MULTI-STATE SYSTEM'S PERFORMANCE ANALYSIS THROUGH RELIABILITY APPROACH UNDER RANDOM OPERATING CONDITIONS

Thesis Submitted For the Award of the Degree of

DOCTOR OF PHILOSOPHY

in

Mathematics

By

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Transforming Education Transforming India

LOVELY PROFESSIONAL UNIVERSITY

PUNJAB

2021

Declaration

I, Pardeep Kumar, declare that the Ph.D. thesis entitled ‘Multi-State System’s Performance Analysis through Reliability Approach under Random Operating Conditions’ has not been previously submitted for a degree or diploma at any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made. This thesis is my own work.

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I hereby affirm as under that:

1. The thesis presented by Pardeep Kumar is worthy of consideration for the award of the degree of Doctor of Philosophy.
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Abstract

The industrial system has completely changed it face, due to the technological advancements in the field of electronics, mechatronics, and mechanics. Many advanced and complex machines have been developed by the engineers, which are extremely efficient. Their performance level is unmatchable compared the old machines. These machines have enabled the manufactures to produce, products in a huge quantity in a very less time and cost. This also increases the profit of the industry. There are many plants and industries like robotic industries, rice mill, and sugar industries in which the machines, worth corers of rupees have been installed by the owner of the plant. Damage or loss to these machines can lead to huge loss to the plant, people working in the plant and environment. Sometimes, machines may also fail due to the failures of its components/subsystems. These components/subsystems could be electronics, mechanical etc. As all these subsystems belong to different engineering streams, therefore, understanding the interaction of these components is very complex task. In these machines, components work with different level of performance level and the performance of the whole system just depends on the performance of its components. These types of system are called multi-state systems. The reliability assessment of the MSS is very important so that proper maintenance of the system could be planned and the downtime of the system could be reduced.

In this thesis, scholar determined the various reliability measures of the wireless communication system, sugar mill, rice mill, robotic arm system, automatic ticket vending machines, flow transmission system etc. The abstract of the each model is given below.

Model-I: In this model, a wireless communication system is considered for the reliability evaluation. The systems' main components are Input Transducer, Transmitter, Communication Channel, and Receiver are taken into consideration. Markov process and mathematical modelling is used to formulate a mathematical model of the considered system (on the basis of various failures/repairs). Reliability of the wireless communication system with respect to its components failure is obtained and explained with the graphs. Also, critical components of the system are identified with the aid of sensitivity analysis. At last, MTTF and MTBF with variation in various failures are also obtained.

Model-II: The aim of this model is to analyze a system, which consists of two components, working under a cost-free warranty policy. Past literature reflects that till now the focus of researchers is on those systems which work without taking rest. But here authors emphasized on an industrial system which takes rest after working for a specific amount of time. This strategy helps the system to run for a long time with less failure. After taking rest, the system starts its working again. During the mathematical modeling of the system various state of the same are critically analyzed. Reliability of the considered system has been obtained for the different combinations of failure and repair rates. Also, the various parameters which affect the system performance have been identified.

Model-III: The present study deals with the analysis of various parameters in view of reliability for a manufacturing plant namely rice manufacturing plant, for considered conditions as well as availability during the season for the regenerating Markov model. The Laplace transformation has been used to simplify and for the analytic expressions of Availability, Reliability, MTTF and MTBF. The numerical illustrations have been carried out. The profit analysis, sensitivity analysis carried out for the considered model.

Model-IV: This model presents the performance analysis of the automatic ticket vending machine (ATVM). One can easily see long queues at the ticket counter of the railway station during train time. It is not easy to get the ticket quickly because of these long queues or sometimes due to laziness of the staff or due to other reasons. Therefore, these machines have been installed at the railway station for the passengers so that dispensation of ticket can be easily done. But frequent failures in the machine have been observed by the passengers, therefore, authors of chapter intend to analyze the performance measures of the automatic ticket vending machine Authors, have developed a model with the help of Markov model and Chapman differential equations are developed. These equations are solved using Laplace transformation and various performance indicators of reliability are calculated of the ticket vending machine. Sensitivity analysis is also performed on the system MTTF and reliability to determine the critical components of the ticket vending machine.

Model-V: The main aim of this model is to evaluate the reliability indices of the four robotic arm which work in a series connection along with one redundant robotic arm at system level using the

Markov model. Whenever any robotic arm fails the redundant robotic arm replaces the failed robotic arm immediately which reduces the downtime of the system. The system is repairable and failure and repair rate are assumed to be constant. The Markov modelling is employed to obtain the Chapman-Kolmogorov equations. Laplace transformation is used to solve the developed differential equations and obtain the transition state probabilities of the system. An explicit expression for the reliability, MTTF and MTBF are obtained in this model. Sensitivity analysis is also performed to determine the most critical components of the system. Graphs are also plotted for a better understanding of the reader.

Model-VI: The aim of this model is to analyze the performance of Wahid sugar mill (situated in Punjab, India) incorporating human error using reliability approach. The sugar mill is a complex system consisting of heavy machines (operated by human operators) which are used for the production of the sugar and other products. Human error and unplanned outages in the mill affects its availability and reliability and increases the downtime of the system which can cost a lot for the same. Keeping the above facts into consideration, a mathematical model is formulated for the same for obtaining the various reliability measures of the system like availability, reliability, MTTF and MTBF. Critical components of the sugar mill which affect its performance are determined with the help of sensitivity analysis. For analyzing the profit from this industry, a profit function is also developed.

Model-VII: The main aim of this model is to show how the Universal generating function technique can be implemented on the Excel software with great ease, which gives us the same results, which one can obtain by the algebraic multiplication of the UGF polynomials for the system components for obtaining the system performance. As without the proper knowledge of computer programming, it is not possible to apply the UGF technique for a system with a large number of components as the computation burden increases drastically as one starts implementing this technique for obtaining the system performance measures. Therefore, it is the need of the hour to have an easy way to implement the UGF technique. In this paper, authors present how the UGF technique can be implemented on the Excel software with ease, which can save an ample amount of time for the research community. An example is also given in this chapter so that one can easily implement it on the Excel software for obtaining the performance measures of the MSS.

Acknowledgement

First of all, I would like to thank the Almighty for granting perseverance. I would like to express my gratitude to Dr. Amit Kumar, Assistant professor, Department of Mathematics, Lovely Professional University, Phagwara, Punjab for his patient, guidance, and support throughout this work. I was truly very fortunate to have the opportunity to work with his as a student. I was both an honor and a privilege to work with him. He also provided help in technical writing and presentation and I found his guidance to be extremely valuable.

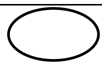
I take this opportunity to express my sincere thanks to Mr. R. P. Dubey (Manager Wahid Sugar Mill, Phagwara), Mr. Anurag Aggarwal (Ex-Engineer in Delhi Metro), Gulab Chand (Indian Railway Engineer, Jalandhar) and many others who helped me and guided me properly whenever I visited them or talked to them over phone.

I am also thankful to all teachers, and all my friends who devoted their valuable time and helped me in all possible ways towards successful completion of this work, I do not find enough words with which I can express my feeling of thanks to the all the faculty members of Department of Mathematics, Lovely Professional University, Phagwara, Punjab, for their help, inspiration and moral support which went a long way in the successful completion of my work. I thank all those who have contributed directly and indirectly to this work.

Lastly, and more importantly, I would like to thank my family for their years of unyielding love and encouragement. They have always wanted the best for me and I admire my parents and wife's determination and sacrifice to put me through Ph.D.

Signature: **Date:**

Nomenclature

	Good state
	Degraded state
	Failed state
$A(t)$	Availability of the system at time t
$R(t)$	Reliability of the system at time t
MTBF	Mean time between failure
MTTF	Mean time to first failure
MTTR	Mean time to repair
H.E.	Human error
$E_P(t)$	Expected profit
K_1 / K_2	Revenue/service per unit time
t	Time scale
s	Laplace transform variable
$P_i(t)$	Probability that the system is in state i at time t
$\overline{P}_i(s)$	Laplace transform of $P_i(t)$
$P_i(x,y,t)$	Probability that the system is in state i at time t and has elapsed repair time y and has elapsed failure time x
$\overline{P}_i(x,y,s)$	Laplace transform of $P_i(x,y,t)$
$P_{up}(t)$	System upstate probability
$P_{down}(t)$	System downstate probability
UGF	Universal generating function

MSS	Multi-state system
ATVM	Automatic ticket vending machine
ARINC	Aeronautic radio, Inc.
AGREE	Advisory group of reliability of electronic equipment

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Chapter 1: Introduction

1. Background of the Study

The development in the field of science and technology has brought a revolution in the industries. Each industry, in this era of competition, wants to perform well in the market by introducing the quality product in the market but it also wants to minimize the cost of the production. Due to this reason, some industries have started using complex and heavy machines for fast production, so that the demand of the customers can be easily fulfilled. Some industries are using robots and automation for the timely completion of assigned tasks. The benefit of using robotics and automation are many and one of them is they work without tiring. These machines can even work with varying levels of performance. This varying level of performance is called the performance rates of the machines. For example, an electrical generator can produce electricity with varying degrees of capacity. A system which can work with varying level of performances is called a Multi-state system (MSS). These systems are very heavy (not all) and occupy a lot of space and having cost in millions of rupees. These heavy systems can be easily seen in the sugar industry, telecommunication industry, rice mill, automotive industries etc. These heavy systems may fail due to unit failure, electrical failures, mechanical failure, common cause failure, catastrophic failures, human error, overstress and many more. Some failures have minor effects and don't cause deadly harm to the system and its components. But some failures have major effects and cause deadly harm to the system, environment, and people working in the organization etc. Some of the major incidents which happened due to some major failures across India are listed below.

- Bhopal gas tragedy, 1984, the death toll was 3,787 and thousands were injured.
- Chasnala mining disaster, 1975, killed 372 miners in Dhanbad, India.
- Jaipur oil deport fire, 2009, killed 12 people and injured over 200.
- Kobra chimney collapse, 2009, at least 45 deaths were recorded.
- Bombay docks explosion, 1944, almost 1300 lives were lost.

There are many other incidents that can be found in the literature which either destroyed the organization asset or people or both. Failure of the machines/components cannot be completely avoided, as machines/components have to fail at some point of time in their life time but the effect of its failure can be mitigated, so that the cost of destruction can be minimized in the plants/mills. Therefore, Reliability engineering is the need of the hour. Reliability engineering provides the solutions to reduce production losses, improvement in the design of the product, and reduce the cost of the high-cost assets. In reliability engineering, the major task is to reduce the downtime of the system so that the system remains available for a longer duration. This can be achieved by proper maintenance strategies and timely inspection of the machines/pieces of equipment. Although, this costs the organization yet it saves a lot of money for the organization and increases the uptime of the system which increases the production of the organization.

1.1 Necessity of Reliable Machines/ Equipment

In these days, when new machines/technologies are entering into the market daily, each one is better than the other in one respect or the other. If these machines/technologies work properly for a given period of time then it is assumed as fully reliable but if they don’t work properly it is called unreliable. Unreliable products of the company completely spoil the reputation of the company. The users of the product these days put their remarks online where every other customer can check the review of the performance of the product.

An unreliable service can cause accidents while traveling. For example, any major faulty component of the train can cause its accident with the other train which may take the lives of hundreds of people. In satellite launching, any fault in the rocket components may cause the loss of billions of rupees and may have deadly effects on the atmosphere.

In the above examples, it is quite clear that unreliable products can cause harm to an individual or to the society or to the environment/plant. Therefore, product reliability is having the utmost importance in system engineering. Reliable devices don’t often quickly fail for a prescribed period of time if they are operated under specified conditions. As failure occurs less frequently in reliable devices, the cost of repair/replacement also remains less for these components.

1.2 Key Definitions

1.2.1 Reliability

Reliability of a product or device is defined as the probability that the device will perform its intended task for a specified period of time under specified operating conditions. Different authors have given different definitions of reliability. Some of them are given below:

- Reduction of things gone wrong.
- Reliability is that ability of the item to stay useful.
- Reliability is the quality over time.

The definition of reliability come out with the following key point.

- The reliability of a device is expressed as a probability. It is actually the function of the time period.
- The device is required to give adequate performance i.e. it is expected from the device to perform its task without failure.
- The duration of the adequate performance is specified i.e. it cannot fail before the time period *t*.
- The environmental or operating conditions are prescribed. It implies that the device must be operated in the prescribed conditions only.

1.2.2 The Reliability Function

Quantitatively, the reliability of a machine is the likelihood that the machine cannot fail before the time period *t* is given by [6]. Define a continuous random variable *T* as the time to the failure of the product, then mathematically it is represented by:

$$R(t) = P(T > t) \tag{1.1}$$

The equation (1.1) implies that the device cannot fail before the time *t*. Here *R(t)* always satisfies the following conditions:

- $R(0)=1$ i.e., Reliability at time $t=0$ is one, as initially the device is as good as new.
- $R(\infty)=0$ i.e., Reliability at time $t\to\infty$ is zero, as no device can work forever.
- $0\le R(t)\le 1$ i.e., Reliability is the non-increasing function of time.

1.2.3 Cumulative Distribution Function (CDF)

The cumulative distribution is defined as the probability that the device can fail before time *t*, Define a continuous random variable *T* as the time to the failure of the product, then mathematically, it is represented by:

$$F(t)=P(T\le t) \tag{1.2}$$

The equation (1.2) implies that the device can fail before the time period *t*. Unreliability always satisfies the following conditions:

- $F(0)=0$ i.e., initially no failure has been observed.
- $F(\infty)=1$ i.e., no component will survive in the long run.
- $0\le F(t)\le 1$ i.e., CDF is the non-decreasing function of time.
- $R(t)+F(t)=1$ i.e., sum of the reliability and cumulative distribution function is unity.

1.2.4 Probability Density Function

When we differentiate the cumulative distribution function w.r.t. time, one can get the following expression

$$f(t)=\frac{d}{dt}\big(F(t)\big)=\frac{d}{dt}\big(1-R(t)\big)=-\frac{d}{dt}\big(R(t)\big) \tag{1.3}$$

This function describes the distribution of the failure of the components. A probability density function always satisfies the following properties.

$$f(t) \geq 0 \tag{1.4a}$$

$$\int_0^{\infty} f(t)\,dt = 1 \tag{1.4b}$$

Reliability using the probability function can be expressed as

$$R(t) = \int_t^{\infty} f(t)\,dt \quad . \tag{1.5}$$

Similarly, the CDF function can also be expressed using the pdf as given below

$$F(t) = \int_0^{t} f(t)\,dt \quad . \tag{1.6}$$

1.2.5 Failure Rate

The failure rate is defined as the ratio of the probability density function to the reliability function. Mathematically, it is represented by

$$\lambda(t) = \frac{f(t)}{R(t)} \tag{1.7}$$

This failure rate $\lambda(t)$ can be an increasing function or decreasing function or it may be constant.

1.2.6 Bathtub Curve

The description of the product's life cycle can be easily explained with the help of the bathtub curve. The bathtub curve is divided into three stages as shown in the following diagram. Initially, when the product is in Stage (I), in this stage the rate of failure of the product is high and it starts decreasing with time. It is also called the infant mortality stage. It occurs due to the design problem of the product, bad assembling of the product, manufacturing defects, and installation problems. These problems can be resolved during the design phase of the product. Stage (II) of the product is called the useful life period of the product. In this stage, the rate of failure of the product is almost constant and only random failures occur in this period of the life of the product. After this, Stage (III) of the product starts. In this stage, the failure rate of the product starts increasing due to aging. This is also called wear-out failures.

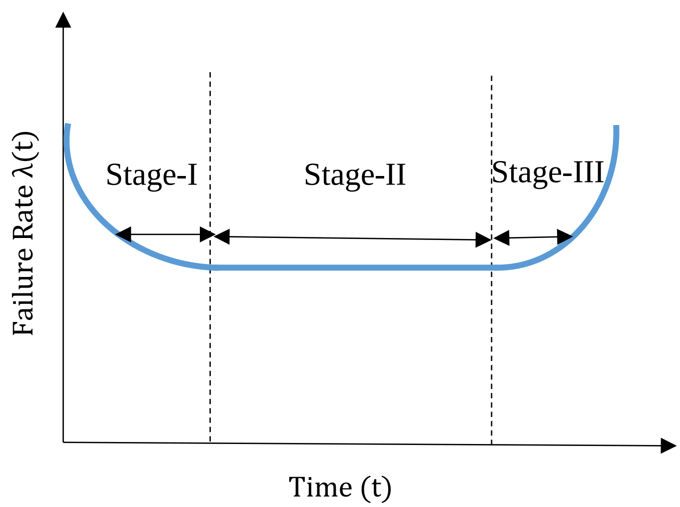


Figure 1.1: Bathtub Curve

1.2.7 Mean Time to Failure (MTTF)

A non-repairable item is one that is immediately removed from the population once it fails. For calculating the mean time to failure a large sample from the population is collected, then the failure of each component is observed and recorded for a specified time duration *T*, then the mean of these values is calculated. The smaller the value of the MTTF, it indicates that the product is not much reliable and vice-versa.

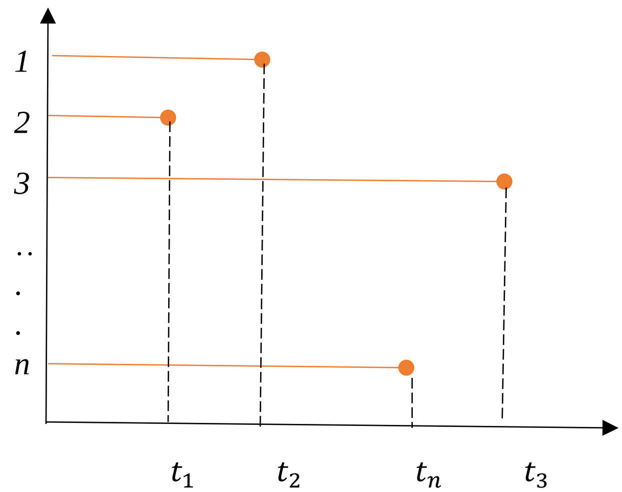


Figure 1.2: Diagram of Failure of Components

Hence, the MTTF can be calculated as given below:

$$MTTF = \frac{t_1 + t_2 + ... + t_n}{n} \tag{1.8}$$

When the system reliability is already known then MTTF will be obtained as follows.

$$MTTF = \int_0^\infty R(t)\, dt \tag{1.9}$$

1.2.8 Mean Time to Repair (MTTR)

The MTTR is defined as the total time taken to overhaul a component divided by the total number of repairs. Mathematically, it can be expressed as,

$$\text{MTTR} = \frac{\text{Total time taken to repair components}}{\text{Total no. of repairs}} \tag{1.10}$$

It is expected that the mean time to repair to be less. For that many companies purchase the spare part of the product so that the timely repair can be done without any delay. It is a measure of maintainability.

1.2.9 Mean Time between Failures (MTBF)

A repairable system is one that can be restored to the working condition by repair. For a non-repairable system, MTTF is calculated and for a repairable system, MTBF is calculated. MTBF is the basic measure of reliability. It is more important for industry and integrators rather than customers [119]. Mathematically, MTBF is outlined as quantitative relation of the total of time of the operation between two failures divided by the amount of failures.

$$\text{MTBF} = \frac{\sum \text{Time between two failures}}{\text{Total no. of failures}} \tag{1.11}$$

MTBF can also be exemplified with the aid of the following Figure 1.3.

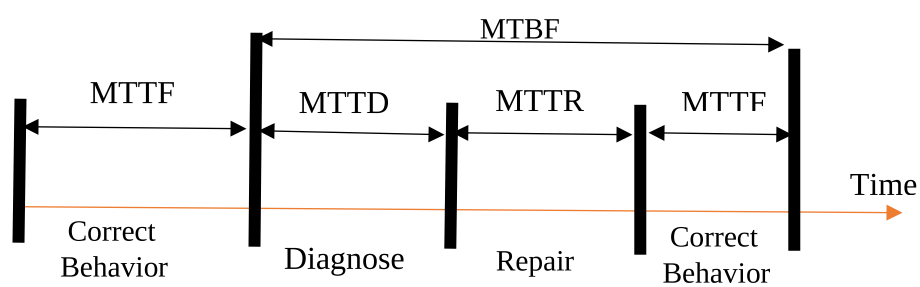


Figure 1.3: A Schematic Diagram of MTTF, MTTR, and MTTD

In this diagram, it is quite clear that initially the product is new and perform its task for some specific time and when it fails for the first time it represents the time to the first failure of the component after this repair action starts, the repairman diagnoses the faulty component. Once diagnose of the component is completed, the repairman repairs or replaces the faulty component after that the component starts its working. Again, after working for some time period the component again fails and this process repeats again.

Therefore, we can say that

$$\text{MTBF} = \text{MTTD} + \text{MTTR} + \text{MTTF} \tag{1.12}$$

In the [7] it has been observed that sometimes the diagnose time is considered to be zero. Hence, (1.12) reduces to

$$\text{MTBF} = \text{MTTR} + \text{MTTF} \tag{1.13}$$

1.2.10 Availability

Availability is another major measure of the system performance. Availability is the probability that the system will perform its intended task when maintained and operated in the prescribed time. Mathematically, it can be expressed as

$$A(t) = \frac{\text{uptime}}{\text{uptime} + \text{downtime}} \tag{1.14}$$

Mainly, in the literature, the use of the point availability and steady-state availability has been observed.

(a) Point availability: Availability is the likelihood that the system will accomplish its planned task at a particular point of time when maintained and run in the prescribed condition. It is denoted by $A(t)$. Clearly point availability is the function of time.

(b) Steady state availability: The steady-state is the stable availability of the system. Mathematically, it is denoted by A and defined as

$$A = \lim_{t\to\infty} A(t) \tag{1.15}$$

Initially, availability may be unstable due to a number of reasons like training/learning issues, decision on the number of repairmen required, lack of good spare parts etc. But as time passes it stabilize at a point. This can be seen in the following Figure 1.4.

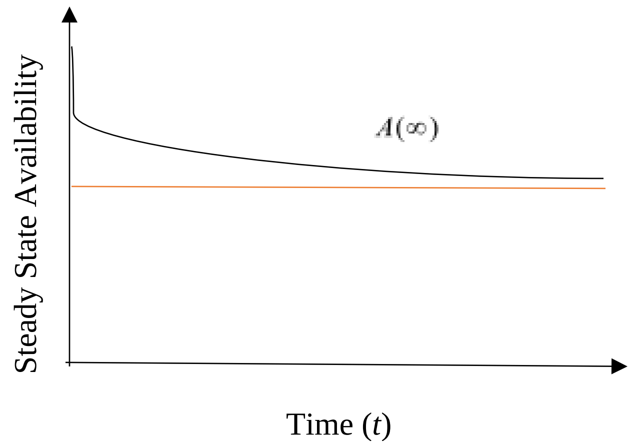


Figure 1.4: Steady State Availability

1.2.10.1 Computation of Availability in Series Configuration

If, in a system, there are n different elements connected in a series configuration, and each element has availability $A_i, 1 \le i \le n,$ respectively, then the availability of the whole system is obtained using the following formula:

$$A_{System} = A_1 \times A_2 \times \ldots \times A_n \tag{1.16}$$

Equation (1.16), gives the availability of the whole system, which is apparently less than the availability of the weakest element in the system. If all the components are identical and have the availability equal to A then the availability of the whole system using equation (1.16) reduces to

$$A_{System} = A^n \tag{1.17}$$

1.2.10.2 Computation of Availability in Parallel Configuration

If, in a system there are n different elements connected in a parallel configuration, and each element has availability $A_i, 1 \le i \le n$ respectively, then the availability of the whole system is obtained using the following formula:

$$A_{System} = 1 - (1 - A_1) \times (1 - A_2) \times \ldots \times (1 - A_n) \tag{1.18}$$

Equation (1.18), gives the availability of the whole system, which is apparently more than the availability of the strongest element in the system. If all the elements are identical and have the availability equal to A, then the availability of the whole system using equation (1.18) reduces to

$$A_{System} = 1-(1-A)^n \tag{1.19}$$

1.2.11 Important Analysis or Sensitivity Analysis of the System

Important analysis or the sensitivity analysis is also called the what-if analysis. It actually, helps to determine what impact of increasing or decreasing the failure rate of a single element has on the system performance. Sensitivity analysis can be performed for the reliability measures like reliability, MTTF. It helps to identify that variation in the failure rate of which component effect the system performance the most.

1.2.12 Mean Downtime

The average time for which the system was unavailable for its use is known as the mean downtime of the system. The reason for the downtime could be administrative delay, preventive or corrective maintenance, self-imposed delay, failures of components etc.

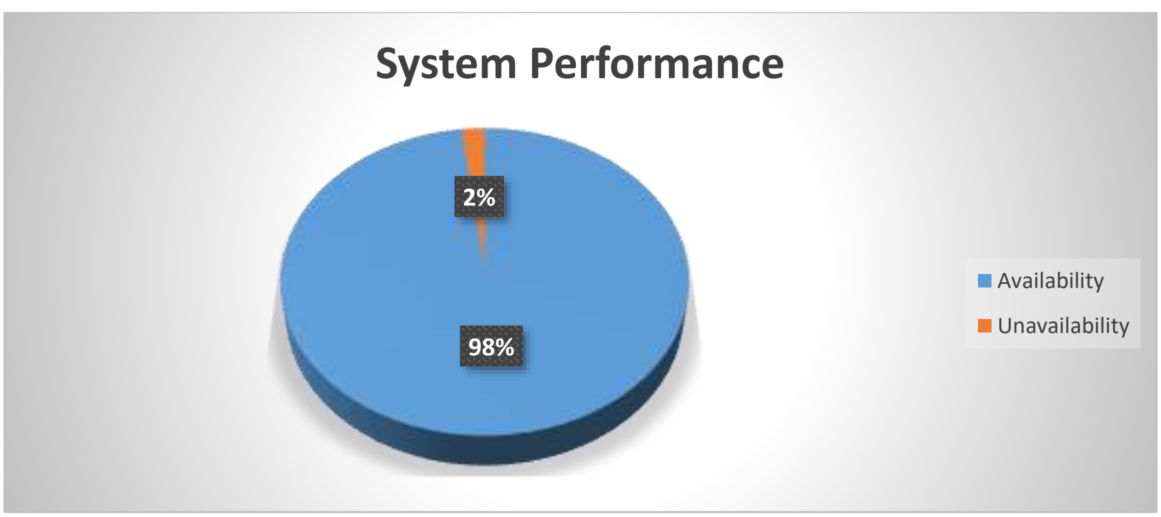


Figure 1.5: System's Availability and Unavailability

From Figure 1.5, it is quite clear that the system availability is 98% and system unavailability is just 2%.

1.2.13 Exponential Distribution

Let X be the continuous random variable, then $X \sim \exp(\lambda)$, if its probability density function is expressed by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{1.20}$$

Where λ is the parameter of the distribution and $\lambda \geq 0$

Majority of electrical components follow exponential distribution. Therefore, a component which follows an exponential distribution, then the reliability is represented by:

$$R(t) = e^{-\lambda t} \tag{1.21}$$

The expression for the MTTF is given by:

$$\text{MTTF} = \frac{1}{\lambda} \tag{1.22}$$

1.3 System's Various Configuration

Every system is composed of various components or subsystems. Each component of the system is the smallest entity which is not further subdivided. Each component performance has a significant contribution in the performance determination of the whole system. So it plays a vital role that how they are interconnected in the system. There are various configurations of the system like series, parallel, mixed/hybrid. In this section, we will discuss each of these configuration one by one.

(i) Series Configuration

The components in the system are said to be connected in the series configuration, if the whole system fails due to the failure of at least one component of the system. The system works as long as all the components of the system remain operational. Suppose that in a system there are n independent components. Let $E_i, 1 \le i \le n,$ denotes the event that i^{th} component is operational. Therefore, $P(E_i), 1 \le i \le n,$ respectively represents the probability that i^{th} component is operational. Hence, the likelihood of the complete system is functioning is given by:

$$P(E) = P(E_1) \times P(E_2) \times ... \times P(E_n) \tag{1.23}$$

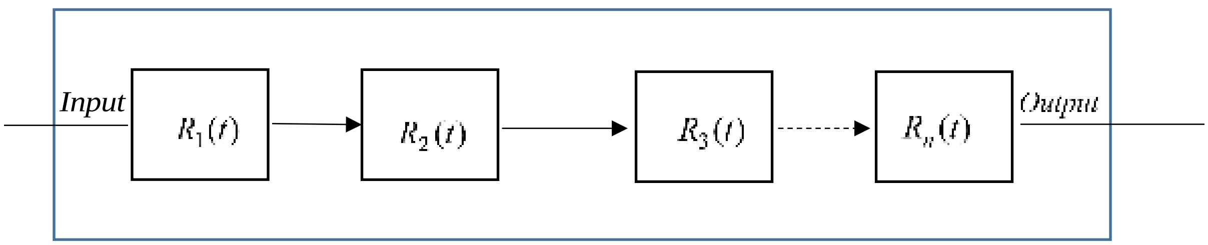


Figure 1.6: Series Configuration

If we replace each $P(E_i), 1 \le i \le n,$ by $R_i(t), 1 \le i \le n$, then the equation (1.23) reduces to

$$R(t) = R_1(t) \times R_2(t) \times ... \times R_n(t) \tag{1.24}$$

Further, assume that each component has failure rate $\lambda_i, 1 \le i \le n$, then using the equation (1.21) in (1.24), we get,

$$R(t) = e^{-\lambda_1 t} \times e^{-\lambda_2 t} \times ... \times e^{-\lambda_n t}$$

$$R(t) = e^{-(\lambda_1 + \lambda_2 + ... + \lambda_n)t} \tag{1.25}$$

The equation (1.25) represents the reliability expression of the series system.

(ii) Parallel Configuration

The components in the system are said to be connected in the parallel configuration, if the whole system fails only when every component of the system has failed. The system works as long as at least one component of the system remains operational. Suppose that in a system there are n independent components. Let $E_i, 1 \le i \le n,$ denotes the event that i^{th} component is operational. Therefore, $P(E_i), 1 \le i \le n,$ represents the probability that i^{th} component is operational. Hence, the probability the whole system is working is given by:

$$P(E) = 1 - \left((1 - P(E_1)) \times (1 - P(E_2)) \times ... \times (1 - P(E_n))\right) \qquad (1.26)$$

If we replace each, $P(E_i), 1 \le i \le n,$ by, $R_i(t), 1 \le i \le n,$ then the equation (1.26) reduces to

$$R(t) = 1 - \left((1 - R_1(t)) \times (1 - R_2(t)) \times ... \times (1 - R_n(t))\right) \qquad (1.27)$$

Further, assume that each component has failure rate $\lambda_i, 1 \le i \le n,$ then using the

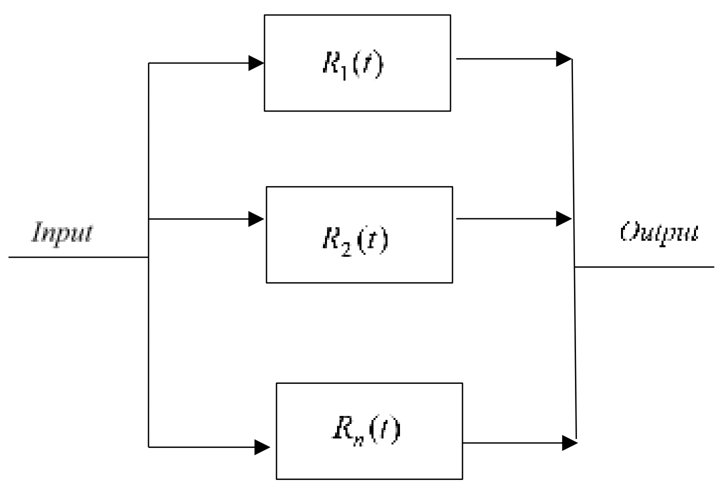


Figure 1.7: Parallel Configuration

Using equation (1.21) in (1.27), we get,

$$R(t) = 1 - \left((1 - e^{-\lambda_1 t}) \times (1 - e^{-\lambda_2 t}) \times \ldots \times (1 - e^{-\lambda_n t})\right) \qquad (1.28)$$

The equation (1.28) represents the reliability expression of the parallel system.

(iii) Mixed/Hybrid Configuration

A system which contains its components in both series as well as in parallel configuration is known as a mixed or hybrid configuration system. The following Figure 1.8 represents a mixed configuration system. It is having six components which are connected in mixed configuration. The system reliability can be calculated by breaking the network into parallel and series configuration. The reliability computation process is also given below.

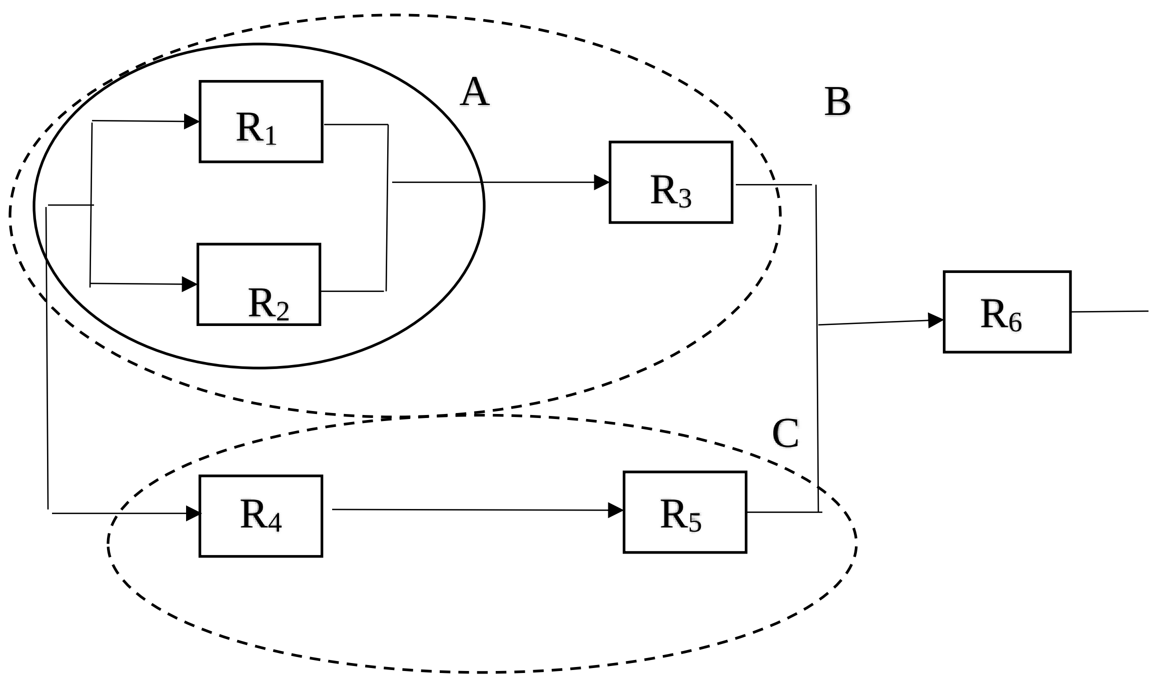


Figure 1.8: Parallel/Series Configuration of Six Components

$$R_A = \left[1 - (1 - R_1)(1 - R_2)\right]$$

$$R_B = R_A \, R_3$$

$$R_C = R_4 \, R_5$$

Since R_B and R_C are in parallel with one another and in series with R_6. Hence the system reliability is given by

$$R_S = [1-(1-R_B)(1-R_C)]R_6 \qquad (1.29)$$

1.4 Redundancy

To improve the reliability and system's uptime the best possible solutions are to use the components of high reliability or to use the redundancy in the system. After achieving certain reliability, it is a very costly affair to improve the reliability of the component further. Therefore, redundancy is the best solution. One can introduce redundancy at the system level or at the component level. There are different types of redundancy like *k-out-of-n:* F/G system, cold/hot/warm standby redundancy, series/parallel redundancy and many more. We will discuss all these redundancies one by one.

(i) ***k-out-of-*n: F** : In this, the system has n active components although we require only k components to be operational for the system to remain operational. On the failure of at least k components the entire system will fail.

(ii) ***k-out-of-n*: G** : In this, the system has *n* active components although we require only k components for the system to remain operational. The whole system works when at least k components work.

The important note here is that a *k-out-of-n*: G system is same as *(n-k+1)-out-of*-n: F system.

These two systems can be easily found in the various industries and defense. The two examples of this system are given below

- A lift shaft has four cables to pull the lift out of which the system requires at least 2 for the safe operation. This is an example of *2-out-of*-4: G system.
- An automobile engine has 8 cylinders out of which the engine requires just 6 to successfully run. This is an example of *6-out-of*-8: G system.

(iii) **Hot redundancy**: In hot standby redundancy, one standby device remains active with the primary device. On the failure of the main unit, the standby unit takes the whole load and the time to switchover is almost negligible. The failure rate of the standby unit and the main unit both remain the same in this redundancy. The standby unit may fail before it is put in use. The hot redundancy is generally installed very near to the primary unit, it could be in the same building, or within the same city.

(iv) **Warm redundancy**: In warm standby redundancy, one standby unit takes the whole load when the primary device fails. The failure rate of the warm stand by unit remains less than the failure rate of the main component. This device too can fail when it is not in use.

(v) **Standby redundancy**: In the case of standby redundancy, the redundant unit remains inactive until the main unit fails. When the main unit fails the standby unit takes over the whole load. The time taken to take over the device is more as compared to warm redundancy.

1.5 Various Failures in the System

Failure is the phenomena which leads a system in a failed/degraded state. In literature different failure can be found, some of them are internal, external, natural or unnatural, mechanical or electrical. In this section, we discuss some of the failures due to which the whole system can fail/degrade.

- **Human error/failure:** A failure which can occur due to a human operator is termed as human error or human failure. As in India due to the economic crisis, it is not possible to use capital intensive techniques. Therefore, many organizations employ different workers for production or for other activities. If the worker who is not trained is given the duty to operate the machine, then system failures are in 99% cases are bound to happen. Sometimes workers may be trained, but due to the negligence of the worker system may fail.
- **Catastrophic failure:** Due to natural calamities like, earthquakes, volcanoes, costly assets can fail. This has a very bad effect on the economic condition of the

plant or mill. These risks cannot be stopped but can be mitigated with the proper planning of the building design.

- **Switch failures**: When one device works in a standby mode then on the failure of the main device the switching device is just used to start the standby component. On the failure of the main system if the switching device fails then the activation of the standby system becomes impossible.
- **Mechanical/Electrical failures**: Other vital failures are mechanical and electrical failures. These failures occur due to the overage, stress, cracks, explosion in the system.
- **Design failures**: The design of the system is a significant factor in the life cycle of the component. Due to bad configuration of the components, bad quality of the components of the system, the wrong casting of the parts design the system may fail.

1.6 Stochastic Process

A stochastic process is the collection of random variables. It is denoted by $[X_n(t)]_{n\in T}$ where T is the indexed set. This set is the general subset of the real line. Each random variable $X_n(t)$ takes its values from a mathematical space know as state-space. Many types of stochastic processes are found in the literature. Based on the classification, the stochastic processes are divided into four categories.

- Discrete time discrete space
- Discrete time continuous space
- Continuous time discrete space
- Continuous time continuous space

1.6.1 Markov Process

Markov process is distinct type of state based process in which the transition of the future state just depends only on the present states, but not on the past states. Due to this property, it is also known as memory less process of the Markov process. These days, many researchers are successfully using Markov process to model the system behavior. Once the model is developed, various measures of the system reliability like availability, reliability, MTTF of the system can be determined. Here, we present the general Markov model for the non-repairable and repairable system.

1.6.1.1 Markov Model for a Non-Repairable System

Consider a single element non-repairable system. Initially, the system is as good as a new one. In the diagram below, S_0 represents the perfect working state of the system. Let λ be the failure intensity of the component. When the component fails, the system reaches in the state S_1. The below Figure 1.9 clearly demonstrate it.

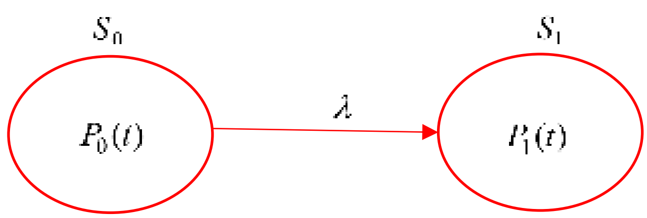


Figure 1.9: Markov Model for Non-Repairable System

From the above model the system of differential equation can be developed as follows:

$$\frac{d(P_0(t))}{dt} = -\lambda P_0(t) \tag{1.30}$$

With initial condition;

$$P_0(0) = 1 \tag{1.31}$$

On solving the above system of equation, we get,

$$P_0(t) = e^{-\lambda t} \tag{1.32}$$

As we know that sum of the probability is equal to unity, therefore,

$$P_0(t) + P_1(t) = 1$$

$$P_1(t) = 1 - P_0(t)$$

$$P_1(t) = 1 - e^{-\lambda t} \tag{1.33}$$

1.6.1.2 Markov Model for a Repairable System

Consider a single element repairable system. Initially, the system is as good as a new one. In the diagram given below, the state S_0 represents the perfect working state of the system. Let λ be the failure intensity of the component. When component fails system reaches in the state S_1, then the system is repaired/ replaced with the good component and it is again brought back in the state S_0 with the repair intensity μ. Below Figure 1.10 clearly demonstrate it.

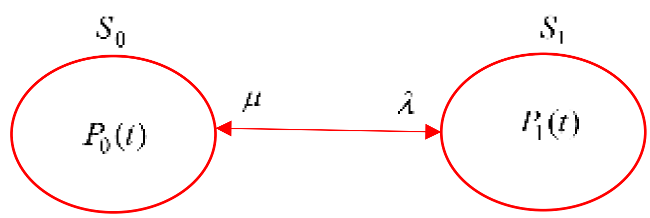


Figure 1.10: Markov Model for Repairable System

From the above model the system of differential equation can be developed as follows:

$$\frac{d(P_0(t))}{dt} = -\lambda P_0(t) + \mu P_1(t) \tag{1.34}$$

$$\frac{d(P_1(t))}{dt} = \lambda P_0(t) - \mu\ P_1(t) \tag{1.35}$$

With initial conditions;

$$P_0(0) = 0,\ P_1(0) = 0 \tag{1.36}$$

On solving the above system of equation, we get,

$$P_0(t) = \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} + \frac{\mu}{\lambda+\mu} \tag{1.37}$$

$$P_1(t) = \frac{-\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} + \frac{\lambda}{\lambda+\mu} \tag{1.38}$$

1.7 Laplace Transformation

Let $f(t)$ be defined for all $t \geq 0$, then the Laplace transformation of $f(t)$ is represented by $L(f(t))$ and given by:

$$L(f(t)) = \int_0^\infty e^{-st} f(t)dt \tag{1.39}$$

Provided the integral exits, is called the Laplace Transform of $f(t)$. It is denotes as

$$L(f(t)) = F(s) = \int_0^\infty e^{-st} f(t)dt \tag{1.39a}$$

1.8 Supplementary Variable Technique

Supplementary variable technique plays a very important role in the stochastic process. It was first of all used by the Cox [8].in 1955. Later, it was used by the Graver [9] in 1963. The benefit of using supplementary variable technique is that it converts the non-Markovian behavior of the system into the Markovian.

1.9 Universal Generating Function

The problem of using the Markov model is that it suffers from state explosion. Even in the smallest system, the number of system states increases very drastically. Sometimes, it becomes very difficult to write all the system states and skipping the states may give the wrong conclusion about the system performance. To overcome this situation, a new technique was introduced by Ushakov [16] in 1986. This technique is based on the algebraic multiplication of the polynomials. In this techniques, the system states reduce drastically. In the UGF technique, each state of the system has various performance rates. Let us assume that there be n components in the system and each component has the various performance level represented by the set $g_j = \{g_{j1}, g_{j2}, ..., g_{jk_j}\}$ where $j \in \{1, 2, ..., n\}$, where g_{ji} represent the performance level of j^{th} component in the i^{th} state. Then the UGF of the j^{th} component of the system can be represented as

$$u_j(z) = \sum_{i=1}^{k_j} p_{ji} z^{g_{ji}} \tag{1.41}$$

Where z is a dummy variable.

In general, if there are n UGF functions then the composition operator is used as given below.

$$\Omega_f\left(u_1(z), u_2(z), ..., u_n(z)\right) = \Omega_f\left(\sum_{i=1}^{k_1} p_{1i} z^{g_{1i}}, \sum_{i=1}^{k_2} p_{2i} z^{g_{2i}}, ..., \sum_{i=1}^{k_n} p_{ni} z^{g_{ni}}\right)$$

$$= \left(\sum_{i=1}^{k_1}, \sum_{i=1}^{k_2}, ..., \sum_{i=1}^{k_n} p_{1i} p_{2i...} p_{ni} z^{f(g_{1i}, g_{2i}, ..., g_{ni})}\right) \tag{1.42}$$

If all the elements of the system are in a series configuration and the system is the task processing system then the function f is replaced by minimization function then the equation (1.42) takes the form

$$\Omega_{series}\left(u_1(z), u_2(z), ..., u_n(z)\right) = \Omega_{series}\left(\sum_{i=1}^{k_1} p_{1i} z^{g_{1i}}, \sum_{i=1}^{k_2} p_{2i} z^{g_{2i}}, ..., \sum_{i=1}^{k_n} p_{ni} z^{g_{ni}}\right)$$

$$= \left(\sum_{i=1}^{k_1} \sum_{i=1}^{k_2}, ..., \sum_{i=1}^{k_n} p_{1i} p_{2i...} p_{ni} z^{\min(g_{1i}, g_{2i}, ..., g_{ni})}\right) \tag{1.43}$$

and if all the elements of the system are in a parallel configuration and the system is the data transmission system then the function f is replaced by maximum function, then the equation (1.42) takes the form

$$\Omega_{parallel}\left(u_1(z),u_2(z),...,u_n(z)\right)=\Omega_{parallel}\left(\sum_{i=1}^{k_1}p_{1i}z^{g_{1i}},\sum_{i=1}^{k_2}p_{2i}z^{g_{2i}},...,\sum_{i=1}^{k_n}p_{ni}z^{g_{ni}}\right)$$

$$=\left(\sum_{i=1}^{k_1},\sum_{i=1}^{k_2},...,\sum_{i=1}^{k_n}p_{1i}\,p_{2i...}\,p_{ni}\,z^{\max(g_{1i},g_{2i},...,g_{ni})}\right) \qquad (1.44)$$

The various performance measures can be calculated for the multi-state system performance analysis like availability, loss of load probability, expected generating capacity deficiency.

1.9.1 Availability

Let w be the constant level of demand required to run the system, then the system availability is defined as the sum of the probabilities of all those states which satisfy their own demand w. Mathematically, it is represented by:

$$A_w(t)=\sum_{g_k\geq w}p_k(t) \qquad (1.45)$$

1.9.2 Loss of Load Probability

The loss of load probability can be obtained from the system's availability using the below formula:

$$LOLP_w(t)=1-A_w(t) \qquad (1.46)$$

1.9.3 Mean Expected Generating Capacity Deficiency

The mean expected generating deficiency is defined as the average deficiency of the system and it is given by:

$$D = \sum p_k (w - g_k) I_{w-g_k} \tag{1.47}$$

Where I_{w-g_k} is an indicator function defined by

$$I_{w-g_k} = \begin{cases} 1 & if \;\; w - g_k \geq 0 \\ 0 & if \;\; w - g_k < 0. \end{cases}$$

Chapter 2: Literature Review

In this chapter, we present the work of various researchers who have given their contribution in the field of reliability theory. Reliability engineering is the branch of engineering which deals with system design, installations, performance, and maintenance and also with its improvement. It also deals with the recognition of the failure pattern of the system and its components and tries to mitigate the failures of the system or to delay the failure. Various theories, algorithms, models have been developed by the researchers to enhance the performance of the many systems.

Initially, the use of reliability theory was found in the insurance company. Particularly, it was used for human survival probabilities. Around 1930s and 1940s, Weibull discovered that things fail due to fatigue and stress. He introduced a distribution after his name "Weibull-distribution". The detailed information about Weibull distribution is available in [4]. Also, in this time period, new theories like queuing theory and renewal theories were developed. The use of exponential distribution provided a good mathematical foundation to the reliability theory. During the world war, the theory of reliability became the most significant subject. In the war, very complex electronic components were used and it was observed that their failure rates were quite high. Vacuum tubes were found to be very unreliable. After the war, to improve the reliability of airborne electronic components one organization ARINC (Aeronautic Radio, Inc.) was established.

Around 1950, U.S Air Force formed the ad-hoc group to improve the reliability of general equipment. In 1952, the defense department established an advisory group on the reliability of electronic equipment (AGREE). In 1970, for the reliability assessment of the system fault tree was developed. Then, around 1980, the Air force introduced a reliability and maintainability program whose objective was to increase the system availability and readiness and reduce the maintenance cost and life-cycle cost through increased reliability and maintainability. After 2000, many researchers made their significant contribution to the advancement of the field. This detail is available in [1], [5].

2.1 Binary State System

In the early theories, structure and its components were considered to have only two possible states, namely perfectly operational and completely abortive. Many authors including [15], [18], [20], [21], [57] worked with these kind of systems and determined the mathematical expression to calculate the various reliability measures for the same. Actually, this does not give us the exact reliability of the systems. In real life, systems are very large and complex. Therefore, now we present in this thesis what do we mean by large and complex systems.

2.2 Complex System and Large System

In these days, industries use very large and complex systems so that production could be done very fast in less time to meet the demand of the market. Hwang et al. [13] explained clearly that what a complex system is. According to him "A complex system is a system that cannot be reduced to a series-parallel system". He also explained that if a system consists of 3 to 5 components then it is called a small system and if consists of 6 to 10 components then it is called a moderate system. If the system has more than 10 components, then it is called a large system. He presented that which technique is suitable for which type of system. He also concluded that there is not a single technique that is suitable for all types of systems. Another type of complex system is the power system. The power system is mainly divided into three parts i.e., generation, transmission and distribution. Theories related to power system reliability are given by Billinton and Allan [19]. Ram and Kumar [59] analyzed the performance of a two-unit complex system out of which one unit works under 2-out-of-3: *F* strategy. The second unit is a single unit. The system is operated by the human. The system fails when the two components of the first unit fail or the second unit fails or due to human error. Markov modeling was utilized for modeling the considered system. The system's performance measures like availableness, reliability and MTTF were calculated. System profit was also computed. In their research, they found that human error has a very less impact on the system's MTTF whereas the third component of subsystem "A" has more influence on the system's MTTF. It is the demand of society that new technologies should be more reliable. Therefore, reliability engineering plays important role in every field. Ram [50] discussed various approaches of reliability in engineering and physical sciences. One of the most common topics in the reliability theory is to identify the critical components of the system. Many researchers have given various methods to determine these critical components. Amrutkar and Kamalja [92] presented an overview of various important measures of system reliability for finding the most critical components of the system. After the identification of the failed component, it is repaired or replaced. In the literature, there are different types of repair and maintenance strategies were offered by e.g. [10], [12], [14], [17], [32] to improve the performance of the system.

2.3 Literature Based on System Redundancy

To increase the system's consistency and readiness and to delay the failures of the system, the redundancy technique can be used. This redundancy technique could be used at the system level or at the component level. There are mainly two types of redundancy which are found in the literature namely active redundancy and standby redundancy. Detail of these redundancies is available in [3]. In active redundancy, all the units remain active and all share the equal load of the system. In standby redundancies, the standby unit only becomes functional when the main unit fails and the whole load is transferred to the standby unit. Ebeling [25] in his book "An introduction to reliability and maintainability engineering" explained hot, warm and cold standby redundancies. The basic difference among these three are: Hot redundancy component may fail even when it is not in operational mode. Warm redundancy may fail even when it is not in operational mode. But its failure probability is smaller than the failure probability of the main unit. Cold standby redundancy cannot fail unless it is in operational mode. It becomes functional only on the failure of the main unit. Ardakan and Hamadani [62] used the concept of mixed redundancy. With a mixed redundancy policy, we mean a system that contains both active and standby redundancy. In this kind of system, every system can have a different number of cold-standby and active redundancy characterized by n_{A_i} and n_{S_i}. Authors determined values of n_{A_i} and n_{S_i} to optimize the reliability of the system. Li [82] presented the comparison between active and standby redundancy. The advantages and disadvantages of both types of redundancies were discussed. For both the redundancies, Markov-modeling was utilized. System reliability and mean time to failure were compared for both types of redundancies. Reliability and MTFS of the parallel-series system and series-parallel system for the arbitrary values of failure rates and operating time period were obtained by Chauhan and Malik [83]. This parallel-series system was represented by the pair (*m*, *n*). By taking specific values of *m* and *n* various results were obtained. It was found the system reliability increases as the number of components in the subsystem are increased and decreases when the number of subsystems are increased.

2.4 Literature Based on *k-out-of-n* Redundancy

The *k-out-of-n* is the most commonly used redundancy in the industries. These are divided into two categories namely *k-out-of-n*: *F* and *k-out-of-n*: *G*. A *k-out-of-n*: *F* implies that the system fails when at least *k* components of the system fail and *k-out-of-n*: *G* implies that the system works if at least *k* components are working. For a better understanding and learning various methods and algorithms, one can refer Kuo and Zuo [28]. Krishnamoorthy and Ushakumari [26] investigated a *k-out-of-n*: *G* system with D-policy for the repair. Under this policy when the workload surpasses a threshold limit D a server is instantly called for repair and starts repair without wasting time. Arulmozhi [27] developed a closed-form equation of a *M-out-of-N* warn standby system with *R* repairmen. Author also showed that under certain restrictions the system reduces to *M-out-of-N* warn standby system with no repairman. Barron et al. [31] proposed an algorithmic approach for an *R-out-of-n* system with several repairmen. The system fails whenever there are only *(R-1)* good components in the system. Expression of point availability, limiting availability were derived. Also, the distribution of system uptime and downtime were determined. Mishra and Jain [53] considered a main *k-out-of-n*: *F* system and a secondary subsystem having the general repair distribution. The main subsystem shut off the secondary subsystem when more than *k* units of the main system fail but if the secondary unit fails then it doesn't have any effect on the working of the main unit. The life distribution of the system follows a negative exponential distribution. The system's steady-state availability with single repairmen was determined by the author. Wu et al. [64] analyzed a *k-out-of-n: G* repairable system with a single repairman who takes a single vacation. In a real-world situation, the repairman can perform other jobs in his idle time. The author determined how long should be the vacation time without affecting the repairman's primary work should be considerable. Yuan and Xu [39] analyzed the performance of a deteriorating system with a repairman who can take multiple vacations. Taghipour and Kassaei [69] analyzed a *k-out-of-n* load-sharing system. On the failure of one component its load is equally divided to the (*n*-1) components. The author developed a model to find the optimal inspection interval for such a system, which reduces the total expected cost incurred over the system's lifetime. Haggag [71] studied a *3-out-of-4*: system involving four types of failures and preventive maintenance. He determined the explicit expression of availability, steady-state availability and reliability. He also carried out the profit analysis of the system. He found that the system gives good performance if a preventive maintenance strategy is used for the system. Grida et al. [95] compared the steady-state availability of a *3-out-of-4*: cold standby system and *6-out-of-8* system. He showed that for low repair-failure ratio *3-out-of-4*: cold standby system performs better than *6-out-of-8* system and for high repair-failure ratio *6-out-of-8* system performs better than *3-out-of-4*: cold standby system. Li [75] also introduced the concept of dormant failure in the system. Dormant failures are those failures that cannot be detected when they occur in the system. These failures can be detected with scheduled periodic inspection, test or maintenance activity. He introduced a methodology to calculate the MTBF of the dormant *k*-out-of-*n* system

2.5 Literature Based on Markov Modeling

The Markov model is widely used for the determination of the reliability indices of an industrial system. This model is a state-based model, in which future states of the system depends only on the present state. In order to use this model, one must have prior knowledge of the transition rates of each state and the probability of each state. A basic and very good explanation of this model is given in [2], [25]. Billinton and Allan [19] also showed the use of the Markov model in the determination of the power system reliability. Kalaiarasi et al. [93] presented a 4-unit system. Markov modeling was used to determine the system reliability, failure probability and mean time to system failure. It was found in the research when the time increases the reliability decreases and the failure rate also increases. Niwas and Garg [105] presented a single unit system that works under rest policy so that the failure of the system can be delayed. This system goes for a rest after operating for some time period. After taking complete rest, system starts its working again. When this system fails, it goes to a repairman who completely analyzes it. If the repairman declares that this system failed not due to the negligence of the user, the system is replaced or repaired by the manufacture otherwise its cost of repair is borne by the user. This model was described with the help of Markov model and system's various performance measures like MTSF, availability and expected profit were calculated. Yusuf et al. [107] analyzed the performance of a single host with three types of heterogeneous software with the help of Markov modeling. Initially, one software is used but when it fails other software of the same type is used. In this way, the failure of software doesn't cause the system to fail. This helps in extending the system's availability and the system becomes more profitable. He also proposed the further extension of his work using multiple hosts with heterogeneous software for the calculating system reliability. Hence, we can say that the Markov model is a very helpful tool which analyze the system performance.

For a system having non-Markovian property can be converted to Markovian by introducing a new variable known as supplementary variable. Cox [8] introduced the concept of supplementary variable technique. Shakuntala et al. [38] presented the reliability analysis of the polytube manufacturing plant by considering the four units of the plant. They used variable repair and failure rates in the modeling of the system. State probabilities were determined using Lagrange's method. After that they considered the constant failure and repair rates and using R-K fourth-order method system reliability was determined for the various choices of the repair and failure rates. They also performed the sensitivity analysis of the subsystems to determine the critical components of the system. Kumar and

Ram [51] use the Markov modeling and supplementary variable technique for the reliability determination of the coal-management division of the thermal power station. They determined the most critical component of the system by performing the sensitivity analysis. The profit of the system was also determined. Ram and Manglik [60] presented a structure consisting of three same components in parallel. This system works as long as at least one component of the system works. Three sorts of failure namely; incomplete failure, human failure, and catastrophic failures were incorporated in the system. With the aid of Markov process theory, supplementary variable technique and Laplace transformation system important reliability measures like availability, reliability, and MTTF were determined. Sensitivity analysis and Cost analysis were also carried out in their research. Zheng et al. [101] performed the sensitivity analysis of an instantaneous computing system with one warm standby component in the presence of the common cause failure (CCF) with the help of a continuous-time Markov chain. The effect of the CCF in case of hot and warm standby redundancy was compared. It was found that CCF reduce the system reliability more in the case of hot standby redundancy. Ram and Manglik [78] applied Markov modeling for the performance analysis of the attendance monitoring system, considering four types of failures: component failure, thoughtful failure, calamitous failure and button failure. Various performance measures like reliability, availability and MTTF of the system were determined. Sensitivity and profit analysis were also performed for the biometric attendance system. Gupta and Kumar [87] developed a stochastic model based on continuous-time and discrete space for the VOIP system. Redundancy at the system level was used so that system become more reliable. Software rejuvenation was also implemented to stop software failure. Chopra and Ram [114] analyzed the performance of a parallel system that has two dissimilar units. The benefit of using two dissimilar units is that the system is cheaper than the system which has identical units (sometimes). Author's used the Gumbel-Hougaard family copula repair facility to quickly repair the system. Various performance measures of the system like availability, reliability, MTBF were determined for the system.

2.6 Literature Based on Indian Ticketing System

Daily millions of people travel on the Indian railway. For traveling from one place to another place they have to purchase the ticket from the railway station. But one can see long queues at the railway stations. Various researchers have presented their ideas to improve the railway ticketing system. Generally, for traveling long-distance journeys, passengers book their tickets using various websites or applications but for the short distance, passengers don't book the ticket. As of now, the following researchers have given their suggestions to improve the ticketing system of the Indian railway. Patel et al. [111] presented the benefits of buying an ordinary ticket online. For this user just need an internet connection. If buying the ordinary tickets becomes online, passengers can easily purchase the ticket from anywhere and the change is not required to purchase the ticket as the fare is directly deducted from the customer bank account. But this research has a few drawbacks also as the internet connection is not easily available in the backward areas and some even don't have a smartphone and some even don't know how to operate a smart mobile phone. Chatterji and Nath [56] discussed that daily 1 million people travel in the reserved compartment and 16 million people travel in the ordinary compartment. They suggested UID-based technology, with this technology reserved tickets can also be booked using the Automatic ticket vending machine (ATVM) machine. Methews and P [65] suggested the existing ticketing system can be substituted with the smart card system. The passengers may travel using these cards and once the amount gets finished in the card they can again recharge it. They suggested this recharge in the card can be done on a monthly basis or on a quarterly basis. Khan et al. [80] proposed the SMS ticketing system. The passenger who wants to travel sends one SMS then in return he gets back a message from the system which can be checked by the ticket checker using a handheld device. This helps in saving the paper also. In this research, he didn't discuss how to book multiple tickets in just one go. Majumder et al. [54] suggested an RFID ticketing system for the passenger. This RFID card can be collected from the railway station after paying some amount. Whenever a passenger wants to purchase a ticket he touches the RFID with the card reader and the amount is deducted from the bank account. Zongjiang [43] introduced an efficient railway ticketing system in which he also introduced the additional features like ticket inquiries, cancellation of ticket and refund of the money and booking the ticket online. Mohod et al. [86] proposed the digitalized ticket checking system. This system reduces the major checking task of the T.T.E. When any passenger occupies his seat he receives a message and can easily check how many seats are unoccupied. This helps him to allot the seat to those passengers who could not book their seats in case they ask for it. Various applications have been developed to book the ticket. Venugopal and Vyas [76] presented the comparison of the various features of the application like Ixigo, Makemytrip, Cleartrip and Irctc. Each application has some advantages and disadvantages. Budhkar and Das [90] presented the trend of the selling of railway tickets. He presented that during the rush period train booking is full within 2-3 days. Generally, the booking of the train starts 60 days prior to the date of travel. This booking is closed by railway 12 hours before the travel of the train. They suggested that it is always better to book the ticket early to avoid the last-minute rush. Gundla and Vishal [91] proposed a device that is installed at the platform and railway crossing. If any obstacle is detected on the railway track, then the signal is sent to the loco pilot to slow down the train or to stop the train completely. This research is useful to save the life of the people. But it was observed that the time to detect the problem is high and it is not always possible to stop the train immediately after detecting the obstacle.

As the popularity of the ATVM is increasing among the passenger and passengers can purchase the ticket on their own, so, these machines should be capable of identifying the original currency notes. Some researchers have also given their contribution in this direction. Zeggeye and Assabie [77] proposed a hardware and software solution to identify the Ethiopian currency notes. The system was tested with genuine Ethiopian currency, counterfeit Ethiopian currency, and currency of other countries. It was found that their system gives an accuracy of up to 96.13%. Sharma et al. [47] suggested an LBP algorithm for the recognition of Indian currency in ATVM machines. This can give results up to 100% if good quality images of notes are stored in the machines. Min et al. [48] proposed a solution that can save the time of passengers if they want to purchase the ticket using ATVM machines. He proposed a specially designed sheet that can be purchased from the railway station. For purchasing the ticket from ATVM passenger has to fill up this sheet manually and after that, he just needs to enter the sheet into the ATVM which automatically reads all the information and prints the ticket. In this way, passengers need not enter any information using the ATVM.

2.7 Literature Based on Sugar Industry

Sugar Industry is a very big and complex system. There are many units in the sugar industry that are highly complex. This industry produces a large amount of sugar which is ultimately supplied in the market. Failures of the main components of the sugar industry may cause a huge loss to the industry and environment. Also, people working inside these industries may lose their lives. Therefore, many researchers have given their contribution to improve the working of the industry and improve the reliability of the industrial components. Nandhani [96] analyzed the year wise production and reason for the change of production in the sugar cane industry from 2000-2010. Zhao and Li [67] presented the effects of climate change on sugar cane production and gave the strategies to mitigate the effects of climate change. Performance analysis of the sugar mill taking various parameters into consideration like the area under sugar cane, sugar cane production, cane usage for sugar production etc. was carried out by Ganeshgouda et al. [73]. Sharma and Vishwakarma [58] applied the Genetic algorithm technique for the performance analysis of the feeding system of the sugar industry. Zaidi [74] analyzed the transient and study state behavior of the feeding unit of the sugar industry. Dahiya et al. [112] investigated the functioning of the A-pan crystallization structure of the sugar industry using fuzzy reliability approach. Saini and Kumar [113] analyzed the performance of the evaporation system of the sugar mill. It was determined in their analysis from reliability point of view sulphited syrup subsystem is highly sensitive. They shared the data with the designer so that the performance of the system can be improved.

2.8 Literature Based on Robotic and Automation in the Industry

As industries are mainly switching towards automation in production. Some industries have started using robotics in the industry. Because unlike humans, robots never get tired and can work continuously. But these robots are just machines, failures in any component of the robot may lead to heavy loss and destruction in the industry. Therefore, the reliability assessment of the robotic system is very much necessary so that failures don't occur frequently in the robots. Various researchers have presented the applications of the robotic system in the industry. Buchs et al. [63] presented the application of a robotic system in a hospital during surgery. He discussed during surgery failure can occur in the robotic system. But with increased experience, these failures can be reduced. Majid and Fudzin [70] presented the application of the robotic system in an automotive assembly plant. He collected the seven-year data for computing the reliability and availability of the plant. After analysis, he recommended that robot with the least reliability should be removed immediately. Majid and Fudzin [88] developed a model for an automotive painting line. This model is helpful for the industry in making decision on the replacement of the robot. In their model, they suggested that the defender robot must be replaced at the end of five years. A robot must be very reliable and safe in its operation. Cheng and Dhillon [40] analyzed the performance of a robot-safety system. The result of the analysis indicates that redundant robots, safety and repair help in the performance improvement of the system. Although robots are very reliable yet failure in robot can occur at any time and is a random phenomenon. Kampa [104] reviewed main reliability factors that helps to mitigate the failure of the robotic system. He recommended the preventive maintenance of the robotic system to prolong its uptime. As exact data for calculating the reliability of the system may not be always available therefore Kumar et.al [108] utilized a hybridized technique for calculating the reliability of the robotic system with available data (containing vagueness, uncertainty, etc.). Similar robotic system reliability analysis can be found in [22], [61], [106].

2.9 Literature Based on Wireless System

With the advancement in the field of science and technology, wireless communication has gained great popularity. With the help of the communication system, this world has become a very small place. It has now become possible to send messages, voice messages or text messages from one place to another place in a matter of seconds. In order to send the message from supply to destination, it's necessary for all the parts of the wireless communication system to be reliable. Failure of any component may result in the loss of important information. Therefore, it is required that the system remains available for a long period of time without any failure. Tambe [68] focused on the implementation of the wireless network and the various problems associated with it. With the advancement, in the field of technology, demand for the fast network has also increased. Sharma et al. [79] compared the 5-G with other generations from 1-G to 4-G. Advantages of the 5-G network over other networks were discussed. Xiao et al. [34] addressed the issue of wireless network security. A statistical method was developed by Ochang et al. [72] to measure the knowledge of technical employees and non-technical employees on how to react to WLAN security threats.

2.10 Literature Based on Multi-State System

For a better understanding of the binary state system, one can refer [20]. Reliability evaluation of the small binary state system is a very easy task. Ram and Kumar [59] presented the performance analysis of the complicated system under human failure which is the simplest engineering system found in the industry but if the number of the components of the binary system increase and Markov modeling is used for the reliability evaluation, then the system suffers from the state explosion. For example, if we have only 10 components in the system then the number of states are $2^{10} = 1024$. It is a Herculean task to draw the state transition diagram of such a system as one may forget some of the possible states. Therefore, a technique is required to reduce the total number of the system state for reliability evaluation. Such a technique was introduced by the Ushakov [16], known as universal generating function technique (UGF). First of all, she used this technique in the redundancy optimization problem. Later this technique was used in multi-state system's reliability evaluation. It was observed by the researchers the system and its component may have more than two performance levels. Levitin [29] contributed a lot to the development of the multi-state system performance analysis. He developed various methods and algorithms for different multi-state system reliability evaluation. Levitin and Lisnianski [24] presented the sensitivity and important analysis of the multi-state system which helps in determining the most critical component of the system. Lisnianski et al. [23] also applied the universal generating function technique in the reliability evaluation of the power system. Levitin and Xing [35] showed in their research paper that the failure propagation in the system may destroy the other components of the system. In the development of the theory of the multi-state system the algorithms for determining the reliability of k-out-of-n were developed. Chaturvedi et al. [42] developed an efficient algorithm for the reliability determination of the k-out-of-n system. This algorithm gives the exact reliability of the large multi-state system. Yingkui and Jing [44] presented the systematic literature review of the all literature available of the multi-state system up to the year 2012. The major problem of the UGF technique is that it determines the performance distribution of the system from the known performance distribution of its components. Lisnianski [45] studied the dynamic multistate system with the help of L_Z-transformation. He presented the various properties of the L_Z-transformation and discussed a few examples.

Nair and Manoharan [100] analyzed the performance of the power station. They collected the data from Kuttiady Hydro Electric Project. This project is being run by Kerala state electricity board. There are three generators in the power station. Availability, mean output performance, Mean output deficiency of the system were calculated using L_Z-transformation. In real-life situation, sometimes the precise and accurate data collection is a great hurdle. Due to the insufficient data precise probabilities of the state cannot be determined. In this case, Dempster–Shafer theory is used to determine the reliability indices of the Multi-state system. The use of the theory was presented by [49]. Meenakshi and Singh [110] further extended the work and applied it to the $((e,f),k,I_C)/(m,n)):F$ system where the system can fail if (e,f) sub matrix fails or any k components fail or consecutive I_C components fail in m rows or in n columns of a (m,n) sub-matrix. Probabilities of the system were obtained with the help of stochastic Markov modeling. After this, reliability, MTTF were obtained for the considered system. Also, sensitivity analysis was performed to determine the most critical components of the system. Meena and Vasanthi [85] discussed the use of the MANETs (Mobile ad hoc network) on the battlefield because it is easy to use wireless communication system in the battlefield rather than using the fixed wire network in the battlefield. They used the new modified Universal generating function method to determine the reliability of the MANETs system. For getting more detail on multi- state system one can refer to [11], [30], [36], [37], [47], [52], [55], [84], [94], [97], [112]. In a multi-state system, if two systems are considered then the performance of the systems can be shared with each other. In these systems, when one system satisfies its own demand and has surplus performance then that surplus performance can be shared with the other system. Levitin [41], Wang et al. [103] in their work showed how the performance of the system can be shared. For this, they used the UGF technique. Dong et al. [116] presented in their research that when the connection between system components is uncertain, then the weighted UGF technique can be used for the reliability evaluation of the system. In their research, they compared the various performance measures of the multi-state system for flow transmission system, task processing system, fixed weighted MSS and variable weighted MSS. Ding and Lisnianski [33] presented the concept of fuzzy universal technique for the reliability evaluation of the multi-state system. For the study of multi-state system with dependence among system's component one can refer [120]. In the system reliability evaluation, it is sometimes very difficult to find the exact value of the reliability. In that case, the reliability of the system can also found in the interval. Kumar et al. [115] determined the interval-valued reliability of a 2-out-of-4 system which consists of two elements connected in a series configuration.

2.11 Meta-Heuristic Approach

These days, many versions of the component are available in the market which may have different weights, performance rates, costs, and reliabilities. Therefore, the system analyst must select the components carefully which give the maximum reliability and optimal cost under the given constraints. For this, many meta-heuristic methods are available like Genetic algorithm, Ant bee colony method, Particle swarm optimization technique, Multi-objective gray wolf optimization algorithm. All these algorithms and techniques provide the best possible solution which gives the maximum reliability of the system. These algorithms are very fast and search for the best possible solution when implemented on a computer. For more detail, about these algorithms, one can refer to [68], [97]-[99], [102], [117], [118].

All the above author's contribution in the field of the multi-state system and the binary state system is magnificent and praiseworthy. But still, there are many systems that have not been discussed in the literature and some models have yet not been studied under certain factors like human error or hardware and software failures. Based on these, the scholar has identified the following research gaps.

2.12 Research Gaps

- Many realistic system's reliability has been ignored and the majority of researchers have considered only general systems. Therefore, we propose to work with realistic systems.
- Very less number of researchers have used variable failure and repair rates of the system components. Therefore, we purpose to use variable failure and repair rates.
- To investigate Human error impact on various system's reliability indices.
- We purpose to use various redundancies in the realistic systems.
- Implementation of the UGF technique has not yet been done on the Excel software in the literature. We propose to implement the UGF technique on Excel software with great ease.

2.13 Objectives of the Research Proposal

Objectives of Research Proposal are given below.

- To analyze and collect information about a system and then convert it in the form of a mathematical model.

- To investigate various state probabilities of the obtained model with the aid of Supplementary variable technique, Markov process, universal generating function.
- To obtain various reliability indexes for the same.
- To find the critical components of the model with the aid of sensitivity analysis.
- To analyze the use of various redundancy in a system to improve the performance of the system.

Chapter3: Application of Markov Process/ Mathematical Modelling in Analyzing Communication System Reliability

The aim of this chapter is to analyze the main components of a wireless communication system e.g. Input Transducer, Transmitter, Communication Channel, and Receiver on the basis of their interconnection for evaluating the various reliability measures for the same. Markov process and mathematical modeling have been used to formulate a mathematical model of the considered system (on the basis of various failures/repairs). Reliability of the wireless communication system with respect to its components failure is obtained and explained with the graphs. Also, critical components of the system are identified with the aid of sensitivity analysis. At last MTTF and MTBF with variation in various failures are also obtained.

3.1Introduction

In this era of technology, wireless communication has gained immense popularity. Because of the communication system, this world has become a smaller place. It has now become possible to send messages, voice messages or text messages from one place to another place in a matter of seconds. In order to send the message from source to destination, it is necessary for all the components of the wireless communication system to be reliable. Failure of any component may result in the loss of important information. Therefore, it is required that the system remains available for a long period of time without any failure.

Some authors [68], [79], [34], [72] have already worked in the field of wireless communication system. But no has ever tried to determine the reliability indices of the wireless communication system.

Keeping this in mind, reliability measures of a wireless communication system have been determined. The next section gives a brief description of the components of a wireless communication system.

3.2 System Description

The description of the components of the wireless communication system is as follows.

- **Source:** First of all, an input is given to the system. This input could be a picture, human voice, text message. The main objective is to send this input to the final destination.
- **Input Transducer:** The main function of the input transducer is to convert the input into an electrical waveform which is also known as a baseband signal.
- **Transmitter:** Transmitter in the system is used to convert the information which is easily understandable by the communication system.
- **Communication Channel:** Transmitter output is sent through a channel to the receiver.
- **Receiver:** The receiver receives the signal which is sent by the transmitter through the communication channel. Here the message is decoded to extract the original information. Two main parts of the receiver are selectivity and sensitivity.
  - **Selectivity:** It is defined as the ability of the receiver to accept the desired band of frequency and reject all other unwanted signals.
  - **Sensitivity:** It is defined as the ability of the receiver to identify and amplify the weak signals at the receiver output.
- **Output transducer:** The receiver output is sent to the output transducer which ultimately converts the electrical signal to the original message.

1-out-of-3: G

Figure 3.1(a): Process Diagram of Communication System

3.3Assumptions

The reliability analysis of the wireless communication system is based on the following assumptions.

Assumption 1: Initially, every component of the system is as good as a new one.

Assumption 2: When any component fails maintenance team starts repairing the component.

Assumption 3: When any system's component fails, the system goes into a degraded state or into an absorbing state

Assumption 4: System's components follow exponential distribution.

Assumption 5: In a degraded state, the system doesn't stop its working.

3.4 Nomenclature and State Narratives

For ease of reference, the major nomenclature and state narratives used are given below in Table 3.1 (a) and Table 3.1 (b) respectively.

t	Time scale
s	Laplace transformation variable
$P_i(t);\ i = 0,1,2,3,4$	Probability of the system being in state S_i at time $'t'$.
$\overline{P}_i(s)$	Laplace transform of $P_i(t)$
$P_j(x,t);$ $j = 5,6,7$	Probability density function of system being in completely failed state at instant t with elapsed repair time x
$\overline{P}_j(x,s);$ $j = 5,6,7$	Laplace transform of $P_j(x,t)$
λ	Failure rate of component of the transmitter
λ_{CTF}	Failure rate of communication channel

η_1 / η_2	Failure rate of selectivity/sensitivity
$\mu_1(x) / \mu_2(x)$	Repair rate of selectivity/sensitivity
$\mu_{CTF}(x)$	Repair rate of communication channel
$\mu(x)$	Repair rate of the transmitter
$\mu_{12}(x)$	Simultaneous repair rate of selectivity and sensitivity

Table 3.1(a): Nomenclature

S_0	Good state: All the components of the communication system are as good as new
S_1	Degraded state: State in which the first component of the transmitter fails
S_2	Degraded state: State in which the second component of the transmitter also fails
S_3	Degraded state: State in which selectivity of the receiver fails
S_4	Degraded state: State in which sensitivity of the receiver fails
S_5	Failed state: State in which transmitter fails
S_6	Failed state: State in which selectivity and sensitivity both fail
S_7	Failed state: State in which the communication channel fails

Table 3.1(b): State narratives

3.5 State Transition Diagram

It is observed that the communication system can break/fail due to many issues including transmitter failure, receiver not working properly, the problem in the communication channel etc. Here, the authors analyze these aspects and identify the various states in which system may be found at any time $'t'$. These states are shown in the following state transition diagram Figure 3.1 (b).

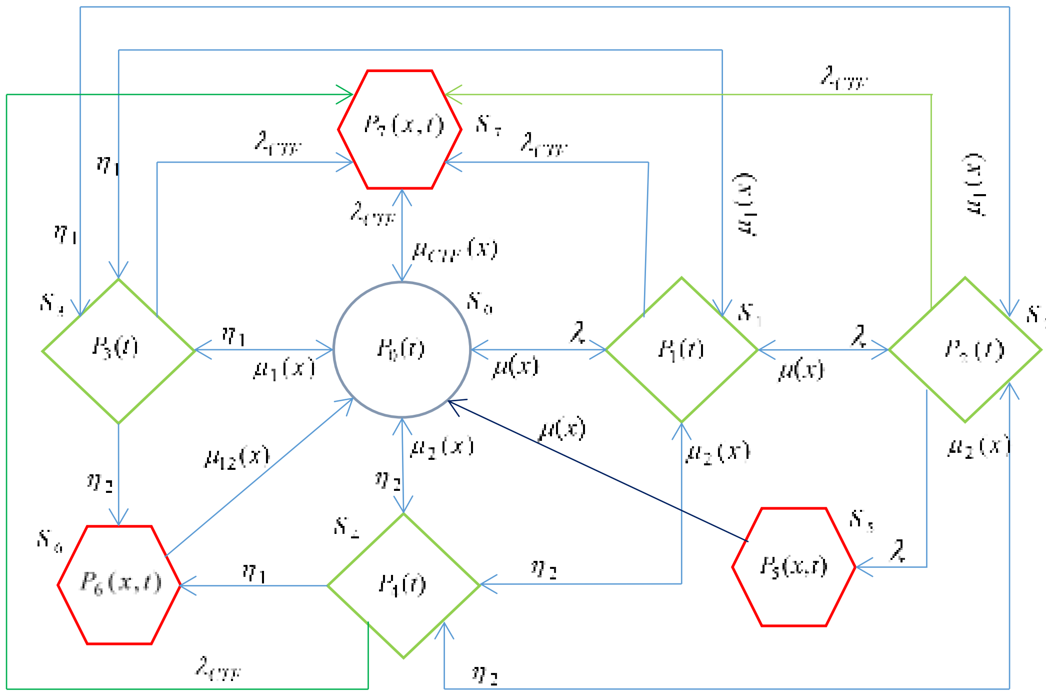


Figure 3.1(b): State Transition Diagram of Communication System

3.6 Mathematical Approach

By considering the above transition state diagram given in Figure 3.1 (b) through the Markov process, during the time $t+\Delta t$ and letting $\Delta t \to 0$, the following set of differential–integral equations are obtained

$$\left(\frac{\partial}{\partial t}+\lambda+\eta_1+\eta_2+\lambda_{CTF}\right)P_0(t)=\mu(x)\,P_1(t)+\mu_1(x)\,P_3(t)+\mu_2(x)\,P_4(t)+\int_0^\infty \mu_3(x)\,P_5(x,t)\,dx$$
$$+\int_0^\infty \mu_{12}(x)\,P_6(x,t)\,dx+\int_0^\infty \mu_{CTF}(x)\,P_7(x,t)\,dx \tag{3.1}$$

$$\left(\frac{\partial}{\partial t}+\mu(x)+\lambda+\eta_1+\eta_2+\lambda_{CTF}\right)P_1(t)=\mu(x)\,P_2(t)+\lambda\,P_0(t)+\mu_1(x)\,P_3(t)+\mu_2(x)\,P_4(t) \tag{3.2}$$

$$\left(\frac{\partial}{\partial t}+\mu(x)+\lambda+\eta_1+\eta_2+\lambda_{CTF}\right)P_2(t)=\lambda\,P_1(t)+\mu_1(x)\,P_3(t)+\mu_2(x)\,P_4(t) \tag{3.3}$$

$$\left(\frac{\partial}{\partial t}+3\mu_1(x)+\eta_2+\lambda_{CTF}\right)P_3(t)=\eta_1\,P_0(t)+\eta_1\,P_1(t)+\eta_1\,P_2(t) \tag{3.4}$$

$$\left(\frac{\partial}{\partial t}+3\mu_2(x)+\eta_1+\lambda_{CTF}\right)P_4(t)=\eta_2\,P_0(t)+\eta_2\,P_1(t)+\eta_2\,P_2(t) \tag{3.5}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_3(x)\right)P_5(x,t)=0 \tag{3.6}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{12}(x)\right)P_6(x,t)=0 \tag{3.7}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_{CTF}(x)\right)P_7(x,t)=0 \tag{3.8}$$

Boundary conditions:

$$P_5(0,t)=\lambda\,P_2(t) \tag{3.9}$$

$$P_6(0,t)=\eta_1\,P_4(t)+\eta_2\,P_3(t) \tag{3.10}$$

$$P_7(0,t)=\lambda_{CTF}\left\{P_0(t)+P_1(t)+P_2(t)+P_3(t)+P_4(t)\right\} \tag{3.11}$$

Initial condition:

$$P_i(t)=\begin{cases}1, & i=0, t=0\\ 0, & otherwise\end{cases} \tag{3.12}$$

$$\left(s+\lambda+\eta_1+\eta_2+\lambda_{CTF}\right)\overline{P}_0(s)=\mu(x)\overline{P}_1(s)+\mu_1(x)\overline{P}_3(s)+\mu_2(x)\overline{P}_4(s)+\int_0^\infty \mu_3(x)\overline{P}_5(x,s)\,dx$$
$$+\int_0^\infty \mu_{12}(x)\overline{P}_6(x,s)\,dx+\int_0^\infty \mu_{CTF}(x)\overline{P}_7(x,s)\,dx \quad (3.13)$$

$$\left(s+\mu(x)+\lambda+\eta_1+\eta_2+\lambda_{CTF}\right)\overline{P}_1(s)=\mu(x)\overline{P}_2(s)+\lambda\overline{P}_0(s)+\mu_1(x)\overline{P}_3(s)+\mu_2(x)\overline{P}_4(s) \quad (3.14)$$

$$\left(s+\mu(x)+\lambda+\eta_1+\eta_2+\lambda_{CTF}\right)\overline{P}_2(s)=\lambda\overline{P}_1(s)+\mu_1(x)\overline{P}_3(s)+\mu_2(x)\overline{P}_4(s) \quad (3.15)$$

$$\left(s+3\mu_1(x)+\eta_2+\lambda_{CTF}\right)\overline{P}_3(s)=\eta_1\overline{P}_0(s)+\eta_1\overline{P}_1(s)+\eta_1\overline{P}_2(s) \quad (3.16)$$

$$\left(s+3\mu_2(x)+\eta_1+\lambda_{CTF}\right)\overline{P}_4(s)=\eta_2\overline{P}_0(s)+\eta_2\overline{P}_1(s)+\eta_2\overline{P}_2(s) \quad (3.17)$$

$$\left(\frac{\partial}{\partial x}+s+\mu_3(x)\right)\overline{P}_5(x,s)=0 \quad (3.18)$$

$$\left(\frac{\partial}{\partial x}+s+\mu_{12}(x)\right)\overline{P}_6(x,s)=0 \quad (3.19)$$

$$\left(\frac{\partial}{\partial x}+s+\mu_{CTF}(x)\right)\overline{P}_7(x,s)=0 \quad (3.20)$$

Boundary conditions:

$$\overline{P}_5(0,s)=\lambda\overline{P}_2(s) \quad (3.21)$$

$$\overline{P}_6(0,s)=\eta_1\overline{P}_4(s)+\eta_2\overline{P}_3(s) \quad (3.22)$$

$$\overline{P}_7(0,s)=\lambda_{CTF}\left\{\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_3(s)+\overline{P}_4(s)\right\} \quad (3.23)$$

The various transition states probability of the wireless communication system are obtained as given below by solving the Equations (3.13)-(3.20) and using initial and boundary conditions:

$$\bar{P}_0(s) = \frac{1}{\left(\begin{array}{l} s+\lambda+\eta_1+\eta_2+\lambda_{CTF} - \left\{\mu(x) + \dfrac{\lambda_{CTF}\ \mu_{CTF}(x)}{s+\mu_{CTF}(x)}\right\}\left\{\dfrac{H_2}{H_4} + \dfrac{H_3(\mu(x)+H_2-\lambda)}{H_1 H_4}\right\} \\ -\lambda_{CTF}\ \dfrac{\mu_{CTF}(x)}{s+\mu_{CTF}(x)} \\ -\left\{\dfrac{\lambda\mu_3(x)}{s+\mu_3(x)} + \dfrac{\lambda_{CTF}\ \mu_{CTF}(x)}{s+\mu_{CTF}(x)}\right\}\left\{\dfrac{H_2^{\ 2}}{H_1\ H_4} + \dfrac{H_2\ H_3(\mu(x)+H_2-\lambda)}{H_1^{\ 2} H_4} + \dfrac{H_3}{H_1}\right\} \\ -H_7\left[\begin{array}{l} \left\{\mu_1(x) + \dfrac{\eta_2\mu_{12}(x)}{s+\mu_{12}(x)} + \dfrac{\lambda_{CTF}\ \mu_{CTF}(x)}{s+\mu_{CTF}(x)}\right\}\dfrac{\eta_1}{H_5} \\ +\left\{\mu_2(x) + \dfrac{\eta_1\mu_{12}(x)}{s+\mu_{12}(x)} + \dfrac{\lambda_{CTF}\ \mu_{CTF}(x)}{s+\mu_{CTF}(x)}\right\}\dfrac{\eta_2}{H_6} \end{array}\right] \end{array}\right)}$$

$$\bar{P}_1(s) = \left\{\frac{H_2}{H_4} + \frac{H_3(\mu(x)+H_2-\lambda)}{H_1 H_4}\right\}\bar{P}_0(s)$$

$$\bar{P}_2(s) = \left\{\frac{H_2^{\ 2}}{H_1 H_4} + \frac{H_2 H_3(\mu(x)+H_2-\lambda)}{H_1^{\ 2} H_4} + \frac{H_3}{H_1}\right\}\bar{P}_0(s)$$

$$\bar{P}_3(s) = \frac{\eta_1 H_7}{H_5}\bar{P}_0(s)$$

$$\bar{P}_4(s) = \frac{\eta_2 H_7}{H_6}\bar{P}_0(s)$$

Where

$$H_1 = s+\mu(x)+\lambda+\eta_1+\eta_2+\lambda_{CTF} - \frac{\mu_1(x)\eta_1}{(s+3\mu_1(x)+\eta_2+\lambda_{CTF})} - \frac{\mu_2(x)\eta_2}{(s+3\mu_2(x)+\eta_1+\lambda_{CTF})}$$

$$H_2 = \lambda + \frac{\mu_1(x)\eta_1}{(s+3\mu_1(x)+\eta_2+\lambda_{CTF})} + \frac{\mu_2(x)\eta_2}{(s+3\mu_2(x)+\eta_1+\lambda_{CTF})}$$

$$H_3 = \frac{\mu_1(x)\eta_1}{(s+3\mu_1(x)+\eta_2+\lambda_{CTF})} + \frac{\mu_2(x)\eta_2}{(s+3\mu_2(x)+\eta_1+\lambda_{CTF})}$$

$$H_4 = s+\mu(x)+\lambda+\eta_1+\eta_2+\lambda_{CTF} - H_3 - \frac{H_2(\mu(x)+H_2-\lambda)}{H_1}$$

$$H_5 = (s+3\mu_1(x)+\eta_2+\lambda_{CTF})$$

$$H_6 = (s+3\mu_2(x)+\eta_1+\lambda_{CTF})$$

$$H_7 = \left\{1+\frac{H_2}{H_4}+\frac{H_3}{H_1}+\frac{H_2^{\,2}}{H_1H_4}+\frac{H_3(\mu(x)+H_2-\lambda)}{H_1H_4}+\frac{H_2H_3(\mu(x)+H_2-\lambda)}{H_1^{\,2}H_4}\right\}$$

The probability that the system is in the up (i.e. in a good or a degraded working state) and down (failed state) (From Figure 3(b)) state at any time is given as:

$$P_{up}(t) = P_0(t)+P_1(t)+P_2(t)+P_3(t)+P_4(t) \tag{3.24}$$

$$P_{down}(x,t) = P_5(x,t)+P_6(x,t)+P_7(x,t) \tag{3.25}$$

Also we have $P_{up}(t)+P_{down}(t)=1$

3.7 Numerical Analysis and Assessment of Various Reliability Measures

3.7.1 Reliability

Reliability investigation is an integral part of ensuring security in a communication system. However, the reliability assessment of this system is complex due to its multistate failure scenarios and multistate functionality. Basically, it is the probability that the system cannot fail before a time period '*t*'. The major components of a communication system that significantly contribute to its reliability characteristics are the transmitter, receiver and communication channel. It is observed from the state transition diagram Figure 3.1(b) and equation (3.24) that the reliability of the communication system is obtained as:

$$R(t)=\left\{\begin{array}{l}\left(\begin{array}{l}\frac{\lambda^2}{2}(\lambda+\eta_2)^2(\lambda+\eta_1)^2(\lambda^2-\eta_1\eta_2)\;t^2+\lambda(\lambda+\eta_2)(\lambda+\eta_1)\\(\lambda^4-4\lambda^2\eta_1\eta_2-2\lambda\eta_1\eta_2{}^2-2\lambda\eta_1{}^2\eta_2-\eta_1{}^2\eta_2{}^2)\;t-\eta_1{}^3\eta_2{}^3\\-3\lambda\eta_1{}^2\eta_2{}^3-9\lambda^3\eta_1{}^2\eta_2-9\lambda^2\eta_1{}^2\eta_2{}^2-3\lambda^2\eta_1{}^3\eta_2-3\lambda\eta_1{}^3\eta_2{}^2\\+\lambda^6-3\lambda^2\eta_1\eta_2{}^3-9\lambda^4\eta_1\eta_2-9\lambda^3\eta_1\eta_2{}^2\end{array}\right)\frac{e^{-(\lambda+\eta_1+\eta_2+\lambda_{CTF})\;t}}{(\lambda+\eta_1)^3(\lambda+\eta_2)^3}\\+\frac{\eta_1(3\lambda^2+3\lambda\eta_1+\eta_1{}^2)\,e^{-(\eta_2+\lambda_{CTF})\;t}}{(\lambda+\eta_1)^3}+\frac{\eta_2(3\lambda^2+3\lambda\eta_2+\eta_2{}^2)\,e^{-(\eta_1+\lambda_{CTF})\;t}}{(\lambda+\eta_2)^3}\end{array}\right\}$$

(3.26)

3.7.1.1 Reliability of Communication System with Respect to Transmitter

The reliability of the communication system is given by equation (3.26). In order to calculate the reliability of the communication system with respect to Transmitter, we vary its failure rate λ as 0.12, 0.17, 0.22 in (3.26) and put remaining failure rates as $\eta_1=0.10, \eta_2=0.09, \lambda_{CTF}=0.02.$ After that changing time unit *t* in the resultant expression of reliability, we obtained the behavior of reliability for the communication system with regard to Transmitter failure as given in Table 3.2 (a) and Figure 3.2(a).

Time (*t*)	**Reliability if** $\lambda=0.12$	**Reliability if** $\lambda=0.17$	**Reliability if** $\lambda=0.22$
0	1.0000000000	1.0000000000	1.0000000000
1	0.9719463888	0.9715566787	0.9708883712
2	0.9307042940	0.9284093269	0.9246571636
3	0.8803324924	0.8746150281	0.8656923815
4	0.8242968079	0.8142638678	0.7992989949
5	0.7654006369	0.7508504697	0.7300777750
6	0.7058136453	0.6870866920	0.6614564631
7	0.6471466649	0.6249263020	0.5957230970
8	0.5905427913	0.5656771842	0.5342391347
9	0.5367682746	0.5101368520	0.4776837591
10	0.4862950318	0.4587207613	0.4262690406

Table 3.2(a): Reliability vs. Time (*t*) w.r.t. variation in failure rate of Transmitter

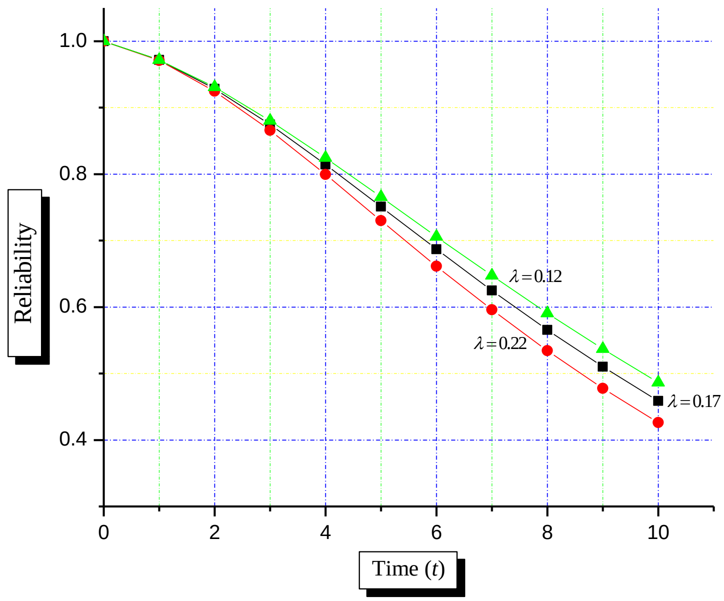


Figure 3.2(a): Reliability (with respect to Transmitter failure) vs. Time (*t*)

3.7.1.2 Reliability of Communication System for Receiver Selectivity

A receiver is an important unit of the communication system. Actually, it is used to compensate for the transmission loss in a message. It consists of two parts namely selectivity and sensitivity. The reliability of the communication system is given by equation (3.26). In order to calculate the reliability of the communication system with respect to selectivity, we vary its failure rate as $\eta_1 =$ 0.05, 0.10, 0.15 in (3.26) and put remaining failure rates as $\lambda = 0.17, \eta_2 = 0.09, \lambda_{CTF} = 0.02.$ Varying time unit *t* as well as the failure rate of selectivity in (3.26). Table 3.2(b) and Figure 3.2(b) can be easily obtained.

Time (t)	Reliability if $\eta_1 = 0.05$	Reliability if $\eta_1 = 0.10$	Reliability if $\eta_1 = 0.15$
0	1.0000000000	1.0000000000	1.0000000000
1	0.9754471826	0.9715566787	0.9678561411
2	0.9417528293	0.9284093269	0.9163408861
3	0.9002292010	0.8746150281	0.8525980338
4	0.8530094388	0.8142638678	0.7826326738
5	0.8023117421	0.7508504697	0.7109773879
6	0.7500947783	0.6870866920	0.6407867127
7	0.6979310935	0.6249263020	0.5740863409
8	0.6469949335	0.5656771842	0.5120470088
9	0.5981041779	0.5101368520	0.4552280923
10	0.5517825866	0.4587207613	0.4037734216

Table 3.2(b): Reliability vs. Time (*t*) w.r.t. variation in failure rate of selectivity

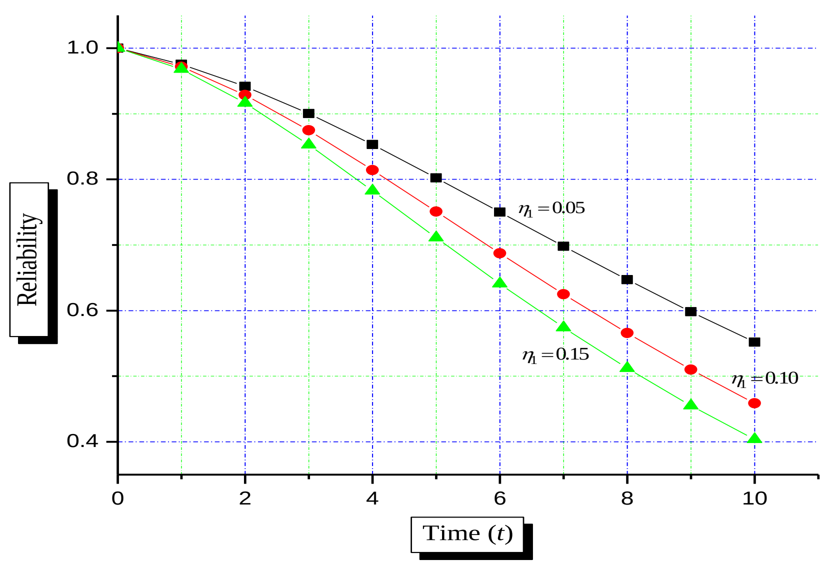


Figure 3.2(b): Reliability vs. Time (*t*) w.r.t. variation in failure rate of selectivity

3.7.1.3 Reliability of Communication System with Regard to Sensitivity

Varying time unit *t* as well as the failure rate of Sensitivity $\eta_2 = 0.05, \eta_2 = 0.09, \eta_2 = 0.13$ in (3.26), and put remaining failure rates as $\lambda = 0.17$, $\eta_1 = 0.10$, $\lambda_{CTF} = 0.02$. we get the subsequent Table 3.2(c) and corresponding Figure 3.2(c) for the reliability of the considered system with regard to sensitivity failure as:

Time (t)	**Reliability if** $\eta_2 = 0.05$	**Reliability if** $\eta_2 = 0.09$	**Reliability if** $\eta_2 = 0.13$
0	1.0000000000	1.000000000	1.000000000
1	0.9750171720	0.9715566787	0.9682320167
2	0.9402986917	0.9284093269	0.9174374355
3	0.8974965556	0.8746150281	0.8543397797
4	0.8489868768	0.8142638678	0.7847333954
5	0.7971389813	0.7508504697	0.7130845718
6	0.7439898063	0.6870866920	0.6425682060
7	0.6911386044	0.6249263020	0.5752741353
8	0.6397533479	0.5656771842	0.5124527015
9	0.5906269656	0.5101368520	0.4547415126
10	0.5442496747	0.4587207613	0.4023525271

Table 3.2(c): Reliability vs. Time (*t*) w.r.t. variation in failure rate of sensitivity

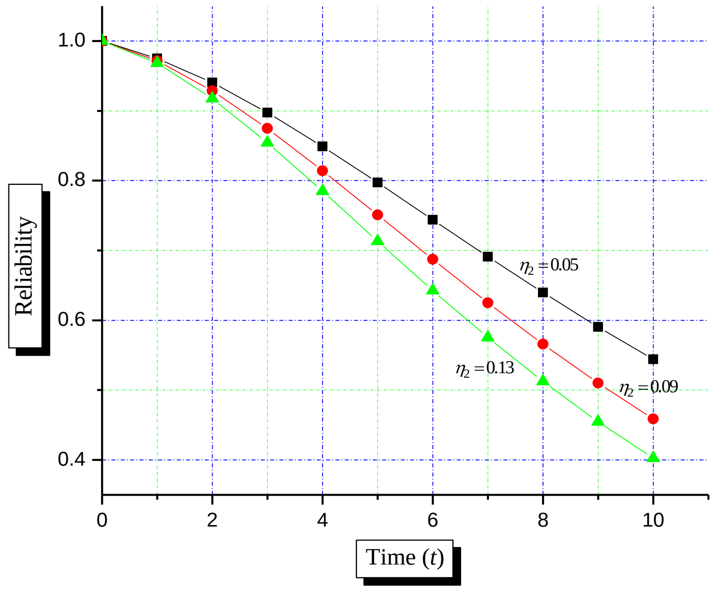


Figure 3.2(c): Reliability vs. Time (*t*) w.r.t. variation in failure rate of sensitivity

3.7.1.4 Reliability of Communication System with Regard to Failure of Communication Channel

Varying time unit t as well as the failure rate of communication channel $\lambda_{CTF} = 0.02, \lambda_{CTF} = 0.07, \lambda_{CTF} = 0.11$ in (3.26), and put remaining failure rates as $\lambda = 0.17,\ \eta_1 = 0.10,\ \eta_2 = 0.10.$ in (3.26), we get the subsequent Table 3.2(d) and corresponding Figure 3.2(d) for the reliability of the considered system with regard to the failure of communication channel as:

Time (t)	**Reliability if** $\lambda_{CTF} = 0.02$	**Reliability if** $\lambda_{CTF} = 0.07$	**Reliability if** $\lambda_{CTF} = 0.11$
0	1.000000000	1.000000000	1.000000000
1	0.9715566787	0.9241733002	0.8879359468
2	0.9284093269	0.8400594982	0.7754726547

3	0.8746150281	0.7527881309	0.6676631778
4	0.8142638678	0.6666628697	0.5680926237
5	0.7508504697	0.5847629337	0.4787633971
6	0.6870866920	0.5090063407	0.4003985690
7	0.6249263020	0.4403781220	0.3328306247
8	0.5656771842	0.3791847563	0.2753446455
9	0.5101368520	0.3252776181	0.2269384935
10	0.4587207613	0.2782282060	0.1865019438

Table 3.2(d): Reliability vs. Time (*t*) w.r.t. variation in failure of communication channel

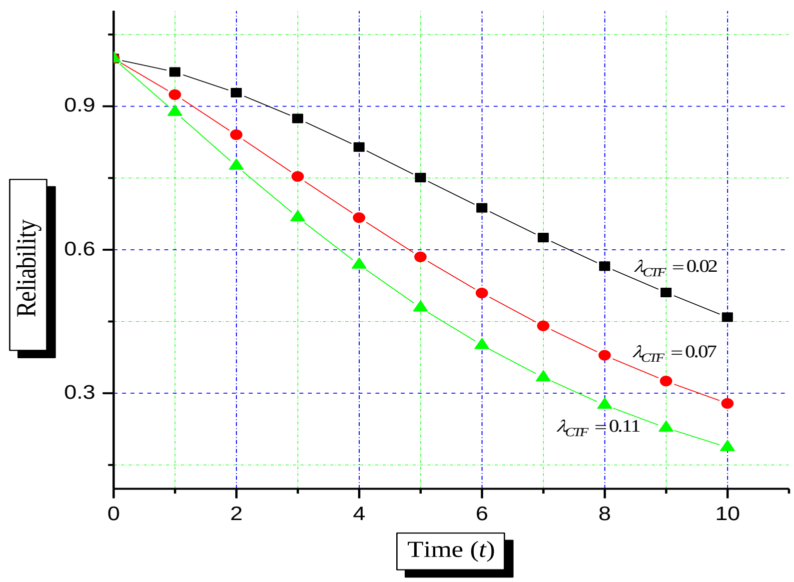


Figure 3.2(d): Reliability vs. Time (*t*) w.r.t. variation in failure of communication channel

3.7.2 Mean Time to Failure (MTTF)

Basically, MTTF is the average failure time of the system. Mathematically, it is calculated as

$$MTTF = \int_0^\infty t f(t)dt = \int_0^\infty R(t)dt = \operatorname*{limit}_{s \to 0} \overline{R}(s) \tag{3.27}$$

Where $f(t)$ is the probability density function and $R(t)$ is system reliability

The MTTF of the considered system can be obtained by using (3.23) in (3.27) as

$$\text{MTTF} = \left\{ \begin{array}{l} \dfrac{1}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})} + \dfrac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2} + \dfrac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^3} \\ + \dfrac{\eta_1\left(1+\dfrac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})} + \dfrac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_2+\lambda_{CTF})} \\ + \dfrac{\eta_2\left(1+\dfrac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})} + \dfrac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_1+\lambda_{CTF})} \end{array} \right\} \tag{3.28}$$

Put $\lambda = 0.17,\ \eta_1 = 0.10,\ \eta_2 = 0.09, \lambda_{CTF} = 0.02$ and changing the failure rates one by one from 0.01 to 0.09 in the MTTF expression obtained in equation (3.28), Table 3.3 and Figure 3.3 are obtained for MTTF of the system as:

Variations in Failure rates	MTTF with respect to failure rates			
	λ	η_1	η_2	λ_{CTF}
0.01	12.66114848	27.22352477	27.79228395	12.72418578
0.02	12.65401198	21.59503367	21.87125866	11.52860707
0.03	12.63760654	18.32847626	18.41812133	10.52935865
0.04	12.61047273	16.22855446	16.18665999	9.682271635
0.05	12.57228349	14.78521757	14.64460784	8.955470755
0.06	12.52338197	13.74498663	13.52711370	8.325375455
0.07	12.46448864	12.96814206	12.68795963	7.774167498
0.08	12.39651631	12.37165248	12.04008976	7.288133370
0.09	12.32045454	11.90327056	11.52860707	6.856547861

Table 3.3: MTTF vs. variation in failure rates

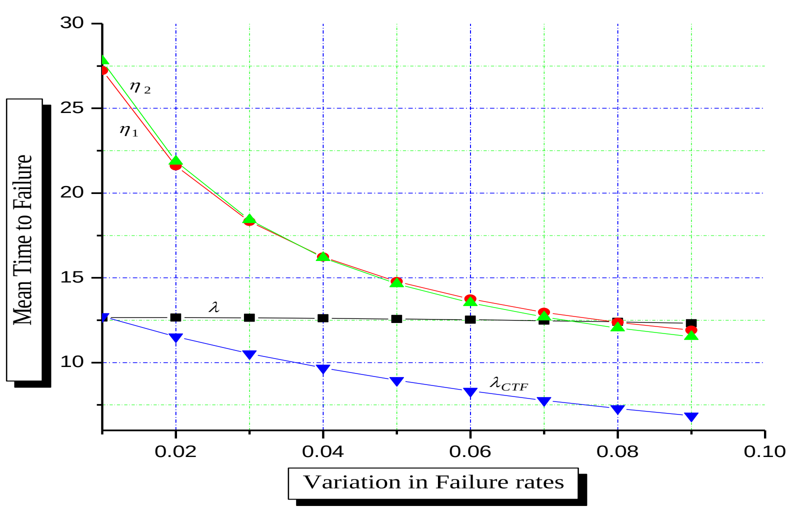


Figure 3.3: MTTF vs. variation in failure rates

3.7.3 Sensitivity Analysis

Sensitivity analysis is a technique that is used to decide how the different values of an independent factor affects a particular dependent factor under some constraints or one can say it's a technique by that one will analyze which factor affects the system's performance most. In this section, sensitivity analysis is performed for the reliability and MTTF to see that which component's failure rate affects system performance.

3.7.3.1 Sensitivity Analysis for Mean Time to Failure

Sensitivity analysis of the communication system with respect to MTTF is performed by differentiating the MTTF expression obtained in equation (3.28) with respect to various failure rates and then put the values of various failure rates as $\lambda = 0.17$ $\eta_1 = 0.10,$ $\eta_2 = 0.09,$ $\lambda_{CTF} = 0.02,$ in these partial derivatives. Now changing the failure rates one by one respectively, one can acquire Table 3.4 and corresponding Figure 3.4 for the sensitivity of MTTF for the communication system as:

Failure rates	**Sensitivity for MTTF with respect to failure rates**			
	$\frac{\partial(MTTF)}{\partial\lambda}$	$\frac{\partial(MTTF)}{\partial\eta_1}$	$\frac{\partial(MTTF)}{\partial\eta_2}$	$\frac{\partial(MTTF)}{\partial\lambda_{CTF}}$
0.01	-.34053654	-760.7880169	-798.7047326	-131.2726201
0.02	-1.14025791	-414.8329124	-437.5225361	-108.8757242
0.03	-2.16397371	-256.3992688	-271.7909812	-91.69963650
0.04	-3.26749091	-171.5144244	-182.7770710	-78.24259860
0.05	-4.36416396	-121.1736175	-129.8350221	-67.50649773
0.06	-5.40383255	-89.11631997	-96.01228652	-58.80684555
0.07	-6.35943298	-67.59688514	-73.22821273	-51.66148799

0.08	-7.218424564	-52.55171961	-57.23974622	-45.72283361
0.09	-7.977272727	-41.68688493	-45.64903304	-40.73517307

Table 3.4: Sensitivity of MTTF vs. failure rates

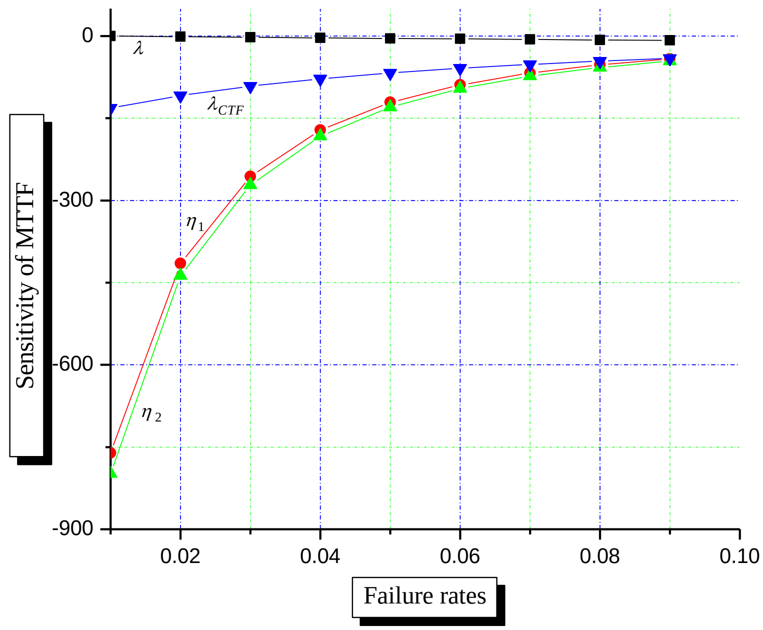


Figure 3.4: Sensitivity of MTTF vs. failure rates

3.7.3.2 Sensitivity Analysis for Reliability

The sensitivity for reliability is performed in the same manner as it was performed for the sensitivity of MTTF i.e. by differentiating the reliability function with regard to the failure rates and then substitute the values of different failure rates as $\lambda = 0.17\ \eta_1 = 0.10,$ $\eta_2 = 0.09,\ \lambda_{CTF} = 0.02,$ in these partial derivatives. Now changing the time unit *t* in these partial derivatives, one can acquire Table 3.5 and corresponding Figure 3.5 for the sensitivity of reliability of communication system as:

Time (t)	Sensitivity for Reliability			
	$\frac{\partial R(t)}{\partial \lambda}$	$\frac{\partial R(t)}{\partial \eta_1}$	$\frac{\partial R(t)}{\partial \eta_2}$	$\frac{\partial R(t)}{\partial \lambda_{CTF}}$
0	0	0	0	0
1	-0.010377236	-0.07587873	-0.08479179	-0.971556678
2	-0.059795214	-0.25369336	-0.28545918	-1.856818654
3	-0.145810975	-0.47450432	-0.53815077	-2.623845084
4	-0.250544815	-0.69898884	-0.79968309	-3.257055471
5	-0.355956704	-0.90358688	-1.04351183	-3.754252347
6	-0.449053281	-1.07615536	-1.25524130	-4.122520152
7	-0.522565557	-1.21219005	-1.42872080	-4.374484113
8	-0.573894217	-1.31191760	-1.56302604	-4.525417475
9	-0.603642929	-1.37822604	-1.66029285	-4.591231666
10	-0.614293829	-1.41528151	-1.72424994	-4.587207613

Table 3.5: Sensitivity of Reliability vs. Time (*t*)

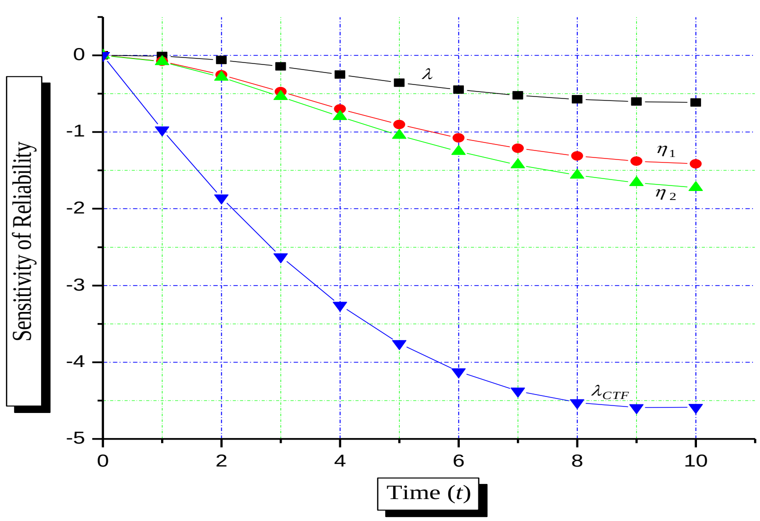


Figure 3.5: Sensitivity of Reliability vs. Time (*t*)

3.7.4 Mean Time Between Failures

For calculating the MTBF of the wireless communication system, initially, we find MTTR of the wireless communication system from the equation (3.25). MTTR is the average time that a system takes to recover from failure. For MTTR, take limit $s \to 0$ in the equation (3.25), then we get

$$MTTR = \lim_{s \to 0} \overline{P}_{down}(s) \qquad (3.29)$$

After finding the MTTR of the system, one can easily find MTBF of the system. MTBF is the average mean time between the two failures. For MTBF, find the sum of MTTF and MTTR. Thus, after adding equation (3.28) and (3.29) we get

$$MTBF = MTTF + MTTR \qquad (3.30)$$

The expression for the MTBF is given below:

$$MTBF = \left[\begin{array}{l} \frac{1}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})} + \frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2} + \frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^3} \\ + \frac{\eta_1\left(1+\frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})}+\frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_2+\lambda_{CTF})} \\ + \frac{\eta_2\left(1+\frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})}+\frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_1+\lambda_{CTF})} + \frac{\lambda^3}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^3\mu} \\ + \frac{\eta_1\eta_2\left(1+\frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})}+\frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_1+\lambda_{CTF})\mu_{12}} \\ + \frac{\eta_1\eta_2\left(1+\frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})}+\frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_2+\lambda_{CTF})\mu_{12}} \end{array} \right]$$

$$
+\frac{\lambda_{CTF}}{\mu_{CTF}}\left[\left(\begin{array}{l}\frac{1}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})}+\frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}+\frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^3}\\
+\frac{\eta_1\left(1+\frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})}+\frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_2+\lambda_{CTF})}\\
+\frac{\eta_2\left(1+\frac{\lambda}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})}+\frac{\lambda^2}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^2}\right)}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})(\eta_1+\lambda_{CTF})}+\frac{\lambda^3}{(\lambda+\eta_1+\eta_2+\lambda_{CTF})^3\mu}\end{array}\right)\right]
$$

(3.31)

Set the failure rates and repair rates as $\lambda = 0.17$, $\eta_1 = 0.10$, $\eta_2 = 0.09$, $\lambda_{CTF} = 0.02$, $\mu = 1$, $\mu_{12} = 1$ $\mu_{CTF} = 1$ in (3.31). In order to obtain the MTBF of the wireless communication system one by one vary each failure rate from 0.01 to 0.09 as shown in the below Table 3.6.

Variation in failure rates	λ	η_1	η_2	λ_{CTF}
0.01	13.66115036	28.22755363	28.79592321	13.72515572
0.02	13.65402513	22.59867293	22.87455697	12.53039778
0.03	13.63764559	19.33177458	19.42111999	11.53184335
0.04	13.61055465	17.23155312	17.18939423	10.68534226
0.05	13.57242573	15.78795180	15.64710784	9.95903497
0.06	13.52360145	14.74748663	14.52940547	9.32935423
0.07	13.46480114	13.97043384	13.69006569	8.77849302
0.08	13.39693617	13.37375853	13.04202962	8.29274738
0.09	13.32099454	12.90521043	12.53039778	7.86140020

Table 3.6: MTBF vs. variation in failure rates

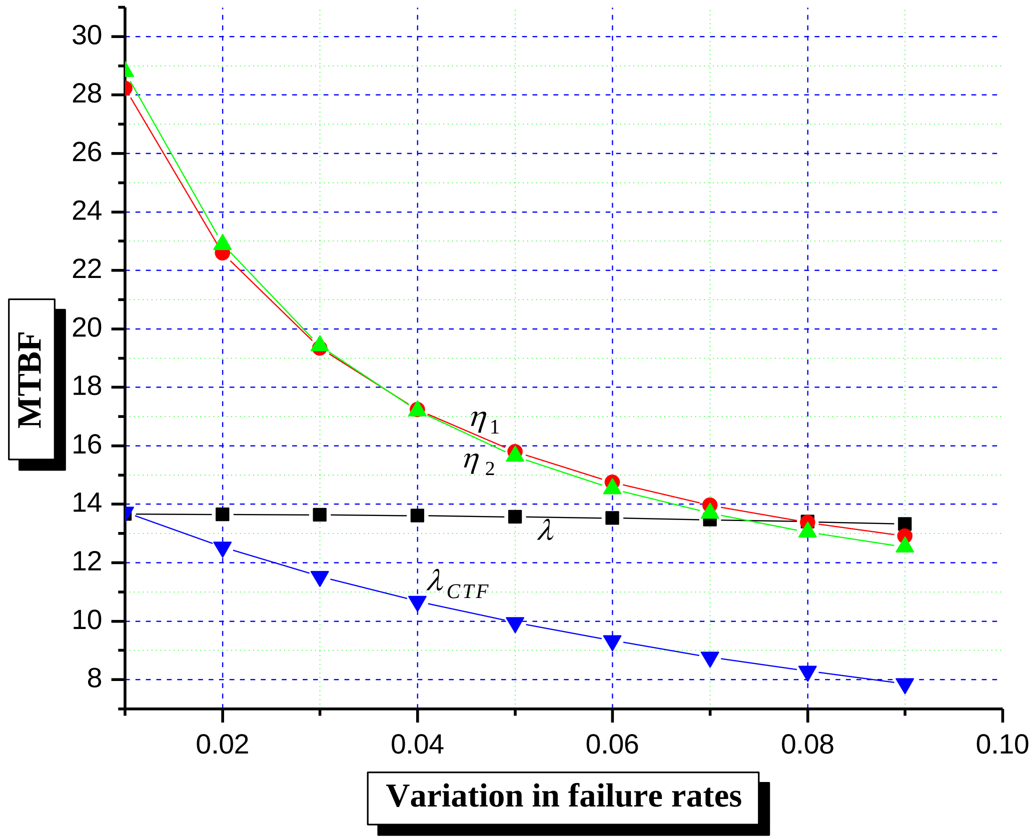


Figure 3.6: MTBF vs. variation in failure rates

3.8 Result and Discussion

Analysis of reliability, MTTF, MTBF, sensitivity analysis for the communication system have been completed in this chapter.

The reliability of the communication system is shown in the Figure 3.2(a), 3.2(b), 3.2(c) and 3.2(d) with respect to the failure of transmitter, selectivity, sensitivity and communication channel respectively. From these figures, one can conclude that the reliability of the communication system is 0.4862950318, 0.4587207613, 0.4262690406 (at 10 units of time) when the failure of transmitter is taken as 0.12, 0.17 and 0.22

respectively. The same with respect to failure/degradation selectively (part of receiver) is found as 0.64699, 0.565677, 0.512047 (at 10 units of time). The reliability of the failure/degradation of sensitivity (part of receiver) is obtained as 0.544294, 0.458720, 0.402352 (at 10 units of time). Last but not the least, the reliableness of the communication system with reference to the failure rate of communication channel is obtained as 0.458720, 0.278228, 0.186501.

The MTTF of the communication system with reference to different failures is presented in Figure 3.3. It is seen from the graph that system's MTTF is highest with reference to the failure rate of sensitivity and selectivity of the receiver and lowest with reference to the failure of the communication channel. It is almost constant with reference to the failure of the Transmitter. Figure 3.4 and Figure 3.5 shows the sensitivity analysis of the communication system with respect to MTTF and reliability. It has been determined from Figure 3.4 that the MTTF of the communication system is most sensitive with reference to the failure rate of the receiver (sensitivity and selectivity). The MTBF of the communication system with reference to different failures is presented in Figure 3.6. It is seen from the graph that system's MTBF is highest with reference to the failure rate of sensitivity and selectivity of the receiver and lowest with reference to the failure of the communication channel. It is almost constant with reference to the failure of the Transmitter. The reliability of the communication system is a lot of stricken by the failure rate of the communication channel .

3.9 Conclusion

This analysis may be helpful for communication system industries. From the on top of result discussion section, one will conclude that the highest/least value of the reliability of the communication system is 1/0.186501. Also, it is discovered that the system's reliability, MTTF and MTBF are far stricken by the failure rate of the communication channel. Also, the system's MTTF is far stricken by the failure rate of the receiver (sensitivity and selectivity). Hence, it is suggested that the performance of the communication system can be enhanced if more attention is given to the failure/degradation of the receiver (sensitivity and selectivity) and communication channel. It is expected that this analysis is of nice facilitate to the wireless communication system industry.

Chapter 4: Reliability Analysis of an Industrial System Equipped with Two Components Which Work under Cost-Free Warranty Policy

In this chapter an industrial system working with two components under a cost-free warranty policy has been investigated. Some authors only deal with systems that work continuously and don't take a rest. The authors in this chapter paid their attention to the industrial system which after working for a random time period takes rest and again starts working. This strategy helps the system to run for a long time without the failure of its machinery. After taking a rest, the system starts working again. In this chapter, during the formulation of model all the failures and repair rates have been taken constant. The reliability of the system is obtained. Also, the various parameters that affect the system performance have been shown.

4.1 Introduction

Industries are the backbone of any country. For the development of each nation, many industries are operating round the clock. Practically, some industries operate 24*7. In these industries complex and heavy machines have been installed for the production of the product. In order to produce the product, companies have to make strategies so that their product may survive in the market against its competitor. The company has to look into various aspects like the design of the product, manufacturing of the product, proper installation of the product, spare parts of the product. Besides this, the consumer of the product is also aware of the market trend. Before purchasing any product, these days, a buyer, first of all, compares the product with other products and checks the features of the product with other products. This attention is also paid at the time of purchasing the product that repair of this product is easily available and spare parts of the product are easily available at less cost. Keeping all these things in mind, the manufacture of the product gives a warranty on the product. This warranty is an assurance to the customer that if the product fails without performing its intended task then the manufacturer will repair or replace the product at his own expense. In this way, the warranty of the product has a significant role in the sale of any product. Here in this chapter authors focused on the concept of free replacement warranty (FRW). Under this warranty, on the failure of the product, the product is properly inspected by the repairman who checks what the cause of the product failure was. If the product fails due to the negligence of the user the repairman declares that the service contract is finished and all the outlays of repair are endured by the user. On the other hand, when the product fails not due to the negligence of the user and the product is in warranty period then the whole expense of repair or replacement is borne by the manufacturer. Here the authors also paid attention to the working of the system. Literature shows that industries work continuously to get the maximum output. But practically, the continuous working of the machine increases the failure rate of the machine also it badly affects the revenue of the manufacturer. In this chapter, we introduce a system with two independent components. These two components work independently. The system takes rest after working for some time period, after taking complete rest, system restarts its working and work with full efficiency. This reduces the failure rate of the components. When either of the components fails it goes for the inspection where the repairman properly inspects the component and checks whether the component is under warranty or not.

Author [105] has already analyzed the performance of a single unit industrial system under cost free warranty policy. No one has ever analyzed the performance of two unit industrial system under cost free warranty policy.

4.2 Description of the System

In this chapter, the authors have paid attention to an industrial system that has two components. The system's components follow exponential distribution. The considered system works under the following assumptions.

4.2.1 Assumptions

(i) **Assumption 1:** The system has two independent components.

(ii) **Assumption 2:** A single repairman is always available for the repair of these components.

(iii) **Assumption 3:** The system works for a certain time period, then it goes to the rest. After rest, the system resumes its working.

(iv) **Assumption 4:** Repair to the user is free of cost during the warranty period provided the failure is not due to the carelessness of the user.

(v) **Assumption 5:** On component failure, it is checked by the repairman that the component is in warranty period or not.

(vi) **Assumption 6:** In the respite, no component will fail, however a failed component may be repaired.

(vii) **Assumption 7:** After repair, the component work with full efficiency.

(viii) **Assumption 8:** The repair and failure rates have been taken constant.

4.2.2 State Depiction

The systems various possible states are listed in the following Table 4.1(a).

State	Description
S_0	Initially, both components work perfectly
S_1	The structure is in a rest state after working for a certain time period
S_2	The structure is in a degraded state when its first component fails
S_3	The structure is in a degraded state and it is declared by the repairman the component has failed due to the negligence of the user
S_4	The structure is in a degraded state. It is declared by the repairman the component has failed due to the negligence of the user and the system is in the rest period

S_5	The structure is in a degraded state and it is declared by the repairman the component has not failed due to the negligence of the user
S_6	The structure works with full efficiency when its first failed component is repaired in warranty period
S_7	The structure is in the rest in the warranty period when its first component has been repaired
S_8	The structure is in a degraded state due to the failure of the first component and it is in the rest period
S_9	The structure is in a failed state when its second component also fails
S_{10}	The structure is in a failed state and it is declared by the repairman the second component has failed due to the negligence of the user
S_{11}	The structure is in a failed state. It is declared by the repairman the second component has failed due to the negligence of the user and the structure is in the rest period
S_{12}	The structure is in a failed state and it is declared by the repairman the second component has not failed due to the negligence of the user
S_{13}	The structure works with full efficiency when its both failed components are repaired in the warranty period
S_{14}	The structure is in the rest in the warranty period
S_{15}	The structure is in the rest when it is in the failed state

Table 4.1(a): State narratives

4.2.3 Nomenclature

The following nomenclature will be used in the throughout paper.

λ	The constant failure rate of the single unit
λ_1	The simultaneous failure rate of both the components
μ	Constant repair rate of the single unit
μ_1	Constant repair rate of both units
a	Transition rate with that the system goes to rest
b	Transition rate with that the system comes back to operating Condition
h	Constant scrutiny rate of the unsucessful unit
p/q	Represents warranty is completed/not completed
$p_0(t)$	Represents the likelihood that at any time *t*, the structure is in the good state
$p_1(t)$	Represents the likelihood that at any time *t*, the structure is in the rest state
$p_2(t)$	Represents the likelihood that at any time *t*, the structure is in the degraded state when its first component fails
$p_3(t)$	Represents the likelihood that at any time *t*, the structure is in the degraded state it is declare by the repairmen structure has failed due to the negligence of the user
$p_4(t)$	Represents the likelihood that at any time *t*, the structure is in a degraded state and it is declared by the repairman the component has failed due to the negligence of the user and the structure is in the rest period

$p_5(t)$	Represents the likelihood that at any time *t*, the structure is in the degraded state and it is declared by the repairman the component has not failed due to the negligence of the user
$p_6(t)$	Represents the likelihood that at any time *t*, the structure works with full efficiency when its first failed component is repaired in warranty period
$p_7(t)$	Represents the likelihood that at any time *t*, the structure is in the rest in warranty period
$p_8(t)$	Represents the likelihood that at any time *t*, the structure is in the degraded state due to the failure of the first component and it is in the rest period
$p_9(t)$	Represents the likelihood that at any time *t*, the structure is in a failed state when its second component additionally fails
$p_{10}(t)$	Represents the likelihood that at any time *t*, the structure is in failed state and it's declared by the repairman the second component has failed due to the negligence of the user
$p_{11}(t)$	Represents the likelihood that at any time *t*, the structure is in the failed state. It is declared by the repairman the second component has failed due to the negligence of the user and structure is in the rest period
$p_{12}(t)$	Represents the likelihood that at any time *t*, the structure is in the failed state and it is declared by the repairman the second component has not failed due to the negligence of the user
$p_{13}(t)$	Represents the likelihood that at any time *t*, the structure works with full efficiency when it's both failed component is repaired in warranty period
$p_{14}(t)$	Represents the likelihood that at any time *t*, the structure is in the rest in warranty period

$p_{15}(t)$	Represents the likelihood that at any time t, the structure is in the rest when it is in the failed state
$\overline{p}(s)$	Represents the Laplace transformation of $p(t)$

Table 4.1(b): Notations

4.3 Model Analysis

The model consists of two freelancer components. One trained worker takes care of the system's performance every time. Initially, both the units are operational. Once one component fails inside the guarantee period then it is inspected by the repairman who declares that the component has failed due to the negligence of the user and the user will have to bear all the cost otherwise company will bear the cost of the failed component. The whole system goes to the respite period after functioning for some time period. The system starts working after taking complete rest. Within the rest period, no component will fail, but the failed component will be repaired. Based on all assumptions the state transition diagram is shown below.

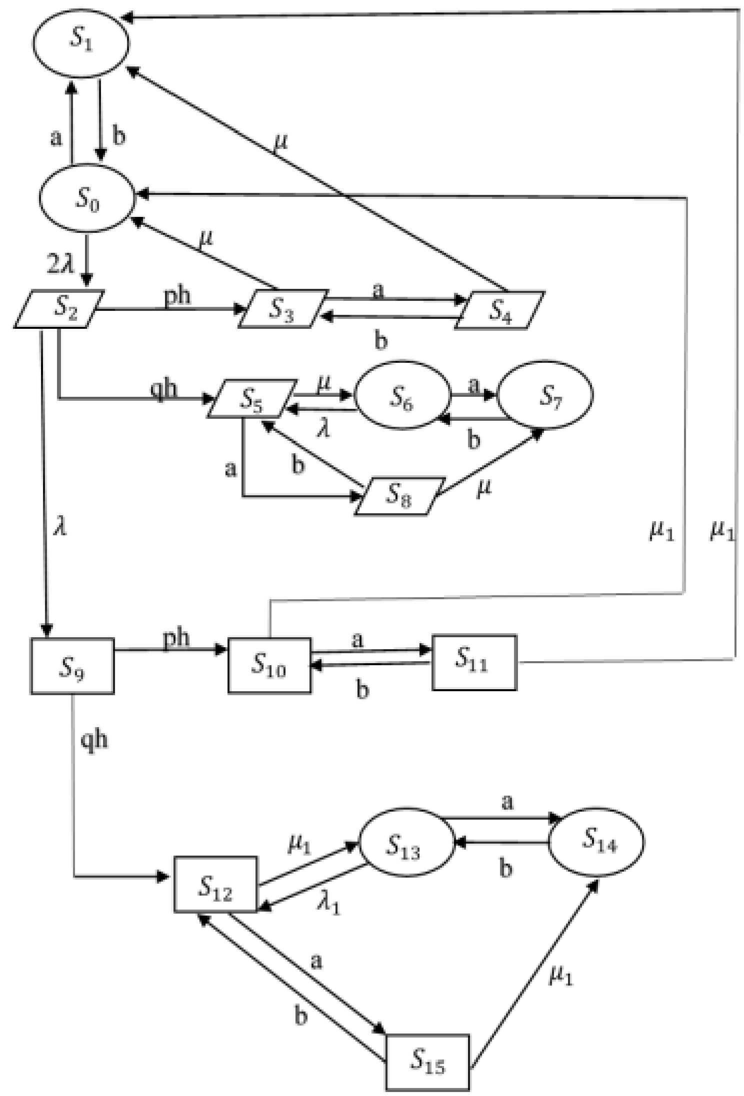


Figure 4.1: State transition diagram of the two unit system

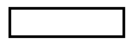
	Indicates a good state.
	Indicates a degraded state.
	Indicates a failed state.

Table 4.2: State representation diagrams

4.4 Mathematical Modeling of the System

$$\left[\frac{d}{dt}+a+2\lambda\right]p_0(t)=b\ p_1(t)+\mu\ p_3(t)+\mu_1\ p_{10}(t) \quad (4.1)$$

$$\left[\frac{d}{dt}+b\right]p_1(t)=a\ p_0(t)+\mu\ p_4(t)+\mu_1\ p_{11}(t) \quad (4.2)$$

$$\left[\frac{d}{dt}+h+\lambda\right]p_2(t)=2\lambda\ p_0(t) \quad (4.3)$$

$$\left[\frac{d}{dt}+a+\mu\right]p_3(t)=ph\ p_2(t)+b\ p_4(t) \quad (4.4)$$

$$\left[\frac{d}{dt}+\mu+b\right]p_4(t)=a\ p_3(t) \quad (4.5)$$

$$\left[\frac{d}{dt}+\mu+a\right]p_5(t)=qh\ p_2(t)+\lambda\ p_6(t)+b\ p_8(t) \quad (4.6)$$

$$\left[\frac{d}{dt}+\lambda+a\right]p_6(t)=\mu\ p_5(t)+bp_7(t) \quad (4.7)$$

$$\left[\frac{d}{dt}+b\right]p_7(t)=a\ p_6(t)+\mu\ p_8(t) \quad (4.8)$$

$$\left[\frac{d}{dt}+\mu+b\right]p_8(t)=a\ p_5(t) \quad (4.9)$$

$$\left[\frac{d}{dt}+h\right]p_9(t)=\lambda\, p_2(t) \tag{4.10}$$

$$\left[\frac{d}{dt}+\mu_1+a\right]p_{10}(t)=ph\, p_9(t)+b\, p_{11}(t) \tag{4.11}$$

$$\left[\frac{d}{dt}+\mu_1+b\right]p_{11}(t)=a\, p_{10}(t) \tag{4.12}$$

$$\left[\frac{d}{dt}+\mu_1+a\right]p_{12}(t)=\lambda_1\, p_{13}(t)+b\, p_{15}(t)+qh\, p_9(t) \tag{4.13}$$

$$\left[\frac{d}{dt}+\lambda_1+a\right]p_{13}(t)=\mu_1\, p_{12}(t)+b\, p_{14}(t) \tag{4.14}$$

$$\left[\frac{d}{dt}+b\right]p_{14}(t)=a\, p_{13}(t)+\mu_1 p_{15}(t) \tag{4.15}$$

$$\left[\frac{d}{dt}+\mu_1+b\right]p_{15}(t)=a\, p_{12}(t) \tag{4.16}$$

and initial conditions are

$$p_i(0)=\begin{cases}1 & ;\ i=0\\ 0 & ;\ i\neq 0\end{cases} \tag{4.17}$$

Taking Laplace transformation of all the equations, we get

$$\left[s+a+2\lambda\right]\overline{p}_0(s)=1+b\,\overline{p}_1(s)+\mu\,\overline{p}_3(s)+\mu_1\,\overline{p}_{10}(s) \tag{4.18}$$

$$\left[s+b\right]\overline{p}_1(s)=a\,\overline{p}_0(s)+\mu\,\overline{p}_4(s)+\mu_1\,\overline{p}_{11}(s) \tag{4.19}$$

$$\left[s+h+\lambda\right]\overline{p}_2(s)=2\lambda\,\overline{p}_0(s) \tag{4.20}$$

$$\left[s+a+\mu\right]\overline{p}_3(s)=ph\,\overline{p}_2(s)+b\,\overline{p}_4(s) \tag{4.21}$$

$$\left[s+\mu+b\right]\overline{p}_4(s)=a\,\overline{p}_3(s) \tag{4.22}$$

$$[s+\mu+a]\bar{p}_5(t) = qh\,\bar{p}_2(s) + \lambda\,\bar{p}_6(s) + b\,\bar{p}_8(s) \tag{4.23}$$

$$[s+\lambda+a]\bar{p}_6(s) = \mu\,\bar{p}_5(s) + b\bar{p}_7(s) \tag{4.24}$$

$$[s+b]\bar{p}_7(s) = a\,\bar{p}_6(s) + \mu\,\bar{p}_8(s) \tag{4.25}$$

$$[s+\mu+b]\bar{p}_8(s) = a\,\bar{p}_5(s) \tag{4.26}$$

$$[s+h]\bar{p}_9(s) = \lambda\,\bar{p}_2(s) \tag{4.27}$$

$$[s+\mu_1+a]\bar{p}_{10}(s) = ph\,\bar{p}_9(s) + b\,\bar{p}_{11}(s) \tag{4.28}$$

$$[s+\mu_1+b]\bar{p}_{11}(s) = a\,\bar{p}_{10}(s) \tag{4.29}$$

$$[s+\mu_1+a]\bar{p}_{12}(s) = \lambda_1\,\bar{p}_{13}(s) + b\,\bar{p}_{15}(s) + qh\,\bar{p}_9(s) \tag{4.30}$$

$$[s+\lambda_1+a]\bar{p}_{13}(s) = \mu_1\,\bar{p}_{12}(s) + b\,\bar{p}_{14}(s) \tag{4.31}$$

$$[s+b]\bar{p}_{14}(s) = a\,\bar{p}_{13}(s) + \mu_1\,\bar{p}_{15}(s) \tag{4.32}$$

$$[s+\mu_1+b]\bar{p}_{15}(s) = a\,\bar{p}_{12}(s) \tag{4.33}$$

$$\bar{p}_0(s) = \frac{1}{(s+a+2\lambda) - bM(s) - \mu E(s) - \mu_1 F(s)} \tag{4.34}$$

$$\bar{p}_1(s) = M(s)\,\bar{p}_0(s) \tag{4.35}$$

$$\bar{p}_2(s) = N(s)\,\bar{p}_0(s) \tag{4.36}$$

$$\bar{p}_3(s) = E(s)\,\bar{p}_0(s) \tag{4.37}$$

$$\bar{p}_4(s) = O(s)\,E(s)\,\bar{p}_0(s) \tag{4.38}$$

$$\bar{p}_5(s) = \frac{C(s)}{B(s)}\,\bar{p}_0(s) \tag{4.39}$$

$$\bar{p}_6(s) = \frac{A(s)\,C(s)}{B(s)}\,\bar{p}_0(s) \tag{4.40}$$

$$\bar{p}_7(s) = D(s)\,\bar{p}_0(s) \tag{4.41}$$

$$\bar{p}_8(s) = \frac{R(s)\,C(s)}{B(s)}\,\bar{p}_0(s) \tag{4.42}$$

$$\bar{p}_9(s) = S(s)\,\bar{p}_0(s) \tag{4.43}$$

$$\bar{p}_{10}(s) = F(s)\,\bar{p}_0(s) \tag{4.44}$$

$$\bar{p}_{11}(s) = T(s)\,F(s)\,\bar{p}_0(s) \tag{4.45}$$

$$\bar{p}_{12}(s) = \frac{I(s)}{G(s)}\,\bar{p}_0(s) \tag{4.46}$$

$$\bar{p}_{13}(s) = \frac{H(s)\,I(s)}{G(s)}\,\bar{p}_0(s) \tag{4.47}$$

$$\bar{p}_{14}(s) = \frac{U(s)\,I(s)}{G(s)}\,\bar{p}_0(s) \tag{4.48}$$

$$\bar{p}_{15}(s) = \frac{T(s)\,I(s)}{G(s)}\,\bar{p}_0(s) \tag{4.49}$$

Where $A(s) = \dfrac{\mu\left[(s+b)(s+b+\mu)+ba\right]}{(s+b+\mu)\left[(s+\lambda+a)(s+b)-ba\right]}$

$$B(s) = \left[(s+\mu+a) - \lambda\,A(s) - \frac{ba}{s+\mu+b}\right]$$

$$C(s) = \frac{2\lambda qh}{s+\lambda+h}$$

$$D(s) = \left[\frac{a}{s+b}\,A(s) + \frac{\mu\,a}{(s+b)\,(s+b+\mu)}\right]\frac{C(s)}{B(s)}$$

$$E(s) = \frac{2\lambda ph\,(s+\mu+b)}{(s+\lambda+h)\left[(s+\mu+a)(s+\mu+b)-ba\right]}$$

$$F(s) = \frac{2\lambda^2\,ph\,(s+\mu_1+b)}{(s+\lambda+h)(s+h)\left[(s+\mu_1+a)(s+\mu_1+b)-ba\right]}$$

$$G(s) = \left[(s+\mu_1+a) - \lambda_1\,H(s) - \frac{ba}{s+\mu_1+b}\right]$$

$$H(s)=\frac{\mu_1(s+b)(s+\mu_1+b)+\mu_1 a b}{(s+\mu_1+b)[(s+\lambda_1+a)(s+b)-ba]}$$

$$I(s)=\frac{2\lambda^2 qh}{(s+h)(s+\lambda+h)}$$

$$M(s)=\frac{1}{(s+b)}\left[\frac{\mu a}{s+b+\mu}E(s)+\frac{\mu_1 a F(s)}{(s+b+\mu_1)}+a\right]$$

$$R(s)=\frac{a}{s+b+\mu}$$

$$T(s)=\frac{a}{s+b+\mu_1}$$

$$U(s)=\left[\frac{a}{s+b}H(s)+\frac{\mu_1 a}{(s+b+\mu_1)(s+b)}+a\right]$$

Hence we obtain this relationship

$$\sum_{j=0}^{15}\overline{p}_j(s)=\frac{1}{s} \tag{4.50}$$

It is very difficult to find the inverse Laplace of these state transition probabilities since expressions of probabilities are very complex and complicated. Therefore, the reliability of the system is obtained for the different combinations of parameters. The system’s up state and downstate probabilities are given by:

$$\begin{aligned}\overline{p}_{up}(s)=\ &\overline{p}_0(s)+\overline{p}_1(s)+\overline{p}_2(s)+\overline{p}_3(s)+\overline{p}_4(s)+\overline{p}_5(s)+\overline{p}_6(s)\\&+\overline{p}_7(s)+\overline{p}_8(s)+\overline{p}_{13}(s)+\overline{p}_{14}(s)\end{aligned} \tag{4.51}$$

$$\overline{p}_{down}(s)=\overline{p}_9(s)+\overline{p}_{10}(s)+\overline{p}_{11}(s)+\overline{p}_{12}(s)+\overline{p}_{15}(s) \tag{4.52}$$

4.5 Assessment of Reliability of the System

In equation (4.51) take all repair rates equal to zero, i.e., $\mu=\mu_1=0$ and set $b=0.6,\ \lambda_1=0.05,\ h=0.9,\ p=q=0.5$. We get an expression of reliability which is a function of λ and taking the inverse Laplace transformation of this fresh equation and fix $a = 0.2$ vary λ from 0.1 to 0.3 in the step size of 0.1, we get tables and graph given below:

Time (*t*)	**Reliability R(*t*)**
0	1.0000000000
1	1.0000000000
2	0.9261366400
3	0.8714180320
4	0.8465212820
5	0.8388996550
6	0.8394256150
7	0.8432583470

Table 4.3(a): Reliability of the system ($\lambda = 0.1$)

Time(*t*)	**Reliability R(*t*)**
0	1.0000000000
1	0.9947360069
2	1.0000000000
3	0.9801073219
4	0.8892734487
5	0.7368182396
6	0.5545020171
7	0.3795646629

Table 4.3(b): Reliability of the system ($\lambda = 0.2$)

Time(t)	Reliability R(t)
0	1.0000000000
1	0.9780219745
2	0.9712457419
3	0.9238154756
4	0.7957831779
5	0.6029108497
6	0.3898205409
7	0.2019382563

Table 4.3(c): Reliability of the system ($\lambda = 0.3$)

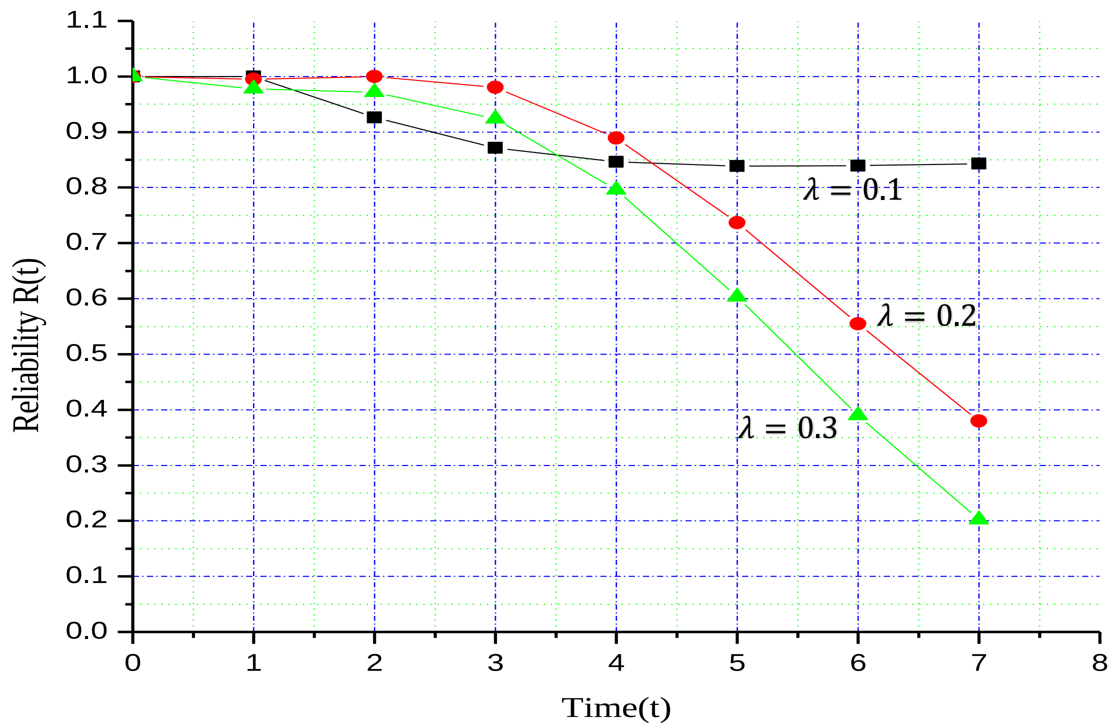


Figure 4.2: Reliability of the system for different values of λ

Now fix $\lambda = 0.1$ and vary a=0.2 to a=0.4 with step size 0.1 we get tables given below and the graph.

Time (*t*)	**Reliability R(*t*)**
0	1.0000000000
1	1.0000000000
2	0.9946729023
3	0.9946729023
4	0.9083449404
5	0.7571288983
6	0.5754577038
7	0.4101022202

Table 4.4 (a): Reliability of System ($a = 0.2$)

Time (*t*)	**Reliability R(*t*)**
0	1.0000000000
1	1.0000000000
2	1.0000000000
3	1.0000000000
4	0.9419648270
5	0.8249911514
6	0.6777189510
7	0.5322932574

Table 4.4(b): Reliability of System ($a = 0.3$)

Time (t)	Reliability R(t)
0	1.0000000000
1	1.0000000000
2	0.9261366400
3	0.8714180320
4	0.8465212820
5	0.8388996550
6	0.8394256150
7	0.8432583470

Table 4.4(c): Reliability of System ($a = 0.4$)

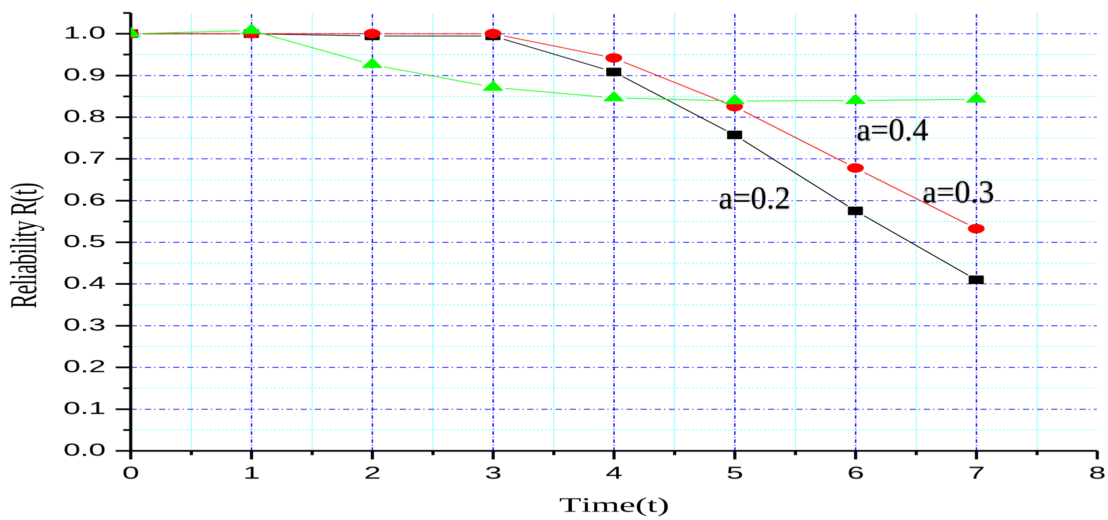


Figure 4.3: Reliability of the system for different values of a

4.6 Result Discussion and Conclusion

In the end, we conclude this chapter. In this chapter, we analyzed the behavior of an industrial system that has two components which work under a free replacement warranty policy. The system rests after working for a specific time period. After taking a rest, the system starts operating again. Our model is an extension of the work done by Niwas and Garg [105], they investigated the performance of a single component system under cost free warranty policy. The present work investigates a two component system under cost free warranty policy. When we compare the reliabilities of the systems, following results are obtained:

When failure rate (λ) is increased:

Authors	Time (t)=7 $\lambda = 0.1$	Time (t)=7 $\lambda = 0.2$	Time (t)=7 $\lambda = 0.3$
Niwas and Garg [105]	0.639605	0.4202751	0.2839525
Kumar and Kumar	0.843258	0.3795646	0.201938
% Difference of Reliability	31.08%	-9.68%	-28.88%

Table 4.5(a): Comparison between existing model and present study

When rest rate (a) is increased

Authors	Time (t)=7 $a = 0.2$	Time (t)=7 $a = 0.3$	Time (t)=7 $a = 0.4$
Niwas and Garg [105]	0.580155	0.612190	0.283952
Kumar and Kumar	0.410102	0.532293	0.843258
% Difference of Reliability	-29.31%	-13.05%	196.97 %

Table 4.5(b): Comparison between existing model and present study

For the lower failure rate $\lambda = 0.1$ reliability of the two unit system is more as compared to a single unit system. But as the failure rate increases $\lambda = 0.2, 0.3$ the reliability of two unit system starts decreasing as compared to a single unit system. Similarly, for the lower rest rate $a = 0.1, 0.2$ reliability of two unit industrial system is less as compared to single unit system. But when the rest rate increases further the two unit system becomes more reliable as compared to single unit system.

Also, we analyze the behavior of different parameter on the system reliability, we firstly vary failure rate λ from 0.1 to 0.3 and we get values given as in Tables 4.3(a), 4.3(b), 4.3(c). From Figure 4.2, it is clear that system reliability is maximum for $\lambda = 0.2$ till the $t = 4$ unit. After $t = 4$ system reliability increases for $\lambda = 0.1$. But the system's reliability continuously decreases for $\lambda = 0.3$. It can be seen that with the increase in the value of λ reliability decreases. In Figure 4.3 the system is equally reliable for $a = 0.2$ and $a = 0.3$ till the time is $t = 4$. System in Figure 4.3 becomes more reliable for $a = 0.4$ after $t = 5$. So, we can see that when the rest rate is increased system reliability increases. So, one can observe when the rest rate is increased the system becomes more reliable also it is more economically beneficial for the industry.

Chapter 5: Analysis of Various Reliability Parameters for Rice Industry

This chapter deals with the analysis of various parameters in view of reliability for a manufacturing plant namely, rice manufacturing plant, for considered conditions as well as availability during the season for the regenerating Markov model. The Laplace transformation has been used to simplify and for the explicit expressions of Availability, Reliability, MTTF and MTBF. The numerical illustrations have been carried out for the data available in the literature. The profit analysis, sensitivity analysis carried out for the considered model.

5.1 Introduction

Rice mill is a food processing industry that converts paddy into rice. Many machines and equipment are installed in the mill for the processing of the paddy. In reliability engineering, a system or a machine is considered as a unique entity that is not further subdivided, although there may be many components in the machine. Failure free operation of the machine is required to get the required output. But the failure of a machine is a random phenomenon that can occur at any point of time in the life period of a machine which may affect the working of the system. This may have bad effects on the environment or on the people working in the mill etc. Therefore, reliability engineering is the thrust area which deals with these problems of the industry and reliability engineers make their efforts to improve the reliability of the product and the engineering system. If the system is reliable and available, then more profit can be earned.

In the present chapter, authors carried out the reliability analysis of the rice mill by taking various important units of the rice mill like cleaning unit, Husking unit, Separation unit, polishing unit and Packing unit. No one in the literature has carried out the reliability analysis of the mill by taking all its units into consideration. Markov Model has been used for modeling the system. For the complete analysis of the system, failure rates and repair rates have been assigned particular values.

5.1.1 Problem Statement

For the failure-free operation of the rice mill, it is necessary to carry out the reliability analysis of the mill. As we know that system is composed of many components. It has been observed that when the system's main components fail then it also affects the system performance. Sometimes failures are so disastrous that they may affect the environment and the health of the people working in the mill. With this problem in mind, we intend to analyze the performance of the rice mill.

5.2 Description of the system

In the present chapter main units of the "Rice mill" have been chosen namely, cleaning unit, husking unit, separation unit, polishing unit, packing unit. All these units are working in a mixed configuration. The system can be in a good state, or in a degraded state or in a failed state when the failure or the repair in the system's units occur. Figure 5.1(a) is the flow diagram of the considered system.

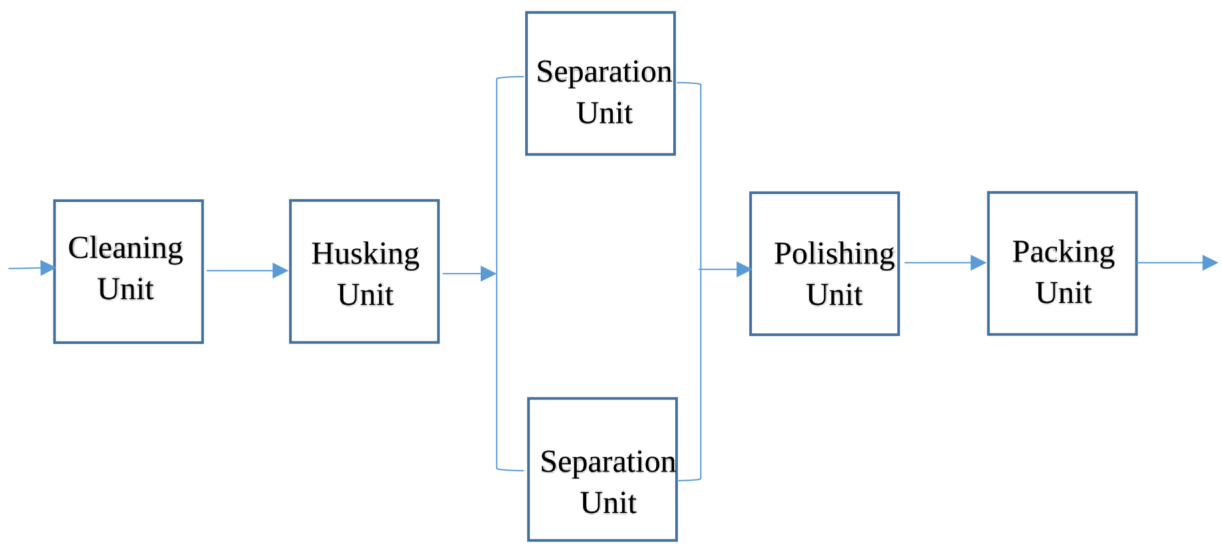


Figure 5.1(a): Configuration of the System

- **Cleaning Unit (A):** A cleaning machine is used to clean the paddy grain. This unit is used to remove all impurities. Due to the failure of this system the whole system fails.
- **Husking Unit (B):** This unit is used to remove the husk from the paddy. When this unit fails then the whole system fails.
- **Separation Unit (C):** This machine is used to separate unhusked paddy from the brain rice. There are two separators. One is an active unit and the other one is in standby redundancy. When the main unit fails the whole load is transferred to the standby unit. This standby unit has the same working capacity as that of the active unit.
- **Polishing Unit (D):** The main function of this unit is to remove the bran layer or germ from the brown rice. Rice is polished white in this unit. When this unit fails then the whole system fails.
- **Packing Unit (E):** The packing of the rice is done by this unit. This process is completed in three steps. First of all, rice is put into plastic bags. After that, the printing machine prints the rice bag. In the third step, the bag is sealed by the machine. When this unit fails then the whole system fails.

5.2.1 Assumptions of the System

- **Assumption 1:** Initially, the system is as good as a new system.
- **Assumption 2:** The system's unit can be only in two states: either good or bad.
- **Assumption 3:** The whole system may be in any of the three states i.e., good, degraded and failed at any time t.
- **Assumption 4:** Repair and failure of units are statistically independent.
- **Assumption 5:** The system units have constant failure and repair rates.
- **Assumption 7:** The repair facility is always available with the system.
- **Assumption 8:** When the main unit of the separation unit fails the whole load is transferred to the standby unit.

5.2.2 Notations and State Description

The notations/state description given in the following Tables 5.1(a) and 5.1(b) will be used in this chapter.

	All components of the system are in good working
	The system is in a degraded state
	The system is in a failed state
$\alpha_i \ (i = 1,2,3,4,5)$	Failure rate of Cleaning unit (A), Husking unit (B), Separation unit (C), Polishing unit (D), Packing unit (E)
$\beta_i \ (i = 1,2,3,4,5)$	Repair rate of Cleaning unit (A), Husking unit (B), Separation unit (C), Polishing unit (D), Packing unit (E)
$P_i(t)$	Probability of the system being in the state S_i at time $'t'$.
$\overline{P}_i(s)$	Laplace transformation of $P_i(t)$

Table 5.1(a): Notations

State	**State description**
S_0	The system is as good as a new one
S_1	The system fails when the cleaning unit of the system fails
S_2	The system fails when the husking unit of the system fails
S_3	The system continues to work when the active separator unit of the system fails and the standby unit takes over the failed unit
S_4	The system fails when the polishing unit of the system fails

S_5	The system fails when the packing unit of the system fails
S_6	The system fails when the cleaning unit of the system fails after the failure of the active separator unit
S_7	The system fails when the husking unit of the system fails after the failure of the active separator unit
S_8	The system fails when the standby separator unit of the system fails after the failure of the active separator unit
S_9	The system fails when the polishing unit of the system fails after the failure of the active separator unit.
S_{10}	The system fails when the polishing unit of the system fails after the failure of the active separator unit.

Table 5.1(b): State descriptions

5.3 Reliability Block Diagram of the System

The block diagram which represents the different state and transition between them throughout its working and non-working condition, given below. The different failures and repairs which may occur during the transition from one state to another state in time $(t, t+\Delta t)$ are shown in the following Figure 5.1(b).

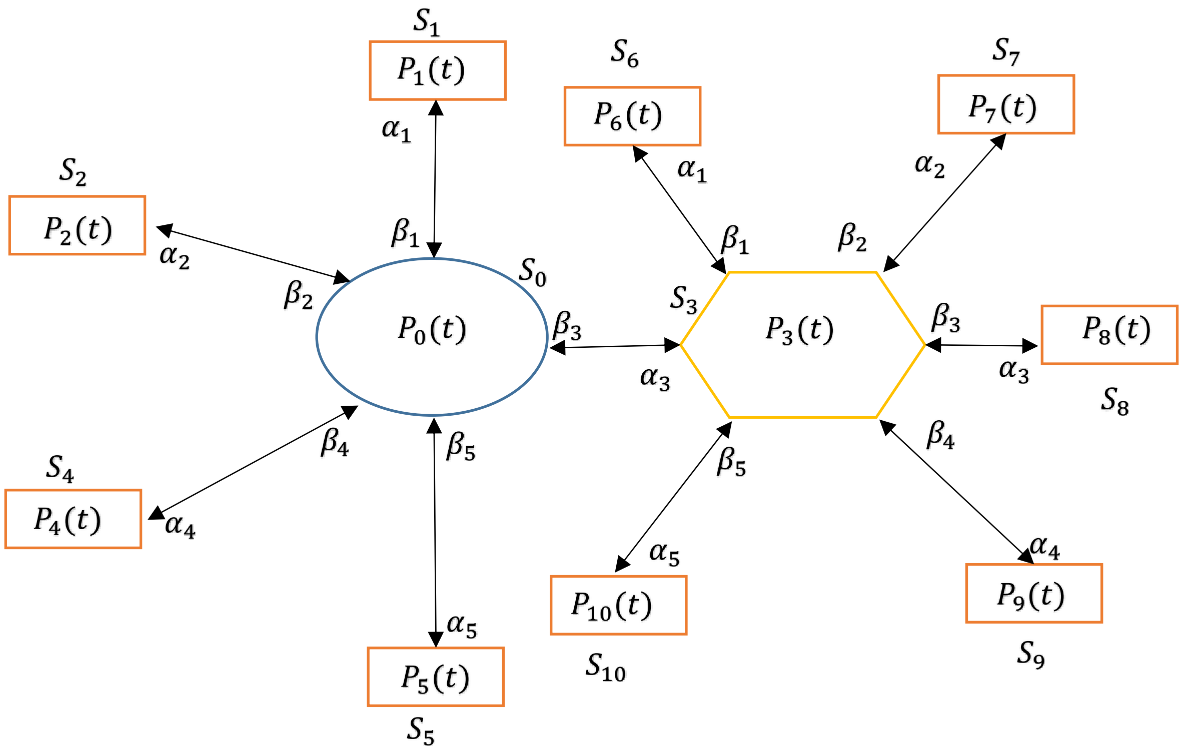


Figure 5.1(b): Block diagram of the system

5.4 Kolmogorov Differential Equations

In this section of the chapter, we develop Kolmogorov differential equation from the block diagram given in Figure 5.1(b) at time $t+\Delta t$ and letting $\Delta t \rightarrow 0$. The equations (5.1)-(5.12) can be written as follows:

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5\right]P_0(t)=\beta_1 P_1(t)+\beta_2 P_2(t)+\beta_3 P_3(t)+\beta_4 P_4(t)+\beta_5 P_5(t) \quad (5.1)$$

$$\left[\frac{d}{dt}+\beta_1\right]P_1(t)=\alpha_1 P_0(t) \quad (5.2)$$

$$\left[\frac{d}{dt}+\beta_2\right]P_2(t)=\alpha_2 P_0(t) \quad (5.3)$$

$$\left[\frac{d}{dt}+\beta_3+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5\right]P_3(t)=\alpha_3\ P_0(t)+\beta_1\ P_6(t)+\beta_2\ P_7(t)+\beta_3\ P_8(t)+\beta_4\ P_9(t)+\beta_5\ P_{10}(t)$$

(5.4)

$$\left[\frac{d}{dt}+\beta_4\right]P_4(t)=\alpha_4\ P_0(t) \quad (5.5)$$

$$\left[\frac{d}{dt}+\beta_5\right]P_5(t)=\alpha_5\ P_0(t) \quad (5.6)$$

$$\left[\frac{d}{dt}+\beta_1\right]P_6(t)=\alpha_1\ P_3(t) \quad (5.7)$$

$$\left[\frac{d}{dt}+\beta_2\right]P_7(t)=\alpha_2\ P_3(t) \quad (5.8)$$

$$\left[\frac{d}{dt}+\beta_3\right]P_8(t)=\alpha_3\ P_3(t) \quad (5.9)$$

$$\left[\frac{d}{dt}+\beta_4\right]P_9(t)=\alpha_4 P_3(t) \quad (5.10)$$

$$\left[\frac{d}{dt}+\beta_5\right]P_{10}(t)=\alpha_5 P_3(t) \quad (5.11)$$

Initial conditions:

$$P_0(0)=1 \text{ and } P_j(0)=0 \text{ at } t=0 \text{ and } j=1,...,10 \quad (5.12)$$

On taking Laplace transformation of equation (5.1)-(5.11), we get,

$$[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5]\overline{P}_0(s)=1+\beta_1\overline{P}_1(s)+\beta_2\overline{P}_2(s)+\beta_3\overline{P}_3(s)+\beta_4\overline{P}_4(s)+\beta_5\overline{P}_5(s)$$

(5.13)

$$[s+\beta_3+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5]\overline{P}_3(s)=\alpha_3\ \overline{P}_0(s)+\beta_1\ \overline{P}_6(s)+\beta_2\ \overline{P}_7(s)+\beta_3\ \overline{P}_8(s)$$
$$+\beta_4\ \overline{P}_9(s)+\beta_5\ \overline{P}_{10}(s)$$

(5.14)

$$[s+\beta_i]\overline{P}_i(s)=\alpha_i\ \overline{P}_0(s) \qquad (i=1,2,4,5) \tag{5.15}$$

$$[s+\beta_i]\overline{P}_j(s)=\alpha_i\ \overline{P}_3(s) \qquad (i=1,2,...,5,\ j=6,7,...,10) \tag{5.16}$$

On solving the above equations, we get

$$\overline{P}_i(s)=\frac{\alpha_i}{[s+\beta_i]}\overline{P}_0(s) \qquad (i=1,2,4,5) \tag{5.17}$$

$$\overline{P}_j(s)=\frac{\alpha_i}{[s+\beta_i]}\overline{P}_3(s) \qquad (i=1,2,...,5,\ j=6,7,...,10) \tag{5.18}$$

$$\overline{P}_3(s)=\frac{\alpha_3}{H_2}\overline{P}_0(s) \tag{5.19}$$

$$\overline{P}_0(s)=\frac{1}{H_1} \tag{5.20}$$

Where

$$H_1=\left[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5-\frac{\beta_1\alpha_1}{s+\beta_1}-\frac{\beta_2\alpha_2}{s+\beta_2}-\frac{\beta_3\alpha_3}{H_2}-\frac{\beta_4\alpha_4}{s+\beta_4}-\frac{\beta_5\alpha_5}{s+\beta_5}\right] \tag{5.21}$$

$$H_2=\left[s+\beta_3+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5-\frac{\beta_1\alpha_1}{s+\beta_1}-\frac{\beta_2\alpha_2}{s+\beta_2}-\frac{\beta_3\alpha_3}{s+\beta_3}-\frac{\beta_4\alpha_4}{s+\beta_4}-\frac{\beta_5\alpha_5}{s+\beta_5}\right] \tag{5.22}$$

After finding state transition probabilities, we can find the system's upstate, which means the probability that the system is in working condition. It may be working in its full capacity or in its reduced capacity. Therefore, the system upstate and downstate are given below:

$$\overline{P}_{up}(s)=\overline{P}_0(s)+\overline{P}_3(s) \tag{5.23}$$

$$\overline{P}_{down}(s)=\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_4(s)+\overline{P}_5(s)+\overline{P}_6(s)+\overline{P}_7(s)+\overline{P}_8(s)+\overline{P}_9(s)+\overline{P}_{10}(s) \tag{5.24}$$

5.5 Reliability Measures of the System

5.5.1 Availability of the System

System availability has a direct relationship with the production of the organization. When the system is available, then more production can be done. This is one of the measures of system reliability. To obtain the time-dependent availability of the system set $\alpha_1 = 0.01$, $\alpha_2 = 0.03$ $\alpha_3 = 0.25$, $\alpha_4 = 0.06$, $\alpha_5 = 0.02$, $\beta_1 = 1$, $\beta_2 = 1$, $\beta_3 = 1$, $\beta_4 = 1$, $\beta_5 = 1$ in (5.23) and taking Inverse Laplace Transform of (5.23), we obtain the time-dependent availability of the system.

$$A(t) = 0.1071428571 \; e^{-1.12000000 \; t} + 0.8928571429 \tag{5.25}$$

Now, vary time t from 0 to 10 in (5.25), we obtain the following Table. 5.2(a)

Time (*t*)	**Availability**
0	1.0000
1	0.9278
2	0.9042
3	0.8965
4	0.8940
5	0.8932
6	0.8929
7	0.8928
8	0.8928
9	0.8928
10	0.8928

Table 5.2(a): Availability of the system

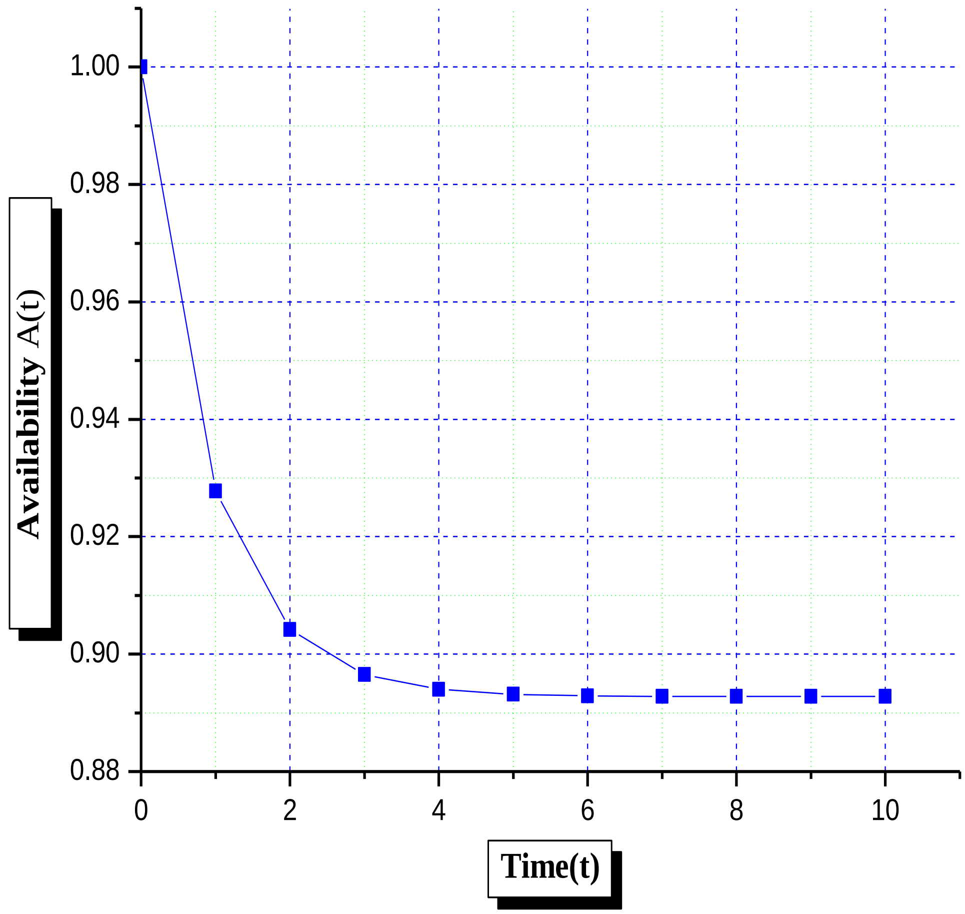


Figure 5.2(a): Availability of the system

5.2.2 Reliability of the System

System reliability is the probability that the system cannot fail before the time period 't'. For the reliability of the system, we set, $\alpha_1 = 0.01, \alpha_2 = 0.03\ \alpha_3 = 0.25$, $\alpha_4 = 0.06$, $\alpha_5 = 0.02$ and repair rate equal to zero in (5.23). Taking the Inverse Laplace of the expression obtained, we get,

$$R(t) = e^{-0.3700000000\ t} + 2e^{-0.2450000000\ t} \sinh(0.1250000000\ )\ t \qquad (5.26)$$

Now, vary time t from 0 to 10 in (5.26), we obtain the following Table 5.2(b).

Time (*t*)	Reliability
0	1.0000
1	0.8869
2	0.7866
3	0.6976
4	0.6187
5	0.5488
6	0.4867
7	0.4317
8	0.3828
9	0.3395
10	0.3011

Table 5.2(b): Reliability of the system

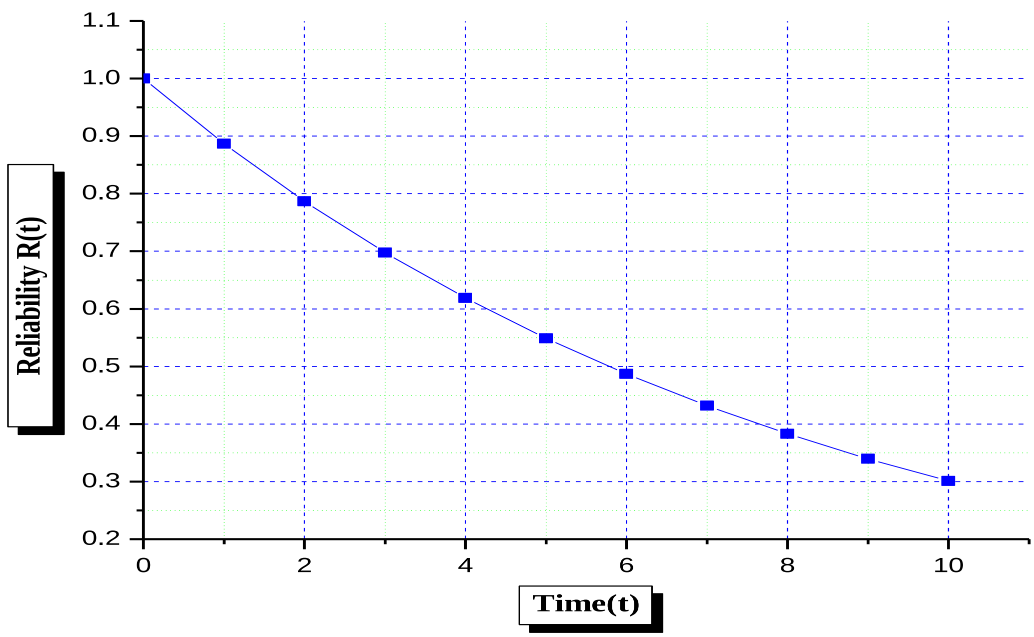


Figure5.2(b): Reliability of the system

5.5.3 MTTF

MTTF is another metric for the reliability of the system. It is the length of time the system, a device or component expected to last in operation. To obtain the MTTF of the system set all repair rates equal to zero in (5.23) and taking limit $s \to 0$ we get the MTTF of the system.

$$MTTF = \int_0^\infty R(t)dt = \lim_{s \to 0} R(s) \tag{5.27}$$

$$MTTF = \frac{(\alpha_1 + \alpha_2 + 2\alpha_3 + \alpha_4 + \alpha_5)}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)^2} \tag{5.28}$$

Now on varying failure rates one by one from 0.01 to 0.09 we obtain the following Table 5.2(c)

Variation in failure rates	α_1	α_2	α_3	α_4	α_5
0.01	4.5288	4.8979	8.2840	5.5664	4.7067
0.02	4.3628	4.7067	8.1632	5.3259	4.5288
0.03	4.2077	4.5288	8.0000	5.1038	4.3628
0.04	4.0625	4.3628	7.8125	4.8979	4.2077
0.05	3.9262	4.2077	7.6124	4.7067	4.0625
0.06	3.7981	4.0625	7.4074	4.5288	3.9262
0.07	3.6776	3.9262	7.2022	4.3628	3.7981
0.08	3.5640	3.7981	7.0000	4.2077	3.6776
0.09	3.4567	3.6776	6.8027	4.0625	3.5640

Table 5.2(c): MTTF of the system

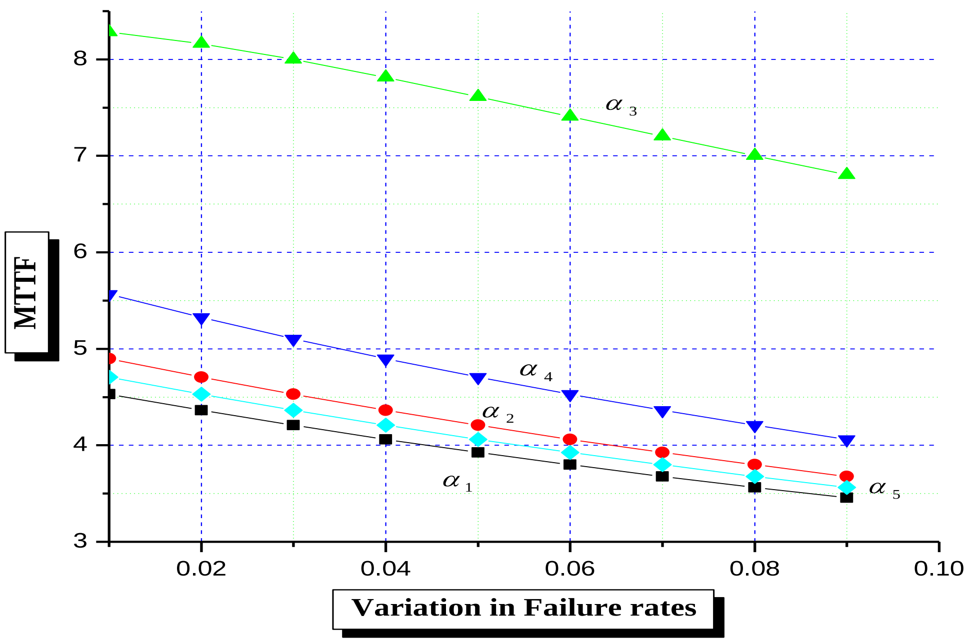


Figure 5.2(c): MTTF of the system

5.5.4 Sensitivity of MTTF

The objective of the sensitivity analysis is to determine the input variables which affect the system most. We perform the sensitivity analysis on the MTTF. Table 5.2(d) shows change in the meantime to failure MTTF of the system resulting from changes in parameters α_1, α_2, α_3, α_4, α_5. Differentiate the equation (5.28) w.r.t failure rates α_1, α_2, α_3, α_4, α_5.Now one by one vary failure rates from 0.01 to 0.09 in these derivatives we obtain the following Table 5.2(d).

Variation in failure rates	$\frac{\partial(MTTF)}{\partial\alpha_1}$	$\frac{\partial(MTTF)}{\partial\alpha_2}$	$\frac{\partial(MTTF)}{\partial\alpha_3}$	$\frac{\partial(MTTF)}{\partial\alpha_4}$	$\frac{\partial(MTTF)}{\partial\alpha_5}$
0.01	-17.1756	-19.8250	-9.1033	-25.0244	-18.4327
0.02	-16.0373	-18.4327	-14.5772	-23.0959	-17.1756
0.03	-15.0036	-17.1756	-17.7777	-21.3718	-16.0373
0.04	-14.0625	-16.0373	-19.5312	-19.8250	-15.0036
0.05	-13.2035	-15.0036	-20.3541	-18.4327	-14.0625
0.06	-12.4176	-14.0625	-20.5761	-17.1756	-13.2035
0.07	-11.6970	-13.2035	-20.4111	-16.0373	-12.4176
0.08	-11.0349	-12.4176	-20.0000	-15.0036	-11.6970
0.09	-10.4252	-11.6970	-19.4363	-14.0625	-11.0349

Table 5.2(d): Sensitivity of the MTTF

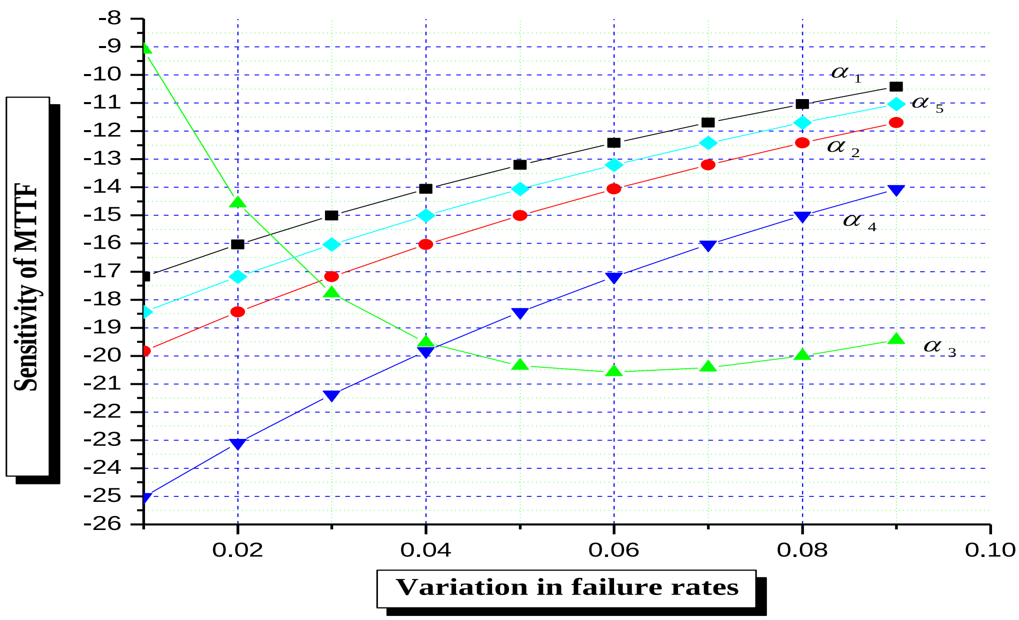


Figure 5.2(d): Sensitivity of the MTTF

5.5.5 Expected Profit

Expected profit of the company, organization is that profit that it can earn at any point of time if the system works as expected. The expected profit of the mill is calculated using the expression given below

$$E_P(t) = K_1 \int_0^t P_{up}(t)\, dt - K_2 t \quad (5.29)$$

Set, $K_1 = 1$, equation (5.29) reduces to,

$$E_P(t) = \int_0^t P_{up}(t)\, dt - K_2 t \quad (5.30)$$

After integration, we get the expression,

$$E_P(t) = -K_2 t - 0.09566326\ e^{-1.20000000\, t} + 0.89285714\ t + 0.095666527 \quad (5.31)$$

In equation (5.31), set service cost $K_2 = 0.1, 0.2, \ldots, 0.5$ and vary t from 0 to 10, we obtain the following Table 5.2(e):

Time	$K_2 = 0.1$	$K_2 = 0.2$	$K_2 = 0.3$	$K_2 = 0.4$	$K_2 = 0.5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8573	0.7573	0.6573	0.5573	0.4573
2	1.6711	1.4711	1.2711	1.0711	0.8711
3	2.4709	2.1709	1.8709	1.5709	1.2709
4	3.2660	2.8660	2.4660	2.0660	1.6666
5	4.0595	3.5595	3.0595	2.5595	2.0595
6	4.8526	4.2526	3.6526	3.0526	2.4526
7	5.6456	4.9456	4.2456	3.5456	2.8456
8	6.4385	5.6385	4.8385	4.0385	3.2385
9	7.2313	6.3313	5.4313	4.5313	3.6313
10	8.0242	7.0242	6.0242	5.0242	4.0242

Table 5.2(e): Expected profit of the system

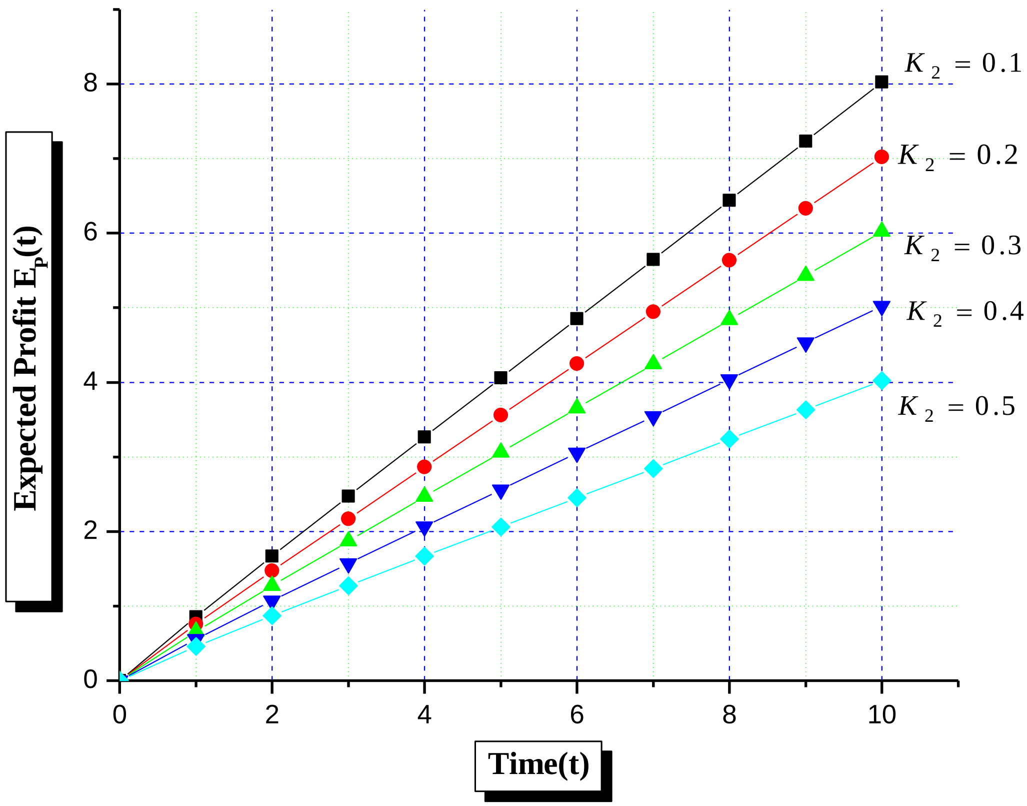


Figure 5.2(e): Expected profit of the system

5.5.6 MTBF

For calculating the MTBF of the Rice mill, initially, we find MTTR of the rice mill from the equation (5.24). MTTR is the average time that a system takes to recover from failure. For MTTR, take limit $s \to 0$ in the equation (5.24), then we get

$$MTTR = \lim_{s\to 0} \overline{P}_{down}(s) \tag{5.32}$$

After finding MTTR of the system, one can easily find MTBF of the system. MTBF is the average mean time between the two failures. For MTBF, find the sum of MTTF and MTTR. Thus, after adding equation (5.28) and (5.32) we get

$$MTBF = MTTF + MTTR \tag{5.33}$$

The expression for the MTBF is given below:

$$MTBF = \left[\begin{array}{l} \frac{1}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5)} + \frac{\alpha_3}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5)^2} \\ + \frac{\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_4}{\beta_4}+\frac{\alpha_5}{\beta_5}}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5)} + \frac{\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}+\frac{\alpha_4}{\beta_4}+\frac{\alpha_5}{\beta_5}\right)\alpha_3}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5)(\beta_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5)} \end{array} \right] \tag{5.34}$$

Set the failure rates and repair rates as $\alpha_1 = 0.01$, $\alpha_2 = 0.03$, $\alpha_3 = 0.25$, $\alpha_4 = 0.06$, $\lambda_5 = 0.02$, $\beta_1 = 1$, $\beta_2 = 1$, $\beta_3 = 1$, $\beta_4 = 1$, $\beta_5 = 1$. In order to obtain the MTBF of the rice mill one by one vary each failure rate from 0.01 to 0.09 as shown in the below Table 5.2 (f).

Variation in failure rates	α_1	α_2	α_3	α_4	α_5
0.01	5.5288	5.8979	9.2840	6.5664	5.7068
0.02	5.3628	5.7067	9.1633	6.3260	5.5288
0.03	5.2077	5.5289	9.0000	6.1038	5.3629
0.04	5.0625	5.3629	8.8125	5.8980	5.2077
0.05	4.9262	5.2077	8.6125	5.7068	5.0625
0.06	4.7982	5.0625	8.4074	5.5289	4.9262
0.07	4.6777	4.9262	8.2022	5.3629	4.7982
0.08	4.5640	4.7982	8.0000	5.2078	4.6776
0.09	4.4568	4.6777	7.8027	5.0625	4.5640

Table 5.2(f): MTBF of the system

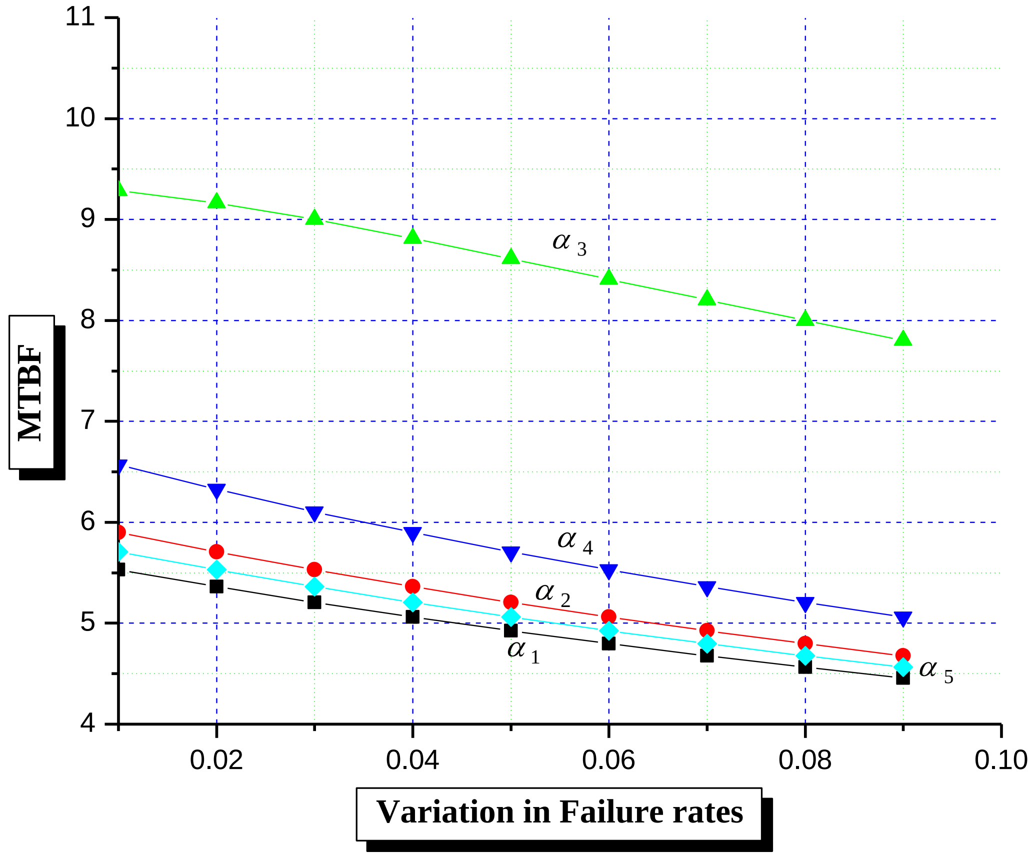


Figure 5.2(f): MTTF of the system

5.6 Results and Discussion

In this chapter, we have developed a model of the rice mill to determine the performance measures of the rice mill. All the main units of the rice mill like the cleaning unit, husking unit, separation unit, polishing unit and packing unit have been taken into consideration. The separation unit has one redundant unit, when the main unit fails the redundant unit takes over the main unit. Markov model has been used to modeling the system and Kolmogorov-differential equations were developed from the transition state diagram of the system and the explicit expression of various performance measures were obtained like

Availability, Reliability, MTTF, Sensitivity of MTTF and Expected profit. Based on the above graphs and tables, we obtain the following important results.

- In Figure 5.2(a), it is observed that system availability decreases very slowly. From t=7 to t=10 System availability is 0.8928
- In Figure 5.2(b), it is observed that as time increases the reliability of the system decreases. At time $t=10$ reliability of the system is 0.3011. This may be due to various factors like aging of the components, corrosion, stress etc.
- In Figure 5.2(c), the MTTF of the system is more w.r.t the variation in the failure rate of the separator unit. This implies that an increase in failure of the third unit doesn't affect the system performance much.
- In Figure 5.2(d), MTTF is more sensitive w.r.t failure rate of separation unit, when we slightly change the value of the failure rate of separation unit, the value of the sensitivity of the MTTF decreases very rapidly and then again starts increasing.
- In Figure 5.2(e), the system expected profit decreases as the service cost of the system increases. So, to earn more profit it is necessary to minimize the expense of the service cost of the system.
- In Figure 5.2(f), the MTBF of the system is more w.r.t the variation in the failure rate of the separator unit. This implies that an increase in failure of the third unit doesn't affect the system performance much.

5.7 Conclusion

In this chapter, we utilized the Markov model to obtain the reliability measure of the rice manufacturing plant. The failure and repair rates have been taken constants. From the above discussion, we conclude the rice mill engineers should pay more attention to the maintenance of the first unit (Cleaning unit) as its MTTF and MTBF are quite low w.r.t variation in the following rate. Also, system's MTTF is very sensitive w.r.t variation in the failure rate of the third unit (Separation unit). Hence, from the above discussion it is concluded that attention should be paid to the working of Cleaning unit and Separation unit. Reliable equipment should be used for this unit for the minimum disruption in the system. Timely preventive maintenance will help to improve the system working. This research is quite useful for the rice mill for improving the performance of the rice mill and for improving their maintenance strategy.

Chapter 6: Reliability Assessment for Multi-State Automatic Ticket Vending Machine (ATVM) through Software and Hardware Failures

This chapter presents the performance analysis of an automatic ticket vending machine (ATVM) through the functioning of its different units. One can easily see long queues at the ticket counter of the railway station during train time. It is not easy to get the ticket quickly because of these long queues or sometimes due to laziness of the staff or due to other reasons. Therefore, these machines have been installed at the railway station for the passengers so that dispensation of the ticket can be easily done. But frequent failures in the ATVM have been observed, therefore in this chapter, we intend to analyze the performance of the ATVM. A precise model of the ATVM has been developed using the Markov process and Kolmogorov differential equations are generated from the model. The developed model has been solved for two kinds of failure/repair rates namely variable and constant. Lagrange’s method and Laplace transformation are used for the solution of the developed model and then different reliability indicators are obtained for ATVM.

6.1 Introduction

In this chapter, the authors’ aim is to analyze the performance of the ATVM which is used for purchasing a ticket/tickets. Every passenger has two options for purchasing the general ticket, the first one is, buy it at the railway ticket counter and another one is to use an automatic ticket vending machine. It is generally observed that people make long queues at the railway ticket counter and wait for their turn. In all this, sometimes they either wait too long in the queue or they may miss their train. So, the government of India has spent a huge amount on installing automatic ticket vending machines at railway stations. Using these machines, one can purchase a ticket/tickets on his own. It reduces the burden on the railway staff and creates a good environment at the railway station. This machine can dispense more than 3000 tickets per day. For purchasing a ticket first of all passenger has to enter the card and then select route and destination using the touch screen monitor. After that, the detail is entered using a touch screen monitor like the total number of passengers, then the machine calculates the amount and the amount is deducted from the card. A ticket is then printed from the ATVM and the passenger carries the ticket with him and takes the train. Sometimes, railway staff is also deployed for issuing the ticket using the ATVM. It is very easy to use ATVM and this service remains available round the clock.

The ticket vending machine is an example of a repairable system. The repairable system is one which when fails, due to any possible failures of the system then through suitable maintenance action, it is brought back to its working condition. On its (ATVM) failure, ATVM components are properly detected by an engineer and after detecting faulty component/components, it is repaired or replaced and ATVM is brought back to service. Thus, the system remains in two periods only that is period of operation and period of failure. This situation can be shown with the help of the following diagram (Figure 6.1(a)).

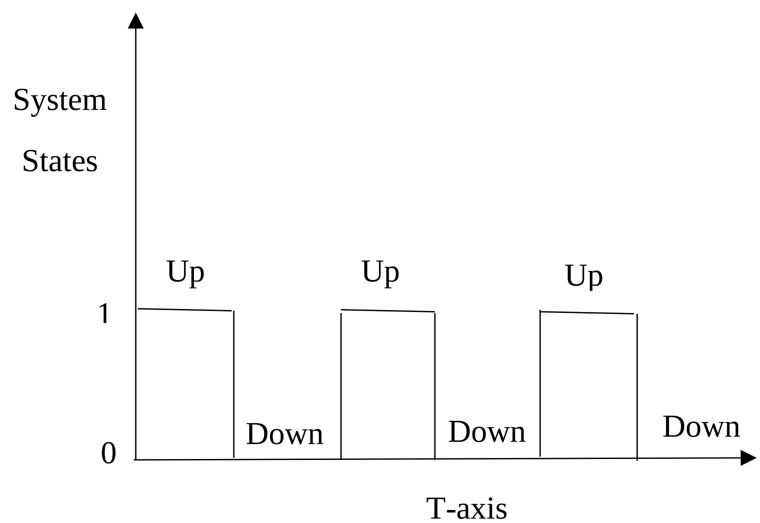


Figure 6.1(a): System's performance history diagram

This diagram given above is called the system history diagram. From this diagram, it is quite obvious that the system oscillates between two states: working and failed. The horizontal lines at level 1 represent the time intervals during which the system is working properly and the horizontal lines at level 0 represent the time intervals during which the system is failed. When the failure rate is less, failures occur less frequently. When the failure rate is more, failures occur quite frequently. In general, the failure rate may be increasing, decreasing, or variable. When the failure rate is decreasing on average the time between two failures increases such a system is known as the “happy system”. When the failure rate is increasing on average then the time between two failures decreases such a system is known as the “sad system”. It is possible that the system failure rate may fluctuate that is it is neither increasing nor decreasing, which is also possible in many engineering systems [81]. In this chapter, the authors considered the variable failure and repair rates of the hardware and software of the ticket vending machine and determined the reliability indices of the ATVM.

Some authors [43], [47], [48], [54], [56], [65], [73], [76], [77], [80], [86], [90], [91] have tried to improve the performance of the railway ticketing system. But none has ever tried to analyze the performance of the ATVM machine incorporating hardware and software failures and repair.

6.2 System Descriptions

The automatic ticket vending machine has the following main hardware and software, description of which is given below. If any of the following hardware or software fails completely then the passenger cannot book the ticket. This is considered as the failure of the ticket vending machine.

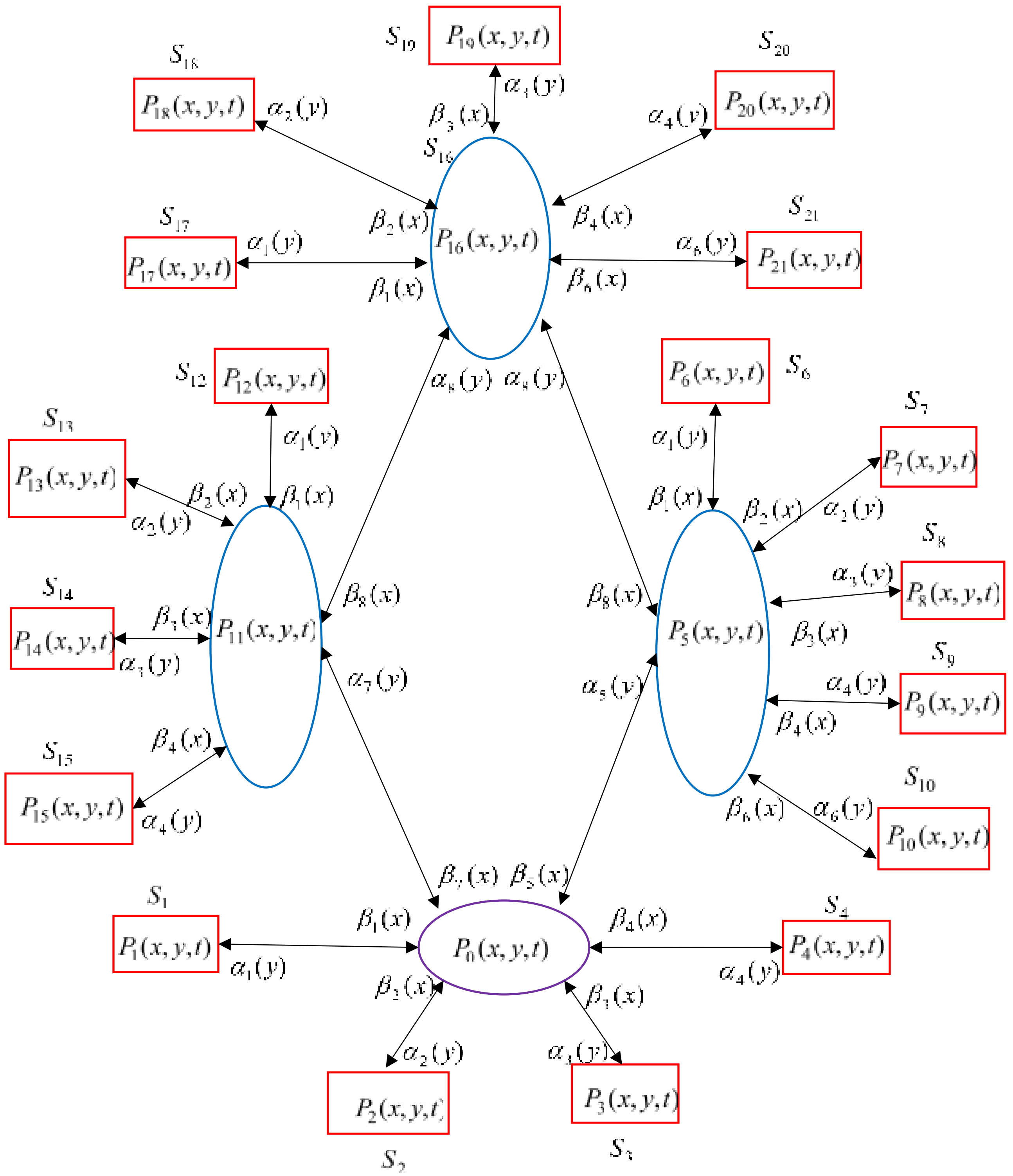
- **Touch screen monitor:** Touch screen monitor is used whenever a passenger wants to book a ticket. Using a touch screen monitor passenger enters the source and the destination and can select any language from the monitor as per his convenience. Nowadays, a platform ticket can be easily purchased using ATVM, for that passenger, enters the instruction with the help of a touch screen monitor. Due to the failure of the touch screen monitor, the ATVM fails completely.
- **Thermal printer:** Thermal printer is used for printing the ticket. This printer supports paper rolls up to the 100-meter in length. From per rol1, 2000 tickets can

be printed. The minimum length of the ticket is 5cm. Due to the failure of the thermal printer, the ATVM fails completely.

- **Card reader sensor:** A card reader sensor is used to read the card. This card can be purchased from the railway ticket counter window. Once the amount is finished, it can be again recharged. When the card reader sensor fails, ATVM fails completely.
- **Software failure:** This machine uses a special software. If it fails, the whole system fails. There may be a minor software failure also and because of this, the system enters into the partial failure state and due to major software failure, the ATVM fails completely.
- **Power supply:** These machines run on the power supply. Generally, the range of the power supply lies in 115-231 volts. Due to the fluctuation of the voltage, there may be the shot circuit then the machine completely fails. When Power goes off, then Power backup UPS automatically takes the whole load of the system and the system runs on the power backup UPS. This state is called the degraded state of the system.
- **Power backup UPS:** Ups in this machine starts its working when the main power goes off. This UPS can work for 30-40 minutes. The life of the UPS is approximately 3-4 years. When the system runs on UPS, the machine continues to work. On the failure of the UPS when the power goes off, the machine fails completely.

In the above discussion, we observe that when the main power supply goes off the power backup UPS system take over and the system enters is in a degraded state in this condition. The different transitions due to failures and repairs are shown in the following state transition diagram (Figure 6.1(b)).

6.1(b) Transition state diagram of ATVM



6.3 Description of the State Transition Diagram

System's transition state diagram is given in Figure 6.1(b). In Figure 6.1(b), the horizontal oval shape represents that, initially, the system is initially as good as a new system. The vertical oval represents, a degraded state due to the power supply failure or minor software failure or due to both the failures simultaneously. All the rectangles in the state transition diagram represent, failed state when any of its main components like touch screen monitor, thermal printer, card reader sensor, Power backup UPS, or software fails. Bi-directional arrows in the diagram represent that system makes its transition to the new state when any component fails or degrades or the power supply goes off and again comebacks to its previous state when the repair activity has been performed correctly by the repairman or the power supply comes back.

6.3.1 Reliability Analysis of the ATVM is based on the Following Assumptions:

This system reliability analysis has been performed on the basis of following assumptions.

- ❖ **Assumption 1:** Initially, all the components of ATVM are in a good working situation.
- ❖ **Assumption 2:** The lifetime of all the components of the ATVM follows the negative exponential distribution.
- ❖ **Assumption 3:** A sufficient repair facility is available.
- ❖ **Assumption 4:** The repairman is called only on the failure of the machine.
- ❖ **Assumption 5:** Failures of hardware, software, and power supply are statistically independent and only one failure can occur at a time.
- ❖ **Assumption 6:** Software failures are of two types' minor software failure and major software failure. In the case of a minor software failure, ATVM enters into a degraded state.
- ❖ **Assumption 7:** There are two types of system failure rates namely: variable and constant.
- ❖ **Assumption 8:** The machine doesn't stop its working in a degraded state and work for only a short period of time.

- **Assumption 9:** Power backup UPS cannot fail in standby mode.
- **Assumption 10:** Whenever UPS fails completely, it is replaced with new UPS.

6.3.2 State Description and Notations of the System

The system is in the following states due to the failure and repair of its components.

S_0	Initially, all the components of the system are as good as a new one
S_1	State in which the touch screen monitor fails and the system fails completely
S_2	State in which thermal printer fails and the system fails completely
S_3	State in which card reader sensor fails and the system fails completely
S_4	State in which software major failure occurs and the system fails completely
S_5	State in which power supply fails and the system runs on ups in a degraded state
S_6	State in which the touch screen monitor fails when the system runs on UPS
S_7	State in which the thermal printer fails when the system runs on UPS
S_8	State in which the card reader sensor fails when the system runs on UPS
S_9	State in which the software fails when the system runs on UPS
S_{10}	State in which UPS of the machine fails completely

S_{11}	State in which minor software failure occur and the system is in a degraded state
S_{12}	State in which the touch screen monitor fails after the minor software failure
S_{13}	State in which the thermal printer fails after the minor software failure
S_{14}	State in which the card reader sensor fails after the minor software failure
S_{15}	State in which software major failure occurs after the minor software failure
S_{16}	State in which, power supply fails after the minor software failures
S_{17}	State in which the touch screen monitor fails after the power supply and software minor failures
S_{18}	State in which thermal printer fails after the power supply and software minor failures
S_{19}	State in which the card reader sensor fails after the power supply and software minor failures
S_{20}	State in which software fails after the power supply and software minor failures
S_{21}	State in which UPS fails after the power supply and software minor failures
t	Time variable
s	Laplace variable

$P_i(x,y,t)$ $i=0,1,\ldots,21$	The system is in state i at time $'t'$ has an elapsed failure time y and elapsed repair time x.
$\overline{P}_i(x,y,s)$ $i=0,1,\ldots,21$	Laplace transformation of $P_i(x,y,t)$.
$P_i(t)$ $i=0,1,\ldots,21$	Probability of the system is in the state S_i at any time $'t'$.
$\overline{P}_i(s)$ $i=0,1,\ldots,21$	Laplace transformation of $P_i(t)$.
$\alpha_i(x); i=1,2,\ldots,8$	The failure rate of Touch screen monitor, Thermal printer, Card reader sensor, Software failure, Power supply failure, UPS failure, Software minor failure, Minor software and power supply failures.
$\beta_i(x); i=1,2,\ldots,8$	Repair rate of Touch screen monitor, Thermal printer, Card reader sensor, Software failure, Power supply failure, UPS failure, Software minor failure, Minor software and power supply failures.

6.1: Notation and state description

6.4 Formulation of the Governing Set of Equations

6.4.1 When Each Failure and Repair Rates is Variable

In this section, we obtain Chapman-Kolmogorov differential equations from (Figure 6.1(b)) when each failure and repair rate is variable. First of all, state transition probabilities at the time $t+\Delta t$ are obtained and when $\Delta t \rightarrow 0$ the following differential equations are obtained

$$\left[\frac{d}{dt}+T_0\right]P_0(t)=C_0 \tag{6.1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_1(x)\right]P_1(x,y,t)=\alpha_1(y)P_0(t) \tag{6.2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_2(x)\right]P_2(x,y,t)=\alpha_2(y)P_0(t) \tag{6.3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_3(x)\right]P_3(x,y,t)=\alpha_3(y)P_0(t) \tag{6.4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_4(x)\right]P_4(x,y,t)=\alpha_4(y)P_0(t) \tag{6.5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+T_1(x,y)\right]P_5(x,y,t)=C_1(x,y,t) \tag{6.6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_1(x)\right]P_6(x,y,t)=\alpha_1(y)P_5(x,y,t) \tag{6.7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_2(x)\right]P_7(x,y,t)=\alpha_2(y)P_5(x,y,t) \tag{6.8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_3(x)\right]P_8(x,y,t)=\alpha_3(y)P_5(x,y,t) \tag{6.9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_4(x)\right]P_9(x,y,t)=\alpha_4(y)P_5(x,y,t) \tag{6.10}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_6(x)\right]P_{10}(x,y,t)=\alpha_6(y)P_5(x,y,t) \tag{6.11}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+T_2(x,y)\right]P_{11}(x,y,t)=C_2(x,y,t) \tag{6.12}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_1(x)\right]P_{12}(x,y,t)=\alpha_1(y)P_{11}(x,y,t) \tag{6.13}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_2(x)\right]P_{13}(x,y,t)=\alpha_2(y)P_{11}(x,y,t) \tag{6.14}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_3(x)\right]P_{14}(x,y,t)=\alpha_3(y)P_{11}(x,y,t) \tag{6.15}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_4(x)\right]P_{15}(x,y,t)=\alpha_4(y)P_{11}(x,y,t) \tag{6.16}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+T_3(x,y)\right]P_{16}(x,y,t)=C_3(x,y,t) \tag{6.17}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_1(x)\right]P_{17}(x,y,t)=\alpha_1(y)P_{16}(x,y,t) \tag{6.18}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_2(x)\right]P_{18}(x,y,t)=\alpha_2(y)P_{16}(x,y,t) \tag{6.19}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_3(x)\right]P_{19}(x,y,t)=\alpha_3(y)P_{16}(x,y,t) \tag{6.20}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_4(x)\right]P_{20}(x,y,t)=\alpha_4(y)P_{16}(x,y,t) \tag{6.21}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\beta_6(x)\right]P_{21}(x,y,t)=\alpha_6(y)P_{16}(x,y,t) \tag{6.22}$$

Boundary conditions:

$$P_1(0,y,t)=\alpha_1(y)P_0(t) \tag{6.23}$$

$$P_2(0,y,t)=\alpha_2(y)P_0(t) \tag{6.24}$$

$$P_3(0,y,t)=\alpha_3(y)P_0(t) \tag{6.25}$$

$$P_4(0,y,t)=\alpha_4(y)P_0(t) \tag{6.26}$$

$$P_5(0,y,t) = \alpha_5(y) P_0(t) \tag{6.27}$$

$$P_6(0,y,t) = \int \alpha_1(y) P_5(x,y,t)\,dx \tag{6.28}$$

$$P_7(0,y,t) = \int \alpha_2(y) P_5(x,y,t)\,dx \tag{6.29}$$

$$P_8(0,y,t) = \int \alpha_3(y) P_5(x,y,t)\,dx \tag{6.30}$$

$$P_9(0,y,t) = \int \alpha_4(y) P_5(x,y,t)\,dx \tag{6.31}$$

$$P_{10}(0,y,t) = \int \alpha_6(y) P_5(x,y,t)\,dx \tag{6.32}$$

$$P_{11}(0,y,t) = \alpha_7(y) P_0(t) \tag{6.33}$$

$$P_{12}(0,y,t) = \int \alpha_1(y) P_{11}(x,y,t)\,dx \tag{6.34}$$

$$P_{13}(0,y,t) = \int \alpha_2(y) P_{11}(x,y,t)\,dx \tag{6.35}$$

$$P_{14}(0,y,t) = \int \alpha_3(y) P_{11}(x,y,t)\,dx \tag{6.36}$$

$$P_{15}(0,y,t) = \int \alpha_4(y) P_{11}(x,y,t)\,dx \tag{6.37}$$

$$P_{16}(0,y,t) = \int \alpha_8(y) P_{11}(x,y,t)\,dx + \int \alpha_8(y) P_5(x,y,t)\,dx \tag{6.38}$$

$$P_{17}(0,y,t) = \int \alpha_1(y) P_{16}(x,y,t)\,dx \tag{6.39}$$

$$P_{18}(0,y,t) = \int \alpha_2(y) P_{16}(x,y,t)\,dx \tag{6.40}$$

$$P_{19}(0,y,t) = \int \alpha_3(y) P_{16}(x,y,t)\,dx \tag{6.41}$$

$$P_{20}(0,y,t) = \int \alpha_4(y) P_{16}(x,y,t)\,dx \tag{6.42}$$

$$P_{21}(0,y,t) = \int \alpha_6(y) P_{16}(x,y,t)\,dx \tag{6.43}$$

Initial conditions:

$$P_i(x,y,0) = 0; \quad i = 1,2,\ldots,21 \tag{6.44}$$

$$P_0(0) = 1 \tag{6.45}$$

Where

$$T_0 = \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y) + \alpha_5(y) + \alpha_7(y)$$

$$T_1(x,y) = \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y) + \alpha_6(y) + \alpha_8(y) + \beta_5(x)$$

$$T_2(x,y) = \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y) + \alpha_8(y) + \beta_7(x)$$

$$T_3(x,y) = \alpha_1(y) + \alpha_2(y) + \alpha_3(y) + \alpha_4(y) + \alpha_6(y) + 2\beta_8(x)$$

$$C_0 = \beta_1(x) P_1(x,y,t) + \beta_2(x) P_2(x,y,t) + \beta_3(x) P_3(x,y,t) + \beta_4(x) P_4(x,y,t) + \int \beta_5(x) P_5(x,y,t) dx + \int \beta_7(x) P_{11}(x,y,t) dx$$

$$C_1(x,y,t) = \beta_1(x) P_6(x,y,t) + \beta_2(x) P_7(x,y,t) + \beta_3(x) P_8(x,y,t) + \beta_4(x) P_9(x,y,t) + \beta_6(x) P_{10}(x,y,t) + \alpha_5(y) P_0(t) + \int \beta_8(x) P_{16}(x,y,t) dx$$

$$C_2(x,y,t) = \beta_1(x) P_{12}(x,y,t) + \beta_2(x) P_{13}(x,y,t) + \beta_3(x) P_{14}(x,y,t) + \beta_4(x) P_{15}(x,y,t) + \alpha_7(y) P_0(t) + \int \beta_8(x) P_{16}(x,y,t) dx$$

$$C_3(x,y,t) = \beta_1(x) P_{17}(x,y,t) + \beta_2(x) P_{18}(x,y,t) + \beta_3(x) P_{19}(x,y,t) + \beta_4(x) P_{20}(x,y,t) + \beta_6(x) P_{21}(x,y,t) + \int \alpha_8(y) P_5(x,y,t) dx + \int \alpha_8(y) P_{11}(x,y,t) dx$$

Solving equations (6.1)-(6.45) with the assistance of Lagrange's method, we obtain:

$$P_0(t) = e^{-T_0 t} \left[1 + \int C_0 \, e^{T_0 t} dt \right] \tag{6.46}$$

$$P_1(x,y,t) = e^{-\int \beta_1(x) dx} \left[\int \alpha_1(y) P_0(t) e^{\int \beta_1(x) dx} dx + \alpha_1(y-x) P_0(t-x) \right] \tag{6.47}$$

$$P_2(x,y,t) = e^{-\int \beta_2(x) dx} \left[\int \alpha_2(y) P_0(t) e^{\int \beta_2(x) dx} dx + \alpha_2(y-x) P_0(t-x) \right] \tag{6.48}$$

$$P_3(x,y,t) = e^{-\int \beta_3(x) dx} \left[\int \alpha_3(y) P_0(t) e^{\int \beta_3(x) dx} dx + \alpha_3(y-x) P_0(t-x) \right] \tag{6.49}$$

$$P_4(x,y,t) = e^{-\int \beta_4(x) dx} \left[\int \alpha_4(y) P_0(t) e^{\int \beta_4(x) dx} dx + \alpha_4(y-x) P_0(t-x) \right] \tag{6.50}$$

$$P_5(x,y,t) = e^{-\int T_1(x,y) dx} \left[\int C_1(x,y,t) e^{\int T_1(x,y) dx} dx + \alpha_5(y-x) P_0(t-x) \right] \tag{6.51}$$

$$P_6(x,y,t) = e^{-\int \beta_1(x)\,dx}\left[\int \alpha_1(y)\,P_5(x,y,t)e^{\int \beta_1(x)\,dx}dx + \int \alpha_1(y-x)\,P_5(x,y-x,t-x)dx\right] \qquad (6.52)$$

$$P_7(x,y,t) = e^{-\int \beta_2(x)\,dx}\left[\int \alpha_2(y)\,P_5(x,y,t)e^{\int \beta_2(x)\,dx}dx + \int \alpha_2(y-x)\,P_5(x,y-x,t-x)dx\right] \qquad (6.53)$$

$$P_8(x,y,t) = e^{-\int \beta_3(x)\,dx}\left[\int \alpha_3(y)\,P_5(x,y,t)e^{\int \beta_3(x)\,dx}dx + \int \alpha_3(y-x)\,P_5(x,y-x,t-x)dx\right] \qquad (6.54)$$

$$P_9(x,y,t) = e^{-\int \beta_4(x)\,dx}\left[\int \alpha_4(y)\,P_5(x,y,t)e^{\int \beta_4(x)\,dx}dx + \int \alpha_4(y-x)\,P_5(x,y-x,t-x)dx\right] \qquad (6.55)$$

$$P_{10}(x,y,t) = e^{-\int \beta_6(x)\,dx}\left[\int \alpha_6(y)\,P_5(x,y,t)e^{\int \beta_6(x)\,dx}dx + \int \alpha_6(y-x)\,P_5(x,y-x,t-x)dx\right] \qquad (6.56)$$

$$P_{11}(x,y,t) = e^{-\int T_2(x,y)\,dx}\left[\int C_2(x,y,t)\,e^{\int T_2(x,y)\,dx}dx + \alpha_7(y-x)\,P_0(t-x)\right] \qquad (6.57)$$

$$P_{12}(x,y,t) = e^{-\int \beta_1(x)\,dx}\left[\int \alpha_1(y)\,P_{11}(x,y,t)e^{\int \beta_1(x)\,dx}dx + \int \alpha_1(y-x)\,P_{11}(x,y-x,t-x)dx\right] \qquad (6.58)$$

$$P_{13}(x,y,t) = e^{-\int \beta_2(x)\,dx}\left[\int \alpha_2(y)\,P_{11}(x,y,t)e^{\int \beta_2(x)\,dx}dx + \int \alpha_2(y-x)\,P_{11}(x,y-x,t-x)dx\right] \qquad (6.59)$$

$$P_{14}(x,y,t) = e^{-\int \beta_3(x)\,dx}\left[\int \alpha_3(y)\,P_{11}(x,y,t)e^{\int \beta_3(x)\,dx}dx + \int \alpha_3(y-x)\,P_{11}(x,y-x,t-x)dx\right] \qquad (6.60)$$

$$P_{15}(x,y,t) = e^{-\int \beta_4(x)\,dx}\left[\int \alpha_4(y)\,P_{11}(x,y,t)e^{\int \beta_4(x)\,dx}dx + \int \alpha_4(y-x)\,P_{11}(x,y-x,t-x)dx\right] \qquad (6.61)$$

$$P_{16}(x,y,t) = e^{-\int T_3(x,y)dx}\left[\begin{array}{r}\int C_3(x,y,t)\,e^{\int T_3(x,y)dx}dx + \int \alpha_8(y-x)\,P_{11}(x,y-x,t-x)\,dx \\ \int \alpha_8(y-x)\,P_5(x,y-x,t-x)\,dx\end{array}\right] \qquad (6.62)$$

$$P_{17}(x,y,t) = e^{-\int \beta_1(x)\,dx}\left[\int \alpha_1(y)\,P_{16}(x,y,t)e^{\int \beta_1(x)\,dx}dx + \int \alpha_1(y-x)\,P_{16}(x,y-x,t-x)dx\right] \qquad (6.63)$$

$$P_{18}(x,y,t) = e^{-\int \beta_2(x)\,dx}\left[\int \alpha_2(y)\,P_{16}(x,y,t)e^{\int \beta_2(x)\,dx}dx + \int \alpha_2(y-x)\,P_{16}(x,y-x,t-x)dx\right] \qquad (6.64)$$

$$P_{19}(x,y,t) = e^{-\int \beta_3(x)\,dx}\left[\int \alpha_3(y)\,P_{16}(x,y,t)e^{\int \beta_3(x)\,dx}dx + \int \alpha_3(y-x)\,P_{16}(x,y-x,t-x)dx\right] \qquad (6.65)$$

$$P_{20}(x,y,t) = e^{-\int \beta_4(x)\,dx}\left[\int \alpha_4(y)\,P_{16}(x,y,t)e^{\int \beta_4(x)\,dx}dx + \int \alpha_4(y-x)\,P_{16}(x,y-x,t-x)dx\right] \quad (6.66)$$

$$P_{21}(x,y,t) = e^{-\int \beta_6(x)\,dx}\left[\int \alpha_6(y)\,P_{16}(x,y,t)e^{\int \beta_6(x)\,dx}dx + \int \alpha_6(y-x)\,P_{16}(x,y-x,t-x)dx\right] \quad (6.67)$$

After the determination of these probabilities one can calculate the availability and reliability of the ATVM by using following mathematical expressions (From Figure 6.1(b)).

$$A(t) = P_0(x,y,t) + P_5(x,y,t) + P_{11}(x,y,t) + P_{16}(x,y,t) \quad (6.68)$$

6.4.2 Mathematical Modeling with Constant Failure and Repair Rates

$$\left[\frac{d}{dt} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_7\right]P_0(t) = \beta_1\,P_1(t) + \beta_2\,P_2(t) + \beta_3\,P_3(t) + \beta_4\,P_4(t) + \beta_5\,P_5(t) + \beta_7\,P_{11}(t) \quad (6.69)$$

$$\left[\frac{d}{dt} + \beta_1\right]P_1(t) = \alpha_1\,P_0(t) \quad (6.70)$$

$$\left[\frac{d}{dt} + \beta_2\right]P_2(t) = \alpha_2\,P_0(t) \quad (6.71)$$

$$\left[\frac{d}{dt} + \beta_3\right]P_3(t) = \alpha_3\,P_0(t) \quad (6.72)$$

$$\left[\frac{d}{dt} + \beta_4\right]P_4(t) = \alpha_4\,P_0(t) \quad (6.73)$$

$$\left[\frac{d}{dt} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_6 + \beta_5 + \alpha_8\right]P_5(t) = \beta_1\,P_6(t) + \beta_2\,P_7(t) + \beta_3\,P_8(t) + \beta_4\,P_9(t) + \beta_6\,P_{10}(t) + \alpha_5\,P_0(t) + \beta_8\,P_{16}(t) \quad (6.74)$$

$$\left[\frac{d}{dt} + \beta_1\right]P_6(t) = \alpha_1\,P_5(t) \quad (6.75)$$

$$\left[\frac{d}{dt} + \beta_2\right]P_7(t) = \alpha_2\,P_5(t) \quad (6.76)$$

$$\left[\frac{d}{dt} + \beta_3\right]P_8(t) = \alpha_3\,P_5(t) \quad (6.77)$$

$$\left[\frac{d}{dt}+\beta_4\right]P_9(t)=\alpha_4\,P_5(t) \tag{6.78}$$

$$\left[\frac{d}{dt}+\beta_6\right]P_{10}(t)=\alpha_6\,P_5(t) \tag{6.79}$$

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\beta_7+\alpha_8\right]P_{11}(t)=\beta_1\,P_{12}(t)+\beta_2\,P_{13}(t)+\beta_3\,P_{14}(t)+\beta_4\,P_{15}(t)+\beta_8\,P_{16}(t)+\alpha_7\,P_0(t) \tag{6.80}$$

$$\left[\frac{d}{dt}+\beta_1\right]P_{12}(t)=\alpha_1\,P_{11}(t) \tag{6.81}$$

$$\left[\frac{d}{dt}+\beta_2\right]P_{13}(t)=\alpha_2\,P_{11}(t) \tag{6.82}$$

$$\left[\frac{d}{dt}+\beta_3\right]P_{14}(t)=\alpha_3\,P_{11}(t) \tag{6.83}$$

$$\left[\frac{d}{dt}+\beta_4\right]P_{15}(t)=\alpha_4\,P_{11}(t) \tag{6.84}$$

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+2\beta_8\right]P_{16}(t)=\beta_1\,P_{17}(t)+\beta_2\,P_{18}(t)+\beta_3\,P_{19}(t)+\beta_4\,P_{20}(t)+\beta_6\,P_{21}(t)+\alpha_8\,P_5(t)+\alpha_8\,P_{11}(t) \tag{6.85}$$

$$\left[\frac{d}{dt}+\beta_1\right]P_{17}(t)=\alpha_1\,P_{16}(t) \tag{6.86}$$

$$\left[\frac{d}{dt}+\beta_2\right]P_{18}(t)=\alpha_2\,P_{16}(t) \tag{6.87}$$

$$\left[\frac{d}{dt}+\beta_3\right]P_{19}(t)=\alpha_3\,P_{16}(t) \tag{6.88}$$

$$\left[\frac{d}{dt}+\beta_4\right]P_{20}(t)=\alpha_4\,P_{16}(t) \tag{6.89}$$

$$\left[\frac{d}{dt}+\beta_6\right]P_{21}(t)=\alpha_6\,P_{16}(t) \tag{6.90}$$

With initial condition;

$$P_i(0)=\begin{cases}1 & i=0\\ 0 & i\neq 0\end{cases} \tag{6.91}$$

Taking Laplace transform of the equations (6.69)-(6.90) we get;

$$\left[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7\right]\overline{P}_0(s)=1+\beta_1\overline{P}_1(s)+\beta_2\overline{P}_2(s)+\beta_3\overline{P}_3(s)+\beta_4\overline{P}_4(s) \\ +\beta_5\overline{P}_5(s)+\beta_7\overline{P}_{11}(s) \tag{6.92}$$

$$\left[s+\beta_1\right]\overline{P}_1(s)=\alpha_1\overline{P}_0(s) \tag{6.93}$$

$$\left[s+\beta_2\right]\overline{P}_2(s)=\alpha_2\overline{P}_0(s) \tag{6.94}$$

$$\left[s+\beta_3\right]\overline{P}_3(s)=\alpha_3\overline{P}_0(s) \tag{6.95}$$

$$\left[s+\beta_4\right]\overline{P}_4(s)=\alpha_4\overline{P}_0(s) \tag{6.96}$$

$$\left[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+\beta_5+\alpha_8\right]\overline{P}_5(s)=\beta_1\overline{P}_6(s)+\beta_2\overline{P}_7(s)+\beta_3\overline{P}_8(s) \\ +\beta_4\overline{P}_9(s)+\beta_6\overline{P}_{10}(s)+\alpha_5\overline{P}_0(s)+\beta_8\overline{P}_{16}(s) \tag{6.97}$$

$$\left[s+\beta_1\right]\overline{P}_6(s)=\alpha_1\overline{P}_5(s) \tag{6.98}$$

$$\left[s+\beta_2\right]\overline{P}_7(s)=\alpha_2\overline{P}_5(s) \tag{6.99}$$

$$\left[s+\beta_3\right]\overline{P}_8(s)=\alpha_3\overline{P}_5(s) \tag{6.100}$$

$$\left[s+\beta_4\right]\overline{P}_9(s)=\alpha_4\overline{P}_5(s) \tag{6.101}$$

$$\left[s+\beta_6\right]\overline{P}_{10}(s)=\alpha_6\overline{P}_5(s) \tag{6.102}$$

$$\left[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_8+\beta_7\right]\overline{P}_{11}(s)=\beta_1\overline{P}_{12}(s)+\beta_2\overline{P}_{13}(s)+\beta_3\overline{P}_{14}(s)+\beta_4\overline{P}_{15}(s) \\ +\beta_8\overline{P}_{16}(s)+\alpha_7\overline{P}_0(s) \tag{6.103}$$

$$\left[s+\beta_1\right]\overline{P}_{12}(s)=\alpha_1\overline{P}_{11}(s) \tag{6.104}$$

$$\left[s+\beta_2\right]\overline{P}_{13}(s)=\alpha_2\overline{P}_{11}(s) \tag{6.105}$$

$$\left[s+\beta_3\right]\overline{P}_{14}(s)=\alpha_3\overline{P}_{11}(s) \tag{6.106}$$

$$[s+\beta_4]\overline{P}_{15}(s)=\alpha_4\,\overline{P}_{11}(s) \tag{6.107}$$

$$[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+2\beta_8]\overline{P}_{16}(s)=\beta_1\,\overline{P}_{17}(s)+\beta_2\,\overline{P}_{18}(s)+\beta_3\,\overline{P}_{19}(s)+\beta_4\,\overline{P}_{20}(s)$$
$$+\beta_6\,\overline{P}_{21}(s)+\alpha_8\,\overline{P}_5(s)+\alpha_8\,\overline{P}_{11}(s)) \tag{6.108}$$

$$[s+\beta_1]\overline{P}_{17}(s)=\alpha_1\,\overline{P}_{16}(s) \tag{6.109}$$

$$[s+\beta_2]\overline{P}_{18}(s)=\alpha_2\,\overline{P}_{16}(s) \tag{6.110}$$

$$[s+\beta_3]\overline{P}_{19}(s)=\alpha_3\,\overline{P}_{16}(s) \tag{6.111}$$

$$[s+\beta_4]\overline{P}_{20}(s)=\alpha_4\,\overline{P}_{16}(s) \tag{6.112}$$

$$[s+\beta_6]\overline{P}_{21}(s)=\alpha_6\,\overline{P}_{16}(s) \tag{6.113}$$

After solving these equations, we get

$$\overline{P_0}(s)=\frac{1}{T_\delta}$$

$$\overline{P_5}(s)=\frac{T_\beta}{H_0}\overline{P_0}(s)$$

$$\overline{P_{11}}(s)=\frac{\beta_8\alpha_8}{H_2H_3T_\alpha}\overline{P_5}(s)+\frac{\alpha_7}{H_2T_\alpha}\overline{P_0}(s)$$

$$\overline{P_{16}}(s)=\frac{\alpha_8}{H_3}\overline{P_5}(s)+\frac{\alpha_8}{H_3}\overline{P_{11}}(s)$$

Where

$$T_\delta=\left[\begin{array}{l} s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7-\frac{\beta_1\alpha_1}{s+\beta_1}-\frac{\beta_2\alpha_2}{s+\beta_2}-\frac{\beta_3\alpha_3}{s+\beta_3}-\frac{\beta_4\alpha_4}{s+\beta_4}-\frac{\beta_5T_\beta}{H_0} \\ -\frac{\beta_7\beta_8\alpha_8T_\beta}{H_2H_3T_\alpha H_0}-\frac{\beta_7\alpha_7}{H_2T_\alpha} \end{array}\right]$$

$$T_\beta=\left[\frac{\alpha_5}{H_1}+\frac{\alpha_7\alpha_8\beta_8}{H_1H_2H_3T_\alpha}\right]$$

$$T_\alpha=\left[1-\frac{\alpha_8\beta_8}{H_2H_3}\right]$$

$$H_0 = \left[1 - \frac{\alpha_8\beta_8}{H_1 H_3} - \frac{\alpha_8^2\,\beta_8^2}{H_1 H_2 H_3^2 T_\alpha}\right]$$

$$H_1 = \left[s + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_6 + \beta_5 + \beta_8 - \frac{\beta_1\alpha_1}{s+\beta_1} - \frac{\beta_2\alpha_2}{s+\beta_2} - \frac{\beta_3\alpha_3}{s+\beta_3} - \frac{\beta_4\alpha_4}{s+\beta_4} - \frac{\beta_6\alpha_6}{s+\beta_6}\right]$$

$$H_2 = \left[s + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_8 + \beta_7 - \frac{\beta_1\alpha_1}{s+\beta_1} - \frac{\beta_2\alpha_2}{s+\beta_2} - \frac{\beta_3\alpha_3}{s+\beta_3} - \frac{\beta_4\alpha_4}{s+\beta_4}\right]$$

$$H_3 = \left[s + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_6 + 2\beta_8 - \frac{\beta_1\alpha_1}{s+\beta_1} - \frac{\beta_2\alpha_2}{s+\beta_2} - \frac{\beta_3\alpha_3}{s+\beta_3} - \frac{\beta_4\alpha_4}{s+\beta_4} - \frac{\beta_6\alpha_6}{s+\beta_6}\right]$$

System up states and down state probabilities are given below.

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_5(s) + \overline{P}_{11}(s) + \overline{P}_{16}(s) \tag{6.114}$$

$$\begin{aligned}\overline{P}_{down}(s) = {} & \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_3(s) + \overline{P}_4(s) + \overline{P}_6(s) + \overline{P}_7(s) + \overline{P}_8(s) + \overline{P}_9(s) + \overline{P}_{10}(s) \\ & + \overline{P}_{12}(s) + \overline{P}_{13}(s) + \overline{P}_{14}(s) + \overline{P}_{15}(s) + \overline{P}_{17}(s) + \overline{P}_{18}(s) + \overline{P}_{19}(s) + \overline{P}_{20}(s) + \overline{P}_{21}(s)\end{aligned} \tag{6.115}$$

6.5 Performance Measures of the Automatic Ticket Vending Machine

6.5.1 Reliability

The system reliability is the probability that system cannot fail before the time period $'t'$. Mathematically, reliability of the system in terms of probability is expressed like

$$R(t) = P(t > T) \tag{6.116}$$

It implies that the system's failure is not possible before the time period T. From this, it is quite clear that reliability is dependent on time t. In order to find the explicit expression of the reliability of the Automatic ticket vending machine set the failure rates of the ticket vending machine equal to

$\alpha_1 = 0.0421356/yr$, $\alpha_2 = 0.0099864/yr$, $\alpha_3 = 0.00178704/yr$, $\alpha_4 = 0.0043800/yr$, $\alpha_5 = 0.00110376/yr$, $\alpha_6 = 0.4537680/yr$, $\alpha_7 = 0.0188340/yr$, $\alpha_8 = 0.00657/yr$ and all repair rates equal to zero, i.e. $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$ in equation (6.114) and on taking inverse Laplace transformation, we get the expression of the reliability.

$$R(t) = 0.9787011461\ e^{-0.4574472\ t} + 0.0050125313\ e^{-0.67764732\ t}\ \sinh(0.022020012\ t) + 2.817824377\ e^{-0.4507633200\ t} \sinh(0.00668388\ t) - 0.0019064154\ e^{-0.8912774400\ t} + 0.0025062656\ e^{-0.897847400\ t} + 0.020699003\ e^{-0.4440794400\ t} \tag{6.117}$$

Now, vary 't' from 0 to 10 in equation (6.117), one can get the Table 6.2(a) and the corresponding Figure 6.2(a) for the reliability of the ATVM.

Time (In Years)	**Reliability R(*t*)**
0	1.00000
1	0.64549
2	0.41651
3	0.26869
4	0.17329
5	0.11175
6	0.07206
7	0.04645
8	0.02994
9	0.01930
10	0.01244

Table 6.2(a): Reliability of the ATVM vs. Time

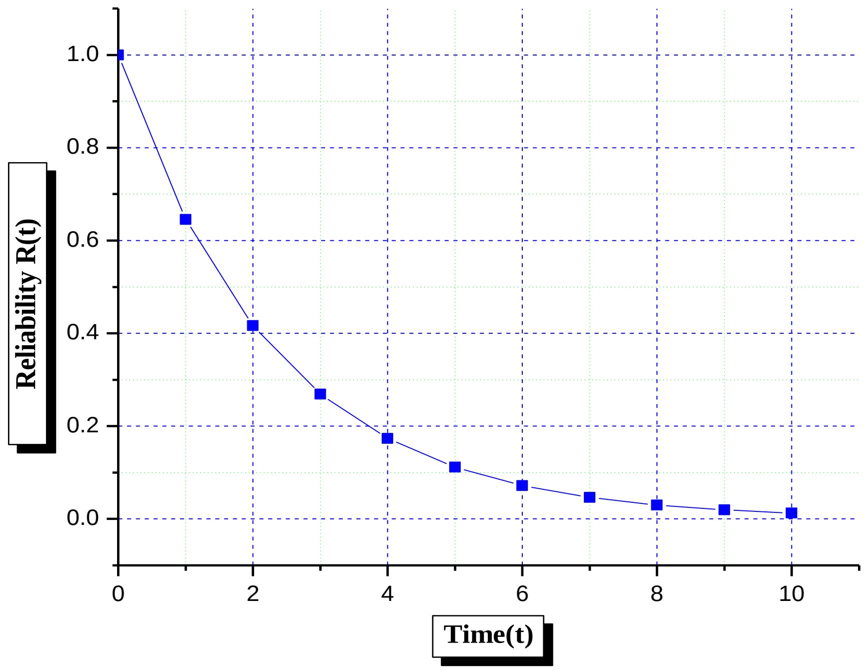


Figure 6.2(a): Reliability of the ATVM vs. Time

6.5.2. Mean Time to Failure (MTTF) of the Automatic Ticket Vending Machine

MTTF is a single value that indicates how long a system will survive on an average when the repair is not allowed. MTTF can be obtained from the system reliability using the formula

$$MTTF = \int_0^\infty R(t)\,dt \tag{6.118}$$

There is another procedure also from which system MTTF can be calculated. For that set all repair rates equal to zero, i.e. $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$ in equation (6.114) and taking the limit $s \to 0$, one can easily get the explicit expression of the MTTF which is given below.

$$\text{MTTF}=\left[\begin{array}{l}\dfrac{1}{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7}+\dfrac{\alpha_5}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+\alpha_8)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)}\\ +\dfrac{\alpha_7}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_8)}\\ +\dfrac{\alpha_8\alpha_5}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+\alpha_8)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6)}\\ +\dfrac{\alpha_8\alpha_7}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_8)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6)}\end{array}\right] \tag{6.119}$$

Now in the expression set failure rates

$\alpha_1 = 0.0421356/yr$, $\alpha_2 = 0.0099864/yr$, $\alpha_3 = 0.00178704\ /yr$, $\alpha_4 = 0.0043800/yr$, $\alpha_5 = 0.00110376\ /yr$, $\alpha_6 = 0.4537680\ /yr$, $\alpha_7 = 0.0188340\ /yr$, $\alpha_8 = 0.00657\ /yr$

and varying each failure rate from 0.1 to 0.9 one by one while keeping the other failure rates fixed, one can easily get Table 6.2(b) and corresponding Figure 6.2(b) for the MTTF of the ATVM.

Variation in the failure rates	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
0.1	8.50292	6.68549	6.33954	6.44502	8.40884	16.73862	16.08443	14.62896
0.2	4.60432	4.01192	3.88442	3.92386	6.07921	16.64446	15.90379	14.10711
0.3	3.15477	2.86466	2.79901	2.81944	4.98506	16.60286	15.82368	13.87656
0.4	2.39894	2.22733	2.18742	2.19989	4.34955	16.57941	15.77844	13.74662
0.5	1.93514	1.82188	1.79508	1.80347	3.93427	16.56437	15.74938	13.66324
0.6	1.62156	1.54126	1.52203	1.52806	3.64166	16.55389	15.72913	13.60518
0.7	1.39541	1.33355	1.32107	1.32560	3.42435	16.54618	15.71422	13.56244

0.8	1.22460	1.17823	1.16696	1.17050	3.25659	16.54027	15.70277	13.52966
0.9	1.09104	1.05408	1.04505	1.04789	3.12317	16.53558	15.69372	13.50372

Table 6.2(b): MTTF of the ATVM w.r.t Variation in the failure rates

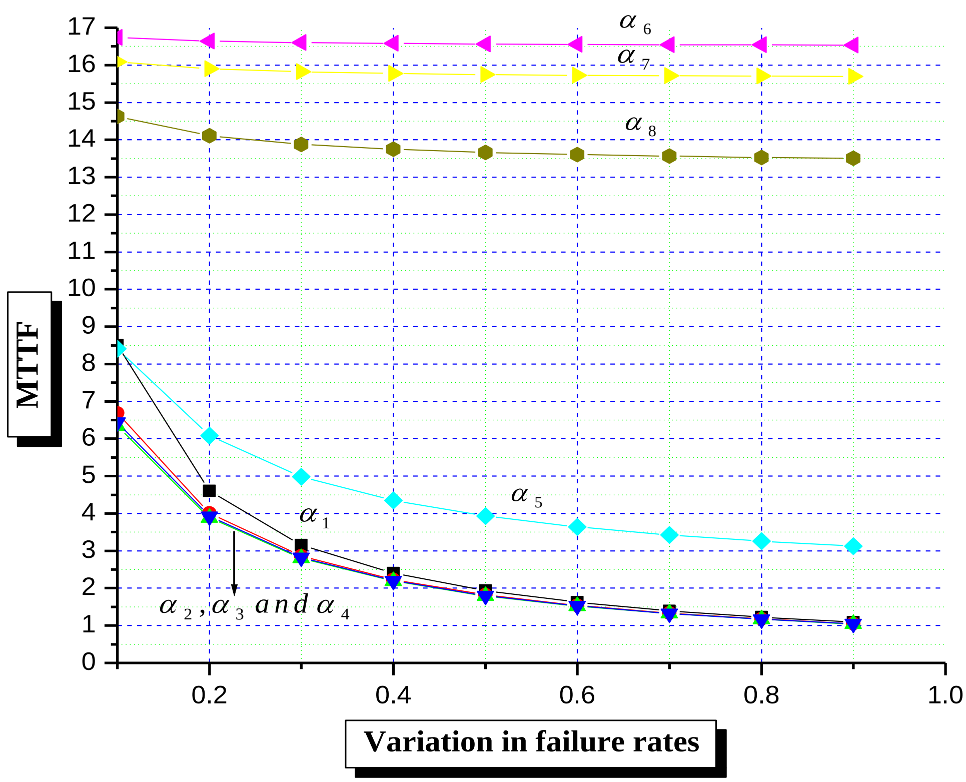


Figure 6.2(b): MTTF of the ATVM w.r.t Variation in the failure rates

6.5.3 Sensitivity of MTTF

Sensitivity analysis is performed to determine the most critical component/ components of the system. It actually determines how actually the system MTTF is affected by changing component's failure rate. Here, the authors perform the sensitivity analysis of the MTTF of the automatic ticket vending machine. For this differentiate equation (6.119) w.r.t all the failure rates one by one and set failure rates as follows

$\alpha_1 = 0.0421356 / yr$, $\alpha_2 = 0.0099864 / yr$, $\alpha_3 = 0.00178704 \ / yr$, $\alpha_4 = 0.0043800 / yr$, $\alpha_5 = 0.00110376 \ / yr$, $\alpha_6 = 0.4537680 \ / yr$, $\alpha_7 = 0.0188340 \ / yr$, $\alpha_8 = 0.00657 \ / yr$

and then varying each failure rate from 0.1 to 0.9 in these derivatives keeping other failure rates value fixed. In this way, one can easily obtain Table 6.2.1(a), 6.2.1(b), and subsequent Figure 6.2(c) for the sensitivity of MTTF for ATVM

Variation in the failure rates	$\frac{\partial(MTTF)}{\partial\alpha_1}$	$\frac{\partial(MTTF)}{\partial\alpha_2}$	$\frac{\partial(MTTF)}{\partial\alpha_3}$	$\frac{\partial(MTTF)}{\partial\alpha_4}$
0.1	-71.82206	-44.48713	-40.01599	-41.35435
0.2	-21.14188	-16.05944	-15.05644	-15.36324
0.3	-9.93676	-8.19490	-7.82393	-7.93848
0.4	-5.74876	-4.95625	-4.78034	-4.83495
0.5	-3.74183	-3.31686	-3.22006	-3.25021
0.6	-2.62787	-2.37415	-2.31532	-2.33368
0.7	-1.94622	-1.78282	-1.74443	-1.75644
0.8	-1.49905	-1.38772	-1.36130	-1.36957
0.9	-1.18997	-1.11073	-1.09179	-1.09773

Table 6.2.1(a): Sensitivity of MTTF for ATVM vs. $\alpha_1, \alpha_2, \alpha_3, \alpha_4$

Variation in the failure rates	$\frac{\partial(MTTF)}{\partial\alpha_5}$	$\frac{\partial(MTTF)}{\partial\alpha_6}$	$\frac{\partial(MTTF)}{\partial\alpha_7}$	$\frac{\partial(MTTF)}{\partial\alpha_8}$
0.1	-36.44888	-1.53651	-2.93965	-8.51531
0.2	-14.88980	-0.57706	-1.10998	-3.19808
0.3	-8.04022	-0.29989	-0.57822	-1.66202
0.4	-5.02312	-0.18329	-0.35388	-1.01583
0.5	-3.43318	-0.12351	-0.23867	-0.68451
0.6	-2.49401	-0.08883	-0.17177	-0.49234
0.7	-1.18935	-0.06695	-0.12950	-0.37105
0.8	-1.48632	-0.05226	-0.10112	-0.28962
0.9	-1.19766	-0.04192	-0.08114	-0.23233

Table 6.2.1(b): Sensitivity of MTTF for ATVM vs. $\alpha_5, \alpha_6, \alpha_7, \alpha_8$

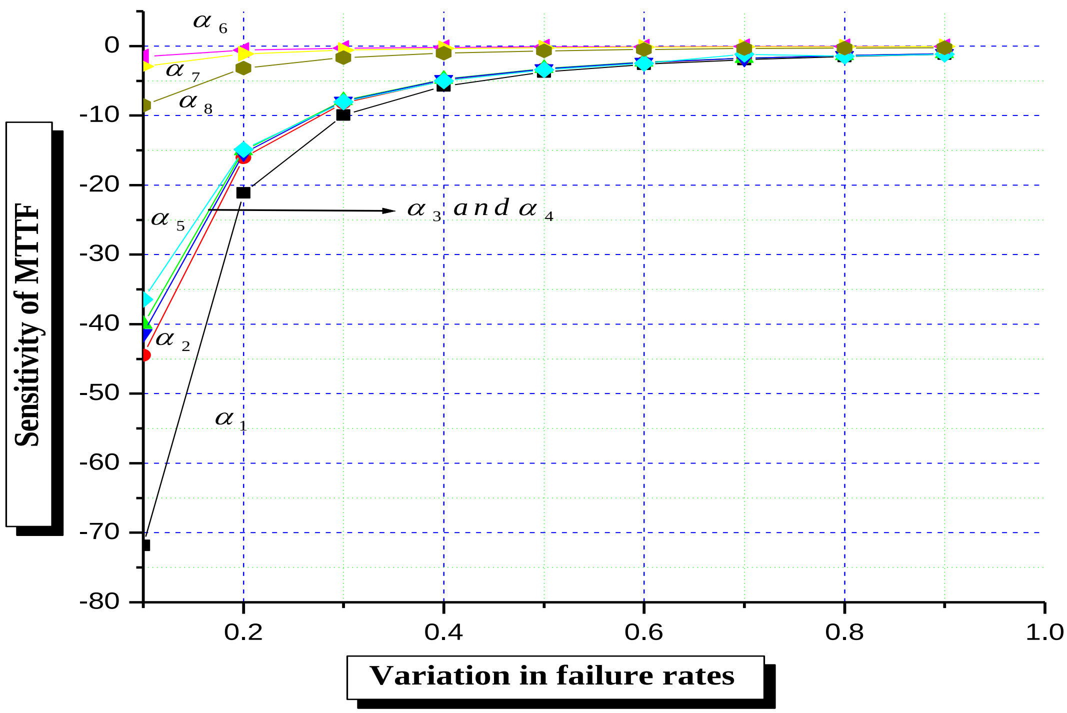


Figure 6.2(c): Sensitivity of MTTF for ATVM vs. Failure rates

6.5.4 Sensitivity of Reliability

The sensitivity of reliability determines how actually the system Reliability is affected by changing the failure rate of the system's component. Here, the authors perform the sensitivity analysis of the reliability of the automatic ticket vending machine. For this set all repair rates equal to zero, i.e., $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$ in equation (6.114) and taking inverse Laplace transformation of the equation. Differentiate the obtained equation w.r.t. all failure rates one by one, in these derivatives substitute the values of the failure rate as given below

$\alpha_1 = 0.0421356\ / yr$, $\alpha_2 = 0.0099864 / yr$, $\alpha_3 = 0.00178704 / yr$, $\alpha_4 = 0.0043800 / yr$,$\alpha_5 = 0.00110376\ / yr$,$\alpha_6 = 0.4537680\ / yr$, $\alpha_7 = 0.0188340\ / yr$, $\alpha_8 = 0.00657\ / yr$

Now changing the time t from 0 to 10 in these derivatives. In this way, one will simply get Table 6.2.2(a), 6.2.2(b) and Figure 6.4(d) for the sensitivity of reliability for ATVM.

Time in years	$\frac{\partial(R(t))}{\partial\alpha_1}$	$\frac{\partial(R(t))}{\partial\alpha_2}$	$\frac{\partial(R(t))}{\partial\alpha_3}$	$\frac{\partial(R(t))}{\partial\alpha_4}$
0	0	0	0	0
1	-0.94316	-0.94316	-0.94316	-0.94316
2	-1.77848	-1.77848	-1.77848	-1.77848
3	-2.51454	-2.51454	-2.51454	-2.51454
4	-3.15956	-3.15956	-3.15956	-3.15956
5	-3.72130	-3.72130	-3.72130	-3.72130
6	-4.20703	-4.20703	-4.20703	-4.20703
7	-4.62347	-4.62347	-4.62347	-4.62347
8	-4.97690	-4.97690	-4.97690	-4.97690
9	-5.27311	-5.27311	-5.27311	-5.27311
10	-5.51745	-5.51745	-5.51745	-5.51745

Table 6.2.2(a): Sensitivity of Reliability for ATVM vs. $\alpha_1, \alpha_2, \alpha_3, \alpha_4$

Time (t) in years	$\frac{\partial(R(t))}{\partial\alpha_5}$	$\frac{\partial(R(t))}{\partial\alpha_6}$	$\frac{\partial(R(t))}{\partial\alpha_7}$	$\frac{\partial(R(t))}{\partial\alpha_8}$
0	0	0	0	0
1	-0.18368	-0.00039	-0.00034	-0.00119
2	-0.60096	-0.00117	-0.00232	-0.00806
3	-1.11655	-0.00197	-0.00663	-0.02311
4	-1.65357	-0.00266	-0.01338	-0.04679
5	-2.16973	-0.00320	-0.02231	-0.07849
6	-2.64319	-0.00360	-0.03323	-0.11707
7	-3.06399	-0.00388	-0.04556	-0.16116
8	-3.42906	-0.00407	-0.05897	-0.20941
9	-3.73915	-0.00419	-0.07306	-0.26053
10	-3.99709	-0.00425	-0.08751	-0.31334

Table 6.2.2(b): Sensitivity of Reliability for ATVM vs. $\alpha_5, \alpha_6, \alpha_7, \alpha_8$

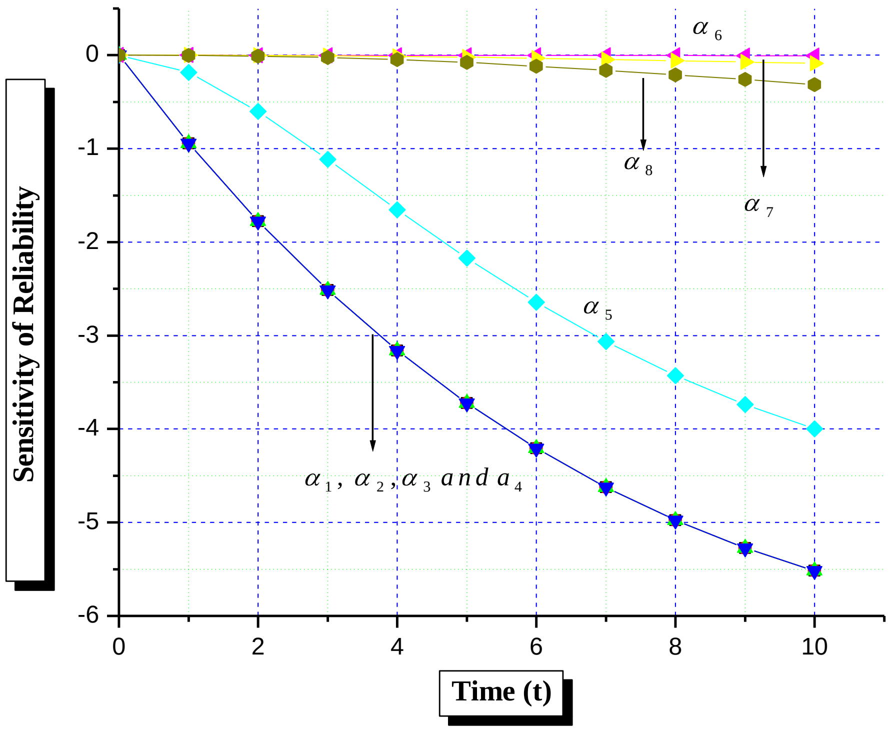


Figure 6.4(d): Sensitivity of Reliability for ATVM vs. Time

6.5.5 MTBF of the Automatic Ticket Vending Machine

For calculating the MTBF of the automatic ticket vending machine, initially, we find MTTR of the automatic ticket vending machine from the equation (6.115). MTTR is the average time that a system takes to recover from failure. For MTTR, take limit $s \rightarrow 0$ in the equation (6.115), then we get

$$MTTR = \lim_{s \rightarrow 0} \overline{P}_{down}(s) \tag{6.120}$$

After finding MTTR of the system, one can easily find MTBF of the system. MTBF is the average mean time between the two failures. For MTBF, find the sum of MTTF and MTTR. Thus, after adding equation (6.119) and (6.120) we get

$$MTBF = MTTF + MTTR \tag{6.121}$$

The expression for the MTBF is given below:

$$MTBF = \left[\begin{array}{l} \frac{1}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)} + \frac{\alpha_5}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+\alpha_8)} \\ + \frac{\alpha_7}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_8)} \\ + \frac{\alpha_8\alpha_5}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+\alpha_8)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6)} \\ + \frac{\alpha_8\alpha_7}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_8)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6)} \\ + \frac{\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}+\frac{\alpha_4}{\beta_4}\right)}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)} + \frac{\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}+\frac{\alpha_4}{\beta_4}+\frac{\alpha_6}{\beta_6}\right)\alpha_5}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+\alpha_8)} \\ + \frac{\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}+\frac{\alpha_4}{\beta_4}\right)\alpha_7}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_8)} \\ + \frac{\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}+\frac{\alpha_4}{\beta_4}+\frac{\alpha_6}{\beta_6}\right)\alpha_8\alpha_5}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6+\alpha_8)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6)} \\ + \frac{\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}+\frac{\alpha_3}{\beta_3}+\frac{\alpha_4}{\beta_4}+\frac{\alpha_6}{\beta_6}\right)\alpha_8\alpha_7}{(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_7)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_8)(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_6)} \end{array} \right] \tag{6.122}$$

Set the failure rates and repair rates as $\alpha_1 = 0.0421356/yr$, $\alpha_2 = 0.0099864/yr$ $\alpha_3 = 0.00178704/yr$, $\alpha_4 = 0.0043800/yr$, $\alpha_5 = 0.00110376/yr$, $\alpha_6 = 0.453768/yr$, $\alpha_7 = 0.0188340/yr$, $\alpha_8 = 0.00657/yr$, $\beta_1 = 36.5/yr$, $\beta_2 = 30.42/yr$, $\beta_3 = 45.62/yr$, $\beta_4 = 91.25/yr$, $\beta_5 = 182.5/yr$, $\beta_6 = 121.67/yr$, $\beta_7 = 365/yr$, $\beta_8 = 243.33/yr$. In order to obtain the MTBF of the automatic ticket vending machine one by one vary each failure rate from 0.1 to 0.9 while keeping the other failure rates fixed, one can easily get the Table 6.2(c) and the corresponding Figure 6.2(e) for the MTBF of the ATVM.

Variation in the failure rates	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
0.1	8.52985	6.71584	6.36320	6.46195	8.42623	16.76510	16.11019	14.65314
0.2	4.63150	4.04331	3.90742	3.93846	6.09406	16.67084	15.92936	14.13072
0.3	3.18203	2.89648	2.82171	2.83301	4.99871	16.62919	15.84916	13.89992
0.4	2.42624	2.25940	2.20996	2.21289	4.36251	16.60572	15.80387	13.76984
0.5	1.96246	1.85409	1.81751	1.81610	3.94678	16.59065	15.77478	13.68636
0.6	1.64889	1.57358	1.54439	1.54044	3.65385	16.58017	15.75451	13.62824
0.7	1.42275	1.36792	1.34336	1.33779	3.43630	16.57245	15.73958	13.58545
0.8	1.25195	1.21068	1.18921	1.18255	3.26836	16.56653	15.72812	13.55264
0.9	1.11840	1.08657	1.06727	1.05982	3.13480	16.56184	15.71905	13.52667

Table 6.2(c): MTBF of the ATVM w.r.t Variation in the failure rates

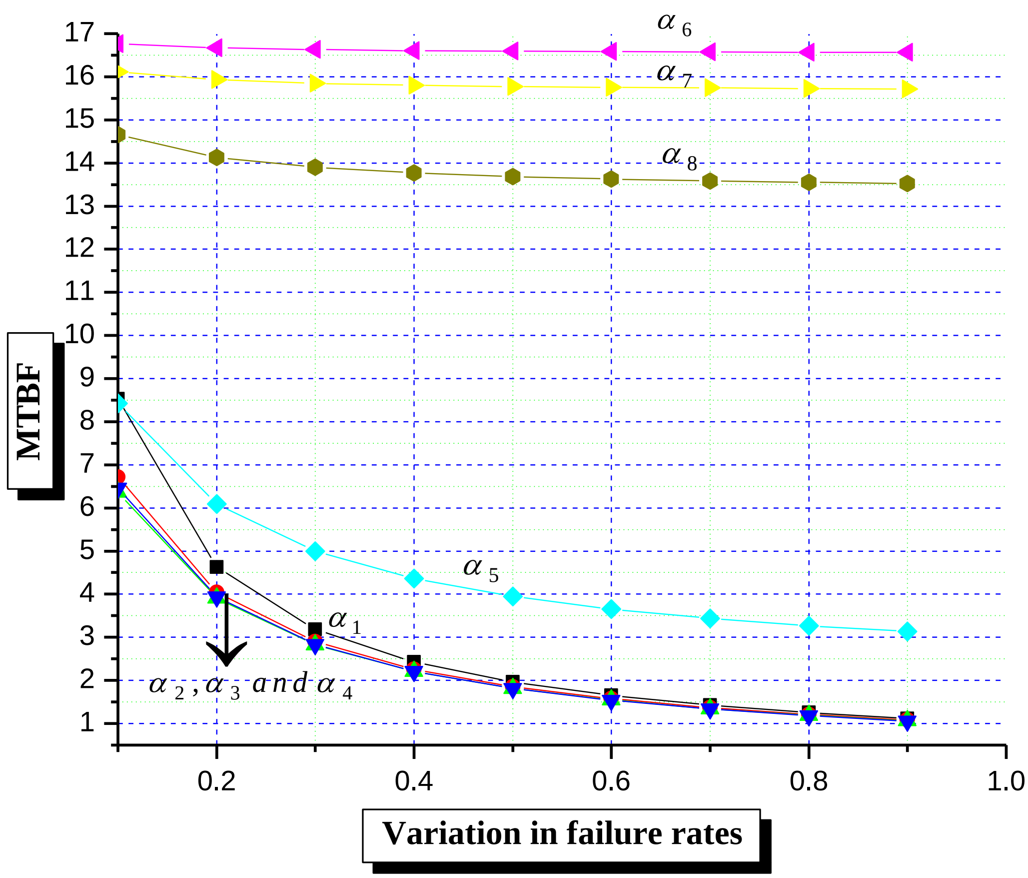


Figure 6.2(e): MTBF of the ATVM w.r.t Variation in the failure rates

6.6 Results Discussion

In this chapter, the authors carried out the performance analysis of the Automatic ticket vending machine. These machines are generally installed at the railway stations and metro railway stations so that passengers can easily get the ticket without waiting in the queues. The results regarding the performance measures of the ATVM are obtained as follows.

❖ Table 6.2(a) and Figure 6.2(a), gives the reliability of the ATVM. From Table 6.2(a), it is quite clear that initially when $t = 0$ the reliability $R(t) = 1$ which shows that initially, all the components of the machine are in good working condition. At $t = 1$ the system reliability is 0.64549. This reliability of the machine keeps on decreasing as the time t increases. This reliability decreases exponentially. At $t = 10$ the reliability of the machine is 0.01244.

- Table 6.2(b) and Figure 6.2(b), gives the MTTF of the ATVM. As an observation, the larger is the value of the MTTF, the less are the chances of machine failures. The smaller is the MTTF the greater are chances of machine failures. The system MTTF is the highest w.r.t the variation in the failure rate of the UPS. Similarly, the machine MTTF is very less affected by the variation in the failure rate of the software minor failure.
- Table 6.2.1(a), 6.2.1(b) and Figure 6.3(c), gives the sensitivity of the system MTTF w.r.t. the variation in the failure rate of ATVM. The system's MTTF is most affected by the variation in the failure rate of the touch screen monitor. As it can be seen from the graph that slightly changes in the value of the failure rate of the touch screen monitor, the system's MTTF changes drastically.
- Table 6.2.2(a), 6.2.2(b) and Figure 6.2(d), gives the sensitivity of the reliability for ATVM w.r.t. time. The system reliability is mainly affected by the touch screen monitor, Thermal printer, card reader and software major failure. These components affect system reliability mainly.
- Table 6.2(c) and Figure 6.2(e), gives the MTBF of the ATVM. The system MTBF the system MTBF is very low w.r.t variation in the failure rates of Thermal printer, card reader sensors and software major failure.

6.7 Conclusion

In this chapter, the authors carried out the performance analysis of the automatic ticket vending machine. This machine is really useful for the passengers as with the help of ATVM, one can easily purchase the ticket without waiting in the long queues at the railway station. If these machines remain operational at the railway station, then it reduces the burden on railway staff and also creates a good environment at the railway station. On the basis of the above result discussion section, it is quite clear that the major components which affect the system reliability are touch screen monitor, Thermal printer, card reader sensors, and software major failure whereas the MTTF is mainly affected by the deviation in the failure rates of the touch screen monitor. Also, the system MTBF is very low w.r.t variation in the failure rates of Thermal printer, card reader, and software major failure. Therefore, their proper maintenance at the right time is very necessary. So the manufacturer of the machine should pay more attention to these components so that they may not get out of order easily. Proper maintenance decreases the downtime of the ATVM machine and increases the revenue of the Indian railways.

Chapter 7: Reliability and Sensitivity Analysis of the Four Robotic Arm System Working in a Series Configuration Along with One Redundant Robotic Arm at System Level Using Markov Model

The performance of a four robotic arm system along with one redundant robotic arm at the system level using the Markov model has been investigated in this chapter. Whenever any robotic arm fails, the redundant robotic arm replaces the failed robotic arm immediately which reduces the downtime of the system. The system is repairable and failure and repair rates have been taken constant. The Markov modeling is employed to obtain the Chapman-Kolmogorov differential equations. Laplace transformation is used to solve the developed differential equations and obtain the state probabilities of the system. An explicit expression for the reliability, MTTF and MTBF are obtained in this chapter. For analyzing the system most critical components sensitivity analysis is also performed. Graphs are also plotted for a better understanding.

7.1 Introduction

Industries have begun using automation and robotization for the production and manufacturing of their products. The benefits of using robotization in the industry are, a robot can work for long hours without taking rest and produce a product of very high quality. Robots can also perform a very dangerous task which human beings cannot perform very easily. Nowadays, the robotic arm is being used in many industries and real -life application. Any robotic arm generally has 6-7 joints. Unlike human arms, robotic arms can also perform the same task. This robotic arm can move in three dimensions. The main application of the robotic arm is in the production line, where it is used to pick up and place the product from one place to another

In this chapter, we present the concept of the non-fixed standby redundancy which on the failure of the main components immediately replaces the failed component and reduces the system downtime. The application of the same system is also given in this chapter.

7.1.1 Problem statement

Nowadays, quick production in the plant or factory is the need of the hour. Therefore, many organizations have started using automation and robotization in their plant or factory. But these robots are just machines and follow the instructions given to them. To accomplish the task, which is given to them needs proper monitoring on them. If during operation they don't perform as expected, it may be quite risky and dangerous for the plant or factory. They may spoil and destroy the costly equipment of the plant. Hence keeping these points in consideration, we plan to assess the reliability of the plant where four robotic arms are working in a series configuration along with one redundant robotic arm.

7.2 System description

Here, we consider four robotic arms connected in a series configuration along with a robotic arm in standby mode. These robotic arms are performing the specific task allotted to them. One robotic arm has been kept in surplus. Once any robotic arm fails, it's immediately changed with the surplus robotic arm. This surplus component is capable of doing all the tasks allotted to the four robotic arms. The diagram of the system is given below.

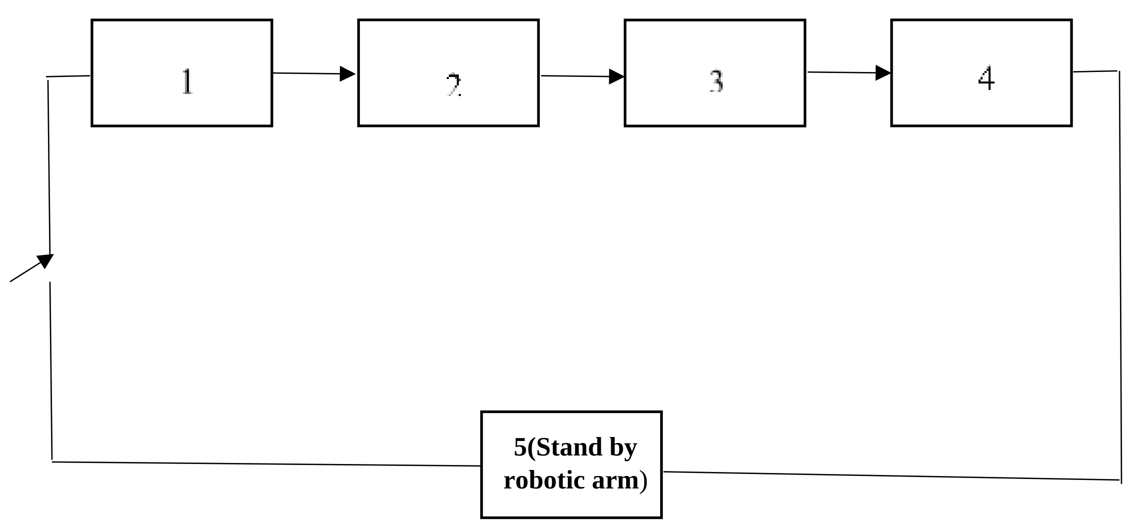


Figure 7.1(a): Diagram of the system

7.2.1 Assumptions of the system

Assumption 1: Initially, all the robotic arms are in good working condition

Assumption 2: Only one robotic arm can fail at a time

Assumption 3: Standby robotic arm cannot fail in standby mode

Assumption 4: Failure and repair rate are statistically independent

Assumption 5: When any main robotic arm fails, it is immediately replaced with standby robotic arm and this replacement time is almost negligible

Assumption 6: Standby robotic arm is capable of doing all the tasks allotted to the four robotic arms.

Assumption 7: Robotic arm failures are independent of each other.

7.2.2 State Description and Notations of the System

The following notations will be used in the throughout chapter. Notations used in this chapter are given in Table 7.1(a) and the various states of the system are given in Table 7.1(b).

Notation	Description
t	Time variable
s	Laplace transform variable
$P_i(t)$	Probability of the system being in the state S_i at any time t
$\overline{P_i}(s)$	Laplace transform of $P_i(t)$
$\lambda_i; i = 1,2,3,4,5$	Failure rate of the i^{th} robotic arm
$\beta_i; i = 1,2,3,4,5$	Repair rate of the i^{th} robotic arm
S_i; $i = 0,1,2...,20$	System's state

Table 7.1(a): Nomenclature

Notation	Description
S_0	All the robotic arms are in good working condition
S_1	The first robotic arm fails and it is immediately replaced with the standby robotic arm
S_2	The second robotic arm fails after the failure of the first robotic arm
S_3	The third robotic arm fails after the failure of the first robotic arm
S_4	The fourth robotic arm fails after the failure of the first robotic arm
S_5	The standby robotic arm fails after the failure of the first robotic arm
S_6	The second robotic arm fails and it is immediately replaced with the standby robotic arm
S_7	The first robotic arm fails after the failure of the second robotic arm
S_8	The third robotic arm fails after the failure of the second robotic arm
S_9	The fourth robotic arm fails after the failure of the second robotic arm
S_{10}	The standby robotic arm fails after the failure of the second robotic arm
S_{11}	The third robotic arm fails and it is immediately replaced with the standby robotic arm
S_{12}	The first robotic arm fails after the failure of the third robotic arm
S_{13}	The second robotic arm fails after the failure of the third robotic arm
S_{14}	The fourth robotic arm fails after the failure of the third robotic arm
S_{15}	The standby robotic arm fails after the failure of the third robotic arm
S_{16}	The fourth robotic arm fails and it is immediately replaced with the standby robotic arm

S_{17}	The first robotic arm fails after the failure of the fourth robotic arm
S_{18}	The second robotic arm fails after the failure of the fourth robotic arm
S_{19}	The third robotic arm fails after the failure of the fourth robotic arm
S_{20}	The standby robotic arm fails after the failure of the fourth robotic arm

Table 7.1(b): System states narratives

7.3 Reliability Block Diagram of the System

The reliability block diagram of the system is given in Figure 7.1(b). The system is found in various states due to the failure and the repair of the robotic arms. Initially, all the robotic arms are in good working condition. On the failure of any robotic arm the system makes its transition to another state and the repairman repairs the robotic arm and after repair action system come backs to its previous state.

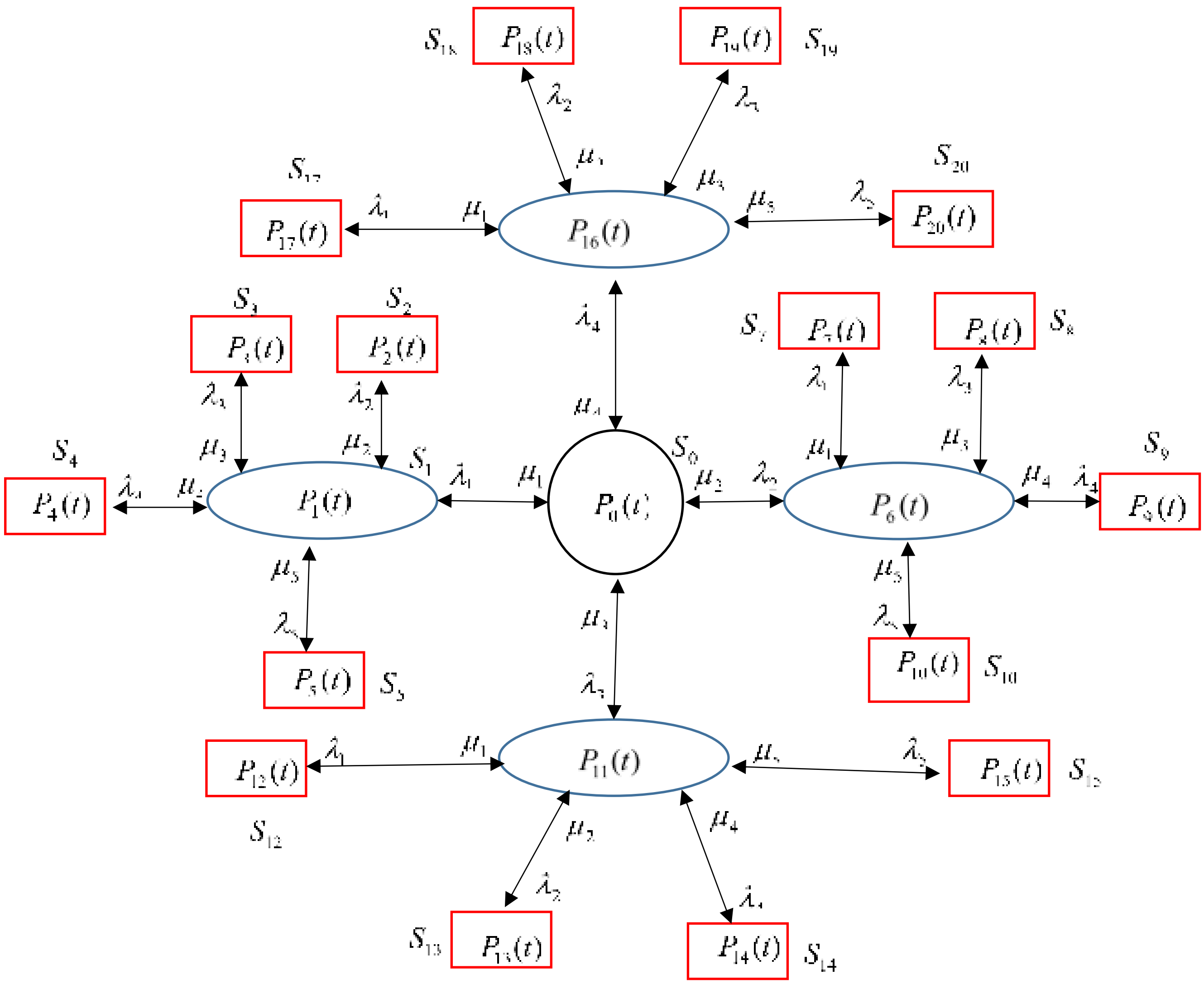


Figure 7.1(b): Transition state diagram of the system

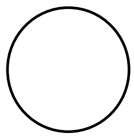
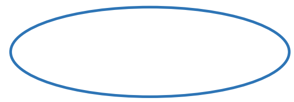
Good state	Degraded state	Failed state
		

Table 7.1(c): State representation diagram

Therefore, from the above Figure 7.1(b) the various states of the system are represented using the above shapes given in Table 7.1(c).

7.4 Mathematical Modelling of the System

For getting the Kolmogorov-Chapman differential equation for the above system, apply Markov birth-death process at time $t + \Delta t$ and letting $\Delta t \rightarrow 0$, we get following differential equations:

$$\left[\frac{d}{dt} + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right] P_0(t) = \mu_1 P_1(t) + \mu_2 P_6(t) + \mu_3 P_{11}(t) + \mu_4 P_{16}(t) \quad (7.1)$$

$$\begin{aligned}\left[\frac{d}{dt} + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \mu_1\right] P_1(t) = \mu_2 P_2(t) + \mu_3 P_3(t) + \mu_4 P_4(t) \\ + \mu_5 P_5(t) + \lambda_1 P_0(t)\end{aligned} \quad (7.2)$$

$$\left[\frac{d}{dt} + \mu_2\right] P_2(t) = \lambda_2 P_1(t) \quad (7.3)$$

$$\left[\frac{d}{dt} + \mu_3\right] P_3(t) = \lambda_3 P_1(t) \quad (7.4)$$

$$\left[\frac{d}{dt} + \mu_4\right] P_4(t) = \lambda_4 P_1(t) \quad (7.5)$$

$$\left[\frac{d}{dt}+\mu_5\right]P_5(t)=\lambda_5\,P_1(t) \tag{7.6}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_3+\lambda_4+\lambda_5+\mu_2\right]P_6(t)=\mu_1\,P_7(t)+\mu_3\,P_8(t)+\mu_4\,P_9(t) + \mu_5\,P_{10}(t)+\lambda_2\,P_0(t) \tag{7.7}$$

$$\left[\frac{d}{dt}+\mu_1\right]P_7(t)=\lambda_1 P_6(t) \tag{7.8}$$

$$\left[\frac{d}{dt}+\mu_3\right]P_8(t)=\lambda_3 P_6(t) \tag{7.9}$$

$$\left[\frac{d}{dt}+\mu_4\right]P_9(t)=\lambda_4 P_6(t) \tag{7.10}$$

$$\left[\frac{d}{dt}+\mu_5\right]P_{10}(t)=\lambda_5 P_6(t) \tag{7.11}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+\lambda_4+\lambda_5+\mu_3\right]P_{11}(t)=\mu_1\,P_{12}(t)+\mu_2\,P_{13}(t)+\mu_4\,P_{14}(t) + \mu_5\,P_{15}(t)+\lambda_3\,P_0(t) \tag{7.12}$$

$$\left[\frac{d}{dt}+\mu_1\right]P_{12}(t)=\lambda_1 P_{11}(t) \tag{7.13}$$

$$\left[\frac{d}{dt}+\mu_2\right]P_{13}(t)=\lambda_2 P_{11}(t) \tag{7.14}$$

$$\left[\frac{d}{dt}+\mu_4\right]P_{14}(t)=\lambda_4 P_{11}(t) \tag{7.15}$$

$$\left[\frac{d}{dt}+\mu_5\right]P_{15}(t)=\lambda_5 P_{11}(t) \tag{7.16}$$

$$\left[\frac{d}{dt}+\lambda_1+\lambda_2+\lambda_3+\lambda_5+\mu_4\right]P_{16}(t)=\mu_1\,P_{17}(t)+\mu_2\,P_{18}(t)+\mu_3\,P_{19}(t) + \mu_5\,P_{20}(t)+\lambda_4\,P_0(t) \tag{7.17}$$

$$\left[\frac{d}{dt}+\mu_1\right]P_{17}(t)=\lambda_1 P_{16}(t) \tag{7.18}$$

$$\left[\frac{d}{dt}+\mu_2\right]P_{18}(t)=\lambda_2 P_{16}(t) \tag{7.19}$$

$$\left[\frac{d}{dt}+\mu_3\right]P_{19}(t)=\lambda_3 P_{16}(t) \tag{7.20}$$

$$\left[\frac{d}{dt}+\mu_5\right]P_{20}(t)=\lambda_5 P_{16}(t) \tag{7.21}$$

With initial condition;

$$P_i(0)=\begin{cases}1 & i=0\\ 0 & i\neq 0\end{cases} \tag{7.22}$$

Taking Laplace of equations from (7.1) to (7.21), we get;

$$[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4]\overline{P_0}(s)=1+\mu_1\,\overline{P_1}(s)+\mu_2\,\overline{P_6}(s)+\mu_3\,\overline{P_{11}}(s)+\mu_4\,\overline{P_{16}}(s) \tag{7.23}$$

$$\begin{aligned}[s+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\mu_1]\overline{P_1}(s)=\mu_2\,\overline{P_2}(s)+\mu_3\,\overline{P_3}(s)+\mu_4\,\overline{P_4}(s)\\ +\mu_5\,\overline{P_5}(s)+\lambda_1\,\overline{P_0}(s)\end{aligned} \tag{7.24}$$

$$[s+\mu_2]\overline{P_2}(s)=\lambda_2\,\overline{P_1}(s) \tag{7.25}$$

$$[s+\mu_3]\overline{P_3}(s)=\lambda_3\,\overline{P_1}(s) \tag{7.26}$$

$$[s+\mu_4]\overline{P_4}(s)=\lambda_4\,\overline{P_1}(s) \tag{7.27}$$

$$[s+\mu_5]\overline{P_5}(s)=\lambda_5\,\overline{P_1}(s) \tag{7.28}$$

$$\begin{aligned}[s+\lambda_1+\lambda_3+\lambda_4+\lambda_5+\mu_2]\overline{P_6}(s)=\mu_1\,\overline{P_7}(s)+\mu_3\,\overline{P_8}(s)+\mu_4\,\overline{P_9}(s)\\ +\mu_5\,\overline{P_{10}}(s)+\lambda_2\,\overline{P_0}(s)\end{aligned} \tag{7.29}$$

$$[s+\mu_1]\overline{P_7}(s)=\lambda_1\overline{P_6}(s) \tag{7.30}$$

$$[s+\mu_3]\overline{P_8}(s)=\lambda_3\overline{P_6}(s) \tag{7.31}$$

$$[s+\mu_4]\overline{P_9}(s)=\lambda_4\overline{P_6}(s) \tag{7.32}$$

$$[s+\mu_5]\overline{P_{10}}(s)=\lambda_5\overline{P_6}(s) \tag{7.33}$$

$$\left[s+\lambda_1+\lambda_2+\lambda_4+\lambda_5+\mu_3\right]\overline{P_{11}}(s)=\mu_1\overline{P_{12}}(s)+\mu_2\overline{P_{13}}(s)+\mu_4\overline{P_{14}}(s)+\mu_5\overline{P_{15}}(s)+\lambda_3\overline{P_0}(s) \quad (7.34)$$

$$\left[s+\mu_1\right]\overline{P_{12}}(s)=\lambda_1\overline{P_{11}}(s) \quad (7.35)$$

$$\left[s+\mu_2\right]\overline{P_{13}}(s)=\lambda_2\overline{P_{11}}(s) \quad (7.36)$$

$$\left[s+\mu_4\right]\overline{P_{14}}(s)=\lambda_4\overline{P_{11}}(s) \quad (7.37)$$

$$\left[s+\mu_5\right]\overline{P_{15}}(s)=\lambda_5\overline{P_{11}}(s) \quad (7.38)$$

$$\left[s+\lambda_1+\lambda_2+\lambda_3+\lambda_5+\mu_4\right]\overline{P_{16}}(s)=\mu_1\overline{P_{17}}(s)+\mu_2\overline{P_{18}}(s)+\mu_3\overline{P_{19}}(s)+\mu_5\overline{P_{20}}(s)+\lambda_4\overline{P_0}(s) \quad (7.39)$$

$$\left[s+\mu_1\right]\overline{P_{17}}(s)=\lambda_1\overline{P_{16}}(s) \quad (7.40)$$

$$\left[s+\mu_2\right]\overline{P_{18}}(s)=\lambda_2\overline{P_{16}}(s) \quad (7.41)$$

$$\left[s+\mu_3\right]\overline{P_{19}}(s)=\lambda_3\overline{P_{16}}(s) \quad (7.42)$$

$$\left[s+\mu_5\right]\overline{P_{20}}(s)=\lambda_5\overline{P_{16}}(s) \quad (7.43)$$

On solving above equations from (7.23) to (7.43), we get;

$$\overline{P_1}(s)=\frac{\lambda_1}{H_1}\overline{P_0}(s) \quad (7.44)$$

$$\overline{P_6}(s)=\frac{\lambda_2}{H_2}\overline{P_0}(s) \quad (7.45)$$

$$\overline{P_{11}}(s)=\frac{\lambda_3}{H_3}\overline{P_0}(s) \quad (7.46)$$

$$\overline{P_{16}}(s)=\frac{\lambda_4}{H_4}\overline{P_0}(s) \quad (7.47)$$

$$\overline{P_0}(s)=\frac{1}{\left[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4-\frac{\mu_1\lambda_1}{H_1}-\frac{\mu_2\lambda_2}{H_2}-\frac{\mu_3\lambda_3}{H_3}-\frac{\mu_4\lambda_4}{H_4}\right]} \quad (7.48)$$

$$H_1 = \left[s + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \mu_1 - \frac{\lambda_2\mu_2}{s+\mu_2} - \frac{\lambda_3\mu_3}{s+\mu_3} - \frac{\lambda_4\mu_4}{s+\mu_4} - \frac{\lambda_5\mu_5}{s+\mu_5} \right]$$

$$H_2 = \left[s + \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \mu_2 - \frac{\lambda_1\mu_1}{s+\mu_1} - \frac{\lambda_3\mu_3}{s+\mu_3} - \frac{\lambda_4\mu_4}{s+\mu_4} - \frac{\lambda_5\mu_5}{s+\mu_5} \right]$$

$$H_3 = \left[s + \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \mu_3 - \frac{\lambda_1\mu_1}{s+\mu_1} - \frac{\lambda_2\mu_2}{s+\mu_2} - \frac{\lambda_4\mu_4}{s+\mu_4} - \frac{\lambda_5\mu_5}{s+\mu_5} \right]$$

$$H_4 = \left[s + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \mu_4 - \frac{\lambda_1\mu_1}{s+\mu_1} - \frac{\lambda_2\mu_2}{s+\mu_2} - \frac{\lambda_3\mu_3}{s+\mu_3} - \frac{\lambda_5\mu_5}{s+\mu_5} \right]$$

From the transition state diagram, we can observe that the system is up when it is either in a good state or in a degraded state and is down when it is in a failed state. Therefore, the upstate probability is the sum of the probabilities of the system being in a good state and in a degraded state. Similarly, the downstate probability, it is the sum of the probabilities of the system being in a failed state. Therefore, the mathematical expression for the system upstate and downstate are given below:

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_6(s) + \overline{P}_{11}(s) + \overline{P}_{16}(s) \tag{7.49}$$

$$\begin{aligned}\overline{P}_{down}(s) = {} & \overline{P}_2(s) + \overline{P}_3(s) + \overline{P}_4(s) + \overline{P}_5(s) + \overline{P}_7(s) + \overline{P}_8(s) + \overline{P}_9(s) + \overline{P}_{10}(s) \\ & + \overline{P}_{12}(s) + \overline{P}_{13}(s) + \overline{P}_{14}(s) + \overline{P}_{15}(s) + \overline{P}_{17}(s) + \overline{P}_{18}(s) + \overline{P}_{19}(s) + \overline{P}_{20}(s)\end{aligned} \tag{7.50}$$

7.5 System's Performance Indicator

For calculating the various performance indicators, following values of failure and repair rates will be used.

Component	**MTTF (In hours)**	**Failure rate/per hour**	**MTTR (In hours)**	**Repair rate/per hour**
1	7000	0.000142	100	0.01000
2	9500	0.000105	200	0.00500
3	8500	0.000117	150	0.00667

4	9000	0.000111	90	0.01111
5(Standby robotic arm)	6000	0.000166	50	0.02

Table 7.2: Failure and repair rate data

7.5.1 Reliability

Reliability of the system is the probability that system cannot fail before the time period $'t'$. For the safe system operation, system components should be very reliable. Now we set all repair rate equal to zero in equation (7.49) and take inverse Laplace transform, we get, a precise expression of reliability of the robotic arm system:

$$R(t) = e^{-0.000475\,t} + 11.83333333\,e^{-0.000487\,t}\sinh(0.000012\,t) + 3.442622951\,e^{-0.0005055\,t}\sinh(0.0000305\,t) + 4.775510204\,e^{-0.0004995\,t}\sinh(0.0000245\,t) + 4.036363636\,e^{-0.0005025\,t}\sinh(0.0000275\,t) \quad (7.51)$$

On changing time unit $'t'$ in the equation (7.51) one will get Table 7.2(a) and Figure 7.2(a) below:

Time (In hours)	**Reliability**
0	1.00000
1000	0.91065
2000	0.73794
3000	0.56093
4000	0.40949
5000	0.29072
6000	0.20225
7000	0.13854
8000	0.09375
9000	0.06282
10000	0.04176

Table 7.2(a): Reliability as a function of time

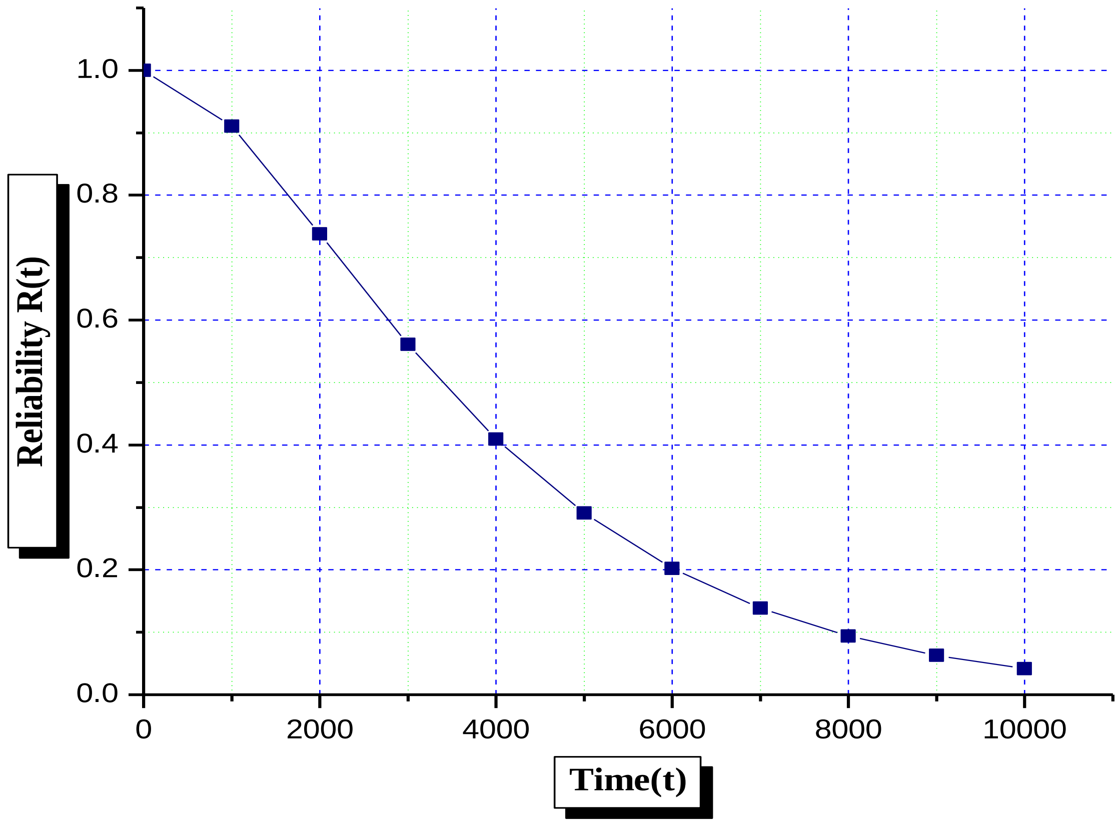


Figure 7.2(a): Reliability v/s Time

7.5.2 MTTF (Mean time to failure)

Mean time to failure is the expected time for the system failure. To find MTTF set each repair rates equal to zero in equation (7.49) and taking limit $s \to 0$. We get,

$$MTTF = \begin{bmatrix} \dfrac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4} + \dfrac{\lambda_1}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4+\lambda_5)} + \\ \dfrac{\lambda_2}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_3+\lambda_4+\lambda_5)} + \dfrac{\lambda_3}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_4+\lambda_5)} \\ + \dfrac{\lambda_4}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_3+\lambda_5)} \end{bmatrix} \quad (7.52)$$

Now, changing the failure rates from 0.0001 to 0.0010 one by one and keeping other failure rates fixed, Table 7.2(b) and Figure 7.2(b) can be obtained.

Variation in failure rates	λ_1	λ_2	λ_3	λ_4	λ_5
0.0001	4349.07	4065.69	4157.49	4111.49	4307.43
0.0002	3691.14	3471.59	3541.88	3506.59	3909.71
0.0003	3294.45	3104.69	3164.81	3134.57	3633.73
0.0004	3034.55	2860.40	2915.15	2887.57	3431.00
0.0005	2853.67	2688.49	2740.13	2714.08	3275.77
0.0006	2721.89	2562.24	2611.96	2586.86	3153.09
0.0007	2622.40	2466.35	2514.82	2490.34	3053.69
0.0008	2545.07	2391.50	2439.12	2415.06	2971.52
0.0009	2483.53	2331.75	2378.74	2359.99	2902.45
0.0010	2433.58	2283.13	2329.67	2306.15	2843.58

Table 7.2(b): MTTF as a function of failure rates

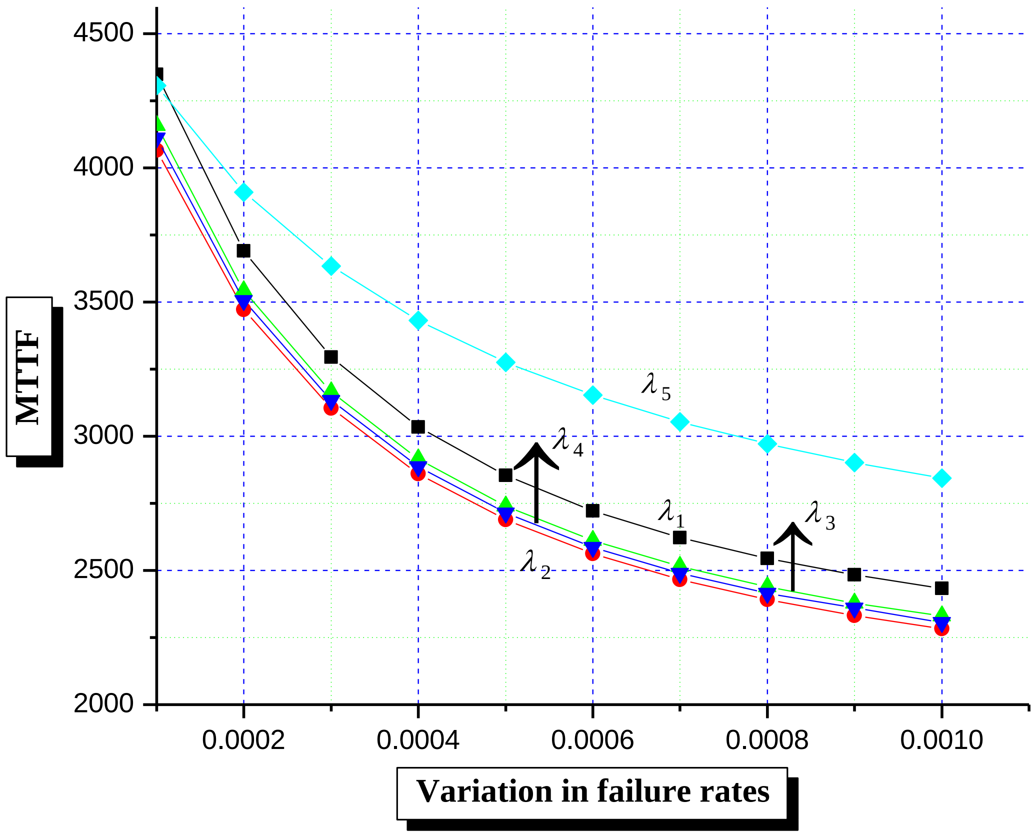


Figure 7.2(b): MTTF v/s Variation in failure rates

7.5.3 Sensitivity of Reliability

Now we perform the sensitivity analysis on the system's reliability. For that we set all repair rates equal to zero in equation (7.49) after this take the inverse Laplace transformation of the equation, differentiate the obtained expression w.r.t failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ respectively and by substituting the values of failure rate given in Table 7.2 and vary the time from 0 to 10000 hrs. in these derivatives, we get the following Table 7.2(c) and Figure 7.2 (c):

Time (In hours)	$\frac{\partial(R(t))}{\partial\lambda_1}$	$\frac{\partial(R(t))}{\partial\lambda_2}$	$\frac{\partial(R(t))}{\partial\lambda_3}$	$\frac{\partial(R(t))}{\partial\lambda_4}$	$\frac{\partial(R(t))}{\partial\lambda_5}$
0	-0.0001	-0.0001	-0.0001	0.0001	-0.0003
1000	-252.7243	-276.0111	-268.5516	-272.2923	-143.2933
2000	-614.2965	-672.8267	-654.3043	-663.6196	-345.8939
3000	-840.1346	-922.7627	-896.9255	-909.9560	-469.7647
4000	-908.0942	-1000.1350	-971.6922	-986.0759	-504.2065
5000	-862.9206	-952.9187	-925.4291	-939.3678	-475.7462
6000	-755.9060	-836.9139	-812.4534	-824.8884	-413.7900
7000	-626.0506	-694.8984	-674.3448	-684.8203	-340.2602
8000	-497.6844	-533.7781	-537.2196	-545.6800	-268.5514
9000	-383.4701	-427.7149	-414.7989	-421.4142	-205.4279
10000	-288.2901	-322.3034	-312.4833	-317.5250	-153.3188

Table 7.2(c): Sensitivity of reliability as a function of time

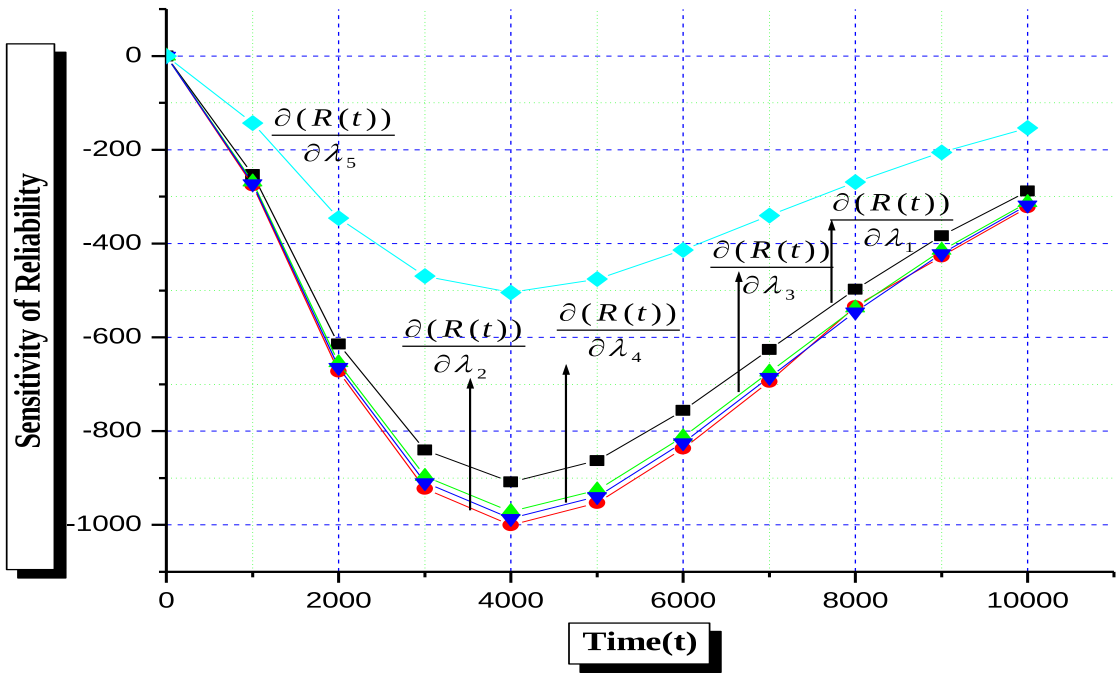


Figure 7.2(c): Sensitivity of Reliability as a function of time

7.5.4 MTBF

For calculating the MTBF of the robotic arm system, initially, we find MTTR of the robotic arm system from the equation (7.50). MTTR is the average time that a system takes to recover from failure. For MTTR, take limit $s \to 0$ in the equation (7.50), then we get

$$MTTR = \lim_{s \to 0} \overline{P}_{down}(s) \tag{7.53}$$

After finding MTTR of the system, one can easily find MTBF of the system. MTBF is the average mean time between the two failures. For MTBF, find the sum of MTTF and MTTR. Thus, after adding equation (7.52) and (7.53) we get

$$MTBF = MTTF + MTTR \tag{7.54}$$

The expression for the MTBF is given below:

$$MTTF = \left[\begin{array}{l} \frac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4} + \frac{\lambda_1}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4+\lambda_5)} + \\ \frac{\lambda_2}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_3+\lambda_4+\lambda_5)} + \frac{\lambda_3}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_4+\lambda_5)} \\ + \frac{\lambda_4}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_3+\lambda_5)} + \frac{\lambda_1\left(\frac{\lambda_2}{\mu_2}+\frac{\lambda_3}{\mu_3}+\frac{\lambda_4}{\mu_4}+\frac{\lambda_5}{\mu_5}\right)}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4+\lambda_5)} + \\ \frac{\lambda_2\left(\frac{\lambda_1}{\mu_1}+\frac{\lambda_3}{\mu_3}+\frac{\lambda_4}{\mu_4}+\frac{\lambda_5}{\mu_5}\right)}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_3+\lambda_4+\lambda_5)} + \frac{\lambda_3\left(\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}+\frac{\lambda_4}{\mu_4}+\frac{\lambda_5}{\mu_5}\right)}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_4+\lambda_5)} \\ + \frac{\lambda_4\left(\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}+\frac{\lambda_3}{\mu_3}+\frac{\lambda_5}{\mu_5}\right)}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)(\lambda_1+\lambda_2+\lambda_3+\lambda_5)} \end{array} \right] \tag{7.55}$$

Set the failure rates and repair rates as $\lambda_1 = 0.000142$, $\lambda_2 = 0.000105$, $\lambda_3 = 0.000117$, $\lambda_4 = 0.000111, \lambda_5 = 0.000166, \mu_1 = 0.01000, \mu_2 = 0.00500, \mu_3 = 0.00667$, $\mu_4 = 0.0111$, $\mu_5 = 0.02$. In order to obtain the MTBF of the robotic arm system one by one vary each failure rate from 0.0001 to 0.0010 keeping the other failure rates fixed as shown in the below Table 7.2 (d) and Figure 7.2(d).

Variation in failure rates	λ_1	λ_2	λ_3	λ_4	λ_5
0.0001	4455.36	4171.89	4263.31	4218.27	4422.44
0.0002	3798.42	3585.12	3651.60	3613.30	4012.96
0.0003	3402.53	3221.07	3276.14	3241.60	3728.83
0.0004	3143.25	2977.64	3027.06	2995.05	3520.11

0.0005	2962.88	2805.63	2852.14	2822.03	3360.29
0.0006	2831.53	2678.84	2723.83	2695.26	3233.99
0.0007	2732.39	2582.20	2626.43	2599.15	3131.65
0.0008	2655.36	2506.51	2550.41	2524.24	3047.06
0.0009	2594.08	2445.90	2489.70	2464.52	2975.95
0.0010	2544.35	2396.43	2440.29	2415.97	2915.35

Table 7.2(d): MTBF vs. variation in failure rates

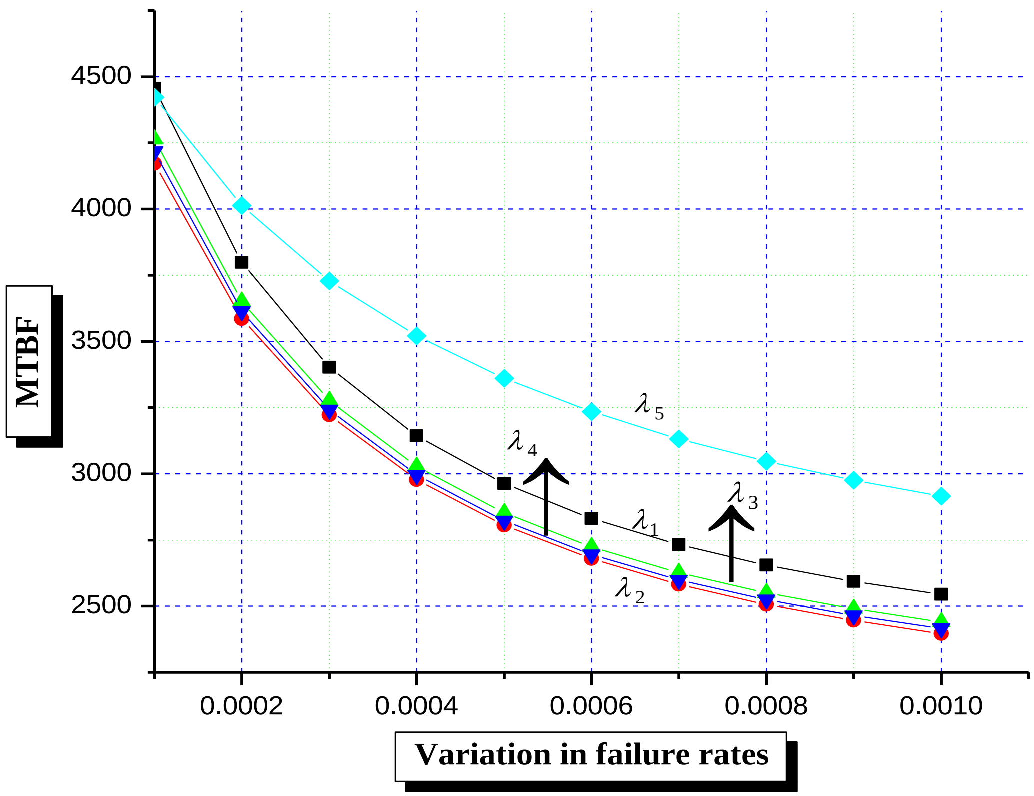


Figure 7.2(d): MTBF vs. Variation in failure rates

7.5 Results Discussion and Conclusion

In this chapter, Robotic arms which work in a series connection along with a standby robotic arm which replaces any main component on its failure have been considered and various performance indicators of the system like reliability, MTTF and MTBF have been evaluated. Sensitivity analysis on the system reliability has been performed for finding the most critical robotic arm which affects the system's performance. On examining the Tables and graphs we obtain the following results:

- ❖ The variation of reliability with respect to time can be observed from Figure 7.2(a). On examining the graph one can observe that reliability decreases as time $'t'$ increases. After 10000 hrs. of system operation, system reliability is 0.04176. As reliability is quite low therefore timely and planned maintenance action can improve the system reliability.
- ❖ The variation in system MTTF on changing the failure rates of the robotic arm can be observed from Figure 7.2(b). For the smooth operation of the plant, MTTF of the system should be more. On critically examining the graph of MTTF one can observe that MTTF of the system is quite low as we vary the failure rate of the second component and fourth component. It decreases very rapidly as we increase the failure rate of the second component and fourth component.
- ❖ On carefully examining the Figure 7.2(c), one can observe that system reliability is more sensitive w.r.t second robotic arm and less sensitive w.r.t fifth robotic arm. As time t increases second robotic arm affects the reliability of the system.
- ❖ The variation in system MTBF on changing the failure rates of the robotic arm can be observed from Figure 7.2(d). On critically examining the graph of MTBF one can observe that MTBF of the system is quite low as we vary the failure rate of the second component and fourth component. It decreases very rapidly as we increase the failure rate of the second component and fourth component.

Authors compare the present work with the work done by Fuzdin and Majid [70]. They investigated an automatic assembly plant which consists of eight robotic arms connected in a series configuration and found the reliability of the each robotic arms after 5000 hours of operation 0.79, 0.62, 0.38, 0.79, 0.62, 0.79, 0.62, 0.62 respectively and the overall system reliability was 0.02768. The reliability of each robotic arm in our proposed model is found to be 0.49(First robotic arm), 0.59(Second robotic arm), 0.55(Third robotic arm), 0.55(Fourth robotic arm), 0.43(Fifth robotic arm) after 5000 hours of operation and the overall system reliability is 0.29072. Here, we conclude that the proposed model is 950.28% more reliable after 5000 hours of operation despite being the less reliable robotic arms. Hence, we conclude that redundancy at the system's level improves the performance of the system and reduces the cost of the plant.

Also, for making the system more safe, reliable and efficient, there are some suggestions for the plant administration. On the basis of the above results, we draw this conclusion that system's second robotic arm and fourth robotic arm should be paid more attention so that the MTTF of the whole system may be improved. To enhance system's reliability, second robotic arm should be paid more attention. Hence it is strongly recommended that plant administration should pay attention to the second robotic arm and Fourth robotic arm. With proper repair facilities, and timely maintenance actions, performance of the second robotic arm and fourth robotic arm can be improved.

Chapter 8: Multi-State Performance Analysis of a Sugar Mill Incorporating Human Error Using Mathematical Modelling and Reliability Approach

The performance analysis of Wahid sugar mill (situated in Punjab, India) incorporating human error using the reliability approach has been analyzed in this chapter. The sugar mill is a complex system consisting of heavy machines (operated by human operators) that are used for the production of sugar and other products. Human error and unplanned outages in the mill affect its availability and reliability and increase the downtime of the system which can cost a lot for the same. Keeping the above facts into consideration, a mathematical model is formulated for the same for obtaining the various reliability measures of the system like availability, reliability, MTTF and MTBF. Critical components of the sugar mill which affect its performance are determined with the help of sensitivity analysis. For analyzing the profit from this industry, a profit function is also developed.

8.1 Introduction

In this age of science and technology, heavy and complex machines are being used for enhancing production in industries but at the same pace, it also increases the complexity of the industry. Not only heavy machines increase production but one has to put an eagle's eye on the components which affect the system's performance. It can be done by obtaining the various performance measures of the system as well as the effects of the component's failure on the same. Hence, keeping all these things into consideration, in this chapter we consider a sugar mill, situated in Punjab, India, which is a complex system comprising of many components like unloader, conveyor, cutter, crusher, bagasse carrying machine and boiler. Failure of any of these components may lead the whole system into a degraded state or in a failed state. These failures may be mechanical, electrical or due to the human operator. Sometimes failures also occur due to power outages, corrosion, and manufacturing defects and wear out, natural calamities like earthquakes or tornados etc. These failures of the system can't be avoided (expect natural calamities) but they can be mitigated with proper repair and maintenance or using the redundancy in the system. In the studies, it has been observed that many industrial systems were investigated by elite researchers through different techniques for finding their various system measures. Also, specific authors [58], [67], [73], [74], [96], [112], [113] investigated some of the sections of the sugar mill. But no one has ever tried to investigate a sugar mill as a whole for performance analysis by taking human error into consideration and also sensitivity analysis of the sugar mill plant regarding its components failure/repair has never been performed. Hence, the authors planned to investigate a sugar mill by taking its various important components along with a human operator. The next section gives a brief description of the components of the sugar mill which are taken into consideration.

8.2 System Description

The description of the components of the sugar mill are as follows

- ***Component*-A:** The unloader is represented by component A. Basically it is used to unload the cane from the means of transport. In the present study two unloaders in a parallel configuration are taken into consideration. If one of them fails then the sugar mill goes into a degraded state.
- ***Component*-B:** The conveyor is represented by component B. Once the cane is unloaded then it is kept on the conveyor for further processing.
- ***Component*-C:** The cutter is represented as a component C. Basically it used to cut the cane into a specific size of pieces.
- ***Component*-D:** The crushing system is represented by component D. It is used to crush the cane and extract the juice from it.
- ***Component*-E:** The bagasse carrying system is represented by component E. After the extraction of juice from canes, Bagasse is used as a fuel in the sugar mill. It is used in the heat-generating system of the mill. A bagasse carrying machine is used to carry the bagasse to the heat-generating system or to the store of the mill.
- ***Component*-F:** The boiler is represented by component F. It is used to generate heat in the various stage of production in the sugar mill. The reliability of the sugar mill will be enhanced if one can optimize the performance of the boiler. In the present study two boilers are taken into consideration for the enhancement of the production of the sugar mill. If one of the boilers fails, then the sugar mill reduced its production

(i.e. work in a degraded state). When both the boilers fail the whole system fails completely.

The interconnection of these component (flow diagram) in sugar mill is represented in the following Figure 8(a).

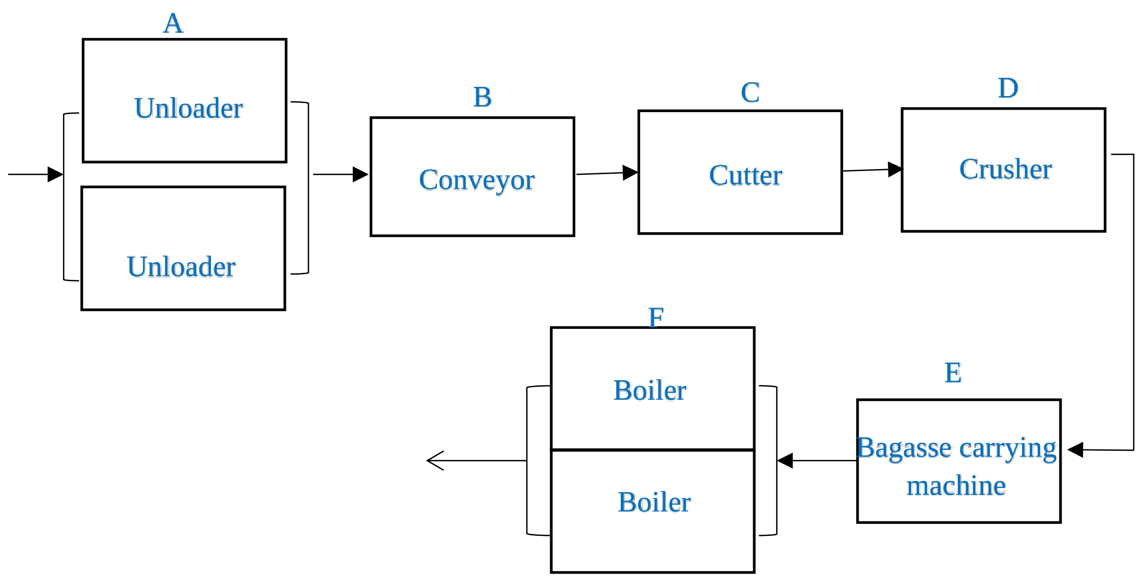


Figure 8(a): Configuration of the System

8.2.1 Assumptions

The reliability analysis of the sugar mill has been done under following assumptions.

- **Assumption 1:** Initially, the whole system is as good as a new one and all the components are working with full efficiency.
- **Assumption 2:** Components of the system can be in working, partially failed or in a failed state.
- **Assumption 3:** The Repair facility is always available with the system.
- **Assumption 4:** Failure and repair rates have been taken as constant and follow negative exponential distribution.
- **Assumption 5:** The human operator always available to operate the system.
- **Assumption 6:** The raw material is always available for production.

8.2.2 Nomenclature and State Description

Following nomenclatures (Table 8.1(a)) and state description (Table 8.1(b)) followed throughout the paper.

	The circle indicates that the system is as good as a new one
	The octagon indicates the system's performance is down due to the failure of the either one component or two components
	The rectangle indicates a failed state of the system
t	Time scale
s	Laplace Transformation variable
$P_i(t); i = 0,1,2,...,27$	Probability of the system being in a state S_i at instant t.
$\overline{P}_i(s); i = 0,1,2,...,27$	Laplace transform of $P_i(t)$
α_i	The failure rate of the i^{th} component of the system
α_{HE}	Human error failure rate
β_i	Repair rate of the i^{th} component of the system
β_{HE}	Human error repair rate

Table 8.1(a): Nomenclature

S_0	Good state: The system is as good as a new system
S_1	Degraded state: State in which the first unloader fails
S_2	Failed state: State in which the second unloader fails after the failure of the first unloader
S_3	Failed state: State in which conveyer of the system fails
S_4	Failed state: State in which cutter of the system fails
S_5	Failed state: State in which crusher of the system fails

S_6	Failed state: State in which bagasse carrying machine of the system fails
S_7	Failed state: State in which the system fails due to human error
S_8	Degraded state: State in which the first boiler of the system fails
S_9	Failed state: State in which the conveyer of the system fails after the failure of the first boiler
S_{10}	Failed state: State in which the cutter of the system fails after the failure of the first boiler
S_{11}	Failed state: State in which the crusher of the system fails after the failure of the first boiler
S_{12}	Failed state: State in which the bagasse carrying machine of the system fails after the failure of the first boiler
S_{13}	Failed state: State in which the second boiler fails after the failure of the first boiler
S_{14}	Failed state: State in which the system fails due to human error after the failure of the first boiler
S_{15}	Degraded state: State in which the first unloader and first boiler fail
S_{16}	Failed state: State in which the second unloader fails after the failure of first unloader and first boiler
S_{17}	Failed state: State in which conveyer fails after the failure of first unloader and first boiler
S_{18}	Failed state: State in which cutter fails after the failure of first unloader and first boiler
S_{19}	Failed state: State in which crusher fails after the failure of first unloader and first boiler
S_{20}	Failed state: State in which bagasse carrying machine fails after the failure of the first unloader and the first boiler
S_{21}	Failed state: State in which the second boiler fails after the failure of the first unloader and the first boiler
S_{22}	Failed state: State in which the system fails due to human error after the failure of first unloader and first boiler

S_{23}	Failed state: State in which conveyer fails after the failure of the first unloader
S_{24}	Failed state: State in which cutter fails after the failure of the first unloader
S_{25}	Failed state: State in which crusher fails after the failure of the first unloader
S_{26}	Failed state: State in which bagasse carrying machine fails after the failure of the first unloader
S_{27}	Failed state: State in which the system fails due to human error after the failure of the first unloader

Table 8.1(b): State description

8.3 Reliability Block Diagram of the system

Critically analyzing the probability of various failure/repair of the components of the sugar mill during its production different possible state and there interconnection are identified and represented in the following state transition diagram (Figure 8.1(b)). All the possible states $S_i : i = 0,1,2,\ldots,27$ are shown in the below diagram. For better understanding of these states Table 8.1(b) is given previously. The Chapman-Kolmogorov differential equations are developed from the state transition diagram in the interval $(t, t+\Delta t)$ as follows.

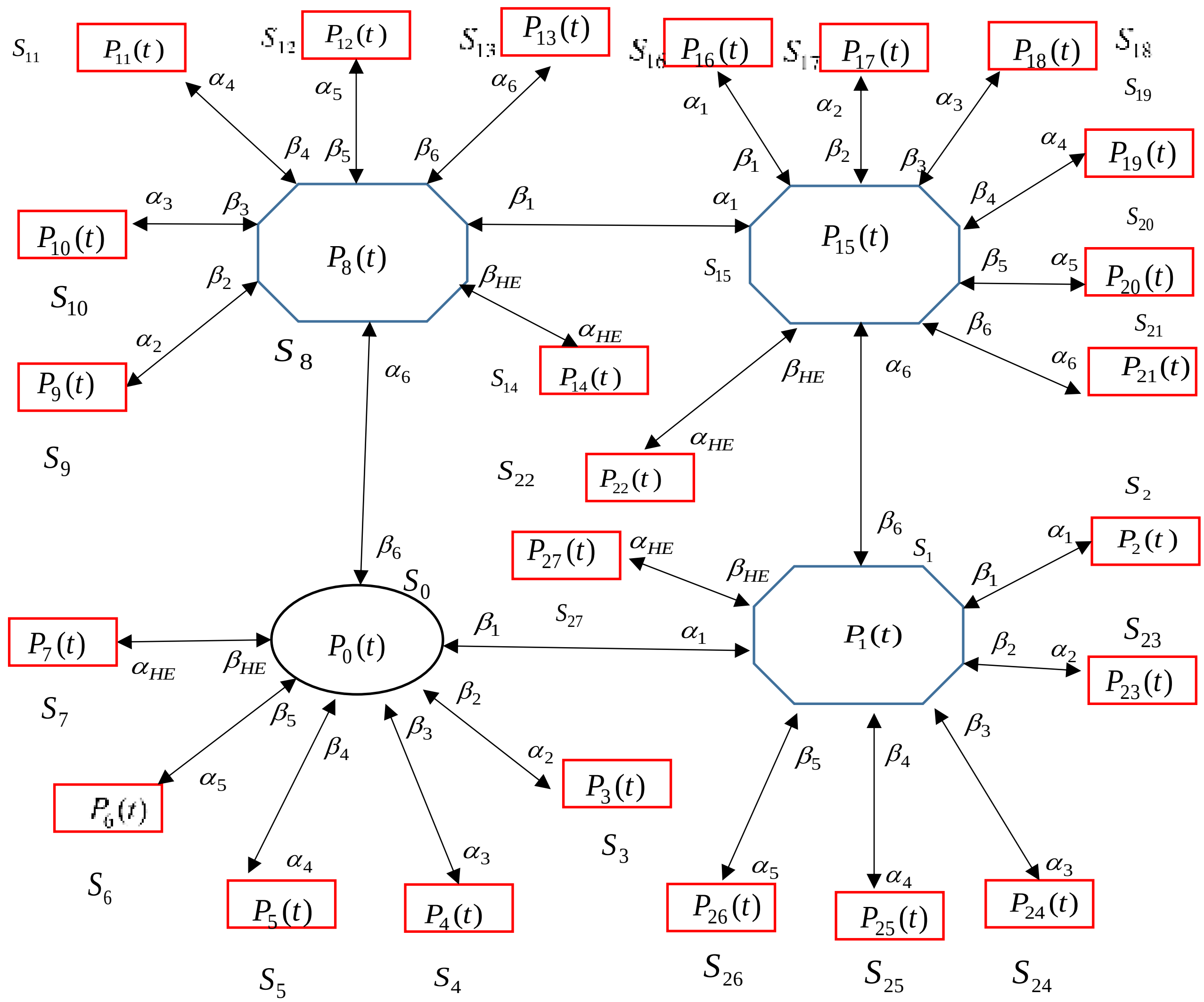


Figure 8.1(b): State Transition diagram

8.4 Differential Equations Formulation and Solution of the Problem

In this section, we develop Chapman-Kolmogorov differential equations from the above transition state diagram given in figure 8.1(b). Suppose that the system makes transition at the time $t+\Delta t$ and letting $\Delta t \rightarrow 0$ the following set of the differential equations can be obtained.

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}\right]P_0(t)=\beta_1 P_1(t)+\beta_2 P_3(t)+\beta_3 P_4(t)+\beta_4 P_5(t)$$

$$+\beta_5 P_6(t)+\beta_6 P_8(t)+\beta_{HE} P_7(t) \qquad (8.1)$$

$$\left[\frac{d}{dt}+\beta_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}\right]P_1(t)=\alpha_1 P_0(t)+\beta_1 P_2(t)+\beta_2 P_{23}(t)+\beta_3 P_{24}(t)$$

$$+\beta_4 P_{25}(t)+\beta_5 P_{26}(t)+\beta_6 P_{15}(t)+\beta_{HE} P_{27}(t) \qquad (8.2)$$

$$\left[\frac{d}{dt}+\beta_1\right]P_2(t)=\alpha_1 P_1(t) \qquad (8.3)$$

$$\left[\frac{d}{dt}+\beta_2\right]P_3(t)=\alpha_2 P_0(t) \qquad (8.4)$$

$$\left[\frac{d}{dt}+\beta_3\right]P_4(t)=\alpha_3 P_0(t) \qquad (8.5)$$

$$\left[\frac{d}{dt}+\beta_4\right]P_5(t)=\alpha_4 P_0(t) \qquad (8.6)$$

$$\left[\frac{d}{dt}+\beta_5\right]P_6(t)=\alpha_5 P_0(t) \qquad (8.7)$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_7(t)=\alpha_{HE} P_0(t) \qquad (8.8)$$

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}+\beta_6\right]P_8(t)=\beta_1 P_{15}(t)+\beta_2 P_9(t)+\beta_3 P_{10}(t)+\beta_4 P_{11}(t)$$

$$+\beta_5 P_{12}(t)+\beta_6 P_{13}(t)+\beta_{HE} P_{14}(t)+\alpha_6 P_0(t) \qquad (8.9)$$

$$\left[\frac{d}{dt}+\beta_2\right]P_9(t)=\alpha_2 P_8(t) \tag{8.10}$$

$$\left[\frac{d}{dt}+\beta_3\right]P_{10}(t)=\alpha_3 P_8(t) \tag{8.11}$$

$$\left[\frac{d}{dt}+\beta_4\right]P_{11}(t)=\alpha_4 P_8(t) \tag{8.12}$$

$$\left[\frac{d}{dt}+\beta_5\right]P_{12}(t)=\alpha_5 P_8(t) \tag{8.13}$$

$$\left[\frac{d}{dt}+\beta_6\right]P_{13}(t)=\alpha_6 P_8(t) \tag{8.14}$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_{14}(t)=\alpha_{HE} P_8(t) \tag{8.15}$$

$$\left[\frac{d}{dt}+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}+\beta_1+\beta_6\right]P_{15}(t)=\beta_1 P_{16}(t)+\beta_2 P_{17}(t)+\beta_3 P_{18}(t)$$

$$+\beta_4 P_{19}(t)+\beta_5 P_{20}(t)+\beta_6 P_{21}(t)+\beta_{HE} P_{22}(t)+\alpha_1 P_8(t)+\alpha_6 P_1(t) \tag{8.16}$$

$$\left[\frac{d}{dt}+\beta_1\right]P_{16}(t)=\alpha_1 P_{15}(t) \tag{8.17}$$

$$\left[\frac{d}{dt}+\beta_2\right]P_{17}(t)=\alpha_2 P_{15}(t) \tag{8.18}$$

$$\left[\frac{d}{dt}+\beta_3\right]P_{18}(t)=\alpha_3 P_{15}(t) \tag{8.19}$$

$$\left[\frac{d}{dt}+\beta_4\right]P_{19}(t)=\alpha_4 P_{15}(t) \tag{8.20}$$

$$\left[\frac{d}{dt}+\beta_5\right]P_{20}(t)=\alpha_5 P_{15}(t) \tag{8.21}$$

$$\left[\frac{d}{dt}+\beta_6\right]P_{21}(t)=\alpha_6 P_{15}(t) \tag{8.22}$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_{22}(t)=\alpha_{HE}\ P_{15}(t) \tag{8.23}$$

$$\left[\frac{d}{dt}+\beta_{2}\right]P_{23}(t)=\alpha_{2}\ P_{1}(t) \tag{8.24}$$

$$\left[\frac{d}{dt}+\beta_{3}\right]P_{24}(t)=\alpha_{3}P_{1}(t) \tag{8.25}$$

$$\left[\frac{d}{dt}+\beta_{4}\right]P_{25}(t)=\alpha_{4}P_{1}(t) \tag{8.26}$$

$$\left[\frac{d}{dt}+\beta_{5}\right]P_{26}(t)=\alpha_{5}P_{1}(t) \tag{8.27}$$

$$\left[\frac{d}{dt}+\beta_{HE}\right]P_{27}(t)=\alpha_{HE}P_{1}(t) \tag{8.28}$$

Initial condition

$$P_i(0)=\begin{cases}1, & i=0\\ 0, & i\neq 0\end{cases} \tag{8.29}$$

On taking the Laplace transformation in equation (8.1)-(8.29), we get,

$$\left[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}\right]\overline{P}_0(s)=1+\beta_1\overline{P}_1(s)+\beta_2\overline{P}_3(s)+\beta_3\overline{P}_4(s)+\beta_4\overline{P}_5(s)$$
$$+\beta_5\overline{P}_6(s)+\beta_6\overline{P}_8(s)+\beta_{HE}\overline{P}_7(s) \tag{8.30}$$

$$\left[s+\beta_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}\right]\overline{P}_1(s)=\alpha_1\overline{P}_0(s)+\beta_1\overline{P}_2(s)+\beta_2\overline{P}_{23}(s)+\beta_3\overline{P}_{24}(s)$$

$$+\beta_4\overline{P}_{25}(s)+\beta_5\overline{P}_{26}(s)+\beta_6\overline{P}_{15}(s)+\beta_{HE}\overline{P}_{27}(s) \tag{8.31}$$

$$\left[s+\beta_1\right]\overline{P}_2(s)=\alpha_1\overline{P}_1(s) \tag{8.32}$$

$$\left[s+\beta_2\right]\overline{P}_3(s)=\alpha_2\overline{P}_0(s) \tag{8.33}$$

$$\left[s+\beta_3\right]\overline{P}_4(s)=\alpha_3\overline{P}_0(s) \tag{8.34}$$

$$\left[s+\beta_4\right]\overline{P}_5(s)=\alpha_4\overline{P}_0(s) \tag{8.35}$$

$$[s+\beta_5]\overline{P}_6(s)=\alpha_5\overline{P}_0(s) \tag{8.36}$$

$$[s+\beta_{HE}]\overline{P}_7(s)=\alpha_{HE}\overline{P}_0(s) \tag{8.37}$$

$$[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}+\beta_6]\overline{P}_8(s)=\beta_1\overline{P}_{15}(s)+\beta_2\overline{P}_9(s)+\beta_3\overline{P}_{10}(s)+\beta_4\overline{P}_{11}(s)$$

$$+\beta_5\overline{P}_{12}(s)+\beta_6\overline{P}_{13}(s)+\beta_{HE}\overline{P}_{14}(s)+\alpha_6\overline{P}_0(s) \tag{8.38}$$

$$[s+\beta_2]\overline{P}_9(s)=\alpha_2\overline{P}_8(s) \tag{8.39}$$

$$[s+\beta_3]\overline{P}_{10}(s)=\alpha_3\overline{P}_8(s) \tag{8.40}$$

$$[s+\beta_4]\overline{P}_{11}(s)=\alpha_4\overline{P}_8(s) \tag{8.41}$$

$$[s+\beta_5]\overline{P}_{12}(s)=\alpha_5\overline{P}_8(s) \tag{8.42}$$

$$[s+\beta_6]\overline{P}_{13}(s)=\alpha_6\overline{P}_8(s) \tag{8.43}$$

$$[s+\beta_{HE}]\overline{P}_{14}(s)=\alpha_{HE}\overline{P}_8(s) \tag{8.44}$$

$$[s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}+\beta_1+\beta_6]\overline{P}_{15}(s)=\beta_1\overline{P}_{16}(s)+\beta_2\overline{P}_{17}(s)+\beta_3\overline{P}_{18}(s)+\beta_4\overline{P}_{19}(s)$$

$$+\beta_5\overline{P}_{20}(s)+\beta_6\overline{P}_{21}(s)+\beta_{HE}\overline{P}_{22}(s)+\alpha_1\overline{P}_8(s)+\alpha_6\overline{P}_1(s) \tag{8.45}$$

$$[s+\beta_1]\overline{P}_{16}(s)=\alpha_1\overline{P}_{15}(s) \tag{8.46}$$

$$[s+\beta_2]\overline{P}_{17}(s)=\alpha_2\overline{P}_{15}(s) \tag{8.47}$$

$$[s+\beta_3]\overline{P}_{18}(s)=\alpha_3\overline{P}_{15}(s) \tag{8.48}$$

$$[s+\beta_4]\overline{P}_{19}(s)=\alpha_4\overline{P}_{15}(s) \tag{8.49}$$

$$[s+\beta_5]\overline{P}_{20}(s)=\alpha_5\overline{P}_{15}(s) \tag{8.50}$$

$$[s+\beta_6]\overline{P}_{21}(s)=\alpha_6\overline{P}_{15}(s) \tag{8.51}$$

$$[s+\beta_{HE}]\overline{P}_{22}(s)=\alpha_{HE}\overline{P}_{15}(s) \tag{8.52}$$

$$[s+\beta_2]\overline{P}_{23}(s)=\alpha_2\overline{P}_1(s) \tag{8.53}$$

$$[s+\beta_3]\bar{P}_{24}(s)=\alpha_3\bar{P}_1(s) \tag{8.54}$$

$$[s+\beta_4]\bar{P}_{25}(s)=\alpha_4\bar{P}_1(s) \tag{8.55}$$

$$[s+\beta_5]\bar{P}_{26}(s)=\alpha_5\bar{P}_1(s) \tag{8.56}$$

$$[s+\beta_{HE}]\bar{P}_{27}(s)=\alpha_{HE}\bar{P}_1(s) \tag{8.57}$$

The system of equations from (8.1)-(8.28) together with initial condition (8.29) are known as Chapman-Kolmogorov differential equations. In order to find the various performance indicator of the considered system, here authors solve the above set of equations and finds the various state probabilities $\bar{P}_i(s); i=0,1,...,27$ for the sugar mill as following along with upstate and downstate of the sugar mill.

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_8(s)+\bar{P}_{15}(s) \tag{8.58}$$

$$\begin{aligned}\bar{P}_{down}(s)&=\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)+\bar{P}_5(s)+\bar{P}_6(s)+\bar{P}_7(s)+\bar{P}_9(s)+\bar{P}_{10}(s)+\bar{P}_{11}(s)\\&+\bar{P}_{12}(s)+\bar{P}_{13}(s)+\bar{P}_{14}(s)+\bar{P}_{16}(s)+\bar{P}_{17}(s)+\bar{P}_{18}(s)+\bar{P}_{19}(s)+\bar{P}_{20}(s)+\bar{P}_{21}(s)+\bar{P}_{22}(s)\\&+\bar{P}_{23}(s)+\bar{P}_{24}(s)+\bar{P}_{25}(s)+\bar{P}_{26}(s)+\bar{P}_{27}(s)\end{aligned} \tag{8.59}$$

$$\bar{P}_0(s)=\frac{1}{H_1}; \qquad \bar{P}_1(s)=\left[\frac{\alpha_1}{H_2}+\frac{\beta_6\alpha_6\alpha_1}{H_2H_3H_4}\right]\bar{P}_0(s);$$

$$\bar{P}_8(s)=\left[\frac{\beta_1\beta_6\alpha_1\alpha_6^2}{H_2H_3^2H_4^2}+\frac{\alpha_6}{H_3}+\frac{\beta_1\alpha_1\alpha_6}{H_2H_3H_4}\right]\bar{P}_0(s)$$

$$\bar{P}_{13}(s)=\left[\frac{\alpha_6^2\beta_6\alpha_1}{H_2H_3H_4^2}+\frac{\beta_1\beta_6\alpha_1^2\alpha_6^2}{H_2H_3^2H_4^3}+\frac{\alpha_1\alpha_6}{H_3H_4}+\frac{\alpha_1\alpha_6}{H_2H_4}+\frac{\alpha_1^2\beta_1\alpha_6}{H_2H_3H_4^2}\right]\bar{P}_0(s)$$

Where

$$H_1=\left[\begin{aligned}&s+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}-\frac{\beta_1\alpha_1}{H_2}-\frac{2\beta_1\beta_6\alpha_1\alpha_6}{H_2H_3H_4}-\frac{\beta_2\alpha_2}{s+\beta_2}-\frac{\beta_3\alpha_3}{s+\beta_3}-\frac{\beta_4\alpha_4}{s+\beta_4}-\\&\frac{\beta_5\alpha_5}{s+\beta_5}-\frac{\beta_{HE}\alpha_{HE}}{s+\beta_{HE}}-\frac{\beta_6\alpha_6}{H_3}-\frac{\beta_1\beta_6^2\alpha_6^2\alpha_1}{H_2H_3^2H_4^2}\end{aligned}\right]$$

$$H_2 = \begin{bmatrix} s+\beta_1+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}-\dfrac{\beta_1\alpha_1}{s+\beta_1}-\dfrac{\beta_2\alpha_2}{S+\beta_2}-\dfrac{\beta_3\alpha_3}{S+\beta_3}-\dfrac{\beta_4\alpha_4}{S+\beta_4} \\ -\dfrac{\beta_5\alpha_5}{S+\beta_5}-\dfrac{\beta_{HE}\alpha_{HE}}{s+\beta_{HE1}}-\dfrac{\beta_6\alpha_6}{H_4}-\dfrac{\beta_1\beta_6\alpha_1\alpha_6}{H_3H_4^2} \end{bmatrix}$$

$$H_3 = \begin{bmatrix} s+\beta_6+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_{HE}-\dfrac{\beta_1\alpha_1}{H_{41}}-\dfrac{\beta_2\alpha_2}{S+\beta_2}-\dfrac{\beta_3\alpha_3}{S+\beta_3}-\dfrac{\beta_4\alpha_4}{S+\beta_4} \\ -\dfrac{\beta_5\alpha_5}{S+\beta_5}-\dfrac{\beta_6\alpha_6}{s+\beta_6}-\dfrac{\beta_{HE}\alpha_{HE}}{s+\beta_{HE}} \end{bmatrix}$$

$$H_4 = \begin{bmatrix} s+\beta_1+\beta_6+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6++\alpha_{HE}-\dfrac{\beta_1\alpha_1}{s+\beta_1}-\dfrac{\beta_2\alpha_2}{S+\beta_2}-\dfrac{\beta_3\alpha_3}{S+\beta_3}- \\ \dfrac{\beta_4\alpha_4}{S+\beta_4}-\dfrac{\beta_5\alpha_5}{S+\beta_5}-\dfrac{\beta_6\alpha_6}{S+\beta_6}-\dfrac{\beta_{HE}\alpha_{HE}}{S+\beta_{HE}} \end{bmatrix}$$

8.5 Numerical Computation and Assessment of Various Reliability Measures

8.5.1 Availability

System availableness is one in all responsibleness measures of a system. It is the probability that the system is working at time t when operated under the prescribed conditions. For the system availability, set the numerical value of various failure/repair as $\alpha_1 = 0.05$, $\alpha_2 = 0.01$, $\alpha_3 = 0.02$, $\alpha_4 = 0.03$, $\alpha_5 = 0.015$, $\alpha_6 = 0.028$, $\alpha_{HE} = 0.04$, $\beta_1 = 1$ $\beta_2 = 1$, $\beta_3 = 1$, $\beta_4 = 1$, $\beta_5 = 1$, $\beta_6 = 1$, $\beta_{HE} = 1$ in (8.58) and take Inverse Laplace Transform of (8.58), the expression of the time-dependent availability is given below:

$$A(t) = \begin{bmatrix} -0.000968379e^{-1.4763761t} - 0.0001949522e^{-1.460089466t} + 0.1073957516 \\ e^{-1.117248193t} + 0.0001833977e^{-0.8655224585t} - 0.090377261e^{-2.327216180t} \\ -0.0026085536e^{-1.525425094t} - 0.15703075e^{-2.387741855t} - 0.0109295171 \\ e^{-2.40425974t} + 0.002032653e^{-0.6969340258t} + 0.0027243855e^{-0.6908052982t} \\ +0.0004439964e^{-0.863320324t} - 0.0009123318e^{-0.85408515t} \\ -0.0009921283e^{-0.68225879t} + 0.8938297613\, e^{.0.00028275031t} \end{bmatrix} \tag{8.60}$$

The behavior of time dependent availability of sugar mill can be obtained by changing time parameter t in (8.60). Subsequent Table 8.2(a) and corresponding Figure 8.2(a) represent availability of the sugar mill.

Time unit (t)	**Availability** $A(t)$
0	1.00000
1	0.90691
2	0.90425
3	0.89853
4	0.89626
5	0.89560
6	0.89553
7	0.89567
8	0.89588
9	0.89611
10	0.89636

Table 8.2(a): Behavior of Availability of the sugar mill with time

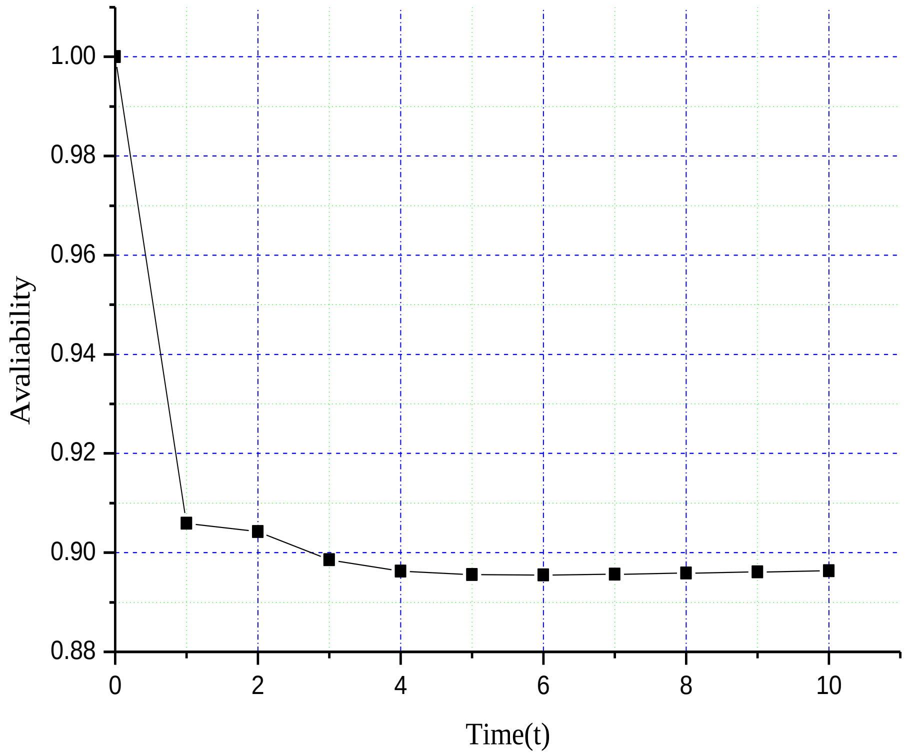


Figure 8.2(a): Behavior of Availability of the sugar mill with time

8.5.2 Reliability

System reliability is the probability that the system cannot fail before a time period 't'. For the reliability of the sugar mill the numerical value of different failure is considered as $\alpha_1 = 0.05, \alpha_2 = 0.01, \alpha_3 = 0.02, \alpha_4 = 0.03, \alpha_5 = 0.015, \alpha_6 = 0.028, \alpha_{HE} = 0.04$ and repair rate considered as zero in (8.58). The expression of sugar mill reliability is obtained as

$$R(t) = \left(0.00020000\ (5000 + 390\,t + 7\,t^2)\ e^{-0.19300000\ t}\right) \tag{8.61}$$

The behavior of time dependent reliability of sugar mill can be obtained by changing time unit t in (8.61). Table 8.2(b) and corresponding Figure 8.2(b) represent reliability of the sugar mill.

Time unit (*t*)	**Reliability R(*t*)**
0	1.00000
1	0.88994
2	0.78962
3	0.69866
4	0.61661
5	0.54290
6	0.47695
7	0.41815
8	0.36589
9	0.31959
10	0.27868

Table 8.2(b): Behavior of reliability of the system with time unit

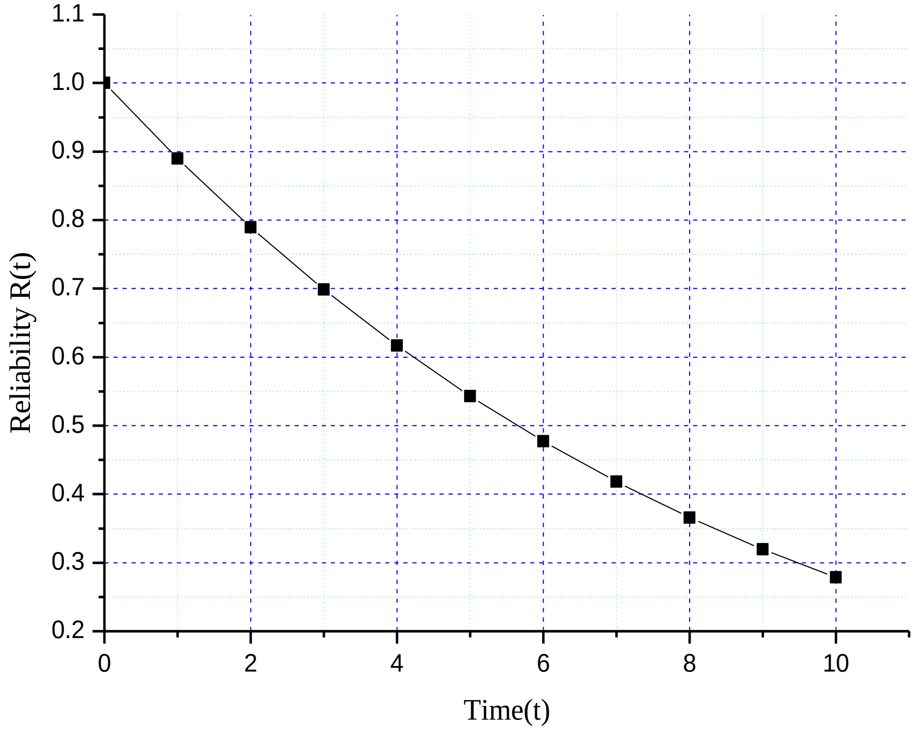


Figure 8.2(b): Behavior of Reliability of the System with time unit

8.5.3 Mean Time to Failure (MTTF)

Mathematically, the MTTF of a system is calculated as

$$MTTF = \int_0^\infty R(t)dt = \lim_{s \to 0} \overline{R}(s) \tag{8.62}$$

Now using equation (8.61) in (8.62), we get the MTTF of the sugar mill:

$$MTTF = \left[\frac{1}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE}} + \frac{\alpha_1 + \alpha_6}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})^2} + \frac{2\alpha_1\alpha_6}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})^3} \right] \tag{8.63}$$

Varying one by one failure rate from 0.01 to 0.09 with an interval of 0.01and fix other failure rates, Table 8.2(c) and Figure 8.2(c) can be easily obtained.

Variations in Failure rates	MTTF with respect to failure rates						
	α_1	α_2	α_3	α_4	α_5	α_6	α_{HE}
0.01	8.31561	7.66484	8.25048	8.92729	7.94742	7.86005	9.71726
0.02	8.20020	7.15360	7.66484	8.25048	7.40081	7.76656	8.92729
0.03	8.04273	6.70381	7.15360	7.66484	6.92172	7.63667	8.25048
0.04	7.86050	6.30529	6.70381	7.15360	6.49869	7.48393	7.66484
0.05	7.66484	5.94995	6.30529	6.70381	6.12266	7.31754	7.15360
0.06	7.46321	5.63130	5.94995	6.30529	5.78639	7.14403	6.70381
0.07	7.26054	5.34404	5.63130	5.94995	5.48403	6.96762	6.30529
0.08	7.06005	5.08387	5.34404	5.63130	5.21081	6.79138	5.94995
0.09	6.86383	4.84719	5.08387	5.34404	4.96279	6.61736	5.63130

Table 8.2(c): MTTF of the Sugar mill with various in failure rate

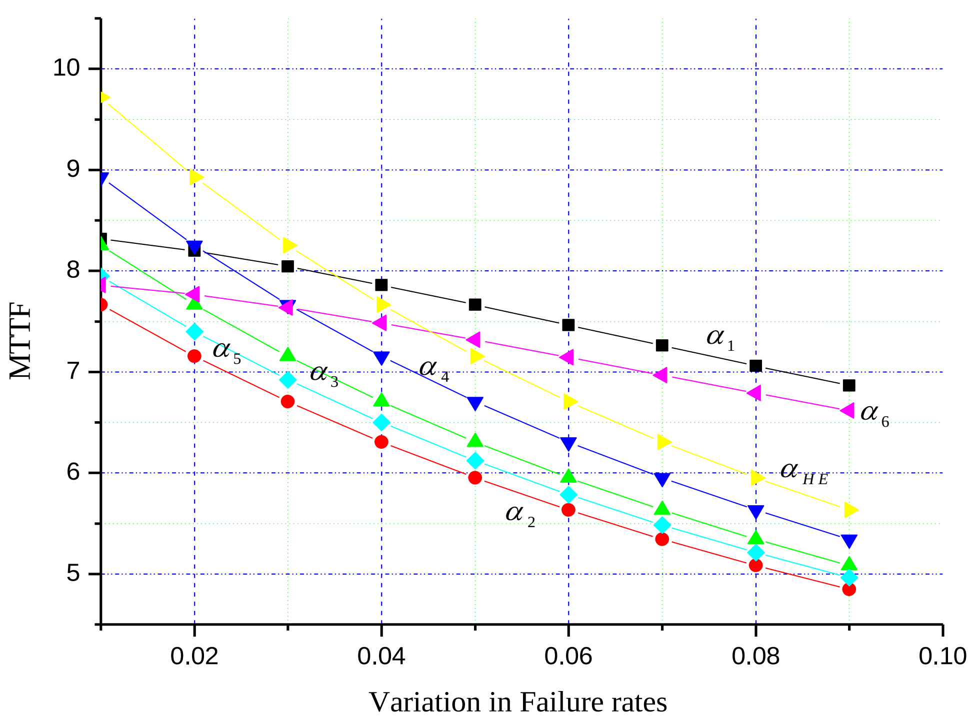


Figure 8.2(c): MTTF w.r.t. failure rate

8.5.4 Sensitivity Analysis of MTTF

The objective of the sensitivity analysis is to determine the input variables which affect the system performance most. Here authors perform the sensitivity analysis on the MTTF of the sugar mill. Table 8.2(d) shows change in the meantime to failure MTTF of the system resulting from changes in parameters $\alpha_1,\ \alpha_2,\ \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_{HE}$.

Variation in failure rates	**Sensitivity with respect of MTTF**						
	$\frac{\partial(MTTF)}{\partial\alpha_1}$	$\frac{\partial(MTTF)}{\partial\alpha_2}$	$\frac{\partial(MTTF)}{\partial\alpha_3}$	$\frac{\partial(MTTF)}{\partial\alpha_4}$	$\frac{\partial(MTTF)}{\partial\alpha_5}$	$\frac{\partial(MTTF)}{\partial\alpha_6}$	$\frac{\partial(MTTF)}{\partial\alpha_{HE}}$
0.01	-8.64993	-54.60011	-62.80532	-72.91918	-58.49511	-6.93044	-85.55885
0.02	-13.99608	-47.86119	-54.60011	-62.80532	-51.06985	-11.43978	-72.91918
0.03	-17.21471	-42.26548	-47.86119	-54.60011	-44.93705	-14.31633	-62.80532
0.04	-19.04569	-37.57298	-42.26548	-47.86119	-39.81881	-16.08062	-54.60011
0.05	-19.96413	-33.60267	-37.57298	-42.26548	-35.50705	-17.08201	-47.86119
0.06	-20.28051	-30.21609	-33.60267	-37.57298	-31.84373	-17.55829	-42.26548
0.07	-20.20068	-27.30604	-30.21609	-33.60267	-28.70721	-17.67328	-37.57298
0.08	-19.86272	-24.78852	-27.30604	-30.21609	-26.00272	-17.54093	-33.60267
0.09	-19.36012	-22.59706	-24.78852	-27.30604	-23.65563	-17.24117	-30.21609

Table 8.2(d): Sensitivity of the MTTF

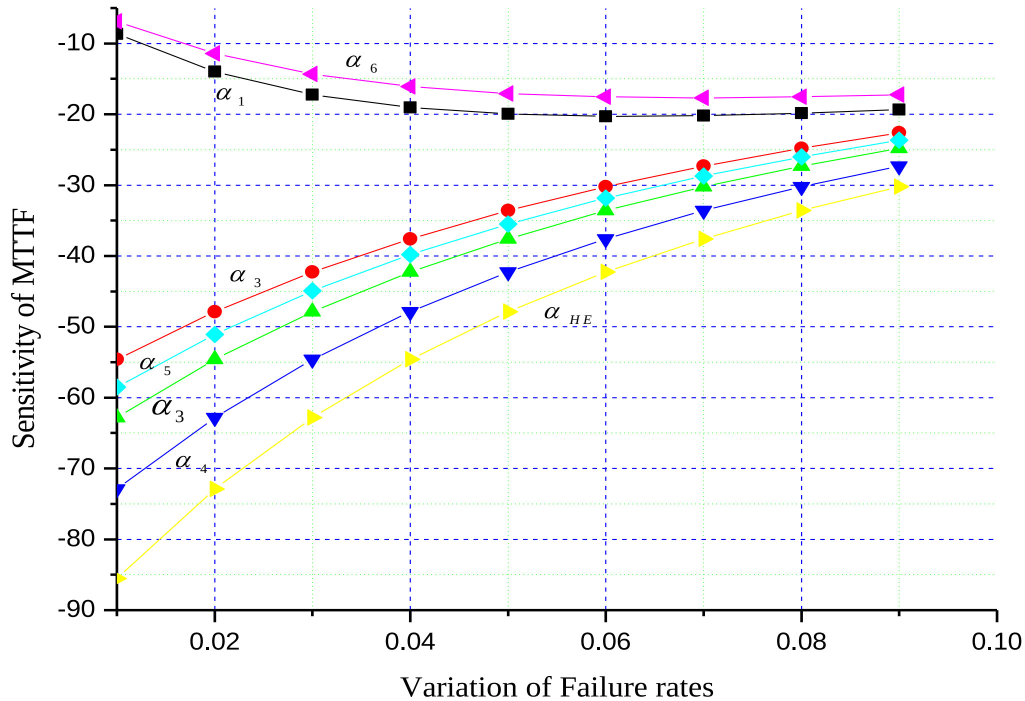


Figure 8.2(d): Sensitivity of the MTTF w.r.t variation in failure rates

8.5.5 Estimated profit from the sugar mill

The estimated profit of the sugar mill in $[0,t)$ is calculated by using

$$E_P(t) = K_1\int_0^t P_{up}(t)dt - K_2 t \tag{8.64}$$

Equation (8.64) will provide the expected profit from the sugar mill for the various choice of revenue and service cost. Using equation (8.60) in equation (8.64), authors obtain the profit function for the considered sugar mill as follow.

$$E_P(t)=\left[K_1\left\{\begin{array}{l}0.0006559165477\,e^{-1.476376154\,t}+0.0001335207330\,e^{-1.460089466\,t}-\\ 0.09612524081\,e^{-1.117248193\,t}-0.0002118925909\,e^{-0.8655224585\,t}\\ +0.03883492317\,e^{-2.327216180\,t}+0.001710050295\,e^{-1.525425094\,t}\\ 0.06576538233\,e^{-2.387741855\,t}+0.004545896979\,e^{-2.40425974\,t}-\\ 0.002916565831\,e^{-0.6969340258\,t}-0.003943782037\,e^{-0.6908052982\,t}\\ -0.005142893452\,e^{-0.8633203241\,t}+0.001068197741\,e^{-0.8540851581\,t}\\ +0.001454181879\,e^{-0.6822587943\,t}+3161.198061\,e^{0.0002827503194\,t}\\ -3161.208517\end{array}\right\}-K_2t\right] \tag{8.65}$$

Now vary the service cost $K_2 = 0.1, 0.2, 0.3, 0.4, 0.5$ fix revenue K_1 as one and vary time unit *t* in (8.65). Table 8.2(e) and corresponding Figure 8.2(e) is obtained as follows:

Time unit (*t*)	**Expected Profit from the system**				
	$K_2=0.1$	$K_2=0.2$	$K_2=0.3$	$K_2=0.4$	$K_2=0.5$
0	0	0	0	0	0
1	0.76029	0.66029	0.56029	0.46029	0.36029
2	1.56722	1.36722	1.16722	0.96722	0.76722
3	2.36828	2.06828	1.76828	1.46828	1.16828
4	3.16547	2.76547	2.36547	1.96547	1.56547
5	3.96133	3.46133	2.96133	2.46133	1.96133
6	4.75687	4.15687	3.55687	2.95687	2.35687
7	5.55246	4.85246	4.15246	3.45246	2.75246
8	6.34824	5.54824	4.74824	3.94824	3.14824
9	7.14424	6.24424	5.34424	4.44424	3.54424
10	7.94048	6.94048	5.94048	4.94048	3.94048

Table 8.2(e): Expected profit of the system

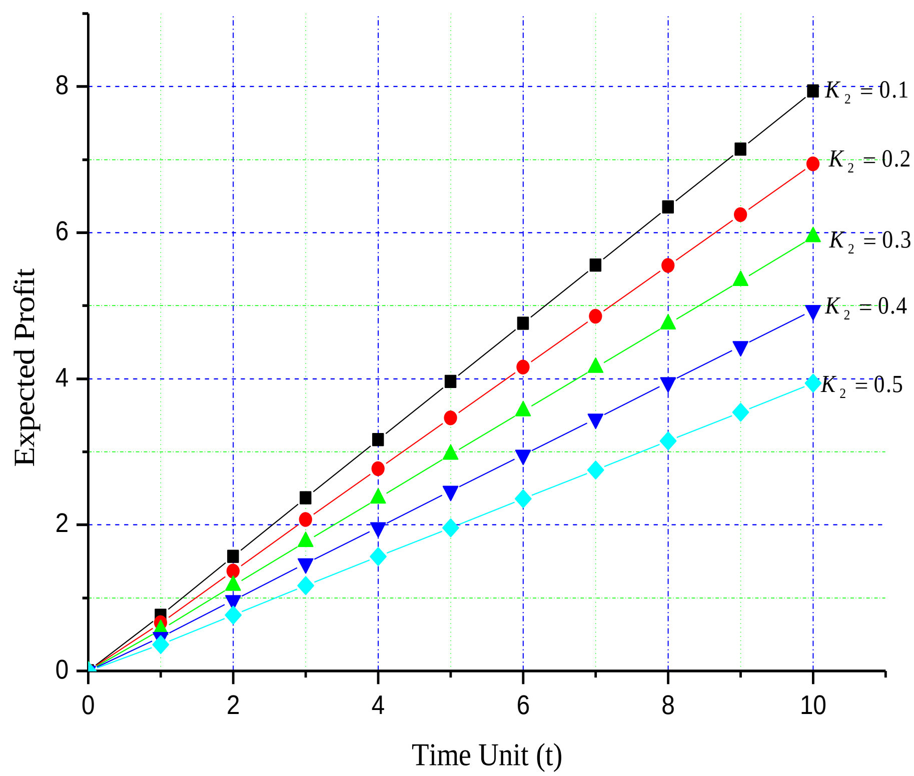


Figure 8.2(e): Expected Profit of the sugar mill vs. time unit *t*

8.5.6 MTBF

For calculating the MTBF of the sugar mill, initially, we find MTTR of the sugar mill from the equation (8.59). MTTR is the average time that a system takes to recover from failure. For MTTR, take limit $s \to 0$ in the equation (8.59), then we get

$$MTTR = \lim_{s \to 0} \overline{P}_{down}(s) \tag{8.66}$$

After finding MTTR of the system, one can easily find MTBF of the system. MTBF is the average mean time between the two failures. For MTBF, find the sum of MTTF and MTTR. Thus, after adding equation (8.63) and (8.66) we get

$$MTBF = MTTF + MTTR \tag{8.67}$$

The expression for the MTBF is given below:

$$MTBF = \left[\begin{array}{l} \frac{1}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})} + \frac{\alpha_1 + \alpha_6}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})^2} \\ + \frac{2\alpha_1\alpha_6}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})^3} + \frac{\left(\frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\alpha_5}{\beta_5} + \frac{\alpha_{HE}}{\beta_{HE}}\right)}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})} \\ + \frac{\left(\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\alpha_5}{\beta_5} + \frac{\alpha_{HE}}{\beta_{HE}}\right)\alpha_1}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})^2} + \frac{\left(\frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\alpha_5}{\beta_5} + \frac{\alpha_6}{\beta_6} + \frac{\alpha_{HE}}{\beta_{HE}}\right)\alpha_6}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})^2} \\ \frac{2\alpha_1\alpha_6\left(\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\alpha_5}{\beta_5} + \frac{\alpha_6}{\beta_6} + \frac{\alpha_{HE}}{\beta_{HE}}\right)}{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_{HE})^3} \end{array} \right] \tag{8.68}$$

Set the failure rates and repair rates as $\alpha_1 = 0.05$, $\alpha_2 = 0.01$, $\alpha_3 = 0.02$, $\alpha_4 = 0.03$, $\alpha_5 = 0.015$, $\alpha_6 = 0.028$, $\alpha_{HE} = 0.04$, $\beta_1 = 1$, $\beta_2 = 1$, $\beta_3 = 1$, $\beta_4 = 1$, $\beta_5 = 1$, $\beta_6 = 1$, $\beta_{HE} = 1$. In order to obtain the MTBF of the sugar mill one by one vary each failure rate from 0.01 to 0.09 as shown in the below Table 8.2 (f).

Variations in Failure rates	MTBF with respect to failure rates						
	α_1	α_2	α_3	α_4	α_5	α_6	α_{HE}
0.01	9.31561	8.66484	9.25048	9.92729	8.94742	8.86005	10.71726
0.02	9.20020	8.15360	8.66484	9.25048	8.40081	8.76656	9.92729
0.03	9.04273	7.70382	8.15360	8.66484	7.92173	8.63667	9.25048
0.04	8.86050	7.30529	7.70382	8.15360	7.49869	8.48393	8.66484
0.05	8.66484	6.94996	7.30529	7.70382	7.12267	8.31759	8.15360
0.06	8.46321	6.63130	6.94996	7.30529	6.78639	8.14403	7.70382
0.07	8.26055	6.34404	6.63130	6.94995	6.48403	7.96762	7.30529
0.08	8.06005	6.08387	6.34404	6.63130	6.21081	7.79138	6.94996
0.09	7.86383	5.84719	6.08387	6.34404	5.96279	7.61736	6.63130

Table 8.2(f): MTTF of the Sugar mill with various in failure rate

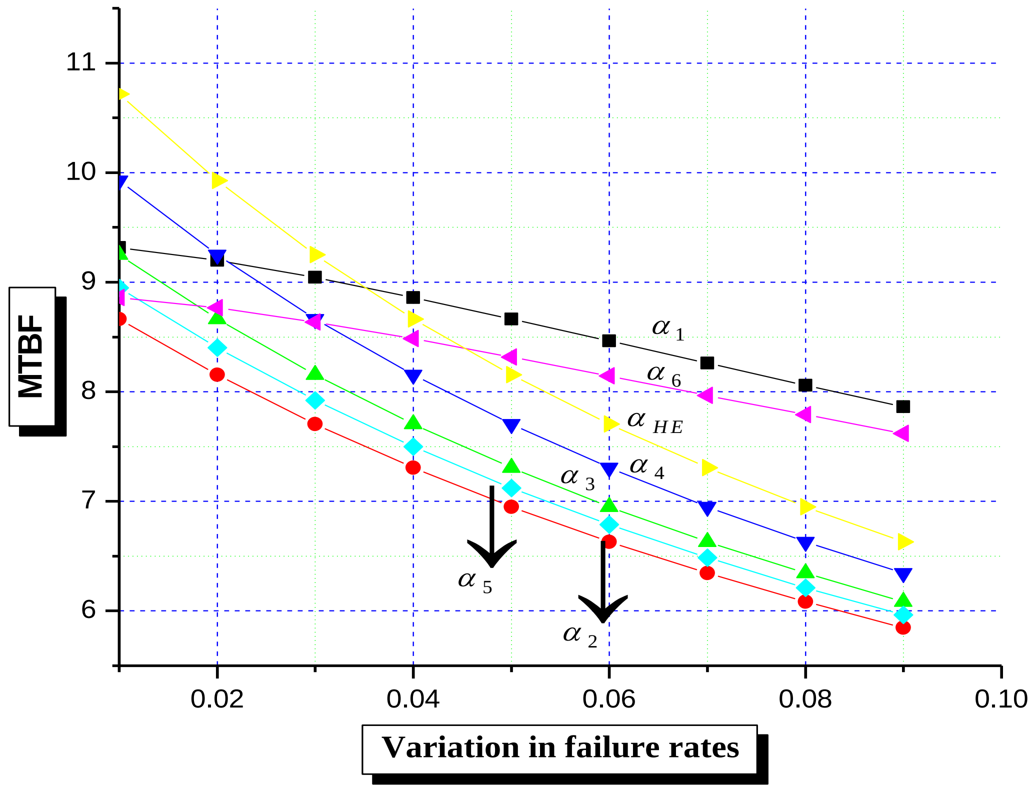


Figure 8.2(f): MTBF w.r.t. variation in failure rate

8.6 Result Discussion

In this chapter, the performance of "Wahid sugar mill" situated in Punjab, India has been analyzed by incorporating human error. For this, the six components of the plant have been taken into consideration. After analyzing the system mathematically, we get the following results.

The sugar mill's availability is shown in Figure 8.2(a). It is determined that system availableness decreases terrible slowly as time passes. Also, the availability of the sugar mill at ten units of time is 0.89636. Figure 8.2(b) reflects the reliability of the sugar mill with respect to the time unit. It is found that the reliability of the sugar mill at ten units of time is 0.27868. This means that with the passage of time unit system's reliability is also decreasing. This may be due to the aging, corrosion, stress etc. in the system's component. Figure 8.3(c) shows the nature of MTTF of the sugar mill with respect to variation in failure rates. It reveals that the MTTF of unloader is the highest. So the performance of the sugar mill less disturbed by the variation in the unloader's failure rate. Despite increasing the failure rate of the unloader MTTF is higher as compared to other components of the system. The graph of sugar mill sensitivity for its MTTF is shown in Figure 8.2(d), it can be seen that MTTF is highly delicate with respect to human error. When the human error rate increases, it adversely affects the system MTTF. Figure 8.2(e) shows the behavior of the expected profit from the sugar mill. It shows that on increasing the service cost expected profit of the system decreases. Hence, to optimize the profit function the management needs to give more attention to the maintenance policy. Figure 8.2(f) shows the MTBF of the sugar mill. MTBF of the mill is quite low w.r.t variation in the failure rate of conveyer belt. Both MTTF and MTBF are low w.r.t variation in the failure rate of the conveyer belt.

8.7 Conclusion

Authors compare the present study with the work done by Sharma and Vishwakarma [58]. They investigated the reliability of the feeding unit which comprises of four units Cutting system(A), Crushing system(B), Bagasse carrying system(C), Heat generating system (D). In their model the reliability of the system after 500 operation hours is 0.021197. In our model, we introduced redundancy at the component level and human error was also incorporated in the model. Our model's reliability after 480 hours of operation is 0.88994 which is 40% higher than their model. Hence, our proposed model is the improved version of their model.

It is clear from the result discussion section that more attention is required on human error and failures in the conveyer belt. Both these can be reduced by employing skilled labor and imparting them training from time to time. It asserts that this research is beneficial for the management of the same for improving its productivity.

Chapter 9: A New Approach for UGF Implementation on the Excel Software for Evaluating the Multi-State System Performance Measures

The main aim of this chapter is to show how the Universal generating function (UGF) technique can be implemented using the Excel software with great ease, which gives us the same results, which one can obtain by the algebraic multiplication of the UGF polynomials for the system's components for obtaining the system's performance. As without the proper knowledge of computer programming, it is not possible to apply the UGF technique for a big system as the computation burden increases drastically as one starts implementing this technique for obtaining the system's performance measures. Therefore, it is the need of the hour to have an easy way to implement the UGF technique. In this chapter, the authors present how the UGF technique can be implemented using the Excel software with ease, which can save an ample amount of time for the research community. An example is also given in this chapter so that one can easily implement it using the Excel software for obtaining the performance measures of the MSS.

9.1 Introduction

With the advent of computer technology, these days many good soft wares are available in the market like MATLAB, MAPLE, etc. which can perform the computation very quickly and precisely. These soft wares are capable of performing very complex calculations in very little time. But for using these soft wares one must have to have sound knowledge of the commands of these soft wares and the computer programming. It is not everyone's cup of tea to learn computer programming. Hence, it is a great hurdle for new researchers or for those people who are not computer savvy. These people spend a great amount of time learning these soft wares to be able to get the research related results. Therefore, this is the need of the hour to have an easy way of implementing the technique on the other software which gives the same results with great ease. It helps the researcher to spend more time learning other research-related activities. Here, in this chapter authors suggest a new approach to implement the UGF technique on Excel software which gives the same result which one can obtain with the help of algebraic multiplication of the UGF polynomials of the system's component.

In every industry, many machines are installed to produce products. Some industrial systems are very complex and have thousands of components. These components work with varying degrees of performance rates. The system which works with a varying level of performance rates is called a multi-state system. In this system, the system and its components both have various performance rates. Many engineering systems like Power systems, Flow transmission systems, Data transmission systems work with varying degrees of performance levels. For example, in a power system, a generator may be capable of producing electricity equal to 150 MW, but due to the degradation in the generator components, it may be capable of producing electricity equal to 100 MW. Further, degradation in the generator components may cause the generator to produce electricity equal to 50 MW. In the end, when components degrade further it fails completely. So, here this generator work with four performance levels $\{150MW, 100MW, 50MW, 0MW\}$. So, in this way, if there are 4 generators installed in the power station, then the whole system may have many performance rates. Therefore, to analyze the performance of the system, four methods have been purposed in the literature for the reliability indices determination: (1) Structure-function approach (2) Stochastic-process approach (3) Universal generating function technique (4) Monte- Carlo simulation technique. The structure-function technique can be used when the state of each component is known. But for a big system, this technique is not preferred. The Stochastic-process technique suffers from dimensionality curse. Even for a small system number of system states increase very drastically and one may easily forget to take any of the system states in the transition state diagram. This is the main disadvantage of the stochastic-process method. Monte-Carlo simulation technique is based on the simulations therefore, sometimes it also takes a long time to get the simulation results. The last technique is the UGF technique, which has been extensively used by the researchers for obtaining various performance measures of the Multi-state system. But the implementation of this technique can be improved.

9.1.1 Problem statement

It is quite clear from the above discussion that the UGF technique is the best technique for the multi-state system reliability evaluation. But its implementation without the proper knowledge of computer programming is very difficult. All the researchers used MATLAB software for the performance evaluation of the MSS using the UGF technique or used the Descartes product rule. In the literature, no one has ever tried to implement the UGF technique using the Excel software which is very easy and can save a lot of time for the research community. Therefore, the authors in this chapter, present how the UGF technique can be implemented using Excel software very easily. This chapter is structured as: In section 9.2, the various notations used in the chapter are given. In section 9.3, the UGF definition and its various composition operators are given. In section 9.4, Multi-state performance indices are described. In section 9.5, Implementation of the UGF technique using Excel software is given. In section 9.6, result and discussion are given.

9.2 Notations

The following notations will be used in the chapter for understanding the mathematical calculations.

N	Number of system's components
$k_i \ (i = 1, 2 ... N)$	Number of states of the i^{th} component
$G_i(t) \ (i = 1, 2 ..., N)$	Represents performance rate variable of the i^{th} component at time t
$g_i \ (i = 1, 2 ... N)$	Random variable representing the performance of the i^{th} component
g_{ij}	Represents the performance rate of the i^{th} component in the j^{th} state

p_{ij}	Represents the probability of the i^{th} component in the j^{th} state
$u(z)$	Represents the UGF of the system
$u_i(z)$	Represents the UGF of the i^{th} component of the system
A	Represents system availability
δ_A	Represents availability operator
E	Represents mean output performance
D	Represents mean output performance deficiency
I	Represents indicator function
k	Total number of system states
f	Structure function
MSS	Multi-state system
UGF	Universal generating function

Table 9.1: Notations

9.3 UGF Definition and Various Composite Operators

Suppose that a system has $'N'$ elements, where the i^{th} element of the system has different performance rates, which is represented by the performance rate variable $G_i(t)$ that takes the value from the set $g_i = \{g_{i1}, g_{i2}, g_{i3} ... g_{ik_i}\}$, where g_{ij} is the performance rate of the i^{th} component in the j^{th} state. The probabilities associated with i^{th} element in the various states is given by the set $p_i = \{p_{i1}, p_{i2}, p_{i3} ... p_{ik_i}\}$, where p_{ij} is the probability of the i^{th} component in the j^{th} state. Therefore UGF polynomial for the i^{th} element is given by:

$$u_i(z) = \sum_{j=1}^{k_i} p_{ij} \, z^{g_{ij}} \tag{9.1}$$

In this way, the UGF polynomial of every element is written. For obtaining the different performance indicators of the multi-state system, the UGF polynomial of the whole system is obtained which is given below.

$$u(z) = \sum_{j_1=1}^{k_1} \sum_{j_2=1}^{k_2} \dots \sum_{j_N=1}^{k_N} p_{ij_1} p_{ij_2 \dots} p_{ij_N} \, z^{f(g_{ij_1}, g_{ij_2} \dots g_{ij_N})} \tag{9.2}$$

This f is called the structure-function. There are basically two types of structure-function: f_{series} and $f_{parallel}$. f_{series} and $f_{parallel}$ give different performance rate value, depending upon, which type of system we are dealing with.

For a Flow transmission system:

$$f_{series} = \min(G_1(t), G_2(t) \dots G_n(t)) \tag{9.3}$$

For a Task processing system:

$$f_{series} = \frac{1}{\frac{1}{G_1(t)} + \frac{1}{G_2(t)} + \dots + \frac{1}{G_n(t)}} \tag{9.4}$$

For a Flow transmission system:

$$f_{parallel} = (G_1(t) + G_2(t). + \dots + G_n(t)) \tag{9.5}$$

For a Task processing system:

$$f_{parallel} = \max(G_1(t), G_2(t) \dots G_n(t)) \tag{9.6}$$

From equation (9.2), it is quite obvious that total number of system states are equal to

$$k = \prod_{i=1}^{N} k_i \tag{9.7}$$

But the fact is that some system states have the same performance rate values, then we combine the terms which have the same performance rate and it just becomes only one state of the system. In this way, the UGF technique reduces the total number of system states. To make things more clear, we consider a Flow transmission system with two elements. Suppose for the first element, the performance rate set is $g_1 = \{g_{11}, g_{12}, g_{13}\}$ and the corresponding probability set is $p_1 = \{p_{11}, p_{12}, p_{13}\}$ and for the second element, the performance rate set is $g_2 = \{g_{21}, g_{22}\}$ and the corresponding probability set is $p_2 = \{p_{21}, p_{22}\}$.Therefore, the UGFs of first and second elements are given by:

$$u_1(z) = p_{11} z^{g_{11}} + p_{12} z^{g_{12}} + p_{13} z^{g_{13}} \tag{9.8}$$

$$u_2(z) = p_{21} z^{g_{21}} + p_{22} z^{g_{22}} \tag{9.9}$$

When these components work in series and the system is Flow transmission system, then the UGF of the whole system is given by:

$$u(z) = u_1(z) \underset{series}{\otimes} u_2(z)$$

$$u(z) = \left(p_{11} z^{g_{11}} + p_{12} z^{g_{12}} + p_{13} z^{g_{13}}\right) \underset{series}{\otimes} \left(p_{21} z^{g_{21}} + p_{22} z^{g_{22}}\right)$$

$$\begin{aligned} u(z) = {} & p_{11} . p_{21} z^{\min(g_{11}, g_{21})} + p_{11} . p_{22} z^{\min(g_{11}, g_{22})} + p_{12} . p_{21} z^{\min(g_{12}, g_{21})} \\ & + p_{12} . p_{22} z^{\min(g_{12}, g_{22})} + p_{13} . p_{21} z^{\min(g_{13}, g_{21})} + p_{13} . p_{22} z^{\min(g_{13}, g_{22})} \end{aligned} \tag{9.10}$$

When these components work in parallel and the system is Flow transmission system, then the UGF of the whole system is given by:

$$u(z) = u_1(z) \underset{parallel}{\otimes} u_2(z)$$

$$u(z) = \left(p_{11} z^{g_{11}} + p_{12} z^{g_{12}} + p_{13} z^{g_{13}}\right) \underset{parallel}{\otimes} \left(p_{21} z^{g_{21}} + p_{22} z^{g_{22}}\right)$$

$$\begin{aligned} u(z) = {} & p_{11} . p_{21} z^{(g_{11}+g_{21})} + p_{11} . p_{22} z^{(g_{11}+g_{22})} + p_{12} . p_{21} z^{(g_{12}+g_{21})} \\ & + p_{12} . p_{22} z^{(g_{12}+g_{22})} + p_{13} . p_{21} z^{(g_{13}+g_{21})} + p_{13} . p_{22} z^{(g_{13}+g_{22})} \end{aligned} \tag{9.11}$$

From equation (9.10) and equation (9.11), it is quite clear that for a small system with two components where component 1 has three performance rates and component 2 has only two performance rates, the whole system has six performance state or less based on the numerical value of the performance rates values of the component1 and component 2. If in the system number of components are more this polynomial terms increase very rapidly and become unmanageable when it is implemented using the MATLAB or solved manually, this is the main drawback of this technique. But its implementation using Excel software is very easy. New research scholars and those who are not computer savvy can easily implement it using the Excel software with ease and can easily obtain the desired results. Here, also notice one more thing in equation (9.10), when the exponent of z is $\min(g_{11}, g_{21})$, then system's state probability is $p_{11}.p_{21}$. Similarly in equation (9.11), when the exponent of z is $(g_{11} + g_{21})$, then system state probability is $p_{11}.p_{21}$. Hence, we establish the relationship between exponents of z and its corresponding probabilities. In general, if the exponent of z is $\min(g_{11}, g_{21},..., g_{N1})$ then its corresponding probability is $p_{11}.p_{21}...p_{N1}$. Similarly, if the exponent of z is $(g_{11} + g_{21} + ... + g_{N1})$ then its corresponding probability is $p_{11}.p_{21}...p_{N1}$. For the computations of this chapter, this argument will be used. In the next section, we give a description of the various performance measures of the multi-state system.

9.4 Multi-State System Performance Indicators

Once the final UGF of the whole system is obtained, then the system may have many performance rates. Only those states are considered good states of the system whose performance rate satisfies a certain demand w. Therefore, the total number of states is divided into two subsets: acceptable and non-acceptable. The states which satisfy $r_i = g_i - w \geq 0$ come in the acceptable subset and the states which satisfy $r_i = g_i - w \prec 0$ come in the unacceptable subset.

9.4.1 Availability

Availability of the multi-state system is defined as the probability that multi-state system performance rate is greater than w. Mathematically, it can be expressed as

$$A = \sum_{r_i \geq 0} p_i \tag{9.12}$$

Using availability operator, availability of the system can be represented as:

$$A = \delta_A(U(z), w) = \sum_{i=1}^{n} p_i . \beta_i \tag{9.13}$$

Where

$$\beta_i = \begin{cases} 1 & r_i \geq 0 \\ 0 & r_i < 0 \end{cases}$$

9.4.2 Mean Output Performance

Mean output performance of the system is defined as the average value of the output performance of the system. Mathematically, it can be expressed as

$$E = \sum_{i=1}^{k} g_i . p_i \tag{9.14}$$

9.4.3 Mean Output Performance Deficiency

The mean output performance deficiency of the MSS is average value when the system output performance doesn't satisfy its demand. Mathematically, it can be expressed as

$$D = \sum_{r_i < 0} p_i (w - g_i) \tag{9.15}$$

9.5 Implementation of UGF Technique on the Excel Software

To show the implementation of the UGF technique using Excel software, we consider here one Flow transmission system having three multi-state elements as shown in the following diagram. The rate of this flow transmission is taken in tons/ per minute. The first element has the performance rates given by the set $g_1 = \{g_{11}, g_{12}, g_{13}\}$ and the corresponding probabilities are given by the set $P_1 = \{P_{11}, P_{12}, P_{13}\}$. The performance rates of the second element are given by the set $g_2 = \{g_{21}, g_{22}, g_{23}\}$ and the corresponding probabilities are given by the set $P_2 = \{P_{21}, P_{22}, P_{23}\}$.Similarly the performance rates of the third element are given by the set $g_3 = \{g_{31}, g_{32}\}$ and the corresponding probabilities are given by the $P_3 = \{P_{31}, P_{32}\}$.The diagram of the system is presented below.

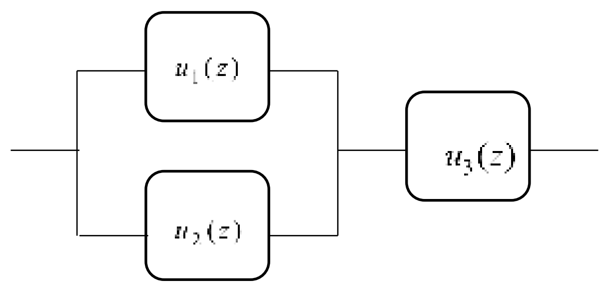
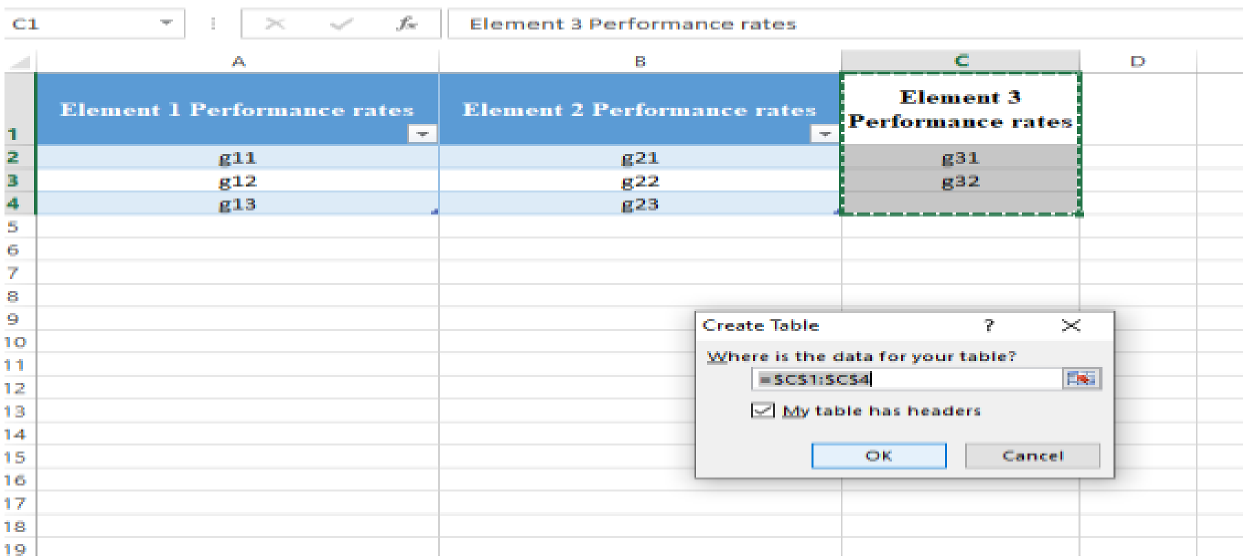


Figure 9.1: A Flow transmission system structure

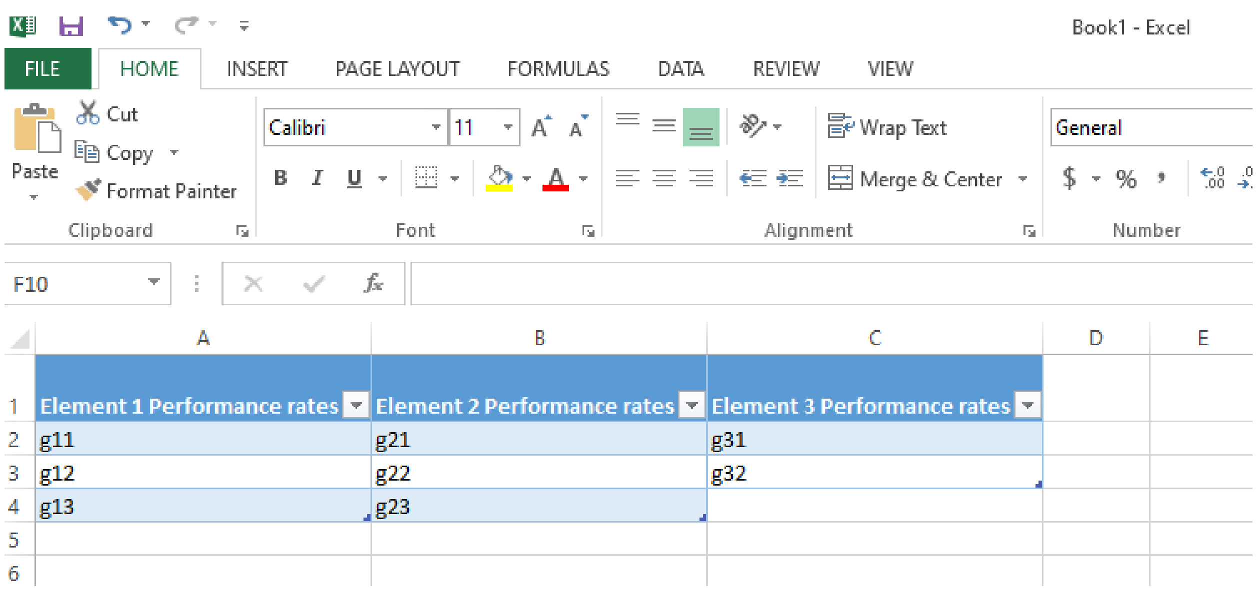
Here, element 1 and element 2 work in the parallel configuration and element 3 works in a series configuration with the first two elements. Enter the data of the first element performance rate in column A, the second element in column B, and performance rate data of the third element in column C. Select the first column A alone and from the keyboard press CTRL+T keys combination then "Create Table box" appears, then check the box "My table has headers" and click the OK button. Do the same step for column B. For column C, the step is shown in the following screenshot

Step 1:



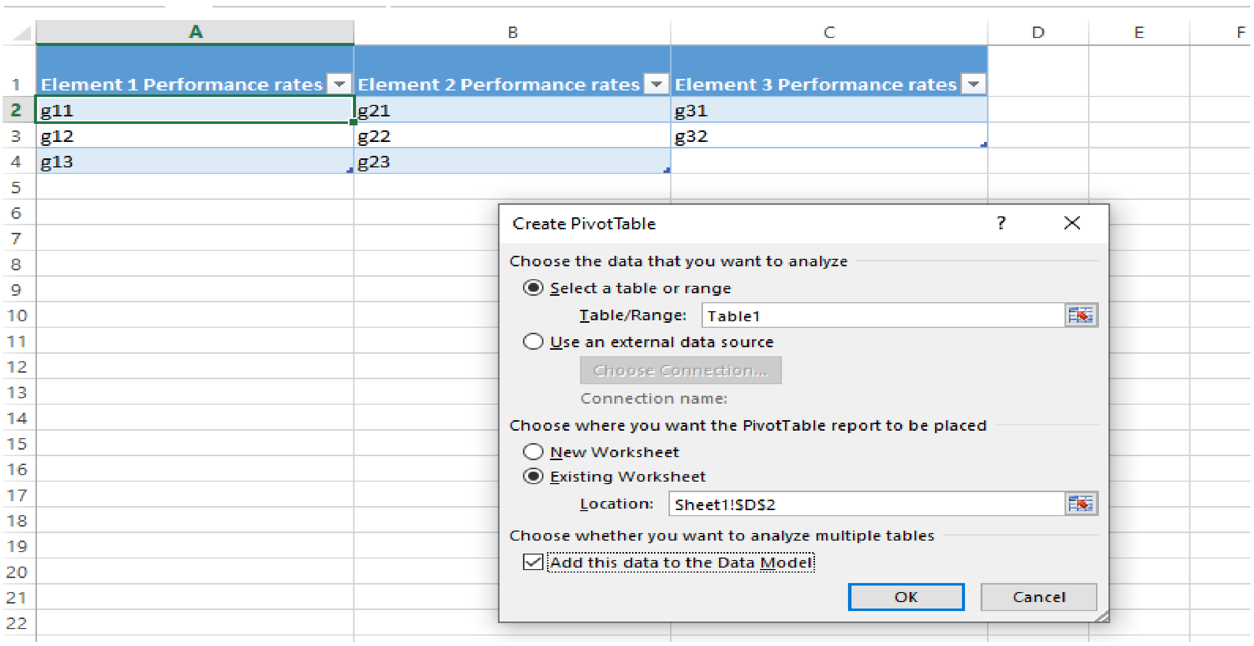
Once it has been done for all three columns, we get the following screen.

Step 2:



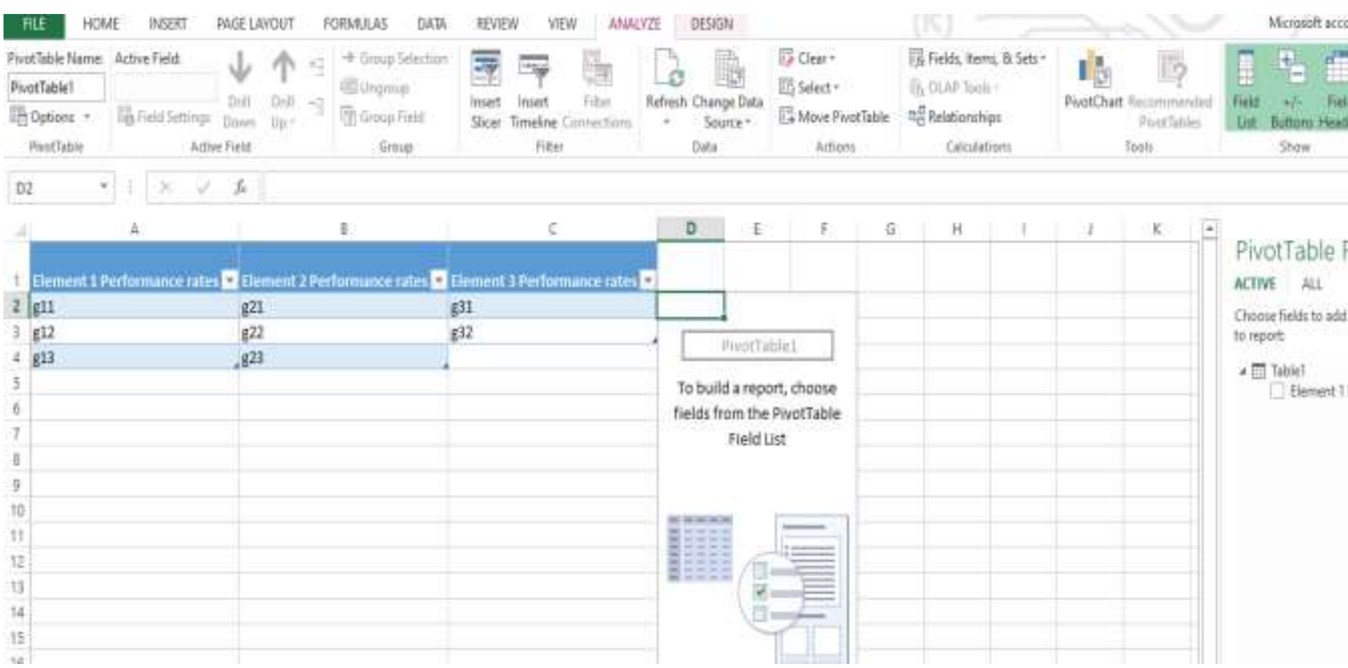
After this step, select any cell of column A except the heading of column A and from the keyboard press ALT+N+V keys combination. The “Create Pivot Table” dialogue box appears, click on the check box “Existing worksheet” and select any blank location of the worksheet. Here authors selected the D2 cell, then this location appears in the “Location box". After this step, tick on the check box "Add this data to the Data Model". The following screenshot explains it completely, then click the OK button

Step 3:



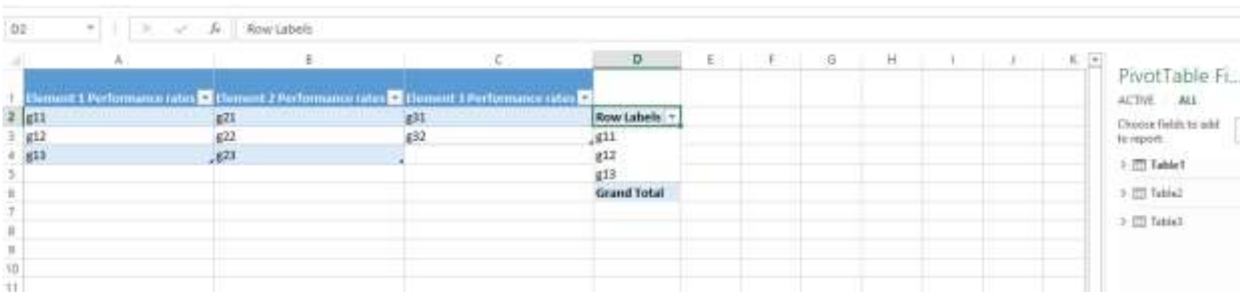
After this the following screen appears.

Step 4:



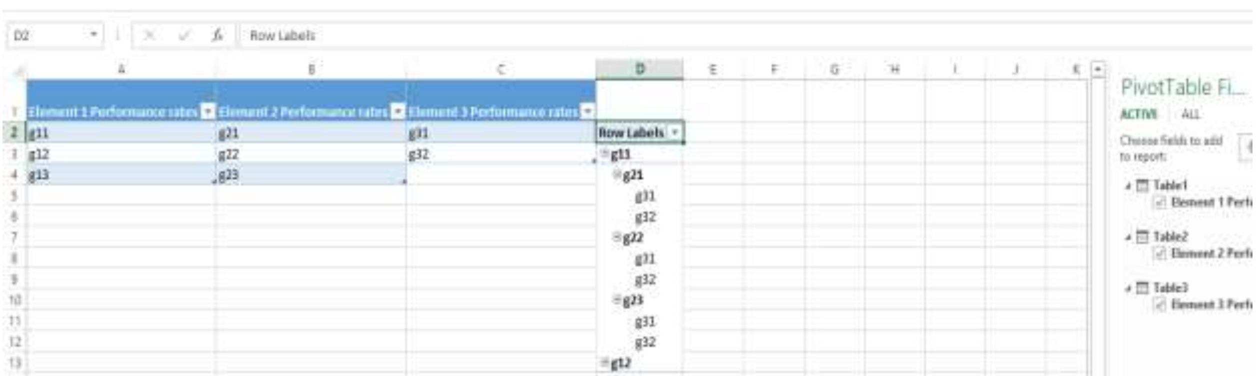
In the above screen on the upper right corner of the screen pivot table field is written. Below it there are two categories: "Active" and "All". By default the "Active" field is active. Below it Table 1 is given. Click the check box of "Element 1 performance rate". In this way Table 1 data is fitted in the model. After this click on the "All" field. The following screen appears.

Step 5:



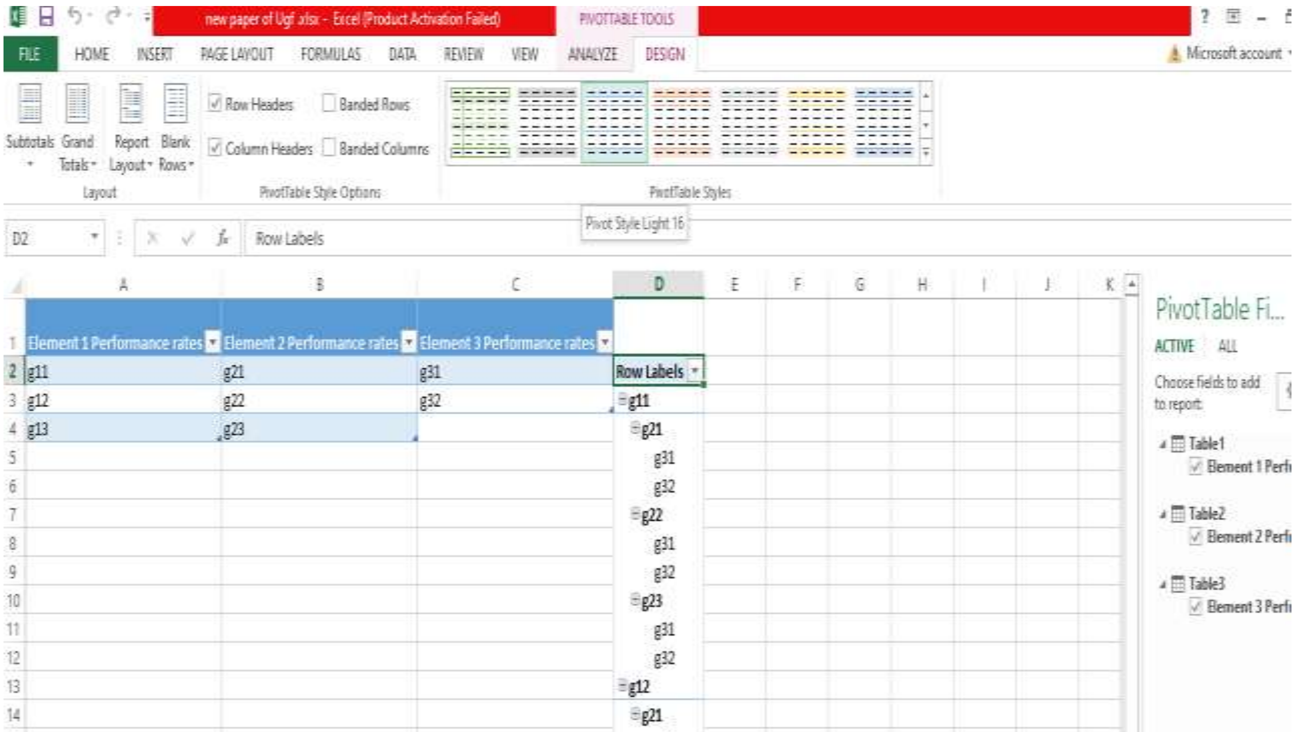
On the right-hand side of the screen Table 2 and Table 3 are written. Click on Table 2 and then tick the check box of element 2 performance rate do the same for Table 3. In this way, Table 2 data and Table 3 data are also added to the model. The following screen appears after this step

Step 6:

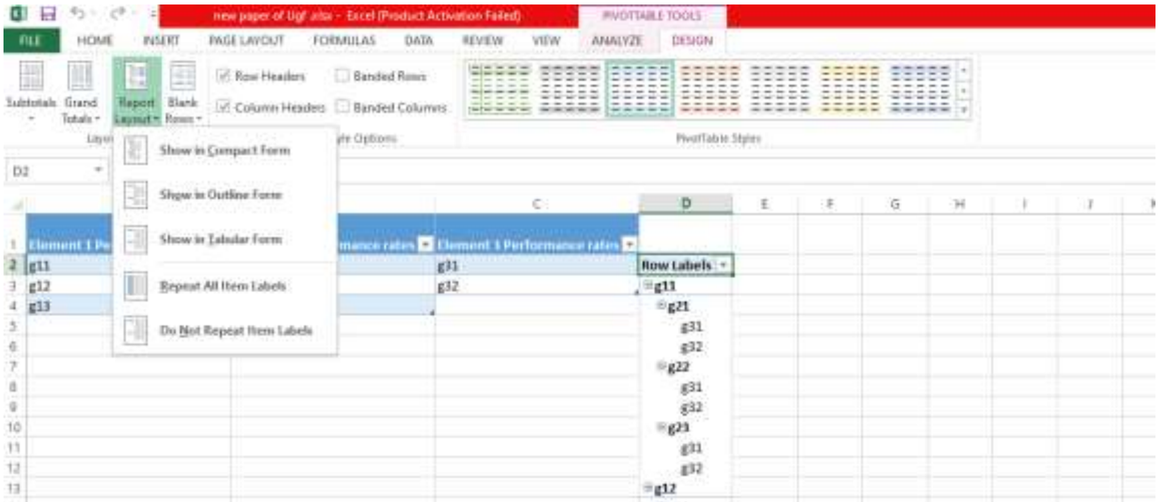


After this click on the Design button from the menu bar. It is shown in the following screenshot.

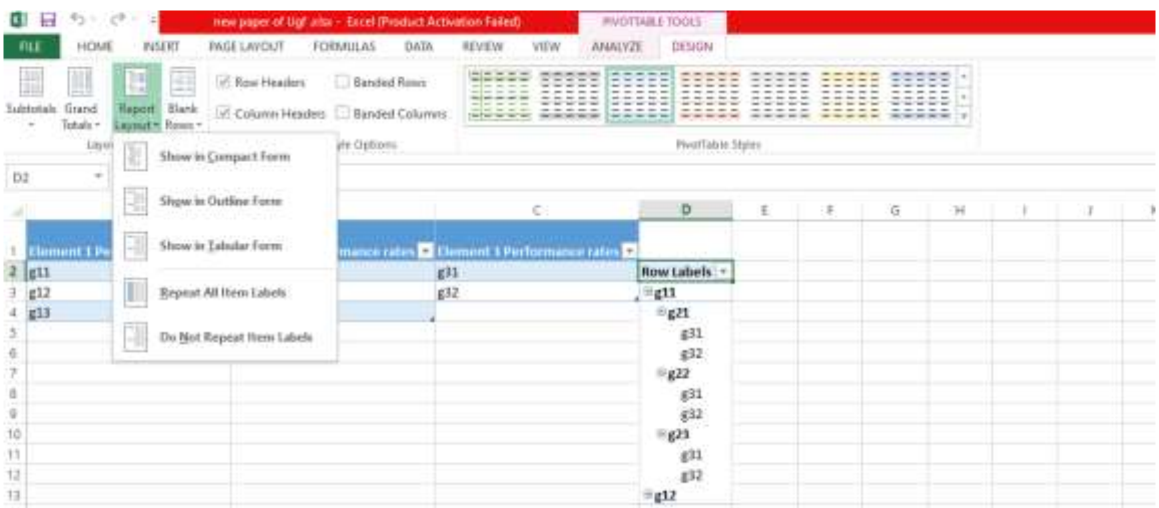
Step 7:



Now click on the “Report Layout”, a drop down menu appears as shown in the following screenshot.

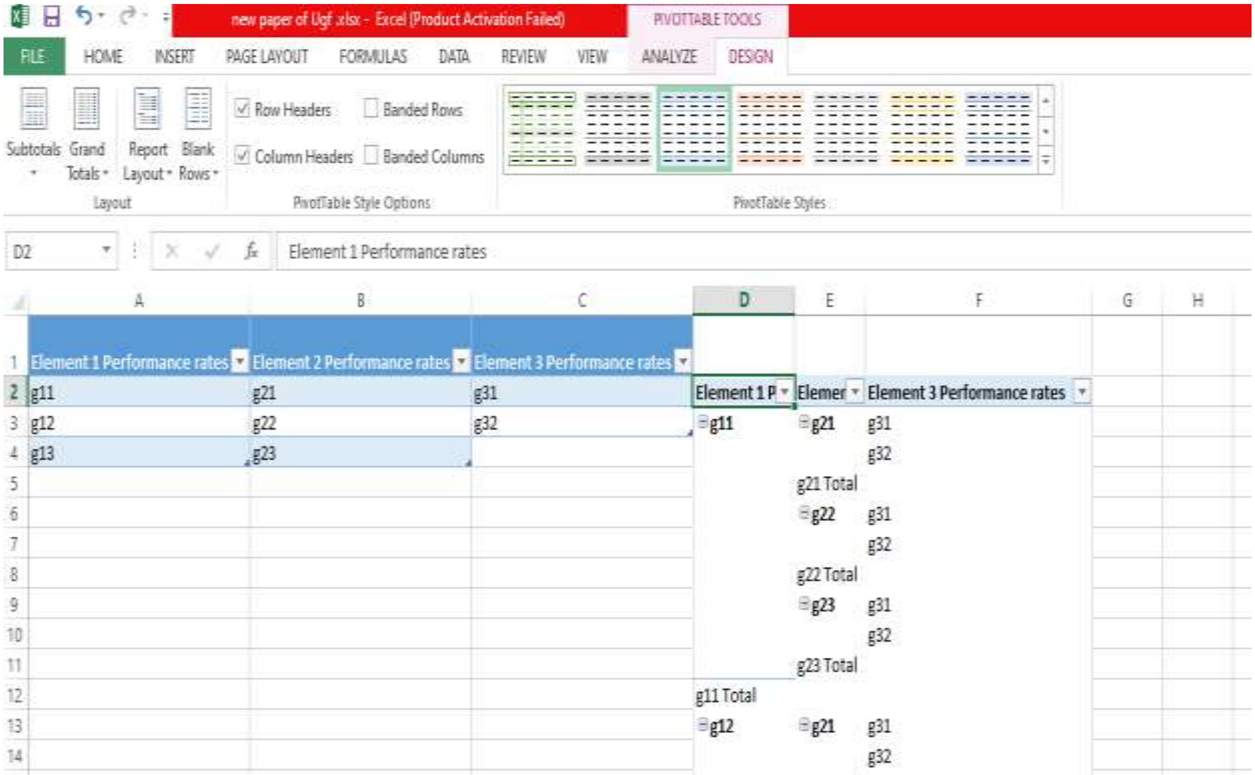


Step 8:



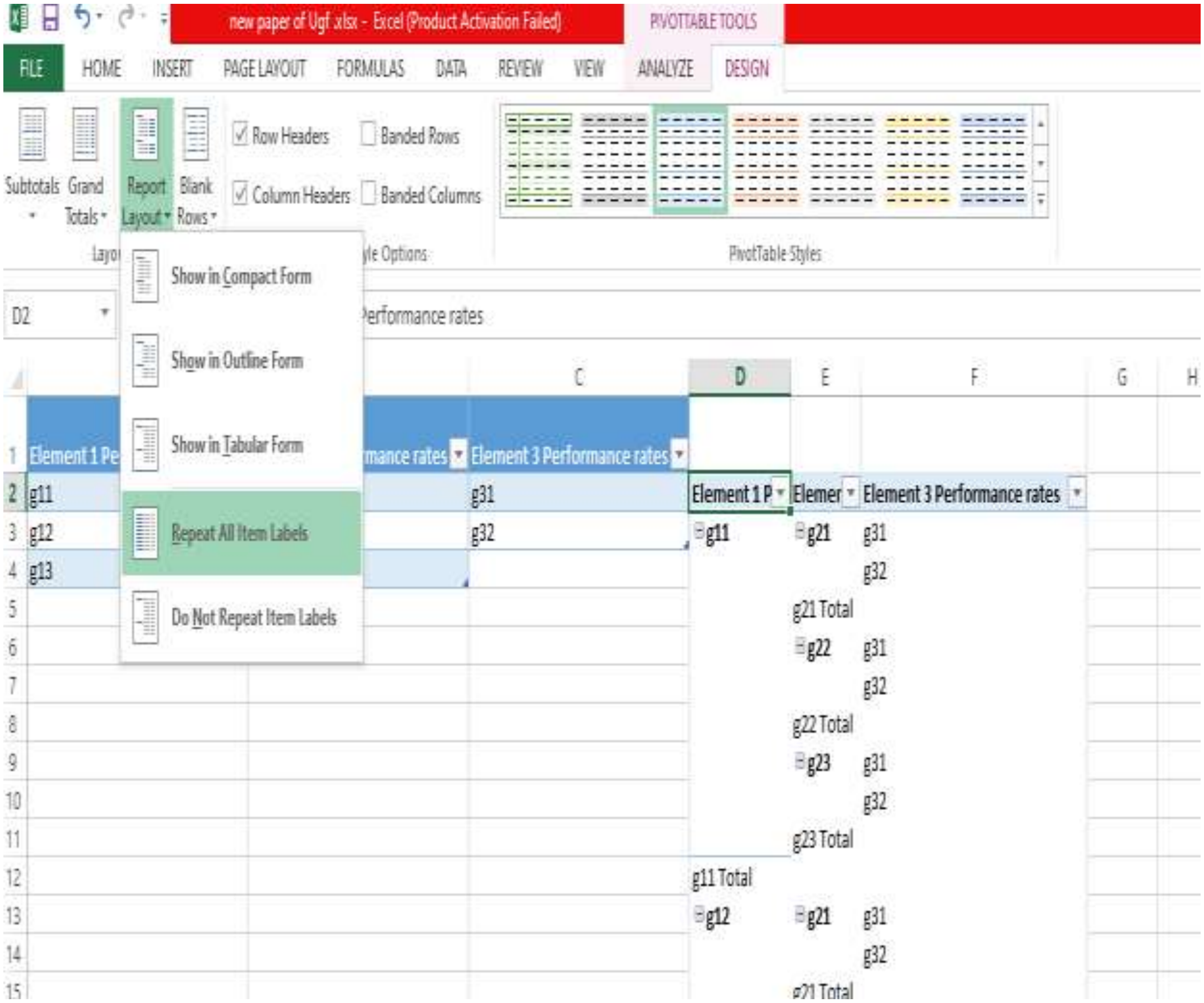
Now click on the show tabular form, the screen appears like this as shown in the following screenshot.

Step 9:



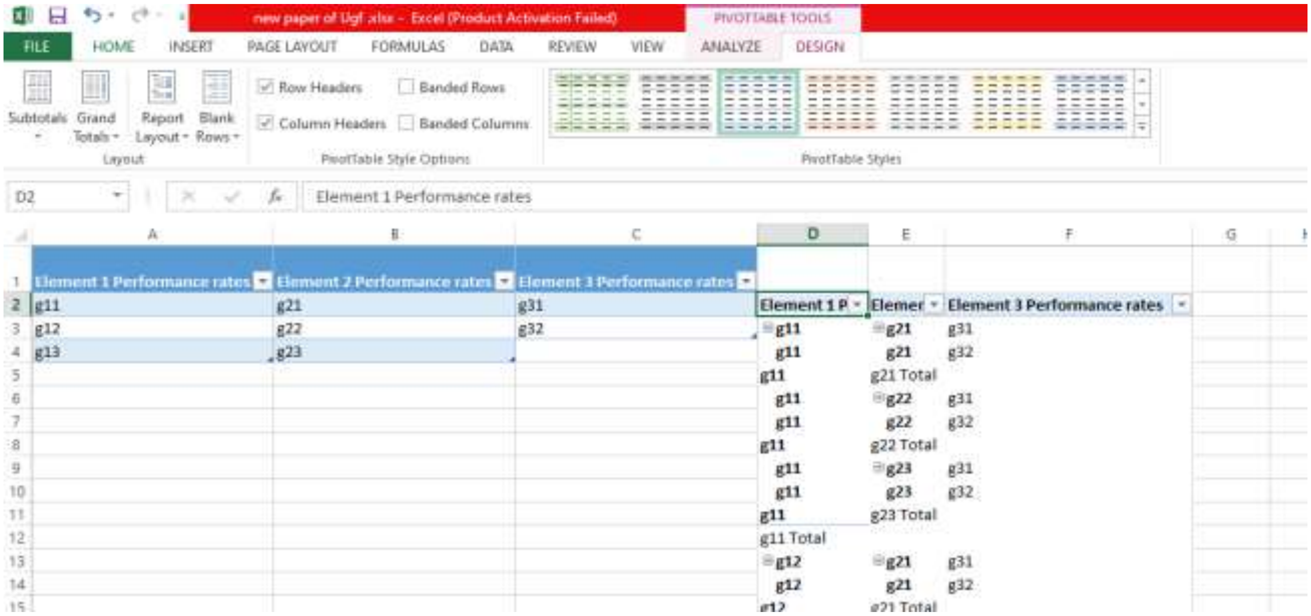
After this step again click on the “Report Layout” and click on the “Repeat All Item Labels”. As it is shown in the following screen

Step 10:



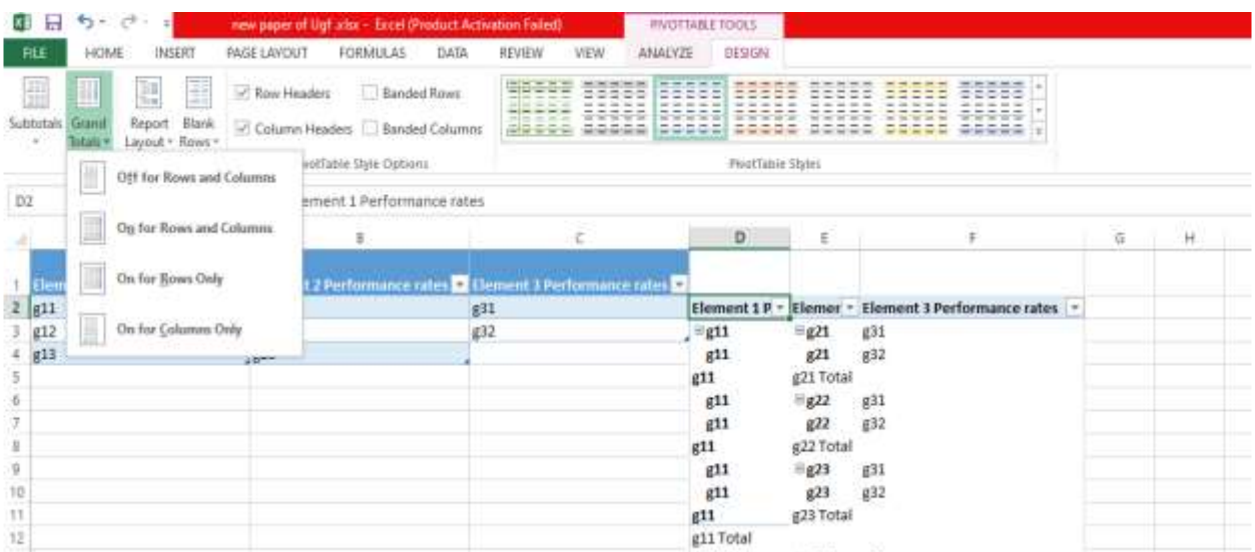
After this step, the following screen appears. The screenshot of which is given below.

Step 11:



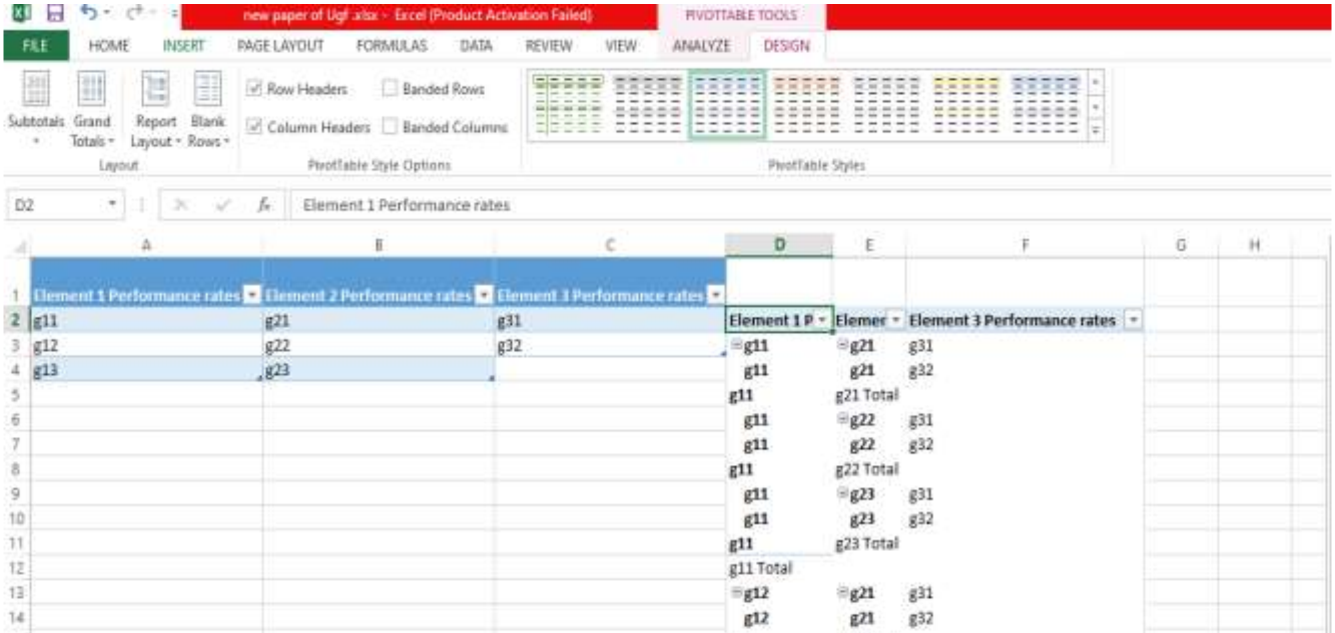
After this step click on the “Grand Totals”. As it is shown in the following screenshot

Step 12:



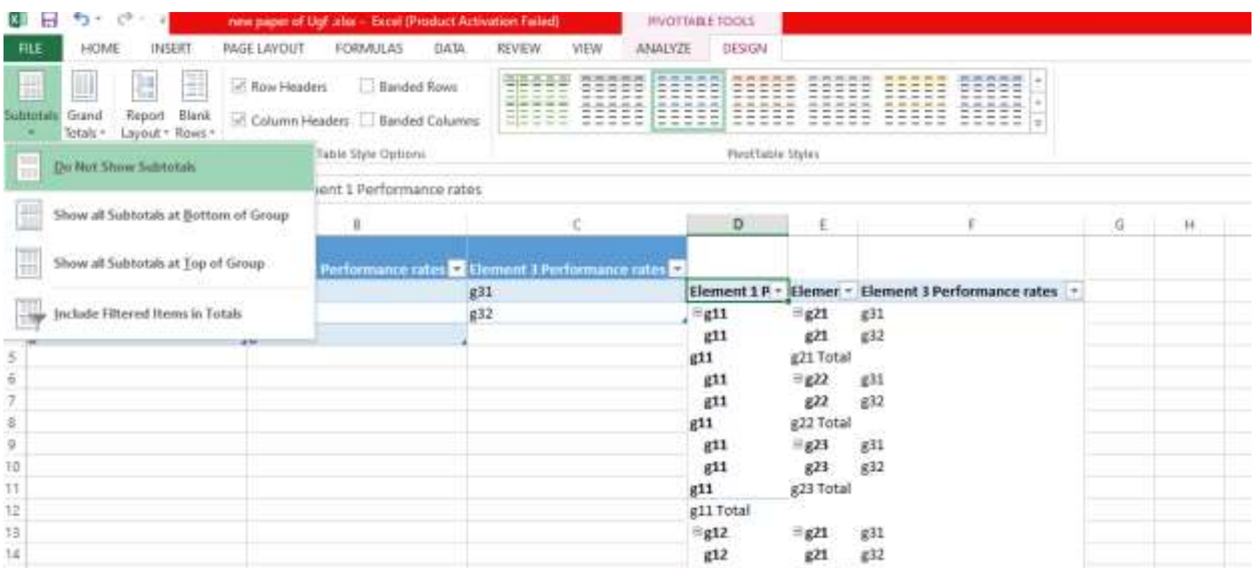
Click on the option “off for Rows and Columns”, the following screen appears.

Step 13:



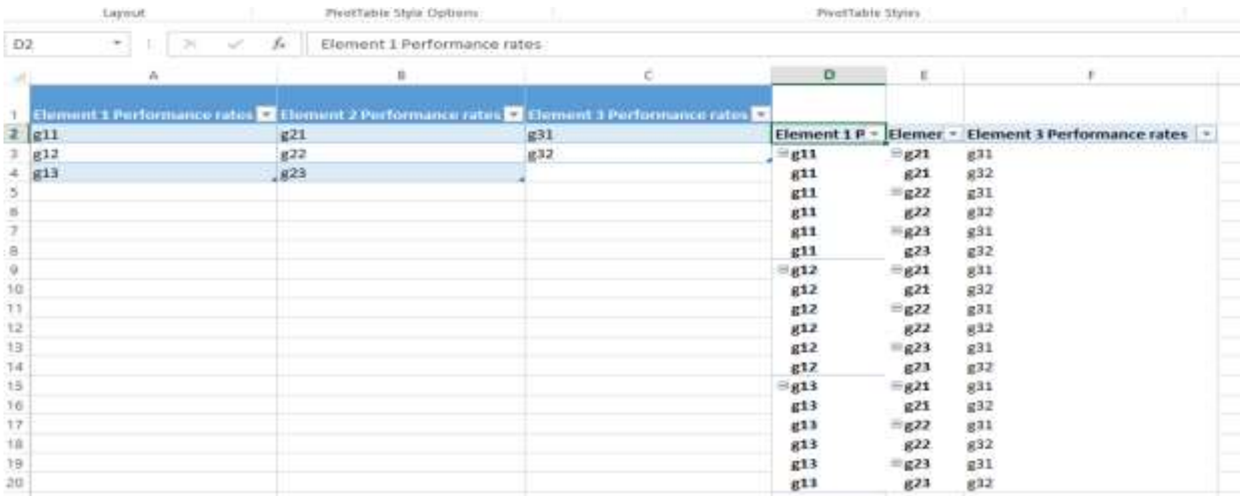
After this step, click on the subtotals as shown in the following screenshot.

Step 14:



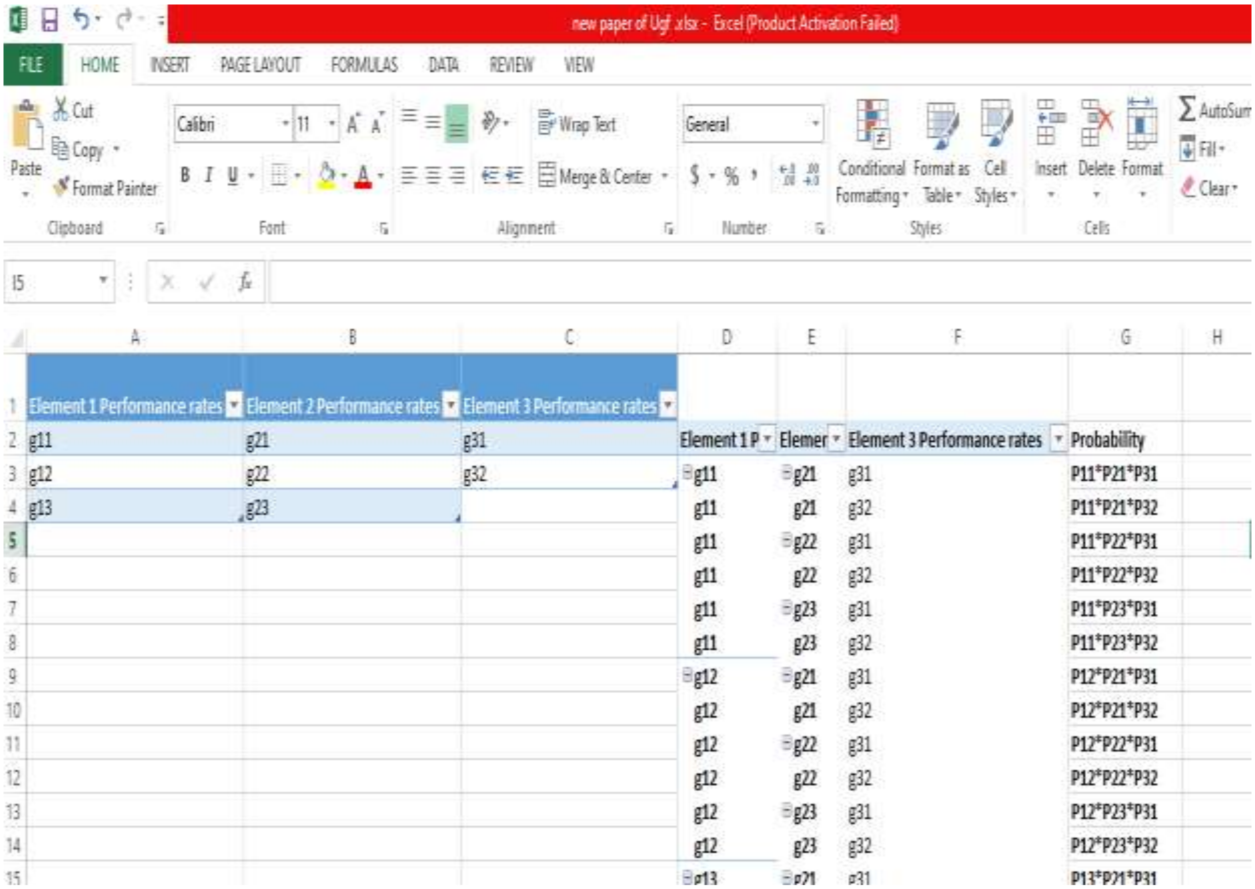
Click on the option "Do Not Show Subtotals". The following screen appears.

Step 15:



Columns D, E and F represent the total possible combination of the performance rates of the system. Once we have a total combination of the performance rates of the system we can apply operations as per the configuration of the system. Also one can easily write the corresponding state probabilities. For example, when the combination of the performance rate of system elements is (g_{11}, g_{21}, g_{31}) then the corresponding probability is $P_{11}.P_{21}.P_{31}$. Similarly, when the combination of the performance rate of the system elements is (g_{11}, g_{21}, g_{32}) then the corresponding probability is $P_{11}.P_{21}.P_{32}$. This is shown in the following screenshot.

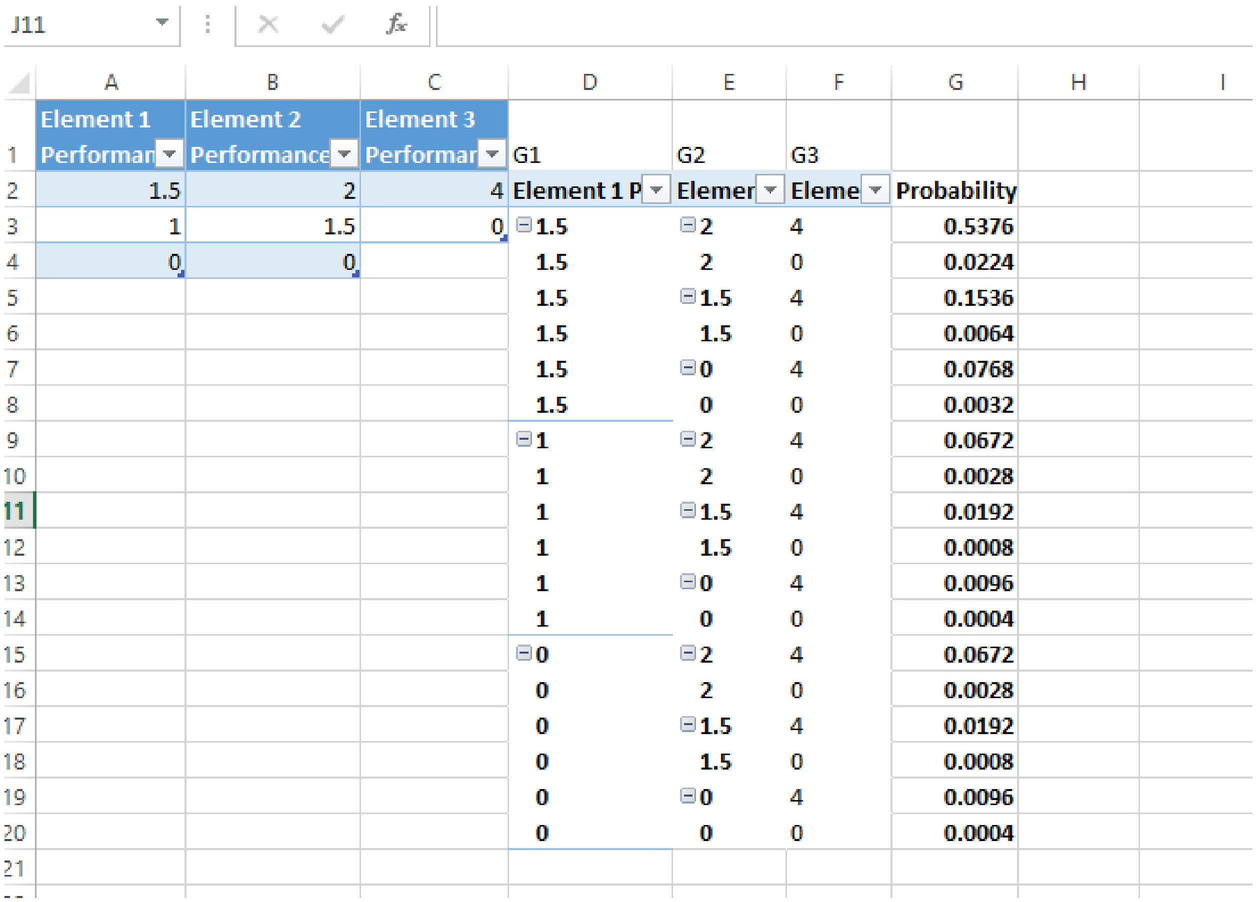
Step 16:



	A	B	C	D	E	F	G
1	Element 1 Performance rates	Element 2 Performance rates	Element 3 Performance rates				
2	g11	g21	g31	Element 1 P	Elemer	Element 3 Performance rates	Probability
3	g12	g22	g32	g11	g21	g31	P11*P21*P31
4	g13	g23		g11	g21	g32	P11*P21*P32
5				g11	g22	g31	P11*P22*P31
6				g11	g22	g32	P11*P22*P32
7				g11	g23	g31	P11*P23*P31
8				g11	g23	g32	P11*P23*P32
9				g12	g21	g31	P12*P21*P31
10				g12	g21	g32	P12*P21*P32
11				g12	g22	g31	P12*P22*P31
12				g12	g22	g32	P12*P22*P32
13				g12	g23	g31	P12*P23*P31
14				g12	g23	g32	P12*P23*P32
15				g13	g21	g31	P13*P21*P31

As these are not numerical values so it is not possible to show the computation procedure. Next, we show you how computation is performed.

Consider the first element has the performance rates $g_{11} = 1.5,\ g_{12} = 1,\ g_{13} = 0$ and the corresponding probabilities are $P_{11} = 0.8,\ P_{12} = 0.1, P_{13} = 0.1$. The second element has the performance rate $g_{21} = 2,\ g_{22} = 1.5,\ g_{23} = 0$ and the corresponding probabilities are $P_{21} = 0.7,\ P_{22} = 0.2, P_{23} = 0.1$. The third element has the performance rates $g_{31} = 4,\ g_{32} = 0$ and the corresponding probabilities are $P_{31} = 0.96,\ P_{32} = 0.04$. Enter this data in the same excel sheet we get the following screen

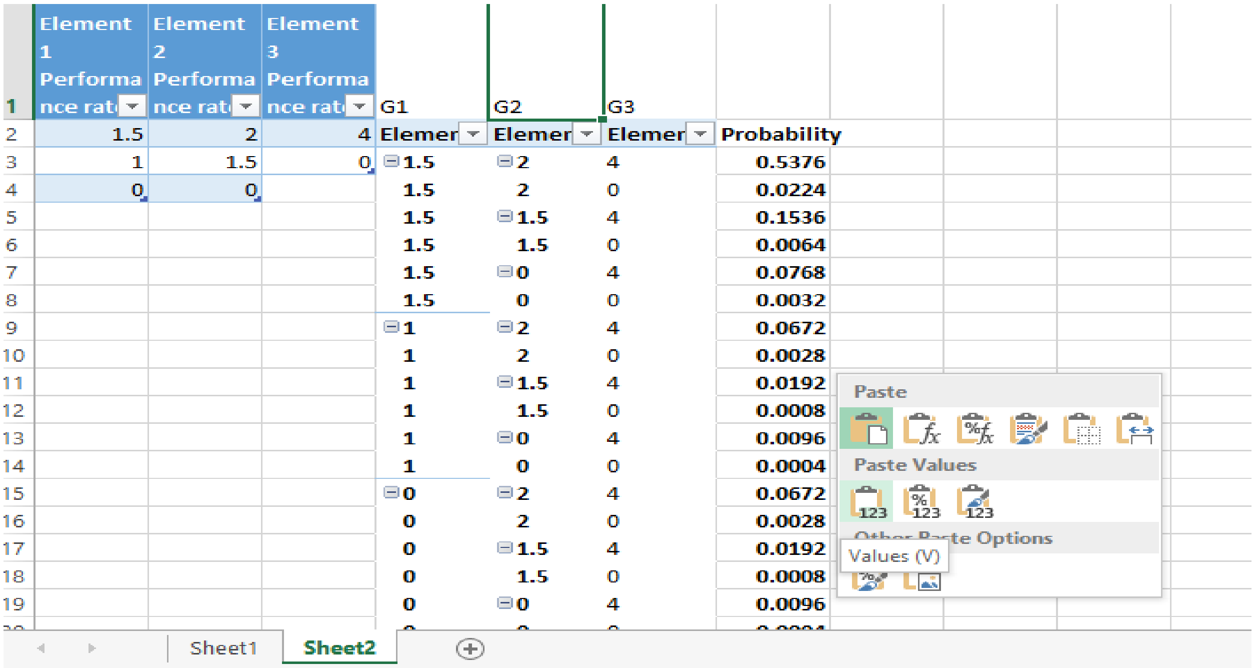


J11

	A	B	C	D	E	F	G	H	I
1	Element 1 Performan	Element 2 Performance	Element 3 Performar	G1	G2	G3			
2	1.5	2	4	Element 1 P	Elemer	Eleme	Probability		
3	1	1.5	0	1.5	2	4	0.5376		
4	0	0		1.5	2	0	0.0224		
5				1.5	1.5	4	0.1536		
6				1.5	1.5	0	0.0064		
7				1.5	0	4	0.0768		
8				1.5	0	0	0.0032		
9				1	2	4	0.0672		
10				1	2	0	0.0028		
11				1	1.5	4	0.0192		
12				1	1.5	0	0.0008		
13				1	0	4	0.0096		
14				1	0	0	0.0004		
15				0	2	4	0.0672		
16				0	2	0	0.0028		
17				0	1.5	4	0.0192		
18				0	1.5	0	0.0008		
19				0	0	4	0.0096		
20				0	0	0	0.0004		
21									

As computation is still not possible because these are not simple data values. Now select the whole data and copy this data to “Sheet 2”. After pasting this data on the second sheet again select the data, then on the right-hand side “Ctrl” appears, click on the down arrow and then select the paste special option as shown in the following screenshot.

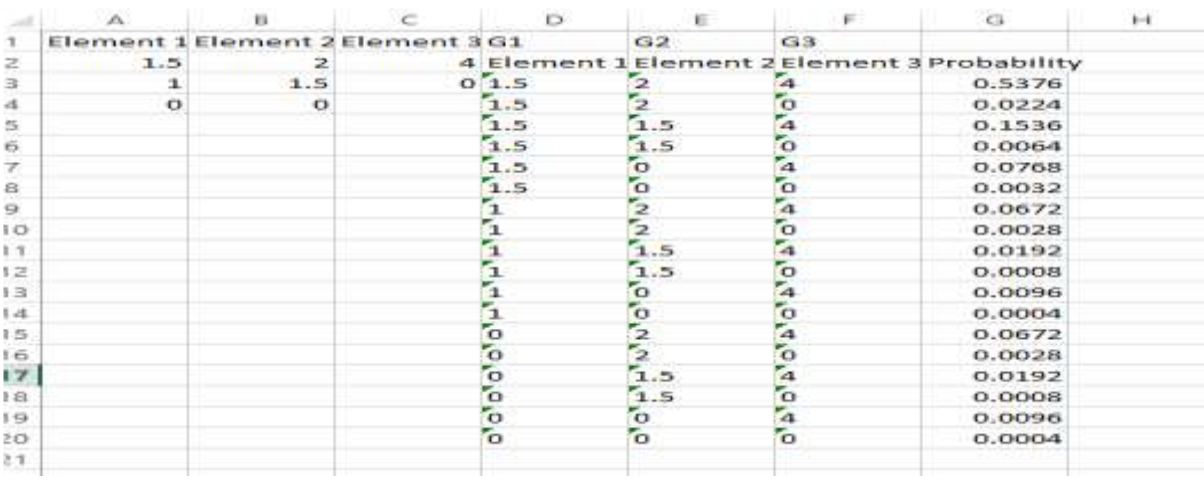
Step 18:



	Element 1 Performance rate	Element 2 Performance rate	Element 3 Performance rate	G1	G2	G3	
2	1.5	2	4	Elemer	Elemer	Elemer	Probability
3	1	1.5	0	1.5	2	4	0.5376
4	0	0		1.5	2	0	0.0224
5				1.5	1.5	4	0.1536
6				1.5	1.5	0	0.0064
7				1.5	0	4	0.0768
8				1.5	0	0	0.0032
9				1	2	4	0.0672
10				1	2	0	0.0028
11				1	1.5	4	0.0192
12				1	1.5	0	0.0008
13				1	0	4	0.0096
14				1	0	0	0.0004
15				0	2	4	0.0672
16				0	2	0	0.0028
17				0	1.5	4	0.0192
18				0	1.5	0	0.0008
19				0	0	4	0.0096

After this the following screen appears.

Step 19:



	A	B	C	D	E	F	G	H
1	Element 1	Element 2	Element 3	G1	G2	G3		
2	1.5	2	4	Element 1	Element 2	Element 3	Probability	
3	1	1.5	0	1.5	2	4	0.5376	
4	0	0		1.5	2	0	0.0224	
5				1.5	1.5	4	0.1536	
6				1.5	1.5	0	0.0064	
7				1.5	0	4	0.0768	
8				1.5	0	0	0.0032	
9				1	2	4	0.0672	
10				1	2	0	0.0028	
11				1	1.5	4	0.0192	
12				1	1.5	0	0.0008	
13				1	0	4	0.0096	
14				1	0	0	0.0004	
15				0	2	4	0.0672	
16				0	2	0	0.0028	
17				0	1.5	4	0.0192	
18				0	1.5	0	0.0008	
19				0	0	4	0.0096	
20				0	0	0	0.0004	
21								

Now select columns D, E and F. A yellow box appears in which an exclamation sign is written. Click on the drop-down arrow and then from these options click on the option "Convert to Numbers" as shown in the following screenshot.

Step 20:

	A	B	C	D	E	F	G	H	I
1	Element 1	Element 2	Element 3	G1	G2	G3			
2	1.5	2	4	Element 1	Element 2	Element 3	Probability		
3	1	1.5		1.5	2	4	0.5376		
4				1.5	2	0	0.0224		
5				1.5	1.5	4	0.1536		
6				1.5	1.5	0	0.0064		
7				1.5	0	4	0.0768		
8				1.5	0	0	0.0032		
9				1	2	4	0.0672		
10				1	2	0	0.0028		
11				1	1.5	4	0.0192		
12				1	1.5	0	0.0008		
13				1	0	4	0.0096		
14				1	0	0	0.0004		
15				0	2	4	0.0672		
16				0	2	0	0.0028		
17				0	1.5	4	0.0192		
18				0	1.5	0	0.0008		
19				0	0	4	0.0096		
20				0	0	0	0.0004		
21									

Number Stored as Text
Convert to Number
Help on this error
Ignore Error
Edit in Formula Bar
Error Checking Options...

Now the whole Excel sheet contains data on which operations are permissible. The screen appears as given below

Step 21:

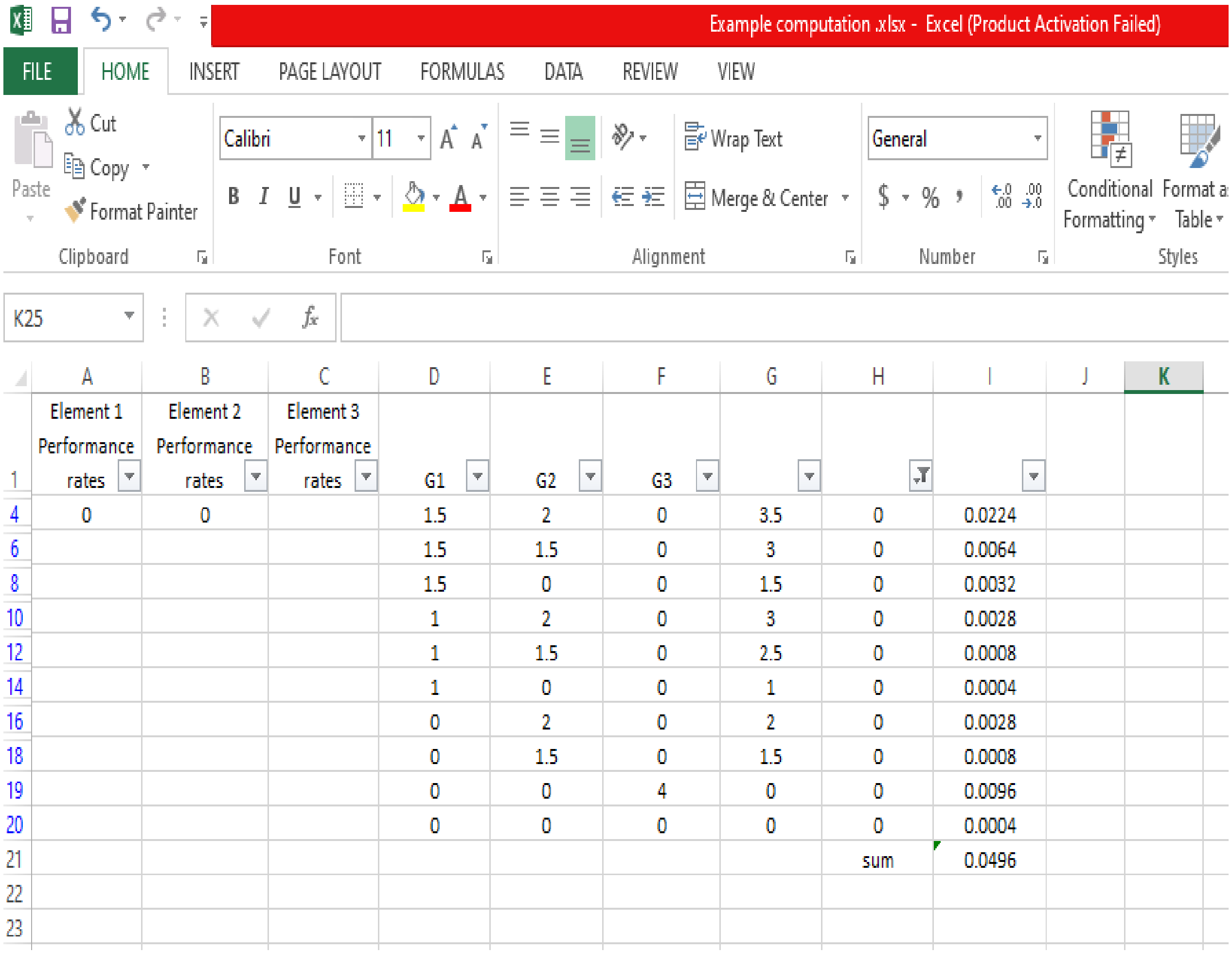
	A	B	C	D	E	F	G	H	I
1	Element 1	Element 2	Element 3	G1	G2	G3			
2	1.5	2	4	Element 1	Element 2	Element 3	Probability		
3	1	1.5	0	1.5	2	4	0.5376		
4	0	0		1.5	2	0	0.0224		
5				1.5	1.5	4	0.1536		
6				1.5	1.5	0	0.0064		
7				1.5	0	4	0.0768		
8				1.5	0	0	0.0032		
9				1	2	4	0.0672		
10				1	2	0	0.0028		
11				1	1.5	4	0.0192		
12				1	1.5	0	0.0008		
13				1	0	4	0.0096		
14				1	0	0	0.0004		
15				0	2	4	0.0672		
16				0	2	0	0.0028		
17				0	1.5	4	0.0192		
18				0	1.5	0	0.0008		
19				0	0	4	0.0096		
20				0	0	0	0.0004		
21									
22									

Now we perform the operations for the configuration given in Figure 13.1. We get the following screenshot.

Step 22:

	A	B	C	D	E	F	G	H	I	J	K
1	Element 1 Performance rates	Element 2 Performance rates	Element 3 Performance rates	G1	G2	G3					
2	1.5	2	4	Element 1 Performance rates	Element 2 Performance rates	Element 3 Performance rates	H=G1+G2	K=MIN(G3,H)	Probability		
3	1	1.5	0	1.5	2	4	3.5	3.5	0.5376		
4	0	0		1.5	2	0	3.5	0	0.0224		
5				1.5	1.5	4	3	3	0.1536		
6				1.5	1.5	0	3	0	0.0064		
7				1.5	0	4	1.5	1.5	0.0768		
8				1.5	0	0	1.5	0	0.0032		
9				1	2	4	3	3	0.0672		
10				1	2	0	3	0	0.0028		
11				1	1.5	4	2.5	2.5	0.0192		
12				1	1.5	0	2.5	0	0.0008		
13				1	0	4	1	1	0.0096		
14				1	0	0	1	0	0.0004		
15				0	2	4	2	2	0.0672		
16				0	2	0	2	0	0.0028		
17				0	1.5	4	1.5	1.5	0.0192		
18				0	1.5	0	1.5	0	0.0008		
19				0	0	4	0	0	0.0096		
20				0	0	0	0	0	0.0004		
21											

Column H gives the system performance rates of the system as one can see that the same value appears in many cells. To get the UGF of the system, find the sum of the probabilities of the same performance rates. For this purpose, one can also use the filter option. It is shown in the following screenshot for the performance level $K = 0$.



	A	B	C	D	E	F	G	H	I
1	Element 1 Performance rates	Element 2 Performance rates	Element 3 Performance rates	G1	G2	G3			
4	0	0		1.5	2	0	3.5	0	0.0224
6				1.5	1.5	0	3	0	0.0064
8				1.5	0	0	1.5	0	0.0032
10				1	2	0	3	0	0.0028
12				1	1.5	0	2.5	0	0.0008
14				1	0	0	1	0	0.0004
16				0	2	0	2	0	0.0028
18				0	1.5	0	1.5	0	0.0008
19				0	0	4	0	0	0.0096
20				0	0	0	0	0	0.0004
21								sum	0.0496
22									
23									

Filter option can be used for obtaining the probability of the system state corresponding to its performance rate. Hence, the expression of the UGF of the whole system can be written as

$$U(z) = 0.5376\ z^{3.5} + 0.2208\ z^{3} + 0.0192\ z^{2.5} + 0.0672\ z^{2} + 0.096\ z^{1.5} + 0.0096\ z^{1} + 0.0496\ z^{0}$$

Suppose that, system demand level is 1.5. Therefore system's availability can be obtained from the system UGF.

$$A(1.5) = \delta_A(U(z), 1.5)$$

$$= 0.5376 + 0.2208 + 0.0192 + 0.0672 + 0.096 = 0.9408$$

Also, expected output of the performance can be calculated from the system UGF using the following formula

$$E = \sum_{i=1}^{k} g_i . P_i$$

$$= 0.5376 \times 3.5 + 0.2208 \times 3 + 0.0192 \times 2.5 + 0.0672 \times 2 + 0.096 \times 1.5 + 0.0096 \times 1 + 0.0496 \times 0$$

$$= 2.88$$

System's mean output performance deficiency can be calculated from the system's UGF expression using the following formula

$$D = \sum_{r_i \prec 0} P_i (w - g_i)$$

$$= 0.5 \times 0.0096 + 1.5 \times 0.0496$$

$$= 0.0792$$

9.6 Result and Discussion

As for the performance evaluation of the multi-state system, the Universal generating function technique is generally employed as it is just based on the simple algebraic multiplication. But its implementation on software like Maple, MATLAB is impossible without the proper knowledge of programming. Learning programming is not an easy task for everyone and it also takes time to learn programming. Also, Descartes product rule is a very slow and not a suitable method for a large system. Hence, the authors of this chapter discussed how UGF can be implemented easily using Excel software. It is very easy to use Excel and one can easily determine the performance of the multi-state system using the Excel software. The authors explained the detailed procedure that how this can be implemented on the Excel software. For that, we took one example from Ding and Lisnianski [33] paper. Authors compared the results, they found that their results are exactly the same as the results obtained by Ding and Lisnianski [33] in their chapter without any error. Thus, we are hopeful that this research can save valuable time for the research fraternity.

Chapter 10: Conclusion and Future Directions of the Research Study

10.1 Conclusion

On the basis of the studied models in this thesis, in each model, critical components of the system have been identified and following conclusions have been drawn.

Model I: Wireless communication has been investigated in this model. Major components of the system like transmitter, communication channel and receiver have been considered for the performance analysis of the wireless communication system. It is identified with the help of sensitivity analysis of reliability and MTTF that receiver and communication channel are the critical components of the wireless communication system. Therefore, more attention must be paid to these two components of the communication so that system's performance can be improved.

Model II: A two-unit system has been investigated in this model under free replacement policy which takes rest after working for some time period. It is seen in this model that on increasing the failure rate of the component from 0.1 to 0.3 reliability decreases as time $'t'$ increases but when the rest rate is increased from 0.2 to 0.4 then the system reliability increases. This model explains it clear that increasing the system's components rest rate improves the system's performance.

Model III: In this model, a rice mill has been investigated for its performance analysis. Major units of the rice mill like cleaning unit, husking unit, separation unit, polishing unit and packing unit have been considered for the reliability evaluation of the system. MTTF and MTBF of the cleaning unit are very low. Using the sensitivity analysis, it's been recognized that the separation unit is the extremely critical unit of the system. Therefore, the mill administration needs to pay more attention to these two units namely (cleaning unit and separation unit) for improving the performance of the rice mill.

Model IV: In this model, the performance of the ATVM has been investigated incorporating hard wares and software. Six major components of the ATVM have been taken into consideration like touch screen monitor, thermal printer, card reader sensor, software failure, Power supply, Power backup UPS. After performing sensitivity analysis on the system MTTF and reliability it is seen that touch screen monitor, thermal printer, card reader sensor and software major failure mainly affect the ATVM performance. Therefore, scheduled maintenance of these components can improve the system's reliability.

Model V: Reliability analysis of a robotic arm system has been carried out in this model. In this model, the performance of four robotic arms that work in series connection along with one standby redundancy has been investigated. MTTF and MTBF of the system is quite low w.r.t variation in the failure rates of second and fourth robotic arm. After performing the sensitivity analysis, it is seen that the second robotic arm is the critical components of the system. Therefore, to improve the overall performance, these two robotic arms (second and fourth) performance must be improved by proper maintenance policy so that these two robotic arms don't fail quickly.

Model VI: In this model, a sugar mill performance analysis has been carried out by incorporating human error. Mill's main components like unloader, conveyor, cutter, crusher, bagasse carrying unit and boiler have been taken into consideration. MTTF and MTBF of the mill is low w.r.t variation in the failure rate of conveyer belt. It is seen after performing sensitivity analysis that system performance is mainly affected by human error. Also, the profit analysis of the system reveals that as the service cost of the system increases, the profit of the mill decreases. Hence, this conclusion is drawn that mill should hire more skilled labor so that system's failure may be mitigated.

Model VII: In this model the implementation of the UGF technique using Excel software has been demonstrated properly. As it is very difficult to perform the calculation of the UGF polynomials manually as the computation burden increases drastically. Also, without the proper knowledge of MATLAB, Maple commands or programming it becomes very difficult to analyze multistate system's performance. Therefore, in this thesis, It has been shown that how the UGF technique can be implemented using Excel software with great ease. The results have been compared with Ding and Lisnianski [33] paper and it has been observed the same result can be obtained without any error.

10.2 Future Directions

This research study can be further carried out in a different aspect. A few of them are listed below:

- In the present research study, the system's performance optimization can be done through Genetic algorithm, Particle-swarm optimization techniques for finding the optimum parameters.
- In this research study, Markov-model has been used to model the system. This model is the state-based model. When the number of components in the system are 5-6, even then the number of system states are too much depending on the number of states of the system's components. It can also challenge computer resources. Therefore, new models must be developed which may reduce the computation burden.
- In the present study, another important measure like busy period of the repairmen has not been calculated for the investigated models of this research. Therefore, one can also carry out further study to determine this measures of the system.
- UGF method is considered as one of the best techniques for the reliability determination of the multi-state system. But it has been noticed that this method is based on algebraic multiplication. One can use the excel software for the reliability determination of *k*-out-of-*n* system and for consecutive *k*-out-of-*n* system.

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