Inventory models with trade credit in a finite planning horizon

A

Thesis

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Mathematics

By

Nitin Kumar Mishra

41500094

Supervised By **Co-Supervised by**

Dr. Sanjay Mishra **Dr. Vikramjeet Singh**
and Dr. Pushpinder Singh

LOVELY FACULTY OF TECHNOLOGY AND SCIENCES
LOVELY PROFESSIONAL UNIVERSITY
PUNJAB

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Declaration of Authorship

I hereby declare that this Ph.D. thesis entitled “Inventory Models with trade credit in a finite planning horizon” was carried out by me for the degree of Doctor of Philosophy in English under the guidance and supervision of Dr.Sanjay Mishra, in Mathematics, Lovely Professional University, Phagwara, Punjab, India.The interpretations put forth are based on my reading and understanding of the original texts and they are not published anywhere in the form of books, monographs or articles. The other books, articles and websites, which I have made use of are acknowledged at the respective place in the text. For the present thesis, which I am submitting to the University, no degree or diploma or distinction has been conferred on me before, either in this or in any other University.

Signed:

Dr. Nitin Kumar Mishra[Candidate]
Date:-

CERTIFICATE

This is to certify that the work incorporated in the thesis titled "Inventory Models with trade credit in a finite planning horizon" submitted by Mr. Nitin Kumar Mishra was carried out by the candidate under my guidance. In my opinion the thesis fulfills the requirements laid down by the Lovely Professional University, Phagwara, Punjab for the award of degree of Doctor of Philosophy in Mathematics.

1. Signed: ____

   Dr. Sanjay Mishra[Supervisor]
   Date:-

2. Signed: ____

   Dr. Vikramjeet Singh[Co-Supervisor]
   Date:-

3. Signed: ____

   Dr. Pushpinder Singh[Co-Supervisor]
   Date:-

Abstract

This thesis deals with the study of a single buyer, single supplier inventory model with time quadratic and stock dependent demand for a finite planning horizon. A single deteriorating item which suffers a shortage with partial backlogging and some lost sales is considered. Model is divided into two scenarios one with no permissible delay in payment and other with permissible delay in payment. Later is called centralized system where the supplier offers the retailer, trade credit. In the centralized system, cost-saving is shared amongst the two. The objective is to study the difference in minimum total cost born by retailer and supplier under two scenarios including above-mentioned parameters. To obtain the optimal solution of the problem the model is solved analytically. A numerical example and a comparative study are then discussed supported by sensitivity analysis of each parameter.

Then a detail solution of re-manufacturing of a product in a supply chain model is discussed. It is a non-traditional model considering time-dependent quadratic demand, Weibull deterioration, shortages, partial backlogging and re-manufacturing of inventory. This paper mainly focuses on remanufacturing and hence an attempt towards reducing the environmental hazard. The process of remanufacturing is completed within one cycle of replenishment. Trade credit between supplier and retailer also had been discussed. Two cases one of a centralized and the other of decentralization for a finite planning horizon in a supply chain model are discussed. An algorithm has been derived for solving a problem in both cases. Some managerial insights are talked about based on sensitivity analysis on the parameters considered.

As per my next objective, I have discussed the recycling of an item within the planning horizon. Recycling of an item has become the natural requirement in inventory handling. It decreases the burden of inventory for defective kind of items. Another obvious phenomenon is deterioration of items in inventory. Hence two-parameter Weibull deterioration of items is considered in this chapter. The idea is to introduce in a supply chain model some greenness through recycling of defective items after the sorting process.

Lastly, a supply chain model which is discussed for fuzzy parameters such as fuzzy deterioration cost, fuzzy holding cost fuzzy inventory carrying cost etcetera is considered for framing of the model which are later defuzzified using Centroid, Signed Distance and Graded Mean Representation method. Centralized replenishment policy in this finite planning horizon model is discussed along with sensitivity analysis.

Appendix A contains the excerpts of the Mathematica program for table formulation submitted for copyright. In appendix B the list of published, accepted and communicated research is provided.

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List of Abbreviations

EOQ	**E**conomic **O**rdering **Q**uantity
SDM	**S**igned **D**istance **M**ethod
GMM	**G**raded **M**ean **M**ethod
GMM	**C**entroid **M**ethod
CO	**C**entralized **O**ptimal
COλ	**C**entralized **O**ptimal with **C**redit period
DO	**D**ecentralized **O**ptimal
TC	**T**otal **C**ost

List of Symbols

Refer 2.3, 3.3 and 4.3 for assumptions and notations.

Dedicated to my father
Late Shri Prathmesh Kumar Mishra

Chapter 1

Introduction

1.1 Introduction to thesis

For ages, people have been maintaining inventory even before the time of barcoding people had a tough time predicting future sale and purchase. The necessity of better and better inventory management has been growing since then. A very expensive tool punch cards were introduced in the early 20th century. In today's scenario, barcoding and microchips have made the collection of inventory data very simple but predicting inventory is still a daunting task. Many models and methods have been given by authors so far. Study of inventory theory is a sub branch of operations research.

1. INTRODUCTION TO OPERATIONS RESEARCH:

   Operations research is a science for solving various real life problems. For different problems there are different models for problem solving in Operations research. To find the solution first the problem is formulated and one of the various methods is adopted.

2. SIGNIFICANCE OF OPERATIONS RESEARCH

   Operations research is used by executives of a company using mathematical analysis for error free decisions and better coordination. It is extensively used by various public and private sectors. Operations research is also used by agriculture, health and defence organizations.

3. GENERAL METHODS FOR SOLVING OPERATIONS RESEARCH MODEL

   Various optimization techniques are adopted to optimize a real world problem with modeling simulation. Some are as follows:

   Manufacturing/Production Optimization
   Network Optimization
   Transportation Optimization

Supply Chain Optimization
Scheduling Optimization etc.

4. Some tools used are as follows:

   probability theory

   numerical methods

   Monte Carlo methods etc.

RESEARCH MODELS
In analytical or detective method techniques such as differentiation integration interpolation and extrapolation and standard graphs are used for solving an operation research model.

In numerical or iterative method as the name suggest problems with large number of variables or parameters iterations on American solution is used to obtain an optimal solution iterations are followed until the final optimal solution is reached which cannot be justified further.

In Monte Carlo method random variables are used to estimate the actual population the simulation is done under certain constraints and random variable that follows a particular probability distribution this probability distribution represents the actual scenario of the problem any test to prove this probability distribution the solution of the model is obtained.

1.1.1 Description of inventory Systems

Types of inventory

There are different types of inventory. Some are mentioned below:

A branch of operations research deals with supply chain management system. One of the essentials of the supply chain management is the management of the goods to be transferred or stored in a storage this is called the inventory of the company or an organisation.

Inventory or stock is the good in the storage Area of the company or an organisation these goods can be of basically three different types raw inventory or goods, semi finished inventory, finished inventory called as the product. The inventory can be stored at different levels and different stages by different players such as manufacturers, distributors, dealers, retailers, buyers as well as customers.

Significance of inventory management

It is significant to store and maintain an inventory, as proper management can save the organisation against the heavy loss. These losses can be because of different reasons popping up during the ongoing business operations.

One of the major losses incurred is due to the demand fluctuation of the market. Heavy demand means more of the inventory in the storage house is needed and low demand means less of the inventory in storage house is required. One should have a track on ups and downs of these market requirement so as to efficiently keepup the inventory otherwise unused inventory may result into high inventory keeping cost and less inventory in the store house can result into customer satisfaction and further into shortage loss.

Other major parameter is deterioration. Deterioration happens when the raw material, semi finished goods or finished goods stored has a certain life time. Beyond that period the goods are no more in a condition to be used hence the purchase cost of those goods are always at risk during the the storage of inventory. Thus inventory management helps in estimating quality manufacturing also.

Inventory model

The most important question which arises is that how much inventory should be stored that means how much inventory should be ordered and at what time . Here time means how much time gap should be there between two orders of the inventory. This period also called as replenishment cycle time varies with different factors such as demand, deterioration etc. To find out the duration of of replenishment cycle a model is to be framed this model is called as inventory model or in other words economic ordering quantity model(EOQ)Hillier (2012).

This model this economic ordering quantity model is formulated mathematically and with the use of mathematical and statistical tools an optimal inventory policy is derived which is also called as scientific inventory management.

1.1.2 Inventory in a finite planning horizon

Finite planning Horizon model is a model which has "n" number or fixed number of replenishment cycles.

Finite planning Horizon model or a fixed planning Horizon was discussed by Giri, Chakrabarty, and Chaudhuri (2000). Considering finite number of replenishment cycles each replenishment cycle. Shortages were considered in each replenishment cycle accept the last cycle. Two different policies were taken into account and were compared. One in which the shortages was there in all cycles except the last one and the other in which the items were completely backlogged for replenishment cycles further Giri, Chakrabarty, and Chaudhuri (2000) stated that the first policy resulted in in average lower cost then second policy.

1.1.3 Deterioration

Deterioration is also understood as partial existence of the item due to damage or decay to the original one. This deteriorated product is either discarded or is recycled. Ghare and Schrader (1963) was the first to study deterioration. There are various ways in which deterioration occurs, like Constant deterioration Papachristos and Skouri (2000) and Pal, Mahapatra, and Samanta (2013), time varying deterioration Sana (2010), Weibull Deterioration Philip (1974), Yang (2012), and Pal, Mahapatra, and Samanta (2014a), exponential Ghare and Schrader (1963) and Jain and Aggarwal (2012) and fuzzy deterioration De Kumar, Kundu, and Goswami (2003), De and Goswami (2006), Roy et al. (2007), and Halim, Giri, and Chaudhuri (2008). They used the two-parameter Weibull distribution to represent the distribution of the time to deterioration. The instantaneous rate function F(t) for a two-parameter Weibull distribution is given by $F(t) = \alpha_1\beta_1 t^{\beta_1-1}$ where α_1 is the scale parameter, $\alpha_1 > 0$; β_1 is the shape parameter, $\beta_1 > 0$; t is time of deterioration, $t > 0$.

1.1.4 Green inventory

As discussed in Zhu and Sarkis (2004) organisations in China dealing with greening of the supply chain has to balance between the cost effectiveness and environmental issues.

Greening of the supply chain is possible by keeping environmental factors into consideration. World and societies are looking for sustainable growth in this direction. Greening of the supply chain also means moving towards more sustainable products. May it be greenhouse emission gases, recyclingM. Klausner and Horvath (1999) of the products, or remanufacturing Wee and Chung c (2009), Wee and Chung (2009), and Saadany and Jaber (2010) of the products the scientist community is exhaustively exploring every such area. There is a very little literature for the green supply chain considering the above factors.

1.1.5 Brief introduction to fuzzy

Many a times the class of elements to be picked from a Universal set doesn't have a fix criteria of precise criteria for the membership.

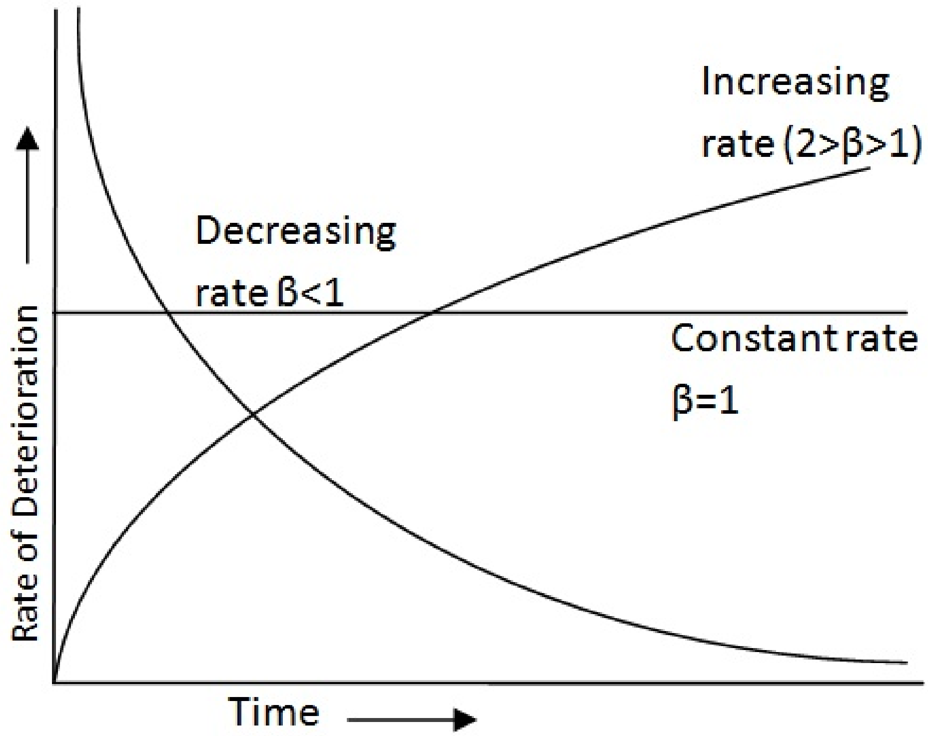

FIGURE 1.1: Weibull deterioration-time graph

For example "the set of good student" in the class to be selected from particular section or class. Other cases are like "the class of short men", "the class of intelligent men" etc. Fuzzy set theory deals with the above problem defining a class with a Continuum of grades of membership. A fuzzy set for class is define in such a way that every element of the set is having a correspondence with a real number from the set of interval between 0 to 1.

High grade of membership of that element is considered when its correspondence is with the real number that is close to 1 and a low grade of membership is considered for that element when the correspondence is with the real number that is close to zero. For other than fuzzy set the element was supposed to be either in the set or outside the set that means only the possibility 1 and 0 one was there Zadeh (1965).

Chang (1999) assumed product quantity "q" as triangular fuzzy number $\tilde{Q}$ such that $\tilde{Q} = (q_1, q_0, q_2)$ and defined a membership function.

Given by Jaggi, Pareek, and Sharma (2013), a fuzzy number (q_0, q_1, q_2) where $q_0 < q_1 < q_2$ and defined on R , is called a triangular fuzzy number by its continuous membership function $\mu_a(x) : x \rightarrow$[0,1] is

$$\mu_a = \begin{cases} \dfrac{x - q_0}{q_1 - q_0}, & q_0 \leq x \leq q_1 \\ \dfrac{q_2 - x}{q_2 - q_1}, & q_1 \leq x \leq q_2 \\ 0, & Otherwise \end{cases}$$

For defuzzification of the total cost function, Graded Mean Representation, Signed Distance and Centroid methods can be used.

The graded mean integration representation of $\tilde{Q}$, for $\tilde{Q} = (q_0, q_1, q_2)$, a triangular fuzzy number, is defined as $P(\tilde{Q})\ \dfrac{q_0 + 4q_1 + q_2}{6}$ The centroid method on $\tilde{Q}$, for $\tilde{Q} = (q_0, q_1, q_2)$, a triangular fuzzy number, is defined as $P(\tilde{Q})\ \dfrac{q_0 + q_1 + q_2}{3}$ The signed distance of $\tilde{Q}$, for $\tilde{Q} = (q_0, q_1, q_2)$, a triangular fuzzy number, is defined as $P(\tilde{Q})\ \dfrac{q_0 + 2q_1 + q_2}{4}$

1.2 Motivation towards the work

A finite planning horizon model with exponentially decreasing demand Lin, Chao, and Julian (2013) discussed that practicing with finite planning horizon model is more proper than infinite model. But Lin, Chao, and Julian (2013) did not considered credit period rate in a supply chain management.

Covert and Philip (1973) used two Parameter Weibull Deterioration since many items in an inventory follows the same such as important drugs, batteries and others.

Fig. 1.1 shows that if two parameter Weibull deterioration when $1<\beta<2$ is for very high deterioration rate in the begining which slows down with respect to time. The rate of deterioration with respect to time is shown in Fig. 1.1 for different values of β. Two Parameter Weibull Deterioration with backlogging is discussed by Rajeswari and Vanjikkodi (2012) but the planning horizon was taken as infinite and is not a supply chain model.

A research paper with Weibull Deterioration, decreasing demand, trade credit, allowing shortages with fuzzy logic by Majumder, Bera, and Maiti (2015) was discussed in an infinite planning horizon and the demand taken was not quadratic.

To the best of our knowledge none of the researchers has considered an inventory model with permissible delay in payments, time quadratic and inventory dependent demand, item deterioration, shortages with partial backlogging and lost sales in all cycles under a finite planning horizon with credit period rate. In this research we have assumed time quadratic and stock dependent demand, time proportional deterioration of item, shortages in every cycle with partially backlogging because of lost sales in a finite planning horizon under permissible delay in payments. The research proposed here is a generalized one as particular cases can be derived by relaxing one or more parameters assumed. Further considering fuzzy logic the scope of generalizing the model will cease to exist.

1.3 Objectives and contribution

To study a generalized inventory model with permissible delay in payments, time quadratic and inventory dependent demand, item deterioration, shortages with partial backlogging and lost sales in all cycles under a finite planning horizon with credit period rate the following objectives are taken into consideration which would satisfy the need of inventory management systems in times to come.

1. To study the model of supplier–retailer inventory coordination with credit term for deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles.

2. Determination of supplier–retailer inventory coordination with credit term for short life-cycle deteriorating product re-manufacturing in a green supply chain inventory control system with inventory dependent and linear-trend demand.

3. To extend the supplier–retailer inventory coordination with credit term for short life-cycle deteriorating product re-manufacturing in a green supply chain inventory control system to time-quadratic demand and time-dependent partial backlogging with shortages in all cycles.

4. To study fuzzyfication of supplier–retailer inventory coordination with credit term for deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles.

1.4 Areas of application

It is by the country Japan that Just In Time (JIT) concept was introduced in the manufacturing units. But De Haan and Yamamoto (1999) established the fact and stated that JIT is a total fiction. The case study dealt with big firms like Toyota, japan for product as cars with 2000 employees keeping inventory of Steel plates and other parts also firms like Shinitetsu Oita, Japan for Steel as product keeping inventory of Coal, lime, iron ore etc. Similar study was made for many more such firms. De Haan and Yamamoto (1999) found that instead of within an outside firm inventory was maintained.

Inventory holding is for different items raw or finalized product such as Fibers, Steel, Power devices, Cars, Shoes, Watertaps, Coal, lime, iron ore, Barley, malt hop, bottles, toys, Textile, rubber, strings, Steel plates, pipes, parts etc.

A good inventory management by a retailer,dealer or manufacturer helps to reduce overcome the inventory crisis situation and thus reduces loss and in return increases profit. Inventory management accounts for timely order placement. Without a proper inventory model considering the type of product into consideration precise quantity required cannot be ordered. Inventory carrying charges for unutilized raw material can be minimized by keeping a check on inventory demand. Similarly for the much favored product in market a shortage can be avoided thus saving time of procurement. Avoiding inventory backlog increases the market goodwill and keep the customers satisfied.

1.5 Basic notions of inventory management/concepts

Rate of demand

It has been observed that for different product different types of demand exists. Some of the major types of demands are constant demand Ghare and Schrader (1963), linear demand Wu and Zhao (2014a) , time-Quadratic demand Sarkar, Ghosh, and Chaudhuri (2012b), exponential demand Ouyang, Wu, and Cheng (2005), demand Price sensitive Dowlatshahi and Heidari (2013), fluctuating Rameswari (2012), demand with trapezoidal rate Hsieh (2013), demand wwhich is inventory Dependent Wu and Zhao (2014a) and many others.

Lead time

The time retailer places and order to the time supplier delivers an inventory is called the lead time. In today's scenerio when there are faster way of communication there happens to be no lead time and therefore in my reserach lead time is considered as zero .

Shortages

Shortage of the goods in an inventory occurs when there are fewer goods available as compared to demand this may occur due to several factors. Factors may vary from suppliers machine non functioning to transportation delay by the retailer during a cycle in a supply chain model Ghiami, Williams, and Wu (2013), Agrawal, Banerjee, and Papachristos (2013), Sarkar, Ghosh, and Chaudhuri (2012a), Ghoreishi, Weber, and Mirzazadeh (2014), and Bhunia, Shaikh, and Gupta (2015).

Rate of deterioration

Deterioration of the product within the system may occur due to various reasons such as wear Klein and Rosenberg (1960), spoilage , rusting, spillage, change or decay Ghare and Schrader (1963) and therefore is important factor to be taken into account as it adds for profit loss.

Different cost associated

The different cost associated with the inventory system of a supply chain model that are considered are as follows:

1. Item or the product purchase cost per unit by retailer
2. Ordering cost of the retailer,
3. Setup cost of the supplier,
4. Opportunity cost
5. deterioration cost
6. remanufacturing cost
7. cost of scrutiny for defected items
8. transportation cost for recycling
9. Holding costs (Carrying Costs)

10. Shortage Costs

11. cost of lost sales

1.5.1 Factors affecting inventory control

Demand: Number of units required throughout the planning horizon is called as demand. Demand can be fluctuating or constant during the finite planning horizon. It may be decreasing or increasing during a cycle.

Replenishment time: The time for ordering the inventory is called as replenishment time or ordering time.

Order period: The length of time between one replenishment to another is called order period or length of the cycle.

Ordering quantity: The number of units to be ordered by retailer in the beginning of each cycle is called ordering quantity.

Credit period rate: The credit period offered to retailer by supplier within each cycle to close the immediate previous account is called the credit period rate.

1.6 Methodology used

Examine the inventory problem and its environment

In inventory optimal values of three types of cost i.e. cost of carrying, cost of shortage of inventory and cost of replenishment is quite very significant. These above mentioned 3 types of cost are often very closely related to each other. When one of the cost is decreased or the cost is increased any one or the rest of the two of the other two cost and sometimes even both may increases there is thus the problem of controlling the cost so that there sum will be lowest. It is very tough and challenging question to control the inventory. Many concept and techniques were proposed by mathematicians for controlling the inventory eectively.

Analyze and define the problem

To control the inventory effectively we should consider two questions mainly.

1. When should inventory be replenished for different cycles during the planning horizon?

2. How much should be replenished during the each cycle for the given planning horizon?.

Thus the time element and quantity element are variable that one subject to control inventory system. Inventory problem is to find the specific values of the variables that minimize the total cost and minimum cost is obtain when the carrying cost and replenishing cost are literally balanced. Analysis of an inventory system consists of the following steps:

1. Determination of the properties of the system.
2. Development of formulation of inventory problem.
3. Development of a model of the system.
4. Derivation of a solution of the system.

To solve the model we shall use the standard techniques available, along with numerical methods, approximation methods or any other suitable method. We shall seek the numerical implication to visualize its practical importance. For these numerical calculations, I have used software Mathematica version 8.0.

General methods for solving operations research models

In general, the following three methods are used for solving OR models. In all these models, values of decision variables are obtained that optimize the given objective function (a measure of effectiveness).

1. Analytical (or Deductive) Method In this method, classical optimization techniques such as calculus, finite difference and graphs are used for solving an OR model. In this case, a general solution have been specified by a symbol and the optimal solution can be obtained in a non iterative manner.

2. Numerical (or Iterative) Method When analytical methods fail to obtain the solution of a particular problem due to its complexity in terms of constraints or number of variables, a numerical (or iterative) method is used to get the solution. In this method, instead of solving the problem directly, a general algorithm is applied to obtain a specific numerical solution. The numerical method starts with a solution obtained by trial and error and a set of rules for improving it towards optimality. The solution so obtained is then replaced by the improved solution and the process of getting an improved solution is repeated until such improvement is not possible or the cost of further calculation cannot be justified.

3. Monte Carlo Method

   With Monte Carlo method estimation of a random variable is calculated. The inventory mathematical model is considered to follow a probability distribution. The random variable which follows the probability distribution of the inventory model is chosen for the estimation under defined criteria and circumstances.

Constructing an numerical model

The decision making model consists of the following basic constituents:

1. Decision variables

   The numerical value of the decision variable has to be obtained which decides the validity of the model the numerical value of the decision variable also provides a decisive alternative for the model so formulated the main aim of the model formulated is to obtain the the correct value numerical value of the decision variables through iterative methods

2. Non-governable variables

   Many a times decision variables are dependent upon some other variables which are which cannot be controlled directly by the the examiner these are called non govern able variables the non-governable variables are are influenced by natural causes of phenomena but these non-governable variables contribute considerable amount it for the the solution of the model

   The objective of the model the objective of the model is to minimise the total cost incurred by the retailer as well as the supplier the total cost constitutes of of the various other cost under consideration

3. Constraints of the model

   The decision variable the non-governable variables and the cost considered in the model comes with restriction for example the the inventory replenishment for the order cannot be negative also that the demand cannot be negative There are several other cost incurred by both retailer and supplier for example holding cost which cannot be considered as negative similarly there are other constraints for the model under examination

   In my research there are four models which has been considered basically the the models can be of two types linear model and nonlinear model a linear model is such that all the variables x_1,x_2, x_3 ......, x_n as well as $f(x)$ and $g(x)$ are of linear form ($f(x)$= criterion or objective function to be optimized, $g(x)$ = constraint) the decision variables non-governable variables for constraints all all are linear within the equations but a nonlinear model which I have considered is is nonlinear in nature in such a way that there are variables which are not linear within the equations these equations. Such as the starting differential equations of the model and the total cost equations.

4. Solution and Screening

In this step the solution of the problems is obtained. With the help of data input this nonlinear model is solved by the iterative method and a numerical solution is obtained with the help of software mathematical version 8.0.

5. Execution of the Solution

   Operations research is further subdivided into many categories. One such is inventory theory. The solution obtained has to be implemented and the conflict between the players has to be reduced. the execution of the solution of the model is necessary to facilitate the retailer and supplier with the theory of conflict free collaboration. Therefore the implementation of the generalized model after relaxing the variables to accomodate the organizational requirements will be the last phase.

Differential equations

Consider now a linear trend in demand over (0,t) given by $f(u) = a + bu$, $b \geq 0$, and a necessarily positive or zero and θ_1 the rate of deterioration of inventory. The inventory level at any time u is given by differential equation

$$\frac{dI(u)}{dt} + (\theta_1)I(u) = -f(u), 0 \leq u \leq t(1.1)$$

Optimization

1. **Local Minima** The function has a local minimum point at x^* if $f(x^*) \leq f(x)$ for all x in domain X within distance of x^*

2. **Extreme Value Theorem** Mathematical Optimization is the technique to find the the minima or a Maxima a minimum value or a maximum value does exist if the function is continuous within the interval as per extreme value theorem.

3. **Hessian Matrix** Definition(Uthayakumar (2015)):- Let f (x) be a function in n variables. The Hessian matrix of f is the matrix consisting of all the second order partial derivatives of f. That is, the Hessian matrix of f at the point x is the nXn matrix. And f is strictly convex $\iff$ H is positive definite. A matrix is positive definite if it's symmetric and all its eigenvalues are positive as in 2.25.

Use of computers mathematica version 8.1

Following an algorithm and writing a program for the software Mathematica version 8.1 the optimal total cost for the retailer can be obtained which is in bold font table below . See appendix A for the program.

	$\rightarrow n_1$						
a_1	1	2	3	4	5	6	7
1	376.68	278.85	**255.7**	261.42	280.42	306.4	336.44

TABLE 1.1: Total cost of retailer in a decentralized system

1.7 Organization of thesis

The thesis is divided into eight chapters. The organization of the thesis is as follows:

Chapter 1 is about the introduction.

Chapter 2 deals with the study of a single buyer, single supplier inventory model with time quadratic and stock dependent demand for a finite planning horizon. Single deteriorating item which suffers shortage with partial backlogging and some lost sales is considered. Model is divided into two scenario one with no permissible delay in payment and other with permissible delay in payment. Later is called centralized system where supplier offers the retailer, trade credit. In the centralized system cost saving is shared amongst the two. The objective is to study the difference in minimum total cost born by retailer and supplier under two scenarios including above mentioned parameters. To obtain optimal solution of the problem the model is solved analytically. Numerical example and a comparative study is then discussed supported by sensitivity analysis of each parameter.

Chapter 3 presents a detail solution of re-manufacturing of a product in a supply chain model. It is a non-traditional model considering time-dependent quadratic demand, Weibull deterioration, shortages, partial backlogging and re-manufacturing of inventory. This paper mainly focuses on remanufacturing and hence an attempt towards reducing the environmental hazard. The process of remanufacturing is completed within one cycle of replenishment. Trade credit between supplier and retailer also had been discussed. Two cases one of a centralized and the other of decentralization for a finite planning horizon in a supply chain model are discussed. An algorithm has been derived for solving a problem in both the cases. Some managerial insights are talked about on the basis of sensitivity analysis on the parameters considered.

Chapter 4 discusses clearly about recycling of an item within the planning horizon. Recycling of an item has become the natural requirement in inventory handling. It decreases the burden of inventory for defective kind of items. Another obvious phenomenon is deterioration of items in inventory. Hence two-parameter Weibull deterioration of items is considered in this chapter. The idea is to introduce in a supply chain model some greenness through recycling of defective items after the sorting process.

Chapter 5 deals with a supply chain model which is discussed for fuzzy parameters such as fuzzy deterioration cost, fuzzy holding cost fuzzy inventory carrying cost etcetera are considered for framing of the model which are later defuzzified using Centroid, Signed Distance and Graded Mean Representation method. Centralized replenishment policy in this finite planning horizon model is discussed along with sensitivity analysis.

Appendix A contains the excerpts of the Mathematica program for table formulation submitted for copyright. In appendix B the list of published, accepted and communicated research is provided followed with bibliography.

1.8 Summary

Existing knowledge about the structure and properties of a specific sub problem can be exploited in solving integrated models. Many more opportunities are still unexplored. This research field thus remains very active.

Chapter 2

Determination of credit period for a model with coordination between supplier and retailer

2.1 Abstract

In this chapter a single buyer, single supplier inventory model with time quadratic and stock dependent demand for a finite planning horizon has been studied. Single deteriorating item which suers shortage, with partial backlogging and some lost sales is considered. Model is divided into two scenarios, one with non permissible delay in payment and other with permissible delay in payment. Latter is called, centralized system, where supplier oers trade credit to retailer. In the centralized system cost saving is shared amongst the two. The objective is to study the dierence in minimum costs borne by retailer and supplier, under two scenarios including the above mentioned parameters. To obtain optimal solution of the problem the model is solved analytically. Numerical example and a comparative study are then discussed supported by sensitivity analysis of each parameter.

2.2 Introduction

The available literature of different inventory models (Wu and Zhao (2014b) etc.) reveals that the relation between retailer and supplier is becoming more stable in today's rapidly changing commercial world. The supplier steps forward and provides retailer, the credit period for the settlement of the amount for quantity purchased. However, the coordination between supplier and retailer depends upon kind of the product, its deterioration, its demand etc. Need is to focus on type of product produced and launched.

It is due to the fact that long term relationship between retailer and supplier is the key to success of both the parties. Many researchers have worked on the models which strengthens the bond between retailer and supplier. Few of the authors like Banerjee

(1986) Goyal (1988), Yang and Wee (2000), Sarker, Jamal, and Wang (2000), Chung (2000) have presented their model for deterioration in a two level supply chain coordinate system.

Goyal and Giri (2001) continued Raafat's review work for permissible delay in payment, price increase and price discount. An EOQ model which assumes that full payment must be done by the retailer immediately after receiving the goods from the supplier is a decentralized system where the supplier replenishes according to retailers optimal quantity requirement on cycle to cycle basis. However, this is not practical, as the supplier may offer the trade credit period to retailer i.e. a delay period for the full payment (purchase cost) of the goods in a centralized system. Some of the related articles with trade credit financing which includes deterioration of item/s can be found in Wu and Wee (2001), Chang, Ouyang, and Teng (2003), Chang (2004), Ouyang, Wu, and Cheng (2005), Chung and Liao (2006), Lo, Wee, and Huang (2007), Chung and Huang (2007), Jaggi and Verma (2009), Chung and Lin (2011), and Hou and Lin (2013) etc. and in their references.

With permissible delays in payments and assuming no shortages Chung and Huang (2007) presented an inventory model. Liao and Huang (2010) investigated a similar model to Chung and Huang (2007) by adopting a different approach. Huang (2007), LiangLiang and Zhou (2011) investigates an inventory model under conditionally permissible delay in payment. Guria et al. (2013) framed an inventory policy for an item with inflation and demand depending on selling price allowing and not allowing shortages with one of the provision being, immediate part payment to the wholesaler.

Shah, Patel, and Shah (1988) extended the model of Goyal (1985) by allowing shortages. Shortages or stock-out situation are likely to occur due to many conditions in any commercial or industrial enterprise. Therefore consideration of stock-out in any inventory model is essential in today's scenario. It is assumed that the retailer on receiving of the ordered quantity from supplier satisfies the customer waiting and then stocks the left over goods for his regular demand. But, all the customers waiting in queue naturally cannot wait for stock to arrive because of their impatience or other sources available in the vicinity. Thus a ratio of customers waiting goes into lost sales. The shortages are then considered as partially backlogged. Abad; (1996) introduced a model for customer's impatience function. But it was Jamal, Sarkar, and Wang (1997) who introduced a model assuming shortages for deteriorating items with permissible delay in payments. Researchers such as Chang and Dye (2001), Ouyang, Teng, and Chen (2006), Jaggi, Khanna, et al. (2010), Jaggi and Mittal (2012), Yang and Chang (2013), and Bhunia et al. (2014) in their models considered deteriorating items with partial backlogging and permissible delay in payments. . Lost sale was considered by Dye, Hsieh, and Ouyang (2007) who modified the model of Abad (1996). Although many authors has considered partial backlogging, deterioration, with permissible delay in payments but, not much literature is found which includes study of lost sales, partial backlogging, deterioration with permissible delay in payments.

As stated in Sarkar, Ghosh, and Chaudhuri (2012b) time-dependent quadratic demand function is better in all sense. Khanra and Chaudhuri (2003) and Ghosh and Chaudhuri (2006) and Manna, Chaudhuri, and Chiang (2007), are also some of the researchers who used the time quadratic demand function in their research work.

Gupta and Vrat (1986) introduced an inventory model for stock-dependent consumption rates. Baker and Urban (1988) introduced that the demand rate decreases with the decrease in inventory where the demand rate being inventory level dependent. developed an EOQ model with partially permissible delay in payments linked to order quantity for deteriorating items. Later, Min et al. (2012) and Sarkar, Ghosh, and Chaudhuri (2012c) studied inventory model for inventory-level-dependent demand and permissible delay in payments.

To the best of our knowledge none of the authors has considered an inventory model with permissible delay in payments, time quadratic and inventory dependent demand, item deterioration, shortages with partial backlogging and lost sales in all cycles under a finite planning horizon with credit period rate. In this model we have assumed time quadratic and stock dependent demand, time proportional deterioration of item, shortages in every cycle with partially backlogging because of lost sales in a finite planning horizon under permissible delay in payments. The model proposed here is a generalized one as particular cases can be derived by relaxing one or more parameters assumed.

The flow of the rest of the chapter is organized as follows: In Section 2 we have provided the assumptions and notations, in section 3 a mathematical model is developed and its solution is provided, section 4 contains optimality conditions for cost equations, in section 5 we have given algorithm, with a numerical example and sensitivity analysis, section 6 is for conclusions.

2.3 Assumptions and notations

2.3.1 Assumptions

1. Inventory level is zero initially.
2. Lead time is zero.
3. The deterioration of items is proportional to time.
4. The ordering cost, holding cost and shortage cost are constant during the planning horizon.
5. Shortages exist for each cycle and are partially backlogged. It is assumed that only a fraction $B(\tau)$of the demand during the stock-out period is backlogged as the customers are impatient. $'\tau'$ is the amount of time for which the customers

waits before receiving goods and the remaining fraction $[1 - B(\tau)]$ is lost. Here $B(\tau) = \dfrac{1}{1+\delta\tau}, \delta > 0.dot$

6. There is no repair or replacement of the deteriorated items.
7. Supply chain consist of single item with single retailer and single supplier.
8. Lot for lot replenishment policy is followed by the supplier.
9. The unit capital cost is same for the retailer and the supplier.
10. The ordering cost of the retailer is less than the set up cost of the supplier.
11. There is no inventory with the supplier as the production rate is infinite and the replenishment is instantaneous.

2.3.2 Notations

For retailer

1. H is planning horizon which is finite and fixed.
2. The demand rate $f(t) = a_1 + b_1 t + c_1 t^2, a_1 \geq 0, b_1 \neq 0, c_1 \neq 0$ at time $t(> 0)$is a continuous function of time, where a_1, b_1,and c_1 are constants.
3. A variable fraction θ_2 =αt of the on-hand inventory deteriorates per unit of time where $0 < \alpha < 1$.
4. $I_{hr}(\$/unit/yr)$, is the inventory holding cost where capital cost is excluded.
5. $C_c(\$/unit/yr)$ is the capital cost for the retailer as well as for the supplier.
6. $h(\$/unit/yr)$ is the inventory cost, where $h = I_{hr} + C_c$.
7. $O_r(\$/order)$ is the ordering cost of the retailer
8. $P_r(\$/unit)$, is the purchasing cost of the retailer.
9. s($\$/unit/yr)$ is the shortage cost of the retailer.
10. l($\$/unit/yr)$ is the cost of lost sales.
11. θ_1 is the stock dependent demand rate.

12. $I_i^D(t)$ is the level of inventory at time t where $t \in [t_i, s_{i+1}]$, $\{i = 1, 2, ..., n_1\}$ for a decentralized system and $I_j^C(t)$ is the level of inventory at time t where t∈[t$_j^C$, s_{j+1}^C], $\{j = 1, 2, ..., n_2\}$ for a centralized system.

13. R_i^D is the total inventory for the interval $[t_i, s_{i+1}]$, $\{i = 1, 2, ..., n_1\}$ for decentralized system and for centralized system R_j^C is the total inventory for the interval $[t_j^C, s_{j+1}^C]$, $\{j = 1, 2, ..., n_2\}$.

14. S_i^D is the total amount of shortages in the interval $[s_i, t_i]$, $\{i = 1, 2, ..., n_1\}$ for decentralized system and for centralized system S_j^C is the total amount of shortages in the interval $[s_j^C, t_j^C]$, $\{j = 1, 2, ..., n_2\}$.

15. $Q_i^D = R_i^D + S_i^D$, is the quantity ordered at the time t_i for i^{th} cycle $\{i = 1, 2, ..., n_1\}$ in decentralize system similarly for centralized system $Q_j^C = R_j^C + S_j^C$, is the quantity ordered at the time t_j^C for j^{th} cycle $\{j = 1, 2, ..., n_2\}$

16. D_i^D is the total amount of deteriorated items in the i^{th} replenishment cycle $\{i = 1, 2, ..., n_1\}$ and D_j^C is the total amount of deteriorated items in the j^{th} replenishment cycle $\{j = 1, 2, ..., n_2\}$ for decentralized and centralized system respectively.

17. L_i^D is the total quantity of lost sales for the interval $[s_i, t_i]$, $\{i = 1, 2, ..., n_1\}$ and L_j^C is the total quantity of lost sales for the interval $[s_j^C, t_j^C]$, $\{j = 1, 2, ..., n_2\}$ for decentralized and centralized system respectively.

18. M_j^C is the credit period offered by the supplier to retailer for j^{th} cycle $\{j = 1, 2, ..., n_2\}$ in centralized system.

19. $B(\tau) = \dfrac{1}{1 + \delta\tau}$,$\delta > 0$, is the backlogging rate and τ is the time that customer has to wait.

20. T_i^D is the length of the i^{th} replenishment cycle, $\{i = 1, 2, ..., n_1\}$ and T_j^C is the length of the j^{th} replenishment cycle, $\{j = 1, 2, ..., n_2\}$ for decentralized and centralized system respectively.

21. TC_r^D and TC_r^C is the total cost of the retailer in decentralized and centralized system repectively during the planning horizon H.

For supplier

1. S_s($/order) is the setup cost per order.

2. P_s($/unit) is the purchasing cost per unit and $P_s < P_r$.

3. TC_s^D and TC_s^C is the total cost for decentralized and centralized system respectively during the planning horizon H.

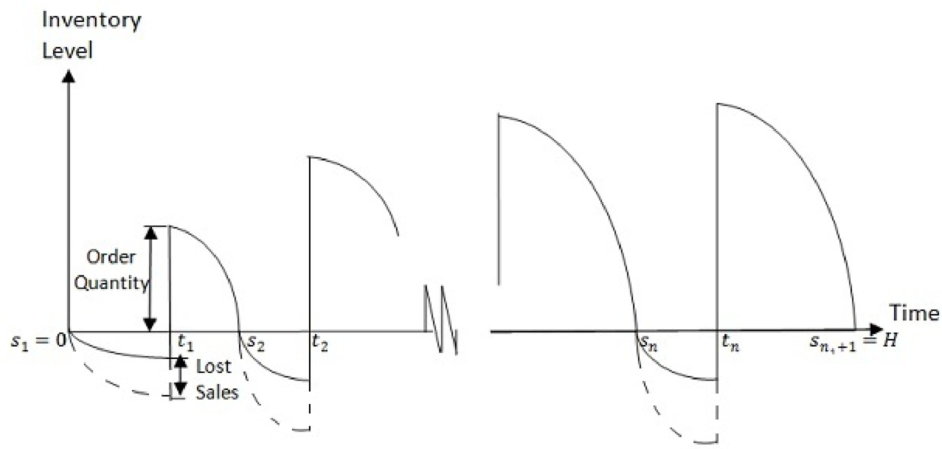

FIGURE 2.1: Graphical representation of Inventory Model

2.3.3 Decision variables

1. t_i $\{i = 1, 2, ..., n_1\}$ and t_j^C $\{j = 1, 2, ..., n_2\}$ are the replenishment time during decentralized and centralized system respectively.

2. s_i $\{i = 1, 2, ..., n_1\}$ and s_j^C $\{j = 1, 2, ..., n_2\}$ are the time for decentralized and centralized system respectively where the inventory level reaches zero. Also that $s_1 = 0$ and $s_{n_1+1} = H$.

3. n_1 and n_2 are the total number of replenishment cycles for decentralized and centralized system respectively for the planning horizon H.

4. λ is the credit period rate.

2.4 Mathematical formulation and solution of the model

Fig. 1 illustrates the carry of inventory during n_1 replenishment cycles. Initially the inventory level is zero and shortages starts accumulating. First replenishment is obtained at time t_1. A portion of which is used to meet the shortages accumulated during the interval $[s_1, t_1]$ and rest is utilized to fulfill the demand and deterioration during the interval $[t_1, s_2]$. The inventory is consumed and falls to zero at s_2. This repeats for every cycle $[s_i, s_{i+1}]$ where $\{i = 1, 2, ..., n_1\}$.

Here we consider two cases, one for decentralized and other for centralized system. Case 1 is for decentralized system. In this case the supplier replenishes the inventory ordered as per the optimum needs of retailer.

Case 1: Decentralized system

In this system the supplier follows the optimal replenishment cycles of the retailer. The instantaneous level of inventory $I_i^D(t)$ with boundary condition $I_i^D(s_{i+1}) = 0$ is given by following differential equation

$$\frac{dI_i^D(t)}{dt} + (\theta_1 + \theta_2)\, I_i^D(t) = -f(t), t_i \leq t \leq s_{i+1}, \{i = 1, 2, \ldots, n_1\}. \tag{2.1}$$

The instantaneous level of shortage $IS_i(t)$ with the boundary condition $IS_i(s_i) = 0$, is given by following differential equation

$$\frac{dIS_i(t)}{dt} = f(t)B(t) = \frac{f(t)}{1 + \delta\,(t_i - t)}, s_i \leq t \leq t_i, \{i = 1, 2, \ldots, n_1\} \tag{2.2}$$

Taking $\theta_2 = \alpha$t $(0 < \alpha < 1)$, the solution of the differential equation 1 is

$$I_i^D(t) = \int_t^{s_{i+1}} e^{\theta_1(u-t)+\frac{\alpha}{2}\left(u^2-t^2\right)} f(u)du, t_i \leq t \leq s_{i+1}, \{i = 1, 2, \ldots, n_1\}. \tag{2.3}$$

The total amount of inventory carried during the interval $[t_i, s_{i+1}]$ is given by

$$R_i^D = \int_{t_i}^{s_{i+1}} \left\{ \int_t^{s_{i+1}} e^{\theta_1(u-t)+\frac{\alpha}{2}\left(u^2-t^2\right)} f(u)du \right\} dt, \{i = 1, 2, \ldots, n_1\}. \tag{2.4}$$

By applying the change in the order of integration and neglecting α^2 and higher powers of α , we get

$$R_i^D = \int_{t_i}^{s_{i+1}} \left[\left(1 + \theta_1 t + \frac{\alpha}{2} t^2\right)(t - t_i) - \frac{\theta_1}{2}\left(t^2 - t_i^2\right) - \frac{\alpha}{6}\left(t^3 - t_i^3\right) \right] f(t)dt,$$
$$\{i = 1, 2, \ldots, n_1\} \tag{2.5}$$

The total amount of shortage during the i^{th} cycle in the interval $[s_i, t_i]$ is $S_i^D = \int_{s_i}^{t_i} IS_i(t)dt = \int_{s_i}^{t_i} \left(\int_{s_i}^{t_i} \frac{f(t)}{1 + \delta\,(t_i - t)} dt \right) dt = \int_{s_i}^{t_i} \frac{(t_i - t)\, f(t)}{1 + \delta\,(t_i - t)} dt,$

$$\{i = 1, 2, \ldots, n_1\}. \tag{2.6}$$

Total quantity during the planning horizon =$Q^D = \sum_{i=1}^{n_1} Q_i^D = \sum_{i=1}^{n_1}\left(R_i^D + S_i^D\right)$.
The total number of items deteriorated during the i^{th}replenishment cycle is

$$D_i^D = \int_{t_i}^{s_{i+1}} \theta_2 I_i^D(t)dt, \{i = 1, 2, \ldots, n_1\}. \tag{2.7}$$

$$D_i^D = \int_{t_i}^{s_{i+1}} \alpha t \left[\left(1 + \theta_1 t + \frac{\alpha}{2}t^2\right)(t - t_i) - \frac{\theta_1}{2}\left(t^2 - t_i^2\right)\right.$$

$- \frac{\alpha}{6}\left(t^3 - t_i^3\right) f(t)dt, \{i = 1, 2, \ldots, n_1\}$.(2.8)

The quantity lost during the ith replenishment cycle in the interval $[s_i, t_i]$ is

$$L_i^D = \int_{s_i}^{t_i} [f(t) - f(t)B(t)]\, dt = \int_{s_i}^{t_i} \frac{\delta(t_i - t) f(t)}{1 + \delta(t_i - t)} dt, \{i = 1, 2, \ldots, n_1\}. \tag{2.9}$$

The total cost for retailer consist of ordering cost, inventory cost (including capital oppurtunity cost), purchasing cost, shortage cost, deterioration cost and the cost of lost sales. Therefore total cost of retailer is

$TC_r^D(n_1, s_1, t_1, \ldots, s_{n_1+1}) = n_1 O_r + h\sum_{i=1}^{n_1}\int_{t_i}^{s_{i+1}} I_i^D(t)dt + P_r\sum_{i=1}^{n_1} Q_i^D + P_r\sum_{i=1}^{n_1} D_i^D + s\sum_{i=1}^{n_1} S_i^D + l\sum_{i=1}^{n_1} L_i^D$

$= n_1 O_r + (h + P_r)\sum_{i=1}^{n_1}\int_{t_i}^{s_{i+1}} \left[\left(1 + \theta_1 t + \frac{\alpha}{2}t^2\right)(t - t_i) - \frac{\theta_1}{2}(t^2 - t_i^2) - \right.$
$\frac{\alpha}{6}(t^3 - t_i^3) f(t)dt$

$+P_r\sum_{i=1}^{n_1}\int_{t_i}^{s_{i+1}} \alpha t \left[\left(1 + \theta_1 t + \frac{\alpha}{2}t^2\right)(t - t_i) - \right.$
$\frac{\theta_1}{2}(t^2 - t_i^2) - \frac{\alpha}{6}(t^3 - t_i^3) f(t)dt$

$$+ (P_r + s + l\delta)\sum_{i=1}^{n_1}\int_{s_i}^{t_i} \frac{(t_i - t) f(t)}{1 + \delta(t_i - t)}dt, \{i = 1, 2, \ldots, n_1\}. \tag{2.10}$$

To minimize TC_r^D the values of t_i and s_i are to be determined. The necessary condition for TC_r^D to be minimum are
$\frac{\partial TC_r^D(t_i, s_i; n_1)}{\partial t_i}$=0 and $\frac{\partial TC_r^D(t_i, s_i; n_1)}{\partial s_i}$=0.
Where $\frac{{}_r^D(t_i, s_i; n_1)}{\partial t_i} = (h + P_r) \int_{t_i}^{s_{i+1}} \left[\frac{\alpha}{2}(t_i^2 - t^2) \right.$
$+\theta_1(t_i - t) - 1 f(t)dt$

$$+P_r \int_{t_i}^{s_{i+1}} \alpha t \left[\theta_1 (t_i - t) - 1\right] f(t)dt + (P_r + s + l\delta) \int_{s_i}^{t_i} \frac{1}{1 + \delta (t_i - t)^2} f(t)dt \quad (2.11)$$

as α is very small $\alpha^{2)}$ and higher terms are neglected.

$$\frac{{}_r^D(t_i, s_i; n_1)}{\partial s_i} = (h + P_r) \left[\left(1 + \theta_1 s_i + \frac{\alpha}{2} {s_i}^2 \right) (s_i - t_{i-1}) - \frac{\theta_1}{2} \left({s_i}^2 - t_{i-1}^2 \right) - \right.$$
$$\frac{\alpha}{6} \left({s_i}^3 - t_{i-1}^3 \right) f(s_i)] + P_r \alpha s_i \left[(1 + \theta_1 s_i)(s_i - t_{i-1}) - \frac{\theta_1}{2} \right.$$
$$\left({s_i}^2 - t_{i-1}^2 \right) f(s_i) - (P_r + s + l\delta) \frac{(t_i - s_i)}{1 + \delta (t_i - s_i)} f(s_i) \quad (2.12)$$

as α is very small α^2 and higher terms are neglected.

Let the optimal solution (see section 2.5 for optimality condition) obtained from equation

$$TC_s^D \left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H \right) = n_1^{DO} S_s + \sum_{i=1}^{n_1^{DO}} P_s Q_i^{DO}$$
$$= n_1^{DO} S_s + \sum_{i=1}^{n_1^{DO}} P_s \left(R_i^{DO} + S_i^{DO} \right). \quad (2.13)$$

And the total optimal order quantity during the planning horizon H is

$$Q^{DO} = \sum_{i=1}^{n_1^{DO}} Q_i^{DO} = \sum_{i=1}^{n_1^{DO}} \left(R_i^{DO} + S_i^{DO} \right). \quad (2.14)$$

Case 2: Centralized system

In this system supplier calculates its replenishment cycle schedule and lures the retailer to follow the same on basis of sharing the cost saving. Since the supplier offers his new optimal number of replenishment cycle n_2^{CO} which is $\leq$ the retailers optimal cycle for decentralized system n_1^D the inventory carrying cost of retailer increases and suppliers set up cost decreases. Since the retailer's ordering cost is much less than supplier's set up cost, centralized system results in extra cost saving which is further shared by both.

The total cost for retailer with n_2 replenishment cycle will be
$TC_r^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C\right) = n_2O_r + h\sum_{j=1}^{n_2} R_j^C + P_r\sum_{j=1}^{n_2} Q_j^C + P_r\sum_{j=1}^{n_2} D_j^C + s\sum_{j=1}^{n_2} S_j^C + l\sum_{j=1}^{n_2} L_j^C$

$= n_2O_r + (h+P_r)\sum_{j=1}^{n_2}\int_{t_j^C}^{s_{j+1}^C}\left[\left(1+\theta_1 t+\frac{\alpha}{2}t^2\right)\left(t-t_j^C\right) - \frac{\theta_1}{2}\left(t^2-\left(t_j^C\right)^2\right) - \frac{\alpha}{6}\left(t^3-\left(t_j^C\right)^3\right)f(t)dt\right.$
$+P_r\sum_{j=1}^{n_2}\int_{t_j^C}^{s_{j+1}^C}\alpha t\left[\left(1+\theta_1 t+\frac{\alpha}{2}t^2\right)\left(t-t_j^C\right) - \frac{\theta_1}{2}\left(t^2-\left(t_j^C\right)^2\right) - \frac{\alpha}{6}\left(t^3-\left(t_j^C\right)^3\right)f(t)dt\right.$

$$+(P_r+s+l\delta)\sum_{j=1}^{n_2}\int_{s_j^C}^{t_j^C}\frac{\left(t_j^C-t\right)f(t)}{1+\delta\left(t_j^C-t\right)}dt, \{j=1,2,\ldots,n_2\}. \quad (2.15)$$

The retailer's increase in cost for centralized system is given by

$$TC_r^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C\right) - TC_r^D(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO}=H). \quad (2.16)$$

The total cost of supplier including the retailers increase in cost is
$TC_s^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C\right) = n_2S_s + \sum_{j=1}^{n_2} P_sQ_j^C + TC_r^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C\right) - TC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO}=H\right) = n_2\left(S_s+O_r\right)$
$+\sum_{j=1}^{n_2}\left(h+P_r+P_s\right)\int_{t_j^C}^{s_{j+1}^C}\left[\left(1+\theta_1 t+\frac{\alpha}{2}t^2\right)\left(t-t_j^C\right) - \frac{\theta_1}{2}\left(t^2-\left(t_j^C\right)^2\right)\right.$
$-\frac{\alpha}{6}\left(t^3-\left(t_j^C\right)^3\right)f(t)dt$
$+P_r\sum_{j=1}^{n_2}\int_{t_j^C}^{s_{j+1}^C}\alpha t\left[\left(1+\theta_1 t+\frac{\alpha}{2}t^2\right)\left(t-t_j^C\right) - \frac{\theta_1}{2}\left(t^2-\left(t_j^C\right)^2\right) - \frac{\alpha}{6}\left(t^3-\left(t_j^C\right)^3\right)f(t)dt + (P_r+s+P_s+l\delta)\right.$
$\sum_{j=1}^{n_2}\int_{s_j^C}^{t_j^C}\frac{\left(t_j^C-t\right)f(t)}{1+\delta\left(t_j^C-t\right)}dt - TC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO}=H\right),$

$$\{j=1,2,\ldots,n_2\}, s_1=0. \quad (2.17)$$

To find the minimum TC_s^C the values of t_j^C and s_j^C are determined, similarly as obtained for TC_r^D in decentralized system.

Let the optimal solution (see section 2.5 for optimality condition) is obtained as in from equation 2.12, 2.11 for Min $TC_s^C\left(n_2, s_1, t_1^C, \ldots, s_{n_2+1}^C = H\right)$ be $n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H$

The total optimal order quantity during the planning horizon H is

$Q^C = \sum_{j=1}^{n_2^{CO}} Q_j^{CO} = \sum_{j=1}^{n_2^{CO}} \left(R_j^{CO} + S_j^{CO}\right)$

The retailer accepts this system only when its total cost is not more than that in centralized system. i.e.

$$TC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) \geq TC_r^C\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO}\right.$$
$$= H - \sum_{j=1}^{n_2^{CO}} C_c\lambda\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO} \tag{2.18}$$

where M_j^C is the length of credit period and λ is the credit period rate, equal for every replenishment cycle.

$\mathrm{M}_j^C = \lambda\left(s_{j+1}^{CO} - s_j^{CO}\right)$

Minimum of credit period rate λ_{min} is such that the new total optimal cost of retailer after receiving of his share equates total optimal cost of decentralized system.

$$TC_r^C\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H\right) - \sum_{j=1}^{n_2^{CO}} C_c\lambda_{min}\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO}$$
$$= TC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) \tag{2.19}$$

Similarly for supplier,

$$n_2^{CO} S_s + \sum_{j=1}^{n_2^{CO}} P_s Q_j^{CO} + \sum_{j=1}^{n_2^{CO}} C_c\lambda_{\max}\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO}$$
$$= TC_s^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right) \tag{2.20}$$

Therefore from equation 2.18 and 2.19, the minimum and maximum values that credit period rate can attain is as follows $\lambda_{\min} =$

$$\frac{TC_r^C\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H\right) - TC_r^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO} = H\right)}{\sum_{j=1}^{n_2^{CO}} C_c\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO}} \tag{2.21}$$

$$\lambda_{\max} = \frac{\begin{array}{c} TC_s^D\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1+1}^{DO}\right. \\ = H - n_2^{CO} S_s + \sum_{j=1}^{n_2^{CO}} P_s Q_j^{CO} \end{array}}{\sum_{j=1}^{n_2^{CO}} C_c\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO}} \tag{2.22}$$

$\bar{\lambda}$, i.e. average of $\lambda_{\max}$ and $\lambda_{\min}$ ensures equal partition of extra cost saving amongst both, incorporating this the final total cost of retailer and supplier are

$$TC_r^{CO\lambda}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H\right) =$$

$$TC_r^{C}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} = H\right) - \sum_{j=1}^{n_2^{CO}} C_c\bar{\lambda}\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO} \tag{2.23}$$

$$TC_s^{CO\lambda}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2+1}^{CO} =\right.$$

$$H = n_2^{CO} S_s + \sum_{j=1}^{n_2^{CO}} P_s Q_j^{CO} + \sum_{j=1}^{n_2^{CO}} C_c\bar{\lambda}\left(s_{j+1}^{CO} - s_j^{CO}\right) Q_j^{CO} \tag{2.24}$$

2.5 Optimality condition for TC_r^D and TC_s^C

The sufficient condition for TC_r^D to be minimum is that the following Hessian matrix $\nabla^2 TC_r^D$ of TC_r^D for a fixed n_1 is positive definite as given by Sarkar, Ghosh, and Chaudhuri (2012b).

$$\nabla^2 TC_r^D = \begin{pmatrix}
\frac{\partial^2 TC_r^D}{\partial t_1^2} & \frac{\partial^2 TC_r^D}{\partial t_1 \partial s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial^2 TC_r^D}{\partial s_1 \partial t_1} & \frac{\partial^2 TC_r^D}{\partial s_1^2} & \frac{\partial^2 TC_r^D}{\partial s_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\partial^2 TC_r^D}{\partial t_2 \partial s_1} & \frac{\partial^2 TC_r^D}{\partial t_1^2} & \frac{\partial^2 TC_r^D}{\partial t_2 \partial s_2} & 0 & 0 & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & \frac{\partial^2 TC_r^D}{\partial t_{n_1-1} \partial s_{n_1-2}} & \frac{\partial^2 TC_r^D}{\partial t_{n_1-1}^2} & \frac{\partial^2 TC_r^D}{\partial t_{n_1-1} \partial s_{n_1-1}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 TC_r^D}{\partial s_{n_1-1} \partial t_{n_1-1}} & \frac{\partial^2 TC_r^D}{\partial s_{n_1-1}^2} & \frac{\partial^2 TC_r^D}{\partial s_{n_1-1} \partial t_{n_1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 TC_r^D}{\partial t_{n_1} \partial s_{n_1-1}} & \frac{\partial^2 TC_r^D}{\partial t_{n_1}^2}
\end{pmatrix} \tag{2.25}$$

Where

$$\frac{\partial^2 TC_r^D\left(n_1, s_1, t_1, \ldots, s_{n_1+1}\right)}{\partial t_i^2} = \left(h_r + w\right) \int_{t_i}^{s_{i+1}} \left(\alpha t_i + \theta_1\right)\left(a_1 + b_1 t + c_1 t^2\right) dt +$$
$$\left(h_r + w\right)\left(a_1 + b_1 t_i + c_1 {t_i}^2\right)$$
$$+w \int_{t_i}^{s_{i+1}} (\alpha t \theta_1 \left(a_1 + b_1 t + c_1 t^2\right) dt + w\alpha t_i \left(a_1 + b_1 t_i + c_1 {t_i}^2\right) - 2(w + s + l\delta)\delta$$
$$\int_{s_i}^{t_i} \frac{\left(a_1 + b_1 t + c_1 t^2\right)}{\left\{1 + \delta\left(t_i - t\right)\right\}^3} dt$$

$$+(w + s + l\delta)\left(a_1 + b_1 t_i + c_1 {t_i}^2\right). \tag{2.26}$$

$$\frac{\partial^2 TC_r^D\left(n_1,s_1,t_1,,s_{n_1+1}\right)}{\partial s_i \partial t_i}$$
$$=\frac{\partial^2 TC_r^D\left(n_1,s_1,t_1,\ldots,s_{n_1+1}\right)}{\partial t_i \partial s_i}$$
$$=-(w+s+l\delta)\frac{\left(a_1+b_1 s_i+c_1 {s_i}^2\right)}{\left(1+\delta\left(t_i-s_i\right)\right)^2}, \tag{2.27}$$

$$\frac{\partial^2 TC_r^D\left(n_1,s_1,t_1,\ldots,s_{n_1+1}\right)}{\partial s_i \partial t_{i+1}}$$
$$=\frac{\partial^2 TC_r^D\left(n_1,s_1,t_1,\ldots,s_{n_1+1}\right)}{\partial t_i \partial s_{i-1}}=0., \tag{2.28}$$

$$\frac{\partial^2 TC_r^D\left(n_1,s_1,t_1,\ldots,s_{n_1+1}\right)}{\partial s_i^2}=\frac{1}{6}\left[-\frac{1}{(1-\delta s_i+\delta}\right.$$
$${t_i}^2 6(s+w+l\delta)\left(-a_1+b_1\left(\delta s_i^2-2s_i\left(1+\delta t_i\right)+t_i\left(1+\delta t_i\right)\right)\right.$$
$$+c_1 s_i\left(2\delta s_i^2+2t_i\left(1+\delta t_i\right)-s_i\left(3+4\delta t_i\right)\right]+3s_i\left(b_1+2c_1 s_i\right)$$
$$\left(s_i-t_{-1+i}\right)\left(2+s_i\theta_1-t_{-1+i}\theta_1\right)$$
$$+6\left(w+h_r\right)\left(a_1+s_i\left(b_1+c_1 s_i\right)\right)\left(1+\left(s_i-t_{-1+i}\right)\left(\alpha s_i+\theta_1\right)\right)$$
$$+\left(w+h_r\right)\left(b_1+2c_1 s_i\right)\left(s_i-t_{-1+i}\right)$$
$$\left(6+2\alpha s_i^2-\alpha t_{-1+i}^2-3t_{-1+i}\theta_1+s_i\left(-\alpha t_{-1+i}+3\theta_1\right)\right)+$$
$$3w\alpha\left(a_1+s_i\left(b_1+c_1 s_i\right)\right)\left(3s_i^2\theta_1+s_i\left(4-4t_{-1+i}\theta_1\right)\right.$$
$$\left.+t_{-1+i}\left(-2+t_{-1+i}\theta_1\right)\right., \tag{2.29}$$

We have that $\nabla^2 TC_r^D$ is a tridiagonal matrix. Therefore using the properties of tridiagonal matrix, equation 2.26, 2.27, 2.28 and 2.29, and also the theorem stated below, it follows that $\nabla^2 TC_r^D$ is positive definite.

Theorem: If t_i and s_i satisfy inequations (i) $\frac{\partial^2 TC_r^D}{\partial t_i^2}$ >0, (ii) $\frac{\partial^2 TC_r^D}{\partial s_i^2}$ >0, (iii) $\frac{\partial^2 TC_r^D}{\partial t_i^2} - \left|\frac{\partial^2 TC_r^D}{\partial t_i \partial s_i}\right|$ >0 and (iv) $\frac{\partial^2 TC_r^D}{\partial s_i^2} - \left|\frac{\partial^2 TC_r^D}{\partial s_i \partial t_i}\right|$ >0 for i=1,2,…,n_1 then $\nabla^2 TC_r^D$ is positive definite.

The same can be used to show that $\nabla^2 TC_s^C$ is a positive definite and $TC_s^C(n_2, s_0, t_1^c, s_1^c, ..., s_{n+1}^c)$ attains a minimum.

Based on the above theorem we propose the algorithm of solution which is as follows:

2.6 Algorithm

1. Allocating the values to the parameters a_1, b_1, c_1, h_r, l, s, α ,δ , O_r, P_r, s_1.

2. Find the optimal ordering schedule for retailer when decentralized.

   (a) Set n_1=1, s_1=0, s_2=H. Initializing the value of the parameter t_1, calculate t_1 from equation 2.12 .

   (b) Set n_1=2.

   (c) Initializing the value of the parameter t_1,and using t_1, and $\{s_1 = 0\}$, calculate s_2 from equation 2.12.

   (d) With the values of t_1 and s_2 calculate t_2 from equation 2.12.

   (e) Proceeding in this way, and using the values of t_2, and s_2, calculate s_3 from equation 2.11, until all unique optimal $t_i's$ and $s_i's$ {i=1, 2, . . . ,$\mathrm{n}_1$are obtained for the value of n_1.

   (f) Repeat step 2.4 and 2.5 for n_1.

   (g) For $n_1 = 1$ and if $TC_r^D\left(n_1\right) < TC_r^D\left(n_1+1\right)$,then $TC_r^D\left(n_1\right)$ $= TC_r^DO\left(n_1\right)$.Stop.

   (h) For $\mathrm{n}_1 \geq 2andif$TC$_r^D\left(n_1\right) < TC_r^D\left(n_1-1\right)$ and $TC_r^D\left(n_1\right) < TC_r^D\left(n_1+1\right)$, then $TC_r^D\left(n_1\right) = TC_r^{DO}\left(n_1\right)$ and stop or else let $\mathrm{n}_1 = n_1+1$, $andgotostep2.3$

3. The optimal replenishment cycle for retailer and supplier when decentralised is $n_1^{DO} = n_1$.

4. Calculate $TC_r^{DO}\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1^{DO}+1}\right)$, $TC_s^{DO}\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, \ldots, s_{n_1^{DO}+1}\right)$ and Q^{DO} from equations 2.10,2.13 and and (14) respectively.

5. Proceeding as in steps 2 to 4 calculate $\mathrm{n}_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots and$ $TC_r^{CO}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2^{CO}+1}\right), TC_s^{CO}\left(n_2^{CO}, s_1, t_1^{CO}, s_2^{CO}, \ldots, s_{n_2^{CO}+1}\right)$ $andQ^{CO}foracentralisedsystem.$

6. Calculate $\lambda_{\min}, \lambda_{\max}, TC_r^{CO\lambda}andTC_s^{CO\lambda}$ from equations 2.21, 2.22, 2.23 and 2.24 respectively.

2.6.1 Numerical example

Example1: {$a_1 = 1, 7, 17$ units/yr}, $b_1 = 5$ units/ yr, $c_1 = 1$ unit/yr, $P_r = 0.3$ \$/unit, s=2 \$/unit, l=12 \$/unit, $\theta_1 = 0.2$, $\delta = 6$, $\alpha = 0.002$, $s_1 = 0$, $O_r = 40$ \$/order, $S_s = 120$ \$/setup, H=4, $P_s = 0.3$ \$/unit, $C_c = 1.2$ \$/unit/yr , $I_{hr} = 1.8$ \$/unit/yr, h=3 \$/unit/yr ($C_c + I_{hr}$). To solve the non linear system of equations we take the help of numerical computation software Mathematica(version 8.0). Table 2.1 comprises of the total cost of the retailer for a={1,7,17} and replenishment cycles n_1={1,2,...7} in decentralized system. Optimal total cost of retailer for a={1,7,17} are \$255.703, \$308.34 and \$371.36 for 3, 4 and 5 replenishment cycles respectively. Total cost decreases from n_1=1, reaches minimum at n_1=3 and again increases gradually for remaining cycles. This convexity of total cost of retailer can be seen in Table 2.1 for all values of a.

Table 2.2 shows the optimal solutions t_i's and s_i's for different values of a and their corresponding optimal cycles. Table 2.5 comprises of suppliers total cost in a centralized system for a ={1,7,17} and for each a different cycles n_2 ranging from 1 to 7. Again as in table 2.3 the total cost shows its convexity for different values of a, here the optimal total cost for a={1,7,17} are \$255.80 \$332.93 and \$429.37 respectively. The optimal replenishment cycles for a={1,7,17} are therefore 1,2,3 respectively. For all the optimal repleshment cycles obtained in Table 2.4, the corresponding optimal t_i's and s_i's are shown in Table for a={1,7,17}.

Table 2.5 is a comparison chart for the total cost, optimal replenishment cycle, optimal quantity for both retailer as well as supplier for decentralized and centralized system. The last two columns shows the cost saving done by retailer and supplier. for a=17 the cost saving by retailer is 24.93% which is less than compared to that of a=7 which 25.83%. The cost saving increase from a=1 to a=7 but shows adecrement in a=17. This implies a=7 will be the more appropriate value for example 1 as far as the profit of retailer and supplier is concerned. It is not difficult to derive that for what values of a_1, b_1, c_1, ...etc. $\frac{\Delta TC}{TC_r^{DO}}$, $\frac{\Delta TC}{TC_r^{DO}}$, would be maximum.

TABLE 2.1: Total cost of retailer in a decentralized system

	$\rightarrow n_1$						
a_1	1	2	3	4	5	6	7
1	376.68	278.85	255.7	261.42	280.42	306.4	336.44
7	545.13	381.68	321.88	308.34	316.59	335.87	361.35
17	819.03	533.65	416.73	377.61	371.36	381.2	400.02

TABLE 2.2: Optimal schedule for retailer in a decentralized system

a_1	n_1^{DO}	s_1	s_2	s_3	s_4	s_5	s_6
1	3	0	2.08	3.13	4.		
7	4	0	1.36	2.38	3.24	4.	
17	5	0	0.99	1.87	2.64	3.35	4.

a_1	n_1^{DO}	t_1	t_2	t_3	t_4	t_5
1	3	0.85	2.18	3.19		
7	4	0.19	1.45	2.44	3.29	
17	5	0.08	1.06	1.92	2.69	3.39

TABLE 2.3: Total Cost of supplier in a centralized system

	$\rightarrow n_2$						
a_1	1	2	3	4	5	6	7
1	255.8	275.82	369.93	493.63	631.21	776.17	925.44
7	377.65	332.93	389.21	492.19	618.04	755.66	899.94
17	599.49	434.24	429.37	504.44	614.51	741.88	878.94

TABLE 2.4: Optimal schedule for supplier in a centralized system

a_1	n_2^{CO}	s_1	s_2	s_3	s_4
1	1	0	4.		
7	2	0	2.56	4.	
17	3	0	1.59	2.89	4.

a	n_2^{CO}	t_1	t_2	t_3
1	1	2.34		
7	2	0.96	2.73	
17	3	0.2	1.72	2.98

TABLE 2.5: Percentage change

	Decent-ralized system							Cent-ralized system					
a_1	TC_r^{DO}	TC_s^{DO}	n_1^{DO}	Q^{DO}	$\lambda_{\min}$	$\lambda_{\max}$	$\bar{\lambda}$	$TC_r^{CO\lambda}$	$TC_s^{CO\lambda}$	n_2^{CO}	Q^{CO}	$\frac{\Delta TC}{TC_r^{DO}}$	$\frac{\Delta TC}{TC_s^{DO}}$
1	255.70	369.93	3.	33.11	0.51	0.99	0.75	198.63	312.86	1.	49.39	22.32	15.43
7	308.34	492.19	4.	40.65	0.47	1.50	0.98	228.71	412.56	2.	65.31	25.83	16.18
17	371.36	614.51	5.	48.36	0.36	1.81	1.08	278.79	521.94	3.	79.97	24.93	15.06

2.6.2 Senstivity analysis

For Example 1 we have observed the sensitivity of each parameter as obtained in Table 2.7, 2.8 and 2.9 . This has been done by changing the value of each parameter by -50%, -25%, 25% and 50%, of the value considered in example 1.

1. $TC_r^{CO\lambda}$ is highly sensitive to O_r and H, moderately sensitive to a_1, b_1, c_1, C_c, I_{hr}, l, S_s and δ. Also $TC_r^{CO\lambda}$ less sensitive to s, P_s, α and θ_1. For all the parameters given in Table 2.6 except S_s, with the increase in parameter's value TC$_r^{CO\lambda}$ increases. For increase in S_s, TC$_r^{CO\lambda}$ decreases.

2. $TC_s^{CO\lambda}$ is highly sensitive to H, moderately sensitive to l,S_s and O_r. Also $TC_r^{CO\lambda}$ less sensitive to a_1, b_1, c_1, C_c, I_{hr}, s, P_r, P_s, α, θ_1 and δ. For all the parameters given in Table 2.6 except S_r, with the increase in parameter's value TC$_s^{CO\lambda}$ increases. For increase in S_r, TC$_s^{CO\lambda}$ decreases.

3. $\overline{\lambda}$ is highly sensitive to C_c, S_s and O_r, moderately sensitive to a_1, b_1, I_{hr}, δ and H. Also that it is less sensitive to c_1, s, l, P_r, P_s, α and θ_1. $\overline{\lambda}$ increases with increase in $I_{hr}, P_r, S_s, \alpha, \theta_1$. With the increase in value of other remaining parameters, $\overline{\lambda}$ decreases.

4. n_1^{DO} is sensitive to O_r and H and n_2^{CO} is sensitive to S_s and H. n_1^{DO} is insensitive to all the parameters except O_r and H while n_2^{CO} is insensitive to all the other parameters except S_s and H. With the increase in O_r, n_1^{DO} decreases but with increase in H, n_1^{DO} increases. Similarly with the increase in S_s, n_2^{CO} decreases but with increase in H, n_2^{CO} increases. This result differs from that of example 1 in Wu and Zhao (2014). In a centralized system the optimal replenishment cycle depends upon the supplier's setup cost and in a decentralized system the optimal replenishment cycle depends upon ordering cost of retailer.

2.6.3 Comparative study

Inferences from example 1 reveals that the present model as a whole is sensitive towards the objective, that is to enhance coordination between supplier and the retailer. For the comparison of our model with that of Wu and Zhao, (2014) we have considered a case where the values of parameters are same as example 1 of their model. But the values of new parameters introduced in the present model are c_1=1, s=1, l=5, α= 0.001 and δ=4, taken in this case.

Following our algorithm we have calculated the cost saving earned by both retailer and supplier as shown in table 2.6.The percentage change in total cost of retailer increases from 2.92 to 21.03 when a_1 is 500. The increment is observed for all the three values of a_1 i.e. 500, 1500 and 2500, both for retailer and supplier.

Although in our model both the retailer and supplier gets hefty profits but the former enjoys greater cost saving. This phenomenon is opposite to what was observed by Wu and Zhao (2014c) . So our model is more effective when it comes to attracting retailer under cost saving schemes.

TABLE 2.6: Comparison chart

Parameters →	a_1	b_1	c_1	P_r	P_s	O_r	S_s	I_{hr}	C_c	θ_1	H	s	l	α	δ	% change in total cost of retailer	% change in total cost of supplier
Wu and Zhao, (2014)	500	1000	–	3	1.2	50	150	0.6	0.4	0.1	1	–	–	–	–	2.92	5.41
	1500															1.53	3.14
	2500															1.19	2.53
Present study	500	1000	1	3	1.2	50	150	0.6	0.4	0.1	1	1	5	0.001	4	21.03	12.61
	1500															31.63	15.73
	2500															13.94	8.00

2.7 Conclusion

The present chapter is an extended model of Wu and Zhao (2014c) , which investigates the effect of parameters such as time quadratic and inventory dependent demand, deterioration, shortages, partial back logging and lost sales on relationship of retailer and supplier under permissible delay in payment for finite planning horizon. An analytical solution of the problem has been discussed for two cases (i) where retailers optimal replenishment schedule is followed by supplier(Decentralized system) and (ii) where retailer follows the optimal replenishment schedule given by supplier(Centralized system). For a Centralized system supplier offers the retailer a credit period rate mechanism for sharing the profit. A completely new algorithm has been discussed. Results are very motivating as the cost saving i.e. some multiple of unit capital cost, under these parameters done by both retailer and supplier is a huge amount given in percentage. In a real life market situation where there are different types of products available in the market this model suites to almost all of such kind as particular cases can be worked out by relaxing one or the other parameter.This model can be extended by assuming a single supplier and multi-retailer integrated system, or by studying quantity discounts and shortages for imperfect items under inflation.

TABLE 2.7: Senstivity Analysis part 1 for the contribution of each parameter in the model

	% change in parameter	a_1	b_1	c_1	C_c
$\frac{\Delta TC_r^{CO\lambda} X100\%}{TC_{rO}^{CO\lambda}}$	-50	-18.81	-21.13	-10.88	-14.21
	-25	-9.25	-10.53	-5.4	-6.92
	25	9.03	10.48	5.33	6.58
	50	17.89	20.93	10.59	12.84
$\frac{\Delta TC_s^{CO\lambda} X100\%}{TC_{sO}^{CO\lambda}}$	-50	-4.74	-4.41	-2.28	-1.75
	-25	-2.34	-2.2	-1.13	-0.81
	25	2.27	2.19	1.1	0.68
	50	4.48	4.37	2.17	1.26
$\frac{\Delta\bar{\lambda} X100\%}{\overline{\lambda_O}}$	-50	14.39	12.16	3.75	64.8
	-25	6.66	5.61	1.79	21.43
	25	-5.75	-4.78	-1.67	-12.71
	50	-10.69	-8.86	-3.23	-21.08
n_1^{DO}	-50	4.	4.	4.	4.
	-25	4.	4.	4.	4.
	25	4.	4.	4.	4.
	50	4.	4.	4.	4.
n_2^{CO}	-50	2.	2.	2.	2.
	-25	2.	2.	2.	2.
	25	2.	2.	2.	2.
	50	2.	2.	2.	2.

TABLE 2.8: Senstivity Analysis part 2 for the contribution of each parameter in the model

	% change in parameter	I_{hr}	S	l	P_r
$\frac{\Delta TC_r^{CO\lambda} X100\%}{TC_{rO}^{CO\lambda}}$	-50	-21.88	-0.35	-8.22	-3.43
	-25	-10.52	-0.17	-9.28	-1.7
	25	9.75	0.17	5.02	1.68
	50	18.8	0.34	8.38	3.35
$\frac{\Delta TC_s^{CO\lambda} X100\%}{TC_{sO}^{CO\lambda}}$	-50	-2.85	-0.15	-29.84	-0.37
	-25	-1.26	-0.08	-4.18	-0.18
	25	0.99	0.07	2.25	0.18
	50	1.75	0.15	3.77	0.35
$\frac{\Delta\bar{\lambda} X100\%}{\overline{\lambda_O}}$	-50	-25.94	0.69	-1.85	-4.48
	-25	-13.3	0.34	5.94	-2.25
	25	13.73	-0.34	-9.83	2.26
	50	27.74	-0.67	-16.18	4.52
n_1^{DO}	-50	4.	4.	3.	4.
	-25	4.	4.	4.	4.
	25	4.	4.	4.	4.
	50	4.	4.	4.	4.
n_2^{CO}	-50	2.	2.	1.	2.
	-25	2.	2.	1.	2.
	25	2.	2.	2.	2.
	50	2.	2.	2.	2.

TABLE 2.9: Senstivity Analysis part 3 for the contribution of each parameter in the model

	% change in parameter	P_s	S_s	O_r	α	θ_1	δ	H
$\frac{\Delta TC_r^{CO\lambda} X100\%}{TC_{rO}^{CO\lambda}}$	−50	−0.54	25.42	−80.54	−0.09	−2.97	−7.36	−68.28
	−25	−0.27	13.12	−39.21	−0.04	−1.46	−2.77	−41.25
	25	0.27	−13.12	13.12	0.04	1.41	1.88	27.35
	50	0.54	−29.58	50.29	0.09	2.77	3.26	89.32
$\frac{\Delta TC_s^{CO\lambda} X100\%}{TC_{sO}^{CO\lambda}}$	−50	−1.3	−44.08	34.95	−0.02	−0.75	−2.25	−53.99
	−25	−0.65	−21.81	16.89	−0.01	−0.36	−0.88	−29.12
	25	0.65	21.81	−2.42	0.01	0.34	0.62	45.64
	50	1.3	41.78	−18.19	0.02	0.66	1.09	79.39
$\frac{\Delta\bar{\lambda} X100\%}{\overline{\lambda_O}}$	−50	0.8	−56.98	94.27	−0.06	−2.71	7.36	30.61
	−25	0.4	−19.61	45.55	−0.03	−1.33	2.76	8.54
	25	−0.4	19.61	−6.54	0.03	1.29	−1.85	20.22
	50	−0.8	16.97	−49.05	0.06	2.55	−3.18	4.31
n_1^{DO}	−50	4.	4.	6.	4.	4.	4.	2.
	−25	4.	4.	5.	4.	4.	4.	3.
	25	4.	4.	4.	4.	4.	4.	6.
	50	4.	4.	3.	4.	4.	4.	7.
n_2^{CO}	−50	2.	3.	2.	2.	2.	2.	1.
	−25	2.	2.	2.	2.	2.	2.	1.
	25	2.	2.	2.	2.	2.	2.	3.
	50	2.	1.	2.	2.	2.	2.	4.

Chapter 3

A green supply chain model for time quadratic inventory dependent demand and partially backlogging with Weibull deterioration under the finite horizon

3.1 Abstract

This chapter presents a detail solution of re-manufacturing of a product in a supply chain model. It is a non-traditional model considering time-dependent quadratic demand, Weibull deterioration, shortages, partial backlogging and re-manufacturing of inventory. This chapter mainly focuses on remanufacturing and hence an attempt towards reducing the environmental hazard. The process of remanufacturing is completed within one cycle of replenishment. Trade credit between supplier and retailer also had been discussed. Two cases one of a centralized and the other of decentralization for a finite planning horizon in a supply chain model are discussed. An algorithm has been derived for solving a problem in both the cases. Some managerial insights are talked about on the basis of sensitivity analysis on the parameters considered.

3.2 Introduction

No doubt improving environmental quality comes at a cost but at the same time, proper disposal of hazardous waste is very costly as in . Due to the market competitiveness retailer and supplier in a supply chain are bound to collaborate by sharing each other's information for mutual benefits such as profit in terms of money and customer satisfaction. For the product's which is electrical, electronic, plastic, glass, jewelry etc., manufacturers are trying to provide a quality product by re-manufacturing and reducing the defects in an item. This results in the greening of a supply chain.

The reverse manufacturing problem for an electronic industry was recently considered and simplified by Chung and Wee (2011) green product design and re-manufacturing. While raising significant concern over environmental initiatives, Zhang, Bi, and Liu (2009) in one of the conclusions mentioned that policymakers should give more heed to employees and nearby communities. Mudgal et al. (2010) identified and analyzed the barriers to green business practices. Considering re-manufacturing in green supply chain Rani, Ali, and Agarwal (2017) discussed a model. Green retailing is now a buzz word amongst retailer due to growing pressure from the eco friendly environment by different stakeholders such as consumers, no profit organizations, government etc. Saha, Nielsen, and Moon (2017) states that continuous investment in green operations is always profitable to the retailer. Remanufacturing a product may include replacement of a worn out part, fixation of breakage occurred due to transportation, software up gradation, remolding or other cosmetic operations.

In some cases, re-manufacturing may limit repairing only but in almost all cases re-manufacturing reduces polluting hazard from environment since the products are neither disposed of nor discarded. The Green supply chain can decrease production cost and environmental problem as supplier manufacturers less number of units and the retailer has not to throw away the defective goods. The 7 product criteria for re-manufacturing given by Jr (2000). Normally model assumes that defective product does not exist in an inventory which is not true practically. The re-manufactured product is latest than previously manufactured products due to latter's technological obsolescence and therefore can be priced higher.
Re-manufacturing cost is less than repair cost was stated by Klausner, Grimm, and Horvath (1999).

Deterioration in an inventory model was introduced by Ghare and Schrader (1963). Permissible delay in payment was at first allowed by Goyal (1985) for the fixed time period in an economic order quantity model. Aggarwal and Jaggi (1995) considered deteriorating items while extending Goyal (1985) work. Two-parameter Weibull distribution rate in an economic ordering quantity model was introduced by Philip (1974). Both constant and two-parameter Weibull distribution deterioration in a production lot size model was first used by Misra (1975). A model taking ramp type demand rate and partially backlogging of product which deteriorates with Weibull deterioration rate was discussed by Skouri and Konstantaras (2009). With two-parameter Weibull's distribution deteriorating rate considering the effect of inflation under finite time horizon of an item was studied by Pal, Mahapatra, and Samanta (2014b). As per Sarkar, Ghosh, and Chaudhuri (2012b) is better than linear demand. Time-dependent quadratic demand function was introduced by Khanra and Chaudhuri (2003).

Authors using the time quadratic demand function in their research work are Ghosh and Chaudhuri (2006), Manna, Chaudhuri, and Chiang (2007), Singh et al. (2017a), Singh et al. (2017b) and others. Shortages were allowed by Jamal, Sarker, and Wang (1997) while extending Goyal (1985) model. Shortage of goods in inventory setup occurs due to many reasons, therefore there is a necessity of considering a stock out situation in any inventory model. The ordered quantity received by the retailer is used to satisfy waiting for customers and the remaining for existing demand. Customer's impatient during waiting time results into lost sales. Shortages are then partially backlogged. Abad (1996) introduced a model for customer impatient function. Later a model assuming shortages with the permissible delay in payment was introduced by Jamal, Sarker, and Wang (1997). Model developed by Abad (1996) was modified by Dye, Hsieh, and Ouyang (2007) introducing lost sales.

However, to the best of our knowledge, a model incorporating an inventory item, which bears parameters such as Weibull deterioration, shortages, partial backlogging, Lost sales, disassembly and re-manufacturing with trade credit in a green supply chain within a finite planning horizon is not yet discussed. Proposed model gives an insight into solving of such problem. Assumptions and notations are given in section 3.3. Model is formulated in section 3.4 and solved for two cases centralized and decentralized. Optimality of the proposed model is given in section 3.5. Model is further explained by an example given in section 3.7. Finally, a table of sensitivity analysis is given in section 3.8. The research reported in this chapter is based on following assumptions. The methodology is mathematical formulation of the model and solving by applying iterative methods through software Mathematica(8.1).

3.3 Assumptions and notations

(a) Shortage with partial backlogging follows with zero initial level of inventory.

(b) Lead time is zero.

(c) Time dependent quadratic demand $f(t) = a + bt + ct^2$ is assumed.

(d) Deterioration is a function of two parameter Weibull distribution of time $\theta_1(t) = \alpha\beta t^{\beta-1}$, $0 < \alpha < 1$, $\beta \geq 1$ where t denotes deterioration time.

(e) The cost such as for purchasing of an item $(P_o(\$/unit))$, deterioration $(D_C(\$/unit))$, lost sale $(L_o(\$/unit))$, setup $(S_s(\$/order))$, ordering $(C_o(\$/order))$, shortage $(S_o(\$/unit))$, holding $(H_o(\$/unit/unittime))$, screening $(S_c(\$/unit))$, transportation $(T_c(\$/unit))$, disassembly $(DsAsm(\$/unit))$, remanufacturing $(Rem(\$/unit))$ and oppurtunity cost $(O_c(\$/unit))$ are constant during the finite planning horizon (H).

(f) There is a shortage of items in each cycle and due to customers impatience, a fraction $B(\tau)$, of the demand is partially backlogged where τ is the amount of time customers wait upto receiving of goods. $1 - B(\tau) = 1 - \dfrac{1}{1+\delta\tau}$, fraction of demand is lost, where $\delta > 0$.

(g) Two cases are considered decentralized and centralized. Decentralized where optimal schedule of ordering n_1^{DO} is dependent on retailer's total cost(TC_r^D) and centralized where both supplier (TC_s^C) and retailer's increased cost($TC_r^C - TC_r^{DO}$) are considered for calculating optimal schedule of ordering n_2^{C-O}.

(h) In both the cases screening of all items i.e. I_{oi}^D for i^{th} cycle, is done. Defected/repairable items which is $P * I_{oi}^D$, after screening by retailer are then transported by supplier for disassembly and re-manufacturing at time $t = t_i^{'}$. All $P * I_{oi}^D$ items at time $t = t_i^{''}$ are then transported back to retailer for sale in the same i^{th} cycle.

(i) t_i $\{i = 1, 2 \dots n_1^D\}$ and t_j $\{j = 1, 2 \dots n_2^C\}$ are the time of replenishment during decentralized and centralized case respectively.

(j) s_i $\{i = 1, 2 \dots n_1^D\}$ and s_j $\{j = 1, 2 \dots n_2^C\}$ are the starting time for shortages in a decentralized and centralized case respectively. Also that $s_1 = 0$ and $s_{n_1+1} = H$.

(k) The total number of orders placed are n_1^D and n_2^C in a decentralized and centralized case respectively during the planning horizon H.

(l) Supply chain is of a single item with a single retailer and single supplier.

(m) Lot for lot replenishment policy is followed by the supplier.

(n) I_{oi} in general or I_{oi}^D is the amount of inventory at time $t = t_i$ in a decentralized case.

(o) I_{si}^D is the amount of inventory left with retailer after removal of items for re-manufacture at time $t = t_i^{'}$ in i^{th} cycle.

(p) I_{fi}^D is the amount of inventory at time $t = t_i''$ just before $p{*}I_{oi}^D$ re-manufactured items are introduced in retailers inventory.

(q) S_i is the total shortage of items occurred during time t where $s_i \leq t < t_i$.

(r) Opportunity cost is same for both retailers as well as the supplier.

(s) Ordering cost of the retailer is less than the setup cost of the supplier.

(t) Supplier holds the inventory during re-manufacturing.

(u) λ is the credit period rate.

3.4 Conceptualization of the proposed model

This section contains two subsections, 3.4.1 and 3.4.2. In the subsection 3.4.1 there is no coordination between retailer and supplier but on the contradiction subsection, 3.4.2 is for a solution of the model when there is coordination between them. The proposed model is as shown in figure 3.1 .

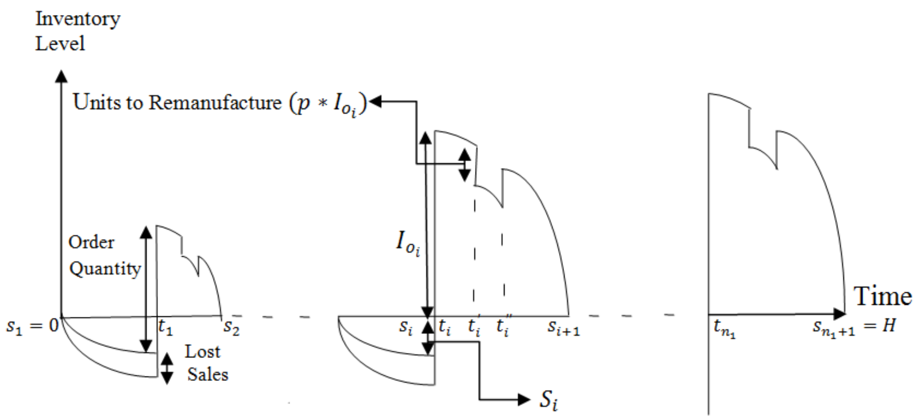

FIGURE 3.1: Inventory Diagram

3.4.1 Decentralized case for finite planning horizon

In a decentralized case scheduled number of replenishments (n_1^D) is dependent upon retailers total cost (TC_r^D). Inventory level (I_{1i}^D) at any time t is dependent on constant rate (θ_1) of inventory level, deterioration rate (θ_2) of item/s of inventory and time-dependent quadratic demand function. Model is represented by the equation below

$$\frac{dI_{1i}^D(t)}{dt} + (\theta_1 + \theta_2)\, I_{1i}^D(t) = -f(t), \{i = 1, 2 \dots n_2^D\} \tag{3.1}$$

where $\theta_2 = \alpha\beta t^{\beta-1}$ is the Weibull deterioration rate. As shown in figure 3.1 screening for defective\repairable is done upto time $t = t_i^{'}$ in an i^{th} cycle and $p * I_{oi}^D$ is deivered to supplier for re-manufacturing in every cycle. Thererfore with boundary conditions $I_{1i}^D\left(t_i^{'}\right) = I_{s_i}^D$ where $t_i \leq t \leq t_i^{'}$ the inventory level at time t is given by

$$I_{1i}^D(t) = I_{s_i}^D e^{\theta_1\left(t_i^{'}-t\right)+\alpha\left(t_i^{'\beta}-t^{\beta}\right)} + e^{-\left(\theta_1 t+\alpha t^{\beta}\right)} \int_t^{t_i^{'}} e^{\theta_1 u+\alpha u^{\beta}} f(u)du. \tag{3.2}$$

With boundary value $I_{1i}^D\left(t_i\right) = I_{o_i}^D$ the above equation reduces to

$$I_{s_i}^D = I_{o_i}^D e^{\theta_1\left(t_i-t_i^{'}\right)+\alpha\left(t_i^{\beta}-t_i^{'\beta}\right)} - e^{-\left(\theta_1 t_i^{'}+\alpha t_i^{'\beta}\right)} \int_{t_i}^{t_i^{'}} e^{\theta_1 u+\alpha u^{\beta}} f(u)du. \tag{3.3}$$

From $t_i^{'}$ to $t_i^{''}$ the defected/repairable items are re-manufactured. At time $t_i^{''}$ all the $p * I_{oi}^D$ items are transported back to the retailer for sale.

Further $I_{2i}^D(t)$ and $I_{3i}^D(t)$ are taken as inventory level at any time t for time duration $t_i^{'} < t \leq t_i^{''}$ and $t_i^{''} < t \leq s_{i+1}$ respectively. For $t_i^{'} < t \leq t_i^{''}$ with boundary conditions $I_{1i}^D\left(t_i^{''}\right) = I_{2i}^D\left(t_i^{''}\right) = I_{f_i}^D$ the inventory level at time t is given by

$$I_{2i}^D(t) = I_{f_i}^D e^{\theta_1\left(t_i^{''}-t\right)+\alpha\left(t_i^{''\beta}-t^{\beta}\right)} + e^{-\left(\theta_1 t+\alpha t^{\beta}\right)} \int_t^{t_i^{''}} e^{\theta_1 u+\alpha u^{\beta}} f(u)du. \tag{3.4}$$

Now since $I_{1i}^D\left(t_i^{'}\right) = I_{2i}^D\left(t_i^{'}\right) = I_{s_i}^D - pI_{o_i}^D$, the above equation will be

$$I_{s_i}^D - pI_{o_i}^D = I_{f_i}^D e^{\theta_1\left(t_i^{''}-t\right)+\alpha\left(t_i^{''\beta}-t^{\beta}\right)} + e^{-\left(\theta_1 t_i^{'}+\alpha t_i^{'\beta}\right)} \int_{t_i^{'}}^{t_i^{''}} e^{\theta_1 u+\alpha u^{\beta}} f(u)du. \tag{3.5}$$

At $t = s_{i+1}$ the inventory level for i^{th} cycle is zero. With boundary conditions for t where, $t_i^{''} \leq t \leq s_{i+1}$,$I_{3i}^D\left(t_i^{''}\right) = I_{f_i}^D + pI_{o_i}^D$ and $I_{3i}^D\left(s_{i+1}\right) = 0$ the inventory level at time t is given by

$$I_{3i}^D(t) = e^{-\left(\theta_1 t+\alpha t^{\beta}\right)} \int_t^{s_{i+1}} e^{\theta_1 u+\alpha u^{\beta}} f(u)du. \tag{3.6}$$

and $I_{f_i}^D = -pI_{o_i}^D + e^{-\left(\theta_1 t_i^{''} + \alpha t_i^{''\beta}\right)} \int_{t_i^{''}}^{s_{i+1}} e^{\theta_1 u + \alpha u^\beta} f(u) du$. Solving above equations we get

$$I_{o_i}^D = \left[a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\right.$$

$$\left(s_{i+1}^3 - t_i^3\right)\left[1 - \theta_1 t_i - \alpha t_i^\beta - p\left\{\theta_1\left(t_i - t_i^{'}\right) + \alpha\left(t_i^{''\beta} - t_i^{'\beta}\right)\right\}\right] + \theta_1\left(\frac{a}{2}\right.$$

$$\left(s_{i+1}^2 - t_i^2\right) + \frac{b}{3}\left(s_{i+1}^3 - t_i^3\right) + \frac{c}{4}\left(s_{i+1}^4 - t_i^4\right) + \alpha\left(\frac{a}{\beta+1}\right.$$

$$\left(s_{i+1}^{\beta+1} - t_i^{\beta+1}\right) + \frac{b}{\beta+2}\left(s_{i+1}^{\beta+2} - t_i^{\beta+2}\right) + \frac{c}{\beta+3}\left(s_{i+1}^{\beta+3} - t_i^{\beta+3}\right), \tag{3.7}$$

$$I_{f_i}^D = -p\left\{\left[a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\right.\right.$$

$$\left(s_{i+1}^3 - t_i^3\right)\left[1 - \theta_1 t_i - \alpha t_i^\beta - p\left\{\theta_1\left(t_i^{''} - t_i^{'}\right) + \alpha\left(t_i^{''\beta} - t_i^{'\beta}\right)\right\}\right] + \theta_1\left(\frac{a}{2}\right.$$

$$\left(s_{i+1}^2 - t_i^2\right) + \frac{b}{3}\left(s_{i+1}^3 - t_i^3\right) + \frac{c}{4}\left(s_{i+1}^4 - t_i^4\right) + \alpha\left(\frac{a}{\beta+1}\right.$$

$$\left(s_{i+1}^{\beta+1} - t_i^{\beta+1}\right) + \frac{b}{\beta+2}\left(s_{i+1}^{\beta+2} - t_i^{\beta+2}\right) + \frac{c}{\beta+3}\left(s_{i+1}^{\beta+3} - t_i^{\beta+3}\right)$$

$$+\left(1 - \theta_1 t_i^{''} - \alpha t_i^{''\beta}\right)\left(a\left(s_{i+1} - t_i^{''}\right) + \right.$$

$$\frac{b}{2}\left(s_{i+1}^2 - t_i^{''2}\right) + \frac{c}{3}\left(s_{i+1}^3 - t_i^{''3}\right) + \theta_1\left(\frac{a}{2}\right.$$

$$\left(s_{i+1}^2 - t_i^{''2}\right) + \frac{b}{3}\left(s_{i+1}^3 - t_i^{''3}\right) + \frac{c}{4}\left(s_{i+1}^4 - t_i^{''4}\right) + \alpha\left(\frac{a}{\beta+1}\right.$$

$$\left(s_{i+1}^{\beta+1} - t_i^{\beta+1}\right) + \frac{b}{\beta+2}\left(s_{i+1}^{\beta+2} - t_i^{\beta+2}\right) + \frac{c}{\beta+3}\left(s_{i+1}^{\beta+3} - t_i^{\beta+3}\right) \tag{3.8}$$

and $I_{s_i}^D = \left(1 - \theta_1 t_i^{'} - \alpha t_i^{'\beta}\right)\left(a\left(s_{i+1} - t_i^{'}\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^{'2}\right) + \frac{c}{3}\right.$

$$\left(s_{i+1}^3 - t_i^{'3}\right) + \left(-p\left\{\theta_1\left(t_i - t_i^{'}\right) + \alpha\left(t_i^{''\beta} - t_i^{'\beta}\right)\right\}\right)$$

$$\left(a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\left(s_{i+1}^3 - t_i^3\right)\right) + \theta_1\left(\frac{a}{2}\right.$$

$$\left(s_{i+1}^2 - t_i'^2\right) + \frac{b}{3}\left(s_{i+1}^3 - t_i'^3\right) + \frac{c}{4}\left(s_{i+1}^4 - t_i'^4\right)$$
$$+ \alpha\left(\frac{a}{\beta+1}\right.$$

$$\left.\left(s_{i+1}^{\beta+1} - t_i'^{\beta+1}\right) + \frac{b}{\beta+2}\left(s_{i+1}^{\beta+2} - t_i'^{\beta+2}\right) + \frac{c}{\beta+3}\left(s_{i+1}^{\beta+3} - t_i'^{\beta+3}\right)\right. \tag{3.9}$$

After substituting the value of $I_{s_i}^D$ in equation 3.2 we get $I_{1i}^D(t) =$

$$\left[a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\left(s_{i+1}^3 - t_i^3\right)\right]\left[1 - \theta_1 t_i - \alpha t_i^\beta\right.$$

$$-p\left\{\theta_1\left(t_i - t_i'\right) + \alpha\left(t_i''\beta - t_i'^\beta\right)\right\}\theta_1\left(\frac{a}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{b}{3}\left(s_{i+1}^3 - t_i^3\right) + \right.$$

$$\frac{c}{4}\left(s_{i+1}^4 - t_i^4\right)$$

$$+\alpha\left(\frac{a}{\beta+1}\left(s_{i+1}^{\beta+1} - t_i^{\beta+1}\right) + \frac{b}{\beta+2}\left(s_{i+1}^{\beta+2} - t_i^{\beta+2}\right) + \frac{c}{\beta+3}\right.$$

$$\left(s_{i+1}^{\beta+3} - t_i^{\beta+3}\right) + \left(1 - \theta_1 t - \alpha t^\beta\right)\left(a\left(t_i - t_i'\right) + \right.$$

$$\frac{b}{2}\left(t_i^2 - t_i'^2\right) + \frac{c}{3}\left(t_i^3 - t_i'^3\right) + \theta_1\left(\frac{a}{2}(t_i^2 - t^2) + \frac{b}{3}(t_i^3 - t^3)\right.$$

$$+\frac{c}{4}(t_i^4 - t^4) + \alpha\left(\frac{a}{\beta+1}\right.$$

$$\left(t_i^{\beta+1} - t^{\beta+1}\right) +$$

$$\frac{b}{\beta+2}\left(t_i^{\beta+2} - t^{\beta+2}\right) + \frac{c}{\beta+3}\left(t_i^{\beta+3} - t^{\beta+3}\right). \tag{3.10}$$

Similarly from equation 3.4 and 3.6 substituting the values of $I_{s_i}^D$ and $I_{f_i}^D$ we obtain $I_{2i}^D(t) =$

$$p\left[\left[a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\left(s_{i+1}^3 - t_i^3\right)\right]\right.$$

$$\left[1 - \theta_1 t_i - \alpha t_i^\beta - p\left\{\theta_1\left(t_i - t_i'\right) + \alpha\left(t_i''^\beta - t_i'^\beta\right)\right\}\right]$$

$$+\theta_1\left(\frac{a}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{b}{3}\left(s_{i+1}^3 - t_i^3\right)\right.$$

$$+\frac{c}{4}\left(s_{i+1}^4 - t_i^4\right) + \alpha\left(\frac{a}{\beta+1}\left(s_{i+1}^{\beta+1} - t_i^{\beta+1}\right) + \frac{b}{\beta+2}\right.$$

$$\left(s_{i+1}^{\beta+2}-t_i^{\beta+2}\right)+\frac{c}{\beta+3}\left(s_{i+1}^{\beta+3}-t_i^{\beta+3}\right)$$
$$+\Bigg[\left(1-\theta_1 t-\alpha t^{\beta}\right)\Bigg(a\left(s_{i+1}-t\right)+\frac{b}{2}\left(s_{i+1}^2-t^2\right)+\frac{c}{3}$$
$$\left(s_{i+1}^3-t^3\right)+\theta_1\Bigg(\frac{a}{2}\left(s_i^2-t^2\right)+\frac{b}{3}\left(t_i^3-t^3\right)+\frac{c}{4}$$
$$\left(t_i^4-t^4\right)+\alpha\Bigg(\frac{a}{\beta+1}\left(t_i^{\beta+1}-t^{\beta+1}\right)+\frac{b}{\beta+2}$$
$$\left(t_i^{\beta+2}-t^{\beta+2}\right)+\frac{c}{\beta+3}\left(t_i^{\beta+3}-t^{\beta+3}\right) \tag{3.11}$$

and

$$I_{3i}^D(t)=\Bigg[a\left(s_{i+1}-t\right)+\frac{b}{2}\left(s_{i+1}^2-t^2\right)+\frac{c}{3}$$
$$\left(s_{i+1}^3-t^3\right)\left(1-\theta_1 t-\alpha t^{\beta}\right)+\theta_1\Bigg(\frac{a}{2}\left(s_{i+1}^2-t^2\right)+\frac{b}{3}\left(s_{i+1}^3-t^3\right)$$
$$+\frac{c}{4}\left(s_{i+1}^4-t^4\right)+\alpha\Bigg(\frac{a}{\beta+1}\left(s_{i+1}^{\beta+1}-t^{\beta+1}\right)+\frac{b}{\beta+2}$$
$$\left(s_{i+1}^{\beta+2}-t^{\beta+2}\right)+\frac{c}{\beta+3}\left(s_{i+1}^{\beta+3}-t^{\beta+3}\right). \tag{3.12}$$

Different cost associated with retailer during the planning horizon where $i=1,2\ldots n_1^D$

Holding cost =

$$\sum_{i=1}^{n_1^D} H_o * R_i^D=\sum_{i=1}^{n_1^D} H_o\left(\int_{t_i}^{t_i'} I_{1i}(t)dt+\int_{t_i'}^{t_i''} I_{2i}(t)dt+\int_{t_i'{}'}^{s_{i+1}} I_{3i}(t)dt\right). \tag{3.13}$$

Ordering cost = $n_1^D * O_r$, Deterioration Cost=
$\sum_{i=1}^{n_1^D} D_C\left(\int_{t_i}^{t_i'}\theta_1 I_{1i}(t)dt+\int_{t_i'}^{t_i''}\theta_1 I_{2i}(t)dt+\int_{t_i''}^{s_{i+1}}\theta_1 I_{3i}(t)dt\right)$

$$=\sum_{i=1}^{n_1^D} D_C\left(\int_{t_i}^{t_i'}\alpha\beta t^{\beta-1}I_{1i}(t)dt+\int_{t_i'}^{t_i''}\alpha\beta t^{\beta-1}I_{2i}(t)dt+\int_{t_i''}^{s_{i+1}}\alpha\beta t^{\beta-1}I_{3i}(t)dt\right), \tag{3.14}$$

Shortage cost =
$\sum_{i=1}^{n_1^D} S_o \int_{s_i}^{t_i} S_i(t)dt$
$= \sum_{i=1}^{n_1^D} S_o \int_{s_i}^{t_i} \left(\int_{s_i}^{t} \frac{f(u)}{1+\delta\left(t_i-u\right)} du \right) dt =$

$$\sum_{i=1}^{n_1^D} S_o \int_{s_i}^{t_i} \frac{\left(t_i-t\right)\left(a+bt+ct^2\right)}{1+\delta\left(t_i-t\right)} dt, \tag{3.15}$$

Cost of lost sales =
$\sum_{i=1}^{n_1^D} L_o \int_{s_i}^{t_i} \left[f(t) - \frac{f(t)}{1+\delta\left(t_i-t\right)} \right] dt$

$$= \sum_{i=1}^{n_1} L_o \int_{s_i}^{t_i} \frac{\delta\left(t_i-t\right)\left(a+bt+ct^2\right)}{1+\delta\left(t_i-t\right)} dt, \tag{3.16}$$

Purchase cost during a planning horizon =

$$\sum_{i=1}^{n_1^D} P_r * Q_i^D = \sum_{i=1}^{n_1^D} P_r \left[I_{o_i}^D + S_i^D \right] \tag{3.17}$$

and Screening Cost=

$$\sum_{i=1}^{n_1^D} S_c * I_{o_i}^D. \tag{3.18}$$

Quantity to be ordered in each cycle

$$= Q_i^D = I_{oi}^D + S_i^D \tag{3.19}$$

Total cost of the retailer during a planning horizon = $\text{TC}_r^D(t_i, s_i, n_1^D)$ = Purchase cost + Holding cost + Detererioration cost + Shortgae cost + cost of Lost sale +Screening Cost =

$\sum_{i=1}^{n_1^D} \{ P_r \left[I_{oi}^D + S_i \right] + H_o \left[\int_{t_i}^{t_i^{'}} I_{1i}(t)dt + \right.$

$\int_{t_i^{'}}^{t_i^{''}} I_{2i}(t)dt + \int_{t_i^{''}}^{s_{i+1}} I_{3i}(t)dt + D_C \alpha\beta \left[\int_{t_i}^{t_i^{'}} t^{\beta-1} I_{1i}(t)dt + \right.$

$\int_{t_i^{'}}^{t_i^{''}} t^{\beta-1} I_{2i}(t)dt + \int_{t_i^{''}}^{s_{i+1}} t^{\beta-1} I_{3i}(t)dt$

$+S_o \int_{s_i}^{t_i} \frac{\left(t_i-t\right)\left(a+bt+ct^2\right)}{1+\delta\left(t_i-t\right)} dt$

$$+L_o \int_{s_i}^{t_i} \frac{\delta\left(t_i-t\right)\left(a+bt+ct^2\right)}{1+\delta\left(t_i-t\right)} dt + S_c * I_{oi}^D \} \tag{3.20}$$

Different total costs associated with supplier during the complete planning horizon for $i = 1, 2 \ldots n_1^D$

Setup cost in a decentralized case $= S_s * n_1^D$, Purchase cost $= \sum_{i=1}^{n_1^D} P_s * (I_{oi}^D + S_i^D)$, Transportation cost $= \sum_{i=1}^{n_1^D} T_c * p * I_{oi}^D$, Cost of Disassembly $= \sum_{i=1}^{n_1^D} DsAsm * p * I_{oi}^D$ and Cost of Remanufacturing $= \sum_{i=1}^{n_1^D} Rem * p * I_{oi}^D$.
Total cost of supplier = $\text{TC}_s^D(t_i, s_i, n_1^D) =$

$$\sum_{i=1}^{n_1^D} \{S_s * n + P_s * (I_{oi}^D + S_i^D) + (T_c * p + DsAsm * p +$$

$$Rem * p + H_o * p) * I_{oi}^D\}, \{i = 1, 2 \ldots n_1^D\} \tag{3.21}$$

where n_1^D is the total number of replenishment cycles.

Solution for decentralized case

For brevity of solution $t_i^{'} - t_i = t_i^{''} - t_i^{'}$ is taken as CT. Considering α, θ_1 very small and neglecting their square and higher order, change in TC_r^D with respect to t_i and s_i is obtained in equation 3.22 and 3.23 as follows.

$$\frac{\delta TC_r^D(t_i, s_i, n_1^D)}{\delta t_i}$$

$$= H_o \left(\int_{t_i}^{t_i^{'}} \left(\left(-a - bt_i - ct_i^2\right) \left(-p\theta_1 CT - p\alpha \left(t_i^{''\beta} - t_i^{'\beta}\right)\right) + \right.\right.$$

$$\left(a \left(s_{i+1} - t_i\right) + \frac{b}{2} \left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3} \left(s_{i+1}^3 - t_i^3\right) \right)$$

$$\left(-p\alpha\beta \left(t_i^{''(\beta-1)} - t_i^{'(\beta-1)}\right)\right) dt + \left(1 - \theta_1 t_i^{'} - \alpha t_i^{'\beta}\right)$$

$$\left(a \left(s_{i+1} - t_i^{'}\right) + \frac{b}{2} \left(s_{i+1}^2 - t_i^{'2}\right) + \frac{c}{3} \left(s_{i+1}^3 - t_i^{'3}\right) \right) +$$

$$\theta_1 \left(\frac{a}{2} \left(t_i^2 - t_i^{'2}\right) + \frac{b}{3} \left(t_i^3 - t_i^{'3}\right) + \frac{c}{4} \left(t_i^4 - t_i^{'4}\right) \right) +$$

$$\alpha \left(\frac{a}{\beta + 1} \left(t_i^{(\beta+1)} - t_i^{'(\beta+1)}\right) + \frac{b}{\beta + 2} \left(t_i^{(\beta+1)} - t_i^{'(\beta+2)}\right) + \frac{c}{\beta + 3} \right.$$

$$\left. \left(t_i^{(\beta+1)} - t_i^{'(\beta+3)}\right) \right)$$

$$- \left(1 - \theta_1 t_i - \alpha \left(t_i^{\beta}\right)\right) \left(a \left(s_{i+1} - t_i\right) + \frac{b}{2} \left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3} \right.$$

$$\left(s_{i+1}^{3}-t_i^{3}\right)+H_o\left((-p)\int_{t_i^{'}}^{t_i^{''}}((1+\right.$$

$$\theta_1\left(t_i^{''}-t_i-t\right)+\alpha\left(t_i^{''\beta}-t_i^{\beta}-t^{\beta}\right)-p\theta_1CT-p\alpha\left(t_i^{''\beta}-t_i^{'\beta}\right)$$

$$(-a-bt_i-ct_i^2)+\left(a\left(s_{i+1}-t_i\right)+\frac{b}{2}\left(s_{i+1}^{2}-t_i^{2}\right)+\frac{c}{3}\left(s_{i+1}^{3}-t_i^{3}\right)\right)$$

$$\left(\alpha\beta\left(t_i^{''(\beta-1)}-t_i{}^{(\beta-1)}\right)-p\alpha\beta\left(t_i^{''(\beta-1)}-t_i^{'(\beta-1)}\right)\right)$$

$$+\theta_1\left(-at_i-bt_i^2-ct_i^3\right)+\alpha\left(-at_i{}^{(\beta-1)}-bt_i{}^{(\beta+1)}-ct_i^{(\beta+2)}\right)dt+$$

$$\left(1-\theta_1t_i^{''}-\alpha t_i^{''\beta}\right)\left(a\left(s_{i+1}-t_i^{''}\right)+\frac{b}{2}\right.$$

$$\left(s_{i+1}^{2}-t_i^{''2}\right)+\frac{c}{3}\left(s_{i+1}^{3}-t_i^{''3}\right)+$$

$$\theta_1\left(\frac{a}{2}\left(s_{i+1}^{2}-t_i^{''2}\right)+\frac{b}{3}\right.$$

$$\left(s_{i+1}^{3}-t_i^{''3}\right)+\frac{c}{4}\left(s_{i+1}{}^{4}-t_i^{''4}\right)+\alpha\left(\frac{a}{\beta+1}\left(s_{i+1}{}^{(\beta+1)}-t_i^{''(\beta+1)}\right)+\frac{b}{\beta+2}\right.$$

$$\left(s_{i+1}{}^{(\beta+2)}-t_i^{''(\beta+2)}\right)+\frac{c}{\beta+3}\left(s_{i+1}{}^{(\beta+3)}-t_i^{''(\beta+3)}\right)$$

$$-\left(1-\theta_1t_i^{'}-\alpha t_i^{'\beta}\right)\left(a\left(s_{i+1}-t_i^{'}\right)+\frac{b}{2}\left(s_{i+1}^{2}-t_i^{'2}\right)+\frac{c}{3}\right.$$

$$\left(s_{i+1}^{3}-t_i^{'3}\right)-\theta_1\left(\frac{a}{2}\left(s_{i+1}^{2}-t_i^{'2}\right)+\frac{b}{3}\right.$$

$$\left(s_{i+1}^{3}-t_i^{'3}\right)+\frac{c}{4}\left(s_{i+1}{}^{4}-t_i^{'4}\right)-\alpha\left(\frac{a}{\beta+1}\right.$$

$$\left(s_{i+1}{}^{(\beta+1)}-t_i^{'(\beta+1)}\right)+\frac{b}{\beta+2}\left(s_{i+1}{}^{(\beta+2)}-t_i^{'(\beta+2)}\right)+\frac{c}{\beta+3}$$

$$\left(s_{i+1}{}^{(\beta+3)}-t_i^{'(\beta+3)}\right)+p\left(a\left(s_{i+1}-t_i\right)+\frac{b}{2}\left(s_{i+1}^{2}-t_i^{2}\right)+\frac{c}{3}\right.$$

$$\left(s_{i+1}^{3}-t_i^{3}\right)\left(\theta_1CT+\alpha\left(t_i^{''\beta}-t_i^{'\beta}\right)\right)$$

$$-H_o\left(\left(1-\theta_1t_i^{''}-\alpha t_i^{''\beta}\right)\left(a\left(s_{i+1}-t_i^{''}\right)+\frac{b}{2}\right.\right.$$

$$\left(s_{i+1}^{2}-t_i^{''2}\right)+\frac{c}{3}\left(s_{i+1}^{3}-t_i^{''3}\right)+\theta_1\left(\frac{a}{2}\right.$$

$$\left(s_{i+1}^2 - t_i^{''2}\right) + \frac{b}{3}\left(s_{i+1}^3 - t_i^{''3}\right) + \frac{c}{4}$$

$$\left({s_{i+1}}^4 - t_i^{''4}\right) + \alpha\left(\frac{a}{\beta+1}\right.$$

$$\left({s_{i+1}}^{(\beta+1)} - t_i^{''(\beta+1)}\right) + \frac{b}{\beta+2}$$

$$\left({s_{i+1}}^{(\beta+2)} - t_i^{''(\beta+2)}\right) + \frac{c}{\beta+3}$$

$$\left({s_{i+1}}^{(\beta+3)} - t_i^{''(\beta+3)}\right)$$

$$+D_C\left(-\int_{t_i^{'}}^{t_i^{''}} \alpha\beta t^{(\beta-1)} p\left(-a - bt_i - ct_i^2\right) dt\right.$$

$$+\alpha\beta t_i^{'(\beta-1)}\left(a\left(s_{i+1} - t_i^{'}\right) + \frac{b}{2}\right.$$

$$\left(s_{i+1}^2 - t_i^{'2}\right) + \frac{c}{3}\left(s_{i+1}^3 - t_i^{'3}\right) - \alpha\beta t_i^{(\beta-1)}$$

$$\left(a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\right.$$

$$\left(s_{i+1}^3 - t_i^3\right) - \alpha\beta t_i^{''(\beta-1)} p\left(a\left(s_{i+1} - t_i\right) + \frac{b}{2}\right.$$

$$\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\left(s_{i+1}^3 - t_i^3\right) +$$

$$\alpha\beta t_i^{'(\beta-1)} p\left(a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right)\right.$$

$$+\frac{c}{3}\left(s_{i+1}^3 - t_i^3\right)$$

$$+S_o\int_{s_i}^{t_i} \frac{a + bt + ct^2}{\left(1 + \delta\left(t_i - t\right)\right)^2} dt$$

$$+L_o\int_{s_i}^{t_i} \frac{\delta\left(a + bt + ct^2\right)}{\left(1 + \delta\left(t_i - t\right)\right)^2} dt$$

$$+P_o\left(\left(a\left(s_{i+1} - t_i\right) + \frac{b}{2}\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3}\right.\right.$$

$$\left(s_{i+1}^3 - t_i^3\right)\left(-\theta_1 - \alpha\beta t_i^{(\beta-1)} - p\alpha\beta\left(t_i^{''(\beta-1)} - t_i^{'(\beta-1)}\right)\right)$$

$$+\left(1 - \theta_1 t_i - \alpha t_i^{\beta} - p\theta_1 CT - p\alpha\left(t_i^{''\beta} - t_i^{'\beta}\right)\right)$$

$$(-a - bt_i - ct_i^2) + \theta_1 (-at_i - bt_i^2 - ct_i^3)$$
$$+\alpha \left(-at_i^\beta - bt_i^{(\beta+1)} - ct_i^{(\beta+2)}\right) + \int_{s_i}^{t_i} \frac{-\delta (a + bt + ct^2)}{(1 + \delta (t_i - t))^2} dt$$
$$+ (a + bt_i + ct_i^2) + S_c \left(\left(a (s_{i+1} - t_i) + \frac{b}{2}\right.\right.$$
$$\left(s_{i+1}^2 - t_i^2\right) + \frac{c}{3} \left(s_{i+1}^3 - t_i^3\right)$$
$$\left(-\theta_1 - \alpha\beta t_i^{(\beta-1)} - p\alpha\beta \left(t_i^{''(\beta-1)} - t_i^{'(\beta-1)}\right)\right)$$
$$+ \left(1 - \theta_1 t_i - \alpha t_i^\beta - p\theta_1 CT - p\alpha \left(t_i^{''\beta} - t_i^{'\beta}\right)\right)$$
$$(-a - bt_i - ct_i^2) + \theta_1 (-at_i - bt_i^2 - ct_i^3)$$
$$+\alpha \left(-at_i^\beta - bt_i^{(\beta+1)} - ct_i^{(\beta+2)}\right) \tag{3.22}$$

$$\frac{\delta TC_r^D(t_i, s_i, n_1^D)}{\delta s_i}$$
$$= H_o \left(\int_{t_{i-1}}^{t_{i-1}'} (((a + bs_i + cs_i^2)\right.$$
$$\left(1 - \theta_1 t - \alpha t^\beta - p\theta_1 CT - p\alpha \left(t_{i-1}^{''\beta} - t_{i-1}^{'\beta}\right)\right)$$
$$+\theta_1 (as_i + bs_i^2 + cs_i^3) + \alpha \left(as_i^\beta + bs_i^{(\beta+1)} + cs_i^{(\beta+2)}\right) dt$$
$$+H_o \left(\int_{t_{i-1}'}^{t_{i-1}''} ((-p) ((a + bs_i + cs_i^2)\right.$$
$$\left(1 + \theta_1 \left(t_{i-1}'' - t_{i-1} - t\right) + \alpha \left(t_{i-1}^{''\beta} - t_{i-1}^\beta - t^\beta\right)\right.$$
$$-p\theta_1 CT - p\alpha \left(t_{i-1}^{''\beta} - t_{i-1}^{'\beta}\right) + \theta_1 (as_i + bs_i^2 + cs_i^3)$$
$$+\alpha \left(as_i^\beta + bs_i^{(\beta+1)} + cs_i^{(\beta+2)}\right)$$
$$+ \left(1 - \theta_1 t - \alpha t^\beta\right) (a + bs_i + cs_i^2) + \theta_1 (as_i + bs_i^2 + cs_i^3)$$
$$+\alpha \left(as_i^\beta + bs_i^{(\beta+1)} + cs_i^{(\beta+2)}\right) dt$$
$$+H_o \left(\int_{t_{i-1}''}^{s_i} \left(\left(1 - \theta_1 t - \alpha t^\beta\right) (a + bs_i + cs_i^2)\right.\right.$$
$$+\theta_1 (as_i + bs_i^2 + cs_i^3) + \alpha \left(as_i^\beta + bs_i^{(\beta+1)} + cs_i^{(\beta+2)}\right) dt$$
$$+D_C \left(\int_{t_{i-1}}^{t_{i-1}'} \left(\alpha\beta t^{(\beta-1)}\right) (a + bs_i + cs_i^2) dt\right.$$

$$+\int_{t'_{i-1}}^{t''_{i-1}} \left(\alpha\beta t^{(\beta-1)}\right)(1-p)\left(a+bs_i+cs_i^2\right)dt+$$
$$\int_{t''_{i-1}}^{s_i} \left(\alpha\beta t^{(\beta-1)}\right)\left(a+bs_i+cs_i^2\right)dt$$
$$-S_o\left(t_i-s_i\right)\frac{a+bs_i+cs_i^2}{1+\delta\left(t_i-s_i\right)}$$
$$-L_o\delta\left(t_i-s_i\right)\frac{a+bs_i+cs_i^2}{1+\delta\left(t_i-s_i\right)}$$
$$+P_o\left(\left(a+bs_i+cs_i^2\right)\left(1-\theta_1 t_{i-1}-\alpha t_{i-1}^{\beta}-p\theta_1 CT\right.\right.$$
$$-p\alpha\left(t''^{\beta}_{i-1}-t'^{\beta}_{i-1}\right)+\theta_1\left(as_i+bs_i^2+cs_i^3\right)$$
$$+\alpha\left(as_i^{\beta}+bs_i^{(\beta+1)}+cs_i^{(\beta+2)}\right)-\frac{a+bs_i+cs_i^2}{1+\delta\left(t_i-s_i\right)}$$
$$+S_c\left(\left(a+bs_i+cs_i^2\right)\left(1-\theta_1 t_{i-1}-\alpha t_{i-1}^{\beta}-p\theta_1 CT\right.\right.$$
$$-p\alpha\left(t''^{\beta}_{i-1}-t'^{\beta}_{i-1}\right)+\theta_1\left(as_i+bs_i^2+cs_i^3\right)+$$
$$\alpha\left(as_i^{\beta}+bs_i^{(\beta+1)}+cs_i^{(\beta+2)}\right) \tag{3.23}$$

After obtaining the values of $t_i's$ and $s_i's$, from equation 3.22 and 3.23, $TC_r^D(t_i, s_i, n_1^D)$ and $TC_s^D(t_i, s_i, n_1^D)$ are calculated for different $n_1^D{'}s$, from equation 3.20 and 3.21. Thus obtaining total optimal cost of retailer $TC_r^{DO}(t_i^{DO}, s_i^{DO}, n_1^{DO})$, optimal cost of supplier $TC_s^{DO}(t_i^{DO}, s_i^{DO}, n_1^{DO})$,Optimal number of replenishment cycles (n_1^{DO}) and optimal ordering quantity (Q^{DO}) from equation 3.20, 3.21 and 4.11 respectively. Refer section 3.7 for an example.

3.4.2 Centralized case for a finite planning horizon

In a centralized case scheduled number of replenishment cycle is dependent upon the suppliers total cost $TC_s^C(t_j, s_j, n_2^C)$ and increase in retailer's cost. Increased retailer's cost is obtained by subtracting total optimal cost of retailer $TC_r^{DO}(t_i^{DO}, s_i^{DO}, n_1^{DO})$ during decentralized system from total cost of retailer in a centralized system $TC_r^C(t_j, s_j, n_2^C)$.

Different cost associated with supplier during the planning horizon for $j = 1, 2 \ldots n_2^C$

With all the cost function assumed to be same as in decentralized case here supplier's total cost $= TC_s^C(t_j, s_j, n_2^C) =$

$$\sum_{j=1}^{n_2^C}\{S_s * n + P_s * (I_{oj} + S_j) + (T_c * p + DsAsm * p+$$

$$Rem * p + H_o * p) * I_{oj}\} + TC_r^C(t_j, s_j, n_2^C) - TC_r^{DO}(t_i^{DO}, s_i^{DO}, n_1^{DO}), \quad (3.24)$$

where $\{j = 1, 2 \ldots n_2^C\}$. $TC_r^C(t_j, s_j, n_2^{CO})$ can be obtained from equation 3.20 by replacing in $TC_s^D(t_i, s_i, n_1^D)$, t_i from optimal t_j, s_i from optimal s_j and n_2^{DO} from n_2^{CO}. Q^{CO} can be obtained with similar substitution in Q^{DO}.

Solution for centralized case

Considering α, θ_1 very small and neglecting their square and higher order, now change in TC_s^C with respect to t_j and s_j is obtained in equation 3.25 and 3.26 as follows.

$$\frac{\delta TC_s^C(t_j, s_j, n_2^C)}{\delta t_j}$$

$$= (Ps + pTc + pDsAsm + pRem)\left(\left(a\left(s_{j+1} - t_j\right) + \frac{b}{2}\right.\right.$$

$$\left(s_{j+1}^2 - t_j^2\right) + \frac{c}{3}\left(s_{j+1}^3 - t_j^3\right)$$

$$\left(-\theta_1 - \alpha\beta t_{j-1}^{(\beta-1)} - \alpha\beta\left(t_j''^{(\beta-1)} - t_j'^{(\beta-1)}\right)\right) +$$

$$\left(1 - \theta_1 t_j - \alpha t_j^\beta - p\theta_1 CT - p\alpha\left(t_j''^\beta - t_j'^\beta\right)\right)\left(-a - bt_j - CT_j^2\right)$$

$$+\theta_1\left(-at_j - bt_j^2 - CT_j^3\right) + \alpha\left(-at_j^\beta - bt_j^{(\beta+1)} - CT_j^{(\beta+2)}\right)$$

$$-Ps\left(\int_{s_j}^{t_j} \frac{\delta\left(a + bt + CT^2\right)}{\left(1 + \delta\left(t_j - t\right)\right)^2} dt\right.$$

$$+\left(a + bt_j + CT_j^2\right) + pHo\left(\int_{t_j'}^{t_j''}\left(\left(a\left(s_{j+1} - t_j\right) + \frac{b}{2}\left(s_{j+1}^2 - t_j^2\right)\right.\right.\right.$$

$$+\frac{c}{3}\left(s_{j+1}^3 - t_j^3\right)\left(-\theta_1 - \alpha\beta t_{j-1}^{(\beta-1)}\right.$$

$$-\alpha\beta\left(t_j''^{(\beta-1)} - t_j'^{(\beta-1)}\right) + \left(1 - \theta_1 t_j - \alpha t_j^\beta\right.$$

$$-p\theta_1 CT - p\alpha\left(t_j''^\beta - t_j'^\beta\right)\left(-a - bt_j - CT_j^2\right) +$$

$$\theta_1\left(-at_j - bt_j^2 - CT_j^3\right) + \alpha\left(-at_j^\beta - bt_j^{(\beta+1)} - CT_j^{(\beta+2)}\right) dt$$

$$+Ho\left(\int_{t_j}^{t_j'}\left(\left(-a - bt_j - CT_j^2\right)\left(-p\theta_1 CT - p\alpha\left(t_j''^\beta - t_j'^\beta\right)\right)\right.\right.$$

$$+\left(a\left(s_{j+1} - t_j\right) + \frac{b}{2}\right.$$

$$\left(s_{j+1}^{2}-t_{j}^{2}\right)+\frac{c}{3}\left(s_{j+1}^{3}-t_{j}^{3}\right)$$

$$\left(-\alpha\beta\left(t_{j}^{''(\beta-1)}-t_{j}^{'(\beta-1)}\right)\right)\,dt+$$

$$\left(1-\theta_{1}t_{j}^{'-\alpha t_{j}^{'\beta}}\right)\left(a\left(s_{j+1}-t_{j}^{'}\right)+\frac{b}{2}\right.$$

$$\left(s_{j+1}^{2}-t_{j}^{'2}\right)+\frac{c}{3}\left(s_{j+1}^{3}-t_{j}^{'3}\right)$$

$$+\theta_{1}\left(\frac{a}{2}\left(t_{j}^{2}-t_{j}^{'2}\right)\right.$$

$$+\frac{b}{3}\left(t_{j}^{3}-t_{j}^{'3}\right)+\frac{c}{4}\left(t_{j}^{4}-t_{j}^{'4}\right)$$

$$+\alpha\left(\frac{a}{\beta+1}\left(t_{j}^{(\beta+1)}-t_{j}^{'(\beta+1)}\right)+\right.$$

$$\frac{b}{\beta+2}\left(t_{j}^{(\beta+2)}-t_{j}^{'(\beta+2)}\right)+\frac{c}{\beta+3}$$

$$\left(t_{j}^{(\beta+3)}-t_{j}^{'(\beta+3)}\right)-\left(1-\theta_{1}t_{j}-\alpha\left(t_{j}^{\beta}\right)\right)\left(a\left(s_{j+1}-t_{j}\right)+\frac{b}{2}\right.$$

$$\left(s_{j+1}^{2}-t_{j}^{2}\right)+\frac{c}{3}\left(s_{j+1}^{3}-t_{j}^{3}\right)+Ho\left((-p)\int_{t_{j}^{'}}^{t_{j}^{''}}\left(\left(1+\theta_{1}\left(t_{j}^{''}-t_{j}-t\right)\right.\right.\right.$$

$$+\alpha\left(t_{j}^{''\beta}-t_{j}^{\beta}-t^{\beta}\right)-p\theta_{1}CT-p\alpha\left(t_{j}^{''\beta}-t_{j}^{'\beta}\right)\left(-a-bt_{j}-CT_{j}^{2}\right)$$

$$+\left(a\left(s_{j+1}-t_{j}\right)+\frac{b}{2}\left(s_{j+1}^{2}-t_{j}^{2}\right)+\frac{c}{3}\left(s_{j+1}^{3}-t_{j}^{3}\right)\right)$$

$$\left(\alpha\beta\left(t_{j}^{''(\beta-1)}-t_{j-1}^{(\beta-1)}\right)-\alpha\beta\left(t_{j}^{''(\beta-1)}-t_{j}^{'(\beta-1)}\right)\right)+\theta_{1}\left(-at_{j}-bt_{j}^{2}-CT_{j}^{3}\right)$$

$$+\alpha\left(-at_{j}^{\beta}-bt_{j}^{(\beta+1)}-CT_{j}^{(\beta+2)}\right)\,dt+\left(1-\theta_{1}t_{j}^{''}-\alpha t_{j}^{''\beta}\right)\left(a\left(s_{j+1}-t_{j}^{''}\right)+\right.$$

$$\frac{b}{2}\left(s_{j+1}^{2}-t_{j}^{''2}\right)+\frac{c}{3}\left(s_{j+1}^{3}-t_{j}^{''3}\right)+\theta_{1}\left(\frac{a}{2}\left(s_{j+1}^{2}-t_{j}^{''2}\right)+\right.$$

$$\frac{b}{3}\left(s_{j+1}^{3}-t_{j}^{''3}\right)+\frac{c}{4}\left(s_{j+1}^{4}-t_{j}^{''4}\right)+\alpha\left(\frac{a}{\beta+1}\left(s_{j+1}^{(\beta+1)}-t_{j}^{''(\beta+1)}\right)+\frac{b}{\beta+2}\right.$$

$$\left(s_{j+1}^{(\beta+2)}-t_{j}^{''(\beta+2)}\right)+\frac{c}{\beta+3}\left(s_{j+1}^{(\beta+3)}-t_{j}^{''(\beta+3)}\right)-\left(1-\theta_{1}t_{j}^{'-\alpha t_{j}^{'\beta}}\right)$$

$$\left(a\left(s_{j+1}-t_{j}^{'}\right)+\frac{b}{2}\left(s_{j+1}^{2}-t_{j}^{'2}\right)+\frac{c}{3}\left(s_{j+1}^{3}-t_{j}^{'3}\right)\right)$$

$$-\theta_1\left(\frac{a}{2}\left(s_{j+1}^2-t_j^{'2}\right)+\frac{b}{3}\left(s_{j+1}^3-t_j^{'3}\right)+\frac{c}{4}\left(s_{j+1}^4-t_j^{'4}\right)\right)$$

$$-\alpha\left(\frac{a}{\beta+1}\left(s_{j+1}^{(\beta+1)}-t_j^{'(\beta+1)}\right)+\frac{b}{\beta+2}\left(s_{j+1}^{(\beta+2)}-t_j^{'(\beta+2)}\right)\right.$$

$$+\frac{c}{\beta+3}\left(s_{j+1}^{(\beta+3)}-t_j^{'(\beta+3)}\right)+p\left(a\left(s_{j+1}-t_j\right)+\frac{b}{2}\right.$$

$$\left(s_{j+1}^2-t_j^2\right)+\frac{c}{3}\left(s_{j+1}^3-t_j^3\right)\left(\theta_1 CT+\alpha\left(t_j^{''\beta}-t_j^{'\beta}\right)\right)$$

$$-Ho\left(\left(1-\theta_1 t_j^{''}-\alpha t_j^{''\beta}\right)\left(a\left(s_{j+1}-t_j^{''}\right)+\frac{b}{2}\right.\right.$$

$$\left(s_{j+1}^2-t_j^{''2}\right)+\frac{c}{3}\left(s_{j+1}^3-t_j^{''3}\right)+\theta_1\left(\frac{a}{2}\right.$$

$$\left(s_{j+1}^2-t_j^{''2}\right)+\frac{b}{3}\left(s_{j+1}^3-t_j^{''3}\right)+\frac{c}{4}\left(s_{j+1}^4-t_j^{''4}\right)$$

$$+\alpha\left(\frac{a}{\beta+1}\left(s_{j+1}^{(\beta+1)}-t_j^{''(\beta+1)}\right)+\frac{b}{\beta+2}\left(s_{j+1}^{(\beta+2)}-t_j^{''(\beta+2)}\right)+\frac{c}{\beta+3}\right.$$

$$\left(s_{j+1}^{(\beta+3)}-t_j^{''(\beta+3)}\right)+DC\left(-\int_{t_j^{'}}^{t_j^{''}}\alpha\beta t^{(\beta-1)}p\left(-a-bt_j-CT_j^2\right)dt\right.$$

$$+\alpha\beta t_j^{'(\beta-1)}\left(a\left(s_{j+1}-t_j^{'}\right)+\frac{b}{2}\left(s_{j+1}^2-t_j^{'2}\right)+\frac{c}{3}\right.$$

$$\left(s_{j+1}^3-t_j^{'3}\right)-\alpha\beta t_{j-1}^{(\beta-1)}\left(a\left(s_{j+1}-t_j\right)+\frac{b}{2}\left(s_{j+1}^2-t_j^2\right)\right.$$

$$+\frac{c}{3}\left(s_{j+1}^3-t_j^3\right)-\alpha\beta t_j^{''(\beta-1)}p\left(a\left(s_{j+1}-t_j\right)\right.$$

$$+\frac{b}{2}\left(s_{j+1}^2-t_j^2\right)+\frac{c}{3}\left(s_{j+1}^3-t_j^3\right)+\alpha\beta t_j^{'(\beta-1)}p\left(a\left(s_{j+1}-t_j\right)\right.$$

$$+\frac{b}{2}\left(s_{j+1}^2-t_j^2\right)+\frac{c}{3}\left(s_{j+1}^3-t_j^3\right)+$$

$$So\int_{s_j}^{t_j}\frac{a+bt+CT^2}{\left(1+\delta\left(t_j-t\right)\right)^2}dt+$$

$$Lo\int_{s_j}^{t_j}\frac{\delta\left(a+bt+CT^2\right)}{\left(1+\delta\left(t_j-t\right)\right)^2}dt+$$

$$Po\left(\left(a\left(s_{j+1}-t_j\right)+\frac{b}{2}\left(s_{j+1}^2-t_j^2\right)+\frac{c}{3}\right.\right.$$

$$\left(s_{j+1}^3 - t_j^3\right)\left(-\theta_1 - \alpha\beta t_{j-1}^{(\beta-1)} - \alpha\beta\left(t_j^{''(\beta-1)} - t_j^{'(\beta-1)}\right)\right)$$
$$+\left(1 - \theta_1 t_j - \alpha t_j^\beta - p\theta_1 CT - p\alpha\left(t_j^{''\beta} - t_j^{'\beta}\right)\right)\left(-a - bt_j - CT_j^2\right)$$
$$+\theta_1\left(-at_j - bt_j^2 - CT_j^3\right) + \alpha\left(-at_j^\beta - bt_j^{(\beta+1)} - CT_j^{(\beta+2)}\right)$$
$$+\int_{s_j}^{t_j} \frac{-\delta\left(a + bt + CT^2\right)}{\left(1 + \delta\left(t_j - t\right)\right)^2} dt +$$
$$\left(a + bt_j + CT_j^2\right) + Sc\left(\left(a\left(s_{j+1} - t_j\right) + \frac{b}{2}\left(s_{j+1}^2 - t_j^2\right) + \frac{c}{3}\right.\right.$$
$$\left(s_{j+1}^3 - t_j^3\right)\left(-\theta_1 - \alpha\beta t_{j-1}^{(\beta-1)} - \alpha\beta\left(t_j^{''(\beta-1)} - t_j^{'(\beta-1)}\right)\right) +$$
$$\left(1 - \theta_1 t_j - \alpha t_j^\beta - p\theta_1 CT - p\alpha\left(t_j^{''\beta} - t_j^{'\beta}\right)\right)\left(-a - bt_j - CT_j^2\right) +$$
$$\theta_1\left(-at_j - bt_j^2 - CT_j^3\right) + \alpha\left(-at_j^\beta - bt_j^{(\beta+1)} - CT_j^{(\beta+2)}\right) \tag{3.25}$$

$$\frac{\delta TC_s^C(t_j, s_j, n_2^C)}{\delta s_j} = \left(P_s + pT_c + pDsAsm + pRem\right)\left(\left(a + bs_j + cs_j^2\right)\right.$$
$$\left(1 - \theta_1 t_{j-1} - \alpha t_{j-1}^\beta - p\theta_1 CT - p\alpha\left(t_{j-1}^{''\beta} - t_{j-1}^{'\beta}\right)\right)$$
$$\left.+\theta_1\left(as_j + bs_j^2 + cs_j^3\right) + \alpha\left(as_j^\beta + bs_j^{(\beta+1)} + cs_j^{(\beta+2)}\right)\right)$$
$$-P_s \frac{a + bs_j + cs_j^2}{1 + \delta\left(t_j - s_j\right)}$$
$$+H_o p\left(\int_{t'_{j-1}}^{t''_{j-1}}\left(\left(a + bs_j + cs_j^2\right)\left(1 - \theta_1 t_{j-1} - \alpha t_{j-1}^\beta - p\theta_1 CT\right.\right.\right.$$
$$\left.-p\alpha\left(t_{j-1}^{''\beta} - t_{j-1}^{'\beta}\right) + \theta_1\left(as_j + bs_j^2 + cs_j^3\right) + \alpha\left(as_j^\beta + bs_j^{(\beta+1)} + cs_j^{(\beta+2)}\right) dt\right.$$
$$+H_o\left(\int_{t_{j-1}}^{t'_{j-1}}\left(\left(\left(a + bs_j + cs_j^2\right)\left(1 - \theta_1 t - \alpha t^\beta - p\theta_1 CT\right.\right.\right.\right.$$
$$\left.-p\alpha\left(t_{j-1}^{''\beta} - t_{j-1}^{'\beta}\right) + \theta_1\left(as_j + bs_j^2 + cs_j^3\right) + \alpha\left(as_j^\beta + bs_j^{(\beta+1)} + cs_j^{(\beta+2)}\right) dt +\right.$$
$$H_o\left(\int_{t'_{j-1}}^{t''_{j-1}}\left((-p)\left(\left(a + bs_j + cs_j^2\right)\left(1 + \theta_1\left(t''_{j-1} - t_{j-1} - t\right)\right.\right.\right.\right.$$
$$+\alpha\left(t_{j-1}^{''\beta} - t_{j-1}^\beta - t^\beta\right) - p\theta_1 CT - p\alpha\left(t_{j-1}^{''\beta} - t_{j-1}^{'\beta}\right) + \theta_1\left(as_j + bs_j^2 + cs_j^3\right)$$
$$+\alpha\left(as_j^\beta + bs_j^{(\beta+1)} + cs_j^{(\beta+2)}\right) + \left(1 - \theta_1 t - \alpha t^\beta\right)\left(a + bs_j + cs_j^2\right)$$
$$+\theta_1\left(as_j + bs_j^2 + cs_j^3\right) + \alpha\left(as_j^\beta + bs_j^{(\beta+1)} + cs_j^{(\beta+2)}\right) dt$$
$$+H_o\left(\int_{t''_{j-1}}^{s_j}\left(\left(1 - \theta_1 t - \alpha t^\beta\right)\left(a + bs_j + cs_j^2\right) + \theta_1\left(as_j + bs_j^2 + cs_j^3\right)\right.\right.$$

$$+\alpha\left(as_j^{\beta}+bs_j^{(\beta+1)}+cs_j^{(\beta+2)}\right)dt+D_C\left(\int_{t_{j-1}}^{t'_{j-1}}\left(\alpha\beta t^{(\beta-1)}\right)\left(a+bs_j+cs_j^2\right)dt\right.$$
$$+\int_{t'_{j-1}}^{t''_{j-1}}\left(\alpha\beta t^{(\beta-1)}\right)(1-p)\left(a+bs_j+cs_j^2\right)dt$$
$$+\int_{t''_{j-1}}^{s_j}\left(\alpha\beta t^{(\beta-1)}\right)\left(a+bs_j+cs_j^2\right)dt-$$
$$S_o\left(t_j-s_j\right)\frac{a+bs_j+cs_j^2}{1+\delta\left(t_j-s_j\right)}-L_o\delta\left(t_j-s_j\right)\frac{a+bs_j+cs_j^2}{1+\delta\left(t_j-s_j\right)}$$
$$+P_o\left(\left(a+bs_j+cs_j^2\right)\left(1-\theta_1 t_{j-1}-\alpha t_{j-1}^{\beta}-p\theta_1 CT-p\alpha\left(t''^{\beta}_{j-1}-t'^{\beta}_{j-1}\right)\right)\right.$$
$$+\theta_1\left(as_j+bs_j^2+cs_j^3\right)+\alpha\left(as_j^{\beta}+bs_j^{(\beta+1)}+cs_j^{(\beta+2)}\right)-\frac{a+bs_j+cs_j^2}{1+\delta\left(t_j-s_j\right)}$$
$$+S_c\left(\left(a+bs_j+cs_j^2\right)\left(1-\theta_1 t_{j-1}-\alpha t_{j-1}^{\beta}-p\theta_1 CT-p\alpha\left(t''^{\beta}_{j-1}-t'^{\beta}_{j-1}\right)\right)+\right.$$
$$\theta_1\left(as_j+bs_j^2+cs_j^3\right)+\alpha\left(as_j^{\beta}+bs_j^{(\beta+1)}+cs_j^{(\beta+2)}\right) \tag{3.26}$$

After obtaining the values of $t'_j s$ and $s'_j s$, $TC_s^C(t_j, s_j, n_2^C)$ and $TC_r^C(t_j, s_j, n_2^C)$ are calculated for different $n'_j s$, from equation 3.25 and 3.26. Thus obtaining total optimal cost of supplier $TC_s^{CO}(t_j^{CO}, s_j^{CO}, n_2^{CO})$, optimal cost of retailer $TC_r^{CO}(t_j^{CO}, s_j^{CO}, n_2^{CO})$, Optimal number of replenishment cycles (n_2^{CO}) and optimal ordering quantity (Q^{CO}).

Now calculating the systems improved cost. The supplier shares the profit, obtained due to reduction of replenishment cycles in centralized case compared to decentralized case, with retailer.

System's improved cost in centralized case

$$= Profit = [TC_s^{DO}+TC_r^{DO}]-[TC_s^{CO}+TC_r^{CO}] \tag{3.27}$$

Improved retailer's cost $= TC_r^{COP} = TC_r^{DO} - \dfrac{TC_r^{DO}}{TC_r^{DO}+TC_s^{DO}}$ and Improved supplier's cost $= TC_s^{COP} = TC_s^{DO} - \dfrac{TC_s^{DO}}{TC_r^{DO}+TC_s^{DO}}$.

Percentage profit of retailer $= \dfrac{TC_r^{DO}-TC_r^{COP}}{TC_r^{DO}} * 100$ and Percentage profit of retailer $= \dfrac{TC_s^{DO}-TC_s^{COP}}{TC_s^{DO}} * 100$.

The profit shared can also be gained by retailer in terms of credit. Where credit period rate $= \dfrac{TC_r^{DO}-TC_r^{COP}}{O_c*(s_{i+1}^{CO}-t_i^{CO})*Q^{CO}}$.

3.5 Optimality condition for TC_r^D and TC_s^C

$\frac{\delta TC_r^D}{\delta t_i^2}$, $\frac{\delta TC_r^D}{\delta s_i^2}$, $\frac{\delta TC_s^C}{\delta t_j^2}$, $\frac{\delta TC_s^C}{\delta t_j^2}$, $\frac{\partial^2 \mathrm{TC}_r^D}{\partial t_i \partial s_i}$ and $\frac{\partial^2 \mathrm{TC}_r^D}{\partial s_i \partial t_i}$ can be obtained through partial differentiation as solved in equation 3.22, 3.23, 3.25 and 3.26. The sufficient condition for TC_r^D to be minimum is that the following Hessian matrix $\nabla^2 \mathrm{TC}_r^D$ of TC_r^D for a fixed n_1 is positive definite Sarkar, Ghosh, and Chaudhuri (2012b). Where

$$\nabla^2 TC_r^D = \begin{pmatrix} \frac{\partial^2 TC_r^D}{\partial t_1^2} & \frac{\partial^2 TC_r^D}{\partial t_1 \partial s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 TC_r^D}{\partial s_1 \partial t_1} & \frac{\partial^2 TC_r^D}{\partial s_1^2} & \frac{\partial^2 TC_r^D}{\partial s_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 TC_r^D}{\partial t_2 \partial s_1} & \frac{\partial^2 TC_r^D}{\partial t_1^2} & \frac{\partial^2 TC_r^D}{\partial t_2 \partial s_2} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 TC_r^D}{\partial t_{n_1-1} \partial s_{n_1-2}} & \frac{\partial^2 TC_r^D}{\partial t_{n_1-1}^2} & \frac{\partial^2 TC_r^D}{\partial t_{n_1-1} \partial s_{n_1-1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 TC_r^D}{\partial s_{n_1-1} \partial t_{n_1-1}} & \frac{\partial^2 TC_r^D}{\partial s_{n_1-1}^2} & \frac{\partial^2 TC_r^D}{\partial s_{n_1-1} \partial t_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 TC_r^D}{\partial t_{n_1} \partial s_{n_1-1}} & \frac{\partial^2 TC_r^D}{\partial t_{n_1}^2} \end{pmatrix}. \tag{3.28}$$

Theorem 1 : If t_i and s_i satisfy inequations (i) $\frac{\partial^2 \mathrm{TC}_r^D}{\partial t_i^2}$ >0, (ii) $\frac{\partial^2 \mathrm{TC}_r^D}{\partial s_i^2}$ >0, (iii) $\frac{\partial^2 \mathrm{TC}_r^D}{\partial t_i^2} - \left| \frac{\partial^2 \mathrm{TC}_r^D}{\partial t_i \partial s_i} \right|$ >0 and (iv) $\frac{\partial^2 \mathrm{TC}_r^D}{\partial s_i^2} - \left| \frac{\partial^2 \mathrm{TC}_r^D}{\partial s_i \partial t_i} \right|$ >0 for $i = 1, 2 \ldots n_1^D$ then $\nabla^2 \mathrm{TC}_r^D$ is positive definite.

The same can be used to show that $\nabla^2 \mathrm{TC}_s^C$ is a positive definite and TC_s^C(n_2, s_0, t_1^c,s_1^c, ..., $s_{n_2+1}^c$) attains a minimum.

Based on the above Theorem 1 the algorithm for the solution is as follows:

3.6 Algorithm for both decentralized and centralized cases

1. The parameters α , θ_1, CT, D_C, P_o, S_o, H_o, L_o, a, b, c, p, S_c, C_o, β , T_c, $DsAsm$, Rem, S_s, P_s and δ are allocated with constant values.

2. In a decentralized case find the optimal ordering schedule.

   (a) Set n_1^D=1, s_1^D=0, s_2=H. Calculate t_1 from equation 3.22

   (b) Set n_1^D=2 in equation 3.22.

(c) With given value of parameter t_1 and $s_1 = 0$, calculate s_2 from equation 3.22

(d) Now with the values of t_1, s_2 and $s_1 = 0$ calculate t_2 from equation 3.23

(e) Similarly for remaining $n_i^{D\prime}s$ repeat 2c, 2d calculate all respective, unique and optimal values of $t_i^{\prime}s$ and $s_i^{\prime}s$ with euations 3.22 and 3.23

(f) For $n_1^D = 1$ and if $TC_r^D\left(n_1^D\right) < TC_r^D\left(n_1^D+1\right)$,then $TC_r^D\left(n_1^D\right)$ $= TC_r^{DO}\left(n_1^D\right)$.Stop

(g) For $n_1^D \geq 2$ and if $TC_r^D\left(n_1^D\right) < TC_r^D\left(n_1^D-1\right)$ and $TC_r^D\left(n_1^D\right) < TC_r^D\left(n_1^D+1\right)$, then $TC_r^D\left(n_1^D\right) = TC_r^{DO}\left(n_1^D\right)$, $n_1^{DO} = n_1^D$ and stop else let $n_1^D = n_1^D + 1$ and goto step

3. In a decentralized case the optimal replenishment cycle for retailer and supplier is $n_1^{DO} = n_1$.
4. Calculate $TC_r^{DO}\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, ..., s_{n_1^{DO}+1}\right)$, $TC_s^{DO}\left(n_1^{DO}, s_1, t_1^{DO}, s_2^{DO}, ..., s_{n_1^{DO}+1}\right)$ and Q^{DO} from equations 3.20, 3.21 and 4.11 respectively.
5. In a centralized case find the optimal ordering schedule.

(a) Set n_2^C=1, s_1=0, s_2=H. Calculate t_1 from equation 3.25

(b) Set n_2^C=2 in equation 3.25.

(c) With given value of parameter t_1 and $s_1 = 0$, calculate s_2 from equation 3.25

(d) Now with the values of t_1, s_2 and $s_1 = 0$ calculate t_2 from equation 3.26

(e) Similarly for remaining $n_j^{C\prime}s$ repeat 5c, 5d calculate all respective, unique and optimal values of $t_j^{C\prime}s$ and $s_j^{C\prime}s$ with euations 3.25 and 3.26

(f) For $n_2^C = 1$ and if $TC_s^C\left(n_2^C\right) < TC_s^C\left(n_2^C+1\right)$,then $TC_s^C\left(n_2^C\right) = TC_s^{CO}\left(n_2^C\right)$.Stop

(g) For $n_2^C \geq 2$ and if $TC_s^C\left(n_2^C\right) < TC_s^C\left(n_2^C-1\right)$ and $TC_s^C\left(n_2^C\right) < TC_s^C\left(n_2^C+1\right)$, then $TC_s^C\left(n_2^C\right) = TC_s^{CO}\left(n_2^C\right)$, $n_2^{CO} = n_2^C$ and stop else let $n_2^C = n_2^C + 1$, and goto step 5c

6. Following steps 2 to 5 calculate t_j^C, s_j^C, n_2^{CO}, TC_s^{CO}, TC_r^{CO}, Q^{CO}.
7. Calculate $Profit$, TC_r^{COP}, TC_s^{COP} and λ.

3.7 Example to distinguish both the cases and obtaining profit in a green supply chain

TABLE 3.1: Total cost of retailer in a decentralized case

	TC_r^D							TC_s^D
$\downarrow a$ $\rightarrow n_1$	1	2	3	4	5	6	7	when $n_1 = \mathbf{3}$
1425.	50369.2	46697.4	**46609.9**	47927.4	50155.2	53125.	56766.2	**34812.7**
1500	52676.9	48810.9	**48653.1**	49935.2	52141.4	55095.7	58724.7	**35583.4**
1575.	54984.7	50924.3	**50696.2**	51943.	54127.5	57066.5	60683.2	**36354.2**

TABLE 3.2: Optimal schedule for retailer in a decentralized case

a	n_1^{DO}	s_1	s_2	s_3	s_4
1425.	3	0	0.8257	1.9838	4.
1500	3	0	0.8237	1.9804	4.
1575.	3	0	0.8219	1.9773	4.

a	n_1^{DO}	t_1	t_2	t_3
1425.	3	0.0002	0.826	1.9843
1500	3	0.0002	0.824	1.9809
1575.	3	0.0002	0.8222	1.9778

TABLE 3.3: Total Cost of supplier in a centralized case

$\downarrow a \rightarrow n_1$	1	2	3	4
1425.	32108.9	**22999.3**	27302.8	38647.8
1500	33604.9	**23744.5**	27722.6	38864.6
1575.	35086.1	**24489.8**	28142.4	39081.4

TABLE 3.4: Optimal schedule for supplier in a centralized case

a	n_2^{CO}	s_1	s_2	s_3
1425.	2	0	2.1551	4.
1500	2	0	2.1531	4.
1575.	2	0	2.1514	4.

a	n_2^{CO}	t_1	t_2
1425.	2	0.0012	2.1563
1500	2	0.0012	2.1544
1575.	2	0.0012	2.1526

Keeping a note of our main assumption that the setup cost of the supplier is greater than ordering cost of retailer an example has been framed. The values of different parameters are taken as follows. $\alpha = 0.0009$, $\theta_1 = 0.0009$, $D_C = 600$, $P_o = 5$, $S_o = 290$, $H_o = 1$, $L_o = 600$, $b = 50$, $c = 15$, $p = 0.005$, $S_c = 0.64$, $C_o = 300$, $\beta = 1.5$, $T_c = 300$, $DsAsm = 35$, Rem=300, $S_s = 2000, P_s = 0.9, \delta = 10, O_c = 0.7, H = 4$ and $t_i'' - t_i' = t_i' - t_i = \dfrac{H}{4}$.

Also three different values of a that is $a = 1425, 1500$ and 1575 are considered for comparision. Shown in the table 5.1 the total optimal cost of the retailer in the decentralized system, $TC_r^{DO} = 46609.9, 48653.1$ and 50696.2 for $a = 1425$, 1500 and 1575 respectively. The corresponding $TC_s^{DO} = 34812.7, 35583.4$ and 36354.2. Table 5.2 is for the optimal schedule when $n_1 = 3$ and $a = 1425, 1500$ and 1575. For centralized case table 5.3 shows $n_2^{CO} = 2$ which is less than $n_1^{DO} = 3$ for all three a's and corresponding $TC_s^{CO} = 22999.3, 23744.5$ and 24489.8 total optimal cost of supplier$TC_s^{CO}(t_j^{CO}, s_j^{CO}, n_2^{CO})$ in the centralized system where the optimal schedule is decided by the supplier. The optimal replenishment schedule so obtained for the corresponding "a" value is given in table 3.4. Column 12 of table **??** shows that profit percentage decreases with increase in the value of a. And with an increase in a, credit period rate λ also decreases.

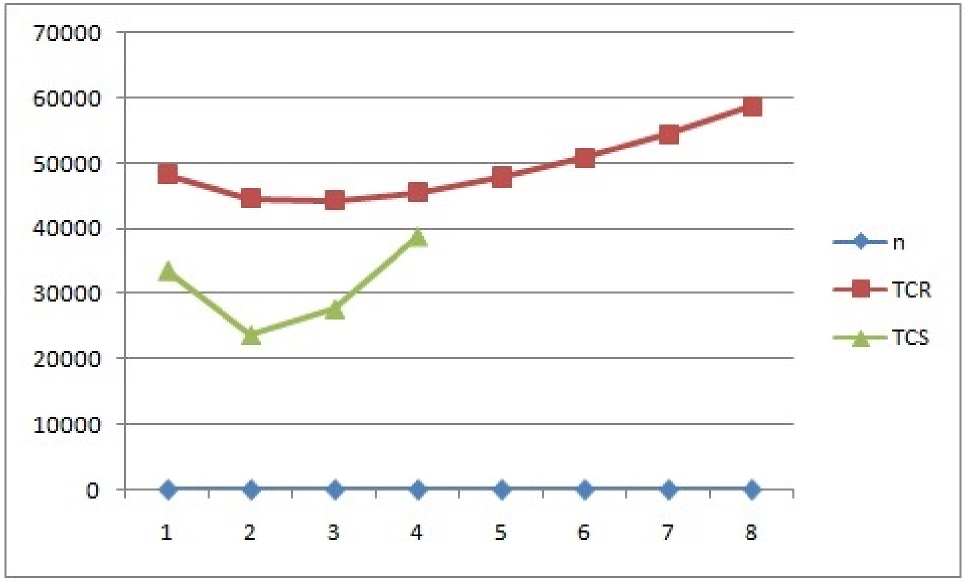

FIGURE 3.2: For Lo

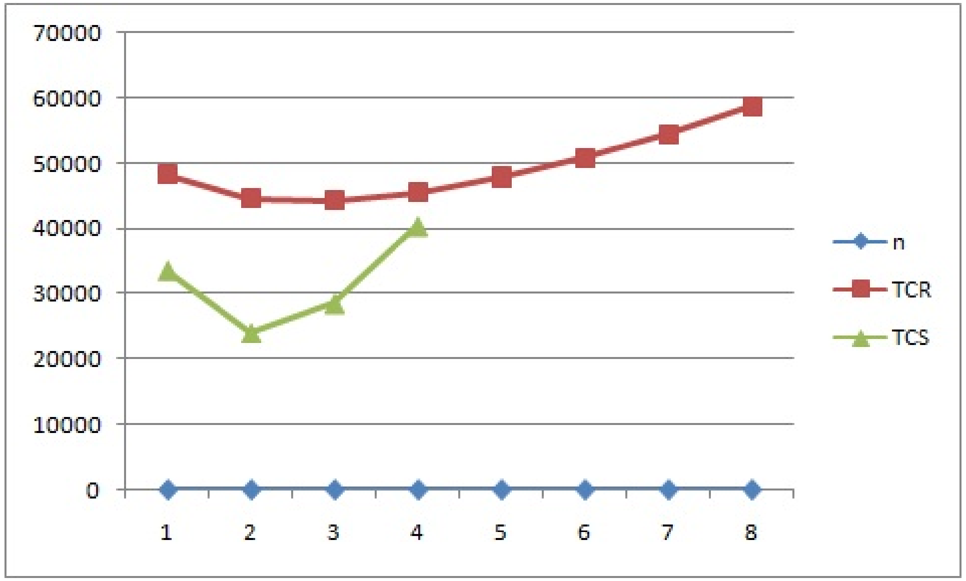

FIGURE 3.3: For Ss

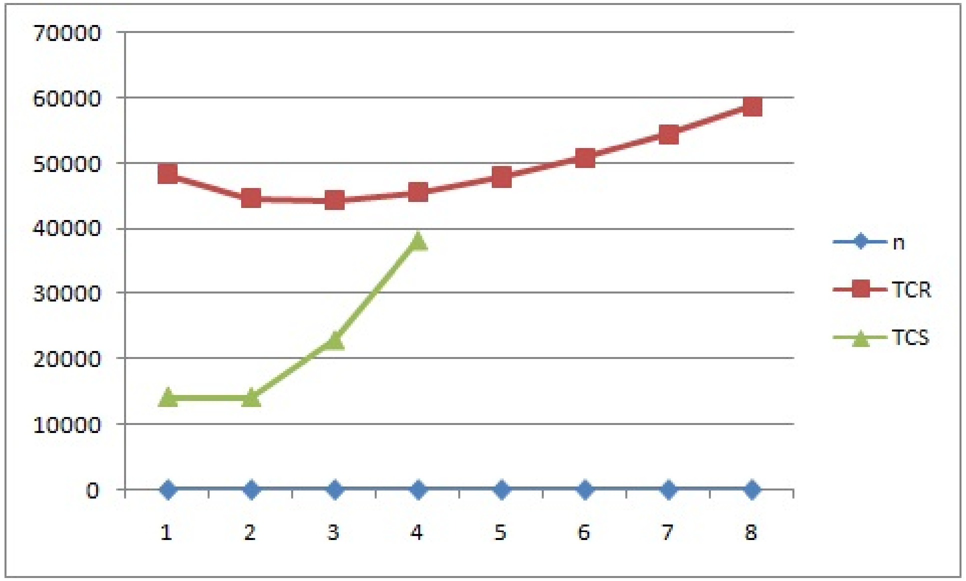

FIGURE 3.4: For Po

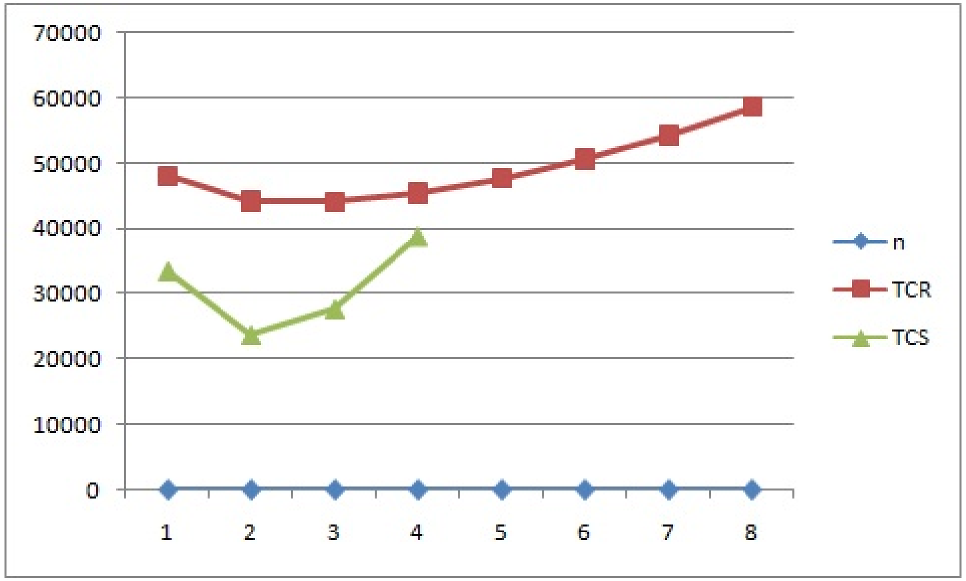

FIGURE 3.5: For α

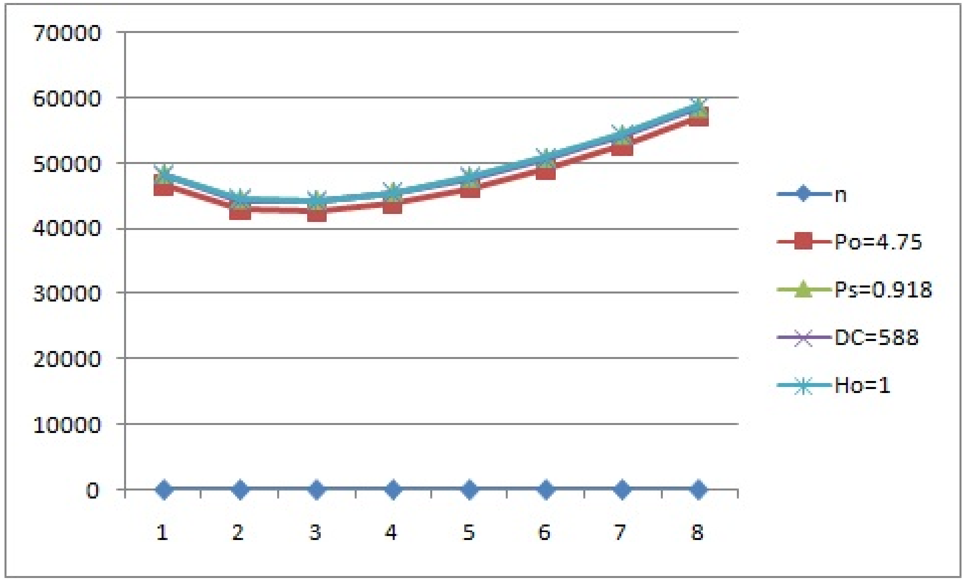

FIGURE 3.6: Convexity

3.8 Conclusion

From example 1, with small percentage change that is 2 and 5, one by one, in all of the parameters value, keeping rest of the values unchanged, percentage profit has been observed. The last column is for percentage profit for both retailer and supplier which will be same for both retailer and supplier. As shown in table 5.5 and 5.6 for different parameters profit increases with the increase in value of the parameters like c, P_o, D_C, L_o, S_s, α, C_o, δ, β, S_o. But the profit decreases with increase in value of parameters a, b, P_s, H_o, p, S_c, T_c, $DsAsm$, Rem, CT and θ_1. Therefore profit decreases with increase in cost related to remanufacturing that is transportation cost(T_c), dissassembly cost($DsAsm$), remanufacturing cost(Rem) and the time taken($CT = t_i^{''} - t_i^{'} = t_i^{'} - t_i = \frac{H}{4}$) for remanufacturing. The more the cost associated with remanufacturing, less is the profit. Profit also decreases with the increase in the amount of inventory delivered for remanufacturing(p) and screening cost(Sc) of inventory for the defective\repairable item.

The proposed model will provide a direction for the suppliers in developing countries like India where the efforts for new startup programs are promoted and initiated. Suppliers are now moving towards greener inventory management due to the government and social pressure. Due to health hazard goods cannot be simply discarded this in turn also reduces supplier and retailer's goodwill and compromises their market reputation. Thus there is a dire need of solving problems related to remanufacturing or repairing. Further, the proposed model can be extended in several ways such as fuzzifying the parameters, increasing the number of retailers, replacing the single item with multi items and applying inflationary conditions to some or all the cost involved in the present model.

TABLE 3.6: For $\delta table1$

δ	TC_r^{DO}	TC_s^{DO}	n_1^{DO}	Q^{DO}	TC_s^{CO}	TC_r^{CO}
9.5	44347.1	35591.7	3	6734.95	20965.2	23738.8
9.8	44344.5	35586.7	3	6734.94	20965.	23741.4
10.2	44340.9	35580.1	3	6734.93	20964.8	23744.9
10.5	44338.2	35575.1	3	6734.93	20964.6	23747.5
10	44342.7	35583.4	3	6734.94	20964.9	23743.1

δ	n_2^{CO}	Q^{CO}	TC_r^{COP}	TC_s^{COP}		$Percentage\ improvement\ in retailer's\ cost$	$Percentage\ improvement\ in supplier's\ cost$
9.5	2	6737.13	35234.7	24800.2	19903.9	44.0772	44.0772
9.8	2	6737.13	35224.8	24802.4	19904.1	44.0689	44.0689
10.2	2	6737.13	35211.4	24805.3	19904.3	44.0577	44.0577
10.5	2	6737.14	35201.2	24807.6	19904.5	44.0492	44.0492
10	2	6737.13	35218.1	24803.8	19904.2	44.0633	44.0633

TABLE 3.7: For b

b	TC_r^{DO}	TC_s^{DO}	n_1^{DO}	Q^{DO}	TC_s^{CO}	TC_r^{CO}
47.5	44215.2	35523.9	3	6714.89	20920.9	23680.2
49.	44291.7	35559.6	3	6726.92	20947.3	23717.9
51.	44393.7	35607.2	3	6742.96	20982.5	23768.3
52.5	44470.3	35643.	3	6754.99	21008.9	23806.1
50	44342.7	35583.4	3	6734.94	20964.9	23743.1

b	n_2^{CO}	Q^{CO}	TC_r^{COP}	TC_s^{COP}		$Percentage\ improvement\ in retailer's\ cost$	$Percentage\ improvement\ in\ supplier's\ cost$
47.5	2	6717.07	35138.	24731.2	19869.8	44.0663	44.0663
49.	2	6729.11	35186.1	24774.8	19890.4	44.0645	44.0645
51.	2	6745.16	35250.1	24832.9	19917.9	44.0621	44.0621
52.5	2	6757.19	35298.2	24876.5	19938.5	44.0604	44.0604
50	2	6737.13	35218.1	24803.8	19904.2	44.0633	44.0633

TABLE 3.8: For Lo

Lo	TC_r^{DO}	TC_s^{DO}	n_1^{DO}	Q^{DO}	TC_s^{CO}	TC_r^{CO}
570.	44342.7	35583.3	3	6734.94	20965.3	23743.3
588.	44342.7	35583.4	3	6734.94	20965.	23743.2
612.	44342.7	35583.5	3	6734.94	20964.8	23743.1
630.	44342.8	35583.5	3	6734.94	20964.6	23743.
600	44342.7	35583.4	3	6734.94	20964.9	23743.1

Lo	n_2^{CO}	Q^{CO}	TC_r^{COP}	TC_s^{COP}		Percentage improvement in retailer's cost	Percentage improvement in supplier's cost
570.	2	6737.13	35217.4	24804.1	19904.4	44.0626	44.0626
588.	2	6737.13	35217.8	24804.	19904.3	44.063	44.063
612.	2	6737.13	35218.3	24803.7	19904.1	44.0636	44.0636
630.	2	6737.14	35218.7	24803.6	19904.	44.064	44.064
600	2	6737.13	35218.1	24803.8	19904.2	44.0633	44.0633

TABLE 3.9: For Ss

Ss	TC_r^{DO}	TC_s^{DO}	n_1^{DO}	Q^{DO}	TC_s^{CO}	TC_r^{CO}
1900.	44342.7	34683.4	3	6734.94	20964.9	23343.1
1960.	44342.7	35223.4	3	6734.94	20964.9	23583.1
2040.	44342.7	35943.4	3	6734.94	20964.9	23903.1
2100.	44342.7	36483.4	3	6734.94	20964.9	24143.1
2000	44342.7	35583.4	3	6734.94	20964.9	23743.1

Ss	n_2^{CO}	Q^{CO}	TC_r^{COP}	TC_s^{COP}		$Percentage$ $improvement$ $in retailer's$ $cost$	$Percentage$ $improvement$ $in supplier's$ $cost$
1900.	2	6737.13	34718.1	24861.9	19446.2	43.9324	43.9324
1960.	2	6737.13	35018.1	24826.9	19721.1	44.0113	44.0113
2040.	2	6737.13	35418.1	24781.	20087.	44.1148	44.1148
2100.	2	6737.13	35718.1	24747.1	20360.9	44.1913	44.1913
2000	2	6737.13	35218.1	24803.8	19904.2	44.0633	44.0633

TABLE 3.10: For p

p	TC_r^{DO}	TC_s^{DO}	n_1^{DO}	Q^{DO}	TC_s^{CO}	TC_r^{CO}
0.000475	44348.7	25156.1	3	6734.96	48374.1	14144.4
0.00049	44348.7	25190.6	3	6734.96	20953.5	14179.4
0.00051	44348.7	25236.7	3	6734.96	20953.5	14221.8
0.000525	44348.7	25271.3	3	6734.96	20953.6	14253.6
0.0005	44348.7	25213.7	3	6734.96	20953.5	14200.6

p	n_2^{CO}	Q^{CO}	TC_r^{COP}	TC_s^{COP}		$Percentage\ improvement\ in retailer's\ cost$	$Percentage\ improvement\ in supplier's\ cost$
0.000475	1	6754.38	6986.32	39891.	22627.5	10.0516	10.0516
0.00049	2	6737.17	34406.5	22406.	12726.9	49.4777	49.4777
0.00051	2	6737.17	34410.1	22418.2	12757.1	49.4501	49.4501
0.000525	2	6737.17	34412.8	22427.4	12779.8	49.4295	49.4295
0.0005	2	6737.17	34408.3	22412.1	12742.	49.4639	49.4639

TABLE 3.11: For α

α	TC_r^{DO}	TC_s^{DO}	n_1^{DO}	Q^{DO}	TC_s^{CO}	TC_r^{CO}
0.000855	43967.	35448.3	3	6734.28	21136.5	23623.9
0.000882	44191.7	35529.9	3	6734.67	21033.	23696.2
0.000918	44494.6	35636.2	3	6735.21	20897.5	23789.
0.000945	44724.2	35714.1	3	6735.61	20797.5	23855.9
0.0009	44342.7	35583.4	3	6734.94	20964.9	23743.1

α	n_2^{CO}	Q^{CO}	TC_r^{COP}	TC_s^{COP}		$Percentage\ improvement\ in retailer's\ cost$	$Percentage\ improvement\ in supplier's\ cost$
0.000855	2	6736.55	34655.	24780.8	19979.5	43.6376	43.6376
0.000882	2	6736.9	34992.4	24794.6	19934.7	43.8932	43.8932
0.000918	2	6737.37	35444.4	24813.3	19873.2	44.2331	44.2331
0.000945	2	6737.73	35784.9	24827.6	19825.8	44.4874	44.4874
0.0009	2	6737.13	35218.1	24803.8	19904.2	44.0633	44.0633

TABLE 3.12: Senstivity Analysis 1

Parameter	*%change*	*value*	n_1^{OP}	Q^{DOP}	n_2^{OP}	Q^{COP}	*%Profit*
$a \rightarrow$	−5%	1425.	3	6434.29	2	6436.39	44.6143
	−2%	1470.	3	6614.68	2	6616.84	44.4466
	+2%	1530.	3	6855.2	2	6857.43	44.2334
	+5%	1575.	3	7035.59	2	7037.88	44.0807
$b \rightarrow$	−5%	47.5	3	6714.89	2	6717.07	44.3387
	−2%	49.	3	6726.92	2	6729.11	44.3386
	+2%	51.	3	6742.96	2	6745.16	44.3385
	+5%	52.5	3	6754.99	2	6757.19	44.3384
$c \rightarrow$	−5%	14.25	3	6718.89	2	6721.08	44.3343
	−2%	14.7	3	6728.52	2	6730.71	44.3369
	+2%	15.3	3	6741.36	2	6743.55	44.3403
	+5%	15.75	3	6750.98	2	6753.19	44.3428
$P_o \rightarrow$	−5%	4.75	3	6734.94	2	6737.13	43.9516
	−2%	4.9	3	6734.94	2	6737.13	44.0192
	+2%	5.1	3	6734.94	2	6737.13	44.1067
	+5%	5.25	3	6734.94	2	6737.13	44.1703

TABLE 3.13: Senstivity Analysis 2

Parameter	*%change*	*value*	n_1^{OP}	Q^{DOP}	n_2^{OP}	Q^{COP}	*%Profit*
$P_s \rightarrow$	−5%	0.855	3	6734.94	2	6737.18	44.411
	−2%	0.882	3	6734.94	2	6737.15	44.2026
	+2%	0.918	3	6734.94	2	6737.12	43.9236
	+5%	0.945	3	6734.94	2	6737.09	43.7134
$D_C \rightarrow$	−5%	570.	3	6734.77	2	6737.1	43.6327
	−2%	588.	3	6734.87	2	6737.12	43.8912
	+2%	612.	3	6735.	2	6737.15	44.2352
	+5%	630.	3	6735.1	2	6737.17	44.4926
,	,	,	,	,	,	,	
$H_o \rightarrow$	−5%	0.95	3	6735.11	2	6737.13	44.2817
	−2%	0.98	3	6735.	2	6737.13	44.1487
	+2%	1.02	3	6734.88	2	6737.13	43.9804
	+5%	1.05	3	6734.78	2	6737.14	43.8606
$L_o \rightarrow$	−5%	570.	3	6734.94	2	6737.13	44.3379
	−2%	588.	3	6734.94	2	6737.13	44.3383
	+2%	612.	3	6734.94	2	6737.13	44.3388
	+5%	630.	3	6734.94	2	6737.14	44.3392

TABLE 3.14: Senstivity Analysis 3

$Parameter$	$\%change$	$value$	n_1^{OP}	Q^{DOP}	n_2^{OP}	Q^{COP}	$\%Profit$
$S_c \rightarrow$	-5%	0.608	3	6734.94	2	6737.13	44.3392
	-2%	0.6272	3	6734.94	2	6737.13	44.3388
	$+2\%$	0.6528	3	6734.94	2	6737.13	44.3383
	$+5\%$	0.672	3	6734.94	2	6737.13	44.3379
$C_o \rightarrow$	-5%	285.	3	6734.94	2	6737.13	44.1957
	-2%	294.	3	6734.94	2	6737.13	44.2815
	$+2\%$	306.	3	6734.94	2	6737.13	44.3956
	$+5\%$	315.	3	6734.94	2	6737.13	44.481
,	,	,	,	,	,	,	
$T_c \rightarrow$	-5%	285.	3	6734.94	2	6737.13	44.4566
	-2%	294.	3	6734.94	2	6737.13	44.3857
	$+2\%$	306.	3	6734.94	2	6737.13	44.2916
	$+5\%$	315.	3	6734.94	2	6737.13	44.2213
$DsAsm \rightarrow$	-5%	33.25	3	6734.94	2	6737.13	44.3523
	-2%	34.3	3	6734.94	2	6737.13	44.3441
	$+2\%$	35.7	3	6734.94	2	6737.13	44.3331
	$+5\%$	36.75	3	6734.94	2	6737.13	44.3248

TABLE 3.15: Senstivity Analysis 4

Parameter	*%change*	*value*	n_1^{OP}	Q^{DOP}	n_2^{OP}	Q^{COP}	*%Profit*
$Rem \rightarrow$	-5%	285.	3	6734.94	2	6737.13	44.4566
	-2%	294.	3	6734.94	2	6737.13	44.3857
	$+2\%$	306.	3	6734.94	2	6737.13	44.2916
	$+5\%$	315.	3	6734.94	2	6737.13	44.2213
$CT \rightarrow$	-5%	0.095	3	6734.94	2	6737.13	44.3636
	-2%	0.098	3	6734.94	2	6737.13	44.3487
	$+2\%$	0.102	3	6734.94	2	6737.13	44.3283
	$+5\%$	0.105	3	6734.94	2	6737.13	44.3127
,	,	,	,	,	,	,	
$\delta \rightarrow$	-5%	9.5	3	6734.94	2	6737.13	44.3379
	-2%	9.8	3	6734.94	2	6737.13	44.3383
	$+2\%$	10.2	3	6734.94	2	6737.13	44.3388
	$+5\%$	10.5	3	6734.94	2	6737.13	44.3392
$\beta \rightarrow$	-5%	1.425	3	6734.94	2	6737.13	43.4384
	-2%	1.47	3	6734.94	2	6737.13	43.9725
	$+2\%$	1.53	3	6734.94	2	6737.13	44.7128
	$+5\%$	1.575	3	6734.94	2	6737.13	45.2897

TABLE 3.16: Senstivity Analysis 5

Parameter	*%change*	*value*	n_1^{OP}	Q^{DOP}	n_2^{OP}	Q^{COP}	*%Profit*
$S_o \rightarrow$	-5%	275.5	3	6734.94	2	6737.13	44.3385
	-2%	284.2	3	6734.94	2	6737.13	44.3385
	$+2\%$	295.8	3	6734.94	2	6737.13	44.3386
	$+5\%$	304.5	3	6734.94	2	6737.13	44.3386
$\theta \rightarrow$	-5%	0.000855	3	6734.7	2	6737.13	44.3408
	-2%	0.000882	3	6734.84	2	6737.13	44.3395
	$+2\%$	0.000918	3	6735.03	2	6737.13	44.3377
	$+5\%$	0.000945	3	6735.18	2	6737.13	44.3363

Chapter 4

Green Supply coordination model

4.1 Abstract

This paper is mainly about re-manufacturing of an item within the planning horizon. Re-manufacturing of a product has become a natural requirement in inventory handling. It decreases the burden of inventory for defective kind of items. Another obvious phenomenon is deterioration of items in inventory. Hence two-parameter Weibull deterioration of items is considered in our model. The idea is greening of a supply chain model through re-manufacturing of defective items after the screening process

4.2 Introduction

Market across the world are looking for a greener management policies in all sectors. recycling of the products have thus begun to become a vital activity. Apart from increasing the profit margin buyer satisfaction has to be claimed with the implementation of environment-friendly models for recycling. We have thus derived a model for recycling of the defective products within a replenishment cycle. Jeganathan et al. (2018) has discussed two-commodity continuous review inventory system with postponed in demands.

Selvi et al. (2017) has derived a replenishment policy for deteriorating items considering sorting price, transportation price for back orders minimizing annual total price. Singh et al. (2017a) is a good model in which the authors have discussed an EOQ model with items which deteriorates with time and are partially backlogged with shortages. Further Singh et al. (2017b) analyzed the inventory replenishment policy under inflation.

A production, remanufacture and waste disposal Economic production quantity model was presented by Kundu and Chakrabarti (2018) concluding that policy of recycling is a better strategy as far as carbon emissions are concerned.

Considering returns with different quality grades Sun et al. (2018) in their study explored the benefits of scheduling the manufacturing and remanufacturing sequence.

Two types of product green (environmental-friendly) product along with the regular product was included in the model studied by Raza et al. (2018) with green (environmental-friendly) product price higher than the regular product. Recently Rani, Ali, and Agarwal (2017) discussed in the green supply the re-manufacturing of items that are deteriorating. First to mention a two-parameter Weibull distribution rate and deterioration in an EOQ model was Philip (1974) and Ghare and Schrader (1963) respectively. Khanra and Chaudhuri (2003) introduced time-dependent quadratic demand function.

The time quadratic demand function was considered by Ghosh and Chaudhuri (2006), Manna, Chaudhuri, and Chiang (2007), Singh et al. (2017a), Singh et al. (2017b) and others.

There is a dire need of deterioration, disassembly and recycling of a product to be considered and the same is presented here. All the notations and assumptions are provided in segment 4.3. For two events the model formulation is provided in segment 4.4. In segment 4.5 the optimality condition for the suggested model is discussed. Finally, the example is given in segment 4.5 further explains the model.

4.3 Assumptions and notations

1. There is no lead time.

2. Demand function is $f(\tau) = a + b\tau + c\tau^2$ where τ is time.

3. Deterioration function of time is $\alpha_1(\tau)$= $\beta\gamma\tau^{\gamma-1}$, $0 < \beta < 1$, $\gamma \geq 1$ is a Weibull distribution.

4. Different price involved in the model are as follows: purchasing of an item holding $(H(\$/unit/unittime))$, $(P_o(\$/unit))$, deterioration $(Det_C(\$/unit))$, lost sale $(L_o(\$/unit))$, setup $(S_s(\$/order))$, shortage $(S_o(\$/unit))$, sorting $(S_c(\$/unit))$, ordering $(C_o(\$/order))$, transportation $(T_c(\$/u\text{-}nit))$, disassembly $(DisAssmb(\$/unit))$, recycling $(Rec(\$/unit))$ and oppurtunity price $(O_c(\$/unit))$. These prices are fixed during the finite planning horizon (H).

5. In first event Optimal plan of ordering n_1^{do} dependents upon total price of buyer (tc_r^d) and second event where both supplier (tc_s^C) and buyer's increased price $(tc_r^C - tc_r^{do})$ are used for optimal ordering schedule n_2^{C-O}.

6. sorting of complete product for the two events i.e. I_{oi}^d for i^{th} cycle, is required. The recyclable items which is $P * I_{oi}^d$, after sorting by buyer are shipped by supplier for recycling and disassembly at time $\tau = \tau_i^{'}$. All $P * I_{oi}^d$ products are then shipped to buyer for selling in the i^{th} cycle at $\tau = \tau_i^{"}$.

7. τ_i $\{i = 1, 2 \ldots n_1^d\}$ and τ_j $\{j = 1, 2 \ldots n_2^c\}$ are the time of replenishment during both the events.

8. n_1^d and n_2^c are number of orders placed in the two events.
9. I_{oi} or I_{oi}^d is the inventory quantity at time $\tau = \tau_i$ for event 1.
10. The quantity of inventory that remains with buyer after recylable items are discarded at time $\tau = \tau_i^{'}$ in i^{th} cycle is I_{si}^d.
11. The quantity of inventory at time $\tau = \tau_i^{"}$ right before $p * I_{oi}^d$ recylabled items are brought in buyers inventory is I_{fi}^d.

4.4 Mathematical conceptualization for the suggested model

There are two segments 4.4.1 and 4.4.2. Firstly the buyer has his own order plan in next segment buyer follows that of the supplier. Suggested model is as demonstrated in figure 4.1.

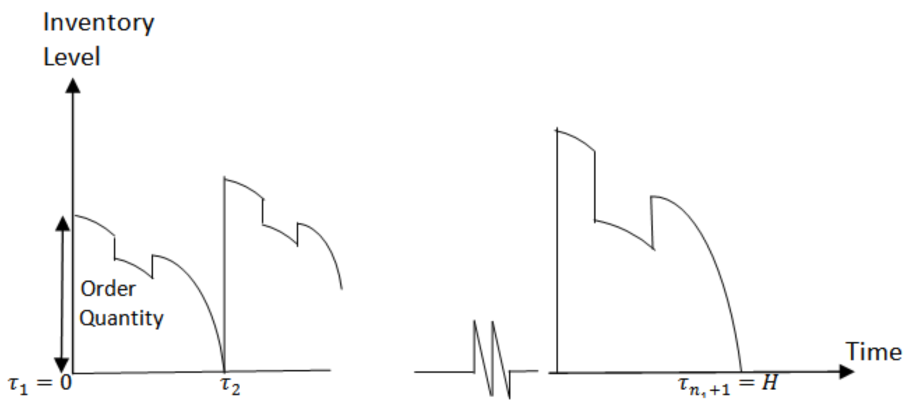

FIGURE 4.1: Pictorial graph of Inventory Model

4.4.1 First event for finite planning horizon

In first event scheduled number of replenishments (n_1^d) is depends on buyers total maximum price (tc_r^d). Constant rate (α_1) of inventory level, deterioration rate (α_2) and quadratic demand function of time dominates the inventory level (I_{1i}^d) at any time τ. Primary equation of the model is as follows:

$$\frac{dI_{1i}^d(\tau)}{d\tau} + (\alpha_1 + \alpha_2)\, I_{1i}^d(\tau) = -f(\tau), \{i = 1, 2 \ldots n_2^d\} \tag{4.1}$$

where $\alpha_2 = \beta\gamma\tau^{\gamma-1}$ is the Weibull deterioration rate. Figure 4.1 shows till time $\tau = \tau_i^{'}$ sorting for defective\recyclable items is completed for i^{th} cycle and $p * I_{oi}^d$ is shipped to supplier for recycling. The inventory level at time t with boundary conditions $I_{1i}^d\left(\tau_i^{'}\right) = I_{\tau_{i-1}}^d$ where $\tau_i \leq t \leq \tau_i^{'}$ can be obtained as follows:

$$I_{1i}^d(\tau) = I_{\tau_{i-1}}^d e^{\alpha_1\left(\tau_i^{'}-t\right)+\beta\left(\tau_i^{'\gamma}-\tau^{\gamma}\right)}+$$

$$e^{-(\alpha_1 t+\beta\tau^{\gamma})}\int_{\tau}^{\tau_i^{'}} e^{\alpha_1 u+\beta u^{\gamma}} f(v)dv. \tag{4.2}$$

With boundary value $I_{1i}^d\left(\tau_i\right) = I_{o_i}^d$ the above equation reduces to

$$I_{\tau_{i-1}}^d = I_{o_i}^d e^{\alpha_1\left(\tau_i-\tau_i^{'}\right)+\beta\left(\tau_i^{\gamma}-\tau_i^{'\gamma}\right)}-$$

$$e^{-\left(\alpha_1\tau_i^{'}+\beta\tau_i^{'\gamma}\right)}\int_{\tau_i}^{\tau_i^{'}} e^{\alpha_1 u+\beta u^{\gamma}} f(v)dv. \tag{4.3}$$

The defected/recyclable items are recycled from time $\tau_i^{'}$ to $\tau_i^{"}$. $p * I_{oi}^d$ products are shipped to buyer for selling at $\tau_i^{"}$. $I_{2i}^d(\tau)$ and $I_{3i}^d(\tau)$ are inventory level at τ for $\tau_i^{'} < t \leq \tau_i^{"}$ and $\tau_i^{"} < t \leq \tau_{i+1}$ respectively. For $\tau_i^{'} < t \leq \tau_i^{"}$ with boundary conditions $I_{1i}^d\left(\tau_i^{"}\right) = I_{2i}^d\left(\tau_i^{"}\right) = I_{f_i}^d$ the inventory level at time τ is given by

$$I_{2i}^d(\tau) = I_{f_i}^d e^{\alpha_1\left(\tau_i^{"}-t\right)+\beta\left(\tau_i^{"\gamma}-\tau^{\gamma}\right)}+$$

$$e^{-(\alpha_1 t+\beta\tau^{\gamma})}\int_{\tau}^{\tau_i^{"}} e^{\alpha_1 u+\beta u^{\gamma}} f(v)dv. \tag{4.4}$$

Now since $I_{1i}^d\left(\tau_i^{'}\right) = I_{2i}^d\left(\tau_i^{'}\right) = I_{\tau_{i-1}}^d - pI_{o_i}^d$, the above equation will be

$$I_{\tau_{i-1}}^d - pI_{o_i}^d = I_{f_i}^d e^{\alpha_1\left(\tau_i^{"}-t\right)+\beta\left(\tau_i^{"\gamma}-\tau^{\gamma}\right)} + e^{-\left(\alpha_1\tau_i^{'}+\right.}$$

$$\left.\beta\tau_i^{'\gamma}\right)\int_{\tau_i^{'}}^{\tau_i^{"}} e^{\alpha_1 u+\beta u^{\gamma}} f(v)dv. \tag{4.5}$$

At $t = \tau_{i+1}$, i^{th} cycle the number of items are nil. $I_{3i}^d\left(\tau_i^{"}\right) - pI_{o_i}^d = I_{f_i}^d$ and $I_{3i}^d\left(\tau_{i+1}\right)$ is zero for $\tau_i^{"} \leq t \leq \tau_{i+1}$ and

$$I_{3i}^d(\tau) = e^{-(\alpha_1 t+\beta\tau^{\gamma})}\int_{\tau}^{\tau_{i+1}} e^{\alpha_1 u+\beta u^{\gamma}} f(v)dv. \tag{4.6}$$

and

$I_{f_i}^d = -pI_{o_i}^d + e^{-\left(\alpha_1\tau_i^{"}+\beta\tau_i^{"\gamma}\right)}\int_{\tau_i^{"}}^{\tau_{i+1}} e^{\alpha_1 u+\beta u^{\gamma}} f(v)dv.$. Solving above equations we get $I_{o_i}^d$ and $I_{\tau_{i-1}}^d$. After substituting the value of $I_{\tau_{i-1}}^d$ in equation 4.2 we get $I_{1i}^d(\tau) =$. Similarly from equation 4.4 and 4.6 substituting the values of $I_{\tau_{i-1}}^d$ and $I_{f_i}^d$ we obtained $I_{2i}^d(\tau)$ and $I_{3i}^d(\tau)$.

Buyer's price are as under

Holding price =

$\sum_{i=1}^{n_1^d} H * R_i^d = \sum_{i=1}^{n_1^d} H \Big(\int_{\tau_i}^{\tau_i'} I_{1i}(\tau)d\tau$

$$+\int_{\tau_i'}^{\tau_i''} I_{2i}(\tau)d\tau + \int_{\tau_i''}^{\tau_{i+1}} I_{3i}(\tau)d\tau. \tag{4.7}$$

Ordering price $= n_1^d * O_r$, Deterioration price=

$\sum_{i=1}^{n_1^d} Det_C \Big(\int_{\tau_i}^{\tau_i'} \alpha_1 I_{1i}(\tau)d\tau + \int_{\tau_i'}^{\tau_i''} \alpha_1 I_{2i}(\tau)d\tau$

$+ \int_{\tau_i''}^{\tau_{i+1}} \alpha_1 I_{3i}(\tau)d\tau$

$= \sum_{i=1}^{n_1^d} Det_C \Big(\int_{\tau_i}^{\tau_i'} \beta\gamma\tau^{\gamma-1} I_{1i}(\tau)d\tau$

$$+\int_{\tau_i'}^{\tau_i''} \beta\gamma\tau^{\gamma-1} I_{2i}(\tau)d\tau + \int_{\tau_i''}^{\tau_{i+1}} \beta\gamma\tau^{\gamma-1} I_{3i}(\tau)d\tau, \tag{4.8}$$

price of purchasing =

$$\sum_{i=1}^{n_1^d} P_r * q_i^d = \sum_{i=1}^{n_1^d} P_r * I_{o_i}^d \tag{4.9}$$

and sorting price=

$$\sum_{i=1}^{n_1^d} S_c * I_{o_i}^d. \tag{4.10}$$

Number of items in i^{th} cycle = q_i^d

$$= I_{oi}^d \tag{4.11}$$

Maximum price of the buyer =

tc$_r^d(\tau_i, \tau_{i-1}, n_1^d)$ = total price of purchase + total price to Hold inventory + total price of Detererioration + sorting price =

$\sum_{i=1}^{n_1^d} \{ P_r \left[I_{oi}^d + S_i \right] + H \Big[\int_{\tau_i}^{\tau_i'} I_{1i}(\tau)d\tau$

$+ \int_{\tau_i'}^{\tau_i''} I_{2i}(\tau)d\tau +$

$\int_{\tau_i''}^{\tau_{i+1}} I_{3i}(\tau)d\tau + Det_C \beta\gamma \Big[\int_{\tau_i}^{\tau_i'} \tau^{\gamma-1} I_{1i}(\tau)d\tau$

$$+\int_{\tau_i'}^{\tau_i''} \tau^{\gamma-1} I_{2i}(\tau)d\tau + \int_{\tau_i''}^{\tau_{i+1}} \tau^{\gamma-1} I_{3i}(\tau)d\tau + S_c * I_{oi}^d \} \tag{4.12}$$

Supplier's price are as under

Setup price in first event $= S_s * n_1^d$, Purchase price $= \sum_{i=1}^{n_1^d} P_s * I_{oi}^d$, Transportation price $= \sum_{i=1}^{n_1^d} T_c * p * I_{oi}^d$, price of Disassembly $= \sum_{i=1}^{n_1^d} DisAssmb * p * I_{oi}^d$ and price of recycling $= \sum_{i=1}^{n_1^d} Rec * p * I_{oi}^d$.

Total price of supplier = $\text{tc}_s^d(\tau_i, \tau_{i-1}, n_1^d) =$

$$\sum_{i=1}^{n_1^d}\{S_s * n + P_s * I_{oi}^d + (T_c * p + DisAssmb * p + Rec * p + H * p) * I_{oi}^d\}, \{i = 1, 2 \ldots n_1^d\} \tag{4.13}$$

frequency refilling the inventory for a finite planning horizon is n_1^d

Solution for first event

Taking $C\tau = \tau_i^{'} - \tau_i = \tau_i^{"} - \tau_i^{'}$. Ignoring exponent 2 and higher for β, α_1, change in tc_r^d with respect to τ_i, $\dfrac{\delta tc_r^d(\tau_i, \tau_{i-1}, n_1^d)}{\delta \tau_i}$ is calculated.

After obtaining the values of $\tau_i' s$ $tc_r^d(\tau_i, \tau_{i-1}, n_1^d)$ and $tc_s^d(\tau_i, \tau_{i-1}, n_1^d)$ are calculated for different $n_1^{d\prime} s$, from equation 4.12 and 4.13. Thus obtaining total optimal price of buyer $tc_r^{do}(\tau_i^{do}, \tau_{i-1}^{do}, n_1^{do})$, optimal price of supplier $tc_s^{do}(\tau_i^{do}, \tau_{i-1}^{do}, n_1^{do})$, Optimal number of replenishment cycles (n_1^{do}), (q^{do}) by 4.12, 4.13 and 4.11 respectively. Example is presented in segment 4.5.

4.4.2 Second event

Second event's frequency of refilling cycle relies on the difference in buyer's price calculated by deducting $tc_r^{do}(\tau_i^{do}, \tau_{i-1}^{do}, n_1^{do})$ by $tc_r^c(\tau_j, \tau_j, n_2^c)$ and suppliers maximum price $tc_s^c(\tau_j, \tau_j, n_2^c)$.

Supplier's prices are as under

Supplier's maximum price as in earlier event $= tc_s^c(\tau_j, \tau_j, n_2^c) =$

$$\sum_{j=1}^{n_2^c}\{(T_c * p + DisAssmb * p + Rec * p + H * p) * I_{oj} + S_s * n + P_s * I_{oj}\} + tc_r^c(\tau_j, \tau_j, n_2^c) - tc_r^{do}(\tau_i^{do}, \tau_{i-1}^{do}, n_1^{do}), \tag{4.14}$$

where $\{j = 1, 2 \ldots n_2^c\}$. $tc_r^c(\tau_j, \tau_j, n_2^{co})$ can be obtained from equation 4.12 by replacing in $tc_s^d(\tau_i, \tau_{i-1}, n_1^d)$, τ_i from optimal τ_j, τ_{i-1} from optimal τ_j and n_2^{do} from n_2^{co}. With q^{do} now to calculate q^{co} .

Solution for second event

Ignoring exponent 2 and higher for β, α_1, now change in tc_s^C with respect to τ_j i.e. $\frac{\delta tc_s^c(\tau_j, \tau_j, n_2^c)}{\delta \tau_j}$ is calculated.

After obtaining the values of $\tau_j' s$, $tc_s^c(\tau_j, n_2^c)$ and $tc_r^c(\tau_j, n_2^c)$ are calculated for different $n_j' s$. Thus obtaining total optimal price of supplier $tc_s^{co}(\tau_j^{co}, n_2^{co})$, optimal price of buyer $tc_r^{co}(\tau_j^{co}, n_2^{co})$, Optimal number of replenishment cycles (n_2^{co}) and (q^{co}) optimal number of units.

Share of profit with buyer is calculated since the systems optimal price is decreased.

System's improved price in second event= $Profit =$

$$[tc_s^{do} + tc_r^{do}] - [tc_s^{co} + tc_r^{co}] \tag{4.15}$$

Improved buyer's price $= tc_r^{cop} = tc_r^{do} - \frac{tc_r^{do}}{tc_r^{do} + tc_s^{do}}$ and Improved supplier's price $= tc_s^{cop} = tc_s^{do} - \frac{tc_s^{do}}{tc_r^{do} + tc_s^{do}}$.

Percentage profit of buyer $= \frac{tc_r^{do} - tc_r^{cop}}{tc_r^{do}} * 100$ and Percentage profit of buyer $= \frac{tc_s^{do} - tc_s^{cop}}{tc_s^{do}} * 100$.

If buyer opts for credit the rate of credit for a period is given by $= \frac{tc_r^{do} - tc_r^{cop}}{O_c * (\tau_{i+1}^{co} - \tau_i^{co}) * q^{co}}$.

4.5 Optimality condition for tc_r^d and tc_s^c

The $\frac{\delta tc_r^d}{\delta \tau_i^2}$, $\frac{\delta tc_r^d}{\delta \tau_{i-1}^2}$, $\frac{\delta tc_s^c}{\delta \tau_j^2}$, $\frac{\delta tc_s^c}{\delta \tau_j^2}$, $\frac{\partial^2 \text{tc}_r^d}{\partial \tau_i \partial \tau_{i-1}}$ and $\frac{\partial^2 \text{tc}_r^d}{\partial \tau_{i-1} \partial \tau_i}$ can be obtained easily through partial differentiation. The sufficient condition for tc_r^d to be minimum is that the following Hessian matrix $\nabla^2 \text{tc}_r^d$ of tc_r^d for a fixed n_1 is positive definite Sarkar, Ghosh, and Chaudhuri (2012b). Where $\nabla^2 \text{tc}_r^d =$

$$\begin{pmatrix} \frac{\partial^2 \text{tc}_r^d}{\partial \tau_1^2} & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_1 \partial \tau_0} & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 \text{tc}_r^d}{\partial \tau_0 \partial \tau_1} & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_0^2} & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_0 \partial \tau_2} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_2 \partial \tau_0} & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_1^2} & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_2 \partial \tau_2} & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_{n_1-1} \partial \tau_{n_1-2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_{n_1-1} \partial \tau_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \text{tc}_r^d}{\partial \tau_{n_1}^2} \end{pmatrix} . \quad (4.16)$$

If τ_i and τ_{i-1} satisfy inequations (i) $\frac{\partial^2 \text{tc}_r^d}{\partial \tau_i^2} > 0$, (ii) $\frac{\partial^2 \text{tc}_r^d}{\partial \tau_{i-1}^2} > 0$, (iii) $\frac{\partial^2 \text{tc}_r^d}{\partial \tau_i^2} - \left| \frac{\partial^2 \text{tc}_r^d}{\partial \tau_i \partial \tau_{i-1}} \right| > 0$ and (iv) $\frac{\partial^2 \text{tc}_r^d}{\partial \tau_{i-1}^2} - \left| \frac{\partial^2 \text{tc}_r^d}{\partial \tau_{i-1} \partial \tau_i} \right| > 0$ for $i = 1, 2 \ldots n_1^d$ then $\nabla^2 \text{tc}_r^d$ is positive definite.

The same can be used to show that $\nabla^2 \text{tc}_s^c$ is a positive definite and $\text{tc}_s^c(\text{n}_2, \tau_0, \tau_1^c, \tau_0^c, ..., \tau_{n_2+1}^c)$ attains a minimum. Where $\nabla^2 \text{tc}_s^c =$

$$\begin{pmatrix} \frac{\partial^2 \text{tc}_s^c}{\partial \tau_1^2} & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_1 \partial \tau_0} & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 \text{tc}_s^c}{\partial \tau_0 \partial \tau_1} & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_0^2} & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_0 \partial \tau_2} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_2 \partial \tau_0} & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_1^2} & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_2 \partial \tau_2} & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_{n_1-1} \partial \tau_{n_1-2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_{n_1-1} \partial \tau_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \text{tc}_s^c}{\partial \tau_{n_1}^2} \end{pmatrix} . \quad (4.17)$$

TABLE 4.2: Refill optimal frequency for buyer in first event is as follows

Sc	n_1^{do}	τ_1	τ_2	τ_3	t_4
0.64	2	0	2	3.3291	4.

Example to distinguish both the events and obtaining profit in a green supply chain To validify the model an example has been solved taking into consideration the assumptions. The values of different parameters are taken as follows. $\beta = 0.0009$, $\alpha_1 = 0.09$, $CT = 0.4$, $DC = 1$, $Po = 1$, $Ho = 5$, $a = 1500$, $b = 500$, $c = 150$, $p = 30$, $Sc = 0.64$, $Co = 300$, $\gamma = 1$, $Tc = 0.01$, $DisAssmb = 0.01$, $Rec = 0.01$, $Ss = 2000$, $Ps = 0.5$, $\delta = 10$, $H = 4$ and $\tau_i^{"} - \tau_i^{'} = \tau_i^{'} - \tau_i = \frac{H}{4}$.

Shown in the table 5.1 the total optimal price of the buyer in the first system, $tc_r^{do} = 6.92112 \times 10^6$. The corresponding $tc_s^{do} = 2.16682 \times 10^7$. Table 5.2 is for the optimal schedule when $n_1 = 1$ and $Sc = 0.64$. For second event table 5.3 shows $n_2^{co} = 1$ which is equal to $n_1^{do} = 1$ for Sc and $tc_s^{co} = 7.07822 \times 10^6$ total optimal price of supplier $tc_s^{co}(\tau_j^{co}, \tau_j^{co}, n_2^{co})$ in the second system where the optimal schedule is confirmed by the supplier. The optimal replenishment schedule so obtained for "Sc" value is given in table 5.4.

TABLE 4.5: Percentage profit

Sc	tc_r^{do}	tc_s^{do}	n_1^{do}	q^{do}	tc_s^{co}	tc_r^{co}
0.64	6.92112×10^6	2.16682×10^7	2	143057.	7.07822×10^6	1.76245×10^7

n_2^{co}	q^{co}	tc_r^{cop}	tc_s^{cop}	*Systems improved price*	*Percentage improvement in supplier's price*
1	142733.	3.88653×10^6	5.98024×10^6	1.87225×10^7	13.5944

Percentage improvement in supplier's price
13.5944

4.6 Conclusion

The 13^{th} and 14^{th} column in table 4.5 is for percentage profit for both buyer and supplier which will be same for both buyer and supplier. Both the buyer and supplier has obtained 13.5944 percentage of profit in the second event where the supplier decides the optimum replenishment schedule which is a huge margin. In table 4.5 12^{th}, the column shows that system has considerable improvement in total price i.e. total price is reduced. This model proposes recycling of items where total setup price of the supplier is more than the ordering price of the buyer. The suggested model can be extended in several ways such as fuzzifying the parameters, using inflation to the price involved in the present model and introducing weibull deterioration as discussed by Singh et al. (2018a), Singh et al. (2019) and others.

Chapter 5

Fuzzy parameters for supplier and retailer coordination in inventory control

5.1 Abstract

A supply chain model is discussed for materials substances such as metals, ceramics, or plastics manufactured which is deteriorating in nature. Fuzzy parameters such as fuzzy deterioration cost, fuzzy holding cost fuzzy inventory carrying cost etcetera are considered for framing of the model which are later defuzzified using Centroid, Signed Distance and Graded Mean Representation method. Centralized replenishment policy in this finite planning horizon model is discussed along with sensitivity analysis.

5.2 Introduction

Along with defined criteria of membership, fuzzy set was introduced by Zadeh (1965). The EOQ formula including the fuzziness was given by Park (1987). Introduction to fuzzy arithmetic theory and operation was provided by Kaufman and Gupta (1991). An operator's approximation for an interval in a fuzzy number system was provided by Grzegorzewski (2002) and Mahata and Goswami (2009) solved using Fuzzy Non-Linear Programming (FNLP) taking different cost as triangular fuzzy numbers. The solution of EPQ with cost fuzzy in nature are solved by special fuzzy technique(PGP) Mahapatra, Mandal, and Samanta (2011). De Kumar, Kundu, and Goswami (2003) derived a methodology for the optimum value of the fuzzy total cost. Roy et al. (2007) allowed payment delay considering fuzzy cost function for fuzzy inflation and deterioration rate. Partial backlogging, demand which is stochastic with deterioration fuzzy, Halim, Giri, and Chaudhuri (2008) examined an EOQ model. Total profit for fuzzy inflation, discount environment with constant product deterioration using method UFM and GRG technique is evaluated by Maity and Maiti (2008).

For a deteriorating item, optimal inventory decision is derived using a genetic algorithm (GA) for an inventory-based demand by Roy et al. (2009). An EOQ model for Pareto optimal solution taking fuzzy total costs including shortage, holding and another cost as triangular fuzzy numbers is investigated by Mahata and Goswami (2009).

With shortages, inventory dependent demand and deteriorating products Valliathal and Uthayakumar (2010) studied a deterministic model which is fuzzified for different cost such as set up cost, opportunity cost etc. considering those as triangular fuzzy numbers and using Signed-distance method to defuzzify.

Product deterioration cost, trade credit, demand rate and other cost considered as fuzzy numbers, Mahata and Mahata (2011) defuzzified using Graded Mean Integration Representation method. Taking lost sales rate as triangular fuzzy number ie fuzzifying the backorder rate, Ouyang and Chang (2001) constructed a new fuzzy number, called as a statistic-fuzzy number, and then developed an algorithm to find the optimal schedule.

Lin (2008) extended Ouyang and Chang (2001) by fuzzyfying the backorder and shortage, defuzzyfying using the signed distance method and compared the fuzzy model with that of the crisp. Other authors discused fuzzy shortages are Gani and Maheswari (2010) and De and Sana (2013) .

Expressing order quantity as a triangular fuzzy number, Yao and Lee (1996) found after defuzzification that cost of the crisp model is on the lower side compared to the fuzzy model.

Triangular fuzzy numbers as input values for an inventory model, the total minimum cost is found by Gen, Tsujimura, and Zheng (1997), along with inventory replenishment quantity, transforming a fuzzy model into crisp.

Backorder quantity as a fuzzy triangular number, Chang, Yao, and Lee (1998) compared fuzzy and crisp model finding the centroid of the cost function.

With item quantity as a triangular fuzzy number, Chang (1999) found the centroid of the fuzzy cost function's membership function.

Authors considering triangular fuzzy number are Yao and Wu (2000), Wu and Yao (2003), Gani and Maheswari (2010), and Valliathal and Uthayakumar (2013) and others.

Graded mean method of defuzzification was used by Gani and Maheswari (2010). Signed Distance Method for defuzzification was considered by Chen and Ouyang (2006), Ameli, Mirzazadeh, and Shirazi (2011), Roy and Samanta (2009), Valliathal and Uthayakumar (2010), Sadi-Nezhad, Nahavandi, and Nazemi (2011), and Valliathal and Uthayakumar (2013) and others. Centroid method for defuzyfication was used by Petrović, Petrović, and Vujošević (1996) and Yao and Chiang (2003).

In this chapter, we have fuzzified all parameters viz. namely demand, holding, ordering, unit cost, excluding cycle length and length of the planning horizon as triangular fuzzy numbers. The arithmetic operations are defined under the function principle. The total cost function and order quantity has been defuzzified using Signed Distance

Method, Centroid Method and Graded Mean Representation Method to obtain the optimal order quantity.

5.3 Assumptions

1. At the starting point of time $s_1 = 0$ there is no inventory.
2. Within each cycle product deterioration is some proportion of 't' time.
3. The model is considered with shortages and backloggging in each cycle with rate $\kappa(\flat) = \frac{1}{1+\rho\flat}, \rho > 0$.
4. Retailer and Supplier has same Capital cost.
5. Set up cost is more than the Ordering cost.

5.4 Notations

Fuzzy point, memership function for level of a fuzzy interval, triangular fuzzy number is defined as in Jaggi et al. (2012). Also Graded Mean Integration Representation, Signed Distance, Centroid Method is followed for defuzzification.

For retailer

H	Total length of planning horizon.
$f(\tilde{a}_1, \tilde{b}_1, \tilde{c}_1; t)$ $= \tilde{a}_1 + \tilde{b}_1 t + \tilde{c}_1 t^2$	time quadratic demand where $\tilde{a}_1, \tilde{b}_1,$ $and \tilde{c}_1$ are constants.
$\theta_2 = \tilde{\alpha} t$	Deterioration of the inventory where $0 < \tilde{\alpha} < 1.$
$\tilde{I}_{hr}$	Holding cost.
$\tilde{C}_c$	capital cost
$\tilde{h}$	inventory cost, where $\tilde{h} = \tilde{I}_{hr} + \tilde{C}_c$
$\tilde{O}_r$	ordering cost
$\tilde{P}_r$	purchasing cost
$\tilde{s}$	shortage cost
$\tilde{l}$	lost sales cost
$\tilde{\theta}_1$	Stock dependent demand rate
$\tilde{I}_i^D(t)$	Inventory level in decentralized system
$\tilde{I}_j^C(t)$	Inventory level in centralized system
$\tilde{R}_i^D$	total inventory for the interval $[t_i, s_{i+1}]$
$\tilde{R}_j^C$	total inventory for the interval $[t_j^C, s_{j+1}^C]$
$\tilde{S}_i^D$	amount of shortages in the interval $[s_i, t_i]$
$\tilde{S}_j^C$	total amount of shortages in the interval $[s_j^C, t_j^C]$ for centralized system
$\tilde{Q}_i^D = \tilde{R}_i^D + \tilde{S}_i^D$	quantity ordered
$\tilde{D}_i^D$	amount of deteriorated items

1. The numer of products deteriorated are $\tilde{D}_j^D$ and $\tilde{D}_j^C$ the two system respectively.
2. Quantity of lost sales is denoted by $\tilde{L}_i^D$ and $\tilde{L}_j^C$.
3. $\kappa(\flat) = \dfrac{1}{1 + \rho\flat}$, $\rho > 0$, is the backlogging rate and $\flat$ is the time that customer has to wait.
4. Retailer has $\widetilde{TC}_r^D$ and $\widetilde{TC}_r^C$ as the total cost for two cases and $\widetilde{TC}_s^D$ and $\widetilde{TC}_s^C$ is for supplier.

For supplier

$\tilde{S}_s$($/order)	Cost for set up
$\tilde{P}_s$($/unit)	per product purchase

5.5 Determining parameters

1. Time of refilling for different cycles are t_i $\{i = 1, 2, ..., n_1\}$.

2. Commencement of shortage in each cycle is at s_i $\{i = 1, 2, ..., n_1\}$ and s_j^C $\{j = 1, 2, ..., n_2\}$ where $s_1 = 0$ and $s_{n_1+1} = H$.

3. Total numer of refilling are n_1 and n_2.

4. The rate of credit is $\tilde{\lambda}$.

5.6 Conceptualization

Fig.1 displays the replenishment, reduction of inventory during the finite planning horizon. Inventory reduces due to sale and deterioration. Also shown is that during shortage period in every cycle lost of sale has occurred. Shortage period is between the interval $[s_i, t_i]$ where $\{i = 1, 2, ..., n_1\}$.

In a decentralized system the retailer decides the number of replenishment cycle while in a centralized system supplier decides for the number of replenishment cycles in a finite planning horizon. We have fuzzyfied the model and have compared the result using three different of defuzzyication.

Since this chapter is an extension Singh et al. (2017a), the differential equation and total cost of the supplier in present model is same as (1), (2) and (17) of Singh et al. (2017a).

The fuzzyfied total cost of retailer is

$\widetilde{TC}_r^D(n_1, s_1, t_1, ..., s_{n_1+1}) = n_1\widetilde{O}_r + \widetilde{h}\sum_{i=1}^{n_1}\int_{t_i}^{s_{i+1}} I_i^D(t)dt + \widetilde{P}_r\sum_{i=1}^{n_1}\widetilde{Q}_i^D + \widetilde{P}_r\sum_{i=1}^{n_1}\widetilde{D}_i^D + \widetilde{s}\sum_{i=1}^{n_1}\widetilde{S}_i^D + \widetilde{l}\sum_{i=1}^{n_1}\widetilde{L}_i^D$

To minimize $\widetilde{TC}_s^C$ we calculate the values of t_j and s_j. Taking $f_i(a_i, b_i, c_i; t) = a_1 + b_1 t + c_1 t^2 i = 1, 2, 3, T = (t_j - t), TS_j = (s_j - t_{j-1}), T_j = (t_j - s_j)$ and applying Centroid Method for defuzzyfication the necessary condition to minimize $\widetilde{TC}_s^C$ are

$$\frac{\partial \tilde{TC}_s^C(t_j, s_j; n_1)}{\partial t_j} = 0 \text{ and } \frac{\partial \widetilde{TC}_s^C(t_j, s_j; n_1)}{\partial s_j} = 0.$$

$$where \frac{\partial \widetilde{TC}_s^C(t_j, s_j; n_1)}{\partial t_j} =$$

$$\Big(\Big((hr_1 + W_1)\int_{t_j}^{s_{j+1}}((\alpha_1/2)$$
$$\Big(\Big(t_j^2\Big) - t^2\Big) + \theta_1 T - 1 f(a_1, b_1, c_1; t)dt + W_1 * \int_{t_j}^{s_{j+1}} \alpha_1 * t(\theta_1 T - 1) f(a_1, b_1, c_1; t)dt$$
$$+ (S_1 + W_1 + l_1 * \rho_1)\int_{s_j}^{t_j} f(a_1, b_1, c_1; t)/\Big((1 + \rho_1 T)^2\Big)\, dt +$$
$$\Big((hr_2 + W_2)\int_{t_j}^{s_{j+1}}\Big((\alpha_2/2)\Big(\Big(t_j^2\Big) - t^2\Big) + \theta_2 T - 1\Big) f(a_2, b_2, c_2; t)dt$$
$$+ W_2 * \int_{t_j}^{s_{j+1}} \alpha_2 * t(\theta_2 T - 1) f(a_2, b_2, c_2; t)dt$$
$$+ (S_2 + W_2 + l_2 * \rho_2)\int_{s_j}^{t_j} f(a_2, b_2, c_2; t)/\Big((1 + \rho_2 T)^2\Big)\, dt + ((hr_3 + W_3)$$
$$\int_{t_j}^{s_{j+1}}\Big((\alpha_3/2)\Big(\Big(t_j^2\Big) - t^2\Big) + \theta_3 T - 1\Big) f(a_3, b_3, c_3; t)dt$$

$+W_3 * \int_{t_j}^{s_{j+1}} \alpha_3 * t\left(\theta_3 T-1\right) f(a_3, b_3, c_3; t) dt$

$$+\left(S_3+W_3+l_3 * \rho_3\right) \int_{s_j}^{t_j} f(a_3, b_3, c_3; t) / \left(\left(1+\rho_3 T\right)^2\right) dt))/3$$

(5.1)

and $\dfrac{\partial \widetilde{TC}_s^C\left(t_j, s_j; n_1\right)}{\partial s_j}$=

$\left(\left(\left(hr_1+W_1\right)\left(\left(1+\theta_1 * s_j+\left(\alpha_1/2\right)\left(s_j^2\right)\right) TS_j\right.\right.\right.$
$-\left(\theta_1/2\right)\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)-\left(\alpha_1/6\right)$
$\left(\left(s_j^3\right)-\left(t_{j-1}^3\right)\right) f\left(a_1, b_1, c_1; s_j\right)+\left(W_1 * \alpha_1 * s_j\right.((1$
$+\theta_1 * s_j TS_j-\left(\theta_1/2\right)\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)$
$\left(a_1+b_1 * s_j+c_1 *\left(s_j^2\right)\right)-\left(W_1+S_1+l_1 * \rho_1\right)\left(T_j/\left(1+\rho_1 * T_j\right)\right)$
$f\left(a_1, b_1, c_1; s_j\right)+\left(\left(hr_2+W_2\right)\left((1+\theta_2 * s_j+\left(\alpha_2/2\right)\right.\right.$
$\left(s_j^2\right) TS_j-\left(\theta_2/2\right)\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)-$
$\left(\alpha_2/6\right)\left(\left(s_j^3\right)-\left(t_{j-1}^3\right)\right)\left(a_2+b_2 * s_j+c_2 *\left(s_j^2\right)\right)$
$+\left(W_2 * \alpha_2 * s_j\left(\left(1+\theta_2 * s_j\right) TS_j-\left(\theta_2/2\right)\right.\right.$
$\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right) f\left(a_2, b_2, c_2; s_j\right)-\left(W_2+S_2+l_2 * \rho_2\right)\left(T_j/(1\right.$
$+\rho_2 * T_j f\left(a_2, b_2, c_2; s_j\right)+\left(\left(hr_3+W_3\right)((1+\right.$
$\theta_3 * s_j+\left(\alpha_3/2\right)\left(s_j^2\right) TS_j-\left(\theta_3/2\right)\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)$
$-\left(\alpha_3/6\right)\left(\left(s_j^3\right)-\left(t_{j-1}^3\right)\right) f\left(a_3, b_3, c_3; s_j\right)$
$+\left(W_3 * \alpha_3 * s_j\left(\left(1+\theta_3 * s_j\right) TS_j-\left(\theta_3/2\right)\left(\left(s_j^2\right)\right.\right.\right.$
$-\left(t_{j-1}^2\right) f\left(a_3, b_3, c_3; s_j\right)-\left(W_3+S_3+l_3 * \rho_2\right)$

$$\left(T_j/\left(1+\rho_2 * T_j\right)\right) f\left(a_3, b_3, c_3; s_j\right) 3 \tag{5.2}$$

Similarly with Graded Mean Representation Method we obtain, $\dfrac{\partial \tilde{TC}_s^C\left(t_j, s_j; n_1\right)}{\partial t_j}$ =0 and

$\dfrac{\partial \widetilde{TC}_s^C\left(t_j, s_j; n_1\right)}{\partial s_j}$ =0.Where $\dfrac{\partial \widetilde{TC}_s^C\left(t_j, s_j; n_1\right)}{\partial t_j}$=

$\left(\left(\left(hr_1+W_1\right) \int_{t_j}^{s_{j+1}}\left(\left(\alpha_1/2\right)\left(\left(t_j^2\right)-t^2\right)+\theta_1 T-1\right) f(a_1, b_1, c_1; t) dt+W_1 * \int_{t_j}^{s_{j+1}} \alpha_1 *\right.\right.$
t$\left(\theta_1 T-1\right) f(a_1, b_1, c_1; t) dt+\left(S_1+W_1+l_1 * \rho_1\right) \int_{s_j}^{t_j} f(a_1, b_1, c_1; t) / \left(\left(1+\rho_1 T\right)^2\right) dt+$
$4\left((hr_2+\right.$
W$_2 \int_{t_j}^{s_{j+1}}\left(\left(\alpha_2/2\right)\left(\left(t_j^2\right)-t^2\right)+\theta_2 T-1\right) f(a_2, b_2, c_2; t) dt+W_2 *$

$\int_{t_j}^{s_{j+1}} \alpha_2 * t\left(\theta_2 T-1\right)\left(a_2+b_2 * t+c_2 * t^2\right) dt+\left(S_2+W_2+l_2 * \rho_2\right)$

$\int_{s_j}^{t_j} f(a_2,b_2,c_2;t)/\left((1+\rho_2 T)^2\right)\, dt+((hr_3+W_3)$
$\int_{t_j}^{s_{j+1}}\left((\alpha_3/2)\left(\left(t_j^2\right)-t^2\right)+\theta_3 T-1\right) f(a_3,b_3,c_3;t)dt+W_3 *$

$\int_{t_j}^{s_{j+1}} \alpha_3 * t\left(\theta_3 T-1\right) f(a_3,b_3,c_3;t)dt+\left(S_3+W_3+l_3 * \rho_3\right)$

$$\int_{s_j}^{t_j} f(a_3,b_3,c_3;t)/\left((1+\rho_3 T)^2\right) dt))/6 \tag{5.3}$$

and $\frac{\partial \widetilde{TC}_s^C\left(t_j,s_j;n_1\right)}{\partial s_j}=$
$(((hr_1+W_1)((1+\theta_1 * s_j+(\alpha_1/2)$
$\left(s_j^2\right) TS_j-(\theta_1/2)\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)-$
$(\alpha_1/6)\left(\left(s_j^3\right)-\left(t_{j-1}^3\right)\right)$
$\left(a_1+b_1 * s_j+c_1 * \left(s_j^2\right)\right)+(W_1 * \alpha_1 * s_j((1+\theta_1 * s_j)$
$TS_j-(\theta_1/2)\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)$
$f(a_1,b_1,c_1;s_j)-(W_1+S_1+l_1 * \rho_1)(T_j/(1+\rho_1 * T_j))$
$f(a_1,b_1,c_1;s_j)+4((hr_2+W_2)((1+\theta_2 * s_j+(\alpha_2/2)$
$\left(s_j^2\right) TS_j-(\theta_2/2)$
$\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)-(\alpha_2/6)\left(\left(s_j^3\right)-\left(t_{j-1}^3\right)\right)$
$f(a_2,b_2,c_2;s_j)+\left(W_2 * \alpha_2 * s_j\left((1+\theta_2 * s_j) TS_j-(\theta_2/2)\left(\left(s_j^2\right)\right.\right.\right.$
$\left.-\left(t_{j-1}^2\right) f(a_2,b_2,c_2;s_j)-(W_2+S_2+l_2 * \rho_2)\right.$
$(T_j/(1+\rho_2 * T_j)) f(a_2,b_2,c_2;s_j)+((hr_3+W_3)$
$\left(\left(1+\theta_3 * s_j+(\alpha_3/2)\left(s_j^2\right)\right) TS_j-(\theta_3/2)\right.$
$\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right)-(\alpha_3/6)\left(\left(s_j^3\right)\right.$
$\left.-\left(t_{j-1}^3\right) f(a_3,b_3,c_3;s_j)+(W_3 * \alpha_3 * s_j((1+\theta_3 * s_j) TS_j-\right.$
$(\theta_3/2)\left(\left(s_j^2\right)-\left(t_{j-1}^2\right)\right) f(a_3,b_3,c_3;s_j)-$

$$(W_3+S_3+l_3 * \rho_2)(T_j/(1+\rho_2 * T_j)) f(a_3,b_3,c_3;s_j)6 \tag{5.4}$$

And with Graded Mean Representation Method we obtain, $\frac{\partial \tilde{TC}_s^C\left(t_j,s_j;n_1\right)}{\partial t_j}$ =0

and $\frac{\partial \widetilde{TC}_s^C\left(t_j,s_j;n_1\right)}{\partial s_j}$ =0.Where $\frac{\partial \widetilde{TC}_s^C\left(t_j,s_j;n_1\right)}{\partial t_j}$=
$\left(\left((hr_1+W_1)\int_{t_j}^{s_{j+1}}((\alpha_1/2)\right.\right.$

$\left(\left(t_j^2\right) - t^2\right) + \theta_1 T - 1 f(a_1, b_1, c_1; t)dt + W_1 * \int_{t_j}^{s_{j+1}} \alpha_1 * t\left(\theta_1 T - 1\right) f(a_1, b_1, c_1; t)dt$
$+ \left(S_1 + W_1 + l_1 * \rho_1\right) \int_{s_j}^{t_j} f(a_1, b_1, c_1; t) / \left(\left(1 + \rho_1 T\right)^2\right) dt + 2\left(\left(hr_2 + W_2\right)\right.$
$\int_{t_j}^{s_{j+1}} \left(\left(\alpha_2 / 2\right)\left(\left(t_j^2\right) - t^2\right) + \theta_2 T - 1\right)$
$f(a_2, b_2, c_2; t)dt + W_2 * \int_{t_j}^{s_{j+1}} \alpha_2 * t\left(\theta_2 T - 1\right) f(a_2, b_2, c_2; t)dt$
$+ \left(S_2 + W_2 + l_2 * \rho_2\right) \int_{s_j}^{t_j} f(a_2, b_2, c_2; t) / \left(\left(1 + \rho_2 T\right)^2\right) dt$
$+ \left(\left(hr_3 + W_3\right) \int_{t_j}^{s_{j+1}} \left(\left(\alpha_3 / 2\right)\left(\left(t_j^2\right) - t^2\right) + \theta_3 T - 1\right) f(a_3, b_3, c_3; t)dt\right.$
$+ W_3 * \int_{t_j}^{s_{j+1}} \alpha_3 * t\left(\theta_3 T - 1\right) f(a_3, b_3, c_3; t)dt +$

$$\left(S_3 + W_3 + l_3 * \rho_3\right) \int_{s_j}^{t_j} f(a_3, b_3, c_3; t) / \left(\left(1 + \rho_3 T\right)^2\right) dt))/4 \tag{5.5}$$

and $\dfrac{\partial \widetilde{TC}_s^C\left(t_j, s_j; n_1\right)}{\partial s_j} =$

$(((hr_1 + W_1)((1 + \theta_1 * s_j + (\alpha_1 / 2)$
$\left(s_j^2\right) TS_j - \left(\theta_1 / 2\right)\left(\left(s_j^2\right) - \left(t_{j-1}^2\right)\right)$
$- \left(\alpha_1 / 6\right)\left(\left(s_j^3\right) - \left(t_{j-1}^3\right)\right) f(a_1, b_1, c_1; s_j)$
$+ (W_1 * \alpha_1 * s_j((1 + \theta_1 * s_j) TS_j - (\theta_1 / 2)$
$\left(\left(s_j^2\right) - \left(t_{j-1}^2\right)\right) f(a_1, b_1, c_1; s_j) - \left(W_1 + S_1 + l_1 * \rho_1\right)$
$\left(T_j / \left(1 + \rho_1 * T_j\right)\right) f(a_1, b_1, c_1; s_j) + 2\left(\left(hr_2 + W_2\right)\right.$
$\left(\left(1 + \theta_2 * s_j + \left(\alpha_2 / 2\right)\left(s_j^2\right)\right) TS_j\right.$
$- \left(\theta_2 / 2\right)\left(\left(s_j^2\right) - \left(t_{j-1}^2\right)\right) - \left(\alpha_2 / 6\right)$
$\left(\left(s_j^3\right) - \left(t_{j-1}^3\right)\right) f(a_2, b_2, c_2; s_j) + (W_2 * \alpha_2 * s_j((1 + \theta_2 * s_j) TS_j$
$- \left(\theta_2 / 2\right)\left(\left(s_j^2\right) - \left(t_{j-1}^2\right)\right) f(a_2, b_2, c_2; s_j)$
$- \left(W_2 + S_2 + l_2 * \rho_2\right)\left(T_j / \left(1 + \rho_2 * T_j\right)\right) f(a_2, b_2, c_2; s_j)$
$+ \left(\left(hr_3 + W_3\right)\left(\left(1 + \theta_3 * s_j + \left(\alpha_3 / 2\right)\left(s_j^2\right)\right) TS_j\right.\right.$
$- \left(\theta_3 / 2\right)\left(\left(s_j^2\right) - \left(t_{j-1}^2\right)\right) - \left(\alpha_3 / 6\right)$
$\left(\left(s_j^3\right) - \left(t_{j-1}^3\right)\right) f(a_3, b_3, c_3; s_j) + (W_3 * \alpha_3 * s_j((1 +$
$\theta_3 * s_j TS_j - \left(\theta_3 / 2\right)\left(\left(s_j^2\right) - \left(t_{j-1}^2\right)\right)$

$$f(a_3, b_3, c_3; s_j) - \left(W_3 + S_3 + l_3 * \rho_2\right)\left(T_j / \left(1 + \rho_2 * T_j\right)\right) f(a_3, b_3, c_3; s_j)))/4 \tag{5.6}$$

As shown in table 5.1 total number of replenishment cycles are 4. And the total number of refilling cycles calculated and suggested by supplier is 2 as shown in table 5.2. Optimal schedule for supplier in a decentralized is shown in table 5.3 by three different defuzzyication methods, Centroid Method, Graded Mean Representation Method and Signed Distance Method. As shown in table 5.4 percentage profit is maximum by

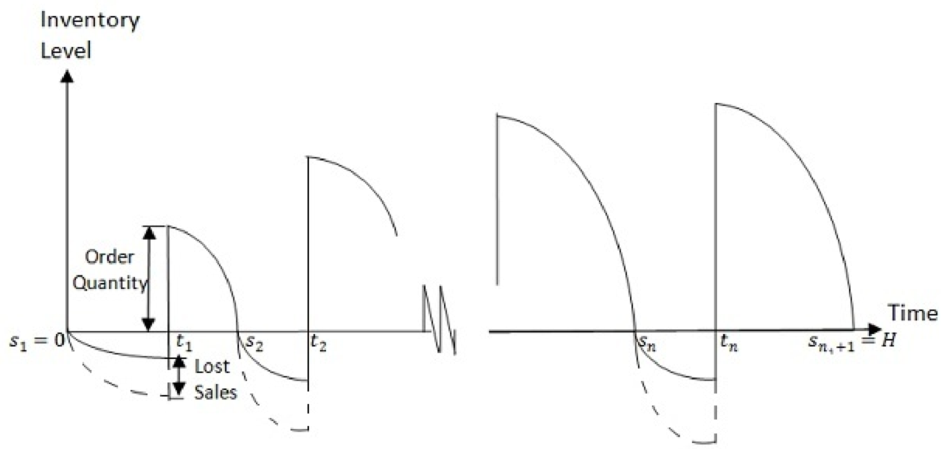

FIGURE 5.1: Model description

centroid method of defuzzyication for retailer and for supplier it is by Graded Mean Method of defuzzyication.

5.7 Optimality condition for $\widetilde{TC}_r^D$ and $\widetilde{TC}_s^C$

The sufficient condition for $\widetilde{TC}_r^D$ to be minimum is that the following Hessian matrix $\nabla^2\widetilde{TC}_r^D of \widetilde{TC}_r^D$ for a fixed n_1 is positive definite Sarkar, Ghosh, and Chaudhuri (2012b).

$$\nabla^2\widetilde{TC}_r^D = \begin{pmatrix} \frac{\partial^2\widetilde{TC}_r^D}{\partial t_1^2} & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_1\partial s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2\widetilde{TC}_r^D}{\partial s_1\partial t_1} & \frac{\partial^2\widetilde{TC}_r^D}{\partial s_1^2} & \frac{\partial^2\widetilde{TC}_r^D}{\partial s_1\partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_2\partial s_1} & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_1^2} & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_2\partial s_2} & 0 & 0 & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_{n_1-1}\partial s_{n_1-2}} & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_{n_1-1}^2} & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_{n_1-1}\partial s_{n_1-1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2\widetilde{TC}_r^D}{\partial s_{n_1-1}\partial t_{n_1-1}} & \frac{\partial^2\widetilde{TC}_r^D}{\partial s_{n_1-1}^2} & \frac{\partial^2\widetilde{TC}_r^D}{\partial s_{n_1-1}\partial t_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_{n_1}\partial s_{n_1-1}} & \frac{\partial^2\widetilde{TC}_r^D}{\partial t_{n_1}^2} \end{pmatrix} \quad (5.7)$$

TABLE 5.1: Optimal total cost suggested by retailer

$\widetilde{TC}_r^D$

↓ $\tilde{c} = 0.8$ → n_1	1	2	3	4	5	6	7
C.M.	525.777	369.339	313.198	**301.936**	311.587	331.785	357.905
Signed. D.M.	525.577	369.198	313.099	**301.863**	311.528	331.734	357.858
G.M.R.M.	525.377	369.058	313.002	**301.791**	311.471	331.685	357.815 height

$\widetilde{TC}_s^D$ when $n_1 = 4$
491.7
491.684
491.692

TABLE 5.2: Optimal total cost when total refilling cycles are suggested by supplier

$\widetilde{TC}_s^C$

↓ $\tilde{c} = 0.8$ → n_1	1	2	3	4	5	6	7
C.M.	363.926	**326.378**	386.355	491.7	619.033	757.641	902.62
S.D.M.	363.797	**326.298**	386.318	491.692	619.041	757.658	902.642
G.M.R.M.	363.666	**326.218**	386.281	491.684	619.049	757.676	902.666 height

$\widetilde{TC}_r^C$ when $n_1 = 2$
369.339
369.058
369.198

TABLE 5.3: Refilling schedule as suggested by supplier

$\tilde{c} = 0.8$	n_2^{CO}	s_1	s_2	s_3
$C.M.$	2	0	2.52914	3.99999
$G.M.R.M$	2	0	2.53169	3.99999
$S.D.M.$	2	0	2.53042	3.99999

$\tilde{c} = 0.8$	n_2^{CO}	t_1	t_2
$C.M.$	2	0.909663	2.70008
$G.M.R.M$	2	0.911067	2.70181
$S.D.M.$	2	0.910364	2.70094

TABLE 5.4: Percentage profit when $\tilde{c} = 0.8$

	Decentralized system							*Centralized system*					
$\tilde{c} = 0.8$	$\widetilde{TC}_r^{DO}$	$\widetilde{TC}_s^{DO}$	n_1^{DO}	Q^{DO}	$\lambda_{\min}$	$\lambda_{\max}$	$\bar{\lambda}$	$\widetilde{TC}_r^{CO\lambda}$	$\widetilde{TC}_s^{CO\lambda}$	n_2^{CO}	Q^{CO}	$\frac{\rho\widetilde{TC}}{\widetilde{TC}_r^{DO}}$	$\frac{\rho\widetilde{TC}}{\widetilde{TC}_s^{DO}}$
C.M.	301.936	491.7	4	38.9321	0.460187	1.59805	1.02912	214.051	414.226	2	63.1288	29.1071	15.7563
G.M.R.M.	301.791	491.684	4	38.9117	0.453455	1.57345	1.01345	216.453	411.538	2	63.1086	28.2773	16.3003
S.D.M.	301.863	491.692	4	38.9218	0.456822	1.58575	1.02129	215.253	412.881	2	63.1187	28.6917	16.0286

TABLE 5.5: Senstivity analysis 1

Parameter	%change	value	CentroidMthd.		GradedMthd.		SignedMthd.	
			%Rtlr.	%S.	%Rtlr.	%S.	%Rtlr.	%S.
$a \rightarrow$	−20%	5.6	30.62	16.53	29.80	17.06	30.21	16.79
	−10%	6.3	29.00	15.81	28.18	16.34	28.59	16.08
	+10%	7.7	25.96	14.39	25.15	14.94	25.56	14.66
	+20%	8.4	24.54	13.69	23.73	14.25	24.14	13.97
$b \rightarrow$	−20%	4.	29.79	15.86	27.86	15.97	28.84	15.90
	−10%	4.5	28.57	15.49	27.22	15.82	27.90	15.65
	+10%	5.5	26.41	14.66	26.12	15.42	26.26	15.04
	+20%	6.	25.44	14.20	25.65	15.18	25.55	14.69
$c \rightarrow$	−20%	0.8	29.11	15.76	28.28	16.30	28.69	16.03
	−10%	0.9	28.27	15.42	27.44	15.97	27.85	15.69
	+10%	1.1	26.66	14.77	25.85	15.31	26.25	15.04
	+20%	1.2	25.89	14.44	25.09	14.99	25.49	14.72
$H_r \rightarrow$	−20%	2.4	32.27	16.56	31.43	17.08	31.85	16.82
	−10%	2.7	29.66	15.79	28.83	16.30	29.25	16.04
	+10%	3.3	25.57	14.51	24.77	15.07	25.17	14.79
	+20%	3.6	23.97	14.02	23.19	14.58	23.58	14.30
,	,	,						
$h_c \rightarrow$	−20%	0.96	28.89	14.19	27.71	14.97	28.39	14.52
	−10%	1.08	27.93	14.79	26.97	15.43	27.47	15.10
	+10%	1.32	27.29	15.19	26.59	15.66	26.95	15.42
	+20%	1.44	27.36	15.15	26.75	15.56	27.10	15.33
$l \rightarrow$	−20%	9.6	30.89	17.17	30.12	17.68	30.50	17.42
	−10%	10.8	29.03	16.05	28.23	16.58	28.63	16.32
	+10%	13.2	26.12	14.27	25.29	14.83	25.70	14.55
	+20%	14.4	24.99	13.57	24.15	14.13	24.57	13.85
$S \rightarrow$	−20%	1900.	27.53	15.14	26.72	15.69	27.12	15.41
	−10%	1960.	27.49	15.12	26.68	15.66	27.08	15.39
	+10%	2040.	27.41	15.07	26.60	15.61	27.00	15.34
	+20%	2100.	27.37	15.04	26.56	15.59	26.96	15.32
$\alpha \rightarrow$	−20%	0.0016	27.47	15.10	26.65	15.64	27.05	15.37
	−10%	0.0018	27.46	15.10	26.64	15.64	27.05	15.37
	+10%	0.0022	27.45	15.09	26.63	15.63	27.04	15.36
	+20%	0.0024	27.44	15.09	26.62	15.63	27.04	15.36

TABLE 5.6: Senstivity analysis 2

Parameter	*%change*	*value*	*C.M.*	*G.M.*	*S.M.*	*C.M.*	*G.M.*	*S.M.*
$S_r \rightarrow$	−20%	32.	27.69	13.98	26.71	15.09	27.34	13.70
	−10%	36.	27.56	14.53	26.67	15.36	27.19	14.53
	+10%	44.	27.34	15.65	26.60	15.91	26.91	16.20
	+20%	48.	27.24	16.21	26.57	16.19	26.80	17.03
		0.24	27.70	15.31	26.88	15.86	27.29	15.59
$C_s \rightarrow$	−20%	0.27	27.57	15.20	26.76	15.75	27.17	15.48
	−10%	0.33	27.33	14.98	26.51	15.52	26.92	15.25
	+10%	0.36	27.20	14.87	26.39	15.41	26.80	15.14
	+20%							
		4.8	28.42	15.61	28.13	16.45	28.02	15.87
$\rho \rightarrow$	−20%	5.4	27.90	15.33	27.55	16.14	27.49	15.60
	−10%	6.6	27.07	14.88	26.63	15.66	26.66	15.16
	+10%	7.2	26.73	14.70	26.26	15.46	26.32	14.98
	+20%							
$\theta \rightarrow$	−20%	0.16	27.94	15.32	27.13	15.87	27.53	15.59
	−10%	0.18	27.69	15.21	26.88	15.75	27.29	15.48
	+10%	0.22	27.21	14.98	26.4	15.53	26.81	15.25
	+20%	0.24	26.98	14.87	26.17	15.42	26.58	15.15

5.8 Conclusion

Mentioned are the two tables of sensitivity analysis. For all the parameters, observations have been recorded with 10% and 20% of the increase and 10% and 20% of decrease of the parameter's actual value. These values are mentioned in column number 3 of the table 5.5 and 5.6.

%profit of the retailer is maximum with the Centroid Method than compared to the other two methods for all the parameters. For -20% of parameters value, the maximum rise is by l that is 30.89% and minimum rise is by α that is 27.47%. %profit of supplier is maximum with Graded Mean Representation Method. The maximum rise is by l and the minimum is obtained by a change in h_c.

%profit of the retailer is minimum with Graded Mean Representation Method. For -20% of parameters value, minimum fall is due to α that is 26.65%. %profit of supplier is minimum with Graded Mean Representation Method. For -20% of the parameters value, the minimum rise is by alpha 26.65%. Except for S_r, %profit of the retailer is minimum with Centroid Method. The study can be extended for the inclusion of Weibull deterioration of the product, inflation of cost and greening of the supply chain as in Singh et al. (2017b), Singh et al. (2018b) and Singh et al. (2019).

Appendix A

Program in software Mathematica version 8.1

The excerpts of the program for table formulation given for copyright is as given below:

FIGURE A.1: Copyright Submission

```
percentage = ( 0.95  0.98  1.02  1.05  1 );
foralpha = Table[alphainitial[[1, 1]] * percentage[[1, i]], {j, 1, 1}, {i, 1, 5}];
{α = 0.0009, θ1 = 0.0009, CT = 0.1, DC = 600, Po = 5, So = 290, Ho = 1, Lo = 600,
 a = 1500, b = 50, c = 15, p = 0.005, Sc = 0.64, Co = 300, β = 1.5, Tcp = 1, Tc = 300,
 DsAsmp = 1.25, DsAsm = 35, Remp = 10, Rem = 300, Ss = 2000, Ps = 0.9, δ = 10}; TCRIND =
 Table[∑_{j=1}^{j=i} i * Co + ∑_{j=1}^{j=i} (Po + Sc) * ((a (Tabforsi_v[[i, j + 1]] - Tabforti_v[[i, j]]) +
        (b / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
        (c / 3) (Tabforsi_v[[i, j + 1]]^3 - Tabforti_v[[i, j]]^3))
      (1 - θ1 * Tabforti_v[[i, j]] - foralpha[[1, v]] * Tabforti_v[[i, j]]^β -
        p * (θ1 * (Tabforti_v[[i, j]] + 2 * CT - Tabforti_v[[i, j]] + CT) + foralpha[[1,
             v]] * ((Tabforti_v[[i, j]] + 2 * CT)^β - Tabforti_v[[i, j]]^β))) +
     θ1 * ((a / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
        (b / 3) (Tabforsi_v[[i, j + 1]]^3 - Tabforti_v[[i, j]]^3) + (c / 4)
         (Tabforsi_v[[i, j + 1]]^4 - (Tabforti_v[[i, j]])^4)) + foralpha[[1, v]]
      ((a / (β + 1)) (Tabforsi_v[[i, j + 1]]^(β + 1) - Tabforti_v[[i, j]]^(β + 1)) +
        (b / (β + 2)) (Tabforsi_v[[i, j + 1]]^(β + 2) - Tabforti_v[[i, j]]^(β + 2)) +
        (c / (β + 3)) (Tabforsi_v[[i, j + 1]]^(β + 3) - Tabforti_v[[i, j]]^(β + 3))) +
     ∫_{Tabforsi_v[[i,j]]}^{Tabforti_v[[i,j]]} ((a + b * y + c * y^2) / (1 + δ * (Tabforti_v[[i, j]] - y))) dy) +
  ∑_{j=1}^{j=i} Ho * (∫_{Tabforti_v[[i,j]]}^{Tabforti_v[[i,j]]+CT} ((1 - θ1 * y - foralpha[[1, v]] * y^β)
         (a (Tabforsi_v[[i, j + 1]] - y) + (b / 2) (Tabforsi_v[[i, j + 1]]^2 - y^2) +
           (c / 3) (Tabforsi_v[[i, j + 1]]^3 - y^3)) +
        θ1 * ((a / 2) (Tabforsi_v[[i, j + 1]]^2 - y^2) + (b / 3) (Tabforsi_v[[
                i, j + 1]]^3 - y^3) + (c / 4) (Tabforsi_v[[i, j + 1]]^4 - y^4)) +
        foralpha[[1, v]] * ((a / (β + 1)) (Tabforsi_v[[i, j + 1]]^(β + 1) -
              y^(β + 1)) + (b / (β + 2)) (Tabforsi_v[[i, j + 1]]^(β + 2) - y^(β + 2)) +
           (c / (β + 3)) (Tabforsi_v[[i, j + 1]]^(β + 3) - y^(β + 3))) +
        (-p (θ1 * ((Tabforti_v[[i, j]] + 2 CT) - (Tabforti_v[[i, j]] + CT)) +
              foralpha[[1, v]] * ((Tabforti_v[[i, j]] + 2 CT)^β - (Tabforti_v[[i,
                      j]] + CT)^β))) (a (Tabforsi_v[[i, j + 1]] - Tabforti_v[[i, j]]) +
           (b / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
           (c / 3) (Tabforsi_v[[i, j + 1]]^3 - Tabforti_v[[i, j]]^3))) dy) +
  ∑_{j=1}^{j=i} Ho * (∫_{Tabforti_v[[i,j]]+CT}^{Tabforti_v[[i,j]]+2 CT} ((-p ((a (Tabforsi_v[[i, j + 1]] - Tabforti_v[[i, j]]) +
                (b / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
                (c / 3) (Tabforsi_v[[i, j + 1]]^3 - Tabforti_v[[i, j]]^3))
              (1 - θ1 * Tabforti_v[[i, j]] - foralpha[[1, v]] * Tabforti_v[[i, j]]^β -
                p (θ1 * (Tabforti_v[[i, j]] + 2 * CT - Tabforti_v[[i, j]] + CT) +
                   foralpha[[1, v]] * ((Tabforti_v[[i, j]] + 2 * CT)^β -
                       Tabforti_v[[i, j]]^β)) + θ1 (Tabforti_v[[i, j]] + 2 * CT - y) +
                foralpha[[1, v]] ((Tabforti_v[[i, j]] + 2 * CT)^β - y^β)) +
             θ1 * ((a / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
```

```
(b / 3) (Tabforsi_v[[i, j + 1]]^3 - Tabforti_v[[i, j]]^3) +
  (c / 4) (Tabforsi_v[[i, j + 1]]^4 - (Tabforti_v[[i, j]])^4)) +
 foralpha[[1, v]] ((a / (β + 1)) (Tabforsi_v[[i, j + 1]]^(β + 1) -
    Tabforti_v[[i, j]]^(β + 1)) + (b / (β + 2)) (Tabforsi_v[[i, j + 1]]^
     (β + 2) - Tabforti_v[[i, j]]^(β + 2)) + (c / (β + 3))
   (Tabforsi_v[[i, j + 1]]^(β + 3) - Tabforti_v[[i, j]]^(β + 3)))) +
((1 - θ_1 * y - foralpha[[1, v]] * y^β) (a (Tabforsi_v[[i, j + 1]] - y) +
   (b / 2) (Tabforsi_v[[i, j + 1]]^2 - y^2) +
   (c / 3) (Tabforsi_v[[i, j + 1]]^3 - y^3)) + θ_1 * ((a / 2)
    (Tabforsi_v[[i, j + 1]]^2 - y^2) + (b / 3) (Tabforsi_v[[i, j + 1]]^3 -
     y^3) + (c / 4) (Tabforsi_v[[i, j + 1]]^4 - y^4)) + foralpha[[1,
  v]] ((a / (β + 1)) (Tabforsi_v[[i, j + 1]]^(β + 1) - y^(β + 1)) +
   (b / (β + 2)) (Tabforsi_v[[i, j + 1]]^(β + 2) - y^(β + 2)) +
   (c / (β + 3)) (Tabforsi_v[[i, j + 1]]^(β + 3) - y^(β + 3)))))) dy) +

Sum_{j=1}^{j=i} Ho * (Integral_{Tabforti_v[[i,j]]+CT}^{Tabforti_v[[i,j]]+2 CT} ((1 - θ_1 * y - foralpha[[1, v]] * y^β)
  ((a (Tabforsi_v[[i, j + 1]] - y) + (b / 2) (Tabforsi_v[[i, j + 1]]^2 - y^2) +
    (c / 3) (Tabforsi_v[[i, j + 1]]^3 - y^3)) +
   θ_1 * ((a / 2) (Tabforsi_v[[i, j + 1]]^2 - y^2) + (b / 3) (Tabforsi_v[[
        i, j + 1]]^3 - y^3) + (c / 4) (Tabforsi_v[[i, j + 1]]^4 - y^4)) +
   foralpha[[1, v]] ((a / (β + 1)) (Tabforsi_v[[i, j + 1]]^(β + 1) - y^(β + 1)) +
     (b / (β + 2)) (Tabforsi_v[[i, j + 1]]^(β + 2) - y^(β + 2)) +
     (c / (β + 3)) (Tabforsi_v[[i, j + 1]]^(β + 3) - y^(β + 3))))) dy) +

Sum_{j=1}^{j=i} DC * (Integral_{Tabforti_v[[i,j]]}^{Tabforti_v[[i,j]]+CT} (foralpha[[1, v]] * β * y^(β - 1)
  ((a (Tabforsi_v[[i, j + 1]] - Tabforti_v[[i, j]]) +
     (b / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
     (c / 3) (Tabforsi_v[[i, j + 1]]^3 - Tabforti_v[[i, j]]^3))
    (1 - θ_1 * y - foralpha[[1, v]] * y^β - p (θ_1 * ((Tabforti_v[[i, j]] + 2 CT) -
         (Tabforti_v[[i, j]] + CT)) + foralpha[[1, v]] *
        ((Tabforti_v[[i, j]] + 2 CT)^β - (Tabforti_v[[i, j]] + CT)^β))) +
   θ_1 * ((a / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
     (b / 3) (Tabforsi_v[[i, j + 1]]^3 - Tabforti_v[[i, j]]^3) +
     (c / 4) (Tabforsi_v[[i, j + 1]]^4 - (Tabforti_v[[i, j]])^4)) +
   foralpha[[1, v]] * ((a / (β + 1)) (Tabforsi_v[[i, j + 1]]^(β + 1) -
        Tabforti_v[[i, j]]^(β + 1)) + (b / (β + 2)) (Tabforsi_v[[i, j + 1]]^
         (β + 2) - Tabforti_v[[i, j]]^(β + 2)) + (c / (β + 3))
     (Tabforsi_v[[i, j + 1]]^(β + 3) - Tabforti_v[[i, j]]^(β + 3))))) dy) +

Sum_{j=1}^{j=i} DC * (Integral_{Tabforti_v[[i,j]]+CT}^{Tabforti_v[[i,j]]+2 CT} (foralpha[[1, v]] * β * y^(β - 1)
  (-p ((a (Tabforsi_v[[i, j + 1]] - Tabforti_v[[i, j]]) +
        (b / 2) (Tabforsi_v[[i, j + 1]]^2 - Tabforti_v[[i, j]]^2) +
```

```
            Tabforti_v[[n1ast[[1]], j]]^β))) +
    θ1 * ((a / 2) (Tabforsi_v[[n1ast[[1]], j + 1]]^2 - Tabforti_v[[n1ast[[1]], j]]^2) +
       (b / 3) (Tabforsi_v[[n1ast[[1]], j + 1]]^3 - Tabforti_v[[n1ast[[1]], j]]^3) +
       (c / 4) (Tabforsi_v[[n1ast[[1]], j + 1]]^4 -
          (Tabforti_v[[n1ast[[1]], j]])^4)) +
    foralpha[[1, v]] ((a / (β + 1)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 1) -
          Tabforti_v[[n1ast[[1]], j]]^(β + 1)) +
       (b / (β + 2)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 2) -
          Tabforti_v[[n1ast[[1]], j]]^(β + 2)) +
       (c / (β + 3)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 3) -
          Tabforti_v[[n1ast[[1]], j]]^(β + 3)))) +
 Ho * p * ((a (Tabforsi_v[[n1ast[[1]], j + 1]] - Tabforti_v[[n1ast[[1]], j]]) +
       (b / 2) (Tabforsi_v[[n1ast[[1]], j + 1]]^2 - Tabforti_v[[n1ast[[1]], j]]^2) +
       (c / 3) (Tabforsi_v[[n1ast[[1]], j + 1]]^3 - Tabforti_v[[n1ast[[1]], j]]^3))
     (1 - θ1 * Tabforti_v[[n1ast[[1]], j]] - foralpha[[1, v]] *
        Tabforti_v[[n1ast[[1]], j]]^β -
       p * (θ1 * (Tabforti_v[[n1ast[[1]], j]] + 2 * CT - Tabforti_v[[n1ast[[1]], j]] +
             CT) + foralpha[[1, v]] * ((Tabforti_v[[n1ast[[1]], j]] + 2 * CT)^β -
             Tabforti_v[[n1ast[[1]], j]]^β))) +
    θ1 * ((a / 2) (Tabforsi_v[[n1ast[[1]], j + 1]]^2 - Tabforti_v[[n1ast[[1]], j]]^2) +
       (b / 3) (Tabforsi_v[[n1ast[[1]], j + 1]]^3 - Tabforti_v[[n1ast[[1]], j]]^3) +
       (c / 4) (Tabforsi_v[[n1ast[[1]], j + 1]]^4 -
          (Tabforti_v[[n1ast[[1]], j]])^4)) +
    foralpha[[1, v]] ((a / (β + 1)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 1) -
          Tabforti_v[[n1ast[[1]], j]]^(β + 1)) +
       (b / (β + 2)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 2) -
          Tabforti_v[[n1ast[[1]], j]]^(β + 2)) +
       (c / (β + 3)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 3) -
          Tabforti_v[[n1ast[[1]], j]]^(β + 3))) ∫_{Tabforti_v[[n1ast[[1]], j]] + CT}^{Tabforti_v[[n1ast[[1]], j]] + 2 CT} dy),
 {i, 1, 1}]; Print[MatrixForm[TCSIND]]; Print[
"QIND"];
QIND =
 Table[
  Sum_{j=1}^{j=n1ast[[1]]} ((a (Tabforsi_v[[n1ast[[1]], j + 1]] - Tabforti_v[[n1ast[[1]], j]]) +
      (b / 2) (Tabforsi_v[[n1ast[[1]], j + 1]]^2 - Tabforti_v[[n1ast[[1]], j]]^2) +
      (c / 3) (Tabforsi_v[[n1ast[[1]], j + 1]]^3 - Tabforti_v[[n1ast[[1]], j]]^3))
    (1 - θ1 * Tabforti_v[[n1ast[[1]], j]] - foralpha[[1, v]] *
       Tabforti_v[[n1ast[[1]], j]]^β - p *
       (θ1 * (Tabforti_v[[n1ast[[1]], j]] + 2 * CT - Tabforti_v[[n1ast[[1]], j]] + CT) +
         foralpha[[1, v]] * ((Tabforti_v[[n1ast[[1]], j]] + 2 * CT)^
```

```
                β - Tabforti_v[[n1ast[[1]], j]]^β))) +
      θ1 * ((a / 2) (Tabforsi_v[[n1ast[[1]], j + 1]]^2 - Tabforti_v[[n1ast[[1]], j]]^2) +
          (b / 3) (Tabforsi_v[[n1ast[[1]], j + 1]]^3 - Tabforti_v[[n1ast[[1]], j]]^3) +
          (c / 4) (Tabforsi_v[[n1ast[[1]], j + 1]]^4 -
             (Tabforti_v[[n1ast[[1]], j]])^4)) +
      foralpha[[1, v]] ((a / (β + 1)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 1) -
             Tabforti_v[[n1ast[[1]], j]]^(β + 1)) +
          (b / (β + 2)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 2) -
             Tabforti_v[[n1ast[[1]], j]]^(β + 2)) +
          (c / (β + 3)) (Tabforsi_v[[n1ast[[1]], j + 1]]^(β + 3) -
             Tabforti_v[[n1ast[[1]], j]]^(β + 3))) +
      ∫_{Tabforsi_v[[n1ast[[1]],j]]}^{Tabforti_v[[n1ast[[1]],j]]} ((a + b * y + c * y^2) /
          (1 + δ * (Tabforti_v[[n1ast[[1]], j]] - y))) dy),
   {i, 1, 1}]; Print[MatrixForm[QIND]]; Print[
  "TCSJT"];
 TCSJT =
  Table[
   Sum_{j=1}^{j=i} (Ss * i +
      Ps * ((a (Tabforsidash_v[[i, j + 1]] - Tabfortidash_v[[i, j]]) +
             (b / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
             (c / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3))
          (1 - θ1 * Tabfortidash_v[[i, j]] - foralpha[[1, v]] *
              Tabfortidash_v[[i, j]]^β - p * (θ1 * (Tabfortidash_v[[i, j]] +
                    2 * CT - Tabfortidash_v[[i, j]] + CT) + foralpha[[1, v]] *
                  ((Tabfortidash_v[[i, j]] + 2 * CT)^β - Tabfortidash_v[[i, j]]^β))) +
         θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
             (b / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3) +
             (c / 4) (Tabforsidash_v[[i, j + 1]]^4 - (Tabfortidash_v[[i, j]])^4)) +
         foralpha[[1, v]] ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]]^(β + 1) -
                Tabfortidash_v[[i, j]]^(β + 1)) + (b / (β + 2)) (Tabforsidash_v[[i,
                  j + 1]]^(β + 2) - Tabfortidash_v[[i, j]]^(β + 2)) + (c / (β + 3))
              (Tabforsidash_v[[i, j + 1]]^(β + 3) - Tabfortidash_v[[i, j]]^(β + 3))) +
         ∫_{Tabforsidash_v[[i, j]]}^{Tabfortidash_v[[i, j]]} ((a + b * y + c * y^2) / (1 + δ * (Tabfortidash_v[[i, j]] - y)))
             dy)) + (Tc + Rem + DsAsm) *
     p * ((a (Tabforsidash_v[[i, j + 1]] - Tabfortidash_v[[i, j]]) +
          (b / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
          (c / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3))
```

```
(1 - θ1 * Tabfortidash_v[[i, j]] - foralpha[[1, v]] * Tabfortidash_v[[i, j]]^β -
  p * (θ1 * (Tabfortidash_v[[i, j]] + 2 * CT - Tabfortidash_v[[i, j]] + CT) +
     foralpha[[1, v]] * ((Tabfortidash_v[[i, j]] + 2 * CT)^β -
        Tabfortidash_v[[i, j]]^β))) +
 θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
    (b / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3) +
    (c / 4) (Tabforsidash_v[[i, j + 1]]^4 - (Tabfortidash_v[[i, j]])^4)) +
 foralpha[[1, v]] ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]]^(β + 1) -
       Tabfortidash_v[[i, j]]^(β + 1)) + (b / (β + 2)) (Tabforsidash_v[[i, j + 1]]^
        (β + 2) - Tabfortidash_v[[i, j]]^(β + 2)) + (c / (β + 3))
     (Tabforsidash_v[[i, j + 1]]^(β + 3) - Tabfortidash_v[[i, j]]^(β + 3)))) +
Ho * p * ((a (Tabforsidash_v[[i, j + 1]] - Tabfortidash_v[[i, j]]) +
    (b / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
    (c / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3))
  (1 - θ1 * Tabfortidash_v[[i, j]] - foralpha[[1, v]] * Tabfortidash_v[[i, j]]^β -
    p * (θ1 * (Tabfortidash_v[[i, j]] + 2 * CT - Tabfortidash_v[[i, j]] + CT) +
       foralpha[[1, v]] * ((Tabfortidash_v[[i, j]] + 2 * CT)^β -
          Tabfortidash_v[[i, j]]^β))) +
   θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
      (b / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3) +
      (c / 4) (Tabforsidash_v[[i, j + 1]]^4 - (Tabfortidash_v[[i, j]])^4)) +
   foralpha[[1, v]] ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]]^(β + 1) -
         Tabfortidash_v[[i, j]]^(β + 1)) + (b / (β + 2))
       (Tabforsidash_v[[i, j + 1]]^(β + 2) - Tabfortidash_v[[i, j]]^(β + 2)) +
      (c / (β + 3)) (Tabforsidash_v[[i, j + 1]]^(β + 3) -
         Tabfortidash_v[[i, j]]^(β + 3))) ∫_{Tabfortidash_v[[i, j]] + CT}^{Tabfortidash_v[[i, j]] + 2 CT} dy) + ∑_{j = 1}^{j = i} i * Co +
∑_{j=1}^{j=i} (Po + Sc) * ((a (Tabforsidash_v[[i, j + 1]] - Tabfortidash_v[[i, j]]) +
      (b / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
      (c / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3))
    (1 - θ1 * Tabfortidash_v[[i, j]] - foralpha[[1, v]] * Tabfortidash_v[[i, j]]^β -
      p * (θ1 * (Tabfortidash_v[[i, j]] + 2 * CT - Tabfortidash_v[[i, j]] + CT) +
         foralpha[[1, v]] * ((Tabfortidash_v[[i, j]] + 2 * CT)^β -
            Tabfortidash_v[[i, j]]^β))) +
   θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
      (b / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3) +
      (c / 4) (Tabforsidash_v[[i, j + 1]]^4 - (Tabfortidash_v[[i, j]])^4)) +
   foralpha[[1, v]] ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]]^(β + 1) -
         Tabfortidash_v[[i, j]]^(β + 1)) + (b / (β + 2)) (Tabforsidash_v[[i,
            j + 1]]^(β + 2) - Tabfortidash_v[[i, j]]^(β + 2)) + (c / (β + 3))
       (Tabforsidash_v[[i, j + 1]]^(β + 3) - Tabfortidash_v[[i, j]]^(β + 3))) +
   ∫_{Tabforsidash_v[[i, j]]}^{Tabfortidash_v[[i, j]]} ((a + b * y + c * y^2) / (1 + δ * (Tabfortidash_v[[i, j]] - y)))
```

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dy) +
∑_{j = 1}^{j = i} Ho * (∫_{Tabfortidash_v[[i, j]]}^{Tabfortidash_v[[i, j]] + CT} ((1 - θ1 * y - foralpha[[1, v]] * y^β)
      (a (Tabforsidash_v[[i, j + 1]] - y) + (b / 2) (Tabforsidash_v[[i, j + 1]]^2 -
            y^2) + (c / 3) (Tabforsidash_v[[i, j + 1]]^3 - y^3)) +
     θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]]^2 - y^2) + (b / 3)
          (Tabforsidash_v[[i, j + 1]]^3 - y^3) +
        (c / 4) (Tabforsidash_v[[i, j + 1]]^4 - y^4)) + foralpha[[1, v]] *
      ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]]^(β + 1) - y^(β + 1)) +
        (b / (β + 2)) (Tabforsidash_v[[i, j + 1]]^(β + 2) - y^(β + 2)) +
        (c / (β + 3)) (Tabforsidash_v[[i, j + 1]]^(β + 3) - y^(β + 3))) +
     (-p (θ1 * ((Tabfortidash_v[[i, j]] + 2 CT) - (Tabfortidash_v[[i, j]] + CT)) +
           foralpha[[1, v]] * ((Tabfortidash_v[[i, j]] + 2 CT)^β -
              (Tabfortidash_v[[i, j]] + CT)^β)))
      (a (Tabforsidash_v[[i, j + 1]] - Tabfortidash_v[[i, j]]) +
        (b / 2) (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
        (c / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3))) dy) +
∑_{j = 1}^{j = i} Ho * (∫_{Tabfortidash_v[[i, j]] + CT}^{Tabfortidash_v[[i, j]] + 2 CT} ((-p ((a (Tabforsidash_v[[i, j + 1]] -
                Tabfortidash_v[[i, j]]) + (b / 2)
              (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) +
             (c / 3) (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3))
           (1 - θ1 * Tabfortidash_v[[i, j]] - foralpha[[1, v]] * Tabfortidash_v[[i,
                 j]]^β - p (θ1 * (Tabfortidash_v[[i, j]] + 2 * CT - Tabfortidash_v[[
                      i, j]] + CT) + foralpha[[1, v]] * ((Tabfortidash_v[[i, j]] +
                        2 * CT)^β - Tabfortidash_v[[i, j]]^β)) +
             θ1 (Tabfortidash_v[[i, j]] + 2 * CT - y) + foralpha[[1, v]]
               ((Tabfortidash_v[[i, j]] + 2 * CT)^β - y^β)) + θ1 * ((a / 2)
               (Tabforsidash_v[[i, j + 1]]^2 - Tabfortidash_v[[i, j]]^2) + (b / 3)
               (Tabforsidash_v[[i, j + 1]]^3 - Tabfortidash_v[[i, j]]^3) + (c / 4)
               (Tabforsidash_v[[i, j + 1]]^4 - (Tabfortidash_v[[i, j]])^4)) +
           foralpha[[1, v]] ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]]^
                  (β + 1) - Tabfortidash_v[[i, j]]^(β + 1)) +
             (b / (β + 2)) (Tabforsidash_v[[i, j + 1]]^(β + 2) -
                Tabfortidash_v[[i, j]]^(β + 2)) + (c / (β + 3)) (Tabforsidash_v[[
                  i, j + 1]]^(β + 3) - Tabfortidash_v[[i, j]]^(β + 3)))) +
      ((1 - θ1 * y - foralpha[[1, v]] * y^β) (a (Tabforsidash_v[[i, j + 1]] - y) +
            (b / 2) (Tabforsidash_v[[i, j + 1]]^2 - y^2) + (c / 3) (Tabforsidash_v[[
                 i, j + 1]]^3 - y^3)) + θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]]^
                2 - y^2) + (b / 3) (Tabforsidash_v[[i, j + 1]]^3 - y^3) +
            (c / 4) (Tabforsidash_v[[i, j + 1]]^4 - y^4)) + foralpha[[1,
           v]] ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]]^(β + 1) - y^(β + 1)) +
            (b / (β + 2)) (Tabforsidash_v[[i, j + 1]]^(β + 2) - y^(β + 2)) +
```

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(c / (β + 3)) (Tabforsidash_v[[i, j + 1]] ^ (β + 3) - y^ (β + 3)))))) ⅆy) +
∑_{j = 1}^{j = i} Ho * (∫_{Tabfortidash_v[[i, j]] + CT}^{Tabfortidash_v[[i, j]] + 2 CT} ((1 - θ1 * y - foralpha[[1, v]] * y^β)
    ((a (Tabforsidash_v[[i, j + 1]] - y) + (b / 2) (Tabforsidash_v[[i, j + 1]] ^2 -
          y^2) + (c / 3) (Tabforsidash_v[[i, j + 1]] ^3 - y^3)) +
      θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]] ^2 - y^2) + (b / 3)
          (Tabforsidash_v[[i, j + 1]] ^3 - y^3) +
         (c / 4) (Tabforsidash_v[[i, j + 1]] ^4 - y^4)) + foralpha[[1, v]]
       ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]] ^ (β + 1) - y^ (β + 1)) +
         (b / (β + 2)) (Tabforsidash_v[[i, j + 1]] ^ (β + 2) - y^ (β + 2)) +
         (c / (β + 3)) (Tabforsidash_v[[i, j + 1]] ^ (β + 3) - y^ (β + 3))))) ⅆy) +
∑_{j = 1}^{j = i} DC * (∫_{Tabfortidash_v[[i, j]]}^{Tabfortidash_v[[i, j]] + CT} (foralpha[[1, v]] * β * y^ (β - 1)
    ((a (Tabforsidash_v[[i, j + 1]] - Tabfortidash_v[[i, j]]) +
         (b / 2) (Tabforsidash_v[[i, j + 1]] ^2 - Tabfortidash_v[[i, j]] ^2) +
         (c / 3) (Tabforsidash_v[[i, j + 1]] ^3 - Tabfortidash_v[[i, j]] ^3))
       (1 - θ1 * y - foralpha[[1, v]] * y^β - p (θ1 *
              ((Tabfortidash_v[[i, j]] + 2 CT) - (Tabfortidash_v[[i, j]] + CT)) +
            foralpha[[1, v]] * ((Tabfortidash_v[[i, j]] + 2 CT) ^β -
               (Tabfortidash_v[[i, j]] + CT) ^β))) +
      θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]] ^2 - Tabfortidash_v[[i, j]] ^2) +
         (b / 3) (Tabforsidash_v[[i, j + 1]] ^3 - Tabfortidash_v[[i, j]] ^3) +
         (c / 4) (Tabforsidash_v[[i, j + 1]] ^4 - (Tabfortidash_v[[i, j]]) ^4)) +
      foralpha[[1, v]] * ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]] ^ (β + 1) -
            Tabfortidash_v[[i, j]] ^ (β + 1)) + (b / (β + 2))
          (Tabforsidash_v[[i, j + 1]] ^ (β + 2) - Tabfortidash_v[[i, j]] ^ (β + 2)) +
         (c / (β + 3)) (Tabforsidash_v[[i, j + 1]] ^ (β + 3) -
            Tabfortidash_v[[i, j]] ^ (β + 3))))) ⅆy) +
∑_{j = 1}^{j = i} DC * (∫_{Tabfortidash_v[[i, j]] + CT}^{Tabfortidash_v[[i, j]] + 2 CT} (foralpha[[1, v]] * β * y^ (β - 1)
    (-p ((a (Tabforsidash_v[[i, j + 1]] - Tabfortidash_v[[i, j]]) + (b / 2)
            (Tabforsidash_v[[i, j + 1]] ^2 - Tabfortidash_v[[i, j]] ^2) + (c / 3)
            (Tabforsidash_v[[i, j + 1]] ^3 - Tabfortidash_v[[i, j]] ^3)) (1 -
          θ1 * Tabfortidash_v[[i, j]] - foralpha[[1, v]] * Tabfortidash_v[[i,
              j]] ^β - p (θ1 * (Tabfortidash_v[[i, j]] + 2 * CT - Tabfortidash_v[[
                  i, j]] + CT) + foralpha[[1, v]] * ((Tabfortidash_v[[i, j]] +
                    2 * CT) ^β - Tabfortidash_v[[i, j]] ^β))) + θ1 * ((a / 2)
            (Tabforsidash_v[[i, j + 1]] ^2 - Tabfortidash_v[[i, j]] ^2) + (b / 3)
            (Tabforsidash_v[[i, j + 1]] ^3 - Tabfortidash_v[[i, j]] ^3) + (c / 4)
            (Tabforsidash_v[[i, j + 1]] ^4 - (Tabfortidash_v[[i, j]]) ^4)) +
        foralpha[[1, v]] ((a / (β + 1)) (Tabforsidash_v[[i, j + 1]] ^ (β + 1) -
               Tabfortidash_v[[i, j]] ^ (β + 1)) + (b / (β + 2)) (Tabforsidash_v[[
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i, j + 1]] ^ (β + 2) - Tabfortidash_v[[i, j]] ^ (β + 2)) +
(c / (β + 3)) (Tabforsidash_v[[i, j + 1]] ^ (β + 3) - Tabfortidash_v[[
i, j]] ^ (β + 3)))) + ((1 - θ1 * y - foralpha[[1, v]] * y^β)
(a (Tabforsidash_v[[i, j + 1]] - y) + (b / 2) (Tabforsidash_v[[i, j +
1]] ^2 - y^2) + (c / 3) (Tabforsidash_v[[i, j + 1]] ^3 - y^3)) +
θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]] ^2 - y^2) + (b / 3)
(Tabforsidash_v[[i, j + 1]] ^3 - y^3) +
(c / 4) (Tabforsidash_v[[i, j + 1]] ^4 - y^4)) + foralpha[[1, v]]
((a / (β + 1)) (Tabforsidash_v[[i, j + 1]] ^ (β + 1) - y^ (β + 1)) +
(b / (β + 2)) (Tabforsidash_v[[i, j + 1]] ^ (β + 2) - y^ (β + 2)) +
(c / (β + 3)) (Tabforsidash_v[[i, j + 1]] ^ (β + 3) - y^ (β + 3)))))) dy) +
Sum_{j = 1}^{j = i} DC * (Integrate_{Tabfortidash_v[[i, j]] + 2 CT}^{Tabforsidash_v[[i, j + 1]]} (foralpha[[1, v]] * β * y^ (β - 1)
((1 - θ1 * y - foralpha[[1, v]] * y^β) ((a (Tabforsidash_v[[i, j + 1]] - y) +
(b / 2) (Tabforsidash_v[[i, j + 1]] ^2 - y^2) + (c / 3) (Tabforsidash_v[[
i, j + 1]] ^3 - y^3)) + θ1 * ((a / 2) (Tabforsidash_v[[i, j + 1]] ^
2 - y^2) + (b / 3) (Tabforsidash_v[[i, j + 1]] ^3 - y^3) +
(c / 4) (Tabforsidash_v[[i, j + 1]] ^4 - y^4)) + foralpha[[1, v]]
((a / (β + 1)) (Tabforsidash_v[[i, j + 1]] ^ (β + 1) - y^ (β + 1)) +
(b / (β + 2)) (Tabforsidash_v[[i, j + 1]] ^ (β + 2) - y^ (β + 2)) +
(c / (β + 3)) (Tabforsidash_v[[i, j + 1]] ^ (β + 3) - y^ (β + 3)))))) dy) +
Sum_{j = 1}^{j = i} So * (Integrate_{Tabforsidash_v[[i, j]]}^{Tabfortidash_v[[i, j]]} ((Tabfortidash_v[[i, j]] - y) (a + b * y + c * y^2) /
(1 + δ * (Tabfortidash_v[[i, j]] - y))) dy) +
Sum_{j = 1}^{j = i} Lo * (Integrate_{Tabforsidash_v[[i, j]]}^{Tabfortidash_v[[i, j]]} ((Tabfortidash_v[[i, j]] - y) * δ *
(a + b * y + c * y^2) / (1 + δ * (Tabfortidash_v[[i, j]] - y))) dy) -
RETTOCIND[[1]], {k, 1, 1}, {i, 1, 4}]; Print[MatrixForm[
TCSJT]];
Print[
"Suppliers
Total
Optimal
Cost
without
Coordination
for
a
="];
Print["SUPTOCWC="]; SUPTOCWC =
Table[
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Appendix B

Conference participation, and Published, accepted, communicated Papers

B.0.1 Conference participation

1. Received a **best paper award** for the research paper title "A supply chain model for time quadratic inventory dependent demand for Weibull deterioration item with a credit term for short life-cycle under the finite horion" presented in the 4^{th} International Conference on computing Sciences(ICCS) "Feynman-100", 30^{th} August, 2018 held at Lovely Professional University, Punjab, India.

2. Presented a research paper title "A supply chain replenishment inflationary inventory model with trade credit" in "ICICC-2019 conference held at **VŠB - Technical University Of Ostrava, Czech Republic, Europe** on $21-22^{nd}$ March, 2019.

3. A research paper with title "A finite planning horizon fuzzy inventory model using hexagonal fuzzy number" has been accepted for oral presentation at the Conference "INTERNATIONAL CONFERENCE ON SCIENCE TECHNOLOGY AND MANAGEMENT" (ICSTM-20) on 1^{ST}-2^{ND} JUNE, 2020, in **NEW YORK, USA**

4. Presented a research paper title "A green supply chain re-manufacturing model for time quadratic inventory dependent demand and partially backlogging with Weibull deterioration under the finite horizon" in the second International Conference on Emerging Trends in Mechanical Engineering (eTIME - 2019), 9^{th} and 10^{th} August, 2019 held at St Joseph Engineering College, Mangaluru, 575028, India

5. Participated in the 106^{th} Indian Science Congress held at Lovely Professional University, Phagwara, Jalandhar from January 3 to 7, 2019. Is a life time member of **THE INDIAN SCIENCE CONGRESS ASSOCIATION**.

6. Presented a research paper title "An EOQ model of time quadratic and inventory dependent demand for deteriorated items with partially backlogged shortages under trade credit" in the International conference on "Recent Advances in Fundamental and Applied Science" RAFAS-2016' from 25-26 November, 2016 held at Lovely Professional University, Punjab, India.

7. Presented a research paper title "Fuzzyfication of supplier–retailer inventory coordination with credit term for deteriorating item with time-quadratic demand and partial backlogging in all cycles" in the International conference on "Recent Advances in Fundamental and Applied Science" RAFAS-2019' from 5-6 November, 2019 held at Lovely Professional University, Punjab, India.

B.0.2 List of papers published are as follows:-

1. Pushpinder Singh, **Nitin Kumar Mishra**, Vikramjeet Singh, Seema Saxena, "An eoq model of time quadratic and inventory dependent demand for deteriorated items with partially backlogged shortages under trade credit" AIP Conference Proceedings 1860 (1), 020037.(**Scopus Indexed**)

2. Vikramjeet Singh, **Nitin Kumar Mishra**, Sanjay Mishra, Pushpinder Singh, Seema Saxena, "A green supply chain model for time quadratic inventory dependent demand and partially backlogging with Weibull deterioration under the finite horizon." AIP Conference Proceedings 2080 (1), 060002.(**Scopus Indexed**)

3. Vikramjeet Singh, **Nitin Kumar Mishra**, Sanjay Mishra, Pushpinder Singh, Seema Saxena, "An inventory model in a green supply chain for inventory dependent and time quadratic demand in a finite horizon" in International Journal of Control and Automation, Vol.12, No.4, (2019), pp. 218-220(**Scopus Indexed**)

4. Sanjay Mishra, **Nitin Kumar Mishra**, Vikramjeet Singh, Pushpinder Singh, and Seema Saxena, "The Fuzzyfied Supply Chain Finite Planning Horizon Model", J. Comput. Theor. Nanosci. 16, 4135–4142 (2019)(**Scopus Indexed**)

An EOQ model of time quadratic and inventory dependent demand for deteriorated items with partially backlogged shortages under trade credit

Pushpinder Singh, Nitin Kumar Mishra, Vikramjeet Singh, and Seema Saxena

An EOQ Model of Time Quadratic and Inventory Dependent Demand for Deteriorated Items with Partially Backlogged Shortages Under Trade Credit

Pushpinder Singh[1,a)], Nitin Kumar Mishra[1,b)], Vikramjeet Singh[2,c)] *and* Seema Saxena[1,d)]

[1]*Department of Mathematics, Lovely Professional University, Phagwara 144 411, Punjab(India).*
[2]*Department of Mathematics, IKGPTU Campus, Hoshiarpur,146001, Punjab(India).*

[a)]First Author:pushpindersnl@gmail.com
[b)]Corresponding author: snitinmishra@gmail.com, nitin.mishra@lpu.co.in
[c)]Vikram31782@gmail.com
[d)]mseemamishra@gmail.com

Abstract. In this paper a single buyer, single supplier inventory model with time quadratic and stock dependent demand for a finite planning horizon has been studied. Single deteriorating item which suffers shortage, with partial backlogging and some lost sales is considered. Model is divided into two scenarios, one with non permissible delay in payment and other with permissible delay in payment. Latter is called, centralized system, where supplier offers trade credit to retailer. In the centralized system cost saving is shared amongst the two. The objective is to study the difference in minimum costs borne by retailer and supplier, under two scenarios including the above mentioned parameters. To obtain optimal solution of the problem the model is solved analytically. Numerical example and a comparative study are then discussed supported by sensitivity analysis of each parameter.
Keywords: Inventory, Deterioration, Time quadratic and inventory dependent demand, Partially backlogged shortages, Supply chain management.

INTRODUCTION

The existing literature of different inventory models (Wu and Zhao[1] and others) reveals that the relation between retailer and supplier is becoming more stable in today's rapidly changing commercial world. The supplier steps forward and provides the retailer, credit period to settle the amount for quantity purchased. However, the coordination between supplier and retailer depends upon kind of the product, its deterioration, its demand etc. Need is to focus on type of product produced and launched.

Ghare and Schrader [2] were the first authors to consider ongoing deterioration of inventory. Goyal[3] introduced a model for EOQ under permissible delay in payments, for fixed time period. Aggarwal and Jaggi[4] extended the work of Goyal[3], for the deteriorating items. Due to the fact that long term relationship between retailer and supplier is key to success of both parties many researchers have worked on the models which strengthen the bond between retailer and supplier. For instance [5, 6, 7, 8, 9, 10, 11, 12] have presented their model for deterioration, in a two level supply chain coordinate system.

Raafat[13] provided a review of the deteriorating inventory literature. Raafat[13] defined deterioration as (i) spoilage, (ii) physical depletion and (iii) decay, and also further classified many mathematical deteriorating inventory models into a number of categories. Goyal and Giri[14] continued Raafat's review work for permissible delay in payment, price increase and price discount. An EOQ model which assumes that full payment must be done by the retailer immediately after receiving the goods from the supplier in a decentralized system where the supplier replenishes according to retailers optimal quantity requirement on cycle to cycle basis. However, this is not practical, as the supplier may offer the trade credit period to retailer i.e. a delay period for the full payment of the goods in a centralized system. Some of the related articles with trade credit financing which includes deterioration of item/s can be found

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A green supply chain model for time quadratic inventory dependent demand and partially backlogging with Weibull deterioration under the finite horizon

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Vikramjeet Singh, Nitin Kumar Mishra, Sanjay Mishra, Pushpinder Singh, and Seema Saxena

A green supply chain model for time quadratic inventory dependent demand and partially backlogging with Weibull deterioration under the finite horizon

Vikramjeet Singh[1,a)], Nitin Kumar Mishra[2,b)], Sanjay Mishra[2,c)], Pushpinder Singh[3,d)] and Seema Saxena[2,e)]

[1]*Department of Mathematics, IKGPTU Campus, Batala, Punjab(India) 143506.*
[2]*Department of Mathematics, Lovely Professional University, Phagwara , Punjab(India) 144 411.*
[3]*Department of Mathematics, Mata Gujri College, Fatehgarh Sahib, Punjab(India) 140407.*

[a)]First Author:Vikram31782@gmail.com
[b)]Corresponding Author: snitinmishra@gmail.com
[c)]drsanjaymishra1@gmail.com
[d)]pushpindersnl@gmail.com
[e)]mseemamishra@gmail.com

Abstract. This paper presents a detail solution of re-manufacturing of a product in a supply chain model. It is a non-traditional model considering time-dependent quadratic demand, Weibull deterioration, shortages, partial backlogging and re-manufacturing of inventory. This paper mainly focuses on remanufacturing and hence an attempt towards reducing the environmental hazard. The process of remanufacturing is completed within one cycle of replenishment. Trade credit between supplier and retailer also had been discussed. Two cases one of a centralized and the other of decentralization for a finite planning horizon in a supply chain model are discussed. An algorithm has been derived for solving a problem in both the cases. Some managerial insights are talked about on the basis of sensitivity analysis on the parameters considered.

Keywords: Inventory, Weibull deterioration, Time quadratic and inventory dependent demand, Partially backlogged shortages, Green supply chain management.

1 INTRODUCTION

No doubt improving environmental quality comes at a cost but at the same time, proper disposal of hazardous waste is very costly as given by Richter [1]. Due to the market competitiveness retailer and supplier in a supply chain are bound to collaborate by sharing each others information for mutual benefits such as profit in terms of money and customer satisfaction as discussed by Wu and Zhao [2]. For the products which is electrical, electronic, plastic, glass, jewelry etc., manufacturers are trying to provide a quality product by re-manufacturing and reducing the defective items as considered in Tiwari et al. [3]. This results in the greening of a supply chain.

The reverse manufacturing problem for an electronic industry was recently considered and simplified by Chung and Wee [4] green product design and remanufacturing. While raising significant concern over environmental initiatives, Zhang et al. [5] in one of the conclusions mentioned that policymakers should give more heed to employees and nearby communities. Mudgal et al. [6] identified and analyzed the barriers to green business practices. Considering re-manufacturing in green supply chain Rani et al. [7] discussed a model. Green retailing is now a buzz word amongst retailer due to growing pressure from the eco friendly environment by different stakeholders such as consumers, no profit organizations, government etc. Saha et al. [8] states that continuous investment in green operations is always profitable to the retailer. Remanufacturing a product may include replacement of a worn out part, fixation of breakage occurred due to transportation, software up gradation, remolding or other cosmetic operations.

In some cases, re-manufacturing may limit repairing only but in almost all cases re-manufacturing reduces polluting hazard from environment since the products are neither disposed of nor discarded. The Green supply chain can

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A green supply chain model for time quadratic inventory dependent demand under the nite horizon.

Vikramjeet Singh Nitin Kumar Mishra Sanjay Mishra Pushpinder Singh Seema Saxena

Abstract

This paper is mainly about re-manufacturing of an item within the planning horizon. Re-manufacturing of a product has become a natural requirement in inven-tory handling. It decreases the burden of inventory for defective kind of items. Another obvious phenomenon is deterioration of items in inventory. Hence two-parameter Weibull deterioration of items is considered in our model. The idea is greening of a supply chain model through re-manufacturing of defective items after the screening process

***Keywords**: Green supply chain management inventory Weibull deterioration Time quadratic inventory dependent demand Partially backlogged shortages*

Introduction

Market across the world are looking for a greener man-agement policies in all sectors. Re-manufacturing of the products have thus begun to become a vital activity. Apart from increasing the pro t margin customer sat-isfaction has to be claimed with the implementation of environment-friendly models for manufacturing. We have thus derived a model for re-manufacturing of the defective products within a replenishment cycle. A two-warehouse partial backlogging inventory model with ramp type demand rate, three-parameter Weibull distribu-tion deterioration under in ation and permissible delay

in payments are discussed Chakraborty et al. [1]. Je-ganathan et al. [4] has discussed two-commodity con-tinuous review inventory system with postponed in de-mands.

Selvi et al. [12] has derived a replenishment pol-icy for deteriorating items considering screening cost, transportation cost for back orders minimizing annual total cost. Singh et al. [14] is a good model in which the authors have discussed an economic ordering quantity model with deteriorating items also including partial backlogging with shortages. Further Singh et al. [15] analyzed the inventory replenishment policy under in-ation.

A production, remanufacture and waste disposal Eco-nomic production quantity model was presented by Kundu and Chakrabarti [6] concluding that policy of remanufacturing is a better strategy as far as carbon emissions are concerned.

Considering returns with di erent quality grades Sun et al. [16] in their study explored the bene ts of scheduling the manufacturing and re-manufacturing se-quence. Two types of product green (environmental-friendly) product along with the regular product was included in the model studied by Raza et al. [10] with green (environmental-friendly) product price higher than the regular product. Recently Rani et al. [9] discussed re-manufacturing in the green supply chain with items that are deteriorating. Deterioration of an inventory model was introduced by Ghare and Schrader [2]. First to mention a two-parameter Weibull distribution rate in an EOq model was Philip [8]. Khanra and Chaud-huri [5] introduced time-dependent quadratic demand function.

Ghosh and Chaudhuri [3], Manna et al. [7], Singh et al. [14], Singh et al. [15] and others. were the authors using the time quadratic demand function in their pa-pers.

However, to the best of our knowledge, a model incorporating an inventory item, which bears param-eters such as Weibull deterioration, disassembly and re-manufacturing with trade credit in a green supply chain within a nite planning horizon is not yet dis-cussed fully. Proposed model gives an insight into solv-ing such problem. Assumptions and notations are given in section 1. Model formulation is done in section 2 and solved

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The Fuzzyfied Supply Chain Finite Planning Horizon Model

Sanjay Mishra[1], Nitin Kumar Mishra[1,*], Vikramjeet Singh[2], Pushpinder Singh[3], and Seema Saxena[1]

[1] *Department of Mathematics, Lovely Professional University, Phagwara 144411, Punjab, India*
[2] *Department of Mathematics, IKGPTU Campus, Batala 143506, Punjab, India*
[3] *Department of Mathematics, Mata Gujri College, Fatehgarh Sahib, 140407, Punjab, India*

RESEARCH ARTICLE

A supply chain model is discussed for materials substances such as metals, ceramics, or plastics manufactured which is deteriorating in nature. Fuzzy parameters such as fuzzy deterioration cost, fuzzy holding cost fuzzy inventory carrying cost etcetera are considered for framing of the model which are later defuzzified using Centroid, Signed Distance and Graded Mean Representation method. Centralized replenishment policy in this finite planning horizon model is discussed along with sensitivity analysis.

Keywords: Supply Chain Management, Inventory, Fuzzy, Time Quadratic, Inventory Dependent Demand, Partially Backlogged Shortages.

1. INTRODUCTION

Along with defined criteria of membership, fuzzy set was introduced by Ref. [1]. The EOQ formula including the fuzziness was given by Ref. [2]. Introduction to fuzzy arithmetic theory and operation was provided by Ref. [3]. An operator's approximation for an interval in a fuzzy number system was provided by Refs. [4, 5] solved using Fuzzy Non-Linear Programming (FNLP) taking different cost as triangular fuzzy numbers. The solution of EPQ with cost fuzzy in nature are solved by special fuzzy technique (PGP) [6]. Reference [7] derived a methodology for the optimum value of the fuzzy total cost. Reference [8] allowed payment delay considering fuzzy cost function for fuzzy inflation and deterioration rate. Partial backlogging, demand which is stochastic with deterioration fuzzy, Ref. [9] examined an EOQ model. Total profit for fuzzy inflation, discount environment with constant product deterioration using method UFM and GRG technique is evaluated by Ref. [10].

For a deteriorating item, optimal inventory decision is derived using a genetic algorithm (GA) for an inventory-based demand by Ref. [11]. An EOQ model for Pareto optimal solution taking fuzzy total costs including shortage, holding and another cost as triangular fuzzy numbers is investigated by Ref. [5].

With shortages, inventory dependent demand and deteriorating products [12] studied a deterministic model which is fuzzified for different cost such as set up cost, opportunity cost etc. considering those as triangular fuzzy numbers and using Signed-distance method to defuzzify.

Product deterioration cost, trade credit, demand rate and other cost considered as fuzzy numbers, Ref. [13] defuzzified using Graded Mean Integration Representation method. Taking lost sales rate as triangular fuzzy number i.e., fuzzifying the backorder rate, Ref. [14] constructed a new fuzzy number, called as a statistic-fuzzy number, and then developed an algorithm to find the optimal schedule.

Reference [15] extended [14] by fuzzyfying the backorder and shortage, defuzzyfying using the signed distance method and compared the fuzzy model with that of the crisp. Other authors discused fuzzy shortages are [16] and [17].

Expressing order quantity as a triangular fuzzy number, Ref. [18] found after defuzzification that cost of the crisp model is on the lower side compared to the fuzzy model.

Triangular fuzzy numbers as input values for an inventory model, the total minimum cost is found by Ref. [19], along with inventory replenishment quantity, transforming a fuzzy model into crisp.

Backorder quantity as a fuzzy triangular number, Ref. [20] compared fuzzy and crisp model finding the centroid of the cost function.

With item quantity as a triangular fuzzy number, Ref. [21] found the centroid of the fuzzy cost function's membership function.

Authors considering triangular fuzzy number are [16, 22–24] and others.

Graded mean method of defuzzification was used by Ref. [16]. Signed Distance Method for defuzzification

*Author to whom correspondence should be addressed.

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B.0.3 List of papers communicated are as follows:-

1. "Fuzzyfication of supplier–retailer inventory coordination with credit term for deteriorating item with time-quadratic demand and partial backlogging in all cycles" in a **Scopus indexed** Journal.

2. "A finite planning horizon fuzzy inventory model using hexagonal fuzzy number" in a **Scopus indexed** journal.

3. "A finite planning horizon fuzzy inventory model using pentagonal fuzzy number" in a **Scopus indexed** journal.

B.0.4 Other relevant articles published during research work (not in thesis):-

1. Pushpinder Singh, **NK Mishra**, Manoj Kumar, Seema Saxena, Vikramjeet Singh, "Risk analysis of flood disaster based on similarity measures in picture fuzzy environment", Afrika Mathematika 29 (7-8), 1019-1038(**Scopus Indexed**).

2. Vikramjeet Singh, Seema Saxena, Pushpinder Singh, **NK Mishra**, "Replenishment policy for an inventory model under inflation", AIP Conference Proceedings 1860 (1), 020035 (**Scopus Indexed**).

3. Seema Saxena, Vikramjeet Singh, Rajesh Kumar Gupta, **Nitin Kumar Mishra**, Pushpinder Singh, "Green Inventory Supply Chain Model with Inflation under Permissible Delay in Finite Planning Horizon", Adv. Sci. Technol. Eng. Syst. J., Volume 4, Issue 5, Page No 123-131, 2019,by .(**Scopus Indexed**)

4. Vikramjeet Singh, Seema Saxena, Rajesh Kumar Gupta, **Nitin Kumar Mishra**, Pushpinder Singh, "A Supply Chain Model with deteriorating items under inflation",
   DOI 10.1109/ICCS.2018.00028, by (**Scopus Indexed**).

5. Seema Saxena, Vikramjeet Singh, Rajesh Kumar Gupta, Pushpinder Singh, **Nitin Kumar Mishra**, "A Supply Chain Replenishment Inflationary Inventory Model with Trade Credit". by . In: Khanna A., Gupta D., Bhattacharyya S., Snasel V., Platos J., Hassanien A. (eds) International Conference on Innovative Computing and Communications. Advances in Intelligent Systems and Computing, vol 1059. **Springer**, Singapore(**Scopus Indexed**).

B.0.5 Other relevant articles communicated and accepted for publication during research work (not in thesis):-

1. Rajesh Kumar Gupta, Seema Saxena, Vikramjeet Singh,Pushpinder Singh and Nitin Kumar Mishra, "An Inventory Ordering Model with Different Defuzzification Techniques under Inflation" in Journal of Computational and Theoretical Nanoscience. (**Scopus Indexed**)

B.0.6 Other relevant articles communicated for publication during research work under revision (not in thesis):-

1. Vendor buyer green inventory model with price sensitive demand under inflation, International Journal of Applied Mathematics and Computation(**Scopus Indexed**)

B.0.7 Other relevant articles communicated for publication during research work (not in thesis):-

1. Affirmable inventory administration for environment preserving under fuzzy logic and inflation, to Mili publication (**Scopus Indexed**)