# MATHEMATICAL MODEL TO PROTECT THE STORED GRAINS FROM INSECTS

Thesis Submitted for the Award of the Degree of

## DOCTOR OF PHILOSOPHY

**In**

**Mathematics**

**By**

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**2023**

# DECLARATION

I, hereby declared that the presented work in the thesis entitled “Mathematical model to protect the stored grains from insects” in fulfilment of degree of **Doctor of Philosophy (Ph. D.)** is outcome of research work carried out by me under the supervision of Dr. Rakesh Yadav working as Professor, in the Mathematics Department/School of Chemical engineering and Physical Sciences of Lovely Professional University, Punjab, India. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

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# CERTIFICATE

This is to certify that the thesis entitled "**MATHEMATICAL MODEL TO PROTECT THE STORED GRAINS FROM INSECTS ''** submitted by **KIRTI BHAGIRATH, ID No. 41900369** for award of Ph.D. Degree of the Institute embodies original work done by her under my supervision.

Signature of the Supervisor

Name: Dr. Rakesh Yadav

Designation: Professor

Date: 18/05/2023

# ABSTRACT

Agriculture occupies an important place in India's economy. About 70% of the country's population's livelihood is provided by agriculture and a contribution around 50 % to the national income. Agricultural development is considered the foundation of the economic growth of India.

The development of technology has led to a steady rise in grain production. But even then we can see that in the viciousness of the introduction of high yielding varieties, the use of new mechanisms in farming has improved irrigation management, but the green revolution has not reached the predictable success. After harvesting, the food grains are stored in different traditional and primitive storage structures for shorter or longer periods where immense losses occur not only in terms of quality but in quantity also which are caused by many abiotic and biotic factors. The different factors that are causing a loss in storage are weevils, beetles, moths and rodents.

Insect infestation is one of the main factors causing immense damage to the stored grains. There is a need to control insect infestation to meet the increasing needs of human beings. There were many ways by which insect infestation was controlled like

- Maintaining a temperature unfavorable for the growth of insects
- The Sterile Insect Release Method
- To model the population redistribution for adults like *C. ferruginous,* a transport (diffusion) equation was used in stored grain.
- Insects exposure to phosphine gas
- Chemicals
- Light emitting diodes were used to catch the insects
- The Superiority of one species over the other to settle and dominate new patches
- An Increase in the concentration of carbon dioxide at particular temperatures leads to a reduction in adult emergence and also the killing of eggs.

The insect's growth mainly depends on different factors like temperature, humidity, suitable location and availability of food. To study the insect's growth and the different factors responsible for its growth, developing a mathematical model is necessary.

In order to research the growth and development of insects, this thesis is built around three main goals. It tried to answer the following questions

- Dependency of damage of grains on insect population
- Does insect growth show the same behavior under a constant and fluctuating environmental ecosystem?
- How Angoumois *grain moth* affects stored grains in an environmental ecosystem?

**Research Objectives are**

1. To develop the model describing small outbreak of a % of damaged grains in storage ecosystem.
2. To develop and validate a mathematical model that describes the population growth of insects in a randomly fluctuating environmental ecosystem.
3. To develop and validate a mathematical model to analyze the effect of *Sitotroga Cerealella* on grains in the environmental ecosystem

The whole work is discussed in different chapters and the abstracts related to each of the chapters are given below:

**Chapter 1**

This chapter deals with the introduction along with the literature review giving a brief background for the upcoming chapters. Different papers were reviewed to get to know the biology of insects, to study different factors responsible for the growth and development of insects, hot spot area, the different methods applied to control the growth and the development of insects and mathematical methods.

**Chapter 2**

A Mathematical model had been proposed and analyzed to study the relation between the percentage of damaged grains and the insect population. It had been observed that the percentage of damaged grains increased with the increase in the insect population. The linear differential equation was formed with the help of some assumptions.

For this, we laid down certain assumptions as follows:

1. The conditions were sufficiently unfavorable that the outbreak never became large.

2. For conciseness, we considered that a single insect damaged the grains at first rapidly.

An experiment was performed on different varieties of paddy under storage conditions, the environmental factors i.e. R.H. temperature and moisture content was maintained 75 % with KOH for two weeks and moisture content varied from 11.3 to 12.0 % after two weeks. At the end of the experiments, % of damaged grains, total population of insects and loss of weight were recorded. We concluded from this model with experimental data and plotted graph that the % damage of grains increased with the increasing number of the insect populations.

**Chapter 3**

The mathematical model had been developed and validated that described the growth in population in a randomly fluctuating environmental ecosystem. A mathematical model was formulated explaining insect population and their growth in an arbitrarily changeable environmental ecosystem. The changes in the net growth rate of insects were discussed using the stochastic models. The changes in birth rate and death rate were studied. It tried to answer different questions like "Did insects show the same behavior under constant and fluctuating environments"? We studied the insect population changed in a successful colonized population in a randomly fluctuating environment. We also discussed the death rate changed using the model that approximated the situation describing a population that fluctuated around an average far from zero.

**Chapter 4**

The most noteworthy pest of stored agricultural products worldwide is the Angoumois grain moth *Sitotroga Cerealella*. Its infestations rise during storage, in pre-harvest or post-harvest

For this problem an entomological data was selected. A Mathematical Model was developed which was based on two varieties *viz*. % grain infestation and % loss in germination. For 3, 6 and 9 months of duration losses in grains weight and germination of rice varieties infested by *Sitotroga Cerealella* were observed. From the data, it was observed that the % of germination value lost was exhibiting oscillating behaviour with regard to (Kernel)% grain infestation. Thus, it was hypothesized that % of germination value lost in terms of grain infestation percentage followed a law with an oscillating behaviour demonstrated by the solution. So, a model had been developed by using differential equations of second order by taking the assumption that the rate of change of % of germination loss with respect to grain infestation was proportional to germination loss. Regression was used. Line of best fit was found.

We concluded from this model that infestation by *Sitotroga Cerealella* brought considerable damage to impact significant variations in germination when different storage periods were taken. As the storage period increased insect infestation increased progressively and the germination decreased following a more or less similar trend. Insect infestation was directly proportional to the number of insects presented inside the store. This mathematical model laid focused on the role of infestation by insects during storage which led to the reduction in germination which proved its efficiency in itself.

**Chapter 5**

In this chapter, a mathematical model had been developed and validated to analyze the growth in population through a nonlinear stochastic process. By employing nonlinear stochastic differential equations for growth and the associated Fokker-Plank equations for the probability densities that were time dependent, the evolution of the probability density of an insect population was examined. It was clear that the variance behaviour was dependent on the beginning conditions, however in the case of the mean, the impact of the early conditions vanished quickly. As the growth of the insects continued, we noticed how the mean and variance responded differently. The behavior of the mean was that it was monotonically increasing but for the case of the variance, at some time the variance rose above the steady-state variance before the process reached the steady-state and then as the growth proceeded and shrank back to zero, the variance would fall back to zero.

**Chapter 6**

Analysis of *Cryptolestes ferrugineus* (Stephens) (Coleoptera: Laemophloeidae) population dynamics showed that population varied in small patches and large patches. The two main variables that controlled the dynamics of population, as determined by key factor analysis, were the temperature and the quantity of insects previously mentioned. The number of insects grew as the total number of degree days increased. In none of the tested scenarios did nine population-based unstructured models suit the bug counts. The optimum equation for this relationship has three variables and was sigmoidal. Temperature, temperature change, and the earlier number of insects were the main factors determining *Cryptolestes ferrugineus* population trends in grain bins. These elements were considered when creating this new model, which accurately predicted the insect population before it reached its peak density

**Chapter 7**

This chapter discussed about the future scope of the research. Different areas where the research could be useful is detailed. The future scope of the research is wide and promising. The development of more sophisticated models, combined with the integration of machine learning algorithms, will likely lead to significant advancements in our understanding of insect behavior and the management of insect pest in stored grain systems.

# *Acknowledgment*

Firstly, I want to express my gratitude to God for guiding me through all of the challenges. Every day, I have felt your guiding.

I would like to express my sincere gratitude to my supervisor, Dr. Rakesh Yadav, a professor in the Department of Mathematics, Lovely Professional University in Phagwara, for his wise counsel, unwavering support, and enthusiastic cooperation, which made this work possible and allowed me to finish all of the stages of writing my thesis. This thesis would not have been possible if it weren't for his unwavering confidence in me and support. I find it difficult to put into words how grateful I am to him. He is not only a scholar par excellence but also a person par excellence— the epitome of commitment, wisdom, and humanity.

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$x(t)$ : integer-valued random variable that served as counter

$t$ : time

M : parameter

$Q_i$ : Integer valued random variable which counted the number of cases of the grains which were damaged by the insects

$f(d_n)$ : probability generating function

$N$ : total number of insects in the zeroth generation

$x$ : size of a biological population

$\bar{a}$ : Malthusian parameter of population growth

G : rate of % germination loss

$I$ : grain infestation

$C_1$ : amplitude of oscillation

$\omega$ : circular frequency

T : period of oscillation

$P_s(x)$ : steady-state probability density

μ : geometric growth factor

k : carrying capacity

$R_0$ : net reproductive rate

$N_t$ : half of the alive adult number at time *t*

$N_0$ : half of the alive adult number at the starting of the experiment

$R^2$ : coefficient of determination

A : slope of the line

$\text{Log}_{10}^{F}$ : intercept

r : survival rate of $N_t$

B : minor factor that influence insect population

$T_t$ : daily temperature at t days

# CHAPTER -1

## 1.1 INTRODUCTION

Grains hog a big share in the plates of Indians. Grains thus play a great role in the country's economy as it caters to the feeding needs of a large population. Moreover, grains can be transported and stored with general care for a long time. Technology is growing by leaps and bounds today which has also given a boost to the production of grains. Owing to this technology boom 2,000,000,000 tonnes of grains are produced per year. India owns the second largest part of the land in the world which is agricultural amounting to 179,900,000 hectares. India produces about 25 % of the total global production of pulses in the world and consumes 27 % of world consumption and imports 14 % of pulses in the world. According to FEO statistics (2010) on world agriculture, wheat, rice, lentils, and millet are all produced in the second-largest quantities by India. In 2017–18 total food grain production was near to 75,000,000 tonnes.The details about production for the years 2016-2022 of rice, wheat, unmilled paddy and coarse grain is shown in tabular form.

| Item | Jan. | Feb. | Mar. | Apr. | May. | June. | July. | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rice | 125.8 | 161.6 | 194.2 | 221.6 | 213.2 | 207.9 | 194.1 | 180.0 | 165.3 | 144.7 | 125.2 | 110.5 |
| Wheat | 237.8 | 203.2 | 168.6 | 145.3 | 314.4 | 326.3 | 301.8 | 275.9 | 242.4 | 213.2 | 188.4 | 164.9 |
| Total | 364.7 | 365.1 | 362.8 | 366.9 | 527.6 | 534.2 | 495.9 | 455.9 | 407.7 | 358.0 | 313.6 | 275.5 |
| Unmilled Paddy | 199.0 | 189.2 | 146.4 | 99.25 | 96.50 | 95.91 | 78.44 | 61.69 | 32.03 | 20.8 | 186.8 | 237.2 |
| Coarse grain | 0.99 | 0.44 | 1.34 | 2.51 | 2.69 | 2.58 | 2.54 | 2.54 | 2.53 | 1.32 | 1.40 | 1.59 |

**Table 1.1 – Representing Year 2016**

| Item | Jan | Feb | Mar | Apr | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rice | 133.7 | 171.2 | 205.0 | 231.8 | 229.2 | 222.0 | 211.4 | 199.5 | 182.8 | 164.0 | 141.2 | 132.2 |
| Wheat | 138.4 | 116.2 | 95.29 | 81.59 | 297.4 | 335.4 | 323.7 | 301.5 | 279.1 | 259.6 | 239.5 | 217.6 |
| Total | 273.2 | 286.8 | 299.3 | 312.4 | 525.6 | 556.4 | 534.1 | 498.7 | 458.9 | 431.7 | 387.7 | 374.9 |
| Unmilled Paddy | 241.1 | 185.1 | 166.6 | 101.0 | 92.30 | 101.6 | 81.96 | 58.38 | 33.98 | 18.36 | 197.4 | 254.8 |
| Coarse grain | 1.48 | 1.21 | 1.38 | 1.62 | 1.69 | 1.51 | 1.52 | 1.52 | 1.53 | 1.35 | 1.39 | 1.39 |

**Table 1.2 – Representing Year 2017**

| Item | Jan | Feb | Mar | Apr | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rice | 163.2 | 199.9 | 236.7 | 249.2 | 256.1 | 246.1 | 235.2 | 219.0 | 215. 7 | 189.3 | 166.5 | 147.8 |
| Wheat | 196.6 | 178.4 | 155.5 | 136.3 | 356.4 | 438.5 | 416.4 | 407.5 | 375.1 | 359.2 | 336.3 | 309.2 |
| Total | 358.1 | 379.4 | 386.3 | 385.1 | 609.1 | 686.8 | 652.2 | 629.1 | 595.8 | 544.5 | 495.8 | 456.0 |
| Unmilled Paddy | 255.4 | 213.0 | 149.4 | 79.1 | 68.4 | 79.7 | 66.2 | 48.1 | 33.4 | 17.56 | 169.6 | 269.5 |
| Coarse grain | 1.65 | 1.79 | 1.39 | 1.19 | 0.96 | 0.99 | 0.96 | 0.82 | 0.65 | 0.49 | 2.12 | 2.18 |

**Table 1.3– Representing Year 2018**

| Item | Jan | Feb | Mar | Apr | May | June | July | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rice | 183.9 | 229.9 | 268.9 | 295.9 | 290.5 | 276.8 | 289.2 | 276.3 | 251.4 | 256.1 | 233.0 | 216.7 |
| Wheat | 273.2 | 238.3 | 202.2 | 168.9 | 335.6 | 468.6 | 459.3 | 438.8 | 416.9 | 398.1 | 375.7 | 356.7 |
| Total | 456.1 | 469.2 | 465.0 | 465.8 | 626.1 | 745.4 | 748.5 | 719.1 | 678.3 | 645.3 | 614.8 | 565.5 |
| Unmilled Paddy | 276.1 | 269.8 | 205.4 | 158.5 | 135.8 | 125.0 | 109.1 | 80.9 | 65.7 | 43.55 | 198.5 | 254.1 |
| Coarse grain | 2.05 | 2.15 | 1.25 | 0.48 | 1.41 | 1.48 | 1.49 | 1.57 | 1.49 | 1.15 | 3.98 | 3.11 |

**Table 1.4 – Representing Year 2019**

| Item | Jan | Feb | Mar | Apr | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rice | 236.1 | 276.5 | 307.7 | 325.3 | 296.5 | 276.4 | 275.7 | 256.4 | 266.4 | 248.6 | 209.0 | 213.6 |
| Wheat | 329.9 | 314.6 | 276.2 | 249.0 | 359.7 | 559.2 | 546.9 | 413.2 | 415.9 | 386.1 | 366.7 | 346.5 |
| Total | 566.1 | 579.2 | 586.9 | 645.7 | 649.2 | 833.6 | 823.6 | 669.6 | 679.3 | 643.7 | 563.7 | 554.4 |
| Unmilled Paddy | 279.8 | 257.9 | 286.0 | 256.3 | 239.2 | 219.9 | 186.9 | 77.92 | 57.7 | 49.55 | 199.5 | 265.1 |
| Coarse grain | 3.44 | 3.14 | 0.23 | 1.37 | 2.16 | 2.16 | 1.46 | 1.37 | 1.46 | 1.13 | 3.76 | 3.31 |

**Table 1.5 – Representing Year 2020**

| Item | Jan | Feb | Mar | Apr | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rice | 186.6 | 243.9 | 282.3 | 291.1 | 304.8 | 299.2 | 296.8 | 291.1 | 268.3 | 253.2 | 229.2 | 213.0 |
| Wheat | 342.9 | 318.3 | 295.4 | 273.0 | 525.6 | 602.9 | 549.9 | 564.8 | 517.8 | 468.5 | 419.8 | 378.5 |
| Total | 364.7 | 365.7 | 362.8 | 366.8 | 527.6 | 534.2 | 495.9 | 455.6 | 786.2 | 721.7 | 649.8 | 591.5 |
| Unmilled Paddy | 404.2 | 387.9 | 345.0 | 310.6 | 262.2 | 286.9 | 289.8 | 229.2 | 176.1 | 140.6 | 254.6 | 358.8 |
| Coarse grain | 3.20 | 3.52 | 4.66 | 7.97 | 7.50 | 7.44 | 6.30 | 5.4 | 2.51 | 1.88 | 1.49 | 1.54 |

**Table 1.6 – Representing Year 2021**

| Item | Jan | Feb | Mar | Apr | May | June | July | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rice | 221.5 | 263. | 295.7 | 323.2 | 332.68 | - | - | - | - | - | - | - |
| Wheat | 330.1 | 282.7 | 234.0 | 189.9 | 303.46 | - | - | - | - | - | - | - |
| Total | 551.6 | 546.0 | 529.7 | 513.12 | 636.14 | - | - | - | - | - | - | - |
| Unmilled Paddy | 473.6 | 492.5 | 441.1 | 339.03 | 266.11 | - | - | - | - | - | - | - |
| Coarse grain | 1.80 | 2.88 | 3.31 | 4.83 | 4.13 | - | - | - | - | - | - | - |

**Table 1.7 – Representing Year 2022**

"**Source**: The Central Pool's stock of food grains from 2016 to 2022 (**Figs.in lakh million tonnes)"**

Every year, 150 million tonnes of food grains are produced in India. India's production of food grains grew by 6.74 million tonnes in 2019–20 compared to 2018–19. Additionally, production in 2019–20 grew by 26.20 million tonnes compared to the average production throughout the years from 2013 to 2018. A record 117.47 million tonnes of rice were produced in 2019–20, a significant increase over the 107.8 million tonnes that were produced on average over the previous five years. Additionally, the output of wheat in 2019–20 has improved and reached a record 106.2 million tonnes, up 2.61 million tonnes from the production in 2018–19. Additionally, it exceeded by 11.60 million tonnes from the 94.61 million tonnes average of the previous five years. Production of nutritive and coarse cereals reached 45.24 million tonnes in 2019–20, beating the 43.06 million tonnes produced in 2018–19 by 2.18 million tonnes.

According to the ministry of agriculture's second advance estimate, India's output of food grains, including wheat, rice, pulses, and coarse cereals, may set a record of 316.06 million tonnes (mt) for the crop year (July-June) 2021–22.

As compared to the last five years' average production of pulse, there is an increase of about 3.14 million tonnes in pulses output for the current crop year with an estimation of 26.96 million tonnes. When compared to the average production over the previous five years, the output for coarse cereals is estimated to be 3.28 million tonnes higher at 49.86 million tonnes. It is expected that the production is estimated to reach level for other crops such as oilseeds, sugarcane, cotton, etc. The total production of oilseeds is anticipated to increase to a record 37.95 million tonnes in 2021–22, up 1.20 million tonnes from the 35.95 million tonnes reported for the previous year.

The total sugarcane production was estimated to be about 414.04 million tonnes for the year 2021-22 which was higher than the average yield of sugar cane of 373.46 million tonnes by 40.59 million tonnes. Oil seed production reached a new high of 37.95 million are expected to be produced in 2021–2022, up 1.20 million tonnes from the 35.95 million tonnes reported for the previous year.

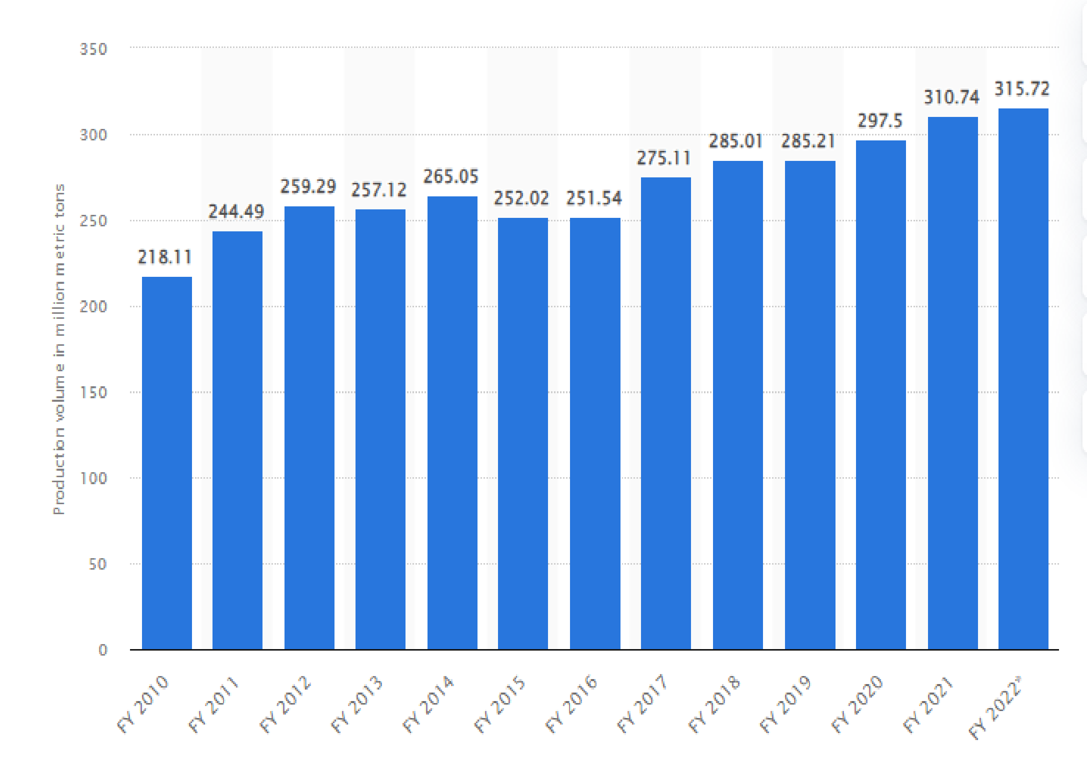


**Figure 1.1-** Showing the amount of food grains produced in India from the 2010–2021 fiscal year, along with projections for 2022 (in million metric tonnes)

Source "Production volume of food grains India FY 2010-2022.Statista. https://www.statista.com/statistics/1140261/india-production-volume-of-food-grains/."

## 1.2 GRAIN LOSSES

Production has been increasing but at the same time, losses remain constant at 10% which is due to improper storage. It is estimated that 6% out of the total 10% loss occurs due to mismanagement during storage. Before reaching the kitchen of the consumers a lot of hard work is done during the harvest which includes threshing, winnowing, bagging, transportation storage et cetera. Storage is the step just before the grains can reach the hands of consumers and it becomes very important to store that properly to ensure that their supply remains constant during the year since they are periodic and produced at specific places only. Storage of food grains is crucial for ensuring that customers receive the right amount of food and that no one in the nation goes hungry. But for this, proper managemental steps have to be taken because both biotic and abiotic factors affect the health of stored grains considerably. Abiotic factors like temperature, carbon dioxide, oxygen and moisture can affect the condition of food grains to a large extent. Biotic factors like fungi, bacteria, arthropods (mainly insects and mites) and vertebrates which include enemies like rodents (rats) and birds can lead to a large - scale destruction of this stored food. Microorganisms and rodents reproduce at a very large rates and can pose a great threat if remain uncontrolled and unchecked. To stop our go downs from rampaging, it is necessary to keep an eye on these variables and how they interact. According to projections, the population of the globe would increase to 9.8 billion in 2050 and 11.2 billion in 2100. In 2030, 8.6 billion people will live on the planet, according to a recent United Nations estimate. And to feed this population global agricultural production has to increase by 60% from 2005-07 level, which is mammoth. Before the harvest, obstacles like weeds, animal pests, and abiotic environmental pressures like temperature damage around 35% of the total biological product that could be produced, or 3,153,000 tonnes, with a loss of 105,500,000 tonnes. A. Mesterhazy et al. [5] concluded that there were also losses after harvest in India which amount to 12 to 16,000,000 tonnes of food grains per annum. During storage, many threats were waiting to destroy the food grains which include insects, rats, microorganisms and other non-living factors like temperature and storage conditions. Improper handling of grains without any prowess could lead to poor grain quality and even a complete quality drop in grains. The monetary worth of these losses exceeded Rs. 50,000 crores annually, according to P. K. Singh [70]. Rats, mice, and bacteria were all blamed for the qualitative and quantitative losses that occured during storage. It had been reported that a large number of insect pests were associated with stored grains.

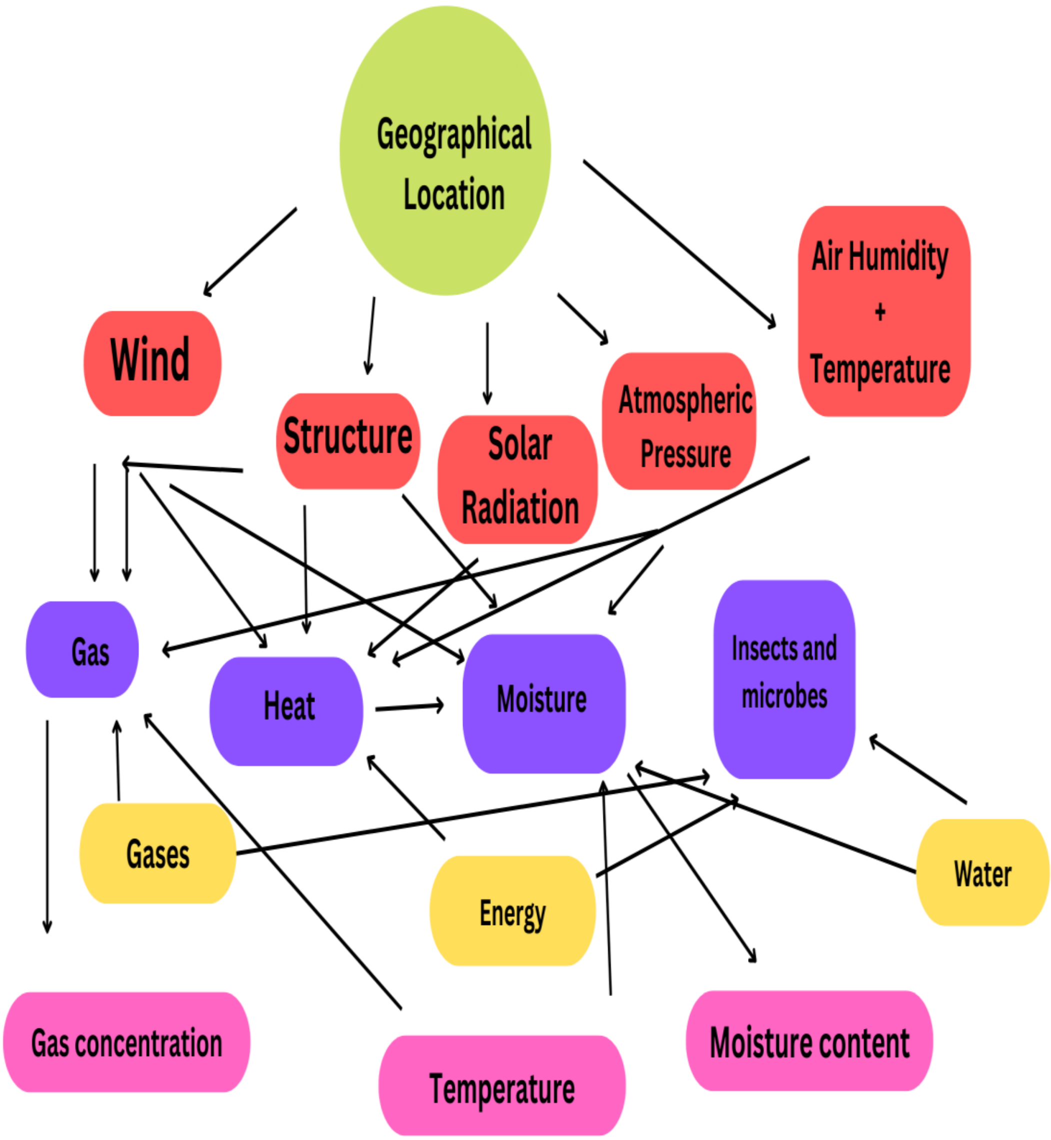


**Figure 1.2-** Showing different Biotic and Abiotic Factors in Grain Storage Ecosystem

## "Global Hunger Index Report for 2019"

According to the 2019 Global Hunger Index, India was rated 102 out of 117 nations, placing it behind South Asian neighbours including Bangladesh, Pakistan, and Nepal. India is plagued by hunger issues, according to a survey on the global hunger index. The irony is that more than a third of the food supply is lost in post-harvest agricultural management at a time when the globe is struggling to meet the needs of every person on Earth.

Developing nations can boost food availability, relieve pressure on natural resources, significantly reduce starvation, and generally improve the conditions of their inhabitants by reducing food quality deterioration and maintaining proper storage management. For high production, it is important to make suitable plans against insects as it one of the factor which is chiefly responsible for the low levels of production and damaging stored grains badly. The aim of this thesis is to develop mathematical model which can be helpful in protecting stored grains from insects.

**According to the Global Hunger Index 2021 report, India has enough data to determine its 2021 GHI scores because it is ranked 101st out of 116 nations. With a score of 27.5, India has a serious level of hunger.**

## 1.3 NEED OF MATHEMATICAL MODELING

What is mathematical modeling?

Models give a description of our beliefs related to the functionality of the world and the translation of these beliefs into mathematical language can be achieved through mathematical modeling. Mathematical modeling is a boon to making integrated strategies like keeping in control the growth of insects, weeds, pests and diseases in stored grains and can be helpful in strategizing the steps in controlling dependency on chemicals. It is a good example of a potentially valuable tool.

Since most trials are conducted with little amounts only, mathematical modelling provides insight into what may occur if enormous quantities of grains were taken into account. Sometimes in most experimental work, it becomes difficult or impossible to include the total effect on an ecosystem caused by the environmental factors but mathematical models can be useful in foreseeing or understanding the combined effect of the factors.

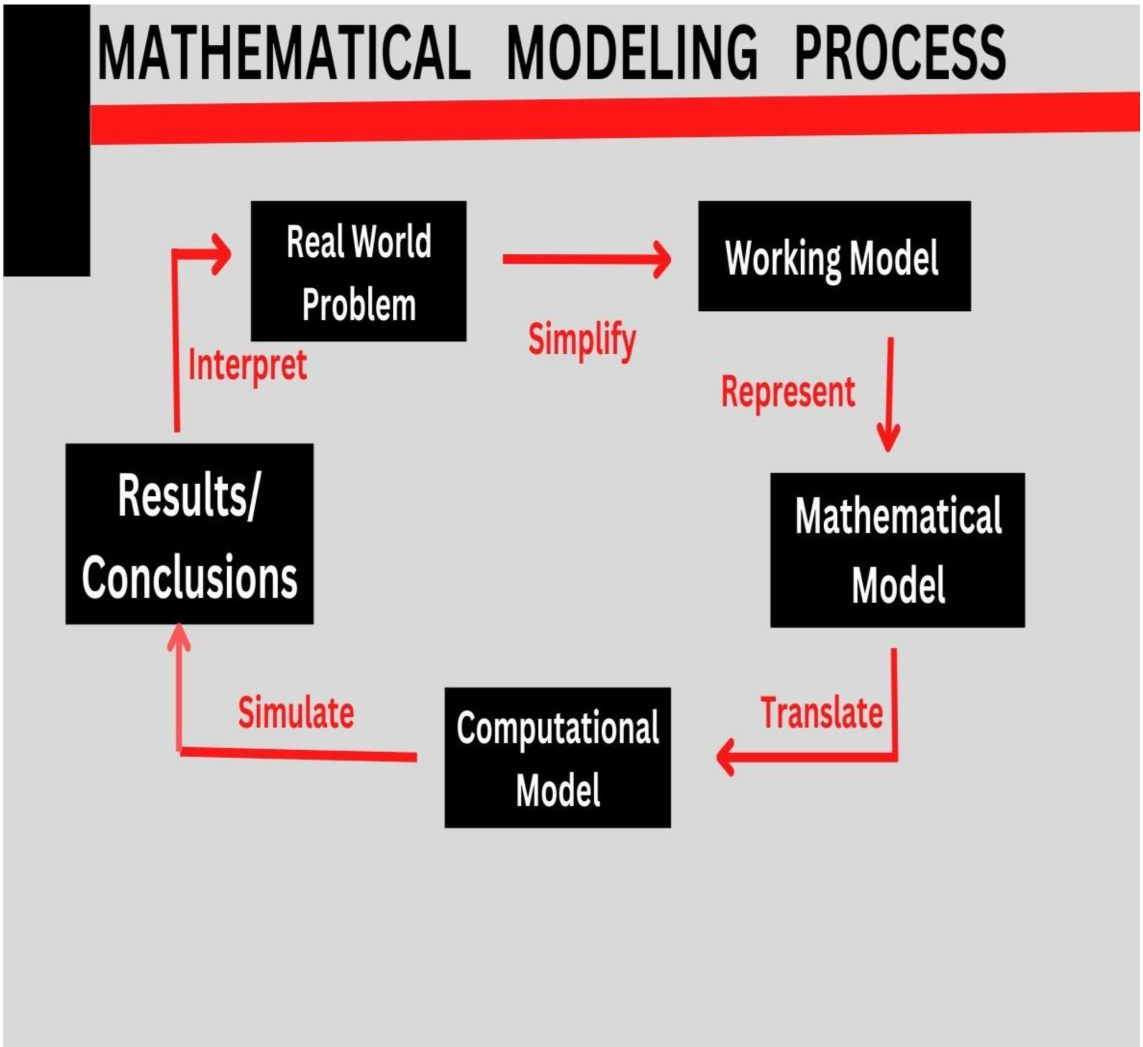


**Figure 1.3-** Showing Mathematical Modeling Process

## Current Food and Agriculture Situation 2021

"The 2030 Agenda for Sustainable Development clearly recognizes the importance of minimizing food waste and loss. Reducing food loss and waste is thought to be crucial for enhancing nutrition and food security, fostering environmental sustainability, and bringing down production costs. However, attempts to prevent food loss and waste won't be successful unless they are supported by a thorough comprehension of the issue. Mathematical Modeling can prove to be of great importance in this regard. We can look forward to reducing dependence upon chemicals and controlling weeds, pests and diseases in store grains.

## 1.4 LITERATURE REVIEW

Grain storage is the bridge between the distribution system of the grains and its procurement so it becomes a necessity to keep the quality and security of the grains during storage. Different organizations have been trying their best to find out a solution for this critical issue related to storage losses. Different researchers approach different methods (chemical, bio-chemical & mathematical) in different times to protect stored grains.F. J. Gay [22] discussed a method that involves the use of a dust barrier to protect an isolated stack of grains and also discussed another method to protect stored wheat from insect damage which made use of the DDT and 666-impregnated dusts. S. Nelson and W. K. Whitney [82] discussed a method to control insects by the use of an electric field at higher frequencies and intensities. S. Hussain and M. Hussain [78] discussed a laboratory experiment to kill the larvae of *Trogoderma granarium* by reducing its pressure to protect stored grains. S. Nelson [83] discussed another method of controlling insects in stored grains by making use of microwaves & other radio frequency energy. A mathematical model was created by B. K. Bala et al. [15] according to the hypothesis of G. Yaciuk et al. [34] that one of the most crucial elements restricting the spread and abundance of insects, mites, and fungus that contaminated and ruined stored grain was temperature. They also proposed model of convergent finite differences that converged to check the grain's temperature as well as changes in the surrounding air temperature during the course of the storage duration and discussed its uses. G. Thorpe and W. B. Elder [35], who found that aeration was very effective in reducing the use of pesticide-treated grains, discussed how " the pace of chemical insecticides' decomposition was directly proportional to the grain's temperature and moisture level " and discussed how transfer of heat and moisture mechanisms were represented mathematically using a finite difference algorithm and chemical pesticides decay in aerated grain was presented. They discussed that the aeration of grains was found to be very helpful in reducing the usage of pesticide methacrifos in tropical and subtropical regions of Australia. Also, G. Thorpe et al. [36] discussed that the community of insect pest *Sitophilus oryzae* (L) might be controlled by handling the microclimates of bulk grains. Considering an aerated bulk of grains, a mathematical model of the heat and transfer phenomenon with a combination of the population model of *Sitophilus oryzae* was presented and predicted that the population growth of weevils could be controlled by blowing cold night air through the bulk of grains. So proper aeration could be helpful to control *Sitophilus orzae* infestation. J. Sutherland et al. [47] discussed about a pneumatic conveyor grain disinfector to disinfect wheat grain by heating it to 70°C which could protect it from all stages of *Rhyzopertha dominica* which was considered to be the most heat-forbearing of Australian grain insects without affecting the grains quality. A mathematical model of the pneumatic heating process was established. M. Ahmed et al. [60] suggested a mathematical framework to investigate the harm insects did to crops in the warehouse environment. He monitored the effect of four insect species namely *Tribolium castaneum, Sitophilus oryzae, Trogoderma granarium, and Rhyzopertha dominica* on the small sample of stored grains. The growth rate of various insect species was estimated using principal component analysis and to examine the community composition with respect to time gradient which was correlated with the loss and damage of grain, they used multiple regression and differential equations for this. C. Jia et al. [16] discussed that insect growth could be prevented by low storage temperature and presented, a two-dimensional heat and transfer mathematical model to explain the transient temperature variations to confirm this, the experiments were performed in a galvanized steel bin taking in account the complex mixed boundary conditions on the surface of grain and around the bin wall. And further C. Jia et al. [17] presented a mathematical model to describe heat and transfer processes accounting for sun radiation, the outside temperature, and air convection which were all mixed boundary conditions taking place during the storage of wheat using the typical meteorological data of a region based on finite element method and obtained the results that grain high temperature were accumulated in different areas of the bin i.e. in the bin center, bin bottom and the top and ventilation was necessary for preventing damage of the grains. A mathematical model, method, and software were proposed by O. Khatchatourian and F. D. Oliveira [65] for modelling the airflow and for the purpose of cooling dynamics of the soya bean mass. Three models were examined for this, and a comparison with experimental data was made. While the second method took into account similar temperatures of the surrounding air and the grain and hypothetically alienated the deep bed into a finite number of thin layers, the first method used PDE system solutions to explain the heat and mass transfer and energy conservation. The third technique, which was based on using homochronous number as an argument, was used to generalise the dimensionless data of the temperature in a deep bed of uniform cross sections with varied velocities in different sections. It was mentioned how this method, which was based on empirical formula, proved to be the best method producing the best outcomes when compared to other methods. The distribution of temperature and moisture transfer of wheat stored in hermetic plastic container or silobag owing to seasonal differences in environmental constraints was estimated, A. Gaston et al. [11] discussed a model linked to three dimensions for transmission of mass and heat. This model calculated the levels of oxygen and carbon dioxide, along with the related Loss of dry matter and grain respiration. To verify the model, measured humidity and temperature values were compared to the anticipated value. A. Tanksale and J. K. Jha [3] developed a mathematical model based on LP to reduce the price of transportation and food grain inventory storage. Cplex optimisation studio was utilised to show the findings that would aid FCI in creating the food movement schedules for each month. Z. M. Isa et al. [90] created a three-dimensional mathematical model to investigate how exposure to phosphine gas caused insects to become extinct within grain. With the use of C code and the computational fluid dynamics [CFD] programme FLUENT, the proposed model to explain sorption and insect death was resolved. The two forms of fumigation distribution considered in this suggested model were fan-forced delivery from the silo's base and tablet delivery from the silo's top. The distribution remained unaltered in the case of tablet fumigation, according to the results, however the spot where the leak was located was particularly crucial for Fumigation using a fan. The outcomes of the half-life pressure test did not reveal phosphine dispersion during tablet fumigation either. According to A. A. Barreto et al. [9], the temperature, moisture content, and insects, mites, and microflora activity were the main contributors to grain spoiling and had a major effect on the grain quality in storage. Consequently, a bi-dimensional coupled momentum heat and mass transport model was described to foretell the distribution of the temperature and moisture movement in a grain bulk., while N. Khuttiyamart and W. Yomsatieankul [64] investigated the causes of moisture movement and loss in moisture in grain storage. It was discovered that oxidation was producing physical changes in the temperature and moisture of the grain in the silo during storage and aeration. This was done by applying a relevant mathematical model. The model's numerical solution, which was the finite difference approach was used to generate the results, which were based on a set of partial differential equations. A Java object-oriented programme was also created to model changes in temperature and humidity on the basis of the proposed mathematical model. A. A. Barreto et al. [10] addressed a mathematical model that had been proven to be accurate for checking grain storage conditions and calculating the change in carbon dioxide concentration in a wheat holder silo bag while accounting for tropical, sub-tropical, and temperate weather conditions in Argentina and found out that insect control was feasible for southern and central regions climatic conditions of Argentina storage. A. Biancolillo et al. [7] created a method to detect an insect infestation in grains that had been stored using NIR spectroscopy in conjunction with discriminant and other conventional modelling techniques. The Indian meal moth (*Plodiainterpunctella*), which was the most prevalent of all the infesting insects, was the main focus. In order to distinguish between the edible and infested rice samples, the conventional techniques Soft Independent Modelling of Class Analogy (SIMCA) and Partial Least Squares Discriminant Analysis (PLS-DA) were applied. These samples of edible and infested rice were obtained from various farmers in six different countries and therefore came to the conclusion that the SIMCA model was specifically for the non-contaminated ones, delivering 97% of the findings properly but not suited for test specimens being insufficiently sensitive. PLS-DA, on the other hand, allowed correctly of roughly 97.5% of infected samples and 95.6% of edible samples. In the range of temperatures best for growth and reproduction, according to W. H. Siddiqui and C. A. Barlow [87], the intrinsic capacity for rise (rm) of the *Drosophila McLanoyastcr Meigen* (DrosMeigen) was more sensitive to temperature variability than to mean constant temperatures. This mismatch was mostly due to the fact that maximal fecundity was only experienced at specific temperatures. S. M. Henson and J. M. Cushing [80] conducted an experiment and came to the conclusion that flour beetles populations(*ribolium*) demonstrated considerable rose in numbers bigger than those when they were cultivated in a fixed volume when grown in flour that was regularly fluctuating. The cubic polynomial model, according to R. T. Arbogast and M. Mullen [75], provided a sufficient depiction of seasonal changes in grain temperature and trapped catches of *Typhaea Stercorea* and *Typhaea Castaneum* infesting maize kept on farms in South Carolina. The small brown planthopper *Laodelphax Striatelluis*, the green rice leafhopper *Nephotettix Cincticeps,* and the rice stem borer Chilo suppressalis, paddy fields in Japan contained three pest bug species. that were the subject of an analysis of the annual light-trap caught trends for the past 50 years by K. Yamamura et al. (49). As the temperature rose, more *Chilo Suppressalis and Nephotettix Cincticeps* were found to be caught in light traps, according to the model of state-space adopted by Akaike's information criteria. In order to study the impact of environmental fluctuation, The Sterile Insect Release Method (SIRM), a mathematical framework that studied how variable environmental changes affected both fertile and sterile insects, was proposed by A. Maiti et al. [5]. By treating the variables of the linearized system as influenced by time and randomly varying, the stochastic version of the model was created. The diffusivity of *Cryptolestes ferruginous* was discussed by F. Jian et al. [25] and was indicated to follow a diffusion pattern while the temperature was steady, the moisture content was dropping, the time it took to move was getting shorter, and the insect population was growing. A model was created by N. Kaliyan et al. [62] to forecast Indian meal moth larvae mortality in varying low-temperature environments. The cumulative lethality index (CLI), which measured a mortality rate accumulation over time, was used to predict total insect population death, and when CLI was equal to 1, total insect population mortality took place. The study by F. Jian et al. [26] showed that the method of finite difference may be utilised to see insect migration of insects and redistribution using the finite difference approach by modelling *Crptolestes ferrugineus* population redistribution using transport equations. F. Jian et al. [29] discussed that at 35°C, patch size affected the dynamics of the insect population and discovered that the number of insects inside a large patch depended more on the number of insects inside small patches than the other way around. *Bactrocera zonata* (Saunders), a dangerous polyphagous pest of horticultural crops, was studied by J. S. Choudhary et al. [42] using growth potential based on temperature at ecologically relevant steady temperatures of 15, 20, 25, 30, and 35 °C; relative humidity of 60 ± 10%. The results showed that temperature played a significant role in determining the climatic suitability for *Bactrocera zonata* in reproduction. In a consistent commensal environment, the two species coexisted for 200 generations, according to A. R. Verdugz and M. Ackermann [6]. Significant environmental changes put a population on the verge of extinction, but in certain circumstances, adaptation through natural selection saved the population and allowed it to survive, according to J. Peniston et al. [43]. According to O. Imura and R. N. Sinha's [66] study of the interaction between *Sitotroga cerealella* and *Sitophilus oryzae* at 28°C and 60% Relative humidity for 22 weeks, the quantity of *Sitophilus oryzae* rose slowly in therapies for both single- and mixed-species but had no impact on other factors. M. Irsad et al. [59] tested various maize varieties against *Sitotroga cerealella* and *Sitophilus oryzae* and found that grains of 'Dehqan' were most susceptible to *Sitotroga cerealella* and 'Shaheen' were most resistant based on loss incurred, and for Sitophilus oryzae grains of 'Shaheen' were most susceptible and 'Azam' was found to be most resistant. Eight other rice varieties were also evaluated, and variety "JP-5" had the greatest index of vulnerability to *Sitophilus oryzae*. *Sitotroga cerealella* caused 'Basmati-370' to sustain a maximum loss ranging from 28.7 to 47.3%. In a study conducted by J. P. Santos et al. [44] to determine the impact of the curculionid *Sitophilus zeamais* and the gelechiid *Sitotroga cerealella* at different developmental stages on the quality of maize seeds, it was discovered that the presence of the 2 insects reduced germination with increasing developmental stage, from 13% at the egg stage for *Sitophilus zeamais* and 10.9% for *Sitotroga cerealella* reached 93% and 85%, respectively, at the adult stage for *Sitophilus zeamais* and *Sitotroga cerealella*. In order to assess the potential impact of seed resistance in combination with an egg parasitoid on the dynamics of the *Sitotroga cerealella* population, R. H. Shukle and L. Wu [74] developed a predictive model. It was discovered that the 16 Soybean trypsin inhibitor (Kunitz inhibitor) had a negative impact on the development of the insect, and protease inhibitor, which might serve as a transgenic resistance factor, was suggested. Under controlled laboratory conditions, L. S. Hansen et al. [52] examined the life history of immature Angoumois grain moths, *Sitotroga cerealella* (Olivier), on dented maize. The temperature was the key determinant of egg incubation time, larval-pupal development time, and egg and larval-pupal survival. 30 C and 75% RH were the ideal conditions for the Angoumois grain moth to develop on maize. Additionally, the effects of four temperatures—20, 25, 30, and 35 C—and two levels of relative humidity—44 and 80%—on the growth rate, age-specific survivorship and fecundity, the sex ratio and intrinsic growth rate of *Sitotroga cerealella* were examined. It was discovered that 30-

degree humidity were the best conditions for *Sitotroga cerealella* population development. The ability of *Sitophilus zeamais* to colonise and monopolise new patches was the mechanism that caused *Sitotroga cerealella* to be quickly eliminated, according to M. N. Larsen et al. [53]. They also discussed the mechanism responsible for selecting the appropriate spatial scale. In their study, S. Ahmed et al. [79] described how, at both 20 °C and 34 °C, the rate of rise in egg mortality and emergence in adult reduction from cured larvae or increased in pupae progressively with increasing exposure time. The most effective MA treatment was that containing 75% CO2 at 34 °C, which destroyed all eggs, larvae within 3 days and all 4 days for the pupae. Y. J. Jeon and H. S. Lee [89] discussed the study in which the scientists carried out an experiment to assess the effects of adult *Sitotroga cerealella* and *Plodia interpunctella* on LEDs trapped in the granary as attractants and compared the BLB with LED. It was discovered that *Sitotroga cerealella* preferred the blue LED over the black LED. By using several grains (Wheat, triticale, sorghum, rye, barley, and other grains), E. Borzoui et al. [21] examined biological and physiological characteristics of *Sitotroga cerealella* (Olivier). The results showed that the fitness of *Sitotroga cerealella* was significantly impacted by various cereals. B. N. et al. [14] talked about the investigation to find out the effects of the Essential oils from *Artemisia khorassanica Podl. and Artemisia sieberi Bess* have fumigant toxicity and sublethal effects on adults of *Sitotroga cerealella* (Olivier). It was discovered that the tested essential oils had a good potential to apply in integrated pest management of *Sitotroga cerealella*. Disrupting sexual communication between the sexes was the most effective way to combat the moth, according to M. Ma et al. [57]. In this study, the external morphology and ultrastructure of the sensilla on the antennae and ovipositor of *Sitotroga cerealella* were examined to ascertain their function. This was done using scanning electron microscopy (SEM) and transmission electron microscopy (TEM). Utilising histopathology, scanning electron microscopy (SEM), and gas chromatography-mass spectrometry (GC-MS), the shape and location of the female sex pheromone gland were also determined. M. Sola et al. [58] detected the innovative multiplex PCR and identifies five internal feeders of grain when treated with $CO_2$ and the sensitivity limit of this gas-based approach was one pupa of insect per kilo of grain.

# CHAPTER-2

## MODEL TO DESCRIBE A SMALL OUT-BREAK OF DAMAGED GRAINS DUE TO INSECTS

### 2.1 INTRODUCTION

This chapter is concerned with a small outbreak of a % of damaged grains in a large susceptible grain. The linear differential equation was formed with the help of some assumptions. The analysis of the result revealed that % of the damage of grains increased with the increasing number of the insect populations.

The presented model described a small outbreak of a % of damaged grains. I assumed that one or more insects arrived in a vacant area and started the outbreak as per the theory of M. Evans [55]. To prevent the situation in which the outbreak got out of control and eventually involved all susceptible in the population. For this we laid down certain assumptions as follows:

1. The conditions were sufficiently unfavorable that the outbreak never became large.

2. For conciseness, we considered that a single insect damaged the grains at first rapidly.

This Process (mode) best described the out-break after the introduction conditions (factors).

We supposed that when the model began there were k insects in the ecosystem. Each of these insects produced one or additional insects according to Michael O. Ashamo et al. [56]. If we perceived the original infective as forming this zeroth generation of the insects, then the amount of grains damaged by the first generation of the insects and so forth for subsequent generations according to the results of R. Varshney and C. R. Ballal [73]. The outbreak ended once all of curves of insects perished or total grains were damaged. The vertices of the graph represented insects and the edge represented the paths of insects from the source to the recipient.

## 2.2 MATERIALS AND METHODS

### 2.2.1 Insect attack probability

The first problem considered was the attack rate of the insect. The simplest reasonable assumption was that each insect for the attack (till damage) length of time t, where t was a continuous random variable. By using the theory of F. Jian et al. [27], we assumed that $x(t)$ be an integer-valued random variable that served as counter thus each time an 'event' occurred $x(t)$ as augmented by one. We supposed that at $t = 0$ the counter was set to zero so $X(0) = 0$.

We supposed that insects occurred independently at random i.e. we supposed that the time was subdivided into small intervals of length $\Delta t$. The probability that one insect occurred in a particular interval of Δt was m Δt, where m was any parameter. Because we made $\Delta t$ as small as we wish. Thus the probability that no insect occurred in an interval of length Δt was $(1 - m\Delta t)$. In general, all non-overlapping time intervals were independent of one another. We calculated the probability density for the random variable x (t).

Let Prob $(\mathrm{x}\ (\mathrm{t}) = \mathrm{n}) = P_n(\mathrm{t})$, $\quad \mathrm{n} = 0, 1, 2, 3 \ldots.$ (1)

We wished to calculate $P_n(\mathrm{t} + \Delta\ \mathrm{t})$, n = 0, 1, 2, 3……

There were two mutually exclusive situations at time t, which were mentioned as follows

1. At time t, $\mathrm{x}\ (\mathrm{t}) = \mathrm{n} - 1$ with probability $P_{n-1}(\mathrm{t})$ and during the next time interval of length Δt an event occurred with probability m Δt.
2. At time t, $\mathrm{x}\ (\mathrm{t}) = \mathrm{n}$ with probability $P_n(\mathrm{t})$ and during the next time interval of length $\Delta t$ no event occurred with probability $1 - \mathrm{m}\ \Delta\mathrm{t}$.

Since above two cases were mutually exclusive they might be arranged in the form

$$P_n(\mathrm{t} + \Delta\ \mathrm{t}) = P_n(\mathrm{t})\ \mathrm{m}\ \Delta\mathrm{t} + P_{n-1}(\mathrm{t})\ (1 - \mathrm{m}\ \Delta\mathrm{t})$$

which simplified to yield

$$\frac{P_n\ (\mathrm{t}+\ \Delta\mathrm{t}\ ) - P_n\ (\mathrm{t})}{\Delta\mathrm{t}} = \mathrm{m}\ [P_{n-1}(\mathrm{t}) - P_n(\mathrm{t})], \mathrm{n} = 0,1,2,3 \ldots$$

under limiting conditions $\Delta t \rightarrow 0$

$$\frac{d(P_n(\text{t}))}{dt} = \text{ m } [P_{n-1}(\text{t}) - P_n(\text{t}) ], \text{n} = 0,1,2,3 \ldots \quad (2)$$

When we took $\text{n} = 0$, then above equation reduced to an event occurring with

$$\frac{d(P_0(\text{t}))}{dt} = -\text{m}P_0 (\text{t}) \quad (3)$$

Since x $(0) = 0$, it followed that $P_0(0) = 0$ and

$P_n(0) = 0,$ n = $1, 2, 3$ ....

Solving the equation (3) for $P_0$ (t)with its initial condition we arrived at

$$P_0(\text{t}) = e^{-mt} \quad (4)$$

Again when we let $n = 1$ in equation (2) and substituted $P_n(\text{t})$ from (3) it was fairly easy to get

$$\frac{d(P_1(\text{t}))}{dt} + \text{m}P_1(\text{t}) = -\text{m}e^{-mt} \quad (5)$$

This was the first order linear differential equation with constant coefficients, so integrating factor i.e $IF = e^{mt}$

Hence this could be written as $d[P_1(\text{t})e^{mt}] = \text{ mdt}$

On integrating this equation with the initial condition $P_1(0) = 0$, we got

$P_1(\text{t}) = mt\, e^{-mt}$

By adopting the parallel procedure for n =1, it inductively leads to the general result

$$P_n(\text{t}) = \frac{(\text{mt})^n e^{-mt}}{\text{n!}},\ n = 0, 1, 2, 3 \ldots. \quad (6)$$

This was well known time dependent Poisson density function which described the probability that by time exactly n insects occur randomly.

We had considered that the length of time t spent by insect for damaging the grains. Thus we had, proba (grain damage for time t) $= \mu e^{-mt}$ (7)

During that time, the damage of grains remained continuous, the insect was being attacked on susceptible grains. We assumed that environment was favorable for the attack of insects on susceptible grains that took place independently at random such that the average number of grains damaged by one insect per unit time was m.

Again number of damaging $j$ grains, given that the damaging period lasts for t units of time $P\left(\frac{j}{t}\right)$ was given by

$$P\left(\frac{j}{t}\right) = \left[\frac{(\mathrm{mt})^j}{\mathrm{j!}}\right] e^{-mt} \tag{8}$$

To eliminate the conditioning in $P\left(\frac{j}{t}\right)$ we made use of the assumption that the length of the infection period was exponentially distributed with parameter j. Thus the probability p, that one insect damaged j grains during its damage period was

$$P_j = \int_0^\infty \left(\frac{(\mathrm{mt})^j}{\mathrm{j!}}\right) \mathrm{e}^{-mt} \ \mu\, \mathrm{e}^{-\mu t}\, \mathrm{dt}$$

$$= \left[\frac{\mu}{\mu+m}\right]\left[\frac{m}{\mu+m}\right]^{\boldsymbol{j}} \tag{9}$$

We had assumed throughout the development of the model, the number of the grains damaged by one particular victim of the insect was independent of the number damaged by any other victim. We considered the $i^{th}$ damaged grain and let $Q_i$ be an integer valued random variable which counted the number of cases of the grains which were damaged by the insect. Since $Q_i$ was distributed according to $P_i$ the expected number of new cases, $E\ (Q_i)$ was given by

$$E\ (Q_i) = \theta = \textstyle\sum_0^\infty \mathrm{j}P_i \tag{10}$$

$$E\ (Q_i) = \frac{m}{\mu}, \mathrm{j} = 0 \tag{11}$$

It was stated in the introduction to this chapter that we imagined conditions to be unfavorable for a major epidemic; thus the insect outbreak died out quickly. We had determined what this means in terms of our variables. We considered one particular infective in the zeroth generation (grain) and had taken $d_n$ as the probability that the portion outbreak developed from the chosen infective had been died out by the nth generation .We assumed that chosen infective in the zeroth (grains) generation called A damaged the $j^{th}$ grains (called B) according to the theory of H. Tripathi and K. C. Garg [39]. We had viewed each B as the head of a curve of the damaged grains. If the A's portion of the outbreak had ended by the $(n+1)^{th}$ grains, then the portion from each of the $j$ B's must independently be ended after $n$ additional grains of the insect had occurred. This occurred with a probability equal to $[d_n]^j$. But since, we did not know j; we had done average over all choices of $j$, weighted of $P_j$, the probability that $A$ had $B_j$. Hence we had

$$d_{n+1} = \textstyle\sum_0^\infty P_j\, [d_n]^j = \mathrm{f}(d_n) \tag{12}$$

We had identified $f(d_n)$ as the probability generating function for this discrete density $P_j$ .The expression found was a recurrence relation for the sequence $\{d_0, d_1, \ldots\ldots\ldots d_n, d_{n+1}\}$. Then we made several observations. Since $d_n$ was the probability that a line of the damaging grains ended by the $n^{th}$ grain i.e.

1. $0 \leq d_0 \leq d_1 \leq, \ldots\ldots\ldots \leq d_n \leq d_{n+1} \leq \ldots . \leq 1$
2. Since the $A$ of line certainly had the disease, $d_0 = P_0$
3. The sequence must approach a limit since it could never exceed unity in numerical values, so

$\lim_{n\to\infty} d_n \to d \leq 1$

## 2.3 RESULTS AND DISCUSSION

The third observation allowed us to rewrite the recurrence relation as a non-recursive equation form, the probability of ultimate extinction of one line of the damaged grain $d = f(d_n)$. This result was true for any insect transmission probability. After equating $d$ to the probability generating function for the geometric density determined earlier yielded

$$d = \frac{1}{1+(1-d)^{\frac{m}{\mu}}} \tag{13}$$

Solved for $d$ provides $d = \begin{cases} \frac{\mu}{m}, & if \ m \geq \mu \\ 1, & if \ m < \mu \end{cases}$

The two choices arose as the roots of a quadratic equation. The proper choice was always the smaller root which turned out to be one satisfying $0 \leq d \leq 1$

Finally, the probability that all of the branches shining damaged grains started by the K individuals was the zeroth grains that had independently died out given by

$$\text{Prob [outbreak ends]} = d^k = \begin{cases} \left(\frac{\mu}{m}\right)^k, & if \ m \geq \mu \\ 1, & if \ m < \mu \end{cases} \tag{14}$$

The condition taken for the outbreak to end with certainty was that $m < \mu$. This was equal to the expected number of $B$'s per $'s \ \theta$, satisfying

$$\theta = \frac{m}{\mu} < 1 \tag{15}$$

Since the insects damaged the grain independently and the number of grains damaged by the $i^{th}$ insect $Q_i$ was distributed with the same geometric pattern as before so we had,

$$\text{Prob}\ (Q_{i=j}) \ = \ P_j \ , E\ (Q_i\ ) \ = \ \theta \quad (16)$$

We defined $W_\lambda \ = \ \sum_{i=1}^{W_{\lambda-1}} Q_i$ (17)

The expected value of $W_\lambda$ followed easily from the conditional expectation given $W_{\lambda-1}$. Hence

$$E(W_\lambda) = \text{E}(\text{E}(W_\lambda|W_{\lambda-1}) = \ E(E(\textstyle\sum_{i=1}^{W_{\lambda-1}} Q_i\ ) \ \ |W_{\lambda-1})) \quad (18)$$

As the expected values of a sum equal the sum of the expected values. We got

$$E(W_\lambda) = E[E(Q_i)\ W_{\lambda-1}\ )] \quad (19)$$

$$E(W_\lambda) = E\big((\theta)W_{\lambda-1}\big) \quad (20)$$

Since θ was taken as constant in the above equation and it was written as

$$E(W_\lambda) = \theta\ E(W_{\lambda-1}\ )$$

$E(W_\theta) = 1$, because the model counted the number of insects which damaged by a single insect in the zeroth damaged generation. Thus

$$E(W_\lambda) = \theta^\lambda, \lambda \ = \ 0,1,2 \dots \dots$$

When we added the expected number of insects in all successive generations the total number of insects in the zeroth generation, $N$ was given by

$$N = k \sum_{\lambda=0}^{\infty} \theta^\lambda \ = \ \frac{k}{1-\theta}$$

Using equation (15) in the above, we arrived at

$$N = \frac{k}{1-\frac{m}{\mu}} = \ \frac{\text{k}\ \mu}{\mu - m} \quad (21)$$

We performed an experiment on different varieties of paddy under storage conditions, the environmental factors i.e. Relative humidity, temperature and moisture content was maintained at 75 % with KOH for two weeks. The moisture content varied from 11.3 to 12.0 % after two weeks. In this experiment, 400 healthy grains were taken in the glass specimen tubes conserved with muslin cloth and ten pairs of adult moths (*S. Cerealella Oliver*) of the same age were introduced in each specimen tube. The experiment was carried out at a constant temperature of 27± 1ºC for three months based on the conditions given by V. Pandey et al. [86]. At the end of the experiments, % of damaged grains, the total population of insects and loss of weight were recorded.

Data regarding the moisture content of seed %, an average insect population and % of damaged grains were presented in the table below.

| **Sr no** | **Moisture content of seed %** | **Average insect population** | **% damaged grains** |
|---|---|---|---|
| 1 | 11.50 | 42.64 | 14.25 |
| 2 | 11.82 | 11.07 | 3.41 |
| 3 | 11.55 | 10.64 | 3.17 |
| 4 | 11.30 | 14.26 | 5.67 |
| 5 | 11.95 | 13.84 | 5.33 |
| 6 | 11.63 | 9.75 | 2.67 |
| 7 | 11.90 | 11.95 | 4.08 |
| 8 | 11.90 | 13.27 | 4.92 |
| 9 | 12.00 | 16.94 | 8.00 |
| 10 | 11.88 | 13.43 | 5.08 |

**Table 2.1-** Representing the data regarding the moisture content of seed %, an average insect population and % of damaged grains.

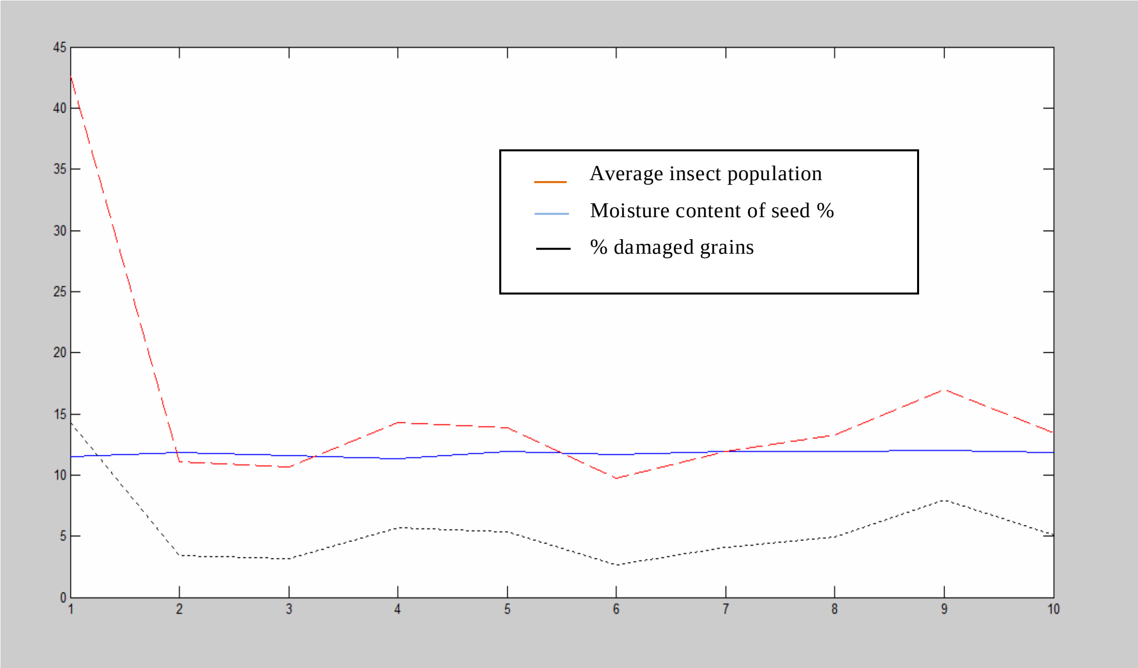


**Figure 2.1 - Graph of Table -2.1**

## 2.4 CONCLUSION

We concluded from this model with experimental data and plotted graph that the % damage of grains increased with the increasing number of the insect populations. We took initially k = 10 pairs, then equation (21) became

$$N = \frac{10\mu}{\mu - m}$$

The quantity $N$ was the one we ordinarily wish to know, it's instructive to work the probability that exactly k insects were involved in the disease outbreak.

The model analysed that the greater the no of insects in the store were damaging a large amount of grains. This model emphasized on the role of insect infestation during storage, in reducing the outbreak of damaged grains due to insects in various conditions, which proved its efficiency in itself.

# CHAPTER 3

# TO DEVELOP AND VALIDATE THE MATHEMATICAL MODEL THAT DESCRIBES THE GROWTH IN POPULATION IN A RANDOMLY FLUCTUATING ENVIRONMENTAL ECOSYSTEM

## 3.1 INTRODUCTION

The most important biotic component of a stored grain ecosystem is the insects. Insect population is greatly influenced due to the fluctuation of abiotic factors in combination with many biotic factors. A mathematical model was formulated explaining insect population and their growth in a randomly fluctuating environmental ecosystem. The changes in the net growth rate of insects were discussed using the stochastic models. Although production has been rising, losses that are the result of faulty storage remain stable at 10%. According to estimates, improper storage management accounts for 6% of the entire 10% loss. Losses in both quality and quantity during storage are brought on by insects, rodents, and microorganisms. It has been reported that a large number of insect pests are associated with stored grains. Insects need on storage conditions to develop and survive, and if the conditions are good, they continue to be active.

A number of variables were taken into consideration, including the temperature, the moisture level, air relative humidity (RH), intergranular gaseous components, broken grains, and dockage that impacted the population dynamics of insects, according to L. Mason [51]. The temperature and humidity levels were among them, and they had an important impact on how long insects were developing and reproducing, as well as how widespread their infestation would be and how long they would be viable. K. Alagsundram et al. [48] conducted research on how changes in external climatic variables induced changes in storage conditions over time. The two significant gradients—temperature and moisture—that form inside the stored grain bulk had a significant impact on insect pest migration, survival, and multiplication as well as grain quality.

In this chapter, a study of change in the insect population in a randomly fluctuating environmental ecosystem was discussed by using a mathematical model. R.Yadav and K. Bhagirath [72] developed a mathematical model to describe the damage caused by insects based on linear differential equation accounting some assumptions and concluded that as the number of insects increased, the % of damaged grains also increased.

## 3.2 MODEL FORMULATION

We had assumed '$x$' as the size of a biological population or the size of an organism . In the absence of fluctuating environment, it was a continuous variable. To describe the population growth of insects, the following differential equation was used

$$\frac{dx}{dt} = \frac{\bar{a}x\ [1-\frac{x}{R}]^n}{n}, \quad \bar{a} > 0 \qquad (1)$$

Here '$\bar{a}$' was the Malthusian parameter of population growth. The second term specified the restriction in the growth which occurred mainly because of crowding effects and competition for the available food. The equation (1) represented here was deterministic, with little consideration given to the influence of chance on population increased. So, in this chapter analysis of the situation when the replacement of this deterministic growth equation with stochastic form was done to note especially the relationship between the process' mean and variance when growth took place. We had considered the case when $n = 1$, we discussed the effects of the fluctuating environment on the growth of insects. Fluctuation in environmental ecosystem brought a change in net growth rate of insects as well as death rate

**Changes in net growth rate**

Incorporation of stochastic components into deterministic model (1) was done. The stochastic form of '$a$' was given by the equation

$$\bar{a} = a + \mu G\ (t) \qquad (2)$$

Where $G\ (t)$ represented Gaussian white noise and μ was its intensity. Considering the study of B. P. Khare [13] an assumption was taken for the fluctuations to be faster than time scale of growth in population. The stochastic general differential equation that explained the population growth became:

$$\frac{dx}{dt} = a\text{x}\,(1 - \frac{\text{x}}{\text{R}}) + \mu\,(1 - \frac{x}{\text{R}})\text{xG (t)} \qquad (3)$$

Also called Stratonovich SDE

Replacing $a$ by $(a + \frac{\mu^2}{2})$

Equivalent to Ito SDE

$$\frac{dx}{dt} = (a + \frac{\mu^2}{2})\,\text{x}\,(1 - \frac{\text{x}}{\text{R}}) + \mu\,(1 - \frac{x}{\text{R}})\text{xG (t)}$$

Comparison with the Fokker Plank equation was done

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}\,[\alpha(\text{x})\text{P}] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[ß(\text{x})\,\text{P}] \qquad (4)$$

We had got,

$$\alpha(\text{x}) = (1 - \frac{x}{\text{R}})\,(a + \frac{\mu^2}{2})\,(1 - \frac{2\text{x}}{\text{R}}) \qquad (5)$$

$$ß(\text{x}) = \mu^2 x^2 (1 - \frac{\text{x}}{\text{R}})^2 \qquad (6)$$

We had considered the singular boundary conditions i.e. $x = 0, x = R$ and approximating $x = 0$ , we obtained

$\alpha(\text{x}) \approx (a + \frac{\mu^2}{2})$ and $ß(\text{x}) \approx \mu^2 x^2$

Since both the Natural boundaries were not attained. This process described a population of insects which was away from extinction and which fluctuated around some average value which was less than R because of the fluctuations in the rate of growth.

The steady state probability density function was evaluated from this:

$$\text{P}\left(\frac{x}{y}, \infty\right) = \frac{A}{ß(\text{x})} e^2 \int_0^x \left(\frac{\alpha(\text{y})}{ß(\text{y})}\right) \text{dy} \qquad (7)$$

Using the condition, $\int \text{P}\left(\frac{x}{y}, \infty\right) \text{dx} = 1$ we could determine '$A$' which was a normalization constant. Putting the values of α(y)and ß(y) from (5) and (6) in (7)

$$\text{P} = \text{A}\left(1 - \frac{\text{x}}{\text{R}}\right).\left(\frac{2a}{\mu^2} - 1\right)\left(\frac{2a}{\mu^2} + 1\right) \qquad (8)$$

Replacing the quantities $\left(\frac{2a}{\mu^2} - 1\right)$ and $\left(\frac{2a}{\mu^2} + 1\right)$ by the quantities $\left(\frac{2a}{\mu^2} - 2\right)$ and $\left(\frac{2a}{\mu^2} + 2\right)$ respectively. When $\left(\frac{2a}{\mu^2}\right) < 1$, showing the population of insects approaching zero or nearly R.

When $\left(\frac{2a}{\mu^2}\right) > 1$ , the density function was monotonically increasing, depicted population accumulation near R according to N. Kant et al. [63].

$\frac{\partial P}{\partial t} = -\frac{\partial J}{\partial x} =- \frac{\partial}{\partial x}\left([\alpha(x)P] - \frac{1}{2}\frac{\partial}{\partial x}[ß(x)P]\right)$ where J was probability current density

$$J = [\alpha(x)P] - \frac{1}{2}\frac{\partial}{\partial x}[ß(x)P]$$

As $\frac{\partial P}{\partial t} = 0$ being independent of time implied $\frac{\partial J}{\partial x} = 0$

$J$= Constant and independent of 'x' but for equilibrium distribution to be normalisable, all derivatives approached to zero and hence $J = 0$ .

$$[\alpha(x)P] - \frac{1}{2}\frac{\partial}{\partial x}[ß(x)P] = 0$$

$$\alpha(x) = \frac{1}{2}\frac{dß}{dx}$$

We introduced the variable '$s$' to derive the time dependent probability density function such that

$$s = \frac{\left(log\frac{x}{1-\frac{x}{R}}\right)}{\mu} \tag{9}$$

$$dx = \mu x \left(1 - \frac{x}{R}\right) ds \tag{10}$$

$$\frac{dx}{ds} = \mu x \left(1 - \frac{x}{R}\right)$$

Using (3)

$$\frac{dx}{dt} = ax\,(1 - \frac{x}{R}) + \mu\,(1 - \frac{x}{R})xG\,(t)$$

$$\frac{dx}{ds}\frac{ds}{dt} = \frac{a}{\mu}\frac{dx}{ds} + \frac{dx}{ds}G\,(t)$$

equation (3) changed to $\frac{ds}{dt} = \left(\frac{a}{\mu}\right) + G(t)$ (11)

where (11) represented the stochastic differential equation for an unrestricted wiener process.

Probability density function $g\left(\frac{s}{s_0}, t\right)$ satisfied the Fokker Plank equation, hence given by:

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial s}\left(\frac{a}{\mu}\right)g + \frac{1}{2}\frac{\partial^2 g}{\partial s^2} \tag{12}$$

where the boundary conditions were given by

$$\lim_{s\to\pm\infty} g\left(\frac{s}{s_0},t\right) = 0 \tag{13}$$

Corresponding to the boundary conditions which were inaccessible i.e.

$x = 0\ (s = -\infty)$ and $x = R\ (s = +\infty)$

$$g\left(\frac{s}{s_0},t\right) = \frac{e^{\left[\frac{-1}{2t}\left(\frac{s-s_0 e^{-at}}{.\mu^2}\right)\right]}}{\sqrt{2\pi t}} \tag{14}$$

We obtained the probability density function

$$P\left(\frac{x}{y},t\right) = \frac{1}{\sqrt{2\pi t}\,\mu y^2 x\left(1-\frac{x}{R}\right)} \frac{e^{\left(\frac{-1}{2t}\left(\frac{1}{\mu}log\frac{x}{y}-\frac{1}{\mu}log\frac{1-\frac{x}{R}}{1-\frac{y}{R}}-\frac{at^2}{\mu}\right)\right)}}{} \tag{15}$$

which determined the behavior of the population.

When $R > -\infty$ then we could say that the food supply was limitless or far from saturation.

From equation (11) we found,

$x = e^{\mu} s$

$< x >= \int_0^R xP\left(\frac{x}{y},t\right) dx$

$$< x > = \int_{-\infty}^{\infty} xg\left(\frac{s}{s_0},t\right) dx \tag{16}$$

$= e^{\left(\frac{\mu^2 t}{2}\right)} e^{\left\{\mu\left(\frac{s_0+at}{\mu}\right)\right\}}$

$$= ye^{at} e^{\left(\frac{\mu^2 t}{2}\right)} \tag{17}$$

Comparing to $x = y\,e^{at}$ (18)

For the deterministic case (in which random fluctuations were absent), we had zero variance.

Now we obtained variance as

$Var(x) =< x^2 > - < x >^2 = y^2 e^{2at}(e^{\mu^2 t} - 1)$

$$= < x >^2 (e^{\mu^2 t} - 1) \tag{19}$$

The coefficient of variation was given by:

$$[Var(x)]^{1/2} = \frac{[(e^{\mu^2 t}-1)]^{1/2}}{<x>} \tag{20}$$

So we could say that when 't' increased, there was an increase in coefficient of variation.

### 3.3 CHANGE IN THE DEATH RATE

According to G. Singh et al. [38], in comparison to the adult population, if the zygote population was more and hence they were less subjected to random fluctuations then random variation would be seen mostly in the death of adults. Derivation of the continuous model was done to approximate the model. We had taken the population size to be $x$ and the deterministic birth rate as $\sigma_x$ .Without taking in account the age of an individual we had assumed that the individual died in time $\Delta t$ with probability $\lambda_x \Delta t$. Since we assumed the number of individuals at time $t$ be $x(t)$and the number of individuals numbers at time $(t + \Delta t)$as $x(t + \Delta t)$ which was a random variable took the value

$$x(t + \Delta t) = x(t) + (\sigma_x - \lambda_x)\Delta t \quad (1)$$

With probability as $\frac{x(t)}{i} (\lambda_x \Delta t)^i \ (1 - \lambda_x \Delta t)^{x-i}$ (2)

Here the number 'i' was Poisson distributed up to first order in $\Delta t$

$$< i >= \lambda_x \Delta t \quad (3)$$

$$< i^2 >= \lambda_x \ (1 - \lambda_x \Delta t) \ + \ (\lambda_x \Delta t)^2 \quad (4)$$

By using equation (3), (4) and (1)

$$< x \ (t + \Delta t) - \ x \ (t) > = < \Delta x \ (t) >= \ (\sigma_x - \lambda_x)\Delta t \quad (5)$$

$$< \ [\Delta x \ (t)] \ ^2 > = \ \sigma_x \ (\Delta t) \ + \ 2 \ \sigma_x \ x\lambda_x (\Delta t)^2 + \ x\lambda_x \Delta t(1 - \lambda_x \Delta t) \ + \ (x\lambda_x \Delta t)^2 \quad (6)$$

Therefore

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \ < \Delta \ (x) >= \ \sigma_x - \lambda_x \quad (7)$$

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \ < \Delta x \ (t) > ^2 = \ x\lambda_x \quad (8)$$

$\sigma_x$ was chosen in the same form as σ (m) of model (9)

$$\sigma \ (m) \ = \sigma \ \left[1 - \left(\frac{m}{R}\right) \alpha\right] \quad \text{with } \alpha \ = 1$$

For the continuous model the Fokker Plank equation (using equation 7 and 8) was given by

$$\frac{\partial P}{\partial t} = - \ \frac{\partial}{\partial x} [\alpha(x) \ P + \ \frac{1}{2} \frac{\partial^2}{\partial x^2} \ ß(x) \ P] \quad (10)$$

With $\alpha(x) = \ a x \ \left(1 - \ \frac{x}{R}\right) = \sigma_x - \lambda_x$ (11)

$$ß(x) \ = \ \lambda_x x \quad (12)$$

Determination of the Steady State Distribution

We had taken the variable $x \geq 0$ with boundary condition $x = 0$ [$ß(0) = 0$ being singular and near $x = 0$],

$ß(x) = \lambda x$ and $\alpha(x) \sim ax$

This was an exit boundary condition that whatever reaches the boundary $x = 0$ was get trapped there forever and led to the extinction of the population and the entrance boundary condition was given by $x = \infty$ and for all $x > 0$ the steady- state probability density function was zero.

Transformation $dz = \frac{dx}{\sqrt{\lambda x}}$ was used to transform the Fokker Plank equation to find the value of $P\left(\frac{x}{y}, t\right)$

$$z = 2\left(\frac{x}{\lambda}\right)^{½} \tag{13}$$

$$P\left(\frac{x}{y}, t\right) = \frac{g\left(\frac{z}{z_0}, t\right)}{\sqrt{\lambda x}} \tag{14}$$

Using 12, 13 and 14 we got

$$\frac{\partial g}{\partial t} = -\left(\frac{1\partial}{2\partial z}\left(a\,z - \frac{a\lambda z^3}{4R} - \frac{1}{z}\right) g + \frac{1}{2}\frac{\partial^2 g}{\partial z^2}\right) \tag{15}$$

By using simple transformation, we obtained,

$$G = z\left(\frac{a\lambda}{4R}\right)^{¼}, \Omega = 2\left(\frac{aR}{\lambda}\right)^{½}, T = \left(\frac{a\lambda}{R}\right)^{½}\frac{t}{4} \tag{16}$$

hence equation (16) changed to

$$\frac{\partial g}{\partial t} = \frac{\partial}{\partial G}\left[\left(\frac{1}{G} - \Omega G + G^3\right)\right] g + \frac{1}{2}\frac{\partial^2 g}{\partial G^2} \tag{17}$$

Equation (10) approximate solutions were derived by using Malthusian regime

1.An initial size was small i.e. $y < R$ or $R \rightarrow \infty$

2.For the Malthus regime we took $y \approx R$ hence $\alpha(x) \approx ax$ so that

$$P\left(\frac{x}{y}, t\right) = \frac{2a}{\lambda} exp\left(\frac{2a x + y e^{-at}}{\mu e^{at-1}}\right) I\left(\frac{4a\sqrt{xy}}{\lambda\left(e^{\frac{at}{2}} - e^{\frac{-at}{2}}\right)}\right) \tag{18}$$

The $I^{th}$ moment of $x^e(t)$ according to the density equation was:

$$\langle x^e(t)\rangle = \int_0^\infty x^e P\left(\frac{x}{y}, t\right) dx$$

$$= y e^{at}\left(\frac{\lambda(e^{at} - 1)}{2a}\right)^{e-1} e^{\left(\frac{-2ay}{\lambda(1 - e^{-at})}\right)} \Gamma(I + 1)\, F\left(I + 1 : 2; \frac{2ay}{\lambda(1 - e^{-at})}\right) \tag{19}$$

With the help of this standard formula $F(b; b; z) = e^z$ (20)

$$F\,(b+1; b; z) = (b + \frac{z}{b})\,F(b;\ b; z) \tag{21}$$

We found $\langle x\,(t)\,\rangle = ye^{at}$ (22)

$$\langle x^2\,(t)\,\rangle - \langle x\,(t)\,\rangle^2 = \frac{\lambda}{a}\,\langle x\,(t)\,\rangle(e^{at} - 1) \tag{23}$$

The average value was similar to the Malthusian deterministic behavior ß (x) $= 0$ and since $\alpha$ (x) was linear in x so this was the expected value. T had been taken as Probability of population growing without limit and hence given by

$$T(\frac{\infty}{y}) = \frac{\int_0^y \Pi(\eta)d\eta}{\int_0^\infty \Pi(\eta)d\eta} \tag{24}$$

Where $\Pi\,(\eta) = e^{-2\int_0^\eta \frac{\alpha(y)}{ß(y)}dy}$ (25)

$$T\left(\frac{\infty}{y}\right) = 1 - e^{-2a\frac{y}{\lambda}} \tag{26}$$

For the case when $a > 0$, we found that with probability $(1 - e^{-2a\frac{y}{\lambda}})$, the population would keep on growing and with probability $(1 - e^{-2a\frac{y}{\lambda}})$ would become extinct for $a < 0$. In the Malthusian regime, an approximation validation was limited to short times and limit of the time t→ ∞ had no meaning.

In the regime $y \approx R$ ; $\frac{x-y}{r} << 1$ and $a(x) \approx a[R - x]$ , $a > 0$ (27)

the parameter was defined as $\eta = \frac{2aR}{\lambda}$

For $\eta > 1$, *the boundary* $x = 0$ was considered as *an entrance boundary* and f*or* the condition when $0 < \eta < 1$, the boundary was a regular boundary and $x = \infty$ was a natural inaccessible boundary when $\eta > 0$.

$P\left(\frac{x}{y}, \infty\right) = Ax^{\eta-1}\,e^{-\eta\frac{x}{R}}$, here A was a normalization constant and its value was given by $A = \frac{(\frac{a}{R})^\eta}{\Gamma(\eta)}$ (28)

The most probable population size in a steady state was given by

$x_0 = R - \frac{\lambda}{2a} = R\left(1 - \frac{1}{\eta}\right)$ which was positive for $\eta > 1$. (29)

*As* $\eta$ increased, the size approaches to R. The moment of the size of the population at steady state was given by

$$< x^e > = \left(\frac{R}{\eta}\right)^e \frac{\Gamma(\eta+1)}{\Gamma(\eta)},\ \langle x\rangle = R > x_0 \tag{30}$$

Probability density depending on time was given by

$$P\left(\frac{x}{y},t\right) = \frac{2a}{\lambda}\left(\frac{x}{y}\right)^{\frac{n-1}{2}} \frac{e^{\frac{\eta at}{2}}}{e^{\frac{at}{2}} - e^{-\frac{at}{2}}} e^{\left(\frac{-2ax + y\exp(-at)}{\lambda(1-\exp(-at))}\right)} I_{n-1}\left(\frac{4a(xy)^{½}}{\lambda(1-e^{\frac{at}{2}})}\right) \quad (31)$$

We found that for all $t > 0$, the probability of having a large population ($x \gg R$) was very small for all $\eta > 0$.While the probability of small size population was low when $\eta > 1$ and very high for $\eta < 1$.Therefore the case when we had $\eta > 1$ described a fluctuation in population around as average far from zero.

$$< x^p >= \left(\frac{\lambda}{2a}\right)^p exp\left(\frac{2ay\ \Gamma(l+n)}{\lambda(e^{at}-1)\Gamma(\eta)}\right) F\left(\eta + p{:}\ \eta;\ \frac{2ay}{\lambda(e^{at}-1)}\right) \quad (32)$$

using the equations (22), (23) for $p = 1, 2 \ldots..$

We obtained from the above equation

$$\langle x\rangle = R\,(1 - e^{-at}) + ye^{-at} = R - (R - y)e^{-at} \quad (33)$$

The deterministic behavior was obtained by solving the deterministic equation

$$\frac{dx}{dt} = r(R - x)$$

$$\langle x^2\rangle - \langle x\rangle^2 = \frac{\lambda}{2a}[\langle x\rangle + ye^{-at}](1 - e^{-at})] \quad (34)$$

We got the comparison to the zero value in this variance.

## 3.4 CONCLUSION

We concluded from this presented mathematical model that protein loss in grains was proportional to the population of the insects and which depended on different environmental factors. Stochastic models were used in studying the change in the net growth rate of insects. We studied that the insect population changed on a successful colonized population in a randomly fluctuating environment. We also discussed the changes in the death rate using the model that approximated the situation described a population that fluctuated around as average far from zero. The research highlighted that protein loss in grains was not only dependent on insect population size but also on various environmental factors considered in the model. Various environmental factors, including temperature, moisture levels, air relative humidity, intergranular gaseous components, broken grains, and dockage, were considered variables influencing insect population dynamics. Temperature and humidity levels were highlighted as crucial factors affecting insect development, reproduction, and infestation. Random changes in the environment could have multifaceted effects on insect populations, influencing their behavior, distribution, and interactions within ecosystems.

# CHAPTER – 4

## TO DEVELOP AND VALIDATE A MATHEMATICAL MODEL TO ANALYZE THE EFFECT OF *S. CEREALELLA* ON GRAINS IN THE ENVIRONMENTAL ECOSYSTEM


**Figure 4.1-** Angoumois grain moth (*Sitotroga Cerealella*)

**Source –** "Angoumois grain moth. Clemson University - USDA Cooperative Extension Slide Series.www.Bugwood.organisation.com https://www.forestryimages.org/browse/detail.cfm?imgnum=1235240. Published June 23, 2003. Updated May 11, 2011. Accessed February 11,2022."

“The Angoumois grain moth (*Sitotroga Cerealella*) is a species of the Gelechiidae moth family, commonly referred to as the "rice grain moth". It is most abundant in the temperate or tropical climate of India, China, South Africa, Indonesia, Malaysia, Japan, Egypt and Nigeria, with its location of origin being currently unknown. It is most commonly associated as a pest of field and stored cereal grains as they burrow within the kernel grains of crop plants, rendering them unusable for human consumption.” (From Wikipedia).

## 4.1 INTRODUCTION

One of the main pests that attack grains stored in storage is the Angoumois grain moth, *Sitotroga cerealella*. Under the premise that the rate of change of % of germination loss with respect to grain infestation was proportional to germination loss, a mathematical model was created using differential equations of second order. An entomological dataset was used for this issue. This model will aid in reducing grain storage losses brought on by many elements like moisture, humidity, etc., which is extremely helpful for managing the stored grains effectively.The most important pest of stored agricultural products worldwide is the Angoumois grain moth. *Sitotroga cerealella* infestations rise during storage, in pre-harvest or post-harvest.

D. A. Ukeh and I. A. Udo [19] concluded that a variety of kernel such as corn, sorghum, wheat, soya bean, rice and paddy was attacked by the larvae of *Sitotroga Cerealella*. And also reported that the development of the pest could possibly be managed by alternating nutritive and the physical properties of cereals and also studied that this pest attack all types of cereal grains especially the wheat where weight loss could reach 50 %. P. Weston and P. L. Rattlingourd [67] studied that the insect grow inside the kernel and affected the grain directly and made the convenient reproduction medium to reproduce their F1 generation. R. T. Arbogast and M. A. Mullen [75] reported that the life cycle of this insect varies with abiotic factors such as temperature and relative humidity. M. Evans [55] reported that plentiful statistics were available due to the agricultural research going on in different agricultural departments of various universities that could provide a basis for analytical study. Mathematical modeling gave an insight that what might happen if large quantities of grains would be considered as generally the experiments were performed with small amounts only and hence the data could be arranged in a systematic order through mathematical models so as to make it more useful for agricultural production. Every year due to high humidity harvested and processed paddy retains high moisture content which caused considerable losses in grains. The most harmful insect that caused intense damage to storage grains is the Angoumois grain moth also called *Sitotroga Cerealella*.

The different factors that affect storage grains are biotic and abiotic where abiotic include the grain moisture, humidity temperature, light and type of storage structure and biotic include insects, etc. A Mathematical model was established by using second order differential equation by assuming that the rate of change of % in germination loss with respect to grain infestation was proportional to germination loss. F. Jian et al. [32] reported that analysis showed that the main factor affecting egg incubation period, larval-pupal development time and adult survivorship were the temperature and the optimum conditions required for its growth and survivorship were 30 C and 70-85 % RH. A. Kumar et al. [12] studied the effect of *Sitotroga Cerealella* infestation on rice grain quality and glycemic index of stored paddy grains and it was found that the glycemic index, glycemic load, total carbohydrate, amylose content and resistant starch were affected to a great extent. M. Muthukumar and K. Ragumoorthi [61] reported that seed quality reduced in all maize hybrids due to the attack of *Sitotroga cerealella* infestation. F. Jian et al. [25] reported that a diffusion equation could be used to model the population redistribution of adult *Cryptolestes ferrugineus* in stored grain and to solve the transport equations finite difference method could be used. T. Akter et al. [84] discussed the biology of the *Sitotroga Cerealella* and reported that the length of male and female was 11.2 ± 0.09 and 12.07 ± 0.06 mm respectively. According to A. R. Verdugz and M. Ackermann [6] concluded that long-term stability of positive pairwise interactions got destabilized due to rapid evolution.

### 4.2 MATERIAL AND METHODS

The experiments were conducted at N.D.U.A.T., crop research station, Faizabad U.P. Twenty-five grams of rice seeds were selected and each of 10 selected varieties, comprising of 5- grain types were kept in 100 × 25 mm size pre sterlised glass specimen tubes. The tubes were placed inside the desiccator with some adult moths and the complete experiment was performed under different environmental conditions for different periods. Based on the information collected from the experiments, a Mathematical Model was developed which was based on two varieties *viz*. % grain infestation and % loss in germination. For 3, 6 and 9 months of duration losses in grains weight and germination of rice varieties infested by *Sitotroga Cerealella* were observed.

| **% Grain infestation** | | | **% Loss in germination** | | |
|---|---|---|---|---|---|
| **3 months** | **6 months** | **9 months** | **3 months** | **6 months** | **9 months** |
| 11.5 | 14.9 | 21.4 | 15.2 | 19.4 | 27.3 |
| 4.6 | 7.3 | 7.3 | 9.9 | 18.5 | 21.2 |
| 13.7 | 16.1 | 16.1 | 23.3 | 26.5 | 31.0 |
| 11.5 | 12.1 | 15.1 | 18.9 | 22.7 | 24.5 |
| 3.3 | 5.4 | 9.8 | 8.0 | 10.5 | 16.5 |
| 6.5 | 7.3 | 7.9 | 8.6 | 13.9 | 16.9 |
| 9.9 | 10.5 | 10.0 | 11.8 | 14.2 | 18.5 |
| 10.5 | 10.5 | 13.2 | 15.3 | 17.2 | 10.9 |
| 10.2 | 14.4 | 17.9 | 17.3 | 19.9 | 25.0 |
| 15.5 | 17.7 | 23.5 | 21.4 | 26.2 | 33.0 |

**Table-4.1** Observation of % grain infestation and % loss in germination for 3, 6 and 9 months

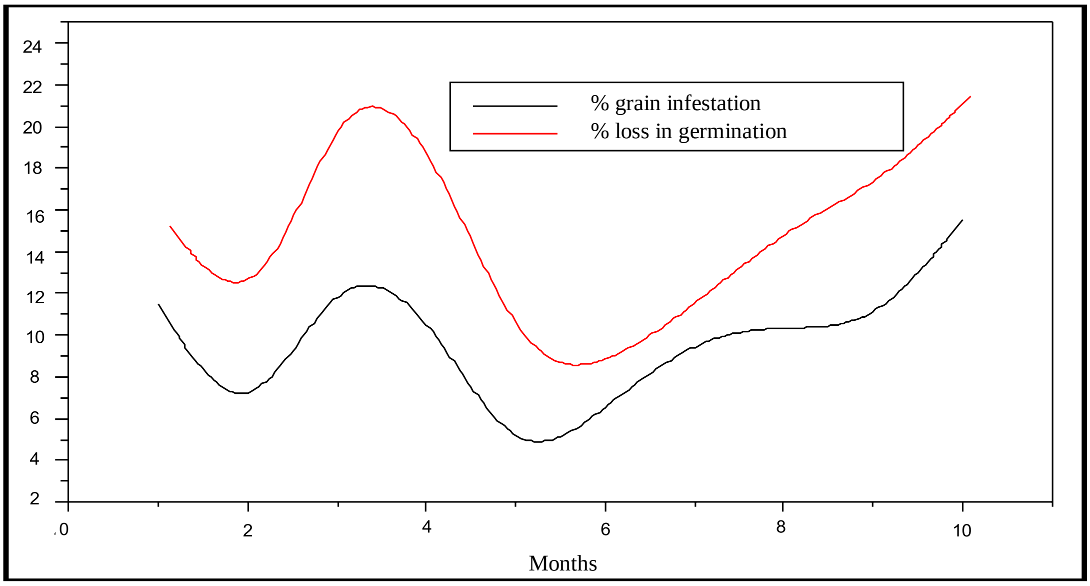


**Figure 4.2-** Showing % of 3 Months germination loss with respect to % grain infestation

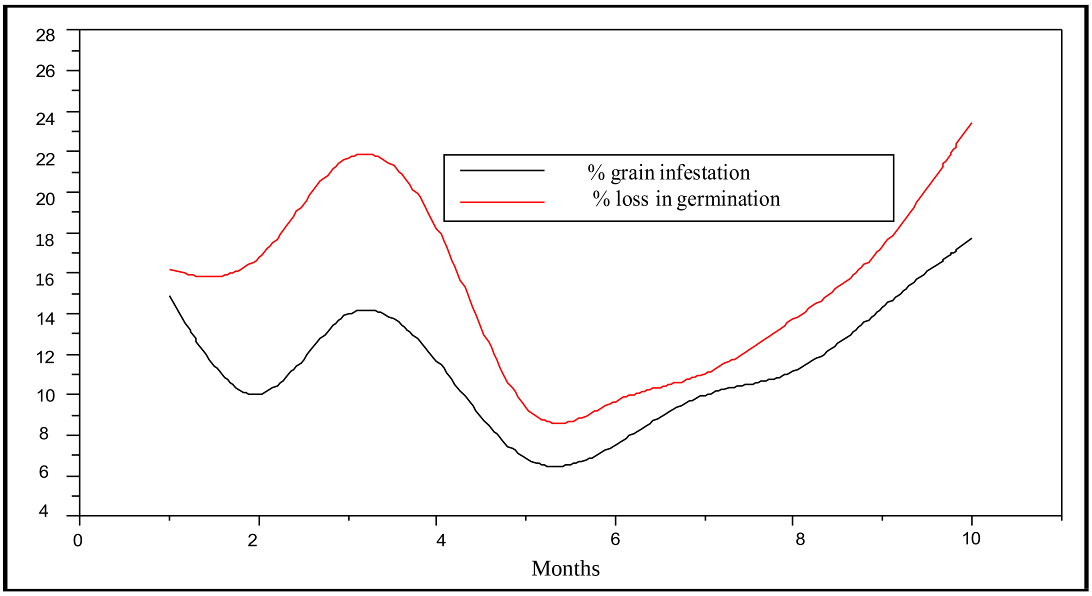


**Figure 4.3-** Showing % of 6 Months germination loss with respect to % grain infestation

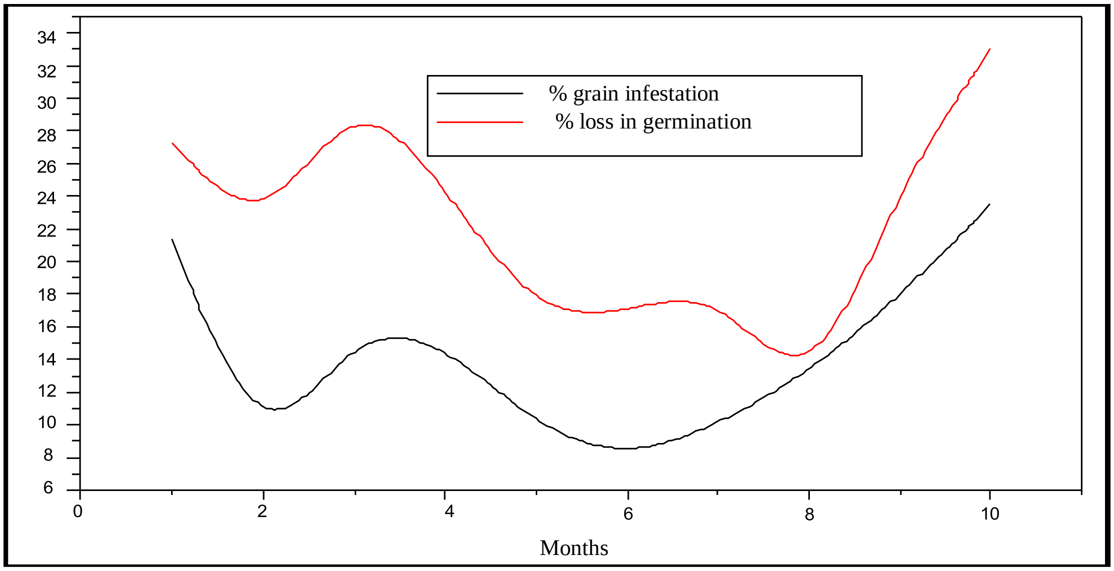


**Figure 4.4-** Showing % of germination loss with respect to % grain infestation 9 Months

The findings showed that the percentage loss in germination value was acting oscillatorily in relation to the percentage grain infection (Kernel). Thus, it was hypothesised that a law governs the percentage loss in germination value in relation to grain infection, and that the solution displayed oscillatory behaviour. We first idealised the data collected in experiment to the extent that this idealized data is included in most of the original data and some averaged data before trying any law to mould this situation. The reasons of idealization the data were as follows:

1) To give a curve of frequency, the curve in Figure was idealized which showed the nature of oscillation.

2) $X_i$ were averaged at averaged kernel (grain infestation) to obtain observation % loss in germination for further use.

| Average experimental data | | Idealized average data | |
|---|---|---|---|
| **Kernel Infestation** | **Germination % loss** | **Kernel Infestation** | **Germination % loss** |
| 15.8 | 20.5 | 16 | 21 |
| 7.3 | 22.1 | 7 | 10 |
| 17.1 | 26.8 | 17 | 27 |
| 18.2 | 24.1 | 19 | 24 |
| 6.2 | 22.0 | 6 | 22 |
| 7.1 | 11.6 | 7 | 12 |
| 10.2 | 14.7 | 10 | 15 |
| 11.4 | 14.4 | 11 | 14 |
| 14.2 | 20.6 | 14 | 21 |
| 18.8 | 26.8 | 19 | 27 |

**Table 4.2 -** Average experimental data and Idealized average data for kernel Infestation and Germination percent loss

One data of germination losses $X_i$ at grain infestation $Y_i$ had been omitted. The main reason behind this was that their behavior was different from the other data of the general trends, might be some other variables which were presented at the time of recording the data were not present here. According to the follow up of the idealized data and corresponding curve other observations of the table were as the same given in table 2.

## 4.3 Results and Discussion

Here the rate of % germination loss be denoted by G and grain infestation by $I$ and it was postulated that the rate of % germination loss with respect to grain infestation was proportional to G

$$\frac{d^2G}{dI^2} \propto G$$

$$\frac{d^2G}{dI^2} = -\omega^2 G \qquad (1)$$

here $\omega^2$ was constant of proportionality

Solution of equation (1) was given by

$$G = A\ Cos\omega I + Bsin\omega I \qquad (2)$$

where $A\ and\ B$ were arbitrary constants

The solution of equation (1) was given by an equivalent expression

$$G = C_1 \sin(\omega I + \lambda) \qquad (3)$$

where $\sin(\omega I + \lambda) = sin\omega I\ cos\lambda + cos\omega I\ sin\lambda \qquad (4)$

Using equation (4) in equation (3) we had obtained

$$G = C_1\ sin\omega I\ cos\lambda + C_1\ cos\omega I\ sin\lambda \qquad (5)$$

On Comparing (2) and (5)

We have got

$$A = C_1 sin\lambda \qquad (6)$$

$$B = C_1\ cos\ \lambda \qquad (7)$$

After squaring and adding above

We had got $A^2 + B^2 = {C_1}^2$ and $\lambda = tan^{-1}\frac{A}{B}$

where $C_1$ was called amplitude of oscillation and the oscillation was $(\omega I + \lambda)$.

Phase angle was $\lambda$, we started from the maximum point and took the angle $\lambda = \frac{\pi}{2}$

$G = C_1 cos\omega I$ (8)

$\omega$ was called the circular frequency and equations (6) and (7) resulted in

$A = C_1, B = 0$

T was the oscillation's period, or the separation between the two points, and ω was another way to express the symbol.

$$\omega = \frac{2\pi}{T} \quad (9)$$

By making use of equations (8) and (9), we got

$$G = C_1 cos\left(\frac{2\pi}{T}\right) I \quad (10)$$

At $\lambda = \frac{\pi}{2}$

We claimed that the data given in table was represented by the equation (10) in the model and to justify this particular claim we proceeded to test the model.

The displacement curve in equation (10) was depicted with respect to a line of symmetry, which we referred to as the "line of best fit" of the data table. As a result, after fitting the data to the line y = mx + c, we obtained the equation y = 0.9 x +11.5.

The inclination of line obtained was 0.90 which gave wonderful effect *i.e.* germination value of grains was directly dependent upon % grain infestation.

Let, $p = x - 10,\ s = y - 20$

Using Idealized data from Table 2 of Kernel infestation (x) and Germination % loss(y) to find different values of p and s, we obtained

$$\sum p = 16\,, \sum s = 19\,, \sum p^2 = 209, \sum ps = 198$$

$$x = 10 + \frac{16}{9} = 11.77 \approx 11.8$$

$$y = 20 + \frac{19}{9} = 22.2$$

$$b_{yx} = \frac{(9*198) - 16*19}{(9*209) - 16^2}$$

$$b_{yx} = 0.90$$

Regression line was therefore provided by

$$y - 22.2 = 0.90\,(x - 11.77)$$

$$y = 0.9x + 11.507$$

## 4.4 Conclusion

From this model, we deduced that *S. cerealella* infestation caused sufficient damage to affect significant changes in germination when varied storage periods were taken. Insect infestation rose over time as storage time increased, and germination reduced over time in a trend that was more or less identical. The amount of insects within the store closely related to the infestation rate. Additionally, the environment, specifically the ecosystem, affected how many insects there were. This mathematical model focused on the part that insect infestation during storage played in reducing germination, which in and of itself demonstrated how effective it was.

# CHAPTER-5

## ANALYSIS OF GROWTH IN POPULATION THROUGH NONLINEAR STOCHASTIC PROCESS

### 5.1 INTRODUCTION

By using related Fokker-Plank equations for the probability densities that were time-dependent along with nonlinear stochastic differential equations for growth, the probability density evolution of an insect population was studied. It was clear that the variance behaviour was dependent on the beginning conditions, however in the case of the mean, the impact of the early conditions vanished quickly. As the growth of the insects continued, we noticed how the mean and variance responded differently. The variation of the process was discovered to have the potential to monotonically rise to a level above the steady state variance before falling back to the steady state variance.

Biologists used a differential equation to describe the growth which was given by

$$dx = (a - bx^{n-1})xdt,\ n \geq 2 \qquad (1)$$

where '$x$' was an insect population or size and '$a$' was the growth parameter whereas second term in (1) represented restriction in growth which was caused due to the reasons like crowding and competition for resources. Another logistic equation was the Gompertz equation which puts a limit on the size

$$dx = (a - blnx)\, xdt \qquad (2)$$

The effect of random fluctuations on the growth of the population was ignored in these deterministic equations by making an assumption that the random fluctuations were independently distributed about the growth path and that growth law remains unaffected from these fluctuations. But in this chapter, these random fluctuations were taken into consideration which was an intrinsic part of the growth process. Equations were formulated which included random fluctuations. The above deterministic equations for the growth were converted into the stochastic form. The main objective was to correlate the mean and the variance of the process as the growth proceeds.

## 5.2 MATHEMATICAL MODEL

The Incorporation of stochastic components into deterministic models could be done by different ways. It was assumed that the random fluctuations were faster than $\mu = \frac{1}{a}$ which specified the time scale of macroscopic variables evolution in the process under consideration. Rapid changes in the environment and other fluctuations affected the system through external parameters.

So the parameter '$a$' was taken as a random variable in equations (1) and (2).

$$a_t = a + \sigma f_t \tag{3}$$

where $a$ was the mean, $f_t$ signified Gaussian white noise and $\sigma$ was the noise intensity. For the random process $X_t$, equation (1) and (2) were replaced by the stochastic differential equation.

$$dX_t = \left(a - bX_t{}^{n-1}\right)X_t dt + \sigma X_t dW_t \tag{4}$$

$$dX_t = (aX_t)\ln\frac{x*}{X_t}dt + \sigma X_t dW_t \tag{5}$$

Here $W_t$ represented the Wiener process. To make it easier $b = a \ln x^*$ had been put in (5).These equations specified the change in the size of the population. For the above SDEs, the choice between the Ito and Stratonovich interpretations depended on the nature of the process which was being installed. According to the theory of K. Alagusundaram et al. [48], there were no qualitative differences in any of the results between the above two approaches for the case which were considered here. By using the theory of P. Holgate [69] and C. Wang [18], to obtain the definite results, Ito stochastic calculus was used.

## 5.3 RESULTS

Each integral of the stochastic differential equation described the growth path or in other words, each solution of an SDE described one realization. This SDE generated an ensemble of realizations that was described by a probability density $P(x,t)$which was transitional in nature, which satisfied the Fokker- Plank Equation.

$$\frac{\partial \mathrm{P}}{\partial \mathrm{t}} = -\frac{\partial}{\partial \mathrm{x}} \mathrm{g(x)P}(x,t) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\sigma^2 x^2 \mathrm{P}(x,t) \tag{6}$$

where $\mathrm{g(x)} = (a - \mathrm{bx}^{\mathrm{n}-1})x$ or $axln\ \frac{\mathrm{x}*}{x}$, respectively. So we had easily evaluated the steady-state probability density for this, which was denoted by$P_s\ (x)$.

$$P_s\ (x) = N_s\, x^{\left(\frac{2a}{\sigma^2}-1\right)} e^{\frac{-2bx^{n-1}}{\sigma^2(n-1)}} \tag{7}$$

and value of $P_s(x)$ for the Gompertz model

$$P_s\ (x) = N_s\, x^{\left(\frac{aln\mathrm{x}*}{\sigma^2}-1\right)} e^{\frac{-aln^2\mathrm{x}}{\sigma^2}} \tag{8}$$

Where $N_s$ was a Normalizing constant and probability density defined in equation (8) was normalized or in other words, steady-state probability existed and had a finite value at any positive value of the parameters. For $a > \frac{\sigma^2}{2}$ the probability density defined in equation (7) was normalized. For the stochastic process, the condition $a > \frac{\sigma^2}{2}$ coincides in the Ito interpretation with the natural boundary condition $x = 0$. And the probability density (7) was divergent when $\frac{\sigma^2}{2} < a < \sigma^2$ at $x = 0$.Hence taking the condition $a > \sigma^2$ for steady state probability density for both (7) and (8).The evolution of the probability density through time was described by the solution of the Fokker-Plank equation(5). All models discussed in equation (1) showed the same quality of behavior allowing the substitution of $y = x^{-n+1.}$ which made the SDEs linear. The logistic SDE changed into the linear SDE when $n = 2$ with the substitution

$$dY_t = [(\sigma^2 - a)\mathrm{Y} + \mathrm{b})Y + b)]dt + \sigma Y_t dW_t \tag{9}$$

Its solution was given by

$$Y_t = \left\{e^{\left(\frac{\sigma^2}{2}-a\right)} + \sigma\mathrm{Wt}\right\}\left\{ Y_0 + \mathrm{b}\int_0^t e^{\ \left\{-\left(\frac{\sigma^2}{2}-a\ \right)s-\sigma\mathrm{Wt}\ \right\}} \mathrm{ds}\right\} ds \tag{10}$$

For Gaussian fluctuations to undergo a linear transformation, we could see that the solution was in the exponential function for the Wiener process. Thus linear SDE solution was not a Gaussian process. The time-dependent solution of the Fokker-Plank Equation (6) couldn’t be determined by using the linearization procedure but could be found numerically and the results of this numerical solution were presented below. For the Gompertz model, the substitution of $y = ln\ x$ in SDE converted the nonlinear process into a process called Ornstein-Uhlenbeck and hence for the Gompertz, stochastic process could be used according to the theory of A. Pleasant et al. [2]. When $\mathrm{P(x, 0)} = \tau(\mathrm{x} - x_0)$ i.e. if initial size of population was known, the solution obtained as

$$\mathrm{P(x,\ t}|x_0) = \frac{1}{\sigma\sqrt{\frac{\pi(1-\mathrm{e}^{-2a\mathrm{t}})}{a}}} e^{\frac{-a(\ln \mathrm{x} - \ln \mathrm{x}* + \frac{\sigma^2}{2a} - \exp(-a\mathrm{t}\ln x_0)^2}{\sigma^2(1-\mathrm{e}^{-2a\mathrm{t}})}} \quad (11)$$

From this time-dependent probability density, the mean and variance both increased monotonically towards the steady–state mean and variance and dependence were on the parameters of the SDE only for these statistics. The analytical solution of Fokker-Plank equations had a complex form of an infinite series over orthogonal polynomials, if only the probability of the initial population sizes range were known.

From the time-dependent probability density $P(x,t)$, it could be very helpful in understanding the behavior of first two moments. By using the Fokker-Planck equation of SDE, the ordinary differential equations for the mean $m_t$ and variance v could be derived.

$$\frac{\partial \mathrm{P}}{\partial \mathrm{t}} = -\frac{\partial}{\partial \mathrm{x}}\,\mathrm{g(x)P}(x,t) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\sigma^2 x^2 \mathrm{P}(x,t) \quad \text{(Fokker Plank's Equation)}$$

$\mathrm{g(x)}$ was the drift coefficient and $\sigma^2 x^2$ was the diffusion coefficient.

**Derivation of the mean $m_t$**

For $m_t$, the drift coefficient was g(x) = $am_t - \mathrm{b}\,({m_t}^2 + \mathrm{v})$

Substituting in Fokker's Plank Equation

$$\frac{\partial \mathrm{P}}{\partial \mathrm{t}} = -\frac{\partial}{\partial m_t}\{(am_t - \mathrm{b}\,(m_t^2 + \mathrm{v}))\mathrm{P}(m_t, \mathrm{v}, t)\} + \frac{1}{2}\frac{\partial^2}{\partial {m_t}^2}\{\sigma^2\,({m_t}^2 + \mathrm{v})\mathrm{P}(m_t, \mathrm{v}, t)\}$$

Integrate the entire equation with respect to v over its entire range.

$$\int_{-\infty}^{\infty} v\,\frac{\partial \mathrm{P}}{\partial \mathrm{t}} = \int_{-\infty}^{\infty}\left[-\frac{\partial}{\partial m_t}\{(am_t - \mathrm{b}\,(m_t^2 + \mathrm{v}))\mathrm{P}(m_t, \mathrm{v}, t)\} + \frac{1}{2}\frac{\partial^2}{\partial {m_t}^2}\{\sigma^2\,(m_t^2 + \mathrm{v})\mathrm{P}(m_t, \mathrm{v}, t)\}\right]\mathrm{dv}$$

Repeated a similar process for the variance v. Substituted the drift coefficient g(x) for v $(2a\mathrm{v} - 4\mathrm{b}m_t\mathrm{v} + \sigma^2 (m_t^2 + \mathrm{v}))$ into the Fokker-Planck's equation, integrated it multiplied by v over the entire range to obtain the variance equation.

$$\int_{-\infty}^{\infty} v\frac{\partial \mathrm{P}}{\partial \mathrm{t}} = \int_{-\infty}^{\infty}\left[-\mathrm{v}\frac{\partial}{\partial m_t}\{(am_t - \mathrm{b}(m_t^2 + \mathrm{v}))\mathrm{P}(m_t, \mathrm{v}, t)\} + \frac{1}{2}v\frac{\partial^2}{\partial m_t{}^2}\{\sigma^2 (m_t^2 + \mathrm{v})\mathrm{P}(m_t, \mathrm{v}, t)\}\right]\mathrm{dv}$$

After Integrating by parts and simplifying, we got the ordinary differential equations for the mean $m_t$ and variance v

$$\frac{\mathrm{d}m_t}{\mathrm{dt}} = am_t - \mathrm{b}(m_t{}^2 + \mathrm{v})$$

$$\frac{\mathrm{dv}}{\mathrm{dt}} = 2a\mathrm{v} - 4\mathrm{b}m_t\mathrm{v} + \sigma^2 (m_t^2 + \mathrm{v}) \qquad (12)$$

To obtain equations for the logistic stochastic process, we used Fokker plank's equation and identified drift coefficient and diffusion coefficient as

$\mathrm{g(v)} = 2\mathrm{av}\ln\left(\frac{y*}{m_t}\right) - 2 + \sigma^2 (m_t{}^2 + \mathrm{v})$ and $\mathrm{b(v)} = \sigma^2 (m_t^2 + \mathrm{v})$ and substituted in Fokker Plank's equation and integrated

$$\int_{-\infty}^{\infty}\frac{\partial \mathrm{P}}{\partial \mathrm{t}} = \int_{-\infty}^{\infty}\left[-\frac{\partial}{\partial v}\left\{\left(2\mathrm{av}\ln\left(\frac{y*}{m_t}\right) - 2 + \sigma^2 (m_t^2 + \mathrm{v})\right)\mathrm{P}(m_t, \mathrm{v}, t)\right\} + \frac{1}{2}\frac{\partial^2}{\partial v^2}\{\sigma^2 (m_t^2 + \mathrm{v})\,\mathrm{P}(m_t, \mathrm{v}, t)\}\right]\mathrm{dv}$$

$$= -\left(2\mathrm{av}\ln\left(\frac{y*}{m_t}\right) - 2 + \sigma^2 (m_t^2 + \mathrm{v})\right)\mathrm{P}(m_t, \mathrm{v}, t)\Bigg| + \int_{-\infty}^{\infty}\left[\left(2\mathrm{a}\ln\left(\frac{y*}{m_t}\right) - \sigma^2\right)P + \sigma^2\mathrm{v}\frac{\partial \mathrm{P}}{\partial \mathrm{v}}\right]dv$$

Boundary terms vanished often, if the probability density function was well-behaved, and we were left with

$$\int_{-\infty}^{\infty}\left[\left(2\mathrm{a}\ln\left(\frac{y*}{m_t}\right)-\sigma^2\right)P+\quad\sigma^2\mathrm{v}\frac{\partial\mathrm{P}}{\partial\mathrm{v}}\right]dv$$

$$Simplified\ the\ remaing\ Integral$$

$$\int_{-\infty}^{\infty}\left[\left(2\mathrm{a}\ln\left(\frac{y*}{m_t}\right)-\sigma^2\right)Pdv+\int_{-\infty}^{\infty}\sigma^2\mathrm{v}\frac{\partial\mathrm{P}}{\partial\mathrm{v}}\right]dv$$

$$\frac{\mathrm{dv}}{\mathrm{dt}}=2\mathrm{av}\ln\left(\frac{y*}{m_t}\right)-2+\sigma^2\left({m_t}^2+\mathrm{v}\right) \tag{13}$$

$$\frac{\mathrm{d}m_t}{\mathrm{dt}}=am_t\ln\left(\frac{y*}{m_t}\right)\mathrm{t}-\frac{av}{2m_t}$$

Different approximation steps would lead to same equations, when the instantaneous fluctuation was proportional to the state $X_t$.

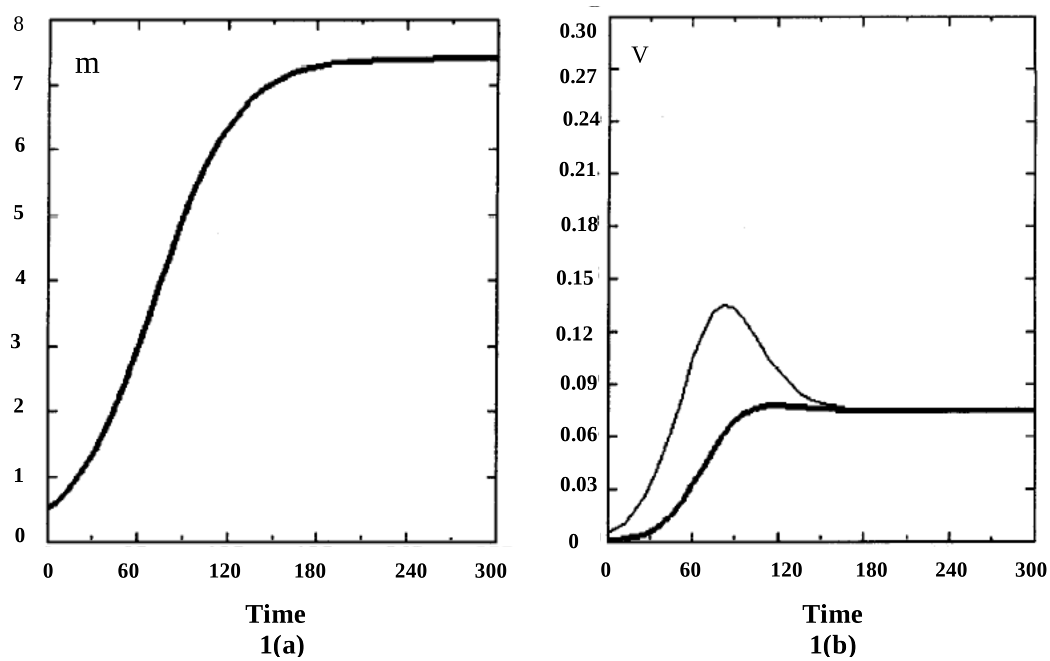


**Figure 5.1-** For the logistic stochastic process, the numerical solution of equation (12) for both the mean and the variance. At $a$ =0.04 , b=0.005 ,$\sigma$=0.01.Bold curves and thin curves correlate with initial conditions m(0)=0.5 , v(0)=0.0001 and m(0)=0.5, v(0)=0.005.Y-axis represents mean value in 1(a) and 2(a) and variance in 1(b) and 2(b) where X-axis represents Time.

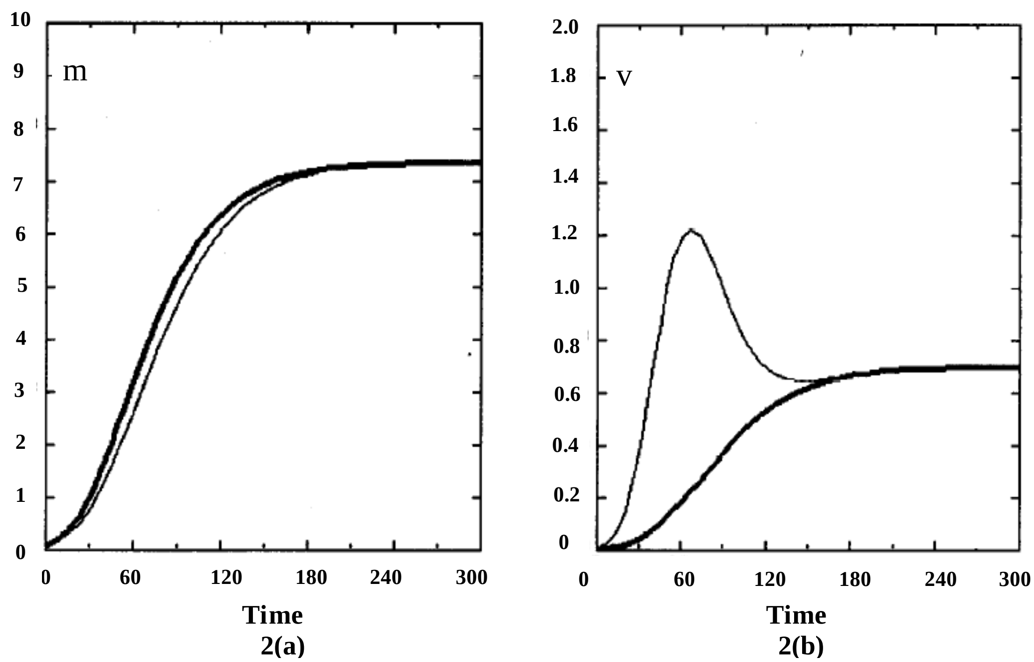


**Figure 5.2-** For the Gompertz Stochastic Process, the numerical solution of equation (13) for both the mean and the variance. At $a$ =0.03 , $x^*$=7.45 ,$\sigma$=0.03. Bold curves and thin curves correlate with initial conditions m (0) =0.07, v (0) =0.0007 and m (0) =0.7, v (0) =0.007 respectively.

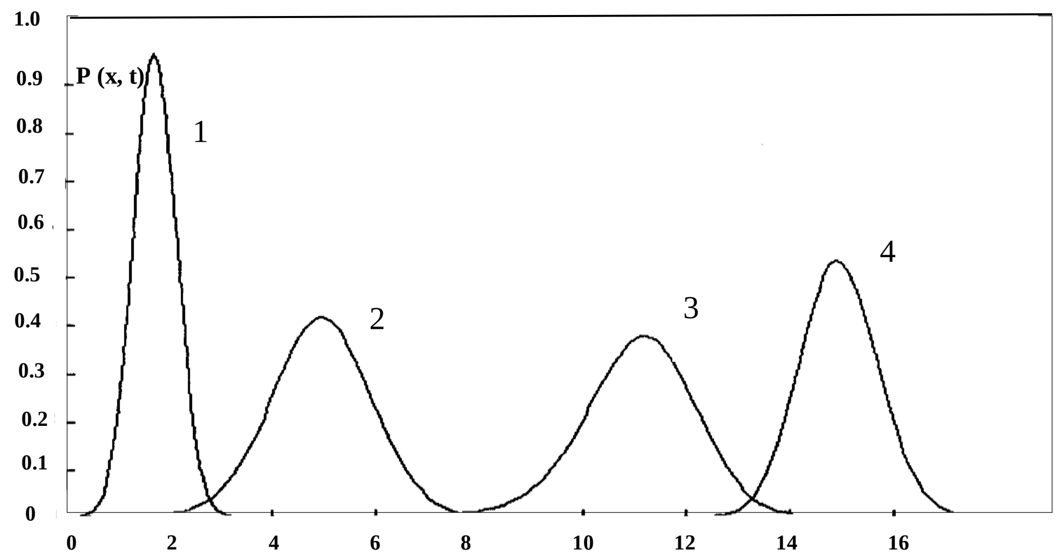


**Figure 5.3 -** For the logistic stochastic process Fokker-Planck equation solution was represented numerically and hence plotted at different times ($t_1, t_2, t_3$) by using parameters $a$= 0.045; b= 0.003; $\sigma$ = 0.005.Curves 1-3 representing the probability density and curve 4 represents the steady state.

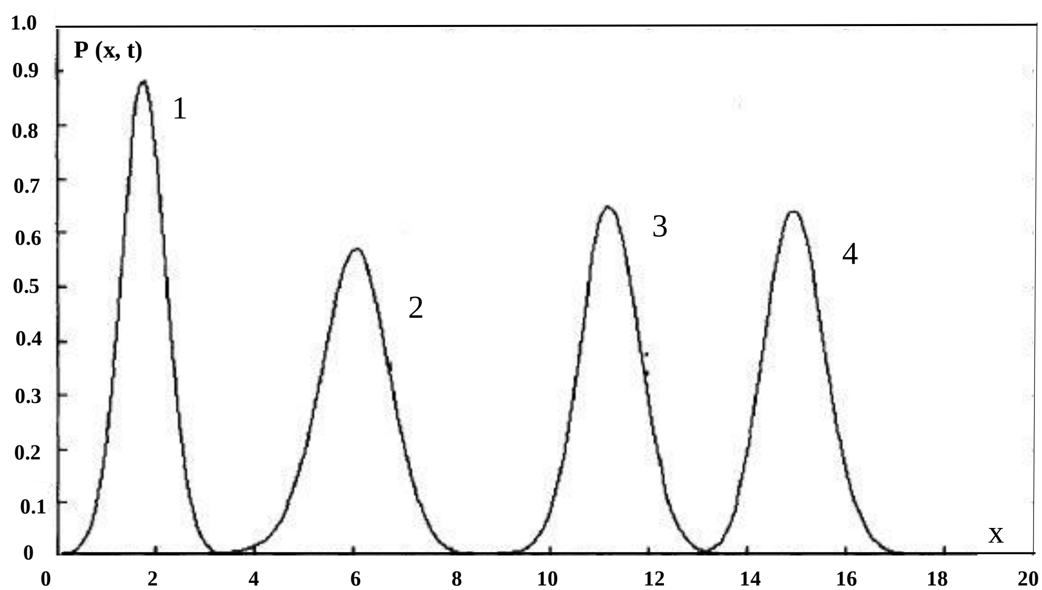


**Figure 5.4-** For the Gompertz process, Fokker-Planck equation solution was represented numerically and hence plotted at different times ($t_1, t_2, t_3$) by using parameters $a$ = 0.03; $x^*$ = 1.5; $\sigma$ =0.01. Curves 1-3 representing the probability density and curve 4 represent the steady state.

The solutions were shown in Figures (5.1) and (5.2) for the set of equations (12) and (13). The evolution of mean and variance to a steady state took place for each case and the probability densities (7) or (8) could be used to obtain the values for the steady-state mean and variance. We could see the difference in the behavior of the mean and the variance while evolving to a steady-state. Initially, variance increased monotonically above its steady-state, attained maximum value at a time when the growth rate was near its maximum and then after some time decreased to a steady state and for the case of mean, it evolved monotonically towards the steady state.

The variance increased monotonically to the steady-state variance initially, when the ratio of the standard deviation to the mean of the initial population size was small. Before shrinking back to the steady-state variance, the variance increased to a point greater than the steady-state variance, when the ratio was initially close to or greater than this ratio at the steady-state. Approximations that were made in deriving (12) or (13) were the main reason for this behavior. Direct numerical solutions of the Fokker-Planck equation for both the process Gompertz and logistic were obtained to solve this which were shown graphically in Figures 5.3 and 5.4. We could see that the Gompertz stochastic process showed the similar behavior which suggested that behavior of the variance was independent on the approximations. The probability density evolution for the increasing time was shown in the graph. The steady-state distribution was shown by the last curve. For the stochastic growth processes, the non-monotonic variations of the variance were clearly described by non-monotonic changes in the amplitudes of the probability density curves. For example, at $t = t_1$ the steady state variance was greater than the process variance and was found to have less value at $t = t_2$.

The description for this different behavior was that as the growth proceeded the mean and variance responded differently. When the ratio of the steady-state standard variation to the steady-state mean was less than the ratio of the initial standard deviation to the initial mean then this scale difference led to the behavior observed. The behavior of the mean was that it was monotonically increasing but for the case of the variance, at some time the variance rose above the steady- state variance before the process reached the steady-state and then as the growth proceeded and shrank back to zero, the variance was fallen back to zero.

## 5.4 CONCLUSION

Biological growth could be described by using the growth equations. Results reflected the complex behavior of the mean and variance when the involvement of the growth processes took place. The behavior would exist in which the variance would rise above the steady-state variance before shrinking back to the steady-state variance. The analysis of this behavior gave the idea of danger for example if the aim was to find the danger to food safety or the quality of water through the growth of micro-organisms. The evolution of mean and the variance of growth processes which were described by using linear differential equations with a random growth rate to their steady-state values took place.

# CHAPTER-6

## TO PREDICT THE POPULATION DYNAMICS OF RUSTY GRAIN BEETLE IN STORED BULK WHEAT BY USING MATHEMATICAL MODELING

**Figure 6.1-** Damage caused by *Cryptolestes ferrugineus*

**Source-** *Cryptolestes ferrugineus*. www.Shutterstock.com. https://www.shutterstock.com/image-photo/rusty-grain-beetle-cryptolestes-ferrugineus-beetles-1236417859. Accessed November 20,2021.

"The lined flat bark beetle species *Cryptolestes ferrugineus* is a native of Europe. The rusty grain beetle is the popular name given to it, and it currently has a global range. (Wikipedia)

## 6.1 INTRODUCTION

Population dynamics of *Cryptolestes ferrugineus* (Stephens) (Coleoptera: Laemophloeidae) revealed that population variation occurred in both small and large patches. The temperature and quantity of insects presented previously were the two main reasons that influenced the population density according to analysis of key factor. The number of insects grew as the total number of degree -days increased. Since consistent temperatures were required for the model related to degree- days, it was impossible to estimate bug populations when temperatures were fluctuating. Based on the results of unstructured population models, an examination of significant variables, and the degree day model, this model provided an explanation for the insect populations under variable.

The population density of *Cryptolestes ferruginous* (rusty grain beetle) were examined under various grain temperature treatments and in containers of varying sizes (small patches, medium patches, and large patches). It was discovered that insect numbers depended more on vast patches than on small patches, as opposed to the previous situation as concluded by F. Jian et al. [30]. If overcrowding was a factor in the decline of populations of insects, then crowding would only had a minimal role in the insect population in grain bins that were being stored because the insects would be unable to reach their maximal density in time inside of wide areas under changing temperatures. There might be limited use for various mathematical models created using the data gathered for the little jars. As a result, it was necessary to develop a mathematical model based on the big patch size.

According to M. Kot [54], Analysing Key variables and models for unstructured population, such as logistic and exponential equations, models for discrete-time, harvested models and delay models were a few examples of broad models that represented the trend of population dynamics. The insect population was difficult to characterize and anticipate the environment of grain storage was a complex system that was also affected by the complexity of insect biology. Consequently, determining the insect population's size was essential. Therefore, constructing complicated models that largely responsible for the calculation of time of egg producing and rate, mortality, and the duration of each stage of an insect has become insect population projection study trend, further increasing the implementation of the model was tricky. The grain depot supervisor needed a generic equation that would help him quickly estimate the bug population. To understand the fundamental elements driving the dynamics of the insect population, R. Morris et al. [76] devised the model for the analysis of key factor. Despite the poor forecasting precision of the key factor analysis-based model created to predict insect population dynamics, analysis of this model was being able to quantify each factor's contribution to population density, and the general model could be created by the quantitative contributions described in the key factor analysis.

Many insect life cycles had been effectively predicted using degree-day models. According to J. Moore et al. [40], these models were based on the finding that an insect's population was closely associated with the local temperature. Ectothermic animals such as insects lack an inbuilt method for regulating body temperature. We had concluded that since their body temperatures were similar to those of their surroundings, accumulated heat units rather than calendar time would be a better predictor of their development and multiplication. This was the primary factor, according to L. Pikington and M. Hoddle [50], behind the degree day models' success in predicting growth rate and insect invasion. Although ambient temperatures predominantly affected the borders of a highly insulated media made of large grains, it was not yet known if population trends could be predicted in this situation using the degree-day model. This project's goal was to list all of the crucial elements that significantly affect the demography of populations of *Cryptolestes ferruginous* and to create a basic model using which population dynamics could be predicted.

## 6.2 MODEL DEVELOPMENT

In the Newly created Key Factor-Degree Day model, degree-day model and the Key-factor analysis only the data pertaining to the big patches were employed. The data pertaining to the Temperature-increase and Temperature-decrease was employed to validate the generated Key Factor-Degree Day and the degree –day models.

The impact of temperature, insect volume, and size of patches on population growth, as well as the inherent birth and mortality rates, were characterized using the half of the adult population counted at the various storage times was estimated using unstructured models. To illustrate the overall pattern of population dynamics, the nine unstructured models were chosen. Only half of the adult population was included because the models, which assumed a one-to-one sex ratio, only took into account females.

The following were the models along with their underlying assumptions (the model's title served as its underlying assumption).

Case I: The carrying capacity and the Geometric development:

$$\frac{dN}{dt} = aN\left(1 - \frac{\mathrm{N}}{\mathrm{K}}\right) \tag{1}$$

Case II: Exponential expansion and ongoing reproduction

$$N_t = N_0\, e^{at} \tag{2}$$

Case III: Dimensional expansion and discrete generations

$$N_t = N_0\, \mu^t \tag{3}$$

Case IV: Growth in population and density dependent, immediate feedback of density on overall growth

$$N_t = \frac{\mathrm{K}}{1+\frac{(\mathrm{k}-N_0)}{N_0}e^{-at}} \tag{4}$$

Case V: Population increase and non-instantaneous, density-dependent feedback on aggregate growth

$$\frac{N_{\mathrm{t+1}}}{N_{\mathrm{t}}} = e^{a}\ \frac{(1-N_{\mathrm{t}})}{k} \tag{5}$$

Case VI: Harvest model

$$\frac{dN}{dt} = aN\left(1 - \frac{\mathrm{N}}{\mathrm{K}}\right) - EN \tag{6}$$

Case VII: Growth model with discrete time and density dependence

$$\frac{N_{t+1}}{N_t} = \frac{R_0}{1+\left(\frac{R_0-1}{K}\right)N_t} \quad (7)$$

Case VIII: Density-independent model for the following time period

$$N_{t+1} = R^{\alpha}N_t \quad (8)$$

Case IX: Density-independent for the upcoming two-period model

$$N_{t+2} = R^{\alpha}N_t \quad (9)$$

Here, t denoted the time interval (in days), the succeeding period (t + 1), the succeeding two periods (t + 2), $a$ denoted the internal rate of growth, and denoted the factor of geometric growth where the carrying capacity was denoted by K and $R_0$ represented the net rate of reproduction rate. Here, $N_t$ and $N_0$ each represented a half of the adult population still living at time t and the beginning of the trial, respectively. The models' fitness was assessed using the coefficient of determination ($R^2$).

The coefficient of determination ($R^2$) of the difference in half of the adult numbers between measured and calculated was used to assess the efficiency of the models. The model wouldn't fit to half of the adult population if $R^2 = 0$ within the therapeutic circumstances.

## 6.3 KEY-FACTOR ANALYSIS

Using an approach proposed by R. Morris [76], the main determinants affecting insect population dynamics were quantified. The regression equations were as follows:

$$\mathrm{Log}_{10}^{N_{t+1}} = A\mathrm{Log}_{10}^{N_t} \quad (10)$$

$$\mathrm{Log}_{10}^{N_{t+1}} = A\mathrm{Log}_{10}^{N_t} + \mathrm{Log}_{10}^{F} \quad (11)$$

$$\mathrm{Log}_{10}^{N_{t+1}} = A\mathrm{Log}_{10}^{rN_t} + \mathrm{Log}_{10}^{F} \quad (12)$$

$$\mathrm{Log}_{10}^{N_{t+1}} = A\mathrm{Log}_{10}^{rN_t} + \mathrm{Log}_{10}^{F} + B \quad (13)$$

where A was the line's slope, $\mathrm{Log}_{10}^{F}$ was its intercept, r was $N_t$ survival rate when exposed to a range of temperatures, and B was a minor factor that affected the population of insects. The values of the $R^2$ were evaluated in order to ascertain the fitting's correctness.

## DEGREE DAY MODEL

Using information from F. Jian et al., the total quantity of adults, including both live and deceased ones, live adults, or young born during the storage period related to the total number of degree days were calculated to assess the influence of temperature on the insect population. [2]

Sum of the degree day = $\sum_{t=0}^{N} \; (T_t - 6.3)$ (14)

where N was the storage time's day and $T_t$ was the daily temperature at t days (°C).

According to F. Jian et al. [26], the number 6.3 of the adult *Cryptolestes ferrugineus* denoted the minimum movement temperature (°C). Because insects' movement signaled the beginning of routine biological activities including feeding, energy use, and mating, the lowest movement temperature was chosen as the threshold development temperature.

The association between both the insect's number and the overall number of degree days with consistent temperature at each test site inside the huge areas was discovered using regression. Only information pertaining to large patches was used because there was little effect of size of patch on the dynamics of insect populations inside of big patches.

An equation that suited the data the best was determined to have the highest $R^2$ value. This best-fit equation was applied to the treatments for changing temperatures in order to predict the insect population.

## 6.5 KEY FACTOR—DEGREE DAY MODEL

The Key Factor-Degree Day model was constructed using the results of the key factor analysis and the created degree day model. Assuming that the following factors had the greatest impact on the insect population at time $t + 1$:

1) The insects number at time t;

2) the total number of degree days during the past four weeks; and

3) variations in temperature.

The fundamental motivation behind this assumption was to create a straightforward equation that could capture the broad population dynamics trend for each insect generation inside grain storage bins. As a result, other variables like crowding were ignored. Only the insect count inside the enormous regions connected to the entire data was used during model construction. The main justification for selecting a 4-week cycle was because this insect's life cycle under the investigated conditions lasted roughly 4 weeks. The best fit was found by the formula corresponding to the total number of degree days over a four-week period. As a result, this chapter just summarised the conclusions drawn from the 4-week data.

Various regression equations based on these hypotheses were adapted to the insect numbers (adults alive, deceased, or offspring) under all the investigated constant temperature circumstances. The following procedures were used to create this model: The first stage was to determine the effective equation to describe the population of insects during each given temperature. In phases two and three, the key factor analysis approach was utilized to gradually incorporate the key factors to the tested equations, and the assumption method was used in step three to change the tested equations.

The best fit was determined to be the equation with the largest $R^2$ that predicted the insects number under both Temperature-increase and Temperature-decrease situations.

## 6.5 RESULTS AND DISCUSSION

### 6.6.1 MODELS OF A NON-STRUCTURED POPULATION

Despite all testing settings, none of the nine models assessed could account for half of the adult populations, which might be related to the interaction effects of insect volume, earlier insect number, temperature, and size of the patch. The main factor restricting population increase would not be temperature under conditions of stable temperature and at a temperature of less than 25 °C. It became evident as a result that the size of the patch and the population trends at 25, 30, and 35°C were significantly influenced by the number of insects presented at the time

| Models | Grain Storage (Patch size) | The grain's temperature (C) | | | | | |
|---|---|---|---|---|---|---|---|
| | | 21C | 25C | 30C | 35C | T-increase[a] | T-decrease[b] |
| Case I | Small Size | 0.0 | 0.30 | 0.10 | 0.02 | 0.0 | 0.80 |
| | Medium Size | 0.0 | 0.44 | 0.0 | 0.0 | 0.77 | 0.35 |
| | Large Size | 0.0 | 0.0 | 0.33 | 0.93 | 0.55 | 0.34 |
| Case II | Small Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | Medium Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | Large Size | 0.0 | 0.59 | 0.0 | 0.0 | 0.0 | 0.0 |
| Case III | Small Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | Medium Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | Large Size | 0.0 | 0.59 | 0.0 | 0.0 | 0.0 | 0.0 |
| Case IV | Small Size | 0.0 | 0.0 | 0.0 | 0.28 | 0.0 | 0.0 |
| | Medium Size | 0.0 | 0.0 | 0.0 | 0.37 | 0.93 | 0.0 |
| | Large Size | 0.0 | 0.0 | 0.0 | 0.74 | 0.95 | 0.25 |
| Case V | Small Size | 0.0 | 0.52 | 0.0 | 0.94 | 0.0 | 0.69 |
| | Medium Size | 0.67 | 0.95 | 0.0 | 0.96 | 0.0 | 0.96 |
| | Large Size | 0.71 | 0.0 | 0.0 | 0.88 | 0.0 | 0.98 |
| Case VI | Small Size | 0.24 | 0.16 | 0.08 | 0.03 | 0.28 | 0.47 |
| | Medium Size | 0.53 | 0.12 | 0.01 | 0.04 | 0.48 | 0.26 |
| | Large Size | 0.56 | 0.52 | 0.01 | 0.30 | 0.25 | 0.30 |
| Case VII | Small Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.27 |
| | Medium Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | Large Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Case VIII | Small Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | Medium Size | 0.0 | 0.55 | 0.11 | 0.20 | 0.70 | 0.0 |
| | Large Size | 0.0 | 0.50 | 0.50 | 0.0 | 0.48 | 0.0 |
| Case IX | Small Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | Medium Size | 0.0 | 0.10 | 0.0 | 0.13 | 0.39 | 0.0 |
| | Large Size | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

**Table 6.2*-**At different storage time intervals, value of the $R^2$ for the unstructured population models fitted to 50% of the live *Cryptolestes ferrugineus* individuals that were counted

At 25 and 30 degrees Celsius, no mathematical equation was able to accurately forecast the bug population when coefficient of determination greater than 0.6 was obtained, Although Case I, IV and V experienced an acceptable suit at 35°C. Beginning with the idea that geometrical development depends on population density, all three of these models with density providing either immediate or delayed feedback on overall growth. This study showed that the prior quantity of *Cryptolestes ferrugineus* would only have an effect on the population at optimal temperatures if overcrowding had not been a limitation. According to F. Jian et al. [28], the non-structural population models that were installed on half of the adult population were counted for $R^2$ at various storage time intervals. Each case was shown graphically.

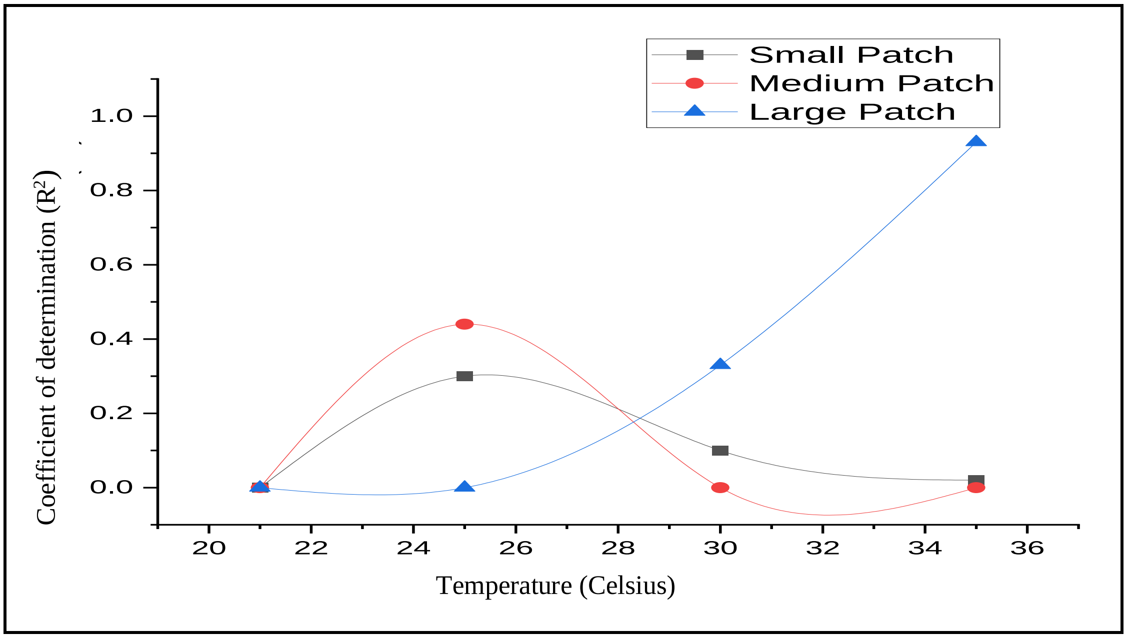


**Figure 6.2**– Showing Unstructured Population Model (CASE 1)

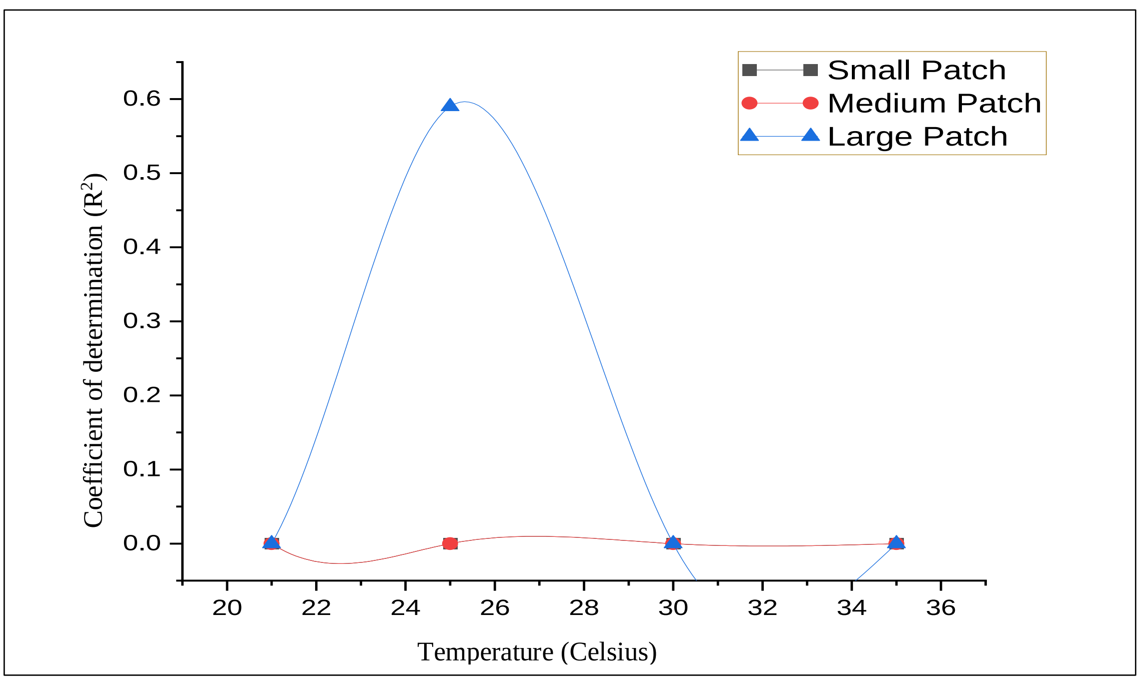


**Figure 6.3** –Showing Unstructured Population Model (CASE 2)

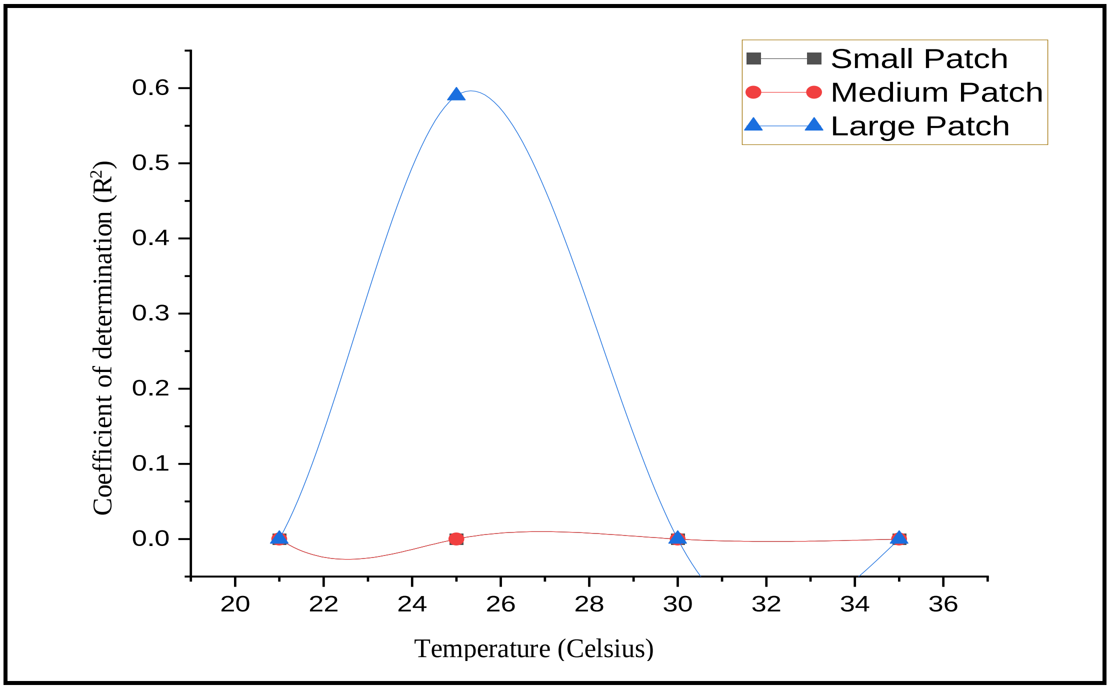


**Figure 6.4** –Showing Unstructured Population Model (CASE 3)

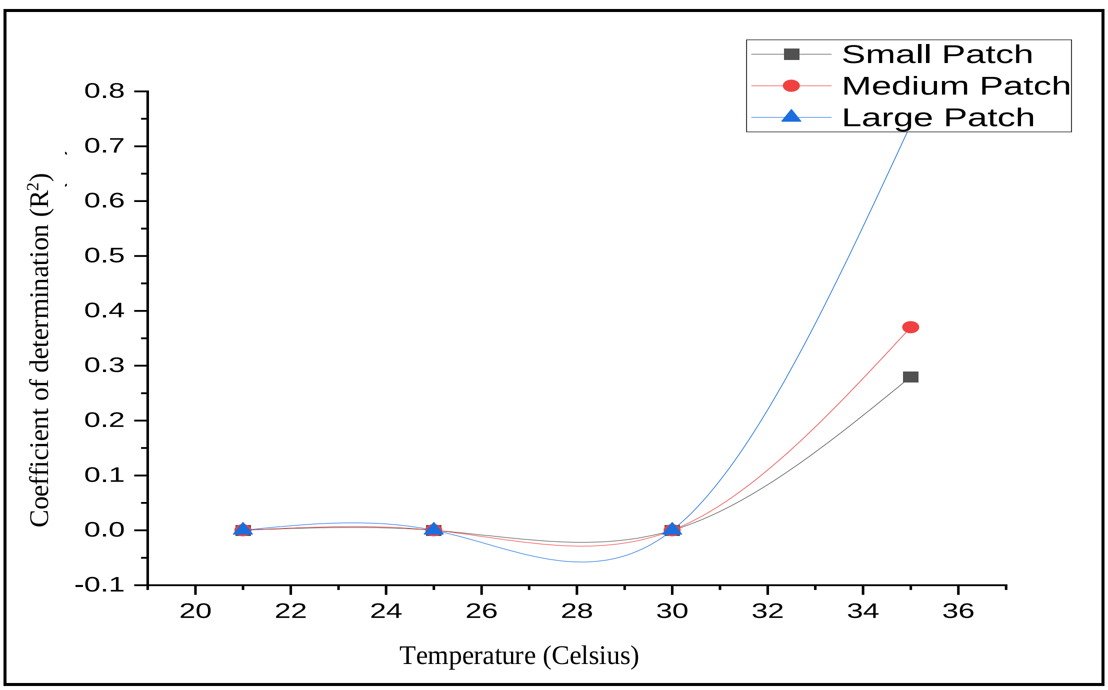


**Figure 6.5** –Showing Unstructured Population Model (CASE 4)

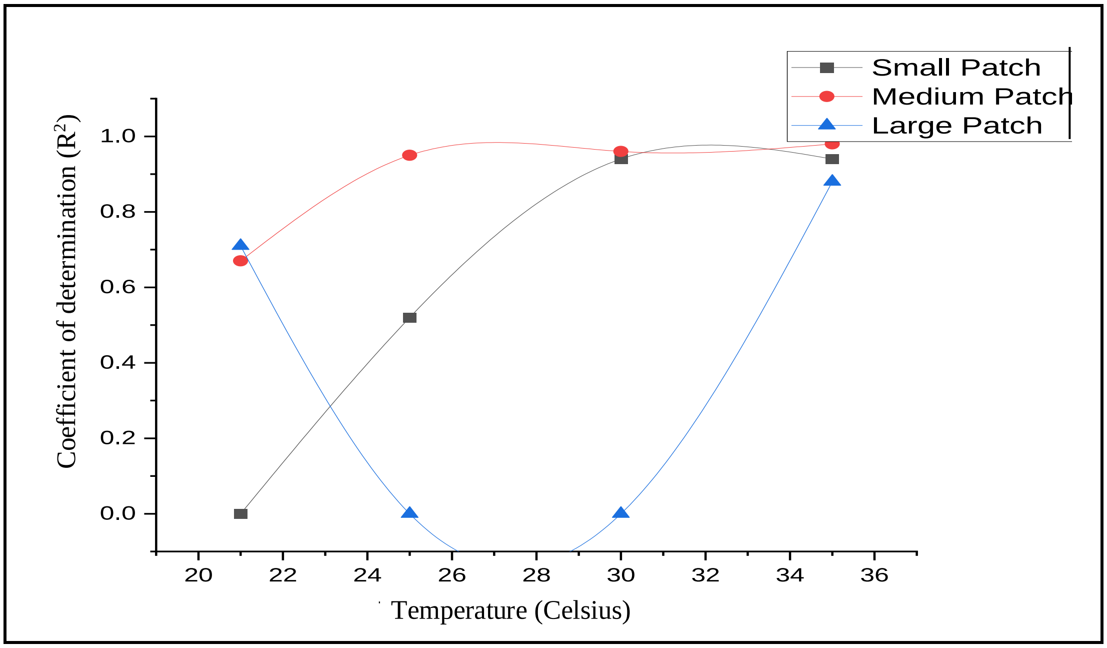


**Figure 6.6** –Showing Unstructured Population Model (CASE 5)

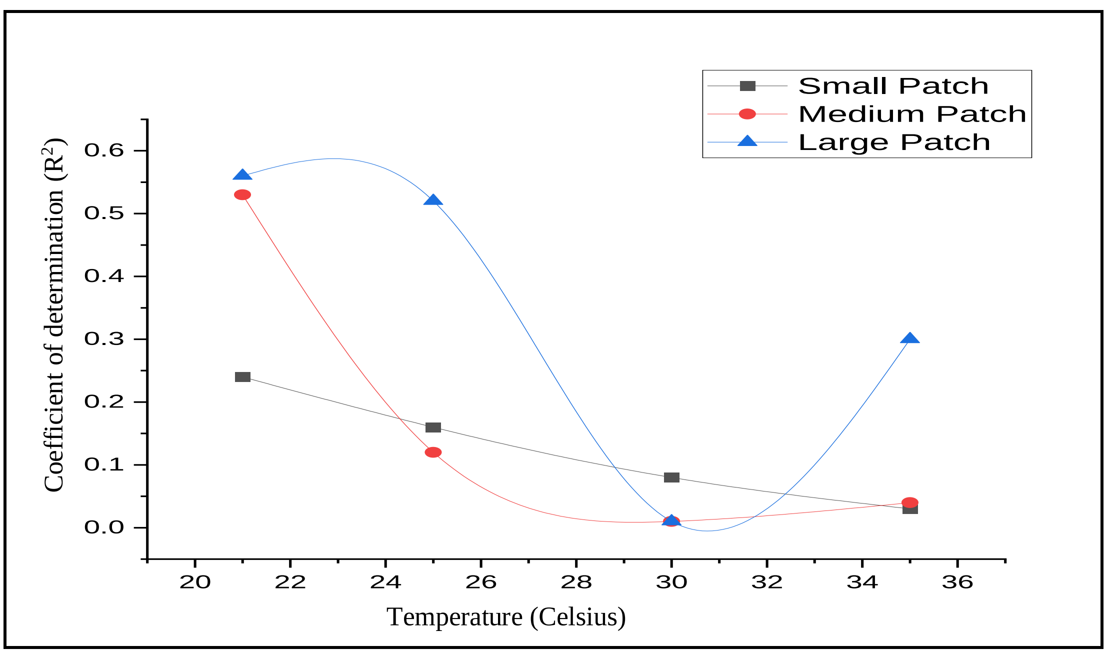


**Figure 6.7** –Showing Unstructured Population Model (CASE 6)

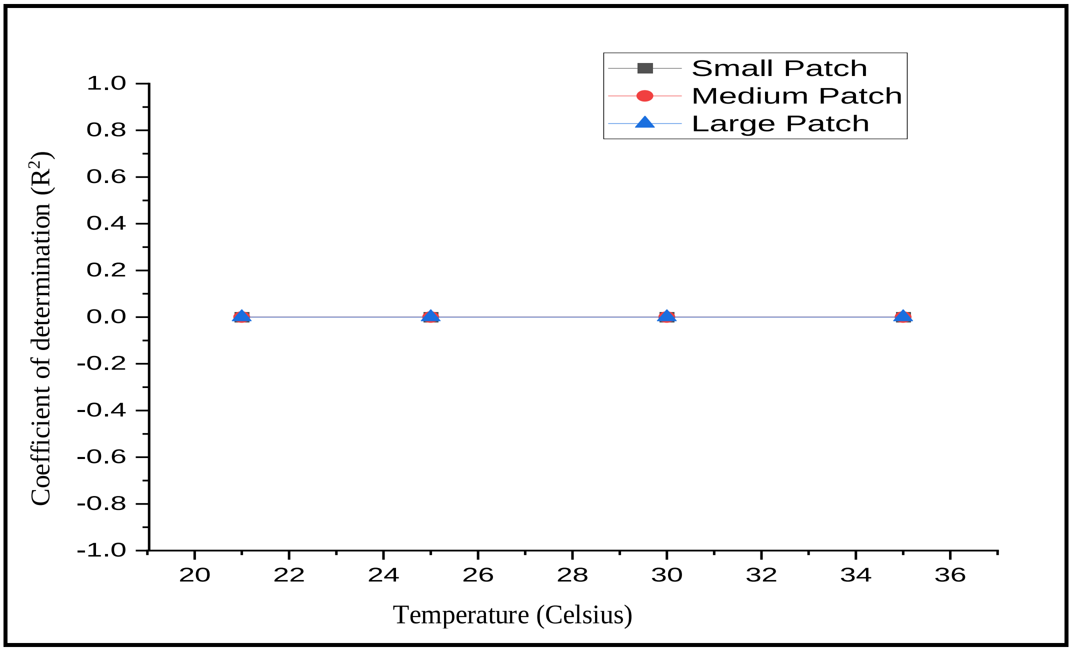


**Figure 6.8** –Showing Unstructured Population Model (CASE 7)

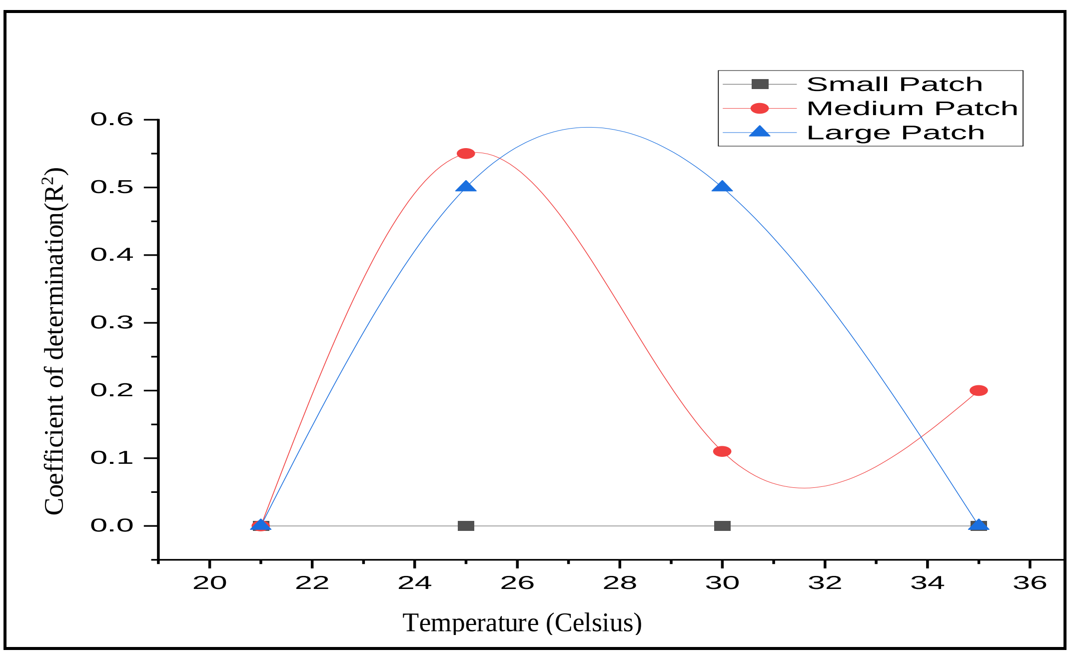


**Figure 6.9** –Showing Unstructured Population Model (CASE 8)

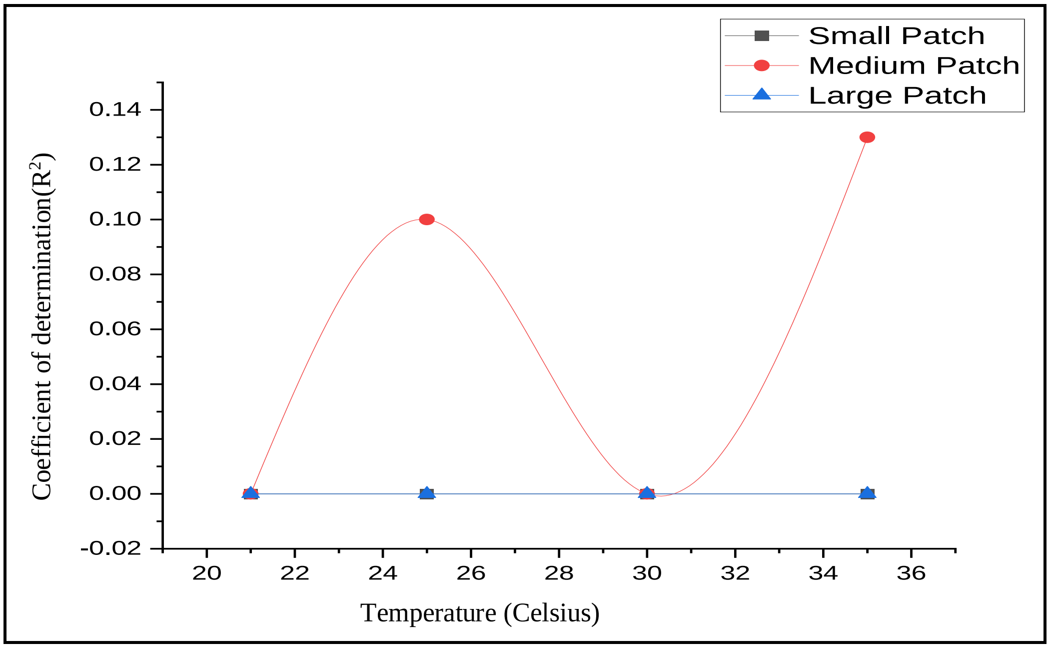


**Figure 6.10** –Showing Unstructured Population Model (CASE 9)

## 6.6.2 KEY FACTORS ANALYSIS

Comparing all of the temperatures considered, equation 11 provided the best fit. The results demonstrated three key points:

1) the population of the insects did not achieve its maximal volume at 30°C;

2) temperature and the prior insect population were the first two important elements impacting the population dynamics; and

3) The consequences of crowding could be numerically evaluated.

## 6.6.3 DEGREE- DAY MODEL

The total number of adults climbed as the sum of degree days increased before the adult population achieved its highest number. Inside the huge spaces, there were roughly as many adults at temperature increased as there were at temperature decreased. As a result, it stands to reason that the infection level would remain constant at a certain number of degree days.

Ukalska et al. [89] offered the most-effective equation for the relationship between the overall amount of adults (or just live grownups or progeny) and the total number of degree days at any tested temperature.

$$\text{Sum numbers of adults} = \frac{C}{1+e^{\frac{-(sum\ degree\ day-P_O)}{A}}} \tag{15}$$

C, A and $P_0$ were constants

When $R^2 \geq 0.76$, this formula could predict the overall number of adults.

The link between the overall number of adults and the overall total of degree days at all verified temperatures that were constant inside the huge regions was best represented by the equation below:

$$\text{Sum numbers of adults} = \frac{C(T-6.3)}{1+e^{\frac{-(sum\ degree\ day-P_O)}{A(T-6.3)^B}}}$$

and $R^2 \geq 0.87$

This study discovered that an important variables impacting the dynamics of the *Cryptolestes ferrugineus* population were temperature, temperature change, and the previous insect population inside grain bins. The number of insects cannot be predicted using only the degree days' sum. The number of degree days should be considered along with other important elements in a basic model that might be used to forecast the number of insects in stored grain. These elements were taken into account in this recently created model, which accurately predicted the number of insects. The most accurate equation to forecast the quantity of insects (living adults alone, all grownups, or progeny) inside the patches of big size at each grain's assessed temperature was

$$\text{Log}_{10}{}^{N}{}_{t+1} = C + AP + \frac{B+DP}{1+EP}\text{Log}_{10}{}^{N}{}_{t} \quad \text{T-increase or constant}$$

$$\text{Log}_{10}{}^{N}{}_{t+2} = C + AP + \frac{B+DP}{1+EP}\text{Log}_{10}{}^{N}{}_{t} \quad \text{T-decrease or constant} \tag{16}$$

P- Total number of effective degree days, A, B, C, D, E were parameters. (16) was obtained by comparing equation (11) and sigmoidal equation (16) as A+BP in equation (16) = $\text{Log}_{10}^{F}\ and\ \frac{C+DP}{1+EP}$ in equation 16 = A in equation 11. This broad equation took into account the important variables previous insect counts, temperatures, temperature swings, and the total number of degree days. In light of the amount of insect populations presented in the samples and the anticipated temperatures at the collecting locations, this Key Factor-Degree Day model could be utilised to foresee insect populations in the upcoming months.

## 6.7 CONCLUSION

In none of the tested scenarios did nine population-based unstructured models suit the bug counts. The optimum equation for this relationship has three variables and was sigmoidal. Temperature, temperature change, and the earlier number of insects were the main factors determining *Cryptolestes ferrugineus* population trends in grain bins. These elements were considered when creating this new model, which accurately predicted the insect population before it reached its peak density.

# CHAPTER -7

# FUTURE SCOPE OF THE RESEARCH

Insects can have a significant impact on stored grains. They can cause damage to the grain kernels, reducing the quality and quantity of the stored grain. This can result in reduced market value and economic losses for grain producers and storage facilities. Some of the most common pests of stored grains include grain beetles, grain mites, grain weevils, and stored product moths. These pests feed on the grain kernels, causing physical damage and reducing the grain's weight and nutritional value. In some cases, insects can also introduce mold and bacteria into the stored grain, further reducing its quality and posing a potential health risk.

To prevent insect infestations in stored grains, it's important to properly store the grain in a clean, cool, and dry environment. This includes sealing grain containers to prevent entry of pests, monitoring the temperature and humidity levels, and regularly inspecting the grain for signs of infestation. In addition, fumigation with approved pesticides can be used to kill insects and their eggs in stored grain.

The stored grain ecosystem is a system inside a system, thus multidisciplinary research should be reinforced and projects should be carried out by coordinated teams with experts in several study fields. The remaining difficulty is in putting the pieces together into a workable whole and filling in the research gaps. Large-scale, multi-factor testing (such those carried out inside storage silos) should be performed to confirm the synthesised whole. It is necessary to widen the scope of the research to include a variety of ecosystem and agroecosystem components as well as regional geography, climate, and agricultural succession.

Mathematical modeling is a valuable tool to study the effects of insects on stored grains. It provides a means to understand complex biological systems and predict their behavior, which can help in developing more effective strategies for insect pest management.

The future scope of the research is wide and promising.

**Predictive modeling**: Developing models that can predict the likelihood of insect infestations based on factors such as temperature, humidity, grain type, and storage conditions. This could allow grain producers and storage facilities to take proactive measures to prevent insect damage.

**Optimization models**: Developing models that can help determine the most effective and efficient methods of controlling insect infestations, taking into account factors such as the type of insect, the type of grain, and the cost and availability of control methods.

**Risk assessment models**: Constructing models that can evaluate the threat of insect infestation causing damage to grain that has been stored depending on a number of variables, such as the grain's vulnerability to infestation, the type and intensity of the infection, and potential negative effects on the economy and environment.

**Monitoring and surveillance models:** Developing models that can monitor and detect insect infestations in real-time, using sensors and other technologies to collect data on temperature, humidity, and other environmental factors that may contribute to insect damage.

**Development of Mathematical Models:** The research aims to develop mathematical models to study the insect population growth, the damage caused by insects and its relationship with the environmental factors and the stored grains. These models will provide a scientific basis for decision making in food storage management and help in predicting the extent of damage.

**Enhanced Food Safety:** The research will help in ensuring food safety by reducing the risk of contamination from insect infestation. This will help in maintaining the quality of food grains and improve consumer confidence in food products.

This study demonstrates how risky such a method would be if it were used, for example, to monitor the growth of microbes in order to assess the risk to water quality or food safety.

**Development of new insect control strategies:** Based on a better understanding of the dynamics of insect populations and the factors that affect their growth and survival, the findings of this research can help design novel tactics for controlling insects in stored grain.

**Sustainable grain storage systems:** This study may aid in the creation of environmentally friendly grain storage methods that require less chemical insecticides and are less susceptible to harm from insects.

**Improved insect control technologies:** The results of this research may inform the development of new technologies for controlling insects in stored grain, such as new traps, baits, and environmental control methods.

**Study of insect populations in other systems:** The results of this research may have implications for the study of insect populations in other systems, such as forests, agricultural systems, and urban environments.

**Optimization of existing insect control methods:** The results of this research can inform the optimization of existing methods for controlling insects in stored grain, making these methods more effective and efficient.

**Improved food security**: By reducing the impact of insect infestation on stored grain, this research can contribute to improved food security and reduce food losses due to grain damage.

**Understanding the role of environmental fluctuations:** The study may offer fresh perspectives on how environmental changes affect insect populations and may contribute to the creation of novel pest management techniques for environments where temperature, humidity, and other environmental factors are constantly changing, such as storage facilities.

**Development of new baits and traps:** The results of this research may inform the development of new baits and traps that are more effective in controlling insect populations in stored grain.

**Study of insect behavior**: The results of this research may provide new insights into the behavior of insects in stored grain and the factors that influence their survival and growth.

**Reduced environmental impact of grain storage:** By reducing the need for chemical insecticides and promoting sustainable grain storage systems, this research can reduce the environmental impact of grain storage and contribute to a more.

By providing a systematic and data-driven approach to understanding and controlling insect infestations, these models can help reduce economic losses and improve the sustainability of grain storage practices.

In conclusion, the future scope of the research is wide and promising. The development of more sophisticated models, combined with the integration of machine learning algorithms, will likely lead to significant advancements in our understanding of insect behavior and the management of insect pest in stored grain systems.

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## LIST OF PUBLICATIONS

[1] Rakesh Yadav and Kirti Bhagirath, *Model to describe a small out-break of damaged grains due to insects,* Plant Archives. **20** (2020), no. 2,1421-1424.

[2] Rakesh Yadav, Kirti Bhagirath and Shivani Bansal, *To develop and validate a mathematical model to analyze the effect of S. cerealella on grains in the environmental ecosystem*, International Journal of Early Childhood Special Education. **14** (2022), no. 5, 7589-7594.

[3] Kirti Bhagirath, *Analysis of growth in population through nonlinear stochastic process,* Journal of Physics: Conference Series. **2267** (2022).

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## LIST OF CONFERENCES

1. Presented paper entitled "Analysis of growth in population through nonlinear stochastic process" in the International Conference on "Recent Advances in Fundamental and Applied Sciences" (RAFAS-2021) held on June 25-26, 2021, organized by School of Chemical Engineering and Physical Sciences, Lovely Professional University, Punjab.
2. Presented paper on "Mathematical modeling to predict the population dynamics of Rusty Grain Beetle in Stored Bulk wheat" in the International Conference on Materials for Emerging Technologies (ICMET-21) held on February 18-19, 2022, organized by Department of Research Impact and Outcome, Division of Research and Development, Lovely Professional University, Punjab.

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*This is to certify that Prof./Dr./Mr./Ms.* ***Ms. Kirti Bhagirath***

*of* ***Lovely Professional University***

*has given poster presentation on* ***Analysis Of Growth In Population Through Nonlinear Stochastic Process***

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This is to certify that **Ms. Kirti Bhagirath** of **Lovely Professional University, Phagwara, Punjab, India** has presented paper on **Mathematical modeling to predict the population dynamics of Rusty Grain Beetle in Stored Bulk wheat** in the **International Conference on Materials for Emerging Technologies (ICMET-21)** held on February 18-19, 2022, organized by Department of Research Impact and Outcome, Division of Research and Development, Lovely Professional University, Punjab.

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