any firm inflation should be considered in determining every cost related to the inventory model. A constant inflation rate for every cost was taken by Buzacott, 1975 and MISRA, 1975. An optimal replenishment model evaluating inflated cost for perishable products was discussed by Thangam and Uthayakumar, 2010. For time varying deterioration and demand, an EOQ simulation model is framed by Singh et al., 2017b. For a supply chain inventory model, where a credit duration is offered to the buyer, Singh et al., 2018b frame the model to figure out the optimal refilling period under inflation with time-dependent deterioration. Primarily, inventory model formulation aims at minimising the overall cost of the system. The focus is on the optimum quantity to be ordered and the optimum time to place an order. In turn optimizing various cost associated with the EOQ model. The products ordered are subject to deterioration with respect to time. An inventory model with exponential deterioration was framed by Aggarwal and Jaggi, 1995b under credit time. This model was later extended by Jamal, Sarker, and Wang, 1997 with the inclusion of shortages. A linear trend for deterioration of the products was considered by Teng, Min, and Pan, 2012. They formulated the economic inventory ordering model with credit term under non-increasing demand. Dye, 2013 in his study showed the consequences of the technological yield for the deteriorating products. Sarkar, Ghosh, and Chaudhuri, 2012a, Wang and Liu, 2018 and Wu et al., 2014 also considered the expiry date of the deteriorating products along with the optimal policy. An important study for the deteriorating and spoiled products is made by Singh et al., 2019. They considered recycling, remanufacturing of the goods and guided the models towards green inventory. They also considered deterioration as Weibull. Until the early 1990's nearly the demand in inventory models is kept certain as it becomes easier to manage the inventories. However this assumption of constant demand was later superseded by time dependent demand and depending on price. While the mathematical formulation becomes simpler with constant demand, it is not practical in reality where the demand fluctuates owing to many constraints prevailing in the market. A linear variation of the demand implying increasing or decreasing was shown by Wu and Zhao, 2014, Ghosh and Chaudhuri, 2006, Dye and Hsieh, 2011. The product's demand rises if there is an abundant supply of the product in the market. But this in turn increases the issue of holding the inventory, as an extra cost is incurred to place the excess inventory. The problem becomes critical when the items are of deteriorating nature. This stock dependent nature of demand is also studied by several researches. Singh et al., 2017a have considered an inventory framework with inventory relying quadratic demand. They have processed a formulation for EOQ with credit delay and partially backlogged shortages. In actuality, demand of any product depends upon the stock and the price prevailing in the market. To have an understanding for this price dependent demand many researchers have made prominent studies. It is easily seen that when the price of the product is reduced, its demand is increased. However it is shown by Chern et al., 2008 that none of the firms can dictate over the prices of the product. In this study an optimal order quantity theory is formulated with demand being price sensible. The work considers two possibilities of permissible delay. First where the credit term offered by the vendor is within the cycle time and second when the buyer gets the credit time which is greater than the cycle. The model is developed under green inventory system. The government's policies towards clean and green environment has led many firms to move in this direction. The researchers develop the model wherein the firms adopting the green inventory measures are making profits too. Srivastava, 2007 in his study have shown how the firms can make remarkable profits adopting to the green supply chain. He considered in his study remanufacturing. The present model is analysed for the convexity of the total cost and the optimal refilling period for the two alternatives with price sensitive demand. Proceeding further with the studies senstivity of certain parameters is exercised and discussed to ascertain the logical insights.The table below presents a review of work done by various reseachers in the field of green technology.

Author	Green Concept	Price Sensitive	Inflation	SupplyChain	Deterioration	Re−manufacturing	Shortages	InspectionRate	%repairable
Moon, Giri andKo(2005)	−−−	−−−	Yes	−−−	Yes	−−−	−−−	−−−	−−−
Ghoshand Chaudhuri(2006)	−−−	−−−	−−−	−−−	Yes	−−−	Yes	−−−	−−−
Chern, Yang, Teng andPapachristos(2008)	−−−	−−−	Yes	−−−	Yes	−−−	Yes	−−−	−−−
Dyeand Ouyang(2011)	−−−	Yes	−−−	Yes	Yes	−−−	−−−	−−−	−−−
Sarkar, Ghosh andChaudhuri(2012)	−−−	−−−	−−−	−−−	Yes	−−−	Yes	−−−	−−−
Dye(2013)	Yes	−−−	−−−	−−−	Yes	−−−	−−−	−−−	−−−
Bhaula andKumar(2014)	−−−	−−−	Yes	−−−	Yes	−−−	Yes	−−−	−−−
Cobb(2016)	Yes	−−−	−−−	Yes	−−−	Yes	−−−	−−−	−−−
Singh, Mishra, Singh andSaxena(2017)	−−−	−−−	−−−	Yes	Yes	−−−	Yes	−−−	−−−
Singh, Saxena, Singh andMishra(2017)	−−−	−−−	−−−	−−−	−−−	−−−	−−−	−−−	−−−
Sarkar(2017)	Yes	−−−	−−−	Yes	Yes	Yes	Yes	−−−	−−−
Singh, Saxena, Gupta MishraandSingh(2018)	−−−	−−−	Yes	Yes	Yes	−−−	Yes	−−−	−−−
Singh, Mishra, Mishra SinghandSaxena(2019)	Yes	−−−	−−−	Yes	Yes	Yes	Yes	−−−	−−−
Thispaper	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

7.2 Theoretical assumptions and Parametric Quantities

7.2.1 Theoretical Assumptions

The following assumptions are followed:

1. Inflation rate g is constant on all inventory costs.
2. Permissible delay period S is offered to the buyer with further two cases as S $<$ cycle time and S $>$ cycle time.
3. Constant deterioration γ and shortages with no lead time is allowed.
4. The model is framed in a finite planning horizon using SFI (Shortages follows Inventory) with I_{oi} units as the demand in the first cycle.
5. The purchasing cost of the buyer and the selling price are different.
6. A portion of the deal is imperfect and is subjected to screening. After screening the imperfect goods are taken back by the vendor for recycling and remanufacturing. The perfect quality units are supplied within the cycle time.
7. The price-sensitive demand function M(p) is x +yp with x,y $>$0.
8. The buyer would pay for the goods received under the permissible delay period $t = S$ given by the vendor. The buyer also pays interest at the rate I_p for the stock held within the time interval [S,s_{i+1}]. Contrary, if S $>=$ s_{i+1} then the interest is not payable by the buyer.
9. The buyer also earns interest at the rate I_e and collects the revenue from the stock during the complete cycle time from $t = 0$ to $t = S$.

7.2.2 Parametric Quantities

1. a: discount rate.
2. g: rate of inflation
3. c_o: Ordering cost
4. c_h: Holding Cost
5. c_c: Purchasing Cost
6. c_d: Deterioration Cost

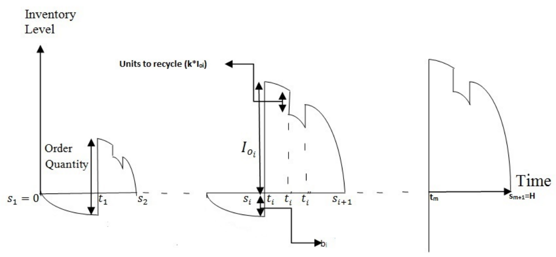

FIGURE 7.1: Graphical creation of Green Inventory Model

7. c_s: Shortage Cost

8. c_r: Screening Cost

7.3 Mathematical Process for Green Inventory Model Formulation

Model Specification

The model is developed for the removal of the imperfect quality items which are present in the lot at a uniform rate k. These imperfect goods pollute the environment and so the lot undergoes full screening and the defectives are picked by the vendor. The screening process continues till $t_i^{'}$. The vendor recycles, repair and remanufactures the defective goods and transports it back to the buyer at time $t_i^{''}$. At s_{i+1} the whole inventory is consumed and at this point of time the demand accumulates for the shortages. The level of change in inventory for various intervals as shown in figure 1 is given by the following differential equations:

$$\frac{d\left(I(t)\right)}{dt} + \gamma I(t) = \text{-M(p)},\ t_i \leq t \leq t_i^{'}, (i = 1, 2, \ldots, m) \tag{7.1}$$

the boundary condition is $I\left(t_i\right) = I_{oi}$ and $I\left(t_i^{'}\right) = I_{si}$

$$\frac{d\left(y(t)\right)}{dt} + \gamma y(t) = \text{-M(p)},\ t_i^{'} \leq t \leq t_i^{''}, (i = 1, 2, \ldots, m) \tag{7.2}$$

the boundary condition is $y\left(t_i^{'}\right) = I_{si} - k.I_{oi}$ and $y\left(t_i^{''}\right) = I_{fi}$

$$\frac{d\left(z(t)\right)}{dt}+\gamma z(t) = \text{-M(p)},\ t_i^{''} \leq t \leq s_{i+1},(i=1,2,\ldots,m) \tag{7.3}$$

the boundary condition is $z\left(t_i^{''}\right)=I_{fi}+k.I_{oi}$ and $z\left(s_{i+1}\right)=0$

The amount of shortage at any point of time is given by :

$$\frac{d\left(b(t)\right)}{dt} = \text{-M(p)},\ s_i \leq t \leq t_i,(i=1,2,\ldots,m) \tag{7.4}$$

the boundary condition is $b\left(s_i\right)=0$ and $b\left(t_i\right)=b_i$

Solving the equations with the boundary conditions, we obtain

$$I_{\text{oi}}=\left(s_{i+1}-t_i\right)*\left(x+y*p\right) \tag{7.5}$$

$$I_{si}=\left(s_{i+1}-t_i-\text{CT}\right)*\left(y*p+x\right) \tag{7.6}$$

$$I_{fi}=\left(y*p+x\right)*\left(s_{i+1}-t_i\right)\left(1+k\right)-\left(y*p+x\right)*\text{CT} \tag{7.7}$$

using the above calculated values, the result for the various inventory level differential equations are

$$I(t)=\left(\gamma\left(-t+t_i\right)+1\right)*\left(s_{i+1}-t_i\right)*\left(y*p+x\right) \\ +(-t+t_i)*\left(y*p+x\right),t_i\leq t\leq t_i{}',\quad \{i=1,2,\ldots,m\}. \tag{7.8}$$

Equation(2) is solved as:

$$y(t)=\left(y*p+x\right)*\left(s_{i+1}-t_i\right)*\left(\left(1-k\right)+\left(t_i-t\right)*\gamma-k*\left(t_i+\text{CT}-t\right)*\gamma\right) \\ +\left(y*p+x\right)*\left(-t+t_i\right),t_i{}'\leq t\leq t_i{}'',\quad \{i=1,2,\ldots,m\} \tag{7.9}$$

Equation(3) is solved as:

$$z(t)=\left(\frac{(y*p+x)}{\gamma}\right)*(s_{i+1}-t)*(1-\gamma*t),t_i{}''\leq t \\ \leq s_{i+1},\{i \\ =1,2,\ldots,m\}. \tag{7.10}$$

The amount of shortages is given by:

$$b(t) = (s_i - t) * (y * p + x), s_i \leq t \leq t_i, \quad \{i{=}\,1, 2, \ldots, m\}. \tag{7.11}$$

The optimal ordering size is given by Qr = $I_{oi} + b_i$

The total cost for the retailer includes:

(i) Net Estimate of Ordering Cost NO$_c = e^{(a-g)*t_i} * m * c_o$

(ii) Net Estimate of Holding Cost NH$_c = c_h \left[\int_{t_i}^{(t_i+\text{CT})} e^{(a-g)*t} * I(t)dt + \right.$ $\int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} e^{(a-g)*t} * y(t)dt + \int_{(t_i+2*\text{CT})}^{s_i} e^{(a-g)*t} * z(t)dt]$

(iii) Net Estimate of Deterioration Cost ND$_c = c_d \left[\int_{t_i}^{(t_i+\text{CT})} e^{(a-g)*t} * \gamma * I(t)dt + \right.$ $\int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} e^{(a-g)*t} * \gamma * y(t)dt + \int_{(t_i+2*\text{CT})}^{s_i} e^{(a-g)*t} * \gamma * z(t)dt]$

(iv) Net Estimate of Screening Cost NS$_c = c_r * e^{(a-g)*t_i} * I_{\text{oi}}$

(v) Net Estimate of Purchasing Cost NP$_c = c_c * e^{(a-g)*t_i} * [I_{\text{oi}} + b_i]$

(vi) Net Estimate of Shortage Cost NS$_h = c_s \int_{s_i}^{t_i} e^{(a-g)*t} * (y * p + x) * (s_i - t)\, dt$

(vii) Interest Payable : The buyer pays for the interest on the amount of stock held with him depending upon the delay period S offered to him by the vendor.

The following two conditions are further considered for paying the interest amount:

Condition(i): (S$<= s_{i+1}$)

For the stock remaining with the buyer after the delay period S,the interest paid by the buyer at the rate I_p on the amount of stock is given as:
IP$_1 = I_p * c_c * \int_S^{s_{i+1}} (y * p + x) * (s_{i+1} - t)\, dt$
IP$_1 = (I_p * c_c * (y * p + x) * (s_{i+1} - S)^2)/2$

Condition(ii): (S$>= s_{i+1}$)

As the delay period is more than the cycle time no interest is payable for the amount of stock in the inventory.

$\text{IP}_2 = 0.$

(viii) Interest Earned : The buyer also earns from the amount of stock held depending upon the delay time S. The interest earned is at the rate I_e for the above mentioned two conditions under consideration.

Condition(i): (S$<= s_{i+1}$)

The buyer not only collects the amount by selling the product, but also earns the interest on the amount at the rate I_e till the cycle period. The interest earned from time $t = 0$ to $t = s_{i+1}$ is given by:

$$\text{IE}_1 = I_e * c_c * \int_0^{s_{i+1}} (y * p + x) * (s_{i+1} - t)\, dt$$

$$\text{IE}_1 = \left(I_e * c_c * (y * p + x) * \left(s_{i+1}^2\right)\right) / \, 2$$

Condition(ii): (S$>= s_{i+1}$)

When the delay period S is more than the cycle duration, the buyer earns interest on the amount of stock from time $t = 0$ to $t = S$ given by:

$$\text{IE}_2 = I_e * c_c * \int_0^{s_{i+1}} (y * p + x) * (S - t) dt$$

$$\text{IE}_2 = I_e * c_c * (y * p + x) * s_{i+1} * (S - (s_{i+1}) / \, 2)$$

Net Value of the total cost with the delayed payement option available is:

$$NTC(t_i, s_i, p) = NO_c + NH_c + ND_c + NS_c + NP_c + IP - IE$$

$NTC(t_i, s_i, p) = NTC_1(t_i, s_i, p), if S <= s_{i+1}$ and

$$NTC(t_i, s_i, p) = NTC_2(t_i, s_i, p), if S >= s_{i+1}$$

$$\text{NTC}_1 = e^{(a-g)*t_i} * m * c_o + (c_h + \gamma * c_d) * \left(\int_{t_i}^{(t_i+\text{CT})} I(t) * e^{(a-g)*t} dt + \right.$$

$$\int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} y(t) * e^{(a-g)*t} dt + \int_{(t_i+2*\text{CT})}^{s_{i+1}} e^{(a-g)*t} * z(t) dt +$$

$c_c * e^{(a-g)*t_i} * (y*p+x) * (s_{i+1} - s_i) + c_r * e^{(a-g)*t_i} * (y*p+x) * (s_{i+1} - t_i) +$

$c_s \int_{s_i}^{t_i} e^{(a-g)*t} * (s_i - t) * (y*p+x)dt + (I_p * c_c * (y*p+x) * (s_{i+1} - S)^2)/2$

$- \left(I_e * c_c * (y*p+x) * \left(s_{i+1}^2\right)\right)/2$

and

$\mathrm{NTC}_2 = e^{(a-g)*t_i} * m * c_o + (c_h + \gamma * c_d) * \left(\int_{t_i}^{(t_i+\mathrm{CT})} I(t) e^{(a-g)*t} dt + \right.$

$\int_{(t_i+\mathrm{CT})}^{(t_i+2*\mathrm{CT})} y(t) * e^{(a-g)*t} dt + \int_{(t_i+2*\mathrm{CT})}^{s_{i+1}} e^{(a-g)*t} * z(t)dt) + c_c * e^{(a-g)*t_i} *$

$(y*p+x) * (s_{i+1} - s_i) + c_r * e^{(a-g)*t_i} * (y*p+x) * (s_{i+1} - t_i) + c_s \int_{s_i}^{t_i} e^{(a-g)*t} *$

$(s_i - t) * (y*p+x)dt - I_e * c_c * s_{i+1} * (y*p+x) * (S - (s_{i+1})/2)$

The optimality of the net total cost is tested for the model in both the conditions of permissible time lag in the payment.

7.4 Optimality Check for Total Cost

In this segment net total cost for both the conditions is optimized by calculating the optimal time intervals. The requisite requirement for the optimality is :

$\partial \mathrm{NTC}_1 / \partial t_i = 0$ and $\partial \mathrm{NTC}_1 / \partial s_i = 0$
where,

$$\begin{aligned}
\partial \mathrm{NTC}_1/\partial t_i &= (a-g)*e^{(a-g)*t_i}*m*c_o+(c_h+\gamma*c_d) \\
&*\left(\int_{t_i}^{t_i^{'}} e^{(a-g)*t}*(s_{i+1}-t)*(y*p+x)*\gamma dt+\int_{t_i^{'}}^{t_i^{"}}(y*p+x)\right. \\
&*e^{(a-g)*t}*((1-k)*(\gamma*(s_{i+1}-t)-2*\gamma*t_i)+k(1+\gamma*\mathrm{CT}))\,dt \\
&+e^{(a-g)*t_i^{"}}*(s_{i+1}-t_i) \\
&*((y*p+x)*((1-k)-2*\gamma*\mathrm{CT}+k*\gamma*\mathrm{CT})-2*(y*p+x)*\mathrm{CT}) \\
&+k*e^{(a-g)*t_i^{'}}*(s_{i+1}-t_i)*(y*p+x)-e^{(a-g)*t_i^{"}} \\
&\left.*((y*p+x)/\gamma)*\left(s_{i+1}-t_i^{"}\right)*\left(1-\gamma*t_i^{"}\right)\right)+c_c*(a-g) \\
&*e^{(a-g)*t_i}*(y*p+x)*(s_{i+1}-s_i)+c_r*(a-g) \\
&*e^{(a-g)*t_i}*(s_{i+1}-t_i)*(y*p+x)-c_r*e^{(a-g)*t_i} \\
&*(y*p+x)+c_s*e^{(a-g)*t_i}*(s_i-t_i)*(y*p+x)
\end{aligned} \tag{7.12}$$

$$\begin{aligned}
&\partial \mathrm{NTC}_1/\partial s_i=(c_h+\gamma*c_d)*\left(\int_{t_{i-1}}^{t_{i-1}^{'}} e^{(a-g)*t}*(y*p+x)*(1+\gamma*(t_{i-1}-t))\,dt\right. \\
&+\int_{t_{i-1}^{'}}^{t_{i-1}^{"}} e^{(a-g)*t}*(y*p+x)*\left(\gamma*(t_{i-1}-t)+(1-k)-k*\gamma*\left(t_{i-1}^{'}-t\right)\right)dt+\int_{t_{i-1}^{"}}^{s_i} e^{(a-g)*t} \\
&*((y*p+x)/\gamma)*(1-\gamma*t)dt+k*e^{(a-g)*t_{(i-1)}^{'}}*\left(s_i-t_{(i-1)}\right)*(y*p+x)+e^{(a-g)*t_{(i-1)}^{"}} \\
&*\left(\left(\left(s_i-t_{(i-1)}\right)*(y*p+x)*((1-k)-2*\gamma*\mathrm{CT}+k*\gamma*\mathrm{CT})-2*(y*p+x)*\mathrm{CT}\right)\right. \\
&\left.\left.-\left(s_i-t_{(i-1)}^{"}\right)*((y*p+x)/\gamma)*\left(1-\gamma*t_{(i-1)}^{"}\right)\right)\right)+c_r*(y*p+x)*e^{(a-g)*t_i} \\
&+c_s*\left(\int_{s_i}^{t_i}(y*p+x)*e^{(a-g)*t}dt+(y*p+x)*e^{(a-g)*t_i}*(s_i-t_i)\right) \\
&+(I_p/2)*(y*p+x)*c_c-I_e*(y*p+x)*c_c*s_{i+1}
\end{aligned} \tag{7.13}$$

Equation(13)and (14) are solved for the optimal values of $t_i^{'}s$ and $s_i^{'}s$ for different cycle m.

Proposition 1: When t_i and s_i are taken as constants,then the net total cost NTC_1 is convex with respect to p.
The first derivative with respect to p is equated to zero.

$\partial \mathrm{NTC}_1/\,\partial p=0$

The optimal values of p are determined from the above equation for which the second order derivative of the net total cost with respect to p is given by:

$$\begin{aligned}\partial^2\text{NTC}_1/\partial p^2 &= c_s * e^{(a-g)*t_i} * y * (s_i - t_i) + (c_h + \gamma * c_d) \\ &* \Big(e^{(a-g)*t_i^{"}} * y * (s_{i+1} - t_i) * ((1-k) - 2*\gamma*\text{CT} + k*\gamma*\text{CT}) + k \\ &* e^{(a-g)*t_i^{'}} * y * (s_{i+1} - t_i) - e^{(a-g)*t_i^{"}} * (y/\gamma) * (s_{i+1} - t_i^{"}) * (1 - \gamma * t_i^{"})\Big) > 0\end{aligned} \tag{7.14}$$

The convexity of the total cost with respect to p is shown by the following numerical illustration followed by the graph.

Proposition 2: When t_i and s_i are taken as constants,then the net total cost NTC_2 is convex with respect to p.
The first derivative with respect to p is equated to zero.

$\partial\text{NTC}_2/\,\partial p = 0$

The optimal values of p for the net cost are determined and the second order derivative of the net total cost with respect to p is given by:

$$\begin{aligned}\partial^2\text{NTC}_2/\partial p^2 &= c_s * e^{(a-g)*t_i} * y * (s_i - t_i) + (c_h + \gamma * c_d) \\ &* \Big(e^{(a-g)*t_i^{"}} * y * (s_{i+1} - t_i) * ((1-k) + (k-2)*\gamma*\text{CT}) + k * e^{(a-g)*t_i^{'}} \\ &* y * (s_{i+1} - t_i) - e^{(a-g)*t_i^{"}} * (y/\gamma) * (-t_i^{"} + s_{i+1}) * (-\gamma * t_i^{"} + 1)\Big) > 0\end{aligned} \tag{7.15}$$

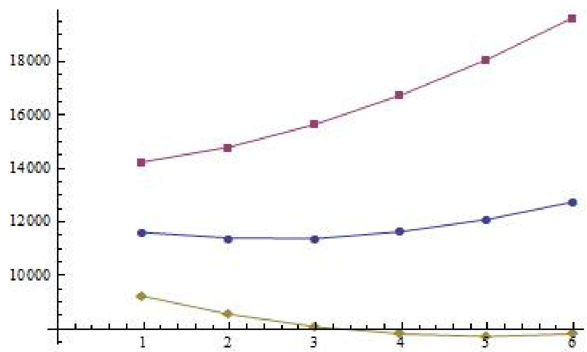

FIGURE 7.2: Net Total Cost for 'p=25,30,20'

7.5 Algorithm

1. Inputting the values to all the parameters.

2. For net total cost in condition (1) of permissible delay NTC_1:

   (a) $t_i^{'}s$ and $s_i^{'}s$ are determined by assigning a value to t_1 and keeping $s_1 = 0$. Calculating s_2 from equation(14).

   (b) Calculating t_2 from equation (13) in the next step as t_1, s_1 and s_2 are now known.

   (c) Repeating the above steps until all the $t_i^{'}s$ and $s_i^{'}s$ are known.

   (d) Calculate NTC_1. If $NTC_1(m) <= NTC_1(m+1)$, then optimal total cost $NTC_1^* = NTC_1(m)$ and optimal time period m* = m.

3. Optimal $t_i^{'}s$, $s_i^{'}s$ and net total cost NTC_1^* are known.

4. Repeat the same solution procedure for the second condition of permissible delay NTC_2. The optimal values of the respective $t_i^{'}s$,$s_i^{'}s$ are determined and the total cost is convex for the above calculated optimal values.

7.6 Numerical illustration for the two conditions of trade credit to minimize net total cost

The numericals solved validate the green model with price varied demand under inflation. The conjectural values to all the parameters are taken. To have the simplicity in the mathematical solvings $-t_i^{'} + t_i^{''} = t_i^{'} - t_i = CT$.

Example:1 Taking the parameter values as:

a=0.12,g=0.05,y=100,p=20,k=0.5,S=0.6

$\gamma = 0.00001,\text{CT} = 0.016, c_o = \$10/unit, c_h = \$35/unit, c_d = \$10/unit,$

$c_c = \$27/unit, c_r = \$1/unit, c_s = \$14/unit, I_e = \$0.05/unit, I_p = \$0.03/unit.$

The numerical is solved with Mathematica 8.0. The following table gives the net total amount for the buyer when the delay period is less than the cycle time. The data shows the convexity of the net total cost.The ordering schedule 5th cycle,4th cycle and

5th cycle are optimal with the optimal value of the cost being \$8370.58,\$9007.58 and \$7708.49 respectively. The optimal ordering quantity for the corresponding optimal replenishment schedules are \$9750,\$10100 and \$9400 respectively. The graph following the table depicts the convex nature of the cost.

TABLE 7.1: Buyer's Inflated total cost for varied values for parameter 'x'

$\downarrow x$ $\rightarrow m$	1	2	3	4	5	6
437.3	9605.73	8986.51	8588.51	8585.17	**8370.58**	8538.81
525	10014.8	9460.16	9133.88	**9007.58**	9075.13	9330.38
350	9203.27	8528.5	8067.81	7796.37	**7708.49**	7798.48

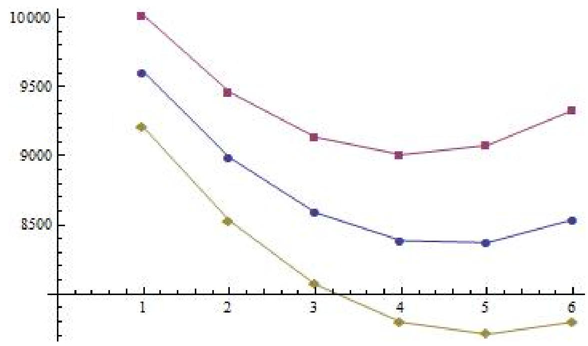

FIGURE 7.3: Net Total Cost for 'x=437.5,525,350'

TABLE 7.2: Buyer's Inflated total cost for varied values for parameter 'p'

$\downarrow p$ $\rightarrow m$	1	2	3	4	5	6
25	11596.	**11365.8**	11390.3	11633.7	12089.2	12749.7
30	**14241.2**	14800.8	15654.5	16753.7	18090.1	19655.7
20	9203.27	8528.5	8067.81	7796.37	**7708.49**	7798.48

The convex nature of the net total cost can be seen from the above graph for a condition of delayed payment. In the similar way the convexity can be proved for the second condition as well.

7.6.1 Sensitivity analysis

The cost parameters as taken in the numerical are changed to certain dimensions and their effect is studied on the net total cost of the green inventory model under both the conditions of delayed payment.Following table shows the change in the parameters and the optimal value of the net total cost.

The optimized net total cost is obtained for a value of the parameter and follows a convex pattern. This is shown by the graph following the table (Figure 4) for one of the conditions of delayed payment. The table also shows the net total cost for the second alternative of trade credit interval which is again convex in nature.

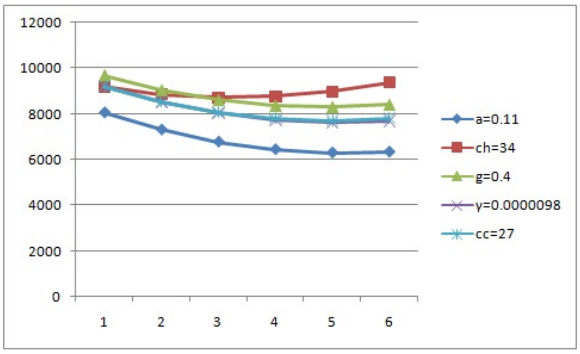

FIGURE 7.4: Sensitivity Analysis for various parameters

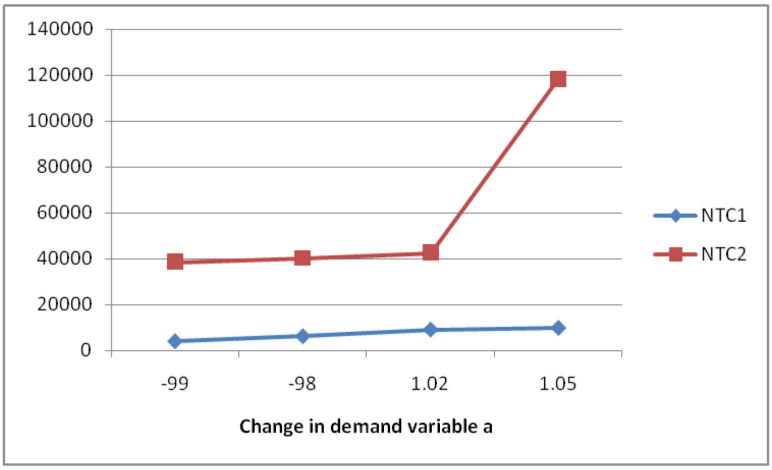

FIGURE 7.5

	% Change in parameter	$Values$	$NetTotalCost(NTC_1)$	$NetTotalCost(NTC_2)$
	−98	0.1176	6299.84	40183.9
	−99	0.1139	4174.29	38468.3
a	1.02	0.1224	9132.72	42526
	1.05	0.1260	9877.45	118381
	−98	0.049	8300.01	41836.4
	−99	0.0495	9192.404	42575.5
g	1.02	0.051	7119.67	40860.5
	1.05	0.0525	18420.15	21043.7
	−98	34.3	8706.19	11383.3
c_h	−99	34.65	9164.94	11360.3
	1.02	35.7	6276.42	11414
	1.05	36.75	4095.44	11437
	−98	26.46	2274.19	2120.58
c_c	−99	25.65	6380.81	9386.95
	1.02	27.54	12744.2	7567.91
	1.05	28.35	20162.8	14834.3
	−98	13.72	12710	20285.3
c_s	−99	13.3	20057.8	29656
	1.02	14.28	2377.63	7791.02
	1.05	14.7	6414.59	1579.7
	−98	0.01568	7783.95	16347.1
CT	−99	0.0152	7867.76	16156.9
	1.02	0.01632	7636.55	16602.8
	1.05	0.0168	7508.42	16796.1
	−98	0.049	8206.49	3335.06
k	−99	0.0475	8691.27	4085.81
	1.02	0.51	7491.59	2488.79
	1.05	0.525	6948.99	1966.49

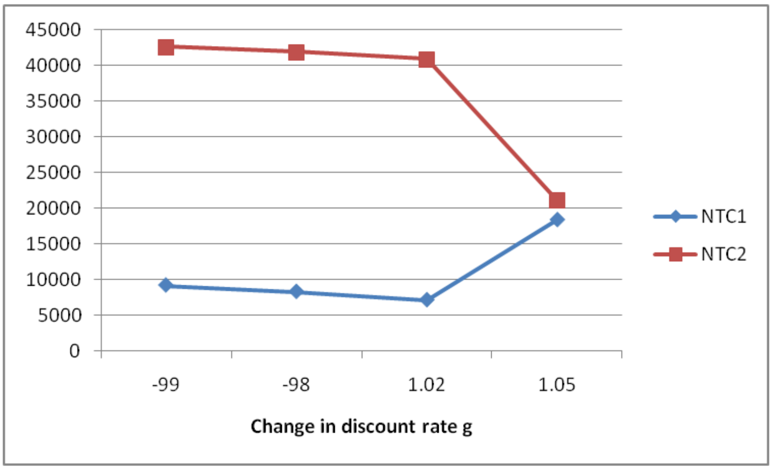

FIGURE 7.6

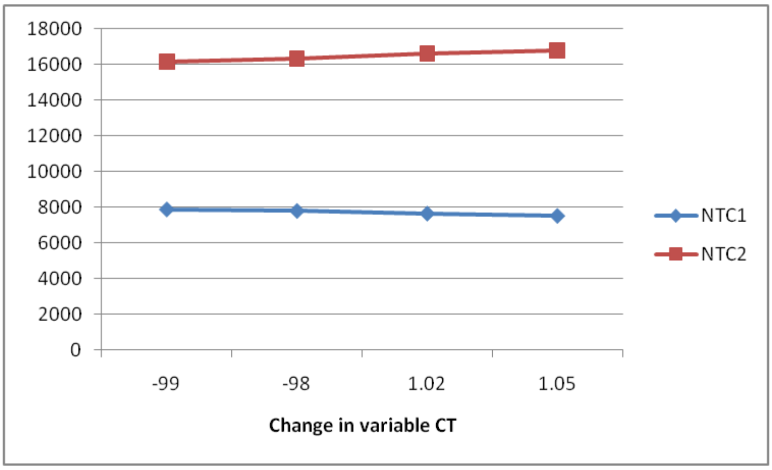

FIGURE 7.7

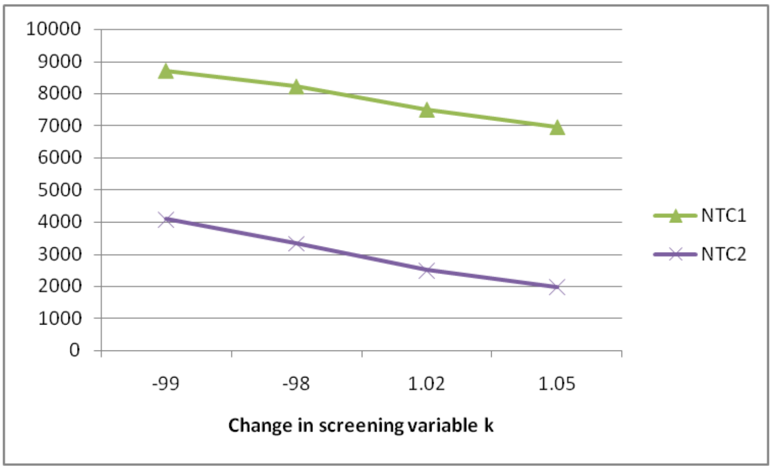

FIGURE 7.8

7.7 Conclusion

An inventory model where the vendor offers delayed payment option to the buyer is framed under two realistic situations. The model incorporates inflation on all inventory costs. Moreover, a green's inventory model considering price dependent demand is studied with the above backdrop. This is a common practice that demand depends upon the price of a product. The model proceeds with the remanufacturing and recycling of the defective goods. An optimal present worth of the net total cost under the two scenarios of the supply chain contract is derived with the help of the numerical. The convexity of the Net total cost is verified graphically also for the two delayed payment options. Also optimal replenishment period is determined for the green's inventory study with inflation. The present study can be extended to fuzzy logic. Fuzzy logics for various parameters are done as their values tend to fluctuate with the market. Singh et al., 2018a used fuzzy logic to analyse the measurements. The paper can also be made to incorporate partially backlogged shortages.

Chapter 8

Affirmable Inventory Administration for Environment Preserving under Fuzzy Logic and Inflation

Abstract

Fuzzy concept focuses on the uncertainty and inexactness of various parameters which have an impact on the decision making. Fuzzy logics are implemented quantitatively on the mathematical models of the realistic problems. The study exercised in this paper investigates the fuzzy theory on the green inventory model. When the products are received by the retailer it is obvious to have some fraction as imperfect goods. Apart from treating these as waste and becoming hazardous to the environment it is preferred to repair and recycle these products to help in saving the environment. The model framed aims at determining the optimal cost when the imperfect goods are remanufactured under inflationary conditions. The model developed is validated through a numerical and managerial perceptions are discussed through sensitivity analysis.

8.1 INTRODUCTION

Zadeh, 1965 demonstrated the concept of Fuzzy logics on the uncertainties arising in different situations. The inexactness prevailing in the real problems can be formulated using fuzzy theory near to precision in a mathematical model. Zimmermann, 2011 in his paper talked about the for fuzzy concepts and various areas where these are applied. Fuzzy Logic brings the unpredictable behavior of the parameters close to a realistic model. The costs are subject to change with respect to time and fuzzifying them can lead to a mathematical model with a more pragmatic solution. Taking the holding cost of inventory in fuzzy nature Park, 1987 formulated and EOQ model using the extension principle and fuzzy numbers. Trapezoidal Fuzzy Logic was applied by Vujošević, Petrović, and Petrović, 1996 to determine the fuzzified total cost of an inventory model without shortages. Chang, Yao, and Lee, 1998 discussed the inventory management by considering shortages taken in fuzzy triangular form. Taking the replenishment measure as triangular fuzzy number Lee and Yao, 1999 formulated a model for inventory without backlogging. Yao and Chiang, 2003b developed an inventory framework fuzzifying total demand using triangular Fuzzy Logics. They defuzzified the total cost using the signed distance method and centroid method. The model so framed was without backlogging. Fuzzifying the lead time and shortages Lin, 2008 developed an inventory model under periodic interval. They used signed distance method to defuzzify the total cost.Risk analysis by fuzzification of various parameters is done in the study by Singh et al., 2018a.

The management of any organizations aims at reducing the cost of production. It is obvious to have unfinished goods when a lot is manufactured. These imperfect products not only lower the income but if unintentionally supplied to the customers lead to bad impression of the commodity resulting in the loss of goodwill. To have a check on these faulty products, the buyer makes an inspection to remove it from finished products. These imperfect goods if not treated properly impose a threat to the environment. To ensure remarkable profitability of an organization an efficient management of the inventory and a check on financial structure is a must. Rosenblatt and Lee, 1986 assumed that if a manufactured lot is of small size than it may have a less percentage of defective goods. They formulated an inventory framework with small order size to avoid defectives. The impact of the defectives on the environment is studied by many researchers and the study made is helpful to the organizations in reducing the waste and leading to a profitable situation. Green inventory model with weibull deterioration and a time quadratic stock dependent demand was studied by Singh et al., 2019. The consequence of the emission of the carbon from the defectives is investigated by Kazemi et al., 2018. To have sustainability in the inventory management, they formulated an inventory model for the defective quality products. Bazan, Jaber, and El Saadany, 2015 in his study not only optimized the total cost but effectively minimized the environmental risk. They formulated a model with less manufacturing cost in turn helping in saving the environment. The influence of the quality of the products on the environment and the role of the defectives on the environmental relationship was studied by Li et al., 2018. To help in reducing the waste Younesi and Roghanian, 2015 design a model for affirmable design of the products to cut down the defectives. They included various factors like quality environment and cost to help the product designers in taking decisions for decreasing the percentage of defective goods.

In today's global scenario to capture the market and be competitive firms extended to work in coordination. They use the permissible delay strategy to accomplish the financial gain. Due to an extension in the payment period, earnings are made through interest and also the opportunity cost. Thus trade credit is an important area for the researchers to formulate inventory models for various realistic situations. For keeping a hold over the market and to capture the potential buyers, supplier use trade credit as a dominant

tool. Goyal, 1985b first formulated an inventory policy for the delayed payment. Various researches as Sana and Chaudhuri, 2008, Khanra, Ghosh, and Chaudhuri, 2011, Sarkar, 2013 etc. have made prominent contribution in this field. Also deterioration is natural to occur for any commodity. The goods start to deteriorate with the passage of time after their arrival in the stock. For this non instantaneous deterioration Ouyang, Wu, and Yang, 2006b studied an inventory policy for the products under permissible delay time. Singh et al., 2017b,Singh et al., 2018b reflected an optimal refilling schedule in a supply chain considering deterioration and delay in paying back. Chang, Teng, and Goyal, 2010 investigated an inventory policy for timely deteriorating products and demand being inventory dependent. Mahata, 2012 developed an optimal refilling schedule for the deteriorating goods with trade credit at two levels. For an uncertainty in the demand Cai, Chen, and Xiao, 2014 developed policy to study the relation between the bank and the delay period for the retailer.
Inflation is the measure of the price rise of any commodity or service. Due to this there is a decrease in the buying amount of the product. With inflation consumers have to pay more for the same amount of the product as compared to the earlier purchases. It is since 1975, researchers have made mathematical models for inventory framework considering inflation. Buzacott, 1975 first Established an inventory formulation for an ordering period considering inflation. Other zestful research works done with inflation are Sarkar, Sana, and Chaudhuri, 2011, Gholami-Qadikolaei, Mirzazadeh, and Tavakkoli-Moghaddam, 2013, Tripathi, Singh, and Mishra, 2014 etc. With every cost subject to inflation and demand price - subjected Ghoreishi, Mirzazadeh, and Weber, 2014 analyze the optimal price and quantity model. They also included the returns made by the consumers in their study.
For demand being inflation stimulated Thangam and Uthayakumar, 2010 examined an inventory framework for perishable goods and partial backlogs being exponential. They implemented discounted cash flow techniques to arrive at a solution to their mathematical model. Singh et al., 2017d in their study determined the optimal refilling period for the retailer considering inflation. Guria et al., 2013 analyze the model for a product with immediacy in a fraction of payment is considered. They examined inflation stimulated demand, selling price and purchasing price for their mathematical analysis. In a finite horizon Gilding, 2014 formulated an optimal refilling time for inventory policy including inflation. Palanivel and Uthayakumar, 2017 formulating a policy for non-instantaneous deteriorating products with two warehouses. Under the effect of inflation, they processed an optimal ordering policy where demand is a function of price and advertising cost. They also considered trade credit for their mathematical analysis.
In practical, demand of any product depends upon its availability that is the lot size and also on the price of the product. There is an interdependency between the availability of the product and the price of the product. This state is examined by different researches. To speculate this realistic nature Maihami and Kamalabadi, 2012b consider demand as price stimulant. Higher price of the product often negatives it's demand, however a

reasonable price gives a positive impact. For a new product launched in the market, the demand is always influenced by its price and the offers available in its purchase. Chaudhary and Sharma, 2015 analyzed the policy for price stimulating demand for products under inflation and weibull deterioration. Maragatham and Palani, 2017 also developed the model for perishable inventory with backorders. The demand was taken as a function of time.
In this present study an inventory policy for decorating goods with delayed payment time is proposed. The demand is taken as a function of price. Shortages are fully backlogged. Since the parameters, price, inflation and discount rate and the percentage of defective variate frequently, these are taken in fuzzy nature in deriving the cost of the model. A comparison with the total cost when all the variables are of crisp nature and the value of total cost when the parameters are fuzzified is done in the study. Numerical examples and sensitivity analysis is done to have in depth perceptions for certain parameters. To author's best ability, an inventory policy for fuzzy parameters considering inflation and speculating a green model by including recycling and remanufacturing for the defective goods under price relying demand has not yet been considered. This study brings new insights of total cost to both the retailers and suppliers and by comparing the cost under the conditions of permissible delay.

8.2 Theoretical assumptions and Symbols

8.2.1 Theoretical Assumptions

The study incorporates following assumptions:

1. Finite planning Horizon inventory policy with no lead time.
2. Demand G(p) = u+v*p is price stimulant and shortages are completely backordered.
3. Inflation rate (a) and discount rate (c) are fuzzified for all the inventory related cost.
4. Rate of deterioration π is constant for the inventory in hand.
5. The percentage for the defective goods in the lot is k in crisp model. Since this is an unpredictable measure based on the previous year data so is fuzzified according to a triangular fuzzy number.

6. The price p dominates the demand in any inventory model. The variation in price of the product should be analyzed while determining the overall cost and profit for the system. is fuzzified price according to triangular fuzzy logic.

7. The delay in paying back time is D less than cycle length is offered by the vendor to the vendee. Throughout this time the vendee earns through the goods sold and also through the interest earned I_e to the amount held. Beyond the delay time the vendee pays the interest I_p for the amount held.

8. Centroid and Signed Distance method are applied to defuzzify the total cost.

8.2.2 Symbols Exercised

1. c: discount rate.

2. a: rate of inflation

3. A_o,A_h,A_c,A_d,A_s,A_r are the Ordering cost, holding cost, purchasing cost, deterioration cost, shortage and screening cost respectively.

4. ZT: Present measure of total cost in crisp nature.

5. $\widetilde{ZT}$: Present measure of the fuzzified total cost.

6. ZT_{dc}: Present measure of the defuzzified total cost using centroid method.

7. ZT_{ds}: Present measure of the defuzzified total cost using signed distance method.

8.3 Mathematical Formulation for Green Inventory Model

Model Specification

The model is framed for utilizing the defective goods so as to protect the environment from the hazards of the waste products. The inventory varies for different time period. At the start of the cycle the inventory measures I_{oi}. The buyer undergoes screening of the inventory from t_i to $t_i^{'}$. The drop in the inventory within this time interval happens due to the removal of the defectives. The measure of inventory at $t_i^{'}$ becomes I_{si}. The defective goods are taken back by the vendor for remanufacturing and recycling and not thrown in the environment. These goods are again added back to the system at time $t_i^{''}$ and the inventory measures I_{fi} at $t_i^{''}$. The inventory becomes nil at s_{i+1} and then the demand of the product adds in the shortage inventory.

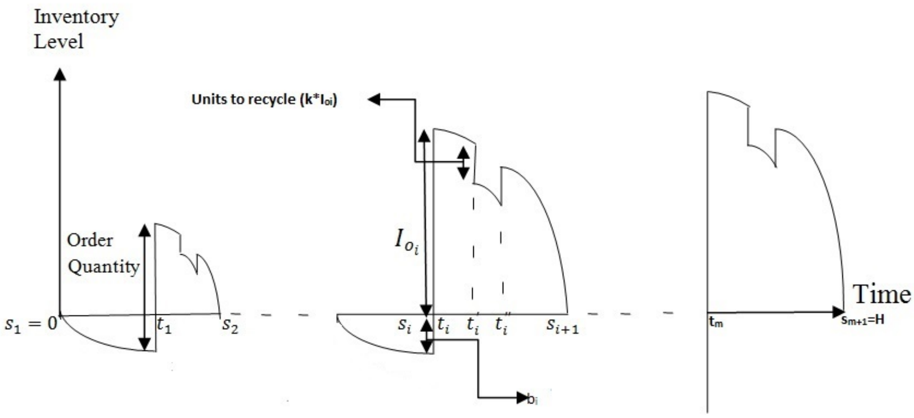

FIGURE 8.1: Graphical creation of Green Inventory Model

Figure below shows the diagramatic flow of inventory followed by the differential equations.

$$\frac{d\left(x(t)\right)}{dt} + \pi x(t) = \text{-G(p)},\ t_i \leq t \leq t_i^{'},(i = 1, 2, \ldots, m) \tag{8.1}$$

the boundary condition is $x\left(t_i\right) = I_{oi}$ and $x\left(t_i^{'}\right) = I_{si}$

$$\frac{d\left(y(t)\right)}{dt} + \pi y(t) = \text{-G(p)},\ t_i^{'} \leq t \leq t_i^{''},(i = 1, 2, \ldots, m) \tag{8.2}$$

the boundary condition is $y\left(t_i^{'}\right) = I_{si} - k.I_{oi}$ and $y\left(t_i^{''}\right) = I_{fi}$

$$\frac{d\left(z(t)\right)}{dt} + \pi z(t) = \text{-G(p)},\ t_i^{''} \leq t \leq s_{i+1},(i = 1, 2, \ldots, m) \tag{8.3}$$

the boundary condition is $z\left(t_i^{''}\right) = I_{fi} + k.I_{oi}$ and $z\left(s_{i+1}\right) = 0$

The amount of shortage at any point of time is given by :

$$\frac{d\left(B(t)\right)}{dt} = \text{-G(p)},\ s_i \leq t \leq t_i,(i = 1, 2, \ldots, m) \tag{8.4}$$

the boundary condition is $B\left(s_i\right) = 0$ and $B\left(t_i\right) = b_i$

Solving the equations with the boundary conditions, we obtain

$$I_{\mathrm{oi}} = (s_{i+1} - t_i) * (u + v * p) \tag{8.5}$$

$$I_{si} = (s_{i+1} - t_i - \mathrm{CT}) * (u + v * p) \tag{8.6}$$

$$I_{fi} = (u + v * p) * (s_{i+1} - t_i)\,(1 + k) - (u + v * p) * \mathrm{CT} \tag{8.7}$$

using the above calculated values, the result for the various inventory level differential equations are

$$x(t) = (\pi\,(-t + t_i) + 1) * (s_{i+1} - t_i) * (u + v * p) \\ +(-t+t_i) * (u + v * p), t_i \le t \le t_i{}', \quad \{i= 1, 2, \ldots, m\}\,. \tag{8.8}$$

Equation(2) is solved as:

$$y(t) = (u + v * p) * (s_{i+1} - t_i) * ((1 - k) + (t_i - t) * \pi - k * (t_i + \mathrm{CT} - t) * \pi) \\ + (u + v * p) * (-t + t_i)\,, t_i{}' \le t \le t_i{}'', \quad \{i = 1, 2, \ldots, m\} \tag{8.9}$$

Equation(3) is solved as:

$$z(t) = ((u + v * p) \div \pi) * (s_{i+1} - t) * (1 - \pi * t), t_i{}'' \le t \le s_{i+1}, \quad \{i= 1, 2, \ldots, m\}\,. \tag{8.10}$$

The amount of shortages is given by:

$$B(t) = (s_i - t) * (u + v * p), s_i \le t \le t_i, \quad \{i= 1, 2, \ldots, m\}\,. \tag{8.11}$$

The optimal ordering size is given by Qr = $I_{oi} + b_i$

$$Qr = \sum_{i=1}^{m} (u + v * p) * (s_{i+1} - s_i) \tag{8.12}$$

The total cost for the retailer includes:

(i)Net Worth of Ordering Cost $W_o = e^{(c-a)*t_i} * m * A_o$

(ii)Net Worth of Holding Cost $W_h = A_h \left[\int_{t_i}^{(t_i+\mathrm{CT})} e^{(c-a)*t} * x(t)dt+\right.$

$\int_{(t_i+\mathrm{CT})}^{(t_i+2*\mathrm{CT})} e^{(c-a)*t} * y(t)dt + \int_{(t_i+2*\mathrm{CT})}^{s_i} e^{(c-a)*t} * z(t)dt]$

(iii)Net Worth of Deterioration Cost $W_d = A_d \left[\int_{t_i}^{(t_i+\mathrm{CT})} e^{(c-a)*t} * \pi * I(t)dt + \right.$

$\int_{(t_i+\mathrm{CT})}^{(t_i+2*\mathrm{CT})} e^{(c-a)*t} * \pi * y(t)dt + \int_{(t_i+2*\mathrm{CT})}^{s_i} e^{(c-a)*t} * \pi * z(t)dt]$

(iv)Net Worth of Screening Cost $W_r = A_r * e^{(c-a)*t_i} * I_{\mathrm{oi}}$

(v)Net Worth of Purchasing Cost $W_p = A_c * e^{(c-a)*t_i} * [I_{\mathrm{oi}} + b_i]$

(vi)Net Worth of Shortage Cost $W_s = A_s \int_{s_i}^{t_i} e^{(c-a)*t} * (u + v * p) * (s_i - t)\, dt$

(vii) Interest Payable : The buyer pays for the interest on the amount of stock held with him depending upon the delay period D offered to him by the vendor.

When (D<= s_{i+1})

For the stock remaining with the buyer after the delay period D,the interest paid by the buyer at the rate I_p on the amount of stock is given as:

IP = $I_p * A_c * \int_D^{s_{i+1}} (u + v * p) * (s_{i+1} - t)\, dt$

IP = $\left(I_p * A_c * (u + v * p) * (s_{i+1} - D)^2\right) / 2$

(viii) Interest Earned : The buyer also earns from the amount of stock held depending upon the delay time D. The interest earned is at the rate I_e for the delay time.

When (D<= s_{i+1})

The buyer not only collects the amount by selling the product, but also earns the interest on the amount at the rate I_e till the cycle period. The interest earned from time $t = 0$ to $t = s_{i+1}$ is given by:

IE = $I_e * A_c * \int_0^{s_{i+1}} (u + v * p) * (s_{i+1} - t)\, dt$

IE = $\left(I_e * A_c * (u + v * p) * \left(s_{i+1}^2\right)\right) / 2$

8.4 Crisp Inventory Model

Net Worth of the total cost with the delayed payment option available is:

$$WT(t_i, s_i, p) = W_o + W_h + W_d + W_r + W_p + W_s + IP - IE$$

$$\begin{aligned}
\text{WT=}&e^{(c-a)*t_i} * m * A_o + (A_h + \pi * A_d) * \left(\int_{t_i}^{(t_i+\text{CT})} I(t) * e^{(c-a)*t} dt\right. \\
&+ \left.\int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} y(t) * e^{(c-a)*t} dt + \int_{(t_i+2*\text{CT})}^{s_{i+1}} e^{(c-a)*t} * z(t) dt\right) \\
&+ A_c * e^{(c-a)*t_i} * (u + v * p) * (s_{i+1} - s_i) + A_r * e^{(c-a)*t_i} \\
&* (u + v * p) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(c-a)*t} * (s_i - t) \\
&* (u + v * p) dt + (I_p * A_c * (u + v * p) * (s_{i+1} - S)^2) / 2 \\
&- \left(I_e * A_c * (u + v * p) * \left(s_{i+1}^2\right)\right) / 2
\end{aligned} \tag{8.13}$$

Optimal values of $t_i' s$ and $s_i' s$ are determined by the following two equations:

$$\frac{\partial (WT)}{\partial t_i} = 0 \tag{8.14}$$

and

$$\frac{\partial (WT)}{\partial s_i} = 0 \tag{8.15}$$

The convexity of total cost is determined by the following two equations. However the convex nature of the cost is shown graphically through figure (2) in the numerical section.

$$\frac{\partial^2 (WT)}{\partial t_i^2} > 0$$

and

$$\frac{\partial^2 (WT)}{\partial s_i^2} > 0$$

8.5 Fuzzy Plan of Action

These are $\tilde{c}, \tilde{a}, \tilde{k}, \tilde{p}$, where $\tilde{c} = (c_1, c_2, c_3)$, $\tilde{a} = (a_1, a_2, a_3)$, $\tilde{k} = (k_1, k_2, k_3)$, $\tilde{p} = (p_1, p_2, p_3)$ with $c_1 < c_2 < c_3$, $a_1 < a_2 < a_3$, $k_1 < k_2 < k_3$ and $p_1 < p_2 < p_3$.

The fuzzified value of the total cost given by equation(13) is shown in the following equation:

$$\begin{aligned}\widetilde{\text{WT}}=&e^{(\tilde{c}-\tilde{a})*t_i} * m * A_o + (A_h + \pi * A_d) * \left(\int_{t_i}^{(t_i+\text{CT})} x(t) * e^{(\tilde{c}-\tilde{a})*t} dt\right.\\ &\left. + \int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} y(t) * e^{(\tilde{c}-\tilde{a})*t} dt + \int_{(t_i+2*\text{CT})}^{s_{i+1}} e^{(\tilde{c}-\tilde{a})*t} * z(t) dt\right)\\ &+ A_c * e^{(\tilde{c}-\tilde{a})*t_i} * (u + v * \tilde{p}) * (s_{i+1} - s_i) + A_r * e^{(\tilde{c}-\tilde{a})*t_i}\\ &* (u + v * \tilde{p}) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(\tilde{c}-\tilde{a})*t} * (s_i - t)\\ &* (u + v * \tilde{p}) dt + (I_p * A_c * (u + v * \tilde{p}) * (s_{i+1} - S)^2)/2\\ &- \left(I_e * A_c * (u + v * \tilde{p}) * \left(s_{i+1}^2\right)\right)/2\end{aligned} \tag{8.16}$$

The fuzzified total cost is defuzzified using two methods namely (i) Centroid Method (ii) Signed Distance Method.

(i) Using, Centroid Method: The defuzzified value of total cost is given by

$$WT_{dc} = \frac{(WT_{dc_1} + WT_{dc_2} + WT_{dc_3})}{3}, \tag{8.17}$$

where

$$\begin{aligned}WT_{dc_1}=&e^{(c_1-a_1)*t_i} * m * A_o + (A_h + \pi * A_d)\\ &* \left(\int_{t_i}^{(t_i+\text{CT})} x(t) * e^{(c_1-a_1)*t} dt + \int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} y(t)\right.\\ &\left.* e^{(c_1-a_1)*t} dt + \int_{(t_i+2*\text{CT})}^{s_{i+1}} e^{(c_1-a_1)*t} * z(t) dt\right) + A_c\\ &* e^{(c_1-a_1)*t_i} * (u + v * p_1) * (s_{i+1} - s_i) + A_r * e^{(c_1-a_1)*t_i}\\ &* (u + v * p_1) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(c_1-a_1)*t} * (s_i - t)\\ &* (u + v * p_1) dt + (I_p * A_c * (u + v * p_1) * (s_{i+1} - D)^2)/2\\ &- \left(I_e * A_c * (u + v * p_1) * \left(s_{i+1}^2\right)\right)/2\end{aligned} \tag{8.18}$$

$$\begin{aligned} WT_{dc_2} = {} & e^{(c_2-a_2)*t_i} * m * A_o + (A_h + \pi * A_d) \\ & * \left(\int_{t_i}^{(t_i+\mathrm{CT})} x(t) * e^{(c_2-a_2)*t} dt + \int_{(t_i+\mathrm{CT})}^{(t_i+2*\mathrm{CT})} y(t) \right. \\ & \left. * e^{(c_2-a_2)*t} dt + \int_{(t_i+2*\mathrm{CT})}^{s_{i+1}} e^{(c_2-a_2)*t} * z(t) dt \right) + A_c \\ & * e^{(c_2-a_2)*t_i} * (u + v * p_2) * (s_{i+1} - s_i) + A_r * e^{(c_2-a_2)*t_i} \\ & * (u + v * p_2) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(c_2-a_2)*t} * (s_i - t) \\ & * (u + v * p_2) dt + (I_p * A_c * (u + v * p_2) * (s_{i+1} - D)^{\,2}) / \, 2 \\ & - \left(I_e * A_c * (u + v * p_2) * \left(s_{i+1}^2 \right) \right) / \, 2 \end{aligned} \tag{8.19}$$

and

$$\begin{aligned} WT_{dc_3} = {} & e^{(c_3-a_3)*t_i} * m * A_o + (A_h + \pi * A_d) \\ & * \left(\int_{t_i}^{(t_i+\mathrm{CT})} x(t) * e^{(c_3-a_3)*t} dt + \int_{(t_i+\mathrm{CT})}^{(t_i+2*\mathrm{CT})} y(t) \right. \\ & \left. * e^{(c_3-a_3)*t} dt + \int_{(t_i+2*\mathrm{CT})}^{s_{i+1}} e^{(c_3-a_3)*t} * z(t) dt \right) + A_c \\ & * e^{(c_3-a_3)*t_i} * (u + v * p_3) * (s_{i+1} - s_i) + A_r * e^{(c_3-a_3)*t_i} \\ & * (u + v * p_3) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(c_3-a_3)*t} * (s_i - t) \\ & * (u + v * p_3) dt + (I_p * A_c * (u + v * p_3) * (s_{i+1} - D)^{\,2}) / \, 2 \\ & - \left(I_e * A_c * (u + v * p_3) * \left(s_{i+1}^2 \right) \right) / \, 2 \end{aligned} \tag{8.20}$$

8.6 Optimality Check for Total Cost By Centroid Method

To optimize the objective function of total cost obtained above, optimal values of $t_i' s$ and $s_i' s$ are determined. The requisite requirement for minimizing the total cost is :

$$\frac{\partial\,(WT_{dc})}{\partial t_i} = 0$$

and

$$\frac{\partial\,(WT_{dc})}{\partial s_i} = 0$$

shown in equation (21) and (22)

The optimal values of $t_i^{'} s$ and $s_i^{'} s$ for the centroid method are determined by Equation(19) and (20).

To have a minimum value for the total cost by centroid method, we have

$$\frac{\partial^2 (WT_{dc})}{\partial t_i^2} > 0$$

and

$$\frac{\partial^2 (WT_{dc})}{\partial s_i^2} > 0$$

$$\frac{\partial (WT_{dc})}{\partial s_i} = (A_h + \pi * A_d) * \left((u + v * p_1) * \left(1 + \pi \left(t_{(j-1)} - \text{CT}\right)\right) * \right.$$
$$\left(\frac{e^{(c_1-a_1)*\text{CT}}}{(c_1 - a_1)}\right) + (u + v * p_1) * \pi * \left(\frac{e^{(c_1-a_1)*\text{CT}}}{(c_1 - a_1)^2}\right) + (u + v * p_1) *$$
$$\left((1 - k_1) + \pi \left(t_{(j-1)} - \text{CT}\right) - k_1 * \pi * t_{i-1}\right) * \left(\frac{e^{(c_1-a_1)*\text{CT}}}{(c_1 - a_1)}\right) +$$
$$(u + v * p_1) * \pi * (1 - k_1) * \left(\frac{e^{(c_1-a_1)*\text{CT}}}{(c_1 - a_1)^2}\right) + ((u + v * p_1)/\pi) *$$
$$\left(\frac{e^{(c_1-a_1)*(s_j - t_{j-1} - 2*\text{CT})}}{(c_1 - a_1)}\right) * \left(-\pi * \left(t_{(j-1)} + 2 * \text{CT}\right) + \left(\frac{\left(t_{(j-1)} + 2 * \text{CT}\right)}{(c_1 - a_1)}\right) - \right.$$
$$\left(\left(\pi * s_j - \frac{2 * \pi * \left(t_{(j-1)} + 2 * \text{CT}\right) - 1}{(c_1 - a_1)^2}\right) - \pi/(c_1 - a_1)\right) +$$
$$A_s * e^{(c_1-a_1)*t_j} * (u + v * p_1) + A_r * (u + v * p_1) * (c_1 - a_1) * e^{(c_1-a_1)*t_j} +$$
$$(A_h + \pi * A_d) * \left((u + v * p_2) * \left(1 + \pi \left(t_{(j-1)} - \text{CT}\right)\right) * \left(\frac{e^{(c_2-a_2)*\text{CT}}}{(c_2 - a_2)}\right) + \right.$$
$$(u + v * p_2) * \pi * \left(\frac{e^{(c_2-a_2)*\text{CT}}}{(c_2 - a_2)^2}\right) + (u + v * p_2) *$$
$$\left((1 - k_2) + \pi \left(t_{(j-1)} - \text{CT}\right) - k_2 * \pi * t_{i-1}\right) * \left(\frac{e^{(c_2-a_2)*\text{CT}}}{(c_2 - a_2)}\right) + (u + v * p_2)$$
$$*\pi * (1 - k_2) * \left(\frac{e^{(c_2-a_2)*\text{CT}}}{(c_2 - a_2)^2}\right) + \left(\frac{(u + v * p_2)}{\pi}\right) * \left(\frac{e^{(c_2-a_2)*(s_j - t_{j-1} - 2*\text{CT})}}{(c_2 - a_2)}\right)$$
$$\left(-\pi * \left(t_{(j-1)} + 2 * \text{CT}\right) + \left(\frac{\left(t_{(j-1)} + 2 * \text{CT}\right)}{(c_2 - a_2)}\right) * (1 - \pi * (s_j - t_{j-1} - \right.$$
$$2 * \text{CT} - \left(\pi * s_j - 2 * \pi * \left(\frac{t_{(j-1)} + 2 * \text{CT} - 1}{(c_2 - a_2)^2} - \pi/(c_2 - a_2)\right) + \right.$$
$$A_s * e^{(c_2-a_2)*t_j} * (u + v * p_2) + A_r * (u + v * p_2) * (c_2 - a_2) * e^{(c_2-a_2)*t_j} +$$

$$(A_h + \pi * A_d) * \left((u + v * p_3) * \left(1 + \pi \left(t_{(j-1)} - \mathrm{CT}\right)\right) * \left(\frac{e^{(c_3-a_3)*\mathrm{CT}}}{(c_3 - a_3)} \right) * \right.$$
$$(u + v * p_3) * \pi * \left(\frac{e^{(c_3-a_3)*\mathrm{CT}}}{(c_3 - a_3)^2} \right) + (u + v * p_3) * \left((1 - k_3) + \pi \left(t_{(j-1)} - \mathrm{CT}\right) - \right.$$
$$k_3 * \pi * t_{i-1} * \left(\frac{e^{(c_3-a_3)*\mathrm{CT}}}{(c_3 - a_3)} \right) + (u + v * p_3) * \pi * (1 - k_3) *$$
$$\left(\frac{e^{(c_3-a_3)*\mathrm{CT}}}{(c_3 - a_3)^2} \right) + \left(\frac{u + v * p_3}{\pi} \right) * \left(\frac{e^{(c_3-a_3)*(s_j - t_{j-1} - 2*\mathrm{CT})}}{(c_3 - a_3)} \right) *$$
$$\left(-\pi * \left(t_{(j-1)} + 2 * \mathrm{CT}\right) + \left(\frac{\left(t_{(j-1)} + 2 * \mathrm{CT}\right)}{(c_3 - a_3)} \right) * \left(\left(t_{(j-1)} + 2 * \mathrm{CT}\right) / (c_3 - a_3)\right) \right.$$
$$* (1 - \pi * (s_j - t_{j-1} - 2 * \mathrm{CT})) - \left(\left(\pi * s_j - 2 * \pi * \left(\frac{t_{(j-1)} + 2 * \mathrm{CT} - 1}{(c_3 - a_3)^2} * \right.\right.\right.$$
$$(c_3 - a_3)^2 - \pi / (c_3 - a_3) + A_s * e^{(c_3-a_3)*t_j} * (u + v * p_3) +$$
$$A_r * (u + v * p_3) * (c_3 - a_3) * e^{(c_3-a_3)*t_j} \tag{8.21}$$

$$\frac{\partial (WT_{dc})}{\partial t_i} = (c_1 - a_1) * e^{(c_1-a_1)*t_j} * m * A_o + (A_h + \gamma * A_d) \tag{8.22}$$
$$* \left((u+v*p_1) * \left(e^{(c_1-a_1)*\mathrm{CT}} / (c_1 - a_1)\right) * (\gamma * (s_{j+1} - 2*\mathrm{CT} - 1 / (c_1 - a_1)) + \mathrm{CT})\right.$$
$$+ ((u+v*p_1) / (c_1 - a_1)) * e^{(c_1-a_1)*\mathrm{CT}} * ((1-k_1) * (\gamma * (s_{j+1} - 2*t_j) - 1) + \mathrm{CT} - 1 -$$
$$1 / (c_1 - a_1)) + e^{(c_1-a_1)*(t_j+2*\mathrm{CT})} * ((u + v * p_1) / \gamma) * (s_{j+1} - (t_j + 2 * \mathrm{CT})) *$$
$$(1 - \gamma * (t_j + 2*\mathrm{CT})) + A_s * \left(e^{(c_1-a_1)*t_j} * (u+v*p_1) * (s_j - t_j)\right) + A_c * (c_1 - a_1) * e^{(c_1-a_1)*t_j}$$
$$* (u+v*p_1) * (s_{j+1} - s_j)\big) + A_r * (c_1 - a_1) * e^{(c_1-a_1)*t_j} * (u+v*p_1) * (s_{j+1} - t_j)$$
$$- A_r * e^{(c_1-a_1)*t_j} * (u + v * p_1) + (c_2 - a_2) * e^{(c_2-a_2)*t_j} * m * A_o + (A_h + \gamma * A_d)$$
$$* \left((u+v*p_2) * \left(e^{(c_2-a_2)*\mathrm{CT}} / (c_2 - a_2)\right) * (\gamma * (s_{j+1} - 2*\mathrm{CT} - 1 / (c_2 - a_2)) + \mathrm{CT}) + \right.$$
$$((u+v*p_2) / (c_2 - a_2)) * e^{(c_2-a_2)*\mathrm{CT}} * ((1-k_2) * (\gamma * (s_{j+1} - 2*t_j) - 1) + \mathrm{CT} - 1 -$$
$$1 / (c_2 - a_2)) + e^{(c_2-a_2)*(t_j+2*\mathrm{CT})} * ((u + v * p_2) / \gamma) * (s_{j+1} - (t_j + 2 * \mathrm{CT})) *$$
$$(1 - \gamma * (t_j + 2*\mathrm{CT})) + A_s * \left(e^{(c_2-a_2)*t_j} * (u+v*p_2) * (s_j - t_j)\right) + A_c * (c_2 - a_2) * e^{(c_2-a_2)*t_j}$$
$$* (u+v*p_2) * (s_{j+1} - s_j)\big) + A_r * (c_2 - a_2) * e^{(c_2-a_2)*t_j} * (u+v*p_2) * (s_{j+1} - t_j) - A_r * e^{(c_2-a_2)*t_j}$$
$$* (u+v*p_2) + (c_3 - a_3) * e^{(c_3-a_3)*t_j} * m * A_o + (A_h + \gamma * A_d) * \left((u+v*p_3) * \left(e^{(c_3-a_3)*\mathrm{CT}} / (c_3 - a_3)\right)\right.$$
$$* (\gamma * (s_{j+1} - 2*\mathrm{CT} - 1 / (c_3 - a_3)) + \mathrm{CT}) + ((u+v*p_3) / (c_3 - a_3)) * e^{(c_3-a_3)*\mathrm{CT}} * ((1 - k_3)$$
$$* (\gamma * (s_{j+1} - 2*t_j) - 1) + \mathrm{CT} - 1 - 1 / (c_3 - a_3)) + e^{(c_3-a_3)*(t_j+2*\mathrm{CT})} * ((u+v*p_3) / \gamma) *$$
$$(s_{j+1} - (t_j + 2*\mathrm{CT})) * (1 - \gamma * (t_j + 2*\mathrm{CT})) + A_s * \left(e^{(c_3-a_3)*t_j} * (u+v*p_3) * (s_j - t_j)\right) + A_c *$$
$$(c_3 - a_3) * e^{(c_3-a_3)*t_j} * (u+v*p_3) * (s_{j+1} - s_j)\big) + A_r * (c_3 - a_3) * e^{(c_3-a_3)*t_j} * (u+v*p_3) *$$
$$(s_{j+1} - t_j) - A_r * e^{(c_3-a_3)*t_j} * (u + v * p_3)$$

(ii) Using Signed Distance Method: The defuzzified value of total cost is given by

$$WT_{ds} = \frac{(WT_{ds_1} + 2 * WT_{ds_2} + WT_{ds_3})}{4} \tag{8.23}$$

where,

$$\begin{aligned} WT_{ds_1} =& e^{(c_1-a_1)*t_i} * m * A_o + (A_h + \pi * A_d) \\ &* \left(\int_{t_i}^{(t_i+\text{CT})} x(t) * e^{(c_1-a_1)*t} dt + \int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} y(t) \right. \\ &\left. * e^{(c_1-a_1)*t} dt + \int_{(t_i+2*\text{CT})}^{s_{i+1}} e^{(c_1-a_1)*t} * z(t) dt \right) + A_c \\ &* e^{(c_1-a_1)*t_i} * (u + v * p_1) * (s_{i+1} - s_i) + A_r * e^{(c_1-a_1)*t_i} \\ &* (u + v * p_1) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(c_1-a_1)*t} * (s_i - t) \\ &* (u + v * p_1) dt + (I_p * A_c * (u + v * p_1) * (s_{i+1} - S)^2)/2 \\ &- \left(I_e * A_c * (u + v * p_1) * \left(s_{i+1}^2 \right) \right)/2 \end{aligned} \tag{8.24}$$

$$\begin{aligned} WT_{ds_2} =& e^{(c_2-a_2)*t_i} * m * A_o + (A_h + \pi * A_d) \\ &* \left(\int_{t_i}^{(t_i+\text{CT})} x(t) * e^{(c_2-a_2)*t} dt + \int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} y(t) \right. \\ &\left. * e^{(c_2-a_2)*t} dt + \int_{(t_i+2*\text{CT})}^{s_{i+1}} e^{(c_2-a_2)*t} * z(t) dt \right) + A_c \\ &* e^{(c_2-a_2)*t_i} * (u + v * p_2) * (s_{i+1} - s_i) + A_r * e^{(c_2-a_2)*t_i} \\ &* (u + v * p_2) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(c_2-a_2)*t} * (s_i - t) \\ &* (u + v * p_2) dt + (I_p * A_c * (u + v * p_2) * (s_{i+1} - S)^2)/2 \\ &- \left(I_e * A_c * (u + v * p_2) * \left(s_{i+1}^2 \right) \right)/2 \end{aligned} \tag{8.25}$$

and

$$\begin{aligned} WT_{ds_3} =& e^{(c_3-a_3)*t_i} * m * A_o + (A_h + \pi * A_d) \\ &* \left(\int_{t_i}^{(t_i+\text{CT})} x(t) * e^{(c_3-a_3)*t} dt + \int_{(t_i+\text{CT})}^{(t_i+2*\text{CT})} y(t) \right. \\ &\left. * e^{(c_3-a_3)*t} dt + \int_{(t_i+2*\text{CT})}^{s_{i+1}} e^{(c_3-a_3)*t} * z(t) dt \right) + A_c \\ &* e^{(c_3-a_3)*t_i} * (u + v * p_3) * (s_{i+1} - s_i) + A_r * e^{(c_3-a_3)*t_i} \\ &* (u + v * p_3) * (s_{i+1} - t_i) + A_s \int_{s_i}^{t_i} e^{(c_3-a_3)*t} * (s_i - t) \\ &* (u + v * p_3) dt + (I_p * A_c * (u + v * p_3) * (s_{i+1} - S)^2)/2 \\ &- \left(I_e * A_c * (u + v * p_3) * \left(s_{i+1}^2 \right) \right)/2 \end{aligned} \tag{8.26}$$

8.7 Optimality Check for Total Cost in Signed Distance Method

Optimal values of $t_i's$ and $s_i's$ are determined by the following two equations:

$$\frac{\partial\left(WT_{ds}\right)}{\partial t_i}=0 \tag{8.27}$$

and

$$\frac{\partial\left(WT_{ds}\right)}{\partial s_i}=0 \tag{8.28}$$

The requisite requirement for minimizing the total cost is

$$\frac{\partial^2\left(WT_{ds}\right)}{\partial t_i^2}>0$$

and

$$\frac{\partial^2\left(WT_{ds}\right)}{\partial s_i^2}>0$$

8.8 Algorithm

1. The parameters are assigned with the numerical values.
2. To determine the present estimate of the total cost in the crisp inventory framework:
   (a) Optimal values of $t_i^{'}s$ and $s_i^{'}s$ are determined from equation (14) and (15). The variable t_1 is assumed and s_1 is taken as zero. s_2 is calculated, then t_2 and so on.
   (b) Repeating the above steps until all the $t_i^{'}s$ and $s_i^{'}s$ are known.
   (c) Net worth of total cost (WT) is determined. If $WT(m) <= WT(m+1)$, then optimal cost value $WT* = WT(m)$ and optimal cycle time m* = m.
3. The parameter fuzzified are assigned certain values. is the fuzzified cost.

4. For defuzzification, net worth of total cost through Centroid method WT_{dc} is determined by calculating the optimal $t_i^{'}s$ and $s_i^{'}s$ through equation(21) and (22).
5. For defuzzification, net worth of total cost through Signed Distance method WT_{ds} is determined by calculating the optimal $t_i^{'}s$ and $s_i^{'}s$ through equation(27) and (28).
6. The net worth of cost through the above two methods are then compared in the sensitivity analysis.

8.9 Numerical illustration for the two conditions of trade credit to minimize net total cost

8.9.1 Crisp Inventory Model

The numerical values for the parameters in crisp nature are as follows. The problem is solved using Mathematica 8.0.

Example:1 c=0.12,a=0.05,y=100,p=20,k=0.5,$\pi = 0.00001$, CT $= 0.016, A_o = \$10/unit, A_h = \$35/unit, A_d = \$10/unit, A_c = \$27/unit, A_r = \$1/unit, A_s = \$14/unit, I_e = \$0.05/unit, I_p = \$0.03/unit, D = 0.6$.

The results to the crisp inventory model when demand is made variable exhibits the convex nature of the total cost. The table below shows the details:

TABLE 8.1: Buyer's Inflated total cost for varied values for parameter 'x'

$\downarrow x \; \rightarrow m$	1	2	3	4	5	6
437.5	9605.73	8986.51	8588.51	8585.17	**8370.58**	8538.81
525	10014.8	9460.16	9133.88	**9007.58**	9075.13	9330.38
350	9203.27	8528.5	8067.81	7796.37	**7708.49**	7798.48

The graph(figure 2) below shows the convexity in crisp model for the total cost with the delayed payment condition. The optimal refilling cycle time are 5th, 4th and 5th respectively.

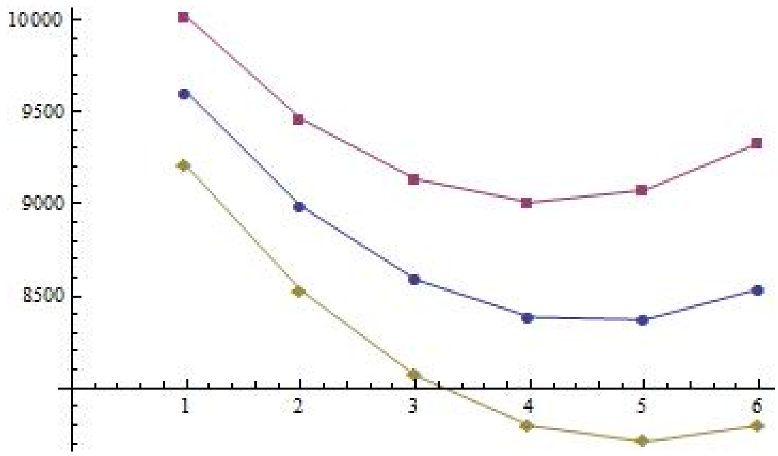

FIGURE 8.2: Crisp Inventory Model : Net Total Cost for 'x=437.5,525,350'

8.9.2 Fuzzy Inventory Model

The parameters fuzzified are as follows:

$\tilde{c} = (0.11, 0.12, 0.13)$, $\tilde{a} = (0.04, 0.05, 0.06)$, $\tilde{p} = (19, 20, 21)$, $\tilde{k} = (0.4, 0.5, 0.6)$

The fuzzified inventory model is solved by the following two methods:

By **Centroid Method**

The following table demonstrates the fuzzified total cost and the fuzzified optimal order quantity.

TABLE 8.2: Fuzzified Inflated total cost for parameter 'x'

x	WT_{dc}	Qr_{dc}
87.5	2.6568 * 10^11	8350
175	2.76816 * 10^11	8700
350	2.99088 * 10^11	9400
437.5	3.10223 * 10^11	9750
525	3.21359 * 10^11	10100

By **Signed Distance Method**

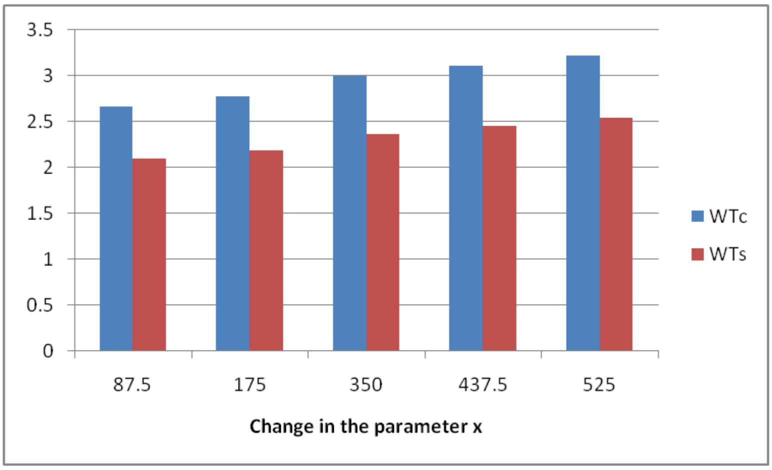

FIGURE 8.3

The following table demonstrates the fuzzified total cost and the fuzzified optimal order quantity.

TABLE 8.3: Fuzzified Inflated total cost for parameter 'x'

x	WT_{ds}	Qr_{ds}
87.5	2.09241*10^11	33050
175	2.18001*10^11	34100
350	2.35521*10^11	36200
437.5	2.44281*10^11	37250
525	2.53041*10^11	38300

The defuzzified optimal cost value as determined by centroid and signed distance method shows that the cost through centroid method is more than the signed distance method. Also the optimal quantity to be ordered is less through the former method as compared to the latter one.

8.9.3 Sensitivity analysis

The green inventory model is analyzed for the changes in the values of certain parameters and the effect of the change is studied on the fuzzified value of the total cost. The table below demonstrates the optimal total cost using the two defuzzification method, centroid and signed distance method.

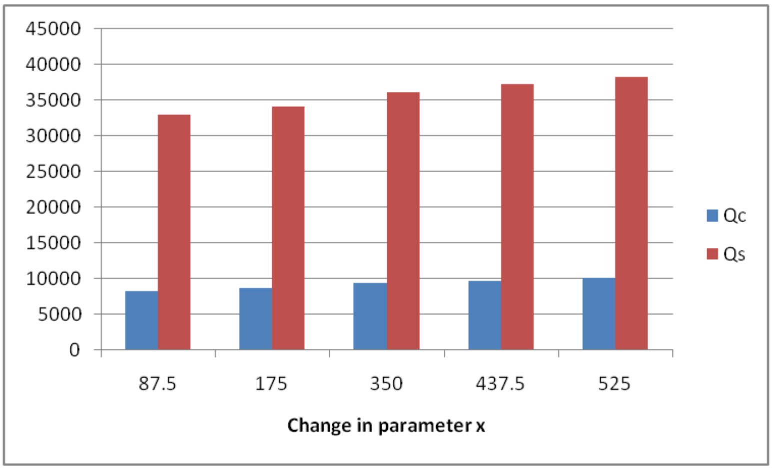

FIGURE 8.4

	% Change in parameter	$Values$	WT_{dc}	WT_{ds}
y	−95 −98 1.02 1.05	95 98 102 105	$3.80682 * 10^{11}$ $3.92682 * 10^{11}$ $4.08682 * 10^{11}$ $4.20682 * 10^{11}$	$2.25004 * 10^{11}$ $2.31005 * 10^{11}$ $2.39006 * 10^{11}$ $2.45006 * 10^{11}$
A_h	−95 −98 1.02 1.05	33.25 34.3 35.7 36.75	$2.4182 * 10^{11}$ $2.49457 * 10^{11}$ $2.59639 * 10^{11}$ $2.67275 * 10^{11}$	$2.23255 * 10^{11}$ $2.30305 * 10^{11}$ $2.39705 * 10^{11}$ $2.46755 * 10^{11}$
A_c	−95 −98 1.02 1.05	25.65 26.46 27.54 28.35	$2.54548 * 10^{11}$ $2.54548 * 10^{11}$ $2.54548 * 10^{11}$ $2.54548 * 10^{11}$	$2.34985 * 10^{11}$ $2.34985 * 10^{11}$ $2.34985 * 10^{11}$ $2.34985 * 10^{11}$
A_s	−95 −98 1.02 1.05	13.72 13.3 14.28 14.7	$3.99805 * 10^{11}$ $3.99805 * 10^{11}$ $3.99805 * 10^{11}$ $3.99805 * 10^{11}$	$2.35522 * 10^{11}$ $2.35522 * 10^{11}$ $2.35522 * 10^{11}$ $2.35522 * 10^{11}$
CT	−95 −98 1.02 1.05	0.0095 0.0098 0.0102 0.0105	$4.69992 * 10^{11}$ $4.69992 * 10^{11}$ $4.69992 * 10^{11}$ $4.69992 * 10^{11}$	$2.35524 * 10^{11}$ $2.35524 * 10^{11}$ $2.35524 * 10^{11}$ $2.35524 * 10^{11}$

The analysis done with the cost parameters suggests that the defuzzified cost by the signed distance method is less than that by centroid method.This is clearly visible through figure 3.

8.10 Results and Discussions

Another important result is derived from the research work "An inventory ordering model with different defuzzification techniques under inflation." The findings listed

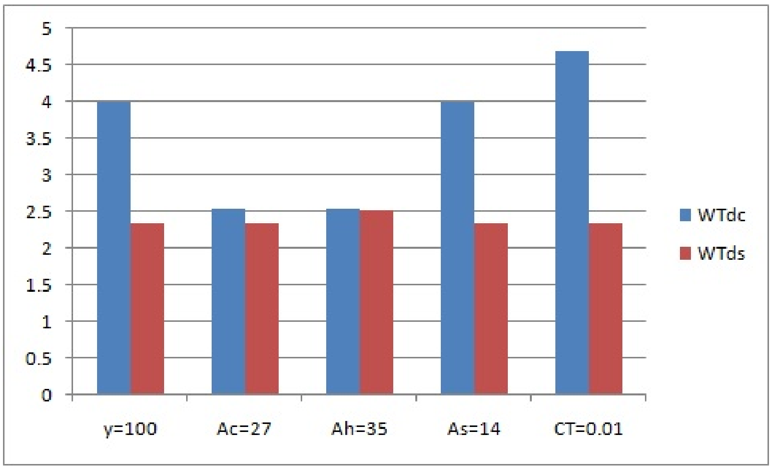

FIGURE 8.5: Sensitivity Analysis of Centroid and Signed Distance Method for various parameters

below proved that the cost derived are liable for a change in a fuzzy environment as compared to crisp sense of dealing the parameters.

1. The cost value in fixed nature is less than the cost value incorporating fuzzy logics. This is due to the fact of missing the uncertainties prevailing in the market and keeping the cost value to the parameters as constant. This also clearly shows that the cost determined with these fixed parameters is not reliable.

2. The use of the two types of fuzzy numbers is done to ascertain more accurately as to which fuuzy logic is more appropriate. It is demonstrated from the numerical that amongst the two, pentagonal fuzzy number reaches to an optimal value of the cost to the inventory system.

3. The numerical solutions demonstrates that the cost value in the crisp problem is less than the cost value obtained in the fuzzy problem.

4. This increase in the cost value shows that the values to the parameters cannot be taken in the fixed nature. The values to these parameters change according to the market situations. Hence the cost derived keeping the parameters in fixed form does not project the correct solution to the problem.

5. Fuzzifying the parameters, gives an increase in the cost, there by showing the realistic approach to the problem. The increase clearly mentions the significance of the fuzzy logics in the inventory problem.

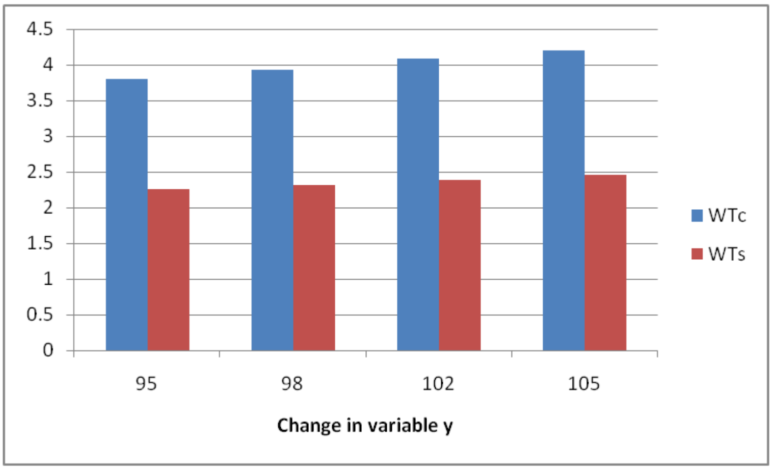

FIGURE 8.6

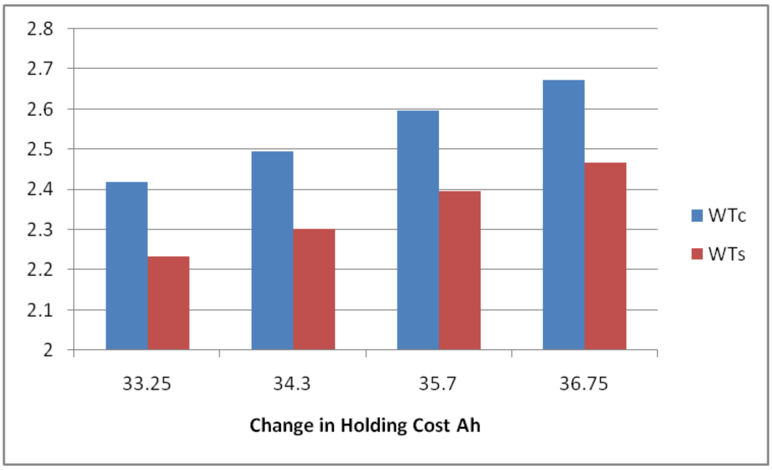

FIGURE 8.7

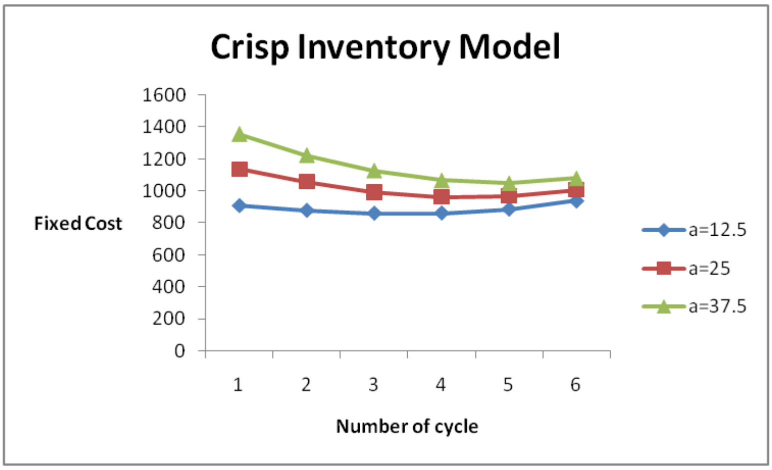

FIGURE 8.8

The results to the crisp inventory model when demand is made variable exhibits the convex nature of the total cost. The table below shows the details:

TABLE 8.4: Fixed inflated cost for the retailer for varied demand parameter

$\downarrow a$ $\rightarrow m$	1	2	3	4	5	6
12.5	910.09	879.49	**861.15**	862.28	887.38	939.51
25	1137.26	1057.47	992.02	**962.82**	966.84	1008.41
37.5	1358.89	1225.81	1127.73	1065.75	**1047.46**	1077.97

Defuzzing the cost value using the signed distance method for the pentagonal fuzzy number :

$$Tpg\,\widetilde{(t_i, s_i, n)} = (a + 2b + 2c + 2d + e)/8$$

The fuzzing cost for varied demand is shown in the following table:

Defuzzing the cost value using the signed distance method for the hexagonal fuzzy number:

TABLE 8.5: Fixed inflated cost for the retailer for varied demand parameter

$\downarrow a \;\; \rightarrow m$	1	2	3	4	5	6
12.5	999.26	945.82	908.69	**895.10**	909.87	956.26
25	1271.13	1159.81	1071.28	1017.88	**1002.48**	1036.66
37.5	1540.45	1366.91	1222.22	1133.83	**1097.34**	1118.24

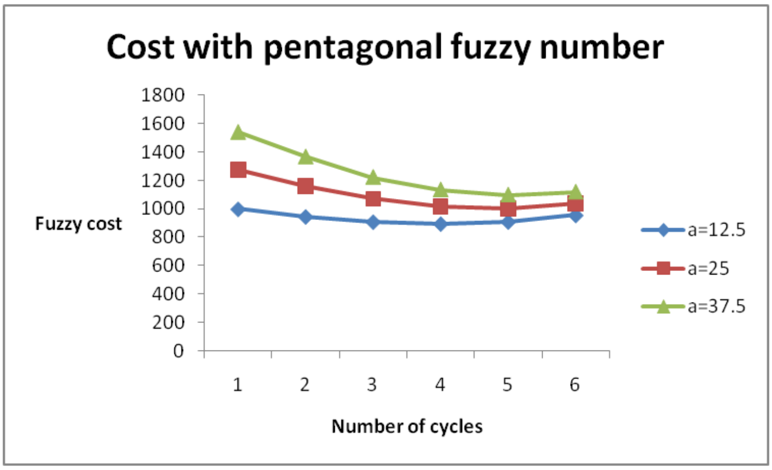

FIGURE 8.9

$$Thg\,\widetilde{(t_i, s_i, n)} = (a + 2b + c + d + 2e + f)/8$$

The fuzzing cost for varied demand is shown in the following table:

The following figure demonstrates the comparison of cost in crisp model and in fuzzy model comparing the two different types of fuzzy numbers hexagonal and pentagonal.

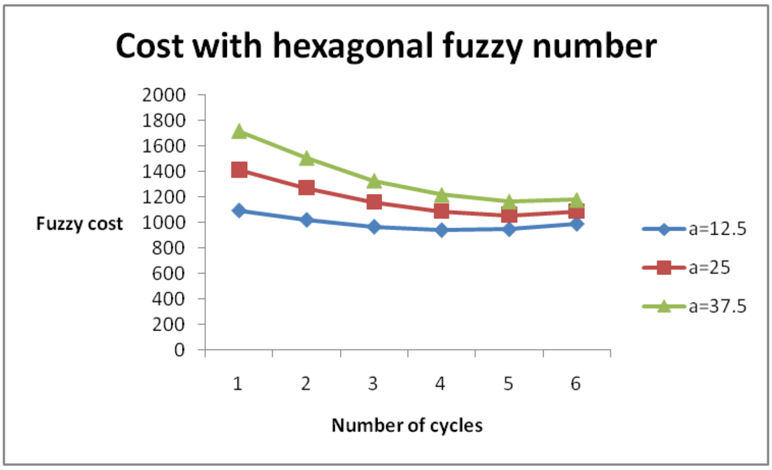

FIGURE 8.10

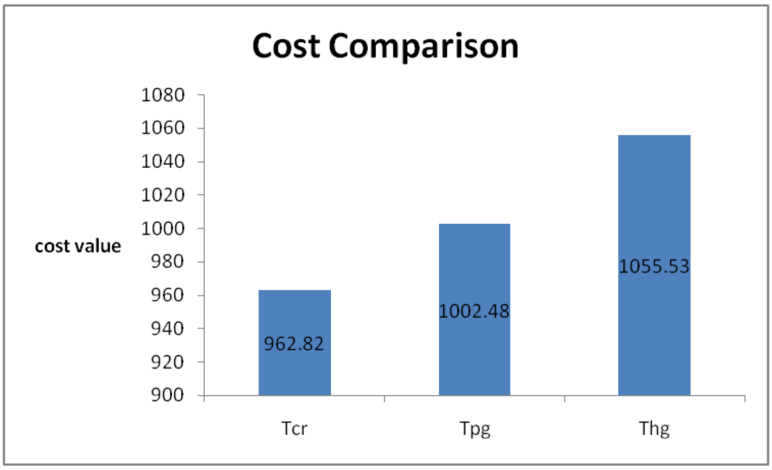

FIGURE 8.11

TABLE 8.6: Fixed inflated cost for the retailer for varied demand parameter

$\downarrow a$ $\rightarrow m$	1	2	3	4	5	6
12.5	1094.94	1022.45	969.17	**943.07**	949.48	992.05
25	1411.78	1268.88	1156.24	1084.19	**1055.53**	1084.16
37.5	1718.46	1506.84	1328.62	1216.66	**1164.28**	1177.62

8.11 Conclusion

The study proposes a backdrop of permissible time length of paying back by the vendee. The model is developed close practicality by fuzzifying the parameters, as discount and inflation rate, inspection rate and the price of the product. These parameters have a high rate of uncertainties and so fuzzifying them leads to a more realistic value of the total cost. All the cost values associated with the inventory are computed by the compounding of the inflation and discount rate. The model progresses with two methods of defuzzification namely centroid and signed distance method. Numericals examine the behaviour of the proposed policies and the results so determined by the two methods are compared with the cost value for the inventory model in crisp nature. It is further noticed that the fuzzy green inventory model with signed distance method shows a decline of total cost as compared to the model with the centroid method. Future scopes includes the fuzzification of all the cost and lead time. Shortages can be partially backlogged for this unpredictable environment.

Chapter 9

Conclusion

This thesis contributes to providing a significant solution to the decision/policymakers for inventory control and planning in the supply echelon mechanisms. The mathematical solution to the various models is important for many big/small scale units in providing a framework to optimize the total cost of the system. The significance in the approach considered in framing the various optimization inventory models for the supply chain dwells in the fact that the admissible delay period provided by the supplier to the retailer brings a win-win strategy to both of them. The delay period provides an opportunity for the retailer to earn from the interest on the money to be paid and also by selling the goods. On the other hand, the supplier earns by the reduction in the set-up cost which is a result of a decrease in the ordering cycles.

This research work elaborately explains the trade credit inventory models with several parameters like deterioration of the inventory, complete and partial stock-outs, time-varying quadratic demand and price varying demand, green technology, and the fuzzy logic. The models are formulated to study the impact of inflation on various parameters of the inventory system. The rate of inflation cannot be avoided while deriving a mathematical solution to any inventory problem. Inflation is defined as a decrease in the money value owing to which there is a reduction in the purchasing power of the money. Every cost included in deriving the total cost of the system is subjected to a constant rate of inflation, which measures its current value, thus providing a more realistic solution to the problem.

The inclusion of green technology is made a compulsory constraint from the government and society. Inventory models are framed using the concept of different R's that is recycling, reusing, remanufacturing, and refurbishing to decrease the waste produced and avoid the reduction of natural resources. The findings of the model show that despite following the green technology, yet in the supply chain both the retailer and the supplier accrue cost benefits. Also on account of the uncertainty prevailing in the market, crisp value to the various factors cannot be taken. Therefore, fuzzifying the inventory system brings the problem closer to reality.

The hypothesis is derived based on the numerical solutions and simplified algorithms are proposed to derive the numerical solutions to various problems. In this study, a single retailer and a single supplier are considered. The numerical solution proposes that there is a reduction in the number of cycles for the supplier when there is coordination in the inventory system. This is due to this savings realized that a profit distribution is made between both the parties under an inflationary environment.

We expect that the research work contributes to the field of ongoing work in inventory framework and management by including and extending our inventory problem models. Numerous prospective extensions are there for the future scope. The research work can be extended to the multi-levels in the supply chain of trade credit. Also, fuzzification of the leftover parameters can be done in a problem. Quantity discounts and price breaks can also be included.

Chapter 10

Conference participation, and Published, accepted, communicated Papers

10.0.1 List of papers published are as follows:-

1. Singh, V., Saxena, S., Singh, P., Mishra, N. K. (2017, July). Replenishment policy for an inventory model under inflation. In **AIP conference proceedings** (Vol. 1860, No. 1, p. 020035). AIP Publishing LLC. (**Scopus Indexed**)

2. Singh, V., Saxena, S., Gupta, R. K., Mishra, N. K., Singh, P. (2018, August). A supply chain model with deteriorating items under inflation. In 2018 **4th International Conference on Computing Sciences (ICCS)** (pp. 119-125). IEEE. (**Scopus Indexed**).

3. Saxena, S., Singh, V., Gupta, R. K., Singh, P., Mishra, N. K. (2020). A Supply Chain Replenishment Inflationary Inventory Model with Trade Credit. In **International Conference on Innovative Computing and Communications** (pp. 221-234). Springer, Singapore. (**Scopus Indexed**).

4. Saxena, S., Singh, V., Gupta, R. K., Mishra, N. K., Singh, P. Green Inventory Supply Chain Model with Inflation under Permissible Delay in Finite Planning Horizon., **Adv. Sci. Technol. Eng. Syst. J.**, Volume 4, Issue 5, Page No 123-131, 2019. (**Scopus Indexed**)

5. An inventory ordering model with different defuzzification techniques under inflation is accepted for publication in a **scopus indexed** journal **Journal of Computational and Theoretical Nanoscience** as a regular article.

Replenishment policy for an inventory model under inflation

Vikramjeet Singh, Seema Saxena, Pushpinder Singh, and Nitin Kumar Mishra

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A Supply Chain Model with deteriorating items under inflation

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***Abstract*—Inventory models are an important part for an organization. This paper presents a model for deteriorating goods within an inflationary environment. The finite planning horizon is considered along with constant demand where credit term is offered to the retailer by the supplier. The model considers completely back ordered shortages. Numerical illustration at the end validate the model.**

***Keywords*-Inflation; deterioration; permissible delay; completely backordered shortages; Inventory;**

I. INTRODUCTION

Deterioration refers to the spoilage or damage to the computer peripherals and other electronic appliances. Deterioration cost affects the total cost of an inventory model. The constant deterioration concept was introduced by [1] [1]. Later many zestful works like time-varying, Weibull deterioration, exponential deterioration is done by researchers [2],[3]. Shortages tend to occur and are completely or partially back ordered. [4] and [5] developed an inventory model with completely back ordered shortages. The customer gets impatient and switches to the other options available in the market. The first model that studied the customer's impatience was developed by [6]. The partial back ordered shortages along with deterioration is also studied by [7],[8] and many others.

Trade credit is a helpful phenomenon in the inventory system which helps both the parties enter into a profitable business. [9] discussed the delay in payment in the inventory model. [10],[11] discussed the permissible delay. Demand is the crucial parameter in determining the inventory cost. The concept of time-varying demand is used by many researchers as [12], [13], [14] and others. many times organizations suffer loss due to neglecting the effects of inflation. [15] was the first to discuss the effect of inflation on an inventory model. [16],[2],[17],[18] studied the inventory model under inflationary environment. The fuzzy concept of the inventory parameters is discussed by [8] and [19].

The research paper progresses with the postulates and terminology followed by the mathematical solution in both independent and dependent system. The optimization for the total cost is done next. In the latter part of the article, an algorithm is unionized with the numerical and conclusion.

II. POSTULATES AND TERMINOLOGY

A. Postulates

1) Single item, single retailer and single supplier are considered with no lead time.
2) Permissible delay(η) is given by the supplier to the retailer in a finite planning horizon.
3) The demand f(t) is constant and deterioration of goods starts when entering the inventory. The rate of deterioration is $\rho\ (t) = \beta\ t, with(\beta > 0)$ and $(t > 0)$.
4) Completely back ordered shortages are considered.
5) The model is for a finite planning horizon and developed under inflationary conditions with the constant inflation rate. All cost are subjected to the same rate of inflation.

B. Terminology

For retailer

1) IC, ID, IH, IS, IP denotes the cost of ordering, deterioration cost, cost of holding, cost of lost sale, shortage cost and purchase cost respectively.
2) $I^I_{1i}(t)$ is the level of inventory during the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system with no co-ordination, $I^D_{1j}(t)$ is the inventory level during the time interval $[t_j, s_{j+1}]$, $[j = 1, 2, 3, \ldots, n_2]$ for the dependent system where credit period is offered by the supplier to the retailer.
3) $R^I_i(t)$ and $D^I_i(t)$ is the total level of inventory and the total quantity deteriorated in the time interval $[t_i, s_{i+1}]$, $[i = 1, 2, 3, \ldots, n_1]$ in an independent system and $R^D_j(t)$ and $D^D_j(t)$ is the total inventory level

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A Supply Chain Replenishment Inflationary Inventory Model with Trade Credit

Seema Saxena, Vikramjeet Singh, Rajesh Kumar Gupta, Pushpinder Singh and Nitin Kumar Mishra

Abstract This research work considers a problem of obtaining the optimal replenishment schedule in a supply chain with a parameter of credit period rate. The model is designed for time-dependent quadratic demand and deterioration. The model is generalized considering partially backlogged shortages under inflation. However, as a peculiar case, an example is aimed for a model without lost sales. Sensitivity analysis is performed to analyze the mathematical formulation, and numerical examples are examined to study the effect of inflation and the time value of money on the economic order quantity model. The model evaluates the optimal replenishment schedules for the single retailer and single supplier for a single product in the supply chain subject to inflation.

Keywords Inflation · Supply chain · Credit term · Inventory

AMS Subject Classification 90B05 · 90B99

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Green Inventory Supply Chain Model with Inflation under Permissible Delay in Finite Planning Horizon

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ABSTRACT

Trade credit is an important cost reduction tool in the inventory management. The effect of trade credit is studied on the integrated system for sharing the cost benefits realized due to the permissible delay. Credit term factor is introduced to divide the cost benefits between the retailer and the supplier. The various costs in the inventory model are subjected to the same inflation rate. This research paper revisits EOQ model for remanufacturing process under green supply chain with the permissible delay available to the retailer. Numerical examples prove that the optimal re-ordering schedule exists and is unique. Also sensitivity analysis is performed on certain parameters to ascertain their logical implications.

1 Introduction

This paper is an extension of the work [1] originally presented in the 4th International Conference on Computing Science(ICCS),2018. This extended research work incorporates the green inventory concept. The harmful effects of the waste and outdated products are imposing a threat to the environment. In present scenario, world is facing pollution as a big hazard to mankind. Every organization is moving towards reducing and reusing the waste/imperfect goods. In this direction this EOQ model aims at a single stage remanufacturing process where the imperfect goods after screening are taken back by the supplier and are remanufactured. This is again transported to the retailer. Replenishment schedule is derived for the retailer and the supplier, considering green product life cycle and the time value of money. Various countries have adopted several measures for waste product management, reusing and remanufacturing programs. International standards as European Union's proposal for Waste Electrical and Electronic equipment (WEEE) introduces the concept of product design, to bring a decrease in the cost of disassembly and remanufacturing [2]. Product, if designed significantly reduces the cost of inspection, disassembly, repair, remanufacturing and recycling. In [3], the author first determine an optimal ordering size with recovery and remanufacturing. He studied the traditional EOQ model with continuous and deterministic demand and return. In [4], the authors probed the changes taking place in the market and in the manufacturing organizations. He studied the changes for the process of product design, with respect to the technology introduced for new material and new production methods, including tools and techniques altered for the manufacturing, inspection, reusing and remanufacturing. In [5], the author provided a comprehensive and immense knowledge for the remanufacturing process from the literature surveyed. The supplier and retailer are stationed far from each other so it is not possible for the supplier to send all perfect goods. Thus to be assure of the brand and quality, retailer screens the lot as it is received. In [6], the authors studied the remanufacturing of the imperfect items to maximize the total profit. The effect of deterioration is dominant and its consequences cannot be ignored while framing an EOQ model. Electronic goods, blood, fruits, grain products, alcohol are some of the deteriorating products. In [7], the authors were the first to study deterioration in an inventory model. Several researchers as [8], [9], [10]. [11] studied different patterns of deterioration in inventory models. In [12], the authors developed an inventory model for the stock dependent demand for deteriorating products under two level credit. In [13], the researcher studied a two-echelon supply chain model for deteriorating products under trade credit. They analyzed two models, one with demand being stock dependent and the second with demand being selling price dependent for the perishable products. In [14], the authors studied

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Your manuscript (CTN-AS-20-0027) entitled "An Inventory Ordering Model with Different Defuzzification Techniques under Inflation" by Rajesh Kumar Gupta, Seema Saxena, Vikramjeet Singh,Pushpinder Singh and Nitin Kumar Mishra

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This is with reference to Your manuscript (CTN-AS-20-0027) entitled "An Inventory Ordering Model with Different Defuzzification Techniques under Inflation" by *Rajesh Kumar Gupta, Seema Saxena, Vikramjeet Singh,Pushpinder Singh and Nitin Kumar Mishra* has been accepted for publication in Journal of Computational and Theoretical Nanoscience as a regular article in our special issue "Internet of Things: Principles, Applications and Security Concerns"

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This is for your kind information

Sincerely,

Dr. Avinash Sharma, Guest Editor
Journal of Computational and Theoretical Nanoscience

10.0.2 List of papers communicated for publication:-

1. Vendor-Buyers Greens Inventory Model with Price Sensitive Demand under Inflation is in process in a **scopus indexed** journal **International Journal of Applied and Computational Mathematics**.

2. Affirmable Inventory Administration for Environment Preserving under Fuzzy Logics and Inflation is submitted for publication in a **scopus indexed** journal.

3. Submitted a research paper titled "Fuzzified inventory ordering model for green environmental sustainability under inflation" in a (**Scopus Indexed**) journal.

10.0.3 Other relevant articles published during research work not in thesis:-

(a) Singh, P., Mishra, N. K., Kumar, M., **Saxena, S.**, Singh, V. (2018). Risk analysis of flood disaster based on similarity measures in picture fuzzy environment. Afrika Matematika, 29(7-8), 1019-1038. (**Scopus Indexed**).

(b) Singh, P., Mishra, N. K., Singh, V., **Saxena, S.** (2017, July). An EOQ model of time quadratic and inventory dependent demand for deteriorated items with partially backlogged shortages under trade credit. In AIP conference proceedings (Vol. 1860, No. 1, p. 020037). AIP Publishing LLC. (**Scopus Indexed**).

(c) Singh, V., Mishra, N. K., Mishra, S., Singh, P., **Saxena, S.** (2019, March). A green supply chain model for time quadratic inventory dependent demand and partially backlogging with Weibull deterioration under the finite horizon. In AIP conference proceedings (Vol. 2080, No. 1, p. 060002). AIP Publishing LLC. (**Scopus Indexed**)

(d) V Singh, NK Mishra, S Mishra, P Singh, **Saxena S.** (2019). An inventory model in a green supply chain for inventory dependent and time quadratic demand in a finite horizon. in International Journal of Control and Automation, 12(4), 218-220 (**Scopus Indexed**)

(e) Mishra, S., Mishra, N. K., Singh, V., Singh, P., **Saxena, S.** (2019). The Fuzzyfied Supply Chain Finite Planning Horizon Model. Journal of Computational and Theoretical Nanoscience, 16(10), 4135-4142. (**Scopus Indexed**)

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