MODELLING ON DUAL CHANNEL SUPPLY CHAIN MANAGEMENT UNDER SUSTAINABILITY

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by

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DECLARATION

I, hereby declared that the presented work in the thesis entitled **"Modelling on dual channel supply chain management under sustainability"** in fulfillment of the degree of **Doctor of Philosophy (Ph.D. Mathematics)** is the outcome of research work carried out by me under the supervision of Dr. Varun Kumar, working as a Associate Professor, in the Department of Mathematics of Lovely Professional University, Punjab, India and under the co-supervision of Assistant professor Dr. Arunava Majumder from Lovely Professional University, Punjab, India. In keeping with the general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigations. This work has not been submitted in part or fully to any other University or Institute for the award of any degree.

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CERTIFICATE

This is to certify that the work reported in the Ph.D. thesis entitled “Modelling on dual channel supply chain management under sustainability” submitted in fulfillment of the requirement for the reward of degree of **Doctor of Philosophy (Ph. D. Mathematics)** in the Department of Mathematics, is a research work carried out by Ruchi Chauhan, (Registration No.: 41700257), is bonafide record of her original work carried out under my supervision and that no part of thesis has been submitted for any other degree, diploma or equivalent course.

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Dedication

This work, is humbly dedicated to my father.

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ABSTRACT

In recent years, many leading manufacturers are combining the traditional retail channel with a direct online channel to reach a wider range of customers. Consequently, a "dual-channel supply chain management" under "price-dependent stochastic demand" is developed in this thesis where the standard product is provided through the offline channel, and "personalized product" is made available through the online channel. The idea of Batarfi et al. (2016) & Modak and Kelle (2018) regarding the dual channel is followed and upgraded in this study. A threshold limit is introduced in Chapter Three, which keeps a check on the selling price difference between the online and offline channels. No shifting of demands takes place if the difference between the selling price falls within a fixed and preassigned limit (Threshold limit). In Chapter Four, the model is expanded by incorporating sustainable pillars such as economic and environmental. As a result, it assists in diminishing carbon emissions during the production and transportation of the finished product. An investment in quality improvement is presented in Chapter Five, which helps the system to transfer from an "out-of-production" to an "in-control" state. In Chapter Six, the uncertainty in demand is addressed with the help of a "triangular fuzzy number". The proposed model of Chapter Six is further enhanced in Chapter Seven by incorporating the third pillar of sustainability namely, social sustainability. All the cost parameters are assumed to be a fuzzy number on lines of Khan et al. (2016) and Sarkar et al. (2019), respectively. In Chapter Eight, a modified "dual-channel supply chain model" is developed in the glass manufacturing industry. Moreover, two types of investment for reducing carbon emission and setup costs are incorporated in the model. The numerical experiments conducted in this work show the efficacy and performance of the proposed methods. In addition, the methods presented in Chapters Fourth, Fifth, and Eighth are successfully applied to handle uncertain demand and exhibit better results than the existing methods.

Notations

Decision variables of the model

L Length of the lead time for the retailer (days)

n Number of lots delivered from the manufacturer to the retailer in one production cycle, a positive integer

k Safety factor (units)

Q_1 Quantity of the standard product ordered by retailer (units)

Q_2 Quantity of the core product ordered for customization (units)

θ Probability that the production process may go out-of-control

Parameters of the model

C_p Production cost for standard product($/unit)

C_i Production cost for customized product (i=1,2,.....N)($/unit)

a_1 Number of customers prefer retail channel

a_2 Number of customers prefer online channel

P_1 Production rate for the standard product ($P_1 > a_1$) (Positive number)

P_2 Production rate for the core product for eventual customization ($P_2 > a_2$) (Positive number)

p_1 Retailer's selling price of the standard product ($p_1 > C_p$) ($/order)

P Total production rate of manufacture (Positive number)

p_i Manufacturer's selling price of the customized product (i=1,2...N) ($/order)

D_1 Variable demand of retail channel (units/year)

D_2 Variable demand of online channel (units/year)

D Total demand put forward to manufacturer (units/year)

ϕ_i Percentage of the core product stock used for customized product (i=1,2...N)

A_r Ordering cost of the retailer per order ($/order)

S_1 Manufacturer's setup cost for standard product ($/setup)

S_2 Manufacturer's setup cost for core product for customized product ($/setup)

r_v Holding cost rate of the manufacturer per unit per unit time ($/unit/unit time)

r_b Holding cost rate of the retailer per unit per unit time ($/unit/unit time)

C_b Unit production cost paid by the retailer (buyer) ($/unit)

C_{vr} Unit production cost paid by the manufacturer (vendor) ($/unit)

C_{ec} Carbon emission tax for single channel

β_1 Price sensitivity in retail channel (customer/day)

β_2 Price sensitivity in online channel (customer/day)

δ_1 Number of customers switching from retail channel to online channel

δ_2 Number of customers switching from online channel to retail channel

h_1 Manufacturer's holding cost which includes financial cost and storage cost ($/unit)

π Unit backlogging cost for the retailer ($/unit)

σ Standard deviation of demand per unit time

m Markup margin(percentage)

R Reorder point of the retailer (units)

M Random lead time demand which has a cumulative distribution function(c.d.f) F with mean DL and standard deviation $\sigma\sqrt{L}$

$E(\cdot)$ Mathematical expectation

s Replacement cost per unit defective item ($/unit)

S_{mc} Social cost parameter for manufacturing core product

S_{mp} Social cost parameter for manufacturing personalized product

E Greenhouse gas (CO_2) emissions from retailing system following single-route (ton/unit)

E' Greenhouse gas (CO_2) emissions from retailing system following dual-route (ton/unit)

$C_{ep,i}$ For exceeding the limit i, emissions penalty ($/year)

E_{li} Emissions limit i (ton/year)

l limit of emission number

CO_{2ae} Carbon emission after investment (tonnes)

x_1, x_2, & x_3 Carbon emission function parameter

$SP_{(CO_2)1}$ Scaling parameter for carbon emission investment function for single-channel

$SP_{(CO_2)}$ Scaling parameter for carbon emission investment function for dual-channel

$RP_{(CO_2)_1}$ Reduction parameter for carbon emission investment function for single-channel

$RP_{(CO_2)}$ Reduction parameter for carbon emission investment function for dual-channel

Transportation parameters

v fuel entailed by one truck in a single trip (gallons)

e_1 Amount of carbon emission from one gallon of diesel truck fuel in case of single-channel (ton/gallon)

e Amount of carbon emission from one gallon of diesel truck fuel in case of dual-channel (ton/gallon)

Cap_1 The capacity of vehicle engaged in the haulage of core products

Cap_2 The capacity of vehicle engaged in the haulage of personalized products

η_1 In an offline route, aggregate trucks of Cap_1 capacity in a single consignment (an integer)

η_2 In an online route, aggregate trucks of Cap_2 capacity in a single consignment (an integer)

E_{tr} During offline dispatch, the quantity of CO_2 emitted due to transportation

$E_{tr}^{'}$ During online dispatch, the quantity of CO_2 emitted due to transportation

Specific energy variables

ς_0' Inverse model's coefficient (KWh/unit)

ς_1' Predictor's coefficient (KWh/year)

SE Amount of specific energy consumed in per use (KWh/unit)

C_e cost of energy (\$/KWh)

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CHAPTER - 1

Introduction

1.1 Introduction

This chapter shed light on problems related to “supply chain management”- enhancement to dual channel, sustainability, the motivation of the study, scope and limitations, methodology, and some basic definitions.

A “supply chain” is a cooperation among various partners implicated in the designing of advanced products and amenity, acquiring resources, modifying them into intermediate and finalized products, and finally delivering to the consumers (Bhattacharya et al. , 2014). To fulfill the customer’s demand, various parties directly or indirectly collaborate. Therefore, the “supply chain” not merely includes supplier and manufacturers, but storehouses, distributors, transporters, and finally consumers are also included in it. Various functions are performed by the supply chain such as the manufacturing, advertising, operations, transportation, funds for new products, and services related to customers. Thus, for efficient operation, the supply chain management needs to take care of all the parameters initiating from product designing or service providing to finally selling, consuming, and ultimately disposing of the product by the consumer. This complete procedure encompasses designing, acquisition, drafting, manufacturing, transporting, and catering after sales (Lu and Swaminathan , 2015). Let’s consider an example where “a customer walks into Wal-Mart for a product. Wal-Mart informs the customer about the price and options he/she has in the product. As a result, a fund transfer from the customer to Wal-Mart is carried out. Wal-Mart communicates the data of sales and replenishment orders to the storehouses, who replenish the order by trucks, back to the Wal-Mart. Wal-Mart transfers funds to the storehouse after the replenishment of the order. The storehouse also conveys the pricing information and sends delivery to Wal-Mart". Thus, information, material, and fund flow are carried out in the entire supply chain. Similarly, when "a customer shops online for Dell computer, then the customer, Dell's Web site, the Dell assembly plant, and all of Dell's suppliers and their suppliers are supply chain partners. The customer gathers information related to the availability, variety, and price of the product from the Web site. Finally, after choice, the customer provides the order information and pays on the Web site". Thus, the supply chain utilizes the information of the order to accomplish the request of the customer which involves "the flow of information, product, and funds" among various partners. The main agenda of the supply chain is to accomplish the demand of customers and generate profit in the process. Additionally, it deduces that the customer is an intrinsic part of the supply chain.

Further, the out-of-the-blue technological boost is forging new chances for the manufacturer besides customers. The high level of globalization, more awareness of environmental issues, and intentions towards preserving the environment influence the choices of the customers. Beforehand, the manufacturer was only paying attention to the situation within the firm through an offline channel demonstrated by figure 1.1. Offline retail channel is any store or distribution house where the customer who purchase the products for immediate consumption. Thus, organizations were paying attention to what was happening within their four walls. Additionally, some customers also prefer offline shopping over online because in offline channels, customers can "personally inspect the products" and "ask for advice and assistance in selecting the product". Moreover, offline channel enables the buyer "to take their purchases home immediately" rather than waiting for delivery and paying for shipping costs. But some customers "dislike shopping in retail stores due to their busy schedules or some inconvenience in retail shops like a long queue, mismanagement, behavior of the retailer, weather conditions, etc". Thus, indicating the customer's channel preference behavior. Also, in the case of an online channel, the product is only described through text or image. Therefore, consumers cannot judge product quality in online stores. Whereas, in the case of the offline channel, the consumers experience the product before deciding whether to purchase it. Henceforth, this leads to the high operating costs of offline channels and makes the competition difficult with online channels. Consequently, manufacturers and distributors are enhancing to dual-channel, combining the traditional offline mode of shopping with the direct online mode, to address all the types of customers.

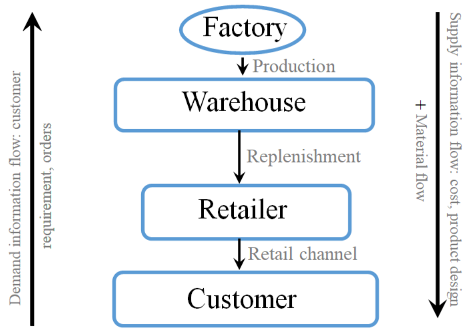


Fig. 1.1: Single-channel centralized system

Furthermore, with the adaptation of online platforms, manufacturers can reach out to their customers, expand their network of sales, and directly promote and sell their products. A business environment for supply chains where manufacturers can sell their "standard products" through retailers and "personalized products" through e-commerce develops by adopting a supply chain having "dual-channel" (Noh et al. , 2019) reflected by figure 1.2. In e-commerce or online channel the producer sells a product to a customer directly without involving any intermediaries. Since the online mode is convincing but the offline retail mode remains the prime source of benefits. Therefore, when a manufacturer enhances to an online mode while maintaining its offline mode independent then a supply chain with dual mode of shopping is formed. Besides, the online channel helps in collecting customer data and analyzing it for marketing and developing products. A dual channel enhances the customer's loyalty to a product. Additionally, the online channel is an effective tool for protecting the manufacturer from a drop in demand in the offline channel (Ranjan and Jha , 2019). Many leading brands and manufacturers, such as "Hewlett-Packard, IBM, Sony, Dell, Nike, and Apple", have upgraded their supply chain operations to a direct channel by looking at its benefit from the "early-to-market" advantage. Also, organizations like Pepsi, Wal-mart, and Adidas have upgraded their supply chain approaches depending on the shopper's decisions. A survey of McKinsey on senior marketing leaders found that with up-gradation to personalization policy, the revenue of firms increased from 5 to 15%[1]. The Wall Street Journal published a report on a renowned market research company Forrester Research, which claimed "more than half of the U.S. population do shopping online"[2]. Thus, organizations

[1] "https://www.mckinsey.com/business-functions/marketing-and-sales/our-insights/the-future-of-personalization-and-how-to-get-ready-for-it"
[2] "www.http://fortune.com/2016/06/08/online-shopping-increases"

are enhancing internet-based and sustainably manufactured products to cater the customers from allover the world. Moreover, new management techniques and framework for the organization is the need of the hour (Piñeiro-Chousa et al. , 2020; Ahmed and Sarkar , 2018). In firms, when manufacturers are following the "one size fits all" strategy, the product is called a "standard product" but when customers can influence the product that is similar to the "standard product" but is individually unique then it is called a "customized product". Thus, customization equips firms to quench the varieties in consumers' options on account of increased product variety (Rajagopalan and Xia , 2012). Consequently, addressing the question that why customization should be implemented by the firm. Additionally, companies upgrading to multiple product variants facilities with specific needs can gain a competitive advantage. Henceforth, it is beneficial for companies to enhance their ability to "meet customers' needs through maximizing individual customization, at a low cost", which can not be accomplished by standard products alone. Also, this increased global competition pressurizes companies to continuously strive for decreasing costs. Although upgrading the supply chain with the "dual-channel" is beneficial but the manufacturer faces various strategic and operational complexities like conflicts, competition, and lack of coordination in pricing policies between manufacturer and retailers. Therefore, restructuring the traditional channel structure and productively engaging the online channel to fulfill the personalized needs of customers and subsequently increase the profit is a pressing priority. Since, customer are inclined to a channel offering a product at minimum purchasing price and product quality. Therefore on the same parameters the performance of "the online and offline channels of supply chain" is analyzed.

Nevertheless, in real life, firms do not offer an unlimited variety because out of all factors (cost, channel competition, etc.) pricing decisions play a pivotal role. In a view of the fact that the customers always draw the comparison between the price information of offline and online stores so that they can buy at the lowest price the most suitable product. And mostly customers acquire "the product at a lower price and get more convenient maintenance services" from the online direct channel. Instinctively, extra product varieties will increase products price. Given this, if the firm increases the price of a "core product or customized product" in such a way that disparity increases beyond a limit (i.e., threshold limit) then shoppers start transferring from e-commerce to the retailer or on another way around depending on the cost of the item. Various academicians have considered uniform transferring of shoppers from e-commerce to retailers in the supply chain model having dual mode of shopping. Whereas some shoppers do not transfer to another channel as they are hesitant to purchase from another. But whenever the gulf between "standard and the customized product" is soared and customers begin switching channels then the idea of the threshold

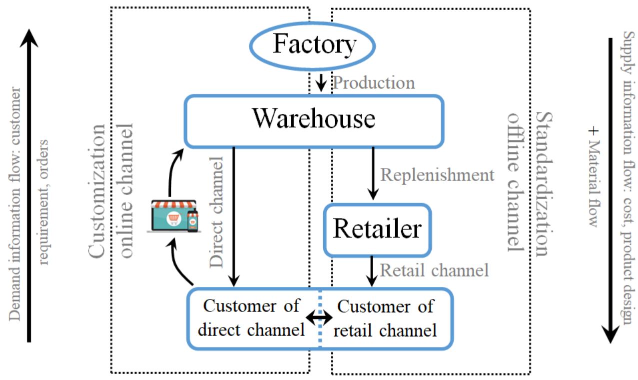


Fig. 1.2: Dual-channel centralized system

limit appears. Henceforth, the switching behavior intensifies the competition and reduces the profits for offline and online channels. "Price competition" is inevitable in the existent offline and online channel. The two mode of shopping can sell the same item at varying costs because consumers perceive the product differently. Besides, price competition is unavoidable between the offline and online channels. Further, the contracts related to price discounts and pricing schemes influence the competition among the channels in a supply chain having dual route. Also, the disturbance in the price of the product sold by "offline and online channels" becomes persistent over time. Although, the manufacturer and retailer can mitigate the conflict between the "online-offline channels" by coordinating and setting the appropriate price of the product (Li et al. , 2016).

Additionally, with the development of information and technology, the global economy is growing thereby shifting from mass production to personalized production and slowly to mass customization, demonstrated by figure 1.3 (Zhang et al. , 2019). Customization demonstrates the production strategy - planning, designing, producing, and servicing a product to the personalized demands of the customer without compromising with the economy leg of the firm. Ray-Ban enhanced the "do it yourself " program to "design your own" program thereby, allowing the customers of being expressive and independent [3]. Also, Oakley adopted "Your style is our style" and Ralph Lauren to "Stitch it" attitude towards customization. Moreover, with the advancement the power is shifting from firms' manufacturers to firms' retailers as personalization of the product is impossible without the information of the customers. In general, firms'

[3] https://configureid.com/2021/09/21/get-inspired-with-these-6-brands-winning-at-product-customization-examples/

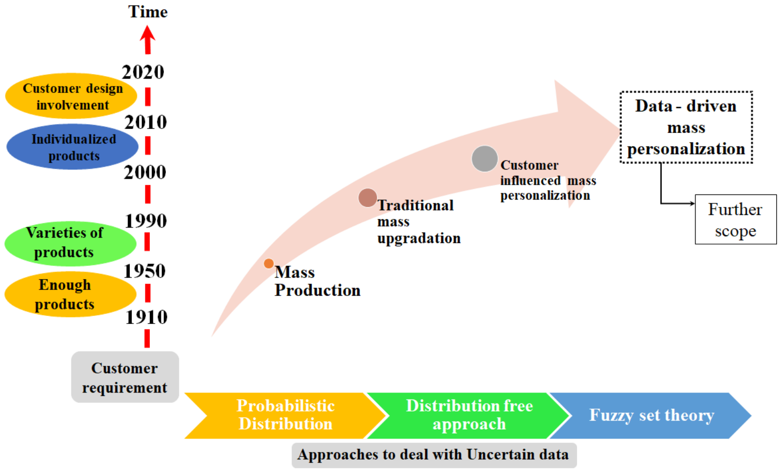


Fig. 1.3: Evolution in the customization facility

retailers accumulate more "demand information" as they are closer to end customers and interact with them more than the manufacturing leg of the firms (Cole and Aitken , 2020). Thus, "retailers tend to have greater power than manufacturers" because of their "enriched information systems and greater knowledge of customers" requirements and choices. As a result, huge retail chains such as "Wal-Mart, Carrefour, and Tesco", occupy a more predominant position in the market than do their suppliers (Gao et al. , 2016). Since they are aware of the "latest information related to the demand" of the market and adjust their strategies governing the product according to the change of market immediately and flexibly. But selling the product through the online channel also aids the firms in building superior customer manufacturer relations and obtaining vital demand information (Chen et al., , 2012). To fasten the integration of "online-offline channels" and enhance the competitiveness in supply chain firms like "Alibaba (online channel) and Suning (offline channel)", "JD.COM (online channel) and Walmart (offline channel)", "Tmall (online channel) and Freshhema (offline channel)" all are switching to cooperative environment from competitive environment. Consequently, this research focuses on improving the coordination and information sharing between manufacturers and retailers by enhancing to centralized supply chain management as it boosts the operational efficiency of the supply chain. Information sharing is the sharing of crucial information within a system, individuals, and businesses. Thus, firms must address the following four questions for enhancing the result of information sharing: (1) "what information should be shared", (2) "with whom information should be shared", (3) "how information should be shared", and (4) "when information should be shared" (Lotfi et al. , 2013). Also sharing the information and data in the supply chain brings many benefits to the firm such as it is easy to anticipate the products according to "customer demand" and "changes in the market". Information facilities alone cannot accomplish the process of “information sharing” smoothly. Firms managing the transparently and accurately the flow of information have good supply chain performance (Nabila et al. , 2022). Subsequently, sharing of the information enhances the supply chain’s optimization in different aspects - planning, controlling inventory, and services related to customers (Kembro et al. , 2017). The availability of historical and current demand data helps the manufacturer in reducing the variance in demand forecast. Although the role of data sharing within the firms is appreciated there are negative aspects also if the collaboration among the various partners of supply chain management is not carried out. For instance, Tesco utilized data of customers collected through customization software and club card data for analyzing the customers and promoting the product, distributing “gift coupons” and “discount vouchers”. Though it welcomed a lot consumers, it was not able to retain its loyal customers. This shows that, without collaborative decision-making, alone analysis of data cannot help the firms in conquering the competition and becoming successful. Because in collaborative supply chains, all the parties in the supply chain will come forward for utilizing the available information through multilevel discussions and formal agreements thus, introducing the concept and importance of “centralized supply chain management”.

In “centralized supply chain management”, there is a “single decision-maker” who tries to improve the entire system and enhance the profit. The decisions concerning the cost parameters of the retailer and the manufacturer are decided by the organization itself (Nair and Sebastian , 2017). Since the retailer and the manufacturer are considered as an integration organization in this policy. Whereas in a decentralized system, each member of the supply chain management tries to improve their structure that is, all the parties engaged in the supply chain are free to take their self-decisions, as well as they, do not have to share their private information. In the case, of a “decentralized supply chain”, several agents at the same echelon compete for limited resources or demand from the same group of customers. Also, the retailer depending on an EOQ policy takes the “replenishment decision” for each item which includes the inventory holding cost and setup costs. Henceforth, it can be observed that in the case of “centralized or integrated supply chain management” the manufacturer is powerful enough as it controls all the decisions. Accordingly, the “centralized supply chain” can increase its financial gain and profit margins. Therefore, the outcome of the two models adopted by supply chain management is different in terms of “efficiency” and “effectiveness”. The “centralized supply chain” might outperform in terms of “efficiency” and “effectiveness”, whereas the decentralized in flexibility and acceptance. Moreover, in reality, “decentralized supply chain management” achieves a lower profit in comparison to the “centralized supply chain” because of a lack of coordination and double marginalized effect. Although “decentralized supply chain management” employing coordination and collaboration can enhance its profitability and even equal it to a “centralized supply chain”. Consequently, “decentralized supply chain management” should deploy incentives and a win-win strategy among different partners of the “supply chain” in terms of trade or a contract (Bendadou et al. , 2021). But the firms adopting decentralized supply chain management may suffer under intense price competition. Centralization furnishes in decreasing variable costs related to “receiving of goods, longer freight distance, or lack of visibility to incoming goods”. Further in this context, centralization can be employed in combining traditional offline channels and online channels. Henceforward, the centralized supply chain is a state of coordination where the firm and its retailer are in good cooperation. Additionally, under uncertain demand, the upper hand of a centralized over a decentralized decision structure is reflected by eliminating the double marginal effect.

Greenness and bio-economy (i.e., “economic and social developments” that rely on sustainable sources (Kendir Cakmak et al. , 2021)) exhibit the concept of sustainability into the mainstream. Since they are interrelated toward efficient utilization of resources within the “cycle of economy” (D’Amato et al. , 2017). During the pandemic of COVID-19, sustainably developing the different networks of supply chain became very crucial (Barbier and Burgess , 2020; Tirkolaee et al. , 2020). In today’s turbulent world, the gravity of a “sustainable supply chain” is not hidden (Mardani et al. , 2019). Additionally, the adaptation of “sustainable development goals” helps the firms in changing the manner of development and use of technologies since they recognize that “action in one area will affect outcomes in others and that growth must be socially, economically, and environmentally balanced”. In 2015, United Nations Member States adopted the 17 Sustainable Development Goals (SDGs), which are an urgent call for action by all countries in a global partnership. These 17 SDGs are: (1) “No poverty”, (2) “Zero hunger”, (3) “Good health and well-being”, (4) “Quality education”, (5) “Gender equality”, (6) “Clean water and sanitation”, (7) “Affordable and clean energy”, (8) “Decent work and economic growth”, (9) “Industry, Innovation and infrastructure”, (10) “Reduced inequalities”, (11) “Sustainable cities and communities”, (12) “Responsible consumption and production”, (13) “Climate action”, (14) “Life below water”, (15) “Life on land”, (16) “Peace and justice strong institutions”, and (17) “Partnerships for the goals”. Figure 1.4 demonstrates the seven sustainable development goals considered and discussed at different stages of this research.

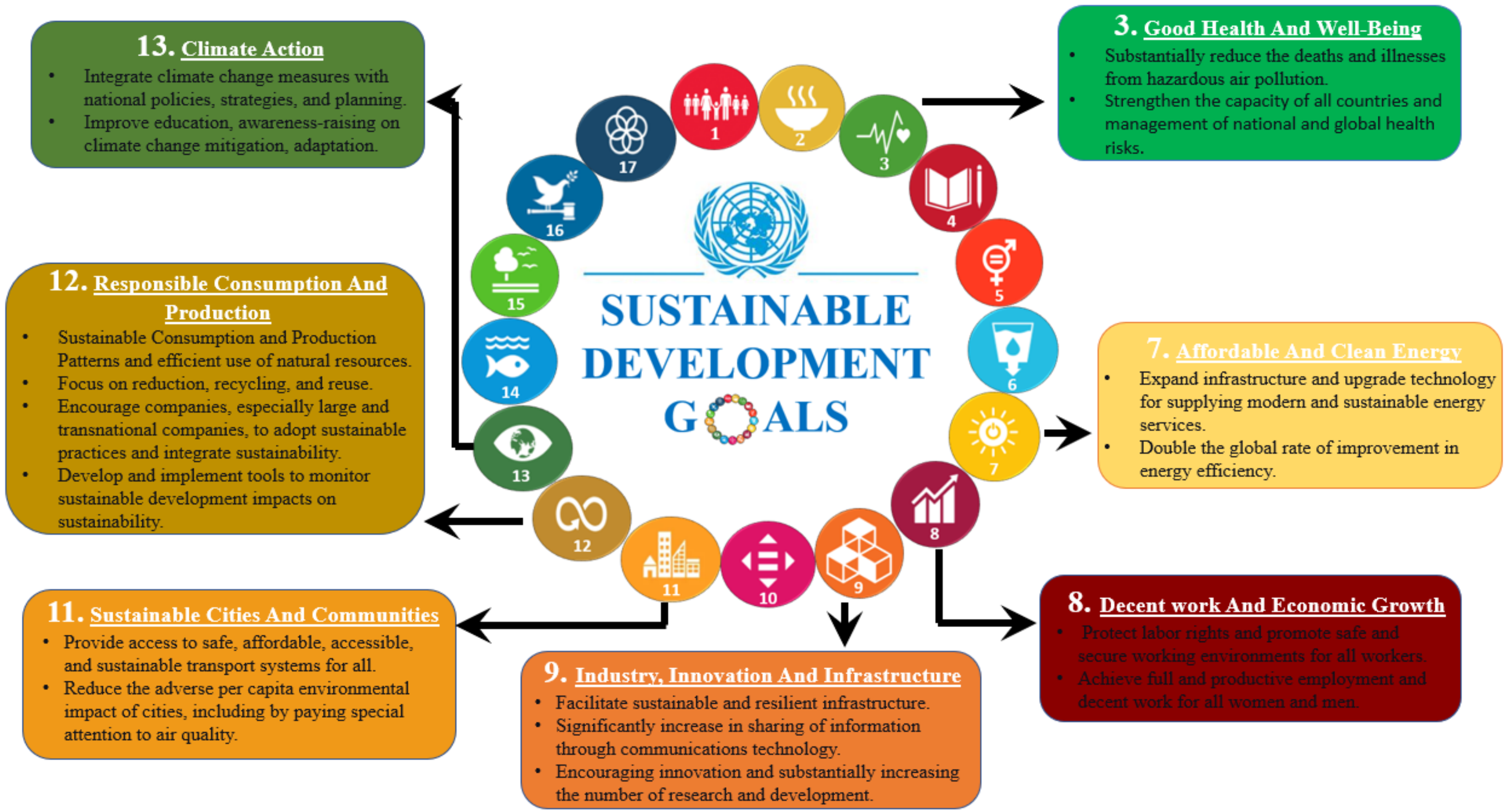


Fig. 1.4: Focused sustainable development goals of research

Environmental toil in supply chain management is broadly concerned with reducing greenhouse gas (GHG) emissions instead of energy consumption. The mother earth demands to finish fossil fuels utility, instead replace them with sustainable resources even during conveyance (Cormack et al. , 2021). Embodying smart technology with supply chains (SSC) helps in sustainably generating a maximum profit by reducing the exploitation of resources and their adverse environmental influence (Sarkar et al. , 2019). Also, "Carbon Trust" surveys reflects that "about 20% of customers prefer to buy green products even if they are more expensive than regular products" [4]. Customer consciousness towards environment is a "market-driven factor" that facilitates the "environmental sustainability" of the supply chain. Customers behavior for actual green purchasing is directly related with their awareness about the green products, which partly depends on the green-marketing efforts implemented by product providers that is manufacturer and retailer. Thus, the infusion of sustainability in the supply chain has become crucial among all players (suppliers, manufacturers, distributors, and retailers). Therefore, players are propelled to not only upgrade customer services by increasing customer loyalty and better customer insights but also simultaneously minimize carbon emissions. There are various sources (e.g., production and transportation) of carbon emission involved at various stages of the supply chain. Manufacturing and transit of the item add carbon stress yet they are never considered simultaneously by the management of the "dual-channel supply chain". Thus, this research explores supply chain management with dual-route by taking into account the energy consumption and energy stress amplified during the transportation of the consignment in the model. Consequently, there are costs related to the environment which influence the supply chain and its profit and thus are unavoidable. Additionally, the enhanced demands for innovative green products from shopkeepers have also transformed the supply chain into highly challenging to manage. Moreover, innovative and sustainably developed products enable a firm to achieve higher profit margins by bringing loyalty and better insight into customers. Thus, adopting green supply chain activities has a lot of boons on the environment (Becerra et al. , 2021). As a result, this research addresses how carbon emissions are influencing customers' choices. Also, according to Dwivedi et al. (2022) technology is an integral element of the industry, but on contrary, it is also viewed as part of the problem by industry and wider society. Firms are enhancing "sustainable supply chain management (SSCM)" as it caters to the economic and non-economic issues in the "supply chain simultaneously" (Mandal et al. , 2021). Thus, every industry and government amalgamate sustainability with "supply chain management (SCM)". government charges penalties to the firms once they cross the carbon emission ceiling. The "European Union Emission Trading System (EU-ETS)" works on a "cap-and-trade policy" where firms are penalized if they cross the carbon emission limit. If emissions are less than the

[4] "http://www.carbontrust.com/news/2011/07/consumer-demand-for-lower-carbon-lifestyles-is-puttingpressure-on-business (accessed on 01.07.2011)"

limit, firms can trade with another firm that surpasses the limit. Thus, many organizations are emphasizing reducing greenhouse gas emissions by investing in green technologies (Reddy et al. , 2020).

According to WHO/ILO, 750,000 people died in their jobs because of the long working hours, 400,000 people died because of respiratory, and approximately 1.9 million people lost their lives in 2016 alone because of work-related disease or injuries [5]. In 2016 itself, approximately 150 million children were victims of child labor, worldwide[6]. The essence of the social pillar is reflected in the development of "human capital, employee care, health and safety, respect for human rights in the workplace, and a ban on child labor". Additionally, there are seven core subjects of social responsibility - "human rights, labor practices, fair operating practices, organizational government, consumer issues, the environment, and community involvement and development" as per the standard, ISO 2600 [7]. So, integrating the social aspect with the supply chain is equally important while considering sustainability in supply chain management. "Socially sustainable development" is achieved once "relationships, structures, and systems dynamically utilize the capability of the current generation to create well-being and healthy communities". Furthermore, Miemczyk et al. (2012) and Meixell and Luoma (2015) manifested that the organizations need to consider the social pillar of sustainability in the operation within the network of the supply chain. "Social sustainability" is the prime focus of the "supply chain management" discussion as there is an increase in its demand because of reputation damage pressure (New , 2015). More attention needs to be paid to the social pillar for reinforcing it same as the environmental and economical dimensions. The largest retailer in the world Walmart enhances all the three sustainable pillars-economic, environmental and social. Simultaneously, committing to a zero waste goal, assisting workers in upgrading their careers, improving energy efficiency and haulage, and taking necessary initiatives for making its supply chain greener. So, investing in social pillars helps the firms in improving their public image or countervailing the negative aspects of their industries during operation. The idea of volunteering lead to social responsibility. The social pillar links the business and society. On the one hand, activities of the businesses influences the life's quality of the people whereas, on the other way, it influences the economic, political, and environmental sectors. Consequently, for creating their images, modern companies work like a responsible behavior for both society and the environment. Thus, the current research integrates the social as well as the environmental pillar of the sustainability in "dual channel supply chain" model.

[5] "https://www.who.int/news/item/16-09-2021-who-ilo-almost-2-million-people-die-from-work-related-causes-each-year"
[6] "International Labour Organization. Global estimates of child labour: results and trends, 2012-2016."
[7] "Ro-Ting Lin, David Koh, in Encyclopedia of Environmental Health (Second Edition), 2019"

In real situations, product quality plays a crucial role in the market economy. Moreover, in the global market, the quality of the product influences the competition between the firms. As a result, to escalate the share of the firm in the market, the focus should be given to quality. Furthermore, the system of production can go from "in-control" to an "out-of-control" state, due to overproduction resulting in the manufacturing of defective products. Accordingly, "managers and decision-makers" need to pay attention to the production system because their ability to control the production of defective products, produced affects the system's performance. Consequently, researchers have proposed many models for analyzing the impact of imperfect products. The quality improvement also influences the demand (Dey et al. , 2021; Sarkar and Giri , 2020). Most shopkeepers segregate the perfect items and imperfect items to be sure of the quality and simultaneously invest in green technology to curtail CO_2 ejection. Further, a sustainable model is exemplified by including out-of-order products and manageable CO_2 ejections from the firm (Mashud et al. , 2020). From years the world is coping with various disasters such as "natural disasters, epidemics, and chemical explosions" (Singh et al. , 2020) and these disasters has disrupted lives of humans and smooth operation of countries (Mitrega and Choi , 2021). Thus , disrupting the management of supply chain (Tirkolaee et al. , 2022). Henceforth, eco-friendly products produced with proper inspection using a sustainable supply chain help in reducing the disruption in the supply chain. Also, in the current situation, the demand function can be either known or unknown. If the product has an existing market, then the previous demand data helps in estimating the product's demand (Basiri and Heydari , 2017). On contrary, if a product is new to a market, then the demand will be uncertain and fuzzy (Kong et al. , 2022). In real situations, unpredictability is an inseparable from the supply chain and imposes serious problems. As per the nature and the circumstances of the parameters which are unpredictable, there are several techniques for tracing uncertainty (Ghelichi et al. , 2018). Based on data availability, uncertainty can be categorized into three types. The first one, due to the insufficient knowledge about data to estimate the probability of plausible future situations is called "deep uncertainty". Thus, to overcome this uncertainty robust optimization approaches are utilized. In second, the randomness unpredictability in the data as it represents the arbitrary nature of the parameters and "robust scenario-based stochastic programming models" are utilized for dealing with it. Lastly, it presumes unpredictability because of inadequate or inconstant information on the parameters and consequently, possibilistic programming is taken into account (Bairamzadeh et al. , 2018). The demand of customers is one of the frequently occurring uncertain parameters. Thus, the demand parameter is identified by randomness uncertainty because of absence of sufficient data about the probability distribution of the demand. Henceforth, the "distribution-free approach" and fuzzy numbers are subsumed as one of "the most prominent tools to model imprecise data". When decision-makers are not able to determine the sharp boundaries of the demand, then fuzziness is used.

1.2 Research gap

The following points illustrates the research gap of the study.

1.2.1 Unequal customer shifting in dual channel

So far in dual channel supply chain management, only equal shifting of customer was considered from one channel to another. But, this assumption is not realistic at all as customers may abstain themselves from purchasing the product. In this study, unequal shifting of customers are considered from online to offline as vice-versa.

1.2.2 Threshold limit

As an unique concept, this study introduces a threshold limit which is an indicator stating that shifting of customer only occurs if the selling price difference between the online and offline channel crosses the specified limit price. This concept was not considered by previous literature.

1.2.3 Uncertainty in dual channel supply chain

Previous literature only concerned about either fuzzy or probabilistic uncertainty whereas, this study uses both fuzzy and probabilistic uncertainty in dual channel supply chain and compares them. In this study distribution free approach is also used where no specific probability distribution is considered.

1.2.4 Introduction of sustainable pillars in dual channel supply chain

This dissertation has concerned about many aspects of sustainability pillars which are not assumed in previous studies. This study analyses the effect of carbon tax, penalty, emission through production and transportation in a dual channel retailing system. No previous literature was found considering all pillars of sustainability in a dual channel supply chain.

Definitions

"A *fuzzy set* 'A' is defined by a membership function $\mu_A(x)$ which maps each and every element of X to [0,1]", that is

$$\text{“}\mu_A(x) \to [0,1]\text{”},$$

"where X is the underlying ground set. So, a fuzzy set is a set whose boundary is not clear and whose elements are characterized by a membership function as defined above".

"*Support* of a fuzzy set is a crisp subset of the ground set X". It is defined as

$$\text{“}Supp(A) = \{x|\mu_A(x) > 0, x \in X\}\text{”}$$

"The *level set* of a fuzzy set A is the index set of A". It is denoted by A and is defined by

$$\text{“}\Lambda_A = \{\alpha|\mu_A(x) = \alpha, \alpha \geq 0, x \in X\}\text{”}$$

"The $\alpha-$*cut of* of a fuzzy set A is a crisp subset of the ground set X, containing all the elements, whose membership grade is greater than or equal to". It is defined by

$$\text{“}A_\alpha = \{x|\mu_A(x) \geq \alpha, x \in X\}\text{”}$$

"A *triangular fuzzy number* is a fuzzy set. It is denoted by $A = (a_1, a_2, a_3)$" and is defined by

$$\mu_A(x) = \begin{cases} 0 & a_1 \leq x, \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3, \\ 0 & x \geq a_3, \end{cases}$$

"where, $a_1, a_2, a_3 \in R$, $A \in F_N$, F_N is the set of triangular fuzzy numbers and are reflected by figure 1.5".

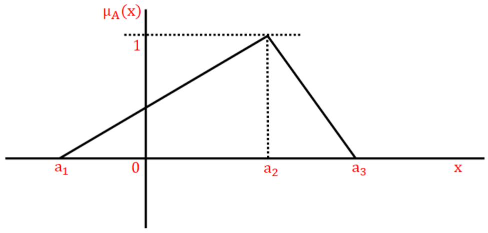


Fig. 1.5: Variation of membership degree triangular fuzzy number $A = (a_1, a_2, a_3)$

"The transformation of a crisp set to appropriate fuzzy set is referred as fuzzification whereas when fuzzy set is converted into an appropriate crisp set then it is called defuzzification".

"For triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$, the $\alpha-$cut of $\widetilde{A}$ is $A(\alpha) = [A_L(\alpha), A_U(\alpha)]$, $\alpha \in [0,1]$, where $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $A_U(\alpha) = a_3 - (a_3 - a_2)\alpha$." The signed distance of $\widetilde{A}$ to $\widetilde{0}_1$ is

$$d(\widetilde{A}, \widetilde{0}_1) = \frac{1}{4}(a_1 + 2a_2 + a_3).$$

"In the current research triangular fuzzy number is considered. Thus, let x be the triangular fuzzy number, $\widetilde{x} = (x - \Delta_1, x, x + \Delta_2)$, where $0 < \Delta_1 < x$ and $0 < \Delta_2 \leq 1 - x$. "Then, the distance of triangular fuzzy number is given as

$$\text{“}d(\widetilde{x}, \widetilde{0}_1) = x + \frac{1}{4}(\Delta_2 - \Delta_1)\text{”}$$

"Where, Δ_1 and Δ_2 are decided by decision-makers".

1.2 Statement of the Problem

Figure 1.6 is demonstrating that the manufacturing and construction sectors add enormously carbon burden to the environment whereas industries and transportation are next in the queue for adding carbon burden. Therefore, the proposed models help the firms in regulating their carbon emissions along with profit. Moreover, enhanced "dual-channel supply chain management" is adopted in the model which helps the firms in catering to the personalized demands of the customers irrespective of traditional supply chain management.

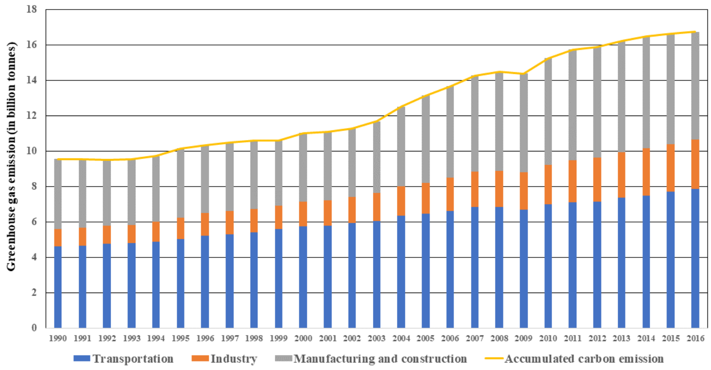


Fig. 1.6: Green house gas emission by transportation, industry, and manufacturing & construction, World

1.3 Motivation of the Study

The government of India proposed a bill on the conservation of energy, in 2022 by calling it the bill of the future. The bill aims to hasten the green transition by regulating energy consumption, mandating the use of non-fossil fuels by industries, and setting efficiency standards for vehicle ships and equipment as part of India's climate action architecture. It specifies a trading scheme for carbon credit. Many companies and industries will be affected by these policies and protocols directly or indirectly. Therefore, we are motivated to implement some sustainable methods with a less cost that is economically feasible and applicable to enhanced supply chain management.

1.4 Scope and Limitations

This research is limited to one manufacturing − one retailer and does not consider the recycling polices. The organization of thesis is as follows. The proposed methods' algorithms will be detailed in Chapters two through seven. In chapter eight, summary and conclusions will be demonstrated.

CHAPTER - 2

Sustainability in a smart supply chain management with improved customization policy

2.1 Definition of the problem

A juxtaposition between a traditional offline and a online channel on “supply chain model with single manufacturer and single retailer” is framed in this chapter. Most of the literature’s used either probabilistic uncertainty or demand variability but a few articles were found which considered both of the characteristics simultaneously in a demand function. In addition, to that this chapter uses “max-min distribution-free approach” to deal with the “stochastic uncertainty” along with price sensitivity. Since demand data or the exact mean and standard deviation is required for estimating a probability distribution of demand which is money as well as time consuming. Therefore, to overcome all these constraints this chapter considers “distribution-free approach” to obtain managerial decisions with the help of known mean and standard deviation.

To implement an online channel, manufacturer and retailer should negotiate properly as a “threshold value” has been incorporated in this chapter. The “threshold value” signifies that if the prices difference between online and offline exceeds beyond a specified limit then customers migrate from retail channel to online channel or vice versa which should influence the profit of the channels individually along with the firm as a whole.

2.2 Assumptions

Following points enlist the various assumptions assumed for formulating the mathematical model.

1. The company adopts "centralized dual-channel supply chain model" is assumed with customization strategy. Hence, the product is available to consumers through a retailer mode and an internet-based direct mode. Further, demand is deemed to be variable as well as random (Modak and Kelle , 2018).

2. A proportion of the number of customer who refuses to purchase items through retail channel choose to purchase "customized product" (Modak and Kelle , 2018).

3. "Single-setup multiple-delivery policy" is used to deliver the product in retail channel whereas "make-to-order policy" is used for online channel.

4. When difference in the online and offline channel's cost falls within a fixed and preassigned limit ("Threshold limit") no customer shifting would occur.

2.3 Mathematical model

This section describes the demand function, profit function, "distribution-free approach" optimal decision of the supply chain, and solution algorithm of this chapter.

2.3.1 Function for demand

Consumers are miscellaneous in nature in their inclination towards the "standard" or the "customized product". Many factors influence customer's intentions such as "price, variety of products, and incapability of customer's to reach out to the retail stores". Zhang et al. (2015) and Huang et al. (2013) used linear functions of demand for the "standard and customized product". Following functions of demand for retailer and online channels are obtained by extending the work of Huang and Swaminathan (2009) and Hua et al. (2010).

The function of demand for offline channel is given by:

$$D_1 = a_1 - \beta_1 p_1 + \delta_1 \left(\sum_{i=1}^{N} p_i - p_1 \right) \tag{2.1}$$

The function of demand for online channel is given by:

$$D_2 = a_2 - \beta_2 \sum_{i=1}^{N} p_i - \delta_2 \left(\sum_{i=1}^{N} p_i - p_1 \right) \tag{2.2}$$

Where, the following points explains the various parameters used in demand function.

• Parameter a_1 and a_2 are demand of customers without having any certain and variable components of offline and online channel.

• The price sensitivity coefficients β_1 of retailer's channel and β_2 of manufacturer's channel, represents the amount of fall/rise in the demand of market when various modes rise/fall the price by one dollar. Thus, $\beta_1 p_1$ and $\beta_2 \sum_{i=1}^{N} p_i$ represents change in customers because of price sensitivity.

• δ_1 and δ_2 represents the shifting of customers from offline channel to the online channel or vice-versa.

• The price of the "standard product" for retailer is given by

$$\text{Markup} = \frac{\text{Selling price (for retailer)} - \text{Cost price (for retailer)}}{\text{Cost price (for retailer)}}$$

$$m = \frac{p_1 - C_r}{C_r}$$

$$\text{markup} \times \text{Cost price} + \text{Cost price} = \text{Selling price} = \text{Cost price}(1 + \text{markup})$$

$$p_1 = mC_r + C_r = C_r(m+1)$$

$$\text{Selling price} = \text{Cost price of production}(1 + \text{markup})^2$$

$$p_1 = C_p(1+m)^2$$

since $C_p = C_r(1+m)$ and the cost of "customized product" for manufacturer is given by $p_i = C_i(1+m)$ where 'm' is a "markup margin". Henceforth, $\delta_1 \left(\sum_{i=1}^{N} p_i - p_1 \right)$ and $\delta_2 \left(\sum_{i=1}^{N} p_i - p_1 \right)$ indicates the change is number of customers because of δ_1 and δ_2

Assuming fixed "markup margin" for "standard as well as customized products" we get the following equations

$$D_1 = a_1 - \beta_1 C_p(1+m) + \delta_1(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right) \tag{2.3}$$

The demand function for online channel is given by:

$$D_2 = a_2 - \beta_2(1+m)\left(\sum_{i=1}^{N} C_i\right) - \delta_2(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right) \tag{2.4}$$

2.3.2 Profit function

This section includes profit equations of manufacturer and retailer for "standard and customized products".

I. **"Manufacturer's profit for core product"**

Per unit of time of the profit of the manufacturer by selling the "core product" from the retail mode is

$$\nabla_1 = \text{Revenue} - \text{Setup cost} - \text{Holding cost} - \text{Manufacturer's production cost}$$

$$\nabla_1 = C_p(1+m)D_1 - \left[\frac{S_1 D_1}{nQ_1} + \frac{r_v C_p Q_1}{2}\left[n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right]\right] - C_{vr}D_1 \tag{2.5}$$

Where, the following points explains the various cost components used in equation 2.5.

• $C_p(1+m)D_1$ is revenue and $C_p(1+m)$ is the selling price of manufacturer for "standard product".

• $\frac{S_1 D_1}{nQ_1}$ is setup cost for retail channel. Figure 2.7 displays that as "Q_1 is the total order quantity of all retailers, manufacturer produces nQ_1 quantity where, n is a positive integer".

• The holding cost of manufacturer is $\frac{r_v C_p Q_1}{2}\left(n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right)$. Thus, the per unit time per unit item expected holding cost is

$$\frac{r_v C_p Q_1}{2}\left(n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right).$$

• $C_{vr}D_1$ represents the production cost of manufacturer for "standard product".

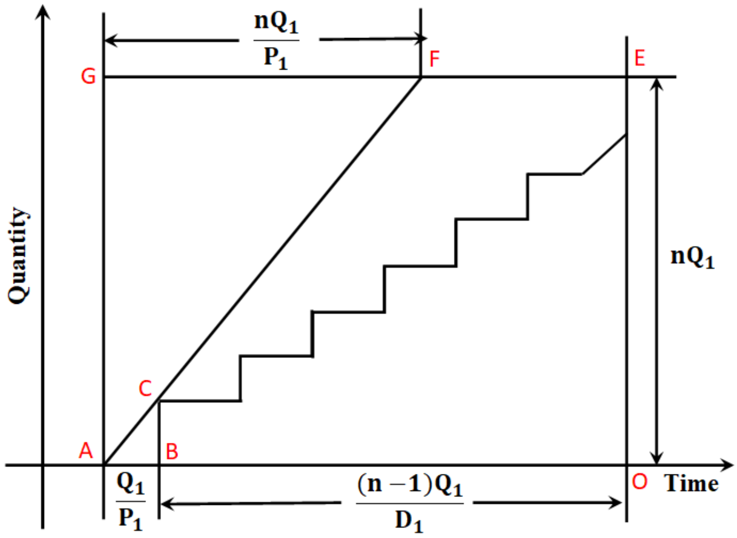


Fig. 2.7: Manufacture's inventory position

II. **"Manufacturer's profit for customized product"**

Figure 2.8 exhibits the online channel inventory of manufacturer's where as the per unit of time manufacturer's profit by selling the "personalized product" from the online mode is

$$\nabla_2 = \text{Revenue} - \text{Setup cost} - \text{Holding cost} - \text{Manufacturing cost}$$

$$\nabla_2 = \sum_{i=1}^{N} C_i(1+m)\phi_i D_2 - \left[\frac{S_2 D_2}{Q_2} + \left(\frac{h_1 Q_2}{2}\right)\left(1 - \frac{D_2}{P_2}\right) + \sum_{i=1}^{N} C_i \phi_i D_2\right] \tag{2.6}$$

Where, the following points explains the various cost components used in equation 2.6.

• $\sum_{i=1}^{N} C_i(1+m)\phi_i D_2$ is revenue and $\sum_{i=1}^{N} C_i(1+m)\phi_i$ is the selling price of manufacturer for "customized product".

• $\frac{S_2 D_2}{Q_2}$ is setup cost as manufacturer follows "make-to-order policy" where he manufacture's the product on demand by customer.

• The holding cost of manufacturer is $\frac{h_1 Q_2}{2}\left(1 - \frac{D_2}{P_2}\right)$.

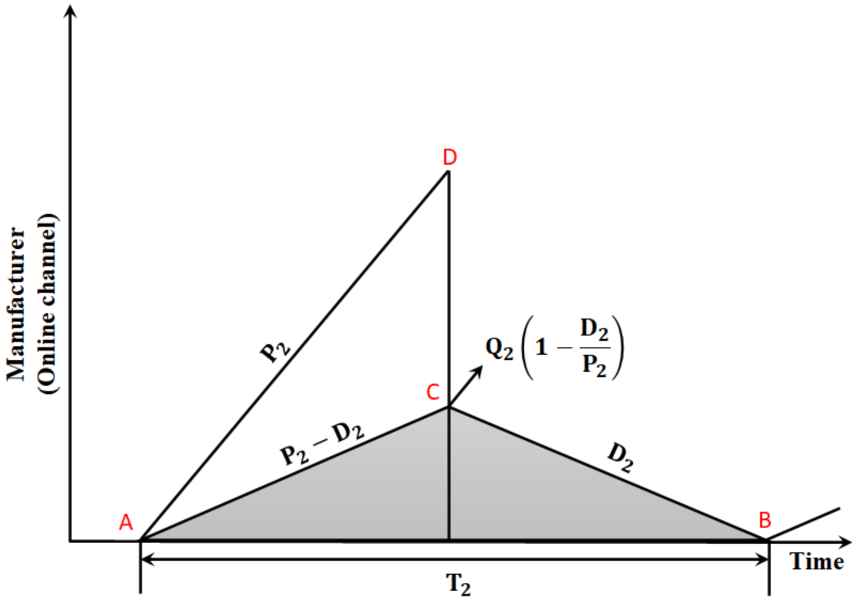


Fig. 2.8: Manufacture's online channel inventory under the EPQ model

Thus, the per unit time per unit item expected holding cost is

$$\frac{h_1 Q_2}{2}\left(1-\frac{D_2}{P_2}\right).$$

• Each customization is composed of a production cost $\sum_{i=1}^{N} C_i$, total production cost of the vendor will be $\sum_{i=1}^{N} C_i \phi_i D_2$.

III. **"Retailer's profit"**

Figure 2.9 manifests the inventory pattern between manufacturer and retailer whereas the per unit time profit of the retailer by selling the "core product" is

$$\nabla_3 = \text{Revenue} - \text{Cost of ordering} - \text{Cost of holding} - \text{Cost of shortage} - \text{Cost of lead time crashing}$$

$$\nabla_3 = C_p(1+m)^2 D_1 - \left[\frac{A_r D_1}{Q_1} + r_b C_b\left(\frac{Q_1}{2} + R - D_1 L\right) + \frac{\pi D_1}{Q_1} E(M-R)^+ + \frac{D_1 CL}{Q_1}\right] \tag{2.7}$$

Where, the following points explains the various cost components used in equation 2.7.

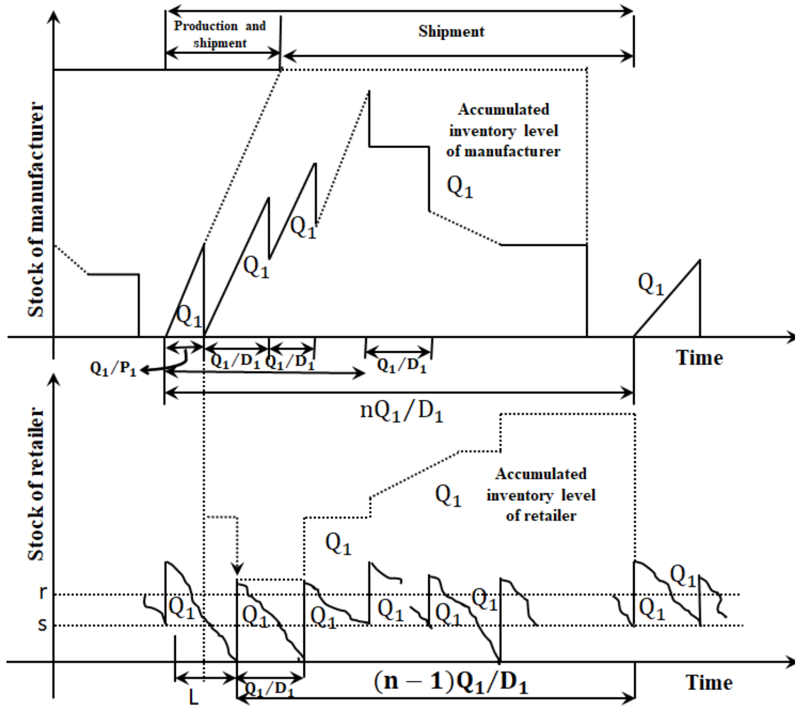


Fig. 2.9: Manufacture's and retailer's inventory pattern

• "$C_p(1+m)^2D_1$ represents revenue of retailer". $C_p(1+m)^2$ is the selling price for retailer for "standard product".

• "The ordering cost per unit time for retailer is $\frac{A_rD_1}{Q_1}$".

• "When the level of inventory reaches the reorder point R, quantity Q is ordered by the retailer. $R-D_1L$ represents the expected level of inventory before the order is received by the retailer where as $Q+R-D_1L$ represents expected inventory level immediately after the delivery of quantity Q. Consequently, $\frac{Q}{2}+R-D_1L$ is the average inventory over a cycle. Henceforth, the retailer's expected holding cost per unit time is $r_bC_b\left(\frac{Q_1}{2}+R-D_1L\right)$."

• "M is the stochastic lead time demand and R is the reorder point for retailer then, the expected shortage at the end of the cycle is expressed as $E(M-R)^+$ for retailer resulting into $\frac{\pi D_1}{Q_1}E(M-R)^+$ as shortage cost."

• "The lead time crashing cost per unit time is $\frac{D_1CL}{Q_1}$."

Sum of profit, ∇_S of the "supply chain with single-channel" is given by adding ∇_1 and ∇_3. We get,

$$\nabla_S = \nabla_1 + \nabla_3 \tag{2.8}$$

$$\nabla_S = C_p(1+m)D_1 - \left[\frac{S_1D_1}{nQ_1} + \frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]\right] - C_{vr}D_1$$

$$+C_p(1+m)^2D_1 - \left[\frac{A_rD_1}{Q_1} + r_bC_b\left(\frac{Q_1}{2}+R-D_1L\right) + \frac{\pi D_1}{Q_1}E(M-R)^+ + \frac{D_1CL}{Q_1}\right] \tag{2.9}$$

2.3.3 "Distribution-free approach"

We elaborate the "distribution-free approach" by using the following points

• Any specific probability distribution should not be considered for any random variable in this approach. A class of "cumulative distribution function(c.d.f)" having "mean D_1L" and "standard deviation $\sigma\sqrt{L}$" is considered.

• The "max-min distribution-free approach" is applied to obtain the values of the decision variables. In this approach, initially the worst possible case (i.e., the expression for minimum profit is obtained which is then maximized for the best profitability).

• In this article we use a max-min method for profit maximization which is the opposite of Moon and Gallego (1993) "min-max distribution-free approach". Moreover, instead of assuming only uncertainty, this article provides demand variability also. Therefore, a modification is added in the inequality used by Moon and Gallego (1993) with a max-min approach and incorporation of variability in demand function.

• The lead time demand M is depending on D_1 which further depends on the cost of the retailer $C_p(1+m)^2$ which leads to variability in the demand. Moreover, randomness in demand is accomplished in additive form (Petruzzi and Dada , 1999; Sarkar at al. , 2018b; Modak and Kelle , 2018).

The underneath inequality is applied for solving the model of the present chapter.

Proposition 2.1.

$$E(M-R)^+ = E((D_1L+X)-R)^+$$

$$\leq \left[\frac{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})}{2}\right] \tag{2.10}$$

where, $R = D_1L+k\sigma\sqrt{L}$ *is reorder point,* D_1L *is the lead time demand,* $k\sigma\sqrt{L}$ *is safety stock, and* k *is a safety factor.*

Proof.

$$E(M-R)^+ = \frac{|M-R|+(M-R)}{2}$$

$$E(M-R)^+ \leq \frac{\sqrt{E(M-R)^2}+E(M-R)}{2}$$

Considering, $M = D_1\sqrt{L}+X$ summation of variability and randomness.

$$E(M-R)^+ \leq \frac{\sqrt{E(D_1\sqrt{L}+X-R)^2}+E(D_1\sqrt{L}+X-R)}{2}$$

where, $R = D_1L + k\sigma\sqrt{L}$ is a safety factor.

$$E(M-R)^+ \leq \frac{\sqrt{E(D_1\sqrt{L}+X-D_1L-k\sigma\sqrt{L})^2}+E(D_1\sqrt{L}+X-D_1L-k\sigma\sqrt{L})}{2}$$

A worst possible case is taken into account for distribution of random variable D_1 having mean D_1L and standard deviation $\sigma\sqrt{L}$. We get,

$$E(M-R)^+ = \frac{\sqrt{E(X+k\sigma\sqrt{L})^2}+E(X+k\sigma\sqrt{L})}{2}$$

$$= \frac{\sqrt{E(X^2+k^2\sigma^2L+2Xk\sigma\sqrt{L})}+E(X+k\sigma\sqrt{L})}{2}$$

$$= \left[\frac{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})}{2}\right]$$

Thus, using the inequality 2.10 the expected sum of profit ∇_S of the "centralized supply chain having single-channel" is given by adding ∇_1 and ∇_3. We get,

$$\nabla_S = C_p(1+m)D_1 - \frac{A_rD_1}{Q_1} - r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right) - \frac{D_1CL}{Q_1}$$

$$-\frac{\pi D_1}{Q_1 2}\left[\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})\right]$$

$$+C_p(1+m)^2D_1 - \left[\frac{S_1D_1}{Q_1n}+\frac{r_vQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]\right] - C_{vr}D_1 \tag{2.11}$$

And expected aggregate profit of the "centralized dual-channel supply chain" ∇_D is given by adding ∇_1, ∇_2, and ∇_3. We get,

$$\nabla_D = C_p(1+m)D_1 - \left[\frac{S_1D_1}{nQ_1}+\frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]\right] - C_{vr}D_1$$

$$+\sum_{i=1}^{N} C_i(1+m)\phi_iD_2 - \left[\frac{S_2D_2}{Q_2}+\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{D_2}{P_2}\right)+\sum_{i=1}^{N} C_i\phi_iD_2\right]$$

$$+C_p(1+m)^2D_1 - \left[\frac{A_rD_1}{Q_1}+r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)+\frac{D_1CL}{Q_1}\right]$$

$$-\frac{\pi D_1}{Q_1 2}\left[\sqrt{\sigma^2 L+(D_1 L)^2+k^2\sigma^2 L-2D_1 Lk\sigma\sqrt{L}}+(D_1 L-k\sigma\sqrt{L})\right] \tag{2.12}$$

2.3.4 Optimal decision of the supply chain

Since, equation 2.12 are non-linear in nature so for a positive definite integer 'm'', we take partial derivative of the profit with respect to k, Q_1, and Q_2 for deriving the solutions which are optimal in nature.

$$\frac{\partial \nabla_D}{\partial k}=\left(-r_b C_b \sigma\sqrt{L}-\frac{\pi D_1}{2Q_1}\left[\frac{k\sigma^2 L-D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L+(D_1 L)^2+k^2\sigma^2 L-2D_1 Lk\sigma\sqrt{L}}}-\sigma\sqrt{L}\right]\right) \tag{2.13}$$

$$\frac{\partial \nabla_D}{\partial Q_1}=\frac{A_r D_1}{{Q_1}^2}-\frac{r_b C_b}{2}+\frac{S_1 D_1}{n{Q_1}^2}+\frac{D_1 CL}{{Q_1}^2}-\frac{r_v C_p}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]+$$

$$\frac{\pi D_1}{2{Q_1}^2}\left[\sqrt{\sigma^2 L+(D_1 L)^2+k^2\sigma^2 L-2D_1 Lk\sigma\sqrt{L}}+(D_1 L-k\sigma\sqrt{L})\right] \tag{2.14}$$

$$\frac{\partial \nabla_D}{\partial Q_2}=\frac{S_2 D_2}{{Q_2}^2}-\frac{h_1}{2}\left(1-\frac{D_2}{P_2}\right) \tag{2.15}$$

Now for definite integer m, the value of k, Q_1, and Q_2 are obtained by equating equations 2.13, 2.14, & 2.15 to zero that is

$$\frac{\partial \nabla_D}{\partial k}=0 \tag{2.16}$$

$$\frac{\partial \nabla_D}{\partial Q_1}=0 \tag{2.17}$$

$$\frac{\partial \nabla_D}{\partial Q_2}=0 \tag{2.18}$$

we get,

$$Q_2^*=\sqrt{\frac{S_2 D_2}{\frac{h_1}{2}(1-\frac{D_2}{P_2})}} \tag{2.19}$$

$$Q_1^*=\sqrt{\frac{-D_1 CL+S_1 D_1+A_r D_1+\frac{\pi D_1}{2}\sqrt{\sigma^2 L+(D_1 L)^2+k^2\sigma^2 L-2D_1 Lk\sigma\sqrt{L}}+D_1 L-k\sigma\sqrt{L}}{\frac{r_b C_b}{2}+\frac{r_v}{2}\left(n\left(1-\frac{D_1}{P_1}\right)-1+2\frac{D_1}{P_1}\right)}} \tag{2.20}$$

$$k^* = \frac{\frac{(-\sigma\sqrt{Q_1 C_b r_b(-Q_1 C_b r_b+\pi D_1))}}{-Q_1 C_b r_b+\pi D_1} + \frac{D_1 \pi\sigma\sqrt{Q_1 C_b r_b(-Q_1 C_b r_b+\pi D_1)L}}{2Q_1 C_b r_b(\pi D_1 - Q_1 C_b r_b)} + LD_1}{\sqrt{L}\sigma} \tag{2.21}$$

It can be clearly seen that the optimal solution for Q_1^*, Q_2^*, and k^* are dependent on each other. Thus, numerical procedure is utilized for finding the optimal values. An iteration method is used with the following algorithm to find the managerial decisions.

2.3.5 Solution Algorithm

For the current model, Matlab software is utilized for solving underneath algorithm.

Step 1 Assign the values to all the parameters.

Step 2 Set n=1.

Step 3 For each value of $L_i, i = 1, 2, \ldots$ execute the underneath steps.

- Step 3a Derive the value of Q_2 from equation 2.19.
- Step 3b Derive the value of Q_1 from equation 2.20.
- Step 3c Derive the value of k from equation 2.21.
- Step 3d Repeat Steps 3a to 3c until there are any notable variation in the values

of Q_2, Q_1, and k upto a specified accuracy level.

Step 4 Use the values of Q_2, Q_1, and k to obtain ∇_S from equation 2.11.

Step 5 Use the value of ∇_S to obtain ∇_D from equation 2.12

Step 6 Putt $n = n+1$ and redo Steps from 3 to 5.

Step 7 If $\nabla_D(n+1) < \nabla_D(n)$ then redo Steps from 2 to 6 or else end the execution of algorithm.

Proposition 2.2. *If we represents Q_1^*, Q_2^*, and k^* as optimal values of Q_1, Q_2 and k, then for definite values of n and $L \in [L_i, L_{i-1}]$, the "dual-channel" profit function ∇_D attains its global maximum at Q_1^*, Q_2^*, and k^* under the condition*

$$\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{D_1\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}$$

$$> \frac{D_1 L^2 \sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3 \sigma^2)}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

Proof. For "dual-channel supply chain", the Hessian matrix H is

$$H_1 = \begin{bmatrix} \frac{\partial^2 \nabla_D}{\partial Q_1^2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} \\ \frac{\partial^2 \nabla_D}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial Q_2^2} & \frac{\partial^2 \nabla_D}{\partial Q_2 \partial k} \\ \frac{\partial^2 \nabla_D}{\partial k \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial k \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial k^2} \end{bmatrix}$$

where,

$$\frac{\partial^2 \nabla_D}{\partial k^2} = -\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} = \frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2 \nabla_D}{\partial Q_2 \partial k} = 0 = \frac{\partial^2 \nabla_D}{\partial k \partial Q_2}$$

$$\frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2} = 0 = \frac{\partial^2 \nabla_D}{\partial Q_2 \partial Q_1}$$

$$\frac{\partial^2 \nabla_D}{\partial Q_2^2} = -\frac{2S_2 D_2}{Q_2^3}$$

$$\frac{\partial^2 \nabla_D}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1(\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]$$

$$\frac{\partial^2 \nabla_D}{\partial k \partial Q_1} = \frac{\pi D_1}{2Q_1^2}\left[\frac{K\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

The principal minor of $|H_1|$ of order 1×1 is

$$|H_{1,1}| = \left|\frac{\partial^2 \nabla_D}{\partial Q_1^2}\right|_{(Q_1^*, Q_2^*, k^*)}$$

$$= -\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1(\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right] < 0$$

The principal minor of $|H_1|$ of order 2×2 is

$$|(H_1)_{2,2}|_{(Q_1^*,Q_2^*,k^*)} = \left(\frac{\partial^2 \nabla_D}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_D}{\partial Q_2^2}\right)$$

$$= \left(-\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L k\sigma\sqrt{L}})\right]\right)\left(-\frac{2S_2 D_2}{Q_2^3}\right) > 0$$

The principal minor of $|(H_1)_{3,3}|_{(Q_1^*,Q_2^*,k^*)}$ of order 3×3 is

$$|(H_1)_{3,3}|_{(Q_1^*,Q_2^*,k^*)} = \frac{\partial^2 \nabla_D}{\partial Q_2^2}\left[\left(\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_S}{\partial k^2}\right) - \left(\frac{\partial^2 \nabla_S}{\partial k \partial Q_1}\right)^2\right]$$

$$= \left[\frac{2S_2 D_2}{Q_2^3}\right]\frac{\pi D_1^2}{Q_1^4}\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$-\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2 - 4D_1\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})} > 0$$

$$\implies \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} > \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2 - 4D_1\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

$$\implies \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{D_1\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}$$

$$> \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2)}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

Since, the Hessian matrix's, all the principal minors are not positive. Hence, the Hessian matrix H_1 is negative definite at (Q_1^*, Q_2^*, k^*). Thus, aggregate expected profit for "dual-channel" gets the global maximum at (Q_1^*, Q_2^*, k^*).

Proposition 2.3. *If we represents Q_1^*, k^* as the optimal values of Q_1, k, then for definite values of $L \in [L_i, L_{i-1}]$ and n, the "single-channel" profit function ∇_S attains its global maximum at Q_1^*, k^* under the condition*

$$\left(A_r + \frac{S_1}{n} + CL\right) \frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{D_1\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}$$

$$> \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2)}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

Proof. For "single-channel supply chain", the Hessian matrix H is

$$H_2 = \begin{bmatrix} \frac{\partial^2\nabla_D}{\partial Q_1^2} & \frac{\partial^2\nabla_S}{\partial Q_1\partial k} \\ \frac{\partial^2\nabla_S}{\partial k\partial Q_1} & \frac{\partial^2\nabla_S}{\partial k^2} \end{bmatrix}$$

where,

$$\frac{\partial^2\nabla_S}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1(\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]$$

$$\frac{\partial^2\nabla_S}{\partial k\partial Q_1} = \frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2\nabla_S}{\partial k^2} = -\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2\nabla_S}{\partial Q_1\partial k} = \frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

The principal minor of $|H_2|$ of order 1×1 is

$$|H_{1,1}|_{(}Q_1^*, k^*) = \left|\frac{\partial^2\nabla_S}{\partial Q_1^2}\right|_{(Q_1^*, k^*)}$$

$$= -\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right] < 0$$

The principal minor of $|H_2|$ of order 2×2 is

$$|H_{2,2}|_{(Q_1^*,k^*)} = \left(\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_S}{\partial k^2}\right) - \left(\frac{\partial^2 \nabla_S}{\partial k \partial Q_1}\right)^2$$

$$= \Big(-\frac{2}{Q_1^3}\Big[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\Big]\Big)$$

$$\Big(-\frac{\pi D_1}{2Q_1}\Big[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}\Big]\Big)$$

$$-\Big(\frac{\pi D_1}{2Q_1^2}\Big[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \sigma\sqrt{L}\Big]\Big)^2$$

$$= \frac{\pi D_1^2}{Q_1^4}\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$-\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2 - 4D_1\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})} > 0$$

$$\implies \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} > \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2 - 4D_1\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

$$\implies \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{D_1\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}$$

$$> \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2)}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

Since, the Hessian matrix's, all the principal minors are not positive. Hence, the Hessian matrix H_1 is negative definite at (Q_1^*, k^*). Thus, aggregate expected profit for "single-channel" gets the global maximum at (Q_1^*, k^*).

Corollary 2.1.

From equation 2.3 it is observed that the terms associated with C_p are negative in D_1 equation which clearly shows that as C_p increases D_1 decreases i.e.,

$$D_1 \propto \frac{1}{C_p}, \implies C_p \propto \frac{1}{D_1}, \tag{2.22}$$

Further from equations 2.11 and 2.12 we get,

$$\nabla_D \propto D_1, \nabla_S \propto D_1, \tag{2.23}$$

Which implies, as D_1 decreases because of increase C_p, ∇_D and ∇_S decreases as fast rate which we can observe from the table 2.3.

Corollary 2.2.

From equation 2.4 it is observed that the terms associated with $\sum_{i=1}^{N} C_i$ are negative D_2 equation which clearly shows that as $\sum_{i=1}^{N} C_i$ increases D_2 decreases i.e.,

$$D_2 \propto \frac{1}{\sum_{i=1}^{N} C_i}, \implies \sum_{i=1}^{N} C_i \propto \frac{1}{D_2} \tag{2.24}$$

Which implies, as $\sum_{i=1}^{N} C_i$ increases D_2 decreases.

Further from equation 2.12 we get,

$$\nabla_D \propto D_2 \tag{2.25}$$

Which implies, as D_2 decreases because of increase C_d, ∇_D increases at slow rate in comparison to otherwise which we can observe from the table 2.4.

2.4 Numerical experimentation and discussion

Following enlisted the input parameters used for numerical experiments C_p =100 $\$/unit$, S_1 =800 $\$/setup$, C_1 =80 $\$/unit$, S_2 =1000 $\$/setup$, C_2 =100 $\$/unit$, C_{vr} =100 $\$/unit$, C_3 =120 $\$/unit$, $C_=b$120 $\$/unit$, r_v =0.2 $\$/unit/$unit time, r_b =0.2 $\$/unit/$unit time, A_r =200 $\$/order$, P_1 =5000 $Units/year$, P_2 =5000 $Units/year$, π =150 $\$/unit$, a_1 =1000 $(-)$, a_2 =1000 $(-)$, β_1 =0.75 $\$/unit$, β_2 =0.80 $\$/unit$, δ_1 =0.2, δ_2 =0.3, ϕ_1 =0.25 %, ϕ_2 =0.3 %, ϕ_3 =0.45 %, h_1 =30 $\$/unit/year$, L =4 weeks, σ =200, m =0.7.

2.5 Profit analysis

We consider an example to compare the profit of the "supply chain management adopting dual mode and single mode of shopping". In case of the "supply chain model having dual mode" manufacturer provides the "standard product" to the customer through offline channel and "customized product" through online channel whereas, in case of "single channel supply chain model", manufacturer provides only "standard product" to the customer through offline channel. The purpose of this section is to examine and analyze the managerial decisions under various circumstances. We assume three types of customization's on "standard product" in case of "dual-channel supply chain model" to the customers.
The value's of parameters to obtain the numerical results are enlisted in input parameters. The "threshold value" is preassigned with the value \$20 for all cases throughout the chapter.
Using the value's of parameters we obtain the optimal values of the decision variables, illustrated by table 2.1. It is observed from table 2.1 that maximum profits for both "single and dual-channel supply chain" are obtained as \$ 782670.28 and \$ 822508.59, respectively. The maximum profits are observed at $n = 2$ and the corresponding values of k, L, Q_1, Q_2, D_1, D_2 are 5.58, 4, 1058.21, 166.22, 490, 456, respectively.

Tab. 2.1: Optimum values of decision variables

n	L(weeks)	k	Q_1	Q_2	D_1	D_2	∇_D	∇_S
1	4	5.59	1060.79	166.22	490	456	822403.97	782565.65
2	**4**	**5.58**	**1058.21**	**166.22**	**490**	**456**	**822508.59**	**782670.28**
3	4	5.59	1055.66	166.22	490	456	822490.54	782652.22

Moreover, table 2.2 shows that if the cost of the "standard product" is greater than that of "customized product" then shifting of customers from retailer shop to online platform is more i.e., $\delta_1 > \delta_2$. Further, if the cost of the "standard product" is smaller than that of "customized product" then shifting of customers from retailer shop to online platform is less i.e., $\delta_1 < \delta_2$. Moreover, it also depicts that in either of case profit of "dual-channel" is more in comparison to single channel.

Tab. 2.2: Analysis of profit for customer shifting

	D_1	D_2	Q_1	Q_2	k	∇_D	∇_S
$\delta_1 > \delta_2$	496.8	418.6	1067.09	159.91	5.66	830690.65	793604.35
$\delta_1 < \delta_2$	454.3	462.8	1026.95	167.32	5.21	804701.27	766334.07

2.5.1 Price sensitivity in demand when selling price difference between online and offline mode is greater than the preassigned threshold limit

To analyze the impact of changing cost of "standard and customized product" on the demand as well as profit for both "dual and single channel supply chain" two cases are considered for price sensitivity analysis. **Case I** represents the effect of changing the selling price of "standard product" on demand and profit (table 2.3) whereas, **Case II** shows the same for varying selling price of "customized product" (table 2.4). Both cases are analyzed when the cost difference of retail and online channel is more than the preassigned "threshold limit" i.e., $|\sum_{i=1}^{3} C_i - C_p| > 20$.

Case I: In this case, the cost of "customized products" are kept as constant and the price of "core product" is varying. The results are shown in table 2.3 as well as in figure 2.10.

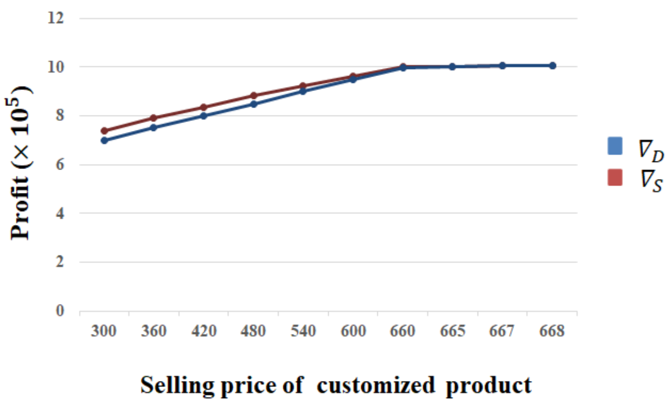


Fig. 2.10: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| > 20$ (Case 1)

Tab. 2.3: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| > 20$ (Case 1)

S.no.	$C_d = \sum_{i=1}^{3} C_i$	C_p	D_1	D_2	Q_1	Q_2	∇_D	∇_S
1.	400	300	439.0	626	1012.0	191.10	739598.1	699808.8
2.	400	360	469.6	524	1041.6	176.80	789601.1	749455.6
3.	400	420	500.2	422	1070.2	160.50	836828.1	799124.1
4.	400	480	530.8	320	1097.8	141.30	881299.4	848811.9
5.	400	540	561.4	218	1124.6	117.90	923055.3	898517.4
6.	400	600	592.0	116	1150.6	86.90	962195.4	948238.7
7.	400	660	622.6	14	1175.9	30.51	999254.5	997974.5
8.	400	665	625.2	5.5	1177.9	19.14	1002417	1002120
9.	400	667	626.2	2.1	1178.8	11.83	1003757	1003778
10.	400	668	626.7	0.4	1179.2	5.20	1004516	1004607

Case II: The cost of "standard product" is kept as constant and the selling price of "customized products" are varying in this case. We consider the variation of the sum of selling prices of all customization i.e. $\sum_{i=1}^{3} C_i$. The results are shown in table 2.4 as well as in figure 2.11.

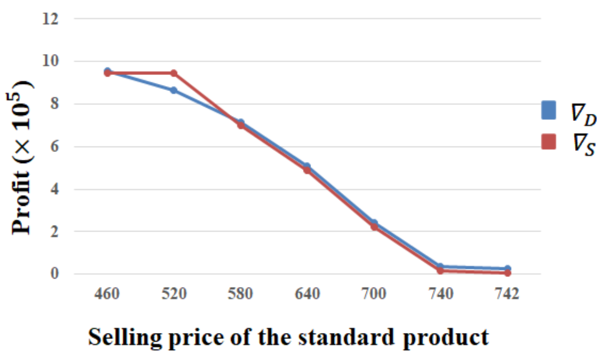


Fig. 2.11: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| > 20$ (Case 2)

Tab. 2.4: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| > 20$ (Case 2)

S.no.	$C_d = \sum_{i=1}^{3} C_i$	C_p	D_1	D_2	Q_1	Q_2	∇_D	∇_S
1.	300	400	456.0	643	1028.59	193.3	768344.7	727387.5
2.	360	400	476.4	530.8	1048.10	177.8	801195.6	760491.2
3.	420	400	496.8	418.6	1067.10	160.0	830983.7	793604.4
4.	480	400	517.2	306.4	1085.70	138.5	857738.4	826726.2
5.	540	400	537.6	194.2	1103.90	111.6	881519.9	859856.1
6.	600	400	558	82.0	1121.70	73.3	902503.2	892993.7
7.	610	400	561.4	63.3	1124.63	64.5	905819.4	898517.4
8.	620	400	564.8	44.6	1127.56	54.3	909015.6	904041.2
9.	630	400	568.2	25.9	1130.47	41.4	912201.7	909565.2
10.	640	400	571.6	7.2	1133.38	21.9	915533.9	915089.4

2.5.2 Price sensitivity in demand when selling price difference between online and offline mode is less than the preassigned threshold limit

Similar analysis is performed by considering two different cases as described in previous section. But, the price difference between online and offline mode is less than the fixed and specified threshold limit i.e., $|\sum_{i=1}^{3} C_i - C_p| < 20$.

Two cases are set out as follows.

Fig. 2.12: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| < 20$ (Case 1)

Case I: The cost of "customized products" are set as constant and the price of core item is changing. The results are shown in table 2.5 and in figure 2.12.

Tab. 2.5: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| < 20$ (Case 1)

Sr. no.	C_p	C_d	$C_p - C_d$	D_1	D_2	Q_1	Q_2	∇_D	∇_S
1.	380	400	-20	515.5	456	1084.15	166.22	816481.1	776642.8
2.	385	400	-15	509.13	456	1078.37	166.22	818399.5	778561.2
3.	390	400	-10	502.75	456	1072.60	166.22	820026.1	780187.8
4.	395	400	-5	496.38	456	1066.70	166.22	821360.9	781522.6
5.	400	400	0	490.00	456	1060.80	166.22	821446.4	782565.7
6.	405	400	5	483.63	456	1054.90	166.22	823155.3	783317
7.	410	400	10	477.25	456	1048.9	166.22	823614.9	783776.6
8.	415	400	15	470.88	456	1042.8	166.22	823658.9	783944.4
9.	420	400	20	464.50	456	1036.8	166.22	823782.7	783820.7

Case II: The cost of "standard product" is kept as constant and the price of "personalized

Fig. 2.13: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| < 20$ (Case 2)

items" are varying in this case (as considered in Case II of previous section). The results are shown in table 2.6 as well as in figure 2.13.

Tab. 2.6: Managerial decisions for $|\sum_{i=1}^{N} C_i - C_p| < 20$ (Case 2)

Sr. no.	C_p	C_d	$C_d - C_p$	D_1	D_2	Q_1	Q_2	∇_D	∇_S
1.	400	380	-20	490	483.2	1060.8	170.59	822558	782565.7
2.	400	385	-15	490	476.4	1060.8	169.51	822407.5	782565.7
3.	400	390	-10	490	469	1060.8	168.40	822029.9	782565.7
4.	400	395	-5	490	462.8	1060.8	167.30	821823.7	782565.7
5.	400	400	0	490	456	1060.8	166.22	821606.0	782565.7
6.	400	405	5	490	449	1060.8	165.1	821376.6	782565.7
7.	400	410	10	490	442.4	1060.8	163.9	821135.7	782565.7
8.	400	415	15	490	435.6	1060.8	162.8	820883.2	782565.7
9.	400	420	20	490	428.8	1060.8	161.7	820619.1	782565.7

2.5.3 Important discussions and significance of results

Table 2.3 and figure 2.10 illustrate that if the cost of the “customized product” increases, it results increase in variation in the cost of the “core and personalized product”. Henceforth, more demand shift from “customized product” to “standard product” and ultimately “dual-channel” behaves like “single-channel” only. Further, table 2.4 and figure 2.11 present that if the cost of the “core product” increases, the difference between the selling prices of the standard and “customized product” also increases. As a result firm slowly moves towards the loss because it leads to increase in price of “customized product” as it requires “standard product” for customization.

Table 2.5 and figure 2.12 characterize that though the cost of the “customized product” increases but if the variation in the cost of “standard and personalized product” is less than preassigned “threshold value” then shifting of customers is inconspicuous. Further, table 2.6 and figure 2.13 reflect that if the cost of the “standard product” increases but if the variation in the cost of “standard and customized product” is less than preassigned “threshold value” then shifting of customers is unspectacular.

Figure 2.14 draws the analysis of profit with markup price for both “dual-channel” (∇_D) and “single channel” (∇_S) when cost of the “standard product” is more than “customized product”. Moreover, figure 2.15 draws analysis of profit with markup price for both “dual-channel” (∇_D) and “single channel” (∇_S) when selling price of the “standard product” is less than “customized product”. It is observed that in both the figures profit increases with the increase in “markup margin”.

2.6 Sensitivity analysis

2.6.1 Sensitivity analysis of offline channel

As similar to the sensitivity analysis of the direct channel, the parameters of cost are differed by -10%, -5%, +5%, and +10% to shift the profit of the firm. The effects of this changes of the key parameters are illustrated in table 2.7 and figure 2.16. Divergence in key parameters C_p, S_1, C_v, and C_b are taken into account. The outcomes of this experiment are discussed as follows.

1. Cost components of manufacturer’s are more sensitive than cost component of retailer’s.
2. “Standard product” selling price is the most sensitive cost whereas that of setup cost of the retailer is the least sensitive.

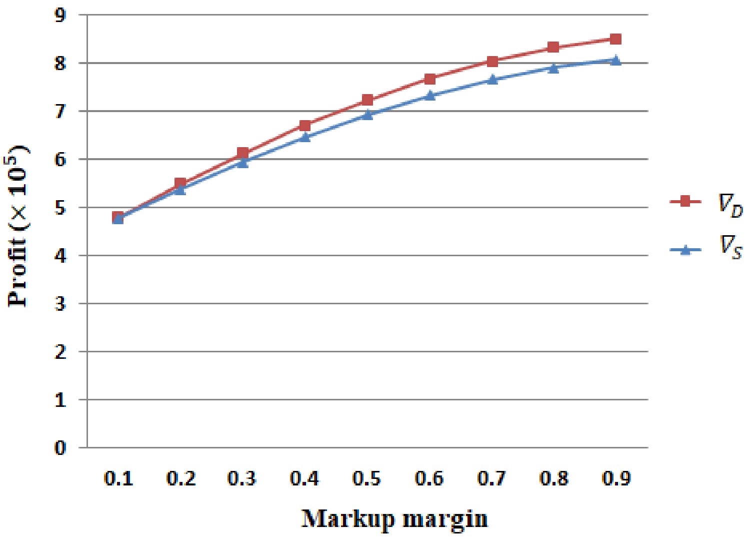


Fig. 2.14: Analysis of profit with markup price when $C_p > C_i$

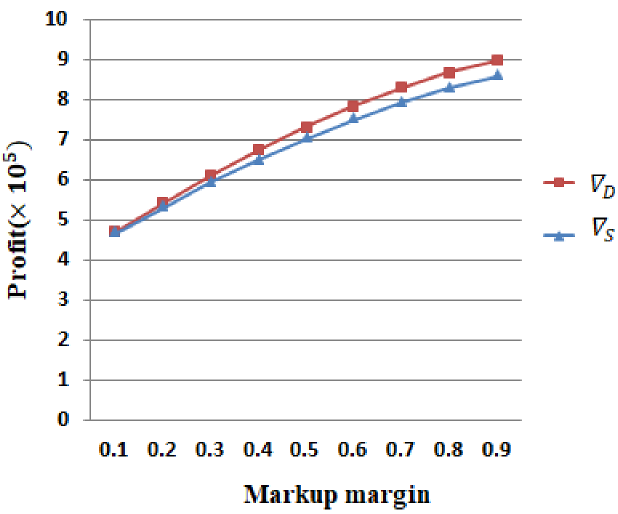


Fig. 2.15: Analysis of profit with markup price when $C_p < C_i$

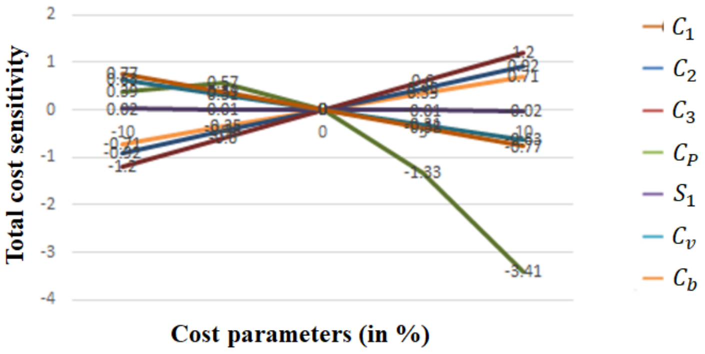


Fig. 2.16: Sensitivity analysis for different key parameters in case of offline channel

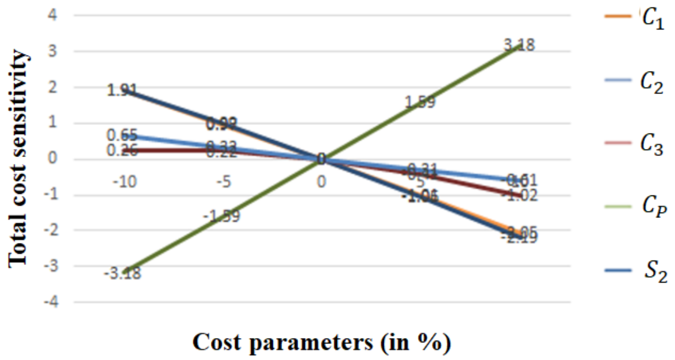


Fig. 2.17: Sensitivity analysis for different key parameters in case of online channel

2.6.2 Sensitivity analysis of online channel

The cost parameters are varied over -10%, -5%, +5%, and +10% and the effects of this changes of the key parameters are illustrated in table 2.7 and figure 2.17.

Variations of key parameters are C_1, C_2, C_3 (since only three customization's are considered), and S_2 considered. For the sake of simplicity, only three types of customization's are considered. Scrutinizing the sensitivity analysis, following points are observed.

1. Cost components of the "standard product" are more sensitive than cost components of "customized product".
2. Cost components of manufacturer's for "standard product" are more sensitive than manufacturer's setup cost.
3. Out of the three customization, third customization cost C_3 for "customized product" is the least sensitive.

Tab. 2.7: "Sensitivity analysis for different key parameters"

Parameters	Variation (in %)	Change in (offline channel)	Change in (online channel)	Parameters	Variation (in %)	Change in (offline channel)	Change in (online channel)
C_1	−10	–	+1.91	S_1	−10	+0.02	–
	−5	–	+0.97		−5	+0.01	–
	5	–	−1.01		5	−0.01	–
	10	–	−2.05		10	−0.02	–
C_2	−10	–	+1.91	C_v	−10	+0.63	–
	−5	–	+0.99		−5	+0.31	–
	5	–	−1.06		5	−0.31	–
	10	–	−2.19		10	−0.63	–
C_3	−10	–	+0.26	C_b	−10	+0.77	–
	−5	–	+0.22		−5	+0.39	–
	5	–	−0.22		5	−0.38	–
	10	–	−0.26		10	−0.77	–
C_p	−10	+3.39	–	S_2	−10	–	+0.65
	−5	+1.57	–		−5	–	+0.32
	5	−1.33	–		5	–	−0.31
	10	−3.41	–		10	–	−0.61

2.7 Managerial Implications

This chapter provides "a dual-channel supply chain model with single manufacturer and single retailer." Managerial decisions shall be more realistic if more than one decision variables are considered instead of single variable or any constant entity. More pragmatic managerial decisions can be obtained if multiple decision variables are assumed. Thus, decisions are made on the basis of quantities such as cost of “core product”, cost of “personalized product”, profit of “single channel” and profit of “dual channel”, and lead time. The managerial implications are enlisted as follows.

• Providing the customization facility to the customers influences hike in demand which may increase the total profitability of the industry. In 1985, Dell took up this strategy i.e., “make-to-order” of computers on demand of customers and this provided Dell a sale of $ 70 million in that year and after five years of adopting the strategy, their revenue climbed to $ 500 million (MaRS , 2021).

• Manager can increase the firm’s gross profit if the “supply chain management” is enhanced to “dual-channel” strategy instead of “single-channel”. As a result of this strategy the Dell’s revenue was able to reach to $ 25 billion within two decades (MaRS , 2021).

• A “centralized dual-channel supply chain model” should bring forth increased profit to the firm due to the exchange of information between manufacturer and retailer.

• An improved managerial decisions can be obtained by assuming a preassigned threshold limit and unequal shifting of customers from one channel to another.

• Since manager is providing “customized product” in addition with “standard product” so price of the customized feature should be arranged in such a way that overall cost of the “customized product” should not overshoot the “threshold limit” otherwise customers will shift back to “standard product”.

• Manager should not increase the cost of the “standard product” beyond a limit otherwise the profit firm will slowly doom. On account of the fact that customization is implemented on the “standard product” and if the prices of the “standard product” increase automatically lead to an increase in the price of “customized product”.

2.8 Conclusions

The most important and original contribution of this chapter is the modifications incorporated on the existing customization strategy with the inclusion of a fixed and specified “threshold limit” and unequal shifting of customers from one channel to another which leads to an improved realistic scenario of “dual-channel supply channel management”. Moreover, due to the speculative fluctuation of demand, estimation becomes difficult task for the decision makers. Some factors such as selling price leads to a variation in demand along with uncertainty. Therefore, the decision maker’s should consider the probabilistic uncertainty with the sensitivity of various factors like selling price in demand expression.

The chapter concluded that implementing the “dual-channel supply chain policy” the company receives the personally customized demands of the customers along with “standard product” which increases the trust of the customers on firm. Moreover, if the firm adopts the “centralized policy” in that case it can manage the complete profit parameters that are related to product after studying the market which helped in increasing the overall profit of the firm. This chapter clearly depicted that if the cost of “standard product” is extravagant in compared to “customized product” then the firm slowly moves towards losses whereas, if the cost of “customized product” is overpriced in comparison to “standard product” then “dual-channel supply chain model” behaves like “single channel model”. As a result, customers start preferring “standard product” over “customized product”. Moreover, if the variation in the cost of the “standard product” and “customized product” is less than preassigned “threshold value” then there is an indistinct shifting of customers. This chapter also revealed that the selling prices of “customized products” are the most sensitive cost parameters for both online and offline channel. Accordingly, the least sensitive parameters are the setup costs for manufacturer for both channels.

The present chapter is limited to some aspects such as absence of multiple retailers, strategy to reduce setup and ordering costs, implementation of vendor managed inventory (VMI) or consignment policy, integrating towards a sustainable development, and many more. Moreover, large cost components like setup or ordering cost may also be diminished by an efficient investment strategy (Majumder et al. , 2017). The study of VMI and consignment contracts has been supervised by many researchers in recent years such as Batarfi et al. (2016) and Sarkar at al. (2018b). This agreement allows manufacturers to take the ownership of its products even after shifting to the retailer’s end. A VMI or consignment contract under “dual-channel centralized supply chain” with the modified customization policy can be an innovative research to be concerned. Furthermore, considering a humanitarian supply chain, social and environmental sustainability like that in Nandra et al. (2020) and Nandra et al. (2021), are one of the virtuous ways of extension of this chapter.

CHAPTER - 3

The impact of adopting customization policy and sustainability for improving consumer service in a dual-channel retailing

3.1 Problem definition

“Smart supply chain management” is an up-gradation in the traditional supply chain. In classical “supply chain management”, a firm manufactures only “core products”, thus not acknowledging the products influenced by customers. Therefore, with the enhancement of smart “dual-route supply chain management”, firms can provide a personification opportunity to customers along with “core products” as considered in this chapter. With the smart supply chain management considered in current model, firms are catering to both “personalized and standard product” demands of the customers thus, the number of customers increases. Consequently, CO_2 emission and social burden due to rise in the production increases. The present article examines the CO_2 emission during production and imposes penalties on the firms if they cross the predefined carbon limit set by the government. Additionally, with an increase in production, the system transfers from an “in-control” to an “out-of-control” state. Therefore, an investment in quality improvement is implemented in the current model. Moreover, the majority of the literature focuses on probabilistic uncertainty or demand variability and very few articles use both of the characteristics simultaneously in a demand function that is presumed in this chapter. Also, a “threshold limit” is incorporated in the model which regulates the variation in the price of the “core and personalized product”. If the difference in the prices of “core and personalized products” overshoots the “threshold limit”, then the shifting of customers between the channels is initiated, influencing the overall firm’s profit. Figure 3.18 demonstrates the contribution of this research.

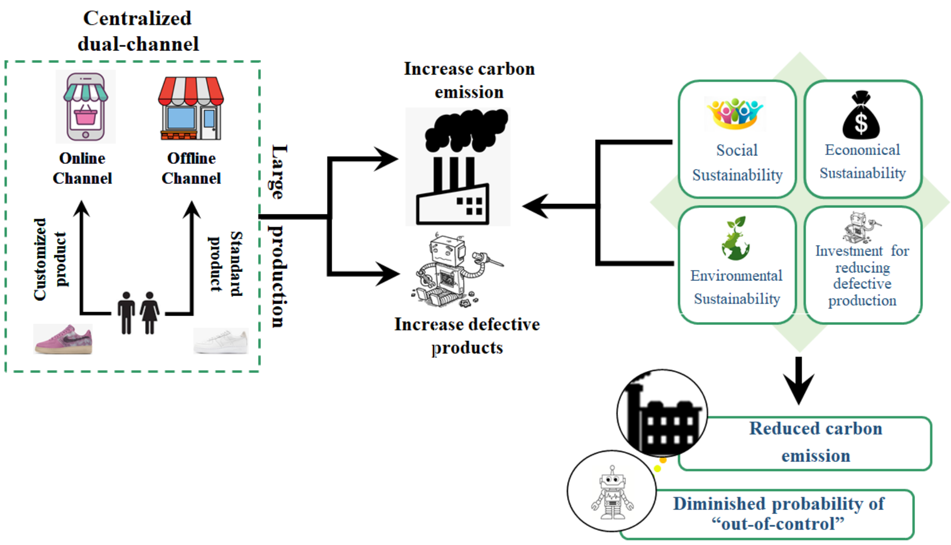


Fig. 3.18: Contribution of this research

3.2 Presumptions

For framing the mathematical model, following points are presumed.

1. The firm espoused a “centralized supply chain model with a dual-route” where trading of a “standard article” is done through a shopkeeper and of a “personalized article” is done through e-commerce with the shopper. Additionally, variable along with random demand is considered.

2. Retailers are not delivered out-of-order articles as the replacement article does not have any out-of-order articles (Majumder et al. , 2017).

3. A presumed ceiling i.e., “threshold limit” on the price’s difference of the item between offline and online channel is considered. If variation in the trading charges proliferates beyond the ceiling then shifting of shoppers from one to another route starts and if the variation is less than the ceiling then there is no transfer of shoppers.

4. The firm also considers the cost related to two pillars of sustainability i.e., environmental (Jaber et al. , 2013) and social (Hutchins and Sutherland , 2008).

3.3 Mathematical model

Following section enlist the functions of demand and profit, "distribution-free approach", and finally supply chain's optimal decision of this chapter.

3.3.1 Function for demand

Customers are miscellaneous in nature therefore many factors influence their preference for "standard and customized product". Enhancing the work of Zhang et al. (2015), a linear function's of demand for the "core and personalized product" is considered. Underneath functions for demand in case of retailer and e-commerce are derived by enhancing the work of Huang and Swaminathan (2009).
The offline channel's function for demand is

$$D_1 = a_1 - \beta_1 p_1 + \delta_1 \left(\sum_{i=1}^{N} p_i - p_1 \right) \tag{3.26}$$

The online channel's function for demand is

$$D_2 = a_2 - \beta_2 \sum_{i=1}^{N} p_i - \delta_2 \left(\sum_{i=1}^{N} p_i - p_1 \right) \tag{3.27}$$

Where, underneath enlist the explanation of parameters utilized in the function of demand.

- Parameter a_1 and a_2 are offline and online demand of product.

- The retailer's channel, price sensitivity coefficients is β_1 and manufacturer's channel is β_2, it demonstrates the decrease/increase in demand of market with increase/decrease in the price by one dollar. Thus, $\beta_1 p_1$ and $\beta_2 \sum_{i=1}^{N} p_i$ represents change in customers because of price sensitivity.

- δ_1 and δ_2 represents the swapping of consumers from offline mode to the online mode or vice-versa. Depending upon price of the product.

- The cost of the "core product" for retailer is given by $p_1 = C_p(1+m)^2$ and the cost of "customized product" for manufacturer is $p_i = C_i(1+m)$ where m is a "markup margin". Henceforth, $\delta_1 \left(\sum_{i=1}^{N} p_i - p_1 \right)$

and $\delta_2\left(\sum_{i=1}^{N} p_i - p_1\right)$ indicates the variation in the customers because of δ_1 and δ_2

Considering constant "markup margin" for "core and customized products" we get

$$D_1 = a_1 - \beta_1 C_p(1+m) + \delta_1(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right) \tag{3.28}$$

The online mode's function for demand is

$$D_2 = a_2 - \beta_2(1+m)\left(\sum_{i=1}^{N} C_i\right) - \delta_2(1+m)\left(\sum_{j=1}^{N} C_i - C_p\right) \tag{3.29}$$

3.3.2 Total cost functions

This section includes total cost equations of manufacturer and retailer for "standard and customized products".

I. **"Manufacturer's total cost for core product"**

The per unit time total cost of the manufacturer by selling the "standard product" through the retail channel is

$$TC_1 = \text{Setup cost} + \text{Holding cost} + \text{Manufacturer's production cost} + \text{cost of}$$

$$\text{imperfect items} + \text{Investment in the quality improvement of the product}$$

$$TC_1 = \frac{S_1 D_1}{nQ_1} + \frac{r_v C_p Q_1}{2}\left[n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right] + C_{vr} D_1 + \frac{sD_1 nQ_1 \theta}{2} + \alpha b(ln\theta_0 - ln\theta) \tag{3.30}$$

where, the following para explains the various cost components used in equation (3.30).

$\frac{S_1 D_1}{nQ_1}$ is setup cost for retail channel. The holding cost of manufacturer is $\frac{r_v C_p Q_1}{2}\left(n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right)$. $C_{vr} D_1$ represents the production cost of the manufacturer for "standard product". $\frac{sD_1 nQ_1 \theta}{2}$ is believed to be the year-long cost of imperfect items. In the course of production of nQ_1 lost size, nearly $\frac{nQ_1\theta}{2}$ defective items are believed to produce. αI_θ is the financing done for improving the quality of the product. The capital expenditure I_θ is presumed for reducing the "out-of-control probability" θ and the capital investment's fraction of annual cost is α. Thus, I_θ can be exhibited as $I_\theta = bln(\frac{\theta_0}{\theta})$ for $0 < \theta \leq \theta_0$, i.e.,$I_\theta = b(ln\theta_0 - ln\theta)$.

II. **"Manufacturer's total cost for personalized product"**

The online channel inventory of manufacturer's where the per unit of time aggregate cost of the manufacturer by selling the "personalized product" through the online mode is

$$TC_2 = \text{Setup cost} + \text{Holding cost} + \text{Manufacturing cost}$$

$$TC_2 = \frac{S_2 D_2}{Q_2} + \left(\frac{h_1 Q_2}{2}\right)\left(1 - \frac{D_2}{P_2}\right) + \sum_{i=1}^{N} C_i \phi_i D_2 \tag{3.31}$$

where, the following para explains the various cost components used in equation 3.31.
$\frac{S_2 D_2}{Q_2}$ is setup cost as manufacturer follows "make-to-order policy" where he manufacture's the product on demand by customer. The holding cost of manufacturer is $\frac{hQ_2}{2}\left(1 - \frac{D_2}{P_2}\right)$. Total production cost of the vendor will be $\sum_{i=1}^{N} C_i \phi_i D_2$.

III. **"Retailer's total cost"**

The per unit time total cost of the retailer by selling the "core product" is

$$TC_3 = \text{Cost of ordering} + \text{Cost of holding} + \text{shortage cost} + \text{Cost of lead time crashing}$$

$$TC_3 = \frac{A_r D_1}{Q_1} + r_b C_b \left(\frac{Q_1}{2} + R - D_1 l\right) + \frac{\pi D_1}{Q_1} E(M - R)^+ + \frac{D_1 CL}{Q_1} \tag{3.32}$$

where, the following para explains the various cost components used in equation 3.32.
The ordering cost per unit time for retailer is $\frac{A_r D_1}{Q_1}$. The retailer's expected holding cost per unit time is $r_b C_b \left(\frac{Q_1}{2} + R - D_1 L\right)$. $\frac{\pi D_1}{Q_1} E(M - R)^+$ is representing the shortage cost. The lead time crashing cost per unit time is $\frac{D_1 CL}{Q_1}$.
Total cost of the "single-channel supply chain" TC_S is given by adding TC_1,TC_3. We get,

$$TC_S = TC_1 + TC_3 \tag{3.33}$$

$$TC_S = \frac{S_1 D_1}{n Q_1} + \frac{r_v C_p Q_1}{2}\left[n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right] + C_{vr} D_1 + \frac{s D_1 n Q_1 \theta}{2} + \alpha b(ln\theta_0 - ln\theta) +$$

$$\frac{A_r D_1}{Q_1} + r_b C_b \left(\frac{Q_1}{2} + R - D_1 L\right) + \frac{\pi D_1}{Q_1} E(M - R)^+ + \frac{D_1 CL}{Q_1} \tag{3.34}$$

3.3.3 "Distribution-free approach"

We elaborate the "distribution-free approach" by using the following points

• Any specific probability distribution should not be considered for any random variable in this approach. A class of "cumulative distribution function (c.d.f)" having "mean D_1L" and "standard deviation $\sigma\sqrt{L}$" is considered.

• The "max-min distribution-free approach" is applied to obtain the values of the decision variables. In this approach, initially the worst possible case (i.e., the expression for minimum profit is obtained which is then maximized for the best profitability).

• In this chapter, a max-min method is utilized for maximization of profit which is the opposite of Moon and Gallego (1993)'s "min-max distribution-free approach". Moreover, instead of assuming only uncertainty, this chapter provides demand variability also. Therefore, a modification is added in the inequality used by with a max-min approach and incorporation of variability in demand function.

• The lead time demand M is depending on D_1 which further depends on the cost of the retailer $C_p(1+m)^2$ which leads to variability in the demand. Moreover, randomness in demand is accomplished in additive form.

The inequality stated by Proposition 3.1 is applied to solve the model.

Proposition 3.1.

$$E(M-R)^+ = E((D_1L+X)-R)^+$$

$$\leq \left[\frac{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})}{2}\right] \tag{3.35}$$

Where, $R = D_1L+k\sigma\sqrt{L}$ is reorder point, D_1L is the lead time demand, $k\sigma\sqrt{L}$ is safety stock, and k is a safety factor.

Proof.

$$E(M-R)^+ = \frac{|M-R|+(M-R)}{2}$$

$$E(M-R)^{+} \leq \frac{\sqrt{E(M-R)^{2}}+E(M-R)}{2}$$

Considering, $M = D_1\sqrt{L}+X$ summation of variability and randomness.

$$E(M-R)^{+} \leq \frac{\sqrt{E(D_1\sqrt{L}+X-R)^{2}}+E(D_1\sqrt{L}+X-R)}{2}$$

where, $R = D_1L+k\sigma\sqrt{L}$ is a safety factor.

$$E(M-R)^{+} \leq \frac{\sqrt{E(D_1\sqrt{L}+X-D_1L-k\sigma\sqrt{L})^{2}}+E(D_1\sqrt{L}+X-D_1L-k\sigma\sqrt{L})}{2}$$

A worst possible case is taken into account for distribution of random variable D_1 having mean D_1L and standard deviation $\sigma\sqrt{L}$. We get,

$$E(M-R)^{+} = \frac{\sqrt{E(X+k\sigma\sqrt{L})^{2}}+E(X+k\sigma\sqrt{L})}{2}$$

$$= \frac{\sqrt{E(X^{2}+k^{2}\sigma^{2}L+2Xk\sigma\sqrt{L})}+E(X+k\sigma\sqrt{L})}{2}$$

$$= \left[\frac{\sqrt{\sigma^{2}L+(D_1L)^{2}+k^{2}\sigma^{2}L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})}{2}\right]$$

Thus, using the inequality 3.35 the expected aggregate cost of the "centralized single channel supply chain" TC_S is given by adding TC_1 and TC_3. We get,

$$TC_S = \frac{A_rD_1}{Q_1}+r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)+\frac{D_1CL}{Q_1}+\left[\sqrt{\sigma^{2}L+(D_1L)^{2}+k^{2}\sigma^{2}L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})\right]$$

$$\frac{\pi D_1}{2Q_1}+\frac{S_1D_1}{Q_1n}+\frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]+C_{vr}D_1+\frac{sD_1nQ_1\theta}{2}+\alpha b(ln\theta_0-ln\theta) \qquad (3.36)$$

And expected aggregate cost of the "centralized dual-channel supply chain" TC_D is given by adding TC_1, TC_2, and TC_3. We get,

$$TC_D = \frac{S_1D_1}{Q_1n}+\frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]+C_{vr}D_1+\frac{sD_1nQ_1\theta}{2}+\alpha b(ln\theta_0-ln\theta)+\frac{S_2D_2}{Q_2}+$$

$$\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{D_2}{P_2}\right)+\sum_{i=1}^{N}C_i\phi_iD_2+\frac{A_rD_1}{Q_1}+r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)+\frac{D_1CL}{Q_1}$$

$$+\frac{\pi D_1}{2Q_1}\left[\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})\right] \tag{3.37}$$

3.3.4 Supply chain's optimal decisions

Since, equation 3.37 is non-linear in nature so for a positive definite integer 'm'', we take partial derivative of the cost with respect to Q_2, Q_1, k, & θ to obtain the solution which is optimal in nature.

$$\frac{\partial TC_D}{\partial k}=r_bC_b\sigma\sqrt{L}+\frac{\pi D_1}{2Q_1}\left[\frac{k\sigma^2L-D_1L\sigma\sqrt{L}}{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}}-\sigma\sqrt{L}\right] \tag{3.38}$$

$$\frac{\partial TC_D}{\partial Q_1}=-\frac{A_rD_1}{{Q_1}^2}+\frac{r_bC_b}{2}-\frac{S_1D_1}{n{Q_1}^2}-\frac{D_1CL}{{Q_1}^2}+\frac{r_vC_p}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]-$$

$$\frac{\pi D_1}{2{Q_1}^2}\left[\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})\right]+\frac{sD_1n\theta}{2} \tag{3.39}$$

$$\frac{\partial TC_D}{\partial Q_2}=-\frac{S_2D_2}{{Q_2}^2}+\frac{h_1}{2}\left(1-\frac{D_2}{P_2}\right) \tag{3.40}$$

$$\frac{\partial TC_D}{\partial \theta}=\frac{sD_1nQ_1}{2}-\frac{\alpha b}{\theta} \tag{3.41}$$

Now for definite value of 'm'', the value of Q_2, Q_1, k, & θ are obtained by equating equations 3.38, 3.39, 3.40, & 3.41 to zero that is

$$\frac{\partial TC_D}{\partial k}=0,\ \frac{\partial TC_D}{\partial Q_1}=0,\ \frac{\partial TC_D}{\partial Q_2}=0,\ \&\ \frac{\partial TC_D}{\partial \theta}=0$$

we get,

$$Q_2^*=\sqrt{\frac{S_2D_2}{\frac{h_1}{2}(1-\frac{D_2}{P_2})}}. \tag{3.42}$$

$$Q_1^* = \sqrt{\frac{-D_1CL + S_1D_1 + A_rD_1 + \frac{\pi D_1}{2}\sqrt{\sigma^2 L + (D_1L)^2 + k^2\sigma^2 L - 2D_1Lk\sigma\sqrt{L}} + D_1L - k\sigma\sqrt{L}}{\frac{r_bC_b}{2} + \frac{r_v}{2}\left(n\left(1 - \frac{D_1}{P_1}\right) - 1 + 2\frac{D_1}{P_1}\right) + \frac{sD_1n\theta}{2}}}. \quad (3.43)$$

$$k^* = \frac{\frac{(-\sigma\sqrt{Q_1C_br_b(-Q_1C_br_b+\pi D_1)})}{-Q_1C_br_b+\pi D_1} + \frac{D_1\pi\sigma\sqrt{Q_1C_br_b(-Q_1C_br_b+\pi D_1)l}}{2Q_1C_br_b(\pi D_1 - Q_1C_br_b)} + LD_1}{\sqrt{L}\sigma}. \quad (3.44)$$

$$\theta^* = \frac{2\alpha b}{sD_1nQ_1} \quad (3.45)$$

Optimal solution for Q_1^*, Q_2^*, k^*, and θ^* so obtained dependents upon each other. Moreover, a closed form expression for centralized total cost function is hard to find. Therefore, numerical procedure is required for evaluating these optimal values. Along with following algorithm an iteration method is utilized to find the managerial decisions.

3.4 Analysis of environmental and social pillars of sustainability in Single Channel

3.4.1 Environmental Pillar

I. *CO_2 emissions throughout the production*

The aggregate of CO_2 emitted (ton/unit) in the production process is:

$$E = E(P_1) = x_1(P_1)^2 - x_2P_1 + x_3 \quad (3.46)$$

where x_1, x_2, and x_3 can be experimentally verified on the lines of Narita (2012). The experiment gives a way to understand that how operating a machine tool adds a carbon emission burden and gave a quadratic equation 3.46 reflecting the equivalent CO_2 emissions. Moreover, it also reflects that increased cutting speed converts tool wears into considerable lofty which shortens its life span and elevates CO_2 emissions. Further, there is also a trade relation with the cutting speed as carbon stress build-up by electricity utilization and the cooling liquid is comparable with time. The quadratic equation 3.46 manifests the behaviour of the corresponding carbon ejection. This is demonstrated by figure 3.19. Understanding the production proficiency of the manufacturer in the current model machine tool is taken into consideration.

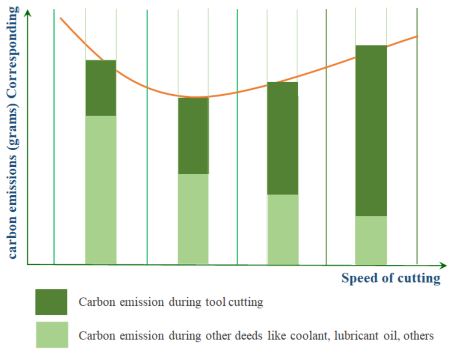


Fig. 3.19: Corresponding carbon emissions from a machine tool at various cutting speeds (recreated from Narita (2012)'s page-257 Figure 6 & Bazan et al. (2015)'s Figure 2).

Therefore, cost of the carbon exude because of production is given as

$$EC_1 = ED_1 C_{ec} \tag{3.47}$$

II *Penalty because of excess of CO_2 emission*

When the carbon burden from the firm surpasses the predetermined ceiling then the penalty cost is collected from it. Thus, the penalty cost as a consequence of carbon emission is given as

$$EC_2 = \sum_{i=1}^{l} Y_i C_{ep,i} \tag{3.48}$$

where

$$Y_i = \begin{cases} 1 & ED_1 > E_{li} \quad (i = 1, 2, ..., l) \\ 0 & otherwise \end{cases} \tag{3.49}$$

3.4.2 Social Pillar

Cumulative social cost as a result of "hard labor, medical maintenance, welfare, and social consciousness" (Hutchins and Sutherland , 2008) for "single channel supply chain model", is

$$SC_1 = nS_{mc} Q_1 \tag{3.50}$$

Therefore, the expected aggregate cost of the "centralized single channel supply chain" TC_S is given by,

$$TC_S = \frac{A_r D_1}{Q_1} + r_b C_b \left(\frac{Q_1}{2} + k\sigma\sqrt{l}\right) + \frac{D_1 CL}{Q_1} + \frac{\pi D_1}{Q_1 2}\left[\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}} + \right.$$

$$\left.(D_1 L - k\sigma\sqrt{L})\right] + \frac{S_1 D_1}{Q_1 n} + \frac{r_v Q_1}{2}\left[n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right] + C_{vr} D_1 + \frac{sD_1 nQ_1\theta}{2} +$$

$$\alpha b(ln\theta_0 - ln\theta) + ED_1 C_{ec} + \sum_{i=1}^{l} Y_i C_{ep,i} + nS_{mc} Q_1 \tag{3.51}$$

3.5 Analysis of environmental and social pillars of sustainability in Dual Channel

3.5.1 Environmental Pillar

I. *CO_2 emissions throughout the production*

The aggregate of CO_2 emitted (ton/unit) in the production process is:

$$E' = E(P) = x_1(P)^2 - x_2 P + x_3 \tag{3.52}$$

where x_1, x_2, and x_3 can be experimentally verified on the lines of Narita (2012). The experiment gives a way to understand that how operating a machine tool adds a carbon emission burden and gave a quadratic equation 3.52 reflecting the equivalent CO_2 emissions. Moreover, it also reflects that increased cutting speed converts tool wears into considerable lofty which shortens its life span and elevates CO_2 emissions. Further, there is also a trade relation with the cutting speed as carbon stress build-up by electricity utilization and the cooling liquid is comparable with time. The quadratic equation 3.52 manifests the behaviour of the corresponding carbon ejection. This is demonstrated by figure 3.19. Understanding the production proficiency of the manufacturer in the current model machine tool is taken into consideration.

Therefore, cost of the carbon exude because of production is given as

$$EC_1' = E' DC_{ec} \tag{3.53}$$

II. *Penalty on firm because of excess of CO_2 emission*

When the carbon burden from the firm surpasses the predetermined ceiling then the penalty cost is collected from it. Thus, the penalty cost as a consequence of carbon emission is given as

$$EC_2' = \sum_{i=1}^{l} Y_i C_{ep,i} \tag{3.54}$$

where

$$Y_i = \begin{cases} 1 & E'D > E_{li} \quad (i = 1, 2, ..., l) \\ 0 & otherwise \end{cases} \tag{3.55}$$

3.5.2 Social Pillar

Cumulative social cost as a result of “hard labor, medical maintenance, welfare, and social consciousness” (Hutchins and Sutherland , 2008) for “dual channel supply chain model”, is

$$SC_2 = n(S_{mc}Q_1 + S_{mp}Q_2) \tag{3.56}$$

Therefore, the expected aggregate cost TC_D of the “centralized supply chain having dual mode” is

$$TC_D = \frac{S_1 D_1}{Q_1 n} + \frac{r_v Q_1}{2}\left[n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right] + C_{vr}D_1 + \frac{sD_1 n Q_1 \theta}{2} + \alpha b(ln\theta_0 - ln\theta) + \frac{S_2 D_2}{Q_2} +$$

$$\left(\frac{h_1 Q_2}{2}\right)\left(1 - \frac{D_2}{P_2}\right) + \sum_{i=1}^{N} C_i \phi_i D_2 + \frac{A_r D_1}{Q_1} + r_b C_b\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) + \frac{D_1 CL}{Q_1}$$

$$+\frac{\pi D_1}{2Q_1}\left[\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}} + (D_1 L - k\sigma\sqrt{L})\right]$$

$$+E'DC_{ec} + \sum_{i=1}^{l} Y_i C_{ep,i} + n(S_{mc}Q_1 + S_{mp}Q_2) \tag{3.57}$$

3.6 Algorithm for obtaining the solution of the model

To solve the current model succeeding algorithm is applied.

Step 1 Values are assigned to all the parameters in accordance to input parameters of the model.

Step 2 Put $n = 1$.

Step 3 Execute the underneath steps for all the values of L_i; i=1,2,...

Step 3(a) From equation 3.42 derive Q_2.

Step 3(b) From equation 3.43 derive Q_1 .

Step 3(c) From equation 3.44 derive k.

Step 3(d) Obtain the value of θ from equation 3.45.

Step 3(e) Repeat Steps 3a to 3d unless there is any variation in Q_2, Q_1, k, and θ upto a level of accuracy specified.

Step 4 Obtain the value of ED_1 and $E'D$ using following steps

Step 4(a) If ED_1 <220 then $C_{ep,i} = 0$ and $Y_i = 0$ else go to step 4(b) and if $E'D$ <220 then $C_{ep,i} = 0$ and $Y_i = 0$ else go to step 4(b).

Step 4(b) If $220 < ED_1 < 330$ then $C_{ep,i} = 1000$ and $Y_i = 1$ else go to step 4(c) and if 220< $E'D < 330$ then $C_{ep,i} = 1000$ and $Y_i = 1$ else go to step 4(c).

Step 4(c) If $330 < ED_1 < 440$ then $C_{ep,i} = 2000$ and $Y_i = 1$ else go to step 4(d) and if 330< $E'D < 440$ then $C_{ep,i} = 2000$ and $Y_i = 1$ else go to step 4(d).

Step 4(d) If $440 < ED_1 < 550$ then $C_{ep,i} = 3000$ and $Y_i = 1$ else go to step 4(e) and if 440< $E'D < 550$ then $C_{ep,i} = 3000$ and $Y_i = 1$ else go to step 4(e).

Step 4(e) If $550 < ED_1 < 660$ then $C_{ep,i} = 4000$ and $Y_i = 1$ else go to step 4(f) and if $550 < E'D < 660$ then $C_{ep,i} = 4000$ and $Y_i = 1$ else go to step 4(f).

Step 4(f) If $660 > ED_1$ then $C_{ep,i} = 4000$ and $Y_i = 1$ and if $660 > E'D$ then $C_{ep,i} = 4000$ and $Y_i = 1$.

Step 5 Obtain the value of EC_1 and EC_1' from equations 3.47 and 3.53.

Step 6 Obtain the value of EC_2 and EC_2' from equations 3.48 and 3.54.

Step 7 Obtain the value of SC_1 and SC_2 from equations 3.50 and 3.56.

Step 8 Use the values of Q_1, θ, k, ED_1, EC_1, EC_2, and SC_1 to derive TC_S from equation 3.51.

Step 9 Use the values of Q_2, Q_1, θ, k, $E'D$, EC_1', EC_2', and SC_2 to obtain TC_D from equation 3.57.

Step 10 Put n=n+1 and redo the Steps from 3 to 11.

Step 11 If $TC_D(n) > TC_D(n+1)$ then redo the steps from 2 to 6 else end the execution.

Proposition 3.2. *If we represents Q_1^*, Q_2^*, k^*, and θ^* as the optimal values of Q_1, Q_2, k, and θ, then for fixed values of n and $L \in [L_i, L_{i-1}]$, the "dual-channel" cost function TC_D attains its global minimum at Q_1^*, Q_2^*, k^*, and θ^* under the condition*

$$\left(\frac{2}{Q_1^3} \left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 C L + \pi D_1 \left(\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2 D_1 L k \sigma \sqrt{L}} \right) \right] \right)$$

$$\left(\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L+(D_1L)^2+k^2\sigma^2 L-2D_1kL\sigma\sqrt{L}}}\right]\right) > \Big(-\frac{\pi D_1}{2Q_1^2}$$

$$\left[\frac{k\sigma^2 L-D_1L\sigma\sqrt{L}}{\sqrt{\sigma^2 L+(D_1L)^2+k^2\sigma^2 L-2D_1Lk\sigma\sqrt{L}}}-\sigma\sqrt{L}\right]\Big)^2 \tag{3.58}$$

Proof. For "dual-channel supply chain", the Hessian matrix H is

$$(H_{TC_D})_1 = \begin{bmatrix} \frac{\partial^2 TC_D}{\partial Q_1^2} & \frac{\partial^2 TC_D}{\partial Q_1\partial Q_2} & \frac{\partial^2 TC_D}{\partial Q_1\partial k} & \frac{\partial^2 TC_D}{\partial Q_1\partial \theta} \\ \frac{\partial^2 TC_D}{\partial Q_2\partial Q_1} & \frac{\partial^2 TC_D}{\partial Q_2^2} & \frac{\partial^2 TC_D}{\partial Q_2\partial k} & \frac{\partial^2 TC_D}{\partial Q_2\partial \theta} \\ \frac{\partial^2 TC_D}{\partial k\partial Q_1} & \frac{\partial^2 TC_D}{\partial k\partial Q_2} & \frac{\partial^2 TC_D}{\partial k^2} & \frac{\partial^2 TC_D}{\partial k\partial \theta} \\ \frac{\partial^2 TC_D}{\partial \theta\partial Q_1} & \frac{\partial^2 TC_D}{\partial \theta\partial Q_2} & \frac{\partial^2 TC_D}{\partial \theta\partial k} & \frac{\partial^2 TC_D}{\partial \theta^2} \end{bmatrix}$$

where,

$$\frac{\partial^2 TC_D}{\partial Q_1^2} = \frac{2}{Q_1^3}\left[A_rD_1+\frac{S_1D_1}{n}+D_1CL+\pi D_1(\sqrt{\sigma^2 L+(D_1L)^2+k^2\sigma^2 L-2D_1Lk\sigma\sqrt{L}})\right]$$

$$\frac{\partial^2 TC_D}{\partial Q_1\partial Q_2} = \frac{\partial^2 TC_D}{\partial Q_2\partial k} = \frac{\partial^2 TC_D}{\partial Q_2\partial \theta} = \frac{\partial^2 TC_D}{\partial k\partial \theta} = 0$$

$$\frac{\partial^2 TC_D}{\partial Q_1\partial k} = -\frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L-D_1L\sigma\sqrt{L}}{\sqrt{\sigma^2 L+(D_1L)^2+k^2\sigma^2 L-2D_1kL\sigma\sqrt{L}}}-\sigma\sqrt{L}\right]$$

$$\frac{\partial^2 TC_D}{\partial Q_1\partial \theta} = \frac{sD_1n}{2}$$

$$\frac{\partial^2 TC_D}{\partial Q_2^2} = \frac{2S_2D_2}{Q_2^3}$$

$$\frac{\partial^2 TC_D}{\partial k^2} = \frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L+(D_1L)^2+k^2\sigma^2 L-2D_1kL\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2 TC_D}{\partial \theta^2} = \frac{\alpha b}{\theta^2}$$

The principal minor of $|(H_{TC_D})_1|$ of order 1×1 is

$$|(H_{TC_D})_{1,1}| = \left| \frac{\partial^2 TC_D}{\partial Q_1^2} \right|_{(Q_1^*, Q_2^*, k^*)}$$

$$= \frac{2}{Q_1^3} \left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2 D_1 L k \sigma \sqrt{L}}) \right] > 0$$

The principal minor of $|(H_{TC_D})_1|$ of order 2×2 is

$$|((H_{TC_D})_1)_{2,2}|_{(Q_1^*, Q_2^*, k^*, \theta^*)} = \begin{vmatrix} \frac{\partial^2 TC_D}{\partial Q_1^2} & \frac{\partial^2 TC_D}{\partial Q_1 \partial Q_2} \\ \frac{\partial^2 TC_D}{\partial Q_2 \partial Q_1} & \frac{\partial^2 TC_D}{\partial Q_2^2} \end{vmatrix}_{(Q_1^*, Q_2^*, k^*, \theta^*)}$$

$$|((H_{TC_D})_1)_{2,2}|_{(Q_1^*, Q_2^*, k^*, \theta^*)} = \left(\frac{\partial^2 TC_D}{\partial Q_1^2} \right) \left(\frac{\partial^2 TC_D}{\partial Q_2^2} \right)$$

$$= \left(\frac{2}{Q_1^3} \left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2 D_1 L k \sigma \sqrt{L}}) \right] \right) \left(\frac{2 S_2 D_2}{Q_2^3} \right) > 0$$

The principal minor of $|((H_{TC_D})_1)_{3,3}|_{(Q_1^*, Q_2^*, k^*, \theta^*)}$ of order 3×3 is

$$|((H_{TC_D})_1)_{3,3}|_{(Q_1^*, Q_2^*, k^*, \theta^*)} = \begin{vmatrix} \frac{\partial^2 TC_D}{\partial Q_2^2} & \frac{\partial^2 TC_D}{\partial Q_2 \partial k} & \frac{\partial^2 TC_D}{\partial Q_2 \partial \theta} \\ \frac{\partial^2 TC_D}{\partial k \partial Q_2} & \frac{\partial^2 TC_D}{\partial k^2} & \frac{\partial^2 TC_D}{\partial k \partial \theta} \\ \frac{\partial^2 TC_D}{\partial \theta \partial Q_2} & \frac{\partial^2 TC_D}{\partial \theta \partial k} & \frac{\partial^2 TC_D}{\partial \theta^2} \end{vmatrix}_{(Q_1^*, Q_2^*, k^*, \theta^*)}$$

$$= \left(\frac{\partial^2 TC_D}{\partial Q_2^2} \right) \left(\frac{\partial^2 TC_D}{\partial k^2} \right) \left(\frac{\partial^2 TC_D}{\partial \theta^2} \right)$$

$$= \frac{\pi D_1}{2 Q_1} \left(\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2 D_1 k L \sigma \sqrt{L}}} \right) \left(\frac{2 S_2 D_2}{Q_2^3} \right) \left(\frac{\alpha b}{\theta^2} \right) > 0$$

The principal minor of $|((H_{TC_D})_1)_{4,4}|_{(Q_1^*, Q_2^*, k^*, \theta^*)}$ of order 4×4 is

$$|((H_{TC_D})_1)_{4,4}|_{(Q_1^*, Q_2^*, k^*, \theta^*)} = \begin{vmatrix} \frac{\partial^2 TC_D}{\partial Q_1^2} & \frac{\partial^2 TC_D}{\partial Q_1 \partial Q_2} & \frac{\partial^2 TC_D}{\partial Q_1 \partial k} & \frac{\partial^2 TC_D}{\partial Q_1 \partial \theta} \\ \frac{\partial^2 TC_D}{\partial Q_2 \partial Q_1} & \frac{\partial^2 TC_D}{\partial Q_2^2} & \frac{\partial^2 TC_D}{\partial Q_2 \partial k} & \frac{\partial^2 TC_D}{\partial Q_2 \partial \theta} \\ \frac{\partial^2 TC_D}{\partial k \partial Q_1} & \frac{\partial^2 TC_D}{\partial k \partial Q_2} & \frac{\partial^2 TC_D}{\partial k^2} & \frac{\partial^2 TC_D}{\partial k \partial \theta} \\ \frac{\partial^2 TC_D}{\partial \theta \partial Q_1} & \frac{\partial^2 TC_D}{\partial \theta \partial Q_2} & \frac{\partial^2 TC_D}{\partial \theta \partial k} & \frac{\partial^2 TC_D}{\partial \theta^2} \end{vmatrix}_{(Q_1^*, Q_2^*, k^*, \theta^*)}$$

$$= \left(\frac{\partial^2 TC_D}{\partial Q_1^2}\right)\left(\frac{\partial^2 TC_D}{\partial Q_2^2}\right)\left(\frac{\partial^2 TC_D}{\partial k^2}\right)\left(\frac{\partial^2 TC_D}{\partial \theta^2}\right) - \left(\frac{\partial^2 TC_D}{\partial Q_2^2}\right)\left(\frac{\partial^2 TC_D}{\partial \theta^2}\right)\left(\frac{\partial^2 TC_D}{\partial k \partial Q_1}\right)^2$$

$$\implies \left(\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]\right)\left(\frac{2S_2 D_2}{Q_2^3}\right)\left(\frac{\alpha b}{\theta^2}\right)$$

$$\left(\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 kL\sigma\sqrt{L}}}\right]\right) - \left(\frac{2S_2 D_2}{Q_2^3}\right)\left(\frac{\alpha b}{\theta^2}\right)$$

$$\left(-\frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]\right)^2 > 0$$

$$\implies \left(\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]\right)$$

$$\left(\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 kL\sigma\sqrt{L}}}\right]\right) > \Big(-\frac{\pi D_1}{2Q_1^2}$$

$$\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]\Big)^2$$

Since, the Hessian matrix's, all the principal minors are not negative. Hence, the Hessian matrix $(H_{TC_D})_1$ is positive definite at $(Q_1^*, Q_2^*, k^*, \theta^*)$. Thus, aggregate expected cost for "dual-channel" gets its global minimum at $(Q_1^*, Q_2^*, k^*, \theta^*)$.

Proposition 3.3. *If we represents Q_1^*, k^*, and θ^* as the optimal values of Q_1, k, and θ then for fixed values of n and $L \in [L_i, L_{i-1}]$, the "single-channel" cost function TC_S attains its global minimum at Q_1^*, k^*, and θ^* under the condition*

$$\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} +_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]$$

$$\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 kL\sigma\sqrt{L}}}\right]\frac{\alpha b}{\theta^2} + \left(\frac{sD_1 n}{2}\right)^2$$

$$\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 l)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}\right] >$$

$$\left(\frac{\pi D_1}{2Q_1^2}\right)^2\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]^2 \frac{\alpha b}{\theta^2} \tag{3.59}$$

Proof. For "single-channel supply chain", the Hessian matrix H is

$$(H_{TC_S})_2 = \begin{bmatrix} \frac{\partial^2 TC_S}{\partial Q_1^2} & \frac{\partial^2 TC_S}{\partial Q_1 \partial k} & \frac{\partial^2 TC_S}{\partial Q_1 \partial \theta} \\ \frac{\partial^2 TC_S}{\partial k \partial Q_1} & \frac{\partial^2 TC_S}{\partial k^2} & \frac{\partial^2 TC_S}{\partial k \partial \theta} \\ \frac{\partial^2 TC_S}{\partial \theta \partial Q_1} & \frac{\partial^2 TC_S}{\partial \theta \partial k} & \frac{\partial^2 TC_S}{\partial \theta^2} \end{bmatrix}$$

where,

$$\frac{\partial^2 TC_S}{\partial Q_1^2} = \frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]$$

$$\frac{\partial^2 TC_S}{\partial Q_1 \partial k} = -\frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 l - d_1 l\sigma\sqrt{l}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2 TC_S}{\partial Q_1 \partial \theta} = \frac{sD_1 n}{2}$$

$$\frac{\partial^2 TC_S}{\partial k^2} = \frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2 TC_S}{\partial k \partial \theta} = \frac{\partial^2 TC_S}{\partial \theta \partial k} = 0$$

$$\frac{\partial^2 TC_S}{\partial \theta^2} = \frac{\alpha b}{\theta^2}$$

The principal minor of $|(H_{TC_S})_2|$ of order 1×1 is

$$|(H_{TC_S})_{1,1}|_{(}Q_1^*, k^*, \theta^*) = \left|\frac{\partial^2 TC_S}{\partial Q_1^2}\right|_{(Q_1^*, k^*, \theta^*)}$$

$$= \frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right] > 0$$

The principal minor of $|(H_{TC_S})_2|$ of order 2×2 is

$$|(H_{TC_S})_{2,2}|_{(Q_1^*,k^*,\theta^*)} = \begin{vmatrix} \frac{\partial^2 TC_S}{\partial k^2} & \frac{\partial^2 TC_S}{\partial k \partial \theta} \\ \frac{\partial^2 TC_S}{\partial \theta \partial k} & \frac{\partial^2 TC_S}{\partial \theta^2} \end{vmatrix}_{(Q_1^*,k^*,\theta^*)}$$

$$= \left(\frac{\partial^2 TC_S}{\partial k^2}\right)\left(\frac{\partial^2 TC_S}{\partial \theta^2}\right) - \left(\frac{\partial^2 TC_S}{\partial k \partial \theta}\right)\left(\frac{\partial^2 TC_S}{\partial \theta \partial k}\right)$$

$$= \frac{\pi D_1 \alpha b}{2Q_1 \theta^2} \frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 k L \sigma \sqrt{L}}} > 0$$

The principal minor of $|(H_{TC_S})_2|$ of order 3×3 is

$$|(H_{TC_S})_{3,3}|_{(Q_1^*,k^*,\theta^*)} = \begin{vmatrix} \frac{\partial^2 TC_S}{\partial Q_1^2} & \frac{\partial^2 TC_S}{\partial Q_1 \partial k} & \frac{\partial^2 TC_S}{\partial Q_1 \partial \theta} \\ \frac{\partial^2 TC_S}{\partial k \partial Q_1} & \frac{\partial^2 TC_S}{\partial k^2} & \frac{\partial^2 TC_S}{\partial k \partial \theta} \\ \frac{\partial^2 TC_S}{\partial \theta \partial Q_1} & \frac{\partial^2 TC_S}{\partial \theta \partial k} & \frac{\partial^2 TC_S}{\partial \theta^2} \end{vmatrix}_{(Q_1^*,k^*,\theta^*)}$$

$$= \Big(\frac{\partial^2 TC_S}{\partial Q_1^2}\Big)\Big(\frac{\partial^2 TC_S}{\partial k^2}\Big)\Big(\frac{\partial^2 TC_S}{\partial \theta^2}\Big) - \Big(\frac{\partial^2 TC_S}{\partial Q_1 \partial k}\Big)\Big(\frac{\partial^2 TC_S}{\partial k \partial Q_1}\Big)\Big(\frac{\partial^2 TC_S}{\partial \theta^2}\Big) +$$

$$\Big(\frac{\partial^2 TC_S}{\partial Q_1 \partial \theta}\Big)\Big(-\frac{\partial^2 TC_S}{\partial \theta \partial Q_1}\Big)\Big(\frac{\partial^2 TC_S}{\partial k^2}\Big)$$

$$\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L k \sigma \sqrt{L}})\right]\left(\frac{\pi D_1}{2Q_1}\right)\left(\frac{\alpha b}{\theta^2}\right)$$

$$\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 k L \sigma \sqrt{L}}}\right] - \left(\frac{\pi D_1}{2Q_1^2}\right)^2 \left[\frac{k\sigma^2 L - D_1 L \sigma \sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 k L \sigma \sqrt{L}}} - \sigma\sqrt{L}\right]^2 \frac{\alpha b}{\theta^2}$$

$$+\left(\frac{sD_1 n}{2}\right)^2 \frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 k L \sigma \sqrt{L}}}\right] > 0$$

$$\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L k \sigma \sqrt{L}})\right]\left(\frac{\pi D_1}{2Q_1}\right)\left(\frac{\alpha b}{\theta^2}\right)$$

$$\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 l)^2 + k^2\sigma^2 L - 2D_1 k L \sigma \sqrt{L}}}\right] + \left(\frac{sD_1 n}{2}\right)^2 \frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 k L \sigma \sqrt{L}}}\right] >$$

$$\left(\frac{\pi D_1}{2Q_1^2}\right)^2 \left[\frac{k\sigma^2 L - D_1 L \sigma \sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 k L \sigma \sqrt{L}}} - \sigma\sqrt{L}\right]^2 \frac{\alpha b}{\theta^2}$$

Since, the Hessian matrix's, all the principal minors are not negative. Hence, the Hessian matrix $(H_{TC_S})_1$ is positive definite at (Q_1^*, k^*, θ^*). Thus, aggregate expected cost for "single-channel" gets the global minimum at (Q_1^*, k^*, θ^*).

3.7 Numerical experimentation and discussion

This segment exemplifies the behavior of the developed model for classical coordination in a dual and single channel. To analyze the consequences of various environmental factors on the supply chain model so that eco-friendly decisions could be workout in the business model which is the sole objective of the numerical examples. To resemble a real manufacturing environment, values of parameters were taken from real world examples Bazan et al. (2015), Jaber et al. (2013) and Chauhan et al. (2021).

3.7.1 Input parameters

Following enlist values of the parameter's considered in developing the model C_p =600 $\$/unit$, γ_1 =0.75 $\$/unit$, γ_2 = 0.80 $\$/unit$, C_1 =350 $\$/unit$, C_2 =150 $\$/unit$, C_3 =100 $\$/unit$, δ_1 =0.2, δ_2 =0.3, S_1 =800 $\$/setup$, S_2 =1000 $\$/setup$, ϕ_1 =0.25 %, ϕ_2 =0.3 %, ϕ_3 =0.45 %, C_b =100 $\$/unit$, C_{vr} =120 $\$/unit$, h_1 =30 $\$/unit/year$, r_b =0.2 $\$/unit/$unit time, L =4 weeks, r_v =0.2 $\$/unit/$unit time, σ =200, A_r =200 $\$/order$, m =0.7, P_1 =5000 $Units/year$, P_2 =5000 $Units/year$, s =400 $\$/unit$, α =0.5, ν =150 $\$/unit$, b =400, a_1 =1000, a_2 =1000 . The aforementioned model has been developed on the lines of Jaber et al. (2013) and Bazan et al. (2015). Consequently, the input parameters regarding the prices of the ejection of CO_2 are taken from his findings. Penalty schedule for emission is represented by table 3.8 is extracted from Jaber et al. (2013).

3.7.2 Results and discussions

An real life illustration is considered to analyze the supply chain management's aggregate cost of the firm with both "dual-channel and single-channel". Moreover, in a supply chain model with "dual-channel" shoppers are provided with "core items and personalized items" through offline and online channels respectively although, in a "supply chain model with single route", "core item" is made available to the shoppers through offline channel. Three sorts of customization are put forward on the "standard products" in "supply chain

Tab. 3.8: Penalty schedule for emission

i	Ceiling of ejection (E_{li})	Fine levied ($C_{ep,i}$)
1	$\chi < 220$	0
2	$220 \leq \chi < 330$	\$ 1000
3	$330 \leq \chi < 440$	\$ 2000
4	$440 \leq \chi < 550$	\$ 3000
5	$550 \leq \chi < 660$	\$ 4000
6	$\chi \geq 660$	\$ 5000

1. For "single-channel supply chain" $\chi = ED_1$
2. For "dual-channel supply chain" $\chi = E'D$

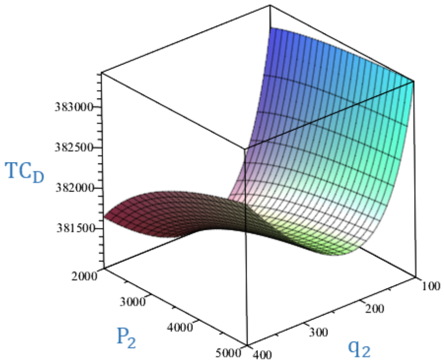
Tab. 3.9: Decision variable's optimum values

n	L	k	Q_1	Q_2	D_1	D_2	θ^1	TC_D^2	TC_S^3
1	3	12.82	1289.4	305.5	1384.5	615.9	8.5	1.898	2.19
2	3	12.94	1060.8	305.5	1384.5	615.9	6.1	1.42	1.64
3	3	12.98	991.1	305.5	1384.5	615.9	4.6	1.29	1.48
4	3	12.99	982.5	305.5	1384.5	615.9	3.5	1.28	1.46[4]
5	3	12.95	1029.8	305.5	1384.5	615.9	2.5	1.39	1.57
6	3	12.87	1195.6	305.5	1384.5	615.9	1.4	1.895	1.95

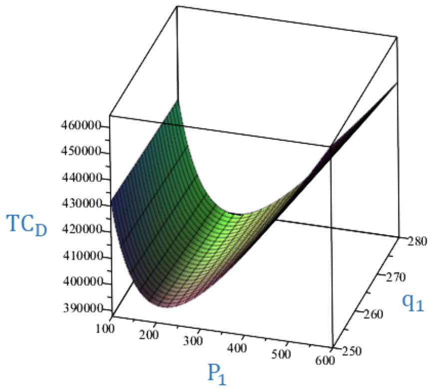
1.$\theta = value \times 10^{-07}$,
2.$TC_D = value \times 10^{10}$,
3.$TC_S = value \times 10^{10}$,
4.Indicates the optimal (minimum) value of the supply chain's aggregate cost

model of dual-rout" to the shoppers are presumed for simplicity. Table 3.9 reflects the numerical values of the input variables utilized to derive the decision variable's optimal value. Further, the table demonstrates the minimum cost of the model for single and dual-route are \$ 1.46×10^{10} and \$ 1.28×10^{10} respectively. The minimum costs are spotted at $n = 4$ and the corresponding values of $k, L, Q_1, Q_2, D_1, D_2, \theta$ are 12.99, 3, 982.5, 305.4571, 1384.608, 615.915, 3.5×10^{-07}, respectively.

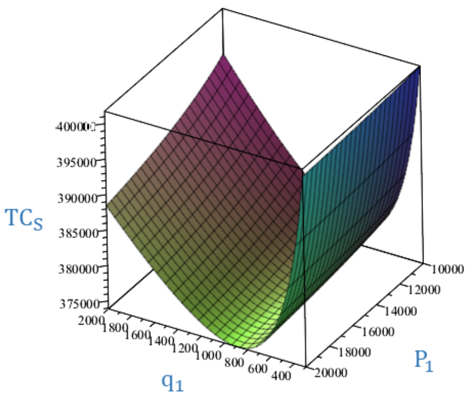
Moreover, in figures 3.20, 3.20a & 3.20b demonstrates the variation of total cost (TC_D) of "supply chain for dual channel" with respect to demand and production of customized (demand $= Q_2$ & production $= P_2$) and standard (demand $= Q_1$ & production $= P_1$) product respectively. Moreover, figure 3.20c manifests the variation of total cost (TC_S) of "single supply chain management" with demand (Q_1) and production (P_1) of "standard product" only.



(a) Graphical representation of total cost of dual channel supply chain TC_D with respect to production rate (P_1) and demand (Q_1)



(b) Graphical representation of total cost of dual channel supply chain TC_D with respect to production rate (P_2) and demand (Q_2)



(c) Graphical representation of total cost of single channel supply chain TC_S with respect to production rate (P_1) and demand (Q_1)

Fig. 3.20: Graphical presentation of total cost of the supply chain of the firm

3.7.3 Out-turn of sensitivity in demand because of difference in the selling price of the product offered by two modes (traditional and online)

For analyzing the price sensitivity, two scenarios (I & II) where examination of the ramification of the varying selling prices of "standard and personalized items" on the demand and aggregate cost of the "dual mode and single mode supply chain" are depicted.

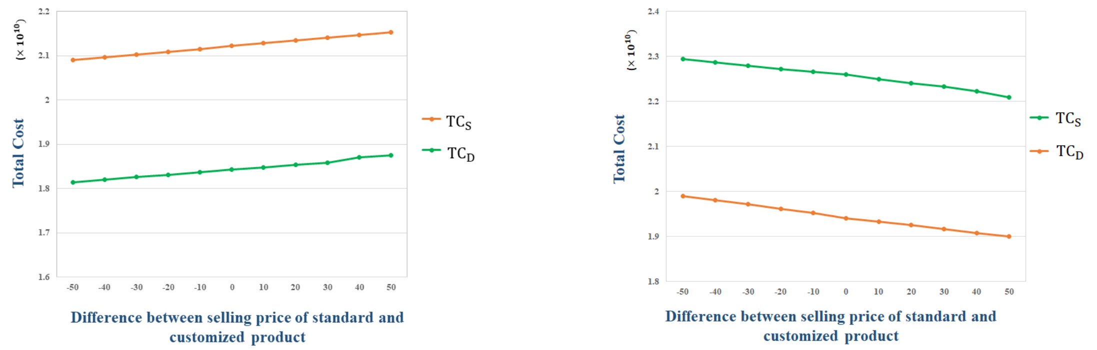
I. In this scenario, the gulf between the purchasing cost offered by the traditional and the online route is less than the ceiling i.e., "threshold limit" ($|\sum_{i=1}^{N} C_i - C_P| <$ Threshold value). Moreover, the graphical depiction of the situation is demonstrated by figure 3.21. Thus, while scrutinizing the situation, underneath two cases are obtained.

Case I: Figure 3.21a demonstrates the variation in the supply chain's aggregate cost with the variation in the selling cost of the "customized product". Furthermore, the figure illustrates that as the cost of "personalized products" increases, there is an increase in the total cost of the firm irrespective of any supply chain. Additionally, there is a 3.1% and 3.4% of increase in the total cost for "single-channel and dual-channel supply chains".

Case II: The deflection of the supply chain's aggregate cost concerning the deflection in the purchasing charges of the "standard product" is reflected by figure 3.21b. Besides, it is also manifesting the decrease in the total cost of the firm with the increase in the cost of the "standard products". Moreover, there is approximately 3.7% and 4.5% of a dip in the firm's total cost having "single-route and dual-route supply chains".

II. In the present scenario, the gulf between the purchasing cost offered by the traditional and the online route exceeds the ceiling i.e., threshold limit ($|\sum_{i=1}^{N} C_i - C_P| >$ Threshold value). Figure 3.22 illustrates the graphical representation of the situation. Henceforth, on the same lines following two cases are obtained.

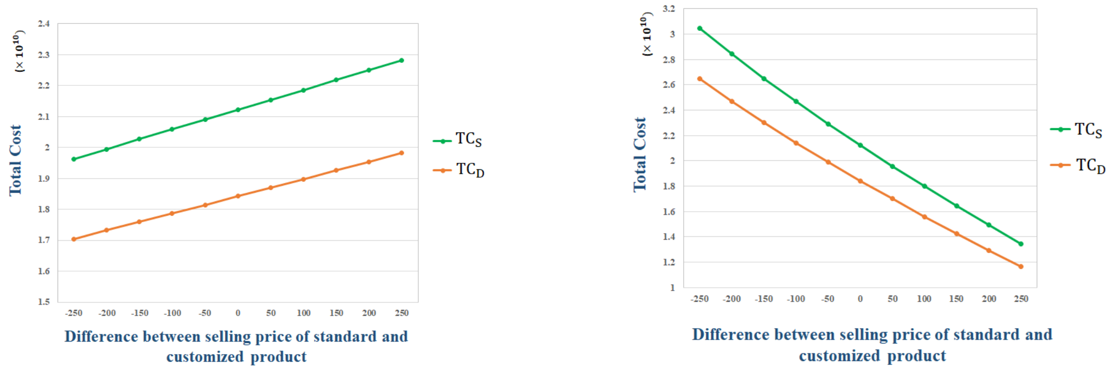
Case I: In figure 3.22a deviation of the total cost of the firm with respect to the deviation in the "customized product's" selling price is illustrated. Additionally, there is a 16.25% and 16.28% increase in the total cost for "single-route and dual-route supply chains". Since the customers are shifting from "customized to standard products" thereby increasing the total cost of the firm.



(a) Variation in the price of customized product

(b) Variation in the price of standard product

Fig. 3.21: Graphical representation of variation of the total cost when $|\sum_{i=1}^{N} C_i - C_P|$ <Threshold value

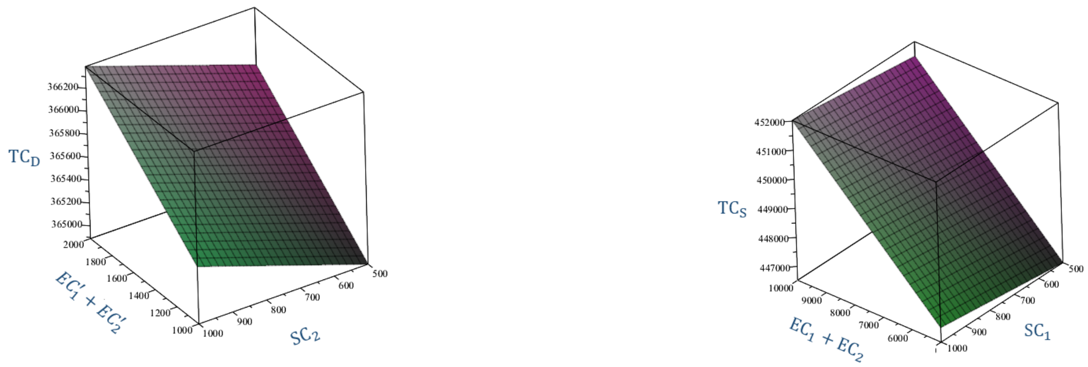


(a) Variation in the price of customized product

(b) Variation in the price of standard product

Fig. 3.22: Graphical representation of variation of the total cost when $|\sum_{i=1}^{N} C_i - C_P|$ >Threshold value

Case II: In figure 3.22b, firms the total cost variation concerning the variation in the selling price of the "standard product" is portrayed. Furthermore, there is a 55.83% and 55.93% decrease in the total cost for "single-route and dual-route supply chains". As the price of "standard products" increases there is a reduction in the demand thereby reducing the total cost of the firm. Moreover, a "standard product" is required for customization thus, increasing the cost of a "customized product".



(a) Dual-channel supply chain

(b) Single-channel supply chain

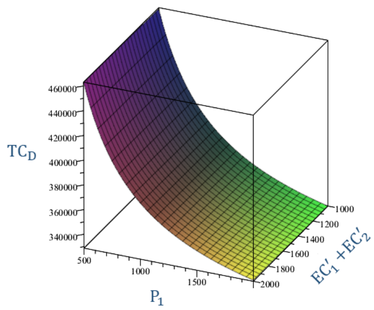
Fig. 3.23: Total cost of the supply chain management of the firm w.r.t environmental and social pillars of sustainability

3.7.4 Analysis of the environmental and social pillars of sustainability

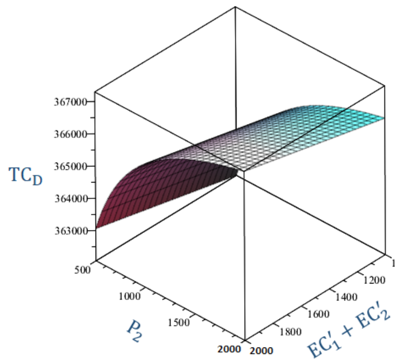
In figure 3.23, an attempt has been made to analyze the variation in the total cost of the firm with that of environmental and social pillars of sustainability. Additionally, figures 3.23a & 3.23b manifests the total cost of the firm having "dual-channel and single-channel supply chains". It can be observed from figures that irrespective of any channel adopted by the firm's supply chain management, the emission of CO_2 cost and social cost charged on the firm increases with the increase in production. Consequently, there is an increase in the total cost of the firm.

Figures 3.24a and 3.24b describe that as the production of "standard and customized products" increases, the overall cost of carbon emission cost charged on the firm increases which is reflected by equation 3.52. Moreover, figure 3.24c manifests that in case of "single-channel supply chain" also, the production of "standard product" is directly proportional to the carbon emissions cost as derived from equation 3.46. Although sustainable development is a key element of the firm and they try to reduce CO_2 emission and social cost but still the burden is increasing on the firm (Dwivedi et al. , 2022). Henceforth, efforts should be made by the firms in using green energy in production and voluntarily compensate for unavoidable emissions by directly funding for certified offsetting projects such as "Saving forests, protecting wildlife, and transforming lives in Zimbabwe, Harnessing clean wind energy to power sustainable development in North China, Cleaner air, renewable electricity and improved well being for communities in Central Vietnam, and Permanent protection for Afognak Island's dense, old growth spruce forest in Alaska"[8] etc.

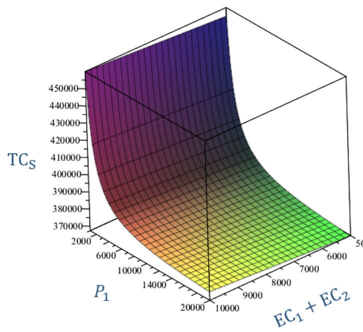
[8] "https://www.porsche.com/uk/aboutporsche/responsibility/porscheimpact/"



(a) Variation in total cost of dual-channel supply chain (TC_D) w.r.t production of standard product (P_1) and environmental pillar $(EC'_1 + EC'_2)$



(b) Variation in total cost of dual-channel supply chain (TC_D) w.r.t production of customized product (P_2) and environmental pillar $(EC'_1 + EC'_2)$



(c) Variation in total cost of single-channel supply chain (TC_S) w.r.t production of standard product (P_1) and environmental pillar $(EC_1 + EC_2)$

Fig. 3.24: Graphical representation of total cost of the supply chain management of the firm w.r.t environmental pillar of sustainability and production of the firm

Various contributors such as "investors, employees, consumers, suppliers, public powers, and organizations of non-government" are increasingly requesting companies to focus and strengthen corporate social responsibilities (Duque-Grisales and Aguilera-Caracuel , 2019). Corporate social responsibilities and social sustainability are complementary (Rameshwar et al. , 2020). Thus, issues such as workplace diversity, health safety, labor strikes, child labor, the impact of operations on community and society, etc., are addressed. Henceforth, figure 3.25 dig into a variation of the aggregate cost of the firm irrespective of supply chain concerning the social pillar of sustainability and production of the firm. figures 3.25a, 3.25b, and 3.25c depict that as the production of products escalates the firm needs to invest more in the social welfare of the workers leading to an escalation in the total cost of the firm.
From equations (3.46), (3.47), (3.52), and (3.53) we get,

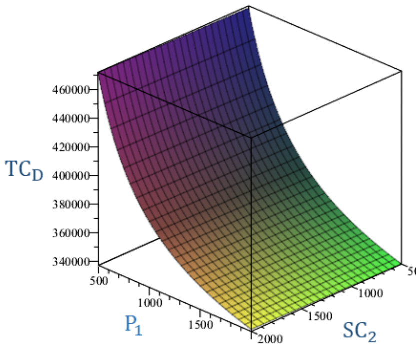
$$EC_1 \propto E \propto P_1 \quad and \quad EC_1' \propto E' \propto P$$

Thus, as the production P_1 and $P = P_1 + P_2$ for "single and dual-channel" respectively proliferates, the cost of the carbon exudes during production EC_1 and EC_1' proliferates thereby increasing the total cost of the firm. Figures 3.26a and 3.26b demonstrates the aforementioned nature for the "dual-channel" whereas figure 3.26c depicts that for "single-channel supply chain".

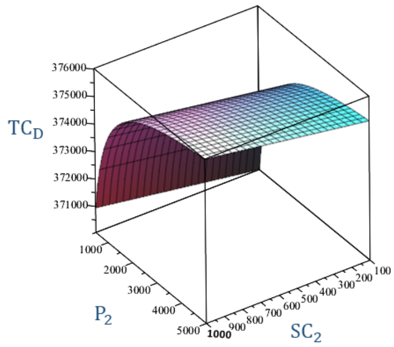
As per the equations 3.46 and 3.52, the cost of CO_2 ejection from production increases when the production rates are low or high. Consequently, the influence of the cost of emission is prominent for the production rate values that deviate from the optimal value. Moreover, the penalty on the firm is generally driven by carbon emissions during production. Thus, in the present context, an optimal solution can neither be obtained at minimum emissions with no penalty nor maximum emissions. Figure 3.27 portrait the diverse production rates from the minimum 1000 units/year to the maximum 3000 units/year and optimal value for the firm.

3.7.5 Analysis of the probability of production going out-of-control

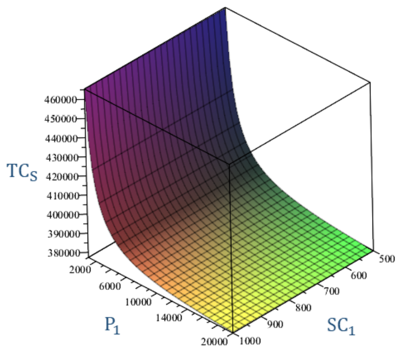
Figures 3.28a, 3.28b, 3.29a, and 3.29b demonstrates the variation of total cost concerning the chances of manufacturing going "out-of-control" and that of variation in the manufacturing of items concerning the possibility of manufacturing flawed products. Moreover, they depict that elevation in production elevates the firm's investment in quality improvement thereby minimizing the chances of "out-of-control" production.



(a) Representation of cost dual-channel supply chain (TC_D) w.r.t production of standard product (P_1) and social pillar (SC_1)

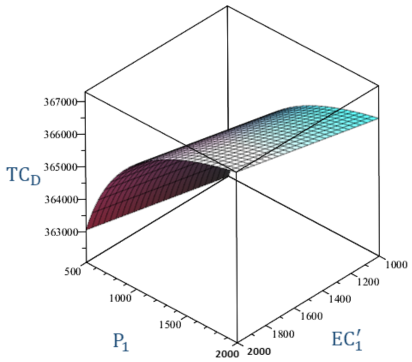


(b) Representation of cost dual-channel supply chain (TC_D) w.r.t production of customized product (P_2) and social pillar (SC_2)

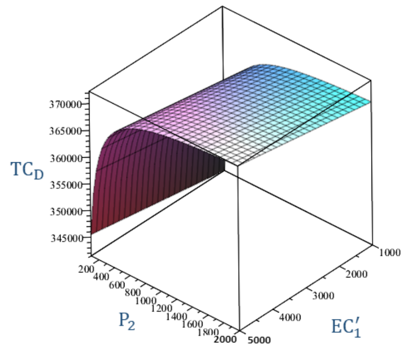


(c) Representation of cost single-channel supply chain (TC_S) w.r.t production of standard product (P_1) and social pillar (SC_1)

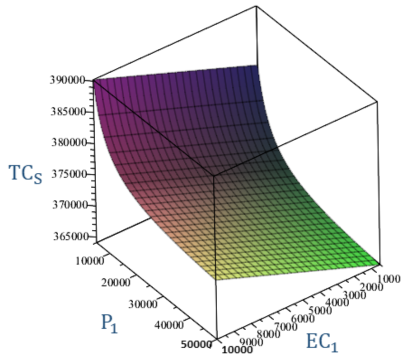
Fig. 3.25: Graphical representation of total cost of the supply chain management of the firm w.r.t social pillar of sustainability and production of the firm



(a) Representation of cost dual-channel supply chain (TC_D) w.r.t production of standard product (P_1) & carbon emission throughout production (EC_1')



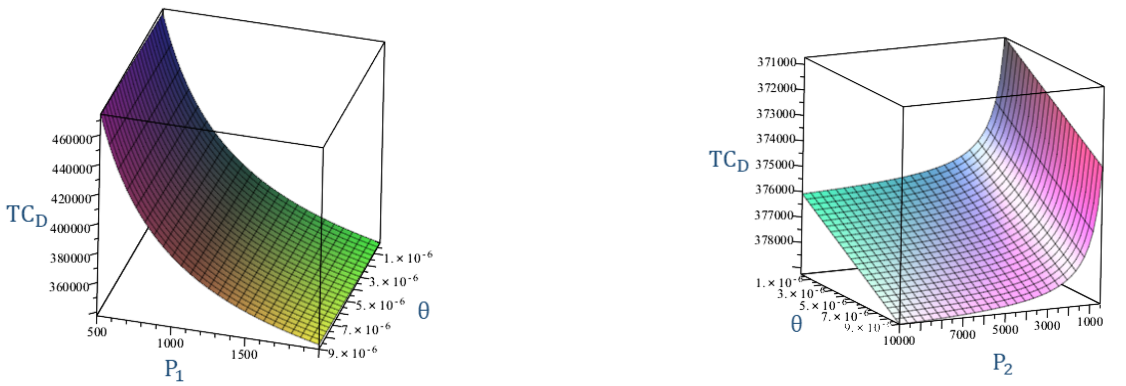
(b) Representation of cost dual-channel supply chain (TC_D) w.r.t production of customized product (P_2) & carbon emission throughout production (EC'_1)



(c) Representation of cost single-channel supply chain (TC_S) w.r.t production of standard product (P_1) & carbon emission throughout production (EC_1)

Fig. 3.26: Graphical representation of total cost of the supply chain management of the firm w.r.t carbon emission throughout production & production of the firm

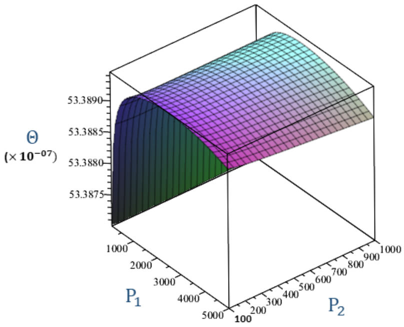
Fig. 3.27: Graphical representation of penalties charged on the firm with the variation in production



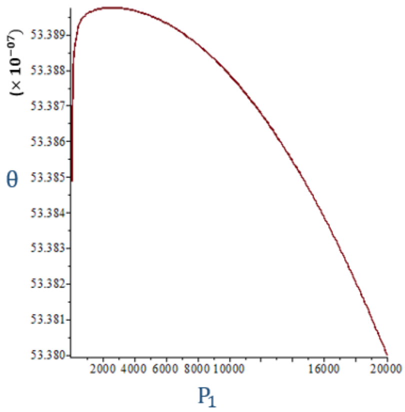
(a) Variation in the total cost of the dual-channel supply chain (TC_D) w.r.t production of standard product (P_1)

(b) Variation in the total cost of the dual-channel supply chain (TC_D) w.r.t production of customized product (P_2)

Fig. 3.28: Graphical representation of total cost of the supply chain management of the firm w.r.t production and probability of production going out of control



(a) Variation in the probability of defective products θ w.r.t production of standard product (P_1) & customized product (P_2) in dual-channel supply chain



(b) Variation in the probability of defective products θ w.r.t production of product (P_1) in single-channel supply chain

Fig. 3.29: Graphical representation of production w.r.t probability of defective products θ

3.8 Comparison with existing literature

We compare the optimal profit of this model with three existing literature such as Dey et al. (2023), Sarkar et al. (2023), and Choi et al. (2023) based on dual channel retailing and customization. Dey et al. (2023) studied dual-channel retailing with traditional and home delivery systems but no customization policy was considered. Sarkar et al. (2023) investigated the bullwhip effect and information sharing in the dual-channel supply chain, whereas Choi et al. (2023) assumed O2O supply chain management with the imperfect production process. Compared to this model the existing literature does not consider customization policy in dual retailing systems. Table 3.10 represents a comparative study with existing literature along with the achieved profits. All three pieces of literature consist of the total system profit. As our model is based on the cost of the system, we have calculated the profit by subtracting the total cost from the revenue. Clearly, we can observe that the total profit achieved from this manuscript is much higher than that of the other literature. This scenario happens due to dual-channel retailing under the customization policy.

3.9 Sensitivity analysis

The effect of change in the total cost of the firm concerning the change of -50%, -25%, $+25\%$, and $+50\%$ in key parameters are are elucidated in table 3.11. Figure 3.30 exhibits the influence of variation in total

Tab. 3.10: Comparison with existing literature

	Dey et al. (2023)	Sarkar et al. (2023)	Choi et al. (2023)	This study
Dual channel retailing	Yes	Yes	Yes	Yes
Customization	No	No	No	Yes
Defect reduction	No	No	Yes	Yes
Total profit	2861.71	1.0163×10^4	1.4550×10^3	2.3810×10^6

cost concerning the key parameters.

The observations obtained from the sensitivity analysis, are illustrated below.

(i) Costs related to carbon ejection during production and social factors are slightly sensitive in dual-route contrast to that of the single-route supply chain. When the carbon ejection cost is varied over $\pm 50\%$ the percentage of the total cost variation is $\pm 6 \times 10^{-6}$. Similarly, for variation of social cost parameter, the same is $\pm 1.2 \times 10^{-5}$.

(ii) Penalties imposed on the firm with a single-route are more sensitive than the dual-route supply chain. From figure 3.30, at $\pm 50\%$ change of penalty, the change of total cost for dual channel becomes $\pm 2 \times 10^{-5}\%$ while the change of total cost for a single channel is $\pm 1.2 \times 10^{-5}\%$.

(iii) The sensitivity analysis depicted in Figure 3.30 highlights that the unit cost of production for standard products exhibits a higher level of sensitivity for retailers compared to manufacturers. According to the graph, at ±50% change in the cost of production paid by the retailer corresponds to an approximate $\pm 6.4 \times 10^{-4}\%$ variation, while the same change in the cost borne by the manufacturer yields a roughly $\pm 5.9 \times 10^{-4}\%$ fluctuation.

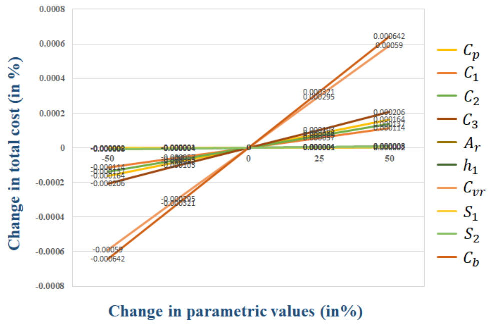
(iv) The cost component associated with the third customization demonstrates a higher level of sensitivity compared to the cost components of the other customizations, excluding the standard product. As shown in Figure 3.30, a $\pm 50\%$ change in the cost of the third customization results in an approximate $\pm 2 \times 10^{-4}\%$ variation, whereas the first customization exhibits a $\pm 1.1 \times 10^{-4}\%$ variation and the second customization shows a $\pm 1.3 \times 10^{-4}\%$ variation.

3.10 Managerial Implications

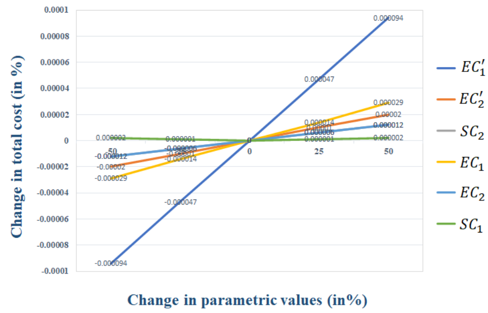
From a managerial perspective, this research has a huge impact on dual-channel retailing industries that offer customization policies to consumers. Adopting a dual-route retailing system benefits the industry

Tab. 3.11: Effect of change in key parameters

Parameters	Variation (in %)	Change in total cost	Parameters	Variation (in %)	Change in total cost
C_p	−50	−0.000164	S_1	−50	−0.000002
	−25	−0.000082		−25	−0.000001
	+25	+0.000082		+25	+0.000001
	+50	+0.000164		+50	+0.000002
C_1	−50	−0.000114	S_2	−50	−0.000008
	−25	−0.000057		−25	−0.000004
	+25	+0.000057		+25	+0.000004
	+50	+0.000114		+50	+0.000008
C_2	−50	−0.000137	C_b	−50	−0.000642
	−25	−0.000069		−25	−0.000321
	+25	+0.000069		+25	+0.000321
	+50	+0.000137		+50	+0.000642
C_3	−50	−0.000206	C_{vr}	−50	−0.00059
	−25	−0.000103		−25	−0.000295
	+25	+0.000103		+25	+0.000295
	+50	+0.000206		+50	+0.00059
A_r	−50	−0.000002	h_1	−50	−0.000008
	−25	−0.000001		−25	−0.000004
	+25	+0.000001		+25	+0.000004
	+50	+0.000002		+50	+0.000008
EC_1'	−50	−0.000094	EC_1	−50	−0.000029
	−25	−0.000047		−25	−0.000014
	+25	+0.000047		+25	+0.000014
	+50	+0.000094		+50	+0.000029
EC_2'	−50	−0.00002	EC_2	−50	−0.000012
	−25	−0.00001		−25	−0.000006
	+25	+0.00001		+25	+0.000006
	+50	+0.00002		+50	+0.000012
SC_2	−50	−0.000012	SC_1	−50	−0.000002
	−25	−0.000006		−25	−0.000001
	+25	+0.000006		+25	+0.000001
	+50	+0.000012		+50	+0.000002



(a) Concerning all cost parameters



(b) Concerning the sustainability cost parameters

Fig. 3.30: Effect of change in parametric values on total cost of the firm

with increased sales. Consumers are provided with the chance to explore the products online. Introducing the customization strategy into online retailing would significantly uplift the choices resulting in consumer demand to be increased. Moreover, the proposed strategy can improve the flexibility and convenience to the customers as consumers can switch between the channels i.e. traditional to online and vice-versa. Moreover, we proposed unequal customer shifting and a threshold limit for switching into the channels. These two assumptions improve the model with a more realistic scenario. Thus, managers can estimate profitability with more accuracy. The outcomes of our model depict that the industry's profitability should increase due to dual-route retailing than that of single-route.

Propositions 3.2 and 3.3 highlight that optimal results can be achieved under specific parameter values, enabling company managers to obtain highly accurate profit and loss estimates. This empowers the firm to make informed decisions on product ordering quantities and timing, maximizing profitability. Moreover, industry managers can estimate the necessary investments required to minimize the occurrence of defective products resulting from extensive customization.

The integration of sustainable investment in the retailing system has emerged as a significant responsibility for industries, aligning with the United Nations' sustainable goals. In this chapter, a sustainable retailing model is proposed that encompasses all pillars of sustainability: economic, environmental, and social. The model addresses carbon emissions during production and incorporates penalties, aiming to reduce carbon stress. The social pillar emphasizes preserving social capital by investing in services and promoting workplace safety, taking into account the alarming statistics provided by the International Labor Organization (ILO)[9], which states that approximately 2.3 million people worldwide succumb to work-related accidents or diseases annually, equivalent to around 6,000 deaths per day. By investing in economic, environmental,

[9] https://www.ilo.org/moscow/areas-of-work/occupational-safety-and-health/WCMS_249278/lang–en/index.htm

and social initiatives, the firm's owners can actively pursue sustainable goals.

3.11 Conclusions

The concept of sustainability utilized in the present chapter. Moreover, sustainability splits into three intertwined categories: "social sustainability", "economic sustainability", and "environmental sustainability". The three pillars of sustainability provide a framework for applying a solution oriented approach to complicated business models like supply chain management. According to the State of global air (2019)[10], air pollution claims more lives than traffic accidents or malaria. Thus, endeavoring to reduce CO_2 emissions and embracing the system in aiding the environment is the need of the hour. As the coronavirus pandemic may result in a 6% drop in greenhouse gas emissions for 2020 because of lock-down yet, there is a shortage of the 7.6% annual reduction needed to limit global warming to $1.5°C$[11]. Therefore, a policy of single-setup-multiple-delivery is remodeled in the present chapter by endorsing the firm with the pillars (i.e., economic, social, and environmental) of sustainability plus adapted customization of the "core products policy". The model reflects that as the production increases, the firm needs to invest more in the form of carbon emission costs and social costs. In case the "dual-channel supply chain" overshoots the carbon emission limits, penalties are levied on the firm. Thereby, efforts are made by the firms to voluntarily compensate for these penalties by funding certified offsetting projects or cap-and-trading carbon certificates. Furthermore, the model demonstrates that the downfall of penalty is 20% more for "single-channel" than dual. Thus, the firm and organizations should take the needful steps towards sustainability because every small resourcefulness will have far-reaching effects.

Additionally, this chapter also improves the quality of the products (i.e., "core and customized") by utilizing different investment finance. Also, an 80% improvement in the quality of products is observed with financial investment along with the reduction in the problem of "out-of-control" probability. The chapter also explores a presumed "threshold limit" plus uneven shifting of shoppers between the channel in a supply chain having dual mode of shopping for a more realistic scenario. The results reflect that as the variation in the cost of the "personalized and core product" elevates, shifting of customers starts depending upon the economic benefit of the customers. There is a decrease of 55.83% and 55.93% in the total cost for "single-route and dual-route supply chains" when charges of "standard products" increase and overshoot the threshold limit. Furthermore, there is a 16.25% and 16.28% increase in the total cost for "single-route and

[10] https://www.stateofglobalair.org/
[11] https://www.trvst.world/environment/sustainability-facts-statistics/

dual-route supply chains" when charges of "customized products" overshoot the "threshold limit". Moreover, it is more economical for the firm to embrace a "centralized dual-route supply chain" since it helps manufacturers in catering shopper's choices. Eventually, the chapter demonstrates that decision-makers should consider the probabilistic uncertainty with the sensitivity of various factors like selling price. Since the evaluation of demand fluctuation becomes a complicated function.

CHAPTER - 4

A green dual channel O2O supply chain management with reduced carbon emission under modified customization strategy: a case study with comparison

4.1 Problem definition

Since the industrial revolution, greenhouse gas emissions have increased immensely. This chapter studies the carbon effect caused by supply chain, exhaustion of energy, manufacturing and shipment under dual channel supply chain management. Henceforth, for analyzing the CO_2 emission while production, quadratic expression of production rate is considered in which emission is directly proportional to production rate. Moreover, the carbon emission from transport vehicles used for online and offline supply chain is considered. Further, an additional element of CO_2 ejection is considered on account that carbon emission might surpass the set emissions limit which leads to the penalties. Irrespective of the above issues industries expect high profitability in their business. Therefore, companies adopt several strategies such as dual channel supply chain management, customization, and lead time reduction. In this respect this chapter assumes online-to-offline (O2O) business model. The model integrates the convenience of online payment of online shopping and the good experience of offline stores and thus brings large development space. Both channels are offered to customer's. Online channel caters the personalized needs of the customers and offline channel caters the "standard products" demand. Moreover, since "threshold value" is subsumed in this chapter so, manufacturers while implementing online channels should negotiate with retailers. Figure 4.31 demonstrates the situations that whenever the gulf between "standard and the customized product" is soared and customers begin switching channels then the idea of a "threshold limit" appears which influences the

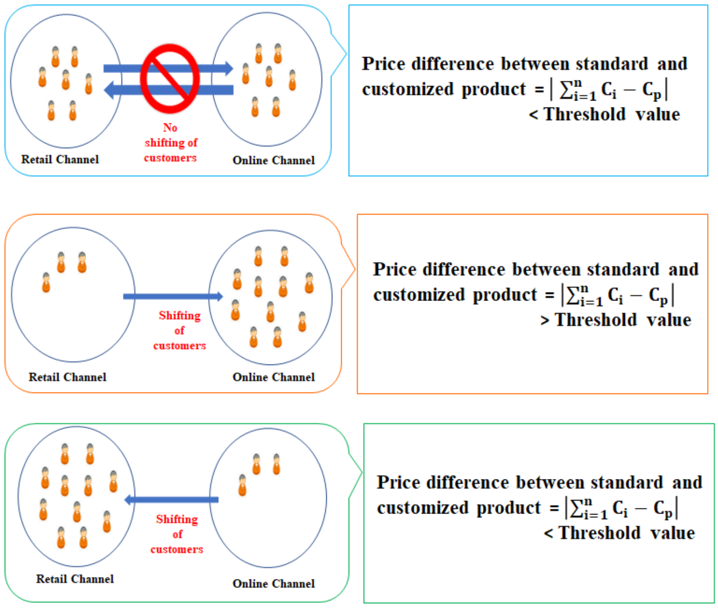


Fig. 4.31: Graphical representation of different situations arising because of the variation in market price of standard and personalized item

individual yield and the firm yield entirely. Managers of the industry need to maximize its profitability and obtain decisions related to administration under the consideration of environmental factors and demand uncertainty. Price dependency and uncertainty on demand are two of the key factors to be dealt with for achieving successful supply chain management. In this uncertain world this chapter will help the firms to maximize their profit while keeping sustainable factor and the cap-and-trade in mind.

4.2 Presumptions

Succeeding module list the assumptions presumed for framing the mathematical model.

1. The article is provided to the customer by the means of shopkeeper route and direct route (internet-based). Thus, by incorporating the make on-hest policy in the model the firm adopts a "centralized supply chain model with dual-route". Moreover, to be more realistic variable as well as random demand is taken into account (Modak and Kelle , 2018).

2. "Standard and make-to-order" articles are furnished by the manufacturer. Although the manufacturer is having its own production house and is providing “customized products”, no investment is required to enhance an arrangement for mass customization (Batarfi et al. , 2016).

3. When the level of inventory drops to reorder point R then the retailer places an order. Further, $R = D_1L + k\sigma\sqrt{L}$, where D_1L, $k\sigma\sqrt{L}$, and k are anticipated demand in the course of lead time, safety reserve, and (Sarkar and Majumder , 2013).

4. Identical cycle time considered in case of both - producer and shopkeeper as the retailer is selling the “standard product” only. Further, different cycle times are considered for the manufacturer in the manufacturing of the elementary product utilized for customization.

5. A segment of the total customers who are not willing to pick up the “standard product” from retailers prefers “personalized products” over “standard products” (Modak and Kelle , 2018).

6. For supplying the product through a retailer “single-setup multiple-delivery strategy” is incorporated whereas a “make-to-order strategy” is endorsed in the case of an online channel.

7. Switching of customers does not take place if the variation in the prices of the core and the “personalized” item falls within the presumed limit (Threshold limit).

8. The lead time ’L’ has ’m’ collectively independent elements. For the jth element, a_j=minimum time span, b_j=normal time span, and c_j=crashing cost per unit time. Practically, we presume $c_1 < c_2 < ... < c_m$ (Sarkar and Majumder , 2013).

4.3 Mathematical model

The present segment elucidates the functions such as demand and profit, a distribution-free technique for finding optimal solutions for the supply chain, various carbon emissions, and an algorithm for obtaining a solution for this chapter.

4.3.1 Demand function

The nature of customers is complex. Therefore, the price of the item, variety of the product, and sometimes ineptitude to reach the retailer govern the shopper's choice between the "core or personalized item". Linear functions for demand for "core and make-to-order items" are considered on the lines of succeeding Liu et al. (2016). Additionally, Huang and Swaminathan (2009) and Hua et al. (2010) work extended for deriving the functions for demand as mentioned underneath.
The offline route's demand function is:

$$D_1 = a_1 - \beta_1 p_1 + \delta_1 \left(\sum_{i=1}^{N} p_i - p_1 \right)$$

The online route's function for demand is:

$$D_2 = a_2 - \beta_2 \sum_{i=1}^{N} p_i - \delta_2 \left(\sum_{i=1}^{N} p_i - p_1 \right)$$

Where, a_1 and a_2 represent the demand, β_1 and β_2 are the coefficient of price sensitivity for the shopkeeper's route and producer's route respectively, reflecting the fluctuation in market demand due to fluctuation in the price by one dollar. Further, δ_1 and δ_2 depicts switching of the customers between offline and online channels or contrariwise hinged with the cost of the product offered by the route. A firm would get an equal amount of return per dollar invested nonetheless the number of products marketed. Consequently, the retailer's marketing charges of the "core item" is $p_1 = c_r(1+m)^2$, and the producer's marketing charges for "personalized item" is $p_i = C_i(1+m)$ where m is a "markup margin". Thus, we get the underneath demand equations by presuming the same "markup margin" for "core and make-to-order item"
For offline channel

$$D_1 = a_1 - \beta_1 C_p(1+m) + \delta_1 (1+m) \left(\sum_{i=1}^{N} C_i - C_p \right)$$

For online channel

$$D_2 = a_2 - \beta_2(1+m) \left(\sum_{i=1}^{N} C_i \right) - \delta_2(1+m) \left(\sum_{i=1}^{N} C_i - C_p \right)$$

4.3.2 Profit function

Present segment enlists the profit of manufacturer and retailer by selling "standard and make-to-order items".

I. **“Profit earned by the manufacturer by selling the standard product”**

By trading the “core item” by offline route, profit earned by the manufacturer per unit of time is given by:

$$\nabla_1 = \text{Revenue} - \text{Setup cost} - \text{Holding cost} - \text{Manufacturing cost for producer}$$

$$\nabla_1 = C_p(1+m)D_1 - \left[\frac{S_1 D_1}{nQ_1} + \frac{r_v C_p Q_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right]\right] - C_{vr}D_1 \tag{4.60}$$

Where $C_p(1+m)D_1$ and $C_p(1+m)$ represent revenue and the trading charges of the standard item. In a “single setup multiple delivery (SSMD)” policy, the manufacturer manufactures a numerical multiple of the retailer’s ordered quantity. As Q_1 is the cumulative ordered quantity by the retailers, thus nQ_1 quantity displayed by figure 2.7 is manufactured by the manufacturer where n is a positive integer. Thus, the length of the cycle for the manufacturer becomes $\frac{D_1}{nQ_1}$. Therefore, the setup cost of the manufacturer is given as $\frac{S_1 D_1}{nQ_1}$. $\frac{r_v C_p Q_1}{2}\left(n\left(1-\frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right)$ reflects the holding cost per unit time for one item. The “standard product’s” production cost for manufacturer is $C_{vr}D_1$.

II. **“Manufacturer’s profit for customized product”**

In EPQ model, inventory of online channel of manufacturer is demonstrated by figure 2.8. By selling the “personalized product” through the online route, profit earned by the manufacturer per unit of time is

$$\nabla_2 = \text{Revenue} - \text{Cost of setup} - \text{Cost of holding cost} - \text{Cost of manufacturing}$$

$$\nabla_2 = \sum_{i=1}^{N} C_i(1+m)\phi_i D_2 - \left[\frac{S_2 D_2}{Q_2} + \left(\frac{h_1 Q_2}{2}\right)\left(1-\frac{D_2}{P_2}\right) + \sum_{i=1}^{N} C_i \phi_i D_2\right] \tag{4.61}$$

Where, $\sum_{i=1}^{N} C_i(1+m)\phi_i D_2$ is revenue and $\sum_{i=1}^{N} C_i(1+m)\phi_i$ is the “personalized item’s” trading charges. Since the producer is catering the personalized demands of the customers so accordingly the setup cost is given as $\frac{S_2 D_2}{Q_2}$. Q_2 is the cumulative orders of the customers. Further, the length of cycle for producer is $\frac{D_2}{Q_2}$ and $\frac{S_2 D_2}{Q_2}$ is the setup cost. The average inventory can be evaluated as

$$\text{Average inventory} = \frac{Q_2}{2}\left(1-\frac{D_2}{P_2}\right)$$

The cost of holding an inventory per unit time for one item by producer is represented by $\frac{h_1 Q_2}{2}\left(1-\frac{D_2}{P_2}\right)$. Manufacturing cost os associated with each customization $\sum_{i=1}^{N} C_i$, thus, the cumulative manufacturing cost for manufacturer is depicted by $\sum_{i=1}^{N} C_i\phi_i D_2$.

III. **Profit earned by retailer**

The inventory between producer and retailer is demonstrated by figure 2.9. The profit earned by the retailer by selling "standard items" per unit time is

$$\nabla_3 = \text{Revenue} - \text{Cost of ordering cost} - \text{Cost of holding cost} - \text{shortage cost} - \text{Cost of lead time crashing}$$

$$\nabla_3 = C_p(1+m)^2 D_1 - \left[\frac{A_r D_1}{Q_1} + r_b C_b\left(\frac{Q_1}{2} + r - D_1 L\right) + \frac{\pi D_1}{Q_1} E(M-R)^+ + \frac{D_1 CL}{Q_1}\right] \tag{4.62}$$

Whereas revenue is reflected by $C_p(1+m)^2 D_1$, $C_p(1+m)^2$ is the selling price of "standard products". $\frac{A_r D_1}{Q_1}$ is the ordering cost per unit time. The retailer's expected cost of holding per unit time is given by $r_b C_b\left(\frac{Q_1}{2} + R - D_1 L\right)$. The cycle expected shortage is given by $E(M-R))^+$ resulting in $\frac{\pi D_1}{Q_1} E(M-R))^+$ as shortage cost. Lastly, $\frac{D_1 CL}{Q_1}$ is the lead time crashing cost per unit time.

Thus, for the "single-channel supply chain" the aggregate profit ∇_S is obtained by summing up ∇_1,∇_3. We get,

$$\nabla_S = \nabla_1 + \nabla_3 \tag{4.63}$$

$$\nabla_S = C_p(1+m)D_1 - \left[\frac{S_1 D_1}{nQ_1} + \frac{r_v C_p Q_1}{2}\left[n\left(1 - \frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right]\right] - C_{vr} D_1$$

$$+C_p(1+m)^2 D_1 - \left[\frac{A_r D_1}{Q_1} + r_b C_b\left(\frac{Q_1}{2} + r - D_1 L\right) + \frac{\pi D_1}{Q_1} E(M-R))^+ + \frac{D_1 CL}{Q_1}\right] \tag{4.64}$$

4.3.3 "Distribution free approach"

Underneath points elucidates the "distribution-free approach"

• "Cumulative distribution function (c.d.f)" with "mean $D_1 L$" and "standard deviation $\sigma\sqrt{L}$" is taken into consideration. As no specific distribution of probability for any random variable should be considered in aforementioned approach.

• A max-min approach for maximizing the yield is embraced in the current research which is contrary to Gallego and Moon's approach. Further, the demand is not just uncertain but variable also in this research which is a new alteration in the existing inequality.

• Variable demand is considered as the value of M i.e., lead time demand is determined by D_1 which is hinged with the selling price $C_p(1+m)^2$ of the retailer. For-bye, the additive form helps in considering random demand.

Proposition 4.1 put forward the inequality utilized in solving the research.

Proposition 4.1.

$$E(M-R))^+ = E((D_1L+X)-R)^+$$

$$\leq \left[\frac{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2Ł-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})}{2}\right] \tag{4.65}$$

Additionally, the upper limit of the aforementioned inequality is tight.

Where, $R = D_1L+k\sigma\sqrt{L}$ is reorder point, D_1L is the lead time demand, $k\sigma\sqrt{L}$ is the stock for safety, and k is a safety factor.

Henceforth, utilizing the inequality 4.65 the expected aggregated yield ∇_S of the "single-route supply chain in the centralized model" is obtained by adding ∇_1 and ∇_3. We get,

$$\nabla_S = C_p(1+m)D_1 - \frac{A_rD_1}{Q_1} - r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right) - \frac{D_1CL}{Q_1}$$

$$-\frac{\pi D_1}{Q_12}\left[\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}}+(D_1L-k\sigma\sqrt{L})\right]$$

$$+C_p(1+m)^2D_1 - \left[\frac{S_1D_1}{Q_1n}+\frac{r_vQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]\right]-C_{vr}D_1 \tag{4.66}$$

And expected aggregated yield of the "centralized supply chain with dual-route" ∇_D is obtained by summing up ∇_1, ∇_2, and ∇_3. We get,

$$\nabla_D = C_p(1+m)D_1 - \left[\frac{S_1D_1}{nQ_1}+\frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right)-1+\frac{2D_1}{P_1}\right]\right]-C_{vr}D_1$$

$$+\sum_{i=1}^{N} C_i(1+m)\phi_i D_2 - \left[\frac{S_2 D_2}{Q_2} + \left(\frac{h_1 Q_2}{2}\right)\left(1-\frac{D_2}{P_2}\right) + \sum_{i=1}^{N} C_i \phi_i D_2\right]$$

$$+C_p(1+m)^2 D_1 - \left[\frac{A_r D_1}{Q_1} + r_b C_b\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) + \frac{D_1 CL}{Q_1}\right]$$

$$-\frac{\pi D_1}{Q_1 2}\left[\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}} + (D_1 L - k\sigma\sqrt{L})\right] \tag{4.67}$$

4.3.4 Optimal values of decision variables of the supply chain

Seeing that the equation 4.67 is non-linear so for a definite integer 'm'', partial derivative of the profit with respect to Q_2, Q_1, & k and equate to zero to obtain the optimal solution. we get,

$$Q_2^* = \sqrt{\frac{S_2 D_2}{\frac{h_1}{2}\left(1-\frac{D_2}{P_2}\right)}}, \tag{4.68}$$

$$Q_1^* = \sqrt{\frac{-D_1 CL + S_1 D_1 + A_r D_1 r + \frac{\pi D_1}{2}\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}} + D_1 L - k\sigma\sqrt{L}}{\frac{r_b C_b}{2} + \frac{r_v}{2}\left(n\left(1-\frac{D_1}{P_1}\right) - 1 + 2\frac{D_1}{P_1}\right)}} \tag{4.69}$$

$$k^* = \frac{\frac{(-\sigma\sqrt{Q_1 C_b r_b(-Q_1 C_b r_b + \pi D_1)})}{-Q_1 C_b r_b + \pi D_1} + \frac{D_1 \pi\sigma\sqrt{Q_1 C_b r_b(-Q_1 C_b r_b + \pi D_1)L}}{2Q_1 C_b r_b(\pi D_1 - Q_1 C_b r_b)} + LD_1}{\sqrt{L}\sigma}. \tag{4.70}$$

Optimal solution obtained for Q_1^*, Q_2^*, and k^* are hinged with each other. Therefore, a closed-form equation for-profit function in the centralized firm is complicated to find. Henceforth, numerical procedures help taken for find the optimal values.

4.4 Analysis of environmental pillar in Single Channel

4.4.1 CO_2 emissions throughout the production

The aggregate of CO_2 emitted (ton/unit) in the production process is:

$$E(P_1) = x_1(P_1)^2 - x_2P_1 + x_3 \tag{4.71}$$

where x_1, x_2, and x_3 can be experimentally verified on the lines of Narita (2012). The experiment gives a way to understand that how operating a machine tool adds a carbon emission burden and gave a quadratic equation 4.71 reflecting the equivalent CO_2 emissions. Moreover, it also reflects that increased cutting speed converts tool wears into considerable lofty which shortens its life span and elevates CO_2 emissions. Further, there is also a trade relation with the cutting speed as carbon stress build-up by electricity utilization and the cooling liquid is comparable with time. The quadratic equation 4.71 manifests the behaviour of the corresponding carbon ejection. This is demonstrated by figure 3.19. For understanding the production proficiency of the manufacturer in the current model, machine tool is taken into consideration.
Therefore, cost of the carbon exude because of production is given as

$$EC_1 = ED_1C_{ec} \tag{4.72}$$

When the carbon burden from the firm surpasses the predetermined ceiling then the penalty cost is collected from it. Thus, the penalty cost as a consequence of carbon emission is given as

$$EC_3 = \sum_{i=1}^{l} Y_iC_{ep,i} \tag{4.73}$$

where

$$Y_i = \begin{cases} 1 & ED_1 > E_{li} \quad (i = 1, 2, ..., l) \\ 0 & otherwise \end{cases} \tag{4.74}$$

4.4.2 CO_2 ejection while transporting goods

This section elucidates the calculation of CO_2 ejection while freightage. Thus, the aggregate carbon stress from all trucks per year is

$$E_{tr} = \eta_1 n \frac{D_1}{nQ_1} ve_1 = \eta_1 \frac{D_1}{Q_1} ve_1 \tag{4.75}$$

Where, $\eta_1 = \left[\frac{Q_1}{Cap_1}\right]$ and η_1 is represented in the Q_1. Thus, the CO_2 ejection cost from freightage is given as

$$EC_2 = E_{tr} C_{ec} \tag{4.76}$$

This additional element of CO_2 ejection is considered on account that it might surpass the licit emissions limit. As a result, equation 4.74 reworked as:

$$Y_i = \begin{cases} 1 & (ED_1 + E_{tr}) > E_{li} \quad (i = 1, 2, ..., l) \\ 0 & otherwise \end{cases} \tag{4.77}$$

4.4.3 Transit Cost

The cost of transition per consignment is considered to be constant per dumper per consignment on presume system of one producer and one shopkeeper (Bozorgi et al. , 2014). Since we are considering one producer and one shopkeeper in the current model, thus the distance is invariant between the manufacturer and a retailer. Consequently, the fuel cost consumed per truck during the delivery of a consignment is constant. With the objective to do analysis of competition between retailer's, chapter is considering two retailers. The transit cost per year is thus enumerated as:

$$SC_{transit} = C_{tr} \eta_1 n \frac{D_1}{nQ_1}$$

$$= \eta_1 C_{tr} \frac{D_1}{Q_1} \tag{4.78}$$

Where C_{tr} reflects the constant price of the dumper per consignment ($/dumper). Henceforth, the cost of transit results in rearrangement in the supply chain's budget.

4.4.4 Specific energy consumption

This module describes how the consumption of energy is considered in the chapter. Henceforth, there are machine tools in the manufacturer's plant whose job is to eliminate material, and thus the material removal rate (MRR) and production rate P_1 are presumed to be identical. Presently, estimating empirically the manufacturer's energy is the only way. Review of the energy exhibits, that more energy is linked with the total operations of machine tools in comparison with the energy needed for verified material elimination. In this approach, the concept of the whole system centralizes to the machine tool. Henceforth, the energy used is associated with the tally of manufactured products with the help of two coefficients ς_0 and ς_1. Li and Kara (2011) determine ς_0 and ς_1 in MRR terminology (cm^3/sec). Further, ς_0 and ς_1) are adjusted to ς_0' and ς_1' to manifest the rate of manufacturing in quantity per year and power in kilowatt-hours terminology. Henceforth, specific energy required for per unit manufacturing is

$$SE(P_1) = \varsigma_0' + \frac{\varsigma_1'}{P_1} \tag{4.79}$$

where as ς_0' and ς_1' are numerical values.
Thus, during manufacturing the budget of power is evaluated as

$$EC_4 = SE(P_1)D_1C_e \tag{4.80}$$

It's worth noting that the power budget of the distributor consists of costs related to carbon ejection plus all other energy acquisitions. Additionally, carbon ejection by machine is not part of this budget.
Therefore, the aggregate yield ∇_S of the supply chain model with "single-route" is

$$\begin{aligned}\nabla_S = & C_p(1+m)D_1 - \frac{A_rD_1}{Q_1} - r_bC_b\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) - \frac{D_1CL}{Q_1} - \frac{\pi D_1}{Q_1 2}\left[\sqrt{\sigma^2L + (D_1L)^2 + k^2\sigma^2L - 2D_1Lk\sigma\sqrt{L}}\right. \\ & \left. + (D_1L - k\sigma\sqrt{L})\right] + C_p(1+m)^2D_1 - \left[\frac{S_1D_1}{Q_1n} + \frac{r_vQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right]\right] - C_{vr}D_1 \\ & - \left[(x_1P_1^2 - x_2P_1 + x_3)D_1C_{ec} + \eta_1\frac{D_1}{Q_1}ve_1C_{ec} + \sum_{i=1}^{l} Y_iC_{ep,i} + \left(\varsigma_0' + \frac{\varsigma_1'}{P_1}\right)D_1C_e + \eta_1C_{tr}\frac{D_1}{Q_1}\right]\end{aligned} \tag{4.81}$$

4.5 Analysis of environmental pillar in Dual Channel

4.5.1 CO_2 emissions throughout the production

The aggregate of CO_2 emitted (ton/unit) in the production process is:

$$E^{'}(P) = x_1(P)^2 - x_2P + x_3 \tag{4.82}$$

where x_1, x_2, and x_3 can be experimentally verified on the lines of Narita (2012). The experiment gives a way to understand that how operating a machine tool adds a carbon emission burden and gave a quadratic equation 4.82 reflecting the equivalent CO_2 emissions. Moreover, it also reflects that increased cutting speed converts tool wears into considerable lofty which shortens its life span and elevates CO_2 emissions. Further, a marketing relationship with the mincing rate as carbon stress build-up by electricity utilization and the cooling liquid is comparable with time. The quadratic equation 4.82 manifests the behaviour of the corresponding carbon ejection. This is demonstrated by figure 3.19. Understanding the production proficiency of the manufacturer in the current model machine tool is taken into consideration.
Therefore, cost of the carbon exude because of production is given as

$$EC_1^{'} = EDC_{ec} \tag{4.83}$$

When the carbon burden from the firm surpasses the predetermined ceiling then the penalty cost is collected from it. Thus, the penalty cost as a consequence of carbon emission is given as

$$EC_3^{'} = \sum_{i=1}^{l} Y_i C_{ep,i} \tag{4.84}$$

where

$$Y_i = \begin{cases} 1 & ED > E_{li} \quad (i = 1, 2, ..., l) \\ 0 & otherwise \end{cases} \tag{4.85}$$

4.5.2 CO_2 ejection while transporting goods

This section elucidates the calculation of CO_2 ejection while freightage. Thus, the aggregate carbon stress from all trucks per year is

$$E'_{tr} = \left(\eta_1 n \frac{D_1}{nQ_1} + \eta_2 n \frac{D_2}{nQ_2} \right) ve$$

$$= \left(\eta_1 \frac{D_1}{Q_1} + \eta_2 \frac{D_2}{Q_2} \right) ve \quad (4.86)$$

As aforementioned η_1 can be expressed as function of Q_1 and η_2 as function of Q_2. Thus, the CO_2 ejection cost from freightage is given as

$$EC'_2 = E'_{tr} C_{ec} \quad (4.87)$$

This additional element of CO_2 ejection is considered on account that it might surpass the licit emissions limit. As a result, equation 4.74 reworked as:

$$Y_i = \begin{cases} 1 & (ED + E'_{tr}) > E_{li} \ \ (i = 1, 2, ..., l) \\ 0 & otherwise \end{cases} \quad (4.88)$$

4.5.3 Transit Cost

The cost of transition per consignment is considered to be constant per dumper per consignment on presume framework of single-manufacturer and a multiple customers in case of online platform. According, Ely X. Col6n-Jimenez[12] thesis average distance between the manufacturer and multiple customers and the fuel cost consumed per truck during the delivery of a consignment is constant. Furthermore, the transit cost for transportation from single-vendor to single-buyer is evaluated same as in the aforementioned section of transit cost. Thus, the total transit cost per year is enumerated as:

$$SC'_{transit} = C_{tr} \eta_1 n \frac{D_1}{nQ_1} + C_{td} \eta_2 \frac{D_2}{Q_2}$$

$$= \eta_1 C_{tr} \frac{D_1}{Q_1} + C_{td} \eta_2 \frac{D_2}{Q_2} \quad (4.89)$$

[12] "https://dspace.mit.edu/handle/1721.1/62766"

where C_{tr} and C_{td} reflects the constant price of the dumper per consignment (\$/dumper). Henceforth, the cost of transit results in rearrangement in the supply chain's budget.

4.5.4 Specific energy consumption

This module describes how consumption of energy is considered in the chapter. Embracing in the same way but with the production rate 'P', specific energy required for per unit manufacturing is

$$SE(P) = \varsigma_0' + \frac{\varsigma_1'}{P} \tag{4.90}$$

Where as ς_0' and ς_1' are numerical values.
Thus, during manufacturing the budget of power is evaluated as

$$EC_4' = SE(P)DC_e \tag{4.91}$$

It's worth noting that the power budget of the distributor consists of costs related to carbon ejection plus all other energy acquisitions. Additionally, carbon ejection by machine is not part of this budget.
Therefore, the aggregate yield of the "supply chain with dual-route" ∇_D is

$$\nabla_D = C_p(1+m)D_1 - \left[\frac{S_1D_1}{nQ_1} + \frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{D_1}{P_1}\right) - 1 + \frac{2D_1}{P_1}\right]\right] - C_{vr}D_1 + \sum_{i=1}^{N} C_i(1+m)\phi_i D_2 -$$

$$\left[\frac{S_2D_2}{Q_2} + \left(\frac{h_1Q_2}{2}\right)\left(1-\frac{D_2}{P_2}\right) + \sum_{i=1}^{N} C_i\phi_i D_2\right] + C_p(1+m)^2D_1 - \left[\frac{A_rD_1}{Q_1} + r_bC_b\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) + \frac{D_1CL}{Q_1}\right.$$

$$\left. - \frac{\pi D_1}{Q_1 2}\left[\sqrt{\sigma^2L + (D_1L)^2 + k^2\sigma^2L - 2D_1Lk\sigma\sqrt{L}} + (D_1L - k\sigma\sqrt{L})\right]\right] - \Big[(x_1P^2 - x_2P + x_3)DC_{ec} +$$

$$\left(\eta_1\frac{D_1}{Q_1} + \eta_2\frac{D_2}{Q_2}\right)\nu eC_{ec} + \sum_{i=1}^{l} Y_iC_{ep,i} + \left(\varsigma_0' + \frac{\varsigma_1'}{P}\right)DC_e + \eta_1C_{tr}\frac{D_1}{Q_1} + C_{td}\eta_2\frac{D_2}{Q_2}\Big] \tag{4.92}$$

Where Q_2, Q_1, and k are given by equations 4.68, 4.69, and 4.70 respectively.
Further, for evaluating managerial decisions, the iteration method along with the underneath algorithm is utilized.

4.6 Algorithm for obtaining solution

Following is the algorithm utilized to solve the model of the chapter.

Step 1 Assign all parameters the values.

Step 2 Set n=1.

Step 3 Carry out the upcoming operations for all the values of Ł$_i$; i=1,2,...

Step 3(a) From equation 4.69 acquire the value of Q_1.

Step 3(b) From equation 4.68 acquire the value of Q_2 .

Step 3(c) From equation 4.70 acquire the value of k .

Step 3(d) Redo the Steps from 3a to 3c till there is no change in Q_2, Q_1, and k upto a level of accuracy as specified.

Step 4 Obtain the value of ED_1 and $E'D$ using following steps

Step 4(a) If ED_1 <220 then $C_{ep,i} = 0$ and $Y_i = 0$ else follow step 4(b) and if $E'D$ <220 then $C_{ep,i} = 0$ and $Y_i = 0$ else follow step 4(b)

Step 4(b) If $220 < ED_1 < 330$ then $C_{ep,i} = 1000$ and $Y_i = 1$ else follow step 4(c) and if $220 < E'D < 330$ then $C_{ep,i} = 1000$ and $Y_i = 1$ else follow step 4(c).

Step 4(c) If $330 < ED_1 < 440$ then $C_{ep,i} = 2000$ and $Y_i = 1$ else follow step 4(d) and if $330 < E'D < 440$ then $C'_{ep,i} = 2000$ and $Y_i = 1$ else follow step 4(d).

Step 4(d) If $440 < ED_1 < 550$ then $C_{ep,i} = 3000$ and $Y_i = 1$ else follow step 4(e) and if $440 < E'D < 550$ then $C_{ep,i} = 3000$ and $Y_i = 1$ else follow step 4(e).

Step 4(e) If $550 < ED_1 < 660$ then $C_{ep,i} = 4000$ and $Y_i = 1$ else follow step 4(f) and if 550< $E'D < 660$ then $C_{ep,i} = 4000$ and $Y_i = 1$ else follow step 4(f).

Step 4(f) If $660 > ED_1$ then $C_{ep,i} = 4000$ and $Y_i = 1$ and if $660 > E'D$ then $C_{ep,i} = 4000$ and $Y_i = 1$.

Step 5 Obtain the value of EC_1 and EC'_1 from equations 4.72 and 4.83.

Step 6 Obtain the value of EC_3 and EC'_3 from equations 4.73 and 4.84.

Step 7 Obtain the value of EC_2 and EC'_2 from equations 4.76 and 4.87.

Step 8 Obtain the value of $SC_{transit}$ and $SC'_{transit}$ from equations 4.78 and 4.89.

Step 9 Obtain the value of EC_4 and EC'_4 from equations 4.80 and 4.91.

Step 10 Use the values of $Q_1, k, ED_1, EC_1, EC_3, EC_2, SC_{transit}$, and EC_4 to obtain ∇_S from equation

Step 11 Use the values of Q_2, Q_1, k, $E'D$, EC'_1, EC'_3, EC'_2, $SC'_{transit}$, and EC'_4 to obtain ∇_D from equation 4.92.

Step 12 Put n=n+1 and redo the Steps from 3 to 11.

Step 13 If $\nabla_D(n+1) < \nabla_D(n)$ then redo the steps from 2 to 6 else end the algorithm.

Proposition 4.2. *If the optimal values of Q_1, Q_2 and k are presented by Q_1^*, Q_2^*, and k^*, then for a definite values of n and $L \in [L_i, L_{i-1}]$, ∇_D the profit function of "dual-channel" attains its maximum value at Q_1^*, Q_2^*, and k^* subjected to the situation*

$$\Longrightarrow \left(A_r + \frac{S_1}{n} + CL\right) \frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{D_1\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}$$

$$> \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2{D_1}^2 L^3\sigma^2)}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

Proof. Underneath mentioned is the Hessian matrix H for the "supply chain model with dual-route".

$$H_1 = \begin{bmatrix} \frac{\partial^2\nabla_D}{\partial Q_1^2} & \frac{\partial^2\nabla_D}{\partial Q_1\partial Q_2} & \frac{\partial^2\nabla_D}{\partial Q_1\partial k} \\ \frac{\partial^2\nabla_D}{\partial Q_2\partial Q_1} & \frac{\partial^2\nabla_D}{\partial Q_2^2} & \frac{\partial^2\nabla_D}{\partial Q_2\partial k} \\ \frac{\partial^2\nabla_D}{\partial k\partial Q_1} & \frac{\partial^2\nabla_D}{\partial k\partial Q_2} & \frac{\partial^2\nabla_D}{\partial k^2} \end{bmatrix}$$

where,

$$\frac{\partial^2\nabla_D}{\partial k^2} = -\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2\nabla_D}{\partial Q_1\partial k} = \frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2\nabla_D}{\partial Q_2\partial k} = 0 = \frac{\partial^2\nabla_D}{\partial k\partial Q_2}$$

$$\frac{\partial^2\nabla_D}{\partial Q_1\partial Q_2} = 0 = \frac{\partial^2\nabla_D}{\partial Q_2\partial Q_1}$$

$$\frac{\partial^2\nabla_D}{\partial Q_2^2} = -\frac{2S_2 D_2}{Q_2^3}$$

$$\frac{\partial^2 \nabla_D}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]$$

$$\frac{\partial^2 \nabla_D}{\partial k \partial Q_1} = \frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

The Hessian matrix's (H_1), principal minor of the order 1×1 at (Q_1^*, Q_2^*, k^*) is given as

$$|H_{1,1}| = \left|\frac{\partial^2 \nabla_D}{\partial Q_1^2}\right|_{(Q_1^*, Q_2^*, k^*)}$$

$$= -\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right] < 0$$

The Hessian matrix's (H_1), principal minor of the order 2×2 at (Q_1^*, Q_2^*, k^*) is given as

$$|(H_1)_{2,2}|_{(Q_1^*, Q_2^*, k^*)} = \left(\frac{\partial^2 \nabla_D}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_D}{\partial Q_2^2}\right)$$

$$= \left(-\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1 (\sqrt{\sigma^2 L + (D_1 L)^2 + k^2 \sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right]\right)\left(-\frac{2S_2 D_2}{Q_2^3}\right) > 0$$

The Hessian matrix's (H_1), principal minor of the order 3×3 at (Q_1^*, Q_2^*, k^*) is given as

$$|(H_1)_{3,3}|_{(Q_1^*, Q_2^*, k^*)} = \frac{\partial^2 \nabla_D}{\partial Q_2^2}\left[\left(\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_S}{\partial k^2}\right) - \left(\frac{\partial^2 \nabla_S}{\partial k \partial Q_1}\right)^2\right]$$

$$= \left[\frac{2S_2 D_2}{Q_2^3}\right]\frac{\pi (D_1)^2}{Q_1^4}\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \frac{D_1 L^2 \sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$-\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2{D_1}^2 L^3 \sigma^2 - 4D_1 \sigma^3 L^2 \sqrt{L})}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})} > 0$$

$$\implies \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} > \frac{D_1 L^2 \sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2{D_1}^2L^3\sigma^2-4D_1\sigma^3L^2\sqrt{L})}{4(\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L})}$$

$$\implies \left(A_r+\frac{S_1}{n}+CL\right)\frac{\sigma^4L^2}{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4L^2+\frac{\pi k\sigma^3L\sqrt{L}}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}+\frac{D_1\sigma^3L^2\sqrt{L}}{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}$$

$$>\frac{D_1L^2\sigma^2}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2{D_1}^2L^3\sigma^2)}{4(\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L})}$$

Since Hessian matrix's every single principal minor is negative, therefore, at (Q_1^*,Q_2^*,k^*), H_1 is negative definite. Moreover, at the same point, the total profit of the firm offering a product online along with an offline platform obtains its global maximum.

Proposition 4.3. *If the optimal values of Q_1 and k are presented by Q_1^*, and k^*, then for a definite values of $L \in [L_i, L_{i-1}]$ and n, ∇_S the profit function of "dual-channel" attains its maximum value at Q_1^*, and k^* subjected to the situation*

$$\implies \left(A_r+\frac{S_1}{n}+CL\right)\frac{\sigma^4L^2}{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4L^2+\frac{\pi k\sigma^3L\sqrt{L}}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}+\frac{D_1\sigma^3L^2\sqrt{L}}{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}$$

$$>\frac{D_1L^2\sigma^2}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2{D_1}^2L^3\sigma^2)}{4(\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L})}$$

Proof. Underneath mentioned is the Hessian matrix H for the "supply chain model with single-route".

$$H_2=\begin{bmatrix}\frac{\partial^2\nabla_D}{\partial Q_1^2} & \frac{\partial^2\nabla_S}{\partial Q_1\partial k}\\ \frac{\partial^2\nabla_S}{\partial k\partial Q_1} & \frac{\partial^2\nabla_S}{\partial k^2}\end{bmatrix}$$

where,

$$\frac{\partial^2\nabla_S}{\partial Q_1^2}=-\frac{2}{Q_1^3}\left[A_rD_1+\frac{S_1D_1}{n}+D_1CL+\pi D_1(\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1Lk\sigma\sqrt{L}})\right]$$

$$\frac{\partial^2 \nabla_S}{\partial k \partial Q_1} = \frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2 \nabla_S}{\partial k^2} = -\frac{\pi D_1}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2 \nabla_S}{\partial Q_1 \partial k} = \frac{\pi D_1}{2Q_1^2}\left[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

The principal minor of $|H_2|$ of order 1×1 is

$$|H_{1,1}|_{(Q_1^*, k^*)} = \left|\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right|_{(Q_1^*, k^*)}$$

$$= -\frac{2}{Q_1^3}\left[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1(\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\right] < 0$$

The principal minor of $|H_2|$ of order 2×2 is

$$|H_{2,2}|_{(Q_1^*, k^*)} = \left(\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_S}{\partial k^2}\right) - \left(\frac{\partial^2 \nabla_S}{\partial k \partial Q_1}\right)^2$$

$$= \Big(-\frac{2}{Q_1^3}\Big[A_r D_1 + \frac{S_1 D_1}{n} + D_1 CL + \pi D_1(\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 Lk\sigma\sqrt{L}})\Big]\Big)$$

$$\Big(-\frac{\pi D_1}{2Q_1}\Big[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}\Big]\Big)$$

$$-\Big(\frac{\pi D_1}{2Q_1^2}\Big[\frac{k\sigma^2 L - D_1 L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \sigma\sqrt{L}\Big]\Big)^2$$

$$= \frac{\pi {D_1}^2}{Q_1^4}\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4 L^2 + \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} - \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}}$$

$$-\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2{D_1}^2 L^3\sigma^2 - 4D_1\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})} > 0$$

$$\implies \left(A_r+\frac{S_1}{n}+CL\right)\frac{\sigma^4L^2}{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4L^2+\frac{\pi k\sigma^3L\sqrt{L}}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}>\frac{D_1L^2\sigma^2}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}$$

$$+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2{D_1}^2L^3\sigma^2-4D_1\sigma^3L^2\sqrt{L})}{4(\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L})}$$

$$\implies \left(A_r+\frac{S_1}{n}+CL\right)\frac{\sigma^4L^2}{\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}$$

$$+\pi\sigma^4L^2+\frac{\pi k\sigma^3L\sqrt{L}}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}+\frac{D_1\sigma^3L^2\sqrt{L}}{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}$$

$$>\frac{D_1L^2\sigma^2}{2\sqrt{\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L}}}+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2{D_1}^2L^3\sigma^2)}{4(\sigma^2L+(D_1L)^2+k^2\sigma^2L-2D_1L\sigma\sqrt{L})}$$

Since Hessian matrix's every single principal minor is negative, therefore, at (Q_1^*, k^*), H_1 is negative definite. Moreover, at the same point, the total profit of the firm offering only standard products obtains its global maximum.

4.7 Numerical experimentation and discussion

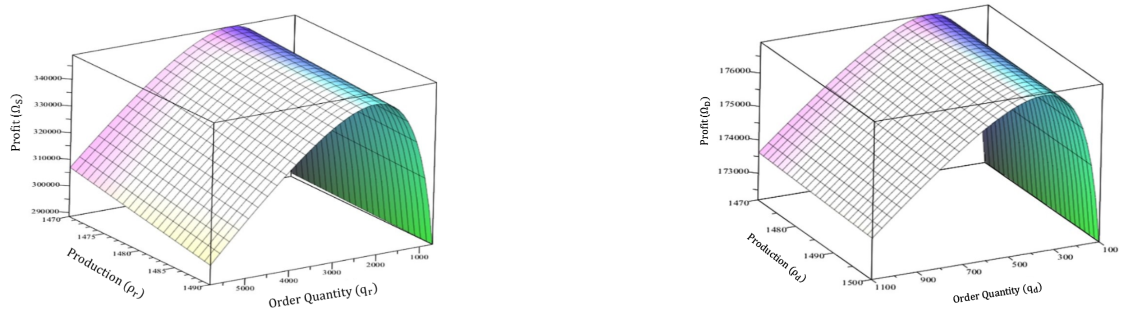
This segment exemplifies the behavior of the developed model for classical coordination in a "dual and single channel". To analyze the consequences of various environmental factors on the "supply chain model" henceforth eco-friendly decisions could be a workout in the business model is the sole objective of the numerical examples. To resemble a real manufacturing environment, values of parameters were taken from real world examples (Bazan et al. , 2015; Jaber et al. , 2013).

4.7.1 Results and discussion

An real life illustration is considered to analyze the profit of the "supply chain management" with both "dual-route and single-route". Moreover, in a "supply chain model with dual-channel" shoppers are provided with "core items and personalized items" through offline and online channels respectively although, in a "supply chain model with single route", core item is made available to the shoppers through offline

Tab. 4.12: Decision variable's optimum values.

n	L	k	Q_1	Q_2	D_1	D_2	∇_D	∇_S
15	4	15.56	342.127	428.9844	1504.219	811.8005	2371020	2356275
16	**4**	**14.3056**	**341.5027**	**426.88**	**1504.219**	**811.8005**	**2381020**	**2358273**
17	4	15.56	340.50	424.144	1504.219	811.8005	2361020	2355275



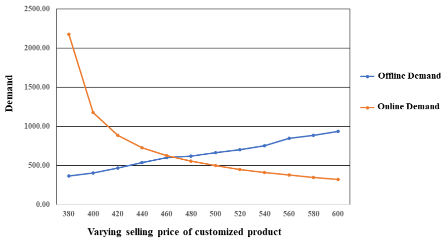
(a) Graphical representation of dual channel supply chain profit ∇_D with respect to production rate (P_1) and demand (Q_1)

(b) Graphical representation of dual channel supply chain profit ∇_D with respect to production rate (P_2) and demand (Q_2)

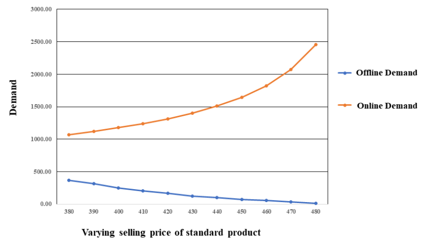
Fig. 4.32: Graphical presentation of dual channel supply chain profit

channel. Three sorts of customization are put forward on the "standard products" in "supply chain model of dual-rout" to the shoppers are presumed for simplicity.

Figure 4.32 demonstrates the variation of supply chain profit for both "single as well as dual channel" with respect to demand and production. Table 4.12 reflects the numerical values of the input variables utilized to derive the decision variable's optimal value. Further, the table demonstrates the maximum yield of a "supply chain model for single and dual-route" are \$ 2358273 and \$ 2381020, respectively. The maximum profits are observed at $n = 16$ and the corresponding values are $k = 14.3056$, $L = 4$, $Q_1 = 341.5027$, $Q_2 = 428.9844$, $D_1 = 1504.219$, $D_2 = 811.8005$.



(a) Case 1 : Varying prices of personalized item



(b) Case 2 : Varying prices of standard item

Fig. 4.33: Graphical presentation of variation of demand when $|\sum_{i=1}^{N} C_i - C_p|$ >Threshold value

4.7.2 Sensitivity in the demand because of prices when the differences in the charges in the two modes (off-line and on-line) exceed the presumed ceiling

For analyzing the consequences of variation in the prices of the “core and personalized items” on demand and profit of the supply chain with “dual-channel” underneath cases are considered.

When the variation in the cost of traditional and direct channels exceeds the “Threshold limit” i.e., $|\sum_{i=1}^{3} C_i - C_p| > 20$ then following situations are examined.

Case I: The trading charges related to the “personalized items” are constant and that of the “standard item” is varied. The results are shown in figure 4.33b which demonstrates that as the prices of the “standard product” increase the sales of the “core item” decreases and that of “personalized item” increases.

Case II: The trading charges of “standard item” are constant and that of the “personalized item” is varied. We consider the variation of the sum of trading charges of all customization’s i.e. $\sum_{i=1}^{3} C_i$. Figure 4.33a reflects the output of the case II and it demonstrates that the sale of “customized product” is decreases as we are increasing the prices beyond the limit.

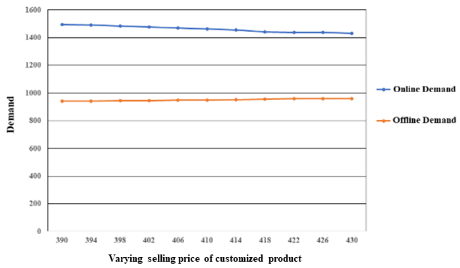
4.7.3 Sensitivity in the demand because of prices when the differences in the charges offered by two modes (off-line and on-line) are lower than the presumed ceiling

On the same lines as the previous section, two cases are taken into account. But, the differences in the product charges offered by the two modes is lower than the presumed threshold ceiling i.e., $|\sum_{i=1}^{3} C_i - C_p| < 20$.

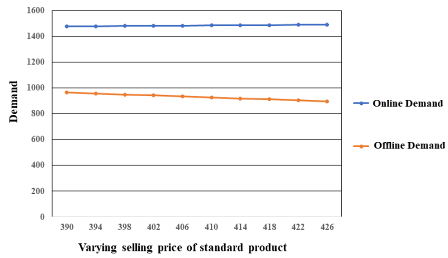
Underneath are two cases set out.

Case I: The trading charges of the "personalized items" are fixed whereas it is fluctuating in the case of core items. The output is reflected by figure 4.34b, which demonstrates that there is not much change or fluctuation in the sales of either channel.

Case II: The trading charges of the "core item" are fixed and that of a "personalized item" is fluctuating. The output of the firm is demonstrated by figure 4.34a which shows that there is not much fluctuation in the shoppers like previous case.



(a) Case 1 : Varying prices of personalized item



(b) Case 2 : Varying prices of standard item

Fig. 4.34: Graphical presentation of variation of demand when $|\sum_{i=1}^{N} C_i - C_p|$ <Threshold value

4.7.4 Profit and carbon emission analysis with respect to production

I. *CO_2 emissions throughout the production*

To examine the aftermath of varying production on carbon emission cost during production (EC_1). Figure 4.35 undeniably manifests that the carbon emission cost during production is directly proportional to the production, as when production increase carbon emission cost also increases. Moreover, initially with the increase in the production and carbon emission cost (EC_1) profit in case of both dual as well as single channel increases to values ∇_D=\$ 2487225 and ∇_S=\$ 2432305 respectively but after this profit starts decreasing which is reflected by figure 4.36.

II. *CO_2 ejection cost from shipment*

In the "single-channel supply chain" manufacturer is reaching out to customers through the retailer, so the distance between manufacturer and retailer is nearly inflexible. Consequently, the cost of CO_2 emission while transportation is also nearly invariable. Moreover, in the supply chain model with "dual-channel" manufacturer is providing standard along with "personalized items" by means of offline (retailer) and online channel. Consequently, the distance between producer and retailer is nearly fixed as aforementioned

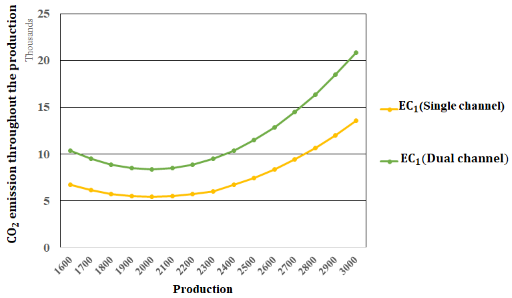


Fig. 4.35: Graphical representation of variation in carbon emission cost (EC_1) during production

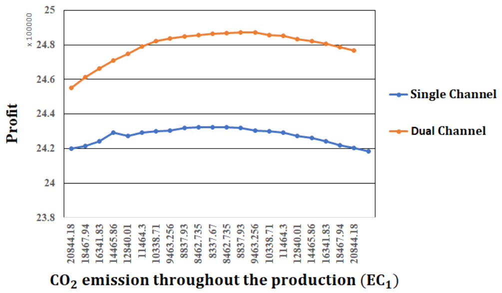


Fig. 4.36: Graphical representation of variation in profit vis-à-vis carbon emission cost (EC_1)

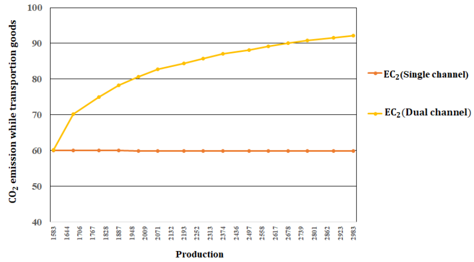


Fig. 4.37: Graphical representation of deviation of CO_2 ejection cost (EC_2) while shipping vis-à-vis production

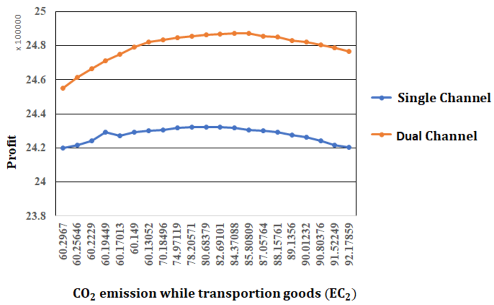


Fig. 4.38: Graphical representation of deviation in profit vis-à-vis CO_2 ejection cost (EC_2)

while in case online channel distance between manufacturer and retailer is not fixed which can be observed from the figures 4.37 and 4.39.

Additionally, in either case figures 4.38 and 4.40 certainly reflects that profit is increasing but after optimal value starts decreasing.

III. *Penalty cost because of CO_2 emission*

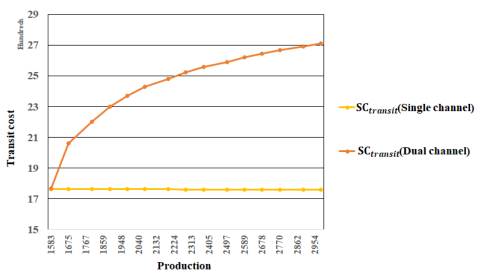


Fig. 4.39: Graphical representation of diversification in transit cost ($SC_(transit)$) with production

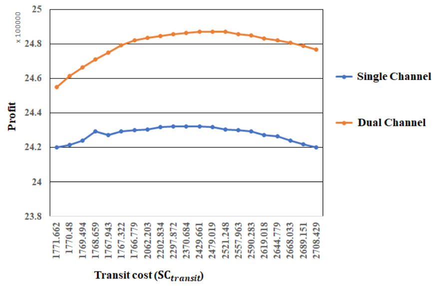


Fig. 4.40: Graphical representation of diversification in profit with transit cost ($SC_{(}transit)$)

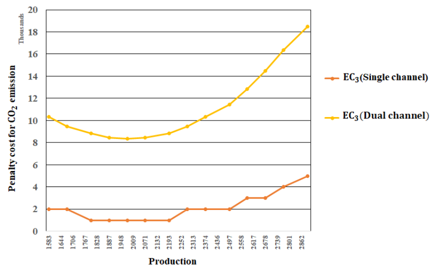


Fig. 4.41: Graphical demonstration of variation in penalty cost (EC_3) vs. production

Figure 4.41 manifests that as the production increases penalty cost (EC_3) also increases. Moreover the increase in EC_3 is more in case of “dual channel” in-comparison to “single channel” as it is offering “standard as well as customized product” to customers. Supplement to this profit in case of both “dual as well as single channel” at first increases but later it start decreasing after attaining optimal value with the increase in penalty cost as demonstrated by figure 4.42.

IV. *Cost of specific energy for production*

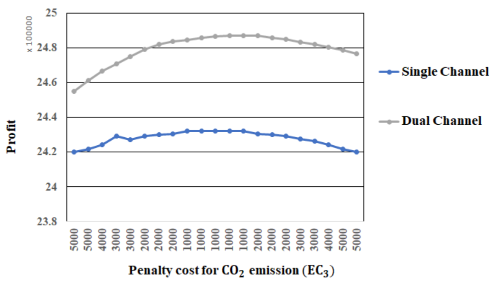


Fig. 4.42: Graphical demonstration of diversification in profit vs. penalty cost (EC_3)

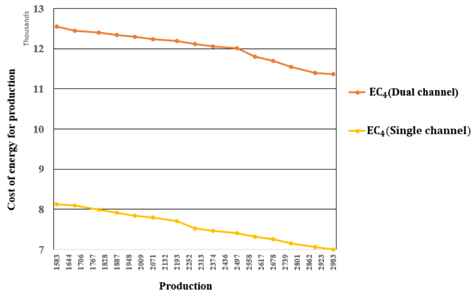


Fig. 4.43: Graphical demonstration of diversification in specific energy cost (EC_4) vis-à-vis production

Equations 4.79 and 4.90 demonstrates that

$$Specific\ \ energy\ \ consumption(SE) \propto \frac{1}{Production}$$

and from equations 4.80 and 4.91 we get,

$$EC_4 \propto Specific\ \ energy\ \ consumption(SE)$$

$$EC_4 \propto \frac{1}{Production} \tag{4.93}$$

Consequently, equation 4.93 manifests that as the production increases cost of specific energy shrinks in both the “supply chain models” (i.e., “single-route” as well as “dual-route”) which is demonstrated by the figure 4.43.

Moreover, figure 4.44 displays that with the drop of the EC_4, profit reflects same behavior as aforementioned that is increases initially but starts decreasing later on.

V. *Effect of transportation vehicle*

The number of trucks utilized per year for delivering the products determines the amount of CO_2 emitted during transportation. Moreover, the capacity of each truck influences the number of trucks. Initially the number of truck for single channel is assumed to be η_1=1 and for dual channel is presumed to be η_1=1 & η_2=1. Henceforth, for various number of trucks the model is solved in both “single and dual supply chain model”. The functioning of the profit for “supply chain model having single channel” is reflected by

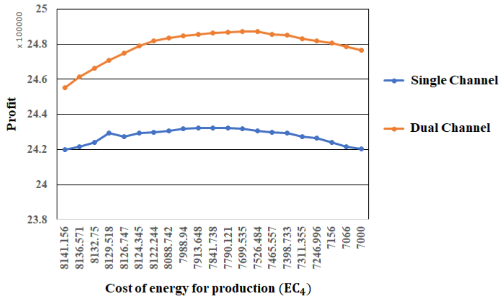
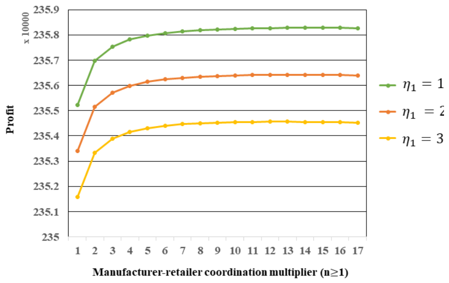
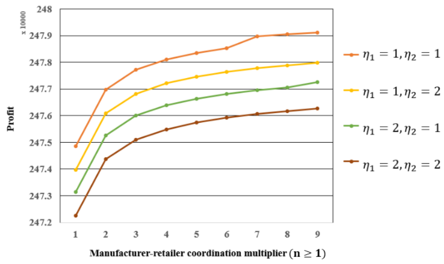


Fig. 4.44: Graphical demonstration of diversification in profit vis-à-vis specific energy cost (EC_4)



(a) Single channel supply chain



(b) Dual channel supply chain

Fig. 4.45: Graphical representation of profit vs manufacturer-retailer coordination multiplier with increasing transportation vehicle

figure 4.45a and that of "dual channel" is depicted by figure 4.45b with respect to the number of trucks. From the graphical representation it can be concluded that as the number of trucks increases the carbon emission increases consequently it leads to penalty on the manufacturer and decrease in its profit. Moreover, CO_2 emission is also influenced by the heaviness of the vehicle which determines the number and varied combination of trucks required for delivering the products. These are some of the open end questions which can be picked up in future research.

4.8 Sensitivity analysis

I. *Effect of setup cost*

Figure 4.46 manifests that as the setup cost of the retailer and manufacture increases from \$ 800 to \$ 1200, profit at the beginning increases with the increasing production rate but from P=2283 units it starts sinking. The gulf in the yield of the channel operated at low manufacturing cost and high manufacturing cost

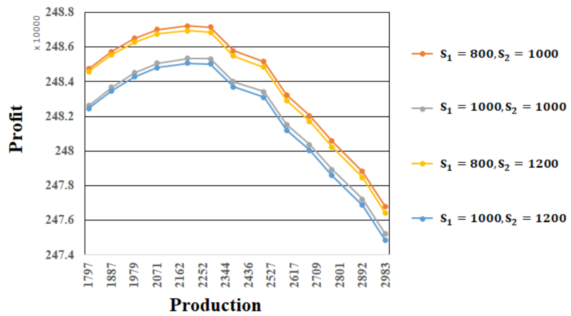


Fig. 4.46: Graphical representation of behavior of the profit with varying production rates and setup cost

demonstrates that it is more lucrative for the firm to operate at a low setup cost.

II. *Effect of holding cost*

Inventory verdicts from beginning to end in the supply chain, are governed by the cost of holding inventory. In the case of low holding cost, the manufacturer produces a large batch of products whereas, in the case of high holding cost, small batches of products are produced for benefit. Inventory cost includes warehouse location, technological resources utilized in the storehouse, and insurance. Figures 4.47a,4.47b, and 4.47c reflects that as the holding cost increases with varying the overall profit decreases. So, if the holding cost is less and manufacturer produces large batches it would be more advantageous. Moreover, from equations 4.86 and 4.87 we get,

$$EC_3^{'} \propto E_{tr}^{'} \propto \frac{1}{Q_2} \propto h_1$$

Thus, as holding cost h_1 increases $EC_3^{'}$ also increases. Henceforth, as the holding cost increases cost of CO_2 ejection from freightage increases.

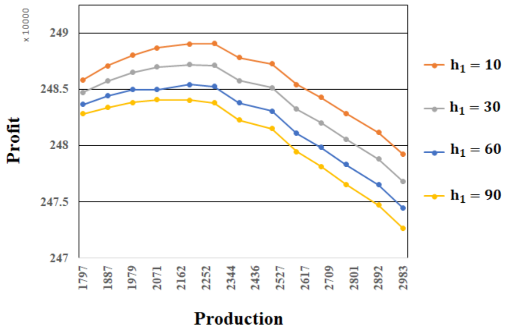
Moreover, from equations 4.86 and 4.87 we also get,

$$EC_3^{'} \propto E_{tr}^{'} \propto \frac{1}{Q_2} \propto \frac{1}{r_v} \propto \frac{1}{r_b}$$

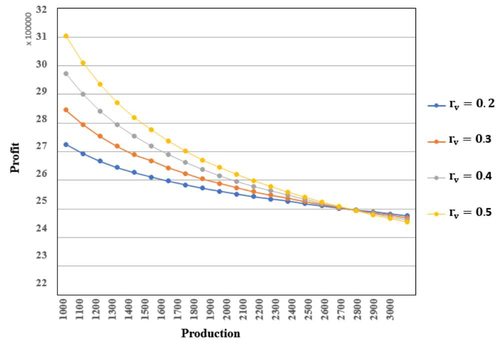
Thus as the r_v and r_b increases cost of CO_2 ejection from freightage decreases.

III. $\widehat{C}$hange in profit of all the cost parameters

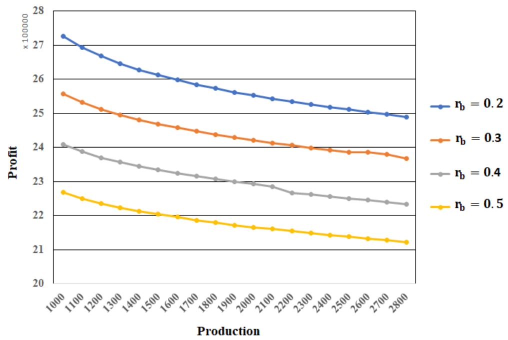
The different parameters of costs are varied over -20%, -10%, $+10\%$, and $+20\%$ to scrutinize the consequences of these parameters on the anticipated yield of the channel encompasses both online as well as



(a) With varying production rates and producer holding cost (h_1)

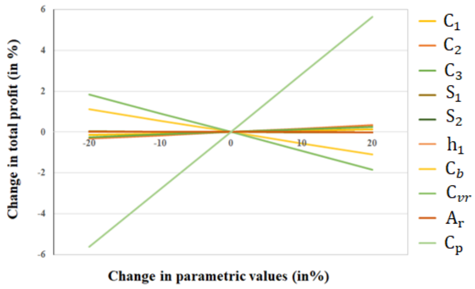


(b) With varying rates of production and holding charges for producer (r_v)

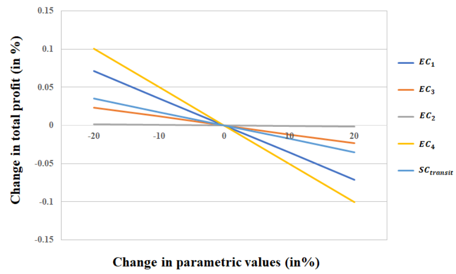


(c) With varying rates of production and holding charges for shopkeeper (r_b)

Fig. 4.47: Graphical representation of behavior of the profit



(a) Concerning all the cost parameters



(b) Concerning the different carbon emission cost parameters

Fig. 4.48: Effect of change in parametric values on total profit of the firm

the offline channel that is “dual-channel”. All the parameters are varied singly while retaining the supplementary parameters constant. Table 4.13 and figure 4.48a elucidates the consequences of aforementioned changes in the essential parameters.

Diversification of cost components C_1, C_2, C_3, C_p, S_1, S_2, C_{vr}, h_1, C_b, and A_r are examined. Further, for clarity and intelligibility manufacturer is incorporating maximum three varieties of make over on “core product”. Hence, underneath essence are obtained.

1. While comparing the elements of cost of “core products” (C_p) with the “customized products” (C_1, C_2, and C_3), parameters of the “standard products” were indeed sensitive in comparison to that of “personalized products”.

2. “Standard product’s” trading elements (C_{vr}) for manufacturer are more sensitive in-comparison to the setup cost (S_1 and S_2) for the manufacturer.

3. “Standard product’s” selling price (C_p) is the most sensitive parameter and ordering cost (A_r) is the least sensitive.

Further, table 4.14 and figure 4.48b reflects that penalty cost for CO_2 emission EC_3' is least sensitive whereas cost of energy for production EC_4' most sensitive while analyzing varied carbon emission cost EC_1, EC_2, EC_3, EC_4, and $SC_{transit}$.

4.9 Comparative study with existing literature

The outcome of the current model validates the investments in sustainable development. The results of the numerical simulation of this chapter shows reduced carbon emission compared to existing literature. In the current chapter, the model is compared with Chauhan et al. (2021) and Modak and Kelle (2018). Both

Tab. 4.13: Cost parameter's sensitivity interpretation.

Parameters	Variation (in %)	Change in profit	Parameters	Variation (in %)	Change in profit
C_1	−20	−0.13712	S_2	−20	+0.010507
	−10	−0.06856		−10	+0.005253
	+10	0.06856		+10	−0.00525
	+20	0.13712		+20	−0.01051
C_2	−20	−0.32909	C_{vr}	−20	+1.113269
	−10	−0.16455		−10	+0.556634
	+10	+0.164547		+10	−0.556634
	+20	+0.32909		+20	−1.11327
C_3	−20	−0.24682	h_1	−20	+0.010507
	−10	−0.12341		−10	+0.005253
	+10	+0.12341		+10	−0.00525
	+20	+0.24682		+20	−0.01051
C_p	−20	−5.63472	C_b	−20	+1.832876
	−10	−2.81736		−10	+0.91644
	+10	+2.817358		+10	−0.91644
	+20	+5.634717		+20	−1.83288
S_1	−20	+0.026785	A_r	−20	+0.006696
	−10	+0.013392		−10	+0.003348
	+10	−0.01339		+10	−0.00335
	+20	−0.02679		+20	−0.0067

Tab. 4.14: Sensitivity analysis of carbon emission cost parameters.

Parameters	Change(in %)	Sensitivity	Parameters	Change(in %)	Sensitivity
EC_1	−20	+0.071017	EC_4	−20	+0.100667
	−10	+0.035509		−10	+0.050333
	+10	−0.03551		+10	−0.05033
	+20	−0.07102		+20	−0.10067
EC_2	−20	+0.001198	$SC_{transit}$	−20	+0.03519
	−10	+0.000599		−10	+0.017595
	+10	−0.0006		+10	−0.0176
	+20	−0.0012		+20	−0.03519
EC_3	−20	+0.023346			
	−10	+0.011673			
	+10	−0.01167			
	+20	−0.02335			

models did not consider carbon emissions during production and transportation. Thus, carbon emissions in both models are evaluated by incorporating the associated sustainable components and compared thereafter. The amount of carbon emission simulated by the current model is contrasted with the models of Chauhan et al. (2021) and Modak and Kelle (2018). Figure 4.49 demonstrates how much carbon is emitted by firm 1, 2, and 3 i.e, 139615.2, 139919.4, and 139259.4 tonnes per year respectively. Thus, as the problems related to climate and the environment take center stage and impact our lives, thus in the present chapter initiative is taken by in reducing their carbon footprints.

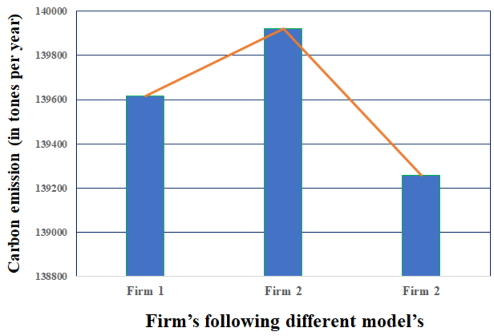


Fig. 4.49: Carbon emission by the firms following different models

Firm 1 = Following Modak and Kelle (2018) model, Firm 2 = Following Chauhan et al. (2021) model, Firm3 = Following present model

4.10 Managerial Implications

In the present chapter single manufacturer, a sole retailer, and carbon emission at different stages of production and supply of the firm is taken into account. Varied decision parameters are presumed for acquiring rational administrative decisions. As a result, multiple inconsistent elements like "core product's" cost, "make-to-order" product's, yields of both - "single and dual channel's", lead time, and carbon emission for considering the decisions. Underneath discusses the managerial implications of this chapter.

CO_2 emission while production is analyzed by taking into account the expression of production rate which is directly proportional to carbon emission. Thus, an increase in production rate leads to rise of carbon emissions. A cap is put on the carbon emission from each firm and if the firm surpasses the set emissions limit then they are subjected to the penalties. Since the manufacturer is also considering carbon emission from transport vehicles used in "online and offline" delivery of the final product so as the distance increases the cost for carbon emission also increases. Enabling "standard products and personalized products" to the customers' works as a pull-up force for demand and ultimately for the gross yield of the firm. To proliferate the overall yield of the industry, in a "supply chain model", "dual-route" is taken into consideration over "single-channel". The reciprocity of information between the parties (manufacturer and retailer) indulged in different stages of the centralized supply chain model improves the performance and profit of the firm. Since manufacturers and retailers exchange information thus while providing customized features managers should arrange the cost of "customized products" in such a way that it does not overshoot the "threshold limit" otherwise the customers will switch the channel. Lead time can be minimized by the manager and by introducing a lead time crashing cost client services can be a boost. Presumed "threshold limit" and variable switching of customers among offline and online channels provide an ameliorate managerial decision. Surpassing the limit while increasing the flash price of the "core product" leads to an adverse influence on the cost of "customized products" and the gross yield of the firm.

4.11 Conclusions

Recently companies are aware that they are not just functioning for the existing generation but they are investing for upcoming generations also. This chapter have constructed based on the idea of two pillars of sustainability such as environmental and economical. Modified "supply chain model with dual-route" while assuming greenhouse gases (largely, CO_2) emissions during the phase of manufacturing and delivering of items is demonstrated in the current chapter. The chapter considers carbon emitted during the production and transportation of the product. The present chapter's model is more sustainable in comparison to the existing model as it took initiative by adopting actions like penalties[13], using upcoming eco-friendly technologies such as green transportation [14] demonstrated by Dhara and Lal (2021). There are real examples which validates the use of penalties in various projects to achieve the "sustainable development goals (SDG)". The penalties are invested in green projects such as saving "forests, protecting wildlife, and transforming lives in Zimbabwe, Harnessing clean wind energy to power sustainable development in North China, Cleaner air, renewable electricity, and improved well being for communities in Central Vietnam, and Permanent protection for Afognak Islands dense, old-growth spruce forest in Alaska"[15], etc are undertaken in the current chapter thereby moving towards carbon neutral. Based on results on environmental pillar, the numerically simulated value is compared with the existing models of Chauhan et al. (2021) and Modak and Kelle (2018). Observation depicts that approximately 4.7% and 2.5% less carbon emission were found in

[13] "https://www.lexology.com/library/detail.aspx?g=da3af3f8-ae61-4396-9352-42f33f7cfbb3"
[14] "https://www.investopedia.com/articles/stocks/07/green-industries.asp"
[15] "https://www.porsche.com/uk/aboutporsche/responsibility/porscheimpact/"

the present model as compared to Chauhan et al. (2021) and Modak and Kelle (2018), respectively. Thus, firm 3 (figure 4.49) demonstrates the present chapter's model can become more sustainable by following the aforementioned actions.

Moreover, from the perspective of economic sustainability, a modified dual channel with customization strategy is considered in more generalized form. The manufacturer provides "core and personalized items" to the customer by employing shopkeepers and e-commerce routes respectively. As compared to Modak and Kelle (2018), a specified "threshold limit" along with uneven switching of shoppers is also considered between the channels. The results depict that reducing energy usage is the crucial component of environmental cost and ultimately profit for supply chain models ("single-route and dual-route"). Administering the "supply chain model with dual-route", the firm can provide more variety ("customized and standard") of products to the customers on their demands. Moreover, the firm can manage all the profit parameters by adopting a "centralized policy". Results show that with the increase in production $EC_1, EC_2, EC_3, SC_{transit}$ initially decreases but finally starts to increase whereas EC_4 decreases. Other results depict that bringing down the holding and setup cost of the manufacturer benefits the firm as it results in a decrease in the cost of CO_2 ejection from freightage and rise in the benefit of the firm. Moreover, the dual channel model presented in the current chapter increased the profit of the firm by 9.6% compared to single channel.

The present chapter has some limitations which could be modified with some extensions. Maintenance in production is an important part of all manufacturing industry. Implementing preventive and corrective maintenance during production can play an important role to achieve a reliable and durable product (Sarkar et al. , 2020). Moreover, the non-existence of multiple retailers lacks manifestation of neutral vying nature between the shopkeepers in the "dual-channel supply chain model" (Sarkar et al. , 2018a; Majumder et al. , 2018). Administering vendor-managed inventory (VMI) enables possession of the items by the producer itself even after transferring to the shopkeeper's end (Batarfi et al. , 2016; Sarkar at al. , 2018b). A VMI model in the firm's "supply chain with centralized, dual-route concepts", and with the enhanced "personalized policy" along with consideration of carbon emission can be innovative research to be concerned. Henceforth, there are noteworthy investigations that require a revisit of this model. Slowing down the depletion of natural resources and pollution associated with it, is a crucial concern in the context of sustainable development and preservation of the environment. This leads to an investigation of product reuse, material recycling and all this comes at a cost and has an impact on the environment (Baranikumar et al. , 2021).

CHAPTER - 5

Involvement of carbon regulation in a smart dual-channel supply chain for customized products under uncertain environment

5.1 Problem definition

The “sustainable dual-channel supply chain” is an up-gradation of the classical supply chain. In a supply chain having single mode of shopping, the manufacturer produces only the “standard product” and does not support the personification of customers’ choices. With the “dual-channel supply chain”, the producer manufactures both “personalized and standard products”. Batarfi et al. (2016) demonstrated the “dual-channel” under a determined environment while in the present chapter supply chain is examined in an uncertain environment. Also, most of the literature either utilizes probabilistic uncertainty or demand variability but very few articles use both of the characteristics simultaneously in a demand function which is considered in this chapter. Additionally, the firm is serving both “online and offline” demand simultaneously thus, the number of customers increases. As a result, the profit and CO_2 emission due to the rise in the delivery of products increases. Therefore, the present article examines the CO_2 emission during transportation and imposes the penalties on the firms if they cross the predefined carbon limit set by the government. Moreover, for incorporating an online channel, the producer-retailer should negotiate properly and a “threshold limit” must be set which regulates the difference between the price of the “core and personalized product”. If the variation in the price of “core and personalized products” overshoots the “threshold limit”, then the shifting of customers between the channels is initiated, influencing the overall firm’s profit. Figure 5.50 manifests the contribution of the present chapter.

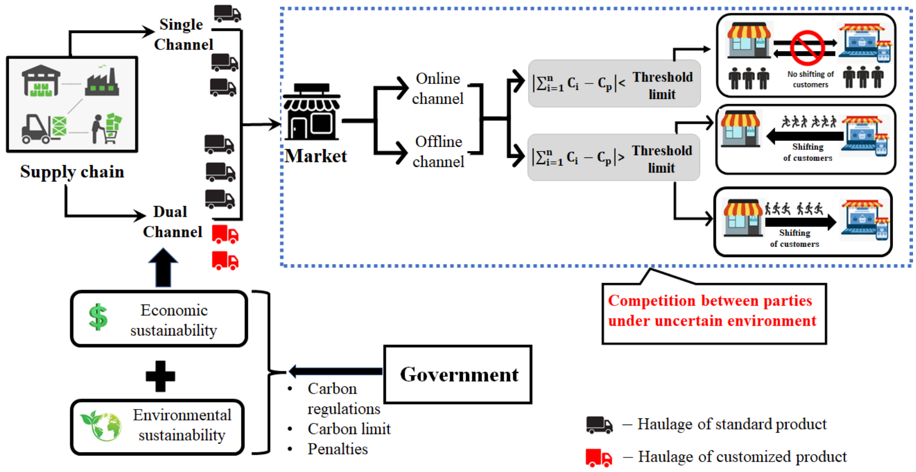


Fig. 5.50: Contribution of this research

5.2 Presumptions

For framing the mathematical model, following points are presumed.

1. A "centralized supply chain model having dual-channel" and a "personalized strategy" is considered. Maneuvering of customers is carried out by offline channels along with an online channel (Wang and He , 2022). Personification is made available through online channel and "standard product" through offline channel only. The number of customers who refuses to purchase items through offline channel chooses online channel.

2. Manufacturing house of the firm manufactures both "standard and personalized products" since it has its own manufacturing house (Batarfi et al. , 2016).

3. The manufacturer works for manufacturing products and transfer by utilizing a "single-setup multiple-delivery strategy" in offline channels whereas a "make-to-order policy" is used for online channels.

4. The lead time 'L' has 'm' collectively independent elements. For the jth element, a_j=minimum time span, b_j=normal time span, and c_j=crashing cost per unit time. Practically, we presume $c_1 < c_2 < ... < c_m$ (Sarkar et al. , 2018a).

5.3 Mathematical model

The underneath segment explains the demand function, profit function, a distribution-free technique for evaluating solutions that are optimal for the model, and a solution algorithm for this chapter.

5.3.1 Demand equation

The nature of the shoppers is miscellaneous since it gets influenced by the product's cost and diversity in products. Underneath offline and e-commerce mode, demand equations are derived by enhancing the model of Chauhan et al. (2021). The offline mode's function for demand is

$$D_1 = a_1 - \beta_1 C_p(1+m) + \delta_1(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right) \tag{5.94}$$

The online mode's function for demand is

$$D_2 = a_2 - \beta_2(1+m)\left(\sum_{i=1}^{N} C_i\right) - \delta_2(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right) \tag{5.95}$$

Where, a_1 and a_2 represent the number of shoppers preferring "offline and online" channels, respectively. The amount of demand fall/rise of the market (offline and online) with the increase/decrease in the price, typified by the price sensitivity coefficients β_1 and β_2 for the offline and online channels, respectively. Thus, by assuming a fixed "markup margin" for "core and personalized products", the change in customers because of price sensitivity is represented by $\beta_1 C_p(1+m)$ and $\beta_2 \sum_{i=1}^{N} C_i(1+m)$. Moreover, the swapping of shoppers from the offline mode of shopping to the online mode of shopping and vice-versa, demonstrated by parameters δ_1 and δ_2, respectively. Additionally, the shifting of customers among the channels is influenced by the factor that which channel is offering the same product at less price. Thus, $\delta_1(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right)$ and $\delta_2(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right)$ represents the variation is number of shoppers as consequence of δ_1 and δ_2, respectively.

Fuzzification

In the real world, anticipating the product's exact demand is troublesome. Consequently, constant demand is taken into consideration by adopting a "distribution-free approach" for lead-time demand. However, researchers have considered fuzzy demand also with the "distribution-free approach". In the existing literature, demand is presumed to be a "triangular fuzzy number". Consequently, a "triangular fuzzy number" is assumed for this chapter because of its simplicity in execution. Moreover, a market investigation on the products manifested that the demand of the market does not follow a specific pattern. Thus, considering a demand to be fixed or following a certain probability distribution is unworkable, justifying the usage of "fuzzy triangular demand". Henceforth, for more realistic solutions "fuzzy triangular demand" is assumed.

This model considers a non-negative "triangular fuzzy number" i.e., fuzzy demand $\widetilde{D_1} = (D_1 - \gamma_1, D_1, D_1 + \gamma_2)$ and $\widetilde{D_2} = (D_2 - \gamma_1, D_2, D_2 + \gamma_2)$. The fuzzy numbers are replaced with crisp values to obtain concluding solutions. The signed distance method is used in the present chapter for converting fuzzy outputs to crisp models. The shopkeeper's and manufacturer's demand can be evaluated by replacing the non-negative "triangular fuzzy number" in the aforementioned equations.

$$\widetilde{D_1} = D_1 + \frac{\gamma_2 - \gamma_1}{4} \tag{5.96}$$

$$\widetilde{D_2} = D_2 + \frac{\gamma_2 - \gamma_1}{4} \tag{5.97}$$

5.3.2 Functions for net profit

In this section, three profit equations are formulated. First, a manufacturer's profit on a "core product" is evaluated. In the second, the manufacturer's profit on a "personalized product" is obtained. In the end, the retailer's profit on the "core product" is derived.

I. **"Manufacturer's net profit on core product"**

The per unit profit of the manufacturer by selling the "core product" by virtue of the offline channel is

$$\nabla_1 = \text{Revenue} - \text{Cost of setup cost} - \text{Cost of holding} - \text{Cost for production cost}$$

$$\nabla_1 = C_p(1+m)\widetilde{D_1} - \left[\frac{S_1\widetilde{D_1}}{nQ_1} + \frac{r_vC_p}{2}\left[n\left(1 - \frac{\widetilde{D_1}}{P_1}\right) - 1 + \frac{2\widetilde{D_1}}{P_1}\right]\right] - C_p\widetilde{D_1} \tag{5.98}$$

Defuzzification

The fuzzy numbers are replaced with crisp values to obtain concluding solutions. The "signed distance" method is used in the present chapter for converting fuzzy outputs to crisp models. Hence, the profit for manufacturer is given by:

$$\nabla_1 = C_p(1+m)\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\Bigg(\frac{S_1}{nQ_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+\frac{r_vC_pQ_1}{2}$$

$$\left(n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)\right)\Bigg)-C_p\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right) \tag{5.99}$$

whereas $C_p(1+m)\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$ exhibits revenue of the firm and $C_p(1+m)$ is the “core product” cost set by the manufacturer. The setup cost of the manufacturer for the “core product” is $\frac{S_1}{nQ_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$. Moreover, $\frac{r_vC_pQ_1}{2}\left(n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)\right)$ is the holding cost and $C_p\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$ is the “core products” cost of production.

II. **“Manufacturer’s net profit on personalized product”**

The per unit profit of the manufacturer by selling the “personalized product” by virtue of the e-commerce is

$$\nabla_2 = \text{Revenue} - \text{Cost for setup} - \text{Holding cost} - \text{Manufacturing cost}$$

$$\nabla_2 = \sum_{i=1}^{N} C_i(1+m)\phi_i\widetilde{D_2}-\left[\frac{S_2}{Q_2}\widetilde{D_2}+\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{\widetilde{D_2}}{P_2}\right)+\sum_{i=1}^{N} C_i\phi_i\widetilde{D_2}\right] \tag{5.100}$$

Again, replacing the fuzzy numbers with crisp values to obtain concluding solutions. Thus, the equation becomes:

$$\nabla_2 = \sum_{i=1}^{N} C_i(1+m)\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{S_2}{Q_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)-\left(\frac{h_1Q_2}{2}\right)$$

$$\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)\right)-\sum_{i=1}^{N} C_i\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right) \tag{5.101}$$

whereas the revenue of the manufacturer on “personalized products” is $\sum_{i=1}^{N} C_i(1+m)\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)$ and the selling price of the “personalized product” is $\sum_{i=1}^{N} C_i(1+m)\phi_i$. The setup cost of the manufacturer for “personalized products” by following a “make-to-order policy” is $\frac{S_2}{Q_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)$. Moreover, $\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)\right)$ and $\sum_{i=1}^{N} C_i\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)$is holding cost and production cost for “personalized product”.

III. **"Retailer's net profit on core product"**

The per unit profit of the retailer by selling the "core product" is

$$\nabla_3 = \text{Revenue} - \text{Cost of ordering} - \text{Cost of holding} - \text{Shortage cost} - \text{Cost of lead time crashing}$$

$$\nabla_3 = C_p(1+m)^2\widetilde{D_1} - \left[\frac{A_r}{Q_1}\widetilde{D_1} + r_bC_b\left(\frac{Q_1}{2} + R - \widetilde{D_1}L\right) + \frac{\pi\widetilde{D_1}}{Q_1}E(M-R)^+ + \frac{\widetilde{D_1}CL}{Q_1}\right] \tag{5.102}$$

where,

$$E(M-R)^+ \leq \left[\frac{\sqrt{\sigma^2L + (\widetilde{D_1}L)^2 + k^2\sigma^2L - 2\widetilde{D_1}Lk\sigma\sqrt{L}} + (\widetilde{D_1}L - k\sigma\sqrt{L})}{2}\right] \tag{5.103}$$

$$\nabla_3 = C_p(1+m)^2\widetilde{D_1} - \frac{A_r\widetilde{D_1}}{Q_1} - r_bC_b\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) - \frac{\widetilde{D_1}CL}{Q_1} -$$

$$\frac{\pi\widetilde{D_1}}{2Q_1}\left[\sqrt{\sigma^2L + (\widetilde{D_1}L)^2 + k^2\sigma^2L - 2\widetilde{D_1}Lk\sigma\sqrt{L}} + (\widetilde{D_1}L - k\sigma\sqrt{L})\right] \tag{5.104}$$

Replacing the fuzzy numbers with crisp values the equation becomes:

$$\nabla_3 = C_p(1+m)^2\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right) - \frac{A_r}{Q_1}\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right) - r_bC_b\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) - \frac{CL}{Q_1}\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right) -$$

$$\frac{\pi}{2Q_1}\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right)\left[\sqrt{\sigma^2L + \left(\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right)L\right)^2 + k^2\sigma^2L - 2\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right)Lk\sigma\sqrt{L}} + \right.$$

$$\left.\left(\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right)L - k\sigma\sqrt{L}\right)\right] \tag{5.105}$$

whereas revenue of the shopkeeper is $C_p(1+m)^2\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right)$ and the cost of the "core product" is $C_p(1+m)^2$. Further, $\frac{A_r}{Q_1}\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right)$ is the cost of ordering and $r_bC_b\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right)$ is the cost of holding for the retailer. Lastly, $\frac{CL}{Q_1}\left(D_1 + \frac{\gamma_2-\gamma_1}{4}\right)$ is the cost of lead time crashing.

Thus, net profit ∇_S in case of the supply chain having "single-channel" is derived by summing $\nabla_1$and ∇_3. We get,

$$\nabla_S = \nabla_1 + \nabla_3 \tag{5.106}$$

$$\nabla_S = C_p(1+m)\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\left[\frac{S_1}{nQ_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+\frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-1+\right.\right.\right.$$

$$\left.\left.\left.\frac{2}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)\right]\right]-C_p\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+C_p(1+m)^2\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{A_r}{Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-$$

$$r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)-\frac{CL}{Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{\pi}{2Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$$

$$\left[\sqrt{\sigma^2L+\left(\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)Lk\sigma\sqrt{L}}+\left(\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L-k\sigma\sqrt{L}\right)\right] \tag{5.107}$$

And expected net profit ∇_D in case of the "centralized supply chain having dual-channel" is derived by summing ∇_1, ∇_2, and ∇_3. We get,

$$\nabla_D = \nabla_1+\nabla_2+\nabla_3 \tag{5.108}$$

$$\nabla_D = C_p(1+m)\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\left[\frac{S_1}{nQ_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+\frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-1+\right.\right.\right.$$

$$\left.\left.\left.\frac{2}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)\right]\right]-C_p\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+C_p(1+m)^2\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{A_r}{Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-$$

$$r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)-\frac{CL}{Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{\pi}{2Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$$

$$\left[\sqrt{\sigma^2L+\left(\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)Lk\sigma\sqrt{L}}+\right.$$

$$\left.\left(\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L-k\sigma\sqrt{L}\right)\right]+\sum_{i=1}^{N}C_i(1+m)\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)-\left[\frac{S_2}{Q_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)+\left(\frac{h_1Q_2}{2}\right)\right.$$

$$\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)\right)+\sum_{i=1}^{N}C_i\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)\Bigg] \tag{5.109}$$

5.3.3 Supply chain optimal solutions

Since, equations of profit are non-linear in nature thus, for a fixed constant 'm'' partial derivative of the profit is evaluated with respect to Q_2, Q_1, & k for obtaining the optimal solution.

$$\frac{\partial \nabla_D}{\partial k}=-r_bC_b\sigma\sqrt{L}-\frac{\pi}{2Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$$

$$\left[\frac{k\sigma^2L-\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L\sigma\sqrt{L}}{\sqrt{\sigma^2L+\left(\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)Lk\sigma\sqrt{L}}}-\sigma\sqrt{L}\right] \tag{5.110}$$

$$\frac{\partial \nabla_D}{\partial Q_1}=\frac{A_r}{Q_1^2}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{r_bC_b}{2}+\frac{S_1}{nQ_1^2}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+\frac{CL}{Q_1^2}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{r_vC_p}{2}$$

$$\left(n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)+\frac{\pi}{2Q_1^2}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$$

$$\left(\left(L\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-k\sigma\sqrt{L}\right)+\sqrt{\sigma^2L+\left(L\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)^2+k^2\sigma^2L-2Lk\sigma\sqrt{L}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)}\right) \tag{5.111}$$

$$\frac{\partial \nabla_D}{\partial Q_2}=\frac{S_2}{Q_2^2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{h_1}{2}\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)\right) \tag{5.112}$$

For fixed integer 'm'', values of Q_2, Q_1,& k are evaluated by equating them to zero.

$$Q_2^*=\sqrt{\frac{S_2\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)}{\frac{h_1}{2}\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)\right)}} \tag{5.113}$$

$$Q_1^*=\left(-CL\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+S_1\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+A_r\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)+L\right.$$

$$\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-k\sigma\sqrt{L}+\frac{\pi\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)}{2}\Bigg)$$

$$\sqrt{\sigma^2L+\left(L\left(D_1+\frac{\gamma_2-\gamma_1}{2}\right)\right)^2+k^2\sigma^2L-2Lk\sigma\sqrt{L}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)}$$

$$\left(\frac{1}{\frac{r_bC_b}{2}+\frac{r_v}{2}\left(n\left(1-\frac{1}{P_1}\left(D_1-\frac{\gamma_2-\gamma_1}{4}\right)\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)}\right)^{\frac{1}{2}} \tag{5.114}$$

$$k^*=\frac{2}{\sqrt{L}\sigma}\left[\frac{-\sigma\sqrt{Q_1C_br_b\left(-Q_1C_br_b+\pi\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)}}{-Q_1C_br_b}+\right.$$

$$\left.\frac{\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\pi\sigma\sqrt{Q_1C_br_b\left(-Q_1C_br_b+\pi\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right)L}}{2Q_1C_br_b\left(\pi\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-Q_1C_br_b\right)}+L\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)\right] \tag{5.115}$$

The Q_1^*, Q_2^*, and k^* optimal solutions are dependent on one another. The numerical procedure is used for finding these optimal values. Thus, for finding the managerial decisions an iteration method is used along with the underneath algorithm.

5.4 Algorithm for obtaining solution-I

In solving the present chapter's model underneath algorithm VRA-I is utilized.

Step 1 Values are assigned to all the parameters in accordance to input parameters of the model.

Step 2 Put n=1.

Step 3 Execute the underneath steps for all the values of L_i; i=1,2,...

- Step 3(a) Evaluate D_1 and D_2 from equations 5.94 & 5.95.
- Step 3(b) Evaluate the value of $\widetilde{D_1}$ and $\widetilde{D_2}$ from equations 5.96 & 5.97.
- Step 3(c) Evaluate the value of Q_2 from equation 5.113.
- Step 3(d) Evaluate the value of Q_1 from equation 5.114.
- Step 3(e) Evaluate the value of k from equation 5.115.
- Step 3(f) Repeat Steps 3a to 3e until there are no variation in the values of the parameters Q_2, Q_1, and k up to a level of accuracy as specified.

Step 4 Derive ∇_S using Q_2, Q_1, k and putting in equation 5.107.

Step 5 Derive ∇_D by putting ∇_S in equation 5.109.

Step 6 Put $n = n+1$ and redo the Steps from 3 to 5.

Step 7 If $\nabla_D(n+1) < \nabla_D(n)$ then redo the Steps from 2 to 6 else end the algorithm.

Proposition 5.1. *If Q_1^*, Q_2^*, and k^* exhibits as the values which are optimal for Q_1, Q_2 and k, then for constant values of n and $L \in [L_i, L_{i-1}]$, the profit function ∇_D for "dual-channel" acquires its maximum value at Q_1^*, Q_2^*, and k^* under the following situation*

$$X + Y > Z$$

Proof. For "dual-channel supply chain", the Hessian matrix H_1 is

$$H_1 = \begin{bmatrix} \frac{\partial^2 \nabla_D}{\partial Q_1^2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} \\ \frac{\partial^2 \nabla_D}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial Q_2^2} & \frac{\partial^2 \nabla_D}{\partial Q_2 \partial k} \\ \frac{\partial^2 \nabla_D}{\partial k \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial k \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial k^2} \end{bmatrix}$$

where,

$$\frac{\partial^2 \nabla_D}{\partial k^2} = -\frac{\pi \widetilde{D_1}}{2Q_1} \left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2 \sigma^2 L - 2\widetilde{D_1} L \sigma \sqrt{L}}} \right]$$

$$\frac{\partial^2 \nabla_D}{\partial k \partial Q_1} = \frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} = \frac{\pi \widetilde{D_1}}{2Q_1^2} \left[\frac{k\sigma^2 L - \widetilde{D_1} L \sigma \sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2 \sigma^2 L - 2\widetilde{D_1} L \sigma \sqrt{L}}} - \sigma \sqrt{L} \right]$$

$$\frac{\partial^2 \nabla_D}{\partial Q_2 \partial k} = \frac{\partial^2 \nabla_D}{\partial k \partial Q_2} = \frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2} = \frac{\partial^2 \nabla_D}{\partial Q_2 \partial Q_1} = 0$$

$$\frac{\partial^2 \nabla_D}{\partial Q_2^2} = -\frac{2S_2 \widetilde{D_2}}{Q_2^3}$$

$$\frac{\partial^2 \nabla_D}{\partial Q_1^2} = -\frac{2}{Q_1^3} \left[A_r \widetilde{D_1} + \frac{S_1 D_1}{n} + \widetilde{D_1} CL + \pi \widetilde{D_1} (\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2 \sigma^2 L - 2\widetilde{D_1} Lk\sigma \sqrt{L}}) \right]$$

The principal minor of $|(H_1)_{1,1}|_{(Q_1^*, Q_2^*, k^*)}$ of 1×1 is

$$|(H_1)_{1,1}|_{(Q_1^*, Q_2^*, k^*)} = \left| \frac{\partial^2 \nabla_D}{\partial Q_1^2} \right|_{(Q_1^*, Q_2^*, k^*)}$$

$$= -\frac{2}{Q_1^3}\left[A_r\widetilde{D_1} + \frac{S_1\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}(\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}})\right] < 0$$

The principal minor of $|(H_1)_{2,2}|_{(Q_1^*,Q_2^*,k^*)}$ of order 2×2 is

$$|(H_1)_{2,2}|_{(Q_1^*,Q_2^*,k^*)} = \left(\frac{\partial^2\nabla_D}{\partial Q_1^2}\right)\left(\frac{\partial^2\nabla_D}{\partial Q_2^2}\right)$$

$$= \left(\frac{-2}{Q_1^3}\left[A_r\widetilde{D_1} + \frac{S_1\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right]\right)$$

$$\left(-\frac{2S_2\widetilde{D_2}}{Q_2^3}\right) > 0$$

The principal minor of $|(H_1)_{3,3}|_{(Q_1^*,Q_2^*,k^*)}$ of order 3×3 is

$$|(H_1)_{3,3}|_{(Q_1^*,Q_2^*,k^*)} = \frac{\partial^2\nabla_D}{\partial Q_2^2}\left[\left(\frac{\partial^2\nabla_D}{\partial Q_1^2}\right)\left(\frac{\partial^2\nabla_D}{\partial k^2}\right) - \left(\frac{\partial^2\nabla_D}{\partial k\partial Q_1}\right)^2\right]$$

$$= \left(\frac{2S_2\widetilde{D_2}}{Q_2^3}\right)\frac{\pi\widetilde{D_1}^2}{Q_1^4}\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} - \frac{\widetilde{D_1}L^2\sigma^2}{2\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}}$$

$$-\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2\widetilde{D_1}^2 L^3\sigma^2 - 4\widetilde{D_1}\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L})} > 0$$

$$\Rightarrow \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} > \frac{\widetilde{D_1}L^2\sigma^2}{2\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}}$$

$$+\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2\widetilde{D_1}^2 L^3\sigma^2 - 4\widetilde{D_1}\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L})}$$

$$\Rightarrow \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} + \frac{\widetilde{D_1}\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}} >$$

$$\frac{\widetilde{D_1}L^2\sigma^2}{2\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2\widetilde{D_1}^2L^3\sigma^2)}{4(\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L})}$$

$$\Rightarrow X+Y>Z$$

Where,

$$X=\left(A_r+\frac{S_1}{n}+CL\right)\frac{\sigma^4L^2}{\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\pi\sigma^4L^2,$$

$$Y=\frac{\pi k\sigma^3L\sqrt{L}}{2\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\frac{\widetilde{D_1}\sigma^3L^2\sqrt{L}}{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}\textit{and}$$

$$Z=\frac{\widetilde{D_1}L^2\sigma^2}{2\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2\widetilde{D_1}^2L^3\sigma^2)}{4(\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L})}$$

Since, the Hessian matrix's, all the principal minors are not positive. Hence, the Hessian matrix H_1 is negative definite at (Q_1^*,Q_2^*,k^*). Thus, aggregate expected profit for "dual-channel" gets the global maximum at (Q_1^*,Q_2^*,k^*).

Proposition 5.2. *If Q_1^*, k^* exhibits the optimal values of Q_1, k, then for constant values of $L\in[L_i,L_{i-1}]$ and L, the profit function ∇_S having "single-channel" acquires its maximum value at Q_1^*, k^* under the following situation*

$$X_1+Y_1>Z_1$$

Proof. For "single-channel supply chain", the Hessian matrix H_2 is

$$H_2=\begin{bmatrix}\frac{\partial^2\nabla_S}{\partial Q_1^2} & \frac{\partial^2\nabla_S}{\partial Q_1\partial k}\\ \frac{\partial^2\nabla_S}{\partial k\partial Q_1} & \frac{\partial^2\nabla_S}{\partial k^2}\end{bmatrix}$$

where,

$$\frac{\partial^2\nabla_S}{\partial Q_1^2}=-\frac{2}{Q_1^3}\left[A_r\widetilde{D_1}+\frac{S_1\widetilde{D_1}}{n}+\widetilde{D_1}CL+\pi\widetilde{D_1}(\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}Lk\sigma\sqrt{L}})\right]$$

$$\frac{\partial^2 \nabla_S}{\partial k \partial Q_1} = \frac{\pi \widetilde{D_1}}{2Q_1^2}\left[\frac{k\sigma^2 L - \widetilde{D_1} L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} Lk\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2 \nabla_S}{\partial k^2} = -\frac{\pi \widetilde{D_1}}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2 \nabla_S}{\partial Q_1 \partial k} = \frac{\pi \widetilde{D_1}}{2Q_1^2}\left[\frac{k\sigma^2 L - \widetilde{D_1} L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

The principal minor $|H_{1,1}|_{(Q_1^*,k^*)}$ of $|H_2|$ of order 1×1 is

$$|H_{1,1}|_{(Q_1^*,k^*)} = \left|\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right|_{(Q_1^*,k^*)}$$

$$= -\frac{2}{Q_1^3}\left[A_r\widetilde{D_1} + \frac{S_1\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}(\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} Lk\sigma\sqrt{L}})\right] < 0$$

The principal minor $|H_{2,2}|_{(Q_1^*,k^*)}$ of $|H_2|$ of order 2×2 is

$$|H_{2,2}|_{(Q_1^*,k^*)} = \left(\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_S}{\partial k^2}\right) - \left(\frac{\partial^2 \nabla_S}{\partial k \partial Q_1}\right)^2$$

$$= \Big(-\frac{2}{Q_1^3}\Big[A_r\widetilde{D_1} + \frac{S_1\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}(\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} Lk\sigma\sqrt{L}})\Big]\Big)$$

$$\Big(-\frac{\pi\widetilde{D_1}}{2Q_1}\Big[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}}\Big]\Big)$$

$$-\Big(\frac{\pi\widetilde{D_1}}{2Q_1^2}\Big[\frac{k\sigma^2 L - \widetilde{D_1} L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} - \sigma\sqrt{L}\Big]\Big)^2$$

$$= \frac{\pi\widetilde{D_1}^2}{Q_1^4}\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} - \frac{\widetilde{D_1} L^2\sigma^2}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}}$$

$$-\frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2\widetilde{D_1}^2 L^3\sigma^2 - 4\widetilde{D_1}\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L})} > 0$$

$$\Rightarrow \left(A_r + \frac{S_1}{n} + CL\right) \frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} > \frac{\widetilde{D_1} L^2\sigma^2}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}}$$

$$+ \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2\widetilde{D_1}^2 L^3\sigma^2 - 4\widetilde{D_1}\sigma^3 L^2\sqrt{L})}{4(\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L})}$$

$$\Rightarrow \left(A_r + \frac{S_1}{n} + CL\right) \frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \frac{\widetilde{D_1}\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}$$

$$> \frac{\widetilde{D_1} L^2\sigma^2}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2\widetilde{D_1}^2 L^3\sigma^2)}{4(\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

$$\Rightarrow X_1 + Y_1 > Z_1$$

where,

$$X_1 = \left(A_r + \frac{S_1}{n} + CL\right) \frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \pi\sigma^4 L^2$$

$$Y_1 = \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \frac{\widetilde{D_1}\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}$$

$$Z_1 = \frac{\widetilde{D_1} L^2\sigma^2}{2\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2\widetilde{D_1}^2 L^3\sigma^2)}{4(\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

We see that all the principal minors of the Hessian matrix are negative. Hence, the Hessian matrix H_2 is negative definite at (Q_1^*, k^*). Therefore, the total expected "single-channel" profit function attains its global maximum at (Q_1^*, k^*).

5.5 Exploration of dual-channel from environment outlook

5.5.1 Carbon ejection during haulage

The present part explains the reckoning of the emission of CO_2 during transportation. Thus, the sum of carbon ejected from all vehicles per year is

$$E'_{tr} = \left(\eta_1 n \frac{D_1 + \frac{\gamma_2 - \gamma_1}{4}}{nQ_1} + \eta_2 n \frac{D_2 + \frac{\gamma_2 - \gamma_1}{4}}{nQ_2} \right) \nu E \tag{5.116}$$

$$= \left(\eta_1 \frac{D_1 + \frac{\gamma_2 - \gamma_1}{4}}{Q_1} + \eta_2 \frac{D_2 + \frac{\gamma_2 - \gamma_1}{4}}{Q_2} \right) \nu E \tag{5.117}$$

Where, $\eta_1 = \left[\frac{Q_1}{Cap_1} \right]$ and $\eta_2 = \left[\frac{Q_2}{Cap_2} \right]$. Therefore, the total cost of CO_2 emission during transportation is

$$C_{ec} = E'_{tr} C_{tax} \tag{5.118}$$

5.5.2 Penalties due to uncurbed carbon emission

When the emission of carbon from the manufacturing house of the firm exceeds the predefined limit, then the collection of penalty costs is done. Therefore, the penalty cost is given as

$$C_{pen} = \sum_{i=1}^{l} Y_i C_{ep,i} \tag{5.119}$$

Where,

$$Y_i = \begin{cases} 1 & E'_{tr} > E_{li} \quad (i = 1, 2, ..., l) \\ 0 & otherwise \end{cases} \tag{5.120}$$

Henceforth,the supply chain management's having dual mode of shopping, aggregate profit is

$$\nabla_D = C_p(1+m) \left(D_1 + \frac{\gamma_2 - \gamma_1}{4} \right) - \left[\frac{S_1}{nQ_1} \left(D_1 + \frac{\gamma_2 - \gamma_1}{4} \right) + \frac{r_v C_p Q_1}{2} \right.$$

$$\left. \left[n \left(1 - \frac{1}{P_1} \left(D_1 + \frac{\gamma_2 - \gamma_1}{4} \right) - 1 + \frac{2}{P_1} \left(D_1 + \frac{\gamma_2 - \gamma_1}{4} \right) \right) \right] \right] - C_p \left(D_1 + \frac{\gamma_2 - \gamma_1}{4} \right)$$

$$+C_p(1+m)^2\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{A_r}{Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-r_bC_b\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)-\frac{CL}{Q_1}$$

$$\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{\pi}{2Q_1}\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)$$

$$\left[\sqrt{\sigma^2L+\left(\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)Lk\sigma\sqrt{L}+}\right.$$

$$\left.\left(\left(D_1+\frac{\gamma_2-\gamma_1}{4}\right)L-k\sigma\sqrt{L}\right)\right]+\sum_{i=1}^{N}C_i(1+m)\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)-\frac{S_2}{Q_2}$$

$$\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)-\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)\right)-\sum_{i=1}^{N}C_i\phi_i\left(D_2+\frac{\gamma_2-\gamma_1}{4}\right)$$

$$-C_{ec}-C_{pen} \tag{5.121}$$

5.6 Algorithm for obtaining solution-II

In solving the present chapter's model underneath VRA-II algorithm is utilized.

Step 1 Values are assigned to all the parameters in accordance to notations.

Step 2 Evaluate the value of E'_{tr}, C_{ec}, and C_{pen} from equations 5.117, 5.118, & 5.119.

Step 3 Use the values of E'_{tr}, C_{ec}, C_{pen}, and old ∇_D to evaluate enhanced ∇_D from equations 5.109 and 5.121.

5.7 Numerical experimentation and discussion

This segment illustrates the developed model for the supply chain management. Examining the consequences of stochastic fuzzy demand on the "supply chain model" and unequal shipment of customers between the channel is the objective of the numerical examples. To resemble a real manufacturing environment, values of parameters were taken from Malik and Sarkar (2019) & Chauhan et al. (2021). The input parameters taken are as follows "$C_p = 100$ (\$/unit); $C_1 = 80$ (\$/unit); $C_2 = 100$ (\$/unit); $C_3 = 120$ (\$/unit); $S_1 = 800$ (\$/setup); $S_2 = 1000$ (\$/setup); $C_b = 120$ (\$/unit); $r_v = 0.2$ (\$/unit/unit time); $r_b = 0.2$

(\$/unit/unit time); $A_r = 200$ (\$/order); $P_1 = 5000$ (Units/year); $P_2 = 5000$ (Units/year); $m = 0.7$; $\pi = 150$ (\$/unit); $a_1 = 1000$; $a_2 = 1000$; $\beta_1 = 0.75$ (\$/unit); $\beta_2 = 0.8$ (\$/unit); $\delta_1 = 0.2$; $\delta_2 = 0.3$; $\phi_1 = 0.25$ (%); $\phi_2 = 0.3(\%)$; $\phi_3 = 0.45$ (%); $h_1 = 30$ (\$/unit/year); $\gamma_1 = 500$; $\gamma_2 = 600$;$\sigma = 200$; $\eta_1 = 1$; $\eta_2 = 1$; $\nu = 75$ (gallons); $E = 0.01008414$ (ton/gallon); $C_{tax} = 10$ (\$/ton);$L = 4$ (weeks)". Further, a presumed "threshold value" i.e., \$20 is considered throughout the chapter. Penalty roaster for carbon emission is exhibited by table 3.8 drawn from Jaber et al. (2013).

5.7.1 Analysis of profit

An example is taken into account to draw a juxtaposition among the supply chain management's profit having "single and dual-channel". In the "supply chain having dual-channel", the firm caters to the need of shoppers of the "core product" through the offline mode of shopping and "customized products" through the e-commerce. In the supply chain having "single-channel", the firm caters to the customer through an offline channel. We consider three types of personalization on the "core product" in the current model. Moreover, this section examines and analyzes managerial decisions under varied situations.

Table 5.15 exemplify the optimal values of the decision variables. It demonstrates that \$ 5144428 and \$ 4926397 is the maximum profits earned by the firm following "single and dual-channel supply chains", respectively. Moreover, it is observed when $n = 2$ and the values of $k, D_1, D_2, Q_1, Q_2,$ and L at $n = 2$ are 19.54463, 2349.938, 2374.938, 1906.6, 1931.6, 2135.499, 905.6477, and 4, respectively.

Tab. 5.15: Decision variables optimum values

n	L	k	D_1	D_2	Q_1	Q_2	∇_D	∇_S
1	4	19.5	2349.9	1906.6	2135.5	905.6	5144099	4926067
2	**4**	**19.5**	**2349.9**	**1906.6**	**2135.5**	**905.6**	**5144428**	**4926397**
3	4	19.5	2349.9	1906.6	2135.5	905.6	5144387	4926356

Further, 5.16 demonstrates that there is more shipment of customers from shopkeepers to an online platform $(\delta_1 > \delta_2)$ if the purchasing cost of the core product is more in comparison to "personalized products".

Similarly, in the case of ($\delta_1 < \delta_2$) i.e., shipment of shoppers from shopkeeper to an online platform is less if the purchasing cost of the "core product" is less in comparison to "personalized products". Although, it also illustrates that the profit of a "dual-channel" is more than a "single channel" irrespective of different approaches.

Tab. 5.16: Influence of shopper's swapping on profit

	k	D_1	D_2	Q_1	Q_2	∇_D	∇_S
$\delta_1 > \delta_2$	20.1	2419.3	1733.3	2170.3	733.8	5298882	5069267
$\delta_1 < \delta_2$	19.8	2384.6	1698.6	2152.9	707.1	5222550	4997675

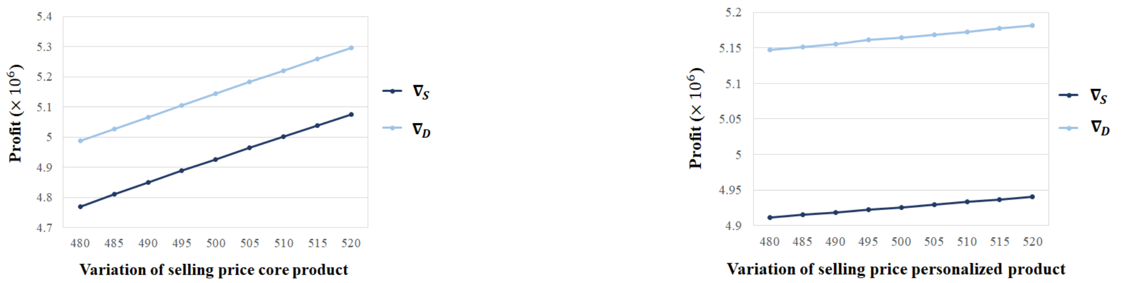
5.7.1.1 The potency of price sensitivity on the profit

Beneath are two parts (A & B), exemplifying the impact of varying selling prices ("core and personalized products") on the choices of shoppers and the "dual-channel's and single-channel's" profit.

Part A. In this part, the divergence in the cost of the item offered by the shopkeeper and the e-channel is less than the "threshold limit" ($|\sum_{i=1}^{N} C_i - C_P| <$Threshold value). Figure 5.51 demonstrates the aforementioned situation. Thus, while scrutinizing the situation, two cases are obtained.

Case I: Figure 5.51a exhibits the variation in the profit of the supply chain with the varying cost of the core product. The figure depicts the increase in the firm's profit with the increase in the cost of core products irrespective of any supply chain. Moreover, there is approximately a 6.41% and 6.19% swapping in the number of shoppers in "single-channel and dual-channel supply chains" from offline channel to online channel.

Case II: The alteration in the profit of the supply chain with the alteration in the selling price of the "personalized product", illustrated in figure 5.51b. The figure demonstrates that the profit of "single-channel and dual-channel supply chains" increases with the increase in the selling cost of the "personalized product". Additionally, there is approximately a 0.58% and 0.67% shifting of customers from online to offline channels in "single and dual-channel".



(a) Concerning varying selling prices of the core product

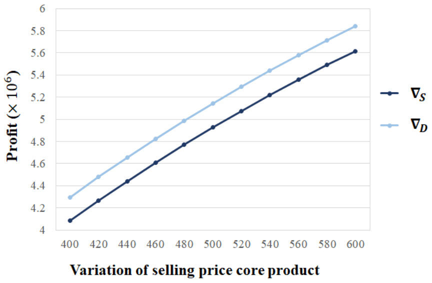
(b) Concerning varying selling prices of the personalized product

Fig. 5.51: Graphical presentation of the total profit of dual-channel vs single channel (Difference of selling price<Threshold value)

Part B. When the alteration in the cost of “core and personalized product” is overshooting the value of “threshold limit” ($|\sum_{i=1}^{N} C_i - C_P| >$Threshold value), exhibited in present part by figure 5.52. Thereby, underneath two cases are obtained.

Case I: Figure 5.52a exhibits the influence of varying selling prices of the core product on the profit of the supply chain. There are roughly 37.39% and 35.95% of shoppers shifted from offline channels to online channels in “single-channel and dual-channel supply chains”. Also, the figure reflects that there is an increase in the profit with the increase in the selling price of core products irrespective of any supply chain.

Case II: The other way around Case I, illustrated in the current case, depicts the impact of variation in the cost of the “personalized item” on the profit of the supply chain. Figure 5.52b represents that the profit of “single-channel and dual-channel supply chains” is directly proportional to the selling price of the “personalized product”. Moreover, there is about 2.95% and 3.64% of swap of customers from online to offline channels in “single and dual-channel”.



(a) Concerning varying selling prices of the core product

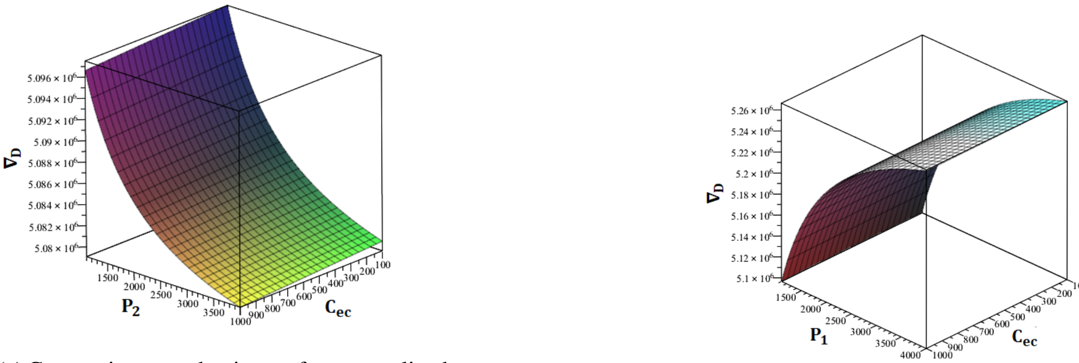
(b) Concerning varying selling prices of the personalized product

Fig. 5.52: Graphical presentation of the total profit of dual-channel vs single channel (Difference of selling price>Threshold value)

5.7.1.2 Influence of carbon emission on profit

This section exhibits the impact of production diversification on the profit of "dual-channel supply chain management" and consequently on carbon emission during haulage. Thereby, firms are charged with penalties for surpassing the set limit. Henceforth, the following points illustrate the concept.
In figure 5.53, 5.53a & 5.53b exemplifies the variation in profit and total cost of carbon emission during transportation concerning the production of the "core product & personalized product", respectively. Additionally, it is reflected from the figure that production increases the profit of the firm, and simultaneously it increases the carbon emission during haulage.

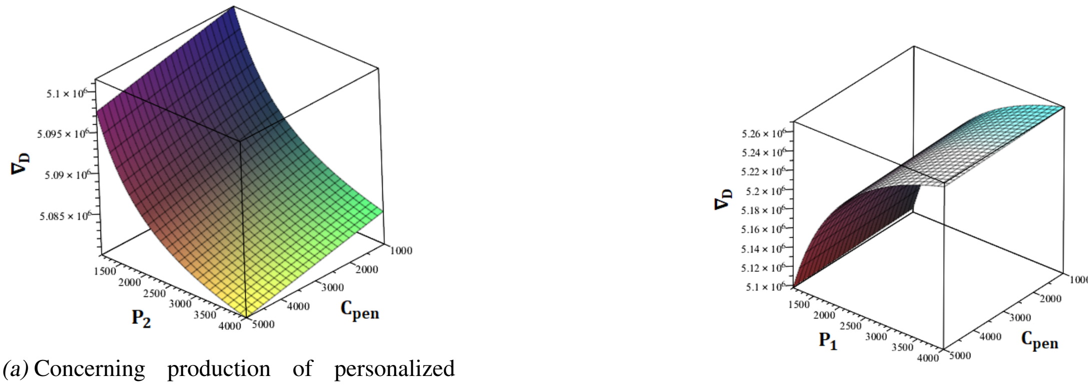
How alteration of product's production influences the profit and penalties charged on the "dual-channel supply chain management" are exhibited in figure 5.54. In figures 5.54a and 5.54b, an increase in the production of "core & personalized products" increases the profit nevertheless, it does increase the penalties charged to the firm.



(a) Concerning production of personalized products

(b) Concerning production of core products

Fig. 5.53: Variation in the profit of the firm following dual-channel supply chain management with respect to production and CO_2 emission during transportation



(a) Concerning production of personalized products

(b) Concerning production of core products

Fig. 5.54: Variation in the profit of the firm following dual-channel supply chain management with respect to production and penalties

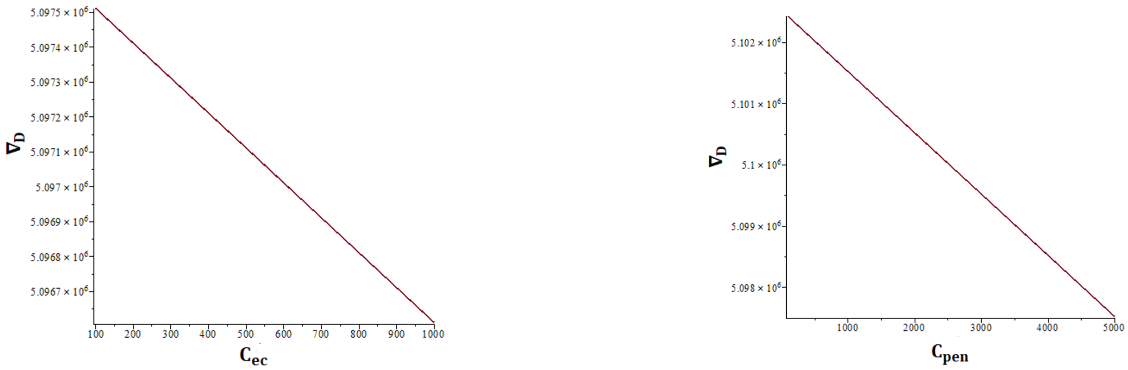
Therefore, figures 5.53 and 5.54 demonstrates that

$$\text{Production} \propto \text{Carbon emission} \propto \text{Penalties} \tag{5.122}$$

Thus,

$$\text{Profit} \propto \frac{1}{\text{Carbon emission cost}} \propto \frac{1}{\text{Penalties}} \tag{5.123}$$

Henceforth, figures 5.55a and 5.55b exhibit that as the carbon emission cost due to haulage increases, there is an increase in the penalties imposed on the firms, which decreases the profit of the firm. Henceforth, to decrease the carbon emission on firms and the environment, investments should be made to mitigate this ejection.



(a) With respect to CO_2 emission throughout transportation

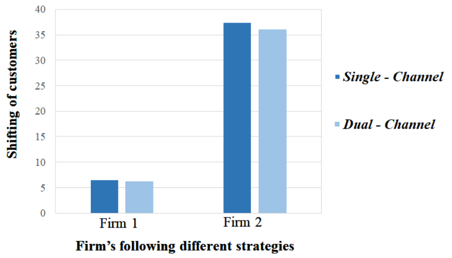
(b) With respect to penalties imposed on the firm

Fig. 5.55: Alteration in the supply chain management's profit having dual-channel

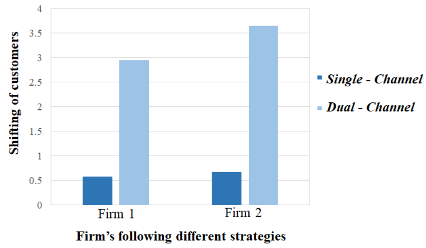
5.7.2 Distinct competition framework

The outcome of the current model validates the investments in personalization along with core products under centralized supply chain management and a fuzzy environment. The following section discusses the three scenarios (I & II) for validating the developed model.

Scenario I: The outcome of the numerical analysis of this chapter reflects that there is approximately less than 10% shifting of customers if the divergence in the selling price is less than the "threshold limit". In the current model, a comparison is drawn by considering two firms. In the case of firm 1, the divergence in the product's cost is below the "threshold limit" while in firm 2, the variation is overshooting the "threshold limit". Thus, the swapping in consumers was evaluated for both firms. In figures 5.56, 5.56a & 5.56b demonstrates that there is an increase in the swapping in customers, irrespective of the fact that whose ("core or personalized product") selling price is varying. Moreover, there is approximately more than an 80% increase in the swap of customers between the channels if the difference between selling price shots is beyond the "threshold limit".



(a) Regarding alteration in selling prices of the core product



(b) Regarding alteration in selling prices of the personalized product

Fig. 5.56: Graphical exhibition of shifting of customers in the firm

Firm 1=Divergence in selling price <Threshold limit, **Firm 2**=Divergence in selling price > Threshold limit

Scenario II: In the current scenario, a competition is organized between multiple retailers. Therefore, retailer 1 follows the "single-channel" whereas retailer 2 adopts a "dual-channel supply chain management" strategy. Moreover, competition is categorized into two parts - in the first part variation in the cost of the product was below the limit whereas in the second part the difference in the selling price was greater than the "threshold limit". Additionally, they further categorized into two cases where in the first case, price is "core product" is varied, and in the second part, that of "personalized product" is varied. Aforementioned competition is demonstrated by figure 5.57. The figure reflects that retailer 2 is earning more profit in comparison with retailer 1. Also, there is about a 4% of increase in the profit of retailer 2 compared to retailer 1.

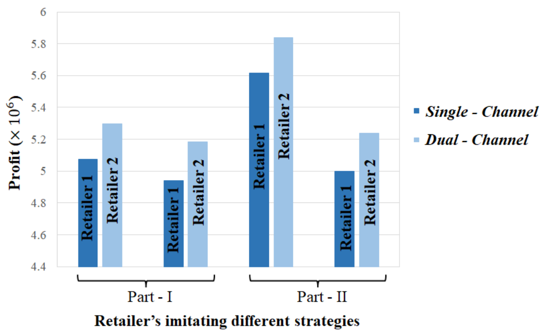


Fig. 5.57: Variation in the retailers profit in supply chain management having single and dual channel

Part 1=Divergence in the core product's selling price, **Part 2**=Divergence in selling price of personalized product

5.8 Case study

The proposed model is validated with the help of a real case study. The real data was shared by the firm installed in Punjab, India. The results exhibit the carbon emission generated by the firm on the environment. The data is collected from the firm through a personal visit. Henceforth, the data is normalized by utilizing statistical tools. Thus, the value of parameters is as follows: the number of trucks used for transporting "standard product" $\eta_1 = 2$ in a single consignment, similarly, a number of trucks utilized in case of transportation of "customized product" $\eta_2 = 1$; quantity of "standard product" ordered, $Q_1 = 2153$ and that of "customized product", $Q_2 = 710$; demand of the "standard product", $D_1 = 2410$ and $D_2 = 1524$ for the "customized product"; fuel consumed by vehicle in one trip, $v = 75$ gallons; carbon emitted per gallon consumption of fuel, $E = 0.02008414$ ton/gallon; tax imposed on the firm by the government for emission of carbon, C_{tax} =\$20/ton. As a result, the carbon ejected by the firm during transportation is 968 tonnes per year. In the proposed model, penalties are imposed on the firm, which obligates the firm to reduce carbon emission. Therefore, carbon emission reduces to 308 tonnes per year in the present model. Henceforth there is approximately 68% more carbon emission from the firm than in the present chapter's model.

5.9 Sensitivity analysis

In the present section, analysis of sensitivity for cost parameters on the overall effect of value changes on the total profit. This sensitivity analysis is executed by changing the parameter values to -5%, -2.5% $+2.5\%$, and $+5\%$ and keeping other parameters unchanged. Figure 5.58 exhibits the influence of variation in total cost concerning the key parameters. From the sensitivity in table 5.17, one can find that

1. C_p, core products production cost is most sensitive to the benefit of the supply chain management with dual mode of shopping compared to the rest of the cost parameters. Since personification is carried out on the "standard product" therefore, a small variation in the production cost of a "standard product" influences the cost parameters of a "customized product". Manufacturer's cost of production is the most significant factor of the supply chain, which depends on the operating cost, energy cost, labor cost etc

2. Comparing carbon emission costs C_{ec} and C_{pen}, the carbon emission penalties is more sensitive. Penalties help firms in reducing carbon emissions during transportation.

3. The second customization cost C_2 in comparison to first (C_1) and third (C_3) customization is most sensitive.

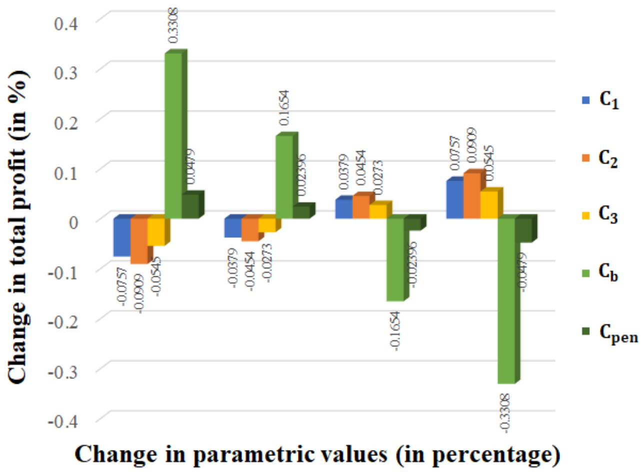


Fig. 5.58: Effect of change in parametric values on profit of the firm

4. The shopkeeper per order cost A_r is the least sensitive in the model. It is found that it have a negligible influence on the supply chain's gross profit, with changes of only $+0.0002\%$ and -0.0002% at the extreme points.

5. The setup cost of the manufacturer S_1 for "standard product" and S_2 for "customized product" reflects only small effects of $\pm 0.0011\%$ and $\pm 0.0028\%$ on the profit respectively.
The "supply chain model" planner should consider the "core products'" production cost as it is a critical factor while making the decisions.

Tab. 5.17: Influence of change on key parameters

Parameters	Variation (in percentage)	Change in profit	Parameters	Variation (in percentage)	Change in profit
C_p	−5%	−5.3900	S_1	−5%	+0.0011
	−2.5%	−2.6900		−2.5%	+ 0.0005
	+2.5%	+2.6900		+2.5%	− 0.0005
	+5%	+5.3900		+5%	−0.0011
C_1	−5%	−0.0757	S_2	−5%	+0.0028
	−2.5%	− 0.0379		−2.5%	+0.0014
	+2.5%	+0.0379		+2.5%	− 0.0014
	+5%	+0.0757		+5%	−0.0028
C_2	−5%	−0.0909	C_b	−5%	+0.3308
	−2.5%	−0.0454		−2.5%	+0.1654
	+2.5%	+0.0454		+2.5%	−0.1654
	+5%	+0.0909		+5%	−0.3308
C_3	−5%	−0.0545	h	−5%	+0.0028
	−2.5%	−0.0273		−2.5%	+0.0014
	+2.5%	+0.0273		+2.5%	−0.0014
	+5%	+0.0545		+5%	−0.0028
A_r	−5%	+0.0002	C_{pen}	−5%	+0.0479
	−2.5%	+0.0001		−2.5%	+0.0239
	+2.5%	−0.0001		+2.5%	−0.0239
	+5%	−0.0002		+5%	−0.0479
π	−5%	+0.0142	C_{ec}	−5%	+0.0009
	−2.5%	+0.0071		−2.5%	+0.0005
	+2.5%	−0.0071		+2.5%	−0.0005
	+5%	−0.0142		+5%	−0.0009

5.10 Managerial Implications

From a managerial perspective, this chapter explores sustainable supply chain management by embracing "dual-channel" having a single manufacturer and retailer in an uncertain environment. For realistic conclusions, more than one decision variable is examined such as the total cost of "single-channel & dual-channel" and selling prices of the products. Additionally, a "triangular fuzzy number" is incorporated for satisfying the demand of the customers. Upcoming enlists the managerial implications of this chapter.

• This is a sustainable dual-supply chain model where the profit is maximized while considering the carbon emission during transportation.
• A penalties are imposed on the firms in case they crosses the carbon emission limit set by government. Thus, this chapter's model helps the firm in curbing the carbon emission in the limit.
• A sharing of information between manufacturer and retailer is carried out. Henceforth, a "centralized supply chain model with dual channel" increases firm's profit.
• To increase the profit, online channel where "personalized product" along with offline channel where "standard product" is incorporated in the chapter. Therefore, increasing the production, variety of products and number of customers.
• The proposed model considered demand as a "triangular fuzzy number" for dealing with the uncertain conditions of the environment.
• Although, the firm offers "customized products" along with the core product but cost of the product enhanced on demand of customers should not overshoot the presumed value, else customers would shift back to the "core product". Henceforth, uneven shifting of customers between the channels and a preassigned limit are assumed.

5.11 Conclusions

Recent trends in the market exemplify that the customers tend to prefer "customized products". In a customization strategy, customers can upgrade the "standard product" according to their options. This scenario may influence increased production, hence generating more carbon emissions. Empirical data collected from an industry located in Punjab, India reflects that the proposed model reduced carbon emission by almost 50% approximately. Profit analysis based on price sensitivity and carbon emission is examined. The results demonstrate that the selling price is inversely proportional to profit. Thus, when the core product's cost rises, initially profit of the firm increase but later on it drops. Whereas, in the case of "customized products", rise in the cost increases the customer's shift to the single channel. Approximately 40% increase in shifting of customers is observed among the channels when the variation in the cost of "customized and standard products" is more than the presumed value. Secondly, it reflects that production is directly proportional to carbon emission and penalties. Henceforth, it depicts that higher environmental cost parameters have a high adverse impact on the profit. Thus, if they are not maintained well, then these costs may make the profit of the supply chain critically low. This model can influence remarkably on various industries and the environment by considering variable demand and carbon emissions. Further, there are two competitions are demonstrated in the model, in the first one, there are approximately 80% of shifting of customers in firm 2 having divergence in the selling price more than threshold limit, comparing to firm 1 is reflected and in second, retailer 2 which is having "dual-channel supply chain policy" earns approximately 4% more profit in comparison to the retailer, having an only single channel. A case study is incorporated for validation of the developed model. The data is collected from the firm and consequently, it is normalized by utilizing statistical tools. Moreover, the results depict that there is approximately a 68% decrease in carbon emission from the firm by using the proposed model. Thus, with the implementation of the proposed model, the firms achieve their environmental goal.

This model can be extended with the application of strategies for diminishing ordering and setup costs and consignment policy under vendor-managed inventory and multiple retailers. Therefore, a supply chain system having dual mode of shopping with multiple retailers can be considered (Majumder et al. , 2018). Further, fair competition between the retailers can also be incorporated into the model (Sarkar et al. , 2018a). In literature, many researchers examined vendor-managed inventory and consignment contracts (Batarfi et al. , 2016). The concept of a variable demand driven by cost of selling and advertising, where all basic costs are considered fuzzy can also be incorporated (Sarkar et al. , 2019).

CHAPTER - 6

A sustainable dual-channel supply chain management under uncertain conditions

6.1 Problem definition

This chapter focus on enhancing the concept of "supply chain management" by incorporating the "environmental and social sustainability". Moreover, there are uncertainties associated with the basic costs and demand of supply chain, which are represented using a "triangular fuzzy number". Additionally, an e-commerce with the retailer is quantified in the model in catering customers. The model deals with the "personalized and standard products" demand through an "online and offline" platform, respectively. A "threshold limit" is introduced, which keeps checking on the variation in the cost of the "personalized and core product" otherwise, swapping of customers begin. The result finds, that there is approximately a 10% increase in the shifting of customers if the variation in the price of the "core and personalized product" overshoot the "threshold limit". Additionally, environmental and social costs are in linear relation with the profit thus, the cost parameters should be maintained well, otherwise, the supply chain profit will turn critically low.

6.2 Presumptions

For framing the mathematical model, following points are presumed.

1. A "centralized supply chain model having dual-channel and a personalized policy" is considered. Maneuvering of customers is carried out by offline channels along with an online channel (Chauhan et al. , 2021).

2. In a firm, both "standard and personalized products" are manufactured since it has a manufacturing house.

3. Consumers prefer e-commerce over offline mode for purchasing the products of their choice.

4. A preassigned limit ("Threshold limit") is considered which limits the variation in the cost of the online and offline channel, thereby there is no customer shifting.

5. The exact value of $E(X-r)^+$ cannot be evaluated because of the lack of information on lead-time demand X distribution. Consequently, the "max-min distribution-free approach" is taken into account for solving this problem (Chauhan et al. , 2021), given as:

$$E(M-R)^+ = E((\widetilde{D_1}L+X)-R)^+ \tag{6.124}$$

$$\leq \left[\frac{\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}Lk\sigma\sqrt{L}}+(\widetilde{D_1}L-k\sigma\sqrt{L})}{2}\right] \tag{6.125}$$

Where, $R=\widetilde{D_1}L+k\sigma\sqrt{L}$ is reorder point, $\widetilde{D_1}L$ is the lead time demand, $k\sigma\sqrt{L}$ is safety stock, and k is a safety factor.

6.3 Mathematical model

The underneath segment extends the profit function derived from Chauhan et al. (2021), a fuzzification technique for evaluating solutions that are optimal for the model, and a solution algorithm for this model.

Fuzzification

For more realistic solutions a non-negative "triangular fuzzy number" $\widetilde{x}=(x-\gamma_1,x,x+\gamma_2)$ is considered. The shopkeeper's and manufacturer's demand and cost parameters can be evaluated by replacing the non-negative "triangular fuzzy number" in the aforementioned equations.

$$\widetilde{x}=x+\left(\frac{\gamma_2-\gamma_1}{4}\right) \tag{6.126}$$

6.3.1 Net profit functions

Present segment enlists the profit of manufacturer and retailer by selling "standard and make-to-order items".

I. **"Profit equation of manufacturer for core product"**

Profit earned by the manufacturer by selling one unit of the "standard product" through offline mode is:

$$\nabla_1 = \text{Revenue} - \text{Setup cost} - \text{Holding cost} - \text{Manufacturer's production cost} \tag{6.127}$$

$$\nabla_1 = C_p(1+m)\widetilde{D_1} - \frac{S_1\widetilde{D_1}}{nQ_1} - \frac{r_vC_p\widetilde{D_1}}{2}\left[n\left(1-\frac{\widetilde{D_1}}{P_1}\right)-1+\frac{2\widetilde{D_1}}{P_1}\right] - C_{vr}\widetilde{D_1} \tag{6.128}$$

Defuzzification

The fuzzy numbers are replaced with crisp values to obtain concluding solutions. The "signed distance" method is used in the present model for converting fuzzy outputs to crisp models. Hence, the profit for manufacturer is given by:

$$\nabla_1 = (1+m)\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right) - \frac{1}{nQ_1}\left(S_1+\frac{\gamma_{2S_1}-\gamma_{1S_1}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right) - \frac{r_vQ_1}{2}$$
$$\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right] - \left(C_{vr}+\frac{\gamma_{2C_{vr}}-\gamma_{1C_{vr}}}{4}\right)$$
$$\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right) \tag{6.129}$$

whereas, $\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)(1+m)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$ represents revenue of the manufacturer and $\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)(1+m)$ is the core product cost coordinated by firm. The setup cost of the manufacturer for the core product is $\frac{1}{nQ_1}\left(S_1+\frac{\gamma_{2S_1}-\gamma_{1S_1}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$. Production of the manufacturer integer multiple of retailer's order quantity. Moreover, $\frac{r_vQ_1}{2}\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right]$ is the holding cost and $\left(C_{vr}+\frac{\gamma_{2C_{vr}}-\gamma_{1C_{vr}}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$ is the core products cost of production.

II. **"Profit equation of manufacturer for customized product"**

Profit earned by the manufacturer by selling one unit of the "customized product" through offline mode is:

$$\nabla_2 = \text{Revenue} - \text{Setup cost} - \text{Holding cost} - \text{Manufacturing cost} \tag{6.130}$$

$$= \sum_{i=1}^{N} \widetilde{C_i}(1+m)\phi_i\widetilde{D_2} - \frac{S_2}{Q_2}\widetilde{D_2} - \frac{h_1 Q_2}{2}\left(1-\frac{\widetilde{D_2}}{P_2}\right) - \sum_{i=1}^{N} C_i\phi_i\widetilde{D_2} \tag{6.131}$$

Again, replacing the fuzzy numbers with crisp values to obtain concluding solutions. Thus, the equation becomes:

$$\nabla_2 = \sum_{i=1}^{N}\left(C_i + \frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)(1+m)\phi_i\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right) - \frac{1}{Q_2}\left(S_2+\frac{\gamma_{2S_2}-\gamma_{1S_2}}{4}\right)$$

$$\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right) - \frac{Q_2}{2}\left(h_1+\frac{\gamma_{2h_1}-\gamma_{1h_1}}{4}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)\right) -$$

$$\sum_{i=1}^{N}\left(C_i + \frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)\phi_i\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right) \tag{6.132}$$

whereas, the revenue of the manufacturer on "personalized products" is $\sum_{i=1}^{N}\left(C_i+\frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}(1+m)\phi_i\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right.\right.$ and the cost of the "personalized product" is $\sum_{i=1}^{N}\left(C_i+\frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)(1+m)\phi_i$. The setup cost of the manufacturer for "personalized products" by following a "make-to-order policy" is $\frac{1}{Q_2}\left(S_2+\frac{\gamma_{2S_2}-\gamma_{1S_2}}{4}\right)\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)$. Moreover, $\frac{h_1Q_2}{2}\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)\right)$ and $\sum_{i=1}^{N}\left(C_i+\frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)\phi_i\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)$ is holding cost and production cost for "personalized product".

III. **"For core product, retailer's profit equation"**

By selling per unit of the "core product" profit earned by the retailer is

$$\nabla_3 = \text{Revenue} - \text{Cost of ordering} - \text{Cost of holding} - \text{shortage cost} - \text{Cost of lead time crashing} \tag{6.133}$$

$$\nabla_3 = \widetilde{C_p}(1+m)^2\widetilde{D_1} - \left[\frac{A_r}{Q_1}\widetilde{D_1} + r_b\widetilde{C_b}\left(\frac{Q_1}{2}+R-\widetilde{D_1}L\right) + \frac{\pi\widetilde{D_1}}{Q_1}E(M-R)^+ + \frac{\widetilde{D_1}CL}{Q_1}\right] \tag{6.134}$$

Replacing the fuzzy numbers with crisp values the equation becomes:

$$\nabla_3 = \left(C_p + \frac{\gamma_{2C_p} - \gamma_{1C_p}}{4}\right)(1+m)^2\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - \frac{1}{Q_1}\left(A_r + \frac{\gamma_{2A_r} - \gamma_{1A_r}}{4}\right)\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - r_b$$

$$\left(C_b + \frac{\gamma_{2C_b} - \gamma_{1C_b}}{4}\right)\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) - \frac{CL}{Q_1}\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - \frac{\pi}{2Q_1}\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)\left[\left[\sigma^2 L +\right.\right.$$

$$\left.\left(\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)L\right)^2 + k^2\sigma^2 L - 2\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)Lk\sigma\sqrt{L}\right]^{\frac{1}{2}} + \left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)L - k\sigma\sqrt{L}\Bigg]$$

whereas, revenue of the shopkeeper is $\left(C_p + \frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)(1+m)^2\left(D_1 + \frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$ and the cost of the "core product" is $\left(C_p + \frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)(1+m)^2$. Further, $\frac{1}{Q_1}\left(A_r + \frac{\gamma_{2A_r}-\gamma_{1A_r}}{4}\right)\left(D_1 + \frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$ is the ordering cost and $r_b\left(C_b + \frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right)$ is the holding cost for the retailer. Lastly, $\frac{CL}{Q_1}\left(D_1 + \frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$ is the lead time crashing cost.

Thus, expected net profit ∇_S of the "centralized supply chain" in case of "single channel" is obtained by summing ∇_1 and ∇_3.

$$\nabla_S = \nabla_1 + \nabla_3 \tag{6.135}$$

$$\nabla_S = \left(C_p + \frac{\gamma_{2C_p} - \gamma_{1C_p}}{4}\right)(1+m)\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - \frac{1}{nQ_1}\left(S_1 + \frac{\gamma_{2S_1} - \gamma_{1S_1}}{4}\right)\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - \frac{r_v Q_1}{2}$$

$$\left(C_p + \frac{\gamma_{2C_p} - \gamma_{1C_p}}{4}\right)\left[n\left(1 - \frac{1}{P_1}\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)\right) - 1 + \frac{2}{P_1}\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)\right] - \left(C_{vr} + \frac{\gamma_{2C_{vr}} - \gamma_{1C_{vr}}}{4}\right)$$

$$\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) + (1+m)^2\left(C_p + \frac{\gamma_{2C_p} - \gamma_{1C_p}}{4}\right)\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - \frac{1}{Q_1}\left(A_r + \frac{\gamma_{2A_r} - \gamma_{1A_r}}{4}\right)$$

$$\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - r_b\left(C_b + \frac{\gamma_{2C_b} - \gamma_{1C_b}}{4}\right)\left(\frac{Q_1}{2} + k\sigma\sqrt{L}\right) - \frac{CL}{Q_1}\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right) - \frac{\pi}{2Q_1}$$

$$\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)\left[\left[\sigma^2 L + \left(\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)L\right)^2 + k^2\sigma^2 L - 2\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)Lk\sigma\sqrt{L}\right]^{\frac{1}{2}} +\right.$$

$$\left.\left(D_1 + \frac{\gamma_{2D_1} - \gamma_{1D_1}}{4}\right)L - k\sqrt{L}\sigma\right] \tag{6.136}$$

And expected net profit ∇_D of the "centralized supply chain in case of dual channel" is obtained by summing ∇_1, ∇_2, and ∇_3.

$$\nabla_D = \nabla_1 + \nabla_2 + \nabla_3 \tag{6.137}$$

$$\nabla_D=\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)(1+m)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{1}{nQ_1}\left(S_1+\frac{\gamma_{2S_1}-\gamma_{1S_1}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{r_vQ_1}{2}$$
$$\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right]-\left(C_{vr}+\frac{\gamma_{2C_{vr}}-\gamma_{1C_{vr}}}{4}\right)$$
$$\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)+(1+m)^2\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{1}{Q_1}\left(A_r+\frac{\gamma_{2A_r}-\gamma_{1A_r}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$$
$$-r_b\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)-\frac{CL}{Q_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{\pi}{2Q_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\left[\left[\sigma^2L+\right.\right.$$
$$\left.\left(\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)Lk\sigma\sqrt{L}\right]^{\frac{1}{2}}+\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L-k\sqrt{L}\sigma\Big]+$$
$$\sum_{i=1}^{N}(1+m)\phi_i\left(C_i+\frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)-\frac{1}{Q_2}\left(S_2+\frac{\gamma_{2S_2}-\gamma_{1S_2}}{4}\right)\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)-$$
$$\frac{Q_2}{2}\left(h_1+\frac{\gamma_{2h_1}-\gamma_{1h_1}}{4}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)\right)-\sum_{i=1}^{N}\left(C_i+\frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)\phi_i\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right) \tag{6.138}$$

6.3.2 Supply chain optimal solutions

Since, equations of profit are non-linear in nature thus, for a fixed constant 'm'', with respect to Q_2, Q_1, and k the partial derivative of the profit are evaluated to obtain the optimal solution and Q_2, Q_1, and k by putting equations equal to zero which is given as

$$Q_2^*=\sqrt{\frac{\left(S_2+\frac{\gamma_{2S_2}-\gamma_{1S_2}}{4}\right)\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)}{\frac{1}{2}\left(h_1+\frac{\gamma_{2h_1}-\gamma_{1h_1}}{4}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)\right)}} \tag{6.139}$$

$$Q_1^*=\left(\left(-\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)CL+\left(S_1+\frac{\gamma_{2S_1}-\gamma_{1S_1}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)+\left(A_r+\frac{\gamma_{2A_r}-\gamma_{1A_r}}{4}\right)\right.\right.$$
$$\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)+\frac{\pi}{2}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\left(\sigma^2L+\left(\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L\right)^2+k^2\sigma^2L-\right.$$
$$\left.2\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)Lk\sigma\sqrt{L}\right)^{\frac{1}{2}}+\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L-k\sigma\sqrt{L}\Big)\left(\frac{r_b}{2}\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)+\right.$$

$$\frac{r_v}{2}\left(n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)\Bigg)^{(-1)}\Bigg)^{\frac{1}{2}} \tag{6.140}$$

$$k^*=\Bigg(\Bigg(-\sigma\left(Q_1r_b\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)\left(-Q_1r_b\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)+\pi\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)\right)^{\frac{1}{2}}\Bigg)$$

$$\frac{1}{-Q_1r_b\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)}\Bigg)+L\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)+\Bigg(Q_1\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)r_b\Bigg(-Q_1\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)r_b$$

$$+\pi\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\Bigg)L\Bigg)^{\frac{1}{2}}\Bigg(2Q_1\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)r_b\left(\pi\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-Q_1\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)r_b\right)\Bigg)^{(-1)}$$

$$\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\pi\sigma\Bigg)\left(\frac{1}{\sigma\sqrt{L}}\right) \tag{6.141}$$

The Q_1^*, Q_2^*, and k^*, optimal solutions are dependent on each other. Moreover, obtaining a closed-form of expression is difficult for a centralized profit function. Therefore, the numerical procedure needs to be used for finding these optimal values. Thus, for finding managerial decisions along with the underneath algorithm an iteration method is utilized .

The emission tax paid by the manufacturer because of CO_2 emission during production of "core product" (Khan et al. , 2016), is given by

$$EC_S=n\widetilde{E_{mc}}Q_1=n\left(E_{mc}+\frac{\gamma_{2E_{mc}}-\gamma_{1E_{mc}}}{4}\right)Q_1 \tag{6.142}$$

Additionally, aggregate of labor salary, health care, and safety i.e., social cost paid by manufacturer for "single channel" (Khan et al. , 2016), is given by

$$SC_S=n\widetilde{S_{mc}}Q_1=n\left(S_{mc}+\frac{\gamma_{2S_{mc}}-\gamma_{1S_{mc}}}{4}\right)Q_1 \tag{6.143}$$

Thus, the "supply chain with single channel's" aggregate profit of the is given by

$$\nabla_S=\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)(1+m)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{1}{nQ_1}\left(S_1+\frac{\gamma_{2S_1}-\gamma_{1S_1}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-$$

$$\frac{r_vQ_1}{2}\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\Bigg[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)-\left(C_{vr}+\frac{\gamma_{2C_{vr}}-\gamma_{1C_{vr}}}{4}\right)$$

$$\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)+(1+m)^2\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{1}{Q_1}\left(A_r+\frac{\gamma_{2A_r}-\gamma_{1A_r}}{4}\right)$$

$$\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-r_b\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)-\frac{CL}{Q_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{\pi}{2Q_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$$

$$\left(\left(\sigma^2L+\left(\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)Lk\sigma\sqrt{L}\right)^{\frac{1}{2}}+\left(\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L-\right.\right.$$

$$\left.\left.k\sigma\sqrt{L}\right)\right)-n\left(E_{mc}+\frac{\gamma_{2E_{mc}}-\gamma_{1E_{mc}}}{4}\right)Q_1-n\left(S_{mc}+\frac{\gamma_{2S_{mc}}-\gamma_{1S_{mc}}}{4}\right)Q_1 \tag{6.144}$$

Consequently, the emission tax paid by the manufacturer in case of "dual channel" is given by

$$EC_D=n(\widetilde{E_{mc}}Q_1+\widetilde{E_{mp}}Q_2)$$

$$=n\left(\left(E_{mc}+\frac{\gamma_{2E_{mc}}-\gamma_{1E_{mc}}}{4}\right)Q_1+\left(E_{mp}+\frac{\gamma_{2E_{mp}}-\gamma_{1E_{mp}}}{4}\right)Q_2\right) \tag{6.145}$$

Additionally, aggregate of labor salary, health care, and safety i.e., social cost paid by manufacturer for "dual channel" (Khan et al. , 2016), is given by

$$SC_D=n(\widetilde{S_{mc}}Q_1+\widetilde{S_{mp}}Q_2)$$

$$=n\left(\left(S_{mc}+\frac{\gamma_{2S_{mc}}-\gamma_{1S_{mc}}}{4}\right)Q_1+\left(S_{mp}+\frac{\gamma_{2S_{mp}}-\gamma_{1S_{mp}}}{4}\right)Q_2\right) \tag{6.146}$$

Thus, the "supply chain with dual channel's" aggregate profit of the is given by

$$\nabla_D=\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)(1+m)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{1}{nQ_1}\left(S_1+\frac{\gamma_{2S_1}-\gamma_{1S_1}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-$$

$$\frac{r_vQ_1}{2}\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-1+\frac{2}{P_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)\right)-\left(C_{vr}+\frac{\gamma_{2C_{vr}}-\gamma_{1C_{vr}}}{4}\right)\right.$$

$$\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)+(1+m)^2\left(C_p+\frac{\gamma_{2C_p}-\gamma_{1C_p}}{4}\right)\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{1}{Q_1}\left(A_r+\frac{\gamma_{2A_r}-\gamma_{1A_r}}{4}\right)\left(D_1+\right.$$

$$\left.\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-r_b\left(C_b+\frac{\gamma_{2C_b}-\gamma_{1C_b}}{4}\right)\left(\frac{Q_1}{2}+k\sigma\sqrt{L}\right)-\frac{CL}{Q_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)-\frac{\pi}{2Q_1}\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)$$

$$\left(\left(\sigma^2L+\left(\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)Lk\sigma\sqrt{L}\right)^{\frac{1}{2}}+\left(\left(D_1+\frac{\gamma_{2D_1}-\gamma_{1D_1}}{4}\right)L-\right.\right.$$

$$k\sigma\sqrt{L}\Big)\Big)+\sum_{i=1}^{N}(1+m)\phi_i\left(C_i+\frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)-\frac{1}{Q_2}\left(S_2+\frac{\gamma_{2S_2}-\gamma_{1S_2}}{4}\right)\Big(D_2+$$

$$\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\Big)-\frac{Q_2}{2}\left(h_1+\frac{\gamma_{2h_1}-\gamma_{1h_1}}{4}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\right)\right)-\sum_{i=1}^{N}\phi_i\left(C_i+\frac{\gamma_{2C_i}-\gamma_{1C_i}}{4}\right)\Big(D_2+$$

$$\frac{\gamma_{2D_2}-\gamma_{1D_2}}{4}\Big)-n\left(\left(E_{mc}+\frac{\gamma_{2E_{mc}}-\gamma_{1E_{mc}}}{4}\right)Q_1+\left(E_{mp}+\frac{\gamma_{2E_{mp}}-\gamma_{1E_{mp}}}{4}\right)Q_2\right)-n\Big(\left(S_{mc}+\frac{\gamma_{2S_{mc}}-\gamma_{1S_{mc}}}{4}\right)Q_1$$

$$+\left(S_{mp}+\frac{\gamma_{2S_{mp}}-\gamma_{1S_{mp}}}{4}\right)Q_2\Big) \tag{6.147}$$

6.4 Algorithm for obtaining solution

For solving the current model, underneath algorithm is used.

Step 1 Allot all parameters with the values defined in numerical analysis section.

Step 2 Putt n=1.

Step 3 Execute the underneath steps for all the values of $L_i, i = 1, 2, \ldots$.

- Step 3a Derive the value of Q_2 from equation 6.139.
- Step 3b Derive the value of Q_1 from equation 6.140.
- Step 3c Derive the value of k from equation 6.141
- Step 3d Redo the Steps 3a to 3c until there is specific accuracy level with no change in the values of Q_2, Q_1, and k.

Step 4 Evaluate the value of EC_S, SC_S, EC_D, & SC_D from equations 6.142, 6.143, 6.145, and 6.146.

Step 5 Evaluate the value of ∇_S and ∇_D using the equations 6.144 and 6.147.

Step 6 Putt $n = n+1$ and redo from step 3 to step 5.

Step 7 If $\nabla_D(n+1) < \nabla_D(n)$ then redo steps from 2 to 6 or else stop.

Proposition 6.1. *For the fixed value of n and* $L \in [L_i, L_(i-1)]$, *if* $Q_1^*, Q_2^*, andk^*$ *are the optimal values of* Q_1, Q_2, *and k, the profit function of "dual-channel"* ∇_D *obtains its global maximum at* $Q_1^*, Q_2^*, andk^*$ *under the condition*

$$X + Y > Z \tag{6.148}$$

Proof: For "dual-channel supply chain", the Hessian matrix H_1 is

$$H_1 = \begin{bmatrix} \frac{\partial^2 \nabla_D}{\partial Q_1^2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} \\ \frac{\partial^2 \nabla_D}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial Q_2^2} & \frac{\partial^2 \nabla_D}{\partial Q_2 \partial k} \\ \frac{\partial^2 \nabla_D}{\partial k \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial k \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial k^2} \end{bmatrix}$$

where,

$$\frac{\partial^2 \nabla_D}{\partial k^2} = -\frac{\pi \widetilde{D_1}}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - \widetilde{D_1}L\sigma\sqrt{L}}}\right]$$

$$\frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} = \frac{\pi \widetilde{D_1}}{2Q_1^2}\left[\frac{k\sigma^2 L - \widetilde{D_1}L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2 \nabla_D}{\partial k \partial Q_2} = 0 = \frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2}$$

$$\frac{\partial^2 \nabla_D}{\partial Q_2^2} = -\frac{2\widetilde{S_2}\widetilde{D_1}}{Q_2^3}$$

$$\frac{\partial^2 \nabla_D}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[\widetilde{A_r}\widetilde{D_1} + \frac{\widetilde{S_1}\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right]$$

The principal minor of $|(H_1)_{1,1}|_{(Q_1^*,Q_2^*,k^*)}$ of order 1×1 is

$$|(H_1)_{1,1}|_{(Q_1^*,Q_2^*,k^*)} = \left|\frac{\partial^2 \nabla_D}{\partial Q_1^2}\right|_{(Q_1^*,Q_2^*,k^*)}$$

$$= -\frac{2}{Q_1^3}\left[\widetilde{A_r}\widetilde{D_1} + \frac{\widetilde{S_1}\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right] < 0$$

The principal minor of $|(H_1)_{2,2}|_{(Q_1^*,Q_2^*,k^*)}$ of order 2×2 is

$$|(H_1)_{2,2}|_{(Q_1^*,Q_2^*,k^*)} = \left(\frac{\partial^2 \nabla_D}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_D}{\partial Q_2^2}\right)$$

$$= \left(-\frac{2}{Q_1^3}\left[\widetilde{A_r}\widetilde{D_1} + \frac{\widetilde{S_1}\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right]\right)\left(-\frac{2\widetilde{S_2}\widetilde{D_1}}{Q_2^3}\right) > 0$$

The principal minor of $|(H_1)_{3,3}|_{(Q_1^*,Q_2^*,k^*)}$ of order 3×3 is

$$|(H_1)_{3,3}|_{(Q_1^*,Q_2^*,k^*)} = \left(\frac{\partial^2 \nabla_D}{\partial Q_2^2}\right)\left[\left(\frac{\partial^2 \nabla_D}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_D}{\partial k^2}\right) - \left(\frac{\partial^2 \nabla_D}{\partial k \partial Q_1}\right)^2\right] > 0$$

$$\left(\widetilde{A_r}+\frac{\widetilde{S_1}}{n}+CL\right)\frac{\sigma^4L^2}{\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\pi\sigma^4L^2+$$

$$\frac{\pi k\sigma^3L\sqrt{L}}{2\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\frac{\widetilde{D_1}\sigma^3L^2\sqrt{L}}{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}>$$

$$\frac{\widetilde{D_1}L^2\sigma^2}{2\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2\widetilde{D_1^2}\sigma^2L^3)}{4(\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L})}$$

$$X+Y>Z$$

Where,

$$X=\left(\widetilde{A_r}+\frac{\widetilde{S_1}}{n}+CL\right)\frac{\sigma^4L^2}{\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\pi\sigma^4L^2,$$

$$Y=\frac{\pi k\sigma^3L\sqrt{L}}{2\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\frac{\widetilde{D_1}\sigma^3L^2\sqrt{L}}{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}},$$

and

$$Z=\frac{\widetilde{D_1}L^2\sigma^2}{2\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L}}}+\frac{\pi(2k^2\sigma^4L^2+\sigma^4L^2+2\widetilde{D_1^2}\sigma^2L^3)}{4(\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}L\sigma\sqrt{L})}$$

Since, the Hessian matrix's, all the principal minors are not positive. Hence, the Hessian matrix H_1 is negative definite at (Q_1^*,Q_2^*,k^*). Thus, aggregate expected profit for "dual-channel" gets the global maximum at (Q_1^*,Q_2^*,k^*).

Proposition 6.2. *For the fixed value of n and* $L\in[L_i,L_(i-1)]$*, if* Q_1^**, and* k^* *are the optimal values of* Q_1 *and k, the profit function of "single-channel"* ∇_S *obtains its maximum value at* Q_1^* *and* k^* *under the following context*

$$X_1+Y_1>Z_1 \tag{6.149}$$

Proof: For "single-channel supply chain", the Hessian matrix H is

$$H_2=\begin{bmatrix}\frac{\partial^2\nabla_D}{\partial Q_1^2} & \frac{\partial^2\nabla_S}{\partial Q_1\partial k}\\ \frac{\partial^2\nabla_S}{\partial k\partial Q_1} & \frac{\partial^2\nabla_S}{\partial k^2}\end{bmatrix}$$

where,

$$\frac{\partial^2 \nabla_S}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[\widetilde{A_r}\widetilde{D_1} + \frac{\widetilde{S_1}\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right]$$

$$\frac{\partial^2 \nabla_S}{\partial Q_1 \partial k} = \frac{\pi\widetilde{D_1}}{2Q_1^2}\left[\frac{k\sigma^2 L - \widetilde{D_1}L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$\frac{\partial^2 \nabla_S}{\partial k^2} = -\frac{\pi\widetilde{D_1}}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - \widetilde{D_1}L\sigma\sqrt{L}}}\right]$$

The principal minor $|(H_2)_{1,1}|_{(Q_1^*,k^*)}$ of order 1×1 is

$$|(H_2)_{1,1}|_{(Q_1^*,k^*)} = \left|\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right|_{(Q_1^*,k^*)}$$

$$\frac{\partial^2 \nabla_S}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[\widetilde{A_r}\widetilde{D_1} + \frac{\widetilde{S_1}\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right]$$

The principal minor $|(H_2)_{2,2}|_{(Q_1^*,k^*)}$ of order 2×2 is

$$|H_{2,2}|_{(Q_1^*,k^*)} = \left(\frac{\partial^2 \nabla_S}{\partial Q_1^2}\right)\left(\frac{\partial^2 \nabla_S}{\partial k^2}\right) - \left(\frac{\partial^2 \nabla_S}{\partial k \partial Q_1}\right)^2 > 0$$

$$\left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \pi\sigma^4 L^2 +$$

$$\frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{D_1\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}$$

$$> \frac{D_1 L^2\sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3\sigma^2)}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

$$X_1 + Y_1 > Z_1$$

where,

$$X_1 = \left(A_r + \frac{S_1}{n} + CL\right)\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \pi\sigma^4 L^2,$$

$$Y_1 = \frac{\pi k\sigma^3 L\sqrt{L}}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{D_1\sigma^3 L^2\sqrt{L}}{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}},$$

and

$$Z_1 = \frac{D_1 L^2 \sigma^2}{2\sqrt{\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L}}} + \frac{\pi(2k^2\sigma^4 L^2 + \sigma^4 L^2 + 2D_1^2 L^3 \sigma^2)}{4(\sigma^2 L + (D_1 L)^2 + k^2\sigma^2 L - 2D_1 L\sigma\sqrt{L})}$$

Since, the Hessian matrix's, all the principal minors are not positive. Hence, the Hessian matrix H_2 is negative definite at (Q_1^*, k^*). Thus, aggregate expected profit for "single-channel" gets the global maximum at (Q_1^*, k^*).

6.5 Numerical experimentation and discussion

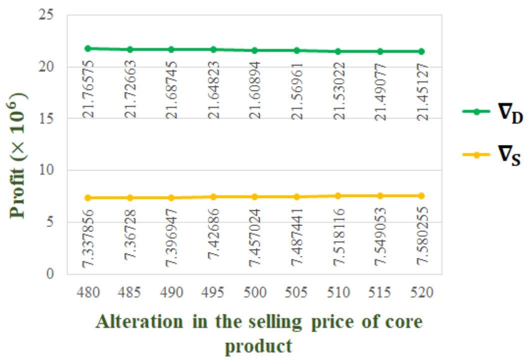
Following enlists the values of input parameters utilized in numerical analysis C_p =500 \$/unit, γ_{1C_p} =400, γ_{2C_p} =600, C_1 =250 \$/unit, γ_{1C_1} =200, γ_{2C_1} =300, C_2 =250 \$/unit, γ_{1C_2} =200, γ_{2C_2} =300, C_3 =100 \$/unit, γ_{1C_3} =50, γ_{2C_3} =150, S_1 =1000 \$/setup, γ_{1S_1} =900, γ_{2S_1} =1100, S_2 =1200 \$/setup, γ_{1S_2} =1100, γ_{2S_2} =1300, C_b =200 \$/unit, γ_{1C_b} =180, $\gamma_{1C_{vr}}$ =220, C_{vr} =100 \$/unit, γ_{2C_b} =80, $\gamma_{2C_{vr}}$ =120, A_2 =200 \$/unit, γ_{1A_r} =180, γ_{2A_r} =220, ϕ_1 =0.25 %, ϕ_2 =0.3 %, ϕ_3 =0.45 %, L =3 weeks, σ =220, m =0.7, π =150 \$/unit, E_{mp} =60 \$, $\gamma_{1E_{mp}}$ =50, $\gamma_{2E_{mp}}$ =70, E_{mc} =50 \$, $\gamma_{1E_{mc}}$ =40, $\gamma_{2E_{mc}}$ =60, S_{mp} =0.668 %, $\gamma_{1S_{mp}}$ =0.561, $\gamma_{2S_{mp}}$ =0.764, S_{mc} =0.426 %, $\gamma_{1S_{mc}}$ =0.382, $\gamma_{2S_{mc}}$ =0.443, r_b =0.2 \$/unit/unit time, h_1 =30 \$/unit/year, P_1 =2500 Unit/year, γ_{1h_1} =20, P_2 =2500 Unit/year, and γ_{2h_1} =40, r_v =0.2 \$/unit/unit time. Moreover, for resembling the real environment, values of parameters were taken from real world examples Bazan et al. (2015), Xu et al. (2017), and Chauhan et al. (2021).

6.5.1 Comparison of profit of single and dual-channel

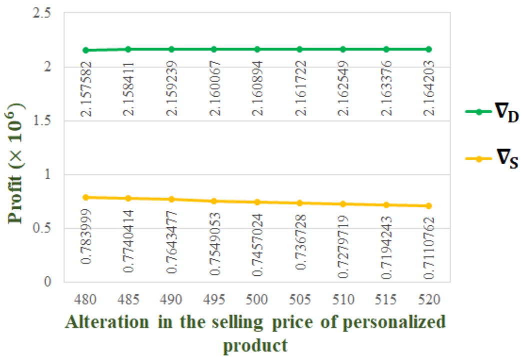
An example is considered to a comparison in the profit of "the supply chain management having dual-channel and single-channel". Additionally, core products are modified with three varieties of customization in case of a "supply chain model with dual-channel". Moreover, throughout the chapter, "threshold value" is preassigned to \$ 20. Thereby, the decision variable's optimal values are obtained by using the input parameters values, which are displayed in table 6.18. Consequently, table 6.18 displays the maximum profit of supply chain with "single-channel and dual-channel" i.e., \$ 5910850 and \$ 6148304, respectively. The maximum profit is obtained at the corresponding values of k, Q_1, Q_2, D_1, D_2 are 21.15291, 2177.773, 598.9205, 2328.269, and 1577.235 respectively. Thereby, reflecting that the profit of firm having "single-channel" is less than the "dual-channel".

Tab. 6.18: Decision variable's optimal values

n	1	**2**	3
k	21.15402	**21.15391**	21.15346
D_1	2328.269	**2328.269**	2328.269
D_2	1577.235	**1577.235**	1577.235
Q_1	2177.398	**2177.773**	2177.499
Q_2	598.9205	**598.9205**	598.9205
∇_S	5902484	**5910850**	5906356
∇_D	6140724	**6148304**	6144387



(a) With respect to the core product



(b) With respect to the personalized product

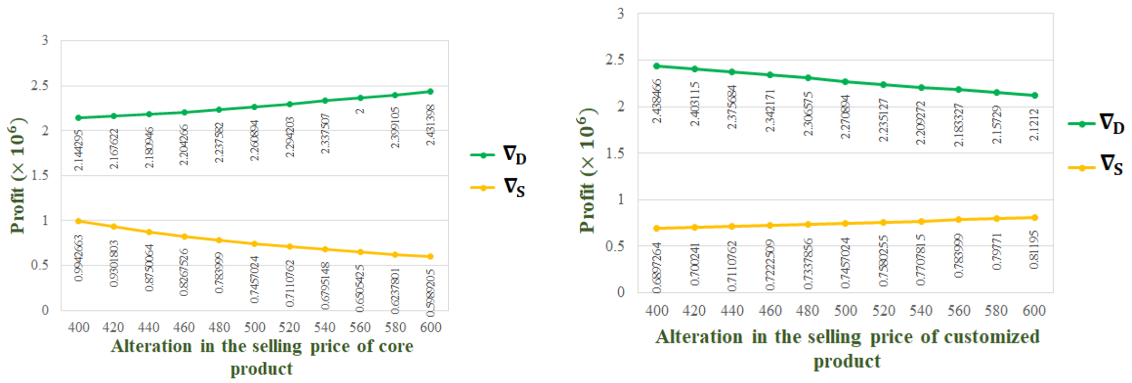
Fig. 6.59: Variation in the profit with regards to the varying selling price when $|C_p - \sum_{i=1}^{N} C_i| <$ Threshold limit

6.5.2 Influence of varying selling price on the profit

In the current section, two cases are taken into account.

Case I: When the variation in the "core and personalized product's" cost is less in comparison to the presumed limit i.e., $|C_p - \sum_{i=1}^{N} C_i| <$ Threshold limit. Leading to two sub-cases demonstrated by figures 6.59a and 6.59b, respectively. In the first case, the cost of the "core product" is varied and of the "personalized product" is kept fixed whereas the second case exhibits the variation in the selling price of the "personalized products" while that of the core products is kept constant. In either of the cases, the shifting of customers and the change in the profit for both channel is approximately 3.03%.

Case II: On the same line, when the variation in the core and "personalized product's" cost overshooting the value of presumed limit i.e., $|C_p - \sum_{i=1}^{N} C_i| >$ Threshold limit. Thereby, giving two



(a) With respect to the core product

(b) With respect to the customized product

Fig. 6.60: Variation in the profit with regards to the varying selling price when $|C_p - \sum_{i=1}^{N} C_i| >$ Threshold limit

scenarios reflected by figures 6.60a and 6.60b, respectively. In case of first scenario, the cost of the "core product" is diversified and the "personalized product" is kept fixed. Moreover, the second case exhibits the divergence in the cost of "personalized product" while that of "core product" is kept constant. Further, in both the scenarios there is approximately 13% of shifting of customers and change in the total profit of the firm.

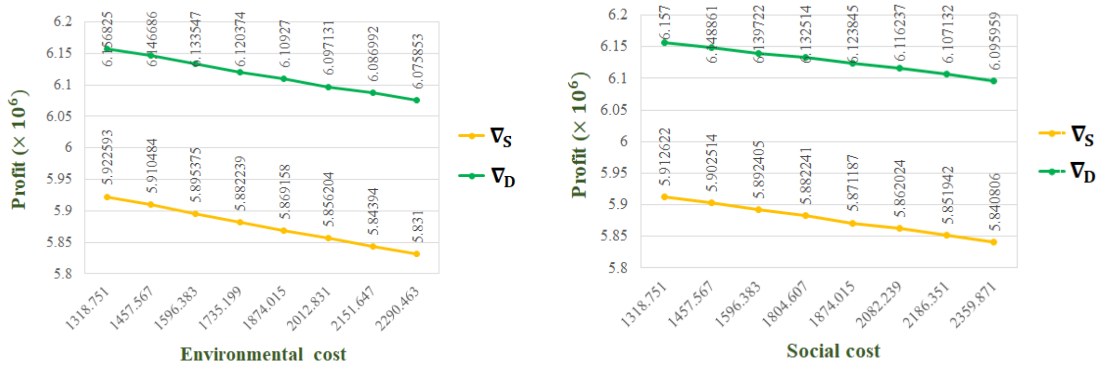
6.5.3 Influence of environmental and social cost on the profit

Equations 6.142, 6.143, 6.145, and 6.146 exhibits that environmental and social costs charged on the firm. Additionally, 6.144 and 6.147 reflects that the profit of "single-channel and dual-channel" are in linear relation to environmental and social cost (Khan et al. , 2016). Therefore, as the environmental and social cost increases the profit of the firm decreases which is demonstrated by figures 6.61a and 6.61b, respectively. Also, profitability and sustainability are often at odds with each other. Henceforth, these cost parameters should be maintained well, otherwise, the supply chain profit will turn critically low.

6.6 Sensitivity analysis

The influence of variation from $-5\%, -2.5\%, +2.5\%, and + 5\%$ of key parameters on the total profit is exemplified by table 6.19 and figure 6.62. Underneath points enlist the sensitivity analysis of the model.

• The selling price of the core product in the profit is most sensitive in the model in comparison to other



(a) With respect to environmental cost

(b) With respect to social cost

Fig. 6.61: Divergence in the profit

parameters.

• Among the manufacturer and shopkeeper, per unit cost paid for production is more effective for the shopkeeper.

• The third customization cost is slightly sensitive to the profit out of all the customization costs.

• Shopkeepers ordering cost is the least sensitive to the profit of the model.

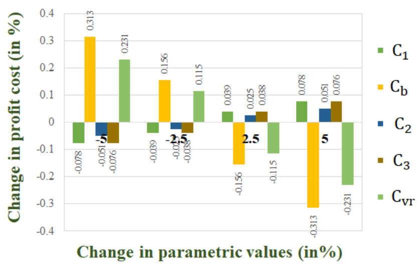


Fig. 6.62: Cost parameters sensitivity interpretation

6.7 "Managerial Implications"

The model in this chapter is an addition to Chauhan et al. (2021) as it incorporates "environmental and social costs" into a "supply chain model" having "dual-channel".

Tab. 6.19: Cost parameters sensitivity interpretation

Parameters	Variation (in %)	Change in profit	Parameters	Variation (in %)	Change in profit
C_p	−5	−5.36	C_3	−5	−0.076
	−2.5	−2.68		−2.5	−0.038
	+2.5	+2.68		+2.5	+0.038
	+5	+5.36		+5	+0.076
C_1	−5	−0.078	C_b	−5	+0.313
	−2.5	−0.039		−2.5	+0.156
	+2.5	+0.039		+2.5	−0.156
	+5	+0.078		+5	−0.313
C_2	−5	−0.051	C_{vr}	−5	+0.231
	−2.5	−0.025		−2.5	+0.115
	+2.5	+0.025		+2.5	−0.115
	+5	+0.051		+5	−0.231

• This is a "sustainable supply chain model" focused on maximizing profit by simultaneously investing in the social and environmental pillars.
• Pillars of sustainability will make the supply chain economically viable and bring the performance of the supply chain in line with regulatory, customer, and community expectations.
• A fuzzy set-based procedure is incorporated to deal with the uncertainties in the demand and cost parameters, which makes it practically applicable.
• To increase the profit and enlarge the circle of customers, both online channels and offline channels are incorporated into the supply chain.

6.8 Conclusions

A sustainable supply chain model under an uncertain environment is developed in this chapter since environmental and social parameters are an important part of the modern framework. In this model, a fuzzy set-based procedure is utilized, for dealing with uncertain parameters thereby accomplishing the objective of a supply chain of maximizing the profit and reducing its adverse impact on workers, communities, and the environment. Thus, this chapter will help managers make smarter decisions based on the specific environmental, health, and safety issues in their industry.
The numerical section proved that with the help of sustainability, the supply chain can focus on environmental-social pillars along with economic pillars. This model can be extended for future perspectives. More specific environmental and social issues can be incorporated by investing in the quality of the product. An investi-

gation on the impact of waste and recyclable products can also be worked on. For dealing with uncertain conditions, stochastic sense can be considered in the data. A similar case can be extended to the supply chain model of multiple retailers.

CHAPTER - 7

An industry application of reducing carbon footprint with economic sustainability

7.1 Problem definition

The environmental pillar of sustainability is one of the important parameters for analyzing the success of supply chain management. Customers are becoming environmentally conscious and manufacturers invest in green technologies, which influence their purchasing behaviour. Since manufacturing industries are the major contributors to the greenhouse gas emission due to large and over-consumption of non-renewable sources. Even though, they produce a huge amount of waste and harm gases in the environment. In the present chapter, a logarithmic investment is considered for reducing the carbon emission from different services - manufacturing and haulage of the final product of a glass manufacturing industry. Moreover, the supply chain of raw materials to manufacture the glass product and final product is considered under dual channel strategy. In an uncertain environment, the rate of demand is taken as a "triangular fuzzy number" in the present chapter's model. The principal concern of the chapter is to escalate the total yield with realistic factors and simultaneously diminishing carbon emissions from the services of the supply. For validating the proposed model numerical examination, comparison, and analysis of sensitivity are carried out. Results reflect that there is approximately a 73% decrease in the carbon emission and about a 15% increase in the profit with the investment in the reduction of CO_2 emission. Moreover, with the investment in setup cost, there is an increase in the profit of the firm in comparison to the literature. Additionally, as the gulf betwixt the "core and personalized products" selling price overshoot the "threshold limit" then there is approximately a 19% increase in the shifting of customers between the channels.

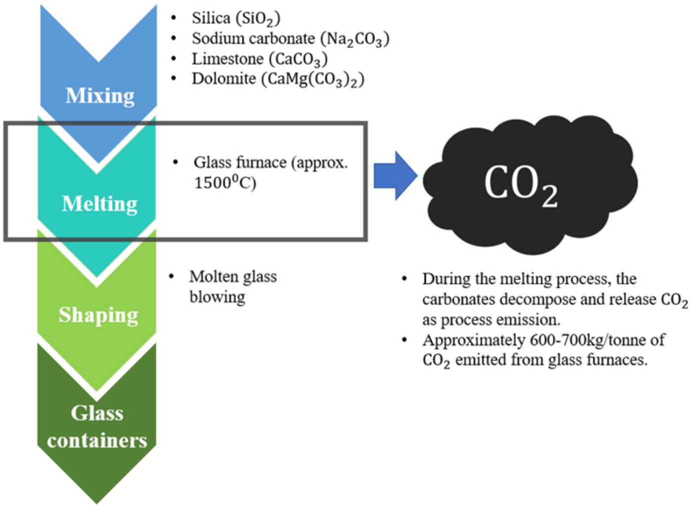


Fig. 7.63: Problem in glass manufacturing process

7.2 Assumptions

Succeeding module list the assumptions presumed for framing the mathematical model.

1. Products rate of demand is considered to be an uncertain parameter.

2. Manufacturing house of the firm manufactures both “standard and personalized products” since it has its own manufacturing house (Batarfi et al. , 2016).

3. In this model, lead time is incorporated, and its duration is controlled by including the crashing cost.

4. A “centralized dual-route supply chain model” with a “personalized strategy” is considered. Catering of customers is carried out by offline channels along with an online channel (Wang and He , 2022). Customization is made available through online channel and “standard product” through offline channel only.

5. The manufacturer works for manufacturing products and convey by adopting a “single-setup multiple-delivery policy” in offline channels whereas a “make-to-order” strategy is utilized for online routes.

6. A preassigned limit (“Threshold limit”) is considered which limits the variation in the cost of the product available on an online and offline route, thereby there is no customer shifting (Chauhan et al. , 2021).

7. To diminish the emission of carbon from the supply chain logarithmic function is considered.

7.3 Mathematical model

7.3.1 Functions of demand

The nature of the shoppers is miscellaneous since it gets influenced by the product's cost and diversity in products. Underneath offline channel and e-commerce, demand equations are obtained by enhancing the model of Chauhan et al. (2021). The function for offline channel's demand is

$$D_1 = a_1 - \beta_1 C_p(1+m) + \delta_1(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right) \tag{7.150}$$

The function for e-commerce's demand is

$$D_2 = a_2 - \beta_2(1+m)\left(\sum_{i=1}^{N} C_i\right) - \delta_2(1+m)\left(\sum_{i=1}^{N} C_i - C_p\right) \tag{7.151}$$

Fuzzification

In the real world, anticipating the product's exact demand is troublesome. Consequently, constant demand is taken into consideration by adopting a "distribution-free approach" for lead-time demand. However, researchers have considered fuzzy demand also with the "distribution-free approach". In the existing literature, demand is presumed to be a "triangular fuzzy number". Consequently, a "triangular fuzzy number" is assumed for this chapter because of its simplicity in execution. Moreover, a market investigation on the products manifested that the demand of the market does not follow a specific pattern. Thus, considering a demand to be fixed or following a certain probability distribution is unworkable, justifying the usage of "fuzzy triangular demand". Henceforth, for more realistic solutions "fuzzy triangular demand" is assumed.

This model considers a non-negative "triangular fuzzy number" i.e., fuzzy demand $\widetilde{D_1} = (D_1 - \varepsilon_1, D_1, D_1 + \varepsilon_2)$ and $\widetilde{D_2} = (D_2 - \varepsilon_1, D_2, D_2 + \varepsilon_2)$. The fuzzy numbers are replaced with crisp values to obtain concluding solutions. The signed distance method is used in the present model for converting fuzzy outputs to crisp models. The shopkeeper's and manufacturer's demand can be evaluated by replacing the non-negative "triangular fuzzy number" in the aforementioned equations.

$$\widetilde{D_1} = D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4} \tag{7.152}$$

$$\widetilde{D_2} = D_2 + \frac{\varepsilon_{2D_2} - \varepsilon_{1D_2}}{4} \tag{7.153}$$

7.3.2 Functions of profit

Present segment enlists the profit of manufacturer and retailer by selling "standard and make-to-order items".

I. **"Manufacture For standard product"**

The costs associated with the "standard product", bore by manufacturer are given as follows.

Setup cost

$$SC_{1m} = \frac{S_1\widetilde{D_1}}{nQ_1}$$

$$SC_{1m} = \frac{S_1}{nQ_1}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) \tag{7.154}$$

Holding cost

$$HC_{1m} = \frac{r_v C_p Q_1}{2}\left[n\left(1 - \frac{\widetilde{D_1}}{P_1}\right) - 1 + \frac{2\widetilde{D_1}}{P_1}\right]$$

$$HC_{1m} = \frac{r_v C_p Q_1}{2}\left[n\left(1 - \frac{1}{P_1}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right)\right) - 1 + \frac{2}{P_1}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right)\right] \tag{7.155}$$

Production cost

$$PC_{1m} = C_p\widetilde{D_1}$$

$$PC_{1m} = C_p\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) \tag{7.156}$$

Revenue

$$Rev_{1m} = C_p(1+m)\widetilde{D_1}$$

$$Rev_{1m} = C_p(1+m)\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) \tag{7.157}$$

Thus, aggregate profit of the manufacturer for "core product" is given as

$$\nabla_1 = \text{Revenue} - \text{Cost of setup} - \text{Cost of holding} - \text{Cost of production}$$

$$\nabla_1 = C_p(1+m)\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) - \frac{S_1}{nQ_1}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) - \frac{r_v C_p Q_1}{2}\left[n\left(1 - \frac{1}{P_1}\right.\right.$$

$$\left.\left.\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right)\right) - 1 + \frac{2}{P_1}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right)\right] - C_p\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) \quad (7.158)$$

II. **"Retailer For standard product"**

The costs associated with the "standard product", bore by retailer are given as follows.

Ordering cost

$$OC_{1r} = \frac{A_r \widetilde{D_1}}{Q_1}$$

$$OC_{1r} = \frac{A_r}{Q_1}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) \quad (7.159)$$

Holding cost

$$HC_{1r} = r_b C_b \left(\frac{Q_1}{2} + R - \widetilde{D_1} L\right)$$

$$HC_{1r} = r_b C_b \left(\frac{Q_1}{2} + R - \left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) L\right) \quad (7.160)$$

Shortage cost

$$SC_{1r} = \frac{\pi \widetilde{D_1}}{Q_1} E(M-R)^+$$

$$SC_{1r} = \frac{\pi}{Q_1}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right) E(M-R)^+ \quad (7.161)$$

where,

$$E(M-R)^+ = E((\widetilde{D_1}L + X) - R)^+$$

$$\leq \left[\frac{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1} L k\sigma\sqrt{L}} + (\widetilde{D_1}L - k\sigma\sqrt{L})}{2}\right]$$

$$\leq \frac{1}{2}\left[\sqrt{\sigma^2 L+((D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4})L)^2+k^2\sigma^2 L-2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)Lk\sigma\sqrt{L}}+\left(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L-k\sigma\sqrt{L}\right)\right]$$

$$SC_{1r} \leq \frac{\pi}{2Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\left[\sqrt{\sigma^2 L+((D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4})L)^2+k^2\sigma^2 L-2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)Lk\sigma\sqrt{L}}+\right.$$

$$\left.\left(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L-k\sigma\sqrt{L}\right)\right] \tag{7.162}$$

Lead time crashing cost

$$LC_{1r} = \frac{\widetilde{D_1}CL}{Q_1}$$

$$LC_{1r} = \frac{CL}{Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right) \tag{7.163}$$

Revenue

$$Rev_{1r} = C_p(1+m)^2\widetilde{D_1}$$

$$Rev_{1r} = C_p(1+m)^2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right) \tag{7.164}$$

Thus, aggregate profit of the retailer for "core product" is given as

$$\nabla_2 = \text{Revenue} - \text{Cost of ordering} - \text{Cost of holding} - \text{Cost of shortage} - \text{Cost of lead time crashing}$$

$$\nabla_2 \leq C_p(1+m)^2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\frac{A_r}{Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-r_bC_b\left(\frac{Q_1}{2}+R-\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L\right)-$$

$$\frac{\pi}{2Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\left[\sqrt{\sigma^2L+(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L)^2+k^2\sigma^2L-2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)Lk\sigma\sqrt{L}}+\right.$$

$$\left.\left(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L-k\sigma\sqrt{L}\right)\right]-\frac{CL}{Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right) \tag{7.165}$$

III. **"Manufacture For personalized product"**

The costs associated with the "customized product", bore by manufacturer are given as follows.

Setup cost

$$SC_{2m}=\frac{S_2\widetilde{D_2}}{Q_2}$$

$$SC_{2m}=\frac{S_2}{Q_2}\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right) \tag{7.166}$$

Holding cost

$$HC_{2m}=\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{\widetilde{D_2}}{P_2}\right)$$

$$HC_{2m}=\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right)\right) \tag{7.167}$$

Manufacturing cost

$$MC_{2m}=\sum_{i=1}^{N}C_i\phi_i\widetilde{D_2}$$

$$MC_{2m}=\sum_{i=1}^{N}C_i\phi_i\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right) \tag{7.168}$$

Revenue

$$Rev_{2m}=\sum_{i=1}^{N}C_i(1+m)\phi_i\widetilde{D_2}$$

$$Rev_{2m}=\sum_{i=1}^{N}C_i(1+m)\phi_i\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right) \tag{7.169}$$

Thus, total profit of the manufacture for "personalized product" is given as

$$\nabla_3 = \text{Revenue} - \text{Setup cost} - \text{Holding cost} - \text{Manufacturing cost}$$

$$\nabla_3 = \sum_{i=1}^{N} C_i(1+m)\phi_i\left(D_2 + \frac{\varepsilon_{2D_2} - \varepsilon_{1D_2}}{4}\right) - \frac{S_2}{Q_2}\left(D_2 + \frac{\varepsilon_{2D_2} - \varepsilon_{1D_2}}{4}\right) - \left(\frac{h_1 Q_2}{2}\right)\left(1 - \frac{1}{P_2}\right.$$
$$\left.\left(D_2 + \frac{\varepsilon_{2D_2} - \varepsilon_{1D_2}}{4}\right)\right) - \sum_{i=1}^{N} C_i\phi_i\left(D_2 + \frac{\varepsilon_{2D_2} - \varepsilon_{1D_2}}{4}\right) \tag{7.170}$$

7.3.3 Carbon emission reduction

I. Single-channel supply chain

Carbon dioxide emitted during the manufacturing of the standard product is given by a production rate (P_1)-dependent function that is $E(P_1) = x_1P_1^2 - x_2P_1 - x_3$. Thus, the total CO_2 emitted by $\widetilde{D_1}$ is $E(P_1) = (x_1P_1^2 - x_2P_1 - x_3)\widetilde{D_1}$. Carbon emitted during haulage of nQ_1 products from the manufacture to the retailer is given as $E_{haul1} = \frac{\widetilde{D_1}ve_1}{Cap_1}$. Henceforth, the total CO_2 emitted throughout the "single-channel supply chain" is

$$E_{tot1} = (x_1P_1^2 - x_2P_1 - x_3)\widetilde{D_1} + \frac{\widetilde{D_1}ve_1}{Cap_1} \tag{7.171}$$

Hence, the total CO_2 emitted cost is given as

$$Emitted_{cost1} = C_{ec}\widetilde{D_1}\left((x_1P_1^2 - x_2P_1 - x_3) + \frac{ve_1}{Cap_1}\right) \tag{7.172}$$

$$Emitted_{cost1} = C_{ec}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right)\left((x_1P_1^2 - x_2P_1 - x_3) + \frac{ve_1}{Cap_1}\right) \tag{7.173}$$

Additionally, in-spite of imposition of the taxes, emission carbon adversely influences the environment. Consequently, for reducing the carbon emissions logarithmic investment is incorporated as

$$I_{(CO_2)1} = \omega_1 \ln \frac{E(P_1)}{(CO_{2ae})_1} \tag{7.174}$$

where, $\omega_1 = SP_{(CO_2)1} \times RP_{(CO_2)_1}$

In equation 7.174, taking the exponents on both sides it gives $\exp^{\frac{I_{(CO_2)1}}{\omega_1}} = \frac{E(P_1)}{(CO_{2ae})_1}$. Thus, after the investment, CO_2 ejected is given as $(CO_{2ae})_1 = \frac{E(P_1)}{exp^{\left(\frac{I_{(CO_2)1}}{\omega_1}\right)}}$.

Therefore, the total CO_2 ejected cost after the reduction investment is given by

$$C_{ec}\left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right)\left(\frac{(x_1P_1^2 - x_2P_1 - x_3)}{exp^{\left(\frac{I_{(CO_2)1}}{\omega_1}\right)}} + \frac{ve_1}{Cap_1}\right) \tag{7.175}$$

II. Dual-channel supply chain

Similarly, CO_2 emitted during the manufacturing of the standard and the "personalized product" is given by a production rate $P = P_1 + P_2$ dependent function that is $E(P) = x_1P^2 - x_2P - x_3$ respectively. Thus, the total CO_2 emitted by $\widetilde{D} = \widetilde{D_1} + \widetilde{D_2}$ is $E(P) = (x_1P^2 - x_2P - x_3)\widetilde{D}$. Carbon emitted during haulage of nQ products from the manufacture to the retailer and the customer is given as $E_{haul2} = \frac{\widetilde{D}ve}{Cap_2}$. Consequently, the total CO_2 emitted throughout the "dual-channel supply chain" is

$$E_{tot} = (x_1P^2 - x_2P - x_3)\widetilde{D} + \frac{\widetilde{D}ve}{Cap_2} \tag{7.176}$$

Hence, the total CO_2 emitted cost is given as

$$Emitted_{cost2} = C_{ec}\widetilde{D}\left((x_1P^2 - x_2P - x_3) + \frac{ve}{Cap_2}\right) \tag{7.177}$$

$$Emitted_{cost2} = C_{ec}\left(D + \frac{\varepsilon_{2D} - \varepsilon_{1D}}{4}\right)\left((x_1P^2 - x_2P - x_3) + \frac{ve}{Cap_2}\right) \tag{7.178}$$

Similarly, for reducing the carbon emissions logarithmic investment is incorporated in "dual-channel supply chain" as

$$I_{(CO_2)2} = \omega \ln \frac{E(P)}{(CO_{2ae})} \tag{7.179}$$

where, $\omega = SP_{(CO_2)} \times RP_{(CO_2)}$

In equation 7.179, taking the exponents on both sides it gives $\exp^{\frac{I_{(CO_2)}}{\omega}} = \frac{E(P)}{(CO_{2ae})}$. Thus, after the investment, CO_2 ejected is given as $(CO_{2ae}) = \frac{E(P)}{exp^{\left(\frac{I_{(CO_2)}}{\omega}\right)}}$.

Therefore, the total CO_2 ejected cost after the reduction investment is given by

$$C_{ec}\left(D+\frac{\varepsilon_{2D}-\varepsilon_{1D}}{4}\right)\left(\frac{(x_1P^2-x_2P-x_3)}{exp^{\left(\frac{I_{(CO_2)}}{\omega}\right)}}+\frac{ve}{Cap_2}\right) \tag{7.180}$$

7.3.4 Setup cost reduction

Equation 7.154 and 7.166 reflects the manufacturer's fixed setup cost per unit time for "standard and personalized product". But practically, this setup cost is variable. A capital investment is incorporated to reduce the manufacturer's setup cost.

Henceforth, an investment in reducing the setup cost for the standard product is given as

$$I_{S1}=A\ln\left(\frac{S_{1_0}}{S_1}\right)=A(\ln S_{1_0}-lnS_1)\quad \text{for}\quad 0<S_1\leq S_{1_0} \tag{7.181}$$

where, S_{1_0} is the original setup cost for standard product, $A=\frac{1}{\delta}$, and $\delta=$ the percentage decrease in S_1 per dollar increase in I_{S1}.

Similarly, an investment in reducing the setup cost for the "personalized product" is

$$I_{S2}=B\ln\left(\frac{S_{2_0}}{S_2}\right)=B(\ln S_{2_0}-lnS_2)\quad \text{for}\quad 0<S_2\leq S_{2_0} \tag{7.182}$$

where, S_{2_0} is the original setup cost for standard product, $B=\frac{1}{\delta}$, and $\delta=$ the percentage decrease in S_2 per dollar increase in I_{S2}.

Total profit ∇_S of the "supply chain with single-channel" is obtained by totaling the ∇_1 and ∇_2. We get,

$$\nabla_S=\nabla_1+\nabla_2 \tag{7.183}$$

$$\nabla_S=C_p(1+m)\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\frac{S_1}{nQ_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\right)\right.$$

$$\left.-1+\frac{2}{P_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\right]\frac{r_vC_pQ_1}{2}-C_p\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)+C_p(1+m)^2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)$$

$$-\frac{A_r}{Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-r_bC_b\left(\frac{Q_1}{2}+R-\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L\right)-\frac{CL}{Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\frac{\pi}{2Q_1}$$

$$\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\left[\sqrt{\sigma^2L+\left(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)Lk\sigma\sqrt{L}}+\right.$$

$$\left.\left(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L-k\sigma\sqrt{L}\right)\right]-C_{ec}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\left(\frac{(x_1P_1^2-x_2P_1-x_3)}{exp^{\left(\frac{I_{(CO_2)1}}{\omega_1}\right)}}+\frac{\nu e_1}{Cap_1}\right)-A(\ln S_{1_0}-lnS_1) \tag{7.184}$$

And expected aggregate profit ∇_D of the supply chain with "dual channel" is obtained by summing ∇_1, ∇_2, and ∇_3. We obtain,

$$\nabla_D=\nabla_1+\nabla_2+\nabla_3 \tag{7.185}$$

$$\nabla_D=C_p(1+m)\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\frac{S_1}{nQ_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\frac{r_vC_pQ_1}{2}\left[n\left(1-\frac{1}{P_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\right)\right.$$

$$\left.-1+\frac{2}{P_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\right]-C_p\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)+C_p(1+m)^2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\frac{A_r}{Q_1}$$

$$\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-r_bC_b\left(\frac{Q_1}{2}+R-\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L\right)-\frac{CL}{Q_1}\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)-\frac{\pi}{2Q_1}$$

$$\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)\left[\sqrt{\sigma^2L+\left(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L\right)^2+k^2\sigma^2L-2\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)Lk\sigma\sqrt{L}}+\right.$$

$$\left(\left(D_1+\frac{\varepsilon_{2D_1}-\varepsilon_{1D_1}}{4}\right)L-k\sigma\sqrt{L}\right)\Bigg]+\sum_{i=1}^{N}C_i(1+m)\phi_i\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right)-\frac{S_2}{Q_2}\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right)$$

$$-\left(\frac{h_1Q_2}{2}\right)\left(1-\frac{1}{P_2}\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right)\right)-\sum_{i=1}^{N}C_i\phi_i\left(D_2+\frac{\varepsilon_{2D_2}-\varepsilon_{1D_2}}{4}\right)-C_{ec}\left(D+\frac{\varepsilon_{2D}-\varepsilon_{1D}}{4}\right)$$

$$\left(\frac{(x_1P^2-x_2P-x_3)}{exp^{\left(\frac{l_{(CO_2)}}{\omega}\right)}}+\frac{ge}{Cap_2}\right)-A(\ln S_{1_0}-lnS_1)-B(\ln S_{2_0}-lnS_2) \tag{7.186}$$

7.3.5 Optimal decision of the supply chain

Equations 7.184 and 7.186 are non-linear in nature therefore, for a positive definite integer 'm'', the optimal values are obtained by equating the partial derivatives of the profit ∇_S and ∇_D with respect to Q_1, Q_2, k, S_1, and S_2 to zero.

$$Q_2^*=\sqrt{\frac{S_2\widetilde{D_2}}{\frac{h_1}{2}\left(1-\frac{\widetilde{D_2}}{P_2}\right)}} \tag{7.187}$$

$$Q_1^*=\sqrt{\frac{-\widetilde{D_1}CL+S_1\widetilde{D_1}+A_r\widetilde{D_1}+\frac{\pi\widetilde{D_1}}{2}\sqrt{\sigma^2L+(\widetilde{D_1}L)^2+k^2\sigma^2L-2\widetilde{D_1}Lk\sigma\sqrt{L}}+\widetilde{D_1}L-k\sigma\sqrt{L}}{\frac{r_bC_b}{2}+\frac{r_v}{2}\left(n\left(1-\frac{\widetilde{D_1}}{P_1}\right)-1+2\frac{\widetilde{D_1}}{P_1}\right)}} \tag{7.188}$$

$$k^*=\frac{\frac{(-\sigma\sqrt{Q_1C_br_b(-Q_1C_br_b+\pi\widetilde{D_1})})}{-Q_1C_br_b+\pi\widetilde{D_1}}+\frac{\widetilde{D_1}\pi\sigma\sqrt{Q_1C_br_b(-Q_1C_br_b+\pi\widetilde{D_1})L}}{2Q_1C_br_b(\pi\widetilde{D_1}-Q_1C_br_b)}+L\widetilde{D_1}}{\sqrt{L}\sigma} \tag{7.189}$$

$$S_1=\frac{AnQ_1}{\widetilde{D_1}} \tag{7.190}$$

$$S_2=\frac{BQ_2}{\widetilde{D_2}} \tag{7.191}$$

where, $\widetilde{D_1} = \left(D_1 + \frac{\varepsilon_{2D_1} - \varepsilon_{1D_1}}{4}\right)$ and $\widetilde{D_2} = \left(D_2 + \frac{\varepsilon_{2D_2} - \varepsilon_{1D_2}}{4}\right)$

From equations 7.187, 7.188, 7.189, 7.190, and 7.191 clearly reflects the dependency of Q_1^*, Q_2^*, k^*, S_1, and S_2 on each other in the optimal solution. Therefore, for evaluating the managerial decisions an iteration method is used with the underneath algorithm.

7.4 Algorithm for finding the solution of the model

In solving the present model underneath algorithm is utilized.

Step 1 Allot all parameters with the values defined in numerical analysis section.

Step 2 Put n=1.

Step 3 Execute the underneath steps for all the values of L_i; i=1,2,...

- Step 3(a) From equation 7.187, get the value of Q_2.
- Step 3(b) Obtain the value of Q_1 from equation 7.188.
- Step 3(c) Obtain the value of k from equation 7.189.
- Step 3(d) Obtain the value of S_1 from equation 7.190.
- Step 3(e) Obtain the value of S_2 from equation 7.191.
- Step 3(f) Redo Steps from 3a to 3e unless there is variation in the values of Q_1, Q_2, k, S_1, and S_2 upto a defined level of accuracy.

Step 4 Obtain the value of ∇_S from equation 7.184, using the Q_1, k, and S_1.

Step 5 Obtain the value of ∇_D from equation 7.186, using the value of ∇_S, Q_2, and S_2.

Step 6 Put n=n+1 and redo the Step 3 to 5.

Step 7 If $\nabla_D(n+1) < \nabla_D(n)$ then redo the algorithm from step 2 to step 6 or else end the algorithm.

Proposition 7.1. *If we represents Q_1^*, Q_2^*, k^*, S_1^*, and S_2^* as the optimal values of Q_1, Q_2, k, S_1, and S_2 then for fixed values of n and $L \in [L_i, L_{i-1}]$, ∇_D the profit function for "dual-channel", acquires its maximum value at Q_1^*, Q_2^*, k^*, S_1^*, and S_2^* under the condition*

$$X > Y + Z \tag{7.192}$$

Proof. For “dual-channel supply chain”, the Hessian matrix H is

$$H_1 = \begin{bmatrix} \frac{\partial^2 \nabla_D}{\partial Q_1^2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial S_1} & \frac{\partial^2 \nabla_D}{\partial Q_1 \partial S_2} \\ \frac{\partial^2 \nabla_D}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial Q_2^2} & \frac{\partial^2 \nabla_D}{\partial Q_2 \partial k} & \frac{\partial^2 \nabla_D}{\partial Q_2 \partial S_1} & \frac{\partial^2 \nabla_D}{\partial Q_2 \partial S_2} \\ \frac{\partial^2 \nabla_D}{\partial k \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial k \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial k^2} & \frac{\partial^2 \nabla_D}{\partial k \partial S_1} & \frac{\partial^2 \nabla_D}{\partial k \partial S_2} \\ \frac{\partial^2 \nabla_D}{\partial S_1 \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial S_1 \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial S_1 \partial k} & \frac{\partial^2 \nabla_D}{\partial S_1^2} & \frac{\partial^2 \nabla_D}{\partial S_1 \partial S_2} \\ \frac{\partial^2 \nabla_D}{\partial S_2 \partial Q_1} & \frac{\partial^2 \nabla_D}{\partial S_2 \partial Q_2} & \frac{\partial^2 \nabla_D}{\partial S_2 \partial k} & \frac{\partial^2 \nabla_D}{\partial S_2 \partial S_1} & \frac{\partial^2 \nabla_D}{\partial S_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

where,

$$a_{11} = \frac{\partial^2 \nabla_D}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[\widetilde{A_1}\widetilde{D_1} + \frac{\widetilde{S_1}\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right]$$

$$a_{12} = \frac{\partial^2 \nabla_D}{\partial Q_1 \partial Q_2} = 0 = a_{21}$$

$$a_{13} = a_{31} = \frac{\partial^2 \nabla_D}{\partial Q_1 \partial k} = \frac{\pi\widetilde{D_1}}{2Q_1^2}\left[\frac{k\sigma^2 L - \widetilde{D_1}L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$a_{14} = a_{41} = \frac{\partial^2 \nabla_D}{\partial Q_1 \partial S_1} = \frac{\widetilde{D_1}}{nQ_1^2}$$

$$a_{15} = a_{51} = \frac{\partial^2 \nabla_D}{\partial Q_1 \partial S_2} = 0$$

$$a_{22} = \frac{\partial^2 \nabla_D}{\partial Q_2^2} = -\frac{2\widetilde{S_2}\widetilde{D_1}}{Q_2^3}$$

$$a_{23} = a_{32} = \frac{\partial^2 \nabla_D}{\partial k \partial Q_2} = 0$$

$$a_{24} = a_{42} = \frac{\partial^2 \nabla_D}{\partial Q_2 \partial S_1} = 0$$

$$a_{25} = a_{52} = \frac{\partial^2 \nabla_D}{\partial Q_2 \partial S_2} = \frac{\widetilde{D_2}}{Q_2^2}$$

$$a_{33} = \frac{\partial^2 \nabla_D}{\partial k^2} = -\frac{\pi \widetilde{D_1}}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2 \sigma^2 L - \widetilde{D_1} L \sigma \sqrt{L}}}\right]$$

$$a_{34} = a_{43} = \frac{\partial^2 \nabla_D}{\partial k \partial S_1} = 0 = a_{35} = a_{53} = \frac{\partial^2 \nabla_D}{\partial k \partial S_2}$$

$$a_{44} = \frac{\partial^2 \nabla_D}{\partial S_1^2} = -\frac{A}{S_1^2}$$

$$a_{45} = a_{54} = \frac{\partial^2 \nabla_D}{\partial S_1 \partial S_2} = 0$$

$$a_{55} = \frac{\partial^2 \nabla_D}{\partial S_2^2} = -\frac{B}{S_2^2}$$

The principal minor of $|(H_1)_{1,1}|_{(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)}$ of order 1×1 is

$$|(H_1)_{1,1}|_{(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)} = \left|a_{11}\right|_{(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)}$$

$$= a_{11} < 0$$

The principal minor of $|(H_1)_{2,2}|_{(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)}$ of order 2×2 is

$$|(H_1)_{2,2}|_{(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \big(a_{11}\big)\big(a_{22}\big) - (a_{12})(a_{21})$$

$$= a_{11} \times a_{22} > 0$$

The principal minor of $|(H_1)_{3,3}|_{(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)}$ of order 3×3 is

$$|(H_1)_{3,3}|_{(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \big(a_{11}\big)\Big[\big(a_{22}\big)(a_{33}) - (a_{32})(a_{23})\Big]$$

$$-(a_{12})\Big[(a_{21})(a_{33}) - (a_{31})(a_{23})\Big] + (a_{13})\Big[(a_{21})(a_{32}) - (a_{31})(a_{22})\Big]$$

$$= \big(a_{22}\big)\Big[\big(a_{11}\big)\big(a_{33}\big) - \big(a_{13}\big)^2\Big] > 0$$

$$\implies (a_{11})(a_{33}) > (a_{13})^2$$

The principal minor of $|(H_1)_{4,4}|_{(Q_1^*,Q_2^*,k^*,S_1^*,S_2^*)}$ of order 4×4 is

$$|(H_1)_{4,4}|_{(Q_1^*,Q_2^*,k^*,S_1^*,S_2^*)} = \begin{vmatrix} a_{22} & a_{23} & a_{24} & a_{25} \\ a_{32} & a_{33} & a_{34} & a_{35} \\ a_{42} & a_{43} & a_{44} & a_{45} \\ a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix}$$

$$= \begin{vmatrix} a_{22} & 0 & 0 & a_{25} \\ 0 & a_{33} & 0 & 0 \\ 0 & 0 & a_{44} & 0 \\ a_{52} & 0 & 0 & a_{55} \end{vmatrix} = (a_{33})(a_{44})\left[(a_{22})(a_{55}) - (a_{25})^2\right] > 0$$

$$\implies (a_{22})(a_{55}) > (a_{25})^2$$

The principal minor of $|(H_1)_{5,5}|_{(Q_1^*,Q_2^*,k^*,S_1^*,S_2^*)}$ of order 5×5 is

$$|(H_1)_{5,5}|_{(Q_1^*,Q_2^*,k^*,S_1^*,S_2^*)} = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & 0 & a_{13} & a_{14} & 0 \\ 0 & a_{22} & 0 & 0 & a_{25} \\ a_{31} & 0 & a_{33} & 0 & 0 \\ a_{41} & 0 & 0 & a_{44} & 0 \\ 0 & a_{52} & 0 & 0 & a_{55} \end{vmatrix}$$

$$= (a_{13})^2(a_{25})^2(a_{44}) + (a_{11})(a_{22})(a_{33})(a_{44})(a_{55}) - (a_{11})(a_{33})(a_{25})^2(a_{44}) - (a_{25})^2(a_{33})(a_{41})^2 -$$

$$(a_{22})(a_{13})^2(a_{55})(a_{44}) - (a_{22})(a_{41})^2(a_{33})(a_{55})$$

$$\implies (a_{13})^2(a_{25})^2(a_{44}) + (a_{11})(a_{22})(a_{33})(a_{44})(a_{55}) > (a_{11})(a_{33})(a_{25})^2(a_{44}) + (a_{25})^2(a_{33})(a_{41})^2 +$$

$$(a_{22})(a_{13})^2(a_{55})(a_{44}) + (a_{22})(a_{41})^2(a_{33})(a_{55})$$

$$X > Y + Z$$

where, $X = (a_{13})^2(a_{25})^2(a_{44}) + (a_{11})(a_{22})(a_{33})(a_{44})(a_{55})$, $Y = (a_{11})(a_{33})(a_{25})^2(a_{44}) + (a_{25})^2(a_{33})(a_{41})^2$, and $Z = (a_{22})(a_{13})^2(a_{55})(a_{44}) + (a_{22})(a_{41})^2(a_{33})(a_{55})$

Since, the Hessian matrix's, all the principal minors are not positive. Hence, the Hessian matrix H_1 is negative definite at $(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)$. Thus, aggregate expected profit for "dual-channel" gets the global maximum at $(Q_1^*, Q_2^*, k^*, S_1^*, S_2^*)$.

Proposition 7.2. *If we represents Q_1^*, k^*, and S_1^* as the optimal values of Q_1, k, and S_1 then for fixed values of n and $L \in [L_i, L_{i-1}]$, ∇_S the profit function for "single-channel", acquires its maximum value at Q_1^*, k^*, and S_1^* in the following condition*

$$X_1 > Y_1 + Z_1 \tag{7.193}$$

where value of X_1, Y_1, and Z_1 can be referred from appendix N.

Proof. For "single-channel supply chain", the Hessian matrix H is

$$H_1 = \begin{bmatrix} \frac{\partial^2 \nabla_S}{\partial Q_1^2} & \frac{\partial^2 \nabla_S}{\partial Q_1 \partial k} & \frac{\partial^2 \nabla_S}{\partial Q_1 \partial S_1} \\ \frac{\partial^2 \nabla_S}{\partial k \partial Q_1} & \frac{\partial^2 \nabla_S}{\partial k^2} & \frac{\partial^2 \nabla_S}{\partial k \partial S_1} \\ \frac{\partial^2 \nabla_S}{\partial S_1 \partial Q_1} & \frac{\partial^2 \nabla_S}{\partial S_1 \partial k} & \frac{\partial^2 \nabla_S}{\partial S_1^2} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

where,

$$b_{11} = \frac{\partial^2 \nabla_S}{\partial Q_1^2} = -\frac{2}{Q_1^3}\left[\widetilde{A_1}\widetilde{D_1} + \frac{\widetilde{S_1}\widetilde{D_1}}{n} + \widetilde{D_1}CL + \pi\widetilde{D_1}\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}Lk\sigma\sqrt{L}}\right]$$

$$b_{12} = b_{21} = \frac{\partial^2 \nabla_S}{\partial Q_1 \partial k} = \frac{\pi\widetilde{D_1}}{2Q_1^2}\left[\frac{k\sigma^2 L - \widetilde{D_1}L\sigma\sqrt{L}}{\sqrt{\sigma^2 L + (\widetilde{D_1}L)^2 + k^2\sigma^2 L - 2\widetilde{D_1}L\sigma\sqrt{L}}} - \sigma\sqrt{L}\right]$$

$$b_{13} = b_{31} = \frac{\partial^2 \nabla_S}{\partial Q_1 \partial S_1} = \frac{\widetilde{D_1}}{nQ_1^2}$$

$$b_{22} = \frac{\partial^2 \nabla_S}{\partial k^2} = -\frac{\pi \widetilde{D_1}}{2Q_1}\left[\frac{\sigma^4 L^2}{\sqrt{\sigma^2 L + (\widetilde{D_1} L)^2 + k^2\sigma^2 L - \widetilde{D_1} L\sigma\sqrt{L}}}\right]$$

$$b_{23} = b_{32} = \frac{\partial^2 \nabla_S}{\partial k \partial S_1} = 0$$

$$b_{33} = -\frac{A}{S_1^2}$$

The principal minor $|(H_2)_{1,1}|_{(Q_1^*,k^*,S_1^*)}$ of $|H_2|$ of order 1×1 is

$$|(H_2)_{1,1}|_{(Q_1^*,k^*,S_1^*)} = \left|b_{11}\right|_{(Q_1^*,k^*,S_1^*)} < 0$$

The principal minor $|(H_2)_{2,2}|_{(Q_1^*,k^*,S_1^*)}$ of $|H_2|$ of order 2×2 is

$$|H_{2,2}|_{(Q_1^*,k^*,S_1^*)} = \begin{vmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{vmatrix} = (b_{22})(b_{33}) - (b_{23})(b_{32})$$

$$\implies (b_{22})(b_{33}) > 0$$

The principal minor $|(H_2)_{3,3}|_{(Q_1^*,k^*,S_1^*)}$ of $|H_2|$ of order 3×3 is

$$|(H_2)_{3,3}|_{(Q_1^*,k^*,S_1^*)} = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}$$

$$= \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & 0 \\ b_{31} & 0 & b_{33} \end{vmatrix} = -(b_{22})(b_{31})^2 + (b_{11})(b_{22})(b_{33}) - (b_{33})(b_{21})^2 > 0$$

$$\implies (b_{11})(b_{22})(b_{33}) > (b_{22})(b_{31})^2 + (b_{33})(b_{21})^2$$

$$X_1 > Y_1 + Z_1$$

where, $X_1 = (b_{11})(b_{22})(b_{33})$, $Y_1 = (b_{22})(b_{31})^2$, and $Z_1 = (b_{33})(b_{21})^2$

Since, the Hessian matrix's, all the principal minors are not positive. Hence, the Hessian matrix H_2 is negative definite at (Q_1^*, k^*, S_1^*). Thus, aggregate expected profit for "single-channel" gets the global maximum at (Q_1^*, k^*, S_1^*).

7.5 Numerical experimentation and discussion

This segment illustrates the developed model for the supply chain management. Examining the consequences of stochastic fuzzy demand on the supply chain model and unequal shipment of customers between the channel is the objective of the numerical examples. To resemble a real manufacturing environment, values of parameters were taken from Malik and Sarkar (2019) & Chauhan et al. (2021). The input parameters taken are as follows $C_p = 120$ (\$/unit); $C_1 = 70$ (\$/unit); $C_2 = 110$ (\$/unit); $C_3 = 130$ (\$/unit); $C_b = 120$ (\$/unit); $r_b = 0.2$ (\$/unit/unit time); $A_r = 250$ (\$/order); $P_1 = 6000$ (Units/year); $P_2 = 6000$ (Units/year); $m = 0.7$; $\pi = 140$ (\$/unit); $a_1 = 1000$; $a_2 = 1000$; $\beta_1 = 0.75$ (\$/unit); $r_v = 0.2$ (\$/unit/unit time); $\beta_2 = 0.8$ (\$/unit); $\delta_1 = 0.2$; $\delta_2 = 0.3$; $\phi_1 = 0.25$ (%); $\phi_2 = 0.3$(%); $\phi_3 = 0.45$ (%); $h_1 = 30$ (\$/unit/year); $\varepsilon_{1D_1} = 600$; $\varepsilon_{2D_1} = 800$; $\varepsilon_{1D_2} = 1400$; $\varepsilon_{2D_2} = 1500$; $g = 375$ (gallons); $\sigma = 200$; $\nu = 275$ (gallons); $e = 0.01008414$ (ton/gallon); $e_1 = 0.01008414$ (ton/gallon); $S_1 = 800$ (\$/setup); $S_2 = 1000$ (\$/setup); $Cap_2 = 500$; $Cap_1 = 400$; $L = 4$ (weeks); $I_{(CO_2)} = 2000$ (\$); $I_{(CO_2)_1} = 1800$ (\$). Further, a presumed "threshold value" i.e., \$20 is considered throughout the chapter.

7.5.1 Comparison of profit of single and dual-channel

An example is considered for comparing the "dual-channel and the single-channel supply chain management's" profit. Also, the standard products are enhanced with three varieties of customization in supply chain model of "dual-channel". Additionally, throughout the chapter, the "threshold value" is assumed to be \$ 20. Finally, the optimal values of the decision variables are obtained, which are displayed in table 7.20. Consequently, table 7.20 displays the maximum value of the profit's for supply chain models with "single and dual-channel" i.e., \$ 2095026 and \$ 2065620, respectively. The maximum profit is obtained at and the corresponding values of $D_1, D_2, Q_1, Q_2, k, S_1$, and S_2 are 1465.22, 770.2, 1921.502, 655.22, 15.59, 12894.12, and 1997.15, respectively. Thereby, reflecting that the profit of firm is more with "dual-channel" in comparison with "single-channel".

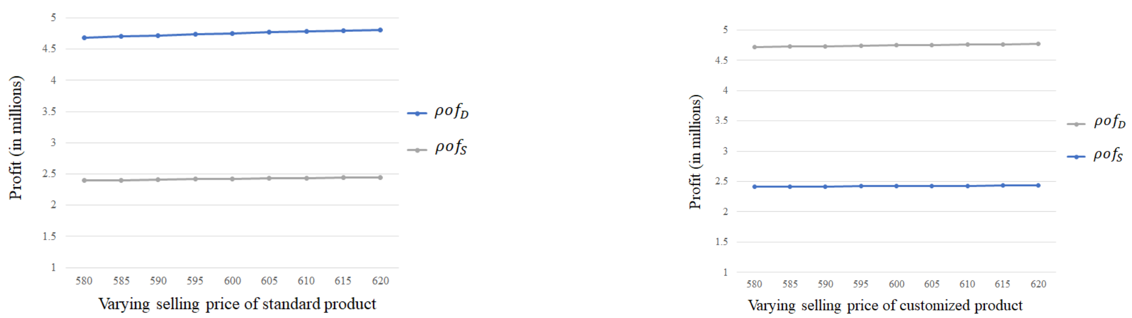
Tab. 7.20: Decision variable's optimal values

n	D_1	D_2	Q_1	Q_2	k	S_1	S_2	∇_D	∇_S
1	1465.22	770.2	1720.271	655.22	15.65	2308.755	1997.15	1988337	1958931
2	1465.22	770.2	1767.953	655.22	15.63	4745.496	1997.15	2013456	1984050
3	1465.22	770.2	1817.369	655.22	15.62	7317.209	1997.15	2039370	2009964
4	1465.22	770.2	1868.545	655.22	15.60	10031.01	1997.15	2066498	2037092
5	**1465.22**	**770.2**	**1921.502**	**655.22**	**15.59**	**12894.12**	**1997.15**	**2095026**	**2065620**
6	1465.22	770.2	1916.253	655.22	15.57	15913.84	1997.15	2085000	2055676

7.5.2 Impact of varying selling price on the profit

In this section, two cases are considered.

Case I: When the variation in the cost of selling for the "core and the personalized item" is less than the presumed value i.e., $|C_p - \sum_{i=1}^{N} C_i| <$ Threshold limit., then it leads to further two sub-cases illustrated by figures 7.64a and 7.64b, respectively. In the first case, the cost of the "core item" is varied whereas cost of the "personalized item" is kept fixed and in the second case the variation in the selling price of the "customized product" is reflected while that of the "standard products" is kept constant. In either of the cases, the shifting of customers and the change in the profit for both channel is approximately 2.19%.

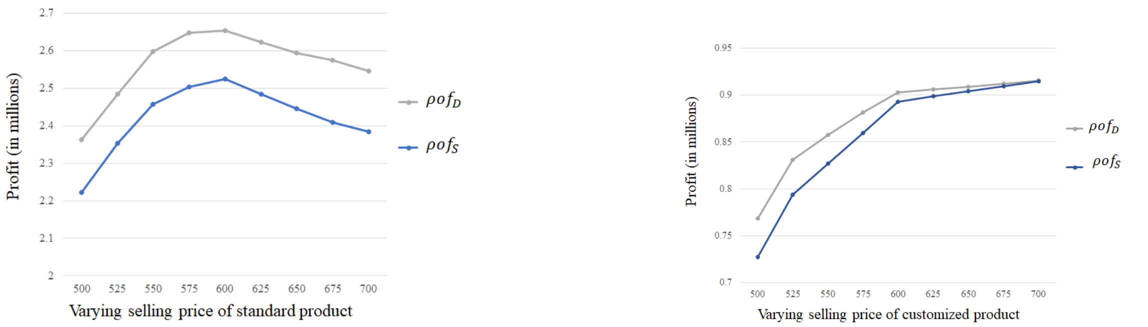


(a) Case 1 : Constant prices of personalized item

(b) Case 2 : Constant prices of standard item

Fig. 7.64: Graphical presentation of variation of demand when $|\sum_{i=1}^{N} C_i - C_p|$ <Threshold value

Case II: Similarly, when variation in the cost of selling of the "core and the personalized item" is more than the presumed value i.e., $|C_p - \sum_{i=1}^{N} C_i| >$ Threshold limit. Thus, giving two scenarios demonstrated by figures 7.65a and 7.65b, respectively. In the first scenario, the cost of selling of the "core



(a) Case 1 : Constant prices of personalized item

(b) Case 2 : Constant prices of standard item

Fig. 7.65: Graphical presentation of variation of demand when $|\sum_{i=1}^{N} C_i - C_p|$ >Threshold value

item" is varied whereas that of the "personalized item" is kept constant. Moreover, the second exhibits the divergence in the cost of "personalized item" while that of standard product is kept constant. Further, in both the scenarios there is approximately 19% of shifting of customers and change in the total profit of the firm.

7.5.3 Analysis of investment in reducing carbon emission

This section demonstrates the benefit of investing in reducing the carbon emission which is reflected by table 7.21. Table exemplifies that in the case of investment in the reduction of CO_2 emission, the profit is $ 2223151 whereas in case the model does not invest in carbon reduction the profit is $ 1932422 i.e., there is approximately 15% increase in the profit of the firm with the investment in the carbon reduction. Moreover, there is about a 73% decrease in carbon emission with investment.

Tab. 7.21: Comparative table-I

	Without investment in reducing CO_2 emission	With investment in reducing CO_2 emission
Carbon emission	21933.15 ton/year	5781.515 ton/year
∇_D	$1932422	$2223151

7.5.4 Comparison with the existing literature

The results of the current model exemplify that investment in the setup cost for both production of "standard and the customized product" is more profitable. The model is compared with the existing literature Chauhan et al. (2021) and Batarfi et al. (2016). The setup cost of S_1 for the "standard product" and S_2 for the "customized product" was not considered to be a decision variable. Table 7.22 summarizes the comparison and reflects that the present model is economically more beneficial in comparison to the rest of the two models.

Tab. 7.22: Comparative table-II

	Chauhan et al. (2021)	Batarfi et al. (2016)	This model
Q_1	1060.79	1384.95	1921.502
Q_2	166.22	354.61	655.22
D_1	490	1100	1465.22
D_2	456	645	770.2
S_1	–	–	12894.12
S_2	–	–	1997.15
∇_D	822509	1337494	2095026

Setup cost S_1 and S_2 are not considered decision variable in the model of Chauhan et al. (2021) and Batarfi et al. (2016)

7.6 Sensitivity analysis

The influence of variation from $-5\%, -2.5\%, +2.5\%, and + 5\%$ of key parameters on the total profit is exemplified by table. Underneath points enlist the sensitivity analysis of the model.

• The cost parameter of the standard product in the model is most sensitive in comparison to other cost parameters. Since customization is carried out on standard products therefore small change in the selling price of "standard products" influences the selling prices of the "customized product".

• Carbon emission tax is the second most sensitive cost parameter in the model after the selling charges of the core item. Since in the chapter carbon emission from all services that is manufacturing and transportation of products is considered thus, a carbon tax is levied on the firms.

• The third customization cost is slightly more sensitive to the profit in comparison to the rest of the customization costs.

• Shopkeepers ordering cost is the least sensitive to the profit of the model.

Tab. 7.23: Cost parameters sensitivity interpretation

Parameters	Variation (in %)	Change in profit	Parameters	Variation (in %)	Change in profit
C_p	−5	−5.65	h_1	−5	−0.005
	−2.5	−2.83		−2.5	−0.002
	+2.5	+2.83		+2.5	+0.002
	+5	+5.65		+5	+0.005
C_1	−5	−0.05	A_r	−5	+0.0003
	−2.5	−0.03		−2.5	+0.0001
	+2.5	+0.03		+2.5	−0.0001
	+5	+0.05		+5	−0.0003
C_2	−5	−0.06	C_b	−5	+0.609
	−2.5	−0.03		−2.5	+0.304
	+2.5	+0.03		+2.5	−0.304
	+5	+0.06		+5	−0.609
C_3	−5	−0.09	C_{ec}	−5	+0.23
	−2.5	−0.04		−2.5	+0.12
	+2.5	+0.04		+2.5	−0.12
	+5	+0.09		+5	−0.23

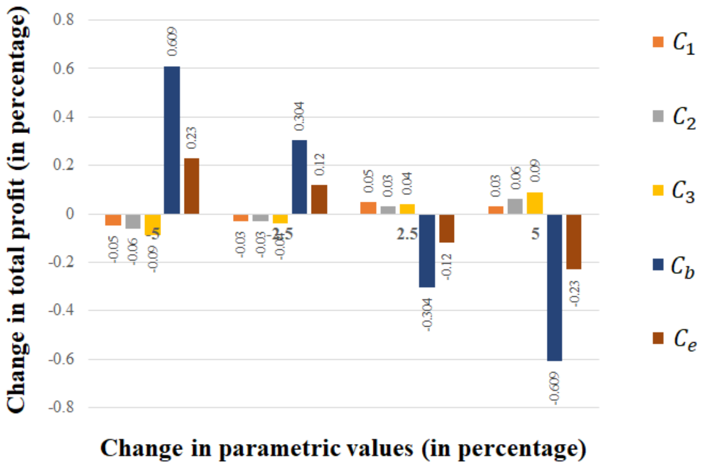


Fig. 7.66: Effect of change in parametric values on profit of the firm

7.7 Managerial Implications

From a managerial perspective, this chapter explores investment in the carbon emission reduction and setup cost for "standard-personalized products" in supply chain management by incorporating "dual-channel" having a single manufacturer and retailer. Upcoming enlists the managerial implications of this chapter.

• An investment is considered in the model for reducing the carbon emission from different services-production and haulage of finished products of the supply chain. Thus it is a sustainable supply chain model focused on maximizing profit by simultaneously investing in the environmental pillar of sustainability.

• Moreover, an investment in setup cost is taken into account which helps the manufacturer in reducing the setup cost in supply chain thus increasing the profit.

• A fuzzy set-based procedure is incorporated to deal with the uncertainties in the demand parameters, which makes it practically applicable.

• For developing an economically beneficial model by enlarging the circle of customers, both online channels and offline channels are incorporated into the supply chain.

• Although, the firm offers "customized products" along with the "core product" but the cost of the influenced item's should not overshoot the presumed limit, else consumers would move back to the core product.

7.8 Conclusions

The chapter considered the modified "dual-channel supply chain" by investigating the reduction of carbon emission and setup costs under fuzzy environment. Since an investment strategy plays an important role in reducing carbon emissions. Therefore, a logarithmic investment is incorporated for reducing carbon emission. In this model, a "triangular fuzzy number" is utilized, for dealing with uncertain demand parameters.

From the comparisons and discussions, we obtained interesting managerial findings. With the investment in reducing the carbon emission from different services - manufacturing and haulages of the supply chain, CO_2 ejection can be reduced by about 73% simultaneously increasing the profit by 15%. Additionally, an investment in setup cost is also considered. Henceforth, a comparison is drawn with Chauhan et al. (2021) and Batarfi et al. (2016), reflecting the more economic benefits of the present model. As the variation in the selling price of the "core and personalized products" overshoot the threshold limit then there is approximately a 19% increase in the shifting of customers between the channels.

This study has several limitations, and opportunities exist to extend this chapter in the future. In this model, "a dual product and single retailer is involved in the supply chain, and discussing a multi-product supply chain with multiple retailer in which regular and green products compete for market share would be interesting" (Zhang et al. , 2020). Additionally, for more realistic output under uncertain environment, all the cost parameters and demand can be considered to be fuzzy number (Sarkar et al. , 2019).

CHAPTER - 8

Summary and Conclusion

8.1 Introduction

The summary and conclusion of the entire work, with suggestions and recommendations for further research, are furnished in this chapter.

8.2 Summary

A "dual-channel supply chain management" under "price-dependent stochastic demand" incorporates the online channel together with the offline channel. In this respect thesis assumes online-to-offline (O2O) business model. The model integrates the convenience of online payment of online shopping and the good experience of offline stores and thus brings large development space. Both channels are offered to customer's. The standard product is provided through the offline channel, and "personalized demand" is catered through the online channel, to cover a wide range of customers.

In the first chapter of this thesis, "supply chain management" is introduced. It demonstrates that the supply chain not only includes the supplier and manufacturers, but storehouses, distributors, transporters, and finally consumers are also included in it. Further, it also exhibits the need for understanding all the parameters related to products starting from raw material to finished product as well as the demands of customers. The second chapter of the thesis highlights the benefits of upgrading to a dual-channel supply chain as the enhancement of the "single-channel supply chain" with the introduction of "personalized products" and online channels is discussed. It also includes a statement of the problem, motivation of the study, scope, and limitations, as well as definitions of some basic terms. Details of the proposed algorithms and analyses, as well as numerical experiments, were given in Chapters three through eight. The numerical experiments conducted in this work show the efficacy and performance of the proposed methods. Models in the thesis also examine the carbon emission carried out during the production and transportation of the products. Additionally, penalties are imposed on the models in case they overshoot the pre-defined limits for carbon emission. Furthermore, the methods presented in Chapters Fourth, Fifth, and Eighth are successfully applied to handle uncertain demand and exhibit better results than the existing methods.

8.3 Conclusion

This thesis incorporates the online channel in traditional supply chain management to cover a wide range of customers. In Chapter Three, a threshold limit was introduced, which moderates the variation in the selling price of the "personalized and standard products". Consequently, if the variation in the price is lying within the presumed limit (Threshold limit) then no swapping in demand takes place. A comparison is drawn between "dual-channel and single-channel supply chains" to exhibit the efficacy of the model. In Chapter Four, the model expanded by incorporating sustainable pillars such as economic and environmental. The results of the model are compared with the results of Chauhan et al. (2021) and Modak and Kelle (2018). Chapter Five presented an investment in quality improvement which helps the system to transfer from an "out-of-production" to an "in-control" state. Thus, improved quality is observed with a financial investment. Chapter Six addressed the uncertainty in demand with the help of a "triangular fuzzy number", which was further expanded by incorporating the third pillar of sustainability namely, "social sustainability" in Chapter Seven. In Chapter Eight, two types of investment for reducing carbon emission and setup costs were incorporated in the modified "dual-channel supply chain model" in the glass manufacturing industry. Finally, a model is compared with the model of Batarfi et al. (2016) and Chauhan et al. (2021). The numerical experiments conducted in this work show the efficacy and performance of the proposed methods. In addition, the methods presented in Chapters Fourth, Fifth, and Eighth are successfully applied to handle uncertain demand and exhibit better results than the existing methods.

The numerical experiments conducted in this work show the efficacy and performance of the proposed methods. In addition, the methods presented in Chapters Fourth, Fifth, and Eighth are successfully applied to handle uncertain demand and exhibit better results than the existing methods.

Future Research

The present model can be further extended by considering multiple retailers and consignment policies or vendor-managed inventory (VMI), amalgamating with sustainable development, and many more. Fair-minded competition between the retailers can be exhibited by taking into account a "multi-retailer dual-channel supply chain system". A consignment strategy or VMI in a "centralized dual-channel supply chain" under enhanced personalization is an innovative research.

There are noteworthy investigations that require a revisit of this thesis. Slowing down the depletion of natural resources and the pollution associated with it is a crucial concern in the context of "sustainable development" and the preservation of the environment. This leads to an investigation of product reuse, and material recycling and all this comes at a cost and has an impact on the environment.

"Customization" of products in a vendor-managed inventory or consignment contract in a "dual-channel supply chain" can be new research. Moreover, the concept of a variable demand driven by selling cost and advertising costs, where all basic costs are considered fuzzy can also be incorporated.

LIST OF PUBLICATIONS

PUBLISHED PAPER

S/N	Title	Journal	Remarks
1.	A modified customization strategy in a dual-channel supply chain model with price sensitive stochastic demand and distribution-free approach	Mathematical Problems in Engineering SCIE journal **(Impact factor: 1.305)**	Published (2021)
2.	Carbon Footprint and Setup Cost Reduction in Glass Manufacturing Industry and its Raw Material Supply Chain	Paper accepted in 3rd International Conference on Functional Materials, Manufacturing and Performances **(Impact factor: 0.402)**	Accepted (2022)
3.	The impact of adopting customization policy and sustainability for improving consumer service in a dual-channel retailing	Journal of Retailing and Consumer Services **(Impact factor: 10.972)**	Published (2023)

COMMUNICATED ARTICLES

S/N	Title	Journal	Remarks
1.	Involvement of carbon regulation in a smart dual-channel supply chain for customized products under uncertain environment	Environment, Development and Sustainability **(Impact factor: 4.080)**	Under second revision
2.	A sustainable dual-channel supply chain management under uncertain conditions	Mathematical Problems in Engineering **(Impact factor: 1.305)**	Under second revision

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