SUPPLY CHAIN INVENTORY MODELS FOR DIFFERENT CARBON EMISSION POLICIES UNDER FINITE PLANNING HORIZON

Thesis Submitted for the Award of the Degree of

DOCTOR OF PHILOSOPHY

in

Mathematics

By

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DECLARATION OF AUTHORSHIP

I, hereby declare that the presented work in the thesis entitled "***Supply chain inventory models for different carbon emission Policies under finite planning horizon***" in fulfilment of degree of **Doctor of Philosophy (Ph. D.)** is the outcome of research work carried out by me under the supervision **Dr. Nitin Kumar Mishra**, working as associate professor, in the School of Chemical Engineering and Physical Sciences Lovely Professional University, Punjab, India. In keeping with the general practice of reporting scientific observations, due acknowledgment has been made whenever work described here has been based on findings of another investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

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CERTIFICATE

This is to certify that the work reported in the Ph. D. thesis entitled ***Supply chain inventory models for different carbon emission Policies under finite planning horizon***" submitted in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy (Ph.D.)** in School of Chemical Engineering and Physical Sciences Lovely Professional University, Punjab, is a research work carried out by Ranu, Registration No- 42000141, is a bonafide record of his/her original work carried out under my supervision and that no part of the thesis has been submitted for any other degree, diploma or equivalent course.

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Dedicated to God and
my Father Shri Randhir Singh

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Table of Contents:

List of Abbreviations:

EOQ- Economic Ordering Quantity

FPH - Finite Planning Horizon

SCM -Supply chain management

CO- Collaborated /centralized Optimal

DO-Without collaboration/without centralized/Decentralized Optimal

TC-Total Cost

List of Notations:

a	The initial annual demand rate.
b	The annual increase in demand.
d_r	The cost of deterioration incurred by the retailer/supplier per unit of the product (\$/unit).
h_r	The holding cost of the stocks per (\$/unit/ year).
O_r	The amount spent in placing an order.
θ	The rate at which the items deteriorate per unit of time at the retailer's site, as well as the supplier's site, where $0 < \theta < 1$.
θ_1	The on-hand inventory of a variable fraction $\theta_1 = \alpha t$ deteriorates with every unit of time, wherein $0 < \alpha < 1$
θ_2	The stock- dependent demand rate is θ_2.
T_{Ret}	Total retailer cost under the FPH (\$/time unit).
T_{sup}	Total supplier cost under the FPH (\$/time unit)
T_{IND}^{r}	Without coordination taking place, the retailer's overall cost is determined. (\$/time /unit)
T_{JT}^{r}	When coordination takes place, the retailer's overall cost is determined. (\$/time /unit)
T_{IND}^{s}	Without coordination taking place, the supplier's overall cost is determined. (\$/time/ unit)
T_{JT}^{s}	When coordination takes place, the supplier's overall cost is determined (\$/time/ unit)
IL_{i+1}	Level of stock at the duration ti with the $(i+1)^{th}$ cycle.
$ILS_{i+1}(t)$	The firm's level of inventory for $^{(i+1)}$th order cycle at time t that goes to the shortage, where $si \le t \le ti$.
$\beta(\varphi)$	$\beta(\varphi) = \frac{1}{\delta\varphi+1}, \quad where\ \delta > 0$ is the backlogging rate when φ is nothing, just the amount of time when the buyer is ready to wait.

Q_{i+1}	Represents the order quantity within (i+1)th cycle at time ti (units).
S_h	The shortage cost for each cycle per unit of time.
L	For each cycle, per unit time's lost sale cost.
S_s	The supplier's setup service charges each cycle. ($/lot).
h_s	The cost of holding the stocks for suppliers per unit per year.
W	Symbolizes the per unit wholesale cost for the retailer (W > P_r)
P_r	Acquisition/purchasing cost for the supplier ($/unit).
H_s	The cost incurred by the supplier for holding inventory.
H_r	The cost of storing goods at the facility of the buyer's agent, is expressed in terms of dollars per unit and per unit of time.
D_p & D_r	The total cost of deterioration incurred by both the retailers and suppliers, measured in dollars per unit.
R	Rate of % at which products are remanufactured.
S	Selling price for each unit.
C_p	Opportunity/penalty cost for each unit.
I_e	Interest earned for each unit of time.
I_c	The interest charge for each unit of time.
M_{i+1}	The trade-credit period's length for $(i + 1)^{th}$ cycle, $M_{i+1} = \delta(t_{i+1})$
δ	the rate of the credit period.
I	The inventory cost of an object in rupees per unit.
N	In units of time, the advance payment period.
A_c	Acquisition cost
r	The amount of a price reduction for making an advance payment.
n	An advance payment's number of equal instalments.
$I_{b_{i+1}}$	The retailer's stock level during the (i+1) cycle at t time.

q_{b_i}	The lot size for the i-th delivery of products from the supplier to the retailer is measured in units.
$I_{P_{i+1}}(t)$	The inventory level of the manufactured products for period t in the (i+1) cycle.
P	The efficiency of manufacturing is expressed as units produced per unit of time.
$Q_1(r,t)$	The total quantity produced for the first r cycles after the initial n-r deliveries.
$Q_1(t)$	The total quantity produced at time t.
$Q_2(r,t)$	The total quantity consumed in the last n-r cycles.
$Q_2(t)$	The total quantity consumed after time t.
A	Advertising and marketing frequency
∀	marketing and advertising demand function fluctuation
Emission-related variables:	
Z	The limit/cap on the amount of carbon emissions that are allowed.
c^	Fixed amount of carbon emission.
$\widehat{P_r}$	The quantity of carbon emissions generated as a result of placing an order is expressed as $ per ton of CO2 per unit.
C_e	The total amount of CO2 released during a replenishing cycle.
ρ	Demand depends upon the amount of carbon emission.
τ	Tax paid to slow the rate of emitting per unit of carbon. ($/ton Co2/unit).
$\widehat{h_r}$	The quantity of carbon emissions due to holding stock in refrigeration is expressed as $ per ton of CO2 per unit.
Co_2	The total amount of carbon dioxide emitted is expressed as units per unit of time per ton.
$\widehat{d_r}$	The total amount of carbon per unit due to product deterioration is expressed in tons of CO2 per unit.
$\hat{P}_r$	The amount of carbon emissions produced due to the act of placing an order by a retailer. (unit/time/ton)

$\widehat{H_r}$	The quantity of carbon emissions produced because of the retailer's inventory holding. (unit/time/ton)
$\widehat{P_s}$	The amount of carbon emissions produced due to the act of placing an order from the supplier site. (unit/time/ton)
$\widehat{H_s}$	The quantity of carbon emissions produced because of the supplier's inventory holding. (unit/time/ton)
Transportation variables:	
v_c	The variable cost of fuel consumed by the retailer during transportation depends upon the fuel consumption. ($/litre)
e_2	The carbon emissions cost associated with transporting a single unit of an item by retailers, particularly considering refrigeration. ($/unit/km)
e_1	Charge of carbon dioxide emissions from retail transportation ($/km)
C_1	Fuel usage of a retailer's vehicle when it has been empty (litres per kilometre)
C_2	Retailer's additional (refrigeration and vehicle services) transportation energy consumption for every per ton of payload (litre/km/ton)
d	The distance travelled between the supplier and the retailer (km)
F_c	The fixed cost of transportation incurred by the retailer upon placing an order ($)
Below are the decision variables:	
T_{i+1}	The length of the $(i+1)^{th}$ replenishment cycle (time/ unit).
H	The time throughout the i^{th} replenishment cycle whenever the stock level reaches zero. $i = 0,1, 2, ..., n_1 - 1$. where $t_0 = 0$ and $t_n = H$ (time unit). A predetermined time interval (4 years).

t_i	The time throughout the i^th replenishment cycle whenever the stock level reaches zero. i = 0,1, 2, ..., $n_1 - 1$, assuming. $t_i = 0$ and t_n = H
s_i	The time for i^th replenishing cycle is when inventory levels approach insufficiency or shortage. i = 0,1… . . ., $n_1 - 1$.
n_1	Represents the total number of replenishing stock rounds when there is no collaboration between supplier and retailer.
n_2	Represents n_2 the total number of replenishing stock rounds in the case of collaboration.

List of Tables:

List of Figures:

Abstract

It is monitored to ensure that it does not exceed the requirements and that occasional stock-outs may occur. Excess inventory is a point of anxiety for management since it ties up capital and can limit the company's ability to generate revenue. Therefore, inventory should be carefully maintained to align with market demand and supply. There are some products in the market that produce a huge amount of carbon and may harm demand. Hence, demand can be assumed to be influenced by mainly carbon emissions, time as well, and pricing, which are always critical components in determining demand. A situation like this is more likely to occur with consumer goods inventory. Therefore, Inventory-managed approaches for decaying goods with carbon emission-dependent, Time-sensitive, and Price-related demand rates should be developed and studied. The suggested study aims to create some mathematical models that will reduce the annual total cost of inventory and carbon emissions (i.e., the total cost function is a convex function) under FPH (finite planning horizon). An optimization model is also discussed in Chapter 5 with three payment options- preliminary, post, and credit payment options in a finite planning horizon. The

The model's primary purpose is to identify the most efficient inventory level and the corresponding total cost that optimizes the overall inventory management. By utilizing this model, companies can make well-informed decisions regarding the total cost and inventory levels, enhancing profitability while also satisfying customer demand.

In this dissertation, our investigation focused on examining a three-echelon supply chain system comprising a single supplier, retailer, and manufacturer. The system's demand is influenced by several factors, including time, inventory, advertising, and carbon. Our analysis primarily aimed to determine the optimality or convexity of the cost function.

The focus of this research dissertation is on two important aspects of supply chain management: deterioration and carbon emission policies within a finite planning horizon. With the growing importance of timely delivery in today's highly competitive environment, we have assumed zero lead time in our study. Our research examines the

effects of investing in preservation or green technology to mitigate carbon emissions and deterioration. We also explore how changes in production rates can impact product quality.

Carbon emissions are a critical factor in supply chain management performance. A sustainable supply chain system should address economic, social, and environmental concerns comprehensively. Our study integrates the supply chain model with carbon emission regulations such as carbon tax and cap, providing an exciting opportunity to make a significant impact on society. In Chapter 9, we discuss the rework process in the production. The key points of each chapter are summarized below.

A Brief Summary of Thesis Chapters

In Chapter 1 of this thesis/dissertation, we provide an introduction to the study. This section covers the basics of operations research, the different components of operations research, inventory supply chain management, and the several parameters that impact the performance of the supply chain process and its participants. We also discuss the importance of inventory in every business operation, along with the associated costs, demand, deterioration, and other components related to supply chain management and carbon emissions.

In Chapter 2, we presented a literature review of remarkable research work by the researcher that has been conducted in the field. We identified and discussed the gaps in the existing literature through a comprehensive study. Furthermore, we explained the software, relevant mathematical concepts, and methodology used to analyze the proposed research project. Finally, we introduced the objectives of the study.

Chapter 3. describes a finite horizon inventory model for supplier-retailer coordination of declining items in quality over time and a demand that is influenced by both their price and carbon emissions within a finite horizon. The model is designed for businesses that face the challenge of reducing carbon emissions while maintaining profitability. We first developed a theoretical model and constructed a mathematical formulation to optimize the overall cost of the retailer and supplier. An algorithm was then created, and numerical iterative approaches were used to solve the optimization problem. Sensitivity analysis was conducted using Mathematica software version 12, and graphical and tabular representations were used to analyze the changing behaviour of various parameters. The proof that the retailer's total cost is positive definite is also presented as a theorem. We compared our results with Wu and Zhao et al. (2014) and found that our model outperformed their model. The study concludes by highlighting the optimal cost and inventory levels determined by the model, as well as the coordinating effect on both the retailer and supplier.

In Chapter 4, we constructed a supply chain inventory model to analyze the coordination effect on the total cost for retailers and suppliers with three payment options for deteriorating items. We also considered carbon emissions and price-sensitive demand in this model, intending to address the problem faced by a retail-supplier business. To validate our model, we cited numerical examples for the three payment options and solved the numerical problem using a numerical iterative method based on an algorithm. We also considered three payment options - advance payment, cash payment, and credit payment - along with a carbon tax and cap policy. To examine the impact of changes in various parameter systems, we presented sensitivity analysis using Mathematica version-12 and used graphical and tabular forms for the analysis.

A lengthier/extended version of Chapter 3 could be discussed in Chapter 5, incorporating quadratic time and inventory-dependent demand under a finite planning horizon with a carbon tax policy and shortages in all cycles. The model has been developed theoretically and mathematically to provide the optimal solution to the problem. A numerical example and comparative evaluation are presented, along with a sensitivity analysis of each parameter. The sensitivity analysis is graphically and tabularly represented using Mathematica version 12. Carbon emissions occurring during transportation are also calculated.

As per my next objective, I have formulated an inventory model for a three-level refrigeration inventory supply chain approach in Chapter 6, taking into account carbon emission-dependent demand for temperature-sensitive items and the implementation of a carbon emission regulation and tax to reduce emissions. First, a theoretical model was constructed and a mathematical formulation was developed to solve the optimization model. An algorithm was created, and a numerical iterative approach was used to tackle the numerical problem. The sensitivity analysis of all parameters was verified using Mathematica software version 12, with graphical and tabular representations used for the analysis.

Chapter 7 deals with a research study that investigates a Mathematical inventory management approach with time, advertising, and inventory-dependent demand patterns. With collaboration and without collaboration, two cases are discussed in this proposed model. Within the first case, retailers and suppliers are not regarded as collaborators, whereas in the second case, collaboration is recognized. The optimality of the planned inventory management model is explained mathematically and theoretically in both situations. The algorithm of the mathematical solution was also properly discussed and the effects of altering various parameters were numerically studied to conduct a sensitivity analysis with the help of Mathematica software version-12. To demonstrate this model, a mathematical illustration, and a tabular and graphical representation, have been also provided. Ultimately, this model reaches a flourishing managerial suggestion and conclusion.

Chapter 8 includes a supply chain inventory model developed for a finite planning horizon. The research study examines the impact of investing in green and preservation technologies. To increase profit and reduce total cost and carbon emissions the impact of the trade credit duration granted by suppliers to retailers is also addressed. Time affects the demand rate in this scenario. To identify the most appropriate solution, a computational approach/algorithm was developed for the supply chain inventory control and management challenge. The optimality and uniqueness of the parameters of the proposed research study are demonstrated through theoretical, mathematical, tabular, and pictorial analysis. Managerial implications are also provided.

In Chapter 9, we focus on sustainable supply chain management for a production system subject to a carbon cap. We also consider time-dependent demand for a rework process of defective production under a finite planning horizon model. We conduct sensitivity analysis using Mathematica software version-12 and discuss an algorithm for solving the optimization problem. Additionally, we prove a theorem which verifies the convexity of the total cost function.

Last but not least, Chapter 10 provides information about the conclusion of the thesis and suggestions for future work.

Chapter 1: Introduction

1.1. Operations research:

Every systematic scientific research requires the implementation of a mathematical model. The mathematical model consists of a set of mathematical and logical relationships that explains the mathematical situation of the real-world phenomenon under investigation. Mathematical Models show the link between system parameters and objective functions to determine possible solutions under some constraints. A model is a simplified version of a real-world situation. It is like an abstract that leaves this information out, a researcher can always hope that the model's solution will include values that are relevant to the original problem. Mathematical models must always be solved and represent the original situation accurately. To explain a system, a mathematical model uses mathematical terminology. Many fields such as biology, economics, earth science, political science, meteorology, engineering, psychology, sociology all can benefit from mathematical models. Mathematical models are widely utilized in economics, physics, engineering sciences, and some other fields. Dynamical systems, Abstract algebra, operations research, statistical models, real analysis, differential equations, complex analysis, game-theoretic models, and some other types of mathematical models are only a few examples. Numerical iteration, Markov decision processes, Monte Carlo simulation, dynamic programming, probability theory, queuing and other stochastic-process models, linear and nonlinear programming data envelopment analysis, econometric methods, expert systems, analytic hierarchy, and the decision analysis process are just a few of the techniques that can be used. Almost all of these methods entail the creation of a mathematical model that describes the area under investigation (Gupta and Hira 1995) and (Taha 2013).

The development and implementation of quantitative research to the solution of real-world issues is the focus of Operations Research. Operational research is concerned with a certain type of military and industrial challenge. The situation in the military and industry is always changing. During World War II, the first formal action of operation research was launched in England when a group of British scientists planned to make more scientific decisions about the optimal use of defence equipment.

Then after that, it is established in many countries. Operational research was founded in India in 1949 with the establishment of a research laboratory in Hyderabad. After that, these ideas developed in the public sector to improve efficiency and productivity. If we consider the problem of industry, where the reality is that every deal with production and supply chain processes is changing due to new technology and competition from other rivals, the issue is how to make a policy that optimizes the entire cost of the system. Operation research is generated from the operation and research of the terms, in which operation keeps referring to business activity. On the other side, research is the activity of analysis and verification that is distinguished by scientific methods. As a result, the term " Operation research" is referred to as scientific decision-making or operations analysis, which is a subfield of applied mathematics concerned with problem-solving (Taha 2013).

Operations research is a field that helps individuals solve problems in the real world. There are numerous problem-solving models in operations research for diverse situations. So, the challenge of supply chain management has been successfully solved by a relatively new development in this domain. The problem is first formulated, and then the solution is determined using one of the many approaches available. One approach is to use optimization models, where decision variables are assigned values that maximize or minimize the objective functions while adhering to the given constraints.

1.1.1. Mathematical general methods for solving operations research problem:

To optimize a real-world issue using modelling simulation, many optimization techniques are used. Here are a few examples:

i. Manufacturing Optimization
ii. Optimization of Transportation
iii. Optimization of the Supply Chain Scheduling
iv. Optimization of the Network
v. Cold chain Optimization
vi. Carbon emission supply optimization

The following are some of the applications that were used:

i. Numerical iteration
ii. probability theory
iii. Monte Carlo approaches

1.2. Inventory:

It translates to "stocks". Inventory can be described as a collection of resources or items which have not been sold. Inventory is widely used in a variety of ways in many industries, such as supply chain management and manufacturing. Inventory is generally stock that is available in any company or organization at a particular point in time for future resale or manufacture. Material inventory is an ideal product, but it is not created for instantaneous use. Warehousing, holding, insurance, deterioration, equipment, and employees are the cost of capital engaged in financing inventory. There is a paradox that says, **“Inventory is a necessary evil”.** This is because maintaining the inventory is not only desirable but also both retailers and suppliers are unable to function without it. Inventory is also described as **“The necessary but idle resource having the economic value”**. Inventory theory came into focus in this form in the middle of the twentieth century, when mathematical models for inventory control were developed.

1.1.2. Various forms of inventory:

There are various kinds of inventories. Listed below are a few examples: -

Producers, distributors, and dealers may keep inventory at various levels and stages. Different kinds of organizations keep various forms of inventory, such as varieties of items in a shopping mall; blood is the inventory in blood banks; books and infrastructures are inventory in schools or colleges; beds, medicines, and other faculty are the inventory in hospitals. Materials, money equipment, and even human beings are examples of inventory. Inventory also known as stock, is kept in a storage facility by a company or other organization. In the case of manufacturing firms, Inventory can be mainly divided into three categories: raw inventory, work in progress, and finished inventory, commonly known as product. The following is a useful taxonomy of inventory three types for business purposes:

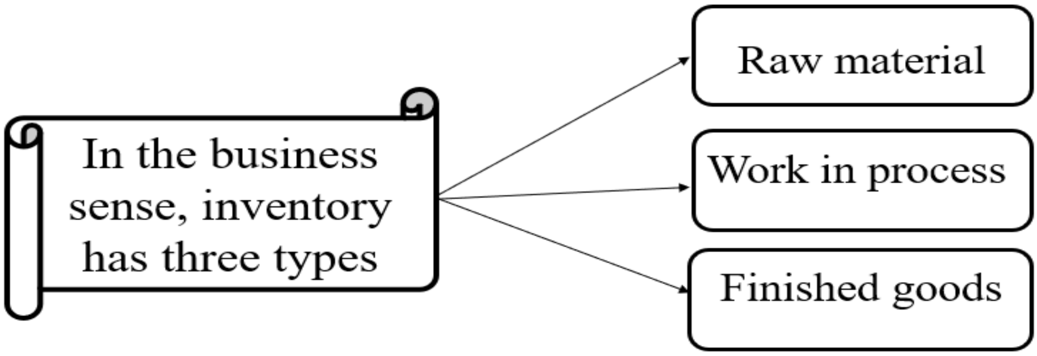

Figure 1.1 Various kinds of inventories

i. **Raw material:**

Raw materials are materials that are used to produce goods or products and have not yet undergone any processing or transformation. They are typically used as inventory items in production or manufacturing businesses, and examples include steel, rubber, timber, tin, copper, lead, and cotton, among others. Raw materials are typically not found in trade businesses since these businesses are involved in buying and selling finished goods rather than manufacturing them. Sugarcane is an example of the raw material in its natural state utilized for the manufacturing of sugar (Plinere and Borisov 2015).

ii. **Work in process:**

The conversion of raw materials into final goods involves several stages of production, including processing, assembly, and finishing. The work in progress section refers to goods that have been partially produced or are in the process of being produced but are not yet ready for sale. For example, in the production of sugar, the sugar may have been processed, but it may not yet be packaged and ready for sale. This sugar would be considered a work in progress and would not be included in the finished goods inventory until it is fully packaged and ready for sale (Plinere and Borisov 2015).

iii. **Finished goods:**

Finished goods are Inventory that is ready to move to the customer. In the example of a sugar factory, the finished goods would be the final packets of sugar that have passed quality inspections and are ready for sale in the market. The inventory is useful in several industrial, business, public sector, trade, banking sector, college-school, and engineering sectors and useful for the transport networks, the construction sectors, control of production, agriculture, networking, etc. Inventory allows these sectors to maintain sufficient stock levels to meet demand and ensure that they can operate efficiently (Plinere and Borisov 2015).

1.2. Management and control of the inventory:

Manufacturing, supply chain management, and other industries all depend on proper inventory control and management. The goal of inventory management is to achieve the most beneficial outcome from the least quantity of inventory. In brief, inventory control and management involve keeping the appropriate amount of stock in such a way that the company can satisfy the greatest number of customers at the lowest possible cost. Inventory management and control means keeping the right amount of stock with optimized total inventory-related costs to meet consumer demand. There are mainly two important costs involved in managing and controlling inventory (Gupta and Hira 1995).

i. ordering and
ii. holding costs.

1.3. Why do we need inventory control?

We often encounter problems when inventory is not properly handled in businesses, which can lead to losses for customers, vendors, suppliers, manufacturers, and others. To avoid these situations and reduce losses while generating profits, inventory levels must be carefully monitored. Inventory refers to unused resources that are stored for future use. Although idle, it is necessary for the smooth operation of any organization. Effective inventory management is critical for any organization; good management can protect the organization from large losses and lead to profits. Inventory management is also essential for limiting demand fluctuations and reducing waste. Given the rapid pace of global industrialization, effective inventory management has become more critical than ever before.

Inventory management has become an important issue among them. Every industry is supposed to run smoothly, hence efficient management of existing resources is necessary for each business organization.

I. To reduce Order Costs:

Ordering costs do include the cost of order placement, as well as inspection, documentation, and other expenses related to a supplier's order process. These costs are accrued with each order placement, regardless of the order size. Therefore, having inventory becomes essential to reduce ordering costs.

II. Demand:

It is not always possible to accurately predict demand over a long period or planning horizon of several years. A company's ability to satisfy customer demand on time is essential to its success and meeting its needs. If a customer is satisfied with your service, they are likely to return, and may even recommend your business to others. Additionally, greater demand usually translates to a higher inventory requirement.

III. Effectively utilize the capital:

The inventory is mostly stored for transactional purposes. Without a sufficient range of inventory, a company cannot sustain a certain level of sales. An enterprise cannot ensure smooth operation in the field of production unless it maintains an appropriate inventory of raw materials. Stocks of inventory can be maintained, but they must be well handled. Then, an enterprise can effectively utilize capital.

IV. Time-saving:

Proper inventory management can help reduce transportation costs and associated emissions by optimizing the storage and distribution of inventory, reducing the need for frequent transportation of goods. This, in turn, saves time and resources for the organization while also promoting sustainability.

V. Efficient running of the business:

Proper inventory management is required for a business to succeed. Inventory control is required to enhance the proper performance of every organization, and therefore inventory control and inventory management are also important. The very rapid pace of industrialization has created lots of management issues. Inventory management is one of the most serious challenges that every industrialist faces. Every industry is expected to run efficiently, hence effective management of available resources is necessary for each business organization. On one side, industries are dealing with rising input costs, but on the other, due to competition, they are limited in their ability to raise the price of finished goods. So, management members in the business sector must effectively manage their resources. A lack of inventory can lead to stockouts, bringing manufacturing operations to a halt, and on another side, a large inventory can result in higher production costs due to the high cost of carrying inventory. So, the inventory management system should ensure that inventories are neither too high nor too low.

VI. Lead time:

lead time refers to the time interval between when we place an order and when the order is received (the time between initiation and completion). Most of the model considers lead time as zero, but in reality, lead time cannot be ignored as it can significantly impact inventory management. The existence of lead time makes it essential to have an inventory to meet customer demand during the lead time and avoid stockouts.

VII. To reduce lost sales:

A lost sale is a loss of earnings, sale or profit. A lost sale occurs when a customer wants to buy something but there is not sufficient stock to fulfil their demand and the customer doesn’t want to put a backorder. To avoid lost sales, inventory should be kept.

VIII. Economic service to customers:

Market demand and supply have an impact on the inventory held by the company. Due to excess market demand, issues such as shortages and backorders have occurred. If market demand falls, the company's goods will remain unsold and may even expire. As a result, the company's inventory should be managed according to market demand. Similarly, supply has an impact on inventories. Supply should also be available when and where it is needed. As a result, inventory control and management are essential.

IX. Price variations and unpredictability:

When the prices of raw materials are very low, companies often purchase them in bulk to keep their operations running smoothly. By lowering the cost of raw materials and obtaining higher prices for their products, companies can optimize their profits. Other factors that can lead to bulk purchasing include quantity discounts, the short shelf life of goods, and the risk of decay or deterioration of stock in inventory.

1.4. Basic terminology used in inventory management/concepts:

I. Demand:

A product's demand refers to the number of units needed to fulfil a business's requirements. The amount of inventory that needs to be maintained is determined by the consumption or usage rate of the product. There are two types of it.

i. Deterministic demand
ii. Non-deterministic demand

The quantity required over repeated periods is known with certainty in a deterministic demand pattern, while in a non-deterministic, probabilistic demand pattern, the demand required over a given period is not known with certainty. Different mathematical methods can be used to classify demand into various categories:

i. **Constant demand:**

Constant demand refers to a situation where the demand for a product remains constant over a given period and is not affected by external factors such as market demand and supply.

ii. **Time-dependent demand:**

Demand is considered time-dependent when the quantity demanded for a product fluctuates over time. For instance, the demand for Rooh-Afza Sharbat increases during the summer season but decreases in winter.

iii. **Price-dependent demand:**

Price is an important component of any product, and when demand fluctuates according to price, this is referred to as price-dependent demand. The relationship between price and demand is an important factor to consider in inventory management and pricing strategies.

iv. **Inventory and carbon-price-dependent demand:**

Inventory and carbon-price-dependent demand are major and important elements of every business's operation. It is controlled and managed to ensure that it does not exceed the requirement and along with, there is no shortage occur. Excess inventory causes worry for management. Additionally, it may be helpful to clarify that excess inventory not only prevents the company from making a profit but also ties up capital that could be used for other business operations- to enhance customer demands. However numerous research studies have traditionally held that product demand is stable or driven only by price, and the impact of carbon emissions cannot be neglected. Several researchers, like those by (Huang 2016) and (Aliabadi, Yazdanparast, and Nasiri 2019) have found a link between the number of carbon emissions connected with a product and its demand. It is well known that consumer awareness of environmental issues may have a major impact on purchasing behaviour within a particular market, i.e., demand may rise or fall as on-hand inventory levels rise or fall. In addition, pricing is always a critical demand factor. A situation like this is more probable to appear with consumer goods inventory. As a result, models for managing inventory of deteriorating items with the demand that is influenced by both carbon emissions and price should be explored. The suggested study's purpose is to create some mathematical models that will contribute to minimizing the overall inventory cost while simultaneously maximizing profits.

v. Advertisement-dependent demand:

(Asoke Kumar Bhunia and Shaikh 2014) and (Khan et al. 2020) investigated that the advertisement of an item is directly associated with demand. Therefore, advertisements will increase the product demand and the product will sell out very soon. For retailers and suppliers to enhance customer demands is a challenging feat. That idea would be great in the case of items that have a short duration of use, their life span is short or will soon reach their deadline for expiring. Due to carbon emissions and advertisements, demand for products will undoubtedly be influenced. Advertisement is one of the most effective promotional approaches to raise awareness about a product's popularity among all classes of consumers. Manufacturers, retailers, or suppliers need to publish advertising in mass media such as print media, visual media, and other advanced technology to stimulate a larger group of customers to buy their products. So, the retailer, supplier, and manufacturer must determine the advertisement process before the sales period.

II. Deteriorating rate:

Deterioration generally refers to the process of decay in which an object gradually becomes ruined or unusable. Every natural thing alters its nature as it progresses through time. The rate of deterioration for any inventory management and control has become a critical concern. Deterioration usually means quality deficiency or utility deficiency. (Ghare and Schrader 1963) were two authors who introduced deterioration in inventory control and management. Everything goes on to deteriorate with time. The depletion of both inventory and manufacturing systems is the most unexpected. The terms broken, evaporated, spoiled, dried out, and damaged are all examples of words that describe the deterioration of objects. There are mainly two types of deteriorating items. The first type of deteriorating items is those which include all items that are destroyed over time such as fruits, flowers, drugs, vegetables, and evaporating liquids such as Perfumes, alcohol, gasoline, and some others depleted over time. Fruits, vegetables, and cereals are declining as a result of direct decaying in storage. The second kind of deteriorating item identifies those items which reduce their value due to technological changes such as computers and mobile phones. Electronic products, photographic film, chemicals, medications, and other items gradually lose their potential or utility over time as well due to technological changes (Jain, Singh, and Singh 2011). Hence, many scholars have made great attention to degradation inventory schemes in the previous few years. The study on decaying stock control & management has grown to be much more extensive. Several publications based on deteriorating items have already been published. Since it is very common to have deterioration of inventory in our everyday life, we should not ignore deterioration in inventory control. The deterioration rate is mostly of two types:

- **i.** Constant deterioration
- **ii.** Time-dependent deterioration

III. Cost function:

The amount of money we spend on items is referred to as cost. Costs come in a variety of forms, based on various mathematical circumstances.

i. Carrying cost or holding cost:

The carrying or holding cost is the cost associated with storing or holding inventory in a warehouse or store. It depends upon quantity and time. As quantity and time increase cost also increases and vice versa. It includes the investment for deterioration, warehouse costs, preserving costs, taxes, stock risk costs, and maintenance costs. It helps us for calculating the estimated profit or forecasting the strategic planning of whether the level of the stock increases or decreases in the store or warehouse.

ii. Cost of shortage or cost of stock-out:

These costs are related to either a delay in satisfying demand or a failure to meet demand at all. Shortage costs occur when the stock of any supplier goes out of stock or diminishes.

iii. Back ordering cost or lost sale cost:

The cost incurred by an organization when it is unable to fulfill an order promptly and informs the customer that it will be delivered at a later time is known as the backordering cost. It can be direct, indirect, or uncertain, and usually depends on the enterprise and the time required for order completion. When a buyer places an order, they are typically charged in advance, and the enterprise notifies the buyer when the item will be received. If the buyer is not willing to wait, the sale is considered lost.

iv. Costs of Depreciation, Depreciation, and Obsolescence:

Depreciation or Obsolescence costs arise when products become out-of-fashion or outdated, and retailers are unable to sell them at their original price. They may need to sell them at a discount or dispose of them, which results in a loss for the business.

v. Cost per unit:

The cost incurred for each unit. is the total cost associated with producing or purchasing that unit, which may include direct costs such as material and labour costs, as well as indirect costs such as overhead expenses?

vi. Set up cost:

Setup costs are the expenses related to setting up the facility to produce goods. This cost includes expenses for the place to set up a system of production as well as the costs of the raw materials, equipment, arrangements, paperwork, and labour. It is a one-time cost and is incurred when a new product is introduced or when changes are made to an existing production process.

vii. Ordering cost:

The cost of placing an order is calculated by multiplying the per-order cost by the total number of orders placed within a given timeframe (e.g., a year or a month). The time when the order is placed may affect the cost if there are seasonal variations or other factors that affect the per-order cost, but it is not a direct factor in the calculation of the ordering cost.

viii. Transportation cost:

A manufacturer must pay moving expenses when transferring its stock or other resources to another venue. Therefore, it is necessary to incorporate these tasks into a system of product movement to save money and give customers the most optimized services possible. By having a well-planned and organized system for moving products, a manufacturer can minimize the time and resources required for transportation, reduce the risk of damage or loss, and ensure that products are delivered to customers on time and in good condition. This can ultimately lead to greater customer satisfaction and loyalty, as well as improved profitability for the manufacturer.

ix. Manufacturing or producing cost:

The costs associated with producing a product or item from start to finish can be divided into fixed and variable costs. When production is dependent on demand, it is referred to as a variable, on the other side, it’s not dependent on demand and is called fixed.

x. Rework cost:

The cost of reworking a thing is the amount of money used to repair or modify or reconstruct it so that it can be sold as a perfect or acceptable item. It is determined during the process of conducting inspections after the final phases of manufacturing. Rework cost is determined by the number of defective goods, which can be an exponential, constant, or proportional proportion of the entire production.

IV. Lead time:

Lead time refers to the time interval between when an order is placed and when the order is received (the time between initiation and completion). The majority of the model is built on the assumption that lead times are zero. sometimes lead time is undoubtedly in inventory control, which we ignore. The uncertainty in a supplier's lead time is a naturally occurring phenomenon. Mostly every supplier suffers such a situation at a certain point within their commercial dealings. Even now, scholars haven't explored deeply enough into this particular issue. (Jain, Singh, and Singh 2011)scenario addresses the realities of real-world business, since the supplier's lead time, which is a random variable, has been observed.

V. Payment options:

(Shi et al. 2021) and (C. Wu and Zhao 2014) explained the role of advance payment in the relationship between distributors and buyers, particularly for high-demand products. Advance payment can be beneficial for products that have a short shelf life, are perishable, or are close to their expiration duration. In a situation that is both very competitive and uncertain and to ensure timely delivery, suppliers often request a particular part of payment in advance from their clients. Customers can benefit from advance payment by securing a lower price for their orders. However, advance payment can also lead to a large carbon footprint due to increased production and carbon emissions. To enhance liquidity, industries have implemented a strategy of providing price discounts to customers who pay in advance. This allows the seller to earn more interest on the liquidity, which in turn stimulates the production of more goods and results in higher carbon emissions. Therefore, making a payment in advance can have a noteworthy effect on both carbon emissions and production.

VI. Rework process:

(B. Sarkar et al. 2015); (B. Sarkar et al. 2018); (B. Sarkar 2019) and (Tiwari, Daryanto, et al. 2018) have proposed models that show the rework process is beneficial for minimizing both carbon emissions and total cost of deterioration. In multi-stage manufacturing processes, the reprocessing of scrapped products is a common problem for production industries.

To ensure customers receive only non-defective products, the purchaser inspects all items delivered and returns any defective ones to the supplier for rework. Reworking scrapped materials in every cycle can reduce unexpected defective items, total cost, and carbon emissions during the production process.

Supply chain management:

The supply chain covers the entire process of producing and selling essential goods. Manufacturers, suppliers, transports, warehouses, retailers, and customers are all part of the supply chain. A process by which natural resources or raw materials are turned into finished items and then sold to end-user or customers.

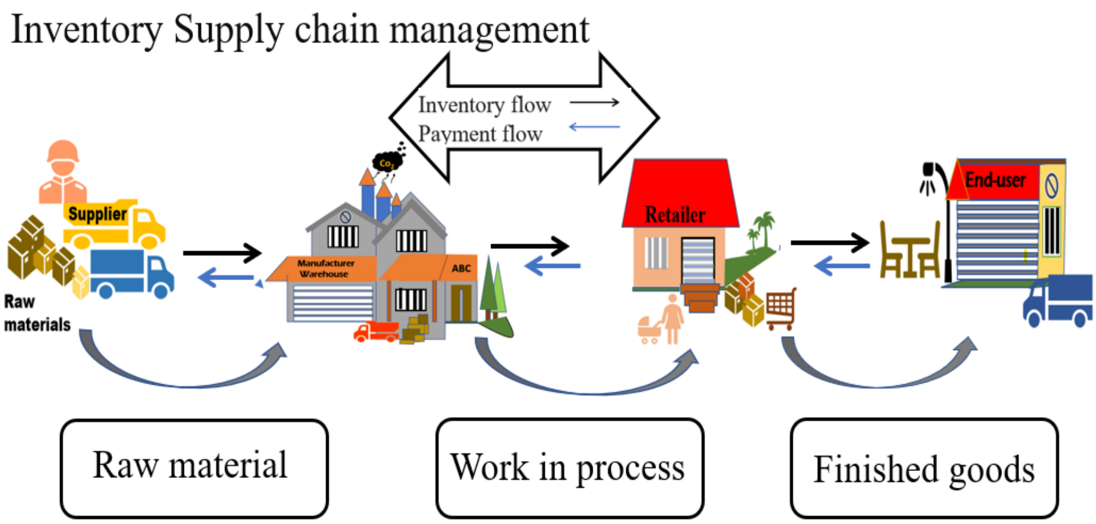

Figure 1.2 Process of supply chain management.

Supply chain management must go through various stages of centralization, decentralization, and a combination of both strategic actions. The followings are the main components of the supply chain process:

i. Planning and forecasting
ii. Procurement and sourcing
iii. Production and operations
iv. Inventory management

v. Warehousing and storage
vi. Transportation and distribution
vii. Customer service and support
viii. Effective management of these components can help optimize the supply chain, reduce costs, improve customer satisfaction, and enhance overall business performance.

The followings are the four main pillars of the supply chain process. They have a crucial role to play in effective inventory management:

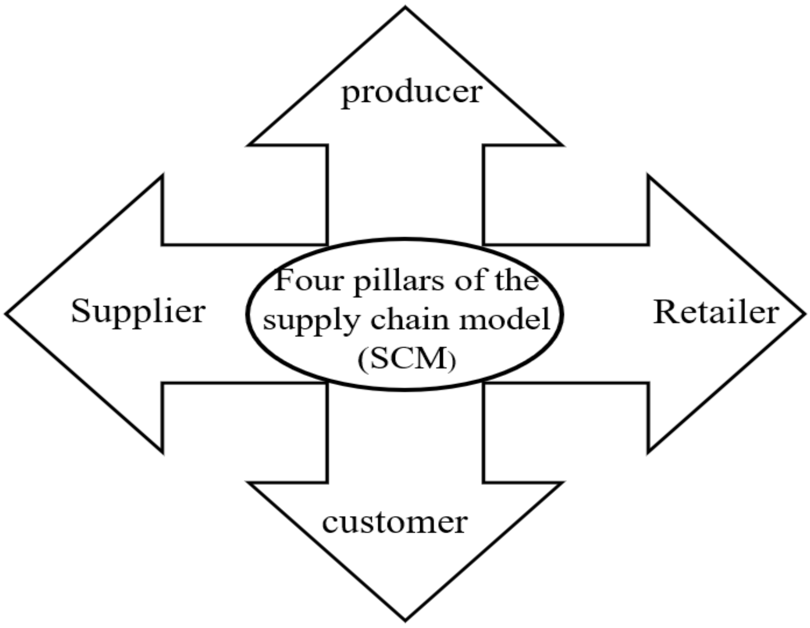

Figure 1.3 Four Pillars of supply chain management

Most of the traditional models considered only the supply chain model without coordination. But supply chain members such as suppliers, retailers, and manufacturers are affected by each other. Collaboration amongst supply chain management is important for efficient supply chain management. In today's global world, an institution's or business owner's primary aim is to optimize overall value simultaneously and emphasize carbon emission reduction that can be achieved by coordinating different strategies among supply chain participants. To extend the entire supply chain, a lot of work has been conducted. Multiple parameters are used to enhance the efficiency of inventory supply. The management of deteriorating goods in the supply chain is currently one of the most critical challenges.

Table 1.1 Comparison between centralized/coordination and decentralized/without collaboration supply chain

Comparison Basis	**Centralized supply chain**	**Decentralized supply chain**
Applying field	Applicable for small-sized businesses.	Applicable for large-sized businesses.
Economically benefit	Comparatively beneficial for business purposes	Not as beneficial
Communication flow	Vertical	Open and Free
Productivity	Increase	Decrease
Decision-planning/ Administration	Fast and quick	Comparatively slow
Cost/profit	Minimum/Maximum	Maximum/ Minimum

I. Manufacturer:

A manufacturer is also called a producer. a person who creates a product through a process is known as a manufacturer. A manufacturer, whether an individual or a company, engages in the process of transforming raw materials into finished products using a diverse range of tools, technology, and production methods. Subsequently, these completed items are delivered to customers.

II. Retailers:

A retailer is a person who sells goods to individual customers. A retailer is a person who buys things from producers or whole sellers and sells them to the end user at a market rate. Retailers keep goods in-store and sell them in small to the general customers.

III. Sellers:

A supplier is a person, organization, or institution that sells or supplies something to consumers, such as commodities or services.

1.4. Carbon emission:

The emission of carbon has been causing global warming for many years. Global warming has been receiving attention over the last few years. The condition that is created due to greenhouse gases (GHG) and some human activity is called global warming. Global warming has been receiving attention over the last few years. Carbon emission gases like methane and carbon dioxide increase the temperature of our Earth and cause global warming (Cause of Climate Change 2022). Global warming causes severe damage to our Earth, as it has destructive, widespread, and long-lasting effects. It rapidly destroys the biodiversity of our world, causing the disappearance of many species of plants and animals. Sea level rise, ozone layer depletion, rising temperatures of the Earth, intensified stormy conditions, droughts, and floods are all effects of global warming. Many studies have shown that the majority of carbon emissions come from the manufacturing of goods. Managing the supply chain for goods that deteriorate is challenging and risky because the usefulness of products that deteriorate can decline due to spoilage, damage, or degradation during storage or transport. Therefore, some countries are now focusing their efforts on reducing carbon emissions to combat global warming.

A group of researchers is currently focusing on reducing their carbon footprint. For example, (Toptal, Özlü, and Konur 2014) also explained that the Kyoto Protocol is a global agreement connected to the UN Framework Convention on Climate Change. The UNFCCC (the United Nations Framework Convention on Climate Change) came into effect on March 21, 1994. On December 11, 1997, The Kyoto Protocol was signed by 37 industrialized countries. India also ratified the Kyoto Protocol in 2002. The United Nations Climate Change holds a yearly conference in a different country, which is called the COP. Encourage the business sector and developing economies to engage in the effort to reduce carbon emissions. The government of any country and some regulatory agencies considered carbon emission schemes to decrease carbon emissions.

For instance, carbon tax and cap, carbon tax, carbon credit, and carbon offset, etc are the main regulation policies.

i. **Carbon tax:**

(U. Mishra et al. 2021) and (Xi Chen et al. 2013) explained carbon tax in their research articles, stating that a carbon tax is levied (imposed) by some government agencies on business firms or industries that produce carbon dioxide during their work process and leading to environmental pollution. The main objective of the government agencies behind the imposition of tax is to control global warming and protect the environment. A carbon tax is levied (imposed) by some government agencies on those business firms or industries that produce carbon dioxide during their work process and lead to environmental pollution. The main objective of the government agencies behind the imposition of tax is to control global warming and protect the environment. In other words, the carbon tax is also a fee is imposed on companies that use the environment-polluting raw material, such as fossil fuels, in their processes, contributing to global warming.

ii. **Carbon cap and tax:**

The carbon cap is slightly different from the carbon tax. A government agency or regulatory authority imposes a carbon cap on the firm. If the firm emits more carbon than this carbon cap, then it is taxed; otherwise, it is not taxed.

iii. **Carbon cap and trade:**

(Benjaafar, Li, and Daskin 2013); and (Qin, Ren, and Xia 2017) defined carbon cap and trade term as a cap-and-trade scheme, in which a government or regulatory body sets an aggregate legal limit on emissions (the cap) for a specified period and a cap-and-trade policy has its own set of advantages, in that emissions credits can be distributed to reduce the policy's adverse effects on industry and predict emissions discharge. A carbon credit is a permit given by a government or regulatory agency for a specific time that allows a company to produce a specific amount of carbon emission. When an organization emits excess carbon over a limit, it is taxed and at the same time, it has to reduce the carbon emission for which it can purchase credit from those organizations that emit less amount from that limit. Less carbon-emitted industries can sell their credit to other organizations that are emitting higher amounts of carbon. One carbon credit equals one ton of carbon emissions.

IV. Carbon offset:

(Benjaafar, Li, and Daskin 2013); (Xi Chen et al. 2013); (Toptal, Özlü, and Konur 2014); and (Dye and Yang 2015) explained that carbon offset is a project-based mechanism that includes a financial investment towards emission reductions. A higher carbon-emitting nation can fund a project in a developed or developing country in exchange for a carbon credit, which goes to the developing country. A carbon cap is a permit given by a government or regulatory agency for a specific time that allows a company to produce a specific amount of carbon emissions. When an organization emits excess carbon over a limit, it is taxed, and at the same time, it has to reduce the carbon emissions, for which it can purchase credits from those organizations that emit a lesser amount within that limit. The credits can be bought and sold on carbon markets, and companies can use them to offset their emissions or to comply with regulations that require them to reduce their emissions. It is a tradable permit that represents a reduction of one ton of carbon dioxide (or equivalent greenhouse gas) emissions. Less carbon-emitting industries can sell their credits to other organizations that are emitting higher amounts of carbon. One carbon credit equals one ton of carbon fuel.

1.5. Cold and refrigerated supply chain:

The demand for refrigeration services to maintain the freshness of products is also experiencing rapid growth. (Hariga, As'ad, and Shamayleh 2017) explained that manufactured cooling is referred to as refrigeration. Energy is transported from a low-temperature resource to a high-temperature resource in the form of heat. Refrigeration and the cold chain have a significant impact on manufacturing, storage facilities, and the supply chain as preservation technologies. There are many technical similarities between space cooling and refrigeration, but refrigeration services commonly provided substantially lower temperatures for the conservation of commodities. Another side, a cold chain refers to cooling facilities that consistently maintain an optimum temperature-controlled environment across an entire supply chain. Certain goods, such as pharmaceuticals product, Frozen Foods, ice cream, Fruits, vegetables, Seafood, Flowers, meat, all dairy items, and Beverage products such as wines and spirits, require an uninterrupted cold chain to save them. Therefore, supply chains must be equipped with facilities, such as refrigerated warehouses and refrigerated trucks so that efficient storage and transportation of temperature-sensitive goods. To reduce degradation, the cold chain is also vital for maintaining the shelf life and quality of the products of perishable foods and temperature-sensitive products.

Many environmental implications are caused by refrigeration and cooling technology, especially in terms of overall contribution to GHG emissions. GHG emissions arise because of liquid refrigerant and emissions from the provision of electricity needed to keep supplying the freezing facility. It has been postulated that some businesses provide refrigerated delivery services to other business owners, such as Walmart and Americold. Manufacturing industries, as well as inventory control and management practices, contribute to situations like global warming. Our cold supply chain represents a large portion of the overall global carbon footprint. (Xi Chen et al. 2013) introduced that in 2016, Walmart has made a new commitment to reduce carbon emissions from the global supply chain by 1 billion metric tons by 2030 to achieve this goal Project Gigaton was launched to address the fact that almost all emissions in the retail sector occurred in product supply chains and transportation rather than stores and distribution centres. The following are websites: (Walmart Sustainability. 2022), (Project Gigaton Accounting Methodology. 2022), and (Glob. Cold Chain Alliance.2023) for more information regarding this.

1.6. Kaya identity:

Global temperature rise is the effect of greenhouse gas emissions and even some living beings. Additionally, the role of individual identity has gained significant attention in the context of climate change in recent years. A worldwide awareness for environmental conservation and protection is encouraging many more researchers, organizations, and other government agencies to create and maintain an eco-friendly as well as a negligible emission management system for supply chains.

$$Co_2 = \left(\frac{GDP}{Ppopulation}\right)\left(\frac{EC}{GDP}\right)\left(\frac{Co_2}{EC}\right) * Population$$

I. Gross domestic product (GDP):

The entire value of all products and services generated in a country in a given year is known as the gross domestic product (GDP). Gross domestic product measures the monetary worth of final goods and services produced in a country over a specified duration, whether it is quarterly or yearly. These products are then purchased by the end users. GDP consists of commodities and services produced for market sale, as well as certain non-monetary production, such as education services, government-provided defence, and security. However, it's important to note that GDP does not encompass all productive activity. For further details, please consult the following source: (International monetary fund. 2023)

II. Energy:

According to (Quaglini et al. 2022), the term "energy" (E) refers to the amount of fossil energy measured in joules (J). All fuels and electricity are assumed to be derived from fossil energy. To access additional information, kindly consult the provided source: (Cause of Climate Change. 2022) and (Final energy consumption. 2014) Everyday activities such as heating, cooling, electricity supply, and transportation are all dependent on reliable and efficient energy services. Energy is essential for the proper and effective functioning of all economic sectors, spanning from business and industry to agriculture. Energy intensity is a ratio of the volume of energy required to produce one unit of GDP and can be used to estimate a country's energy efficiency. The total energy consumed by end-users, such as households and businesses, industry, and modern agriculture, is referred to as final energy consumption. The energy consumed by the end-user, except for energy used by the energy sector.

1.7. Planning horizon:

In general, the inventory management approach can be categorized into two major cases built upon their time horizons: finite and infinite planning horizons. It depends on the nature of demand and coordination between two organizations whether it is finite or infinite. The time horizon is the amount of time during which every firm or organization engages in prospective analysis to envision and anticipate future conditions as part of the formulation of a strategic plan. The time horizon refers to the length of time that the inventory level will be monitored. Infinite horizon planning refers to replenishment cycles that remain constant, whereas a finite planning horizon indicates variability in replenishment cycles that is dependent upon various factors, including demand, lead times, and supply chain coordination. Therefore, the time it takes to replenish stocks can fluctuate from time to time.

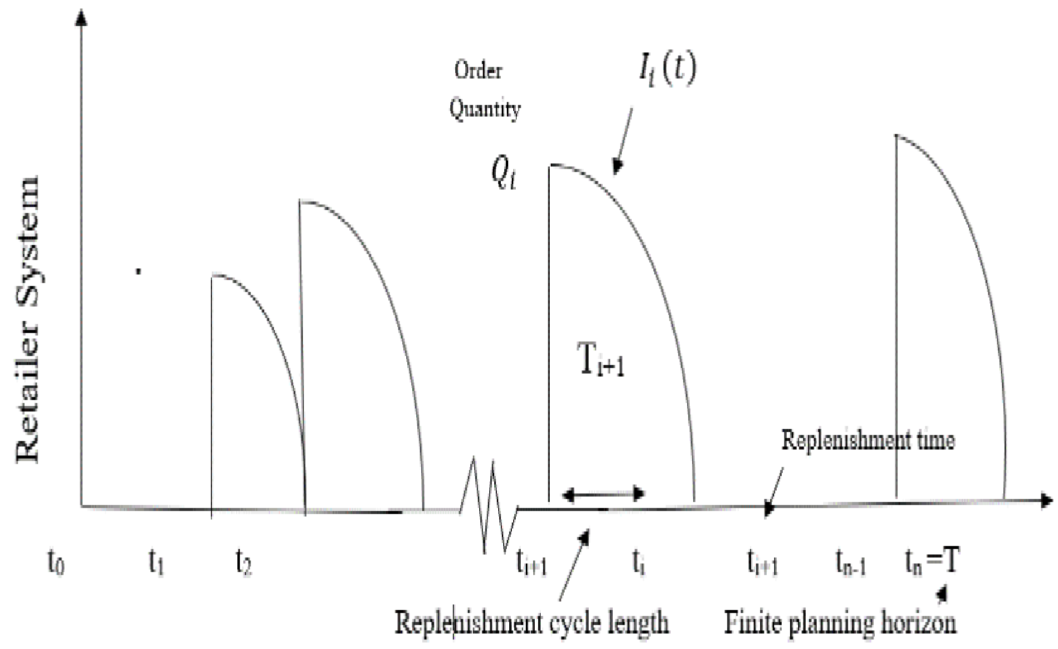

Figure 1.4 Finite planning horizon graph

Table 1.2 Comparison between Finite and Infinite Planning horizons in a supply chain

Comparison Basis	**Infinite Planning horizon**	**Finite Planning horizon**
Simulations	Steady	Terminating
Possibility of becoming obsolete and wastefulness	High	Low
Ending time or condition	Not defined well	Defined well
Overstocking hazard	The tendency for ordering a greater amount of stock than required leads to overstocking.	Reduces the danger of overstocking.
Replenishment cycle time	Repeating only	Repeating or non-repeating
Cycle length	Usually, equal	May be equal or unequal
Holding costs of inventory	Increased holding costs including insurance, and opportunity storage costs due to excess inventory holding	Minimizing excess supply along with related holding costs.

Chapter 2: Literature Review

2.1. Literature review:

In today's fast-changing economic world, becoming more stable is essential. The existing literature on different carbon emission policy supply chain inventory models (B. Sarkar et al. 2021); (U. Mishra et al. 2021); (Shi et al. 2019); (Shi et al. 2021) and others reveal that with some carbon emission policies relations between retailers, manufacturers, and suppliers are becoming more stable in today's fast-changing economic world. However, coordination between retailers, suppliers, and manufacturers depends on factors such as the types of goods, their demand, their deterioration, and certain carbon emission policies. Many scholars have paid great attention to degradation inventory schemes in the last few years. Indeed, due to the deterioration, the inventory of the commodity continues to decline. It is necessary to focus on the deteriorating product that is produced and delivered. Deterioration of inventory is a common occurrence in our everyday lives, so we should not overlook it in inventory control. In recent years, governments and some national or international carbon emissions regulatory agencies are heading towards a way that the product and its inventory management should be free from environmental impacts. This is particularly relevant for items that are prone to deterioration. Several publications focusing on deteriorating items have already been released.

For instance, both (Ghare and Schrader 1963) were the first to study the deterioration rate for inventory lot-sizing problems with a constant deterioration rate and constant demand rate. Afterwards, (Covert, Philip, and Philip 1973) enhanced Ghare and Schrader's study by considering two parameters of Weibull distribution for deteriorating items. (Misra 1975) produced the first concept of output lot size for both the constant and variable deterioration rate. (Tadikamalla 1978) made slight changes to their model. They consider a variable deterioration rate instead of a constant one. (Goyal 1985) Goyal (1985) studied the EOQ model while considering the permitted extension in payment, for the fixed time. (Aggarwal and Jaggi 1995) prolonged Goyal's

(1985) work by considering an extra parameter deterioration. (Benkherouf 1995) eliminated the assumption of decreasing time-dependent demand and constant deterioration rate for an inventory model under a shortage. (Hariga 1996) studied the lot-size problem for deterioration with shortage and time-varying demand. Many researchers, including (Chakrabarti and Chaudhuri 1997); (Giri and Chaudhuri 1997); (Ghosh et al. 2006); (Sabahno 2008); (Sett, Sarkar, and Goswami 2012); (Yadav and Vats 2014); (Santhi and Karthikeyan 2015); (Chowdhury et al. 2017); (P. N. Singh et al. 2017) and some others, have studied in this area. Additionally, this study examines how the demand rate is affected by both the available stock and the selling price. Full and partial shortages are permitted. Investing in preservation lead to a decrease in the deterioration process. The research study by (U. Mishra et al. 2018) investigates the replenishment strategy of a retailer for items that are prone to deterioration in the context of a trade credit policy. Also, this study explores the investment in preservation technologies to lower the pace of deterioration. (Sepehri et al. 2021) present a price-dependent demand for deteriorated goods, with permitted late payments which enable the buyer to control or manage the inventory and generate demand. Moreover, a carbon cap and trade are also considered. It has been noted that there is a trade-off between investing in carbon emission reduction technologies and the profit derived from emission reduction.

Supply chain management for deteriorated products has become one of the most well-known pressing issues today. Supplier-retailer relation goes through various stages, including with coordination or without coordination, and a combination of both strategic actions. Most traditional models consider only supply chain models without coordination. Since supply chain members, such as suppliers and retailers, mutually influence each other, it is necessary to develop a system that enhances collaboration among supply chain management. Models have been developed by several researchers to enhance the relationship between suppliers & retailers. For some illustration, the work of (Banerjee 1986); (Goyal 1988); (Wee, Wee, and Wee 1998); (Chu et al. 1998); (Chung 2000); (Liao, Tsai, and Su 2000); (Yang and Wee 2000); (Teng and Chang 2005); have presented deterioration model in a Supplier-retailer coordination system with two levels. (Wahab et al. 2011) studied three distinct situations: a national, and international supplier-retailer coordination system that considers environmental impact in its decisions. They proposed two-level coordinated supplier-retailer models to find the Policy for optimized production–supply. (Kim and Sarkar 2017) developed a two-level supplier-retailer model that assumes a trade credit scheme and transport concessions. This model provides a system for coordination between transport concessions, trade finance, the number of deliveries, the increase of quality goods, and minimal operating costs, in order to minimize the total cost of the system. In this research approach, the seller provides the trade credit duration to the customer. (Barman, Das, and De 2020) presented a multi-decision-making approach to address a supplier-retailer model for the decaying items. Additionally, demand is influenced by the price of sale, item stock, and time under a finite time horizon. Furthermore, this study addresses the shortages of inventory along with a time-dependent backlogging rate.

For retailers and suppliers, developing the business to enhance customer demands is a challenging feat. In recent years, many researchers have focussed on the field of carbon emission-dependent demand. Some of the related articles have explored the supply chain, including the topics such as the deterioration of items and carbon emission-dependent demand.

(Hovelaque and Bironneau 2015) investigated a unique model that considers the connection between an inventory model (EOQ), total greenhouse gas emissions, and demand that is both price and environmentally dependent. (Huang 2016) developed a two-level inventory supply decision model for only one product that includes a retailer and a producer that is dealing with selling price-and-carbon dependent demand. Product carbon emissions have an impact on both demand and manufacturing costs. When a manufacturer's carbon emissions decrease, demand for the goods rises, and the cost rises. On the other hand, when the product's carbon emissions increase, demand for the product begins to fall, and the cost also starts to decrease. The EOQ model created by (Aliabadi, Yazdanparast, and Nasiri 2019) aims to minimize emissions by implementing a carbon tax policy, while also mitigating default risk. The study takes into account factors such as the demand rate for degradable goods, credit terms, and selling price, with a particular focus on trade credit, along with backlogging. (V. Singh et al. 2019) develops a supply chain coordination inventory model with deterioration for finite horizon planning.

So far, the previously surveyed literature reveals that carbon emissions have been used in a supply chain coordination model with deterioration and several parameters. However, all parameters under infinite or finite horizon planning have been studied only for a single length of a cycle. It seems there are a handful (few) studies that specifically address emissions-dependent demand in inventory and supplier-retailer inventory models with carbon emission and deterioration items. There has been no study before it with deterioration and carbon emission demand in the supply chain inventory model under finite horizon planning for a single length of the cycle.

(Benjaafar, Li, and Daskin 2013) suggest that, in some circumstances, simple operational modifications might result in considerable emission reductions without major cost increases. Furthermore, the study examined how coordination between different entities within a single supply chain affects both costs and carbon emissions. Moreover, the research offers several observations: it is difficult to measure the complete impact of various regulatory policies without considering the operational changes that organizations may make in response to the regulation. Additionally, operational models are valuable tools for assessing the effects of different regulatory policies and the benefits of investing in low-carbon-emitting technologies. (Xi Chen et al. 2013) proposes a requirement in the EOQ model under which emissions can be decreased by adjusting order inventory.

Carbon emissions Regulatory laws were also studied, such as a tight cap-and-trade system, carbon taxes, cap-and-price mechanism, and carbon caps. (U. Mishra, Wu, and Sarkar 2020) explored a sustainable economic production quantity (SEPQ) approach with the carbon tax and the cap under distinct situations with and without shortages by investing in green technologies, which can help reduce carbon emissions. (Q. Zhang, Tsao, and Chen 2014); (U. Mishra, Wu, and Sarkar 2020); (U. Mishra, Wu, and Sarkar 2021); and some other researchers also considered carbon tax and cap in their models.

A supplier-retailer inventory model assumes that full or partial advance payment must be made by the buyer/retailer or customer before the retailer receives the goods from the supplier. The following is a list of related articles that discuss advance payment and the deterioration of items. (Hsu et al. 2006) conducted a research study that explored a stock maintenance approach for degrading goods with an expiration date, which was based on advance payment. (Hariga, As'ad, and Shamayleh 2017) examine an inventory supply network where the supplier offers a complete trade credit option for payments to the retailer, while the retailer provides their customers with only a partial trade credit period. Some retailers offer a price reduction if you pay in advance. The prepaid payment method is an effective way to reduce demand estimation risk and boost sales volume. (Lashgari, Taleizadeh, and Ahmadi 2016) are researchers who investigate an inventory issue in supplier-retailer coordination, including a retailer, a supplier, and several consumers with partial payment delay and partially prepaid. (Teng et al. 2016) addressed advance payments in EOQ models, in which a supplier often demands a buyer to pay a proportion of the acquisition cost in advance as a guarantee. (J. Wu, Teng, and Chan 2018) in this model, for essential commodities, the seller usually also requires advance payment. (Shi et al. 2019) have proposed an EOQ inventory coordination model that takes into account a carbon taxation policy aimed at reducing carbon emissions. In this model, the supplier offers three different payment choices to the retailer, namely advance, cash, and instalment payment, as well as credit options for payment. (U. Mishra, Wu, and Sarkar 2021) researchers introduce an inventory control model with deterioration and carbon emission rate that can be controlled under carbon emission planning including a carbon tax and carbon cap with advance payment. (He et al. 2015) also described advance payment in their model. (Marchi and Zanoni 2023) provided this study. Beatrice Marchi and Simone Zanoni 2023 extend a previous model developed by Huang et al. (2020) to investigate how carbon regulations and green technology affect integrated inventory management in a supply chain. For analysis, three carbon emission strategies are considered: carbon emissions, carbon taxes, and cap-and-trade.

The three-level supply chain covers the entire process of producing, holding, transporting, and selling essential goods. Manufacturers, suppliers, retailers, transports, warehouses, and consumers are all part of the inventory supply chain. A process by which natural resources or raw materials are turned into finished items and then sold to end-users or customers after transportation. The researcher (He et al. 2015) discuss a firm's production lot-sizing problems within the cap-and-trade mechanism and tax approach. (B. Sarkar et al. 2016) explored a three-echelon supply coordination model in which the supplier manufactures semi-finished goods and transports them to the other manufacturer for finished goods. The model's objective is to decrease supply chain costs by including variable transportation and carbon emissions costs from many deliveries. According to (Daryanto et al. 2019), emitting carbon from warehousing and transportation and the disposal of decayed goods are all part of a three-level supply chain.

(S. Wang et al. 2017) focused on the optimization of the Transportation Problem (VRP) with service times to maintain the cold-supply chain in China's carbon tax-based supply networks to minimize the cost pressure on freezing supply chains carried on by the carbon tax policy. Researchers developed a low-cost, low-carbon cold supply chain distribution route for optimization. Additionally, there are fixed costs associated with the vehicle's distribution process, such as damage costs, transportation costs, associated with fresh goods produced by agriculture, the cost of carbon emissions, and freezing costs, to consider. In their study, (Hariga, As’ad, and Shamayleh 2017) examined a multi-stage supplier-retailer chain model consisting of a factory, a distribution centre, a storage facility, and a retailer. The researcher introduces a capital expenditure minimization model, a composite economic and environmental risks reduction model, and a carbon emissions minimization model, all of which aim to determine the best order size and shipping volumes, including the number of vehicles that will be used in supply management, directions and the number of refrigerated units to be used during the entire process and reducing carbon footprint through the use of carbon tax policy. (M. Wang et al. 2018) the research investigates a carbon trading framework in a fresh food supply that uses refrigerated delivery service with the carbon cap-and-trade regulation. It also examines the relationship between refrigerated shipping services and carbon trading, as well as their combined impact on the collaboration between suppliers and retailers. (Hu et al. 2021) used a Model for reducing carbon footprint, the relationship between the expected rate of freshness, the frequency of the refrigerated truck, carbon emission, and the distance coefficient are discussed in this chapter. This study hopefully finds a way to reduce the total cost of a business, control carbon emissions, and benefit the development of the cold chain industry by studying the relationship between such main factors and carbon emissions and overall cost.

(Dye 2013) and (Dye and Yang 2015) considered greenhouse gas strategy tax, a cap-and-trade system, and carbon offsets in their study.

(Asoke Kumar Bhunia and Shaikh 2014) formed a supplier-retailer coordination planning for items that are prone to deterioration with fluctuating demand that is affected by both the selling price and terms of advertising. Shortages are not permitted in the first model but are allowed in the second. The deterioration rate in both models follows a three-parameter Weibull distribution model. (Khan et al. 2020), state that demand is affected by both the price of sale and the advertisement.

Some of the related research work with a three-level supply chain that considers the deterioration of items and rework process can be found in (B. Sarkar et al. 2015); (B. Sarkar 2019); and (B. Sarkar 2019). The study by (Tiwari, Daryanto, et al. 2018) presents an inventory model that integrated a supply chain consisting of a single supplier and a single retailer with decaying goods of inferior quality, considering carbon emissions. The objective of this approach is to reduce overall inventory costs as well as carbon emissions costs. (B. Sarkar 2019) found that the vendor’s setup value is variable and delivery inventory levels are different and unpredictable. After collecting the lot, the customer performs an observation of the delivered products, and if imperfect goods are found, they are returned to the supplier for the rework process. The major objective of considering this model is to show how to reduce the expansion of carbon pollution for the retailer and buyer model. The major purpose of (Tiwari, Daryanto, et al. 2018) in this analysis was the optimization of green production lots or backed-up goods in keeping with trade credit policies to reduce total annual costs. At the end of every sustainable manufacturing cycle, the rework aims to reduce environmental damage by preventing the retailer from passing on defective items. Trade credit policies can act as a source of stock reduction and sales growth. (Rout et al. 2020) look at a supplier-retailer integrated model for a single retailer- single buyer production stock, taking into consideration imperfect items for the rework process, different regulatory policies for emission reduction such as carbon tax, and deterioration. (B. Sarkar et al. 2021) developed a plan that included a three-level supply network approach that includes only one supplier, one producer, and several retailers. The main objective of this plan was to consider coordinated sustainable supply chain management to lower total supply chain costs. Simultaneously, the intention was to control deterioration and reduce emissions. To avoid excessive holding expenditures, the retailer keeps the order according to customer requirements. The demand is considered to be constant throughout the supply chain and the majority of a retailer's expenditure is spent on ordering and holding inventory. (De-la-Cruz-Márquez et al. 2021) provide a supplier and retailer inventory framework for growing low-grade commodities when the demand for products is price dependent, relating to carbon emissions. The impact of a carbon tax, as well as the amount of carbon footprint, is analysed. After the screening procedure, all objects of poor quality are sold in a single collection. In supply chain management, (Ahmed et al. 2022) propose mending defective products locally to save resources and decrease the environmental effect. By merging product repair and multi-trade credit rules, it offers a multi-trade credit period for global purchasing and produces an inventory model to optimise supply chain profit. Sensitivity assessments evaluate the model under various supply chain characteristics. (Maheshwari et al. 2023) develops a resource-efficient rework and remanufacturing model to solve the matter of decreasing supply chain costs and impact on the environment in the three-layered supply chain. Using Lingo and Mathematica software, analytical optimisation techniques are used to minimise system costs across the most efficient planning horizon. The determination of parameters to the objective function is investigated via sensitivity analysis.

2.2. Motivation and background (research gap):

Numerous academics, scientists, and researchers have made significant contributions to the field of supply chain inventory models. The majority of research in this area has focused on developing models for either infinite planning horizons or finite planning horizons with equal cycle lengths.

As far as we know, no researchers have yet created a model for inventory management of items experiencing degradation with associated demand dependent on price or emissions, specifically designed for supplier-retailer scenarios under a finite planning horizon.

Additionally, no research has been conducted on the coordination of supplier-retailer inventory management in the context of carbon emission policies, such as carbon taxes or caps, for deteriorating products with the demand that depends on both time and carbon emissions and may involve partial or full advance payment under finite planning horizon. This represents a significant research gap, and the proposed work is unique in that it considers issues that are relevant to both economic and environmental concerns.

Another gap in current research is the absence of any discussion by researchers on a three-level supply network for managing inventory of items that are subject to deterioration with refrigeration, carbon emissions dependent demand, and subject to carbon tax regulations, all within a finite planning horizon.

Numerous authors have delved into various aspects of inventory management, ranging from carbon emissions and advertising to defective items, rework processes, carbon offsets, and inspections. However, to date, no literature has explored the cumulative impact of carbon and advertisement-dependent demand, defective items, rework processes, and carbon offset regulations on a supply network model operating within a finite planning horizon.

After conducting a thorough literature review, I have identified four main objectives that I propose to pursue in my research.

In supply chain control and management with carbon emission under a finite planning horizon, the optimal cycle length dependent on total cost is nowhere to be found in the academic literature review. Therefore, based on the research gap, this study tries to focus on the parameters such as two-level and three-level supply chains, management with deterioration items, advance payment, refrigeration, carbon emission, and advertisement-dependent demand and rework parameters. some carbon emission policies will be used to show how these policies affect carbon emission and supply chain management. The supplier-retailer inventory for deterioration items with different carbon emission planning such as the carbon cap, carbon offset, carbon tax, and cap credit, can be modelled in a finite planning horizon.

Nowadays, the world economic scenario has turned up with one highly essential aspect: trying to minimize carbon emissions. Organizations or industries all over the entire globe are now expected to attain their long-term or sustainable (economic, environmental, and social) targets. They have to maintain a balance between profitability and long-term growth. Therefore, this research study is motivated by the idea of establishing a carbon-reduction environment under finite planning while keeping all sustainable targets.

To our knowledge, this is the first study that takes into account inventory control with different policies of carbon emission under finite planning. Therefore, the proposed research topic is ***"Supply chain inventory models for different carbon emission policies under finite planning horizon".***

Figure 2.1 Problem Identify.

2.3. Research objectives of the proposed research study:

The following are the objectives of this study:

i. A supplier-retailer inventory coordination model for deteriorating items with price and carbon emission-dependent demand in a finite horizon would be examined.
ii. A supply chain inventory model for deteriorating items with carbon emission-dependent demand and advanced payment including a carbon tax and cap policy will be studied.
iii. To analyze a three-level refrigeration supply chain model including carbon emission-dependent demand for temperature-sensitive items considering carbon tax regulations.
iv. To analyze carbon offset for a production system with carbon emission dependent and advertisement dependent demand for rework process of defective production under a sustainable supply chain management.

2.4. Proposed methodology:

As part of the proposed research, a mathematical model was developed through a planned methodology to determine the optimal level of costs and inventory based on the literature review. The resulting model directly incorporates factors such as time, costs, and optimal profit or cost within the firm.

The issue of carbon emission in inventory control and management has become an important and current topic of discussion among researchers and scientists. Its potential applications are wide-ranging, spanning fields such as marketing and management, economics, geography, biology, physics, finance, and chemistry.

In this study, numerical iterative methods are described to check the correctness and truthfulness of the mathematical model. I am using Mathematica software which helps me to solve numerical calculation work. Mathematica is the most widely used software for research. Mathematica software is very helpful to solve mathematics problems and is time-saving too.

Few Mathematica commands such as Find Root[{f1,f2,…},{{x,x0},{y,y0},…}] seek to find a numerical solution to the set of simultaneous equations.

Several steps were used to develop a better understanding of the information requirements to develop a model for inventory management. These steps include:

- A literature reviews.
- Development of a theoretical model mathematically.
- The model analysis.
- A numerical example and Theorems were also developed after the theoretical model has been created.
- A sensitivity analysis has been done along with graphical representation and even in tabular form.
- comparative study of the model results with existing literature.

A mathematical model was also derived from the reviewing of the literature by using planned Methodology to identify the optimal level of cost and inventory. Within the firm, the model directly reflects the time, various costs, amount of carbon emission and the optimal profit or cost.

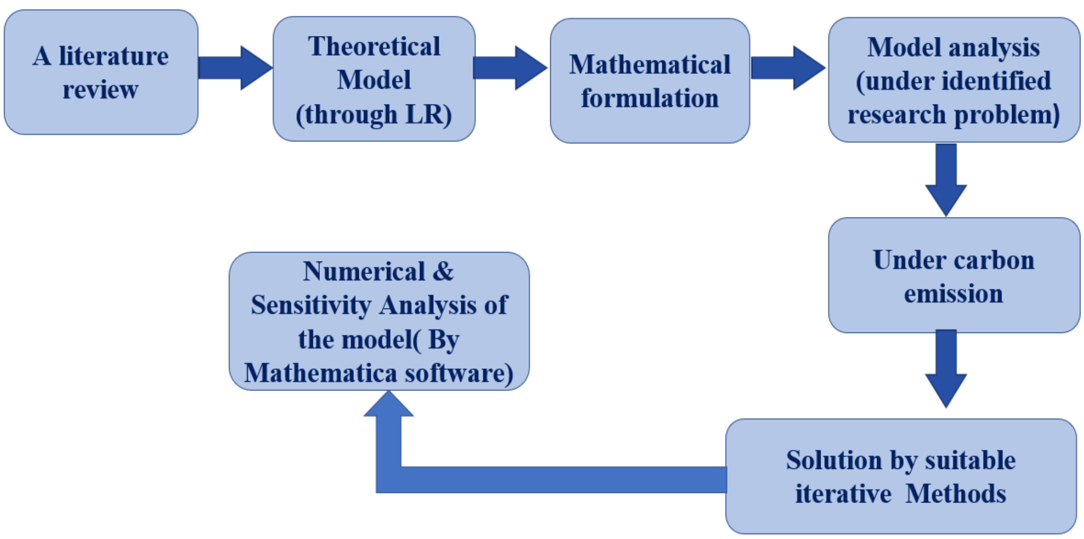

Figure 2.2 Research Methodology

I. Differential equations:

Now, consider a D(t) demand over the time interval $t_i < t < t_{i+1}$ given by D(t) = a+ bt, where a and b are positive or zero and θ the rate of deterioration of inventory. level the inventory level at any time t is given by differential equation.

$$\frac{dI_{b_i}(t)}{dt} = -D(t) - \theta * I_{b_i}(t) \quad \text{where } t_i < t < t_{i+1} \tag{2.1}$$

The inventory level is given by the

$$I_{b_i}(t) = e^{-\theta t} \int_t^{t_{i+1}} (a + bu)e^{\theta * u}\, dt \tag{2.2}$$

The order quantity for ith cycles

$$Q_{i+1} = I_{b_i}(t_i) = \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\, dt \tag{2.3}$$

II. Optimization:

i. Extreme value theorem:

In the Mathematical Optimization Model, the Extreme Value theorem is used to show the G(x) real continuous function obtained maximum or minimum value within any interval. Let G(x) be a continuous function with the interval [a,b] and then the graphical representation of extremal values.

ii. Hessian matrix definition:

Now, let a G(x) be a function in n variables. The hessian matrix of G is square and consists of the second order partially derivative w.r.t to independent variables. Function G(x) is a convex function if the Hessian matrix is positive definite. Moreover, the Hessian matrix had to be positive definite since it contains positive diagonal members and has strictly diagonal dominating features as shown below.

The fact that the Hessian matrix of G(t) is positive definite is sufficient for G(t) to be minimal\maximum or Optimum (Mishra et al. (2012) and Sarkar et al. (2012)).

$$\nabla^2 G(t) = \begin{bmatrix} \frac{\partial^2 G(t)}{\partial t_1^2} & \frac{\partial^2 G(t)}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 G(t)}{\partial t_2 \partial t_1} & \frac{\partial^2 G(t)}{\partial t_2^2} & \frac{\partial^2 G(t)}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 G(t)}{\partial t_3 \partial t_2} & \frac{\partial^2 G(t)}{\partial t_3^2} & \frac{\partial^2 G(t)}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 G(t)}{\partial t_{n_1-1} \partial t_{n_1-2}} & \frac{\partial^2 G(t)}{\partial t_{n_1-1}^2} & \frac{\partial^2 G(t)}{\partial t_{n_1-1} \partial t_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 G(t)}{\partial t_{n_1} \partial t_{n_1-1}} & \frac{\partial^2 G(t)}{\partial t_{n_1}^2} \end{bmatrix}$$

Figure 2.3 Hessian matrix

III. Use of computers Mathematica software version 12.0:

Following an algorithm and writing a program for the software Mathematica version 12.0 to calculate the optimal total cost for the retailer as well as supplier and order quantity, replenishment time, and replenishment cycles can be obtained which is in the table below. Mathematica software is very helpful to solve mathematics problems and is time-saving too. Few Mathematica commands such as.

1. Find Root[{f1, f2, …},{{x,x0},{y,y0},…}], which seeks to find a numerical solution to the set of simultaneous equations.

2. $t_{j+1} = t_{j+1}/.$ FindRoot[EQ12, $\{t_{j+1}, 0\}$, AccuracyGoal → 4, PrecisionGoal → 4];

3. Print[MatrixForm[Tabforti_v]];

See appendix B for the Mathematica program.

Table 2.1 Total cost of the retailer, supplier, order quantity, replenishment time and replenishment cycle in a supply chain system

↓ a → t_i	t_0	t_1	t_2	t_3	t_4	t_5	n	T_{Ret}	T_{sup}	Q_{nt}
0.001	0	1.42	2.0827	2.6329	3.1244	4	5	30942.0	15428.31	18092.29

2.5. Areas of application:

i. This study aims to offer organizations guidance on making inventory decisions that align with various environmental requirements while effectively utilizing technologies.
ii. By providing more flexibility across the production process, this study can enable better resource utilization, potentially leading to increased profits.
iii. This study has been conducted that may offer insights for governments and regulatory agencies to create policies that promote economic growth while reducing carbon emissions.
iv. The study could provide valuable information for businesses seeking to forecast demand.
v. This research study must be available to optimize supply chain management in real time, which can help control inventory and increase working capacity. It also suggests that industry collaboration can help support firms in reducing costs using this information.
vi. Proposed research outcomes contribute significantly to creating a more sustainable environment by reducing carbon emissions from production processes, supply chains, or services. This research study suggests that this reduction will be achieved by using carbon footprint information and identifiable analysis.

2.6. Sources of data collection:

i. **Primary Data:** I have planned to use Primary data which consists mainly of personal interaction &Telephonic, data analysis, field visits (visits to the industry), photocopies, and inspections are utilized as required.
ii. **Secondary Data:** secondary information is also used in addition to primary sources. Externally and internally secondary data sources can be used. Data and activities mentioned in Annual reports, as well as the sample's sites on the internet, are used as internal sources. Data from publications and research papers can be used in external.

Table 2.2 Methods of Data Collection

S.No.	Primary Data	Secondary Data
1.	Fetched data directly from sources through specific research methods.	Fetched data from existing sources that were collected by other researchers for different analysis objectives.
2.	Survey: a. Interview. b. Observations.	Internal Sources: a. Annual Reports. b. Authentic Websites.
3.	Field Visit. Data Analysis.	External Sources: a. Published Research, b. Journals.

2.7. Industrial visit report:

Visit Sugar Factory: Kaithal Cooperative Sugar Mill

Sugar industry E-mail ID: ktlsugar@gmail.com

Date & Time of Visit: 12TH August- 2021

Duration: 1 Day

Kaithal Cooperative Sugar Industries

Kaithal Cooperative Sugar Industries is registered under, the Haryana State Federation of Coop. Sugar Mills Ltd., Panchkula. it was established in 1991. It is one largest sugar industries in Haryana. The sugar plant has a capacity of 25,00 TCD (ton cane per day).

Working process:

On the 12th of August 2021, we went to Kaithal Cooperative Sugar Mill. First, we went to the Sugar Industry. we arrived at Sugar Industries at 9.15 am. First of all, we met Mr. Sumit Kumar, a general worker of a sugar mill who guides me on where we should go, to get proper information about the working process. After that, we went to the Chief.

Engr. A.A.Siddiqui office and Chief Chemist K.K.Tiwari introduces my working process. Then I visited the whole industry with Chief Chemist K.K. Tiwari. Sugar is produced using the Double Sulphitation Process, and a Sugar Syrup Clarification System has been built in the facility to meet international standards. The manufacturing process follows three steps.

- SUGAR
- BAGASSE
- MOLASSES

The Bagasse is left after crushing the sugarcane, that is used to produce energy. after producing energy only 3-4% of Bagasse is left that later sell to other industries.

Chief Chemist told me about their warehouse. they had four warehouses and also talked about their transportation system. The retailer has a transportation system during the supply of sugar. The industry just hires transportation to purchase raw materials.

The supply process is done online only. Online porter has been provided by the government and a website **https://etenders.hry.nic.in** has been provided on which the employees of sugar mills put their daily sugar rates, according to which the retailer will place their demand. After that, The Ch. chemist took us to the sugar sale manager Mr Jagdish Chander who gave me information about the tax. He said that within the states 5% GST on sugar with which 2.5% CGST and 2.5% SGST. Out of state only 5% IGST. He also said that the tax imposed on Molasses is 28% and 5% at Bagasse.

After getting all the tax information, we went to Mr. Balwan Singh the accountant of the Sugar mill who provided me with all the working process data. He provided me with a manufacturing and trading report, a profit and loss report, a report of the cost of sugar production, and a balance sheet for the financial year 2019-20.

Summary:

The visit was aimed to enhance knowledge about the working process, to understand the operations that are used in the SUGAR INDUSTRY, Kaithal, and to understand the different processes implemented to prepare sugar from sugarcane. At Industry we were able to see and understand different mechanical operations carried out in the industry for instance cutting, shredding, screening, centrifuging, etc. The aim of the visit was fulfilled at the end of the "SUGAR INDUSTRY" visit and last, we came back to my home.

Some pictures were taken during the industry visit.

Bagasse left after production

Manufacturing area

Bagasse used to produce energy

Boiler house

Chapter 3: An Inventory Model for the Association Between Retailer and Supplier for a Finite Planning Horizon with Carbon Emission-Dependent Demand.

3.1. Abstract:

As people's concern about environmental issues grows, the influence of emissions on demand becomes increasingly evident in the field of biomaterials engineering and the manufacturing of other materials. Due to carbon emissions and some other factors some of the items may deteriorate. develop. In both the centralized and decentralized cases, we are solving the model. This study supports retailers and suppliers in reducing total inventory costs and carbon emissions by computing the optimal amount of the order and optimal order interval. Finally, we are presenting numerical examples of the suggested method and its optimal results. A sensitivity analysis has also been conducted with the help of Mathematica version-12. In addition, several managerial insights are also highlighted. Further, this chapter is extended with the carbon tax and advance payment in the next chapter.

3.2. Introduction:

Deterioration for inventory control and management, the rate of deterioration has become a big issue. Inventory deterioration is a very common occurrence in our daily lives. Everything continues to deteriorate with carbon emissions and price-dependent demand. Deterioration is represented by the term’s dryness, brokenness, evaporation, rotting,spoilage, and damage. There are many types of deteriorating items. The first type of deteriorating items is those whichinclude all items that are destroyed with time, price, and due to carbon emissions like vegetables, flowers fruit, Biomaterials (drugs and pharmaceuticals), and so on. The second category of deteriorating items refers to those goods which fall their value due to technological changes such as computers and mobile phones. Due to the increasing recognition of the impact of deterioration on inventory control and management, scholars have dedicated significant attention to studying degradation inventory schemes in recent years. So, the research study on decaying inventory control and management has grown to be much more extensive. Several publications such as (Ghare and Schrader 1963) and (Covert, Philip, and Philip 1973) based on deteriorating items have already beenpublished. Recognizing the importance of environmental considerations, governments and some national or international carbon emissions regulatory agenciesare heading toward a way that the product and its inventory management should be free from environmental impacts.

A critical aspect of the inventory supply chain system is stock management and controlof deteriorated goods. Management of the supply network for deteriorated products becomes one of the most well-known pressing issues today. Supplier-retailer coordination has to go through various stages with coordination or without coordination, and a combination of both strategic actions. Most traditional models are considered only supplychain models without coordination. But supply chain members such as suppliers and retailers, affect each other, making it necessary to develop a system that enhances collaboration among supply chain management with carbon emission regulation.

3.3. Literature review:

Deterioration has become a significant concern in inventory control and management. To draw attention to this type of phenomenon, (Ghare and Schrader 1963) the first two famous authors looked at the deterioration rate with a constant demand rate and a constant degradation rate for inventory control and management. Further, (Covert, Philip, and Philip 1973) extended the model byconsidering two parameters of Weibull distribution for deteriorating items. (Misra 1975) developed the first model of out-put lot size for both the variable and constant deterioration rate. (Tadikamalla 1978) modified the conceptual framework of (Ghare and Schrader 1963) with a few changes. Who took a variable deterioration rate instead of a constant. (Goyal 1985) studied the EOQ model considering the sufficient amount of time for payment. Goyal’s work was advanced by (Aggarwal and Jaggi 1995) by considering an extra parameter deterioration. Various other researchers such as (Hariga 1996)6); (Giri and Chaudhuri 1997); (Chakrabarti and Chaudhuri 1997); (Ghosh et al. 2006); (Sabahno 2008); (Sett, Sarkar, and Goswami 2012); (Yadav and vats 2014); (Santhi and Karthikeyan 2015);

(Chowdhury et al. 2017); (V. Singh et al. 2017) and (V. Singh et al. 2018) and some others also have considered deterioration in their research work. (U. Mishra et al. 2017) considered the demand rate as a function of selling price and stock, allowing for full and partial Shortage. Investing in preservation was found to slow down the deterioration process. (U. Mishra et al. 2018) considers a retailer's Credit policy for items that are degrading and invest in preservation technologies to slow down the deterioration process. (Sepehri et al. 2021) presented a model for deterioration commodities, considering price dependent demand, with permitted late payments for the customers to manage the inventory and generate demand, as well as a carbon cap and trade, is also considered. It is noteworthy that there must be a trade-off between the investment in carbon emission reduction technologies and the profit derived from reducing emissions.

Models have been developed by several researchers to enhance the relationship between the supplier and retailers. For someillustration, (Banerjee 1986); (Goyal 1988); (Haleem et al. 2021); (Wee, Wee, and Wee 1998); (Chu et al. 1998); (Chung 2000); (Liao, Tsai, and Su 2000); (Yang and Wee 2000); (Teng and Chang 2005) work has been presented deterioration model and Supplier-retailer coordination system. (Wahab et al. 2011) Studied three distinct situations: a national, and international supplier-retailer coordination system that considers environmental impact in its decisions, and two-level coordinated supplier-retailer models are proposed to find the policy for optimized production and supply. (Kim and Sarkar 2017) a two-level supplier-retailer model, assuming a trade credit scheme, and transport concessions to provide such a system for coordination between transport concessions, trade finance, the number of deliveries, the increase of qualitygoods, and minimal operating costs. The objective was to bring the minimum level of the total cost, where the seller provides the customer trade credit duration. (Barman, Das, and De 2020) presented a multi-decision-making approach for deteriorating items with shortages. A supplier-retailer model is used, where demand is determined by the price of sale, item stock, and time under a finite time horizon. Shortages of inventory are addressed along with a time-dependent backlogging rate. Enhancing customer demands is a challenging feat for retailers and suppliers in today's competitive market. The selling price of an item is one of the most essential elements in choosing any product. It is usually observed that demand falls as a result of a slightly higher price, while a lower selling price has the opposite impact. Besides pricing, another commercial factor that influences demand is carbon emission. Climate change has become increasingly dangerous in recent time, leading people to purchase more eco-conscious items as a result of this idea. This indicates that the demand for some particular item will be affected by environmental issues. The purpose of this type of demand is to decrease carbon emissions and result in increased demand for the product. Therefore, it can be stated that an item's demand is a function of carbon emissions and its selling price. In the last few years, many researchers have focused on the field of carbon emission-dependent demand. Some of the related study on a supply chain include the deterioration of item/s and carbon emission-dependent demand. These findings were also supported by several studies. For instance, (Hovelaque and Bironneau 2015) investigated a unique model that considers the connection between an inventory model, total greenhouse emissions, and demand that is both environmentally and price interdependent. (Huang 2016) developed a two-level supply decision model based on a particular product that includes a producer and a retailer that is dealing with selling price-and-carbon dependent demand. Product carbon emissions have an impact on both demand and manufacturing costs. When a manufacturer's carbon emissions are decreased, demand for the goods rises, and the cost rises as well. On the other hand, when the product's carbon emissions are increased, demand for the product begins to fall, and the cost also starts to decrease. (Aliabadi, Yazdanparast, and Nasiri 2019) developed an EOQ inventory model to reduce emissions through a policy that is a carbon tax, lowering the hazard of default. The demand rate is influenced by greenhouse emissions, credit term, as well as selling price. (Li 2016) investigates the impact of carbon emissions and price on-demand, as well as the coordination between supplier and retailer. From the above literature review, it is evident that very few publications in existing research took greenhouse gas emissions-dependent demand. None of the authors reviewed have considered carbon and price-dependent demand for deteriorating items under a finite planning horizon. Therefore, we are considering all these parameters in this chapter. The main purpose of this research work is to lower carbon emissions and achieve a lower total cost through coordination compared to without coordination. Furthermore, to investigate the supply chain, we consider demand dependent on greenhouse gas emissions under a finite planning horizon in this chapter.

3.4. Assumptions:

In this chapter, we have taken the following assumptions and applied some additional assumptions and ratings as appropriate.

1. The effects of carbon emissions on demand are expressed in the form: D (p, G) = a-b p-c G. The firm's per-unit pricing is p, and the quantity of emissions produced per unit of product is G in this function. if the initial demand of the market is a market demand depends upon price will be b, and c is the consumer's sensitivity to carbon emissions per unit. That is, when the price rises by one unit, demand falls by b, and when carbon emissions per unit product rise by one unit, demand falls by c.
2. There is no lead time.
3. It will not be allowed to have any shortages in this model.
4. The time horizon for planning is finite.
5. The quantity of replenishment is not constant but instantaneous.

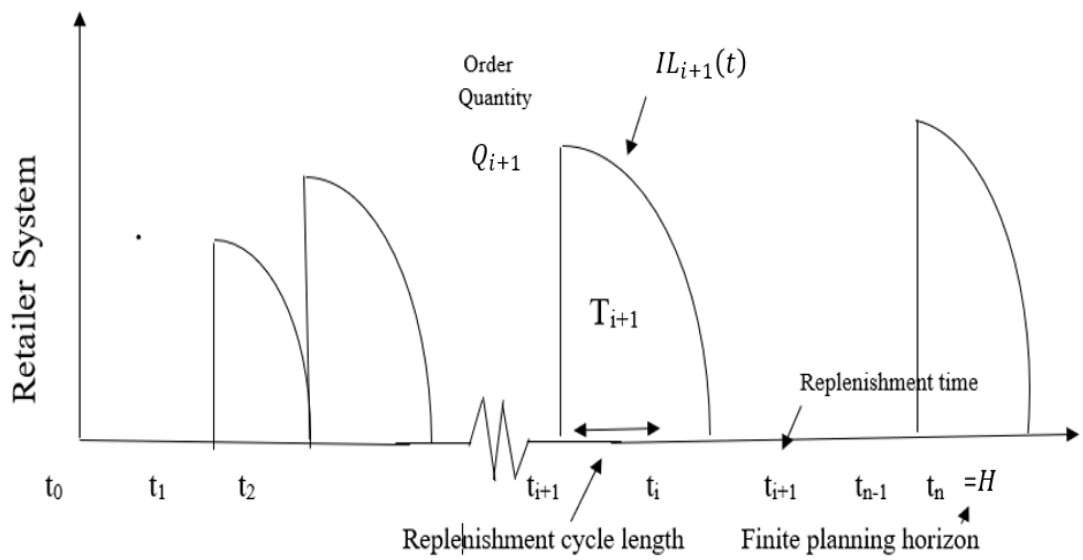

Figure 3.1 Inventory level graph

3.5. Mathematical formulation and analysis of the model:

3.5.1. Decentralized Strategy:

Model of decentralized strategic planning under finite planning horizon: - Assume that H is the planning period that is partitioned into m equal parts H_i = H/n_1 segments. As a result, across the planning horizon T, the ordering times are J. At t_0 = 0, the initial replenishment quantity size of Q_{i+1} has been finished. According to the definition, the Stock levels are declining due to demand alone within the period [0, t_i].

carbon emissions put an effect on demand that can be represented in the way: D (P, G) = a–b∗P−c∗G. The firm's per-unit pricing is p, and the quantity of emissions produced per unit of product is G in this function. if the initial demand of the market is a market demand depends upon price will be b, and c is the consumer's sensitivity to carbon emissions per unit. That is, when the price rises by one unit, demand falls by b, and when carbon emissions per unit product rise by one unit, demand falls by c. Therefore, the Stock level is represented by using a mathematical differential equation is:

$$\frac{dIL_{i+1}(t)}{dt} = -\mathrm{D}(\mathrm{P},\mathrm{G}) - \theta IL_{i+1}(t) \qquad t_i < \mathrm{t} < t_{i+1} \qquad (3.1)$$

$$IL_{i+1}(t) = e^{-\theta t}\int_{t}^{t_{i+1}} D(P,G)e^{\theta u}\,\mathrm{du} \qquad (3.2)$$

$$IL_{i+1}(t_{i+1}) = 0 \quad \text{and} \quad IL_{i+1}(t_i) = Q_{i+1} \qquad (3.3)$$

$$IL_{i+1}(t) = \int_{t}^{t_{i+1}} D(P,G)e^{\theta(u-t)}\,\mathrm{du} \qquad (3.4)$$

$$IL_{i+1}(t) = \left[\frac{D(P,G)e^{\theta(u-t)}}{\theta}\right]_t^{t_{i+1}}$$

$$IL_{i+1}(t) = \frac{D(p,g)\left\{e^{\theta(t_{i+1}-t)}-1\right\}}{\theta} \qquad (3.5)$$

The order quantity for i^th cycles

$$Q_{i+1} = IL_{i+1}(t_i) = \frac{D(p,g)\left\{e^{\theta(t_{i+1}-t_i)}-1\right\}}{\theta} \quad (3.6)$$

Retailer's overall cost

Total cost =Ordering cost + Inventory cost + purchasing cost + Deteriorating cost

$$T_{Ret}{}^{IND} = n_1 * O_r + \sum_{i=0}^{n_1-1} h_r \int_{t_i}^{t_{i+1}} IL_{i+1}(t)\, dt + \sum_{i=0}^{n_1-1} W * Q_{i+1} + \sum_{i=0}^{n_1-1} d_r * \theta \int_{t_i}^{t_{i+1}} IL_{i+1}(t)\, dt \quad (3.7)$$

$$T_{Ret}{}^{IND} = n_1 * O_r + \sum_{i=0}^{n_1-1} W * Q_{i+1} + \sum_{i=0}^{n_1-1}\{d_r * \theta + h_r\} \int_{t_i}^{t_{i+1}} IL_{i+1}(t)\, dt \quad (3.8)$$

$$T_{Ret}{}^{IND} = n_1 * O_r + \sum_{i=0}^{n_1-1}\{d_r * \theta + h_r\} \int_{t_i}^{t_{i+1}} \frac{D(p,g)\left\{e^{\theta(t_{i+1}-t)}-1\right\}}{\theta}\, dt + \sum_{i=0}^{n_1-1} W * \frac{D(p,g)\left\{e^{\theta(t_{i+1}-t_i)}-1\right\}}{\theta} \quad (3.9)$$

$$T_{Ret}{}^{IND} = n_1 * O_r + \{d_r * \theta + h_r\} \int_{t_{i-1}}^{t_i} \frac{D(p,g)}{\theta}\left\{e^{\theta(t_i-t)} - 1\right\} \mathrm{dt} + W * \int_{t_{i-1}}^{t_i} D(p,g)\left(e^{\theta(t-t_{i-1})}\right) \mathrm{dt} + \{d_r * \theta + h_r\} \int_{t_i}^{t_{i+1}} \frac{D(p,g)}{\theta}\left(e^{\theta(t_{i+1}-t)} - 1\right) \mathrm{dt} + W * \int_{t_i}^{t_{i+1}} D(p,g)\left(e^{\theta(t-t_i)}\right) \mathrm{dt} + \cdots \quad (3.10)$$

Where i=1,2,3…………n_1-1

$$\frac{\partial\left(T_{Ret}{}^{IND}\right)}{\partial t_i} = \left\{\frac{h_r}{\theta} + d_r\right\}(D(p,g)\left\{\frac{\partial}{\partial t_i}\int_{t_{i-1}}^{t_i}\left(e^{\theta(t_i-t)} - 1\right) \mathrm{dt}\right\} + W * \left(D(p,g)\right) * \left\{\frac{\partial}{\partial t_i}\int_{s_{i-1}}^{t_i}\left(e^{\theta(t-s_{i-1})} - 1\right) \mathrm{dt}\right\} + \left\{\frac{h_r}{\theta} + d_r\right\}\left\{\frac{\partial}{\partial t_i}\int_{t_i}^{t_{i+1}}\left(D(p,g)\right)\left(e^{\theta(t_{i+1}-t)} - 1\right) \mathrm{dt}\right\} + W * \left\{\frac{\partial}{\partial t_i}\int_{t_i}^{t_{i+1}} D(p,g)\left(e^{\theta(t-t_i)}\right) \mathrm{dt}\right\} \dots\dots\dots\dots\dots \quad (3.11)$$

$$\frac{\partial\{T_{Ret}{}^{IND}\}}{\partial t_i} = \left\{\frac{h_r}{\theta} + d_r\right\}(D(p,g) * \theta\ \left\{\int_{t_{i-1}}^{t_i} e^{\theta(t_i - t)}\ \mathrm{dt} + \left\{1 - e^{\theta(t_{i+1} - t_i)}\right\}\right\} + W * \left(D(p,g)\right)\left\{\ e^{\theta(t_i - t_{i-1})}\ - 1\right\} - \left\{\theta \int_{t_i}^{t_{i+1}}\left(e^{\theta(t - t_i}\right)\ \mathrm{dt}\right\} \tag{3.12}$$

Min $(T_{Ret}{}^{IND})$ $\{n_1, t_0, t_1 \dots \dots \dots \dots \dots . . t_{n_1-1}\}$ at the optimal solution $\{n_1{}^*, t_0{}^*, \dots \dots \dots \dots \dots \dots \dots\ . t_{n_1-1}{}^*\}$

Again, the second time derivative of Eqn (3.12) w.r.t $t\ _i$.

$$\frac{\partial^2\{T_{Ret}{}^{IND}\}}{\partial t_i{}^2} = \left\{\frac{h_r}{\theta} + d_r\right\}(D(p,g) *\ \left\{\theta^2 \int_{t_{i-1}}^{t_i} e^{\theta(t_i - t)}\ \mathrm{dt} + 1 + \theta^2 * e^{\theta(t_{i+1} - t_i)}\ \right\} + W * \left(D(p,g)\right)\ \left\{\ e^{\theta(t_i - t_{i-1})}\ \right\}\left\{\theta^2 \int_{t_i}^{t_{i+1}}\left(e^{\theta(t - t_i}\right)\ \mathrm{dt}\right\} + \theta \tag{3.13}$$

Second-time derivative of Eqn. (3.12) w. r. t. t_{i-1}.

$$\frac{\partial^2\{T_{Ret}{}^{IND}\}}{\partial(t_i)(\ t_{i-1})} = -\theta\left\{\frac{h_r}{\theta} + d_r\right\}(D(p,g)\left\{\ e^{\theta(t_i - t_{i-1})}\right\} - \theta\ W\ (D(p,g)\left\{\ e^{\theta(t_i - t_{i-1})}\ \right\} \tag{3.14}$$

$$\frac{\partial^2\{T_{Ret}{}^{IND}\}}{\partial(t_i)(\ t_{i-1})} = -\theta\left\{\frac{h_r}{\theta} + d_r + W\right\}(D(p,g)\left\{\ e^{\theta(t_i - t_{i-1})}\ \right\} \tag{3.15}$$

$\text{Again second} - \text{time Differentiate of Equ.}\,(3.12)\ \text{w.r.t to}\ t_{i+1}\ \text{then we have}$

$$\frac{\partial^2\{T_{Ret}{}^{IND}\}}{\partial(t_i)(\ t_{i+1})} = -\theta^2\left\{\frac{h_r}{\theta} + d_r\right\}(D(p,g)\left\{\ e^{\theta(t_{i+1} - t_i)}\ \right\} - \theta^2\ W\ (D(p,g)\left\{\ e^{\theta(t_{i+1} - t_i)}\ \ \right\} + \theta^2\ W\ (D(p,g) \tag{3.16}$$

$$\frac{\partial^2\{T_{Ret}{}^{IND}\}}{\partial(t_i)(t_{n_1})} = 0 \tag{3.17}$$

For all n is different from i-1, i+1 and i. The fact that the Hessian matrix of $T_{Ret}{}^{IND}$ is positive definite is sufficient for a total cost to be minimal. (T. Sarkar et al. 2012) and (V. Singh et al. 2017)

$$\nabla^2 T_{Ret}{}^{IND} = \begin{pmatrix} \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_1^2} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_2 \partial t_1} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_2^2} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_3 \partial t_2} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_3^2} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\ 0 & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_{n_1-1} \partial t_{n_1-2}} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_{n_1-1}^2} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_{n_1-1} \partial t_{n_1}} \\ 0 & 0 & 0 & 0 & 0\ 0 & 0 & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_{n_1} \partial t_{n_1-1}} & \frac{\partial^2 T_{Ret}{}^{IND}}{\partial t_{n_1}^2} \end{pmatrix}$$

Figure 3.2 Hessian matrix

Condition to check positive definite: -

$$\left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1, t_0, t_1 \ldots\ldots\ldots\ldots\ldots . t_{n_1})}{\partial t_i{}^2}\right| \geq \left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1, t_0, t_1 \ldots\ldots\ldots\ldots\ldots . t_{n_1})}{\partial(t_i)(t_{i-1})}\right| + \left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1, t_0, t_1 \ldots\ldots\ldots\ldots\ldots . t_{n_1})}{\partial(t_i)(t_{i+1})}\right|$$

(3.18)

$$\left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1,t_0,t_1 \ldots\ldots\ldots t_{n_1})}{t_i{}^2}\right| - \left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1,t_0,t_1 \ldots\ldots\ldots t_{n_1})}{\partial(t_i)(t_{i-1})}\right| - \left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1,t_0,t_1 \ldots\ldots\ldots t_{n_1})}{\partial(t_i)(t_{i+1})}\right| \geq 0$$

(3.19)

$$\left\{\frac{h_r}{\theta} + d_r\right\}(D(p,g) * \left\{\theta^2 \int_{t_{i-1}}^{t_i} e^{\theta(t_i-t)}\ \mathrm{dt} + 1 + \theta^2 * e^{\theta(t_{i+1}-t_i)}\right\} + W * \left(D(p,g)\right)\ \left\{e^{\theta(t_i-t_{i-1})}\right\}\left\{\theta^2 \int_{t_i}^{t_{i+1}}\left(e^{\theta(t-t_i)}\ \mathrm{dt}\right\} + \theta - \theta\left\{\frac{h_r}{\theta} + d_r + W\right\}(D(p,g)\left\{e^{\theta(t_i-t_{i-1})}\right\} - \theta^2\left\{\frac{h_r}{\theta} + d_r\right\}(D(p,g)\left\{e^{\theta(t_{i+1}-t_i)}\right\} - \theta^2\ W(D(p,g)\left\{e^{\theta(t_{i+1}-t_i)}\right\} + \theta^2\ W\ (D(p,g)\ \geq\ 0 \quad (3.20)$$

3.1. Theorem: - If t_i satisfies the inequations

(i) $\left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1,t_0,t_1 \ldots\ldots\ldots t_{n_1})}{t_i{}^2}\right| \geq 0$

(ii) $\left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1,t_0,t_1 \ldots\ldots\ldots t_{n_1})}{t_i{}^2}\right| \geq \left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1,t_0,t_1 \ldots\ldots\ldots t_{n_1})}{\partial(t_i)(t_{i-1})}\right| + \left|\frac{\partial^2 T_{Ret}{}^{IND}(n_1,t_0,t_1 \ldots\ldots\ldots t_{n_1})}{\partial(t_i)(t_{i+1})}\right|$

For all i = 1, 2, 3,………….. $n_1 - 1$. Then $\nabla^2\ T_{Ret}{}^{IND}$ is positive and definite.

In the absence of collaboration, the supplier's overall costs are determined by the retailer's replenishment schedule. As a result, during the planning horizon T, the supplier's total costs consist of set-up and manufacturing costs.

Supplier total cost:

$$T_{sup}{}^{IND} = n_1 * S_s + \sum_{i=0}^{m_1^*-1} P_r * Q_{i+1}^* \quad (3.21)$$

Furthermore, throughout the T planning horizon, the optimum order quantity is determined by

$$Q_{i+1} = \sum_{i=0}^{n_1^*-1} P_r Q_{i+1}^* \quad (3.22)$$

3.5.2. Centralized Strategy:

Model of centralized strategic planning under finite planning horizon: - The supplier initiates the coordination process intending to reduce the overall cost of the inventory supply chain process. Collaboration and coordination between the supplier and the retailer are crucial to achieving this objective. The supplier and the retailer work together as a single entity to optimize the cost of the entire inventory supply chain process, which is typically achieved through supply chain collaboration and coordination, taking into account factors such as the type of goods, their deterioration, and their demand. The business needs a Collaboration to strike a balance between cost and environmental effects. It is possible to optimize replenishment schedules to reduce total expenses while considering environmental issues. The objective of the centralized inventory decision model in a two-echelon inventory supply chain process is to reduce the overall cost and carbon emissions across the inventory supply chain process. The unit product carbon emissions (g) and the sales price (p) must be determined to achieve this objective. As a result, the overall supply chain cost function is influenced by a centralized system.

$$T_{sup}{}^{JT} = n_2 * S_s + n_2 * O_r + \sum_{j=0}^{m_2-1}\{d_r * \theta + h_r\} \int_{s_j}^{s_{j+1}} IL'_{j+1}(t)dt + \sum_{j=0}^{m_2-1}\{W + P_r).\} Q'_{j+1} - Min\, T_{Ret}{}^{IND} \quad (3.23)$$

$$T_{sup}{}^{JT} = n_2\{S_s + O_r\} + \sum_{j=0}^{m_2-1}\{d_r * \theta + h_r\} \int_{s_j}^{s_{j+1}} IL'_{j+1}(t)dt + \sum_{j=0}^{m_2-1} W * Q'_{j+1} + \sum_{j=0}^{m_2-1} P_r).* Q'_{j+1} - Min\, T_{Ret}{}^{IND} \quad (3.24)$$

$$With\ t'_0 = 0\ and\ t'_{n_{2-1}} = T$$

Min $T_{sup}{}^{JT}\{n_2, t'_0, t'_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots . t'_{n_2-1\}}\ \}$ at the optimal solution

$\{n_2, t_0^{*\prime}, t_1^{*\prime} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . t *'_{n_2-1\}}\}$

Min $T_{sup}{}^{JT}\{n_1, t_0, t_1 \ldots\ldots\ldots\ldots\ldots . . t_{n_1-1}\}$ at the optimal solution

$\{n_1{}^*, t_0{}^*, \ldots\ldots\ldots\ldots\ldots\ldots\ldots .\ t_{n_1-1}{}^*\}$

Furthermore, throughout the H planning horizon, the optimum order quantity is determined.

$$Q_{j+1} = \sum_{j=0}^{n_2^*-1} Q_{j+1}^{*\prime} \tag{3.25}$$

$$T_{sup}{}^{JT} = n_2 * O_r + \sum_{j=0}^{n_2-1} W * Q_{j+1} + \sum_{j=0}^{n_2-1}\{d_r * \theta + H\}\int_{t_j}^{t_{j+1}} IL_{i+1}(t)\ \ dt \tag{3.26}$$

$$T_{sup}{}^{JT} = n_2 * O_r + \sum_{j=0}^{n_2-1}\{d_r * \theta + h_r\}\int_{t_j}^{t_{j+1}} \frac{D(p,g)\left\{e^{\theta(s_{i+1}-t)}-1\right\}}{\theta}\ \ dt + \sum_{j=0}^{n_2-1} W * \frac{D(p,g)\left\{e^{\theta(t_{i+1}-t_i)}-1\right\}}{\theta}$$

The cost of the system has been reduced, and in the association scenario, the retailer and supplier share the profits earned from shorter replenishment cycles instead of the disassociation scenario.

$$P = \left\{\{T_{Ret}{}^{IND} + T_{sup}{}^{IND}\} - \{T_{Ret}{}^{JT} - T_{sup}{}^{JT}\}\right\}$$

Gained Retailer's cost $T_{Ret}{}^{JT^P} = \left\{T_{Ret}{}^{IND}(n_1) - \frac{T_{Ret}{}^{IND}(n_1)}{T_{Ret}{}^{IND}(n_1)+T_{sup}{}^{IND}(n_1)}\right\} * P$

Gained Supplier's cost $T_{sup}{}^{JT^P} = \left\{T_{sup}{}^{IND}(n_1) - \frac{T_{sup}{}^{IND}(n_1)}{T_{Ret}{}^{IND}(n_1)+T_{sup}{}^{IND}(n_1)}\right\}$

Now percentage gain in the Retailer's cost = $\frac{T_{Ret}{}^{JT^P}}{T_{Ret}{}^{IND}(n_1)} \times 100$

Now percentage gain in Supplier's cost = $\frac{T_{sup}{}^{JT^P}}{T_{sup}{}^{IND}(n_1)} \times 100$

3.6. Algorithm and procedures for resolving the problem:

1. First of all, constant values to all given parameters; W_h; P_r; dr ; O_r; a; b; c; θ, G.
2. To find the optimal ordering pattern in a decentralized situation.

a. If we will set, $n_1 = 1$ then $t_0 = 0$ and $t_1 = H$.

b. If we take $n_1 = 2$, then by initializing the value of $t_0 = 0$ and $t_2 = H$. After that, we can calculate t_1 by using eq.(3.12).

c. From eq.12, find t_2 using the calculated values of t_1 in the previous step.

d. Again, taking the values of t_2, calculate t_3 from equation (3.12). Continuing in the same way till t_{n_1-1} is obtained.

e. Having the values of t_{n_1-1} nearly equal to H (horizon planning), and the values of t_i is satisfying the theorem of the Hessian matrix.

f. For each n_1= 1; 2; 3; :::: We will calculate the unique and optimal values of t_i .

g. By using equation (3.10), $T_{Ret}{}^{IND}(n_1)$ is obtained to determine the optimal value of the total cost of the retailer $T_{Ret}{}^{IND}(n_1)$ by using the following conditions:

If n_1=1, then $T_{Ret}{}^{IND}(n_1) = T_{Ret}{}^{JT}(n_1)$ and stop. For $n_1 \geq 2$, and if $T_{Ret}{}^{IND}(n_1) \leq T_{Ret}{}^{IND}(n_1 - 1)$ and $T_{Ret}{}^{IND}(n_1) \leq T_{Ret}{}^{IND}(n_1 + 1)$ then $T_{Ret}{}^{IND}(n_1) =$ optimal T_{Ret} and stop otherwise go to the previous step. Similarly, $T_{sup}{}^{IND}(n_1)$ and $Q_{i+1}(n_1)$ system by using eq. (3.21) and (3.22).

3. To find the optimality for the association case.

   a. After that, with new replenishment n_2 we can find $t_{i.}$

   b. Again, Set $n_2 = 1;\ t_0 = 0; t_1 = H$.

   c. Then take $n_2 \geq 2,$ Calculate t_i from equation (3.24).

   d. Calculate ${T_{sup}}^{JT}(n_2), {T_{Ret}}^{JT}(n_2),$ and $({Q^*}_{i+1})(n_2)$ for the centralized case. from equation (3.24) to (3.26).

4. Now, also calculate profit, gained retailer cost and gained supplier cost.

3.7. Numerical illustration for the proposed model:

All of the different parameters and replacement cycles, i.e., for $n_1 - 1 = 1, 2, 3,$ and so on. Numerical data illustration: - This section gives a piece of information about numerical data; we'll look at numerical data to illustrate how various factors affect the overall cost of retailers and suppliers. Let a=0.15, h_r =12, d_r = 10, a = 0.15, b = 0.000001, W_h = 4, P_r= 50, O_r =2, S_s =1, θ=0.1, p=5, G=1, c = 0.00001, H=4 with the appropriate units. We will use Mathematica software (version 12) to solve Non-linear Equations (3.10) and (3.12). we can see tabulation and graphical presentation of the overall cost of suppliers and retailers and optimal quantity.

3.8. Sensitivity analysis and findings:

This section uses reliable information to present numerical examples for performing sensitivity analysis. The goal is to understand the effects of variations in the main parameter on other parameters in the proposed model, which can provide valuable managerial information. Sensitivity analysis measures the impact by modifying each of the elements: G, a, c, and b. Only one element is modified at a time while the others remain constant. By adjusting various parameters, numerical data is used to explain the sensitivity of the aforementioned parameters. Based on these criteria, certain important findings have been mentioned. Sensitivity analysis leads to the following conclusions: the initial demand of the market (a), price-consciousness in the market (b), the amount of carbon (G), and customer responsiveness to carbon emissions per unit (c) all have an effect on the ideal order quantity, overall retailer and supplier costs. The optimal overall cost, retailer, supplier, and quantity are all affected by these parameters. These parameters also impact the demand. In summary, the initial demand (a), price consciousness in the market (b), the amount of carbon, and consumer environmental consciousness (c) influence the demand. Therefore, this study considers the price and carbon-dependent demand.

Following are the effects of all these parameters.

1. The parameters ‘a’ have a significant impact on both ${T_{Ret}}^{IND}$ (total cost of the retailer without collaboration). Table [1] and Figure [3] demonstrate that ${T_{Ret}}^{IND}$ is very sensitive to changes in 'a,' and their values increase with an increase in the value of 'a.' The optimal total cost for the retailer occurs at the 4^{th} replenishment cycle. Conversely, a decrease in 'a' results in a decrease in $Q_{i+1}(n_1)$.

   Tables [2] demonstrate that there are no changes in the optimal cycle time with increases or decreases in 'a'. As a result, the optimal order point is not influenced by the parameter a.

   The values of ${T_{sup}}^{IND}$ (total cost of the supplier without collaboration) in Tables [3] increase with an increase in 'a' and decrease with a decrease in 'a. The optimal supplier cost is obtained at the 1^{st} replenishment cycle for all values, and there is a positive relationship between ${T_{sup}}^{IND}$ and 'a'

2. In Tables [4], the Optimal order quantity is dramatically increased or decreased by some increases or decrease parameter changes a’’. Replenishment quantity is also sensitive to parameter a.

   Moreover, In Table [5] and Figure [4] the total cost of ${T_{sup}}^{JT}$ increases with an increase in 'a' and vice versa.

   The values of ${T_{sup}}^{IND}$ in Tables [6] increase with an increase in 'a' and decrease with a decrease in 'a.' The optimal supplier cost is obtained at the 1^{st}

replenishment cycle for all values, and there is a positive relationship between ${T_{sup}}^{IND}$ and 'a'.

3. In Table [7] and Figure [5] values of ${T_{Ret}}^{IND}$ increase when 'b' decreases and vice versa. The optimal total cost for the retailer occurs at the 2nd and last replenishment cycle values for 'b'. However, there is a negative relationship between ${T_{Ret}}^{IND}$ and 'b'.

   Also, the optimal order time in Table [8], is not significantly change in the optimal cycle time with increases or decreases in 'b'. As a result, the optimal order point is not influenced by the parameter a.

   In Tables [9] ${T_{sup}}^{IND}$ decreases with an increase in 'b' and increases with a decrease in 'b'. whereas the optimal order quantity in Table [10] is significantly oppositely affected by a certain decrease or increase in the parameter b'. Replenishment quantity is low sensitive to the parameters b.

   Also, ${T_{sup}}^{JT}$ in Table [11] is significantly oppositely affected by certain decreases or increases in the parameter 'b'. Replenishment quantity is also sensitive to the parameters b. The optimal supplier cost is obtained at the 1st replenishment cycle for all values, and there is an opposite relationship between ${T_{sup}}^{JT}$ and 'b'.

4. Table [12] and Figure [7] demonstrate a clear pattern: as the value of 'c' increases, the values of ${T_{Ret}}^{IND}$ increase, and vice versa. The optimal total cost for the retailer is observed during the 6th replenishment cycle values of 'c'. There exists a positive relationship between ${T_{Ret}}^{IND}$ and 'c'.

   Table [13] demonstrates that there are no changes in the optimal cycle time with increases or decreases in 'c'. As a result, the optimal order point is not influenced by the parameter 'c'.

   The optimal order quantity in Tables [14], is significantly oppositely affected by certain decreases or increases in the parameter ‘c'. Replenishment quantity is also sensitive to the parameters c.

Table [15] demonstrates a consistent trend: as the value of 'c' increases, ${T_{sup}}^{IND}$ decreases, and conversely, as 'c' decreases, ${T_{sup}}^{IND}$ increases. The optimal supplier cost is obtained during the 1st replenishment cycle for all values of 'c'. There exists an inverse relationship between ${T_{sup}}^{IND}$ and 'c

Moreover, In Table [16] and Figure [8] the total cost of ${T_{sup}}^{JT}$ increases with an increase in 'c' and vice versa.

5. The variable 'G' greatly influences the total cost of the retailer without collaboration, as shown in Table [17] and Figure [9]. It is evident that ${T_{Ret}}^{IND}$ (total cost of the retailer without collaboration) is highly responsive to variations in 'G,' with its values decreasing as 'G' increases. The optimal total cost for the retailer is observed during the 6th replenishment cycle.
According to Table [18], it is evident that the optimal cycle time remains constant regardless of changes in the parameter 'G.' As a result, the optimal order point is not influenced by this parameter ‘G’. However, replenishment quantity is also sensitive to the parameter G.

3.9. Tables and Graphical Formulation:

Table 3.1. Total cost for the retailer when not in coordination with five different values of a

↓→ n_1 a	1	2	3	4	5	6
0.072	22.309217	21.362434	20.595100	20.522287	20.968959	21.775582
0.0735	22.732414	21.724227	20.899228	20.783223	21.197527	21.979282
0.075	23.155611	22.086020	21.203356	21.044158	21.426094	22.182983
0.0765	23.578808	22.447813	21.507484	21.305094	21.654662	22.386683
0.078	24.00200	22.809605	21.811612	21.566029	21.8832	22.590384

Table 3.2. Lowest total cost for the retailer and optimal number of replenishment cycles when not in coordination

a	n_1^*	t_0	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	T_{Ret}^{IND}
0.072	4	0	0.8	1.6	2.4	3.2	-	-	20.522287
0.0735	4	0	0.8	1.6	2.4	3.2	-	-	20.783223
0.075	4	0	0.8	1.6	2.4	3.2	-	-	21.044158
0.0765	4	0	0.8	1.6	2.4	3.2	-	-	21.305094
0.078	4	0	0.8	1.6	2.4	3.2	-	-	21.305094

Table 3.3. Total cost for a supplier when not in coordination for five different values of a

a	T_{sup}^{IND}
0.072	49.146117
0.0735	50.170208
0.075	51.194299
0.0765	52.218390
0.078	53.242481

Table 3.4. Optimal replenishment quantity when not in coordination

a	*Quantity*
0.072	0.289095
0.0735	0.295119
0.075	0.301143
0.0765	0.307167
0.078	0.313191

Table 3.5. Total cost for supplier with coordination for five different values of a

$\downarrow a$ / $\rightarrow n_1$	1	2	3	4	5	6
0.072	1.690419	3.463660	5.852332	8.486184	11.242357	14.068326
0.0735	1.829831	3.577510	5.953444	8.579666	11.330757	14.153100
0.075	1.969244	3.691360	6.054555	8.673148	11.419158	14.237875
0.0765	2.108657	3.805210	6.155666	8.766629	11.507559	14.322649
0.078	2.24807	3.91906	6.25678	8.86011	11.596	14.4074

Table 3.6. Optimal replenishment quantity when retailer and supplier are in Co-ordination for a

a	*Quantity*
0.072	0.290839
0.0735	0.296899
0.075	0.302959
0.0765	0.309020
0.078	0.315080

Table 3.7. Total cost for the retailer when not in coordination for five different values of b

$\downarrow b \rightarrow n_1$	1	2	3	4	5	6
$4.8 * 10^{-7}$	11.731999	11.277049	12.053669	13.320944	14.833003	16.484740
$4.9 * 10^{-7}$	11.731995	11.277046	12.053667	13.320942	14.833002	16.484738
$5. * 10^{-7}$	11.731992	11.277044	12.053665	13.320940	14.833000	16.484737
$5.1 * 10^{-7}$	11.731989	11.277042	12.053663	13.320939	14.832999	16.484735
$5.2 * 10^{-7}$	11.731986	11.277039	12.053661	13.320937	14.832997	16.484734

Table 3.8. Optimal no of replenishment cycle and minimum total cost for retailer and supplier when not in coordination for different values of b

b	n_1^*	t_0	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	T_{Ret}^{IND}
$4.8 * 10^{-7}$	2	0	1.33333	2.66667	-	-	-	-	20.522287
$4.9 * 10^{-7}$	2	0	1.33333	2.66667	-	-	-	-	20.783223
$5. * 10^{-7}$	2	0	1.33333	2.66667	-	-	-	-	21.044158
$5.1 * 10^{-7}$	2	0	1.33333	2.66667	-	-	-	-	21.305094
$5.2 * 10^{-7}$	2	0	1.33333	2.66667	-	-	-	-	21.305094

Table 3.9. Total cost forsupplier for b when not in coordination for five different values of b

b	${T_{sup}}^{IND}$
$4.8 * 10^{-7}$	50.330644
$4.9 * 10^{-7}$	50.330627
$5.0 * 10^{-7}$	50.330610
$5.1 * 10^{-7}$	50.330594
$5.2 * 10^{-7}$	50.330577

Table 3.10. optimal replenishment quantity for b when not in coordination

b	${T_{sup}}^{IND}$
$4.8 * 10^{-7}$	0.603968
$4.9 * 10^{-7}$	0.603968
$5.0 * 10^{-7}$	0.603967
$5.1 * 10^{-7}$	0.603967
$5.2 * 10^{-7}$	0.603967

Table 3.11. Total cost for supplier and optimal quantity with coordination for five different values of b

b	$n_2 = 1$	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 6$	$Q_{j+1}(JT)$
$4.8 * 10^{-7}$	1.754454	32.198386	33.924627	36.161723	38.653684	41.291074	0.605990
$4.9 * 10^{-7}$	31.754444	32.198376	33.924617	36.161713	38.653675	41.291065	0.605989
$5.0 * 10^{-7}$	31.754433	32.198366	33.924608	36.161704	38.653665	41.291056	0.605989
$5.1 * 10^{-7}$	31.75442	32.198356	33.924598	36.161694	38.653656	41.291047	0.605989
$5.2 * 10^{-7}$	31.754411	32.198346	33.924588	36.161685	38.653647	41.291038	0.605989

Table 3.12. Total cost for the retailer when not in coordination with five different values of c

$c \quad \rightarrow n_1$	1	2	3	4	5	6
0.072	45.556357	41.822656	38.258508	36.060324	34.904130	34.474901
0.0735	45.556359	41.822658	38.258510	36.060325	34.904131	34.474902
0.075	45.556362	41.822660	38.258512	36.060327	34.904133	34.474904
0.0765	45.556364	41.822662	38.258513	36.060328	34.904134	34.474905
0.078	45.556366	41.822663	38.258515	36.060330	34.904135	34.474906

Table 3.13. Retailer total overall costs when not in coordination for five different values of c

c	n_1^*	t_0	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	T_{Ret}^{IND}
0.072	6	0	0.571428	1.14286	1.71429	2.28571	2.28571	3.42857	34.474901
0.0735	6	0	–	–	-	-	-	-	34.474902
0.075	6	0	–	–	-	-	-	-	34.474904
0.0765	6	0	–	–	-	-	-	-	34.474905
0.078	6	0	–	–	-	-	-	-	34.474906

Table 3.14. Optimal replenishment quantity When retailer andsupplier are not in co-ordination for c.

c	*Quantity*
0.072	0.312873
0.0735	0.306855
0.075	0.300838
0.0765	0.294821
0.078	0.288804

Table 3.15. The total costof supplier when re-tailer and supplier are not in coordination forc.

c	T_{sup}^{IND}
0.072	2.234862
0.0735	2.191884
0.075	2.148905
0.0765	2.105926
0.078	2.062948

Table 3.16. Total cost for supplier with coordination for five different values of c

$C \rightarrow n_1$	1	2	3	4	5	6
0.072	8.560008	9.003722	10.729854	12.966885	15.458803	18.096163
0.0735	8.560009	9.003723	10.729855	12.966886	15.458804	18.096163
0.075	8.560009	9.003723	10.729856	12.966886	15.458804	18.096164
0.0765	8.560010	9.003724	10.729856	12.966887	15.458805	18.096165
0.078	8.560011	9.003725	10.729857	12.966888	15.458806	18.096165

Table 3.17. Total cost for the retailer when not in coordination for five different values of g

↓G → n_1	*1*	*2*	*3*	*4*	*5*	*6*
0.48	39.450838	37.751187	34.897364	33.027623	32.049753	31.728021
0.49	39.450813	37.751165	34.897345	33.027607	32.049738	31.728008
0.50	39.450788	37.751142	34.897326	33.027590	32.049723	31.727995
0.51	39.450763	37.751120	34.897306	33.027573	32.049709	31.727981
0.52	39.450738	37.751097	34.897287	33.027557	32.049694	31.727968

Table 3.18. Retailer total overall costs when not in coordination with five different values of G

G	n_1^*	t_0	t_1^*	t_2^*	t_3^*	t_4^*	t_5^*	t_6^*	${T_{Ret}}^{IND}$
0.48	6	0	0.571428	1.14286	1.71429	2.28571	2.28571	3.42857	31.728021
0.49	6	0	–	–	-	-	-	-	31.728008
0.50	6	0	–	–	-	-	-	-	31.727995
0.51	6	0	–	–	-	-	-	-	31.727981
0.52	6	0	–	–	-	-	-	-	31.727968

Table [19] illustrates the total cost incurred by the retailer and supplier, taking into account factors such as the order quantity, and number of replenishment cycles, for both coordination and non-coordination scenarios. Additionally, Table [20] shows the percentage of improvement achieved through coordination between the retailer and supplier.

Table 3.19. Percentage gain for five different values of a

a	${T_{Ret}}^{IND}$	${T_{sup}}^{IND}$	n_1	Q_{i+1}^*	${T_{Ret}}^{JT}$	${T_{sup}}^{JT}$	n_2	${Q_{i+1}^*}'$
0.072	20.522287	49.146117	4	0.289095	6.67077	1.690419	1	0.290839
0.0735	20.783223	50.170208	4	0.295119	6.7681	1.829831	1	0.0.296899
0.075	21.044158	51.194299	4	0.301143	6.86543	1.969244	1	0.302959
0.0765	21.305094	52.218390	4	0.307167	6.96276	2.108657	1	0.309020
0.078	21.566029	53.242481	4	0.313191	7.06008	2.24807	1	0.315080

Table 3.20. Percentage gain for retailer and supplier

The overall gain in the cost of supply chain	${T_{Ret}}^{JT^P}$	${T_{sup}}^{JT^P}$	Percentage gain in retailer's cost	Percentage gain in supplier's cost
61.3072	2.46296	5.89823	12.0014	12.0014
62.3555	2.51845	6.07948	12.1177	12.1177
63.4038	2.57367	6.26100	12.2299	12.2299
64.4521	2.62865	6.44277	12.3381	12.3381
65.5004	2.68338	6.62477	12.4426	12.4426

Figure Formulation:

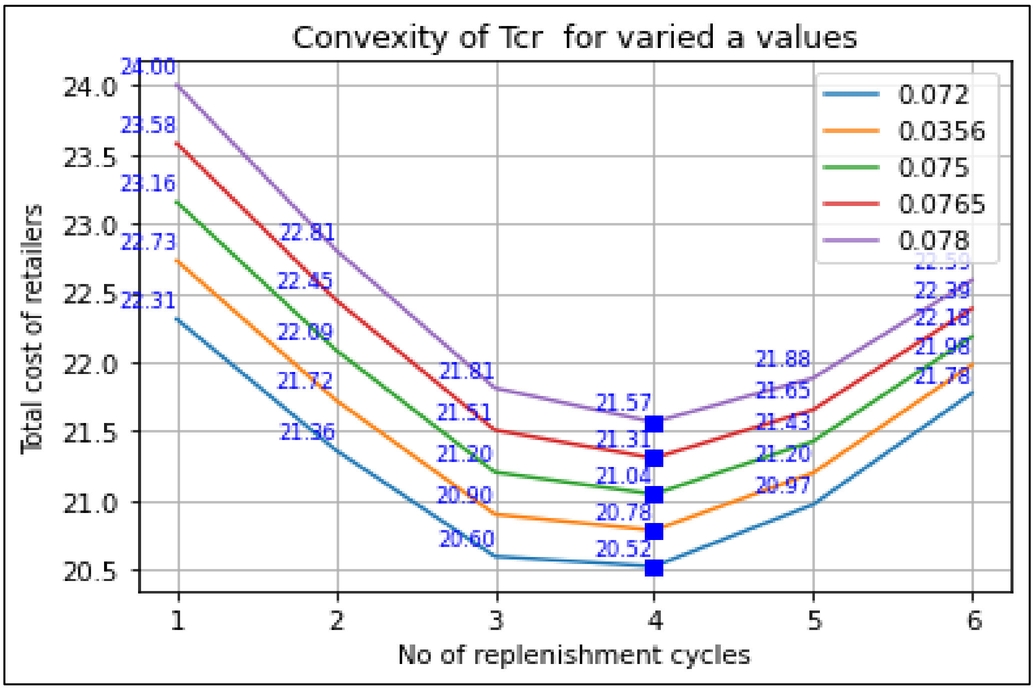

Figure 3.3 Total retailer cost graph for different values of a

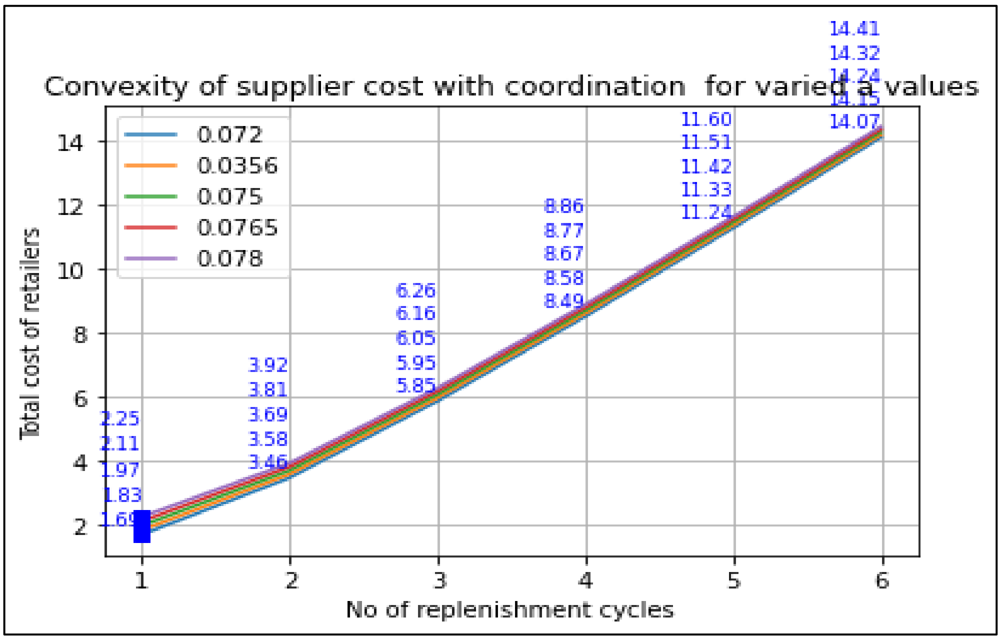

Figure 3. 4 Total supplier cost graphs for different values of a

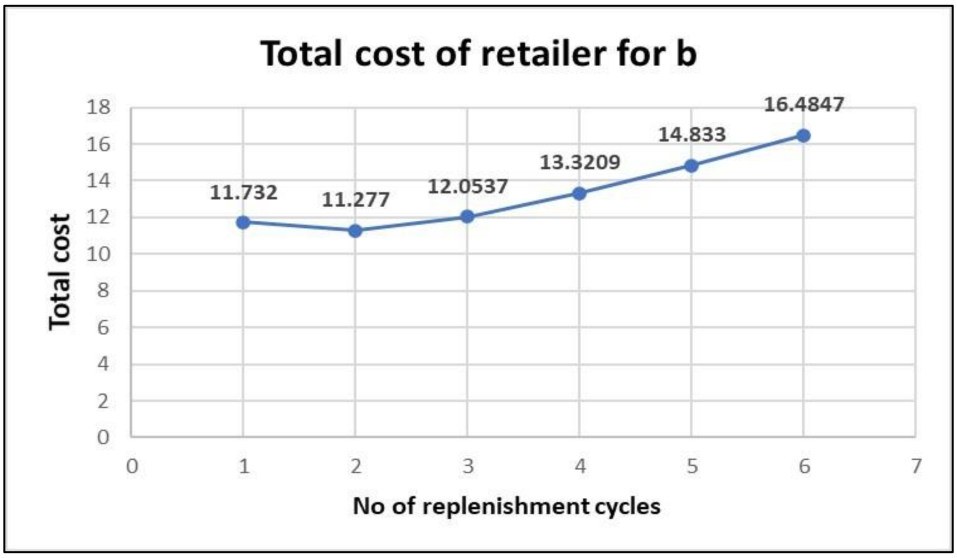

Figure 3.5 Total retailer cost graph for the values of b

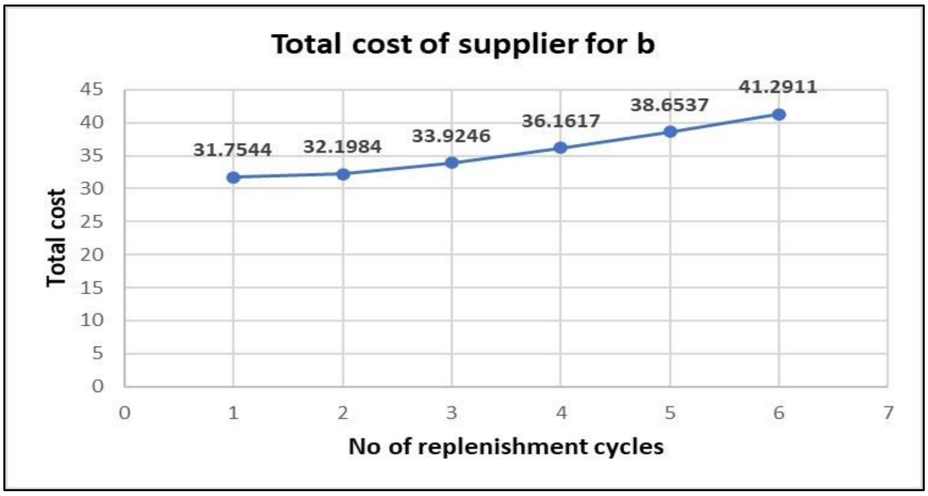

Figure 3.6 Total supplier cost graph for the values of b

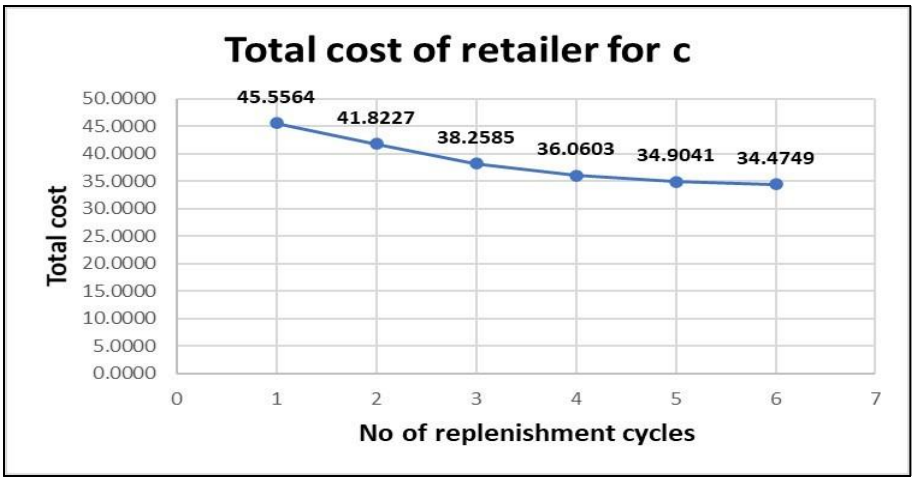

Figure 3.7 Total retailer cost graph for the values of c

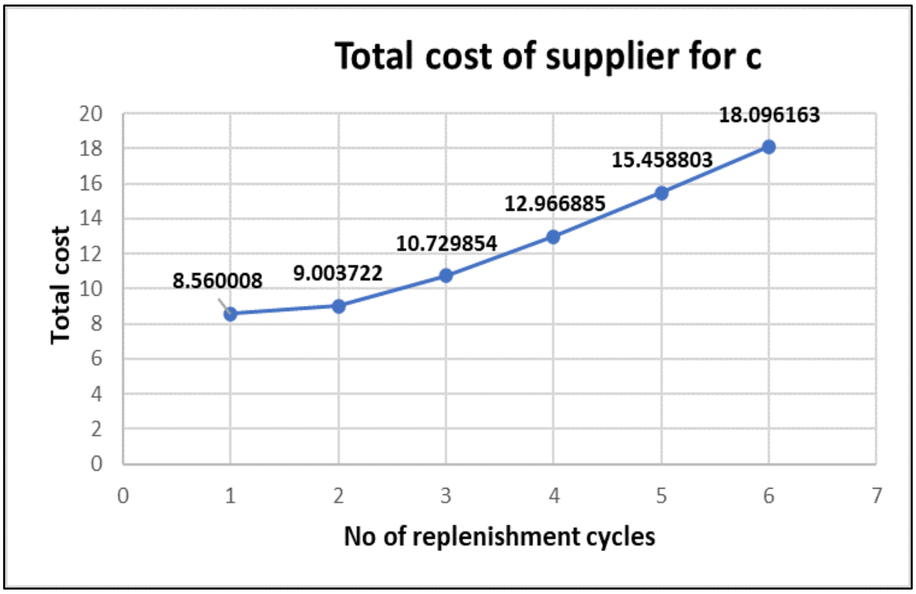

Figure 3.8 Total supplier cost graph for the values of c

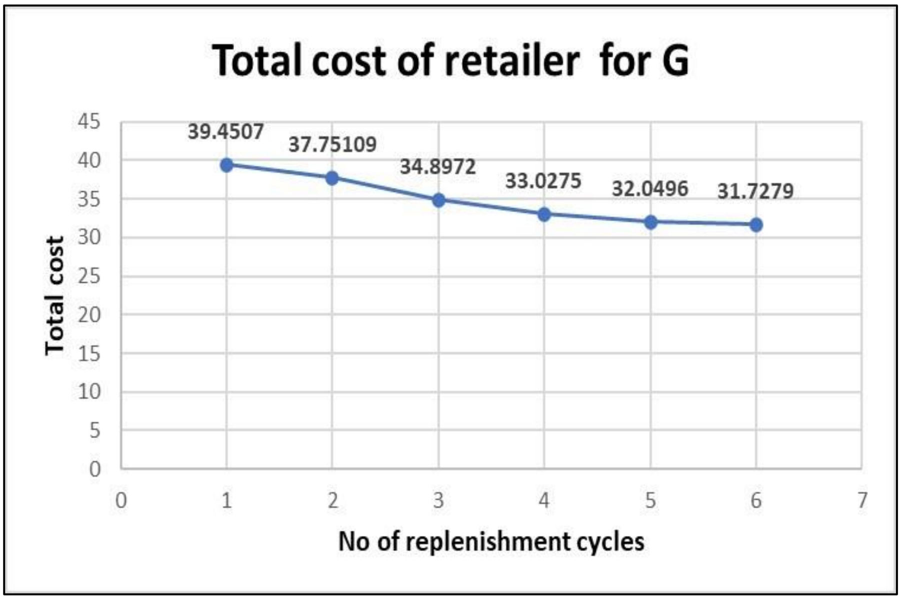

Figure 3.9 Total retailer cost graph for the values of g

3.10. Comparative Study:

This work has led Wu and Zhao's model by looking at the impact of new characteristics like carbon-dependentand price demand, as well as deterioration, on the relationship between retailer and supplier over a finite planning horizon. An analytical solution to the problem has been studied for two cases: I where the supplier follows the retailer's optimal replenishment schedule (de-association system) and (ii) where the retailer follows the supplier's optimal replacement schedule (association system). Suppliers and retailers are motivated by cost savings and profit sharing in a centralized system. A brand-new algorithm has been proposed. To compare the proposed model toWu and Zhao's a situation is explored in which the parameter values are the same as in Example 1 of their model. Furthermore, all of the new parameters introduced in this model have values p=0.4, G=16, and c=6. The cost savings achieved by both the retailer and the supplier as a result of the suggested algorithm are listed in Table 3.20. As a result, the present approach is more effective for both retailers and suppliers under profit strategies.

Table 3.21. Comparison Chart

Parameters →	*a*	*b*	*c*	*C*	*P*	*O*	*S*	*I*	d_r	θ	h_r	W_h	*T*	*G*	*p*	% Cost saving of retailer	% Cost saving of supplier
Wu and Zhao (1), model	500	1000	-	0.4	1.2	50	150	1	-	0.1	0.6	3	1	-	-	2.92	5.41
	1500	□	□	□	□	□	□	□	□	□	□	□	□	□	□	1.53	3.14
	2500	□	□	□	□	□	□	□	□	□	□	□	□	□	□	1.19	2.53
Proposed model	500	1000	6	0.4	1.2	50	150	1	1.3	0.1	0.6	3	1	16	0.4	99.66	99.66
	1500	□	□	□	□	□	□	□	□	□	□	□	□	□	□	98.40	98.40
	2500	□	□	□	□	□	□	□	□	□	□	□	□	□	□	98.38	98.38

Note: -

□: The box indicates the same values have been considered as in the immediate cell above.

– : Parameters are not considered by (C. Wu and Zhao 2014).

3.11. Conclusion and future study of proposed work:

With the growing popularity of living a green lifestyle, there is a higher need for carbon-free products. In line with this perspective, we focus on developing a two-echelon inventory model for coordination that considers carbon emissions and deteriorating items in this research study. It is an inventory model that allows people to make informed decisions on which product to manufacture, sell or purchase. Moreover, the supplier-retailers are encouraged to select products with a low rate/amount of carbon emissions to optimise the overall cost of the supply chain. We have discussed the selection of optimum order quantity and the optimal total cost by using advanced software Mathematica, algorithm, and numerical examples. In addition, a comparative study was conducted concerning the main parameters, providing important managerial insights. Graphical representations are also provided to easily understand the model. The presented model encompasses realistic scenarios that are relevant to specific types of inventories. The approach proves particularly beneficial in supplier retailer businesses dealing with household items, electronic equipment, bio-materials such as drugs and pharmaceuticals (Amato 2015) fashionable garments, and for few other goods that are available for sale. Furthermore, there is potential for extending this study in the future.

3.11.1. Future study:

This chapter is further explored in the next chapter 4. Moreover, this planned research could be expanded in the future to include advanced payment, fuzzy, and carbon emission tax and cap. It's also possible to add criteria like trade credit and demand based on pricing and advertisement-dependent demand. This work can be also extended for multiple retailers (Nandra, Majumder, and Mishra 2020).

3.12. Recommendations for stakeholders:

To minimize total costs, managers and stakeholders prioritize the reduction of both the deterioration rate and carbon emissions. The constraint should be kept as low as possible to avoid deterioration. Many companies, as well as real-life circumstances, would benefit from a thorough focus on the deteriorating rate and carbon emissions. By adopting this approach, any company can effectively lower its total cost and increase profit in the supply chain process.

This approach is particularly advantageous and beneficial for supplier-retailer businesses, dealing with household items, electronic equipment, fashionable garments, environmental-friendly products (organic and sustainable food items, energy-efficient appliances, eco-friendly cleaning products, renewable energy systems, and recycled) and other available goods for sale.

Chapter 4: A supply chain inventory model for deteriorating products with carbon emission-dependent demand and advanced payment that incorporates a carbon tax and cap policy

4.1. Abstract:

Climate change is predominantly driven by emissions, making them a significant factor of concern. Several countries have shifted their attention towards reducing carbon emissions, employing carbon taxes and caps as primary mechanisms to accomplish this objective. The majority of inventory retailer-supplier models assume that the retailer's order cost should be paid to the supplier when they receive the order. In practical scenarios, certain suppliers may require retailers to make a full or partial payment of the total cost in advance, while others may offer the option of prepayment in multiple equal instalments. The option of advance payment provides customers with the lowest price for their order; however, it also leads to the largest carbon footprint, significantly affecting both carbon emissions and the overall production process. Therefore, this study examines a carbon tax and cap supply chain inventory model for deterioration with carbon emission-dependent demand, considering three payment options: preliminary, cash, and post-payment. Finally, we present numerical examples of the proposed approach and its outcomes, as well as a sensitivity analysis using Mathematica version 12.

4.2. Introduction:

The phenomenon known as global warming is the result of greenhouse gases (GHG) and certain human activities. The emission of carbon has been causing global warming for many years, and it has been receiving attention over the last few years. Carbon emission gases like methane and carbon dioxide increase the temperature of our Earth leading to global warming. Global warming causes severe damage to our planet, with destructive, widespread, and long-term effects. The rising global temperature rapidly destroys the biodiversity of our world leading to the disappearance of many species of plants and animals. Natural phenomena such as sea-level rise, ozone layer depletion, rising earth temperatures, intensively stormy conditions, dryness, and flooded conditions are all effects of global warming. Today, global warming has become a significant challenge, greatly affecting the living beings around us. Reducing and lowering carbon emissions is a worldwide issue. Several countries and regulatory agencies are currently focusing their efforts on reducing their carbon footprints, such as (Toptal, Özlü, and Konur 2014) and (Rout et al. 2020) have also explained that the Kyoto Protocol is a global agreement connected to the UN Framework Convention on Climate Change. The UNFCCC came into effect on March 21, 1994. On December 11, 1997, the Kyoto Protocol was signed by 37 industrialized countries. India also ratified the Kyoto Protocol in 2002. The United Nations Climate Change holds a yearly conference in a different country called the COP. It encourages the business sector and developing economies to engage in reducing carbon emissions. Our supply chain plays a significant role in the global carbon footprint. Manufacturing fields, as well as inventory control and management, contribute to situations such as global warming. (Xi Chen et al. 2013) introduced that in 2016, Walmart decided to avoid 1 billion metric tons of carbon emissions from the global supply chain by 2030. To achieve this goal, (Project Gigaton Accounting Methodology. 2022) was launched. It aims to address the fact that almost all emissions in the retail sector occurred in product supply chains and transportation rather than stores and distribution centres. Furthermore, one of the most significant economic benefits of reducing carbon emissions and deterioration is the reduction of carbon emissions during the entire supply chain process. Carbon emissions and deterioration from economic sectors are causing serious rising temperatures. One of the most pressing issues today is supply chain management for deteriorating goods with carbon reduction regulations, which is becoming a serious concern for urban areas. Consequently, some national or international agencies, governments, and businesses are facing increasing pressure to reduce carbon emissions. Manufacturers can lower their carbon footprint by using modern carbon-reducing techniques. For instance, carbon tax and cap are the main regulation policies. The first phase focuses on carbon emissions. (Xu Chen and Hao 2015) and (U. Mishra, Wu, and Sarkar 2021) explained carbon tax in their study as a levy imposed by some government agencies on those business firms or industries that produce carbon dioxide during their work process leading to environmental pollution. The main objective of the government agencies behind the imposition of tax is to control global warming and protect the environment. In other words, the carbon tax is also a fee that is imposed on those companies that utilize environment-polluting raw material (fossil fuels) during their working process, thereby contributing to global warming.

(Benjaafar, Li, and Daskin 2013) and (Qin, Ren, and Xia 2017) defined the term "carbon cap and trade" as a scheme in which a government or regulatory body sets an aggregate legal limit on emissions (the cap) for a specified period. A cap-and-trade policy has its own set of advantages, in that emissions credits can be distributed to reduce the policy's adverse effects on industry and predict emissions discharges. A carbon credit is a permit issued by a government or regulatory agency for a specific time that allows a company to produce a specific amount of carbon emissions. When an organization exceeds the designated carbon limit, it incurs taxes and must simultaneously reduce carbon emissions. To compensate, organizations can purchase credits from those organizations that emit less than the limit. Less carbon-emitting industries can sell their credits to other organizations that emit higher amounts of carbon. One carbon credit equals one ton of carbon fuel.

Every natural thing change as time progresses. The rate of deterioration for inventory management and control has become a critical concern. Deterioration usually implies a deficiency in quality or utility. Inventory deterioration is a common occurrence in our daily lives, and it should not be overlooked in inventory control. Inventory deterioration and carbon-price-dependent demand are major and important elements of every business's operation. They need to be carefully controlled and managed to meet requirements and prevent shortages. Excess inventory is a cause for worry for management since it prevents the company from making a profit. For retailers and suppliers, developing the business to enhance customer demands is a challenging feat. Although most researchers assume that product demand is constant or price-sensitive, due to carbon emissions, demand for products will undoubtedly be influenced. Several research studies, such as (Aliabadi, Yazdanparast, and Nasiri 2019) and (Huang 2016) have confirmed that demand for goods also depends upon carbon emissions. It is generally recognized that consumer awareness of the environment affects customers in a market, i.e., demand. Additionally, pricing is always important, demand may rise or fall according to the price of items.

(J. Wu, Teng, and Chan 2018) and (Shi et al. 2019) have devoted their attention to the relationship between distributors and buyers in terms of advance payments, which are a significant aspect of high-demand product management. Advance payment is beneficial for items that evaporate quickly, perishable items, and those near their expiration date. In a highly competitive and unpredictable environment, it is usual for suppliers to request some kind of advance payment from their customers. Customers make advance payments to ensure that the order will be delivered on time. Although advance payment offers the customer the lowest price for the order, it is associated with large carbon emissions. To enhance liquidity, industries have implemented a strategy of providing price discounts to customers who pay for their orders in advance. The seller will be able to make more money from interest on liquidity, stimulates the production of more goods and leads to a large carbon footprint. Advance payment has a significant impact on carbon emissions and production.
Similarly, both cash and credit payments can offer benefits in inventory management.
The time horizon refers to the period in which a firm or organization looks into the future while developing a strategic plan. In a finite planning horizon, the replenishment cycle does not repeat itself and is dependent on various factors, including demand. In other words, the time required to restock fluctuates from time to time.

The rest of this chapter is structured as follows. Section 4.3 discusses the relevant studies in the literature review. Section 4.4 introduces the symbols and assumptions that will be used throughout the research study. The theoretical results and mathematical models for various scenarios are presented in Section 4.5. The algorithm is described in Section 4.6. The numerical examples and sensitivity analysis are summarized in Sections 4.7 and Tables and figures are formulated in Sections 4.7.1 and 4.7.2. Sections 4.8 to 4.11 provide managerial suggestions, government implications, and conclusions.

4.3. Literature review:

The first phase focuses on degradation inventory problems, with more and more scholars constantly expanding inventory models for deteriorating goods to reflect more authentic inventory features. The supply chain for deteriorating commodities is critical in the storing industry to stay competitive. Most of the fresh or trendy items fade and deteriorate with time because of evaporation, expiration, spoilage, and depreciation, among other factors. In a pioneering publication, (Ghare and Schrader 1963) proposed an EOQ model with a constant deterioration rate. (Covert, Philip, and Philip 1973) extended the constant degradation rate to a two-parameter Weibull distribution. (Dave and Patel 1981) developed the model for deteriorating products considering linearly growing demand patterns instead of constant demand. Investigation into deteriorating goods was done by (Dye 2013); (Singh et al. 2017); (V. Singh et al. 2018); (V. Singh et al. 2018) and (V. Singh et al. 2019). They developed a model for deteriorating goods and also explained how technology investments impact decaying products. (Pahl and Voß 2014) also researched deteriorating items. From the above review, it is clear that the issues of deteriorating items with finite planning horizons and carbon emission policies have been ignored.

The second phase focuses on carbon emission policies concerning deterioration. Furthermore, one of the most significant economic benefits of reducing carbon emissions and deterioration is the reduction of carbon emissions throughout the entire supply chain process. Carbon emissions and deterioration from economic sectors are causing a serious rise in temperatures. One of the most pressing issues today is supply chain management for deteriorating goods with carbon reduction regulations, which is becoming a serious concern for urban areas. As a result, some national or international agencies, governments, and businesses are increasingly under pressure to reduce carbon emissions. Manufacturers can lower their carbon footprint by using modern carbon-reducing techniques. In today's emerging economy, increasing rates of environmental degradation and the deterioration of inventory are major challenges. Carbon dioxide emissions and the degradation of inventory mostly occur due to different human activities. These human activities are directly or indirectly responsible for the degradation of the environment, and our deteriorating rate cannot be avoided in the supply chain system, which is a key parameter of the Supply Chain in an inventory system. In inventory control, deterioration can be reduced only by reducing global warming and carbon emissions, which is a very difficult task. For that, we have to take the help of carbon emission regulations. We have learned from here that reducing carbon emissions and deterioration will have a positive impact on global warming as well as profit. Reducing carbon emissions in the supply chain is an effective way to reduce greenhouse emissions. It will only be achievable if we implement a carbon tax and carbon cap approach. We can only effectively increase carbon emission reduction activities if we tackle the problem of carbon emissions reduction collaboration and coordination across supply chain firms. Otherwise, achieving the aim of reducing carbon emissions will be challenging. The government of any country and some regulatory agencies consider carbon emission schemes to decrease carbon emissions. For instance, carbon tax and cap, carbon tax are the main regulatory policies. The majority of the existing research on inventory models, on the other hand, has focused on maximizing profit or lowering costs. Only a few of them consider environmental issues, such as minimizing carbon emissions and deterioration. (Dye and Yang 2015) examines a deteriorating inventory system under various carbon emission regulations, as well as the influence of trade credit risk, in this work. Researchers show how environmental restrictions may be included in a decision-making issue for a deteriorating item that involves both trade credit and inventory replenishment. (Rout et al. 2020) explained a carbon cap and tax scheme, where a government or intergovernmental body sets an aggregate legal limit on emissions (the cap) for a specified period, and after that limit, a tax will be imposed. A carbon tax is the most important policy for limiting and ultimately eliminating the use of fossil fuels, which are damaging and destroying our environment. Carbon dioxide and other greenhouse gas emissions are altering the atmosphere. A carbon tax places a price on such emissions, encouraging individuals, companies, and governments to generate less. (U. Mishra, Wu, and Sarkar 2020) explains a sustainable carbon tax and cap-based production inventory model with three cases: sustainable carbon tax and cap-based production inventory model without shortages; partial backorder; and full back-ordering without and with green technology investment. This chapter has explained how carbon emissions and deteriorating items can be controlled in a sustainable supply chain model and what their effects are. Price-Dependent Linear and Non-Linear Demand are used here to reduce carbon emissions and deterioration rate. Backordering and non-back ordering are both cases considered. We have learned from here that reducing carbon emissions and deterioration will have a positive impact on global warming as well as profit. Carbon emission-reducing policies like carbon caps and carbon tax have been considered in this sustainable inventory control model to control carbon emissions and maximize profits. (Sepehri et al. 2021) introduced a price-dependent demand model for deteriorating commodities, including a carbon cap and trade, and allowed late payments for the buyer to manage inventories and build demand. They pointed out that there must be a trade-off between the investment in carbon emission reduction technologies and the profit generated by lowering emissions. With this study, we have established that there is a lack of research considering deteriorating items with finite planning horizons along with carbon tax and cap.

The third phase focuses on preliminary payment, cash, and post-payment. An essential component of inventory management is an advance payment. Due to difficult economic conditions and customer uncertainty, it is common for suppliers to demand some advance payment from their retailers. The customer makes the advance payment to ensure that the order can be delivered on time. In exchange for the advance payment, the supplier offers a reduction in price, a credit facility, or some other type of opportunity to encourage business. Similarly, cash and post-payments are also beneficial for both retailers and suppliers.

Some of the researchers investigated an inventory model for defective items in which the producer provides free transportation to the retailer in return for an advance payment. Production and transportation decisions from the producer to the retailer are linked to carbon emissions. According to (A. Zhang and Zhang 1996), the optimal advance payment approach for saving money and time is to make a larger amount of payment in advance. (Taleizadeh and Taleizadeh 2014) developed an advanced-cash payment model for an evaporating commodity with partial backordering. This research work is slightly different from Zhang (1996). Simultaneously, (Q. Zhang, Tsao, and Chen 2014) investigated an advance payment schedule in which the retailer pays a proportion of the purchase price in advance to gain a discount and then takes sufficient time to pay the remaining balance.

(Teng et al. 2016) modified the EOQ model to include advance payments and account for the fact that a product's expiration date affects the demand rate. However, most advance payment researchers assume that the product can be marketed indefinitely and fail to account for the fact that numerous products, such as baked goods, meat, milk, vegetables, fruits, etc., cannot be sold once they have passed their expiration dates. (J. Wu, Teng, and Chan 2018) described an EOQ inventory model for perishable products with expiration dates and advance-cash-credit payment systems. Barman studied a multi-cycle vendor-buyer supply chain production inventory model with carbon emission regulations such as carbon tax in this study. The system's products are assumed to deteriorate at a constant rate. The buyer pays the vendor the purchase cost in advance in some instalments before the order quantity is replenished. (Shi et al. 2019) created an EOQ inventory coordination model in which the seller provides three payment options to the buyer: cash, advance, instalment payment, or credit payment, as well as setting a carbon taxation policy to reduce carbon emissions. (Mashud et al. 2021) considered non-instantaneous deterioration, advance payments, and partial backorder approaches in this research study. This study provides appropriate green technology investment and preservation technology to reduce both carbon emissions and product deterioration, as well as illustrating the impacts of deterioration and carbon emissions on the total inventory model profit. (U. Mishra, Wu, and Sarkar 2021) researchers introduced an inventory control model with deterioration and carbon emission rate that can be controlled under carbon emission policies such as carbon tax and carbon cap. Various payment approaches can be considered for full, partial, and no backlogging, where demand is based on price and trade credit. This research examined how greenhouse operators enhance their investment in preservation and green technologies, as well as introducing trade credit to enhance their earnings.

From the above studies, it is clear that the discussion on deteriorating items with finite planning horizons has not been adequately covered under carbon tax and cap policies with different payment options.

The fourth phase focuses on carbon-dependent demand. (Li 2016) investigated a model that explains the impact of carbon emissions and pricing on demand, as well as supplier-retailer coordination. The main goal of this approach is to reduce carbon emissions. (Pang et al. 2018) examined the centralized decision process and determined the criteria for carbon emission reduction and order quantity management in the supply chain. Second, it presents a supply chain cooperation and coordination based on a revenue-sharing contract that uses an economic order quantity policy. Furthermore, the study presents techniques for determining the best order amount and the ideal level of carbon emissions through model optimization, considering that market demand is influenced by buyer environmental consciousness. Finally, it also looks into the effects of carbon trading prices on supply chain carbon emissions. (Aliabadi, Yazdanparast, and Nasiri 2019) established an EOQ model to lower the risk of default by reducing emissions through a carbon tax policy. The demand rate is related to the number of carbon emissions, the credit duration, and the selling price, which is used as a trade credit for deteriorating commodities with backlogging. (Lu et al. 2020) explained a sustainable counter-productive model in this study, which also considered carbon tax laws and joint investment in carbon emission reduction technology. The level of raw material inventory and price-dependent demand were also recognized. In this phase, we identified the necessity of examining an inventory model that takes into account carbon tax and cap policies and carbon-dependent demand for deteriorating items, along with different payment options under a finite planning horizon.

The fifth phase focuses on a finite planning horizon in inventory control and management. (C. Wu and Zhao 2014) investigated a model in which the researcher includes time and inventory-dependent demand under a finite planning horizon. (V. Singh et al. 2017) proposed an EOQ inventory supply chain model with deteriorating items in a finite planning horizon for two scenarios: with and without payment delays. The re-manufacturing of inventories was covered in (V. Singh et al. 2019) work under centralized and decentralized planning horizons with deteriorated products. A trade credit policy was also incorporated under a finite planning horizon. (Xu et al. 2020) developed inventory models with partial backlogs for deteriorating items, investigated the effects of carbon emission controls on the inventory system, and also considered time-varying demand in the system because customer demands in a real deteriorating inventory system commonly vary with time. In inventory control and management, a supply chain model has yet to be proposed for deteriorating items along with carbon tax and cap policy, different payment options, and carbon-dependent demand under a finite planning horizon.

4.3.1. Research gap:

There has been very little study on carbon emissions in the supply chain for deteriorating items and carbon emission policies. To the best of our knowledge, no prior research has developed a supplier-retailer inventory model for deteriorating items with price or emission-dependent demand under a finite planning horizon. Additionally, no researchers have discussed supplier-retailer inventory coordination under carbon emission policies such as carbon tax and carbon cap for deteriorating products with price-carbon dependent demand and three payment options. This is a significant research gap, and the work is unique in that the problems discussed are relevant to both economic terms and the environment. This research provides a joint decision on inventory control and carbon emissions, considering various parameters such as carbon emission demand and carbon emission, considering various policies such as carbon tax, and cap to design a long-term sustainable supply chain. The study also considers deterioration with carbon emission regulations and three payment options: preliminary, cash, and post-payment over a finite horizon in inventory control and management.

Table 4.1. Comparison of review of existing literature

Research work	Carbon dependent demand	Deterioration	Preliminary payment	Cash payment	Post payment	Carbon Emissions Regulations	Finite Planning horizon
(Ghare and	×	✓	×	×	×	×	×

Schrader 1963)							
(Xi Chen et al. 2013)	×	×	×	✓	×	✓	×
(Toptal, Özlü, and Konur 2014)	×	×	×	×	×	✓	×
(Taleizadeh and Taleizadeh 2014)	×	✓	✓	×	×	×	×
(Dye and Yang 2015)	×	✓	×	×	×	✓	×
(Teng et al. 2016)	×	✓	✓	×	×	×	×
Rout et al. (2020)	×	✓	×	×	×	✓	×
Sephri et al. (2021)	×	✓	×	×	×	✓	×
Mashud et al. (2021)	×	✓	✓	×	×	✓	×
Shi et al. (2019)	×	×	✓	✓	✓	✓	×
Aliabadi et al. (2019)	✓	✓	×	×	×	✓	×
Wu and Zhao (2014)	×	×	×	×	×	×	✓
Chunming Xu (2020)	×	✓	×	×	×	×	✓
This chapter	✓	✓	✓	✓	✓	✓	✓

4.3.2. Problem defining:

Therefore, the focus of this research is to address the problem of carbon emissions in inventory control and management, while considering three payment options: Preliminary, Cash, and Post payment over a finite horizon with deterioration and carbon emission regulations.

4.4. Assumptions:

In this chapter, we have adopted certain assumptions and applied relevant criteria as necessary.

1. The effects of carbon emissions on demand are expressed in the form: $D(P, G) = a - b*P - c*G$. In this function, the firm's per-unit pricing is represented by p, and the quantity of emissions produced per unit of product is represented by G. The initial demand of the market is represented by a, while market demand's dependence on price is represented by b, and c represents the consumer's sensitivity to carbon emissions per unit. In other words, when the price of the product increases by one unit, the demand for the product decreases by b units. Similarly, when the carbon emissions per unit of product increase by one unit, the demand for the product decreases by c units. There is no lead time because the supplier has a buffer inventory.
2. The planning horizon is finite, and the replenishment cycles are of different lengths.
3. Inventory depreciates at a constant rate θ in this model.
4. Shortages are not taken into consideration due to technical advancements, and buyers generally don't want to delay.
5. For the model's construction, one supplier and one retailer are used, but the same can be extended for multiple retailers. Several products can be included in the framework.

6. Inventory replenishment orders are not constant and instantaneous due to different demands in each cycle.

4.5. Mathematical formulation and analysis of the model:

Due to the absence of shortages, the retailer places an order for products from the supplier before the previous inventory level reaches zero. The supplier then replenishes the order immediately, with the same quantity as the products ordered. The length of the cycle (T) varies in the finite planning horizon, and the amount of quantity (R_{i+1}) ordered is also not constant.

The differential equation 4.1 that represents the changes in inventory level is presented in Figure 4.1 of inventory level.

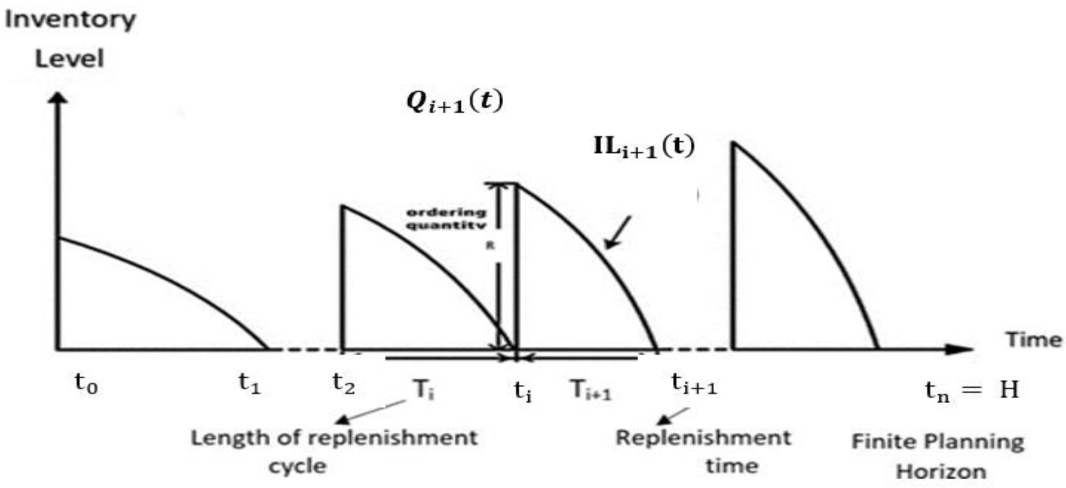

Figure 4.1. Pictorial representation of inventory level

$$\frac{(IL_{i+1}(t))}{dt} = -D(P,G) - \theta * IL_{i+1}(t) \tag{4.1}$$

Where $t_i \leq t \leq t_{i+1}$ (4.2)

See Appendix- 4. A_1

Where $IL_{i+1}(t_{i+1}) = 0$ and $IL_{i+1}(t_i) = Q_{i+1}$

$$\mathrm{IL}_{i+1}(t) \ = \int_{t}^{t_{i+1}} (P, G) e^{\theta(u-t)} du \tag{4.3}$$

$$\mathrm{IL}_{i+1}(t) = \frac{D(P,G)\left(e^{\theta(t_{i+1}-t)}-1\right)}{\theta} \tag{4.4}$$

$$Q_{i+1} \ = IL_{i+1}(t_i) = \ e^{-\theta t_i} \int_{t_i}^{t_{i+1}} D(P,G) e^{\theta t} dt \tag{4.5}$$

$$Q_{i+1} \ = IL_{i+1}(t_i) = D(P,G)(e^{\theta(t_{i+1}-t_i)} - 1 \tag{4.6}$$

Ordering cost

$$O_c = n_1 \ * \ O_r \tag{4.7}$$

Holding cost

$$H_c = \sum_{i=0}^{n_1-1} h_r \ \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du \ dt \tag{4.8}$$

Deterioration cost

$$\mathrm{Dc} = \sum_{i=0}^{n_1-1} \theta * d_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du \ dt \tag{4.9}$$

carbon emission cost

$$Ce = \sum_{i=0}^{n_1-1} \left(\hat{c} \ + \ \widehat{h_r} * \ Q_{i+1} \ + \ \hat{P}_r \ \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du \ dt \right) \tag{4.10}$$

4.5.1. Model for preliminary payment under finite planning horizon:

Acquisition cost

$$\sum_{i=0}^{n_1-1} (1 \ - \ \mathrm{r}) \ * P_r \int_{t_i}^{t_{i+1}} D(P,G) \ e^{\theta(t-t_i)} \ dt \tag{4.11}$$

Capital cost.

$$Cc_1 = \sum_{i=0}^{n_1-1} \frac{\mathrm{I} * \mathrm{N}\ (\mathrm{n}+1)(1-\mathrm{r}) * P_r}{2n} \int_{t_i}^{t_{i+1}} D(P,G) \ e^{\theta(t-t_i)} \ dt \tag{4.12}$$

Interest charges

$$\mathrm{Ic}_1 = \sum_{i=0}^{n_1-1} \mathrm{I}\,(1 - \mathrm{r})\,\mathrm{P_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du \; dt \tag{4.13}$$

Retailers overall cost

Total cost $T_{Ret(1)}$ = Ordering cost+ holding Cost + deterioration cost + Acquisition cost + capitalcost + charges interest + carbon cost

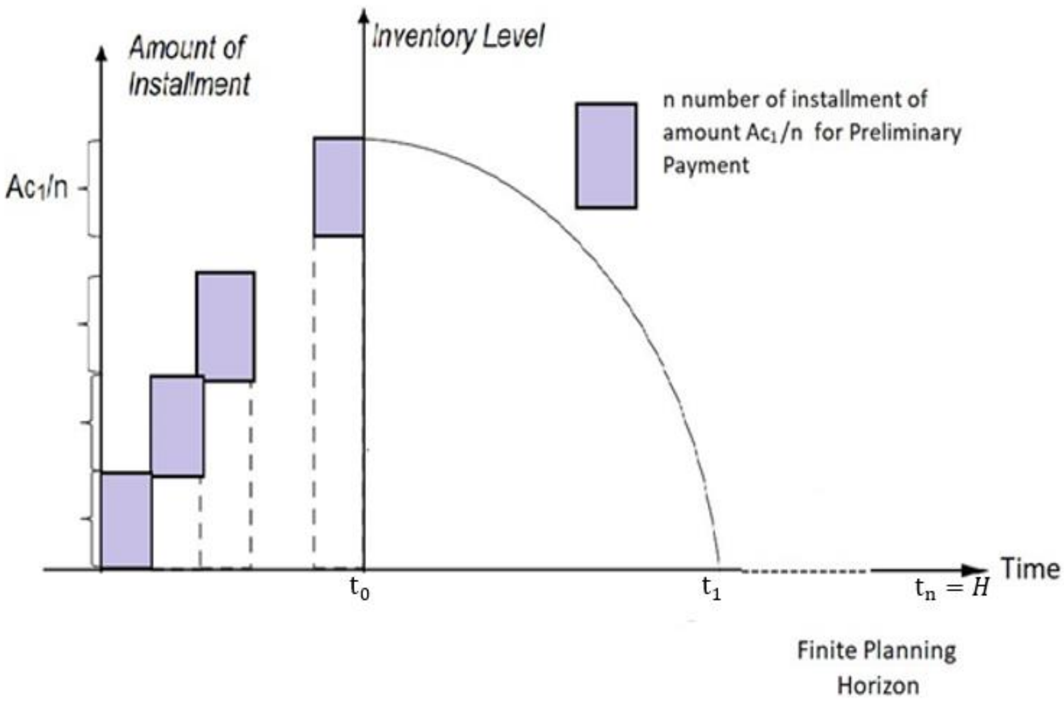

Figure 4.2. Preliminary payment.

$$T_{Ret}(1) = \mathrm{n_1 * O_r} + (1 - \mathrm{r}) * P_r \int_{t_i}^{t_{i+1}} D(P,G)\ e^{\theta(t-t_i)}\ dt$$

$$+ \sum_{i=0}^{n_1-1} \frac{\mathrm{I * N\,(n+1)(1-r)} * P_r}{2n} \int_{t_i}^{t_{i+1}} D(P,G)\ e^{\theta(t-t_i)}\ dt$$

$$+ \sum_{\mathrm{i=0}}^{\mathrm{n_1-1}} \mathrm{I}\,(1$$

$$- \mathrm{r})\,\mathrm{P_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du\ dt$$

$$+ \sum_{\mathrm{i=0}}^{\mathrm{n_1-1}} \mathrm{h_c} \int_{\mathrm{t_i}}^{\mathrm{t_{i+1}}} \int_{\mathrm{t}}^{\mathrm{t_{i+1}}} D(\mathrm{P,G})\mathrm{e^{\theta(u-t)}du\ dt}$$

$$+ \sum_{\mathrm{i=0}}^{\mathrm{n_1-1}} \theta * \mathrm{d_r} \int_{\mathrm{t_i}}^{\mathrm{t_{i+1}}} \int_{\mathrm{t}}^{\mathrm{t_{i+1}}} D(\mathrm{P,G})\mathrm{e^{\theta(u-t)}du\ dt} + \sum_{\mathrm{i=0}}^{\mathrm{n_1-1}} \tau\,(\mathrm{Z} - (\ \hat{\mathrm{c}} + \widehat{\mathrm{h_r}}$$

$$* \mathrm{Q_{i+1}} + \widehat{\mathrm{P_r}} \int_{\mathrm{t_i}}^{\mathrm{t_{i+1}}} \int_{\mathrm{t}}^{\mathrm{t_{i+1}}} D(\mathrm{P,G})\mathrm{e^{\theta(u-t)}du\ dt}\,))$$

(4.14)

$$\frac{\partial T_{Ret}(1)}{\partial t_i} = \left\{\left\{(1-r) * P_r + \frac{\mathrm{I * N\,(n+1)(1-r)} * P_r}{2n} - \tau\,\widehat{h_r}\right\} + \frac{\{I\,(1-r)P_r - \widehat{h_c}\tau + h_c + \theta d_c\}}{\theta}\right\} D(P,G) \sum_{i=0}^{n_1-1}\left\{e^{\theta(t_i-t_{i-1})} - e^{\theta(t_{i+1}-t_i)}\right\} \quad (4.15)$$

$$\frac{\partial^2 T_{Ret}(1)}{\partial {t_i}^2} = \left\{\left\{(1-r) * P_r + \frac{\mathrm{I * N\,(n+1)(1-r)} * P_r}{2n} - \tau\,\hat{h}_r\right\} + \frac{\{I\,(1-r)P_r - \widehat{h_c}\tau + h_c + \theta d_c\}}{\theta}\right\} D(P,G)\theta \sum_{i=0}^{n_1-1}\left\{e^{\theta(t_i-t_{i-1})} + e^{\theta(t_{i+1}-t_i)}\right\} \quad (4.16)$$

$$\begin{pmatrix} \frac{\partial^2 T_{Ret}}{\partial {t_1}^2} & \frac{\partial^2 T_{Ret}}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 T_{Ret}}{\partial t_2 \partial t_1} & \frac{\partial^2 T_{Ret}}{\partial {t_2}^2} & \frac{\partial^2 T_{Ret}}{\partial t_2 \partial t_3} & \ddots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \ddots & 0 & \frac{\partial^2 T_{Ret}}{\partial t^2_{n_1-1}} & \frac{\partial^2 T_{Ret}}{\partial t_{n_1-1} \partial t_{n_1}} \\ 0 & \vdots & \vdots & \cdots & \frac{\partial^2 T_{Ret}}{\partial t_{n_1-1} \partial t_{n_1-2}} & & \\ 0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 T_{Ret}}{\partial t_{n_1} \partial t_{n_1-1}} & \frac{\partial^2 T_{Ret}}{\partial {t_{n_1}}^2} \\ 0 & 0 & 0 & & \cdots & & \end{pmatrix}$$

Figure 4.3. Hessian matrix (T. Sarkar et al. 2012)

$$\frac{\partial^2 T_{Ret}(1)}{\partial (t_i)(t_{i-1})} = \left\{ \left\{ (1-r) * P_r + \frac{\mathrm{I} * \mathrm{N}\,(\mathrm{n}+1)(1-\mathrm{r}) * P_r}{2n} - \tau\,\hat{h}_r \right\} + \frac{\{I\,(1-r)P_r - \widehat{h_c}\tau + h_c\ + \theta d_c\}}{\theta} \right\} D(P,G) \sum_{i=0}^{n_1-1} -\theta\{e^{\theta(t_i - t_{i-1})}\} \quad (4.17)$$

$$\frac{\partial^2 T_{Ret}(1)}{\partial (t_i)(t_{i+1})} = \left\{ \left\{ (1-r) * P_r + \frac{\mathrm{I}\ *\ \mathrm{N}\,(\mathrm{n}+1)(1-\mathrm{r}) *\ P_r}{2n} - \tau\,\hat{h}_r \right\} + \frac{\{I\,(1-r)P_r - \widehat{h_c}\tau + h_c\ + \theta d_c\}}{\theta} \right\} D(P,G) \sum_{i=0}^{n_1-1} \theta\{e^{\theta(t_{i+1} - t_i)}\}$$

(4.18)

$\frac{\partial^2 T_{Ret}(1)}{\partial (t_i)(t_{n_1})} = 0$ for all n_1 = i, i+1, i-1 (4.19)

By (T. Sarkar et al. 2012); (V. Singh et al. 2017); (V. Singh et al. 2019) and (Saxena et al. 2020)the fact that the Hessian matrix of TCR is positivedefinite is sufficient for a total cost to be min. Condition to check positive definite

4.1. Theorem: - If t_i satisfies the inequations

(i) $\left| \frac{\partial^2 T_{Ret}(n_1, t_0, t_1 \ldots\ldots\ldots\ldots t_{n_1})}{\partial {t_i}^2} \right| \geq 0$

(ii) $\left|\frac{\partial^2 T_{Ret}(n_1,t_0,t_1 \ldots\ldots\ldots\ldots t_{n_1})}{\partial t_i^{\,2}}\right| \geq \left|\frac{\partial^2 T_{Ret}(n_1,t_0,t_1 \ldots\ldots\ldots\ldots t_{n_1})}{\partial (t_i)(t_{i-1})}\right| + \left|\frac{\partial^2 T_{Ret}(n_1,t_0,t_1 \ldots\ldots\ldots\ldots t_{n_1})}{\partial (t_i)(t_{i+1})}\right|$

For all i = 1, 2, 3,………….. $n_1 - 1$. Then $\nabla^2 T_{Ret}$ is positive and definite. (4.20)

4.5.2. Model for cash payment under finite planning horizon:

Acquisition cost

$$Ac_2 = \sum_{i=0}^{n_1-1} P_r \int_{t_i}^{t_{i+1}} D(P,G)\ e^{\theta(t-t_i)}\ dt \tag{4.21}$$

Interest charges

$$Ic_2 = \sum_{i=0}^{m_1-1} I * P_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du\ dt \tag{4.22}$$

Retailer's overall cost

Total cost $T_{Ret}(2)$

= *Ordering cost* + *holding Cost* + *deterioration cost* + Acquisition cost+ *charges interest* + *carbon cost*

$$T_{Ret}(2) = n_1 * O_r + P_r \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} D(P,G)\ e^{\theta(t-t_i)}\ dt + I * P_r \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} IL_{i+1} dt + h_r \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} IL_{i+1} dt + \theta * d_c \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} IL_{i+1} dt + \sum_{i=0}^{n_1-1} \tau(Z - (\ c\hat{}\ +\ \widehat{h_r}\ *\ Q_{i+1}\ + \widehat{P}_r \int_{t_i}^{t_{i+1}} IL_{i+1}(t)\ dt\ )) \tag{4.23}$$

$$T_{Ret}(2) = n_1 * O_r + \left(P_r - \widehat{h_r} * \tau\right) \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} D(P,G)\ e^{\theta(t-t_i)} dt + \left\{I * P_r + h_c + \theta\, d_c + \widehat{P_r} * \tau\right\} \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du\ dt + Z * \tau - c\hat{}\ \tau \tag{4.24}$$

$$\frac{\partial T_{Ret}(2)}{\partial t_i} = \left\{P_r + \frac{\mathrm{I} * P_r \;\; -\widehat{p_r}\tau + h_c \; +\theta d_r}{\theta} \;\; - \tau \;\hat{h}_r\right\} \; D(P,G) \sum_{i=0}^{n-1}\left\{e^{\theta(t_i - t_{i-1})} - e^{\theta(t_{i+1} - t_i)}\right\}$$

(4.25)

4.5.3. Model for post-payment under finite planning horizon:

In general, according to the market norm, the simplest method is to purchase a product and pay for it. Yet, Trade Credit is a practical choice for the majority of firms. In this case, the supplier gives the retailer a set amount of time to pay for the purchases. within this permissible limit, thereare no losses for the supplier. Sometimes, to reduce the risk of default, the supplier usually offersa discount to promote early payment. Therefore, the retailer is given a specified credit period M_r by the supplier to the retailer in this situation.

The realtor's acquisition cost per replenishment cycle time T_{i+1} is shown clearly by

Acquisition cost

$$Ac_3 = \sum_{i=0}^{n_1-1} P_r \int_{t_i}^{t_{i+1}} D(P,G) \;\; e^{\theta(t-t_i)} \;\; dt \qquad (4.26)$$

Interest charges

$$Ic_3 = \sum_{i=0}^{n_1-1} I * \; P_r \int_{M}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du \;\; dt \qquad (4.27)$$

4.5.3.1. Sub-case 1: where $M_c \leq T^s$

Since $M_c \leq T^s$, the retailer in this sub-case must pay interest on the products in stock over time M_c

the interest charged per cycle:

$$IE_{31} = I * I_e * s * D(P,G) M_c^2 \qquad (4.28)$$

$$T_{Ret}(31) = n_1 * O_r + \left(P_r - \widehat{h_r} * \tau\right) \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} D(P,G) \;\; e^{\theta(t-t_i)} \, dt + \left\{I * P_r + \; h_r \; + \right.$$
$$\left. \theta \; * d_r \; - \; \widehat{P_r} * \tau\right\} \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du \;\; dt + Z * \tau - \hat{c} * \; \tau -$$
$$\{\mathrm{I} \; * I_e \; * \; \mathrm{s} \; * \; \mathrm{D(P,G)} \; * M^2\}$$

(4.29)

4.5.3.2. Sub-case 2: where $M_c \geq T^s$

the interest charged per cycles:

$$IE_{32} = \sum_{i=0}^{n_1-1} I_e * s * D(P,G)T_{i+1}\left\{M_c - \frac{1}{2}T_{i+1}\right\} \tag{4.30}$$

Interest charges (Ic_{32}) = 0 (4.31)

Retailers overall cost

Total cost ($T_{Ret}(32)$) =Ordering Cost+ Holding Cost + Deterioration Cost + Acquisition cost +Charges Interest- Interest Earn + Carbon Cost

$$T_{Ret}(32) = n_1 * O_r + P_r \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} D(P,G)\, e^{\theta(t-t_i)}\, dt + I * P_r \sum_{i=0}^{n_1-1} \int_{M_c}^{t_{i+1}} IL_{i+1} dt + h_r \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} IL_{i+1} dt + \theta * d \sum_{i=0}^{n_1-1} \int_{s_i}^{s_{i+1}} IL_{i+1} dt + \sum_{i=0}^{n_1-1} \tau(Z - (c\hat{} + \hat{h}_r * Q_{i+1} + \widehat{P_r} \int_{t_i}^{t_{i+1}} IL_{i+1}(t)\, dt)) - \sum_{i=0}^{n_1-1} I_e * s * D(P,G)T_{i+1}\left\{M_c - \frac{1}{2}T_{i+1}\right\}$$

(4.32)

$$T_{Ret}(32) = n_1 * O_r + \left(P_r - \widehat{h_r} * \tau\right) \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} D(P,G)\; e^{\theta(t-t_i)} \; dt + \left\{ h_r + \theta * d_r - \widehat{P_r}\tau\right\} + \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(P,G) e^{\theta(u-t)} du\; dt + Z * \tau - c\hat{} * \tau - \sum_{i=0}^{n_1-1} I_e * s * D(P,G)\{t_{i+1} - t_i\}\left\{M_c - \frac{1}{2}\{t_{i+1} - t_i\}\right\}$$

(4.33)

$$Q_{(i+1)} = \sum_{i=0}^{n_1^*-1} Q_{i+1}^* \tag{4.34}$$

Supplier total cost is given by the following equation.

$$T_{sup} = n_1 * S_s + \sum_{i=0}^{n_1^*-1} P_r * Q_{i+1}^* \tag{4.35}$$

4.6. Algorithm and procedures for resolving the problem:

Step 1: Create a new set of inputs: Or, d_r, h_r, S_s, A_c, C_c, I_c, I_e, P_r, ĉ, $\hat{P}_r$, $\hat{h}_r$, N, r, n, M_c, θ, … …. …. …τ, C_e, T_{i+1}, H etc.

Step 2: The forecasted total cost T_{Ret} (cost of retailer) and T_{sup} (total cost of supplier) is a function of si, which may be calculated numerically. Now, using the given input values in step1, compute the values of t_i from the preceding equations.

$\frac{\partial T_{Ret}}{\partial t_i}$= 0 using the given input values in step 1.

Step3: After calculating the s_i values in step 2, verify the condition:

$$\left|\frac{\partial^2 T_{Ret}}{\partial {t_i}^2}\right| \geq \left|\frac{\partial^2 T_{Ret}}{\partial (t_i)(t_{i-1})}\right| + \left|\frac{\partial^2 T_{Ret}}{\partial (t_i)(t_{i+1})}\right|$$

Step 4: Determine the most appropriate T_{Ret} values which will be the optimal expected averagecost by Using the values of t_i obtained in step 3.

Step 5: Using the values of t_i obtained in step 2 and the equation is given below, by which we willdetermine the optimal order quantity.

$$Q_{(i+1)} = \sum_{i=0}^{n_1^*-1} Q_{i+1}^*$$

Step 6: After the 5th step we can find T_{sup} (Supplier total cost) which depends upon the retailer's replenishment policy.

4.7. Numerical illustration for the proposed model:

This section provides information about the numerical data. We will illustrate how various factors affect the overall cost of the retailer and supplier using the following values: Let a = 0.15, n = 10, r = 0.5, h_r = 10, b = 0.000001, c = 0.0001, W = 4, I = 0.1, O_r = 350, S_s = 400, θ = 2, $\hat{c}$ = 1, h_c = 2, $\hat{h}_r$ = 0.2, Cc = 5, P = 1, G = 1, N = 0.17, M_r = 0.17, I_e = 0.08, s = 50, τ = 0.5, g = 0.00001, d_r = 1, Z = 0.002, P_r = 0.02, $\hat{P}_r$ = 10 with the appropriate units. We will use Mathematica software (version 12) to solve non-linear Equations (4.20) and (4.24). We present a tabulation and graphical representation of the overall cost of suppliers and retailers and optimal quantity for different parameters and replacement cycles, i.e., for $n_1 - 1$ = 1, 2, 3, and so on.

4.8. Tabular formulation for Sensitivity analysis and findings:

1. As per Table 4.2 and Figure 4.2, it can be observed that when the value of r (the discount rate for preliminary payment) increases, the overall cost of the retailer and supplier remains constant. This suggests that with an increasing discount rate for advance payment, the ordering quantity decreases. Therefore, there is no correlation between the overall cost of the retailer and supplier and a direct correlation between the number of replenishment cycles.

2. In Tables 4.3 and 4.4, an increase in initial demand results in an increase in the retailer, supplier, and order quantity. Higher initial demand leads to higher orders, as well as the higher total cost of the retailer (T_{Ret}) and the total cost of the supplier (T_{sup}) for all types of payment.

3. According to Tables 4.3 and 4.4, an increase in the value of θ (the deterioration rate of inventory) for the case of advance payment results in a decrease in the total cost of the retailer, whereas for credit payment in the case of Mc ≥ Ti′ s, it leads to an increase in the total cost of the retailer. However, an increase in the value of θ and the replenishment quantity leads to a decrease in the total cost, implying that if the inventory is replenished more frequently, the deterioration rate is less and so is the total cost. Therefore, θ is directly related to the total supplier cost and order

quantity, and indirectly related to the total retailer cost. Similarly, for cash payment and the case Mc ≤ Ti ′ s, θ is directly related to the total retailer, supplier cost, and order quantity.

Table 4.2. Table for changes in r (discount rate) for Preliminary Payment under finite planning horizon

r	Q_{i+1}	T_{Ret}	T_{sup}
0.240	3170.76	$3.6054 * 10^7$	751125.74
0.245	3170.10	$3.6054 * 10^7$	751125.74
0.250	3169.43	$3.6054 * 10^7$	751125.74
0.255	3168.77	$3.6054 * 10^7$	751125.74
0.260	3168.11	$3.6054 * 10^7$	751125.74

Table 4.3. Preliminary and cash Payment under finite planning horizon

	Preliminary payment			Cash payment		
	$Q_{(i+1)}$	T_{Ret}	T_{sup}	$Q_{(i+1)}$	T_{Ret}	T_{sup}
a=240	360546.74	2799.04	9610.93	360533.82	18095.7	9610.68
a=245	368057.87	2805.52	9761.16	368044.94	18421.9	9760.90
a=250	375569.01	2812.00	9911.38	375556.07	18748.0	9911.12
a=255	383081.49	2822.67	10061.60	383067.19	19074.2	10061.30
a=260	390592.60	2829.06	10211.90	390578.32	19400.4	10211.60
c=0.48	3136.23	$3.6054*10^7$	751125.81	751112.36	35057.21	$3.60534 * 10^7$
c=0.49	3136.23	$3.6054*10^7$	751125.81	751112.36	35057.21	$3.60534 * 10^7$
c=0.50	3136.23	$3.6054*10^7$	751125.81	751112.36	35057.21	$3.60534 * 10^7$
c=0.51	3136.23	$3.6054*10^7$	751125.81	751112.36	35057.21	$3.60534 * 10^7$
c-0.52	3136.23	$3.6054*10^7$	751125.81	751112.36	35057.21	$3.60534 * 10^7$
θ =0.96	28455.16	4672.37	$1.36585 * 10^6$	28399.4	27189.4	$1.36317 * 10^6$
θ =0.98	29963.13	4644.08	$1.43823 * 10^6$	29909.7	27316.6	$1.43567 * 10^6$
θ =1.00	31573.39	4616.31	$1.51552 * 10^6$	31522.2	27444.8	$1.51307 * 10^6$
θ =1.02	33307.18	4657.70	$1.59875 * 10^6$	33244.0	27573.7	$1.59571 * 10^6$
θ =1.04	35143.25	4626.32	$1.68688 * 10^6$	35082.9	27703.5	$1.68398 * 10^6$
τ= 0.240	751139.32	20001.30	$3.60547*10^7$	751112.29	35140.5	$3.60547 * 10^7$
τ =0.245	751138.79	19674.40	$3.60547 * 10^7$	751112.29	35138.90	$3.60547 * 10^7$
τ =0.250	751138.27	19347.70	$3.60547 * 10^7$	751112.29	35137.30	$3.60547 * 10^7$

τ =0.255	751140.46	19037.70	$3.60547 * 10^7$	751112.29	35135.70	$3.60547 * 10^7$
τ =0.260	751139.88	18710.60	$3.60547 * 10^7$	751112.29	35134.10	$3.60547 * 10^7$
pˆr =4.8	751139.26	19922.60	$3.60547 * 10^7$	751112.29	35062.80	$3.60534 * 10^7$
pˆr =4.9	751138.73	19597.30	$3.60547 * 10^7$	751112.29	35062.70	$3.60534 * 10^7$
pˆr =5.0	751138.22	19272.10	$3.60547 * 10^7$	751112.29	35062.50	$3.60534 * 10^7$
pˆr =5.1	751140.39	18963.50	$3.60547 * 10^7$	751112.29	35062.40	$3.60534 * 10^7$
pˆr =5.2	751139.81	18637.90	$3.60547 * 10^7$	751112.29	35062.30	$3.60534 * 10^7$
hˆc = 20	751125.76	3214.06	17422.51	751112.29	35134.70	17422.20
hˆc = 20	751125.76	3212.56	17422.51	751112.29	35133.20	17422.20
hˆc = 20	751125.76	3211.06	17422.51	751112.29	35131.70	17422.20
hˆc = 20	751125.76	3209.58	17422.51	751112.29	35130.20	17422.20
hˆc = 20	751125.76	3208.08	17422.51	751112.29	35128.70	17422.20
S r =20	751125.73	1862.23	17422.51	751112.29	33783.20	17422.24
S r =20	751125.73	1886.73	17422.51	751112.29	33807.70	17422.24
S r =20	751125.73	1911.23	17422.51	751112.29	33832.20	17422.24
S r =20	751125.73	1935.73	17422.51	751112.29	33856.70	17422.24
S r =20	751125.73	1960.23	17422.51	751112.29	33881.20	17422.24
S s =20	751125.73	3136.23	16174.50	751112.29	35057.20	16174.24
S s =20	751125.73	3136.23	16198.50	751112.29	35057.20	16198.24
S s =20	751125.73	3136.23	16222.50	751112.29	35057.20	16222.24
S s =20	751125.73	3136.23	16246.50	751112.29	35057.20	16246.24
S s =20	751125.73	3136.23	16270.50	751112.29	35057.20	16270.24
pr =0.0096	751125.71	3101.69	9610.81	751112.29	35055.63	9610.68
pr = 0.0098	751125.71	3102.36	9761.03	751112.29	35055.66	9760.90
pr =0.0100	751125.71	3103.02	9911.26	751112.29	35055.69	9911.12
pr =0.0102	751125.71	3103.68	10061.50	751112.29	35055.72	10061.30
pr =0.0104	751125.71	3104.34	10211.70	751112.29	35055.75	10211.60
b=4.8*10−7	3136.23	3.6054*107	751125.73	35057.20	$3.60534*10^7$	751112.29
b=4.9*10−7	3136.23	3.6054*107	751125.73	35057.20	$3.60534*10^7$	751112.29
b=5.0*10−7	3136.23	3.6054*107	751125.73	35057.20	$3.60534*10^7$	751112.29
b=5.1*10−7	3136.23	3.6054*107	751125.73	35057.20	$3.60534*10^7$	751112.29
b=5.2*10−7	3136.23	3.6054*107	751125.73	35057.20	$3.60534*10^7$	751112.29

4. Table 4.4 explains the link between M_c (trade credit period) and the finite planning horizon (T). Because the credit duration is longer than T_i, the retailer will have more time to settle the entire payment, which will extend the interest earned period.

5. As a result, the retailer's earnings will rise, resulting in a reduction in total cost.

and vice versa in the case of $M_c \le T_i^{'}s$. Table 4.4 shows the sensitivity analysis for different parameters.

6. As c is the consumer's sensitivity to carbon emissions per unit and when carbon emissions per unit product rise by one unit demand falls by c. As an increase in the value of c in Table 4.4 andTable 4.3 there are no changes in R_{i+1}, TCR and TCS.

Table 4.4. Table for Post Payment under finite planning horizon

Parameters	Post payment for $M_c \le T_i^s$			Post payment for $M_c \ge T_i^s$		
	$Q_{(i+1)}$	T_{Ret}	T_{sup}	$Q_{(i+1)}$	T_{Ret}	T_{sup}
a=240	360533.82	18081.80	9610.68	$1.30478 * 10^7$	49355.27	263356.50
a=245	368044.94	18407.70	9760.90	$1.33197 * 10^7$	50335.45	268793.10
a=250	375556.07	18733.60	9911.12	$1.35915 * 10^7$	51315.63	274229.70
a=255	383067.19	19059.50	10061.30	$1.38633 * 10^7$	52295.80	279666.29
a=260	390578.32	19385.40	10211.60	$1.41351 * 10^7$	53275.98	285102.89
k=0.48	751125.81	3107.33	$3.6054 * 10^7$	$2.7183 * 10^7$	100324.45	$1.30478 * 10^9$
k=0.49	751125.81	3107.33	$3.6054 * 10^7$	$2.7183 * 10^7$	100324.45	$1.30478 * 10^9$
k=0.50	751125.81	3107.33	$3.6054 * 10^7$	$2.7183 * 10^7$	100324.45	$1.30478 * 10^9$
k= 0.51	751125.81	3107.33	$3.6054 * 10^7$	$2.7183 * 10^7$	100324.45	$1.30478 * 10^9$
k=0.52	751125.81	3107.33	$3.6054 * 10^7$	$2.7183 * 10^7$	100324.45	$1.30478 * 10^9$
θ =0.96	28399.40	27160.50	$1.36317 * 10^6$	56941.20	48815.80	$2.73318 * 10^6$
θ =0.98	29909.70	27287.70	$1.36317 * 10^6$	61934.60	49388.30	$2.97286 * 10^6$
θ =1.00	31522.20	27415.90	$1.36317 * 10^6$	67479.40	49971.40	$3.23901 * 10^6$
θ =1.02	33244.00	27544.80	$1.36317 * 10^6$	67718.50	49244.1	$3.25049 * 10^6$
θ =1.04	35082.90	27674.6	$1.36317 * 10^6$	73836.00	49821.6	$3.54413 * 10^6$
$\hat{P}r$ =4.8	751112.29	35033.86	$3.60534 * 10^7$	$8.49346 * 10^6$	74212.8	$4.07686 * 10^8$
$\hat{P}r$ =4.9	751112.29	35033.75	$3.60534 * 10^7$	$8.62455 * 10^6$	74519.80	$4.13978 * 10^8$
$\hat{P}r$ =5.0	751112.29	35033.64	$3.60534 * 10^7$	$8.75935 * 10^6$	74831.60	$4.20449 * 10^8$
$\hat{P}r$ =5.1	751112.29	35033.54	$3.60534 * 10^7$	$8.89799 * 10^6$	75148.30	$4.27103 * 10^8$
$\hat{P}r$ =5.2	751112.29	35033.43	$3.60534 * 10^7$	$9.04062 * 10^6$	75469.90	$4.3395 * 10^8$
τ = 0.240	751112.29	35111.56	$3.60534 * 10^7$	$8.69148 * 10^6$	74841.00	$4.17191 * 10^8$
τ =0.245	751112.29	35109.96	$3.60534 * 10^7$	$8.82956 * 10^6$	75155.90	$4.23819 * 10^8$
τ =0.250	751112.29	35108.36	$3.60534 * 10^7$	$8.97165 * 10^6$	75475.70	$4.30639 * 10^8$
τ =0.255	751112.29	35106.76	$3.60534 * 10^7$	$8.4267 * 10^6$	74204.00	$4.04482 * 10^8$
τ =0.260	751112.29	35105.16	$3.60534 * 10^7$	$8.55724*10^6$	74509.10	$4.10747 * 10^8$
S_r = 168.0	751112.29	33754.30	$3.60534 * 10^7$	$2.7183 * 10^7$	99050.40	$1.30478 * 10^9$
S_r =171.5	751112.29	33778.80	$3.60534 * 10^7$	$2.7183 * 10^7$	99074.90	$1.30478 * 10^9$
S_r =175.0	751112.29	33803.30	$3.60534 * 10^7$	$2.7183 * 10^7$	99099.40	$1.30478 * 10^9$

S_r =178.5	751112.29	33827.80	$3.60534 * 10^7$	$2.7183 * 10^7$	99123.90	$1.30478 * 10^9$
S_r = 182.0	751112.29	33852.30	$3.60534 * 10^7$	$2.7183 * 10^7$	99148.40	$1.30478 * 10^9$
S_s =192	751112.29	35028.30	16174.2	$2.7183 * 10^7$	100324.44	544812.51
S_s =196	751112.29	35028.30	16198.2	$2.7183 * 10^7$	100324.44	544836.51
S_s =200	751112.29	35028.30	16222.2	$2.7183 * 10^7$	100324.44	544860.51
S_s =204	751112.29	35028.30	16246.2	$2.7183 * 10^7$	100324.44	544884.51
S_s = 208	751112.29	35028.30	16270.2	$2.7183 * 10^7$	100324.44	544908.51
p_r =0.0096	751112.29	35026.73	9610.67	25.8932	346.288	400.518
p_r =0.0098	751112.29	35026.76	9760.90	26.4383	346.437	400.529
p_r =0.0100	751112.29	35026.79	9911.12	26.9835	346.586	400.540
p_r =0.0102	751112.29	35026.82	10061.34	27.5287	346.735	400.551
p_r =0.0104	751112.29	35026.85	10211.56	28.0739	346.884	400.561
b= $4.8 * 10^{-7}$	751112.29	35028.31	$3.60534 * 10^7$	$2.7183 * 10^7$	100324.44	$1.30478 * 10^9$
b= $4.9 * 10^{-7}$	751112.29	35028.31	$3.60534 * 10^7$	$2.7183 * 10^7$	100324.44	$1.30478 * 10^9$
b=$5.0 * 10^{-7}$	751112.29	35028.31	$3.60534 * 10^7$	$2.7183 * 10^7$	100324.44	$1.30478 * 10^9$
b= $5.1 * 10^{-7}$	751112.29	35028.31	$3.60534 * 10^7$	$2.7183 * 10^7$	100324.44	$1.30478 * 10^9$
b= $5.2 * 10^{-7}$	751112.29	35028.31	$3.60534 * 10^7$	$2.7183 * 10^7$	100324.44	$1.30478 * 10^9$

7. unit demand falls by c. as an increase in value of c in Table 4.4 and Table 4.3 then there are no changes in Q_{i+1}, T_{Ret} and T_{sup}.
8. An increase in O_r (ordering cost) no changes in order cost as well supplier cost. But we cansee a little change in retailer cost with the help of Table 4.4 and Table 4.3. Retailer total cost willbe increased.
9. Table 4.4 and b) gives the detail of the relationship between S_s (supplier set-up cost) and Q_{i+1}, T_{Ret} and T_{sup}. This case is directly opposite of O_r. Here, changes occur in supplier costonly.
10. Table 4.4 and Table 4.3 show that independently of the choice of parameter G; $\hat{c_h}$; c_h;τ; $\hat{P_r}$; P_r;b; for a different value, the optimal solutions for Preliminary, post and payments on delivery retain the following relationship: $T_{Ret}(1) \leq T_{Ret}(2) \leq T_{Ret}(3)$ and Q_{i+1}

Table 4.5. Comparison Table.

		r		Q_{i+1}	$T_{Ret}(t_i)$	$T_{sup}(t_i)$	Ce
	t_i →	0.6681	1.3356	2.0026	2.6689	3.3347	4.
Proposed model		0.05		751126.	831.071	22533.8	0.331594
		0.10		751126.	831.005	22533.8	0.331595
		0.20		751126.	830.872	22533.8	0.331598
		0.30		751126.	830.739	22533.8	0.331618
		0.40		751126.	830.607	22533.8	0.331603
Shi et al. (2019) model		r	T_2	Q_2	TC_2	-	Ce
		0.05	0.248	978.25	37300	-	21875
		0.10	0.253	999.14	35687	-	21917
		0.20	0.264	1045.30	32451	-	22014
		0.30	0.276	1098.80	29198	-	22130
		0.40	0.290	1161.50	25926	-	22271

In summary, the proposed supply chain inventory model is compared to Shi et al. (2019) model in a scenario with consistent parameter values. The proposed model introduces all variables within finite planning horizon, making it more realistic. The inclusion of a finite planning horizon further enhances its practicality. In addition, we incorporated additional parameters, including carbon-dependent demand and deterioration rate, to determine the optimal order quantity and cost for retailers or suppliers in our research.

4.9. Graphical representation for Sensitivity findings and analysis:

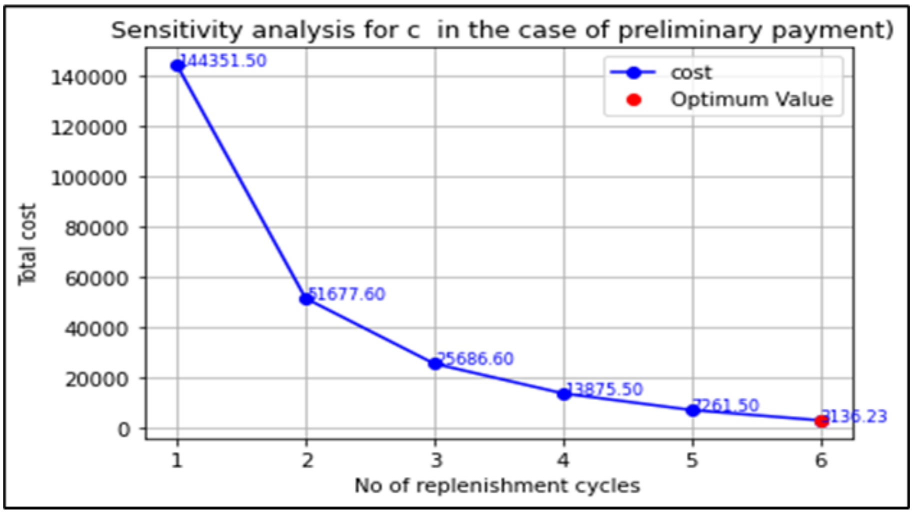

Figure 4.4. Graph presentation for Table 4.2

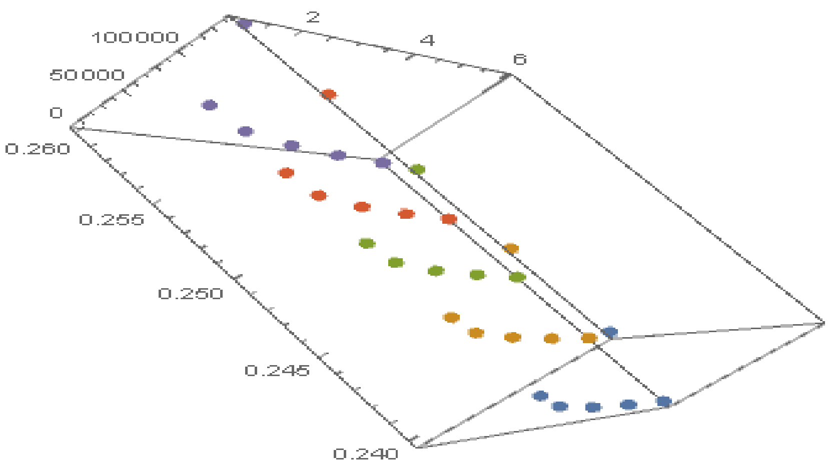

Figure 4.5. Graph presentation for Table 4.3.

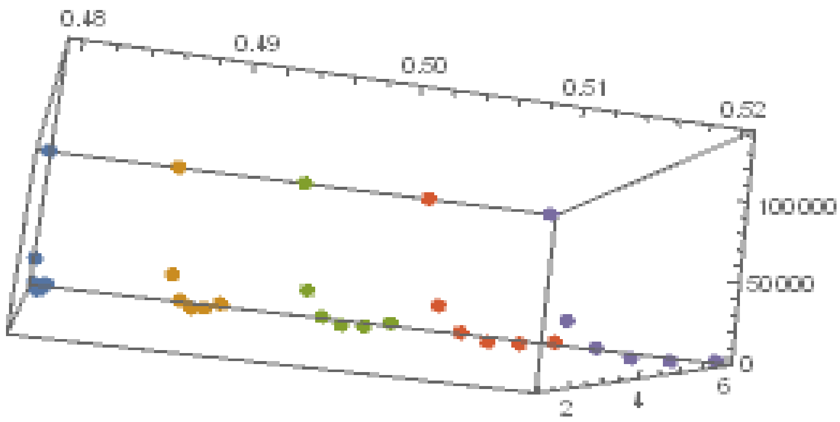

Figure 4.6. 3-D graphical presentation for table 4.3 in the case of preliminary cost

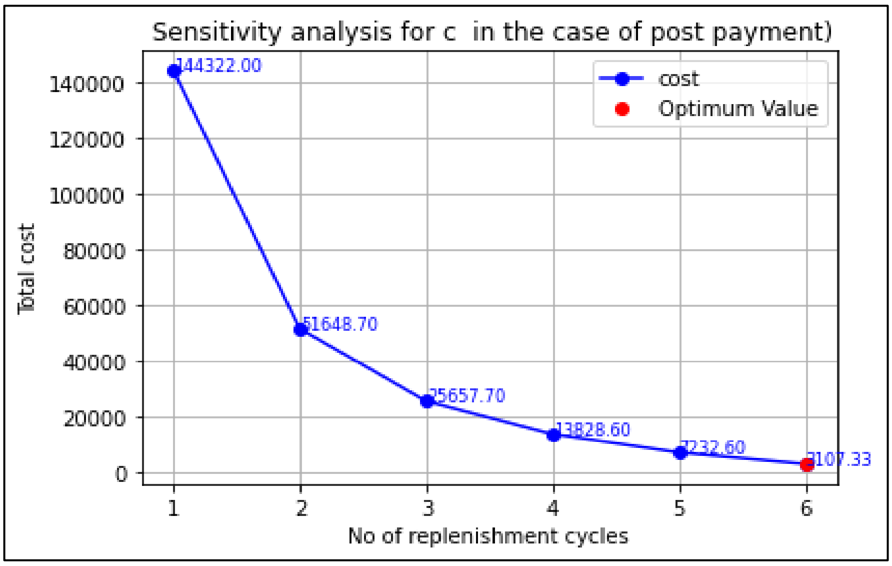

Figure 4.7. Graph presentation for Table 4.3

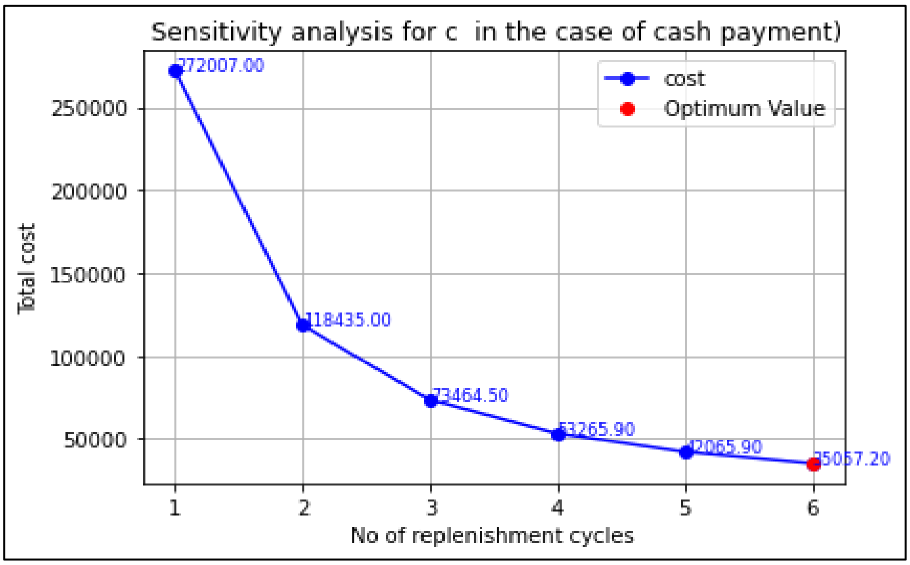

Figure 4. 8. Graph presentation for Table 4.4

4.10. Conclusion and future study of proposed work:

This research proposes a supplier-retailer inventory model that considers three key factors: (1) the seller providing the retailer with three payment schemes (P), (2) some products deteriorating at a fixed rate, and (3) a fixed carbon tax to encourage enterprises to minimize carbon emissions and combat global climate change. Theoretical findings demonstrate that each of these parameters has a unique optimal solution, and numerical examples illustrate the concept under different scenarios. Additionally, sensitivity studies are conducted to investigate the effects of critical parameters on ordering behaviours and the optimal overall retailer and supplier cost per unit of time. The study also examines the correlations between an important parameter and the optimal solution.

4.10.1. Future study:

Future studies could take this research in a variety of directions. First, the scope of the carbon tax policy under consideration could be broadened to include additional carbon tax rules, such as cap-and-trade or carbon offset, which may have a different impact on ordering behaviour. Second, this study could be expanded in the future to include fuzzy parameters, as well as demand based on price and advertising. Third, this study only focuses on reducing the overall retailer and supplier cost per unit of time. A multi-criteria decision analysis that considers both total relevant cost and carbon emissions reduction at the same time would be an interesting and important area for further research. Lastly, supplier-retailer-manufacturer coordination is only conducted to determine the trade credit period. Three-echelon is further explored in Chapter 5, which serves as an extension of the research idea presented in this chapter.

4.11. Recommendations for stakeholders:

This study bases its demand analysis on carbon emissions, which reflects the real-world scenario. Product demand may not always remain constant, and a decrease in demand for items with high carbon production can result in a reduction in carbon emissions. To achieve significant carbon emission reduction in the supply chain, retailers need to pay attention to carbon awareness within the circular economy, while suppliers and manufacturers should strive to increase their carbon emission reduction efficiency. For items that deteriorate with time, such as medications, suppliers and manufacturers benefit from early payment for various reasons. To ensure timely order fulfilment, manufacturers provide an advanced economy, and supply chain members such as retailers, suppliers, and manufacturers must understand the determinants of their payment strategies. Providers offer discounts, credits, or other incentives in exchange for early payment. The proposed model suggests that in a supply chain with advance payment and carbon-dependent demand, an increase in the supplier and manufacturer's carbon emission reduction efficiency and buyers' carbon awareness can encourage greater carbon emission reduction. To promote business growth and circular economy development, supply chain members should understand the determinants of their judgment strategies during the supply chain process.

The government plays a critical role in promoting and managing the circular economy across the supply chain, and its actions significantly influence supply chain partners' decision-making. Our proposed model focuses on three different approaches that use carbon tax and cap regulations, enabling the government to assign carbon emission caps to industries and incentivize them to reduce carbon emissions. Our findings can help the government create carbon reduction regulations that support companies in reducing carbon emissions and promote a sustainable business strategy.

Chapter 5: Three-level refrigeration supply chain inventory model including linear time-dependent demand for temperature-sensitive items considering carbon tax regulations

5.1. Abstract:

This inventory model explains a production process system. In this research, we investigate the combined inventory replenishment decisions of a retailer, supplier, and manufacturer, as well as the incentive to invest in reducing carbon emissions under the greenhouse gas emission regulation strategy. This research looks at temperature-sensitive items that deteriorate at a constant rate. A "break-even" point is established in this chapter where Q_1+Q_2 is the optimal quantity produced by the manufacturer until the "break-even" point is reached. Thereafter, quantity Q_2 is to be distributed in the remaining ongoing unequal cycles until the total produced quantity Q_2 is transported to the retailer. Q_1 is required to be larger than Q_2. Furthermore, the rate of demand varies linearly with time during a finite period. To satisfy customer demand, the manufacturing rate is postulated to be larger than the demand rate. To demonstrate the proposed model, a numerical instance, scenario analysis, and tabular and graphical representations are furnished. Using the Mathematica software, optimality is also discussed for unequal replenishment time during the finite planning horizon. The present research in this area has been thoroughly analyzed and summarized. Finally, managerial suggestions and future recommendations are also provided.

5.2. Introduction:

Global warming is a situation caused by greenhouse gases (GHG) and some human activities. Carbon emissions have been contributing to global warming for many years. In recent years, global warming has received significant attention. This worldwide consciousness of environmental conservation has inspired many researchers, businesses, and organizations to establish an eco-friendly and low-carbon management system for supply chains, with the Kaya identity playing a crucial role. The Kaya identity is a combination of four factors: emissions per unit of energy expenditure, population, per capita income, and energy intensity per unit of manufacturing output. Kaya (1989) presented this identity as follows:

$$CO_2 = \left(\frac{GDP}{Population}\right)\left(\frac{EC}{GDP}\right)\left(\frac{Co_2}{EC}\right) * Population$$

The Kaya identity is a mathematical formula that links economic, social, and environmental factors to estimate the global atmospheric concentration. The accuracy of the model's predictions is evaluated by comparing them to real data, which also assesses the effectiveness of the Kaya identity in forecasting emissions. Over time, the conclusions drawn from the Kaya observations have become more accurate, and the data indicate a decrease in both energy and carbon intensity.

Companies impose a carbon emissions tax on each unit of their products that generate carbon dioxide. Among all policies aimed at minimizing emissions, a carbon tax is considered the most significant policy choice. Governments and regulators in some countries are now focusing on carbon reduction, as evidenced in the research work of (N. K. Mishra and Ranu 2022) and (Toptal, Özlü, and Konur 2014). The Kyoto Protocol, which is a global initiative associated with the UN Framework Convention on Climate Change, introduced control of greenhouse gas (GHG) emissions, and concerns for environmental stability have grown at all levels of society. The Nations Framework Convention on Climate began to take effect on March 21, 1994, and the international agreement, Kyoto Protocol, was signed by 37 developed countries on December 11, 1997. India also signed an international agreement in 2002 that encouraged both developing and developed nations to curb emission levels through industrialization. Organizations can significantly reduce their CO2 emissions by investing in carbon-lowering initiatives such as cleaner and more efficient transportation fleets, energy-efficient warehousing, and sustainable manufacturing processes, as demonstrated by (Bae, Sarkis, and Yoo 2011); (Ilic, Staake, and Fleisch 2009) and (Liu, Anderson, and Cruz 2012).

Encourage enterprises and emerging economies to join the efforts to minimize carbon emissions. International bodies and national governments have established several regulations to lower GHG emissions, such as carbon taxes. (U.

Mishra, Wu, and Sarkar 2021) explained in their research ideas that some government entities have imposed a carbon tax on companies or industries that emit carbon dioxide throughout their operations, causing environmental pollution. These industries, which produce emissions, are responsible for rising temperatures as they utilize fossil materials throughout their production flow. Therefore, governments impose taxes on those industries to control global climate change and protect the environment.

In their research work, (Xi Chen et al. 2013) introduced Walmart (Walmart Sustainability. 2022), which made a new decision in 2016 to avoid 1 billion metric tonnes of carbon emissions from the global supply chain by 2030 through Project Gigaton (Project Gigaton Accounting Methodology. 2022). This initiative aims to mitigate the entire supply chain greenhouse gas emissions by one billion metric tonnes by 2030. Project Gigaton was developed to address the fact that almost all emissions in the supply chain process occur in transportation and manufacturing instead of stores and distribution centres.

The remainder of this research study is structured into various sections. Section 5.3 provides a comprehensive review of the existing literature. In Section 5.4, the notations and assumptions employed are outlined. Section 5.5 outlines the mathematical model and its calculations, while Section 5.6 focuses on algorithms and techniques. Section 5.7 elaborates on numerical simulations, while Section 5.8 presents sensitivity analysis in both tabular and graphical formats. Section 5.9 discusses academic and management implications. Additionally, Section 5.10 offers conclusions, and 5.11 suggestions for future research.

5.3. Literature review:

Researchers have begun including carbon pollution in supply chain models that focus on control and management. The transportation, storage, order management, and waste disposal processes in the supply chain all contribute to greenhouse gas emissions. (Zanoni et al. 2014) created a model that combined stock for a commodity with price and environmental criteria. Earlier literature, such as that of (Taleizadeh et al. 2018), implemented a system to transform the climate component of CO2 emissions into the market price. In their recent research, (Hariga, As'ad, and Shamayleh 2017) investigated the emissions from the supply of cold items and found that coordinating vendor-managed inventory (VMI) can save money and minimize emissions. They also incorporated the cost of CO2 and energy that emit from the cold truck and freezer unit into their approach. (Yakavenka et al. 2019) analysed the total CO2 emissions and transit time to highlight the environmental and social impacts. Supply chain management has received significant attention in both research and business, with CO2 emissions being a major issue. Many governments have imposed carbon taxes to encourage the reduction of greenhouse gas emissions and promote ecological sustainability. Therefore, the effect of emission taxes on the inventory supply network and the decision-making process has become a topic of discussion.

Governments and regulatory agencies face increasing pressure to establish laws aimed at reducing carbon emissions. To tackle this issue, both governments and firms across the globe are taking measures to reduce their carbon footprint. The adoption of carbon emission plans and policies is increasingly popular in the effort to reduce greenhouse gas emissions. According to (Xi Chen et al. 2013), carbon pricing policies were first introduced by Finland and Sweden, making them the pioneering countries in this regard, and since then, several other countries, including India, the United Kingdom, Ireland, British Columbia, Canada, Australia, and China, have followed suit. In their study, (Benjaafar, Li, and Daskin 2013) integrated the total cost of production and carbon emission costs, considering emission taxation policies. The primary aim of carbon tax policies is to impose a fee on industries that exceed their carbon footprint and limit their emissions.

The objective of this study is to examine the carbon emissions associated with transportation, warehousing, and waste disposal, as well as the cost of the system. The analysis considers the regulation of carbon tax. Large companies such as IBM, Pepsi Co, and Johnson require their suppliers to document and report their GHG emissions. (Saif and Elhedhli 2016) reported that Wal-Mart reduced GHG emissions from its supply chain by 18 million tonnes in 2015. (Xi Chen et al. 2013) investigated the impact of the EOQ model under a cap policy on carbon emissions and costs and demonstrated how their findings can be applied to tax, cap-and-offset programs to minimize the emissions. (He et al. 2015) used the EOQ model to discuss a firm's production lot-sizing issues following the carbon tax and the cap-and-trade system norms. There are four primary carbon emission policy options for lowering carbon emissions: mandated capacity for carbon emissions, tax for emitting carbon dioxide emissions, cap-and-trade, and carbon offset investment, as stated by (Song and Leng 2012). Preservation, resources, and recycling are important aspects of green and sustainable supply chain management, according to (Bae, Sarkis, and Yoo 2011).

Based on the previous review, it can be inferred that a planning model that considers both deteriorating items and carbon emission policies has not been utilized. Our research study covers the topic of supply chain management with competing organizations at three different levels. While there is an abundance of literature available on this subject, previous research has mainly focused on two-level inventory frameworks. Therefore, we proposed a three-level inventory supply chain model in this study. The multi-echelon inventory model has been a subject of great interest in recent years, and our proposed model includes a supplier, retailer, and producer. (Kreng and Chen 2007) have already developed a three-tier supply chain inventory model including a distributor, a single producer, and a retailer. The three-tiered inventory supply chain encompasses the complete process of producing, holding, transporting, and selling essential goods, involving manufacturers, suppliers, retailers, transporters, warehouses, and consumers. It is a process that involves converting natural resources or raw materials into finished products, transporting them, and then selling them to end-users or customers. (Kumar et al. 2012) conducted a study on the three-tiered model of inventory in the context of the supply chain that considered highly restricted storage capacity during inflationary periods. (B. Sarkar et al. 2016) have studied a conceptual framework for supply chain coordination consisting of three tiers, where the seller produces semi-finished goods and transports them to a manufacturer for further processing, and then the finished goods are sold to the retailer. The model aims to reduce inventory costs by including variable shipping and greenhouse gas emission fees in multiple deliveries. In a similar vein, (Daryanto et al. 2019) explain that carbon emissions resulting from warehousing, transportation, and disposal of decayed goods are all part of a three-level supply chain. Meanwhile, (B.

Sarkar et al. 2021) propose a coordinated sustainable supply chain management approach for a three-level network consisting of one supplier, one producer, and several retailers. Their main objective is to reduce total supply chain costs while controlling deterioration and emissions. To avoid excessive holding costs, the retailer fulfils orders based on customer requirements, and demand is assumed to be constant throughout the supply chain. Most of the retailer's expenses are related to ordering and holding inventory. Therefore, there is a need to investigate an inventory model that considers carbon costs for three-echelon models over a finite planning horizon.

The cold chain logistics system is responsible for both direct and indirect greenhouse gas emissions, making it the most significant source of pollution in the inventory supply chain network. As a result, reducing carbon dioxide emissions in this sector is crucial for energy conservation and cost reduction. (Quaglini et al. 2022) focus on examining one of the largest underutilized energy sources, the final energy consumed in space cooling systems in the European transportation network. (Hu et al. 2021) developed a carbon emissions optimization model to investigate how to reconcile carbon restrictions with cost reductions in the cold supply chain. (S. Wang et al. 2017) optimized the Transportation Problem (VRP) with service times to maintain the cold supply chain in China's carbon tax-based supply networks, reducing cost pressures on the cold supply chain. The researchers developed a low-cost, low-carbon cold supply chain distribution route to optimize the system.

In addition to variable costs, the distribution process of fresh goods also involves fixed costs such as transportation costs, damage costs, carbon emissions costs, and freezing costs. The carbon emissions in this context are divided into two types: fuel usage and cooling energy consumption. (M. Wang et al. 2018) researched a carbon trading framework for the delivery of fresh goods using refrigerated trucks with a pollution cap-and-trade scheme. The study examines the use of cap-and-trade mechanisms to reduce greenhouse gas emissions and explores the impact of refrigerated shipping services and carbon trading on the coordination between suppliers and retailers. A research analysis by (Matskul et al. 2021) aimed to optimize the management of a frozen supply distribution network by developing a mathematical model for a fixed logistics network of frozen supply chain processes involving refrigerated vehicles while taking into account the associated environmental emission costs. A model for Integrated Routing and Inventory Planning (IRP) was developed by (Stellingwerf et al. 2018) to investigate the advantages of collaboration in a temperature-controlled supply chain in terms of sustainability.

Refrigeration is the process of manufacturing cooling by transferring heat from a low-temperature resource to a high-temperature resource. Its impact is significant in manufacturing, storage facilities, and the supply chain, especially for perishable goods such as food, medicine, vegetables, and flowers. Refrigeration technology plays a crucial role in preserving the safety and quality of perishable goods concerning temperature sensitivity during storage and transportation. Some businesses, such as Walmart and Americold, provide refrigerated delivery services to other business owners to maintain the integrity of the products.

(Hariga, As'ad, and Shamayleh 2017) examined a multi-stage supplier-retailer chain model that includes a factory, distribution, storage facility, and retailer. The study evaluated the impact of carbon dioxide emissions from storage and transportation tasks in the frozen product supply chain. The authors proposed three models to minimize capital expenditure, economic and environmental risks, and carbon emissions, respectively. These models aim to determine the optimal order size, shipping volumes, number of vehicles, directions, and refrigerated units to use throughout the entire supply chain, as well as to reduce carbon footprint through carbon tax policies. However, no existing inventory control and management model accounts for carbon cost, temperature-sensitive items, and three-tier supply chains with a finite planning horizon. Therefore, the authors developed such a model.

(C. Wu and Zhao 2014) and (J. Wu, Teng, and Chan 2018) have noted that previous literature has focused on stock control and management with the demand that is dependent on emissions or demand that fluctuates over time. In addition, it is common to observe that consumer sales volume increases significantly with time and stock levels in two echelons within a finite horizon. (Babagolzadeh et al. 2020) investigated uncertain demand during a cold supply chain over a finite planning horizon, while (Daryanto et al. 2019) examined a constant demand across three echelons along with carbon emissions.

This work presents a model for the supply chain of cold items, which includes a manufacturer, a supplier, and a retailer over a finite period. Linear time-dependent demand is also taken into consideration in this study.

The findings of the previous study indicate that the issue of temperature-sensitive goods has not been tackled in the context of the three-tier inventory model with carbon cost over finite periods and linear demand.

5.3.1. Define the research problem:

There is a significant gap in the literature concerning the three-tier inventory model for a supply chain network that involves refrigeration for deteriorating items with carbon emissions under a finite planning period. Additionally, no research has been found that addresses supply chain inventory management with carbon emissions under a finite period for the optimal length of cycle that depends on the total cost or other factors (such as different cycle lengths). Therefore, this study aims to investigate the management of deteriorating items, refrigeration, and emissions during a three-level supply process. The study will also analyze how different carbon emission tax policies affect carbon emissions and supply chain management. This model has been proposed based on a literature review and will be the first of its kind. It is also applicable for real life. In a real-world, a pharmaceutical business optimises manufacturing, distribution, and retailing operations by managing a three-level refrigeration supply chain for temperature-sensitive vaccines.

5.4. Assumptions:

Here are the assumptions that underpin the mathematical model of the inventory manufacturing and replenishment scenario with constant deterioration and linear time-dependent demand:

i. The demand is a time-dependent linear that varies with time.
ii. The inventory level is continuously monitored and replenished.
iii. The lead time for replenishment is negligible.
iv. In the three-level supply chain approach, we did not maintain a constant time interval for ordering.

v. The demand is fulfilled entirely without any delay.

vi. The production rate of the manufacturer is constant.

vii. Deterioration can only happen when the commodities are properly stored and restocked.

viii. In supply chain management, a constant deterioration rate (θ) is assumed for all players, including retailers, suppliers, and manufacturers.

ix. Items that have deteriorated are not repaired or replaced during the planned horizon.

x. The replenishment cost and the holding cost are constant.

xi. The transportation cost is included in the cost associated with placing an order.

xii. The cost of carbon emissions due to holding and transportation is also considered. A single item that is temperature-sensitive is retained in stock within a finite planning horizon.

xiii. Assuming the production (P) is greater than the required amount by the purchaser (D), as per the given scenario.

xiv. In the single setup single delivery model, we have fixed the emission amount to incorporate emissions from refrigeration setup during transportation and holding. The retailer places 'n' orders for different quantities, and the transportation cost is included in the ordering cost.

xv. This study does not allow for shortages.

xvi. The transportation of products, inventory storage, and disposal of deteriorating items all contribute to carbon emissions.

5.5. Mathematical formulation and analysis of the model:

A cold/refrigerated supply chain model with a single supplier, producer, and retailer has been developed in this study. The supplier sends semi-finished goods or raw materials to the producer in batches of various sizes. The producer, with a production rate of P, completes the items using unfinished products from the supplier and distributes them to the retailer. The producer considers carbon emission costs, which are included in setup costs. The retailer also considers carbon emission costs, which include fuel consumption of vehicles and refrigeration during storage and transportation. In the model, the costs of each supply chain player are derived specifically, and new features such as carbon emissions from holding inventory and deterioration are examined over a limited planning horizon. The model is divided into two sections, in which the producer produces the number of items required for the remaining planning time horizon by simultaneous replenishment in each cycle to the retailer. An analytical solution to the problem has been studied, where the producer reaches the "break-even point" at time t, after which no production occurs, but replenishment persists for the specified intervals as shown in the graphical representation. For the retailer's cost, only ordering, degrading, holding, and carbon emission costs are considered, and transportation costs are included in the retailer's ordering cost. This model extends Rao's model and examines the outcomes of new characteristics over a limited planning horizon.

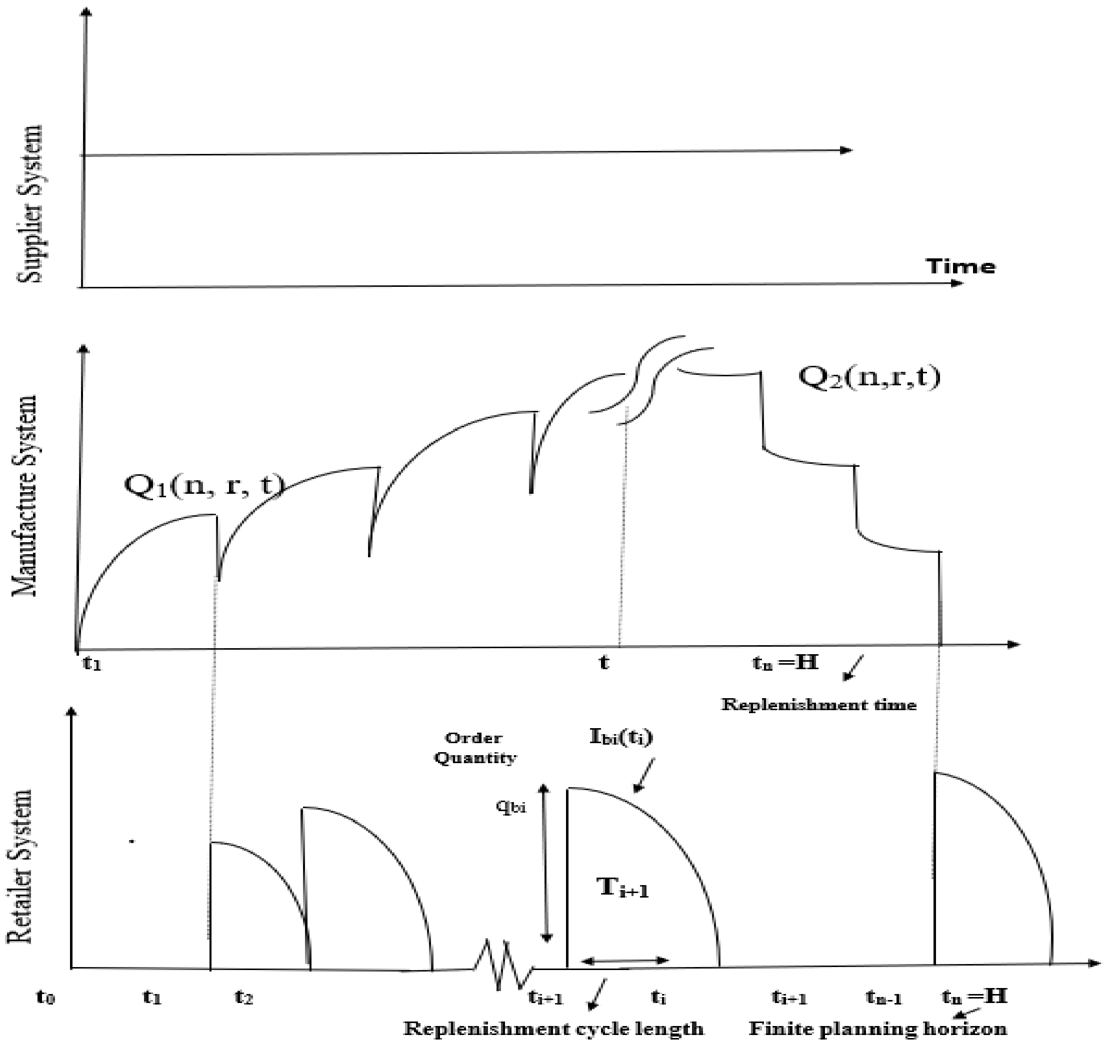

Figure 5.1. Three-echelon supply chain management

$$\frac{dI_{b_{i+1}}(t)}{dt} = -\mathrm{D} - \theta * I_{b_{i+1}} \tag{5.1}$$

$$t_i < \mathrm{t} < t_{i+1}$$

$$I_{b_{i+1}}(t) = e^{-\theta t} \int_t^{t_{i+1}} (a + bt) e^{\theta_b * u}\, \mathrm{du} \tag{5.2}$$

$$I_{b_{i+1}}(t_{i+1}) = 0 \quad \text{and} \quad I_{b_{i+1}}(t_i) = q_{b_i} \tag{5.3}$$

$$I_{b_{i+1}}(t) = \int_t^{t_{i+1}} (a + bu) e^{\theta(u-t)}\, \mathrm{du} \tag{5.4}$$

$$I_{b_{i+1}}(t) = \left[\frac{(a+bu)e^{\theta(u-t)}}{\theta} - \frac{b}{\theta^2} e^{\theta(u-t)} \right]_t^{t_{i+1}}$$

$$I_{b_{i+1}}(t) = \frac{(a+bt_{i+1})e^{\theta(t_{i+1}-t)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t)} - \frac{(a+bt)}{\theta} + \frac{b}{\theta^2} \tag{5.5}$$

The order quantity for ith cycles

$$q_{b_i} = I_{b_{i+1}}(t_i) = \int_{t_i}^{t_{i+1}} (a + bt) e^{\theta(t-t_i)}\, \mathrm{dt}$$

$$q_{b_i} = I_{b_{i+1}}(t_i) = \frac{D(p,g)\left\{e^{\theta(t_{i+1}-t_i)} - 1\right\}}{\theta} \tag{5.6}$$

BUYER SOLUTION:

Ordering costs of the buyer including carbon costs due to transportation:$= n * O_b$

Holding cost: $h_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} I_{b_{i+1}}(t)\, dt$

$$h_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu)e^{\theta(u-t)}\, \mathrm{du}\, dt \qquad (5.7)$$

Deteriorating costs according to Sarkar et al.2012 $d_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \theta * I_{b_{i+1}}(t)\, dt$

$$d_r * \theta \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} \left\{ \int_{t}^{t_{i+1}} (a + bu)e^{\theta(u-t)}\, \mathrm{du} \right\} dt$$

$$d_r * \theta \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu)e^{\theta(u-t)}\, \mathrm{du}\ dt \qquad (5.8)$$

Carbon emission cost due to refrigeration:

Transportation-related emissions can be significant for some products, while emissions related to storage (such as for packed goods) may be small and negligible. However, in the case of refrigeration, it is the opposite. For refrigerated items, emissions due to storage can be substantial. Walmart has recognized the importance of refrigerant emissions in its supermarkets, which generate more greenhouse gases than its truck service. As a result, we need to consider carbon emissions related to refrigeration in the retailer's total cost.

Another major retailer, Tesco, has found that refrigerant emissions account for about 26% of its direct emissions (Tesco-climate change. 2023). According to (Shi et al. 2019), the carbon emissions related to holding or refrigeration costs are as follows:

$$\widehat{P_r} * Co_2 \int_{t_i}^{t_{i+1}} I_{b_{i+1}}(t)\, dt$$

$$\widehat{P_r} * Co_2 \int_{t_i}^{t_{i+1}} \left\{ \int_{t}^{t_{i+1}} (a + bu)e^{\theta(u-t)}\, \mathrm{du} \right\} \mathrm{dt} \qquad (5.9)$$

carbon emission cost due to deterioration

$$\widehat{d_r} * Co_2 \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \theta * I_{b_{i+1}}(t)\, dt$$

$$\widehat{d_r} * \theta * Co_2 \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \left\{ \int_{t}^{t_{i+1}} (a + bu) e^{\theta(u-t)}\, \mathrm{du} \right\} dt \tag{5.10}$$

Retailer's overall cost

Total cost = Inventory holding cost + Ordering cost +Deteriorating cost +Carbon emission cost from deterioration + Carbon emission cost from Inventory holding

$$T_{Ret}(t_i, n) = n * O_r + h_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} I_{b_{i+1}}(t)\, dt + d_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \theta * I_{b_{i+1}}(t)\, dt + \widehat{P_r} * Co_2 \int_{t_i}^{t_{i+1}} I_{b_{i+1}}(t)\, dt + \widehat{d_r} * Co_2 \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \theta * I_{b_{i+1}}(t)\, dt$$

$$T_{Ret}(t_i, n) = n * O_r + \left\{ h_r + d_r * \theta + \widehat{P_r} * Co_2 + \widehat{d_r} * Co_2 * \theta \right\} \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} I_{b_{i+1}}(t)\, dt$$

$$T_{Ret}(t_i, n) = n * O_r + \left\{ h_r + d_r * \theta + \widehat{P_r} * Co_2 + \widehat{d_r} * Co_2 * \theta \right\} \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu) e^{\theta(u-t)}\, \mathrm{du}\, dt$$

$$T_{Ret}(t_i, n) = n_1 * O_r + \left\{ h_r + d_r * \theta + \widehat{P_r} * Co_2 + \widehat{d_r} * Co_2 * \theta \right\} \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \frac{(a + bt_{i+1}) e^{\theta(t_{i+1}-t)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t)} - \frac{(a + bt)}{\theta} + \frac{b}{\theta^2}\, dt$$

$$T_{Ret}(t_i,n) = \mathrm{n} * O_r + \{h_r + d_r * \theta + \widehat{P_r} * Co_2 + \widehat{d_r} * Co_2 * \theta\} \sum_{i=1}^{n} \left\{ (a+bt_{i+1}) \frac{e^{\theta_b(t_{i+1}-t_i)}}{\theta^2} - \frac{b}{\theta^3} e^{\theta(t_{i+1}-t_i)} - \frac{(a+bt_{i+1})}{\theta^2} + \frac{b}{\theta^3} - \frac{2b}{\theta} \right\}$$

$$T_{Ret}(t_i,n) = n_1 * O_r + \{h_r + d_r * \theta + \widehat{P_r} * Co_2 + \widehat{d_r} * Co_2 * \theta\} \sum_{i=1}^{n} \left\{ (a+bt_{i+1}) \frac{e^{\theta(t_{i+1}-t_i)}}{\theta^2} - \frac{b}{\theta^3} e^{\theta(t_{i+1}-t_i)} - \frac{(a+bt_i)}{\theta^2} + \frac{b}{\theta^3} - \frac{2b}{\theta} - \frac{b(t_{i+1}-t_i)}{\theta^2} \right\} t_{i+1}$$

$$T_{Ret}(t_i,n) = n * O_r + \{h_r + d_r * \theta + \widehat{P_r} * Co_2 + \widehat{d_r} * Co_2 * \theta\} \sum_{i=1}^{n} \left\{ \frac{1}{\theta} \left\{ (a+bt_{i+1}) \frac{e^{\theta(t_{i+1}-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{(a+bt_i)}{\theta} + \frac{b}{\theta^2} \right\} - \frac{2b}{\theta} - \frac{b(t_{i+1}-t_i)}{\theta^2} \right\}$$

$$Where \qquad t_{i+1} - t_i = H$$

$$T_{Ret}(t_i,n) = n * O_r + \{h_r + d_r * \theta + \widehat{P_r} * Co_2 + \widehat{d_r} * Co_2 * \theta\} \sum_{i=1}^{n} \left\{ \frac{1}{\theta} \left\{ (a+bt_{i+1}) \frac{e^{\theta(t_{i+1}-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{(a+bt_i)}{\theta} + \frac{b}{\theta^2} \right\} - \frac{2b}{\theta} - \frac{b * H}{\theta^2} \right\}$$

$$T_{Ret}(t_i, n) = \text{n} * O_r + \left\{\frac{h_r + \widehat{P_r} * Co_2}{\theta} + d_r + \widehat{d_r} * Co_2\right\} \sum_{i=1}^{n} \left\{\left\{(a + bt_{i+1})\frac{e^{\theta(t_{i+1}-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{(a + bt_i)}{\theta} + \frac{b}{\theta^2}\right\} - \frac{2b}{\theta} - \frac{b * H}{\theta^2}\right\}$$

$$T_{Ret}(t_i, n) = n * O_r + \left\{\frac{h_r + \widehat{P_r} * Co_2}{\theta} + d_r + \widehat{d_r} * Co_2\right\} \sum_{i=1}^{n} \left\{\left\{(a + bt_{i+1})\frac{e^{\theta_b(t_{i+1}-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{(a + bt_i)}{\theta} + \frac{b}{\theta^2}\right\} - \frac{2b}{\theta} - \frac{b * H}{\theta^2}\right\}$$

Where $q_{b_i} = I_{b_{i+1}}(t_i) = \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,\text{dt} = \left\{(a + bt_{i+1})\frac{e^{\theta(t_{i+1}-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{(a+bt_i)}{\theta} + \frac{b}{\theta^2}\right\}$

$$T_{Ret}(t_i, n) = \text{n} * O_r + \left\{\frac{h_r+\widehat{P_r}*Co_2}{\theta} + d_r + \widehat{d_r} * Co_2\right\} \sum_{i=1}^{n} \left\{\int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,\text{dt} - \frac{2b}{\theta} - \frac{b*H}{\theta^2}\right\} \tag{5.11}$$

$$\frac{\partial T_{Ret}(t_i,n)}{\partial t_i} = \left\{\frac{h_r+\widehat{P_r}*Co_2}{\theta} + d_r + \widehat{d_r} * Co_2\right\}\left\{(a + bt_i)\left(e^{\theta(t_i-t_{i-1})} - 1\right) - \theta \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,\text{dt}\right\} \tag{5.12}$$

Manufacturer solution:

$$\frac{dI_{P_{i+1}}(t)}{dt} = \mathrm{P} - \theta * I_{P_{i+1}}(t) \tag{5.13}$$

$$\frac{dI_{P_{i+1}}(t)}{dt} + \theta * I_{P_{i+1}}(t) = \mathrm{P} \quad \text{where} \quad t_i < \mathrm{t} < t_{i+1}$$

$$I_{P_{i+1}}(t) = e^{-\theta t} \int_{t_i}^{\mathrm{t}} P * e^{\theta * u}\, \mathrm{du}$$

$$I_{P_{i+1}}(t) = \int_{t_i}^{\mathrm{t}} P * e^{\theta_P(u-t)}\, \mathrm{du}$$

$$I_{P_{i+1}}(t_{i+1}) = q_{P_i} - q_{b_i} \quad \text{and} \quad I_{P_{i+1}}(t_i) = q_{P_{i-1}} - q_{b_{i-1}}$$

$$I_{P_{i+1}}(t) = \frac{P * \left\{1 - e^{\theta(t_i - t)}\right\}}{\theta}$$

$$q_{P_i} - q_{b_i} = I_{P_{i+1}}(t_{i+1}) = \frac{P}{\theta}\left\{1 - e^{\theta(t_i - t_{i+1})}\right\}$$

$$q_{P_i} - q_{b_i} = I_{P_{i+1}}(t_{i+1}) = \frac{P}{\theta}\left\{1 - e^{-\theta(t_{i+1} - t_i)}\right\}$$

$$Q_1(r,t) = \sum_{i=1}^{r} \frac{P}{\theta} * \left\{1 - \mathrm{e}^{\theta(\mathrm{t}_i - \mathrm{t}_{i+1})}\right\} + \sum_{i=1}^{r} q_{b_i} + \frac{P}{\theta} * \sum_{i=r-1}^{r} \left\{1 - \mathrm{e}^{\theta(\mathrm{t}_i - \mathrm{t}_{i+1})}\right\}$$

$$Q_1(t) = \sum_{i=1}^{r} \frac{P}{\theta} * \left\{1 - \mathrm{e}^{\theta(\mathrm{t}_i - \mathrm{t}_{i+1})}\right\} + \sum_{i=1}^{r} q_{b_i} \tag{5.14}$$

According to Ghare and Schrader (1963), and Rau (2003) the ending inventory after time t can be stated as Opening inventory = Ending inventory $(1-\theta)^{-(t_{i+1}-t_i)}$ under the fixed deterioration rate. The finished product inventory is Q2; after (Ti-t), the inventory is Q2; this is determined from the above equation. shown in Figure 5.2.

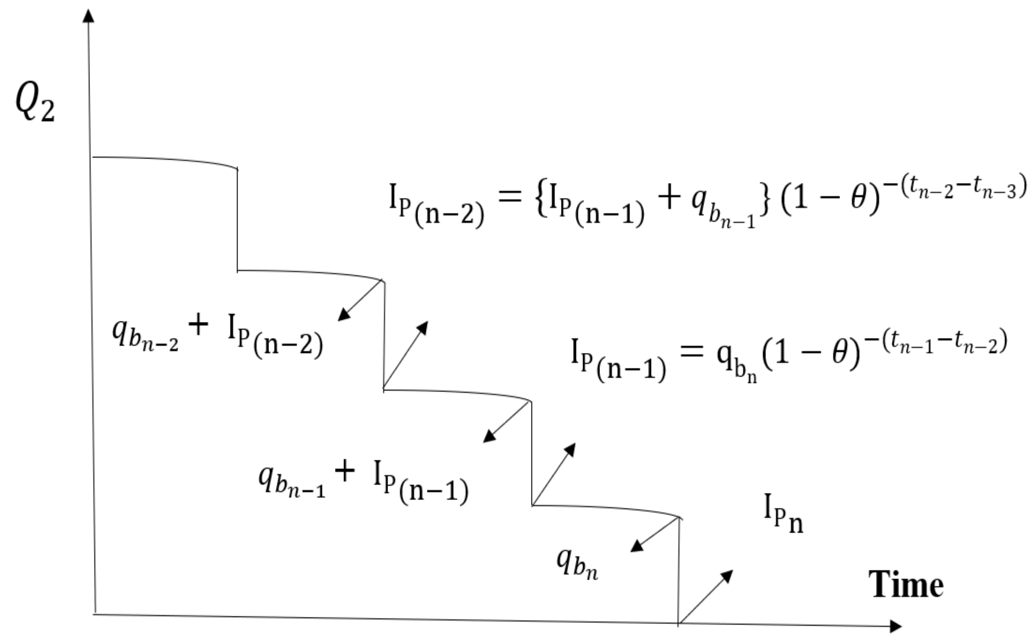

Figure 5.2 Replenished inventory in different cycles.

$$Q_2(n,r,t) = \sum_{i=1}^{r+1} q_{b_{n-i-1}} * (1-\theta)^{(t_{n-i-1}-t_{n-r})} + \left\{\sum_{i=n-r}^{n} q_{b_{n-i-1}} * (1-\theta)^{(t_{n-i-1}-t_{n-r})} + q_{b_i}\right\}(1-\theta)^{(t_{n-r}-t)}$$

$$Q_2(n,r,t) = \sum_{i=1}^{r+1} q_{b_{n-i-1}} * (1-\theta)^{(t_{n-i-1}-t_{n-r})} \quad (5.15)$$

$$T_{sup}(n^*, t_1^*, t_2^* \dots\dots\dots\dots t_n^*) = n^* * S_s + P_r \sum_{i=1}^{n^*} \frac{P}{\theta} * \left\{1 - e^{\theta(t_i - t_{i+1})}\right\} \quad (5.16)$$

5.6. Algorithm and procedures for resolving the problem:

The following techniques can be used to find the solution:

1. Firstly, the values for all specified parameters, including W, h_r, P, a, b, and c, should be fixed.
2. To determine the optimal solution in this three-echelon scenario, we first need to identify the optimal ordering time pattern. This involves finding the values of t_i. Continuous in the following ways to find t_i as

a. If we assign n=1, we get t_0=0 and t_1=H.

b. If we consider n=2, then we can initialise t_0=0 and t_2=H. Then, using Eq. (5.12), we can get t_1.

c. To use the obtained values of t_1 in the preceding step, similarly, find t_2 from Eq. (5.12).

d. By using values of t_2, get t_3 from equation (5.12).

e. Working in this approach until t_{n-1} is achieved. Having t_{n-1} values that are approximately equivalent to H (finite horizon planning), and t_i values that satisfy the Hessian matrix theorem.

f. We will determine the unique and optimum values of t_i for each m 1 = 1; 2; 3;:::.

3. After calculating the value of ti by using Eqn (5.12). Then derived the values of Q_1 and Q_2.
4. Then we can easily find out the value of t by putting Q_1-Q_2=0.
5. The value of $T_{Ret}(t_i, n)$ can be determined by Eq. (5.11).
6. By using equation (5.11), $T_{Ret}(t_i, n)$ (n) is obtained to calculate the optimal value of the total cost of the retailer $T_{Ret}(t_i, n)$ by using the following conditions.

a. For $n = 1, then\ T_{Ret}(t_i, n) = T_{Ret}(t_i, n)$(n) $and\ stop.$ For $n \geq 2,$ And if $T_{Ret}(t_i, n) \leq T_{Ret}(t_i, n-1)$ and $T_{Ret}(t_i, n) \leq T_{Ret}(t_i, n+1)$ then $T_{Ret}(t_i, n)$(n)=Optimal ($T_{Ret}(t_i, n)$) And stop otherwise go to the previous step. Similarly, $T_{Ret}(t_i, n)(n)$ system by using Eqn. (5.11).

7. The aforementioned solution approach will be also used to derive the optimal n1, Q1, Q2, $T_{Ret}(t_i, n)$ and $T_{Ret}(t_i, n)$values.

5.7. Numerical illustration for the proposed model:

For n = 1, 2, 3, ……… replacement cycles and various parameters, this section would contribute information on numerical output; we'll examine numeric values to show how different factors affect the overall cost of retailers, suppliers, time intervals, and replenishment cycles. Let h_r=0.9 ($/unit/unit duration), d_r =0.01 ($/unit), D_{eb}=0.07 (ton CO2/unit), P=400 (unit/unit time), CO_2=0.2 (unit/time/ton), C_{hb}=0.01 ($/ton CO2/unit), $\boldsymbol{\theta}$ =0.2 (unit/time), H=4, a=400 (unit/time), b=100 (unit/time), S_s= 1

(\$/lot), O_r=480 (\$/lot), P_r =0.5 (\$/unit), with the appropriate units. Using the Mathematica application (version 12), we have solved nonlinear Eqn. (5.11) and Eqn (5.12). The optimal quantity, and total cost of the retailer and supplier, are presented graphically and in the tabular form that is given below.

Table 5.1. Total cost for the retailer for five different values of a

↓ a / → n_1	1	2	3	4	5	6	7	8	9
400	7555.22	5112.3	4280.96	**4051.15**	091.56	4273.98	4539.68	4858.18	5212.21
500	10487.00	7661.7	6627.3	6279.70	**6244.24**	6373.98	6601.03	6890.02	7220.78
600	13410.60	10207.4	8971.53	8506.88	**8395.94**	8473.22	8661.79	8921.37	9228.95
800	19242.40	15291.9	13656.1	12958.7	12697.5	**12670.4**	12782.2	12983.2	13244.6
1000	25062.00	20370.7	18337.4	17408.4	16997.7	**16866.4**	16901.8	17044.4	17259.6

Table 5.2. The lowest total cost of the supplier, and retailer and the optimal number of replenishment cycles

a	n_1	t_0	t_1	t_2	t_3	t_5	t_5	t_6	t_7	Q_{bi}	Tcs	Tcr
400	4	0	0.90766	1.74775	2.5367	3.28503	4.			2017.51	2059.71	**4051.15**
500	5	0	0.74887	1.45637	2.13033	2.77633	3.39849	4.		2440.74	2263.78	**6244.24**
600	5	0	0.73787	1.44085	2.11473	2.7638	3.39135	4.		2797.27	2480.83	**8395.94**
800	6	0	0.62188	1.22364	1.80763	2.37579	2.92972	3.47075	4.	3583.66	2895.44	**12670.4**
1000	6	0	0.61299	1.20987	1.79221	2.36135	2.9184	3.46434	4.	4310.01	3323.44	**16866.4**

Preposition 5.1:

$$(a + b\, t_{i+1})e^{\theta(T_{i+1})} < \; b(\, e^{\theta(T_i)} - 1)/\,\theta \; + (a + bt)e^{\theta(T_i)}$$

$$F(t_i) - F(t_{i+1}) < \frac{F'(t_i)}{F(t_i)} \int_{t_i}^{t_{i+1}} F(t)\, dt$$

Let F(t)= $(a + bt)e^{\theta(t - t_i)}$ is a log convex function. By putting the value of F(t) in the above equation. we have

$$(a + b\, t_{i+1})e^{\theta(t_{i+1} - t_i)} - \; (a + bt_i) < \; \frac{b + \theta(a + bt_i)}{(a + bt_i)} \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t - t_i)}\, dt$$

$$(a + b\, t_{i+1})e^{\theta(t_{i+1} - t_i)} - \; (a + bt_i) < \; (\frac{b}{(a + bt_i)} + \theta) \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t - t_i)}\, dt$$

$$\frac{\partial T_{Ret}(t_i,n)}{\partial t_i} = \left\{\frac{H_b + C_{H_b} * Co_2}{\theta} + D_{c_b} + D_{e_b} * Co_2\right\}\left\{(a + b\,t_i)\left(e^{\theta(t_i - t_{i-1})} - 1\right) - \theta\int_{t_i}^{t_{i+1}} (a + \quad bt)e^{\theta(t - t_i)}\,dt\right\} == 0 \tag{5.12}$$

By Eqn. (5.12) we have $\frac{(a+b\,t_i)\left(e^{\theta(t_i - t_{i-1})}-1\right)}{\theta} = \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt$

$$(a + b\,t_{i+1})e^{\theta(T_{i+1})} - (a + bt_i) < \left(\frac{b}{(a + bt_i)} + \theta\right)\frac{(a + b\,t_i)e^{\theta(T_i)} - (a + b\,t_i)}{\theta}$$

$$(a + b\,t_{i+1})e^{\theta(T_{i+1})} - (a + bt_i) < \frac{b\,e^{\theta(T_i)}}{\theta} - \frac{b}{\theta} + (a + b\,t_i)e^{\theta(T_i)} - (a + b\,t_i)$$

$$(a + b\,t_{i+1})e^{\theta(T_{i+1})} < \frac{b\,(e^{\theta(T_i)}-1)}{\theta} + (a + b\,t_i)e^{\theta(T_i)}$$

Lemma 5.1:

t_i increase where i=1,2,3………… n-1 strictly monotonic increase function of last replenishment cycle t_n

Proof: See Appendix 5. A$_1$.

In this lemma, we can see a bonding among replenishment time, last replenishment time, length of replenishment time and time horizon.

$T_{i+1} = t_{i+1} - t_i$ and $t_n =$ H-T_n

Theorem 5.1 The optimal ordering period for a fixed n1 can be determined by finding the unique solution to the non-linear system of Eqn (5.12). In addition, the Hessian matrix of $T_{Ret}(t_i, n)$ must be positive and definite for $T_{Ret}(t_i, n)$ to be minimized for a fixed n. Moreover, as shown in Appendix 5. A$_2$, the theorem proves that $T_{Ret}(t_i, n)$ is always positive. Therefore, the optimal value for t_i with a given constant non-negative integer n can be calculated using numerical iterative techniques and Mathematica programs.

Theorem 5.2 T_{Ret} (n_1, t_0, t_1,………., tn) is a convex function in a finite horizon planning n_1.

See Appendix-5. A_3 for the proof

5.8. Comparative study:

Table 5.3. Comparison Table.

Parameters →	n	a	b	P	O_r	Ss	d_r	D_{eb}	θ	hr	CO_2	C_{hb}	P_r
Rao 2003, model	5	12000	0	24000	50	500	110	-	0.08	15	-	-	20
Proposed model	5	12000	100	24000	50	500	110	0.07	0.08	15	0.2	0.01	20

Parameters →	t_i	H		Retailer's total cost T_{Ret}	Supplier's total cost T_{sup}
Rao 2003, model	t_i=t=fixed	nt (fixed)	Non-optimal	29,138	42,246
Proposed model	Optimal t_i's are derived	$\sum_{i=0}^{n} t_i$	Non-optimal	17126025.0	2884335.87

Observations:

1 To compare the proposed model with Rao's model, a scenario was investigated where the parameter values were the same as in the example of Rao's model. Furthermore, the values of newly added variables in the proposed model, namely C_{o2}=0.2, D_{eb}=0.07, and C_{hb}=0.01, were also taken into consideration.

2 The optimal t_i's are derived in the proposed model, whereas in Rao's model, t_i is fixed.

3 The proposed model has higher total costs for both the retailer and the supplier compared to Rao's model. This is due to different optimization objectives or constraints. Also, in Rao's model, factors not explicitly considered are C_{o2}, D_{eb}, and C_{hb}.

4 Additionally, from this perspective, we can conclude that our model is more realistic than the Rao's model because we are working with a finite planning horizon.

5.9. Sensitivity analysis and findings:

Due to the unpredictable nature of market conditions, it is important to examine how changes in certain parameters can affect the values of key decision-making variables in a hypothetical scenario. Sensitivity analysis allows us to investigate how changes in these parameters impact optimal production quantity, total cost, optimal replenishment quantity, and optimal replenishment cycle. This section will encompass a thorough sensitivity analysis to examine the level of sensitivity of a previous example to changes in various system components. This analysis includes graphical and tabular illustrations to demonstrate how different parameters, such as a, b, h_r, d_r, D_{eb}, P, C_{O2}, C_{hb}, $\boldsymbol{\theta}$, H, S_s, O_r, and P_r, affect the optimality of the inventory approach. The values of all parameters were changed by +25%, +12.5%, 0%, -12.5%, and -25%. To examine the effects of these changes on the optimality of the inventory approach, a sensitivity analysis was conducted. Table 5.1, Figure 5.3, Figure 5.4, and Figure 5.5 demonstrate the findings of the sensitivity analysis, where each variable was individually examined while keeping the other variables constant. The optimal solutions were determined for each scenario using the "Mathematica" computing application version 12.

Table 5.4 explains the T_{Ret} and T_{sup} values for multiple and different shipping n: Figures 5.3, 5.4, and 5.5 show the show's comparison of various values of a. Figures 5.3, 5.4, and 5.5 show a graphical explanation of the convexity of the T_{Ret} function.

The following are the important facts drawn from the numerical example:

1. As a result, the above scenario in Table no 5.5 and Table no 5.6 happened only when one participant (supplier, manufacturer, or Retailer) occupies the industry. on the other hand, maintaining good collaboration is essential for the supply chain process. Table 5.6 shows the integrated view for different parameters. All the while, according to the integrated view, the total cost for, the producer(supplier), or retailer will be minimal. If all supply chain members are considered equal, the integrated method is the most appropriate choice for each one.

2. With the help of Table, no 5.4, we observed that the value of n* is slightly flexible in the collection of a parameter like 'a' and θ, it is almost completely insensitive to other variables including b, c, O_r, S_s, C_{o2}, P, P_r, d_r, and C_{hb}.
3. The total cost of retailer T_{cr} is extremely dependent on the input parameters a, b, and

   θ, moderately reactive to the set C_{o2}, O_r, D_{eb} and C_{hb} parameters and almost unresponsive P, P_r to any change in the rest.
4. Total quantity that is greater sensitivity to the input parameters a, b, less reactive to the parameters θ and almost unresponsive to any change in the parameters like P, P_r, set C_{o2}, O_r, D_{eb} and C_{hb} in Table no 5.4.
5. In Table no 4, the Supplier's total cost T_{cs} is extremely dependent on the input parameters a, b, P, and P_r.

   moderately reactive to the set θ, O_r, and S_s parameters and almost unresponsive to C_{o2}, D_{eb} and C_{hb} any change in the rest.
6. We define the "break-even" point" **t** Such that here t is the point where the total quantity produced becomes equal to the total quantity Q_2 to be consumed in the forthcoming remaining cycles until all of the quantity produced up to the "break-even" point '**t'** is exhausted. Q_1 should be greater than Q_2. Q_1 is the total quantity produced before the "break-even" point" **t**. For different values of 'a', different "break-even point" **t** are shown in Table 5.5 and Table 5.6 also, diagrammatic representation for different values of 'a' 450 and 300 are shown in the Figures 5.6, Figure 5.7 and Figure 5.8. Table 5.3 contains the required quantity produced in each cycle used for replenishment in retailer inventory for meeting linear demand.

5.10. Tabular formulation and graphical representation:

Table 5.4. reveals the following facts of the comprehensive and detailed sensitivity analysis.

Parameters	%Changes	Optimal Replenish cycle	The total cost of system TC	The total cost of the Retailer T_{Ret}	Total cost of supplier/ Manufacturer T_{sup}
a=400	+25	5	2440.74	6244.24	2263.78
	+12.5	4	2191.04	5165.64	2170.55
	0	4	2017.51	4051.15	2059.71
	−12.5	4	1843.97	2936.14	1948.84
	−25	4	1670.44	1820.44	1837.94
b	+25	4	2174.81	2353.52	2170.75
	+12.5	4	2096.16	3202.56	2115.24
	0	4	2017.51	4051.15	2059.71
	−12.5	4	1938.85	4899.21	2004.17
	−25	4	1860.20	5746.67	1948.62
θ	+25	4	2060.56	5233.0	2078.07
	+12.5	4	2038.88	4806.49	2068.66
	0	4	2017.51	4051.15	2059.71
	−12.5	5	2066.25	2693.66	2040.49
	−25	5	2048.47	194.899	2034.58
O_r	+25	4	2017.51	4531.15	2059.71
	+12.5	4	2017.51	4291.15	2059.71
	0	4	2017.51	4051.15	2059.71
	−12.5	5	2017.51	3791.56	2046.68
	−25	5	2017.51	3491.56	2046.68
CO_2	+25	4	2017.51	4068.53	2059.71
	+12.5	4	2017.51	4059.84	2059.71
	0	4	2017.51	4051.15	2059.71
	−12.5	4	2017.51	4042.46	2059.71
	−25	4	2017.51	4033.77	2059.71
P	+25	4	2017.51	4051.15	2240.87
	+12.5	4	2017.51	4051.15	2150.29
	0	4	2017.51	4051.15	2059.71
	−12.5	4	2017.51	4051.15	1969.13
	−25	4	2017.51	4051.15	1878.55
P_r	+25	4	2017.51	4051.15	2573.64
	+12.5	4	2017.51	4051.15	2316.67
	0	4	2017.51	4051.15	2059.71
	−12.5	4	2017.51	4051.15	1802.74
	−25	4	2017.51	4051.15	1545.78
S_s	+25	4	2017.51	4051.15	2060.71
	+12.5	4	2017.51	4051.15	2060.21
	0	4	2017.51	4051.15	2059.71
	−12.5	4	2017.51	4051.15	2059.21
	−25	4	2017.51	4051.15	2058.71
$\widehat{P}_r$	+25	4	2017.51	4058.39	2059.71
	+12.5	4	2017.51	4054.77	2059.71
	0	4	2017.51	4051.15	2059.71
	−12.5	4	2017.51	4047.53	2059.71
	−25	4	2017.51	4043.91	2059.71

$\widehat{d_r}$	$\begin{cases} +25 \\ +12.5 \\ 0 \\ -12.5 \\ -25 \end{cases}$	4 4 4 4 4	2017.51 2017.51 2017.51 2017.51 2017.51	4061.29 4056.22 4051.15 4046.08 4041.01	2059.71 2059.71 2059.71 2059.71 2059.71

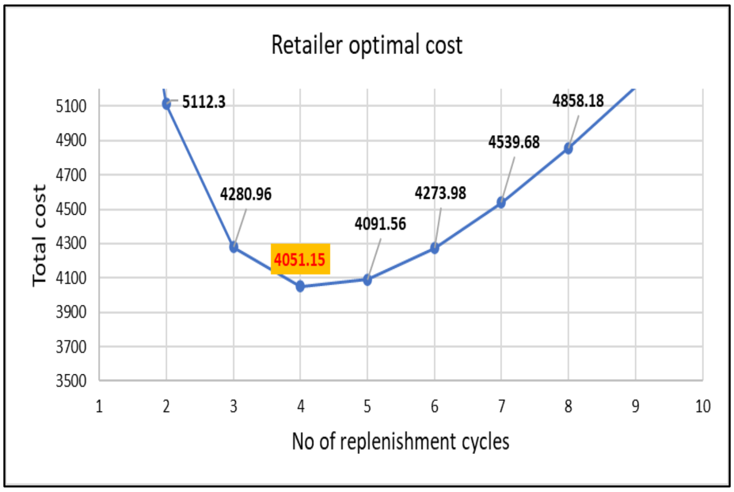

Figure 5.3. The figure provides a graphical representation of the convexity

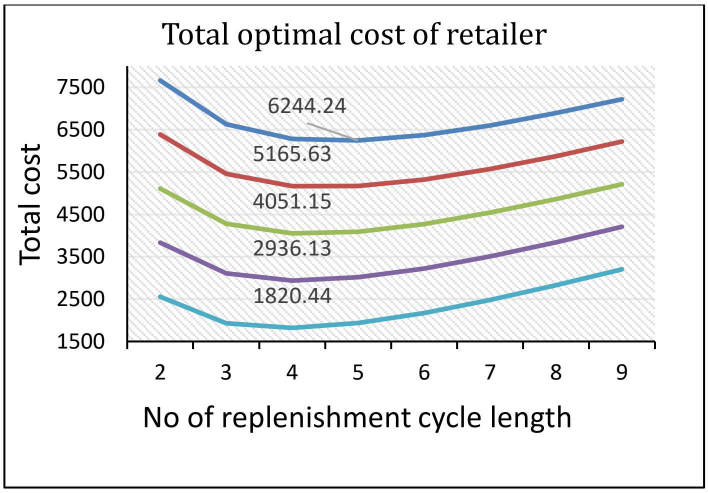

Figure 5.4 This figure presents a visual depiction of the convex nature of T_{Ret} concerning various values of the parameter.

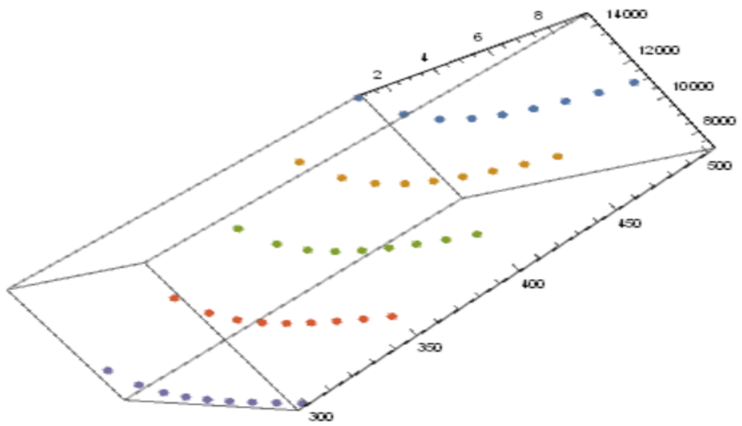

Figure 5.5 This figure shows a three-dimensional visual representation of the convex nature of T_{Ret} for various values of parameter a.

Table 5.5 The comprehensive and detailed examination has yielded the following findings.

Q1 − Q2 =	a	t_0	t_1	t_2	**t**	t_3	t_4	t_5	t_6	t_7	t_8	n	r
115.67	500	0	0.567	1.109	**1.2**	1.630	2.133	2.619	3.092	3.551	4	8	4
146.85	450	0	0.573	1.118	**1.2**	1.641	2.143	2.628	3.098	3.555	4	8	4
178.46	400	0	0.580	1.129	**1.1**	1.652	2.154	2.638	3.105	3.559	4	8	4
Q1 − Q2 =	a	t_0	t_1	**t**	t_2	t_3	t_4	t_5	t_6	t_7	t_8	n	r
14.78	350	0	0.773	**1.1**	1.490	2.163	2.802	3.413	4	-	-	6	3
62.318	300	0	0.785	**1.08**	1.506	2.178	2.814	3.419	4	-	-	6	3

Table 5.6 The thorough examination yielded the following results for Q_1 and Q_2

Q_2							n	r
936.06	1164.37	1353.63	**1505.08**	1619.53			8	4
873.657	1085.56	1260.14	**1398.31**	1500.53			8	4
811.279	1006.81	1166.74	**1291.65**	1381.58			8	4
915.112	1098.49	**1222.63**					6	3
837.042	1001.38	**1108.41**					6	3
Q_1								
532.64	1072.92	**1620.76**	2176.01	2738.5	3308.07	3884.55	8	3
508.036	1023.08	**1545.16**	2074.19	2610.06	3152.63	3701.75	8	3
483.754	973.737	**1470.12**	1972.91	2482.08	2997.52	3519.12	8	3
612.759	**1237.41**	1873.95	2522.14	3181.63			6	2
580.273	**1170.73**	1771.8	2383.47	3005.57			6	2

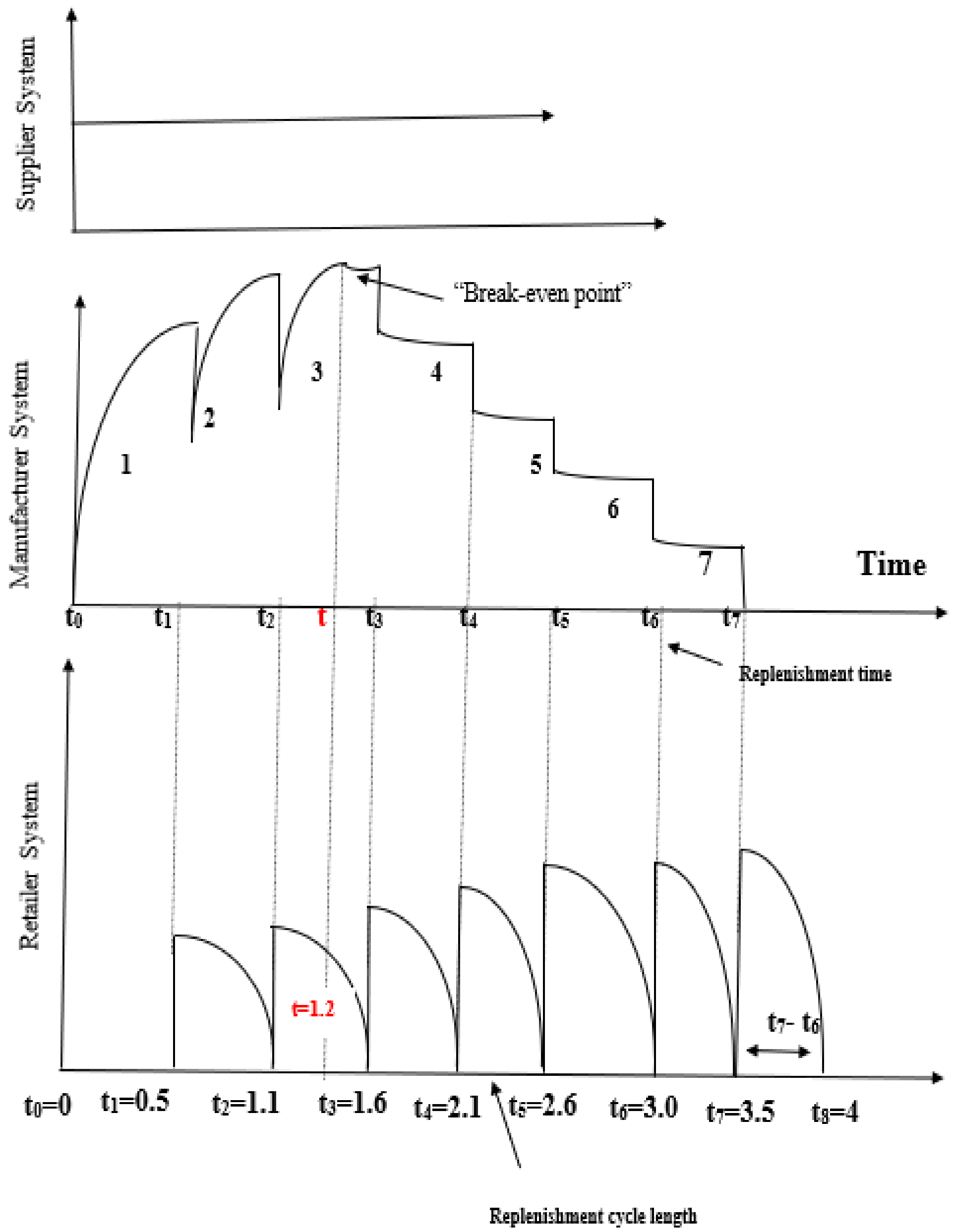

Figure 5.6 illustrates the Break Even Point across various values of t_i for Table 5.5

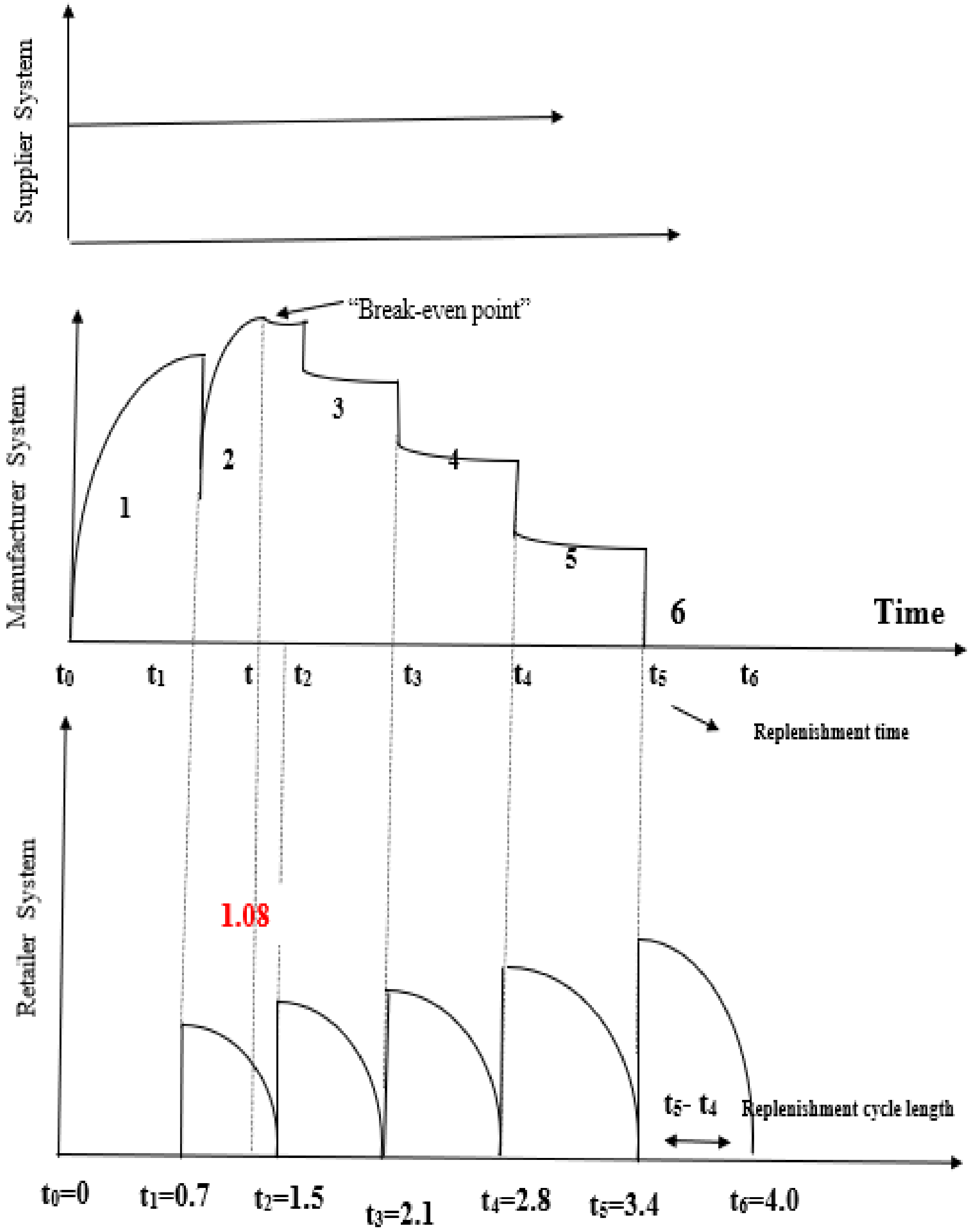

Figure 5.7 Demonstrates the Break Even Point across various values of t_i for table 5.6

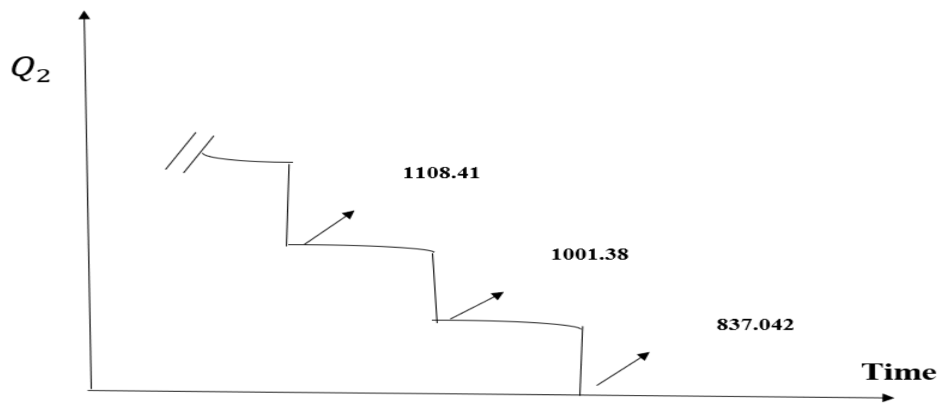

Figure 5.8 Replenishment quantity across different cycles

5.11. Conclusion and future study of proposed work:

This chapter introduces a new supply chain model for cold items, which involves a supplier, a manufacturer, and a retailer. over a finite period. Inventory levels are influenced by three key factors: demand rate, deterioration, and production rate. Production is a linear function that is dependent on demand, which in turn is proportional to carbon emissions, population, and time. The production process is meticulously controlled to ensure equilibrium in the industry and fulfil demand, preventing shortages due to $P(t) > D(t)$. The principal significance of this study is to assist manufacturing unit managers in making informed decisions to achieve optimal inventory levels, optimal replenished quantity, optimal replenishment time, and production rates while fulfilling retailer demand. Additionally, it aims to aid managers in mitigating the risk of shortages and efficiently managing the costs of retailers, suppliers and producers. The study's recommendations have the potential to enable manufacturing unit managers to make better-informed decisions and enhance the overall efficiency of their operations, potentially leading to increased productivity, minimized waste, and improved profitability.

5.11.1. Future study:

To enhance the model's accuracy, potential improvements could include integrating an imperfect manufacturing system and considering a degradation rate for the Weibull distribution. Additionally, incorporating lead time uncertainty would further enhance the realism of the model.

5.12. Recommendations for stakeholders:

This type of study can provide valuable insights for strategic planning of research and development in the economy. Additionally, the findings of this investigation can have practical implications for government and regulatory bodies in formulating effective policies and regulations to foster economic growth while reducing carbon emissions. Analyzing conceptual and scientific ideas is a critical element in the development of any business enterprise, and the research has yielded several conclusions that may be particularly beneficial for industry management. The following paragraphs outline some of these key findings.

Based on the existing research, industry management can make numerous critical decisions to optimize the industry's cost-to-profit ratio. This applies to any production industry, including smart mobile production using a smart production system within a supply chain. The production of high-quality goods is essential for increasing system profitability, but management must also minimize investments in defective manufacturing. Making informed decisions can help managers reduce overall costs, minimize carbon emissions, and increase system profits. By reducing setup costs in the production system, managers or business owners can enhance system profitability while maintaining product quality.

A flexible production rate is usually helpful to the manufacturing business. A profitable business strategy requires the reduction of unit manufacturing costs. The development costs have a significant impact on unit manufacturing costs. The investment in development costs helps in product quality improvement and competition survival. This concept can be adopted as a forecasting strategy by industry management to guide the company's future direction.

In some cases, perishable items can decrease overall profits while increasing overall costs. To avoid such losses, managers should implement cold chain logistics when transporting these items from the manufacturing process to retailers or purchasers to ensure the quality of the products. However, in implementing cold chain logistics with manufacturing processes, the number of perishable items produced may exceed the actual demand. This research can guide managers on how to select a production rate that can effectively increase system profits.

This research provides insights that can assist managers in making important decisions regarding the rate of production, demand, and investment. These factors are crucial in determining the system's profit or total cost.

Chapter 6: A Supply Chain Inventory Model for a deteriorating item under a Finite planning horizon with the Carbon Tax and shortage in all cycles

6.1. Abstract:

We have framed an inventory replenishment model under a finite planning horizon in which replenishment cycle time and replenishment cycle length are different and don't repeat. The finite planning horizon for different replenishment times and cycle lengths is a real-life scenario. Nowadays, every manufacturing industry wants to achieve maximum profit at a low cost. It is very difficult to maintain the optimal level of inventory, total cost, replenishment time, and replenishment cycle. Along with the health of people, increasing carbon emission also has a dangerous effect on today's business environment. Therefore, this research approach analyses an optimal inventory replenishment policy and carbon emission due to deteriorating material and refrigeration while taking into account time, emission-dependent, and inventory-dependent quadratic demand. Materials deterioration affects a wide and varied spectrum of business. Therefore, Material that suffers deterioration is considered. Shortage, some lost sales, and partial backlogging are also considered. The model has been developed theoretically. Also, a mathematical formulation has been obtained to find the unique and optimal solution to the problem. An algorithm, a numerical illustration and a comparative evaluation are explained, along with a sensitivity analysis of each parameter. The tabular and graphical representations of sensitivity analysis were addressed using the Mathematica application version 12. In the previous chapters, this research approach is explored with an additional parameter addressing the shortage of items.

6.2. Introduction:

In recent years, there has been a lot of discussion about inventory systems that emit carbon and degrade the environment. Carbon emissions have been triggering global warming for many years, which has garnered considerable attention. Carbon emission gases, such as carbon dioxide and methane, heat our planet and lead to global warming.

Global warming produces considerable environmental harm because of its destructive, pervasive, and long-term consequences. It is quickly destroying our planet's biodiversity, ultimately causing the extinction of countless plant and animal species. Global warming also causes sea-level rise, ozone layer depletion, rising global temperatures, extreme weather conditions, drought, and flooding. Due to increasingly powerful and frequent severe weather events, global climate change and the emission effect are receiving a lot of attention. The Paris Protocol also referred to as the Cap-and-Trade system, was established to mitigate the impact of greenhouse gas emissions. Under the carbon tax approach, companies are paid a predetermined payment for each metric ton of emissions they produce, whereas the cap-and-trade policy issue a defined quantity of emission allowances annually under the cap-trade program (Toptal, Özlü, and Konur 2014) and (Noh and Kim 2019).

The subsequent sections of this chapter are structured in the following manner: Section 6.3 give a piece of information about the literature of research. Section 6.4 presents a list of the assumptions used in the chapter. In Section 6.5, a mathematical framework is presented. The criteria for optimal cost function are discussed in section 6.6, with the help of a theorem and the Mathematica software. Section 6.7 includes a numerical example, an algorithm for finding ti, si, and total cost in the Mathematica tool, as well as sensitivity analysis, and a comparison discussion with graphs and tables. In Section 6.8, managerial suggestions are provided. Finally, the conclusions of the proposed model are discussed.

6.3. Literature review:

Enterprise economic endeavours are increasingly being blamed for serious climate change and global warming. Furthermore, reducing a polluted environment is one of the most considerable economic benefits of lowering carbon emissions, and it is becoming a big worry for nations worldwide. Only a few of them consider environmental issues in supply chain management, which include reducing emissions. To decrease carbon, the government of any nation and several regulatory bodies have developed carbon emission programs. The primary regulatory policy is a carbon tax. (Xi Chen et al. 2013) and (U. Mishra et al. 2021) mentioned in their writings that a carbon tax is charged or imposed by various government agencies on commercial enterprises or businesses that create carbon dioxide throughout their production process and generate environmental destruction. The primary goal of the government entities responsible for taxation is to prevent global warming and safeguard the environment. In other terms, the carbon tax is imposed on corporations that use harmful raw materials, such as fossil fuels, in their manufacturing process and transportation, consequently attributing to global warming. Also, a carbon tax can prevent environmental degradation and the total cost of the inventory model.

(Xi Chen et al. 2013) investigated an emissions inventory issue using the EOQ model and carbon schemes. (Dye and Yang 2015) analyzed the influence of the trade credit and carbon policies within inventory management, whenever the credit period has an impact on the demand rate. (He et al. 2015) determined the optimum emissions and lot size with two of the most commonly used carbon policies to reduce emissions: cap-and-trade and carbon tax. (Xu et al. 2020) investigated the combined price and production of various commodities in the context of the emission legislation. This report considers the joint decisions of retailers concerning inventory supplies and the investment model for reducing carbon emissions, taking into consideration the three carbon regulatory policies. In particular, it expands the economic quantitative order model to take into account the supply of carbon decreasing expenditure, in addition to complying with the carbon cap, price, and trade policies (Toptal, Özlü, and Konur 2014). Based entirely on the EOQ model, the researcher discusses production lot-sizing problems under the carbon tax, cap, and carbon trade norms. From the existing research, it is found that there is very little study on carbon emission in finite planning. Therefore, we have decided to focus on it in our work.

Obsolescence, damage, and depreciation can all cause on-hand inventory to deteriorate over time, resulting in lost sales, decreased earnings, and poorer customer satisfaction. The variable fraction value of on-hand inventory represents the proportion of total inventory that is prone to deterioration over time. Managing the supply chain for goods that deteriorate and emit carbon is difficult and risky because the utility of such products can decline due to spoilage, damage, or degradation during storage or transport. Carbon emissions and deterioration mostly occur due to global warming.

Many countries are increasingly focusing on reducing greenhouse gas emissions and environmental devastation, employing pivotal methods such as carbon caps and taxes to accomplish this task. (Shi et al. 2021) examined an integrated supply chain model for a single supplier and a single buyer manufacturing stock, taking into consideration imperfect output, including reworked products, various governmental mechanisms for decarbonization, and item deterioration, all at the same time. They developed a multi-objective framework to minimize the overall cost and emissions simultaneously. The demand for a product is generally influenced by several uncontrollable factors such as pricing, season, time, accessibility, and so on. However, addressing a fuzzy demand rather than a static demand is considered. Through an EOQ model, we know that retailers or buyers pay a constant carbon tax, and buyers can use one of three forms of payment: cash payment, advance payment, or credit payment. Credit transactions seem to be the most economical and effective of the multiple payment alternatives for lowering carbon emissions and maintaining the environment. (Shi et al. 2021) also, state that items deteriorate over time and thus can't be sold after their expiry. (Shen et al. 2019) addressed a production planning problem of deteriorating goods, taking into account the implementation of a carbon taxation policy alongside investments in preservation technology. This study also focuses on two carbon pollution schemes: carbon cap policies and the cap-and-trade system. (Tiwari, Ahmed, et al. 2018) provide a joint supply chain approach for decaying goods of inferior quality, considering carbon emissions introduced microbes initially forming a film on materials such as metals and organic biomaterials, which leads to the final degradation of the compound. A hot and humid climate catalyzes the process. Due to carbon emissions, the deterioration of anything is more obvious and can’t be ignored. This chapter mainly focuses on materials that deteriorated because of corrosion, rust, rotting along carbon emission. corrosion or rust is directly related to deterioration but indirectly related to emission. When metal products are manufactured or transported, they often require energy to be produced or moved. Additionally, there is a limited amount of research that specifically examines the relationship between carbon emission policies and asset deterioration over time in inventory control and management, and nobody has considered carbon emission policies with deterioration over a finite planning horizon with unequal cycle length and carbon emission policies.

(U. Mishra, Wu, and Sarkar 2021) study, the main objective is to control carbon emissions and find the optimal solution by considering three models: the carbon cap and tax model, excluding shortage, with partial backlogging, and full backlogging. Carbon emissions carbon caps and taxes are the fundamental points of this chapter. (Abad 1996) explain that shortages or stock-out scenarios arise in any business or industrial company due to many factors, such as pricing and a high rate of deterioration. In today's scenario, taking stock-out into account in any inventory model is necessary. It is anticipated that when one merchant receives the requested amount from the other merchant, they will satisfy the customers who have been waiting and then stock the remaining items for their regular demand. However, due to their impatience or other sources accessible in the region, not all buyers waiting in line can wait for the supply to arrive, resulting in a proportion of consumers waiting leading to lost sales. Shortages are therefore defined as partially backlogged. (Deb and Chaudhuri 1987) model permits shortages in all iterations of replenishment except for the final cycle. Each cycle, during which shortages are created, begins with replenishment and finishes with a backlogged shortage in the final cycle. Furthermore, (Goyal et al. 1992), and (Giri and Chaudhuri 1997) presented a novel restocking approach where each cycle starts with shortfalls and ends with a positive inventory. Most researchers have considered shortage for a single cycle, and there was no literature on a shortage of carbon emission and deterioration under finite planning horizons for unequal cycle length.

(Khanra and Chaudhuri 2003) included a quadratic time-dependent demand function for both finite and infinite horizons in their inventory management system. (Ghosh et al. 2006) employed the quadratic time demand function in their research study under finite planning of equal replenishment cycles. According to (T. Sarkar et al. 2012), incorporating a time-dependent quadratic demand mechanism with shortage and backlogging yields significant results. (P. N. Singh et al. 2017) introduced a supplier-retailer EOQ stock supply model with quadratic demand dependent on time and stock, as well as partial backlog throughout all cycles during a finite planned period. It is noticeable from the previous analysis that the discussion on deteriorating goods in nature has not been addressed for carbon tax cost and shortage along with carbon-dependent, time-dependent quadratic, and inventory-dependent demand under finite planning horizons for unequal cycle lengths.

Table 6.1. For a review of the above literature

Research work	Carbon dependent Demand	Deterioration	Inventory dependent	Time-dependent Quadratic demand	Carbon Emissions cost	Shortages	Finite Planning horizon
(Xi Chen et al. 2013)	×	×	×	×	✓	×	×
(Toptal, Özlü, and Konur 2014)	×	×	×	×	✓	×	×
(U. Mishra, Wu, and Sarkar 2021)	×	✓	×	×	✓	✓	×
(Ghosh et al. 2006)	×	✓	×	✓	×	✓	✓
(Xu et al. 2020)	×	✓	×	×	✓	×	×
(T. Sarkar et al. 2012)	×	✓	×	✓	×	✓	✓
(P. N. Singh et al. 2017)	×	✓	✓	✓	×	✓	✓
This chapter	✓	✓	✓	✓	✓	✓	✓

6.3.1. Research gap and problem defining:

So far, various studies have been on material deterioration and emission have already been conducted. However, none of these studies have considered an inventory management model that incorporates carbon emission measures including a carbon tax, covering degrading materials with emission-dependent, nonlinear time-dependent, and inventory-dependent demand, along with shortage. This research gap is significant and presents a unique opportunity as it combines economic and environmental considerations.

Based on the aforementioned research, we argue that none of the other researchers has developed a model for managing deteriorating products, incorporating emission-dependent, time-dependent quadratic, and inventory-dependent demand over a finite horizon. Finite planning horizons refer to the period during which decisions about inventory replenishment are made. Longer planning horizons allow for more time to make decisions, potentially leading to longer replenishment cycles, while shorter planning horizons can result in shorter replenishment cycles.

6.4. Assumptions:

1. There is no lead time.
2. Carbon emissions have effects on demand: D = a +b*t + c*t^2+$\theta_1 I$. The firm's preliminary market demand is a which is dependent on carbon emissions; market demand is b, which is dependent on time; and c is the consumer's awareness of carbon emissions for each unit. That is, as time increases by one unit, demand increments by c, and θ_1 is inventory-dependent demand. A stock-dependent demand level is a demand for a particular item that is influenced by its current available stock amount. Stock-dependent demand level directly influences the quantity of stock and ordering procedures.
3. The stock level is initially zero or starts with the shortage.
4. Throughout the finite planning horizon, the ordering, holding, and shortages costs are constant but emission costs are constant as well as depend on quantity and holding.
5. Damaged products are neither rectified nor repaired nor are they replaced. Therefore, due to the lack of inventory, shortages may occur.
6. Each cycle has shortages that are partially backlogged. Because consumers are impatient, it is believed that a portion of demand given by $\beta(\varphi)$ during the stock-out phase is backlogged. φ is indeed the timeframe the customer needs to wait until receiving orders, the rest $[1-\beta(\varphi)]$ is no longer available and lost. where $\beta(\varphi)=1/(\delta\varphi+1)$ and $\delta>0$

6.5. Mathematical formulation and analysis of the model:

As seen in the inventory graph for finite planning along with shortages. Differential Eqn. of inventory level besides shortages given by Eqn (6.1) and Eqn (6.5)

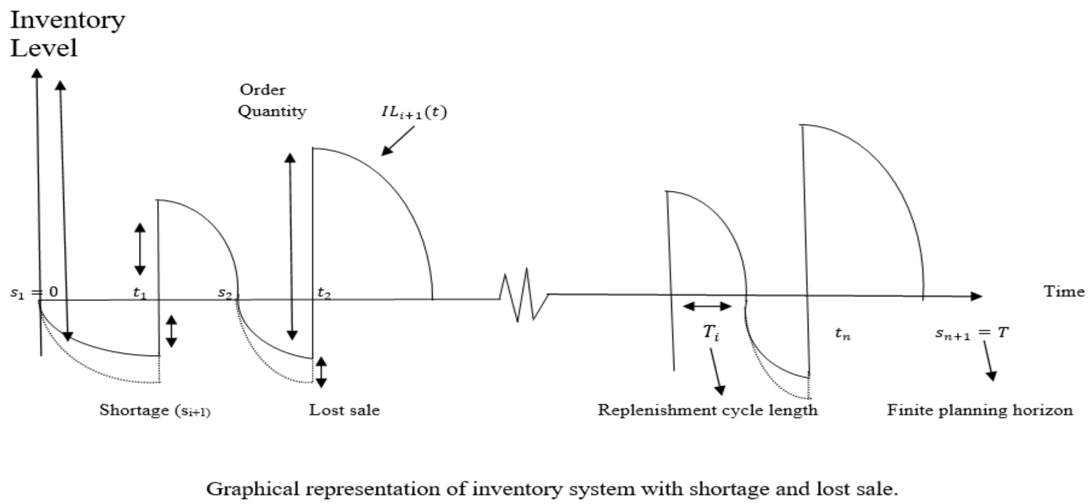

Figure 6.1. Inventory level with the shortage

Inventory level with boundary condition $IL_{i+1}(s_{i+1}) = 0$

$$\frac{d(IL_{i+1}(t))}{dt} + (\theta_2)\, IL_{i+1}(t) = -\mathrm{D} \qquad t_i < \mathrm{t} < s_{i+1} \tag{6.1}$$

$$\frac{d(IL_{i+1}(t))}{dt} + (\theta_2)\, IL_{i+1}(t) = -\mathrm{D(t)} - \theta_1\, IL_{i+1}(t) \qquad t_i < \mathrm{t} < s_{i+1}$$

$$\frac{d(IL_{i+1}(t))}{dt} + (\theta_2 + \theta_1)\, IL_{i+1}(t) = -\mathrm{D(t)} \qquad t_i < \mathrm{t} < s_{i+1}$$

Where $\{1,2,3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots . n_1\}$

Taking $\theta_2 = \alpha\mathrm{t}$ and the differential equation's solution is given by

$$\frac{dIL_{i+1}(t)}{dt} = -\mathrm{D(t)} - (\alpha t + \theta_1) IL_{i+1}(t) \qquad t_i < \mathrm{t} < s_{i+1} \tag{6.2}$$

$$IL_{i+1}(t) = e^{-\frac{\alpha t^2}{2} - t\theta_1} \int_t^{s_{i+1}} \mathrm{D(u)}\, e^{\frac{\alpha u^2}{2} + t}\, \mathrm{du} \tag{6.3}$$

$$IL_{i+1}(t) = \int_{t}^{s_{i+1}} \mathrm{D(u)}\ e^{\theta_1(u-t)+\frac{\alpha(u^2-t^2)}{2}}\,\mathrm{du} \qquad (6.4)$$

$$IL_{i+1}(t) = \left[\left(1 + t\theta_1 + \frac{\alpha}{2}t^2\right)(t - t_i) - \frac{\theta_1}{2}(t^2 - {t_i}^2) - \frac{\alpha}{6}(t^3 - {t_i}^3)\right] D(t)$$

The shortage within the condition $ILS_{i+1}(s_i) = 0$, is given by the following differential equation:

$$\frac{dILS_{i+1}(t)}{dt} = D(t)\beta(t) \qquad where \qquad s_i < \mathrm{t} < t_i \qquad (6.5)$$

$$\frac{dILS_{i+1}(t)}{dt} = \frac{D(t)}{\delta(t_i - t) + 1}$$

$$ILS_{i+1}(t) = \int_{s_i}^{t_i} \frac{D(t)}{\delta(t_i - t)+1}\, dt = \frac{D(t)(t_i - t)}{\delta(t_i - t)+1}$$

So total amount of inventory held during the interval $[t_i, s_{i+1}]$

$$R_{i+1} = \int_{t_i}^{s_{i+1}} \left\{\int_{t}^{s_{i+1}} \mathrm{D(u)}\ e^{\theta_1(u-t)+\frac{\alpha(u^2-t^2)}{2}}\,\mathrm{du}\right\} \mathrm{dt}$$

We may express the above Equation as below by changing the position of integration and skipping the higher powers of α^2.

$$R_{i+1} = \int_{t_i}^{s_{i+1}} \left\{\left(1 + \theta_1 t + \frac{\alpha}{2}t^2\right)(t - t_i) - \frac{\theta_1}{2}(t^2 - {t_i}^2) - \frac{\alpha}{6}(t^3 - {t_i}^3)\right\} D(t)dt \qquad (6.6)$$

The total amount of that quantity for which customers are waiting i.e., the amount of shortage during the interval $[s_i, t_i]$

After rearranging the ordering, S_{i+1} can be given as

$$S_{i+1} = \int_{s_i}^{t_i} ILS_{i+1}(t)\quad dt = \int_{s_i}^{t_i} \left\{\int_{s_i}^{t_i} \frac{D(u)}{\delta(t_i - u)+1} du\right\} dt = \int_{s_i}^{t_i} \frac{(t_i - t)D(t)}{\delta(t_i - t)+1} dt \qquad (6.7)$$

The total order quantity for a finite planning horizon $\mathrm{Q} = \sum_{i=1}^{n} Q_{i+1} = \sum_{i=1}^{n}\{ R_{i+1} + S_{i+1}\}$

$$Q_{i+1} = \int_{t_i}^{s_{i+1}} \left\{ \left(1 + \theta_1 t + \frac{\alpha}{2} t^2\right)(t - t_i) - \frac{\theta_1}{2}(t^2 - t_i{}^2) - \frac{\alpha}{6}(t^3 - t_i{}^3) \right\} D(t) dt + \int_{s_i}^{t_i} \frac{(t_i - t) D(t)}{\delta(t_i - t) + 1} dt \qquad (6.8)$$

The total number of deteriorated components throughout each replenishment is as follows:

$$D_{i+1} = \int_{s_i}^{t_i} \theta_2\, IL_{i+1}(t)\, dt \; = \alpha t \left\{ \int_{t_i}^{s_i} \left(1 + \theta_1 t + \frac{\alpha}{2} t^2\right)(t - t_i) - \frac{\theta_1}{2}(t^2 - t_i{}^2) - \frac{\alpha}{6}(t^3 - t_i{}^3)\, D(t) dt \right\} \qquad (6.9)$$

In the cases of some materials, the Buyer can generally not wait for some product, so only a proportion $\beta(\varphi)$. φ is the buyer waits time for the quantity of negative inventory. Therefore, the leftover proportion $\left(1 - \beta(\varphi)\right)$ is lost. Sarkar et al. (2012), the amount has become lost during the interval $[s_i, t_i]$ is given as:

The amount that has become lost during the interval $[s_i, t_i]$ is given as:

$$L_{i+1} = \int_{s_i}^{t_i} \{D(t) - D(t)\beta(\varphi)\}\, dt = \int_{s_i}^{t_i} \left\{\left(1 - \beta(\varphi)\right) D(t)\right\} dt = \int_{s_i}^{t_i} \left\{ \delta \frac{(t_i - t) D(t)}{\delta(t_i - t) + 1} \right\} dt \qquad (6.10)$$

Amount of Carbon emission cost during the interval $[t_i\, , s_{i+1}]$ can be expressed as:

$$Ce = \sum_{i=0}^{n_1 - 1} \hat{c} \; + \; \hat{P}_r \, * \, Q_{i+1} \; + \; \widehat{h_r} \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\, dt$$

$$Ce = \sum_{i=0}^{n_1 - 1} \hat{c} \; + \; \hat{P}_r \, * \, Q_{i+1} \; + \; \widehat{h_r} \int_{t_i}^{s_{i+1}} \left\{ \int_t^{s_{i+1}} \mathrm{D(u)} \; e^{\theta_1 (u - t) + \frac{\alpha(u^2 - t^2)}{2}} \mathrm{du} \right\} dt \qquad (6.11)$$

The amount of CO2 emissions from refrigeration systems is influenced by factors like refrigerant type, energy efficiency, usage frequency, system size, and power source. Usage of fossil fuels leads to higher emissions while energy-efficient technologies and smaller systems result in lower CO2 emissions. According to Mishra, N.K., & Ranu. (2022), the total cost of Carbon emission cost/carbon tax during the interval $[t_i\, , s_{i+1}]$ can be expressed as:

$$Ce = \tau\left\{\sum_{i=0}^{n_1-1} \hat{c} + \hat{P}_r \int_{t_i}^{s_{i+1}}\left\{\left(1+\theta_1 t+\frac{\alpha}{2}t^2\right)(t-t_i)-\frac{\theta_1}{2}(t^2-{t_i}^2)-\frac{\alpha}{6}(t^3-{t_i}^3)\right\} D(t)dt + \int_{s_i}^{t_i}\frac{(t_i-t)D(t)}{\delta(t_i-t)+1}dt + \widehat{h_r} \int_{t_i}^{s_{i+1}}\left\{\int_{t}^{s_{i+1}} \mathrm{D(u)}\ e^{\theta_1(u-t)+\frac{\alpha(u^2-t^2)}{2}} \mathrm{du}\right\}dt\right\}$$

(6.12)

Total cost =Replenishment cost +Stock holding cost + purchasing cost + Deteriorating cost + Storage cost +Lost sale cost+ Carbon emission cost+ Transportation cost

$$T_C(t_i,s_i,n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} h_r \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\,dt + \sum_{i=0}^{n_1-1} W * Q_{i+1} + \sum_{i=0}^{n_1-1} d_r * \theta_2 \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\ \ dt + \sum_{i=0}^{n_1-1} S_h * \int_{s_i}^{t_i} ILS_{i+1}(t)\ \ dt + \sum_{i=0}^{n_1-1} L * \int_{s_i}^{t_i}\left\{\delta\frac{(t_i-t)D(t)}{\delta(t_i-t)+1}\right\}dt + \sum_{i=0}^{n_1-1} \hat{c} + \hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\,dt \qquad (6.13)$$

$$\begin{aligned} T_C(t_i,s_i,n_1) = {} & n_1 * O_r \\ & + \sum_{i=0}^{n_1-1} h_r \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\,dt + \sum_{i=0}^{n_1-1} W * Q_{i+1} \\ & + \sum_{i=0}^{n_1-1} d_r * \alpha\, t \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\ \ dt + \sum_{i=0}^{n_1-1} S_h \int_{s_i}^{t_i} ILS_{i+1}(t)\ \ dt \\ & + \sum_{i=0}^{n_1-1} L \int_{s_i}^{t_i}\left\{\delta\frac{(t_i-t)D(t)}{\delta(t_i-t)+1}\right\}dt + \sum_{i=0}^{n_1-1} \hat{c} + \hat{P}_r \left\{Q_{i+1}\right. \\ & \left. + \int_{s_i}^{t_i} ILS_{i+1}(t)\,dt\right\} + \widehat{h_r} \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\,dt \end{aligned}$$

$$T_C(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \{h_r + \tau * \widehat{h_r}\} \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\, dt + \sum_{i=0}^{n_1-1} d_r * \alpha\, t \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\ \ dt + \{W + \tau * \hat{P}_r\} \sum_{i=0}^{n_1-1} Q_{i+1} + \{S_h + L * \delta\} \sum_{i=0}^{n_1-1} \int_{s_i}^{t_i} ILS_{i+1}(t)\ \ dt + \tau * \hat{c}$$

$$T_C(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \left(h_r + \tau * \widehat{h_r}\right) \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\, dt + \sum_{i=0}^{n_1-1} d_r * \alpha\, t \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\ \ dt + \{W + \tau\hat{P}_r\} \sum_{i=0}^{n_1-1} (R_{i+1} + S_{i+1}) + \left(S_h + L * \delta + \tau\hat{P}_r\right) \sum_{i=0}^{n_1-1} \int_{s_i}^{t_i} ILS_{i+1}(t)\ \ dt + \tau\hat{c}$$

$$T_C(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \left(h_r + \tau * \widehat{h_r}\right) \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\, dt + \sum_{i=0}^{n_1-1} d_r * \alpha\, t \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\ \ dt + \{W + \tau\hat{P}_r\} \sum_{i=0}^{n_1-1} R_{i+1} + \left(S_h + L * \delta + \tau\hat{P}_r + W\right) \sum_{i=0}^{n_1-1} \int_{s_i}^{t_i} ILS_{i+1}(t)\ \ dt + \tau\hat{c} \qquad (6.14)$$

$$T_C(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \left(h_r + \tau * \widehat{h_r}\right) \int_{t_i}^{s_{i+1}} \int_{t}^{s_{i+1}} \mathrm{D(u)}\ e^{\theta_1(u-t)+\frac{\alpha(u^2-t^2)}{2}} \mathrm{du}\, dt + \sum_{i=0}^{n_1-1} d_r * \alpha\, t \int_{t_i}^{s_{i+1}} \int_{t}^{s_{i+1}} \mathrm{D(u)}\ e^{\theta_1(u-t)+\frac{\alpha(u^2-t^2)}{2}} \mathrm{du}\, dt + \{W + \tau\hat{P}_r\} \sum_{i=0}^{n_1-1} R_{i+1} + \left(S_h + L * \delta + \tau\hat{P}_r + W\right) \sum_{i=0}^{n_1-1} \int_{s_i}^{t_i} ILS_{i+1}(t)\quad dt + \tau \mathrm{c}\hat{}$$

$$T_C(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \left(h_r + \tau * \widehat{h_r} + W + \tau\hat{P}_r\right) \int_{t_i}^{s_{i+1}} \left[\left(1 + t\theta_1 + \frac{\alpha}{2}t^2\right)(t - t_i) - \frac{\theta_1}{2}(t^2 - t_i{}^2) - \frac{\alpha}{6}(t^3 - t_i{}^3)\right] D(t)dt + \sum_{i=0}^{n_1-1} d_r \int_{t_i}^{s_{i+1}} \alpha\, t \left\{\left(1 + \theta_1 t + \frac{\alpha}{2}t^2\right)(t - t_i) - \frac{\theta_1}{2}(t^2 - t_i{}^2) - \frac{\alpha}{6}(t^3 - t_i{}^3)\right\} D(t)dt + \left(S_h + L * \delta + \tau\hat{P}_r + W\right) \sum_{i=0}^{n_1-1} \int_{s_i}^{t_i} \frac{(t_i - t)D(t)}{\delta(t_i - t) + 1} dt + \tau \mathrm{c}\hat{} \qquad (6.15)$$

The objective is that the fundamental values of ti and si must be determined to reduce the total variable cost T_C of the stock control and management. The requirements of show the T_C to be minimum are given below:

$$\frac{\partial T_C(t_i, s_i, n_1)}{\partial t_i} = 0$$

$$\frac{\partial T_C(t_i, s_i, n_1)}{\partial s_i} = 0$$

By neglecting $\alpha^2$and the higher terms of α, because have a negligible value then we get:

$$\frac{\partial T_C(t_i,s_i,n_1)}{\partial s_i} = \left(h_r + \tau * \widehat{h_r} + W + \tau\hat{P}_r\right)\left[\left(s_i\,\theta_1 + \frac{\alpha}{2}\,{s_i}^2 + 1\right)\left(s_i - t_{i-1}\right) - \frac{\theta_1}{2}\left({s_i}^2 - t_{i-1}^2\right) - \frac{\alpha}{6}\left({s_i}^3 - t_{i-1}^3\right)\right] D(s_i) + d_r\,\alpha\,s_i\,\left[(s_i\,\theta_1 + 1)\left(s_i - t_{i-1}\right) - \frac{\theta_1}{2}\left({s_i}^2 - t_{i-1}^2\right)\right] D(s_i) + \left(S_h + L*\delta + \tau\hat{P}_r + W\right)\left[\frac{(t_i - s_i)}{\delta(t_i - s_i) + 1}\right] D(s_i) \quad (6.16)$$

$$\frac{\partial T_C(t_i,s_i,n_1)}{\partial t_i} = \left(h_r + \tau * \widehat{h_r} + W + \tau\hat{P}_r\right)\int_{t_i}^{s_{i+1}}\left[\theta_1(t_i - t) + \frac{\alpha}{2}({t_i}^2 - t^2) - 1\right] D(t)dt + d_r\int_{t_i}^{s_{i+1}}\alpha\,t[\theta_1(t_i - t) - 1]\,D(t)dt + \left(S_h + L*\delta + \tau\hat{P}_r + W\right)\left[\int_{s_i}^{t_i}\frac{D(t)}{\delta(t_i - t)^2 + 1}dt\right] \quad (6.17)$$

$$\frac{\partial^2 T_C(t_i,s_i,n_1)}{\partial {s_i}^2} = \frac{1}{6}\left\{\left(h_r + \tau * \widehat{h_r} + W + \tau\hat{P}_r\right)\left(s_i - t_{i-1}\right)[6(\theta_1 + \alpha s_i)(a + bs_i + cs_i^2) + (3(2s_i\theta_1 + \alpha s_i^2 + 2) - 3\theta_1\left(s_i + t_{i-1}\right) - \alpha(s_i^2 + t_{i-1}^2 + s_i t_{i-1}))(a + bs_i + cs_i^2)\right] + 3d_r * \alpha(s_i - t_{i-1})[(2 - \theta_1 t_{i-1} + \theta_1 s_i)(b + 2cs_i) + (2 - \theta_1 t_{i-1} + \theta_1 s_i)(a + bs_i + cs_i^2)] + 6\left(S_h + L*\delta + \tau\hat{P}_r + W\right)[\frac{(t_i - s_i)}{1 + \delta(t_i - s_i)}(b + 2cs_i) - \frac{1}{1 + \delta(t_i - s_i)^2}(a + bs_i + cs_i^2)]\Big\} \quad (6.18)$$

$$\frac{\partial^2 T_C(t_i,s_i,n_1)}{\partial {t_i}^2} = \left(h_r + \tau * \widehat{h_r} + W + \tau\hat{P}_r\right)\int_{t_i}^{s_{i+1}}(\theta_1 + \alpha t_i)(a + bt + ct^2)dt + \left(h_r + \tau * \widehat{h_r} + W + \tau\hat{P}_r\right)(a + bt_i + ct_i^2) + d_r\int_{t_i}^{s_{i+1}}\alpha\,t\theta_1\,(a + bt + ct^2)dt + d_r * \alpha * t_i * (a + bt_i + ct_i^2) - 2\left(S_h + L*\delta + \tau\hat{P}_r + W\right)\left[\int_{s_i}^{t_i}\frac{(a + bt + ct^2)}{(\delta(t_i - t)^2 + 1)^2}dt\right] \quad (6.19)$$

Theorem proves that T_C is positive. Hence, by utilizing the iterative method and Mathematica software, the optimal values of t_i and s_i for a given positive integer n_1 may be calculated from the above Eqn. 7.16 and 7.17

$$\nabla^2 \mathrm{T}_C = \begin{bmatrix} \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{t}_1{}^2} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{t}_1\, \partial \mathrm{s}_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{s}_2\, \partial \mathrm{t}_1} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{s}_1{}^2} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{s}_1\, \partial \mathrm{t}_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{t}_2\, \partial \mathrm{s}_1} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{t}_2{}^2} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{t}_2\, \partial \mathrm{s}_2} & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots. & \dots & \dots & \dots & \dots. & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{t}_{n_1-1}\, \partial \mathrm{s}_{n_1-1}} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{s}_{n_1-1}{}^2} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{s}_{n_1-1}\, \partial \mathrm{t}_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \mathrm{T}_C}{\partial t_{n_1}\, \partial s_{n_1-1}} & \frac{\partial^2 \mathrm{T}_C}{\partial \mathrm{t}_{n_1}{}^2} \end{bmatrix}$$

Figure 6.2 Hessian matrix

Theorem 6.1:- if t_i and s_i satisfy the inequality,(i) $\frac{\partial^2 T_C(t_i,s_i,n_1)}{\partial t_i{}^2} \geq 0$, (ii) $\frac{\partial^2 T_C(t_i,s_i,n_1)}{\partial s_i{}^2} \geq 0$, (iii) $\frac{\partial^2 T_C(t_i,s_i,n_1)}{\partial t_i{}^2} - \left|\frac{\partial T_C(t_i,s_i,n_1)}{\partial t_i\ \partial s_i}\right| \geq 0$ and (iv)$\frac{\partial^2 T_C(t_i,s_i,n_1)}{\partial s_i{}^2} - \left|\frac{\partial T_C(t_i,s_i,n_1)}{\partial s_i\ \partial t_i}\right| \geq 0$ for all i = 1, 2, . . ., n then T_C will be positive definite.

6.6. Algorithm and procedures for resolving the problem:

1. The initial step involves assigning constant values to all the given parameters, namely W; P; a; b; c; θ, G, L, $\widehat{P_r}$ etc.

2. The proposed model aims to identify the optimal cost and ordering replenishment strategy, which can be achieved through the following steps:

   a. If we will set, $n_1 = 1$ then $s_1 = 0, s_2 = \mathrm{T}$. Initializing with the parameter's value t1, determine t1 with the help of Eq. (6.17).

   b. If we take $n_1 = 2$, then by initializing the value of the t_i taking s_0 =0 and s_2=T. After that, we can calculate s_2 by using Eq. (6.10).

   c. From Eq. (6.17), find t_2 using the calculated values of t_1 and s_2 in the previous step.

   d. By using Eq. (7.16), Again, taking the values of t_2 and s_2, calculate s_3. Trying to continue in this manner until all unique and optimal values of ti and si are obtained for all n_1.

e. Having the values of s_{n_1-1} and t_{n_1-1}nearly equal to T (horizon planning), and the values of si and ti are satisfying the theorem of the Hessian matrix.

f. For each n_1 = 1; 2; 3; :::: We will calculate the unique and optimal values of ti and si.

g. By using Eq. (6.10), T_c (n_1) is collected to calculate the optimum total cost value T_c (n_1) by using the following conditions.

h. For n_1=1 then T_C= T_C (n_1) and stop. For n1>1, and if T_c $(n_1) \leq T_c$ $(n_1 - 1)$ and T_c $(n_1) \leq T_c$ $(n_1 + 1)$ then T_c (n_1) = Optimal T_c and stop otherwise go to the previous step. Similarly, we can calculate the cost of emissions and the total quantity of the system by using Eqn. (6.8) and (6.12).

6.7. Numerical illustration for the proposed model:

Example1.$\mathrm{b} = 10$ unit/yr, $\mathrm{c} = 5$unit/yr, $\mathrm{a} = 25$ unit/yr, $\alpha = 0.001, \mathrm{S} = 2\$/$ unit, $\tau = 0.003$ \$/ton emission, $\hat{h_r} = 0.1$ton CO2/unit, $\hat{P_r} = 0.030.1$ton CO2/ unit, $d_r = 0.01, W_h = 0.3\$/$unit, $l = 10\$/$unit, $H = 4\$/$unit/yr, $S_s = 2\$/$unit, , $\alpha = 0.001, \delta = 4, \theta_1 = 0.002, O_r = 60\$/$order, $H = 4$. The mathematical calculation tool Mathematica version-12 is used to solve the nonlinear equation systems Eq. (6.16) and Eq. (6.17). The optimality of the total system cost and replenishment cycles time can be observed in Table 6. 2, Table 6.3, Figure 6.2, and Figure 6.3 for all values mentioned in the example1, respectively, for all the values mentioned in Example 1. 6a finite planning horizon with unequal cycles length that is a real-time scenario. The finite planning horizon for unequal cycle length can be observed in Table 6.3.

Table 6.2. Table for TC for example.1

↓ n ⟶	Q_{i+1}	CE	TC
1	145.964	0.0504865	1525.31
2	140.436	0.0521384	1079.27

3	121.986	0.0465045	862.14
4	101.422	0.0390947	756.45
5	41.5876	0.0056345	711.40
6*	71.8061	0.0279307	**700.19**
7	62.2117	0.0242669	709.07

(Where * represent the optimal value)

Table 6.3. the optimal strategy for Example 1.

n	t_i	s_i
1	0.174496	0.979947
2	1.10192	1.76505
3	1.85751	2.4267
4	2.50212	3.00571
5	3.07019	3.52535
6*	3.58222	**4.000000**

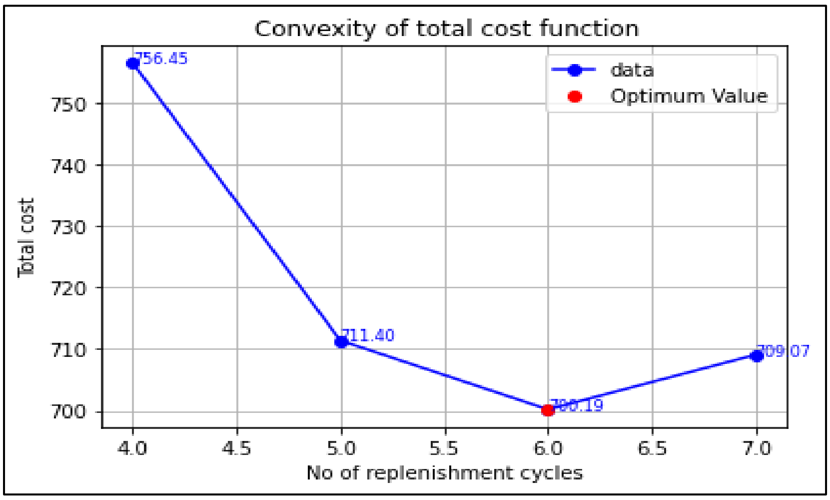

Figure 6.3 Graphic Representation of total cost

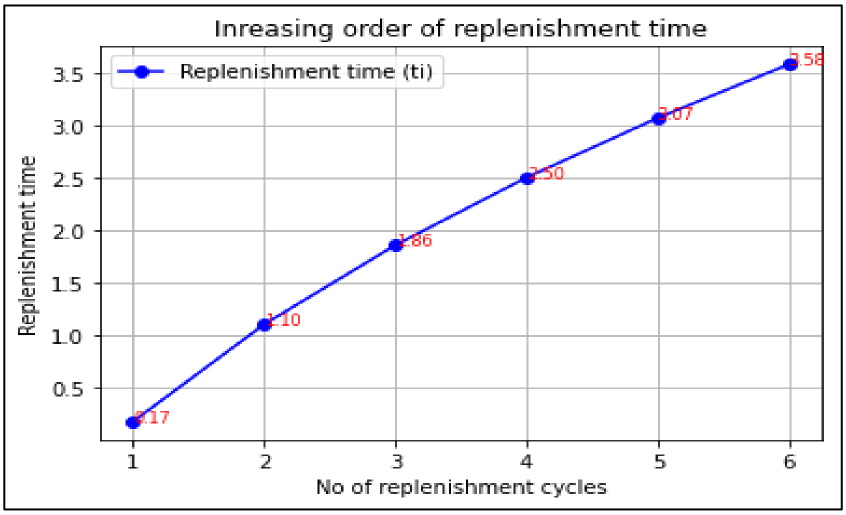

Figure 6.4 Replenishment time occurs increasing the order.

Some exceptional cases:

The following are the important exceptional circumstances that impact the optimum current value of total cost:

1. When inventory-dependent demand is not included, that is $\theta_1 = 0$.

2. The absence of consideration for inventory item deterioration is indicated by $\alpha = 0$.

3. When carbon emission cost is neglected that is, $\tau = 0$.

Table 6.4. Comparison Chart for some expectational cases.

Some expectational condition	Replenishment cycle(n^*}	Carbon emission cost	Time intervals (years)		Q^* Order Quantity	Total Cost of the System (TC^*)
			ti	s_i		
θ_1= 0	6	0.0279	0.174335	0.980015,	71.7926	716.013
			1.10189	1.76513		
			1.85753	2.42678		
			2.50215	3.00577		
			3.07021	3.52538		
			3.58222	4.0000		
$\alpha = 0$	6	0.0279	0.174395	0.979762	71.7904	716.0054
			1.10166	1.76484		
			1.85724	2.42653		
			2.5019	3.00559		
			3.07003	3.52529		
			3.58212	4.00000		
$\tau = 0$	6	0	0.174476	0.979942	71.8081	716.103
			1.1019	1.76504		
			1.85749	2.4267		
			2.50211	3.00571		
			3.07018	3.52535		
			3.5820	4.00000		

To compare the proposed model with Rao's model, a scenario was investigated where the parameter values were the same as in the example of Rao's model. Furthermore, the values of newly added variables in the proposed model, namely C_{o2}=0.2, D_{eb}=0.07, and C_{hb}=0.01, were also taken into consideration. From this perspective, we can conclude that our model is more realistic than the Rao's model because we are working with a finite planning horizon.

6.8. Sensitivity analysis and findings:

As we are aware, uncertainties and unpredictable market conditions can lead to variations in some parameters' values in decision-making scenarios. Therefore, it is essential to examine the resulting changes in the total cost, emission values, and optimal replenishment cycle. Table 6.5 presents a comprehensive sensitivity analysis that illustrates how alterations in parameter values can impact the results or outcomes. This thorough sensitivity analysis displays the effects of changing parameter values. Hence, in this section, we will analyze the sensitivity level of the total cost and carbon emission cost-optimal solution of the previous Example 1 by changing various system component values. This analysis is carried out using graphical illustrations. Each parameter's value is modified by varying a, b, c, τ, $\delta, \alpha\ and\ \theta_1$ in -25%, -50%, +25%, and +50%, focusing on one parameter at a particular time and keeping the remaining parameters constant. The optimal solutions for n, TC, and CE are determined in each scenario with the aid of Mathematica software version-12.

1. As a result of the study, it was shown that the value of n_1*(optimal number of cycles) is more flexible in the selection of a parameter such as 'a', slightly sensitive to those of subset τ, and almost insensitive to other parameters such as b, c, $\delta, \alpha\ and\ \theta_1$.

2. The level of emissions produced (CE) appears to be more sensitive to changes in the parameters of collection τ and a, comparatively low sensitivity to variations in selection parameters, and virtually insensitive to changes in the remaining parameters.

3. The total cost of the entire supply chain TC is highly sensitive to the selection parameters a, b, and c, moderately reactive to the subset P parameters, including τ and δ, and practically insensitive to any changes in the remaining parameters.

Table 6.5 shows the comprehensive study of sensitivity analysis.

Parameters	% Changes	Optimal Replenish cycle	Carbon emission cost	Total cost

a	−50	5	0.0257227	628.6262
	−25	6	0.0251208	666.5544
	+25	6	0.0306681	733.2927
	+50	7	0.0289431	765.3499
B	−50	6	0.0242241	653.7005
	−25	6	0.0260827	676.9800
	+25	6	0.0297682	723.3610
	+50	6	0.0315958	746.4673
c	−50	5	0.0273340	641.0577
	−25	6	0.0256152	671.4946
	+25	6	0.0302138	728.5855
	+50	6	0.0324689	756.7115
τ	−50	6	0.0139655	700.1862
	−25	6	0.0209481	700.1928
	+25	6	0.0349130	700.2059
	+50	6	0.0418954	700.2125
δ	−50	6	0.0230894	668.4622
	−25	6	0.0260249	688.3219
	+25	6	0.0292711	708.1282
	+50	6	0.0302667	713.8058
α	−50	6	0.0279279	698.5433
	−25	6	0.0279293	699.3714
	+25	6	0.0279321	701.0273
	+50	6	0.0279335	701.8553
θ_1	−50	6	0.0279283	709.6992
	−25	6	0.0279295	704.9493
	+25	6	0.0279319	695.4496
	+50	6	0.0279331	690.7000

Table 6.6. Comparison between proposed and existing model table throughout the tabular as well as graphical:

Parameters →	a	b	c	H	P_r	O_r	S_s	L	d_r	θ_1	α	$\hat{P_r}$	τ	$\hat{h_r}$	δ	Total Cost of the system
Sarkar et al. [2012]	25	10	5	4	1.2	60	2	10	–	–	0.001	–	–	–	4	**513.40**
Proposed model	25	10	5	4	1.2	60	2	10	0.01	0.002	0.001	0.03	0.003	0.1	4	**502.19**

Where: − Parameters didn't consider in the Sarkar model.

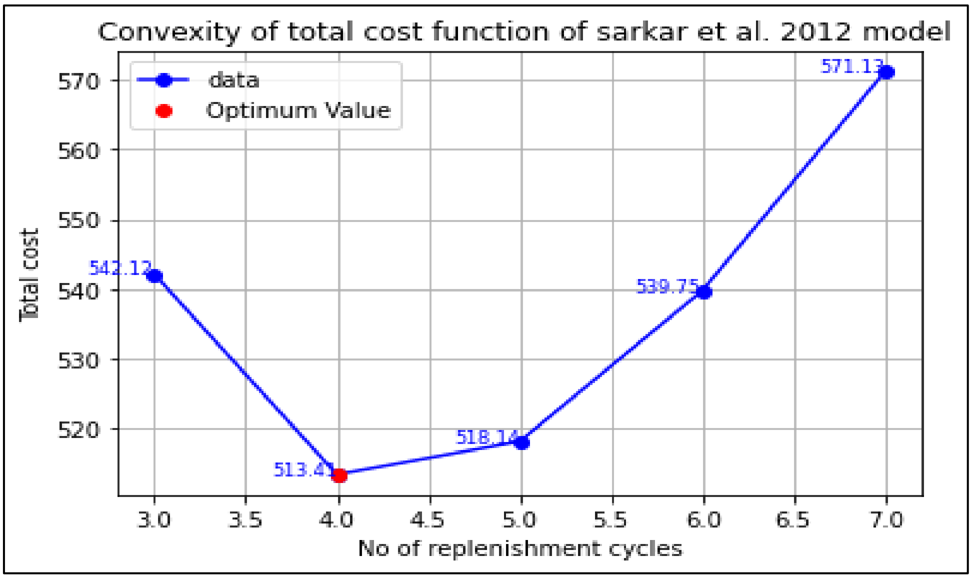

Figure 6.5 Sarkar model in diagrammatic form.

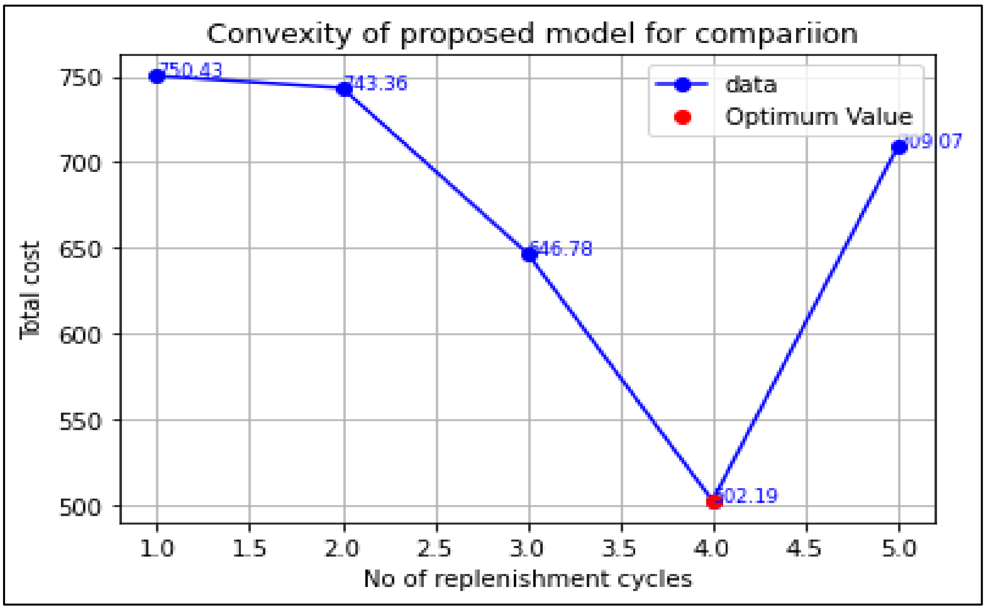

Figure 6.6 Proposed model in diagrammatic form.

Further, Table 6.6, Figure 6.5 and Figure 6.6 reveal that this supply chain inventory model can help the manufacturing or retail industries to reduce costs by optimizing inventory levels and optimizing replenishment time while optimizing replenishment cycles to remain the same. Reduced costs ultimately lead to an increase in the profits.

6.9. Conclusion and future study of proposed work:

The focus of the study is on a supply chain inventory system that deals with goods that deteriorate at a constant rate. This system includes practical and realistic characteristics that are often associated with various inventory types for a manufacturing company or retail company. This study has discovered carbon emission costs that put an impact on the overall system cost. Moreover, the study highlights the impact of carbon emission costs on the overall system cost. The proposed approach is particularly relevant to the retail industry or manufacturing industry. It may be utilized for metals, organic biomaterials, household items, and other things that have the preceding features.

The study provides an analytical framework for addressing the aforementioned issue and presents an optimal solution approach for determining the optimal replenishment strategy and total cost. The findings reveal that carbon dioxide emissions put a significant impact on the total cost. Additionally, this study examines the sensitivity of the solution to variations in various parameter values.

6.9.1. Future study:

There is potential for further expansion of this model by incorporating an inventory system with alternative carbon emission policies. Additionally, the degradation rate of the Weibull distribution may be taken into account. The cost of the manufacturing process, transportation, and refrigeration could also be considered. This model can be easily modified to account for lead time uncertainties.

6.8. Recommendations for stakeholders:

The management has the flexibility to find the optimal timing for the replenishment of orders and to halt the ordering process. Upon cessation of replenishment or order, the customer's demands are met by utilizing the existing inventory. The management understands the ideal moment, denoted as H, where the level of the inventory reached zero. Therefore, it is necessary to maintain inventory to avoid stockouts.

Chapter 7: A collaboration supply chain inventory model including linear time-dependent, inventory and advertisement-dependent demand considering carbon regulations

7.1. Abstract:

For carbon emission, this research study investigates a Mathematical inventory model with time, advertising, and inventory-dependent demand patterns. The main objective of this research study is to keep the total cost of retailers as well as suppliers and carbon emissions as low as possible. With collaboration and without collaboration, two cases are discussed in this proposed model. Within the first case, retailers and suppliers are not regarded as collaborators, whereas in the second case, collaboration is recognized. The optimality of the planned inventory management model is explained mathematically and theoretically in both situations. The algorithm of the mathematical solution was also properly discussed and the effects of altering various parameters are numerically studied to conduct a sensitivity analysis with the help of Mathematica software version 12. To demonstrate this model, a mathematical illustration, and a tabular and graphical representation, have been also provided. Ultimately, this model reaches a flourishing managerial suggestion and conclusion.

7.2. Introduction:

Global temperature rise is the effect of greenhouse gas emissions and even some living beings. A worldwide awareness for environmental conservation and protection is encouraging many more researchers, organizations, and other government agencies to create and maintain an eco-friendly as well as a negligible emission management system for supply chains. Kaya's identity plays a role to calculate the impact of different factors of the supply chain on carbon emission (Kaya 1989). Therefore, we rely on the kaya identity equation in calculating the amount of carbon emission.

$$Co_2 = \left(\frac{GDP}{Population}\right)\left(\frac{EC}{GDP}\right)\left(\frac{Co_2}{EC}\right) * Population$$

The gross domestic product (GDP) represents the total monetary value of all goods and services produced within a country during a specific year.

Everyday activities like-Heating, cooling, electricity supply, and transportation are all dependent on efficient and reasonable energy services. Energy intensity is a ratio of the volume of energy required to produce one unit of GDP and can be used to estimate a country's energy efficiency. It includes energy derived from fossil fuels and other sources. However, it does not include energy used by the energy sector itself, such as for the production of electricity.

The proposed research work focuses on carbon emissions resulting from supply chain processes, such as those involved in heating and refrigeration during storage, as well as emissions resulting from transportation. These are significant sources of carbon emissions that contribute to climate change. By identifying and addressing these sources of emissions, our research can help to promote more sustainable and eco-friendly practices in the supply chain.

(Khan et al. 2020) argue that advertising plays a crucial role in inventory management, as it increases product awareness among customers, retailers, suppliers, and manufacturers. Whenever a new product is launched or an existing product is modified, advertising is used to promote it. Advertising is an important factor in driving demand for products. It can be said that the demand for a product on a given day is heavily dependent on its advertising. Effective advertising can increase demand for a product and lead to faster sales. However, enhancing customer demand for products can be a challenging task for retailers and suppliers. Advertising can work well for perishable items, fashionable products, and those nearing their expiration date, where the product's lifespan is limited. Advertising, along with carbon emissions, can influence the demand for products. It is one of the most effective promotional approaches to increase awareness about a product's popularity among all classes of consumers. There are various ways to advertise products, but with the advent of new technology, people use mass media to promote their products and reach a larger customer base.

In this regard, companies raise customer awareness about new products, their prices, and additional information about their quality. All players in supply chain management use different advertising methods to increase the demand for their products, such as through newspapers, posters, and television. Additionally, advertisements showcase the quality of the product, which can attract customers to purchase the item. In the case of carbon emissions, it has become crucial to advertise carbon-free products to encourage customers to buy them and thereby reduce the amount of carbon emitted. Hence, our proposed work considers the interplay between carbon emissions and the demand generated through advertising as an essential aspect.

Our proposed model aims to reduce carbon footprint and optimize the total cost for both suppliers and retailers. Through our model, we have found that the average credit rate, total cost of suppliers, and total cost of retailers are greatly affected by the deterioration rate. Additionally, we have also found that advertisement-dependent demand plays a significant role in determining the average credit rate.

The majority of the rest of this chapter is structured as follows: Section 7.3 presents a review of the existing literature. In Section 7.4 we present the assumptions and notations used throughout this chapter. The mathematical analysis and calculations are provided in Section 7.5. Section 7.6 describes the sensitivity study presented through graphical and tabular forms. Finally, in Section 7.7, we present the managerial findings and section 7.8 conclusions of our study on supply chain management in the context of carbon taxes.

7.3. Literature review:

Kaya's identity, first introduced by (Kaya 1989) is a widely accepted concept that combines four factors to estimate carbon emissions: emissions per unit of energy expenditure, population, per capita income, and energy intensity per unit of manufacturing output. Kaya, 1989 identity can be expressed as follows.

$$Co_2 = \left(\frac{GDP}{Ppopulation}\right)\left(\frac{EC}{GDP}\right)\left(\frac{Co_2}{EC}\right) * Population$$

In recent years, there has been a growing concern for sustainability. As a result, many governments around the world have enacted carbon taxes to promote energy efficiency and reduce emissions. This has led to discussions on the effects of carbon taxes on supply chains. (Zhou et al. 2020) note that traditional supply chain models have focused solely on economic gains, but modern technology has shifted towards sustainability. Carbon taxes have a significant impact on supply chains, as they require supply chain participants to consider local, national, and global ecosystems, as well as the increasing environmental awareness of the general public. (Liu, Anderson, and Cruz 2012) found that producers were motivated to adopt low-emission techniques when the carbon tax was raised to a certain level. Carbon taxes can also benefit manufacturers, suppliers, and retailers by promoting environmentally friendly practices. According to the (Benjaafar, Li, and Daskin 2013) study, the implementation of a carbon tax in a supply chain can effectively accomplish both cost reduction and emissions reduction objectives. Therefore, a well-designed carbon tax technique can encourage social, economic, and ecological collaboration within the supply chain. (Cheng et al. 2017) proposed a collaborative model approach in a two-level supply chain with a carbon tax, in which the retailer and manufacturer jointly made decisions on carbon reduction activities. The study found that the carbon tax encouraged joint efforts by supply chain participants and improved environmental and economic performance. (Park et al. 2015) argued that a carbon tax is more effective in balancing public welfare, environmental conservation, and economic growth. Overall, these studies suggest that carbon taxes can play an important role in promoting sustainability.

(A. K. Bhunia and Shaikh 2011) and (Khan et al. 2020) have investigated the direct association between advertising and product demand. In inventory management, advertising new or modified products plays a crucial role in raising product awareness and increasing demand among customers. Retailers, suppliers, and manufacturers all advertise their new or modified products, which greatly impacts the demand for those products. This makes advertising an important factor in increasing product demand, and ultimately, driving sales. However, enhancing customer demand can be a challenging feat for retailers and suppliers. This is especially true for perishable items or those nearing their expiration date, as their lifespan is limited. Carbon emissions and advertising have a significant influence on product demand. Advertising is an effective promotional tool that raises product awareness and popularity among all consumer classes. Nowadays, with the advent of new technologies, people advertise their new or modified products through mass media to reach a wider audience. In this context, advertising helps to inform customers about new products, their prices, and other relevant information about their quality.

All players in supply chain management use various advertising methods to increase product demand, including advertising in newspapers, posters, and on television. The quality of the product is often highlighted in advertising, as good quality attracts customers to purchase the item. Researchers such as (Goyal and Gunasekaran 1995); (Shah, Pandey, and Pin 2009); (Bhunia and Shaikh 2011); (Shah, Soni, and Patel 2013); (Asoke Kumar Bhunia and Shaikh 2014); (Shaikh 2017); (U. Mishra 2018); (Panda, Khan, and Shaikh 2019); (Khan et al. 2020); and others have developed inventory models that take into account the effect of advertising on sales. Concerning carbon emissions, it is crucial to advertise carbon-free products to encourage customers to make environmentally conscious choices and reduce the amount of carbon emitted. Retailers, suppliers, and manufacturers must plan their advertising strategies before the sales period to achieve this goal. No researcher has yet considered advertisement, time, and inventory-dependent demand in a finite planning horizon, making this proposed work, which considers advertisement, time, carbon, and stock -dependent demand with carbon emission a finite planning horizon, unique.

The supply chain encompasses the entire process of producing and selling essential goods, involving manufacturers, suppliers, transporters, warehouses, retailers, and customers. It is a process that converts natural resources or raw materials into finished products and then sells them to end-users or customers. However, most of the conventional literature available did not analyze credit terms coordination and collaboration. Nevertheless, supply chain participants such as suppliers, retailers, and manufacturers affect each other. Therefore, a collaborative approach to supply chain management is necessary for effective and beneficial supply chain management. In recent years, a few researchers such as (C. Wu and Zhao 2014); (P. N. Singh et al. 2017); (Kim and Sarkar 2017); and (Barman, Das, and De 2020) have focused their attention on supply chain collaboration with credit terms as a beneficial framework for retailers and suppliers. In today's global world, the primary objective of institutions or business owners is to optimize overall value and emphasize carbon emission reduction, which can be achieved by coordinating different strategies among supply chain participants. A lot of work has been done to extend the entire supply chain, and multiple parameters are used to enhance the efficiency of supply chain for coordination and without coordination for Advertisement, Time, Inventory, and Carbon Emission demand under a finite planning horizon.

According to (Daryanto et al. 2019), transportation-related carbon emissions are determined by factors such as the amount of fuel used by the vehicle, the amount of fuel discharged, and the distance travelled. The authors also mention that fuel consumption is affected by transportation and emission strategies, as well as the total cost and amount of emissions. In inventory control, reducing global warming and carbon emissions through green farming alone is a challenging task, and therefore, we need to rely on advertisement technology and carbon regulations to achieve this goal. These strategies can help reduce the total cost, carbon emissions, and global warming, which will have a positive impact on profit. Despite these challenges, there have been very few studies that have considered carbon emissions in the supply chain for advertisement, time, and emission-dependent demand within a finite planning horizon. This research proposes a joint decision-making approach for inventory control and carbon emissions, considering various parameters such as advertisement, time, carbon emissions, and refrigeration technology to design a long-term sustainable supply chain.

Table 7.1 Comparison of Literature

Research study	Inventory Dependent Demand	Advertisement dependent demand	Collaboration	Carbon Emissions cost	Transportation cost	Finite Planning horizon
(Benjaafar, Li, and Daskin 2013)	×	×	✓	✓	×	×
(C. Wu and Zhao 2014)	✓	×	✓	×	×	✓
(Park et al. 2015)	×	×	×	✓	×	×
(P. N. Singh et al. 2017)	✓	×	✓	×	×	✓

(Cheng et al. 2017)	×	×	×	✓	×	×
(U. Mishra 2018)	×	✓	×	✓	✓	×
(Daryanto et al. 2019)	×	×	×	✓	✓	×
(Khan et al. 2020)	×	✓	×	✓	×	×
This Chapter	✓	✓	✓	✓	✓	✓

7.3.1. Research gap and problem defining:

The chapter considers various factors such as time-dependent, inventory-dependent, and advertisement-dependent demand along with carbon emission regulation to analyze the economic and market scenario for specific goods. The focus is on newly launched products, clothes, permissible items, fashionable items, and fast-moving consumer goods with expiration dates. The study takes into account the trade-credit period offered by suppliers to retailers for their purchases and considers collaboration between suppliers and retailers with carbon regulation. The novelty of the study lies in its application to a finite planning horizon, which has not been explored before in the literature.

7.4. Assumptions

Assumptions for a single item, single retailer, and single manufacturer scenario are:

1. Replenishment order timing will be finite.
2. There is no stock-out or back-ordering allowed.
3. The retailer incurs a predetermined cost for placing an order and also bears a set cost for holding the item in inventory.
4. lead time will not be considered zero but it will be almost neglected.
5. The supplier faces a fixed setup cost.
6. The carbon emissions from transportation are calculated based on fixed emission factors and distance travelled.
7. The setup cost is higher than the ordering cost.
8. Transportation fixed cost when an order is placed by the retailer.
9. Demand is taken in this chapter advertisement, inventory, carbon and time-dependent. Where $\mathrm{D}\ (\mathrm{t}) = A^{\gamma}(\ a +\ \mathrm{b}\ \mathrm{t}\ + \rho\ Co_2) +\ \theta\ \mathrm{I}\ (\mathrm{t}),\ \mathrm{a}, \mathrm{b}, \text{and}\ \rho \geq$

0 and t is positive. a denote the initial market demand, b is time sensitive, ρ is sensitive about carbon awareness and θ demand that's dependent upon inventory.

7.5. Mathematical formulation and analysis of the model:

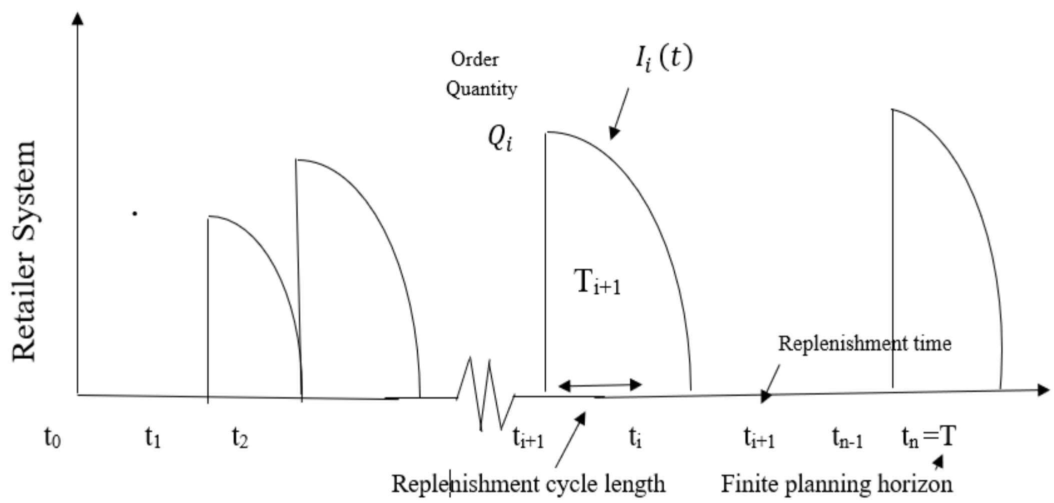

Figure 7.1. Graphical representation of inventory level

In this Phase, a mathematical explanation with no shortages of this research approach for a finite planning period is formulated. Demand is considered advertisement, inventory, carbon and time dependent. The inventory declines eventually, usually to meet the demand and by advertising the product. Now, Eqns. (2), (3), (4) and (5) and represent the demand rate, inventory level, order quantity and holding cost given below. Labelled Figure 1 depicts the rise and fall of inventory. The shift level of inventory during the time interval $\boldsymbol{t_i \le t \le t_{i+1}}$ is provided by a differential equation. Defines it as:

$$D\,(\mathrm{t}, \theta, \mathrm{A}, Co_2) = A^{\gamma}(\,a + \,\mathrm{b\,t}\, - \rho\, Co_2) + \,\theta \mathrm{I}_i(t), \text{where c} \,>\, 0, \mathrm{d}\, \ge 0, \text{and t is positive}$$

$$\frac{d\,IL_{i+1}(t)}{\mathrm{dt}} = -A^{\gamma}(\,a + \,\mathrm{b\,t}\, - \rho\, Co_2) - \theta\, IL_{i+1}, \;\text{where}\quad t_i \le t \le t_{i+1} \qquad (7.1)$$

When $IL_{i+1}(t_{i+1}) = 0$ and $I_i(t_i) = Q_{i+1}$

$$IL_{i+1}(t) = \int_{t}^{t_{i+1}} A^{\gamma}(\, a + \text{ b } * \text{u } - \rho\, Co_2)e^{\theta(u-t)}\, \text{du} \tag{7.2}$$

The order quantity for ith cycles

$$Q_{i+1} = IL_{i+1}(t_i) = \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \text{ b t } - \rho\, Co_2)e^{\theta(\text{t}-\text{t}_i)}\, \text{d}t \tag{7.3}$$

Ordering cost:

$$O_c = \text{n} * O_r \tag{7.4}$$

Holding cost:

$$H_c = \sum_{i=0}^{n-1} h_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} A^{\gamma}(\, a + \text{ b t } - \rho\, Co_2)e^{\theta(u-t)} du\ dt \tag{7.5}$$

According to Benjaafar et al. (2013), Xiang and Lawley (2018), and Shi et al. (2019), the carbon emission cost includes: the carbon emissions associated with order placement (specifically, transportation-related emissions) are represented by the fixed value of c^, In addition, the management of each unit, denoted as $\hat{P}_r$, contributes to varying carbon emissions and carbon emissions associated with the energy consumed by the refrigerator in the storage of each unit $\widehat{h_r}$. Hence, the total carbon emissions attributed to each replenishment process can be expressed as:

$$E = \text{c}\hat{} + \sum_{i=0}^{n-1} \hat{P}_r * Q_i + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} A^{\gamma}(\, a + \text{ b t } - \rho\, Co_2)e^{\theta(u-t)} du\ dt$$

$$CE_c = \tau \left\{ \text{c}\hat{} + \sum_{i=0}^{n-1} \left\{ \hat{P}_r \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \text{ b t } - \rho\, Co_2)e^{\theta(\text{t}-\text{t}_i)}\, \text{d}t \ + \right.\right.$$
$$\left.\left. \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} A^{\gamma}(\, a + \text{ b t } - \rho\, Co_2)e^{\theta(u-t)} du\ dt \right\}\right\} \tag{7.6}$$

The retailer's transportation cost, including the carbon emissions during refrigeration. Therefore,

$$Tc = F_c + 2\mathrm{d}v_c C_1 + \mathrm{d}*v_c C_2 \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \rho \, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t + 2\mathrm{d}e_1 + \mathrm{d}$$
$$e_2 \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \rho \, Co_2) \, e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t \qquad (7.7)$$

where $t_i \leq t \leq t_{i+1}$

The retailer's total cost function can be expressed as follows.

$$T_{IND}^{r} = n_1 * O_r + \sum_{i=0}^{n_1-1} h_r \int_{t_i}^{t_{i+1}} I_i\,(t)\,dt + \sum_{i=0}^{n_1-1} W\,Q_i + \sum_{i=0}^{n_1-1} \tau * \mathrm{c}\hat{} + \widehat{\tau P_r} *$$
$$Q_{i+1} + \sum_{i=0}^{n_1-1} \tau\widehat{h_c} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} A^{\gamma}(a + \mathrm{bt} - \rho Co_2) e^{\theta(u-t)} du \; dt + F_c + 2\mathrm{d}\, v_c C_1 +$$
$$\mathrm{d} * v_c C_2 \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \rho \, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t + 2\mathrm{d}\, e_1 +$$
$$\mathrm{d}\, e_2 \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \rho \, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t$$

$Where \quad T_{i+1} = t_{i+1} - t_i \;\; and$

$$\Big[T_{IND}^{r} = n_1 * O_r + \sum_{i=0}^{n_1-1} h_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \rho \, Co_2) e^{\theta(u-t)} du \; dt +$$
$$W \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \, \rho \, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t + \tau\mathrm{c}\hat{} + \tau\hat{P}_r *$$
$$\sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \, \rho \, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t \; + \sum_{i=0}^{n_1-1} \tau\, \widehat{h_c} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} A^{\gamma}(\, a +$$
$$\mathrm{b\,t} \, - \rho \, Co_2) e^{\theta(u-t)} du \; dt + F_c + 2\mathrm{d}\, v_c C_1 + \mathrm{d} * v_c C_2 \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, -$$
$$\rho \, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t + 2\mathrm{d}\, e_1 + \mathrm{d}\, e_2 \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(\, a + \, \mathrm{b\,t} \, - \rho \, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t \Big]$$

$$\Big[T_{IND}^{r} = n_1 * O_r + \mathrm{c}\hat{} + \Big\{\frac{(h_r + \tau\, \widehat{h}_c)}{\theta} + W_p + \hat{P}_r + \mathrm{d} * v_c C_2 +$$
$$\mathrm{d}\, e_2\Big\} \sum_{i=0}^{n_1-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(a + \mathrm{bt} - \rho Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \, \mathrm{d}t - \frac{A^{\gamma}(h_c + \tau\, \widehat{h}_c)}{\theta}\Big(aH + \frac{1}{2} bH^2\Big) + F_c +$$
$$2\mathrm{d}v_c C_1 + 2\mathrm{d}e_1\Big] \qquad (7.8)$$

$$T^{s}_{IND} = n^{*}_{1} * S_{s} + \sum_{i=0}^{n^{*}_{1}-1} P_{r} * \; Q^{*}_{i+1}$$

$$T^{s}_{IND} = n^{*}_{1} * S_{s} + P_{r} \; * \sum_{i=0}^{n^{*}_{1}-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(a + \mathrm{b\,t} - \rho\, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{d}t \qquad (7.9)$$

$$Q_{IND} = \sum_{i=0}^{n^{*}_{1}-1} Q^{*}_{i+1} \qquad (7.10)$$

$$\Big[T^{s}_{JT} = n_2 * (S_s + O_r) + C_p \; * \sum_{j=0}^{n_2-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(a + \mathrm{b\,t} - \rho\, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{d}t + \left\{\frac{(h_c+\tau\, \widehat{h}_c)}{\theta} + W + \widehat{P}_r + \mathrm{d} * v_c C_2 + \mathrm{d}\, e_2\right\} \sum_{i=0}^{n_2-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(a + \mathrm{b\,t} - \rho\, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{d}t - \frac{A^{\gamma}(h_c+\tau\, \widehat{h}_c)}{\theta}(aT + \tfrac{1}{2} bT^2) + F_c + 2\mathrm{d}\, v_c C_1 + 2\mathrm{d}\, e_1 - T^{r}_{IND}(n^{*}_{1}, t_0, t^{*}_{1}, \ldots\ldots\ldots. t^{*}_{n_1}) \Big] \qquad (7.11)$$

$$Q_{JT} = \sum_{j=0}^{n^{*}_{2}-1} Q'^{*}_{j+1} \qquad (7.12)$$

$$\Big[T^{r}_{JT} = n_2 * O_r + \mathrm{c}\hat{} + \left\{\frac{(h_c+\tau\, \widehat{h}_c)}{\theta} + W_p + \widehat{P}_r + \mathrm{d} * v_c C_2 + \mathrm{d}\, e_2\right\} \sum_{i=0}^{n_1-1} \int_{t_j}^{j_{i+1}} A^{\gamma}(a + \mathrm{b\,t} - \rho\, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_j)}\, \mathrm{d}t - \frac{A^{\gamma}(h_c+\tau\, \widehat{h}_c)}{\theta}(aH + \tfrac{1}{2} bH^2) + F_c + 2\mathrm{d}\, v_c C_1 + 2\mathrm{d}\, e_1 - T^{r}_{IND}(n^{*}_{1}, t_0, t^{*}_{1}, \ldots\ldots\ldots. t^{*}_{n_1}) \Big] \qquad (7.13)$$

$$\delta = T^{r}_{JT}(n'^{*}_{2}, t_0, t'^{*}_{1}, \ldots\ldots\ldots. t'^{*}_{n_2}) - T^{r}_{IND}(n^{*}_{1}, t_0, t^{*}_{1}, \ldots\ldots\ldots. t^{*}_{n_1}) / \sum_{j=0}^{n^{*}_{2}-1} h(t'^{*}_{\mathrm{j}+1} - t'^{*}_{\mathrm{j}})\; Q'^{*}_{j}$$

(7.14)

$$\delta_{max} = T^{s}_{IND}(n^{*}_{1}, t_0, t^{*}_{1}, \ldots\ldots\ldots. t^{*}_{n_1}) - n^{*}_{1} * S_c - C_p \; * \sum_{i=0}^{n^{*}_{1}-1} \int_{t_i}^{t_{i+1}} A^{\gamma}(a + \mathrm{b\,t} - \rho\, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{d}t \Big/ \sum_{j=0}^{n^{*}_{2}-1} h(t'^{*}_{\mathrm{j}+1} - t'^{*}_{\mathrm{j}})\; Q'^{*}_{j} \qquad (7.15)$$

$$\delta_{min} = T_{JT}^{r}(n_2'^{*}, t_0, t_1'^{*}, \dots\dots\dots t_{n_2}'^{*}) - T_{IND}^{r}(n_1^{*}, t_0, t_1^{*}, \dots\dots\dots t_{n_1}^{*}) / \sum_{j=0}^{n_2^{*}-1} h(t_{j+1}'^{*} - t_j'^{*})\, Q_j'^{*} \tag{7.16}$$

We know that $\bar{\delta} = \frac{\delta_{min}+\delta_{max}}{2}$ (7.17)

$$\bar{T}_{JT}^{r} = T_{JT}^{r}(n_2'^{*}, t_0, t_1'^{*}, \dots\dots\dots t_{n_2}'^{*}) - \sum_{j=0}^{n_2^{*}-1} \bar{\delta} h(t_{j+1}'^{*} - t_j'^{*})\, Q_j'^{*} \tag{7.18}$$

$$\bar{T}_{JT}^{s} = n_1^{*} * S_s + \sum_{i=0}^{n_1^{*}-1} P_r\, Q_i^{*} + \sum_{j=0}^{n_2^{*}-1} \bar{\delta} h(t_{j+1}'^{*} - t_j'^{*})\, Q_j'^{*} \tag{7.19}$$

$$\frac{\partial T_{IND}^{r}}{\partial t_i} = A^{\gamma}(a + \mathrm{b}t_i + \rho\, Co_2)\left(e^{\theta(t_i - t_{i-1})} - 1\right) - \theta \int_{t_i}^{t_{i+1}} A^{\gamma}(a + \mathrm{b}\,\mathrm{t} - \rho\, Co_2) e^{\theta(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{d}t \tag{7.20}$$

Where i=1,2,3…………n

$$\nabla^2 T_{IND}^{r} = \begin{bmatrix} \frac{\partial^2 T_{IND}^{r}}{\partial t_1^2} & \frac{\partial^2 T_{IND}^{r}}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 T_{IND}^{r}}{\partial t_2 \partial t_1} & \frac{\partial^2 T_{IND}^{r}}{\partial t_2^2} & \frac{\partial^2 T_{IND}^{r}}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 T_{IND}^{r}}{\partial t_3 \partial t_2} & \frac{\partial^2 T_{IND}^{r}}{\partial t_3^2} & \frac{\partial^2 T_{IND}^{r}}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 T_{IND}^{r}}{\partial t_{n_1-1} \partial t_{n_1-2}} & \frac{\partial^2 T_{IND}^{r}}{\partial t_{n_1-1}^2} & \frac{\partial^2 T_{IND}^{r}}{\partial t_{n_1-1} \partial t_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 T_{IND}^{r}}{\partial t_{n_1} \partial t_{n_1-1}} & \frac{\partial^2 T_{IND}^{r}}{\partial t_{n_1}^2} \end{bmatrix}$$

Figure 7.2. Hessian matrix

The value of the Hessian matrix as discussed in (P. N. Singh et al. 2017) and (T. Sarkar et al. 2012) has to be shown as positive definite after solving, for T_{IND}^{r} to be minimum. Now, replenishment time intervals are obtained with the help of Eq. (7.20) which is the derivative of Eq. (7.8).

Theorem 7.1. If t_i satisfy inequations (i) $\frac{\partial^2 T^r_{IND}}{\partial t_i^2} \geq 0$ (ii) $\frac{\partial^2 T^r_{IND}}{\partial t_i^2} \geq |\frac{\partial^2 T^r_{IND}}{\partial t_i t_{i-1}}| + |\frac{\partial^2 T^r_{IND}}{\partial t_i t_{i+1}}|$ for all i= 1, 2, 3 ……….n_1 then $\nabla^2 T^r_{IND}$ is positive definite.

$$\frac{\partial^2 T^r_{IND}}{\partial {t_i}^2} = \Big\{\Big(A^\gamma b\big(e^{\theta(t_i - t_{i-1})} - 1\big) + \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta(t_i - t_{i-1})} + \theta A^\gamma(a + \mathrm{b\,t} - \rho\, Co_2) + \theta^2 \int_{t_i}^{t_{i+1}} A^\gamma(a + \mathrm{b\,t} - \rho\, Co_2)e^{\theta(t - t_i)}\,\mathrm{dt}\Big)\Big\}$$

$$\frac{\partial^2 T^r_{IND}}{\partial {t_i}^2} = A^\gamma b\big(e^{\theta T_i} - 1\big) + \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i} + \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2) + \theta A^\gamma(a + \mathrm{b}t_{i+1} + \rho\, Co_2)e^{\theta(t_{i+1} - t_i)} - A^\gamma b\, e^{\theta(t_{i+1} - t_i)} - \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2) + b$$

$$\frac{\partial^2 T^r_{IND}}{\partial {t_i}^2} = A^\gamma b\big(e^{\theta T_i}\big) + \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i} + \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2) + \theta A^\gamma(a + \mathrm{b}t_{i+1} + \rho\, Co_2)e^{\theta T_{i+1}} - A^\gamma b\, e^{\theta T_{i+1}} - \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2) \quad (7.21)$$

$$\frac{\partial^2 T^r_{IND}}{\partial {t_i}^2} = \theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i} + \theta A^\gamma(a + \mathrm{b}t_{i+1} + \rho\, Co_2)e^{\theta T_{i+1}} + A^\gamma b(e^{\theta T_i} - e^{\theta T_{i+1}}) \quad (7.\ A)$$

$$\frac{\partial^2 T^r_{IND}}{\partial t_i\, \partial t_{i-1}} = -\,\theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta(t_i - t_{i-1})}$$

$$\frac{\partial^2 T^r_{IND}}{\partial t_i\, \partial t_{i-1}} = -\,\theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i} \quad (7.\ B)$$

Similarly

$$\frac{\partial^2 T^r_{IND}}{\partial t_i\, \partial t_{i+1}} = -\,\theta A^\gamma(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_{i+1}} \quad (7.\ C)$$

$$\frac{\partial^2 T^r_{IND}}{\partial t_i\, \partial t_m} = 0 \quad \text{for all } m \neq i, i+1, i-1 \quad (7.D)$$

$\boldsymbol{T_{cr}}$ is positive definite if Eq. A, B, C and D satisfy the given inequality.

$$\frac{\partial^2 T_{IND}^r}{\partial {t_i}^2} > \left|\frac{\partial^2 T_{IND}^r}{\partial t_i\, \partial t_{i-1}}\right| + \left|\frac{\partial^2 T_{IND}^r}{\partial t_i\, \partial t_{i+1}}\right| + \left|\frac{\partial^2 T_{IND}^r}{\partial t_i\, \partial t_{i+1}}\right|$$

$$\theta(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i} + \theta(a + \mathrm{b}t_{i+1} - \rho\, Co_2)e^{\theta T_{i+1}} + A^\gamma b\left(e^{\theta T_i} - e^{\theta T_{i+1}}\right) > \left|- \theta(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i}\right| + \left|- \theta(a + bt_{i+1})e^{\theta T_{i+1}}\right| + |0|$$

$\theta(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i} + \theta(a + \mathrm{b}t_{i+1} - \rho\, Co_2)e^{\theta T_{i+1}} + A^\gamma b\left(e^{\theta T_i} - e^{\theta T_{i+1}}\right) > \theta(a + \mathrm{b}t_i - \rho\, Co_2)e^{\theta T_i} + \theta(a + \mathrm{b}t_{i+1} - \rho\, Co_2)e^{\theta T_{i+1}}$ that is true for all $i = 1, 2, . . ., n1$

7.6. Algorithm and procedures for resolving the problem:

The following steps are available to find the solution.

Step1: First of all, a new and unique set-up of parameters is considered: O_r, d_r, h_r, S_s, A, P_r, cˆ, Pˆr, hˆr, θ, … …. …. …τT_{i+1}, H etc. The values of these parameters are assigned based on already existing literature.

Step 2: We will determine the values of t_i by the following ways.

a) Then, start with considered n=1. Then t_0 =0 and t_1=T

b) If we take n=2, then then we assumed t_0=0 and t_2=T. Then, to find t_1 arrange the partial derivative of the function T_{IND}^r in terms of t_i is equal to zero. It's identical to $\frac{\partial T_{IND}^r}{\partial t_i} = 0$

c) Afterwards, by using values of t_2, get t_3 and then t_4……so on. Similarly, by this above process, we found all t_i.

d) After degerming the t_i values, verify the result $\left|\frac{\partial^2 T_{IND}^r}{\partial {\mathrm{t_i}}^2}\right| \geq \left|\frac{\partial^2 T_{IND}^r}{\partial (\mathrm{t_i})(\mathrm{t_{i-1}})}\right| + \left|\frac{\partial^2 T_{IND}^r}{\partial (\mathrm{t_i})(\mathrm{t_{i+1}})}\right|$ that prove the convexity of function. Based on this convexity we can calculate the optimal value of t_i.

Step 3: Based on optimal t_i, the total cost of the retailer T_{IND}^r can be calculated by Eq. (7.8). The following steps are available to find the T_{IND}^r.

a) We Start by considering n=1 and $T_{IND}^{r}(\text{n1}) \leq T_{IND}^{r}(\text{n1}+1)$, then $T_{IND}^{r}(\text{n1}) = T_{IND}^{ro}(\text{n1})$ and stop here.

b) we set n ≥ 2 and $T_{IND}^{r}(\text{n1}) \leq T_{IND}^{r}(\text{n1}-1)$ *and* $T_{IND}^{r}(\text{n1}) \leq T_{IND}^{r}(\text{n1}+1)$ then $T_{IND}^{r}(\text{n1}) = T_{IND}^{ro}(\text{n1})$ and stop. If n1=n1+1 then go to step 2(b).

Step 4: Using the values of t_i obtained in the previous step, by which we can determine the optimal replenishment order quantity. $\text{Q}_{(\text{i}+1)} = \sum_{\text{i}=0}^{\text{n}_1^*-1} \text{Q}_{\text{i}+1}^*$.

Step 5: Subsequently, by Eq. (7.9) we find T_{IND}^{s} (Supplier total cost) which depends upon the retailer's ordering process.

Step 6: Similarly, the same process is in the case of coordination.

7.6. Numerical illustration for the proposed model:

Here are the parametric fundamental values with their appropriate units: $O_r =$ 80 \$/order, h_r=1 \$/unit/time, $\Theta = 3$, $W = 3$, a= 50, 60, 70 units, b=15 units, $S_s =$ 100, $\hat{P}_r = 0.02\ (in\ kg/order)$, $Co_2 = 0.02\ (in\ kg/order)$,$\tau = 6$ (in dollars/kg), $P_r = 0.01$,$\delta = 0.2$, $\alpha = 1.3$, $\text{v}_\text{c} = 8$, $\text{e}_2 = 2.31 \times 10^{-6}$, $\text{e}_1 = 0.043$, $\text{C}_1 = 25$, $\text{C}_2 = 0.36$, d=25, $\text{F}_\text{c} = 0.01$, A=2, H=2, h=60. The solution of the nonlinear partial differential Eq. (7.20) is obtained using the numerical iterative Mathematica (version 12.0) software.

7.7. Tabular and graphical illustrations:

For no collaboration, Table 7.2, Table 7.3, Figure 7.3 and Figure 7.4 show optimal retailer's overall cost for a= 40,50,60,70,80 is \$124188, \$135541, \$144159, \$151166 and \$158282 reach its minimum at 7, 8, 8 and 8 optimal ordering cycles respectively. After reaching its minimum at n_1=7, 7, 8, 8 and 8 then again started increasing gradually for all upcoming cycles. Table 2 shows the convexity of the retailer's cost.

In the case of collaboration, Table 7.4, Table 7.5, Figure 7.5 and Figure 7.6 show us the optimal supplier's overall cost for different values of a (40, 50, 60, 70, 80) to be \$896, \$946, \$1103, \$1150, and \$ at 7, 8, 8, and 8 optimal ordering cycles, respectively. However, after reaching its minimum at n2=7, 7, 8, 8, and 8, it starts increasing gradually for all upcoming cycles. Table 7.2, Table 7.3, Table 7.4, Table 7.5, Table 7.6, Figure 7.2, Figure 7.3, Figure 7.4, Figure 7.5 and Figure 7.6 provide us with information on convexity when suppliers and retailers are not collaborating and when they are collaborating.

Table 7.2. The total cost incurred by a retailer without collaborating with a supplier can vary based on the values of 'a' and n_1.

↓ a → n	1	2	3	4	5	6	7	8	9
40	1382870	367427	203246	152401	132840	125552	**124188**	126095	129983
50	160150	421552	230447	170745	147239	137958	**135541**	136910	140572
60	185790	484481	260902	189908	160987	148596	144179	**144159**	146801
70	2107720	545702	290439	208455	174276	158872	152523	**151166**	152830
80	2359060	607653	320403	227303	187791	169324	161006	**158282**	158945

Table 7.3. Optimal retailer's cost and optimal replenishment cycles with no collaboration different value of a and n_1

↓ a → t_i	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	T_{IND}^{r}
40	0	0.5688	1.1105	1.6303	2.1318	2.6177	3.0900	3.5503	4.		**124188**
50	0	0.5590	1.0957	1.6138	2.1158	2.6040	3.0799	3.5449	4.		**135541**
60	0	0.4939	0.9717	1.4354	1.8869	2.3274	2.7581	3.1799	3.5937	4.	**144159**
70	0	0.4884	0.9628	1.4249	1.8760	2.3173	2.7496	3.1737	3.5903	4.	**151166**
80	0	0.4840	0.9557	1.4164	1.86721	2.3090	2.74255	3.16854	3.58752	4.	**158282**

In the case of collaboration, Table 7.4, Table 7.5 and Figure 7.4. give us the optimal supplier's overall cost for different values of a= 40,50,60,70,80 are \$896, \$946, \$1103, \$1150 and \$ at 7, 8, 8 and 8 optimal ordering cycles respectively. Again, after attaining its minimum at n_2=7, 7,8, 8 and 8 then again started increasing gradually for all next upcoming cycles. Table 7.3 and Table 7.4 proved us with information on convexity whenever supplier and retailer are in collaboration.

Table 7.4 Total cost of the supplier in the case of collaboration for different values of a and n_2

↓a →n_2	1	2	3	4	5	6	7	8	9
40	1259810	243811	79609	28827	19351	2159	**896**	2906	6900
50	1467320	286674	95520	35871	12449	3263	**946**	2420	6189
60	1715340	341075	117420	46470	17629	5332	**1017**	1103	3854
70	1958350	395364	140004	58057	23956	8647	2400	**1150**	2925
80	2202780	450279	162908	69839	3040	12029	3815	**1200**	1974

Table 7.5 Optimal supplier's cost and optimal replenishment cycles with collaboration for different values of a and n_2

↓a →t_i	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	T_{JT}^{S}
40	0	0.5689	1.1105	1.6303	2.1318	2.617	3.0901	3.4	4.		896
50	0	0.5591	1.0958	1.6138	2.1159	2.6041	3.0800	3.5449	4.		946
60	0	0.4940	0.9717	1.4355	1.8869	2.3274	2.7582	3.180	3.5937	4.	1103
70	0	0.4884	0.9628	1.4250	1.8761	2.3173	2.7496	3.1737	3.5903	4.	1150
80	0	0.4840	0.9557	1.4164	1.8672	2.309	2.7425	3.1685	3.5875	4.	3815

Table 7.6. Optimality with collaboration and without collaboration different values of a

		with collaboration					without collaboration					
Parameters	n_1^*	T_{IND}^r	T_{IND}^s	Q_{IND}	π_{min}	π_{max}	n_2^*	T_{JT}^r	T_{JT}^s	Q_{JT}	% T_{IND}^r	% T_{IND}^r
40	7	124188	137511	1378.8	0.00168	0.00160	7	127440	140763	1378.8	2.61	2.36
50	7	135541	172653	1580.2	0.00171	0.00182	7	130399	167512	1580.2	3.7	2.9
60	8	144159	242246	1633.0	0.00178	0.00234	7	115525	213612	1781.3	19.8	11.8
70	8	151166	280562	1817.2	0.00042	0.00239	8	36393	165791	1817.2	75.9	40.9
80	8	158282	320269	2001.2	0.00043	0.00252	8	26203	188190	2001.2	83.4	41.2

Table 7.7. Sensitivity findings and analysis by variation in different parameters

%Change in Parameters		a	b	θ	O_r	CO 2	A	S_s	A^γ	h_r	τ	W
$\frac{\triangleleft\, \overline{Tc}_r^{c\rho}}{\overline{Tc}_{rO}^{c\rho}} \times 100\%$	−20	120.938	−5.994	−24.794	−0.164	2.555	−18.833	−3.485	−13.135	0.205	−3.215	−4.340
	−10	104.812	−1.773	−12.177	0.	0	−10.473	0.078	−6.790	0.153	0.311	−0.574
	0	70.0103	−1.155	−9.003	0.	0	−10.712	0.052	−8.579	0.102	0.207	−0.383
	+10	35.064	−0.564	−1.275	0.	0	−5.429	0.026	−4.482	0.051	0.103	−0.191
	+20	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$\frac{\triangleleft\, \overline{Tc}_s^{c\rho}}{\overline{Tc}_{sO}^{c\rho}} \times 100\%$	−20	−32.186	−18.438	−878.38	−5.142	7.392	115.770	2.245	176.399	−0.174	2.677	1.471
	−10	−24.402	−15.536	−1140.69	0.	0	144.072	−0.596	193.323	−0.130	−0.259	−1.168
	0	−16.274	−10.412	−501.20	0.	0	−21.002	−0.397	−16.307	−0.087	−0.172	−0.778
	+10	−8.139	−5.232	−616.63	0.	0	−10.644	−0.198	−8.520	−0.043	−0.086	−0.389
	+20	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
$\frac{\triangleleft\, \bar{\delta}}{\bar{\delta}_0} \times 100\%$	−20	−15.196	−4.162	−38692.4	−5.204	7.335	288.762	2.820	322.398	−0.176	2.713	1.491
	−10	−11.028	−4.936	−20318.6	0.	0	277.491	−0.202	309.718	−0.132	−0.262	−1.184
	0	−6.945	−3.216	−2929.2	0.	0	−0.127	−0.134	0	−0.088	−0.175	−0.789
	+10	−3.290	−1.572	−1477.7	0.	0	−0.057	−0.067	0	−0.044	−0.087	−0.394
	+20	0.	0.	0.	0.	0.	0.	0.	0	0.	0.	0.
n_1^*	−20	8	8	7	8	9	7	8	7	8	8	8
	−10	8	8	7	8	9	7	8	7	8	8	8
	0	8	8	8	8	9	8	8	8	8	8	8
	+10	8	8	8	8	9	8	8	8	8	8	8
	+20	8	8	9	8	9	8	8	8	8	8	8
n_2^*	−20	8	8	7	8	9	7	8	7	8	8	8
	−10	8	8	7	8	9	7	8	7	8	8	8
	0	8	8	8	8	9	8	8	8	8	8	8
	+10	8	8	8	8	9	8	8	8	8	8	8
	+20	8	8	9	8	9	8	8	8	8	8	8

7.8. Sensitivity analysis and findings:

Table 7.7 presents the sensitivity levels of each parameter in the given Example. To analyze the sensitivity, the value of each component used in the Example was varied by -10%, -20%, 20%, and 10%. Real economic conditions are subject to uncertainty and unpredictable nature, leading to fluctuations in the values of some variables in a decision-making scenario. Therefore, it's crucial to examine resulting changes when altering different parameters, which can be achieved through a comprehensive sensitivity analysis illustrating the effects of variable alterations. Each parameter, including O_r, h_c, Θ, W, a, b, S_s, c^,$\hat{P}_r$, Co_2 ,τ, P_r ,δ , A^{γ} , v_c, e_2, e_1, C_1 , C_2 , d, F_c , a, T, and h, were changed individually, while the remaining parameters remained fixed. The sensitivity analysis and observations for $\overline{Tc}_r^{C\rho}$, $\overline{Tc}_s^{C\rho}$, $\bar{\delta}$, n_1* and n_2* were completed by varying their values by -10%, -20%, +20%, and +10%, while focusing on one component at a time and keeping the remaining values fixed. These observations were carried out using the "Mathematica" numerical iterative computation application version-12, and the results are detailed in Table 7.7.

1. As a result of the study, the value of n_1* and n_2* is shown in **Table 7.7.** to be highly reactive for parameters like A, α, and, Θ and insensitive to other left parameters such as S_s, Co_2 ,τ, P_r , O_r, h_c, W, a, and b.
2. The average credit is given by the supplier to the retailer $\bar{\delta}$ is very sensitive to the parameters Θ, and A, moderately sensitive to τ, h_r, and practically insensitive to S_s, Co_2 , P_r , O_r, W_p, and b.
3. Retailer's total cost $\overline{Tc}_r^{C\rho}$ is very sensitive to the parameters, a, moderately sensitive to Θ and practically insensitive to S_s, Co_2 ,τ, P_r , O_r, h_r, W, and b.
4. Supplier's total cost $\overline{Tc}_s^{C\rho}$ is very sensitive to the parameters, a, Θ, and A. moderately sensitive to S_s , τ, P_r , h_r, W, and b. and practically insensitive to Co_2 , and O_r

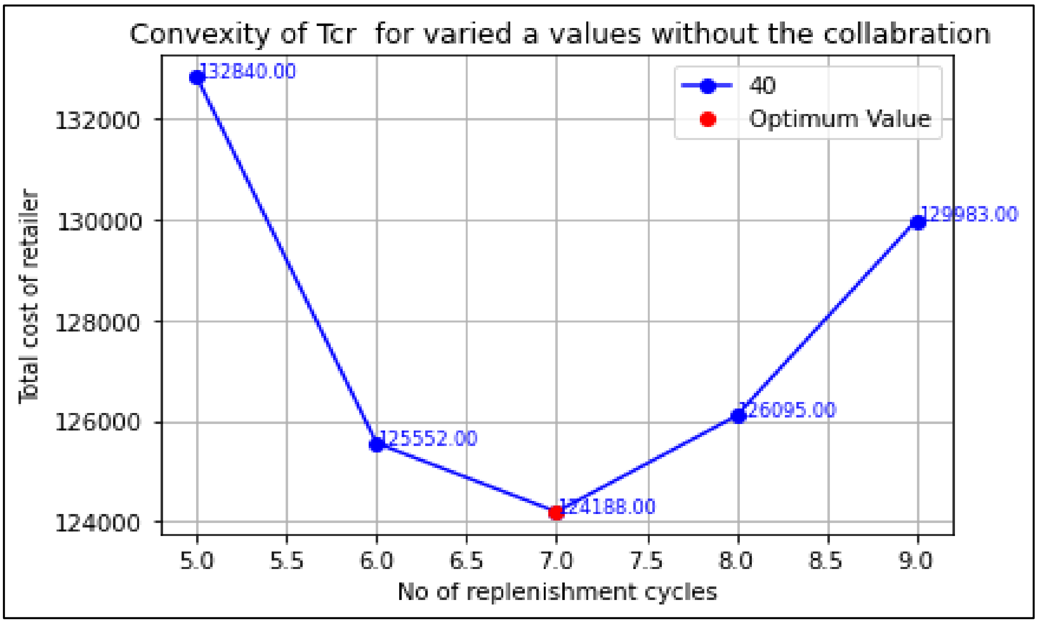

Figure 7.3 Pictorial presentation for Table 7.2

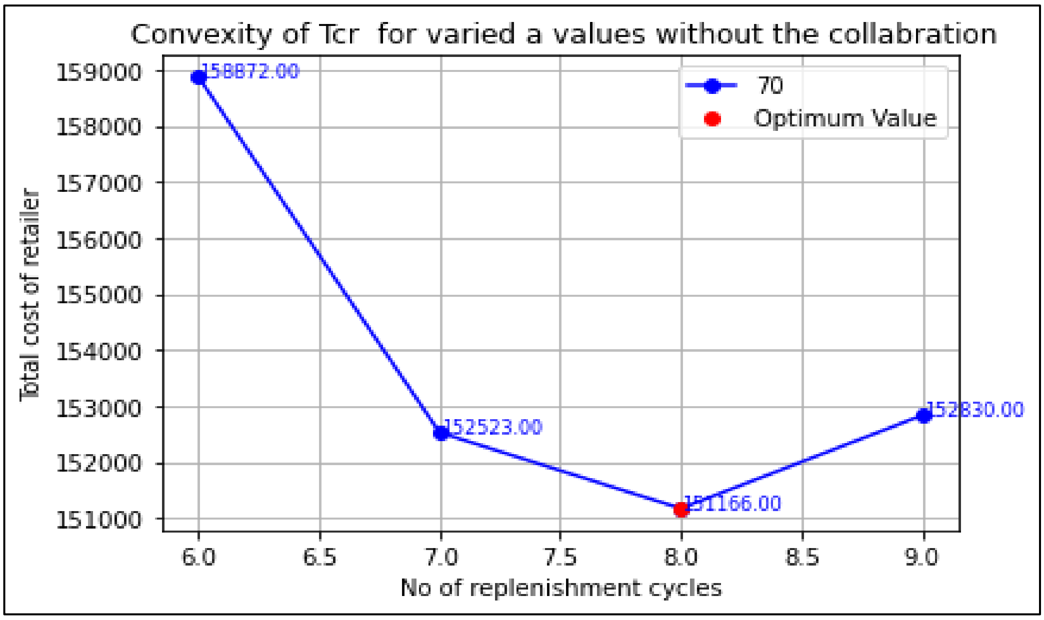

Figure 7.4 Convexity representation for Table 7.2

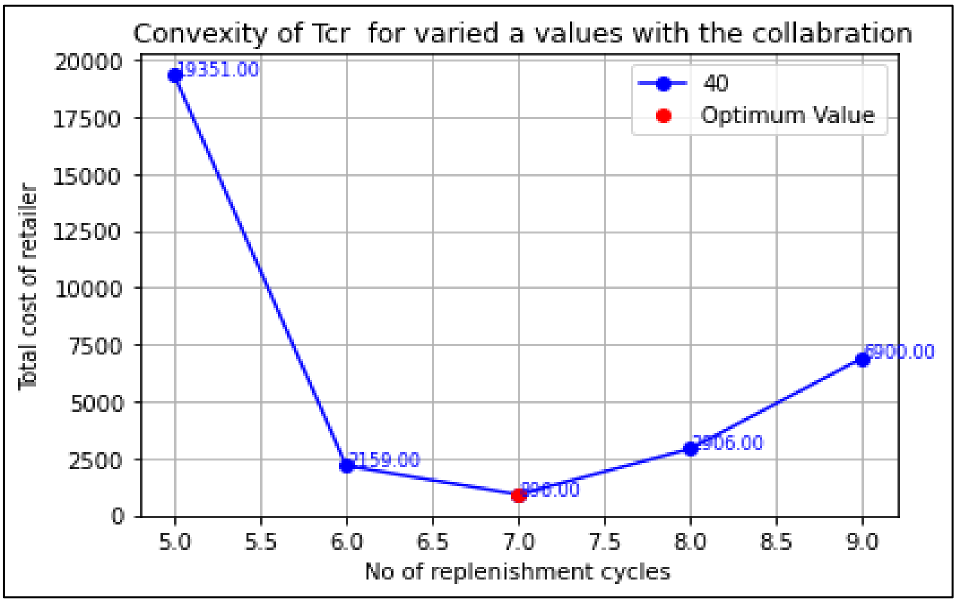

Figure 7.5 Pictorial presentation for Table 7.4

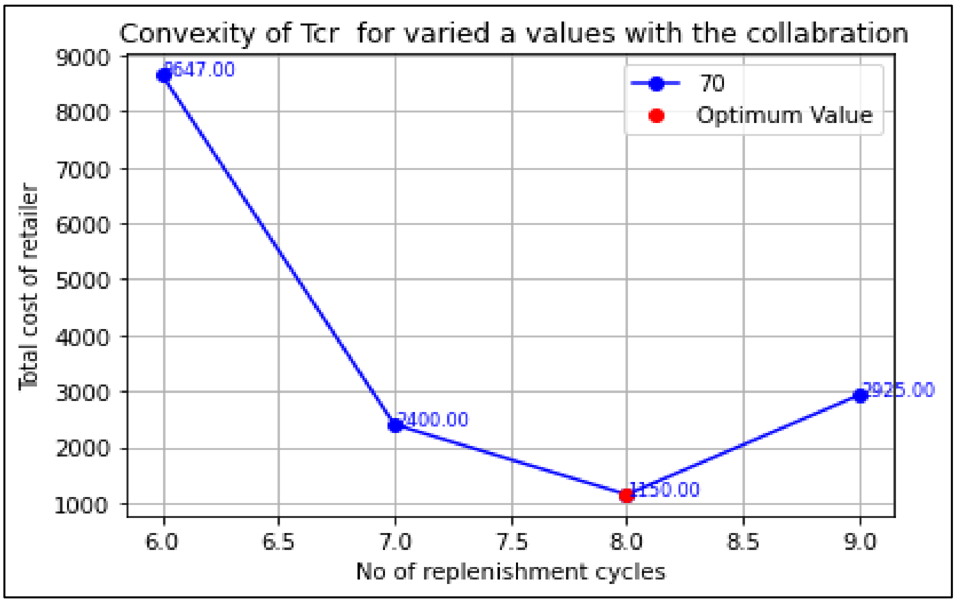

Figure 7.6 Pictorial presentation for Table 7.4

7.9. Conclusion and future study of proposed work:

This chapter presents an optimization technique to determine the best ordering replenishment policies for managing a supply chain with time, carbon, and advertisement-based demands that were not discussed in previous chapters. The proposed framework comprises an optimization module that utilizes an algorithm for both coordinated and uncoordinated scenarios. The study focuses on a -retailer and supplier system with an item chain, and it is the first to explore centralized and decentralized controls for an inventory system using an advertisement demand-based optimization setting. In the case of collaboration, the retailer is offered credit by the supplier to reduce their costs. As a result, collaboration leads to lower costs compared to the scenario without collaboration.

7.9.1. Future study:

In the future, this research study aims to expand by incorporating additional parameters such as multi-echelon and carbon offset, and by addressing potential shortages. Further extensions could include investigating multi-item and multi-echelon models with time, inventory, and advertising-dependent demand. Additionally, the research idea suggests exploring the reworking process during the production of defective items.

7.10. Recommendations for stakeholders:

The findings of this work can be applied in various ways to improve business deals. Firstly, they can guide us in determining cost-effective terms and conditions in existing markets. Secondly, by utilizing carbon footprint measurements and analysis, manufacturers can leverage their current resources to make significant contributions towards a more sustainable environment. This includes lowering carbon emissions caused by the manufacturing process, supply, or service, and determining optimal product generation levels to remain within GHG emissions caps while fully utilizing available resources. Thirdly, producers can perform sensitivity analyses to evaluate the long-term profitability of investing in production enhancements. Such information can also aid manufacturers in reducing costs, managing inventory, and enhancing working capacity. Additionally, this research promotes flexibility throughout the supply chain process, which leads to better resource utilization and increased profit potential.

Chapter 8: Inventory replenishment model with trade credit, green, and preservation technology under a finite planning horizon

8.1. Abstract:

Maintaining sustainability in a competitive business environment is a daunting task. To address this challenge, we have proposed a supply chain inventory replenishment model with a finite planning horizon. Our research study aims to enhance profits, reduce total costs and carbon emissions by examining investments in green (carbon offset) and preservation technologies, and analysed trade credit duration granted by suppliers to retailers. Carbon offsets/green technology represent a prevalent and significant measure to reduce carbon emissions. Time is crucial in the demand rate in this context, and material degradation affects several business sectors. We have calculated the cost of investing in preservation or green technology (carbon offset) to control material deterioration and reduce environmental emissions, as well as the cost of ordering, holding, and replenishing cycle duration.

Moreover, we have developed an algorithm to identify the optimal solution for inventory control and management challenges in the supply chain approach. The parameters of our proposed research study are analysed theoretically, mathematically, and pictorially to demonstrate their optimality and uniqueness. Additionally, we have provided managerial implications of our proposed research study. Our research indicates that our proposed study can be applied to real-world scenarios.

8.2. Introduction:

The growing importance of sustainable supply chain management in today's competitive business environment cannot be ignored. Sustainable development is crucial for the growth of enterprises. The increase in carbon emissions has contributed to the worsening of the global climate, making environmental issues a top priority today. To reduce carbon emissions, most developed countries are implementing innovative technologies and policies such as carbon taxes and cap policies. The Kyoto

Protocol, which came into effect on February 16, 2005, aimed to reduce global warming and its disastrous effects on humankind by setting emission caps for developed countries. The protocol proposed three adaptable approaches to emission reduction. Enterprises are given the right to emit or discharge a specific number of designated pollutants in the form of emissions permits (emission caps) by a central authority or government agency. If they exceed the limit or cap on emissions, they will be taxed, as stated by (Xi Chen et al. 2013), (Lin 2018) and (M. Wang et al. 2018).

Collaboration in the supply chain allows individuals to generate increased profits, lower costs, and reduce carbon emissions. Trade credit periods are a vital component of this collaboration. In today's competitive environment, both retailers and suppliers can benefit from trade credit periods. Suppliers can enhance their market sales and earn additional profit by offering a permissible delay in payment to retailers. In return, retailers can earn interest during the delay in payment and ensure they never face the problem of being out of stock. Trade credit periods create a win-win situation for both parties and contribute to a more efficient and profitable supply chain.

In response to the growing concern over carbon emissions, organizations and governments are increasingly focusing on reducing carbon emissions in supply chain operations. As a result, it has become crucial to adopt emission-reduction technologies (carbon offset) while considering supply chain emission trading regulations, as stated by (U. Mishra, Wu, and Sarkar 2020).

It is a well-known fact that everything gradually deteriorates over time, as noted by (Pospíšil et al. 1999). The commercial and economic importance of various materials, such as metals, polymer blends, and organic biomaterials, necessitates investigating their deterioration behaviour concerning emission concerns. Deterioration is an inevitable part of daily life, and natural processes, including the deterioration of goods and emissions, decrease the usefulness of products. The deterioration of goods such as vegetables, flowers, and metals gradually occur over time. Green technology investments (carbon offset) can also be made to reduce carbon emissions. By investing in high-quality and fast-moving preservation approaches (PRA) and green approaches

(GRA), inventory degradation and greenhouse gas emissions can be reduced, as stated by (U. Mishra et al. 2021).

Several researchers have developed inventory models with various features, such as demand being dependent on time, price, and trade credit duration, incorporating pricing with default risk with backorders, green approaches, preservation approaches, and different payment options (delay or advance payment), as noted by (U. Mishra et al. 2017). However, most of these researchers have developed inventory models with different parameters under infinite planning horizons. Until now, there have been no studies that have investigated the influence of preservation technology and carbon emission reduction policies, under a finite planning horizon with different replenishment cycle times. Therefore, the proposed research aims to develop a model that takes preservation and carbon emission reduction policies into account for planning scenarios with finite planning horizons, with unequal replenishment lengths.

The rest of this research chapter is structured into several sections. Section 8.3 conducts a literature review that identifies a research gap. Section 8.4 outlines the assumptions used in the study. Section 8.5 delves into the derivation of the mathematical model, while Section 8.6 presents a numerical example, analyzes the findings, and conducts sensitivity analysis. Lastly, in Section 8.7, the study's conclusion is explained, including its limitations, section 8.8 represents the managerial implications, and suggestions for future research innovations.

8.3. Literature review:

Inventory control of perishable materials such as vegetables, flowers, chemical compounds, and gasoline is affected by their deterioration. Material deterioration and depletion can result from anthropogenic activities, technological, and environmental factors. (Xu Chen and Hao 2015) developed an approach in which demand depends on credit and time, and material deterioration occurs at a constant rate. (Bakker, Riezebos, and Teunter 2012) further reviewed deterioration rates from 1990 to 2011 and incorporated credit rates in their approach. (Taghizadeh-Yazdi, Farrokhi, and Mohammadi-Balani 2020) investigated an integrated multi-tier framework that deals with the deterioration of items, incorporating the effect of trade credit policy and price on demand, controlled material deterioration, and emissions rates in the inventory model.

A critical aspect that producers must consider to minimize costs and maximize profits is the controllable deterioration of materials. (Huang 2016) explored a scenario where seasonal materials were preserved through investments in advanced preservation technology to manage costs. (Hsieh and Dye 2013) analyzed the impact of preservation investments on the total inventory costs, supply chain risks, and maximum profit in the supply chain process using the (Dye 2013) model. (Liu, Anderson, and Cruz 2012) built upon (Hsieh and Dye's 2013) approach by considering dynamic pricing and preservation investment strategies for perishable items that are sensitive to price and quality. (U. Mishra et al. 2017) investigated a manufacturing system of degrading items and assessed the effectiveness of preservation investments. (Bardhan et al. 2019) further extended the model by incorporating preservation investments, replenishment policies, and material deterioration.

The financial aspects of trading for a system of decaying materials with preservation technologies were examined by (Mohanty et al. 2018) but they did not take into account emissions regulation policy. On the other hand, (Kumar et al. 2012) investigated a manufacturing framework that includes a trade credit system, advanced preservation technologies, and a market regulation approach for emissions. While they considered trade credit, they did not take into account the demand that depends on credit. Several recent studies in supply chain management have examined the use of technology to slow down the rate of material deterioration.

In numerous sectors of the economy and business, supply chain processes contribute significantly to environmental emissions. Green companies need to invest in their systems to make improvements that will reduce emissions and the deterioration of materials. As environmental carbon emissions are being controlled, many researchers and practitioners are focused on sustainable inventory control and management.

(Dye and Yang 2015) conducted an extensive analysis of sustainable inventory control and management using credit terms, cycle, and emission restrictions. They investigated a sustainable instantaneous stock model and evaluated the consequences of emissions under various environmental regulations.

(Qin, Ren, and Xia 2017) developed a sustainable inventory control and supply chain management approach that considers cap-trade, carbon tax, and credit terms. They examined endogenous and exogenous credit terms in their study. (Hovelaque and Bironneau 2015) and developed a replenishing inventory approach that calculates the optimal profit by reducing carbon emissions, incorporating the cost of technology equipment and carbon taxation. (Ahmed, Ahmed, and Sarkar 2018) created an eco-sustainable supply chain approach that considers carbon emissions. (Tiwari, Daryanto, et al. 2018) studied sustainable depreciating inventory systems and emission rates for optimality. Lin (2018) investigated cost-reduction strategies for sustainable inventory systems that reduce transportation emissions with back-ordering. Furthermore, (Tiwari, Ahmed, et al. 2018) created a sustainable production model for multi-item under trade credit and shortages.

(Mohanty et al. 2018) looked at credit terms in an approach to decaying products using preservation techniques, they neglected to take into account an emissions regulation policy. The use of technology to slow down the pace of deterioration and emissions has been examined in several recent supply chain management studies. Several researchers have studied material deterioration by emissions or material degradation by emissions. For example, (U. Mishra, Wu, and Sarkar 2020) considered an emissions regulatory approach and a preservation new approach with inventory dependent on demand, but they did not consider the inventory model under FPH (finite planning horizon). (U. Mishra et al. 2021) extended preservation technology and an emissions regulatory policy by linking credit to demand, but all parameters were not considered under a finite planning horizon. (C. Wu and Zhao 2014) and (Datta 2017) explained the model for trade credit for a finite planning horizon but did not consider preservation technology and emissions regulations. Similarly, (P. N. Singh et al. 2017) and (V. Singh et al. 2019) considered a model with deterioration and trade credit under FPH.

8.3.1. Research gap and problem defining:

This research aims to explore the inventory management strategies for products that continuously deteriorate over time and are subjected to time-dependent demand, while also incorporating preservation techniques and emissions regulatory policies. Table 8.1

presents a comparison of the proposed study with other relevant work in the field of inventory management.

Table 8.1. Summarises the literature referenced above.

Research study	Time Demand	Deterioration	Preservation technology	Green technology	Carbon Emissions cost	Credit Time	Finite Planning horizon
(Toptal, Özlü, and Konur 2014)	×	×	×	×	✓	×	×
(Lin 2018)	×	×	×	×	✓	×	×
(Xi Chen et al. 2013)	×	×	×	×	✓	×	×
(Dye and Yang 2015)	×	✓	✓	×	×	×	×
(U. Mishra, Wu, and Sarkar 2020)	×	✓	✓	×	✓	✓	×
(U. Mishra et al. 2021)	×	✓	✓	×	✓	×	×
(Lu et al. 2020)	×	×	✓	×	✓	×	×
(Bardhan et al. 2019)	×	✓	✓	×	✓	×	×
Hovelaque et al. (2014)	×	×	×	✓	✓	×	×
(Tiwari, Daryanto, et al. 2018)	×	✓	×	✓	×	✓	×
(Mohanty et al. 2018)	×	✓	×	×	✓	×	×
(Datta 2017)	×	✓	×	✓	×	✓	×
Shi et al. (2020)	×	×	×	×	✓	✓	×
(C. Wu and Zhao 2014)	✓	×	×	×	×	✓	✓
(P. N. Singh et	×	✓	×	×	×	✓	✓

al. 2017)							
(V. Singh et al. 2019)	×	✓	×	×	×	×	✓
(N. K. Mishra and Ranu 2022)	×	✓	×	×	×	✓	✓
This research work	✓	✓	✓	✓	✓	✓	✓

8.4. Assumptions and Notations:

1. The model assumes no shortages or back-ordering, as customers can easily find substitute products for routine items.
2. Inventory replenishment is instantaneous.
3. The model considers a finite planning horizon with variable replenishment cycle lengths.
4. The model is designed for a single supplier and retailer.
5. Two cases are explored with varying trade-credit periods: (a) a credit period that exceeds the length of the replenishment cycle, and (b) a credit period that is shorter than the length of the replenishment cycle.
6. The model assumes zero lead time, meaning inventory is replenished immediately.
7. Demand is time-dependent, and a function of time t is given below $\mathrm{D(t)} = (\mathrm{a+bt})$, a>0, and also b>0 where a is the initial demand of the market, and b is time-dependent demand.
8. The equation for the increase in emissions is $E = \emptyset\,(1 - e^{-mG})$, where G represents the GRA charge of carbon emission per unit of time decreasing and Φ represents the carbon emission proportion following GRA investment (0 < Φ < 1). The parameter m, where (m>0), represents investment (carbon offset) sensitivity concerning carbon emission rates. Various methods of utilizing green technology exist, including renewable energy, green transportation technologies, and energy efficiency, that can be used to achieve carbon offset goals, particularly in the context of inventory control and management. The

notations used in the research study are mentioned independently after the equation.

9. The cost of carbon emissions comprises three components: replenishment, handling, and environmental deterioration. The formula used to calculate the cost is $P(\Psi) = (1 - e^{-\lambda\Psi})$, where λ represents the preservation investment cost and the rate of deterioration is determined by the effectiveness of the preservation approach investment, represented by the variable Ψ. The function P(Ψ) is continuous, twice differentiable, and concave, representing the retailer's expenditure related to greenhouse emissions. The first derivative of the function, $P'(\Psi) = \lambda e^{-\lambda\Psi}$, indicates that the retailer should invest, while the second derivative, $P''(\Psi) = -\lambda^2 e^{-\lambda\Psi}$ <0, shows that the function is concave. These concepts were explored in studies by (Mishra 2016), (Bardhan et al. 2019) and as well as (V. Singh et al. 2019)

8.5. Mathematical formulation and analysis of the model:

The retailer initiates an order to the supplier to replenish the stock before the depletion of the initial stock level. The supplier promptly fulfils the retailer's order, thereby excluding any instances of shortages or lost sales in this study. The alteration in stock levels during the $(i+1)^{th}$ cycle is expressed through the following differential equation:

$$\frac{(IL_{i+1}(t))}{dt} = -D(t) - (1 - P(\Psi))\theta \quad * IL_{i+1}(t) \tag{8.1}$$

were, $t_i \leq t \leq t_{i+1}$

$$IL_{i+1}(t) = e^{-\theta(1-P(\Psi))t} \int_{t}^{t_{i+1}} D(t)\, e^{\theta(1-P(\Psi))u} du \tag{8.2}$$

where Boundary conditions are given below

$IL_{i+1}(t_{i+1}) = 0$ and $IL_{i+1}(t_i) = OQ_{i+1}$

$$Q_{i+1} = IL_{i+1}(t_i) = e^{-\theta(1-P(\Psi))t_i} \int_{t_i}^{t_{i+1}} D(t, M)\, e^{\theta(1-P(\Psi))t} dt$$

$$Q_{i+1} = IL_{i+1}(t_i) = \int_{t_i}^{t_{i+1}} (a + bt) e^{\theta(1-P(\Psi))(t-t_i)} dt \tag{8.3}$$

The cost of replenishment of an Order is $n * O_r$ (8.4)

Purchasing cost: $\sum_{i=0}^{n-1} P_r * Q_{i+1}$

$$\sum_{i=0}^{n-1} P_r \int_{t_i}^{t_{i+1}} (a+bt)\, e^{\theta(1-P(\Psi))(t-t_i)} dt \quad (8.5)$$

Cost of Hold and Stock:

$$\sum_{i=0}^{n-1} h_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} D(t) e^{\theta(1-P(\Psi))(u-t)} du\ dt \quad (8.6)$$

$$\sum_{i=0}^{n-1} h_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu) e^{\theta(1-P(\Psi))(u-t)} du\ dt$$

Cost of Deteriorating Inventory:

$$(1-P(\Psi)) \sum_{i=0}^{n-1} \theta\, d_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bt) e^{\theta(1-P(\Psi))(u-t)} du\ dt$$

$$e^{-\lambda\Psi} \sum_{i=0}^{n-1} \theta\, d_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bt) e^{\theta(1-P(\Psi))(u-t)} du\ dt \quad (8.7)$$

The implementation of green technology is crucial in achieving sustainability and reducing carbon emissions, as emphasized by the United Nations Environment Programme (UNEP). Carbon offsets serve as compensation for emissions rather than a replacement. The variable ĉ represents the fixed carbon emissions related to order placement, which includes transportation emissions. Meanwhile, the dynamic carbon emissions for each unit ordered are denoted by $\widehat{P_r}$, and the carbon emissions associated with refrigeration during warehousing are represented by $\widehat{h_r}$. These concepts have been explored in various studies, including (Benjaafar, Li, and Daskin 2013), (Xiang and Lawley 2019), (Shi et al. 2019), and (N. K. Mishra and Ranu 2022). Thus, the following equation calculates the total carbon emissions for each replenishment cycle:

Amount of Carbon emission during holding, placing an order and transportation:

$$Ce = \sum_{i=0}^{n-1} \hat{c} + \hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bt)e^{\theta(1- P(\Psi))(u-t)} du\ dt$$

The following presented for investment in the green approach.

$$(1 - \emptyset\,(1 - e^{-mG}))\ Ce$$

The per year carbon emission tax for a cycle is $CT = \tau\,(1 - \emptyset\,(1 - e^{-mG}))\ Ce$

$$= \tau\,(1 - \emptyset\,(1 - e^{-mG})) \sum_{i=0}^{n-1} \hat{c} + \hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bt)e^{\theta(1- P(\Psi))(u-t)} du\ dt \quad (8.8)$$

The usual practice in the market is that buyers purchase goods or materials from suppliers and pay for their purchases. However, many businesses offer a trade credit period, allowing the retailer to pay with a permissible delay. This arrangement benefits both the supplier and the retailer, as no losses are incurred. In some cases, suppliers may offer discounts to encourage early payment. The retailer is granted a credit period $M_{i+1} = \delta(t_{i+1})$ by the supplier. Generally, the greater the amount of the order, the longer the credit duration provided by the supplier.

8.5.1. First case:

We will consider two cases. In the first case, the credit period M_{i+1} lies within the cycle length (t_i, t_{i+1}). This occurs when the credit duration provided by the supplier is not greater than the inventory replenishment length T_{i+1}. In this situation, the retailer earns interest on their sales revenue for the duration of the credit period, while also incurring interest charges on the items they have already stocked. Interest charges $M_{i+1} \le T_{i+1}$ In this case $M_{i+1} = \delta(t_{i+1})$ lies into the interval $t_i \le t \le t_{i+1}$.

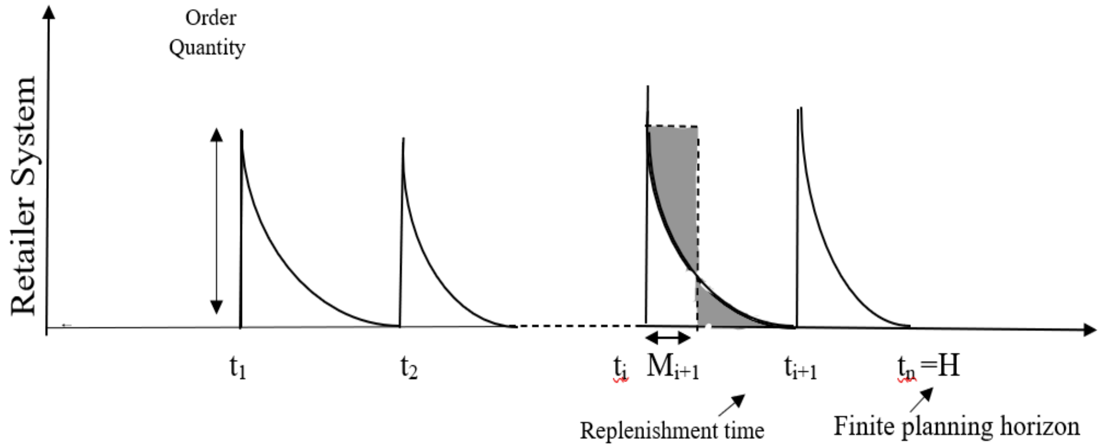

Figure 8.1. When credit period $M_{i+1} \leq T_{i+1}$

So, the interest earned by the retailer is as follows:

$$\sum_{i=0}^{n-1} I_e * s \int_{t_i}^{t_i+\delta(t_{i+1}-t_i)} (a + bt)[t_i + \delta(t_{i+1} - t_i) - t]dt \quad (8.9)$$

Interest payable by the retailer is given by:

$$\sum_{i=0}^{n-1} I_c * W \int_{t_i+(t_{i+1}-t_i)\delta}^{t_{i+1}} (a + bt)[t - t_i - (t_{i+1} - t_i)\delta\,]dt \quad (8.10)$$

Retailers are given below:

holding cost + Cost of placing an order + purchasing cost +carbon preservation cost +deterioration preservation technology cost + Interest charges- Interest Earned

$$T_{Ret} = \mathrm{n} * O_r + \sum_{i=0}^{n-1} h_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu)\, e^{\theta(1-P(\Psi))(u-t)} du\ dt + \sum_{i=0}^{n-1} P_r * Q_{i+1} + \tau(1 - \emptyset\,(1 - e^{-mG}))\ \sum_{i=0}^{n-1} \Big(\mathrm{c}^{\wedge} + \widehat{P_r} * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu) e^{\theta(1-P(\Psi))(u-t)} du\ dt\Big) + e^{-\lambda\Psi} \sum_{i=0}^{n-1} \theta\, d_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a +$$

$bu)e^{\theta(1-P(\Psi))(u-t)}du\ dt\ + \sum_{i=0}^{n-1} I_e * s \int_{t_i}^{t_i+(t_{i+1}-t_i)\delta} (a+bt)[t_i + (t_{i+1}-t_i)\delta -$

$t]dt - \sum_{i=0}^{n-1} I_c * W \int_{t_i+\delta(t_{i+1}-t_i)}^{t_{i+1}} (a+bt)[t - (t_{i+1}-t_i)\delta - t_i]dt$

$$
\begin{aligned}
T_{Ret} = \mathrm{n} * O_r + \sum_{i=0}^{n-1} h_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta\left(1-P(\Psi)\right)(u-t)}du\ dt \\
+ \tau\left(1-\emptyset(1-e^{-mG})\right) \sum_{i=0}^{n-1} \mathrm{c}\hat{} + \widehat{P_r} * Q_{i+1} \\
+ \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta\left(1-P(\Psi)\right)(u-t)}du\ dt + e^{-\lambda\Psi} \sum_{i=0}^{n-1} \theta\, d_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a \\
+ bu)e^{\theta\left(1-P(\Psi)\right)(u-t)}du\ dt \\
+ \sum_{i=0}^{n-1} s * I_e \int_{t_i}^{t_i+(t_{i+1}-t_i)\delta} (a+bt)\,[t_i + (t_{i+1}-t_i)\delta - t]dt \\
- \sum_{i=0}^{n-1} I_c * W \int_{t_i+(t_{i+1}-t_i)\delta}^{t_{i+1}} (a+bt)[t - t_i - (t_{i+1}-t_i)\delta\,]dt
\end{aligned}
$$

$$T_{Ret} = \mathrm{n} * O_r + \sum_{i=0}^{n-1} h_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu)e^{\theta(1- P(\Psi))(u-t)} du\ dt$$

$$+ \sum_{i=0}^{n-1} P_r \int_{t_i}^{t_{i+1}} (a + bt)\ e^{\theta(1- P(\Psi))(t- t_i)} dt$$

$$+ \tau\ (1 - \emptyset\ (1 - e^{-mG}))\ \sum_{i=0}^{n-1} \mathrm{c}\hat{} + \widehat{P}_r$$

$$* \int_{t_i}^{t_{i+1}} (a + bt)\ e^{\theta(1- P(\Psi))(t- t_i)} dt$$

$$+ \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu)e^{\theta(1- P(\Psi))(u-t)} du\ dt + e^{-\lambda\Psi} \sum_{i=0}^{n-1} \theta\ d_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a$$

$$+ bu)e^{\theta(1- P(\Psi))(u-t)} du\ dt$$

$$+ \sum_{i=0}^{n-1} s * I_e \int_{t_i}^{t_i+(\delta * t_{i+1} - \delta * t_i)} (a + bt)\ [t_i + (\delta * t_{i+1} - \delta * t_i) - t] dt$$

$$- \sum_{i=0}^{n-1} I_c$$

$$* W \int_{t_i+(\delta * t_{i+1} - \delta * t_i)}^{t_{i+1}} (a + bt)[t - t_i - (\delta * t_{i+1} - \delta * t_i)\ ] dt$$

$$T_{Ret} = \mathrm{n} * O_r + \sum_{i=0}^{n-1} \{h_r + \tau\,(1 - \emptyset\,(1 - e^{-mG}))\widehat{h_r} + \theta\, d_r\, e^{-\lambda\Psi}\} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + b * u)\, e^{\theta P(\Psi)-1)(t-u)} du\ dt + \sum_{i=0}^{n-1} \left(\{P_r + \hat{P}_r * \tau(1 - \emptyset\,(1 - e^{-mG}))\} \int_{t_i}^{t_{i+1}} (a + bt)\, e^{\theta(1-P(\Psi))(t-t_i)} dt \right) + \hat{c} * \tau\,(1 - \emptyset\,(1 - e^{-mG})) + \sum_{i=0}^{n-1} s * I_e \int_{t_i}^{t_i+(\delta * t_{i+1} - \delta * t_i)} (a + bt)[t_i + (\delta * t_{i+1} - \delta * t_i) - t] dt - \sum_{i=0}^{n-1} I_c * W \int_{t_i+(\delta * t_{i+1} - \delta * t_i)}^{t_{i+1}} (a + bt)\,[t - t_i - (\delta * t_{i+1} - \delta * t_i)\,] dt$$

$$T_{Ret} = \text{n} * O_r + \sum_{i=0}^{n} \left\{ \frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau(1- \emptyset (1 - e^{-mG}))\widehat{h_r}}{\theta(1- P(\Psi))} + d_r \, e^{-\lambda\Psi} \right\} \int_{t_i}^{t_{i+1}} (a + bt) e^{\theta(1- P(\Psi))(t- t_i)} dt$$

$$+ \sum_{i=0}^{n} \left\{ P_r + \hat{P}_r * \tau (1 - \emptyset (1 - e^{-mG})) \right\} \int_{t_i}^{t_{i+1}} (a + bt) \, e^{\theta(1- P(\Psi))(t- t_i)} dt + \text{c}\hat{} * \tau (1 - \emptyset (1 - e^{-mG}))$$

$$- \left\{ \frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau(1- \emptyset (1 - e^{-mG}))\widehat{h_r}}{\theta(1- P(\Psi))} + d_r \, e^{-\lambda\Psi} \right\} (a\, H + 0.5 * b * H^2)$$

$$+ \sum_{i=0}^{n-1} s * I_e \int_{t_i}^{t_i + (\delta * t_{i+1} - \delta * t_i)} (a + bt) \, [t_i + (\delta * t_{i+1} - \delta * t_i) - t] dt$$

$$- \sum_{i=0}^{n-1} I_c * W \int_{t_i + (\delta * t_{i+1} - \delta * t_i)}^{t_{i+1}} (a + bt) [t - t_i - (\delta * t_{i+1} - \delta * t_i)] dt$$

$$\Big[T_{Ret} = \text{n} * O_r + \sum_{i=0}^{n} \Big(\Big\{ \frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau(1- \emptyset (1 - e^{-mG}))\widehat{h_r}}{\theta(1- P(\Psi))} + d_r \, e^{-\lambda\Psi} \Big\} + \Big\{ P_r + \hat{P}_r * € (1 - \emptyset (1 - e^{-mG})) \Big\} \Big) \int_{t_i}^{t_{i+1}} (a + bt) \, e^{\theta(1- P(\Psi))(t- t_i)} dt + \text{c}\hat{} * \tau (1 - \emptyset (1 - e^{-mG})) - \Big\{ \frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau(1- \emptyset (1 - e^{-mG}))\widehat{h_r}}{\theta(1- P(\Psi))} + d_r \, e^{-\lambda\Psi} \Big\} (a\, H + 0.5 * b * H^2) + \sum_{i=0}^{n-1} s * I_e \int_{t_i}^{t_i + (\delta * t_{i+1} - \delta * t_i)} (a + bt) \, [t_i + (\delta * t_{i+1} - \delta * t_i) - t] dt - \sum_{i=0}^{n-1} I_c * W \int_{t_i + (\delta * t_{i+1} - \delta * t_i)}^{t_{i+1}} (a + bt) \, [t - t_i - (\delta * t_{i+1} - \delta * t_i)] dt \Big] \quad (8.11)$$

$$\frac{\partial(T_{Ret})}{\partial t_i} = \Bigg(\Bigg\{\frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau(1- \emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1- P(\Psi))} + d_r\, e^{-\lambda\Psi}\Bigg\} + \{P_r + \hat{P}_r * \tau\,(1- \emptyset\,(1-e^{-mG}))\}\Bigg)\Bigg((a + b\,t_i)\big(e^{\theta(1- P(\Psi))(t_i-t_{i-1})} - 1\big) - \theta(1 - P(\Psi))\int_{t_i}^{t_{i+1}}(a+bt)e^{\theta(1- P(\Psi))(t-t_i)}\,dt\Bigg) + s * I_e\int_{t_i}^{t_i+(\delta*t_{i+1}- \delta*t_i)}(a+bt)(1- \delta)\,dt - (a+b\,t_i)\,(\delta*t_{i+1} - \delta*t_i) + * W\int_{t_i+(\delta*t_{i+1}- \delta*t_i)}^{t_{i+1}}(a+bt)\,(1- \delta)dt$$

$$\frac{\partial(T_{Ret})}{\partial t_i} = \Bigg(\Bigg\{\frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau\big(1- \emptyset\,(1-e^{-mG})\big)\widehat{h_r}}{\theta(1- P(\Psi))} + d_r\, e^{-\lambda\Psi}\Bigg\} + \{P_r + \hat{P}_r * \tau\,(1- \emptyset\,(1- e^{-mG}))\}\Bigg)\Bigg((a + b\,t_i)\big(e^{\theta(t_i-t_{i-1})(1- P(\Psi))} - 1\big) + \theta(P(\Psi) - 1)\int_{t_i}^{t_{i+1}}(a + bt)e^{\theta(1- P(\Psi))(t-t_i)}\,dt\Bigg) + s * I_e\Bigg\{\int_{t_{i-1}}^{t_{i-1}+(\delta*t_{i+1}- \delta*t_i)}(a+bt)\ \delta\,dt + \int_{t_i}^{t_i+(\delta*t_{i+1}- \delta*t_i)}(a+bt)\,(1- \delta)\,dt - (a+b\,t_i)\,(\delta*t_{i+1} - \delta*t_i)\Bigg\} - I_c * W\Bigg\{\int_{t_{i-1}+(\delta*t_i- \delta*t_{i-1})}^{t_i}(a+bt)\,\delta\,dt + \int_{t_i+(\delta*t_{i+1}- \delta*t_i)}^{t_{i+1}}(a+bt)\,(1- \delta)dt - (a + bt_i)\,(1-\delta)[t_i - t_{i-1}]\Bigg\} \quad (8.12)$$

By taking a partial derivative of T_{Ret} w.r.t to t_i equal to zero. We can find the value of t_i.

$$\frac{\partial(T_{Ret})}{\partial t_i} = 0$$

8.5.2. Second case:

M_{i+1} lies outside the cycle length (t_i, t_{i+1}). when the credit duration is higher than the inventory replenishment lengthT_{i+1} .In the second case we assume, $M \geq T_{i+1}$, in this case M_{i+1} lies outside the interval.

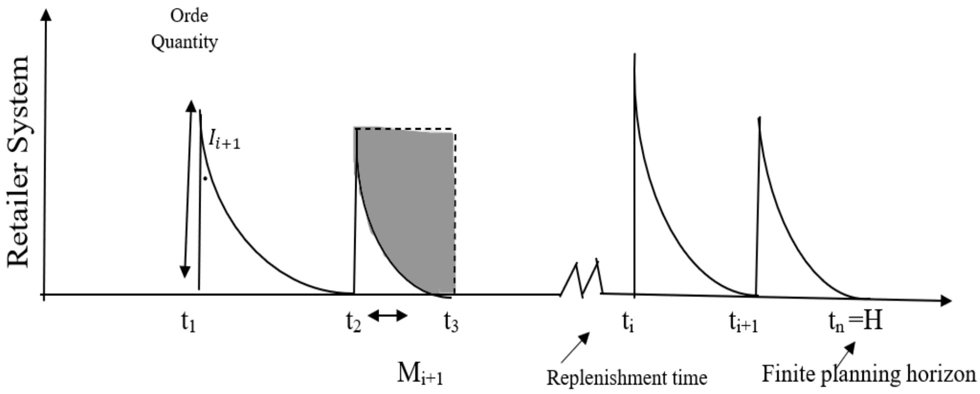

Figure 8.2. When the credit period is $M \geq T_{i+1}$

$t_i \leq t \leq t_{i+1}$. So, the interest earned by the retailer is as follows:

$$\sum_{i=0}^{n-1} I_e * s \int_{t_i}^{t_{i+1}} (a+bt)\,[t_i + \delta(t_{i+1} - t_i) - t]dt \quad (8.13)$$

Interest payable by the retailer is zero. Then:

$$T_{Ret} = \text{n} * O_r + \sum_{i=0}^{n} \left(\left\{ \frac{h_r}{\theta(1-P(\Psi))} + \frac{\tau(1-\emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))} + d_r\, e^{-\lambda\Psi} \right\} + \left\{ P_r + \hat{P}_r * \tau(1-\emptyset\,(1-e^{-mG})) \right\} \right) \int_{t_i}^{t_{i+1}} (a+bt)\; e^{\theta(1-P(\Psi))(t-t_i)} dt + \hat{c} * \tau(1-\emptyset\,(1-e^{-mG})) - \left\{ \frac{h_r}{\theta(1-P(\Psi))} + \frac{\tau(1-\emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))} + d_r\, e^{-\lambda\Psi} \right\} (a\,H + 0.5 * b * H^2) + \sum_{i=0}^{n-1} s * I_e \int_{t_i}^{t_{i+1}} (a + b * t)\,[t_i + (\delta * t_{i+1} - t_i * \delta) - t]dt \quad (8.14)$$

$$\frac{\partial(T_{Ret})}{\partial t_i} = \left(\left\{ \frac{h_r}{\theta(1-P(\Psi))} + \frac{\tau\,(1-\emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))} + d_r\, e^{-\lambda\Psi} \right\} + \left\{ P_r + \hat{P}_r * \tau\,(1-\emptyset\,(1-e^{-mG})) \right\} \right) \left((a + b\, t_i)\left(e^{\theta(1-P(\Psi))(t_i - t_{i-1})} - 1\right) - \theta(1-P(\Psi)) \int_{t_i}^{t_{i+1}} (a + bt) e^{\theta(1-P(\Psi))(t-t_i)}\, \text{d}t \right) + s * I_e \left\{ \int_{t_{i-1}}^{t_i} (a+bt)\;\delta\, dt + \int_{t_i}^{t_{i+1}} (a+bt)(1-\delta)\, dt + \right.$$

$(a + b\,t_i)\,(t_i - t_{i-1} - \delta * t_i - \delta * t_{i-1}) - (a + b\,t_i)\,(t_{i+1} - t_i - \delta * t_{i+1} + \delta * t_i)\Big\}$

(8.15)

By taking the partial derivative of T_{Ret} w.r.t to t_i equal to zero. We can find the value of t_i.

$$\frac{\partial(T_{Ret})}{\partial t_i} = 0$$

$$T_{Sup} = n^* * S_r + \sum_{i=0}^{n^*-1} C_p \delta(t_{i+1} - t_i) I_c * Q_i^* \quad (8.16)$$

$$Q_i = \sum_{i=0}^{n^*-1} Q_i^* \quad (8.17)$$

8.6. Algorithm and procedures for resolving the problem:

Step 1: To begin, let's establish initial values for all parameters with appropriate units.

a = 0.0001 unt., O_r = 20$/setup/year, b = 1000 units, M = 0.5year, h_r = 0.4 $/unt. /Annually, τ = 0.6$/kg/ annually, m = 0.5 unt., $\emptyset$ =0.4 unt., $\theta = 4$, $\alpha = 0.02$ unt., $\Psi =$ 0.5 , λ =0.8 unt., P_r =2$/unt. /Year, c^ = 10kg/year, $\hat{P}_r$ = 40 kg/order/ annually, $\widehat{h_r}$ =8 kg/ annually, G=, I_e = 0.08 $/unt. / Annually, I_c = 0.1 $/unt. /Year, s= 50 $/unt., S_s=120$/setup/annually.

Step 2: Find the Root $\left[\frac{\partial(T_{Ret})}{\partial t_i} = 0\right]$

Step 3: The final results are optimal solutions t_i.

Step 4: Insert optimal values of t_i into the equations (8.3), (8.11) and (8.14) to find the value of total cost and order quantity.

8.7. Numerical illustration for the proposed model:

Let us consider parametric values such as a = 0.0001 unt., O_r = 20\$/setup/year, b = 1000 units, M = 0.5year, h_r = 0.4 \$/unt. /Annually, τ = 0.6\$/kg/ annually, m = 0.5 unt., $\emptyset$ =0.4 unt., $\theta = 4$, $\alpha = 0.02$ unt., $\Psi = 0.5$, λ =0.8 unt., P_r =2\$/unt. /Year, $\hat{c}$ = 10kg/year, $\hat{P_r}$ = 40 kg/order/ annually, $\widehat{h_r}$ =8 kg/ annually, G=, I_e = 0.08 \$/unt. / Annually, I_c = 0.1 \$/unt. /Year, s= 50 \$/unt., S_s= 120\$/setup/ annually. Solve this numerical problem with the help of Mathematica software.

8.8. Tabular Form and Graphical Representation:

Table 8.2. Total cost for the retailer when $M_{i+1} \leq T_{i+1}$ for five different combinations of

↓ a \ → n	1	2	3	4	5	6	7	8
0.001	807408.5	212851.2	101532.77	65367.62	49743.41	74762.86	155208.69	516831.09
0.0008	807408.7	212851.2	101532.79	65367.62	55494.86	93719.13	216591.70	942996.74
0.0009	807408.6	212851.2	101532.78	65367.62	50832.96	83209.24	181501.34	675694.18
0.0011	807408.5	212851.1	101532.76	65367.61	49743.41	67850.95	134956.30	409364.16
0.0012	807408.42	212851.17	101532.76	65367.61	49743.41	62106.30	118989.62	333520.31

Table 8.3. Replenishment time for T_{Ret} , T_{sup}, and Q_{nt} when $M_{i+1} \leq T_{i+1}$

↓ a \ → t_i	t_0	t_1	t_2	t_3	t_4	t_5	n	T_{Ret}	T_{sup}	Tc	Q_{nt}
0.001	0	1.4259	2.0827	2.6329	3.1244	4	**5**	49743.41	23977.87	73721.28	18092.3
0.0008	0	1.4719	2.1499	2.7178	3.2251	4	5	55494.86	29425.50	84920.36	20168.3
0.0009	0	1.4350	2.0960	2.6496	3.1443	4	5	50832.96	25604.07	76437.03	18486.1
0.0011	0	1.4259	2.0827	2.6329	3.1244	4	5	49743.41	23085.97	72829.38	18092.3
0.0012	0	1.4259	2.0827	2.6329	3.1244	4	5	49743.41	22324.44	72067.85	18092.3

The optimal level for the overall cost incurred by the retailer is shown in Figure 8.3. At the 5th replenishment cycle, the overall cost incurred by the retailer is optimized and equals 49743.41. The optimal values for the overall cost incurred by the retailer corresponding to various of 'a' can be seen in Tables 8.2 and 8.3. For each value of 'a', the optimal value is achieved at the 5^{th} replenishment cycle. Table 8.3 provides information on the replenishment time, optimal values for the overall cost incurred by the retailer and supplier, and the order quantity.

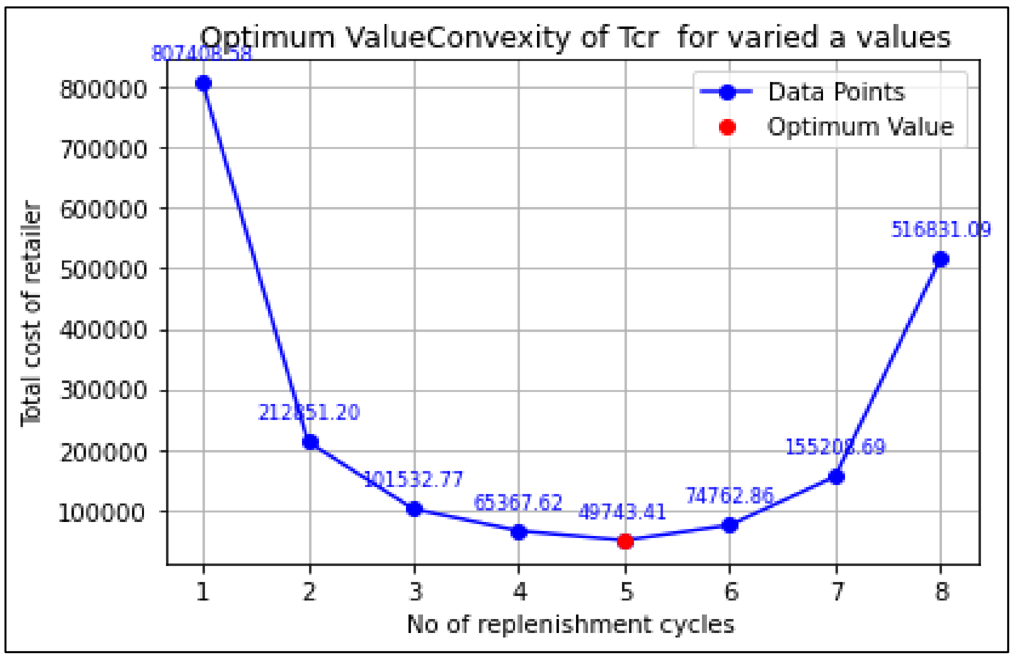

Figure 8.3. An optimal level of the total cost for the retailer

Table 8.4. Represent the following facts of sensitivity analysis for the main parameters.

Parameters	%Changes	Optimal Replenish Cycle	Total order Quantity Q_{nt}	The cost incurred by the retailer T_{Ret}	The cost incurred by the supplier T_{sup}
A	+20	5	18092.3	49743.41	22324.44
	+10	5	18092.3	49743.41	23977.87
	0	5	18092.3	49743.41	23085.97
	−10	5	20168.3	55494.86	29425.50
	−20	5	18486.1	50832.96	25604.07
B	+20	4	28206.8	78286.61	3218901.1
	+10	4	25856.2	71789.39	2950659.02
	0	4	23505.6	65292.17	2682416.88
	−10	4	21155.1	58794.94	2794658.59
	−20	4	18804.5	52297.7	2575774.22
θ	+20	5	24276.9	68347.01	4267545.71
	+10	5	20870.7	58057.29	3583507.45
	0	5	18092.2	49668.03	3033108.27
	−10	5	15812.2	42789.90	2588149.43
	−20	5	13929.1	37118.60	2226602.26
O_b	+20	5	211.42	859.78	1860.18
	+10	6	211.42	834.78	1860.18
	0	6	211.42	809.78	1860.18
	−10	5	211.42	781.85	1860.18
	−20	5	211.42	751.85	1860.18
S_s	+20	4	211.42	809.78	723.10
	+10	4	211.42	809.78	663.10
	0	4	211.42	809.78	603.10
	−10	4	211.42	809.78	543.10
	−20	4	211.42	809.78	483.10

Table 8.5. Total cost for the retailer when $M_{i+1} > T_{i+1}$ for five different values of 'a'

↓ a →n	1	2	3	4	5	6	7	8
0.001	762631.00	179074.16	74765.77	43266.14	**30942.01**	46862.71	102241.42	388303.80
0.0008	762631.16	179074.19	74765.79	43266.15	**34815.61**	60016.82	148908.53	744379.65
0.0009	762631.08	179074.18	74765.78	43266.15	**31670.30**	52660.24	121929.16	525295.82
0.0011	762630.92	179074.15	74765.76	43266.14	30942.01	42201.82	87445.92	298127.06
0.0012	762630.85	179074.13	74765.75	43266.14	30942.01	38389.75	76045.01	236187.31

Table 8.6. Replenishment time for T_{Ret} , T_{sup}, and Q_{nt} when $M_{i+1} > T_{i+1}$

↓ a →t_i	t_0	t_1	t_2	t_3	t_4	t_5	n	T_{Ret}	T_{sup}	Tc	Q_{nt}
0.001	0	1.4259	2.0827	2.6329	3.1244	4	**5**	**30942.01**	15428.31	46370.32	18092.29
0.0008	0	1.4719	2.1499	2.7178	3.2251	4	5	**34815.61**	18896.41	53722.02	20168.36
0.0009	0	1.4350	2.0960	2.6496	3.1443	4	5	**31670.30**	16463.58	48133.88	18486.1
0.0011	0	1.4259	2.0827	2.6329	3.1244	4	5	30942.01	14860.51	45802.52	18092.3
0.0012	0	1.4259	2.0827	2.6329	3.1244	4	5	30942.01	14375.68	45317.69	18092.3

As we can see, Figure 8.4 show the optimal level of the total cost of retailer for $M_{i+1} > T_{i+1}$.
At n=5 replenishment cycle, the total optimal value incurred by the retailer is 30942.01 similarly for various values of 'a' we can see in Table 8.5 and Table 8.6 the Optimal level of retailer total cost. For the different values of 'a' optimal value arrived at the 5th replenishment cycle. Table 8.6 Shows the replenishment time, optimal value of retailer, supplier and order quantity.

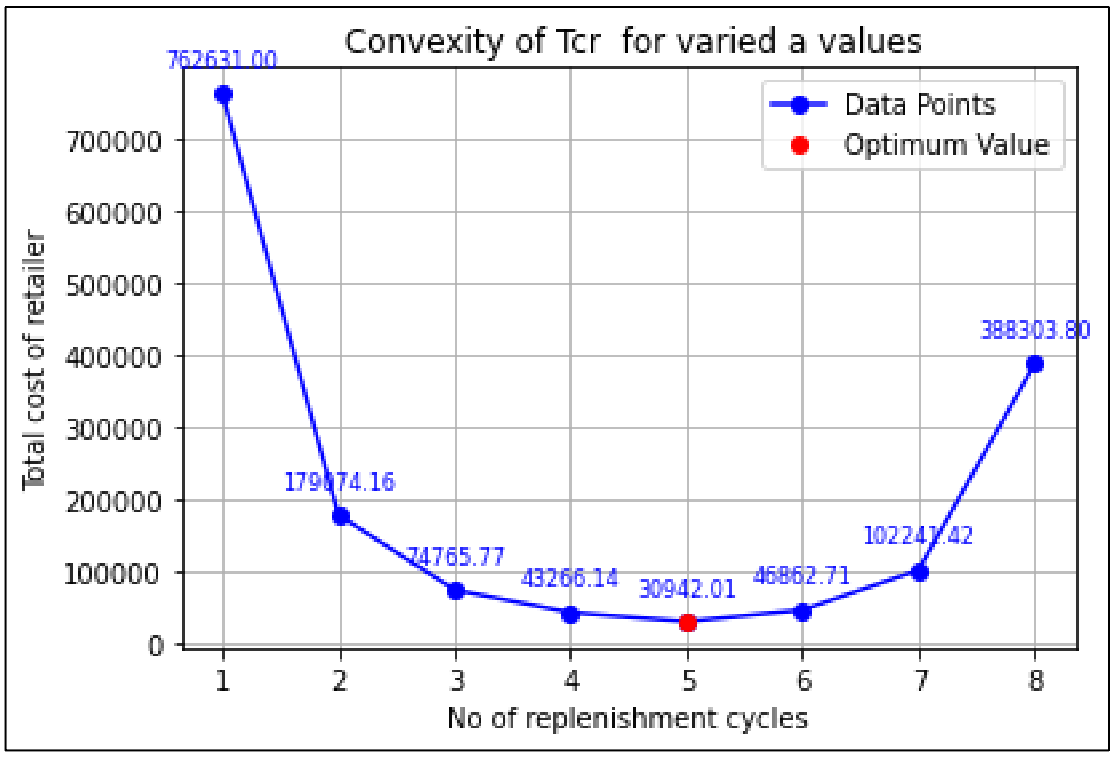

Figure 8.4. The optimal level of the cost incurred by the retailer, when $M_{i+1} > T_{i+1}$

Table 8.7. Represent the following facts of sensitivity analysis for the main parameters.

Parameters	% Changes	Optimal Replenish cycle	Total order Quantity Q_{nt}	Retailer's total cost T_{Ret}	Supplier's total cost T_{sup}
A	+20	6	246961.56	290818.70	37108.96
	+10	6	229788.07	271067.58	32219.83
	0	6	214125.64	253278.14	28003.36
	−10	6	199814.24	237205.04	24358.73
	−20	6	186711.34	222635.75	21201.25
B	+20	4	627510476	186533.33	723033.14
	+10	4	575278343	171006.99	662817.30
	0	4	523043328	155479.80	602686.98
	−10	4	470804689	1399515.29	603342.10
	−20	5	418864929	124512.83	542508.00
θ	+20	4	221117363	662621004	254786691720
	+10	4	107218244	320461137	123541769573
	0	4	52304332	155479805	60268698111
	−10	4	25694820	75525839	29607391885
	−20	4	12726308	36553824	14664173720
O_b	+20	6	214125.64	253374.14	14830218.5
	+10	6	214125.64	253278.14	14830218.5
	0	6	214125.64	253230.14	14830218.5
	−10	6	214125.64	253326.14	14830218.5
	−20	6	214125.64	253182.14	14830218.5
S_s	+20	6	214125.64	253278.14	28111.36
	+10	6	214125.64	253182.14	28057.36
	0	6	214125.64	253230.14	28003.36
	−10	6	214125.64	253326.14	27949.36
	−20	6	214125.64	253374.14	27895.36

Lemma 8.1:

t_i increase where i=1,2,3………… n-1 strictly monotonic increase function of last replenishment cycle t_n. $T_{i+1} = t_{i+1} - t_i$ and t_n = H-T_n

Theorem 8.1: The unique solution only exists for the non-linear system of Eqn (8.11) is the optimal replenishment period for a fixed replenishment cycle n.

The Hessian matrix of T_{Ret} must be positive definite for t_i to be minimum for a fixed n. Therefore, in **Appendix- 8. A1**, the theorem establishes that T_{Ret} is positive definite. As a result, the optimum value of t_i for a given fixed n +ve integer can be computed by using the numerical iterative technique and Mathematica programs version 12.0. Given the optimal of $t_{i,}$, the total cost function also will be optimal. **Or**

Theorem 8.2: If t_i satisfy inequations (i) $\frac{\partial^2 T_{Ret}}{\partial t_i^2} \geq 0$ (ii) $\frac{\partial^2 T_{Ret}}{\partial t_i^2} \geq |\frac{\partial^2 T_{Ret}}{\partial t_i t_{i-1}}| + |\frac{\partial^2 T_{Ret}}{\partial t_i t_{i+1}}|$ for all i= 1, 2, 3 ……….n_1 then $\nabla^2 T_{Ret}$ is positive definite.

8.9. Sensitivity findings and analysis:

The purpose of this study is to optimize sustainability in the inventory replenishment approach by examining how inventory system parameters are operated. Both retailers and suppliers can benefit from this study by gaining a better understanding of these parameters. Retailers can increase or reduce certain parameters to determine the minimum total cost and maximum profit. The study also aims to identify significant management implications. To examine the influence of all parameters, the study modifies one parameter individually while holding the remaining parameters constant. Tables 8.4 and 8.7 present some of the key parameter analyses.

1. Table 8.4 and Table 8.7 demonstrate that even a slight increase or decrease in the deterioration rate of the stock (represented by θ) has a direct impact on the retailer's total cost, without affecting the replenishment cycles. This implies that θ is positively related to the order quantity, as well as the total costs of the retailer and supplier. However, the replenishment cycles remain constant and are not affected by changes in θ.

2. Table 8.4 shows that an increase in the initial market demand parameter 'a' has little effect on the replenishment quantity, retailer's total cost, and supplier's total cost, as they are practically insensitive to changes in 'a'. However, if 'a' decreases, the order quantity becomes moderately reactive, affecting the retailer's and supplier's total cost but not the replenishment cycles per year. Therefore, we can conclude that the initial market demand parameter has a negligible effect on the order quantity, retailer, and supplier cost.

   On the other hand, parameter 'b' (demand that depends on time) is highly sensitive to the order quantity and the total cost of both the retailer and supplier. Table 8.7 provides more detailed information, showing that the replenishment quantity, retailer’s, and supplier's costs are highly sensitive to increases in both 'a' (initial market demand) and 'b' (demand that depends on time).

3. The analysis in Table 8.4 and Table 8.7 demonstrates that changes in the ordering cost have a moderate impact on the retailer's total cost function, but do not significantly affect the supplier's total cost or the ordering quantity. When the ordering cost increases or decreases, the retailer's total cost also increases or decreases during each replenishment cycle. This relationship between the ordering cost and the retailer's total cost is consistent.

4. According to the data presented in Tables 8.4 and 8.7, changes in setup costs have a moderate effect on the supplier's total cost function but do not have a significant impact on the retailer's total cost or the ordering quantity. When setup costs increase or decrease, the supplier's total cost also increases or decreases correspondingly during each replenishment cycle. This relationship between setup costs and the supplier's total cost remains consistent.

8.10. Comparison study:

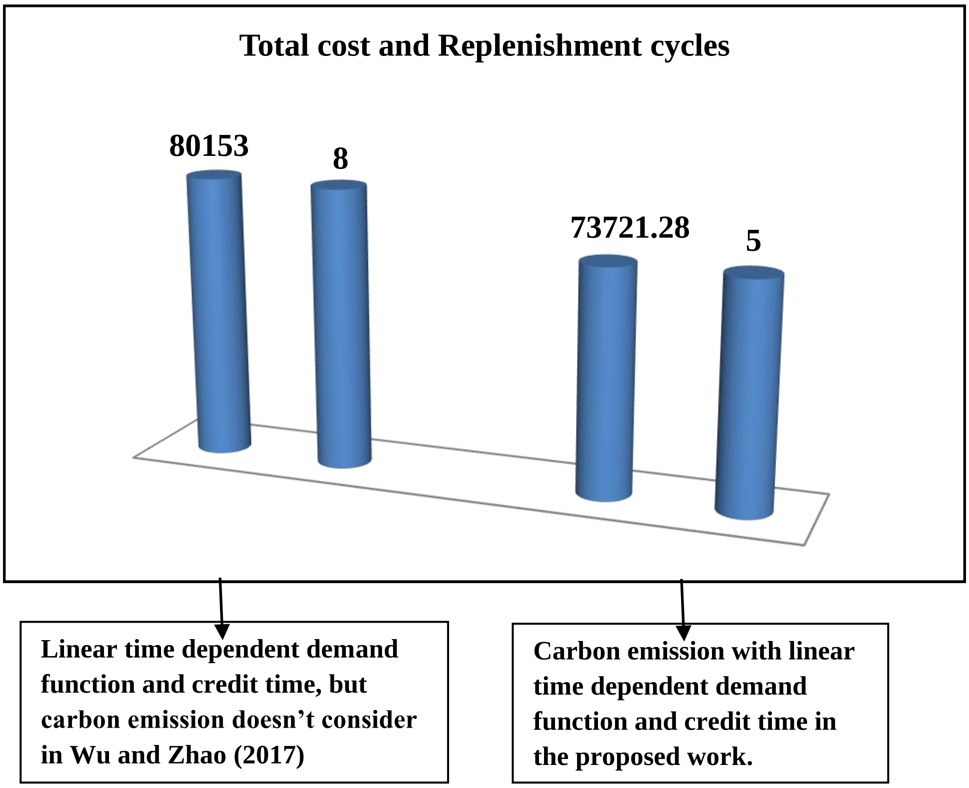

Figure 8.5 Comparison between existing literature and proposed model

8.11. Conclusion and future study of proposed work:

Retailers and suppliers have a common goal of reducing their overall costs while also minimizing the deterioration of materials and environmental impact. They may utilize trade credit policies to increase profitability. This study considers the impact of time and editing period on-demand rate. A specialized algorithm is developed to identify the optimal solution for the inventory problem in the supply chain. Further, sensitivity analysis is performed to evaluate the effects of various parameters on the solution.

The proposed research study is unique in comparison to other inventory models due to several distinctive aspects, including:

- The demand in this study is time-dependent and has a linear correlation with time.
- Investment in environmentally friendly/ green technology.
- A special duration of the trade credit is provided by the supplier to the retailer.
- Implementation of preservation technology to mitigate material deterioration.

- The use of a finite planning horizon.
- Analysis of two credit phases with different terms and conditions.

8.11.1 Future extension of proposed work:

This approach explored an extension of previous chapters by considering green and preservation technology. The proposed approach can also be expanded in future studies using other various methods. For example, incorporating different types of demand functions such as exponential and quadratic functions, time-varying demand, carbon, and inventory-dependent demand can be explored. Additionally, the approach can be modified to consider instantaneous material deterioration, shortages, and carbon regulations. Therefore, future research can evaluate the influence of these additional factors.

8.12. Recommendations for stakeholders:

The study's findings suggest that implementing continuous technology investments to reduce material deterioration and environmental pollution can have a significant positive impact on inventory management and control accuracy. To combat the negative effects of material deterioration and environmental pollution, both retailers and suppliers should consider investing in preservation technology.

The research study yielded the following outcomes:

- The optimal cost is obtained by the convex function set of replenishment time cycle solutions that provide a unique optimal solution.
- Emphasis on carbon control to find the optimal solution.

  Providing both economic and environmental information is essential for running a business sustainably and effectively.

Chapter 9: Analyze the carbon regulations that apply to a production system that integrates a process for reworking defective items within a finite planning horizon (FPH)

9.1. Abstract:

Manufacturing companies are increasingly recognizing the significance of implementing sustainable supply chain management (SSCM) and the need to address the issue of reworking defective items in their operations, both for their business goals and for the environment. To achieve sustainability goals, it is essential to design supply chain networks while taking ecological and environmental factors into account. This research work's focus is on developing an environmentally friendly supply chain system within the framework of the emission trading mechanism. A non-linear programming model is mathematically formulated to address the problem, to minimize both the total cost and carbon emissions. The approach is applied to analyse two distinct regulations: the cap and tax scheme under a finite planning horizon (FPH). Additionally, the study includes a sensitivity analysis, along with graphical or tabular representations, to enhance its effectiveness. The numerical results confirm that the solution obtained from the problem is optimal, and a reduction in carbon emissions is observed. To solve the problem, a programming model in the Mathematica software version -12.0 is employed.

9.2. Introduction:

Maintaining a reduction in carbon emissions while optimizing total costs is essential for supply chain operations. In reality, meeting customer demand is crucial for both systems and processes. With increasing awareness among people, minimizing carbon emissions has become imperative. This research aims to demonstrate the impact of reducing carbon dioxide emissions while simultaneously optimizing costs and order quantity. As stated by (Lovell 2010), emission policies were established under international climate change sovereignty. Businesses and individuals can offset their carbon dioxide emissions by participating in projects or initiatives aimed at reducing greenhouse gases worldwide. These initiatives could involve the implementation of clean or renewable energy technologies or direct carbon capture from the atmosphere, such as through reforestation or tree planting efforts. In 1997, the establishment of the Kyoto Protocol took place. The protocol introduced carbon offset regulations, which are referred to as project-based or Clean Development Mechanism (CDM) processes under the International Agreement. (Dye and Yang 2015) have provided further explanations on this topic. The emission of greenhouse gases from supply chain processes is a critical concern, as stated by (Zhou et al. 2020). In response to the critical concern of greenhouse gas emissions from supply chain processes, many governments or regulatory agencies have imposed carbon taxes on industries, individuals, or organizations that exceed a predetermined emission cap. These measures aim to encourage energy conservation and decarbonization, leading to significant financial impacts on the supply chain and subsequent management strategies. Consequently, several national governments have attempted to reduce carbon dioxide emissions through various carbon reduction strategies, including carbon offsets, carbon taxes, cap-and-trade systems, carbon caps, and eco-sustainable technological requirements. The reduction of carbon emissions and optimization of total costs are essential considerations for any supply chain operation. However, in the real world, both systems and processes are not fully realized without customer demand. As environmental awareness continues to grow, the need to reduce carbon emissions becomes more pressing. This study investigates the impact of reducing carbon dioxide emissions and optimizing costs and order quantities in supply chain operations. The development of emission policies was initiated under international climate change governance, as discussed by (Lovell 2010). This research focuses on analyzing an inventory supply chain problem under the carbon emission regulations mentioned earlier to enhance the understanding of this research area.

In reality, the assumption that reworked processes are completely perfect may not be valid due to several factors such as deterioration of items, machinery wear and tear, and low-quality raw materials. These imperfections in the production process result in the production of defective products that cannot be ignored. To ensure that the production process meets the required quality standards, defective products undergo a reworking process. However, it is assumed that after reworking, the quality of the defective products is the same as that of newly manufactured products. RMISCP must operate effectively to increase revenue and remain competitive in an increasingly globalized world. SMMR system refers to an inventory supply chain problem that considers multiple retailers and a single manufacturer/supplier. While several studies have examined inventory management problems in SMMR systems, none have yet explored the rework process under FPH with carbon emission regulations.

Additionally, the objective of the defective item rework process is to manage the inventory level during production and meet the demands of multiple retailers while minimizing the total cost of the system and complying with emission regulations. This research aims to examine the impact of carbon emission regulations on the inventory problem of a single manufacturer/supplier and multiple retailers (RMISCP) that includes the rework process under FPH.

The remaining parts/sections/portions of the research study are structured as follows. Section 9.3 provides a literature review. subsequent section 9.4. followed by a description of assumptions. Section 9.5 focuses on the model expressed in mathematical terms, including the derivation of a non-linear differential equation. Section 9.6 presents a suitable numerical example while section 9.7 offers sensitivity analysis. Finally, Section 9.8 concludes the chapter with Section 9.9 managerial observations and a list of possibilities for future extensions.

9.3. Literature review:

In recent years, there has been a greater focus on studying inventory control or management concerning carbon emissions. Governments have imposed limits on carbon emissions for individuals and companies, known as carbon caps. Several research studies have been conducted in the context of inventory models with emissions policies. (Wahab et al. 2011) were the pioneers in examining the impact of carbon emissions on the supply chain inventory system, creating a mathematical framework for identifying optimal replenishment/shipping cycles and manufacturing costs. (Benjaafar, Li, and Daskin 2013) introduced a model that amended traditional models to include strategic planning that considered both cost and environmental impact by incorporating carbon pollution variables into decision parameters. (Dye and Yang 2015) formulated a model that discussed carbon regulation with credit-dependent demand. (Hovelaque and Bironneau 2015) evaluated an economic quantity order model, where the deterministic demand rate of the product was determined by the carbon emissions and price for each year. (Micheli and Mantella 2018) constructed a model that considered emission policies such as carbon caps, taxes, and carbon offsets, and discussed their environmental and economic impacts. According to their research, carbon emissions produced by transportation contribute significantly to the carbon footprint. (Tao et al. 2019) investigated the influence of carbon regulatory policies and customers' awareness of low-carbon practices on optimized order quantity, overall costs, and CO2 emissions through the utilization of an EOQ model. (Ramudhin, Chaabane, and Paquet 2010) incorporated carbon emissions and the total cost of the supply chain into supply chain planning, solving them. (Cheng et al. 2017) examined the impact of carbon emission laws on the traditional stock transportation problem and implemented linearization methods and a hybrid genetic algorithm to build and run mixed integer nonlinear programming models within both infinite and finite planning horizons. Carbon emissions have become a topic of greater concentration. Therefore, carbon emissions are considered in this research work within a finite planning horizon (FPH).

The literature on product rework during production is vast, particularly regarding supply chain inventory control models. In this context, we will mention only a few studies that suggest new and different approaches to the rework process. (Benkherouf, Skouri, and Konstantaras 2016) investigated an inventory management system that included manufacturing, remanufacturing, and renovating activities over a finite planning horizon, assuming demand would vary over time, and that the buyer would return used goods, which would then be categorized as "manufacturable" or "renovative" after inspection. (Zouadi, Yalaoui, and Reghioui 2018) presented a manufacturing or remanufacturing approach that reduces emissions. The model incorporates emissions from production, remanufacturing, and transportation and examines their effects. (V. Singh et al. 2019) provided a comprehensive solution for re-manufacturing a product with shortages, Weibull deterioration, partial inventory backlog, and trade credit. Although there exists a vast body of literature on inventory control and management models, including those that incorporate a product rework process, very few research studies explore the implications of carbon emissions on the rework or remanufacturing process in production FPH. This study focuses on remanufacturing as a way to reduce environmental problems in the context of carbon tax policies. The RMISCP approach was also used to investigate the impact of emission reduction measures on inventory decisions. (Modak and Kelle 2019) developed a single-manufacturer and multiple-retailer inventory model with a carbon tax scheme. Further, (Samuel et al. 2020) investigated an inventory model with a supplier and several retailers and the effect of emissions on the inventory supply chain activities. The goal is to optimize the total profit, aiming for maximum returns by simultaneously evaluating the investment in recycling, item cost, purchasing quantity, and contribution quantity. The environmental and economic attributes of the supply chain are the most researched aspects throughout the literature review. However, only a few studies address remanufacturing, carbon cap and offset, and all the aforementioned components of supply chain sustainability with multiple retailers and a single manufacturer under a finite planning horizon (FPH).

9.3.1. Problem defining:

The present research study introduces a finite planning horizon lot sizing SMMR inventory control model that incorporates manufacturing, rework, or remanufacturing activities under carbon constraints. The main target of this carbon mechanism is to evaluate the production, rework, and repair scheme that reduces both producer and retailers' total costs, including production order costs, set-up costs, production costs, penalty costs, deteriorating costs, and emission costs. This is achieved through a nonlinear programming problem (NLPP) solution.

9.3.2. Research gap

In the literature review, the environmental and economic dimensions of the supply chain have received significant attention and have been extensively researched. Only a handful of research studies accommodate all the components of supply chain sustainability, including remanufacturing, carbon cap, and offset, within a finite planning horizon (FPH). However, very few researchers have examined the rework process with carbon emission policies for an SMMR inventory supply chain system over the finite planning horizon with unequal cycle lengths. Previous chapters didn't consider the rework process.

9.4. Assumptions:

The analysis was conducted based on the following assumptions.

1. It is assumed that a collection or set contains only one item or element in cases where a single-item collection is presumed.
2. The study has a well-defined planning horizon that is finite but not fixed.
3. The study assumes that the lead time is zero for the entire duration.
4. According to the framework, neither retailers nor producers are allowed to have shortages.
5. Production of stock is based on demand in this context.
6. This research involves a single manufacturer and multiple retailers.
7. The rate of deterioration of the inventory remains constant.
8. Shipments from the producer to the retailer happen at irregular intervals.

9.5. Mathematical formulation and analysis of the model:

In this section, a mathematical model for inventory management is presented for a single-manufacturer and multiple-retailers (SMMR) system that incorporates remanufacturing and carbon regulations. A carbon cap policy is considered in each period, which means that carbon dioxide emissions must not exceed the specified limit (cap) within a finite planning horizon (FPH). This limit may be imposed by an external regulatory agency or government to comply with specific emission limits. The model consists of three production phases, as shown in Figure.

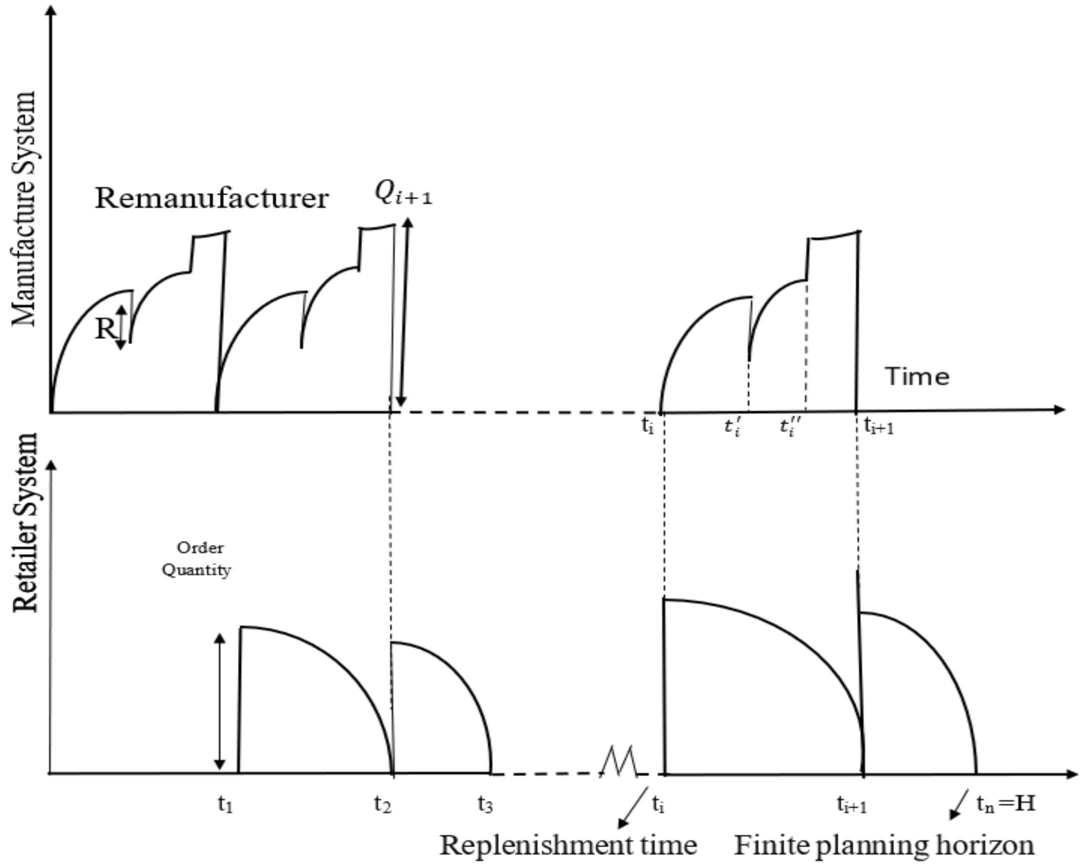

Figure 9.1 Production Level of Supplier and Replenishment to Retailers

9.5.1. Phase 1. $t_i < t < t_i'$

$$\frac{dI_{P_i}(t)}{dt} = \phi \sum_{j=1}^{5} D_j(t) - \theta * I_{P_i}(t) \quad (9.1)$$

$$\sum_{j=1}^{5} D_j(t) = D(t) = a + b*t$$

$$\frac{dI_{P_i}(t)}{dt} = \phi D(t) - \theta * I_{P_i}(t)$$

$$\frac{dI_{P_i}(t)}{dt} + \theta * I_{P_i}(t) = \phi (a + b * t) \quad (9.2)$$

$$t_i < t < t_i'$$

$$I_{P_i}(t) = e^{-\theta t} \phi \int_{t_i}^{t} (a + b * u) * e^{\theta * u} \quad (9.3)$$

$$I_{P_i}(t) = \phi \int_{t_i}^{t} (a + b * u) * e^{\theta(u-t)}\, du \qquad (9.4)$$

$$I_{P_i}(t_i') = I_{P_{(1,i)}} \quad \text{and} \quad I_{P_i}(t_i) = 0 \qquad (9.5)$$

$I_{P_{(1,i)}} = Inventory\ level\ at\ point\ 'A'\ is\ I_{P_i}(t_i') = \phi \int_{t_i}^{t_i'} (a + b * u) * e^{\theta(u-t_i')}\, du$

$$I_{P_{(1,i)}} = \phi \left\{ \frac{-b + a\theta + b\theta t_i' + e^{\theta(-t_i' + t_i)}(b - a\theta - b\theta t_i)}{\theta^2} \right\} \qquad (9.6)$$

9.5.2. Phase 2. for $t_i' < t < t_i''$

After Remanufacturing, the inventory level at point B is $= I_{P_i}(t_i') - R\%\ of\ I_{P_i}(t_i')$

$I_{P_{(2,i)}}$ = Inventory level at point 'c' is $I_{P_i}(t_i'') = I_{P_i}(t_i') - R\%\ of\ I_{P_i}(t_i') + \phi \int_{t_i'}^{t_i''} (a + b * u) * e^{\theta(u-t_i'')}\, du I_{P_{(2,i)}} = I_{P_i}(t_i') - R\%\ of\ I_{P_i}(t_i') + \phi \int_{t_i'}^{t_i''} (a + b * u) * e^{\theta(u-t_i'')}\, du$

$$I_{P_{(2,i)}} = I_{P_i}(t_i') - R\%\ of\ I_{P_i}(t_i') + \phi \int_{t_i'}^{t_i''} (a + b * u) * e^{\theta(u-t_i'')}\, du$$

$$I_{P_{(2,i)}} = \phi \int_{t_i}^{t_i'} (a + b * u) * e^{\theta(u-t_i')}\, du - R\%\ of \phi \int_{t_i}^{t_i'} (a + b * u) * e^{\theta(u-t_i')}\, du + \phi \int_{t_i'}^{t_i''} (a + b * u) * e^{\theta(u-t_i'')}\, du \qquad (9.7)$$

9.5.3. Phase 3. For $t_i'' < t < t_{i+1}$

Inventory level at the point D= $I_{P_i}(t_i'') + R\%\ of\ I_{P_i}(t_i')$

$$I_{P_i}(t_i') - R\%\ of\ I_{P_i}(t_i') + \phi \int_{t_i'}^{t_i''} (a + b * u) * e^{\theta(u-t_i'')}\, du + R\%\ of\ I_{P_i}(t_i')$$

$$\phi \int_{t_i}^{t_i'} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i')}\, \mathrm{du} - R\%\, of\, \phi \int_{t_i}^{t_i'} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i')}\, \mathrm{du}$$

$$+ \phi \int_{t_i'}^{t_i''} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i'')}\, \mathrm{du} + R\%\, of \phi \int_{t_i}^{t_i'} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i')}\, \mathrm{du}$$

$$\phi \int_{t_i}^{t_i'} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i')}\, \mathrm{du} + \phi \int_{t_i'}^{t_i''} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i'')} \qquad (9.8)$$

The inventory level at point 'E' is

$$I_{P_{(3,i)}} = \text{Inventory level at 'D'} + \phi \int_{t_i''}^{t_{i+1}} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t)}\, \mathrm{du}$$

$$I_{P_{(3,i)}} = \phi \int_{t_i}^{t_i'} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i')}\, \mathrm{du} + \phi \int_{t_i'}^{t_i''} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_i'')}\, \mathrm{du}$$

$$+ \phi \int_{t_i''}^{t_{i+1}} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t_{i+1})}\, \mathrm{du}$$

$$I_{P_i}(t_i < \mathrm{t} < t_{i+1}) = \Big(ab(-3 + e^{\theta(t_i'-t_i'')} + e^{\theta(-t_i'+t_i)} + e^{\theta(t_i''-t_{1+i})} - \theta t_i'(-1 + e^{\theta(t_i'-t_i'')}) - \theta t_i''(-1 + e^{\theta(t_i''-t_{1+i})}) - \theta t_i e^{\theta(t_i-t_i')} + \theta t_{1+i})\Big)/\theta^2 \qquad (9.9)$$

Total cost of production= setup +holding +penalty + purchasing +carbon emission cost + deterioration cost

$$\Big[T_{sup} = n * S_s + H_s \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} I_{p_i}(t)\, dt + c_p * R\%\, of\, I_{P_i}(\boldsymbol{t_i'}) + \mathrm{Ps} * \sum_{i=1}^{n} I_{P_i}(t_i < \mathrm{t} < t_{i+1}) + \sum_{i=0}^{m_1-1} D_p * \theta \int_{t_i}^{t_{i+1}} I_{p_i}(t)\, dt + \sum_{i=0}^{n-1} \tau \Big(Z - \Big(\hat{P}_s * I_{P_i}(\boldsymbol{t_i'}) + \widehat{H_s} \int_{t_i}^{t_{i+1}} \phi \int_{t_i}^{\mathrm{t}} (\mathrm{a} + \mathrm{b} * \mathrm{u}) * e^{\theta(u-t)}\, \mathrm{du} t\Big)\Big)\Big]$$

$$\left[T_{sup} = n * S_s + \sum_{i=1}^{n}\left((H_s + \theta D_p)\phi \int_{t_i}^{t_{i+1}} \int_{t_i}^{t} (a + b*u) * e^{\theta(u-t)}\, du\ dt + c_p * R\%\ of \phi \int_{t_i}^{t_i'} (a + b*u) * e^{\theta(u-t_i')}\, du + Ps * (ab(-3 + e^{\theta(t_i'-t_i'')} + e^{\theta(-t_i'+t_i)} + e^{\theta(t_i''-t_{1+i})} - \theta t_i'(-1 + e^{\theta(t_i'-t_i'')}) - \theta t_i''(-1 + e^{\theta(t_i''-t_{1+i})}) - \theta t_i e^{\theta(t_i-t_i')} + \theta t_{1+i}))/\theta^2\right)\right]$$

$$\sum_{i=0}^{n-1} \tau \left(Z - \left(\widehat{P_s} * \int_{t_i}^{t_i'} (a + b*u) * e^{\theta(u-t_i')}\, du + \widehat{H_s} \int_{t_i}^{t_{i+1}} \phi \int_{t_i}^{t} (a + b*u) * e^{\theta(u-t)}\, du\, dt\right)\right)$$

$$\left[T_{sup} = n * S_s + \sum_{i=1}^{n}\left((H_s + \theta D_p - \tau * \widehat{H_s})\phi \int_{t_i}^{t_{1+i}} \frac{-b+a\theta+b\theta t+e^{\theta(-t+t_i)}(b-a\theta-b\theta t_i)}{\theta^2} dt + \left(c_p * R\% - \widehat{P_s} * \tau\right) \phi \int_{t_i}^{t_i'} (a + b*u) * e^{\theta(u-t_i')}\, du + Ps * \left(\phi \int_{t_i}^{t_i'} (a + b*u) * e^{\theta(u-t_i')}\, du + \phi \int_{t_i'}^{t_i''} (a + b*u) * e^{\theta(u-t_i'')}\, du + \phi \int_{t_i''}^{t_{i+1}} (a + b*u) * e^{\theta(u-t_{i+1})}\, du\right)\right)\right]$$

$$T_{sup} = n * S_s + \sum_{i=1}^{n}\left((H_s + \theta D_p)\phi \int_{t_i}^{t_{1+i}} \frac{-b+a\theta+b\theta t+e^{\theta(-t+t_i)}(b-a\theta-b\theta t_i)}{\theta^2} dt + (c_p\ of R\% + Ps - \widehat{P_s} * \tau)\phi \int_{t_i}^{t_i'} (a + b*u) * e^{\theta(u-t_i')}\, du + Ps * \left(\phi \int_{t_i'}^{t_i''} (a + b*u) * e^{\theta(u-t_i'')}\, du + \phi \int_{t_i''}^{t_{i+1}} (a + b*u) * e^{\theta(u-t_{i+1})}\, du\right)\right) + \tau * Z$$

$$t_i' = 0.1 + t_i\ and\ t_i'' = 0.1 + t_i' = 0.2 + t_i$$

$$T_{sup} = n * S_s + \sum_{i=1}^{n}\Bigg((H_s + \theta D_p - \tau * \widehat{H_s})\phi \int_{t_i}^{t_{1+i}} \frac{-b+a\theta+b\theta t+e^{\theta(-t+t_i)}(b-a\theta-b\theta t_i)}{\theta^2}dt + (c_p\ of R\% + \mathrm{Ps} - \widehat{P_s} * \tau)\phi \int_{t_i}^{0.1+t_i}(\mathrm{a} + \mathrm{b}*\mathrm{u}) * e^{\theta(u-0.1-t_i)}\,\mathrm{du} + \mathrm{Ps} * \Big(\phi \int_{0.1+t_i}^{0.2+t_i}(\mathrm{a} + \mathrm{b}*\mathrm{u}) * e^{\theta(u-0.2-t_i)}\,\mathrm{du} + \phi \int_{0.2+t_i}^{t_{i+1}}(\mathrm{a} + \mathrm{b}*\mathrm{u}) * e^{\theta(u-t_{i+1})}\,\mathrm{du}\Big)\Bigg) + \tau * Z \qquad (9.10)$$

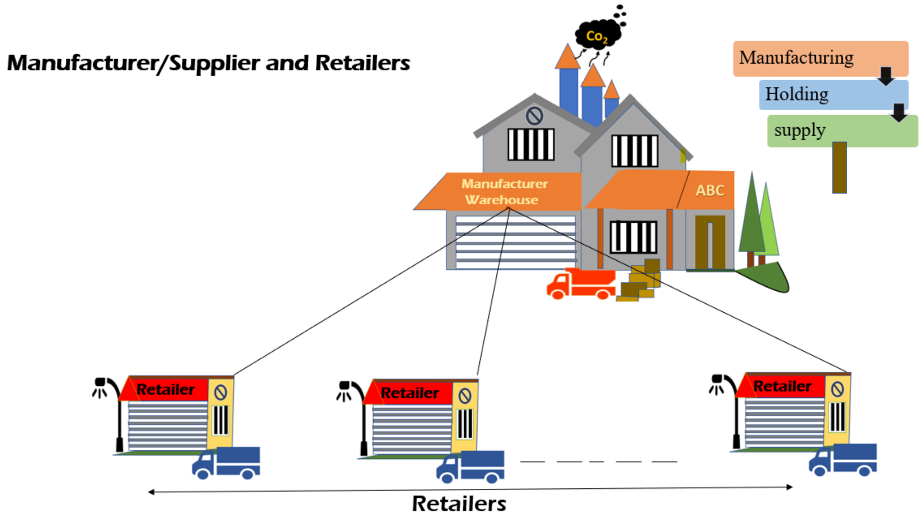

Figure 9.2 Graphical abstract for single supplier replenished to multiple retailers.

$$\frac{\partial T_{sup})}{\partial t_i} = (H_s + \theta D_p - \tau * \widehat{H_s})\phi\left\{\frac{-b+a\theta+b\theta t_1+e^{\theta(-t_1+t_{i-1})}(b-a\theta-b\theta t_{i-1})}{\theta^2} + \int_{t_i}^{t_{i+1}}\frac{e^{\theta(-t+t_i)}(-bt_i)+(b-a\theta-b\theta t_i)e^{\theta(-t+t_i)}}{\theta}dt\right\} + (c_p\ of R\% + \mathrm{Ps} - \widehat{P_s} * \tau)\phi * \left\{(-\theta)\int_{t_i}^{0.1+t_i}(\mathrm{a} + \mathrm{b}*\mathrm{u}) * e^{\theta(u-0.1-t_i)}\,\mathrm{du} + [\mathrm{a} + \mathrm{b}*(0.1+t_i)] - (\mathrm{a} + \mathrm{b}*t_i) * e^{-\theta(0.1)}\right\} + \mathrm{Ps} * \phi\left\{(-\theta)\int_{0.1+t_i}^{0.2+t_i}(\mathrm{a} + \mathrm{b}*\mathrm{u}) * e^{\theta(u-0.2-t_i)}\,\mathrm{du} + [\mathrm{a} + \mathrm{b}*(0.2+t_i)] - [\mathrm{a} + \mathrm{b}*(0.1+t_i)] * e^{-\theta(0.1)} - [\mathrm{a} + \mathrm{b}*(0.2+t_i)]e^{\theta(0.2+t_i-t_{i+1})}\right\}$$

(9.11)

It is an approach to running a business in which the Supplier/manufacturer supplies their items to end users through several different retailers.

Retailer's solution:

$$\frac{dI_{b_i}(t)}{dt} = -\sum_{j=1}^{5} D_j(\mathrm{t}) - \theta * I_{b_i}(t)$$

$$\frac{dI_{b_i}(t)}{dt} = -D(t) - \theta * I_{b_i}(t) \quad (9.12)$$

$$\sum_{j=1}^{5} D_j(\mathrm{t}) = \sum_{j=1}^{5} a_j + b_j t = a + bt$$

$$t_i < \mathrm{t} < t_{i+1}$$

$$I_{b_i}(t) = e^{-\theta t} \int_t^{t_{i+1}} (a + bu) e^{\theta * u}\, \mathrm{dt}$$

$$I_{b_i}(t_{i+1}) = 0 \quad \text{and} \quad I_{b_i}(t_i) = \int_{t_i}^{t_{i+1}} (a + bu) e^{\theta *(u-t)}\, \mathrm{dt}$$

$$I_{b_i}(t) = \int_t^{t_{i+1}} (a + bu) e^{\theta(u-t)}\, \mathrm{du}$$

$$I_{b_i}(t) = \left[\frac{(a + bu)e^{\theta(u-t)}}{\theta} - \frac{b}{\theta^2} e^{\theta(u-t)} \right]_t^{t_{i+1}}$$

$$I_{b_i}(t) = \frac{(a+bt_{i+1})e^{\theta(t_{i+1}-t)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t)} - \frac{(a+bt)}{\theta} + \frac{b}{\theta^2} \quad (9.13)$$

The order quantity for i^{th} *cycles*

$$Q_{i+1} = I_{b_i}(t_i) = \int_{t_i}^{t_{i+1}} (a + bt) e^{\theta(t-t_i)}\, \mathrm{dt}$$

$$Q_{i+1} = I_{b_i}(t_i) = \left[\frac{(a+bu)e^{\theta(u-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(u-t_i)} \right]_{t_i}^{t_{i+1}}$$

$$Q_{i+1} = I_{b_i}(t_i) = \frac{(a+bt_{i+1})e^{\theta(t_{i+1}-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{(a+bt_i)}{\theta} + \frac{b}{\theta^2} \quad (9.14)$$

$$O_r = \sum_{j=1}^{5} O_{rj}\,, H_r = \sum_{j=1}^{5} H_{rj}\,, D_r = \sum_{j=1}^{5} D_{rj}\,,\ C_e = \sum_{j=1}^{5} C_{ej}\,, F_c = \sum_{j=1}^{5} F_{cj}\,, d = \sum_{j=1}^{5} d_j\,, v_c = \sum_{j=1}^{5} v_{c\,j}\,, C_1 = \sum_{j=1}^{5} C_{1\,j}\,, C_2 = \sum_{j=1}^{5} C_{2\,j}\,, e_1 = \sum_{j=1}^{5} e_{1\,j}\,, e_2 = \sum_{j=1}^{5} e_{2\,j}$$

Ordering costs of the buyer$= n * O_r$

Holding cost = $H_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} I_{b_i}(t)\, dt$

$$H_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t_i}^{t_{i+1}} e^{-\theta t} \int_{t}^{t_{i+1}} (a_i + b_i t) e^{\theta * u} du\ dt \tag{9.15}$$

Deteriorating costs according to Sarkar et al.2012

$$D_r = \sum_{j=1}^{5} D_{rj} = D_{r1} + D_{r2} + D_{r3} + D_{r4} + D_{r5}$$

$$D_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \theta * I_{b_i}(t)\, dt$$

$$D_r * \theta \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \left\{ \int_{t}^{t_{i+1}} (a + bu) e^{\theta(u-t)}\, \mathrm{du} \right\} \mathrm{dt}$$

$$D_{c_b} * \theta \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu) e^{\theta(u-t)}\, \mathrm{du}\ \ dt \tag{9.16}$$

Amount of carbon emission due to transportation:

$$T_c = F_c + 2\mathrm{d} v_c C_1 + \mathrm{d} * v_c C_2 \int_{t_i}^{t_{i+1}} (\, a + \mathrm{b\,t}\,) e^{\theta(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{d}t + 2\mathrm{d}\, e_1 + \mathrm{d}\, e_2 \int_{t_i}^{t_{i+1}} (\, a + \mathrm{b\,t}\,) e^{\theta(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{d}t \tag{9.17}$$

Amount of Carbon emission according to (Shi et al. 2019) and (N. K. Mishra and Ranu 2022). Which include, the first T_c amount of carbon emission due to transportation and other factors) associated with replenishment order, second $\widehat{P_r}$ associated with inventory replenished and third $\widehat{h_r}$ associated with handling of inventory (refrigeration effect).

$$C_e = \sum_{i=0}^{n-1} \hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} du \ dt + F_c + 2\mathrm{d}v_c C_1 + \mathrm{d} * v_c C_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t + 2\mathrm{d}\, e_1 + \mathrm{d}\, e_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t$$

Cost of carbon emission:

$$\sum_{i=0}^{n-1} \tau \left(C - \left(\hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} du \ dt + F_c + 2\mathrm{d}v_c C_1 + \mathrm{d} * v_c C_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t + 2\mathrm{d}\, e_1 + \mathrm{d}\, e_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t \right) \right)$$

(9.18)

Retailer’s overall cost

Total cost =Ordering cost + Inventory holding cost + Deteriorating cost +Carbon emission cost + transportation cost

$$T_{Ret} = n * O_r + H_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} I_{b_i}(t)\, dt + D_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \theta * I_{b_i}(t)\, dt + \sum_{i=0}^{n-1} \tau \left(C - \left(\hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} du \ dt + F_c + 2\mathrm{d}v_c C_1 + \mathrm{d} * v_c C_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t + 2\mathrm{d}\, e_1 + \mathrm{d}\, e_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t \right) \right) \quad (\mathbf{9}.19)$$

$$T_{Ret} = n * O_r + H_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} \,\mathrm{du}\, dt + \theta D_r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} \,\mathrm{du}\, dt + \sum_{i=0}^{n-1} \tau \left(C - \left(\hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} du \ dt + F_c + 2\mathrm{d}v_c C_1 + \mathrm{d} * v_c C_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t + 2\mathrm{d}\, e_1 + \mathrm{d}\, e_2 \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t \right) \right)$$

$$T_{Ret} = n * O_r + \left\{ H_r + \theta * D_r + \widehat{h_r} \right\} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} \,\mathrm{du}\, dt + \left\{ \mathrm{d}\tau\, e_2 - \hat{P}_r * \tau - \mathrm{d} * v_c C_2 \tau \right\} \int_{t_i}^{t_{i+1}} (a + \mathrm{b\,t}) e^{\theta(\mathrm{t}-\mathrm{t}_i)} \,\mathrm{d}t + \tau\, C - \tau F_c - 2\tau \mathrm{d} v_c C_1 - 2\tau \mathrm{d}\, e_1 \quad (9.20)$$

$$T_{Ret} = n * O_r + \left\{\frac{H_r + \theta * D_r + \widehat{h_r}}{\theta} + \mathrm{d}\tau\, e_2 - \widehat{P_r} * \tau - \mathrm{d} * v_c C_2 \tau\right\} \int_{t_i}^{t_{1+i}} ((a + \mathrm{bt}) e^{\theta(t - t_i)}$$
$$+ \tau\, C - \tau F_c - 2\tau \mathrm{d} v_c C_1 - 2\tau \mathrm{d}\, e_1$$
$$- \left(\frac{H_r + \theta * D_r + \widehat{h_r}}{\theta}\right) (a * H + 0.5\, b * H^2)$$

Lemma 9. 1:

t_i increase where i=1,2,3………… n-1 strictly monotonic increase function of last replenishment cycle t_n. In this lemma, we can see a bonding among replenishment time, last replenishment time, length of replenishment time and time horizon. Where $T_{i+1} = t_{i+1} - t_i$ and $t_n =$ H-T_n.

Theorem 9.1: The nonlinear system of Eqn (9.11) possesses a unique solution that is the optimal replenishment period for a fixed replenishment cycle n. The Hessian matrix of TC_s must be positive definite for ti to be the minimum for a fixed n. Therefore, the theorem states that TC_s is positive definite. As a result, the optimal value of ti for a given fixed non-negative integer n can be computed using numerical iterative techniques and Mathematica version 12.0 programs. Based on the optimal value of t_i, the total cost function will also be optimal.
Proof: See Appendix:

Theorem 9.2:

T_{sup} (n, t_0, t_1………, tn) is a convex function for no replenishment cycles in a finite horizon planning H.

9.6. Methodology for solving problems:

In this subsection, we introduce an approach for determining the option solution with the minimum retailer cost. Looking at the preceding section's challenge, the minimal total cost of the retailer occurs at the point t_i simultaneously, satisfied the $\frac{\partial (T_{sup})}{\partial t_i} = 0.$ We derive the following equations by the first partial derivative of TC_s w.t.r to t_i, respectively.

$$\frac{\partial(T_{sup})}{\partial t_i} = (H_s + \theta D_p - \tau \ * \widehat{H_s})\phi\left\{\frac{-b+a\theta+b\theta t_1+e^{\theta(-t_1+t_{i-1})}(b-a\theta-b\theta t_{i-1})}{\theta^2} + \int_{t_i}^{t_{i+1}} \frac{e^{\theta(-t+t_i)}(-bt_i)+(b-a\theta-b\theta t_i)e^{\theta(-t+t_i)}}{\theta} dt\right\} + \ (c_p \ ofR\% \ + \text{Ps} - \ \widehat{P_s} * \tau)\phi * \left\{(-\theta)\int_{t_i}^{0.1+t_i} (\text{a} + \text{b} * \text{u}) * e^{\theta(u-0.1-t_i)} \,\text{du} + [\text{a} + \text{b} * (0.1 + t_i)] - (\text{a} + \text{b} * t_i) * e^{-\theta(0.1)}\right\} + \text{Ps} * \phi\left\{(-\theta)\int_{0.1+t_i}^{0.2+t_i} (\text{a} + \text{b} * \text{u}) * e^{\theta(u-0.2-t_i)} \,\text{du} + [\text{a} + \text{b} * (0.2 + t_i)] - [\text{a} + \text{b} * (0.1 + t_i)] * e^{-\theta(0.1)} - [\text{a} + \text{b} * (0.2 + t_i)]e^{\theta(0.2+t_i-t_{i+1})}\right\} == 0$$

To solve the defined problems, we construct an iterative approach that is based on the method developed by (C. Wu and Zhao 2014).

$$\frac{\partial^2 T_{sup})}{\partial {t_i}^2} = (c_p \ ofR\% \ + \text{Ps} - \ \widehat{P_s} * \tau)\phi\left(b - be^{-0.1\theta} - \frac{b(1.\theta-1.e^{-0.1\theta}\theta)}{\theta}\right) + \text{Ps} * \phi\left(b - be^{-0.1\theta} - be^{\theta(0.2+t_i-t_{i+1})} - \frac{b(1.\theta-1.e^{-0.1\theta}\theta)}{\theta} - e^{\theta(0.2+t_i-t_{i+1})}\theta(a + b(0.2 + t_i)\right) + (H_s + \theta D_p - \tau \ * \widehat{H_s})\phi\left(\frac{b\theta-e^{\theta(t_{i-1}-t_i)}\theta(b-a\theta-b\theta t_{i-1})}{\theta} - \frac{b(1-e^{\theta(t_i-t_{1+i})}+\theta-e^{\theta(t_i-t_{i+1})}\theta t_i+2a\theta^3 t_i-a\theta^3 t_{i+1})}{\theta}\right) \quad (9.\ A_1.1)$$

$$\frac{\partial^2 T_{sup}}{\partial t_i \, \partial t_{i-1}} = \frac{(H_s+\theta D_p-\tau * \widehat{H_s})*\phi*(-be^{\theta(t_{i-1}-t_i)}\theta+e^{\theta(t_{i-1}-t_i)}\theta(b-a\theta-b\theta t_{i-1}))}{\theta^2} \quad (9.\ A_1.2)$$

Similarly

$$\frac{\partial^2 T_{sup}}{\partial t_i \, \partial t_{i+1}} = \frac{b(H_s+\theta D_p-\tau * \widehat{H_s})*\phi(-\theta+(e^{\theta(t_i-t_{i+1})}\theta-a\theta^3)t_i)}{\theta^2} + e^{\theta(0.2+t_i-t_{i+1})}\text{Ps} * \phi * \theta(a + b(0.2 + t_i)) \quad (9.\ A_1.3)$$

$$\frac{\partial^2 T_{sup}}{\partial t_i \, \partial t_n} = 0 \quad (9.\ A_1.4)$$

for all n ≠ i, i+1, i-1

Furthermore,

$$\nabla^2 T_{sup} \quad =$$

$$\begin{bmatrix} \frac{\partial^2 T_{sup}}{\partial t_1^2} & \frac{\partial^2 T_{sup}}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 T_{sup}}{\partial t_2 \partial t_1} & \frac{\partial^2 T_{sup}}{\partial t_2^2} & \frac{\partial^2 T_{sup}}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 T_{sup}}{\partial t_3 \partial t_2} & \frac{\partial^2 T_{sup}}{\partial t_3^2} & \frac{\partial^2 T_{sup}}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 T_{sup}}{\partial t_{n_1-1} \partial t_{n_1-2}} & \frac{\partial^2 T_{sup}}{\partial t_{n_1-1}^2} & \frac{\partial^2 T_{sup}}{\partial t_{n_1-1} \partial t_{n_1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 T_{sup}}{\partial t_{n_1} \partial t_{n_1-1}} & \frac{\partial^2 T_{sup}}{\partial t_{n_1}^2} \end{bmatrix}$$

Figure 9.3 Hessian matrix

T_{sup} is positive definite if Eq. (9. A_1.1), (9. A_1.2), (9. A_1.3) and (9. A_1.4) satisfy the given inequality.

$$\frac{\partial^2 T_{sup}}{\partial t_i^2} \geq \left|\frac{\partial^2 T_{sup}}{\partial t_i t_{i-1}}\right| + \left|\frac{\partial^2 T_{sup}}{\partial t_i t_{i+1}}\right| \qquad \text{or}$$

$$\frac{\partial^2 T_{sup}}{\partial t_i^2} - \left|\frac{\partial^2 T_{sup}}{\partial t_i t_{i-1}}\right| - \left|\frac{\partial^2 T_{sup}}{\partial t_i t_{i+1}}\right| \geq 0$$

$$\begin{aligned}(H_s + \theta D_p - \tau & \\ & * \widehat{H_s})\phi\big(b\theta - e^{\theta(t_{i-1}-t_i)}(\theta - 1)(b - a\theta - b\theta t_{i-1}) \\ & - b(-e^{\theta(t_i - t_{i+1})}\theta t_i + 2a\theta^3 t_i - a\theta^3 t_{i+1}) - (-be^{\theta(t_{i-1}-t_i)}) \\ & - b(-a\theta^2)t_i)\big)/\theta \ - (c_p \ of R\% \ + \text{Ps} - \ \widehat{P_s} \\ & * \tau)\phi\left(b - be^{-0.1\theta} - \frac{b(1.\theta - 1.e^{-0.1\theta}\theta)}{\theta}\right) + \text{Ps} \\ & * \phi\left(b - be^{\theta(0.2 + t_i - t_{i+1})} - \frac{b(1.\theta)}{\theta}\right) > 0\end{aligned}$$

that is true for all $i = 1, 2, \ldots, n$

Moreover, the Hessian matrix had to be positive definite since it contains positive diagonal members and has strictly diagonal dominating features. As a result, the optimal replenishment interval to the nonlinear system of Equation (9.11) is obtained. now we need to show the non-linear equation (9.11) has a unique optimal solution and also $\mathrm{TC_s}(\mathrm{t_i}, \mathrm{n})$ is optimal function throughout the optimal value of t_i in a finite horizon planning H.

Furthermore, because it had strictly diagonal dominating characteristics and positive diagonal members, the Hessian matrix required to be positive definite. As a result, the optimum replenishment interval for nonlinear system Equation (9.11) is established. Now we need to demonstrate the convexity of $\mathrm{TC_s}(\mathrm{t_i}, \mathrm{n})$ throughout the optimal value of ti in the finite horizon planning Horizon.

9.7. Numerical illustration for the proposed model:

Fundamental values of all parameters with their appropriate units are listed here. $O_b =$ 80 \$/order, H_r=4 \$/unit/time, $\Theta = 3, H_s =, Ps = 3$, a= 50, 60, 70 , b=15, $D_p = 0.04, \widehat{h_r} =$ 0.2, $D_r = 0.04$, $Ss = 100$,c^ = 4, $\hat{P_r} = 0.02, \widehat{P_s} = 0.02, R =, \tau = 6$, $C_p = 0.01$, $\tau = 1.3$, $\mathrm{v_c} = 8$, $\mathrm{e_2} = 2.31 \times 10^{-6}$, $\mathrm{e_1} = 0.043$, $\mathrm{C_1} = 25$, $\mathrm{C_2} = 0.36$, d=25, $\mathrm{F_c} = 0.01$, C=, $\phi = \mathbf{0.02}$ To find the values of t_i, the cost function, solved the non-linear solution is solved by an iterative numerical approach in "Mathematica" mathematical problem-solving software. Table 9.1, Table 9.2, Figure 9.2 and Figure 9.3 Give comprehensive information about the optimal supplier/manufacturer's overall cost for a= 0.5, 0.125, and 0.625 are \$ 299171.9, \$299146.0, and \$ 299182.3 are reach its optimal level at 2, 2, and 3 optimal replenishment cycles respectively. After achieving its minimum at n=2, 2, and 3 then again goes upward gradually for all upcoming cycles. Table 9.1, and Table 9.2 reveal the convexity behaviour of supplier/manufacturer's overall cost function. This is also informed with the help of a graphical explanation.

Table 9.1 Optimal Planning Time for Replenishment

↓ a → t_i	t_0	t_1	t_2	t_3	n	Q_{nt}	Tc_s	Tc_r
0.5	0	0.34754	4.		2	12.8670	235.542	299171.9
0.125	0	0.34769	4.		2	11.0627	227.561	299146.0
	0	0.26658	2.38275	4.	3	12.1633	234.438	299182.3

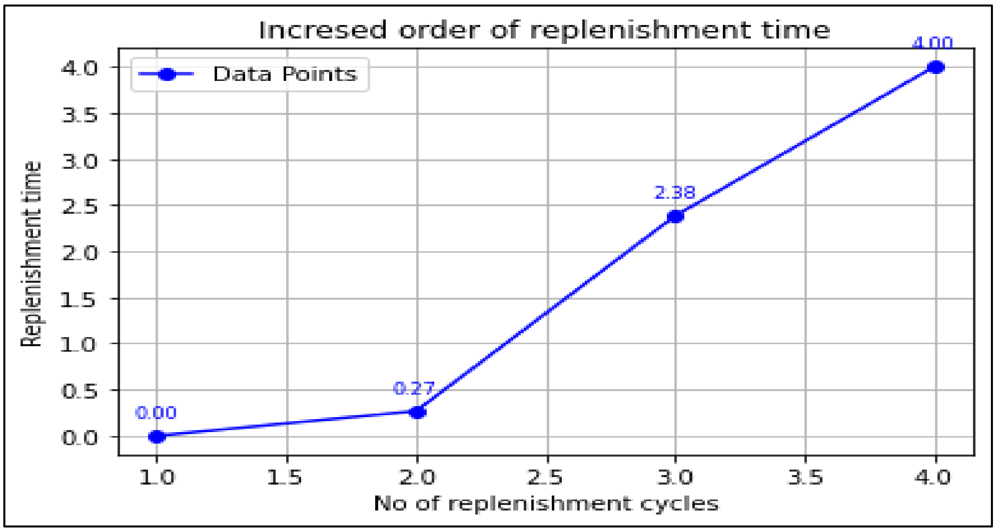

Figure 9.4. Demonstrates the increasing duration for replenishment orders to be placed.

Table 9.2. Revel the optimal value of the Tc_r, Tc_s *and* Q_{nt} .

↓ a → m_1	1	2	3	4	5	6	7
0.5	482240.9	299171.9	299374.0	299519.7	299615.8	299669.6	299701.7
0.125	386245.8	299146.0	299356.0	299508.6	299610.4	299668.7	299698.5
0.625	439306.7	299295.5	299182.3	299523.3	299617.5	299669.9	299702.9

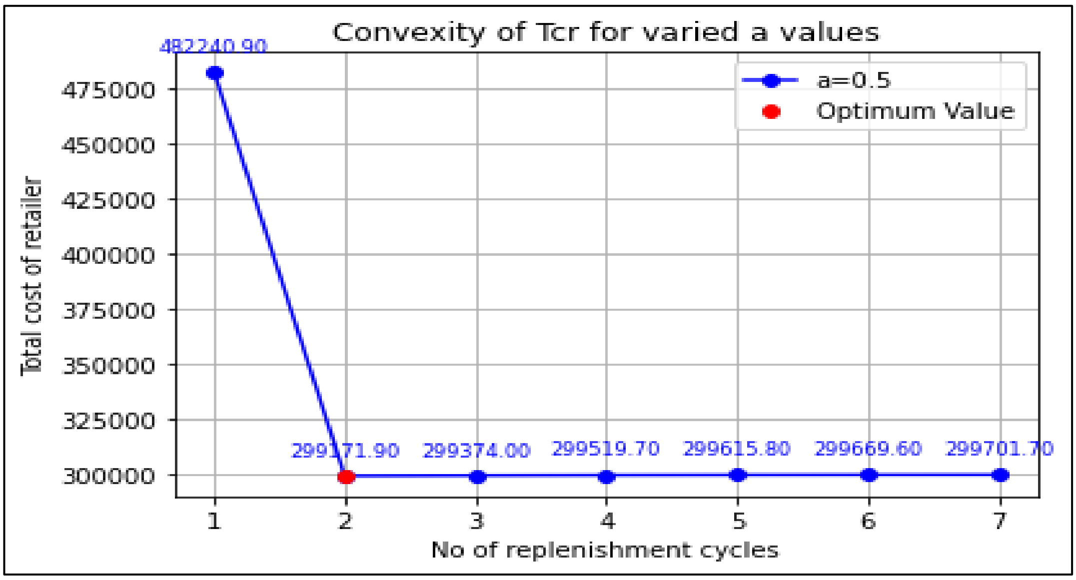

Figure 9.5. Demonstrates the supplier cost level that is considered ideal for the second replenishment cycle.

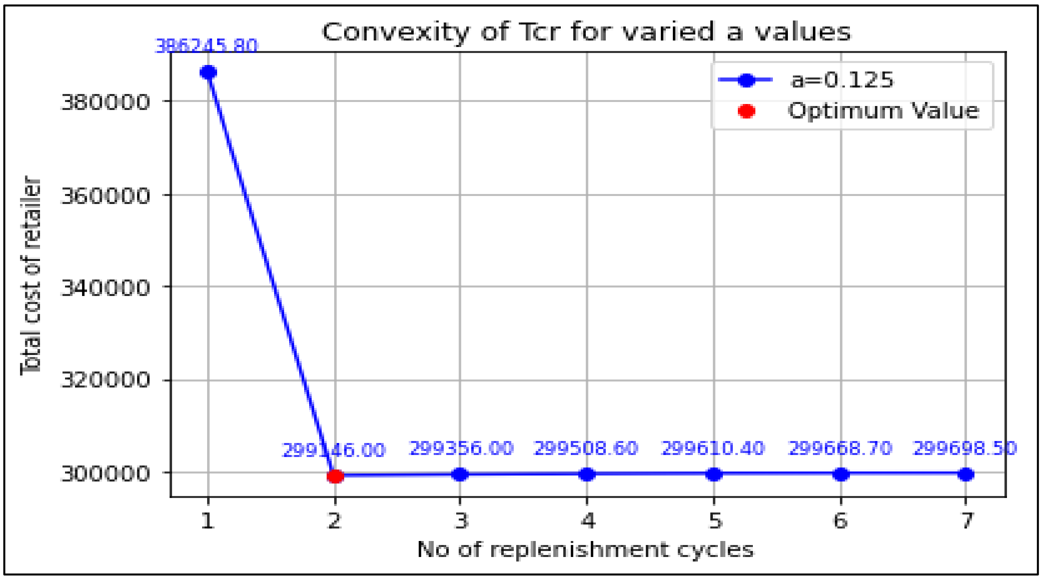

Figure 9.6 displays the ideal level of supplier cost during the second replenishment cycle.

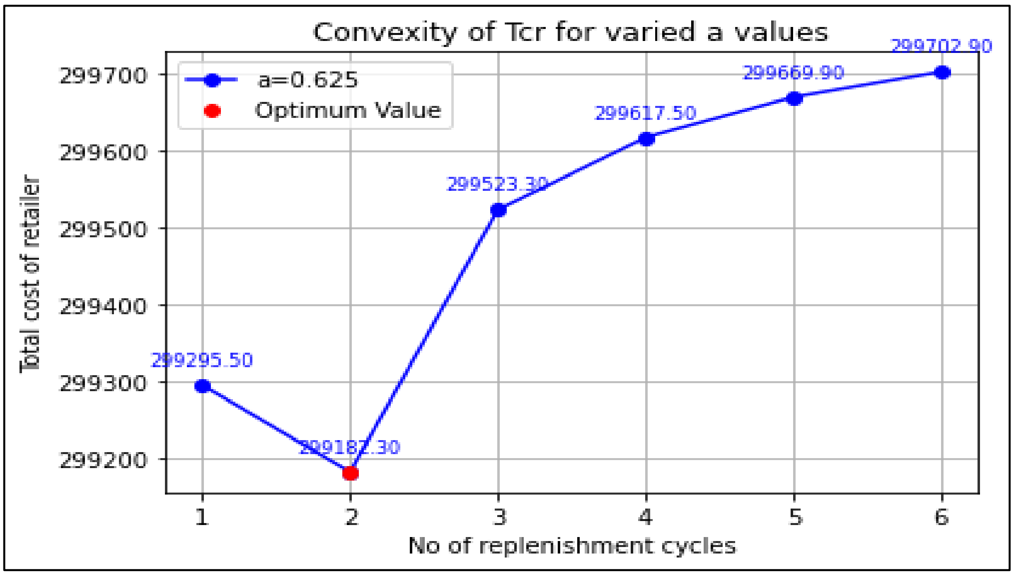

Figure 9.7 shows the supplier cost level that is considered optimal for the third replenishment cycle.

Table 9.3 Identify the following results of the detailed sensitivity analysis

Parameters	%Changes	Optimal Replenis h cycle	Total order Quantity Q_{nt}	The total cost of the Retailer Tc_r	Total cost of supplier Tc_s
a	−50	2	11.0627	238.438	299154.8
	−25	2	11.0627	227.563	299146.0
	0	2	12.8670	235.542	299171.9
	+25	2	12.1533	234.438	299180.3
	+50	2	11.4236	233.289	299188.6
b	−50	2	16.5956	241.454	704181.5
	−25	2	11.4701	239.055	731207658.4
	0	2	12.8670	235.542	142299.34
	+25	2	13.9324	231.081	2057841.7
	+50	2	14.6733	225.692	3279.5
O_r	−50	2	12.8729	115.559	139382.69
	−25	2	12.8729	155.559	139382.69
	0	2	12.8729	235.559	139382.69
	+25	2	12.8729	295.559	139382.69
	+50	2	12.8729	355.559	139382.69
Ss	−50	2	12.8729	235.559	142274.3
	−25	2	12.8729	235.559	142261.8
	0	2	12.8729	235.559	142299.3
	+25	2	12.8729	235.559	142311.8
	+50	2	12.8729	235.559	142324.3

Ps	−50 −25 0 +25 +50	2 2 2 2 2	6.5223 9.4323 12.8729 14.5015 15.5373	225.711 225.711 235.559 240.221 243.186	3325482.3 3773490.4 142299.3 267132.5 19467.1
R	−50 −25 0 +25 +50	2 2 2 2 2	12.8560 12.8588 12.8729 12.8869 12.9007	235.511 235.519 235.559 235.599 235.639	2298744.9 92515.8 142299.3 93256.9 216987.4
τ	−50 −25 0 +25 +50	2 2 2 2 2	4.8525 9.3831 12.8729 12.4557 12.4557	241.543 225.570 235.559 234.365 234.365	81420066113.8 120296869840.5 142299.3 18203511.0 18203511.0
ϕ	−50 −25 0 +25 +50	2 2 2 2 2	25.9945 25.9945 25.9945 25.9945 25.9945	235.559 235.559 235.559 235.559 235.559	1987220.9 993620.9 142299.3 4968020.7 5961620.7
D_p *and* D_r	−50 −25 0 +25 +50	2 2 2 2 2	19.9083 17.4554 25.9945 29.6750 33.7843	229.403 226.769 235.559 239.086 242.906	172261836.5 199438281.3 142299.3 90732502.2 63556057.4
H_s	−50 −25 0 +25 +50	2 2 2 2 2	11.2515 12.0282 25.9945 13.7306 14.6546	230.918 233.142 235.559 238.014 240.659	12227015.9 2902677.4 142299.3 1884162.5 3477456.4
H_r	−50 −25 0 +25 +50	2 2 2 2 2	12.8670 12.8670 12.8670 12.8670 12.8670	225.532 229.287 235.559 236.542 238.423	142299.4 142299.4 142299.4 142299.4 142299.4
$\widehat{h_r}$	−50 −25 0 +25 +50	2 2 2 2 2	12.8670 12.8670 12.8670 12.8670 12.8670	225.032 230.287 235.559 240.798 265.790	142299.4 142299.4 142299.4 142299.4 142299.4

9.8. Sensitivity analysis and findings:

In this paragraph, we provide information about the sensitivity analysis to explore the performances of the proposed framework when a set of different parameters is altered. At a particular time, only one parameter alters while all remain constant. The most important parametric value of the $O_r, H_r, \Theta, H_s, Ps$, a, b, D_p , D_r, Ss,cˆ,$\widehat{P}_r, \widehat{P}_s, R, \tau$, τ , and ϕare altered one by one. We are focusing on analysing the fluctuation in all parameters on order quantity, total retailer cost, and supplier cost. The evaluation is based on the numerical example and prepared algorithm described earlier. Table 3 performs an evaluation and pictorial representation of the Replenishment order quantity, retailer and supplier cost parameters are changed from -50% to +50%, and also fluctuations in replenishment quantity, and expected total cost of retailer and supplier are observed.

The main motive of this research analysis is to identify significant management applications and

It is important to assess the applicability of the model's solutions. This evolution is verified with the mathematical problem-solving software "MATHEMATICA" version-12 and the iterative numerical mathematical approach for non-linear differential equations. Table 10.3 reveals the comprehensive analysis in detail.

1. As **Table 9.3** identifies, the supplier's total cost TC_s is very sensitive to the parametric value of b, R, τ, H_s, D_p, P_s, ϕ and D_s. moderately sensitive to S_s , a. and practically insensitive to H_r, $\widehat{h_r}$ and O_r

2. **Table** 9.3 gives an idea of a detailed analysis of that retailer's total cost TC_r is very sensitive to the parameters O_r, moderately sensitive to a, b, R, τ, H_s, D_p, P_s, ϕ and $D_s$and practically insensitive to S_s.

3. The comprehensive and very clear study in **Table 9.3** shows that he orders quantity replenished by the supplier to the retailer's Q_{nt} is very sensitive to the parameters b, τ, H_s, D_p, P_s, ϕ and D_s, moderately sensitive to a, R and practically insensitive to O_r, ϕ, $\widehat{h_r}$, S_s and H_r .

4. No of replenishment cycles 'n' is shown in **Table 9.3** to be constant reactive for the parameters like a, b, τ, H_s, D_p, P_s, ϕ, H_r, D_s, τ, $\widehat{h_r}$, S_s and O_r,

9.9. Conclusion and future study of proposed work:

In this chapter, we begin by developing an inventory model under the assumption that the demand for a single item follows a linear function of time with unequal cycle lengths over a finite planning horizon (FPH). We also highlight collaborative supply chain management, where the framework is implemented to reduce total supply chain costs while controlling CO2 emissions. The primary aim of the model is to minimize costs for both retailers and suppliers, while concurrently ensuring efficient emission management to safeguard the environment. To illustrate the efficacy and characteristics of the proposed framework, we employ a numerical example and perform sensitivity analysis. Finally, we provide managerial recommendations to demonstrate the practical application of our proposed model.

In the future, we plan to extend this model to consider multi-item, inflation, shortage, and quadratic demand, among other factors, if necessary.

9.10. Recommendations for stakeholders:

Based on my understanding of the situation, my managerial suggestion would be to conduct a thorough analysis of the carbon regulations that apply to a production system that involves a rework process for defective products within a finite planning horizon (FPH). This would involve examining the potential environmental impact of the production process, identifying areas where improvements could be made to reduce carbon emissions, and developing strategies to ensure compliance with relevant regulations. It may also be useful to explore the use of more sustainable materials and production methods, as well as implement measures to reduce waste and increase energy efficiency. The ultimate objective is to establish an economically viable production system that is also environmentally responsible.

The availability of this research study is essential for optimizing real-time supply chain management, which can aid in inventory control and enhance operational capacity. The study also recommends industry collaboration to support firms in utilizing this information to reduce costs.

Chapter 10: Concluding Remarks and Future Scope

10.1. Concluding remarks:

During the creation of the "Supply Chain Inventory Model for Carbon Emission Policy under Finite Planning Horizon," we expect our model to aid in the implementation of supply chain management with diverse carbon emission policies under a limited planning horizon. The suggested models will have extensive applicability and prove valuable in managing inventory throughout the supply chain process while adhering to varying carbon emission policies.

This research study explored numerous practical implications for supply chain inventory management in the current business scenario. The study specifically focused on different carbon emission strategies in supply chain management with a limited planning horizon, which is more practical in today's business setting. Additionally, the study aimed to optimize total cost or profit, while addressing carbon-dependent demand and developing a cold supply chain strategy to enhance customer satisfaction and reduce deterioration. Moreover, preserving the environment and sustainability in the face of rework processes that increase carbon emissions is vital. This thesis work is arranged into ten chapters.

- Chapters 1 and 2 of the thesis/dissertation comprise the introduction, literature review, methodology, and discussion of the research gap.
- Chapter 3 presents a finite horizon model for studying supplier-retailer inventory coordination for deteriorating products whose demand is affected by both carbon emissions and price. This model is developed to address the challenge of coordinating inventory between retailers and suppliers for products that have a high carbon footprint. An algorithm is created to solve the optimization model, and numerical iterative approaches are used to tackle a numerical problem. The sensitivity analysis of all parameters is verified using Mathematica software version 12. A mathematical model is constructed following a specified methodology to determine the optimal cost and inventory level and identify the

coordinating effect on both the retailer and the supplier. This research study is inspired by the challenges faced by the industries such as textile, electric vehicles, sustainable packaging materials, organic clothing, biodegradable cleaning products and energy-efficient appliances.

- My next objective, as described in Chapter 4, is to present a supply chain inventory model that analyzes the coordinating effect on the total cost of the retailer and supplier for three payment options for deteriorating items, while also considering carbon emission and price-sensitive demand. This model is developed specifically for the retail-supplier business problem. To solve the optimization problem, an algorithm is created, and the numerical iterative method is employed. The three payment options considered are Payment in advance, payment in cash, and payment on credit, along with a carbon tax and cap policy. Sensitivity analysis is performed using Mathematica version 12, and the results are presented in both graphical and tabular form. Perishable food items (fruits, vegetables, meat, etc.), pharmaceuticals and drugs, fresh flowers, and plants.

- Chapter 5 presents an extended version of Chapter 3, incorporating a quadratic time and inventory-dependent demand under a finite planning horizon with carbon tax policy, shortage, and backlogging in all cycles. The model has been developed both theoretically and mathematically to provide an optimal solution to the problem. This chapter includes a numerical example and a sensitivity analysis for each parameter. This model is intended for large stockiest who sell fresh fruits, fresh/frozen seafood, and dairy products (milk, ice cream stores, cheese, yogurt, and vegetables to make informed managerial decisions.
- In Chapter 6, we framed a comprehensive inventory model for a three-level refrigeration supply chain with carbon emission-dependent demand for temperature-sensitive items. We also consider a carbon emission regulation and carbon tax to reduce carbon emissions. Firstly, we construct a theoretical model after studying the literature, followed by a mathematical formulation to solve the optimization model. We then develop an algorithm and use numerical iterative approaches to tackle a numerical problem. Finally, we use Mathematica software version 12 to perform sensitivity analysis of all parameters and present the results

using both graphical and tabular representations. Our research approach applies to a wide range of items, including food, beverages, pharmaceuticals, organs, tissues, insulin, blood, some blood products, fresh flowers and plants vaccines and medical supplies and more.

- Chapter 7 addresses a mathematical inventory model that examines the impact of time, advertising, and inventory-dependent demand patterns. The model considers two cases: with and without collaboration. The numerical algorithm used to solve the problem is discussed in detail, and sensitivity analysis is conducted by varying different parameters using Mathematica software version 12. The model leads to practical managerial recommendations and conclusions. The research study is relevant to seasonal stockists and items approaching their expiration date, such as perishable foods, electric vehicles, organic food products, and sustainable Clothing.
- Furthermore, In Chapter 8, we have developed a supply chain inventory model for a finite planning horizon that explores the impact of investing in green and preservation technologies and the impact of Suppliers' trade credit terms to retailers on profit enhancement, total cost reduction, and carbon emission reduction. An algorithm was formulated to identify the optimal solution for the supply chain inventory management challenge. To verify the effectiveness of the indicated model, a sensitivity evaluation was carried out. This research study is relevant to businesses seeking such as organic food products to reduce their carbon footprint and enhance profitability through the adoption of green and preservation technologies and improved trade credit management.
- Subsequently, In Chapter 9, a sustainable supply chain management model is framed for a production system with carbon caps. The model also considers the rework process of defective products with linear time-dependent demand under a finite planning horizon. An algorithm is prepared to solve the problem and sensitivity analysis is verified using Mathematica software version 12. Additionally, the convexity of the total cost function is discussed and verified using a theorem. This research study can be applied to industries that produce goods with carbon emission regulations, such as the automotive and electronics vehicles industries.

- Further, Chapter 10 gives us a piece of information about the conclusion of the thesis and suggestions for future study.

10.2. Future scope of this study includes:

Future research studies can leverage the comprehensive scope of our current research to uncover additional insights and applications. Building on our existing study, superior results may be achieved by incorporating diverse suggestions for further analysis. Some potential areas for future research include:

1. Due to a shortage of resources, this research relies on secondary data. To enhance the study in the future, data collection from primary databases could be pursued, which would make the approach more realistic, economical, and effective.
2. This supply chain inventory model can be effectively implemented for addressing more complex and challenging structural problems in the future. Additionally, it could incorporate other relevant technological tools, such as MATLAB, BARON, and CPLEX, alongside the Mathematica software. Furthermore, we are exploring the utilization of mixed-integer programming through Python's Pyomo framework and the SCIP solver.
3. This research study primarily focuses on a two- or three-player supply chain, namely the retailer, supplier, and manufacturer, for a single item. In the future, these two- or three-echelon supply chain inventory models can be expanded to encompass two or three echelons for multiple items. Additionally, various supply chain parameters, including inflation, and lead time, can be considered in future studies. To enhance customer satisfaction and increase retention, new types of deterioration and shortage costs can also be incorporated.
4. In future implementations, we could extend our inventory models by incorporating both crisp sets and fuzzy sets.

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Appendix-A: Chapters Appendix

4.1. Appendix 4.A1

Solution of Eqation2.

$$\frac{(IL_{i+1}(t))}{dt} = -D(P,G) - \theta * IL_{i+1}(t) \qquad (4.A_1.1)$$

$$Where \quad t_i \leq t \leq t_{i+1}$$

Integrating factor = $e^{\int \theta dt} = e^{\theta t}$

Thus, the solution will be given as:

$$- IL_{i+1}(t)\ e^{\theta t} = -\int_t^{t_{i+1}} D(P,G) e^{\theta u} du$$

$$IL_{i+1}(t) = e^{-\theta t} \int_t^{t_{i+1}} D(P,G) e^{\theta u} du \qquad (4.A_1.2)$$

5.2. Appendix: 5.A1

Proof of Lemma 5.1.

Since t_n = H-T_n

T_i to Follow Hariga (1996) and Wu and Zho (2014), we also used the principle of mathematical induction to prove that Following Hariga (1996), we also adopt the principle of mathematical induction to show that T_i increase with the T_n =n-1, n-2……….2.1 Put i=n-1 in Eqn. (5.20), We get

$$\frac{\partial(T_{Ret})}{\partial t_i} = [a + b\,(\mathrm{H} - T_n)]e^{\theta(T_{n-1})} - [a + b\,(\mathrm{H} - T_n)] - \theta \int_{\mathrm{H}-T_n}^{\mathrm{H}} (a + bt)e^{\theta(\mathrm{t}-\mathrm{H}+T_n)}\,\mathrm{d}t = 0$$

Then differentiate Eqn. (5.12) w.r.t T_n

$$[a + b\,(\mathrm{H} - T_n\,)]e^{\theta(T_{n-1}\,)}\frac{d(T_{n-1})}{dT_n} - b\,e^{\theta(T_{n-1}\,)} + b - [a + b\,(\mathrm{H} - T_n\,)] -$$

$$\theta^2 \int_{\mathrm{H}-T_n}^{\mathrm{H}} (a + bt)e^{\theta(\mathrm{t}-\mathrm{H}+T_n\,)}\,\mathrm{d}t = 0 \qquad (5.\mathrm{A}_1.1)$$

$(a + b\,t_i)\big(e^{\theta(\mathrm{t}_i - t_{i-1})} - 1\big) = \theta\int_{t_i}^{t_{i+1}}(a + bt)e^{\theta(\mathrm{t}-\mathrm{t}_i)}\,\mathrm{d}t$ we get by Eqn. (12) and then put in Eqn. (5.A$_1$)

$$\theta[a + b\,(\mathrm{H} - T_n\,)]e^{\theta(T_{n-1}\,)}\frac{d(T_{n-1})}{dT_n} = b(\,e^{\theta(T_{n-1}\,)} - 1) + \theta[a + b\,(\mathrm{H} - T_n\,)]e^{\theta(T_{n-1}\,)}$$

$$\theta[a + b\,(\mathrm{H} - T_n\,)]e^{\theta(T_{n-1}\,)}\left\{\frac{d(T_{n-1})}{dT_n} - 1\right\} = b(\,e^{\theta(T_{n-1}\,)} - 1)$$

$$\left\{\frac{d(T_{n-1})}{dT_n}\right\} = \frac{b(\,e^{\theta(T_{n-1}\,)} - 1)}{\theta[a + b\,(\mathrm{H} - T_n\,)]e^{\theta(T_{n-1}\,)}} + 1$$

$$\frac{d(T_{n-1})}{dT_n} = \frac{b(\,e^{\theta(T_{n-1}\,)} - 1) + \theta[a + b\,(\mathrm{H} - T_n\,)]e^{\theta(T_{n-1}\,)}}{\theta[a + b\,(\mathrm{H} - T_n\,)]e^{\theta(T_{n-1}\,)}} \geq 0 \qquad (5.\mathrm{A}_1.2)$$

After that, let's us take that $\frac{\mathrm{d}(\mathrm{T_m})}{\mathrm{dT_n}} > 0$ for m = i + 1, i + 2, . . ., n1 − 1 And then again differentiate Eqn. (5.20) w.r.t $\mathrm{T_n}$, we have

$$b(e^{\theta(\mathrm{T}_i)} - 1)\frac{d(t_i)}{dT_n} + \theta(a + b\,t_i)e^{\theta(\mathrm{T}_i)}\frac{d(T_i)}{dT_n} - \theta(a + b\,t_{i+1})e^{\theta(\mathrm{T}_{i+1})}\frac{d(t_{i+1})}{dT_n} + \theta(a +$$

$$b\,t_i)\frac{d(t_i)}{dT_n} + \theta^2\frac{d(t_i)}{dT_n}\int_{t_i}^{t_{i+1}}(a + bt)e^{\theta(\mathrm{t}-\mathrm{t}_i)}\,\mathrm{d}t \qquad (5.\mathrm{A}_1.3)$$

By using prepositions and Eqn. (5.12)

$$\frac{d(t_i)}{dT_n} \leq \frac{d(t_{i+1})}{dT_n} = -\sum_{m=i+2}^{n-1}\frac{d(T_m)}{dT_n} - 1 \leq 0 \qquad (5.A_1.4)$$

This implies $\frac{d(T_i)}{dT_n} \geq 0$ where i=1,2,3………… n-1.

Moreover, as we know that $T_n =$ H- t_{n-1} this implies $\frac{d(T_i)}{dT_{n-1}} \leq 0$ where i=1,2,3………… n-1. Now, we take $t_i = H - \sum_{m=i+1}^{n-1} T_u - T_n = t_{n-1} - \sum_{m=i+1}^{n-1} T_u$ from this, it is concluded that $\frac{d(t_i)}{dt_n} \geq 0$ for all i=1,2,3………… n-1.

5.2. Appendix: 5.A2

Proof of the theorem 5.1.

The purpose of the calculation of the values of ti is to decrease the total variable cost T_{cr} of the system. First and foremost, the primary requirement to find t_i is to ensure that the $\frac{\partial T_{Ret}(\boldsymbol{t_i},,n)}{\boldsymbol{\partial t_i}} = \boldsymbol{0}$

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial {t_i}^2} = \left\{\left(\frac{H_b + C_{H_b} * Co_2}{\theta} + D_{c_b} + D_{e_b} * Co_2\right)\left(b\left(e^{\theta(t_i - t_{i-1})} - 1\right) + \theta(a + bt_i)e^{\theta(t_i - t_{i-1})} + \theta(a + bt_i) + \theta^2 \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t - t_i)}\,\mathrm{dt}\right)\right\}$$

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial {t_i}^2} = b\left(e^{\theta T_i} - 1\right) + \theta(a + bt_i)e^{\theta T_i} + \theta(a + bt_i) + \theta(a + bt_{i+1})e^{\theta(t_{i+1} - t_i)} - b\,e^{\theta(t_{i+1} - t_i)} - \theta(a + bt_i) + b$$

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial {t_i}^2} = b\left(e^{\theta T_i}\right) + \theta(a + bt_i)e^{\theta T_i} + \theta(a + bt_i) + \theta(a + bt_{i+1})e^{\theta T_{i+1}} - b\,e^{\theta T_{i+1}} - \theta(a + bt_i)$$

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial {t_i}^2} = \theta(a + bt_i)e^{\theta T_i} + \theta(a + bt_{i+1})e^{\theta T_{i+1}} + b(e^{\theta T_i} - e^{\theta T_{i+1}}) \qquad (5.A_2.1)$$

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial t_i\, \partial t_{i-1}} = -\,\theta(a + bt_i)e^{\theta(t_i - t_{i-1})}$$

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial t_i\, \partial t_{i-1}} = -\,\theta(a + bt_i)e^{\theta T_i} \qquad (5.A_2.2)$$

Similarly

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial t_i\, \partial t_{i+1}} = -\,\theta(a + bt_{i+1})e^{\theta T_{i+1}} \qquad (5.A_2.3)$$

$$\frac{\partial^2 T_{Ret}(t_i,n)}{\partial t_i\, \partial t_m} = 0 \qquad \text{for all } m \neq i, i+1, i-1 \qquad (5.A_2.4)$$

Additionally,

$$\nabla^2 T_{Ret}(t_i, n) =$$

$$\begin{bmatrix}
\frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_1^2} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_2 \partial t_1} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_2^2} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_3 \partial t_2} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_3^2} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_{n_1-1} \partial t_{n_1-2}} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_{n_1-1}^2} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_{n_1-1} \partial t_{n_1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_{n_1} \partial t_{n_1-1}} & \frac{\partial^2 \boldsymbol{T_{cr}}(t_i,n)}{\partial t_{n_1}^2}
\end{bmatrix}$$

Figure 5.9. Hessian matrix Sarkar et al. (2012)

T_{Ret} is positive definite if Eq. A, B, C and D satisfy the given inequality

$$\frac{\partial^2 T_{Ret}(t_i, n)}{\partial {t_i}^2} > \left|\frac{\partial^2 T_{Ret}(t_i, n))}{\partial t_i\, \partial t_{i-1}}\right| + \left|\frac{\partial^2 T_{Ret}(t_i, n)}{\partial t_i\, \partial t_{i+1}}\right| + \left|\frac{\partial^2 T_{Ret}(t_i, n)}{\partial t_i\, \partial t_m}\right|$$

$$\theta(a + bt_i)e^{\theta T_i} + \theta(a + bt_{i+1})e^{\theta T_{i+1}} + b\left(e^{\theta T_i} - e^{\theta T_{i+1}}\right) > \left|-\,\theta(a + bt_i)e^{\theta T_i}\right| + \left|-\,\theta(a + bt_{i+1})e^{\theta T_{i+1}}\right| + |0|$$

$\theta(a+bt_i)e^{\theta T_i}+\theta(a+bt_{i+1})e^{\theta T_{i+1}}+b\left(e^{\theta T_i}-e^{\theta T_{i+1}}\right)>\theta(a+bt_i)e^{\theta T_i}+$
$\theta(a+bt_{i+1})e^{\theta T_{i+1}}$ that is true for all $i = 1, 2, \ldots, n1$
-1.

As a result of the positive diagonal elements and strictly diagonal dominating features present in the Hessian matrix, it must be positive definite, which allows for the determination of the optimal replenishment interval for the nonlinear Equation (5.12) system. Next, it is necessary to demonstrate that the optimal solution to Equation (5.12) is unique and that T_{Ret} is a convex function throughout the optimal value of ti in a finite horizon planning n_1.

5.3. Appendix: 5.A_3

Proof- of Theorem 5.2.

Through the above process, we have

$$T_{Ret}(t_i,n)= n_1 * O_b + \left\{\frac{H_b + C_{H_b} * Co_2}{\theta} + D_{c_b} + D_{e_b} * Co_2\right\}\sum_{i=1}^{n}\left\{\int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,\mathrm{dt} - \frac{2b}{\theta} - \frac{bT}{\theta^2}\right\}$$

Where, t0= 0 and tn= T.

Now Let us assume,

$$T_{Ret}(t_i, n) = n_1 * O_b + \sum_{i=1}^{n} \left\{ \frac{H_b + C_{H_b} * Co_2}{\theta} + D_{c_b} + D_{e_b} * Co_2 \right\} \left\{ \int_{t_i}^{t_{i+1}} (a + bt) e^{\theta(t - t_i)} \,\mathrm{dt} - M \right\}$$

were $\frac{2b}{\theta} + \frac{bT}{\theta^2} = M$

$$g(n, 0, T) = \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} (a + bt) e^{\theta(t - t_i)} \,\mathrm{dt}$$

$g(n+1, 0, T) - g(n, 0, T) = \int_{t_{n_1-1}}^{t_{n_1}} (a + bt) e^{\theta(t - t_{n-1})} \,\mathrm{dt} + \int_{n_1}^{\mathrm{T}} (a + bt) e^{\theta(t - t_n)} \,\mathrm{dt} - \int_{t_{n_1-1}}^{\mathrm{T}} (a + bt) e^{\theta(t - t_{n-1})} \,\mathrm{dt}$

$$= \int_{t_{n-1}}^{t_n} (a + bt) e^{\theta(t - t_{n-1})} \,\mathrm{dt} + \int_{t_n}^{\mathrm{T}} (a + bt) e^{\theta(t - t_n)} \,\mathrm{dt} - \int_{t_{n-1}}^{t_n} (a + bt) e^{\theta(t - t_{n-1})} \,\mathrm{dt} - \int_{t_n}^{\mathrm{T}} (a + bt) e^{\theta(t - t_{n-1})} \,\mathrm{dt}$$

$g(n+1, 0, T) - g(n, 0, T) = \int_{t_{n_1}}^{\mathrm{T}} (a + bt) \left\{ e^{\theta(t - t_n)} - e^{\theta(t - t_{n-1})} \right\} \mathrm{dt} < 0$

$$g(n+1, 0, T) - g(n, 0, T) < 0$$

$$g(n+1, 0, T) < g(n, 0, T)$$

Now, we will take $g(n, 0, T) - g(n-1, 0, T) - [g(n+1, 0, T) - g(n, 0, T)]$

$$= \int_{t_{n-1}}^{T} (a+bt)\left[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}\right] \mathrm{dt} - \int_{t_n}^{T} (a+bt)\left[e^{\theta(t-t_n)} - e^{\theta(t-t_{n-1})}\right] \mathrm{dt}$$

$$= \int_{t_{n-1}}^{T} (a+bt)\left[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}\right] \mathrm{dt} + \int_{t_n}^{T} (a+bt)\left[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}\right] \mathrm{dt}$$
$$- \int_{t_n}^{T} (a+bt)\left[e^{\theta(t-t_n)} - e^{\theta(t-t_{n-1})}\right] \mathrm{dt}$$

$$g(n,0,T) - g(n-1,0,T) - [g(n+1,0,T) - g(n,0,T)]$$
$$= \int_{t_{n-1}}^{T} (a+bt)\left[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}\right] \mathrm{dt}$$
$$+ \int_{t_n}^{T} (a+bt)\left[2e^{\theta(t-t_{n-1})} - e^{\theta(t-t_n)} - e^{\theta(t-t_{n-2})}\right] \mathrm{dt} < 0$$

$$g(n,0,T) - g(n-1,0,T) - [g(n+1,0,T) - g(n,0,T)] <$$

$$g(n,0,T) - g(n-1,0,T) < g(n+1,0,T) - g(n,0,T)$$

e^t is a convex function, as a result, g (n, 0, H) is also convex in n1. Therefore, $Tcr(t_i, n)$ is essentially convex function.

8.1. Appendix 8.A$_1$:

Proof of the theorem 8.1.

To calculate the total variable cost T_{Ret} of the system by computing the values of ti. First of all, they find ti by putting the $\frac{\partial\, (T_{Ret}\,(t_i,,n))}{\partial t_i} = \mathbf{0}$

$$\frac{\partial(T_{Ret})}{\partial t_i} = \left(\left\{\frac{h_r}{\theta(1-P(\Psi))} + \frac{\tau(1-\emptyset(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))} + d_r\, e^{-\lambda\Psi}\right\} + \left\{P_r + \hat{P}_r * \tau\,(1-\emptyset\,(1-e^{-mG}))\right\}\right)\Big((a + b\,t_i)\left(e^{\theta(1-P(\Psi))(t_i - t_{i-1})} - 1\right) - \theta(1 - P(\Psi))\int_{t_i}^{t_{i+1}}(a+bt)e^{\theta(1-P(\Psi))(t-t_i)}\,dt\Big) + s * I_e\left\{\int_{t_{i-1}}^{t_{i-1}+(\delta * t_i - \delta * t_{i-1})}(a+bt)\ \delta\, dt + \int_{t_i}^{t_i+(\delta * t_{i+1} - \delta * t_i)}(a+bt-\delta(a+bt))\,dt - \delta(t_{i+1}(a+b\,t_i) - t_i(a+b\,t_i))\right\} - I_c * W\left\{\int_{t_{i-1}+(\delta * t_i - \delta * t_{i-1})}^{t_i}(a+bt)\,\delta\, dt + \int_{t_i+(\delta * t_{i+1} - \delta * t_i)}^{t_{i+1}}(a+bt-\delta(a+bt))dt - (a + bt_i - \delta(a + bt_i))[t_i - t_{i-1}]\right\}$$

$$\frac{\partial^2 T_{Ret}}{\partial {t_i}^2} = \left(\left\{\frac{h_r}{\theta(1-P(\Psi))} + \frac{\tau(1-\emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))} + d_r\, e^{-\lambda\Psi}\right\} + \left\{P_r + \hat{P}_r * \tau\,(1-\emptyset\,(1-e^{-mG}))\right\}\right)\left(b\left(e^{\theta(1-P(\Psi))(t_i-t_{i-1})} - 1\right) + \theta(1-P(\Psi))(a+b\,t_i)e^{\theta(1-P(\Psi))(t_i - t_{i-1})} + (\theta(1-P(\Psi)))^2\,(a + b\,t_i)\right) + I_e * s\{(a + b(t_{i-1} + (\delta * t_i - \delta * t_{i-1})))\,\delta^2 + (a + b(t_i + (\delta * t_{i+1} - \delta * t_i)))\,(1-\delta) - (t_{i+1} * b * \delta - b * \delta * t_i) + (\delta * a + \delta * b * t_i)\} - I_c * W\{(\delta * a + \delta * b * t_i) + (a + b(t_{i-1} + (\delta * t_i - \delta * t_{i-1})))\delta^2 + \left(a + b * t_i + (\delta * t_{i+1} - \delta * t_i)\right)(1-\delta)^2 - (b - b * \delta)[t_i - t_{i-1}] - \left(a + bt_i - \delta(a + bt_i)\right)\} \qquad \textbf{(8.A1.1)}$$

$$\frac{\partial^2 T_{Ret}}{\partial t_i\, \partial t_{i-1}} = \Bigg(\Bigg\{\frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau(1- \emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1- P(\Psi))} + d_r\, e^{-\lambda\Psi}\Bigg\}$$
$$+ \left\{P_r + \hat{P}_r * \tau\,(1 - \emptyset\,(1-e^{-mG}))\right\}\Bigg)\Big(-\theta(1- P(\Psi))(a+b$$
$$* \; t_i)\left(e^{\theta(1- P(\Psi))(t_i - t_{i-1})}\right)\Big) + I_e$$
$$* \; s\{(a + b(t_{i-1} + (\delta * t_i - \delta * t_{i-1}))) \; (\delta - \delta * \delta) - (a * \delta + b * \delta$$
$$* \; t_{i-1}) \} - I_c$$
$$* \; W\{(a + b(t_{i-1} + (\delta * t_i - \delta * t_{i-1}))) \, (\delta - \delta * \delta) + (a + bt_i$$
$$- \delta(a + b * t_i))\}$$

(8.A1.2)

Similarly

$$\frac{\partial^2 T_{Ret}}{\partial t_i\, \partial t_{i+1}} =$$
$$-\Bigg(\Bigg\{\frac{h_r}{\theta(1- P(\Psi))} + \frac{\tau(1- \emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1- P(\Psi))} + d_r\, e^{-\lambda\Psi}\Bigg\} + \Big\{P_r + \hat{P}_r * \tau\,(1 - \emptyset\,(1 -$$
$$e^{-mG}))\Big\}\Bigg)\left(\theta(1- P(\Psi))(a + b\, t_{i+1})e^{\theta(1- P(\Psi))(t_{i+1} - t_i)}\right) + I_e * s\{(a + (b * t_i + (\delta *$$
$$b * t_{i+1} - \delta * b * t_i))) \, (\delta - \delta * \delta) - (a * \delta + b * \delta * t_i)\, \delta\} - I_c * W\{ (a * \delta + b *$$
$$\delta * t_{i+1})(1 - \delta) - (a + (b * t_i + \delta * b(t_{i+1} - t_i))) \, (\delta - \delta * \delta)\}$$

(8.A1.3)

$$\frac{\partial^2 T_{Ret}}{\partial t_i\, \partial t_n} = 0$$ **(8.A1.4)**

for all $n \neq i, i+1, i-1$

The Hessian matrix was found to be positive definite, as it contains positive diagonal elements and strictly diagonal dominating features. Consequently, the optimal replenishment interval for the nonlinear system described by Equation (8.11) was obtained. To demonstrate the uniqueness of the optimal solution for the nonlinear equation (8.11), we also need to show that $T_{Ret}(t_i, n)$ is the optimal function over the optimal value of ti in a finite horizon planning H.

Furthermore, since the Hessian matrix had positive diagonal elements and strictly diagonal dominant features, it needed to be positive definite. This enabled us to obtain the optimal replenishment interval for the nonlinear system of Equation (8.12). Now, we need to demonstrate that the function $T_{Ret}(t_i, n)$ is convex throughout the optimal value of ti in the finite horizon planning H.

Furthermore,

$$\nabla^2 T_{Ret} = \begin{pmatrix} \frac{\partial^2 T_{Ret}}{\partial t_1^2} & \frac{\partial^2 T_{Ret}}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 T_{Ret}}{\partial t_2 \partial t_1} & \frac{\partial^2 T_{Ret}}{\partial t_2^2} & \frac{\partial^2 T_{Ret}}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 T_{Ret}}{\partial t_3 \partial t_2} & \frac{\partial^2 T_{Ret}}{\partial t_3^2} & \frac{\partial^2 T_{Ret}}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 T_{Ret}}{\partial t_{n-1} \partial t_{n-2}} & \frac{\partial^2 T_{Ret}}{\partial t_{n-1}^2} & \frac{\partial^2 T_{Ret}}{\partial t_{n-1} \partial t_n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 T_{Ret}}{\partial t_n \partial t_{n-1}} & \frac{\partial^2 T_{Ret}}{\partial t_n^2} \end{pmatrix}.$$

Figure 8.6 Hessian matrix.

T_{Ret} is positive definite if Eq. $(\mathbf{8.A_1.1})$, $(\mathbf{8.A_1}.2)$ $(\mathbf{8.A_1.3})$ and $(\mathbf{8.A_1.4})$ satisfy the given inequality.

$$\frac{\partial^2 T_{Ret}}{\partial t_i^2} \geq \left|\frac{\partial^2 T_{Ret}}{\partial t_i t_{i-1}}\right| + \left|\frac{\partial^2 T_{Ret}}{\partial t_i t_{i+1}}\right| \qquad \text{or}$$

$$\frac{\partial^2 T_{Ret}}{\partial t_i^2} - \left|\frac{\partial^2 T_{Ret}}{\partial t_i t_{i-1}}\right| - \left|\frac{\partial^2 T_{Ret}}{\partial t_i t_{i+1}}\right| \geq 0$$

$$\Bigg(\left\{\frac{h_r}{\theta(1-P(\Psi))} + \frac{\tau(1-\emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))} + d_r\, e^{-\lambda\Psi}\right\} + \left\{P_r + \hat{P}_r * \tau\,(1-\emptyset\,(1-e^{-mG}))\right\}\Bigg)\Big((a+b\,t_i)\big(e^{\theta(1-P(\Psi))(t_i-t_{i-1})} - 1\big) - \theta(1-P(\Psi))\int_{t_i}^{t_{i+1}}(a+bt)e^{\theta(1-P(\Psi))(t-t_i)}\,dt\Big) + I_e * s\left\{\int_{t_{i-1}}^{t_{i-1}+\delta(t_i-t_{i-1})}(a+bt)\;\delta\,dt + \int_{t_i}^{t_i+(\delta * t_{i+1}-\delta * t_i)}(a+b*t-\delta(a+bt))\,dt - (a+b\,t_i)\,(\delta * t_{i+1} - \delta * t_i)\right\} -$$

$$I_c * W\left\{\int_{t_{i-1}+(\delta * t_i-\delta * t_{i-1})}^{t_i}(a*\delta+b*\delta*t)\,\delta\, dt + \int_{t_i+(\delta*t_{i+1}-\delta*t_i)}^{t_{i+1}}(a+bt)\,(1-\delta)dt - (a+bt_i)\,(1-\delta)[t_i - t_{i-1}]\right\} - \left(\left\{\frac{h_r}{\theta(1-P(\Psi))}+\frac{\tau(1-\emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))}+d_r\,e^{-\lambda\Psi}\right\}+\left\{P_r+\hat{P}_r*\tau\,(1-\emptyset\,(1-e^{-mG}))\right\}\right)\left(-\theta(1-P(\Psi))(a+b\,t_i)\left(e^{\theta(1-P(\Psi))(t_i-t_{i-1})}\right)\right) + I_e * s\{(a+(b*t_{i-1}+(\delta*b*t_i-\delta*b*t_{i-1})))\,(\delta-\delta*\delta)-(a*\delta+b*\delta*t_{i-1})\} - I_c * W\{(a+(b*t_{i-1}+(b*\delta*t_i-b*\delta*t_{i-1})))\,(\delta-\delta*\delta)+(a+bt_i-\delta(a+bt_i))\} - \left(\left\{-\frac{h_r}{\theta(P(\Psi)-1)}+\frac{\tau(1-\emptyset\,(1-e^{-mG}))\widehat{h_r}}{\theta(1-P(\Psi))}+d_r\,e^{-\lambda\Psi}\right\}+\left\{P_r+\hat{P}_r*\tau\,(1-\emptyset\,(1-e^{-mG}))\right\}\right)\left(\theta(1-P(\Psi))(a+b\,t_{i+1})e^{\theta(1-P(\Psi))(t_{i+1}-t_i)}\right) + I_e * s\{(a+(b*t_i+(b*\delta*t_{i+1}-b*\delta*t_i)))\,(\delta-\delta*\delta)-(a*\delta+\delta*b*t_i)\} - I_c * W\left\{(a*\delta+b*\delta*t_{i+1})\,(\delta-\delta*\delta)-(a+(b*t_i+(\delta*b*t_{i+1}-\delta*b*t_i)))\,(\delta-\delta*\delta)\right\} \geq 0$$

that is true for all $i = 1, 2, \ldots, n$

Appendix -B: Intellectual Property Rights: Diary No. and Registration No of Copyright Applications

B.1. Diary number and Registration no for graphical abstracts:

5 copyright applications have been filed, 3 graphical abstracts for published articles, 1 for a Mathematica program and 1 graphical abstract for the thesis. The Diary Number and Registration no of Copyright Applications for the latter has been received as shown below.

1. A program in Mathematica to find an optimal inventory replenishment schedule with a "break-even" point between producer and retailer has been granted with diary number 3622/2023-CO/L and Registration Number: L-125636/2023. http://www.copyright.gov.in/CopyrightROC_Details.aspx?DiaryNo=3622/2023-CO/L&RocNo=L-125636/202

2. The inventory management model for deteriorating items considers both payment options and a policy of carbon tax and cap has been granted with diary number: 10698/2023-CO/L and Registration Number: L-127806/2023 http://www.copyright.gov.in/CopyrightROC_Details.aspx?DiaryNo=10698/2023-CO/L&RocNo=L-127806/2023

3. A model for carbon regulations and leverages advertisement efforts to enhance the efficiency of supply chain management has been granted with diary number 11222/2023-CO/L and Registration Number L-127808/2023. http://www.copyright.gov.in/CopyrightROC_Details.aspx?DiaryNo=11222/2023-CO/L&RocNo=L-127808/2023

4. “A Mathematical Inventory Model for Sustainable Credit Period Rates in Supply Chain Management” Not Available.

5. “Inventory supply chain management incorporating carbon emission policies within the finite planning horizon” diary number 18504/2023-CO/L and Registration Number L-134211/2023 http://www.copyright.gov.in/CopyrightROC_Details.aspx?DiaryNo=18504/2023-CO/L&RocNo=L-134211/2023

B. 2. Copyright for Mathematica Program new version 12.0

1. Mathematica Program has given for copyright that was not included above all tables or figure formulation of the Mathematica version 12. The dairy no and Registration Number for the copyright of the program is as shown below: http://www.copyright.gov.in/CopyrightROC_Details.aspx?DiaryNo=3622/2023-CO/L&RocNo=L-125636/202

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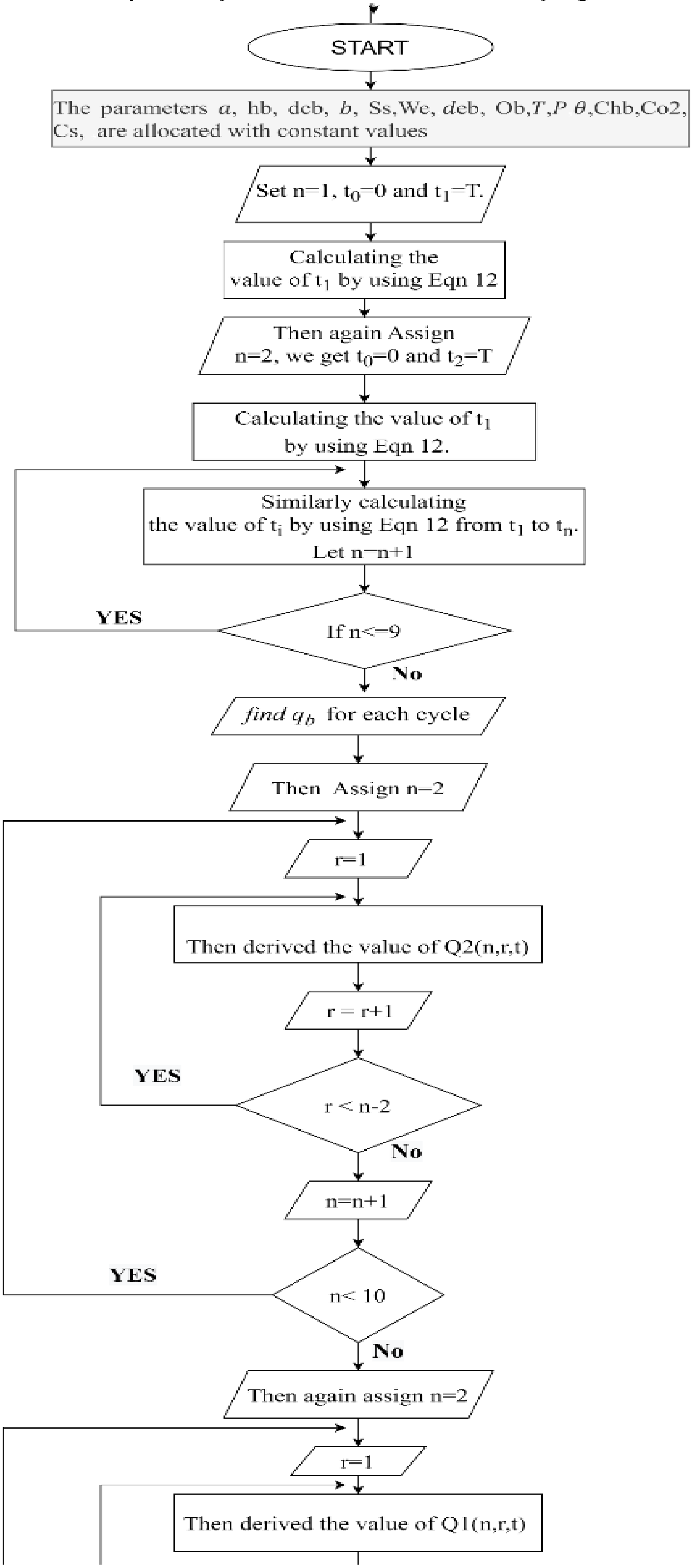
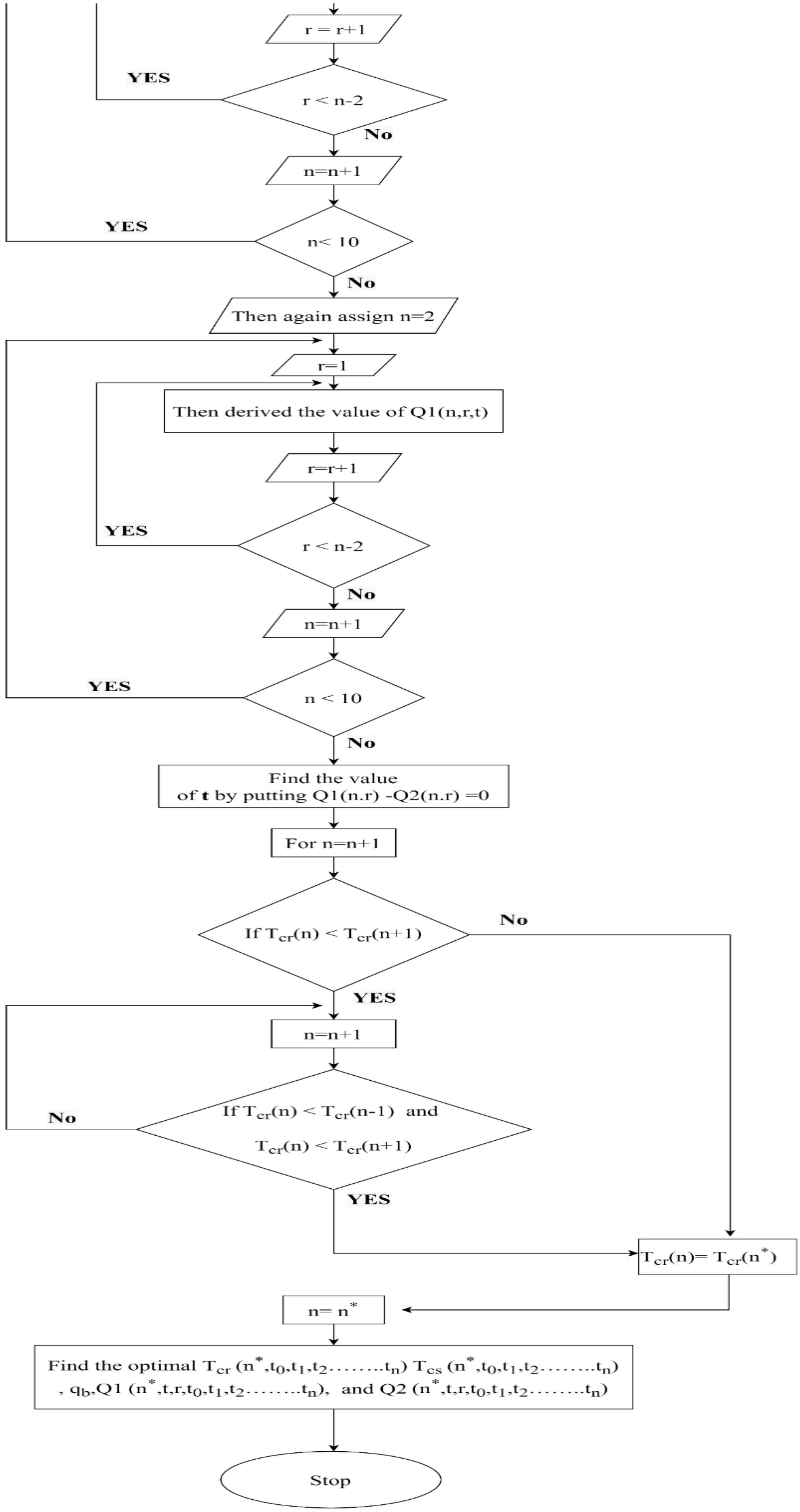
Flow chart for Mathematica program:

Graphical representation of the flow of the program:




r = r+1
YES
r < n-2
No
n=n+1
YES
n< 10
No
Then again assign n=2
r=1
Then derived the value of Q1(n,r,t)
r=r+1
YES
r < n-2
No
n=n+1
YES
n < 10
No
Find the value of t by putting Q1(n.r) -Q2(n.r) =0
For n=n+1
If Tcr(n) < Tcr(n+1)
No
YES
n=n+1
No
If Tcr(n) < Tcr(n-1) and Tcr(n) < Tcr(n+1)
YES
Tcr(n)= Tcr(n*)
n= n*
Find the optimal Tcr (n*,t0,t1,t2........tn) Tcs (n*,t0,t1,t2........tn) , qb,Q1 (n*,t,r,t0,t1,t2........tn), and Q2 (n*,t,r,t0,t1,t2........tn)
Stop

B. 3. Copy right for graphical abstract:

2. Graphical Abstract entitled "The inventory management model for deteriorating items considers both payment options and a policy of carbon tax and cap" applied for a copyright. Dairy no for the copyright of the graphical abstract is obtained as shown below: http://www.copyright.gov.in/CopyrightROC_Details.aspx?DiaryNo=10698/2023-CO/L&RocNo=L-127806/2023

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1	Form-XIV	18504/2023-CO/L	95258	Inventory supply chain management incorporating carbon emission policies within the finite planning horizon.	500
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Online	C-0000107496	1307230028605

Appendix-C: Conference, workshop or seminar participation and presentations

C. 1. Conference presentations are as follows:

1. Presented a research article titled " Single Supplier- Retailer Inventory Model or Deteriorating Items with Linear Decreasing Demand” at the 3rd International IOP Conference on Recent Advances in Fundamental and Applied Sciences (RAFAS 2021) 25-26 June 2021 held at Lovely Professional University, Punjab, India.
2. Presented a research article titled " An Inventory Model for the Association Between Retailer and Supplier for a Finite Planning Horizon with Carbon Emission-Dependent Demand” at the International AIP Conference on Materials for Emerging Technologies-2021 (ICMET-2021) hosted by the Department of Research Impact and Outcome, Division of Research and Development, Lovely Professional University, Punjab, India on 18th-19th February 2022.
3. A supply chain inventory model for deteriorating products with carbon emission-dependent demand and advanced payment that incorporates a carbon tax and cap policy in the International AIP Conference on Computational Applied Sciences &Its Applications, organized by the Department of Applied Sciences, University of Engineering & Management Jaipur, on 28- 29 April 2022.
4. I have given an oral presentation on the “Analyze the carbon regulation for a production system with a rework process of defective production under a finite planning horizon (FPH) “at the 4th International Conference on “Recent Advances in Fundamental and Applied Sciences” (RAFAS 2023) held on March 24-25, 2023, organized by School of Chemical Engineering and Physical Sciences, Lovely Faculty of Technology and Sciences, Lovely Professional University, Punjab.

C. 2. Workshop, webinar, FDP, or Mathematica software online tutorial participation are as follows:

1. Graduate Aptitude Test in Mathematics (MA) – Passed (With Registration Number MA21S58021073, Result Date 23-05-2021) GATE - 2021:: Welcome (iitb.ac.in)
2. One Week FDP on **Research Methodology and Scientific Components** for an Empirical Study in Research arranged by Genesin of Educational Impressions Roorkee, Uttarakhand, India on April 15, 2021, to April 21, 2021, with ISO: 9001:2015 Certified Co.
3. One-week International FDP (**Faculty Development Program**) Advanced Tools and Techniques for Scientific Research Writing and Publication (ATTSRWP-2022) organized by the Department of Applied Sciences, University of Engineering & Management Jaipur, on 12-18 September 2022.
4. One-week e-Workshop on "Materials and Manufacturing: Insights to Modern Technologies (MMIMT-2022)" organized by the **Department of Production & Industrial Engineering**, BIT Sindri, Dhanbad from 1st August to 5th August 2022. (MMIMT-2022)
5. National webinar on an integrated approach in science and technology for a **sustainable** future by Gandhi memorial national college Ambala Cant (Hry)on 11th February 2022.
6. Join the **Wolfram Mathematica version-12** training tutorial on 27 July 2022.
7. The certification course "**Optimization with Python: Solved Operations Research Problems"** was completed and organized by Udemy with 12.5 hours on 27 January 2023. https://www.udemy.com/
8. The certificates of STC on Data Science using R from 17-21 July 2023 were organized by the **National Institute of Technical Teachers Training and Research** (NITTTR), Chandigarh under the Ministry of Education,

Government of India. http://www.nitttrchd.ac.in/

9. **Swayam** Certification course **(three credits-12 week)** "Essential Mathematics for Machine Learning" by Prof. S. K. Gupta, Prof. Sanjeev Kumar has been completed, organized by IIT Roorkee, oct. 2023.

10. **Swayam** Certification course **(three credits-12 week)** "Machine Learning" by Prof. Anubha Gupta IIT Delhi has been completed, oct. 2023.

Appendix-D: Published, accepted, and communicated Papers

D. 1. The list of articles published is as follows (are part of the thesis):

1. Mishra, N.K., **Ranu**. (2022). “A supply chain inventory model for deteriorating products with carbon emission-dependent demand, advanced payment, carbon tax and cap policy.” Mathematical Modelling of Engineering Problems, Vol. 9, No. 3, pp. 615-627. https://doi.org/10.18280/mmep.090308. **ISSN:** 2369-0739 (print); 2369-0747 (online) Q2 Scopus-Indexed.

2. **Ranu**, Mishra, N.K. (2023). “A collaborating supply chain inventory model including linear time-dependent, inventory, and advertisement-dependent demand considering carbon regulations.” **Mathematical Modelling of Engineering Problems**, Vol. 10, No. 1, pp. 227-235. https://doi.org/10.18280/mmep.100126. **ISSN:** 2369-0739 (print); 2369-0747 (online) Q2 Scopus-Indexed.

3. Mishra, N.K., **Ranu**. (2023). “Analyse the carbon regulation for a production system with rework process of defective production under a finite planning horizon (FPH)”. European Chemical Bulletin, Vol. 12, No. 3, pp. 1718-1748. ISSN 2063-5346, Q3 Scopus-Indexed. DOI 10.31838/ecb/2023.12.3.126

4. Mishra, N.K., Ranu. (2023). A supply chain inventory model for a deteriorating material under a finite planning horizon with the carbon tax and shortage in all cycles. Journal Européen des Systèmes Automatisés , Vol. 56, No. 2, pp. 221-230. https://doi.org/10.18280/jesa.560206 ISSN:1269-6935 and E-ISSN:2116-7087, Q3 Scopus-Indexed.

5. Nitin Kumar Mishra and **Ranu.** "An Inventory Model for the Association Between Retailer and Supplier for a Finite Planning Horizon with Carbon Emission-Dependent Demand" at the **International AIP Conference** on Materials for Emerging Technologies-2021 (ICMET-21). https://doi.org/10.1063/5.0162943

D. 2. Other relevant articles communicated and accepted during research work are as follows (are part of the thesis):

1. Communicated and under review a research article entitled "A three-level refrigeration supply chain model including carbon emission-dependent demand for temperature-sensitive items considering carbon tax regulations" in the **Scopus-indexed Operations Research Forum journal**. E-ISSN:2662-2556
2. Communicated and under review, a research article entitled "Inventory replenishment model with trade credit and preservation technology under a finite planning horizon" in the **Scopus-indexed** International Journal of Sustainable Development and Planning. ISSN:1743-7601E-ISSN:1743-761X
3. Communicated and under review a research article entitled "Developing an Inventory Management Strategy that Incorporates Several Carbon Policies within a finite time planning" in the **Q1 Scopus indexed and SCIE (Web of Science)** International Journal of Industrial Engineering Computations. Cite Score (2021)- 6.00; SJR (2020) - 0.79; SNIP (2020) - 1.424; h-Index (2020) – 30; Impact Factor (Q2) - 3.271

D. 3. Other relevant articles published during research work are as follows (not in the thesis but relevant to the thesis):

1. **Ranu**; Nitin Kumar Mishra and Meenakshi Sharma (2023). "An Inventory Model for Perishable Items with Exponential Demand." in the **International**

AIP Conference on Materials for Emerging Technologies-2021 (ICMET-21). https://doi.org/10.1063/5.0162923

2. Nitin Kumar Mishra and **Ranu**. “Single Supplier- Retailer Inventory Model for Deteriorating Items with Linear Decreasing Demand” in the **International AIP Conference** on Materials for Emerging Technologies-2021 (ICMET-21). https://doi.org/10.1063/5.0162939

3. Nitin Kumar Mishra, Ajay Pratap Singh, **Ranu**, and Sakshi Sharma “To analyse Weibull demand in the production inventory model under a finite planning horizon” European Chemical Bulletin, Vol. 12, No. 3, pp. 1571-1583. ISSN 2063-5346. doi: 10.31838/ecb/2023.12.3.117

4. Nitin Kumar Mishra, Anushka Sharma, **Ranu**, and Renuka S Namwad, “Formulating a finite planning horizon model for Inventory Management, incorporating Linear Demand, Exponential Deterioration, and Trade Credit Policy.” European Chemical Bulletin, **ISSN 2063-5346**. doi: 10.48047/ecb/2023.12.4.186

D. 4. Other relevant published Book Chapters during research work are as follows (relevant to thesis):

1. **Ranu** and Nitin Kumar Mishra (2023). Carbon Emission Policy Integration: A Comprehensive Review of Supply Chain Management Strategies, The book “Advancements in Computational Mathematics” ISBN: 978-81-19334-65-0.

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A Supply Chain Inventory Model for Deteriorating Products with Carbon Emission-Dependent Demand, Advanced Payment, Carbon Tax and Cap Policy

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ABSTRACT

Emissions are a major contributor to climate change. Some nations are now concentrating their efforts on lowering carbon emissions. In many nations, carbon taxes and caps are the main tools that are used to attain this goal. The majority of the inventory retailer-supplier model assumed that the retailer's order cost should be paid to the supplier at that time when he gets their order. Few suppliers can expect to receive the entire or a portion of the total cost in advance from retailers in this real-life situation, and others will offer prepayment in numerous equal installments. The advance payment offers the customer the lowest price for the order, but it has the largest carbon footprint. The advance payment has a great impact on carbon emissions and production. Therefore, this study looked at a carbon tax and cap supply chain inventory model for deterioration with carbon emission-dependent demand, and Three payment options: Preliminary, cash, and post-payment have been considered. The model was constructed by first assessing the overall cost of supply chain participants with carbon tax regulation. Finally, we illustrate numerical examples of the proposed approach and its outcomes. The implications of adjusting the various parameters on the optimal total cost are also graphically and tabularly discussed in depth. With the help of Mathematica version-12, a sensitivity analysis was also performed. Several management takeaways are also emphasized. These findings are incredibly managerial and enlightening for enterprises seeking profitability while still fulfilling their environmental duties, and this study is extremely useful for any country's government policy.

1. INTRODUCTION

The condition that is created due to greenhouse gases (GHG) and by some human activity, we call it global warming. The emission of carbon has been causing global warming for many years. Global warming has been receiving attention over the last few years.

Carbon emission gases like methane carbon dioxide, increase the temperature of our Earth and cause global warming, visit the page to see Climate Change Indicators: Atmospheric Concentrations of Greenhouse Gases. It causes severe damage to our earth as it has destructive, widespread, and lifelong effects. The global temperature rapidly destroys the biodiversity of our world, causing the disappearance of many species of plants and animals. Some natural phenomena such as Sea level rise, ozone layer depletion, the rising temperature of the earth, intensively stormy conditions, dryness, flooded conditions are all effects of global warming. In today's time, global warming has become a big challenge. It greatly affects the living life around us. Reducing and lowering carbon emissions is a worldwide issue. Some countries or regulatory agencies are now concentrating their efforts on lowering their carbon footprints such as Toptal et al. [1] and Rout et al. [2] also explained that Kyoto Protocol is a global agreement connected to the UN Framework Convention on Climate Change. The UNFCCC came into effect on March 21, 1994. On December 11, 1997, The Kyoto Protocol was signed by 37 industrialized countries.

India also ratified the Kyoto protocol in 2002. The United Nations Climate Change holds every yearly conference in a different country that is called the COP. The Paris Agreement operationalizes the United Nations Framework Convention on Climate Change (UNFCCC) into action by committing developed and developing countries to reduce greenhouse gas (GHG) emissions. Encourage the business sector and developing economies to engage in the effort to reduce carbon emissions. Our supply chain represents a large portion of the overall global carbon footprint. Situations such as global warming are promoted by manufacturing fields as well as by inventory control and management. Chen et al. [3] introduced that in 2016, Walmart has taken a new decision to avoid 1 billion metric tons of carbon emissions from the global supply chain by 2030 to achieve this goal a Project Gigaton (https://www.walmartsustainabilityhub.com/project-gigaton/emissions- targets) was launched to address the fact that almost all emissions in the retail sector occurred in product supply chains and transportation rather than stores and distribution centers. Furthermore, one of the most significant economic benefits of reducing carbon emissions and deterioration is the reduction emission of carbon during the entire supply chain process Carbon emissions and deterioration from economic sectors are causing serious rising temperatures. One of the most pressing issues today is supply chain management for deteriorating goods with carbon reduction regulations, which is becoming a serious concern for urban areas. As a result, some national or international

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A Collaborating Supply Chain Inventory Model Including Linear Time-Dependent, Inventory, and Advertisement-Dependent Demand Considering Carbon Regulations

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Keywords:
collaboration, advertising demand, and carbon emission

ABSTRACT

For carbon emission, this research study investigates a Mathematical inventory model with time, advertising, and inventory-dependent demand patterns. The main objective of this research study is to keep the total cost of retailers as well as suppliers and carbon emissions as low as possible. With collaboration and without collaboration, two cases are discussed in this proposed model. Within the first case, retailers and suppliers are not regarded as collaborators, whereas in the second case, collaboration is recognized. The optimality of the planned inventory management model is explained mathematically and theoretically in both situations. The algorithm of the mathematical solution was also properly discussed and the effects of altering various parameters are numerically studied to conduct a sensitivity analysis with the help of Mathematica software version 12. To demonstrate this model, a mathematical illustration, and a tabular and graphical representation, have been also provided. Ultimately, this model reaches a flourishing managerial suggestion and conclusion.

1. INTRODUCTION

Global temperature rise is the effect of greenhouse gas emissions and even some living beings. Kaya identity also plays a suitable role in the direction of Climate change that had received a great deal of attention in past few years. A worldwide awareness about environmental conservation and protection is encouraging many more researchers, organizations, and other government agencies to create and maintain an eco-friendly as well as a negligible emission management system for supply chains. Kaya identity plays a role to calculate the impact of different factors of the supply chain on carbon emission [1]. Therefore, we rely on the kaya identity equation to calculate the amount of carbon emission.

$$Co_2 = \left(\frac{GDP}{Population}\right)\left(\frac{EC}{GDP}\right)\left(\frac{Co_2}{EC}\right) * Population$$

The entire value of all products and services generated in a country in a given year is known as the gross domestic product (GDP). GDP quantifies the monetary worth of final goods and services produced in a country over a specified duration, it may be quarterly or yearly.

Everyday activities like-Heating, cooling, electricity supply, and transportation are all dependent on efficient and reasonable energy services. All economic sectors, from business and industry to agriculture, rely on energy to function properly and effectively. Energy intensity is a ratio of the volume of energy required to produce one unit of GDP and can be used to estimate a country's energy efficiency.

The total energy consumed (EC) by end-users, such as households and businesses, industry, and modern agriculture, is referred to as final energy consumption in cooling, heating, and fossil fuel. It is the energy that achieves by the final consumer, except for energy used by the energy sector that produced emissions in large quantities. Our research work mainly focuses on carbon emission during supply chain processes such as holding (heating, refrigeration), and carbon emission due to transportation.

In inventory management, Khan et al. [2] advertising a new product or modified goods plays an extremely important role because it raises product awareness for customers to buy something, retailer, supplier, and even manufacturer, all advertise their new products, and also when a modified product from an old to new, they also advertise it. In this way, the advertisement plays an important role in increasing the demand for any product and from which we can say that the demand for any product on the day depends absolutely on its advertisement. Therefore, the advertisement will increase the product demand and the product will be sold out very soon. For retailers and suppliers to enhance customer demands is a challenging feat. That idea would be great in the case of perishable items, fashionable products and those items that are reaching their expiring date, where their life span is short. Due to carbon emissions and advertisements, demand for products will undoubtedly be influenced. Advertisement is one of the most effective promotional approaches to raise awareness about a product's popularity among all classes of consumers. There are many different ways to do advertising, in today's new technology, people advertise their new products or old to new through mass media to stimulate a larger group of customers to buy their products.

In this regard they aware customer about new products, about their price and extra information about their quality. All the players in supply chain management advertise in different ways to increase the demand for their products, for this, they advertise through different media like newspapers, posters, and television. Moreover, in the advertisement, they show the quality of the product. and at the same time, the good quality attracts the customer to buy the item very much. In the case of

Analyze the carbon regulations that apply to a production system that integrates a process for reworking defective items within a finite planning horizon(FPH)

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3. Abstract:

Manufacturing companies are increasingly recognizing the importance of sustainable supply chain management (SSCM) and the need to address the issue of reworking defective items in their operations, both for their business goals and for the environment. To achieve sustainability goals, it is essential to design supply chain networks while taking ecological and environmental factors into account. This article's focus is on developing an environmentally friendly supply chain system within the framework of the emission trading mechanism. The problem at hand is formulated mathematically as a non-linear programming model with the objective of minimizing the total cost and carbon emissions. The approach is applied to analyse two distinct regulations: the cap and tax scheme under a finite planning horizon (FPH). Additionally, the study includes a sensitivity analysis, along with graphical or tabular representations, to enhance its effectiveness. The numerical results indicate the optimal solution for the decision variables, and a decrease in carbon emissions is observed. To solve the problem, a programming model in the Mathematica software version -12.0 is employed.

1. Introduction:

It is imperative for supply chain operations to maintain a reduction in carbon emissions while optimizing total costs. In reality, meeting customer demand is crucial for both systems and processes. With an increasing awareness among people, it has become essential to minimize carbon emissions. This research aims to demonstrate the impact of reducing carbon dioxide emissions while simultaneously optimizing costs and order quantity. As per Heather Lovell's

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A Supply Chain Inventory Model for a Deteriorating Material under a Finite Planning Horizon with the Carbon Tax and Shortage in All Cycles

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ABSTRACT

We have framed an inventory replenishment model under a finite planning horizon in which replenishment cycle time and replenishment cycle length are different and don't repeat. The finite planning horizon for different replenishment times and cycle lengths is a real-life scenario. Nowadays, every manufacturing industry wants to achieve maximum profit at a low cost. It is very difficult to maintain the optimal level of inventory, total cost, replenishment time, and replenishment cycle. Along with the health of people, increasing carbon emission also has a dangerous effect on today's business environment. Therefore, this article analyses an optimal inventory replenishment policy and carbon emission due to deteriorating material and refrigeration while taking into account time, emission-dependent, and inventory-dependent quadratic demand. Materials deterioration affects a large and varied spectrum of business. Therefore, Material that suffers deterioration is considered. Shortage, some lost sales, and partial backlogging are also considered. Backlogging is dependent on the frequency of the waiting period for the next replenishment over a given finite time horizon and fluctuating replenishment cycle. The model has been developed theoretically. Also, a mathematical formulation has been obtained to find the optimal solution to the problem. Following the algorithm, a numerical illustration and a comparative evaluation are explained, along with a sensitivity analysis of each parameter. The tabular and graphical representations of sensitivity analysis were addressed using the Mathematica application version 12.

1. INTRODUCTION

In recent years, there has been a lot of discussion about inventory systems that emit carbon and degrade the environment. Carbon emissions have been triggering global warming for many years, which has garnered considerable attention. Carbon emission gases, such as carbon dioxide and methane, heat our planet and lead to global warming. To learn more about this, we can visit the page on Climate Change Indicators: Atmospheric Concentrations of Greenhouse Gases. Global warming produces considerable environmental harm because of its destructive, pervasive, and long-term consequences. It is quickly destroying our planet's biodiversity, ultimately causing the extinction of countless plant and animal species. Global warming also causes sea-level rise, ozone layer depletion, rising global temperatures, extreme weather conditions, drought, and flooding. Due to increasingly powerful and frequent severe weather events, global climate change and the greenhouse effect have received a lot of attention. The Paris Protocol, also known as Cap-and-Trade, was created to lessen the greenhouse effect. Under the carbon tax, companies are paid a fixed sum for every tonne of emissions they generate, while the cap-and-trade policy issues a specified number of emissions allowances per year under the cap-trade program [1, 2].

There is growing agreement that Carbon emissions from enterprise economic activities are increasingly being blamed for serious climate change and global warming. Furthermore, reducing a polluted environment is one of the most considerable economic benefits of lowering carbon emissions, and it is becoming a big worry for nations worldwide. Only a few of them consider environmental issues in supply chain management, which include reducing carbon emissions. To reduce carbon emissions, the government of any nation and several regulatory bodies have developed carbon emission programs. The primary regulatory policy is a carbon tax. A carbon tax is charged or imposed by various government agencies on commercial enterprises or businesses that create carbon dioxide throughout their production process and generate environmental destruction [3]. The primary goal of the government entities responsible for taxation is to prevent global warming and safeguard the environment. In other terms, the carbon tax is a cost levied on corporations that use harmful raw materials, such as fossil fuels, in their manufacturing process and transportation, and they are blamed for global warming. Also, a carbon tax can prevent environmental degradation and total cost of Inventory model [4].

Chen et al. investigated an emissions inventory issue using the EOQ model and carbon schemes such as carbon tax, cap, and cap-and-offset [4]. Analyzed the influence of the credit period and environmental policies on inventory management, whenever the credit period has an impact on the demand rate [5]. Determined the optimum lot size and emissions with two of the most commonly used carbon policies to reduce carbon emissions: cap-and-trade and carbon tax [6]. It is investigated that the combined price and production of various