# THEORETICAL INVESTIGATION ON SELF ACTION EFFECTS AND STIMULATED RAMAN SCATTERING OF INTENSE LASER BEAMS IN PLASMAS WITH TEMPERATURE AND DENSITY RAMPS

Thesis Submitted for the Award of the Degree of

**DOCTOR OF PHILOSOPHY**

**In**

**PHYSICS**


**By**

**SANJEEV KUMAR**

**Registration Number: 42000185**

**Supervised by**

**Dr. Naveen Gupta (UID: 21559)**

**Assistant Professor**

**Department of Physics**

**Lovely Professional University**

**Phagwara, Punjab, India**

**Co-Supervised by**

**Dr. Shri Bhagwan Bhardwaj**

**Assistant Professor**

**Department of Physics**

**S.U.S. Government College**

**Matak Majri, Indri, Karnal, (HR)**



**LOVELY PROFESSIONAL UNIVERSITY, PUNJAB**

**2023**

# DECLARATION

I hereby declared that the presented work in the thesis entitled **"Theoretical Investigation on Self Action Effects and Stimulated Raman Scattering of Intense Laser Beams in Plasmas with Temperature and Density Ramps"** in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy (Ph.D.) in Physics** is the outcome of original research work carried out by me under the supervision of **Dr. Naveen Gupta**, working as Assistant Professor in the Department of Physics, Lovely Professional University, Phagwara, Punjab, India and co-supervision of **Dr. Shri Bhagwan Bhardwaj**, working as Assistant Professor in the Department of Physics, S.U.S. Government College, Matak Majri, Indri, Karnal, Haryana, India. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

**Sanjeev Kumar**

Registration No.- 42000185

Department of Physics

Lovely Professional University

Phagwara, Punjab, India

# CERTIFICATE

This is to certify that the work reported in the Ph.D. thesis entitled **"Theoretical Investigation on Self Action Effects and Stimulated Raman Scattering of Intense Laser Beams in Plasmas with Temperature and Density Ramps"** submitted in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy (Ph.D.)** in **Physics** is a research work carried out by **Mr. Sanjeev Kumar, Registration No. 42000185**, is bonafide record of his original work carried out under my supervision and that no part of the thesis has been submitted for any other degree, diploma or equivalent course.

**Supervisor**

Dr. Naveen Gupta
Assistant Professor
Department of Physics
Lovely Professional University
Phagwara, Punjab
India

**Co-Supervisor**

Dr. Shri Bhagwan Bhardwaj
Assistant Professor
Department of Physics
S.U.S. Government College
Matak Majri, Indri, Karnal
Haryana, India

## *Abstract*

The main focus of this thesis on nonlinear propagation of $q$-Gaussian and Cosh-Gaussian (ChG) laser beams in nonlinear medium such as plasmas. Further, the impacts of self action effects and stimulated Raman scattering (SRS) of laser beams in plasmas with temperature and density ramps, have been studied theoretically. In the midst of different existing semi-analytical approaches, variational theory approach has been used to investigate the propagation of $q$-Gaussian and Cosh-Gaussian (ChG) laser beams in nonlinear medium such as underdense plasmas under nonlinear regime.

Theoretical investigation has been conducted on the interaction between a $q$-Gaussian laser beam profile and underdense plasmas with temperature ramp and axial density ramp. The study incorporates the influence of self-focusing and focuses specifically on SRS. The nonlinear propagation of $q$-Gaussian laser beam in plasma is influenced by two factors: DP '$q$' and the ellipticity of the beam. It has been seen that the self-focusing is most pronounced for $q = 3$, while it decreases in the transverse direction where the beam is more elliptical. Increasing the value of $q$, which moves the $q$-Gaussian beam closer to an ideal Gaussian distribution, leads to reduced cross-focusing. Consequently, the effect of off-axial rays diminishes, resulting in a decrease in the intensity of the pump and, subsequently, a decrease in SRS. However, SRS increases when the slope of the temperature and density ramps is increased. This is due to the self-focusing of the laser beam intensifying with higher slope values, combined with a longer propagation distance.

Theoretical investigations have been conducted to explore the phenomenon of SRS in the context of intense self-focused ChG laser beams interacting nonlinearly with plasmas featuring an axial density ramp. The study focuses on the impact of the laser beam's irradiance profile on its nonlinear interaction with the plasma. It has been observed that the flatness of the laser beam's irradiance profile plays a crucial role in enhancing the coupling of laser energy with the plasma by promoting self-focusing. However, this simultaneous increase in self-focusing also leads to an elevation in SRS reflectivity of the plasma.

One key factor influencing the results is the parameter $b$ associated with the cosh function, which is commonly known as the cosh factor or decentered parameter of the laser beam. The investigation reveals that as the nonlinear refraction of the laser beam reaches a state of equilibrium for self-focusing, an increase in the value of $b$ within the range of $0 \leq b \leq 1$ enhances the extent of self-focusing. At $b = 1$, the beam width is minimized as the contributions from off-axial rays become balanced. However, for $b > 1$, the presence of a central dark region prevents the ChG laser beam from receiving contributions from the axial part of the wave fronts, resulting in reduced nonlinear refraction due to the growing size of the central dark region.

*# Dedication*

***"Every challenging work needs self-efforts***

***As well as***

***Guidance, support and motivation of some special peoples"***

*I dedicate my humble efforts to special persons of my life*

*Who were very close to my heart during this research work journey*

***My Mother** (Mrs. Raj Kumari) and **Father** (Mr. Tara Chand)*

*And*

***My Wife** (Mrs. Kiran) and **Lovely Son** (Yash Panwar)*

*And*

***My Brother** (Mr. Sunil Kumar) and **His Wife** (Mrs. Seema Devi)*

*And*

***My Sisters** (Mrs. Monika Devi, Mrs. Sonia Devi) and **Their Life Partners***

*And*

***My Shining Stars** (Dimple, Mayank, Sushant, Chirag, Yash, Divyanshi, Vani, Raghav, Vansh)*

*And*

***My In Laws** and **All Relatives***

*And*

***Along With Hard Working** and **Respected Teachers***

*Who had made my base strong and ignite the spark for doctorate degree,*

*Due to which I am able to survive in education field.*

## *Acknowledgment*

*It is a best opportunity to acknowledge some special persons of my life for their unconditional kind support, guidance and motivation throughout this research degree awarded in physics.*

*First and foremost, I am delighted to express my gratitude and thanks to my research supervisor* ***Dr. Naveen Gupta*** *and co-supervisor* ***Dr. Shri Bhagwan Bhardwaj*** *for their irreplaceable guidance, consistent encouragement, immeasurable care and critical review throughout this research work. Their mastery of scientific questioning, interpretation, imaginations and suggestions enhanced my cognitive awareness and helped me remarkably in the fulfillment of my proposed research objectives.*

*From the bottom of my heart, I deeply express my gratitude to my family for all their unlimited love, care and remarkable support for pursuit of my scientific career. They had encouraged me to follow my passion. Especially, my parents, my wife and my loving son had become my backbone during these years. I value their faith and patience for such a long period. The love and affection, faith, sacrifices of my all family members always strengthen me.*

*I am so thankful to my close friends and colleagues who have always supported me. Especially, sincere thanks to* ***Dr. Durgesh*** *and* ***Dr. Harneet Kaur*** *for their amazing support and encouragement. It is my pleasure to acknowledge my companion* ***Ms. Suman Choudhry*** *for her invaluable support during this period.*

*I am glad to express my gratitude to* ***Department of Physics*** *and* ***Department of Research and Development of Lovely Professional University*** *for their general support. Especially, I am grateful to Dr. Kailash Chandra Juglan and Dr. Niti Kant for their kind encouragement during these years.*

***Sanjeev Kumar***

## *Preface*

The main focus of this thesis especially on the self-action effects and SRS of intense laser beams ($q$-Gaussian and Cosh-Gaussian (ChG) profile) in plasmas with temperature and density ramps. The present research work has been divided into total nine chapters. The nonlinear Schrodinger wave equation for the evolution of beam envelope is solved numerically by using variational theory. In particular,

**Chapter- 1** incorporates the introduction part which involves the scattering of light and its brief history, types of scattering, Raman scattering of light and its types, SRS, difference between laser action and SRS, mechanism of SRS in plasma, role of SRS in inertial confinement fusion (ICF), self-focusing of laser beam, role of self-focusing in SRS, and mechanisms of optical nonlinearity (ponderomotive nonlinearity, ohmic heating and relativistic nonlinearity) in plasma.

C**hapter- 2** includes the literature review of different research papers which are based on self-action effects, stimulated Raman scattering and variational theory approach.

C**hapter- 3** involves the description of the amplitude structures of $q$-Gaussian and cosh-Gaussian (ChG) laser beam profiles and their effective beam width with different laser-plasma parameters in detail, variational theory approach and research objectives.

**Chapter- 4** centers on the examination of self-action effects exhibited by elliptical $q$-Gaussian laser beams in plasmas featuring an axial temperature ramp. These effects are further explored under the influence of the ponderomotive force. Additionally, significant attention is given to investigating the axial phase and dynamics of the beam width of these elliptical $q$-Gaussian laser beams. Various laser-plasma parameters are considered, and the corresponding differential equations are numerically solved through the application of variational theory.

**In Chapter- 5**, the emphasis is placed on examining the self-action effects of cosh-Gaussian (ChG) laser beams. This analysis is conducted within the context of axially inhomogeneous underdense plasmas featuring an upward density ramp.

**In Chapter- 6**, the focus is on studying the SRS of an elliptical $q$-Gaussian laser beam in plasma. This investigation takes into account the presence of an axial density ramp, as well as the impact of the ponderomotive force on the system. Additionally, the chapter explores the changes in the width of the $q$-Gaussian laser beam, the electron plasma wave (EPW), the scattered wave, and the SRS reflectivity of plasma. Various laser-plasma parameters and density profiles are considered in this analysis.

**Chapter- 7** explores the examination of the excitation of electron plasma waves (EPW) in collisionless underdense plasmas. The investigation focuses on the self-focusing of cosh-Gaussian laser beams, the density ramp that increases upward, and the evolution of the cosh-Gaussian laser beam width. The variational theory method is employed to analyze the effects of different laser-plasma parameters.

**Chapter- 8** explores the examination of SRS in plasma by analyzing self-focused cosh-Gaussian laser beams. The investigation specifically focuses on the axial density ramp and the changes in the width of the cosh-Gaussian laser beam, as well as the electron plasma wave (EPW), scattered wave, and SRS reflectivity of the plasma. This exploration is conducted using the variational theory method while considering various laser-plasma parameters and density profiles.

**Chapter- 9** includes conclusions and future scope.

# List of Research Paper Publications

1. Naveen Gupta, Suman Choudhry, S. B. Bhardwaj, Sanjeev Kumar and Sandeep Kumar, "Relativistic effects on Stimulated Brillouin Scattering of self-focused *q*-Gaussian laser beams in plasmas with axial density ramp", Journal of Russian Laser Research 42, 418-429 (Published online: 5 July 2021).
2. Naveen Gupta, Sandeep Kumar, A Gnaneshwaran, Sanjeev Kumar, Suman Choudhry, "Self-focusing of cosh-Gaussian laser beam in collisional plasma: effect of nonlinear absorption", Journal of Optics 50, 701-711 (Published online: 11 Aug. 2021).
3. Naveen Gupta, Sanjeev Kumar, S. B. Bhardwaj, "Stimulated Raman scattering of self focused elliptical *q*-Gaussian laser beam in plasma with axial temperature ramp: effect of ponderomotive force", Journal of Electromagnetic waves and Applications 36 (6), 767-786 (2022) (Published online: 29 Sept. 2021).
4. Naveen Gupta, Sandeep Kumar, A Gnaneshwaran, S. B. Bhardwaj, Sanjeev Kumar, Suman Choudhry, "Nonlinear interaction of quadruple Gaussian laser beams with narrow band gap semiconductors", Journal of optics 51, 269-282 (2022) (Published online: 07 Oct. 2021).
5. Naveen Gupta, Suman Choudhry, Sanjeev Kumar, S. B. Bhardwaj, Sandeep Kumar, Gyanesh, Siddhanth Shishodia and Kishore B, "Scattering of Laser Light in Dielectrics and Plasmas: A Review", NLOQO 55 (1-2), 1-44 (01 Jan. 2022).
6. Naveen Gupta, Sanjeev Kumar, S. B. Bhardwaj, "Stimulated Raman scattering of self-focused elliptical *q*-Gaussian laser beam in plasma with axial density ramp: effect of ponderomotive force", Journal of Optics 51, 819–833 (2022) (Published online: 29 Jan. 2022).
7. Naveen Gupta, Rohit Johari, Sanjeev Kumar, S. B. Bhardwaj, Suman Choudhry, "Optical guiding of q-Gaussian Laser beams in radial density plasma channel created by two prepulses: Ignitor and Heater", Journal of Optics 51, 749-760 (Published online: 06 March 2022).
8. Naveen Gupta, Sanjeev Kumar, Suman Choudhry, S. B. Bhardwaj, Sandeep Kumar, "Potential Well Dynamics of Self Focusing of Quadruple Gaussian Laser Beams in Thermal Quantum Plasma", NLOQO 55, 3-4, 281-308 (01 April 2022).

9. Naveen Gupta, Suman Choudhry, S. B. Bhardwaj, Sanjeev Kumar, "Excitation of ion acoustic waves by self-focused q-Gaussian laser beam in plasma with axial density ramp", Journal of Optics 52, 269–280 (Published online: 07 May 2022).
10. Naveen Gupta, Sanjeev Kumar, SB Bhardwaj, Rohit Johari, Suman Choudhry, "Electron plasma wave excitation by self-focused cosh gaussian laser beams in axially inhomogeneous plasma: effect of density ramp", Journal of Optics 1-10, (Published online: 28 July 2022).
11. Naveen Gupta, Suman Choudhry, Sanjeev Kumar, SB Bhardwaj, "Self-focusing of laser-driven ion acoustic waves in plasma with axial density ramp" Journal of Optics 1-7, (Published online: 03 Nov 2022).
12. Naveen Gupta, SB Bhardwaj, Sanjeev Kumar, Suman Choudhry, Rishabh Khatri, Siddhanth Shishodia, Rohit Johari, "Second-harmonic generation of two cross-focused *q*-Gaussian laser beams by nonlinear frequency mixing in plasmas", Journal of Optics 1-12, (Published online: 05 Nov. 2022).
13. Naveen Gupta, Sanjeev Kumar, SB Bhardwaj, "Stimulated Raman Scattering of Cosh-Gaussian Laser Beams in Plasma with Axial Density Ramp: Effect of Self-Focusing", Journal of Russian Laser Research 43, 667–677 (21 Nov. 2022).
14. Naveen Gupta, Sanjeev Kumar, SB Bhardwaj, "Excitation of Electron Plasma Wave by Self Focused Cosh-Gaussian Laser Beams in Collisionless Plasmas: Effect of Density Ramp", Journal of Applied Spectroscopy 90, 160–169 (13 March 2023).
15. Naveen Gupta, Alex AK, Rohit Johari, Sanjeev Kumar, S. B. Bhardwaj, A. Saini, "Stimulated Raman scattering of self-focused Laguerre–Gaussian laser beams in axially inhomogeneous plasma", Journal of Optics 1-12, (Published online: 08 May 2023).
16. Naveen Gupta, A. K. Alex, Rohit Johari, Suman Choudhry, Sanjeev Kumar, Aatif Ahmad & S. B. Bhardwaj, "Formation of elliptical *q*-Gaussian breather solitons in diffraction managed nonlinear optical media: effect of cubic quintic nonlinearity", Journal of Optics (Published online: 18 August 2023).
17. Naveen Gupta, Sanjeev Kumar, S. B. Bhardwaj, "Effect of Self Focusing on Stimulated Raman Scattering of Elliptical q-Gaussian Laser Beam in Underdense

Plasma with Axial Density Ramp", NLOQO 58, 1-2, 99-126 (Published online: 12 Sept. 2023).

**18.** Naveen Gupta, Rohit Johari, Sanjeev Kumar, Suman Choudhry, S. B. Bhardwaj, A. K. Alex, "Excitation of upper hybrid wave by cross focused *q*-Gaussian laser beams in graded index plasma channel", Journal of Optics (Published online: 23 Sept. 2023).

## <u>Conferences/Seminars</u>

**1.** Presented a research paper titled "Excitation of Electron Plasma Wave by Cosh-Gaussian Laser Beams in Underdense Plasma Targets: Effects of Self Focusing and Axial Density Ramp" in International Conference on "Frontiers in Physics, Materials Science and Nanotechnology" organised by Department of Physics, Chaudhary Devi Lal University, Sirsa on 25-26 March, 2022.

**2.** Presented a research paper titled "Stimulated Raman Scattering of Self Focused Cosh-Gaussian Laser Beams in Underdense Plasma Targets: Effect of Density Ramp" in The 23rd International Young Scientists Conference Optics and High Technology Material Science - SPO 2022 organised by Taras Shevchenko National University of Kyiv & Shizuoka University on 24-26 November 2022.

**3.** Presented a research paper titled "Stimulated Raman Scattering of Intense Cosh-Gaussian Laser Beams Interacting with Axially Non-Uniform Underdense Plasmas: Effect of Density Ramp and Self Focusing" in International Symposium on Recent Trends in Optical Materials and Photonic Devices organised by Department of Pure & Applied Physics, Guru Ghasidas Vishwavidyalaya, Bilaspur, INDIA on Dec. 7, 2022.

**4.** Presented a research paper titled "Stimulated Raman Scattering of self focused elliptical *q*-Gaussian laser beam in plasma: Effect of Density Ramp" in the International Conference of Students and Young Researchers in Theoretical and Experimental Physics "HEUREKA-2023" organised by Faculty of Physics, Ivan Franko National University of Lviv on May 16-18, 2023. Lviv (Online), Ukraine.

# TABLE OF CONTENTS

## List of Figures

## Chapter- 1

## Scattering of Light

*And God said: Let there be light*

*Thus there was Optics.*

*Einstein said: Let it be quantized;*

*thus there was LASER.*

*Whose light is so exotic*

*Converting conventional Optics to Nonlinear Optics.*

### 1.1 Introduction

Ever since the human civilization, light has continuously fascinated human beings and the investigations on light matter interactions are as old as human civilization[1]. However, laser[2] as an extraordinary source of light changed the entire scenario of light matter interactions. Laser light's distinctive properties like high intensity, very narrow beam of light with very high spectral purity**,** helped in revealing true beauty of light interaction with matter[3-5]. Resulting from the chirped pulse amplification technique[1,6], the revolution in laser technology has given a new birth to the field of interaction of light with matter by pushing linear optics to nonlinear optics.

Prior to the advent of lasers, transparent optical materials were traditionally regarded as inert, meaning they were expected to remain unaffected by the passage of light through them. However, the development of lasers and their ability to generate high power densities unveiled a fascinating phenomenon: the intense light produced by lasers can actually modify the optical characteristics of the material they traverse. This includes alterations in properties such as the index of refraction or the absorption of an optical beam within the medium. Traditionally an optical medium respond nonlinearly to an optical beam when the motion of its valence electrons becomes an harmonic under the influence of incident optical beam[7,8]. In this situation, the dielectric susceptibility $\chi$ needs to be written as power series of the electric field of the optical beam as (fig.1.1) $\chi = \chi^{(1)} + E\chi^{(2)} + E^2\chi^{(3)} + \cdots$


Fig. 1.1: Nonlinear susceptibility.

The $\chi^{(1)}$ term is dielectric susceptibility corresponds to the susceptibility at ordinary power densities. The $\chi^{(2)}$ term which depends linearly to the optical field, gives the second order polarization of the medium which has quadratic dependence on field strength (as $P = \epsilon_0 \chi E$). The second order polarization of the medium results in excitation of second order over tones of the incident beam (a phenomenon known as second harmonic generation), appearance of static voltages across the medium i.e., optical rectification. If more than one beam propagate simultaneously through the medium then the second order polarization results in two more peculiar effects. It can result into third beam whose frequency is sum or difference of the incident frequencies. Similarly, higher order terms result in excitation of higher overtones of incident beam[9-12] (fig.1.2).

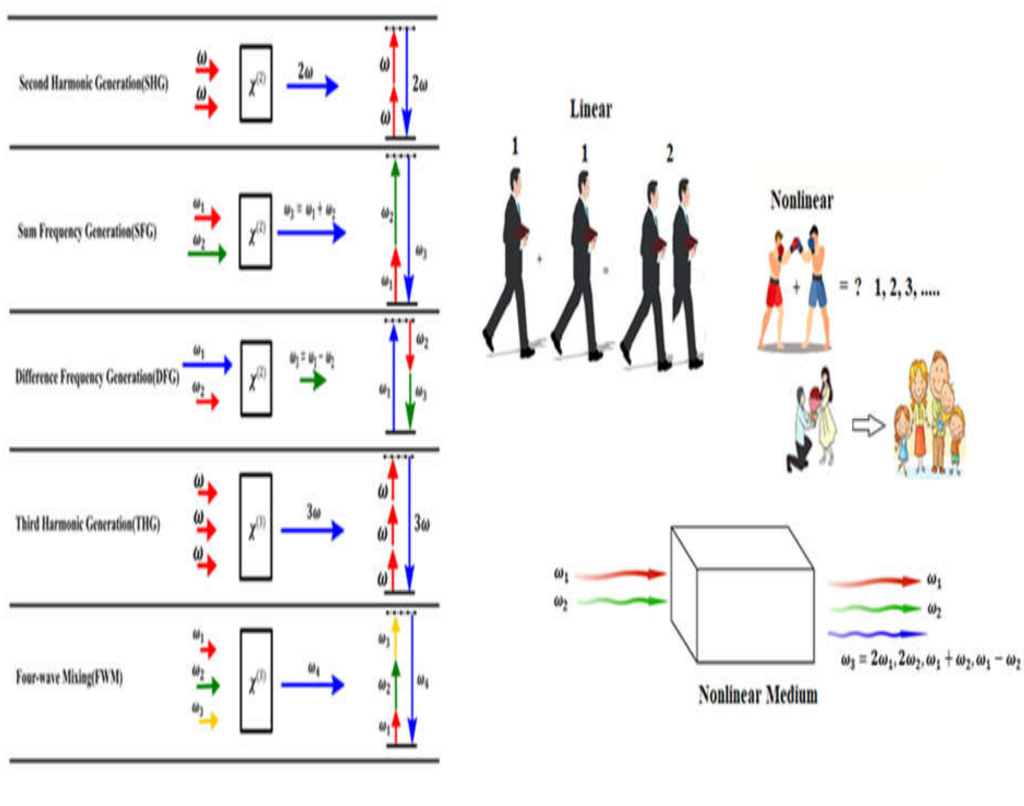



Fig. 1.2: Frequency mixing processes in nonlinear media

Apart from frequency mixing processes, nonlinear optics is rich in copious nonlinear phenomena like self focusing[13], SRS[14], SBS[15] etc. (fig.1.3).

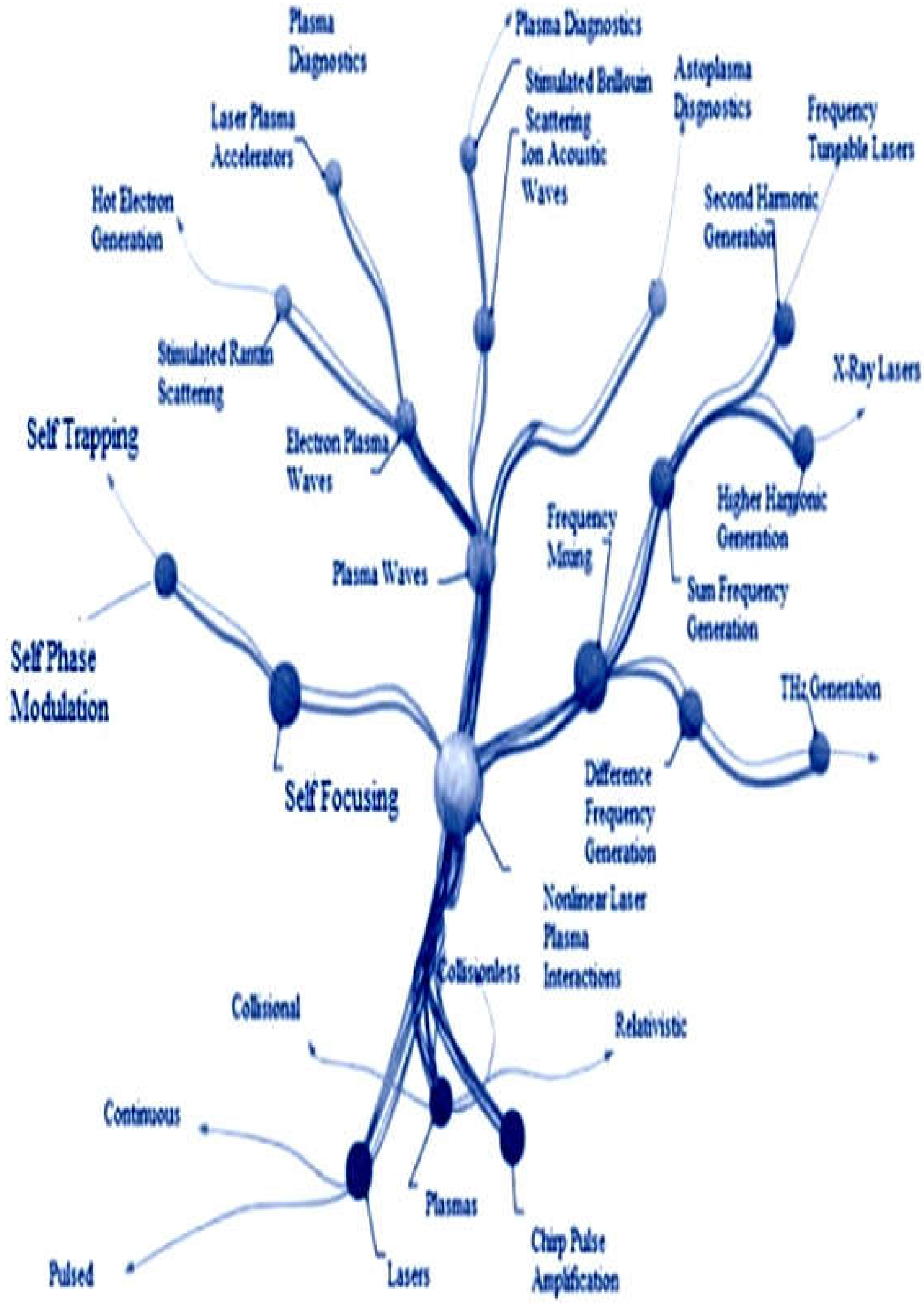



Fig. 1.3: Tree diagram for various nonlinear optical phenomena.

### 1.2 Scattering of Light

Scattering of light is the phenomenon by virtue of which the path of light gets randomized or its frequency gets changed by small amount upon its encounter with some material object[15] (fig. 1.4).

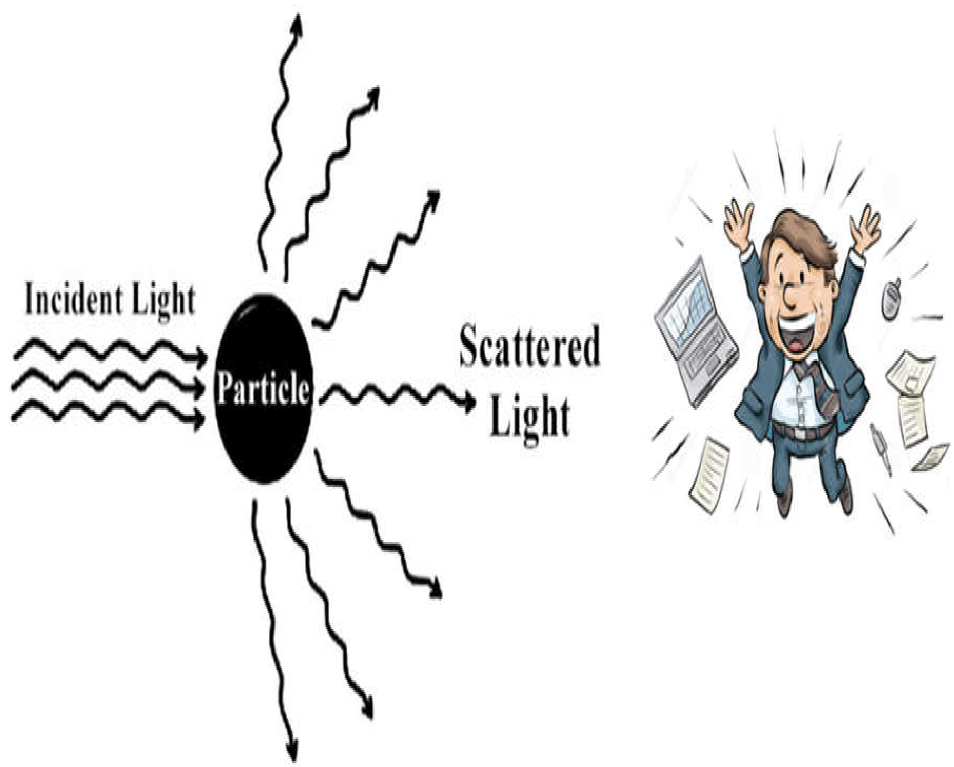



Fig. 1.4: Scattering of light.

It occurs as a consequence of fluctuations in the refractive properties of the medium or due to interaction of light with bosonic excitations of the medium.

#### 1.2.1 Types of Scattering

Scattering of light can be inelastic or elastic depending on whether its frequency changes or not.

Elastic scattering: Rayleigh scattering.

Inelastic scattering: Raman scattering, Brillouin scattering, Rayleigh wing scattering.

The spectrum of different types of scattering processes is shown in fig.1.5.

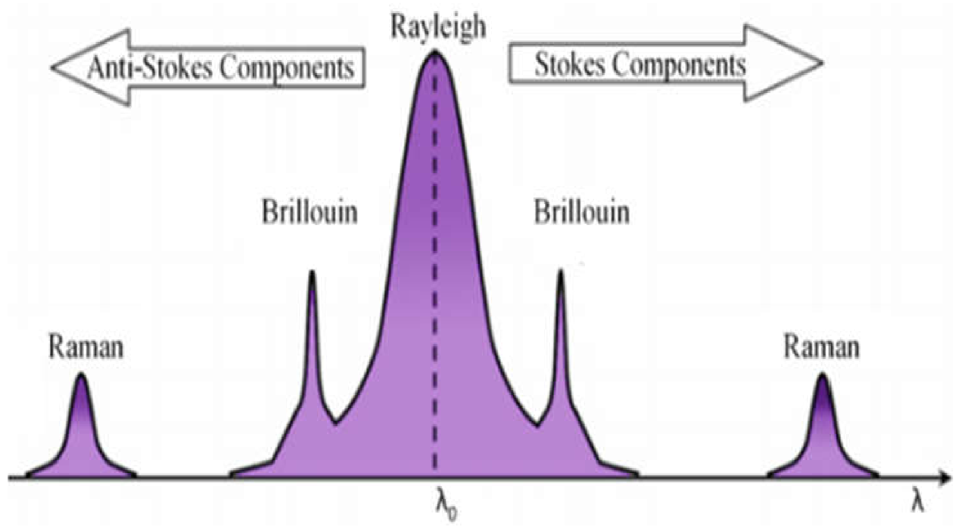



Fig. 1.5: Spectrum of scattering

**1.2.2 Spontaneous vs Stimulated Scattering**

Similar to emission of light that occurs due to electronic transitions, scattering can be spontaneous or stimulated depending upon whether it occurs by its own or is triggered by external photon. Scattering of light is said to be spontaneous if the medium remains unaffected during the propagation of light i.e., the bosonic excitations of the medium are neither damped nor amplified due to their interaction with light. In case of stimulated scattering, the optical properties of the medium get modified due to presence of light[15]. In other words, in the stimulated version of scattering, the bosonic excitations of the medium get amplified. Stimulated scattering differs from spontaneous scattering in several other aspects like:

1. It is observed at high intensity i.e., there exists a certain threshold intensity below which scattering will be spontaneous.
2. It occurs for radiations with very narrow spectral width i.e., for quasi monochromatic radiations.
3. It is highly coherent process in contrast to spontaneous scattering which is totally non coherent.
4. It does not involve any anti Stoke's component.

### 1.3 Brief History of Scattering of Light

The major milestones in the history of scattering are shown in fig.1.6

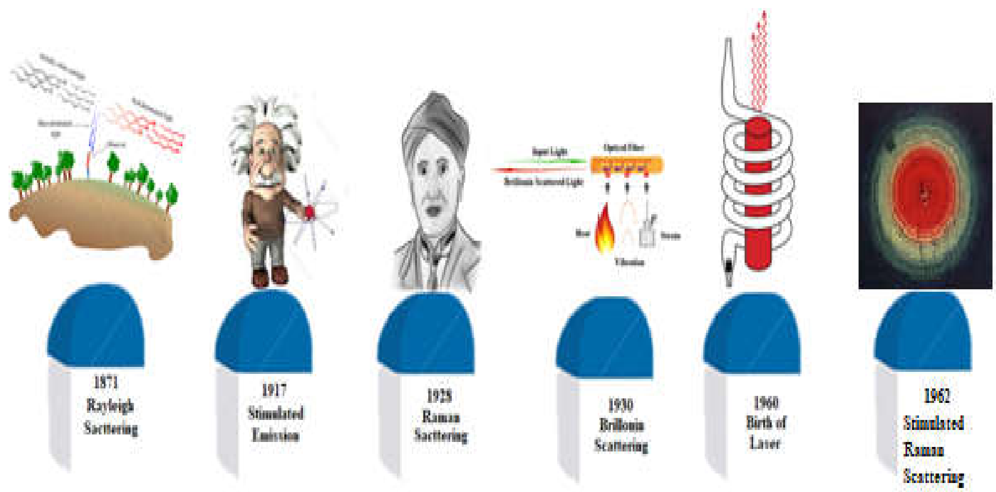



Fig. 1.6: Milestones in the history of scattering.

The most common scattering known as Rayleigh scattering is the scattering of sunlight by the atoms or molecules of the gases present in the earth's atmosphere. It was observed by Lord Rayleigh in 1871 that explains the blue colour of sky and red colour of sunset. Raman scattering occurs due to interaction of light with optical phonons which was discovered in 1928 by Sir C.V. Raman[16] who got Nobel Prize in Physics for his discovery in 1930. Brillouin predicted light scattering[17] with acoustic phonons in 1922 which was verified experimentally in 1930 by Gross. In 1960, T. H. Maiman gave the first experimental demonstration of working laser at a wavelength of 6943 Å and negative temperature were reported by applying optical pumping technique to a ruby as active medium in the laboratory. Within two years of laser's demonstration, E. J. Woodbury of Hughes Aircraft in 1962 accidently observed another version of Raman scattering known as SRS[18]. He was working with a pulsed laser whose shutter consisted of nitrobenzene Kerr cell. He observed that in addition to the ruby wavelength 6,943 Å, the output beam contained additional wavelengths shifted towards the red side of the electromagnetic spectrum. The interesting fact about these new wavelengths was their coherence and directionality like the incident laser beam i.e., these new wavelengths possessed the characteristics of lasers.

### 1.4 Raman Scattering

Raman scattering occurs due to interaction of light with optical phonons.

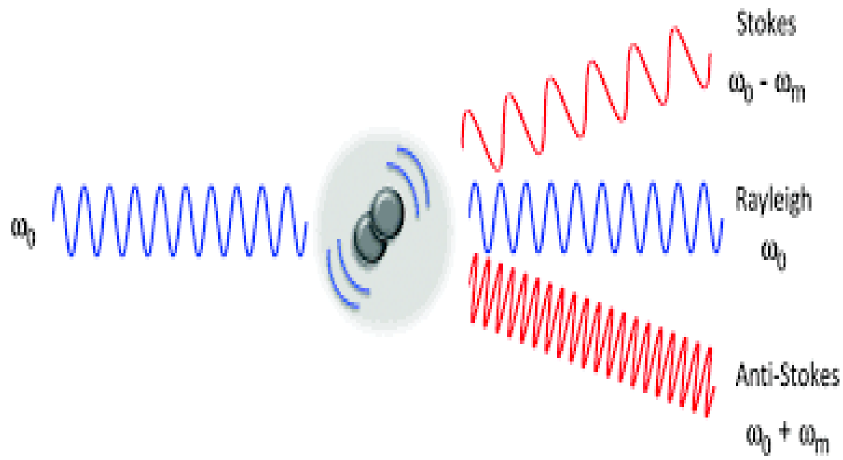
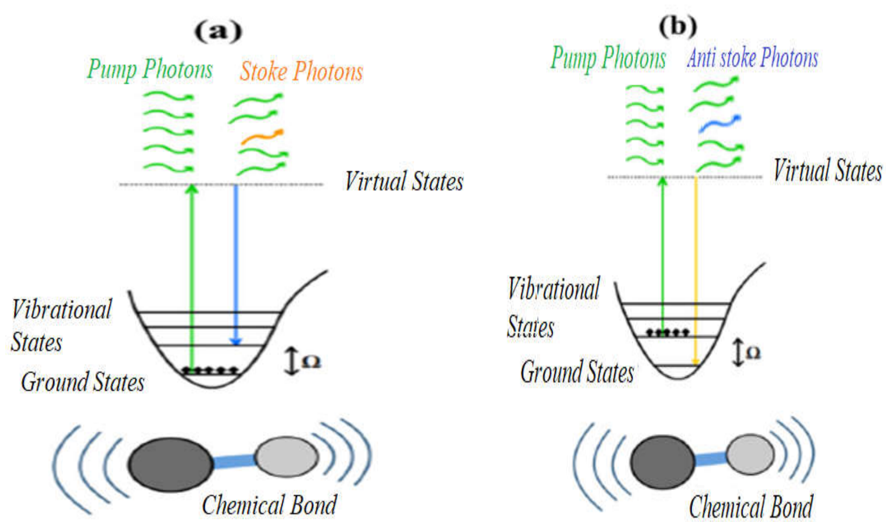



Fig. 1.7: Raman scattering from vibrational modes of molecules.

Equivalently, we can see that it is the scattering of light due to quantized molecular vibrations of the medium (Fig. 1.8).



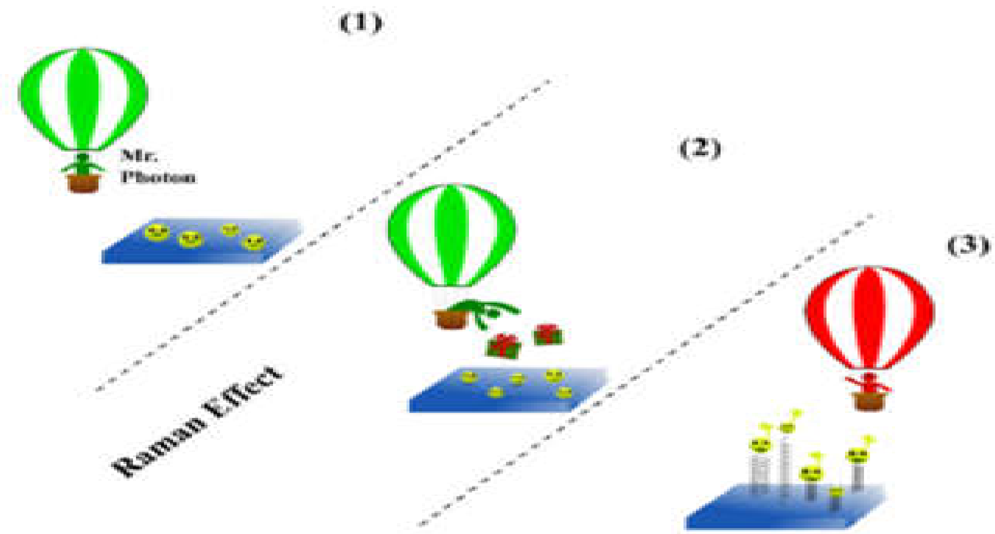



Fig. 1.8: (a) Stoke's and (b) Anti Stoke's components of Raman Scattering.

It can also be considered as scattering of light due to optical phonons. When light is scattered by a medium, most of the scattered photons remain in same energy state and same frequency as the incident photons. Depending on how the light interacts with matter, such an elastic scattering can occur through a variety of mechanisms. However, often a very small fraction of the incident light (less than one photon in $10^6$) is scattered at energies that differs from those in the incident beam. The wavelengths of these photons are determined by changes in the energy states of the scattering molecule. This type of inelastic scattering is called as Raman scattering and the effect is called as Raman Effect.

**1.5 Stimulated Raman Scattering (SRS)**

SRS is coherent Raman scattering processes that, compared to spontaneous Raman scattering processes, provides an amplification of about $10^7$. SRS occurs when low power beam (probe beam) gain energy from high power beam (pump beam). In particular, this exchange of energy occurs when frequencies of pump and stokes beam differ by the vibrational frequency of the molecules of the sample under consideration.

In stimulated version of Raman scattering, a beam of light at frequency $\nu_s$ is incident on the molecule along with a beam at $\nu_L$. As a consequence of scattering, a quantum $h\nu_s$ is added to the beam at $\nu_s$, which as a result gets amplified, while the incident beam losses a quantum $h\nu_L$by lifting the molecule to an excited state differing in energy by $h(\nu_L - \nu_s)$.

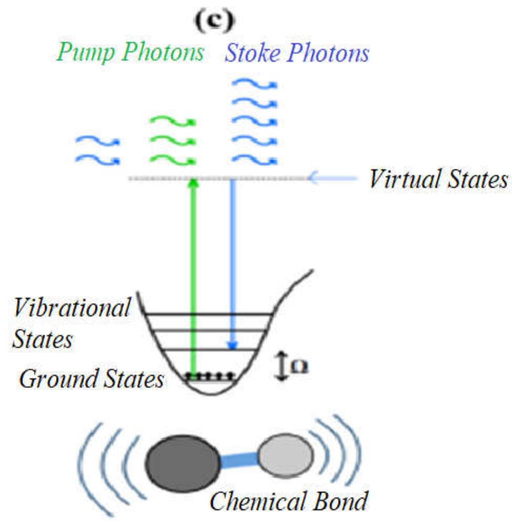



Fig. 1.9: Stimulated Raman Scattering.

SRS was detected accidentally by E. J. Woodbury[18] working at Hughes Aircraft. He was experimenting on interaction of a giant laser pulse with a Kerr cell containing nitrobenzene.

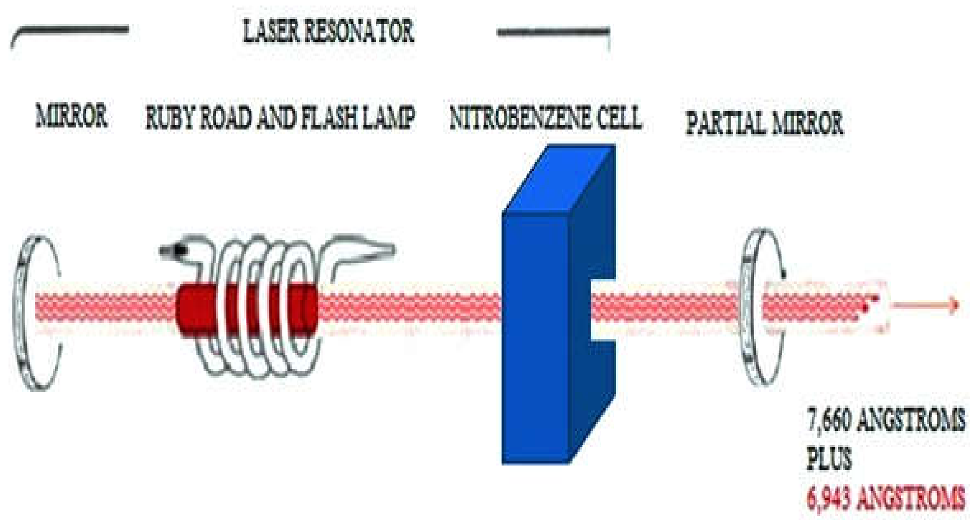



Fig. 1.10: Experimental setup for SRS

In response to an electric field of light, the shutter either allows light to pass or blocks it. Woodbury observed that in addition to the incident wavelength of 6,943 Å from Ruby laser, the output beam contained additional red shifted wavelengths. These additional wavelengths possessed the same properties as that of laser beam i.e., coherence and directionality.

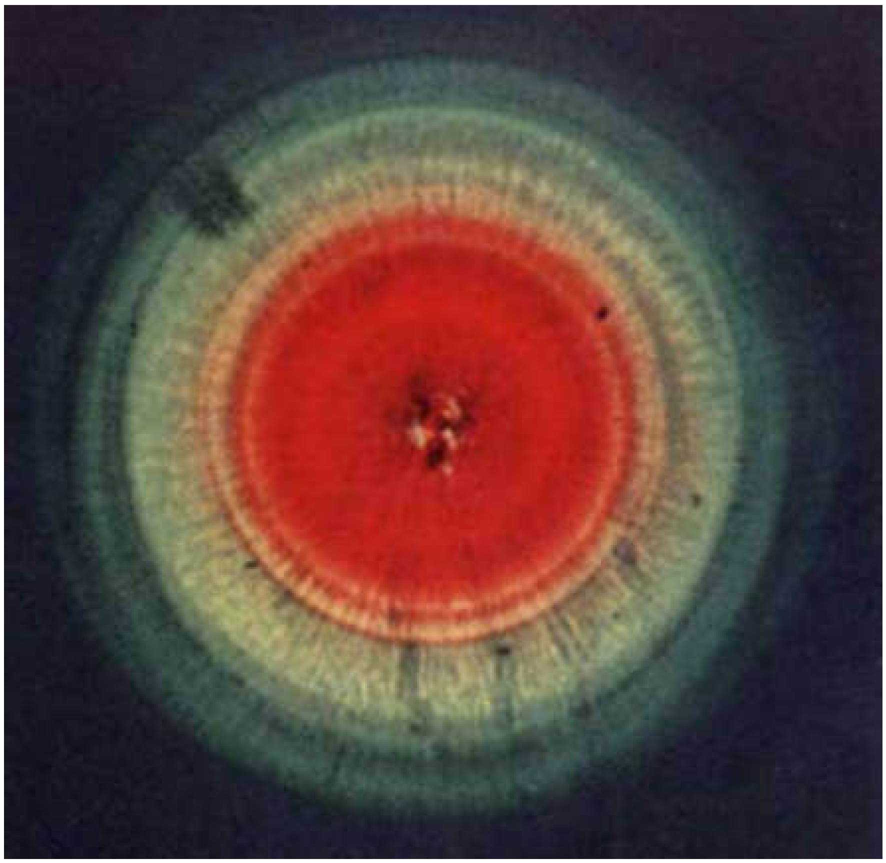


Fig. 1.11: Stimulated Raman Scattering

These new wavelengths were found to be shifted by the quantized vibrational frequencies of the nitrobenzene molecule. Such a shift in frequency of light was reported by C. V. Raman as Raman effect earlier also.

It has recently been discovered that when intense laser pulses are focused in solids, liquids and high-pressure gases, the stimulated Raman process can occur even without a resonator cavity. Near the focus, where the light intensity can exceed $10^9$ Wcm$^{-2}$, the amplification of Raman light is as high as $10^{50}$cm$^{-1}$, enough to amplify stray background light up to the $10^5$ W level in one pass through the focus.

### 1.6 Difference between Laser Action and SRS

The SRS scattered light is having same properties as that of emitted by a laser system i.e., SRS scattered light is coherent, directional as well as highly intense. However, there are some fundamental differences between SRS and laser action:

1. Laser action requires population inversion however SRS occurs without requiring any population inversion.
2. During light amplification in LASER through population inversion, the signal pulse extracts more and more energy from the medium as it gets more and more amplified. However, in the case of SRS gain, the scattered wave gains energy only if the pump is present (fig1.12).

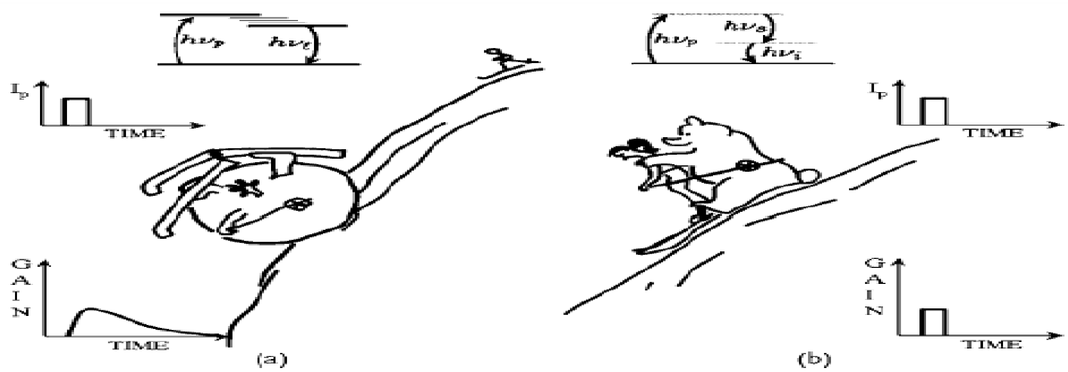



Fig. 1.12: Difference between laser action and SRS.

### 1.7 SRS in Plasmas

Plasmas are known to be 4$^{th}$ state of matter due to their collective behaviour and quasi neutrality. These properties enable plasma to posses different modes of wave propagation[19,20] like EPWs, IAWs etc.

EPWs play the role of optical phonons, as played by the vibrational/rotational modes of the molecules. Hence, along with liquids and solids[21], it is obvious to have SRS in plasmas as well. Mechanism of SRS in plasmas can be visualized as follows (fig.1.13):

1. Due to its inherent property of collective behaviour, plasma contains a natural mode of wave propagation: EPW (as explained in section 1.7).
2. Due to the presence of this seed wave, the density of plasma is rippled.

3. The pump beam interacts with these density ripples to produce a nonlinear transverse current density. This transverse current density is the source for SRS scattered wave.
4. The SRS scattered wave beats with the pump and thus produce density variations at beat frequency which is equal to EPW frequency and thus the EPW gets amplified.
5. The amplified EPW further enhances SRS.
6. This leads to a feedback loop where pump amplifies SRS scattered wave, SRS scattered wave amplifies EPW and then amplified EPW further amplified SRS scattered wave leading to an instability.

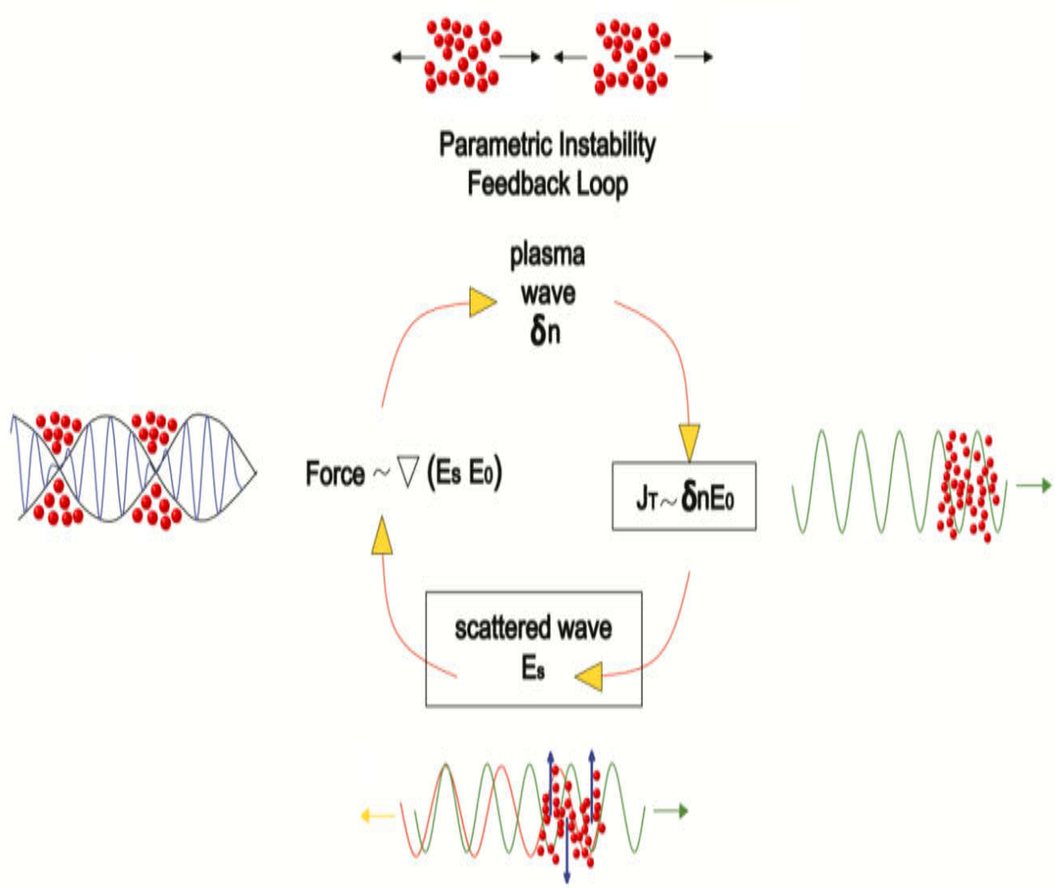



Fig. 1.13: Feedback mechanism of SRS instability.

## 1.8 Role of SRS in Inertial Confinement Fusion

The EPW excited during SRS plays a significant role in various applications based on laser plasma interactions like plasma wake field accelerators and inertial confinement fusion (ICF). During ICF, the SRS excited EPW can produce super thermal electrons, those produce burning of the pellet before it is compressed sufficiently to ignition point.

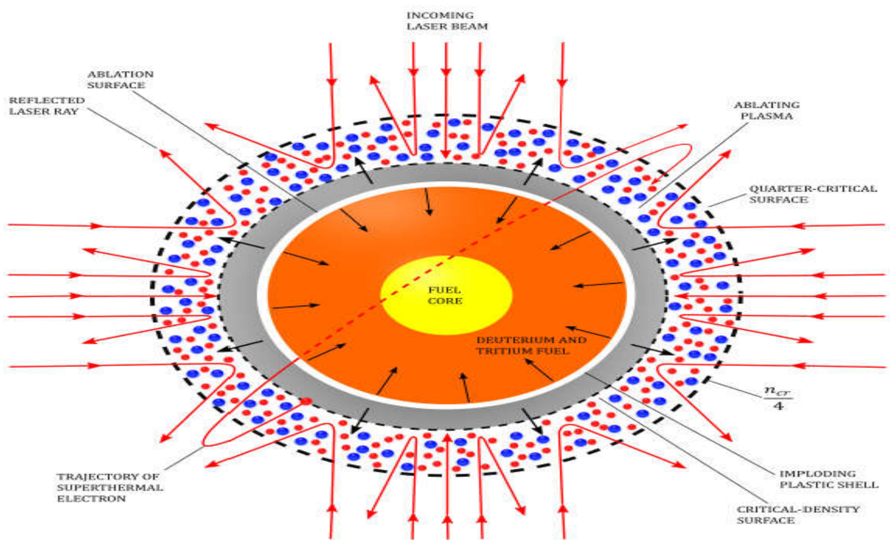



Fig. 1.14: Role of SRS in ICF.

These high-energy electrons can also collide with ions, transferring a portion of their energy to the ions and increasing their temperature. This process is known as electron-ion energy exchange or electron-ion equilibration. Super thermal electrons, due to their higher temperatures, emit bremsstrahlung radiation. Bremsstrahlung radiation carries away energy from the plasma, influencing the overall energy balance and optimization of fusion devices.

The presence of SRS can also have other negative effects on nuclear fusion experiments. It can cause energy loss from the laser beam, reducing the efficiency of the heating process. Additionally, the scattered light from SRS can interfere with the measurements and diagnostics used to monitor and control the plasma conditions. Thus, In context of laser plasma interactions investigations on various aspects of SRS are very much essential.

### 1.9 Self Focusing of Laser Beam

The phenomenon by virtue of which an intense laser beam gets focused by its own, without requiring any convex lens is known a self focusing[23, 24].

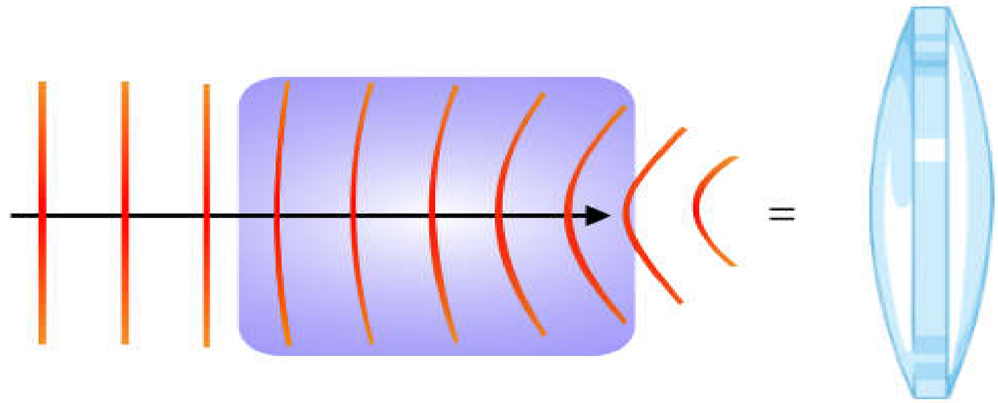


Fig. 1.15: Self focusing of laser beam.

In inertial confinement fusion, the laser beams have a crucial role in compressing and heating the fuel to thermonuclear conditions. Self-focusing is a natural occurrence in these beams and can help to increase the laser intensity at the target. Thus, present study is focused on effects of beam profiles on self focusing in plasmas.

### 1.10 Role of Self Focusing in SRS

When these stimulated Raman processes were recognized and understood, it appeared that nonlinear optics could be treated in a reasonable way. However, it turns out to be much more complicated than was thought. One important effect, which influences all the others, is the self-focusing. Because of the ponderomotive force effect, the refractive index in the plasma is greater where it is traversed by intense light than elsewhere. This implies a gradient of refractive index at the edges of the beam, which focuses the outer portion of the beam inward. The result is that the beam necks down to a very narrow diameter, of the order of a few microns. What happens from there on is rather complex. Usually, a large initial beam breaks up into a number of filaments, and these may contain within them at still finer filaments of diameters not much greater than the wavelength of the light. In these filaments, the light intensity is very much higher than the average throughout geometric spread, and so all sorts of nonlinear optical effects such as SRS and SBS take place primarily in these filaments. Thus, in the investigation of SRS of laser beams in plasmas, the incorporation of the effect of self-focusing becomes essential[25].

### 1.11 Mechanisms of Optical Nonlinearity in Plasma

In the linear regime, a highly collimated optical beam starts spreading along the transverse axes due to diffraction property of light. More tightly we try to confine the beam, more it will spread. Before the invention of laser, it was thought that diffraction broadening of optical beam is unavoidable as it arises inherently from uncertainty principle of quantum mechanics. Conventional way to avoid diffraction of an optical beam is to use optical fibers. But, in laser plasma interaction experiments, intensities of the order of $10^{16} W/cm^2$ are being used. However, a glass fiber gets damaged only at intensity of $10^{12} W/cm^2$. Hence, for ultra high intensity laser beams, optical fibers are not an appropriate solution for optical guiding. In 1964, it was shown by Chiao et al. that the diffraction of an optical beam passing through a material medium could in principle be avoided if the medium start responding nonlinearly (i.e., if the index of refraction of the medium becomes a function of the intensity of light) to the incident beam. The electric breakdown strength of the normal materials limits the use of high intensity lasers. But, plasma has shown possibility to use these high intensity lasers. Plasma by definition is a quasi-neutral gas of charged particles that possess collective behaviour. However, being already ionized, plasmas (fig. 1.16) show almost infinite immunity against such kind of damages. Due to their inherent properties of quasi-neutrality and collective behaviour, they also respond nonlinearly to intense optical beams.

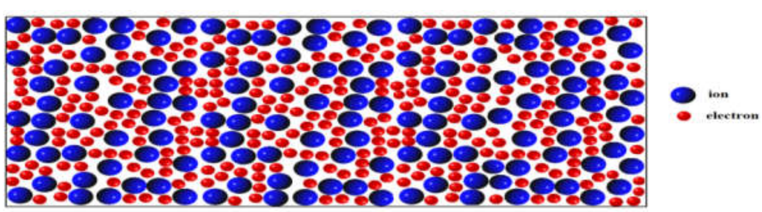



Fig. 1.16: Plasma

The dielectric response of plasma to an incident optical beam of frequency $\omega_0$ is given by

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega_0^2} \tag{1.1}$$

where,

$$\omega_p = \sqrt{\frac{4\pi e^2}{m_e} n_e} \tag{1.2}$$

is the characteristic frequency of plasma which is known as plasma frequency. When perturbed from their equilibrium position, plasma electrons oscillate with this particular frequency. Here, $e, m_e, n_e$ are the electronic charge, mass and density, respectively. Thus any mechanism by which laser beam can modify $n_e$ or $m_e$ will make plasma nonlinear.

There are mainly three mechanisms (fig.1.17) by which plasma can interact nonlinearly with laser. These mechanisms are:

1. Ponderomotive Force[26, 27]
2. Ohmic Heating[28, 29]
3. Relativistic increase in electron mass[30, 31]

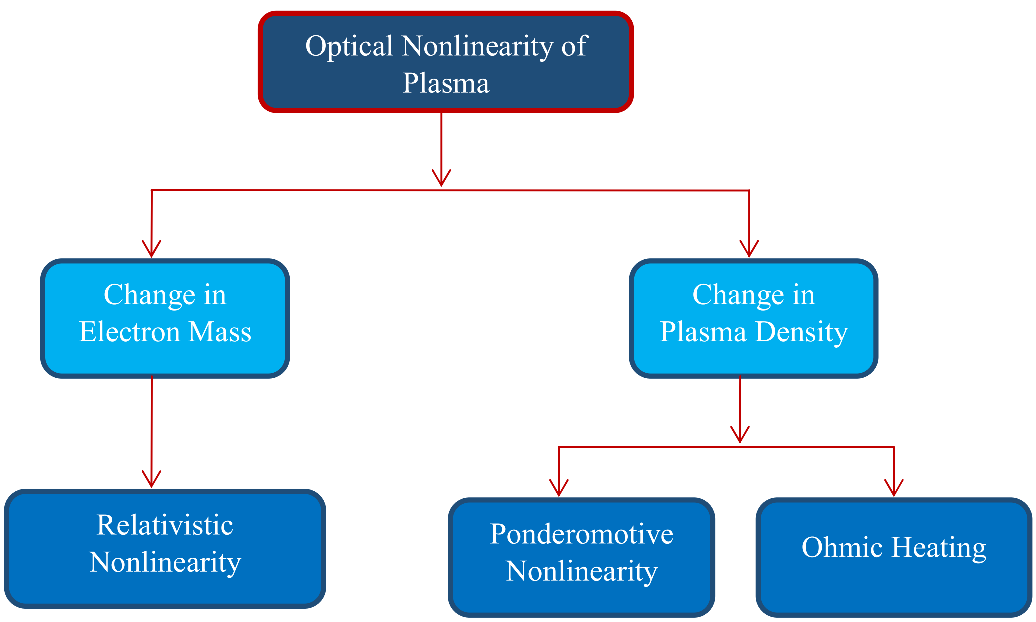



Fig. 1.17:Various optical nonlinearities of plasma

First two mechanisms show transient behaviour because they modify the electron density due to the physical movement of the plasma electrons under the effect of laser beam. Whereas, the third mechanism does not involve any modification of electron density and thus does not show any transient behaviour. It comes into picture alsmost instantaneously as soon as the intensity of the beam crosses the threshold intensity.

### 1.11.1 Collisionless Modification

Generally, the laser beams do not possess uniform amplitude structure over the cross sections. Due to their finite cross section, laser beams are having some amplitude structure over their cross sections. The amplitude profiles of the laser beams depend upon the mode of laser cavity. The lowest order mode of the cavity results in Gaussian beam described as

$$E_0(r) = E_{00} e^{-\frac{r^2}{2r_0^2}} \tag{1.3}$$

where,

$r_0$ is the beam radius and $E_{00}$ is the axial amplitude.

Such beams produce ponderomotive force on plasma electrons.

$$F_{Pond} \propto -\nabla E_0 E_0^\star \tag{1.4}$$

The DC component of this ponderomotive force pushes the plasma electrons from beam axis towards its wings (fig.1.18). Thus, the laser dugs a density channel into the plasma for its propagation. If $n_0$ is the equilibrium electron density of the plasma, then its modified electron density is given by

$$n_e = n_0 e^{-\frac{e^2}{8 m_e \omega_0^2 K_0 T_0} E_0 E_0^\star} \tag{1.5}$$

where,

$K_0$ is the Boltzmann cosntant,

$T_0$ is the temperature of plasma electrons.

The resulting permittivity of plasma can be written as

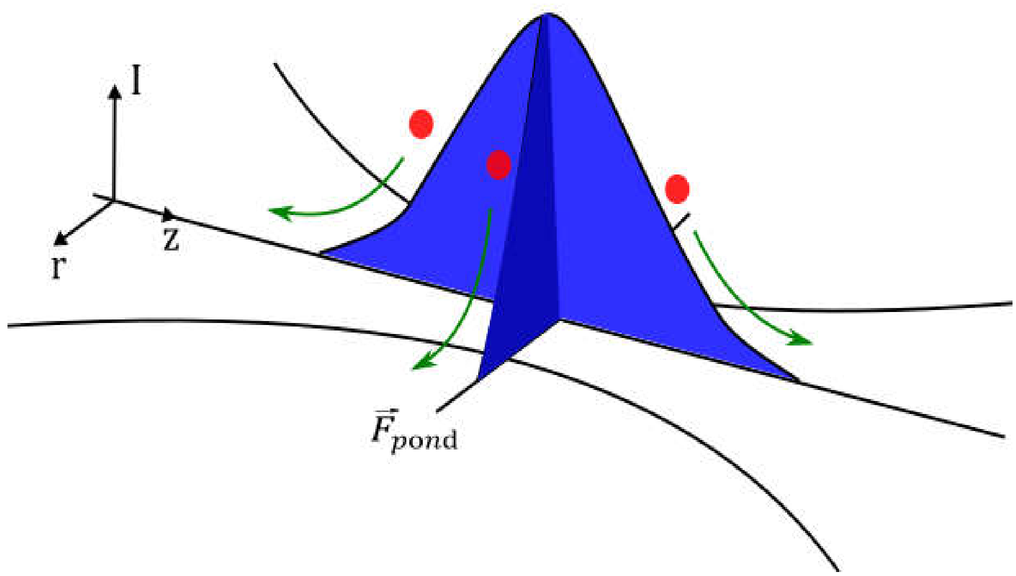



Fig. 1.18: Ponderomotive force acting on plasma electrons

$$\epsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2} e^{-\frac{e^2}{8m_e\omega_0^2 K_0 T_0} E_0 E_0^\star} \tag{1.6}$$

Here, $\omega_{p0} = \sqrt{\frac{4\pi e^2}{m_e} n_0}$ is the natural frequency of the oscillaions of plasma e$^-$ s.

### 1.11.2 Collisional Heating

During the propagation of intense laser beam through plasma, the non-uniform intensity distribution over its cross section produces redistribution of plasma electrons through ohmic heating that changes index of refraction of medium.

Therefore, in this mechanism, the nonuniform amplitude structure of an optical beam results in nonuniform heating of plasma electrons due to their other species like ions and neutral particles. The resulting temperature of plasma electrons is given by

$$T_e = T_0 \left( 1 + \frac{e^2 M}{6 K_0 T_0 m_e^2 \omega_0^2} E_0 E_0^\star \right) \tag{1.7}$$

where, $T_0$ is the equilibrium plasma temperature, $M$ is the mass of ions. The resulting electron density of plasma is given by

$$n_e = n_0 \left(\frac{2T_0}{T_0 + T_e}\right)^{1-\frac{s}{2}} \tag{1.8}$$

where, the phenominological parameter $s$ describes the nature of collisions of plasma electrons.

- $s = -3$ => electron ion collisions
- $s = 2$ => collisions of plasma electrons with diatomic molecules
- $s = 0$ => velocity independent collisions of plasma electrons

. Hence, resulting permittivity of plasma can be derived as

$$\epsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}\left(1 + \frac{e^2 M}{6K_0 T_0 m_e^2 \omega_0^2} E_0 E_0^\star\right)^{\frac{s}{2}-1} \tag{1.9}$$

**1.11.3 Relativistic Modifications**

If the plasma electrons move at such high speeds due to the intense laser beam that their trembling velocity becomes similar to the speed of light, they acquire relativistic mass. In equation (1.2), the mass of electrons should be substituted with $m_0\,\gamma$, where $m_0$ represents the electron's rest mass, and γ is the well known Lorentz factor of special relativity, which is related to the laser beam intensity.

$$\gamma = \left(1 + \frac{e^2}{m_0^2 c^2 \omega_0^2} E_0 E_0^\star\right)^{\frac{1}{2}} \tag{1.10}$$

Thus, using eqs.(1.1) and (1.2) the intensity dependent permittivity can be written as

$$\epsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}\left(1 + \frac{e^2}{m_0^2 c^2 \omega_0^2} E_0 E_0^\star\right)^{-\frac{1}{2}} \tag{1.11}$$

The presence of equation (1.11) reveals that, just like ponderomotive nonlinearity or thermal nonlinearity, the phenomenon of relativistic nonlinearity also causes the laser beam to exhibit maximum index of refraction at its peak intensity and vice versa. This gradient in plasma refraction leads to the laser beam's self-focusing, commonly referred to as relativistic self-focusing.

## Chapter- 2
## Review of Literature

### 2.1 Introduction

This chapter unravels the contribution of different researchers in the discipline of laser matter interactions. They are trying to enhance the efficiency of inertial confinement fusion (ICF).

### 2.2 Self-Action Effects

**V. P. Nayyar and N. K. Verma (1977)[32]** studied about the self-focusing of laser beams in the paraxial region and extra-paraxial region operating in the $TEM_{00}$ mode and the $TEM_{l0}$ mode. For the same value of $E_0^2$ on the x-axis at different points, described the effects of self-focusing and self-defocusing of the high intense laser beam.

**V. S. Soni and V. P. Nayyar (1980)[33]** studied about the self-focusing, defocusing and self-trapping of intense laser beams that have elliptically shapes in magneto plasma. Along x-dimension and y-dimension of the beam, self-trapping required different values of critical power.

**J. S. Bakos et. al (1983)[34]** performed a crossed beam experiment to understand the self-focusing mechanism in a laser induced spark. It has been shown by experimental results that the non-uniform heating mechanism causes self-focusing. This served as a pioneering experiment for laser plasma experimentalists.

**A. M. Rubenchik et. al (1987)[35]** studied about the self-focusing in plasma medium described by the Zakharov model which provides solutions for the wave equation by using the thermal and ponderomotive effects.

**P. Vyas and M. P. Srivastava (1990)[36]** studied about the motion of plasma particles in cylindrical object causing by the self-focusing which reduce the velocity and enhance the density of plasma particles in overdense region and vice versa in underdense region.

**B. I. Cohen et. al (1991)[37]** presented the nonlinear self-focusing behaviour of laser beam in plasmas under the effect of dominant nonlinearity of ponderomotive force and calculated over space time evolution. In the paraxial limit, a simple nonlinear wave equation has been solved by using saturable and exponential nonlinearity. Also shown that in case of intense and electron pulsed free lasers, self-focusing plays crucial role for heating of plasmas that is magnetically confined and for ICF implementations.

**X. L. Chen and R. N. Sudan (1993)[38]** studied about a simplified set of three-dimensional equations when a high intense laser beam propagates in cold underdense plasma medium. Also shown that the dispersion effect in the self-focusing causing by the axisymmetric two-dimensional equations.

**G. Bonnaud et. al (1994)[39]** investigated about the self-focusing of a highly intense laser beam via two dimensional simulation through inhomogeneous plasma under the ponderomotive and relativistic nonlinearity.

**P. Chessa et. al (1998)[40]** described the laser plasma interaction in terms of the particle code (WAKE) by using the nonlinearity of ponderomotive force, and interpreted existing experimental results of relativistic self-focusing in plasma targets.

**R. Bharuthram et. al (1999)[41]** developed a model by using paraxial theory approximation for self-focusing of high intense Gaussian laser beam having step function temporal profile while passing through collisionless plasma medium.

**V. Malka et. al (2000)[42],** it has been found that during the interaction of laser beam with pre-ionized gas-jet plasma, the propagation and self-focusing processes investigated by using Collective Thomson scattering imaging.

**L. M. Chen and H. Lin (2001)[43]**, it has been studied about the anisotropic self focusing of ultra-relativistic intense laser light beam by using a variational principle approach in the under-critical plasma and observed the importance of isotropic profile parameters for the possibility of anisotropic unstabilization by numerical analysis.

**R. W. Boyd (2002)[44]** studied about the self-action effects which impact the laser beam propagation through the nonlinear material medium and stability degradation of laser beam due to quantum noises.

**J. Faure et. al (2002)[45]** reported self-focusing of highly intense laser beam in underdense plasma targets. They reported that when the time scales of motion of plasma particles is comparable with the duration of laser pulse then relativistic self-focusing having critical power is not only in consideration.

**B. Eliasson and P. K. Shukla (2005)[46]** presented simulation and theoretical studies of electron whistlers (frequency modulated) in plasmas in terms of the linear self-focusing and found result analytically for the comparison of a direct simulation of the whistler wave equation and a wave kinematic model which describes the self-focusing of the whistler energy caused by dispersion.

**P. Jha et. al (2006)[47]**, it has been studied about the evolution of laser beam having a Gaussian profile in magnetized, underdense, cold plasma and found that laser beam self-focusing can be enhanced with reduction in required critical power by transverse magnetization of plasma.

**Amrita and A. K. Sharma (2006)[48]** studied about the heating of electrons caused by Gaussian laser beam while passing in collisional plasma medium and found that the increase in laser beam intensity leads to decrease in electron temperature due to which there is a loss of thermal conduction which leads self focusing in periodic manner.

**A. M. Rubenchik et. al (2009)[49]** demonstrated the self focusing of laser beam while propagating from orbit state to ground state in atmosphere by using numerical modeling technique, when self focusing length and atmosphere height are comparable, the spot size of the laser beam on the ground state can be decrease below the diffraction limits without quality degradation laser beam.

**J. Parashar (2009)[50]** studied the third harmonic generation caused by nonlinear behaviour of electron when high intense laser beam having Gaussian profile passing through clustered gas which assists laser beam self-focusing. It was reported that the propagation dynamics of the laser beam are highly dependent on the intensity of the laser beam.

**R. Singh and A. K. Sharma (2010)[51]** investigated about the electrons acceleration caused by ponderomotive force under the effect of relativistic self-focusing in plasma medium. It was found that energy gained by the electron is significantly affected by the extent of self focusing of the laser beam. To analyze the propagation dynamics of the laser beam paraxial theory was used under WKB approximation.

**D. N. Gupta and H. Suk (2011)[52]** observed that thermal self-focusing (by using paraxial theory) of laser beam passing through collisionless plasma can be enhanced by localized plasma density ramp. Along the distance of propagation of the laser beam, axial inhomogeneity of the plasma density was also taken into picture.

**P. Sati et. al (2012)[53]** studied about the self-focusing of laser beam having a quadruple Gaussian profile while propagating through in collisionless plasma and described by paraxial ray theory approximation.

**V. V. Semak and M. N. Shneider (2013)[54]** presented about the self-focusing and self-defocusing of laser pulse while propagating through plasma by using paraxial theory and shown that Kerr focusing affects the divergence of laser beam.

**D. N. Gupta et. al (2013)[55]** investigated about the self-focusing of a laser beam in plasma medium under the effect of weak nonlinearity for some appropriate simulation parameters through simulation results and theoretical results.

**B. Bokaei and A. R. Niknam (2014)[56]** investigated about the thermal self-focusing mechanism causing by ponderomotive nonlinearity and furthermore studied about the thermal self-focusing in plasma under the effect of upward density profile due to which there is increase in self-focusing thermal value as well as in most extreme intensity.

**M. E. Abari et al. (2015)[57]** studied about the characteristics of a laser beam having Gaussian profile while propagating through collisional magnetized plasmas within the ponderomotive and ohmic heating nonlinearities. The effect of self-focusing can be occurred intensively with increase in the cyclotron frequencies (magnetic strength) and plasma density.

**G. Purohit et. al (2016)[58]** studied about the second harmonic generation (SHG) and electron plasma wave (EPW) excitation under the effect of self-focusing of hollow Gaussian beam while propagating through collisionless plasma.

**A. A. Balakin et al. (2017)[59]** studied about the femto-second laser pulses in nonlinear media and its effective self compression possibility with a typical scattering of group velocity during the self-focusing of wave packets having more power as compared to the critical self focusing power. The consequences of subjective examination for the betterment of 3D wave packets with the quasi soliton field circulation by using computer simulation.

**S. A. Kozlov et al. (2018)[60],** it has been discussed about an amazement of few-cycle waves in nonlinear optics like the vanishing of the high power radiation self-focusing phenomenon. Because of the predominance of the course of dispersion over diffraction, an equation is determined and examined under the conditions of a critical power for self-focusing when it loses its actual significance.

**C. Hazarika et al. (2019)[61]** investigated about the paraxial and non-paraxial self focusing of laser beam having Gaussian profile propagates through bulk chalcogenide glass by using a variational method and discussed about the stationary self-focusing for paraxial propagation with distinct singularity manner and focusing defocusing cycles for non-paraxial propagation of that beam. Emphasis is based on the propagation distance and variation in laser beam.

**K. Virk (2020)[62]** discussed about the high intense laser beam self-focusing when beam passing through within nonlinear material medium i.e. plasma. Self-focusing induces when the index of refractive varies with beam. Also, it has been discussed about the different nonlinear processes like SHG by using the properties of laser polarization and plasma.

**P. P. Nikam et. al (2022)[63]** studied about the evolution of self-focusing of laser beam having Gaussian profile in magnetized, underdense plasmas by using a parabolic equation approach. It has been assumed to apply the uniform magnetic field with beam direction as well as with its opposite direction and found that self-focusing increases (decreases) as per applied magnetic field in forward (reverse) manner. By using WKB and paraxial approximations, for dimensionless laser beam width parameter (BWP), the nonlinear differential equation is derived.

### 2.3 Stimulated Raman Scattering

**E. J. Woodbury (1962)[18]** observed about SRS almost accidentally in a giant pulse laser in which the shutter (Kerr cell) containing nitro-benzene and found additional wavelengths having few hundred angstroms (coherent and collimated as main beam) in addition of ruby (6,943 angstroms) wavelength near the its red side contained in the output laser beam.

**N. Bloembergen (1967)[64],** It has been investigated about the theoretical and experimental developments in SRS of light since 1962 and described that this effect cannot be discussed without related to other non linear effects such as Rayleigh and SBS and self-focusing of light etc.

**R. A. Smith (1971)[65]** discussed about the lasers as light sources, photon counting, Rayleigh scattering from isolated molecules, Tyndall, Brillouin and Raman scattering from dense medium, from liquid media, from other collective motions in solids and in gaseous plasmas as well as by electrons with spin reversal.

**A. L. Schawlow (1982)[66]** observed about the spectroscopic investigations possibilities of already discussed unimagined precision by the laser characteristics such as monochromaticity, intensity and directionality etc. Also, it has been discussed about the sharpening of the lines, Doppler free spectra, simplifying complex spectra and resonant scattering.

**C. J. McKinstrie and M. Yu (1991)[67]**, it has been studied that the momentum of time averaged Langmuir wave is not conveyed by the electric field or by the electrons in cold electron-ion plasma; it conveys by an established ion drifts which displace the frequency of EPW just yet don’t ruin the SRS advancement.

**H. C. Barr et. al (1994)[68]**, it has been observed that by utilizing an integrated global model, convective enhancement and supreme development of SRS which are determined for a inhomogeneous (hot) collisional plasma that incorporates the impacts of wave propagation and both collisional and Landau damping in a density ramp. They fostered a code which acquires the time asymptotic linear characteristics of SRS in inhomogeneous (hot) collisional plasmas without response to encompassing WKB technique.

**V. Y. Bychenkova et. al (1997)[69]** studied the impact of a non-Maxwellian function of electron distribution delivered by backwards bremsstrahlung heating caused on SRS threshold in homogeneous plasma. This prompts an alteration of collisional Langmuir wave and its Landau damping coefficients, which can bring about changes in the SRS threshold and frequency spectrum of scattered light.

**K. L. Baker et al. (1997)[70]** studied that Langmuir wave spectrum driven by SRS was quantified by using of Thomson scattering. Thomson scattering signals estimated through experimental way and described Langmuir waves having components parallel and anti-parallel both to the wave vector of incident laser beam. The parallel and anti-parallel components were attributed to the SRS and LDI.

**N. Bloembergen (1999)[71]** observed that lasers have caused progressive changes in the fields of science and technology and found that how lasers given admittance to estimations of short time periods since 1960. Some transient Raman scattering examples, molecules and crystals having impulsive and displacive excitations present the handiness of femto-second and picoseconds pulse techniques.

**J. C. Fernandez et al. (2000)[72]** studied SRS in plasma which access the conditions expected through ignition hohlraum designs for the National Ignition Facility (NIF) and assessed the potential threat of high laser beam reflectivity from SRS to ICF in quasi-homogeneous plasmas.

**S. T. Mahmoud and R. P. Sharma (2001)[73]** investigated about the impact of electron mass variation in relativistic way on a intense laser beam having very high power when it engenders in unmagnetized plasmas. The intense laser light beams can be propagated without convergence or divergence (self-trapped mode) due to varying parameters of beams and plasmas. SRS and SBS have been discussed in the presence of relativistic nonlinearity.

**E. S. Dodda and D. Umstadter (2001)[74]** described a unique method to control the SRS and production of hot electron in short pulse laser beam and plasma interactions which relies on a linear frequency chirping in non band-width pulses in limited manner. Also discussed about the enormous chirped bandwidth in a laser pulse can effect dramatically on the development of SRS.

**H. A. Salih et. al (2004)[75]** observed that SRS happens in two counter-propagating laser beats when the frequency difference between the lasers exceeds $2\omega_p$. SRS defines a valuable strategy to conclude the belongings of engendered plasma waves.

**H. A. Salih et al. (2005)[76]** investigated SRS in unmagnetized plasma when relativistic laser beam passing through it. The impact of nonlinear coupling has been assimilated between the pump beam and scattered beam. Also studied the effect of self-focusing of the laser light beams pumping in case of SRS and obtained semi analytical solutions for SRS in terms of back reflectivity.

**J. Parashar (2009)[77],** it has been discussed about the lattice of nano particles (both periodically and non-periodically spaced) supports an electromagnetic mode and growth time (picoseconds range) for Raman backscattering instability at laser intensities of $10^{12}$ W/cm$^2$. This paper analyzed a sensible physical insight into SRS in equally spaced nano particles.

**R. Singh and V. K. Tripathi (2012)[78]** studied about the decay instability coupled with non SRS in a plasma channel. Raman backscattering process generated secondary EPW having longer wavelength from decay of primary EPW and an IAW. This diversion of energy slows down the Raman process towards with linear Landau damping of the primary wave.

**Z. J. Liu et al. (2016)[79]** presented the simulation of SRS for single speckle light spot by using LAP3D code. The higher intense energy as compared to the average intensity ought to be diminished throughout of Continuous phase plate (CPP) design. The reflectivity increases with increase in high intensity fraction.

**Y. X. Wang et al. (2017)[80]** discussed about the temporal growth rate of backward SRS by Vlasov simulation technique and kinetic theory under the effect of density modulation of a static sinusoidal wave. In terms of density modulation wave-number, with increasing $\varepsilon$, the temporal growth rate decreasing faster because of the creation of more number of modes, which make Landau damping of Langmuir waves more powerful.

**A. P. Misra and D. Chatterjee (2018)[81]** studied about the instabilities of stimulated scattering in case of an intense linearly polarized e.m. wave (EMW) in relativistic plasma with degenerate electrons and derived non-linear (coupled) equations by using Maxwell's equations and a relativistic hydrodynamic model for ion plasma oscillations and low frequency electrons.. The results of non-linear dispersion relations are obtained which reveal modulational instabilities (MIs), SRS and SBS of EMWs.

**In their study, K. Walia et al. (2019) [82]** explored the phenomenon of SRS of a high-power, intense laser beam in a hot quantum plasma. They observed the back-scattered beam resulting from the interaction between the pump beam and the EPW. The researchers noted that the laser beam, EPW, and scattered beam were influenced by relativistic nonlinearity, which led to an increase in the effective mass of electrons. **A. Bierstedt et al. (2020)[83],** it has been shown that while enlightening a levitated droplet with a solitary wavelength (fixed), a narrow signal (bright spectrally) is observed which could be clearly defined as SRS. Such SRS signal amplified the normally weak spontaneous stokes emission.

**In their study, Feng et al. (2020) [84]** investigated the interaction between backward SRS and SBS in areas of high electron density. They employed relativistic particle-in-cell (PIC) simulation and Vlasov-Maxwell simulation to examine the phenomenon of re-scattering. Consequently, the SBS of BSRS emerged as a crucial saturation mechanism for BSRS in addition to laser energy absorption and Langmuir decay instability, particularly in regions with high electron density.

**Y. Ji et al. (2021)[85]** studied about the laser short pulse duration of a few picoseconds and presented particle-in-cell (PIC) simulations of laser plasma instabilities (LPIs).

**O. Kamboj et. al (2022)[86]** observed about the suppression of SBRS in presence of density rippled plasma and magnetic field. Also found that the SBRS is decayed for different modes of the growth rate significantly due to the ripple and local effects. Raman process has been impacted significantly by the magnetized density ripple for radial eigen-mode number. The growth rate attains at its maximum values with increasing in the applied magnetic field, reaches at peaks for some optimal value then suppression goes on.

**K. Brzozowski et. al (2022)[87]** studied about the SRS process, advantages of SRS images collection and methods of signal detection to overcome some limitations especially in spatial resolution and image capture speeds in biological system studies to diagnose of diseases at large scale. Also discussed SRS microscopy uses in comparison with other techniques such as SHG microscopy, fluorescence microscopy, CARS and Raman microscopy; and finally proposed perspectives for its development.

### 2.4 Variational Theory Approach

**F. Cooper et. al (1992)[88]** discussed about the non-linear Schrodinger equations (NSE) by using of Dirac's variational method principle and blowup qualitative information is obtained by using Gaussian (trial variational wave) function at finite number of times as a utility of d (spatial dimensions) and K (arbitrary non-linearity parameter) and found the results of mathematical problems (for both 1D and 2D).

**D. Anderson et. al (2001)[89]** studied about the variational method approach which is based on the Rayleigh Ritz optimization and found numerically estimated solutions to diverse nonlinear equations of different kind of problems (in stationary as well as dynamical situations). Also, they described the practical uses of this approach in correlation with the non-linear Schrodinger wave equation (NLSE) with some examples.

**G. Bouchitte and I. Fragala (2002)[90]** presented a survey report of outcomes for various problems which are having a common feature such as weak geometric structures by using variational theory approach and studied about the measures and applications of this method as built-in with appropriate properties at a tangent and adaptable spaces.

**E. Arevalo and A. Becker (2005)[91]** studied about the electrical field of an intense laser beam having Gaussian profile while propagating through a medium of gas and derived its expression by using of perturbation theory. Variational theory approach described quasi analytical results for the self-focusing expanse, intensity and thickness of laser beam by using that expression as a trial solution.

**J. Zhang et. al (2005)[92]** studied about variational principles (with liberated parameters) in terms of partial differential equations (PDEs) with irregular coefficients by using of J. H. He's semi inverse technique in the field of non-linear fiber optics.

**C. P. Jisha et. al (2006)[93]** studied about a high power pulsed laser beam while transmitting through an bulk cubic quintic medium which shows $3^{rd}$ and $5^{th}$ order terms non-linearity and Self-defocusing effect considered which is caused by the free electrons generated by formation of plasma. Spatio temporal 3D light solitons have been observed analytically as well as numerically by using variational theory approach and method of finite difference beam propagation.

**C. P. Jisha et. al (2009)[94]** studied about a low power laser beam (He-Ne) propagating through a photo-polymer described by a nonlinear Schrodinger wave equation and found solutions of that wave equation theoretically and experimentally by using variational theory method.

**M. Syafwan et. al (2018)[95]** studied about the inter-site pattern solitons (out of phase fluctuation) in distinct non-linear Schrödinger wave equation by using variational theory approximation with numerical results. Such approximated solitons are based on exponential function of projected ansatz in the environs of the anti gamut limit.

**E. I. Duque et. al (2018)[96]** introduced a variational method approach which is based on the principle of Rayleigh Ritz optimization for predicting 2-D solitons in non-linear medium in terms of self-trapped laser beams.

**N. Rivera et. al (2019)[97]** developed a variational theory approach in case of light matter coupling having general non-relativistic QED systems that can illustrate the ground state of multi-electron systems which are united to several optical modes. By using ansatz, they found semi analytical formula by which energies of ground state and excited state can be described well. Their formulation developed a theory of Casimir-Polder forces and Lamb shifts in non-perturbative manner.

**M. F. Ferreira and S. C. Latas (2021)[98]** found the estimated solutions of the dissipative solitons which are defined by the Ginzburg–Landau equation (cubic quintic complex equation) by using of variational method approach in similar manner as by method of moment theory. In both approaches, found the solutions and described the existence of solitons (stationary as well as pulsating) with several mathematical problems.

**A. Djazet et. al (2021)[99]** studied about the electromagnetic field in a laser cavity while interacting with the matter described by the 2-level Maxwell Bloch equations. With nonlinear perturbative investigation up to laser threshold, Ginzburg–Landau equation (vectorial cubic quintic complex) has been derived. Dissipative solitons are found which are having propagation trajectories and stability conditions while trapping in an efficient potential well by using the variational theory approximation.

**Y. Tian (2022)[100]** studied about the variational method approach which is based on the semi-inverse method and found numerically estimated solutions of some non-linear problems by using of Ritz method. In this method, Groebner bases theory has been applied to solve huge categorization of algebraic equations. The outcome shows that this approach is more proficient in simplified manner.

**Y. Li et. al (2022)[101]** studied about the evolution of PQSs (pure-quartic solitons) theoretically by using the variational theory approach in a system which is based on the Lagrangian. By using Gaussian trial solution, the evolution equations provide a logical result of the stable PQSs (pure-quartic solitons).

## Chapter- 3

## Amplitude Structures of Laser Beams

### 3.1 Introduction

The spatial amplitude structure of laser beams plays a crucial role in determining their propagation characteristics in nonlinear optical media. Previous theoretical studies examining the impact of self-action phenomena on SRS in nonlinear media predominantly focused on ideal Gaussian beam profiles, assuming that the laser operates in its fundamental mode, known as TEM00 mode. However, experimental investigations, such as those conducted with the Vulcan laser, have revealed that even when the laser system operates in $TEM_{00}$ mode, the amplitude structure across the laser beam's cross-section does not exhibit an ideal Gaussian profile. A considerable amount of energy extends beyond the full-width half maximum (FWHM) of the distribution, resulting in expanded wings in the intensity profile compared to the Gaussian profile. Through fitting experimental data, it has been determined that the actual amplitude structure across the laser beam's cross-section can be modeled using Tasali's $q$-Gaussian function. Therefore, in the current study, aimed at obtaining more realistic results regarding the self-focusing effects of laser beams on SRS in plasmas, the $q$-Gaussian distribution is employed to represent the amplitude structure across the beam's transverse spread.

Contrary to ideal Gaussian and $q$-Gaussian beams, a new type of laser beams called flat top laser beams has gained significant attention from researchers in recent times. These beams exhibit a consistent distribution of power across their cross section, resulting in greater power and reduced divergence when compared to Gaussian and $q$-Gaussian beams. Consequently, these laser beams offer the potential for uniform heating of targets during laser plasma interactions. Mathematically, the amplitude structure of such beams is represented by the super Gaussian function. However, it is important to note that the super Gaussian approximation is an ideal concept. In laboratory settings, laser beams with flat top irradiance only maintain uniform irradiance up to a certain extent. Beyond that point, the irradiance starts decreasing radially, similar to Gaussian or $q$-Gaussian irradiance. To accurately describe the amplitude structure of such beams, the Cosh-Gaussian (ChG) function can be used as a model.

Thus, in the present investigation two types of amplitude profiles for the laser beams have been investigated:

1. $q$-Gaussian[23,26]
2. Cosh-Gaussian (ChG)[28-30]

Following sections describe the physical characteristics (amplitude profile, effective beam width, spectral width etc.) of these laser beams.

### 3.2 $q$-Gaussian Laser Beams

A $q$-Gaussian laser beam is a type of laser beam that exhibits a non-Gaussian intensity distribution. It is based on the $q$-Gaussian distribution, which is a generalization of the Gaussian distribution that includes a parameter $q$ that controls the degree of non-Gaussian behavior.

In a $q$-Gaussian laser beam, the intensity distribution is characterized by a power-law behavior at the tails, which leads to a more slowly decaying profile than a Gaussian distribution. This non-Gaussian behavior can have significant effects on the propagation and interaction of the laser beam with matter.

$q$-Gaussian laser beams have been studied extensively in recent years, both theoretically and experimentally. They have been shown to exhibit a range of interesting and useful properties, including self-focusing, self-trapping, and soliton formation. They have also been used in a variety of applications, such as high-intensity laser-matter interactions, optical trapping, and optical communication.

One of the most important applications of $q$-Gaussian laser beams is in the field of laser fusion, where they have been shown to improve the efficiency and quality of the laser-induced fusion process. They have also been used in the development of high-power laser systems, where their non-Gaussian behavior can help to mitigate the effects of damage and distortion caused by the laser beam.

Mathematically, the amplitude profile of the $q$-Gaussian laser beam is described by the function

$$E_0(x,y) = E_{00}\left\{1+\frac{1}{q}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\right\}^{-\frac{q}{2}} \tag{3.1}$$

where, $E_{00}$ is the amplitude at the axis of the beam and $a, b$ are the equilibrium semi major and semi minor axis of the beam. The key parameter $q$ is a phenomenological parameter that dictates how much the beam is deviating from ideal Gaussian beam. Thus, the parameter $q$ can be called as DP.

Using L Hospital rule of limits, it can be seen that $q = \infty$ corresponds to ideal Gaussian profile i.e.

$$\lim_{q\to\infty} E_0(x,y) = E_{00}e^{-\left(\frac{x^2}{2a^2}+\frac{y^2}{2b^2}\right)} \tag{3.2}$$

Figs. 3.1 and 3.2 illustrate the changes in irradiance across the cross section of a circular $q$-Gaussian beam for various values of $q$. This analysis aims to observe the impact of $q$ on the irradiance within the beam's cross section.

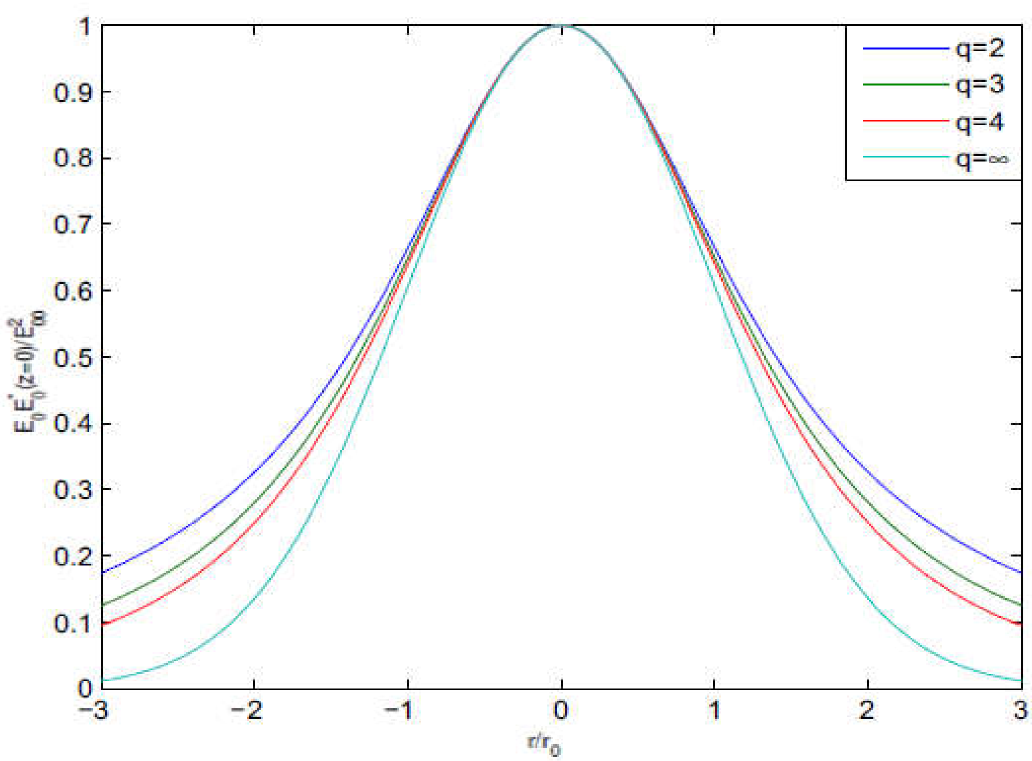



Fig. 3.1: Impact of DP $q$ on beam profile.

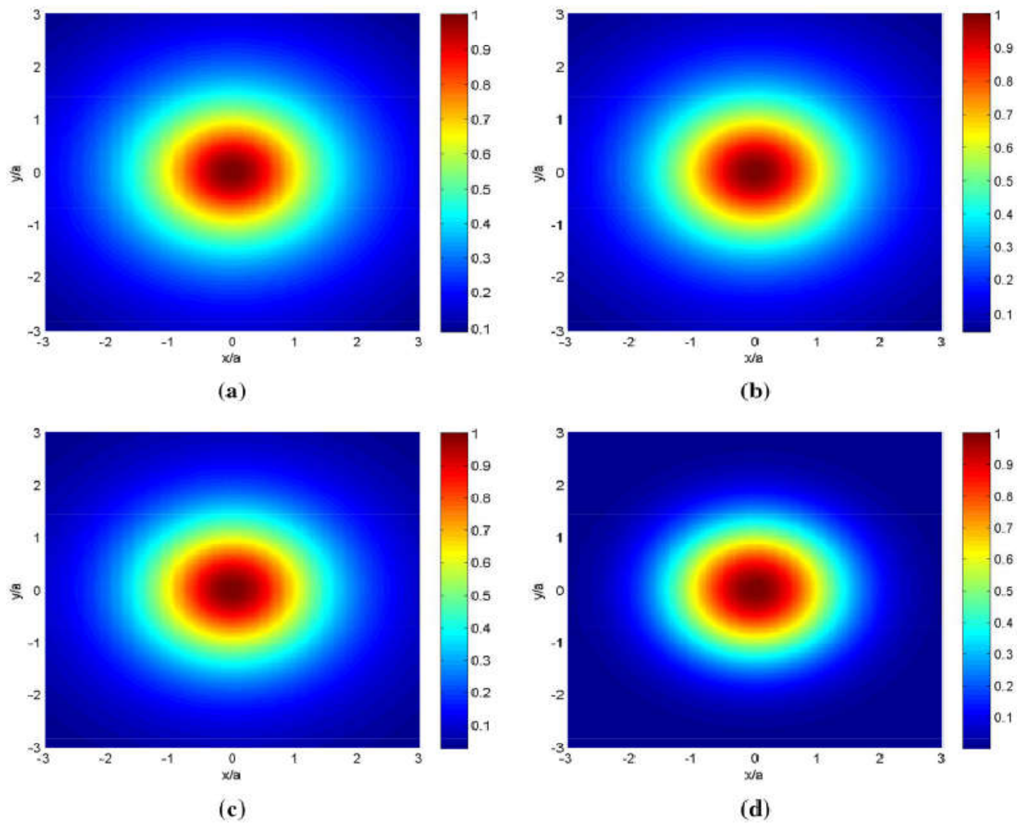



Fig. 3.2: Irradiance profile of laser beam. The parameters used are $\frac{a}{b} = 1.1$ and

$$(\boldsymbol{a})\ q = 2\ \ (\boldsymbol{b})\ q = 3\ \ (\boldsymbol{c})\ q = 4\ \ (\boldsymbol{d})\ q = \infty$$

Since, during the propagation of the beam through the medium, its beam width does not necessarily remains constant, we can write the instantaneous amplitude structure of the beam at a given point inside the medium as

$$E_0(r,\ z) = \frac{E_{00}}{f_x(z)f_y(z)}\left\{1 + \frac{1}{q}\left(\frac{x^2}{f_x^2 a^2} + \frac{y^2}{f_y^2 b^2}\right)\right\}^{-\frac{q}{2}} \tag{3.3}$$

where, the functions $f_x(z)$ and $f_y(z)$ are currently undetermined parameters and are termed as a dimensionless BWPs of the laser beam along $x$ and $y$ axis respectively. BWP $f$ refers to the spatial extent or diameter of the laser beam. It is a measure of the size of the beam at a particular point along its propagation path. The beam width is commonly characterized by the beam's diameter or radius, which can be quantified in various ways depending on the specific definition used.

### 3.2.1 Effective Beam Width of $q$-Gaussian Laser Beams

Any laser beam is not having a sharp boundary i.e., its intensity extends up to infinity; however after some finite distance from the beam axis it becomes insignificant. Hence, by method of moments, the effective spread of beam can be written as

$$\langle a^2 \rangle = \frac{1}{I_0} \iint E_0\, x^2 E_0^* d^2 r \tag{3.4a}$$

$$\langle b^2 \rangle = \frac{1}{I_0} \iint E_0\, y^2 E_0^* d^2 r \tag{3.4b}$$

where,

$$I_0 = \iint E_0\, E_0^* d^2 r \tag{3.5}$$

$$d^2 r = dxdy$$

RMS beam width is a statistical measure of the size of a laser beam, calculated by taking the square root of the mean of the square of the distances of each point in the beam from the beam's center axis. It is a useful metric for characterizing the spatial distribution and intensity profile of a laser beam.

Hence, the effective laser beam widths of $q$-Gaussian beam is obtained as

$$\langle a^2 \rangle = \frac{1}{2} a^2 f_x^2 \left(1 - \frac{1}{q}\right)^{-q} \tag{3.6.1}$$

$$\langle b^2 \rangle = \frac{1}{2} b^2 f_y^2 \left(1 - \frac{1}{q}\right)^{-q} \tag{3.6.2}$$

For a similar Gaussian beam, the beam width is $r_0 f$. Hence, the ratio between the r.m.s beams width of the $q$-Gaussian and Gaussian beams can be expressed as follows:

$$\Sigma = \left(1 - \frac{1}{q}\right)^{-q} \tag{3.7}$$

### 3.3 Cosh-Gaussian (ChG) Laser Beams

A ChG beam is a type of laser beam that has a specific intensity profile characterized by a hyperbolic cosine function multiplied by a Gaussian function. The intensity distribution of this beam can be described as a narrow central lobe surrounded by two wider lobes that gradually decrease in intensity towards the edges. The mathematical expression for the cosh Gaussian beam is given by:

$$E_0(r,\ z) = \frac{E_{00}}{f} e^{-\frac{r^2}{2r_0^2 f^2}} \cosh\left(\frac{b}{r_0 f} r\right) \tag{3.8}$$

In this context, the parameter "$b$" linked to the cosh function is referred to as the "cosh factor". Eq.(3.8) can also be written as

$$E_0(r,\ z) = \frac{E_{00}}{2f} e^{b^2} \left[ e^{-\left(\frac{r}{r_0 f} - b\right)^2} + e^{-\left(\frac{r}{r_0 f} + b\right)^2} + 2e^{-\left(\frac{r^2}{r_0^2 f^2} + b^2\right)} \right]^{\frac{1}{2}} \tag{3.9}$$

Thus, the factor $b$ is associated with the displacement of the intensity maxima of the constituting beams from their axes. Thus, the parameter $b$ is also known as decentered parameter.

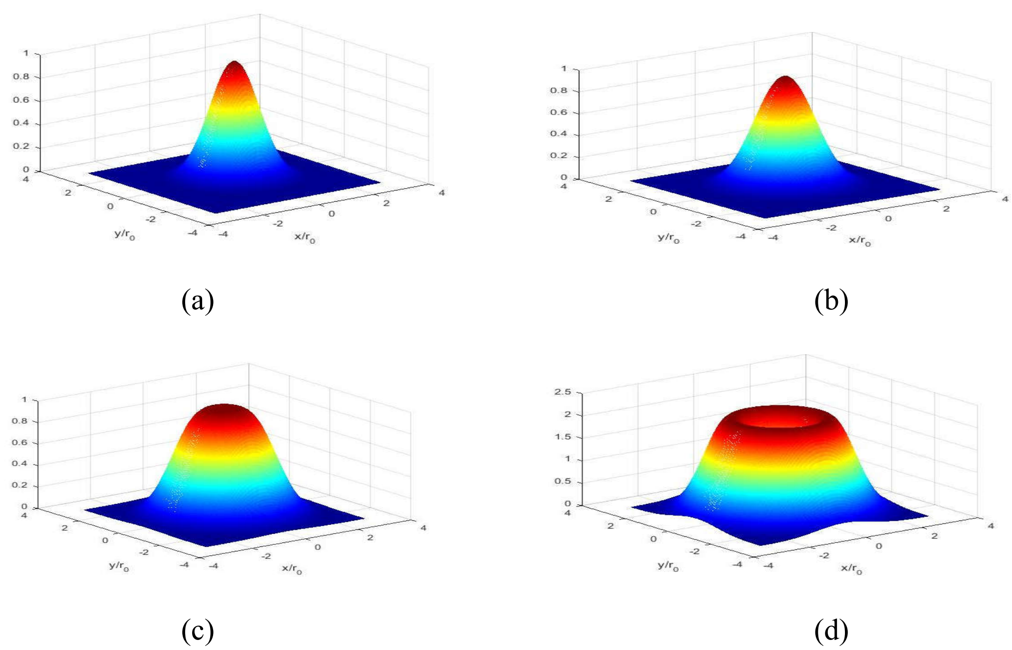



Fig. 3.3: 3D profile of ChG laser beam for (a) $b$=0 (b) $b$=0.5 (c) $b$=1.0 (d) $b$=1.45

To observe the impact of the cosh factor ($b$) on the irradiance across the laser beam's cross-section, which represents its intensity profile, I generated a 3-D intensity profile of the ChG laser beam for various $b$ values, as illustrated in Fig. (3.3). Additionally, Fig. (3.4) displays the corresponding intensity profile projections along the transverse beam axes. It can be seen that for $0 \leq b \leq 1$, with increase in the value of cosh factor $b$, the irradiance over the beam cross section becomes more and more uniform. However, for $b > 1$, a central dark region appears in the beam profile. Thus, the parameter $b$ acts as a control over the beam profile i.e., by optimizing cosh factor $b$, one can obtain desired irradiance over the beam cross section.

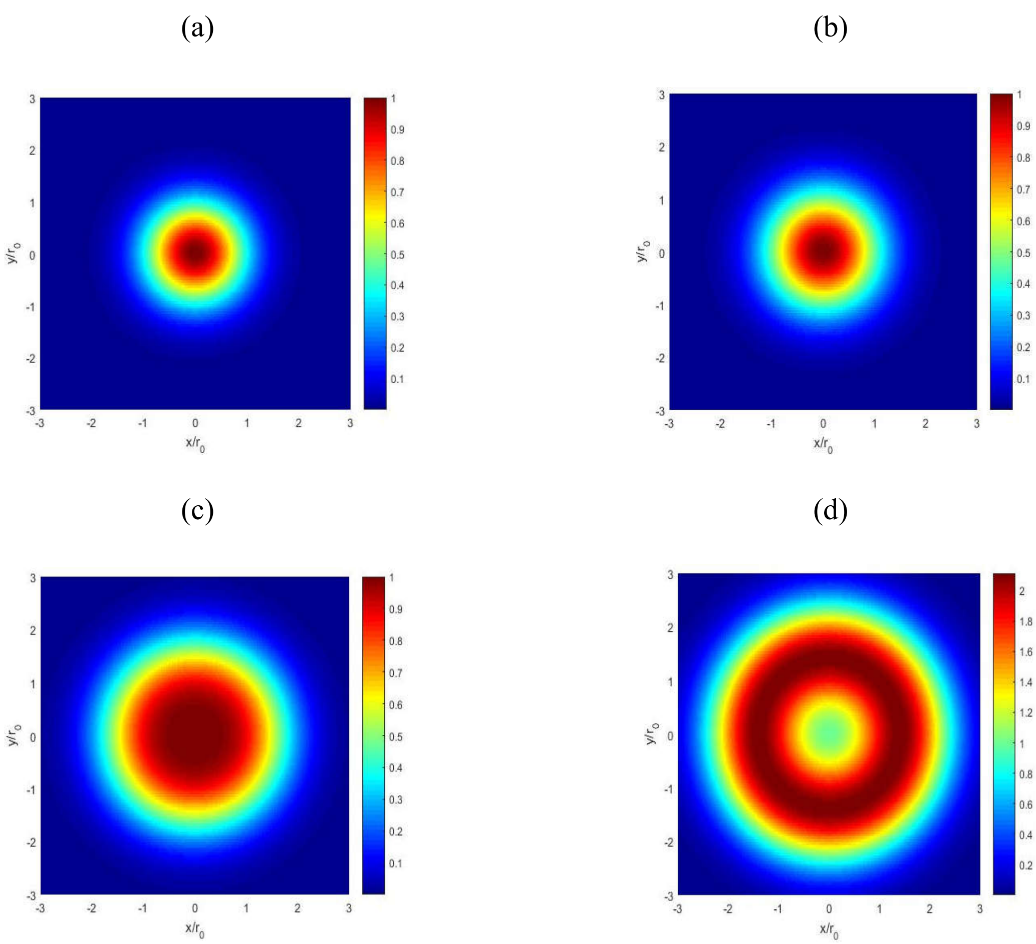



Fig. 3.4: The transverse view of ChG laser beam for

(a) $b$=0 (b) $b$=0.5 (c) $b$=1.0 (d) $b$=1.45.

The cosh Gaussian beam is commonly used in laser applications such as optical trapping, micromachining, and laser cutting, due to its unique intensity profile and the ability to maintain a narrow central lobe over a relatively long distance.

### 3.3.1 Effective Beam Width of ChG Laser Beams

Using eq.(3.8) in eq.(3.4) and (3.5), the effective beam width of ChG laser beam has been obtained as

$$< a^2 > = r_0^2 f^2 (1 + b^2) \tag{3.10}$$

Thus, the ratio of effective beam width of ChG beam to that of Gaussian beam can be written as

$$\Sigma = (1 + b^2) \tag{3.11}$$

### 3.4 Variational Theory

The propagation of an electromagnetic wave through a medium with effective dielectric function $\epsilon$ is governed by wave equation

$$\nabla^2 \vec{E} - \vec{\nabla}(\vec{\nabla}.\vec{E}) + \frac{\omega^2}{c^2} \varepsilon \vec{E} = 0 \tag{3.12}$$

The electric displacement vector can be written as $\vec{D} = \varepsilon \vec{E}$. In the absence of free carriers, the Maxwell's 1st equation gives

$$\vec{\nabla}.\vec{D} = 0 \quad \Rightarrow \vec{\nabla}(\varepsilon \vec{E}) = 0$$

$$\vec{\nabla}.\vec{E} = -\frac{1}{\varepsilon} \vec{E} \tag{3.13}$$

Thus, the wave equation for $\vec{E}$ becomes

$$\nabla^2 \vec{E} - \vec{\nabla}\left(\vec{E}.\frac{\vec{\nabla}\,\mathrm{E}}{\varepsilon}\right) + \frac{\omega^2}{c^2} \varepsilon \vec{E} = 0$$

This equation can be written as

$$\nabla^2 \vec{E} - \varepsilon \frac{\omega^2}{c^2}\left[1 + \frac{c^2}{\varepsilon\omega^2} \nabla^2 \log \varepsilon\right] \vec{E} = 0 \tag{3.14}$$

For

$$\frac{c^2}{\varepsilon\omega^2} |\nabla^2 \log \varepsilon| \ll 1$$

the wave eq.(3.14) becomes

$$\nabla^2\vec{E} + \frac{\varepsilon\omega^2}{c^2}\vec{E} = 0$$

In cylindrical coordinates, this equation can be written as

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{\omega_0^2}{c^2}\varepsilon(EE^*)E = 0 \tag{3.15}$$

Taking $E = E_0(r,z)e^{i(\omega_0 t - k_0 z)}$, one can obtain

$$\frac{\partial E}{\partial z} = \left(\frac{\partial E_0}{\partial z} - ik_0 E_0\right)e^{i(\omega_0 t - k_0 z)}$$

$$\frac{\partial^2 E}{\partial z^2} = \left(\frac{\partial^2 E_0}{\partial z^2} - 2ik_0\frac{\partial E_0}{\partial z} - k_0^2 E_0^2\right)e^{i(\omega_0 t - k_0 z)} \tag{3.16}$$

$$\frac{\partial E}{\partial r} = \frac{\partial E_0}{\partial r}e^{i(\omega_0 t - k_0 z)} \tag{3.17}$$

$$\frac{\partial^2 E}{\partial r^2} = \frac{\partial^2 E_0}{\partial r^2}e^{i(\omega_0 t - k_0 z)} \tag{3.18}$$

Using eqs.(3.16)-(3.18) in eq.(3.15), one can obtain

$$2ik_0\frac{\partial E_0}{\partial z} = \frac{\partial^2 E_0}{\partial z^2} + \left(\frac{\partial^2 E_0}{\partial r^2} + \frac{1}{r}\frac{\partial\, E_0}{\partial r}\right) + \frac{\omega_0^2}{c^2}\,\emptyset(EE^*)E_0 \tag{3.19}$$

Neglecting $\frac{\partial^2}{\partial z^2}$ under slowly varying envelope approximation

$$2ik_0\frac{\partial E_0}{\partial z} = \nabla_\perp^2 E_0 + \frac{\omega_0^2}{c^2}\,\emptyset(EE^*)E_0 \tag{3.20}$$

Equation (3.20) is well known wave equation that governs the evolution of laser beams through nonlinear media. The physical significance of various terms in eq. (3.20) is as follows:

1. $\frac{\partial E_0}{\partial z}$ => Evolution of beam amplitude
2. $\nabla_\perp^2 E_0$ => Diffraction broadening of laser beam
3. $\emptyset(EE^*)$ =>Nonlinear refraction

The equation (3.20) lacks a closed-form analytical solution due to its nonlinear nature, as the superposition principle does not apply to this equation. Hence, dynamics of laser beam can be investigated by solving this equation either by numerical techniques or by semi analytical techniques. However, numerical techniques for this equation are very time consuming and require heavy work stations. Thus, semi analytical techniques are preferred over numerical techniques to solve eq. (3.20). The variational approach provides a powerful tool for investigating eq. (3.20) as it allows us to approximate solutions and gain insights into their properties.

Literature review reveals that almost the base of the nonlinear dynamics of laser beams have built on the paraxial theory[32,33,52,53]. Paraxial theory only takes into account the axial portion of the laser beam means it does not involve the effect off-axial part of laser beam. This approach is not properly suitable when we need to deal with situations related to non-Gaussian beams such as $q$-Gaussian, Cosh-Gaussian, super-Gaussian, Quadruple beams in which off-axial fields play a crucial role in propagation dynamics. In present work, I have proposed the application of variational theory[89-94] which removes the shortcomings of paraxial theory. This approach is derived by investigating certain integral relationships derived from the NLSE. From a fundamental perspective, this method is more favorable, as it allows for the examination of the entire wavefront of the beam during the interaction process. Variational theory is based on variational calculus. It is a semi analytical technique to obtain approximate solutions to PDEs which cannot be solved analytically. This technique replaces a set of PDEs with a set of coupled ODEs those can be solved either analytically or by using simple numerical techniques like Runge Kutta fourth-order method etc. It can be used successfully to have physical insight into number of nonlinear systems like propagation of waves in various nonlinear media, super conductors, Bose Einstein condensates etc. In present investigation, variational technique has been used for the mathematical analysis. The details of variational analysis are as follows:

We first write eq. (3.20) in the form:

$$\hat{F}[\Psi] = 0$$

We then define a Lagrangian density£ $[\Psi, \Psi^*]$, such that:

$$\frac{\partial £}{\partial \Psi^*} = \hat{F}[\Psi]$$

According to variational method, we need to solve the following set of extended Euler-Lagrange equation for the variational parameter $g_i(z)$ with i=1, 2, 3…..N:

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial g_i}{\partial z}\right)}\right)-\frac{dL}{dz}=0 \qquad (3.21)$$

Here, L is the average Lagrangian obtained by integration of £ over the transverse coordinates x and y. $L = \iint £\ dxdy$

For our particular problem, the Lagrangian density[94] corresponding to eq. (3.20) is

$$£ = \mathrm{i}k_0\left(\Psi\frac{\partial\Psi^*}{\partial z}-\Psi^*\frac{\partial\Psi}{\partial z}\right)+\left|\frac{\partial\Psi}{\partial x}\right|^2+\left|\frac{\partial\Psi}{\partial y}\right|^2-\frac{\omega_0^2}{c^2}\int_0^{E_0E_0^*}\Phi(\,E_0E_0^*)d(E_0E_0^*) \qquad (3.22)$$

The variational method investigates the dynamic characteristics of the laser beam through medium under nonlinear regime by using different trial functions for laser beam.

It is important to note that the variational approximation provides an approximate solution and may not capture all the features of the exact solution. However, it can be a useful method to obtain a reasonable approximation for certain nonlinear systems.

### 3.5 Research Objectives

In the present research work, it is proposed to investigate the following problems theoretically as well as through numerical simulations.

1. To study self focusing, self trapping and self phase modulation of $q$-Gaussian laser beams in plasmas with axial temperature ramp.
2. To study self focusing, self trapping and self phase modulation of Cosh-Gaussian laser beams in axially inhomogeneous plasmas.
3. To study the effect of self focusing of $q$-Gaussian laser beams on stimulated Raman scattering in under dense plasma targets with different density profiles.
4. To study the effect of self focusing of Cosh-Gaussian laser beams on stimulated Raman scattering in under dense plasma targets with different density profiles.

## Chapter- 4

## $q$-Gaussian Laser Beams in Collisionless Plasmas: Effect of Axial Temperature Ramp

### 4.1 Introduction

Maiman's invention of the laser marked a significant resurgence in the realm of light matter interactions, leading to the emergence of various new areas of study such as nonlinear optics and laser plasma interactions. Among these, the field of laser plasma interactions has garnered substantial attention from researchers due to its potential applications, including laser-driven nuclear fusion, compact particle accelerators, and coherent radiation sources. The effectiveness of these applications hinges on the laser beam's interaction with plasma, which in turn relies on several nonlinear phenomena like self-focusing, formation of spatial solitons, and self-modulation of the axial phase.

During past few decades, several investigations on these phenomena have been reported by researchers as depicted in literature review. However, all these investigations were focused either on ideal Gaussian beams or on homogeneous plasmas. But our main goal was to investigate the collective effects of non Gaussian intensity and temperature inhomogeneity on propagation behaviour of laser beam in plasma dominated by ponderomotive nonlinearity. Thus in the present investigation, nonlinear propagation dynamics of elliptical $q$-Gaussian laser beams in plasmas with axial temperature ramp has been studied.

### 4.2 Dielectric Function of Plasma with Axial Temperature Ramp

Let's analyze the behavior of a laser beam as it propagates through collisionless plasma. The laser beam is characterized by its electric field vector $E(r,t)$, which can be represented as

$$E(r,t) = A_0(x,y,z)e^{-i(k_0 z-\omega_0 t)}e_x \tag{4.1}$$

The plasma has an equilibrium electron temperature $T_e(z)$, which increases longitudinally with the distance. The temperature profile can be described as $T_0\,(1 +$

$tan(dz))$, where $T_0$ represents the electron temperature at z=0, and the constant d reflects the rate of temperature increase and is known as the slope of the temperature ramp. The laser beam's cross-section exhibits an amplitude structure, as indicated by eqs. 3.3 and 3.8. This amplitude structure results in the generation of a ponderomotive force that acts on the plasma electrons.

$$F_P = -\frac{e^2}{4m\omega_0^2}\nabla(A_0A_0^*) \quad (4.2)$$

Here, the symbols have their usual meaning as defined in chapter- 1. Thus, in the presence of laser, the electron density can be expressed as

$$n = n_0 e^{-(\frac{e^2}{8m\omega_0^2 T_e(z) K_0} A_0 A_0^*)} \quad (4.3)$$

The resulting permittivity of plasma can be derived as

$$\epsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2} e^{-(\frac{e^2}{8m\omega_0^2 T_0(1+\tan(dz))K_0} A_0 A_0^*)} \quad (4.4)$$

Where, $\omega_{P0}^2 = \frac{4\pi e^2}{m} n_0$ is the unperturbed plasma frequency. $\epsilon$ can be expressed as

$$\epsilon = \epsilon_0 + \phi(A_0A_0^*) \quad (4.5)$$

where

$$\epsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega_0^2} \quad (4.6)$$

and

$$\phi(A_0A_0^*) = \frac{\omega_{p0}^2}{\omega_0^2}\left\{1 - e^{-(\frac{e^2}{8m\omega_0^2 T_0(1+\tan(dz))K_0} A_0 A_0^*)}\right\} \quad (4.7)$$

The intensity dependent part $(A_0A_0^*)$ dictates the nonlinear character of plasma.

**4.3 Propagation of Laser Beam**

The wave equation for the beam in plasma with permittivity given by eq. (4.7) is given by

$$2ik_0\frac{\partial A_0}{\partial z} = \nabla_\perp^2 A_0 + \frac{\omega_0^2}{c^2}\phi(A_0A_0^*)A_0 \quad (4.8)$$

The Lagrangian density for eq. (4.8) is

$$\mathcal{L} = i\left(A_0\frac{\partial A_0^*}{\partial z} - A_0^*\frac{\partial A_0}{\partial z}\right) + |\nabla_\perp A_0|^2 - \frac{\omega_0^2}{c}\int^{A_0A_0^*}\phi(A_0A_0^*)d(A_0A_0^*) \qquad (4.9)$$

Using eq.(3.3) in eq.(4.9), we get the reduced Lagrangian as $L = \int \mathcal{L}\, \mathrm{d}^2\mathrm{r}$

using

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\frac{\partial f_{x,y}}{\partial z}}\right) - \frac{\partial L}{\partial f_{x,y}} = 0 \qquad (4.10)$$

One can obtain

$$\frac{d^2 f_x}{dz^2} = \frac{1}{2k_0^2 a^4}\frac{1}{f_x^3}\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left[\left(1+\frac{1}{q}\right)^{-1} + \left(\frac{<L_1>}{E_{00}^2}f_x f_y + \frac{2E_{00}^2}{f_x^2 f_y}\frac{\partial <L_1>}{\partial f_x}\right)\right]$$

(4.11)

$$\frac{d^2 f_y}{dz^2} = \frac{1}{2k_0^2 b^4}\frac{1}{f_y^3}\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left[\left(1+\frac{1}{q}\right)^{-1} + \left(\frac{<L_1>}{E_{00}^2}f_x f_y + \frac{2E_{00}^2}{f_x f_y^2}\frac{\partial <L_1>}{\partial f_y}\right)\right]$$

(4.12)

where

$$<L_1> = \frac{\omega_0^2}{c^2}\int\left(\int^{A_0A_0^*}\phi(A_0A_0^*)d(A_0A_0^*)\right) d^2r$$

Eqs. (4.11) & (4.12) can be written as

$$\frac{d^2 f_x}{dz^2} = \frac{1}{2k_0^2 a^4}\frac{1}{f_x^3}\frac{(1-\frac{1}{q})(1-\frac{2}{q})}{(1+\frac{1}{q})} + \frac{1}{2}\frac{(1-\frac{2}{q})}{a^2\epsilon_0 I_0}\int xA_0A_0^*\frac{\partial\phi}{\partial x}d^2r \qquad (4.13)$$

$$\frac{d^2 f_y}{dz^2} = \frac{1}{2k_0^2 b^4}\frac{1}{f_y^3}\frac{(1-\frac{1}{q})(1-\frac{2}{q})}{(1+\frac{1}{q})} + \frac{1}{2}\frac{(1-\frac{2}{q})}{b^2\epsilon_0 I_0}\int yA_0A_0^*\frac{\partial\phi}{\partial y}d^2r \qquad (4.14)$$

Using eqs. (3.3) and (4.7) in eqs. (4.13) & (4.14), we get

$$\frac{d^2 f_x}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)} \frac{1}{f_x^3} - \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)^{-1} \frac{\beta E_{00}^2}{f_x^2 f_y} I$$

(4.15)

$$\frac{d^2 f_y}{d\xi^2} = \left(\frac{a}{b}\right)^4 \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)} \frac{1}{f_y^3} - \left(\frac{a}{b}\right)^2 \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)^{-1} \frac{\beta E_{00}^2}{f_x f_y^2} I$$

(4.16)

Where

$$\beta = \frac{e^2}{8m\omega_0^2 T_0 K_0}$$

$$d' = dk_0 a^2$$

$$\xi = \frac{z}{k_0 a^2}$$

$$I = \int_0^\infty t\left(1+\frac{t}{q}\right)^{-2q-1} e^{-\frac{\beta E_{00}^2}{f_x f_y\left(1+\tan\left(d'\xi\right)\right)}\left(1+\frac{t}{q}\right)^{-q}} dt$$

In the present investigation, sets of coupled eqs.(4.15) and (4.16) have been solved for following set of parameters:

$$\omega_0 = 1.78 \times 10^{15}\ \frac{rad}{sec}, \quad a = 10\ \mu m, \qquad \beta E_{00}^2 = 3,$$

$$\frac{\omega_{P0}^2 a^2}{c^2} = 9, \qquad T_0 = 10^6\ K$$

for different values of $q, d' and\ \frac{a}{b} viz.$,

$$q = (3, 4, \infty),\ \ d' = (0.025, 0.035, 0.045)$$

$$and$$

$$\frac{a}{b} = (1, 1.1, 1.2)$$

under the boundary conditions

$$f_{x,y} = 1 \text{ and } \frac{df_{x,y}}{d\xi} = 0 \text{ at } \xi = 0$$

Fig. 4.1 illustrates that how the BWPs $f_x$ and $f_y$ of the laser change as it propagates. The graph reveals that within the plasma medium, both $f_x$ and $f_y$ exhibit oscillatory behavior along the longitudinal direction.

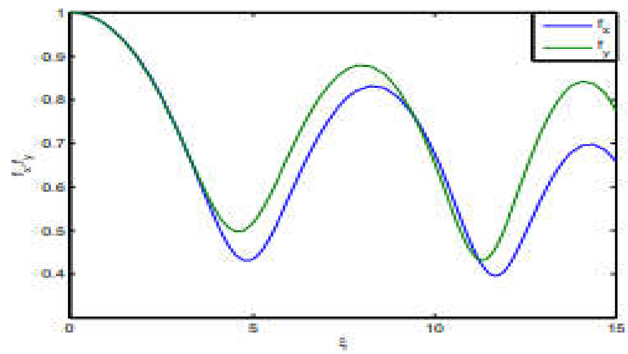



Fig. 4.1: Variations of beam widths over distance. The parameters used are

$q = 3$, $\frac{a}{b} = 1.1$ and $d' = 0.025$.

Reduction in depth of self focusing with increase in DP $q$ is also observed from present study (fig.4.2). This is due to the irradiance profile of $q$-GLB. Laser beams with higher $q$-values exhibit a concentrated energy distribution around the central beam axis, resulting in limited involvement of off-axis rays in nonlinear refraction. The self-focusing phenomenon, which relies on the interplay between optical nonlinearity and nonlinear refraction, is influenced by the $q$-value. Specifically, an increase in $q$ reduces the degree of self-focusing observed in the laser beam.

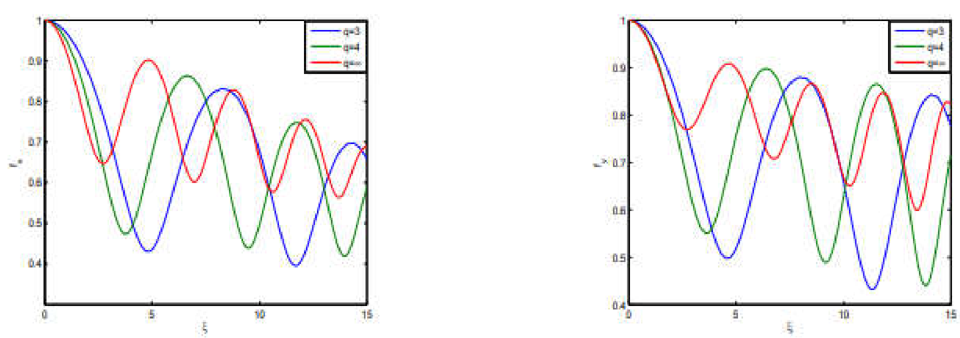



Fig. 4.2: Effect of DP $q$ on BWPs $f_x$ and $f_y$. The parameters used are $\frac{a}{b} = 1.1$ and $d' = 0.025$.

Fig. 4.3 depicts the impact of beam ellipticity in the $y$ direction on the self-focusing of a laser beam. The illustration reveals that as the beam ellipticity in the $y$ axis increases, the degree of self-focusing along the same direction diminishes.

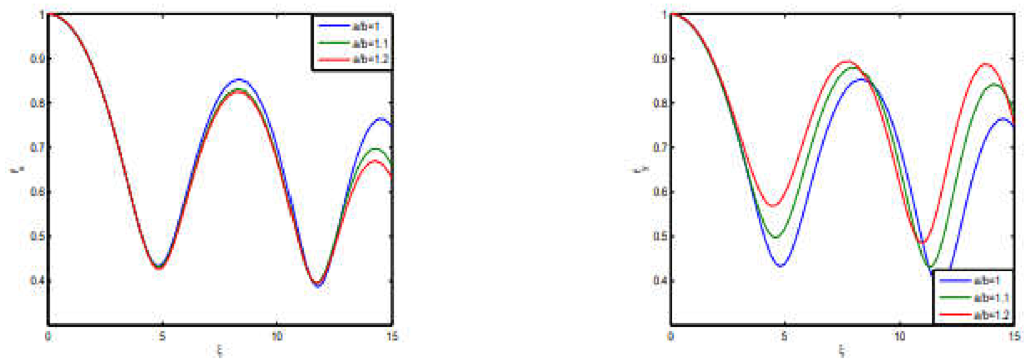



Fig.4.3: Effect of beam ellipticity on BWPs $f_x$ and $f_y$. The parameters used are

$q = 3$ and $d' = 0.025$.

In order to study the effect of density ramp on propagation of laser beam, the equations for beam width are solved for three different values of $d'$ and the resulting behaviour of the beam widths has been depicted in fig. 4.4. It has been seen that effect of density ramp is to make self-focusing of the laser beam tighter and tighter.

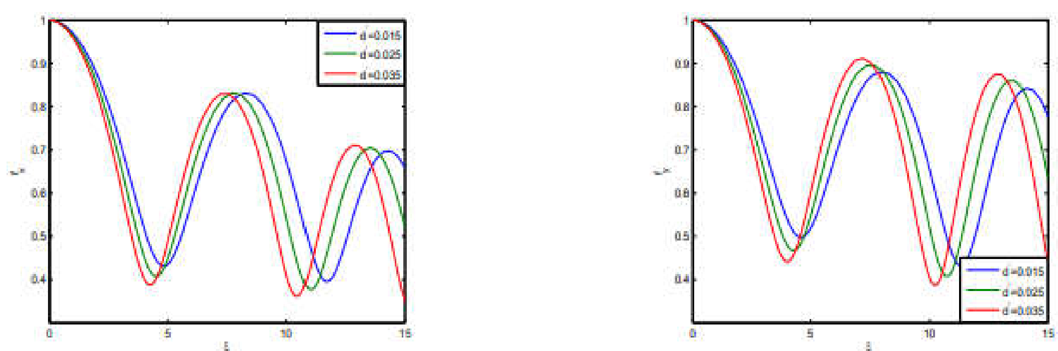


Fig. 4.4: Effect of density ramp on BWPs $f_x$ and $f_y$. The parameters used are

$\frac{a}{b} = 1.1$ and $q = 3$.

### 4.4 Self channeling of laser beam

The beam is said to be self-trapped if its BWPs $f_x$ and $f_y$ remain intact during its propagation. This condition can be obtained by using the conditions $\frac{df_{x,y}}{d\xi} = \frac{d^2 f_{x,y}}{d\xi^2} = 0$, in eqs. (4.15) and (4.16).

Thus, relation between beam intensity and self trapped dimensions of the beam can be derived as

$$r_{a,b}^2 = \left(\frac{q}{1+q}\right)\frac{1}{I_{(f_x f_y)=1}\,\beta E_{00}^2} \tag{4.17}$$

where

$$r_a = \frac{\omega_{p0} a}{c}$$

$$r_b = \frac{\omega_{p0} b}{c}$$

Abrupt decrease in $r_{a,b}$ at lower laser intensity ($\beta E_{00}^2 \ll 1$) has been observed from present study (fig.4.5). Also it can be seen that at sufficiently high laser intensity ($\beta E_{00}^2 \gg 1$), $r_{a,b}$ becomes independent of laser intensity.

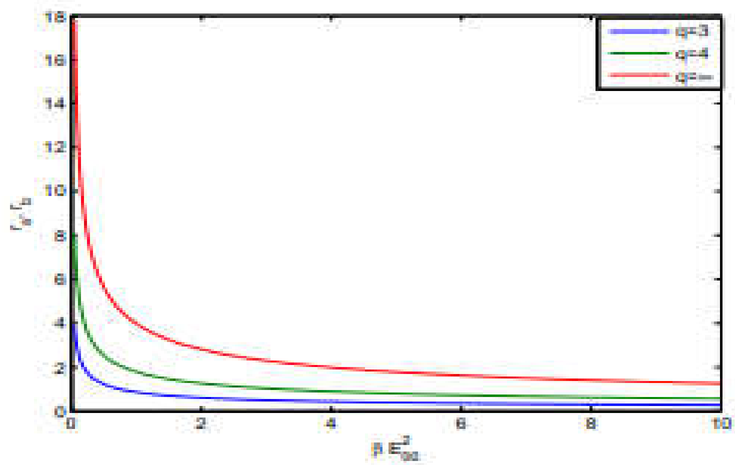



Fig. 4.5: Effect of $q$ on critical curves.

It is also observed that beams with higher DP $q$ possess lesser area for self-focusing in the critical plane.

### 4.5 Self Focusing as a Mechanical Problem

Now it order to get further insight into the self-focusing and self-trapping dynamics of the laser beam, we can relate the self-focusing problem to a simple mechanical problem of an oscillator of unit mass. For that we consider the case of a circular beam ($f_x = f_y = f; a = b = r_0$) propagating through plasma without temperature ramp ($d' = 0$). In that case, eqs. 4.15 and 4.16 reduced to

$$\frac{d^2f}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f^3} - \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 r_0^2}{c^2}\right)\frac{\beta E_{00}^2}{f^3} I' \tag{4.18}$$

$$I' = \int_0^\infty t\left(1+\frac{t}{q}\right)^{-2q-1} e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}} dt$$

Upon multiplication with $\frac{df}{d\xi}$ and then integrating with respect to $f$, eq. 4.18 yields

$$\frac{1}{2}\left(\frac{df}{d\xi}\right)^2 + U(f) = E$$

(4.19)

where, $E$ is constant of integration and

$$U(f) = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{2f^2} - \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 r_0^2}{c^2}\right)\frac{\beta E_{00}^2}{2f^2} I'$$

(4.20)

Fig. (4.6) depicts the variation of potential $U(f)$ with BWP $f$ of the laser beam for different values of DP $q$.

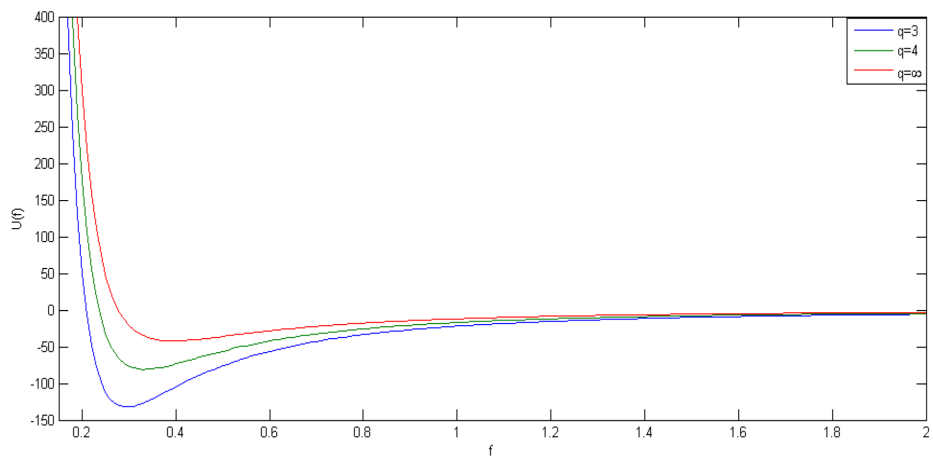



Fig. 4.6: Effect of DP $q$ on potential well for self focusing.

It can be seen that under self-focusing condition ($f < 1$), as the radius of the laser beam shrinks it starts experiencing an attractive potential ($U(f) < 0$) and then after obtaining minimum possible value of BWP $f$, it starts experiencing a repulsive potential. This is because as the radius of the laser beam shrinks, diffraction effects start dominating and diffraction effects correspond to repulsive potential. It can also be seen that $\lim_{f\to 0} U(f) \to \infty$. This is because $f \to 0$ leads to beam collapse and for such a beam, only diffraction effects will dominate. Further it is observed that $\lim_{f\to\infty} U(f) = 0$. The reason behind this is that a laser beam with infinite radius consists of a single wave vector unlike a laser beam with finite radius that consists of superposition of several wave vectors. Thus, laser beam with infinite radius is free from diffraction effects. Also, such a laser beam can't produce any nonlinearity in the supporting medium. Thus, the potential corresponding to laser beam with infinite radius vanishes. The plots in fig.(4.6) also indicate that increase in DP $q$ shifts the bottom of the potential well upwards. This is due to the fact that lasers with larger value of $q$, emit significant number of photons away from the beam axis and their axial part becomes weaker. Thus, laser beams with larger value of $q$ produce lesser nonlinearity into the medium and thus experience lesser attractive potential.

The term $\frac{1}{2}\left(\frac{df}{d\xi}\right)^2$ in eq.(4.19) is equivalent to the kinetic energy of a particle of unit mass. Hence, the constant $E$ can be considered as total energy of the corresponding particle and thus the velocity equivalent of the BWP $f$ can be obtained as

$$\frac{df}{d\xi} = \pm\sqrt{2(E - U(f))} \tag{4.21}$$

Thus, depending on the values of $E$ two possibilities can be there

1. **E>0**

In this case, the line $E$=constant intersects the potential energy curve $U(f)$ only once and at positive value of $U(f)$. Thus, velocity $\frac{df}{d\xi}$ can be real but that will imply defocusing of the laser beam since in this case diffraction will be the dominating phenomenon.

2. **E<0**

In this case, there can be further two possibilities i.e., the line $E$ =constant can intersect the $U(f)$ curve at two different points or it can touch the bottom of the well. In case the line $E$ =constant intersects the potential energy curve ay two points say $f_m$ and $f_M$ ($f_M > f_m$), the laser beam undergoes periodic focusing and defocusing between these two points. In case the $E$ =constant line touches the bottom of the potential well, the laser beam beams propagates while maintaining its BWP $f$ intact. This mode corresponds to self-trapped mode.

Further dynamics of laser beam can be investigated with the help of phase-space plots (figs.4.7, 4.8). These are the plots of the history of the changing variables that in present case are $f, \frac{df}{d\xi}$. Phase space trajectories are a useful concept for visualizing the behavior of a dynamical system. For an oscillatory system like a simple pendulum, the circular closed phase space trajectories indicate purely simple harmonic oscillations and the circular spiraling trajectories indicate damped oscillations. Similarly, the distorted phase space trajectories means oscillations are chaotic in nature.

From the plots in fig. 4.7, it can be seen that the phase space trajectories of the laser beam are closed but distorted for negative values of the energy equivalent $E$. The distortion of the phase space trajectories indicates that the oscillations of the beam width of the laser beam are chaotic in nature. Thus, it can be concluded that the self-focusing phenomenon of laser beams in plasmas is chaotic in nature. The open phase space curves for $E > 0$ indicate diffraction of the laser beam as in this case no potential well exists. From the plots in fig. 4.7, it can also be seen that as we move away from the bottom of the potential well, the area as well as distortion of the phase space trajectories increases. This is due to reduced focusing of the laser beam nearer to the top of the potential well.

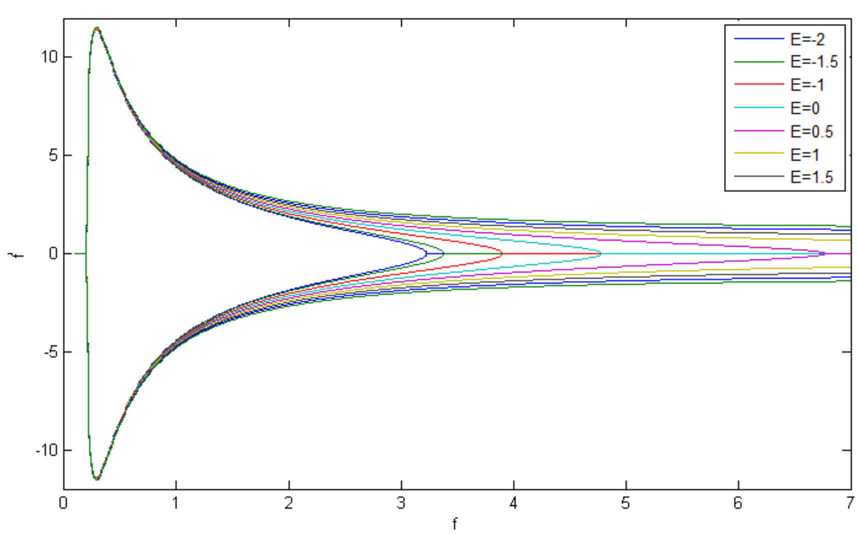



Fig. 4.7: Phase space trajectories of laser beam for different values of energy equivalent $E$.

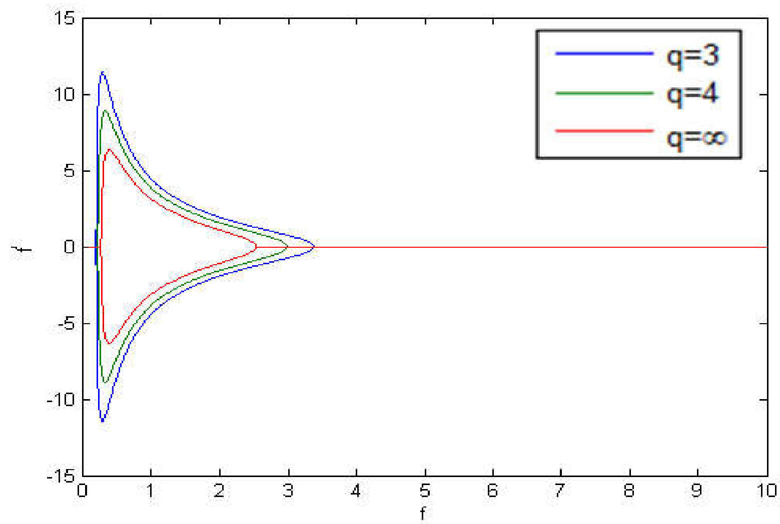



Fig. 4.8: Phase space trajectories of laser beam for different values of DP $q$.

**4.6 Evolution of axial phase of laser beam**

The wave number $k_0$ is related to its components

$$k_0^2 = k_x^2 + k_y^2 + k_z^2 \tag{4.22}$$

Where the longitudinal component $k_z$ arises due to finite beam widths. The effective value of $k_z$ can be expressed as[84]

$$k_z = k_0 - \frac{<k_x^2>}{k_0} - \frac{<k_y^2>}{k_0} \tag{4.23}$$

Thus, the axial phase $\theta_p$ evolves according to

$$\frac{d\theta_p}{dz} = k_0 - \frac{<k_x^2>}{k_0} - \frac{<k_y^2>}{k_0} \tag{4.24}$$

The first term in this equation corresponds to phase of an infinite plane wavefront. Thus, for beam with finite wave front, the axial phase evolve according to

$$\frac{d\theta_p}{d\xi} = -a^2(< k_x^2 > + < k_y^2 > \tag{4.25}$$

Where

$$< k_{x,y}^2 > = \frac{1}{I_k}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_{x,y}^2 A_0 A_0^* dk_x dk_y \tag{4.26}$$

$$I_k = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_0 A_0^* dk_x dk_y \tag{4.27}$$

$$\widetilde{A_0}(k_x, k_y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_0(x,y,z) er^{-i(k_x x + k_y y)} dxdy \tag{4.28}$$

The axial phase of the laser beam evolves according to eq.(4.25). I have solved this equation in accordance with eqs. (4.15) and (4.16).

Fig.4.9 depicts the evolution of axial phase of the laser beam with distance of propagation for different values of DP $q$. It is observed that in 15 Rayleigh lengths ($\xi = 15$), the axial phase of laser beam for $q$=3 decreases by $\theta_p = -80$. However, for that of ideal Gaussian beam, it decreases only by $\theta_p = -45$.

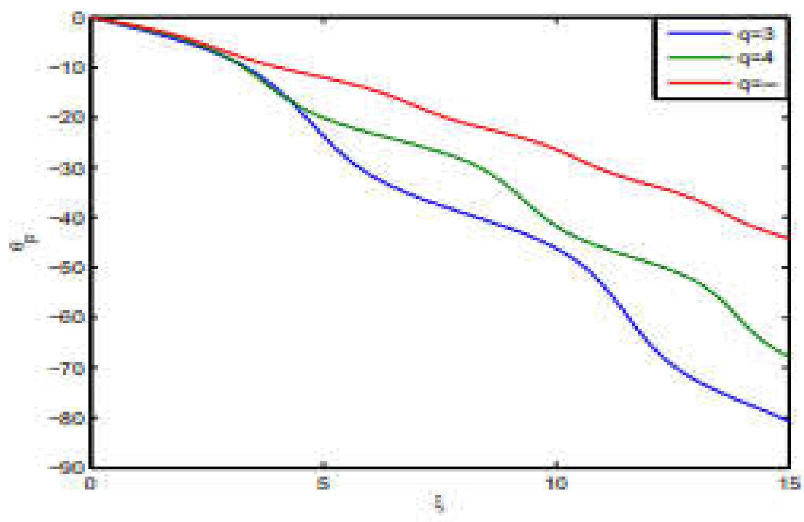



Fig. 4.9: Effect of DP $q$ on axial phase. The parameters used are

$\frac{a}{b} = 1.1$ and $d' = 0.025$.

Thus, the major conclusions about $q$-Gaussian laser beams drawn from the present study are:

- **Distribution of Entropy**: $q$-Gaussian Laser Beams have a broader, flatter emission profile with a higher entropy than traditional Gaussian beams.
- **Focusing Behaviour**: $q$-Gaussian Laser Beams exhibit an anomalously slow focusing behavior compared to Gaussian beams.
- **Self-focusing**: $q$-Gaussian Laser Beams can self-focus and generate stable solitons on propagation, thus maintaining an unchanged spatial profile.
- **Nonlinear Effects**: $q$-Gaussian Laser Beams exhibit a range of distinct nonlinear effects compared to Gaussian beams, such as accelerating dynamics and self-trapping behavior.

## Chapter- 5

## Cosh-Gaussian Laser Beams in Axially Inhomogeneous Plasmas

### 5.1 Introduction

Self action effects i.e., self-focusing, self-channeling and self-modulation of axial phase of optical beams in various nonlinear media have gained significant interest among researchers since the invention of laser. As already explained in chapter-1, self-action effects occur due to modification of optical properties of the supporting medium by an intense optical beam. These modifications in turn affect the propagation characteristics of the optical beam. In one such phenomenon, laser beam focuses by its own without requiring any converging lens or any optical element and is thus referred to as self-focusing. When the self-focusing of the laser beam precisely offsets the diffraction broadening, the beam propagates without any alteration in its width. This phenomenon is referred to as self-trapping or self-channeling of the laser beam.

Several investigations on these phenomena have been reported in the past[32-37]. In context of laser driven nuclear fusion, the goal is to uniformly heat the fuel pellet. In this context, ChG laser beam has gained a great attention of the researchers working on laser plasma interactions. Thus, the aim of this chapter is to give theoretical investigation on self-action effects of ChG laser beams in collisionless plasmas.

### 5.2 Permittivity of Inhomogeneous Plasma

Considering the propagation of a laser beam with electric field vector

$$\boldsymbol{E}(r,t) = E_0(r,z)e^{-\iota(k_0 z-\omega_0 t)}\boldsymbol{e_x} \tag{5.1}$$

through a plasma whose equilibrium electron density is having a upward ramp shape i.e.,

$$n_0(z) = n_0^0(1+\tan(dz)) \tag{5.2}$$

The nonuniform amplitude structure over the cross section of the laser beam results in a transverse force known as ponderomotive force $\left(F_p = -\frac{e^2}{4m\omega_0^2}\nabla(\mathrm{E_0 E_0^\star})\right)$

acting on the plasma electrons. Due to this force, the plasma electrons start migrating from high intensity regions of the illuminated portion of plasma towards the low intensity regions.

The resulting electron density of plasma is given by

$$n(r,z) = n_0(z)e^{-\frac{e^2}{8m\omega_0^2 T_0 K_0}E_0E_0^\star} \tag{5.3}$$

Here, $T_0$ is the temperature of plasma electrons.Thus, the dielectric function ($\epsilon = 1 - \frac{4\pi e^2 n}{m\omega_0^2}$) of plasma can be expressed as

$$\epsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}(1 + \tan(dz))e^{-\frac{e^2}{8m\omega_0^2 T_0 K_0}E_0E_0^\star}$$

Here, ($\omega_{p0}^2 = \frac{4\pi e^2}{m}n_0^0$) corresponds to the unperturbed plasma frequency. Separating $\epsilon$ into linear $\epsilon_0$ and nonlinear $\phi(E_0E_0^\star)$ parts as

$$\epsilon = \epsilon_0 + \phi(E_0E_0^\star)$$

we get

$$\epsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega_0^2} \tag{5.4}$$

$$\phi(E_0E_0^\star) = \frac{\omega_{p0}^2}{\omega_0^2}\left\{1 - (1 + \tan(dz))e^{-\frac{e^2}{8m\omega_0^2 T_0 K_0}E_0E_0^\star}\right\} \tag{5.5}$$

**5.3 Evolution of Beam Envelope**

The model equation for the beam envelope is

$$2\iota k_0 \frac{\partial E_0}{\partial z} = \nabla_\perp^2 \mathrm{E}_0 + \frac{\omega_0^2}{c^2}\phi(E_0E_0^\star)E_0 \tag{5.6}$$

Lagrangian density corresponding to eq.(5.6) is

$$\mathcal{L} = \iota\left(E_0\frac{\partial E_0^\star}{\partial z} - E_0^\star\frac{\partial E_0}{\partial z}\right) + |\nabla_\perp \mathrm{E}_0|^2 - \frac{\omega_0^2}{c^2}\int \phi(E_0E_0^\star)d(E_0E_0^\star) \tag{5.7}$$

Substituting the Cosh Gaussian trial function (as described in chapter 3) for the laser beam in eq. (5.7), we get the reduced Lagrangian as

$$L = \int \mathcal{L}(E_0, E_0^\star, \phi) d^2 r$$

The corresponding Euler-Lagrange equation

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial f}{\partial z}\right)}\right) - \frac{\partial L}{\partial f} = 0 \tag{5.8}$$

gives

$$\frac{d^2 f}{d\xi^2} = \left(\frac{1+e^{-b^2}(1-b^2)}{2(1+b^2)}\right)\frac{1}{f^3} - \left(\frac{e^{-b^2}}{1+b^2}\right)\left(\frac{\omega_{p0}^2 c^2}{c^2}\right)\frac{\beta E_{00}^2}{f^3}(1+\tan(d'\xi))(T_1 - bT_2) \tag{5.9}$$

where,

$$T_1 = \int_0^\infty x^3 e^{-2x^2} \cosh^4(bx) e^{--\frac{\beta E_{00}^2}{f^2} e^{-x^2}\cosh^2(bx)} dx$$

$$T_2 = \int_0^\infty x^2 e^{-2x^2} \cosh^3(bx) \sinh(bx) e^{--\frac{\beta E_{00}^2}{f^2} e^{-x^2}\cosh^2(bx)} dx$$

$$x = \frac{r}{r_0 f}$$

$$\xi = \frac{z}{k_0 r_0^2}$$

For initially collimated laser beam (i.e., laser beam having a plane wave front), eq.(5.9) is subjected to initial conditions:

$$f(0) = 1 \text{ and } \frac{df}{d\xi}\Big|_{\xi=0} = 0$$

In present investigation, eq.(5.9) has been solved for the following set of parameters

$$\omega_0 = 1.78 \times \frac{10^{15}\text{rad}}{\text{sec}};\ \ \mathrm{r_0} = 15\mu\text{m};\ \beta\mathrm{E}_{00}^2 = 3;\ \left(\frac{\omega_{p0} r_0}{c}\right)^2 = 9;$$

$$\mathrm{T_0} = 10^6\text{K}, d' = 0.025$$

and for different values of cosh factor i.e.,

$$b = (0, 0.5, 0.75, 1, 1.1, 1.2, 1.3)$$

and the corresponding evolutions of normalized intensity of the laser beam have been shown in figs 5.1 and 5.2. It can be seen that similar to $q$-GLB, the ChG laser beam also possess oscillatory focusing behaviour in plasma.

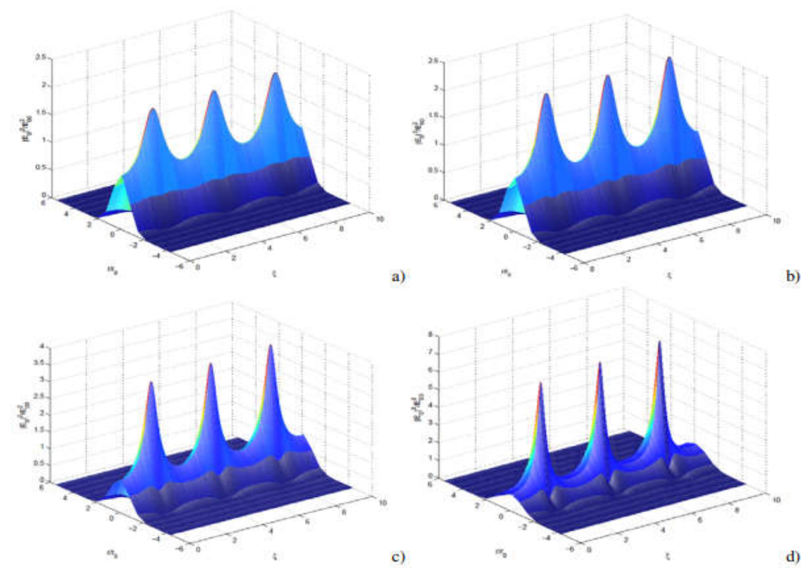



Fig. 5.1: Effect of $b$ on self-focusing of the laser beam for $0 \leq b \leq 1$: $(a) b = 0, (b) b = 0.50, (c) b = 0.75, (d) b = 1$.

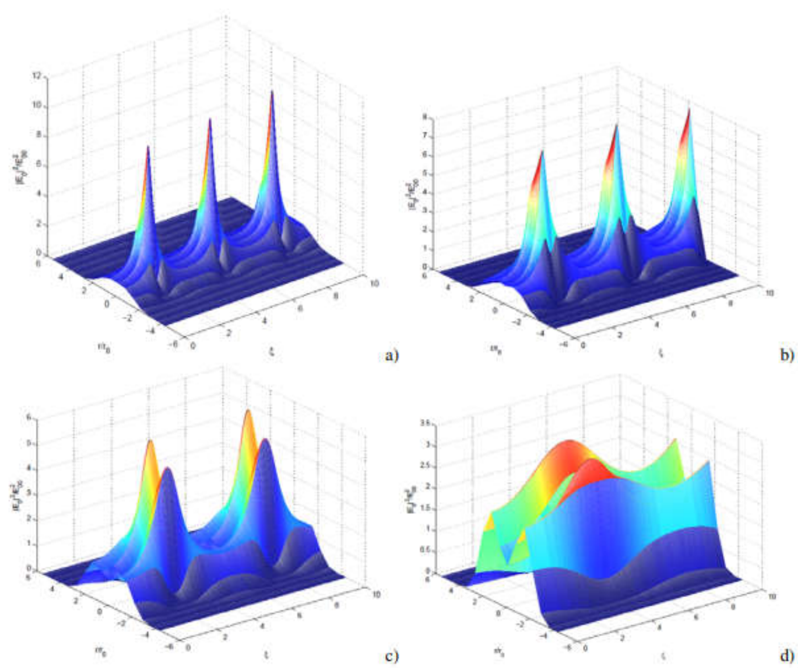



Fig. 5.2: Effect of $b$ on self-focusing of the laser beam for $b > 1$: $(a) b = 1.1, (b) b = 1.2, (c) b = 1.3$.

The effect of density ramp on the evolution of beam envelope can be seen from the tighter focusing of the laser beam with distance i.e., maxima of peak intensity are shifting upwards. The reason for this occurrence is that the equilibrium electron density increases as the longitudinal distance progresses. Consequently, the plasma's refractive index steadily decreases as the laser beam goes deeper into the plasma. As a result, the self-focusing effect intensifies, causing both the maximum and minimum widths of the beam to continuously shift downward after each focal spot. Additionally, it has been observed that the frequency of beam oscillations rises with distance. This can be attributed to the physics underlying this phenomenon: denser plasma leads to a higher phase velocity of the laser beam. Therefore, in denser plasma, the laser beam requires less time to self-focus.

Figs. 5.1 and 5.2 depict the effect of cosh factor $b$ on self focusing. It can be seen that for $0 \leq b \leq 1$, with increase in the value of $b$, self-focusing increases where as the scenario gets reversed after $b > 1$ i.e., for $b > 1$.

Now if along with $\frac{df}{d\xi}|_{\xi=0} = 0$, the second derivative of beam with parameter $f$ with respect to $\xi$ also vanishes at $\xi = 0$ then the beam maintains its beam width as it propagates, experiencing no alterations i.e., beam gets self-trapped.

Thus condition can be derived from eq.5.9 as

$$r_e^2 = \frac{1+e^{-b^2}(1-b^2)}{\beta E_{00}^2 e^{-b^2}(T_1-bT_2)|_{f=1}} \tag{5.10}$$

Fig. 5.3 depicts the effect of decentered parameter $b$ on self-trapping.

It can be seen that with increase in the value of $b$, the $r_e\ vs.\ \beta E_{00}^2$ curves shift downwards. i.e., these beams can be self-trapped at lesser beam power.

This helps to keep the beam power below the threshold of other parametric instabilities.

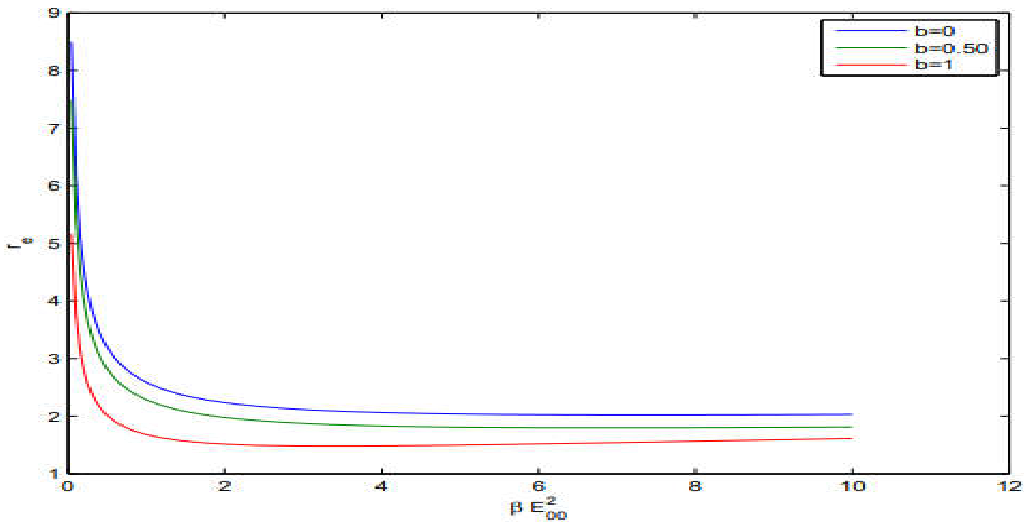



Fig. 5.3: Effect of $b$ on self trapping.

SPM of ChG laser beam has been investigated by following the procedure of section 4.6 and corresponding behaviour of axial phase of the ChG laser beam for different values of decentered parameter $b$ has been depicted in fig.5.4. It can be seen that axial phase of the ChG laser beam shows same behaviour as depicted by $q$-Gaussian laser beam i.e., it decreases monotonically with distance with abrupt jumps at focal spots of the beam. Also, with increase in decentered parameter $b$, the rate of phase modulation increases. This is due to the enhancement of self-focusing of the laser beam with increasing $b$.

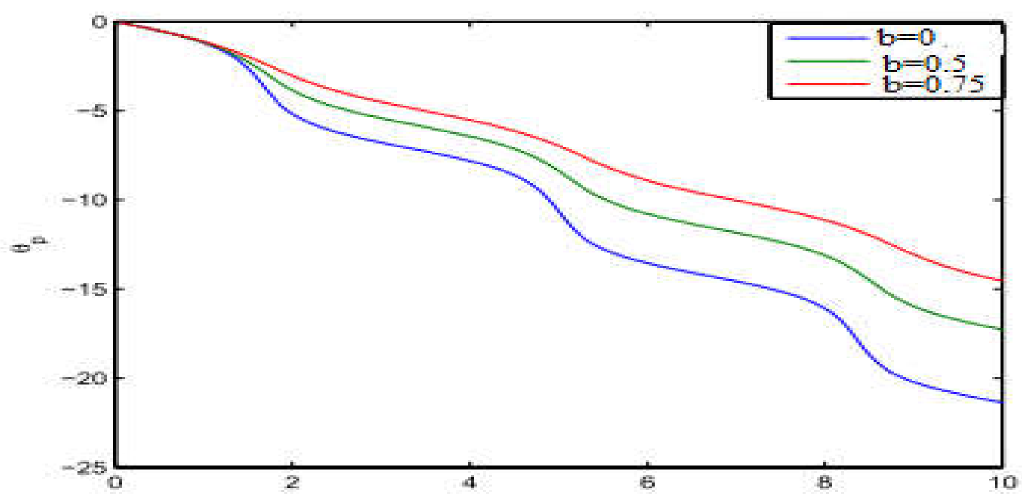



Fig.5.4: Effect of $b$ on self-phase modulation.

# Chapter- 6

## Effect of Plasma Density Ramp on SRS of $q$-Gaussian Laser Beams

### 6.1 Introduction

Ever since the proposal of thermonuclear fusion by using laser beams, the investigations on coupling of intense laser beams with plasmas are at the vanguard of research. The efficiency of laser plasma coupling is decided by various nonlinear processes, those occur during laser plasma interactions. These processes range from collisional absorption to excitation of instabilities i.e., resonant coupling of the incident laser beam with two daughter waves. In the absence of external magnetic field, these daughter waves can be EPWs and IAWs along with a scattered electromagnetic wave. In addition to above mentioned instabilities, there is one more possibility where the laser beam produces density modulation of plasma that in turn leads to self-focusing or filamentation of the laser beam.

For laser driven inertial confinement fusion (ICF), these instabilities are serious threat as they make it difficult to achieve high gain if they are operative to a significant degree. In context of SRS, the pump decays into an EPW and a scattered EM wave, is of more serious concern as it occurs at a density much less than the critical density($\sim\frac{1}{4}n_{cr}$). The consequence is the wastage of a notable quantity of laser energy that must otherwise be combined with the fusion plasma.

When SRS was recognized and understood, it appeared that nonlinear optics could be treated in a reasonable way. However, it turns out to be much more complicated than was thought. One important effect, which influences all the others, is the self-focusing. Because of the ponderomotive force effect, the index of refraction is greater where it is traversed by intense light than elsewhere. This implies a gradient of refractive index at the edges of the beam, which focuses the outer portion of the beam inward. The result is that the beam necks down to a very narrow diameter, of the order of a few microns. What happens from there on is rather complex. Usually, a large initial beam breaks up into a number of filaments, and these may contain within them still finer filaments of diameters not much greater than the wavelength of the light. In these filaments, the light intensity is very much higher than the average beam intensity, and so all sorts of nonlinear optical effects such as SRS and SBS take place primarily in these filaments. Thus, in the investigation of SRS of laser beams in plasmas, the incorporation of the effect of self-focusing becomes essential.

The aim of present study is to investigate the effects of both geometry and irradiance profile of the laser beam as well as of plasma density profile on its self-focusing in collisionless plasma. The study is further extended to investigate the effect of self-focusing on SRS of laser beam.

### 6.2 Equations of Motion of Beam Widths

An optical beam through an axially inhomogeneous plasma with electron density modeled as $n_0(z) = n_0(1 + \tan(dz))$, evolves according to the wave equation

$$2ik_0\frac{\partial A_0}{\partial z} = \nabla_\perp^2 A_0 + \frac{\omega_0^2}{c^2}\phi(A_0A_0^*)A_0 \tag{6.1}$$

where, the symbols have their usual meaning as given in chapter 5 and

$$\phi(A_0A_0^*) = \frac{\omega_{p0}^2}{\omega_0^2}\left\{1 - \left(1 + tan(dz)\right)e^{-\left(\frac{e^2}{8m\omega_0^2 T_0 K_0}A_0A_0^*\right)}\right\} \tag{6.2}$$

Eq. (6.2) has been obtained under the assumption that the plasma is collisionless and thus its nonlinearity is dominated by ponderomotive force.

In present study, the irradiance over the laser beam is modeled by elliptical $q$-Gaussian profile which is given by eq.(3.3). Using the analysis of chapter- 4, one can get the equations of motion of beam widths of the laser beam as

$$\frac{d^2 f_x}{dz^2} = \frac{1}{2k_0^2 a^4}\frac{1}{f_x^3}\frac{(1-\frac{1}{q})(1-\frac{2}{q})}{(1+\frac{1}{q})} + \frac{1}{2}\frac{(1-\frac{2}{q})}{a^2\epsilon_0 I_0}\int xA_0A_0^*\frac{\partial\phi}{\partial x}d^2r \tag{6.3}$$

$$\frac{d^2 f_y}{dz^2} = \frac{1}{2k_0^2 b^4}\frac{1}{f_y^3}\frac{(1-\frac{1}{q})(1-\frac{2}{q})}{(1+\frac{1}{q})} + \frac{1}{2}\frac{(1-\frac{2}{q})}{b^2\epsilon_0 I_0}\int yA_0A_0^*\frac{\partial\phi}{\partial y}d^2r \tag{6.4}$$

Using eqs.(3.3) and (6.2) in eqs.(6.3) and (6.4), one can obtain

$$\frac{d^2 f_x}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)(1-\frac{2}{q})}{(1+\frac{1}{q})}\frac{1}{f_x^3} - \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_x^2 f_y} I \tag{6.5}$$

$$\frac{d^2 f_y}{d\xi^2} = \left(\frac{a}{b}\right)^4 \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f_y^3} - \left(\frac{a}{b}\right)^2\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_x f_y^2} I \tag{6.6}$$

where

$$\beta = \frac{e^2}{8m\omega_0^2 T_0 K_0}$$

$$d' = dk_0 a^2$$

$$\xi = \frac{z}{k_0 a^2}$$

$$I = \iint_0^\infty t\left(1+\frac{t}{q}\right)^{-2q-1} e^{-\frac{\beta E_{00}^2}{f_x f_y}\left(1+\frac{t}{q}\right)^{-q}} dt$$

In this chapter, eqs. 6.5 and 6.6 have been solved numerically for following parameters:

$$\omega_0 = 1.78\times 10^{15}\ rad/sec, \quad a = 10\ \mu m,$$

$$\beta E_{00}^2 = 3, \qquad \frac{\omega_{P0}^2 a^2}{c^2} = 9$$

and for different values of $q, d' and\ \frac{a}{b} viz.$.,

$$q = (3, 4, \infty),\ \ d' = (0.25, 0.35, 0.45)$$

$$and\ \ \frac{a}{b} = (1, 1.1, 1.2)$$

under the boundary conditions

$$f_{x,y} = 1 \qquad and \qquad \frac{df_{x,y}}{d\xi} = 0; \text{at } \xi = 0$$

To examine the impact of DP $q$ on the changes in laser beam widths within a plasma medium, eqs. (6.5) and (6.6) have been solved for various $q$ values. By considering the presence of the plasma medium, resulting evolutions of the beam widths along the $x$ and $y$ directions with respect to the longitudinal distance, have been plotted in fig. (6.1). Similar to the case of plasma with axial temperature ramp, oscillatory focusing of the laser beam can be seen in plasma with density ramp.

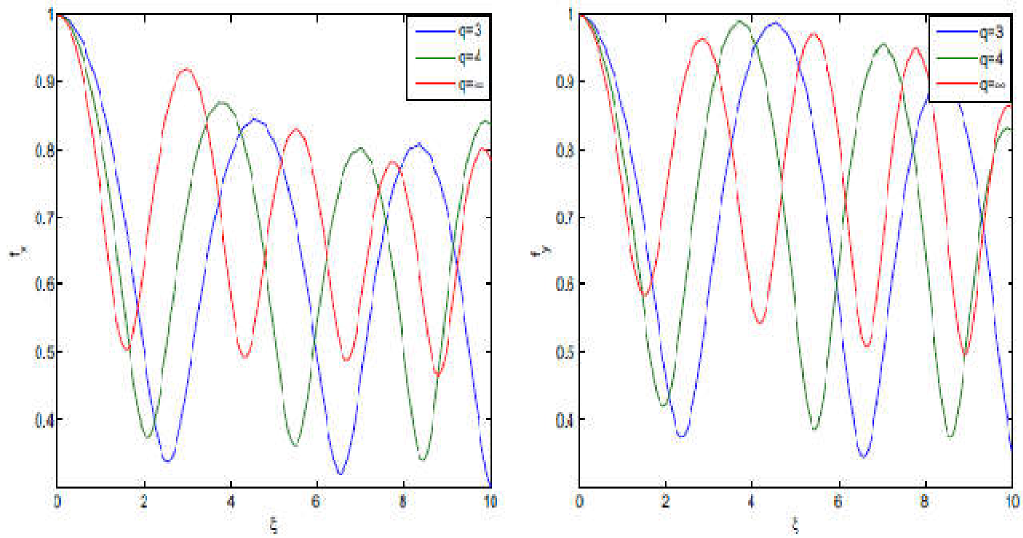



Fig. 6.1: Effect of DP $q$ on beam widths. The parameters used are

$$q = (3, 4, \infty),\ d' = 0.25\ and\ \frac{a}{b} = 1.1.$$

The plots in fig. 6.1 also illustrate that as the DP $q$ increases, the degree of self-focusing decreases in both transverse directions. The physics behind this fact is same as that explained in chapter- 4.

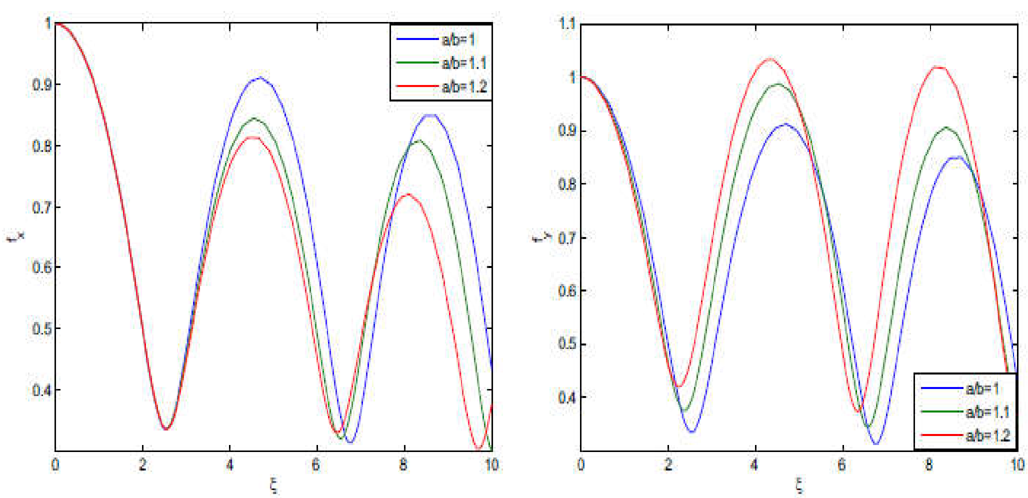



Fig. 6.2: Effect of beam ellipticity on beam widths. The parameters used are

$$q = 3,\ d' = 0.25\ and\ \frac{a}{b} = (1, 1.1, 1.2).$$

Further the effect of beam geometry i.e., its ellipticity on self-focusing has been investigated by considering three different values of $\frac{a}{b}$. One of these values has been taken to be unity (corresponding to circular beam). This is just to compare the dynamics of elliptical beam with that of circular beam. Reduction in self-focusing with increase in beam ellipticity has been observed from the present investigation.

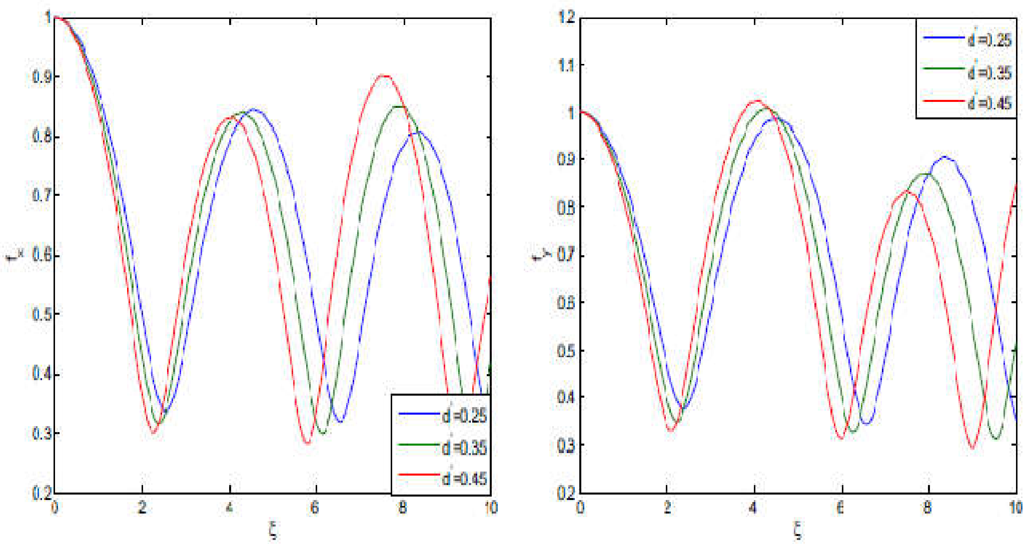



Fig. 6.3: Effect of density ramp on beam widths. The parameters used are

$$q = 3,\ d' = (0.25, 0.35, 0.45)\ and\ \frac{a}{b} = 1.1.$$

Fig. (6.3) demonstrates that how the self-focusing varies along x and y axes in plasma with different density ramps. As the slope of the density ramp increases, the self-focusing of the laser beam extends further along both transverse directions.

### 6.3 Evolution of electron plasma wave

The spatiotemporal evolution of the density perturbation $n_{ep}$ associated with EPW is governed by wave equation

$$2ik_{ep}\frac{\partial n_{ep}}{\partial z} = \nabla_{\perp}^{2} n_{ep} + \frac{\omega_{ep}^{2}}{v_{th}}(1 + \tan(dz))\left(1 - \frac{1}{(1 + \beta A_0 A_0^*)}\right) n_{ep} \qquad (6.7)$$

where, $v_{th} = \sqrt{\frac{2KT_0}{m}}$ is the thermal velocity of the electrons.

Considering Gaussian profile of the EPW

$$n_{ep} = \frac{n_{00}}{\sqrt{f_{xe} f_{ye}}} e^{-\left(\frac{x^2}{2a^2 f_{xe}^2} + \frac{y^2}{2b^2 f_{ye}^2}\right)} \quad (6.8)$$

and using the variational calculus

$$\frac{d^2 f_{xe}}{d\xi^2} = \frac{1}{f_{xe}^3} - \left(\frac{\omega_{ep}^2\ a^2}{v_{th}^2}\right)^2 (1 + \tan(d'\xi)) \frac{1}{f_{xe}^2 f_{ye}} T \quad (6.9)$$

$$\frac{d^2 f_{ye}}{d\xi^2} = \left(\frac{a}{b}\right)^4 \frac{1}{f_{ye}^3} - \left(\frac{a}{b}\right)^2 \left(\frac{\omega_{ep}^2 a^2}{v_{th}^2}\right)^2 (1 + \tan(d'\xi)) \frac{1}{f_{ye}^2 f_{xe}} T \quad (6.10)$$

where

$$T = \int_0^{2\pi} \int_0^{\infty} e^{-x\left(\frac{f_x^2}{f_{xe}^2}\cos(\theta) + \frac{f_y^2}{f_{ye}^2}\sin(\theta)\right)} \left(1 + \frac{x}{q}\right)^{(-q-1)} e^{-\frac{\beta E_{00}^2}{f_x f_y}\left(1+\frac{x}{q}\right)^{-q}} x\, dx\, d\theta$$

For initially plane wave front, eqs.(6.9) and (6.10) are imposed to conditions $f_{xe,ye} = 1$ and $\frac{df_{xe,ye}}{d\xi} = 0\ \ at\ \xi = 0$. Eqs. (6.9) and (6.10) demonstrate the connection between EPW and the pump beam, specifically the $q$-Gaussian beam. The impact of the beam's self-focusing on the density perturbations related to EPW is apparent. By utilizing Poisson's equation, it is possible to derive the field of the EPW.

$$E_P = E_{ep} e^{-i(k_0 z - \omega_0 t)} e_z \quad (6.11)$$

$$E_{ep} = \frac{im\omega_{ep}^2}{ek_{ep}\sqrt{f_{xe} f_{ye}}} e^{-\left(\frac{x^2}{2a^2 f_{xe}^2} + \frac{y^2}{2b^2 f_{ye}^2}\right)} \quad (6.12)$$

The strength of the EPW can be determined using eq. (6.12).

To obtain the average strength of this field, the cross-section of the laser beam is considered, and the averaging is done in conjunction with eqs. (6.5), (6.6), (6.9) and (6.10).

The variations of the strength of EPW with respect to the distance are illustrated in figs. 6.4, 6.5 and 6.6.

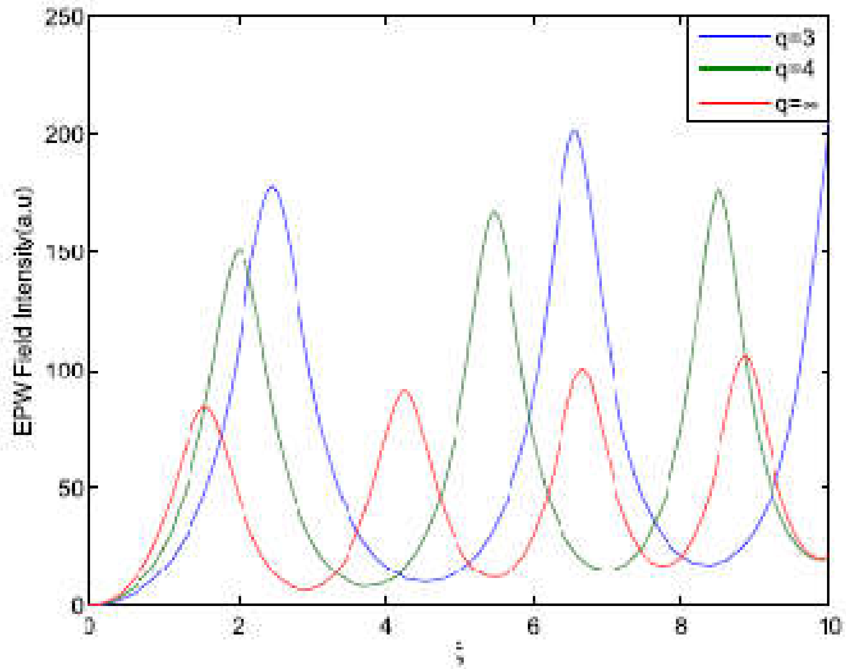



Fig. 6.4: Effect of DP $q$ on strength of EPW. The parameters used are

$$q = (3, 4, \infty),\ d' = 0.25\ and\ \frac{a}{b} = 1.1.$$

The oscillating nature of the field strength of the EPW becomes apparent as it propagates, with the highest field intensity observed precisely at the focal spots of the pump.

The plots shown in fig. 6.4 also demonstrate that as the DP $q$ increases, the field strength of the excited EPW decreases.

This decrease can be attributed to two factors:

- First as the DP $q$ of the pump increases, the focusing of the pump reduces, leading to reduced field strength.
- Second, as $q$ increases, the pump's shifts towards its axis, resulting in a decrease in the impact of the off-axis portion on the plasma electrons.

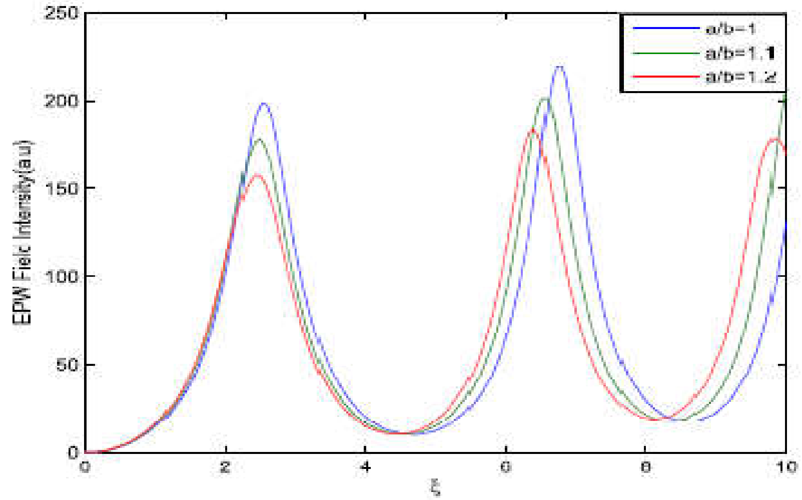



Fig. 6.5: Effect of beam ellipticity on strength of EPW. The parameters used are

$$q = 3, d' = 0.25 \; and \; \frac{a}{b} = (1, 1.1, 1.2)$$

Effect of beam ellipticity on power of EPW is shown in fig. 6.5. It can be seen that higher is the beam ellipticity, lesser will be the power of excited EPW i.e., elliptical beams are less efficient in exciting EPWs.

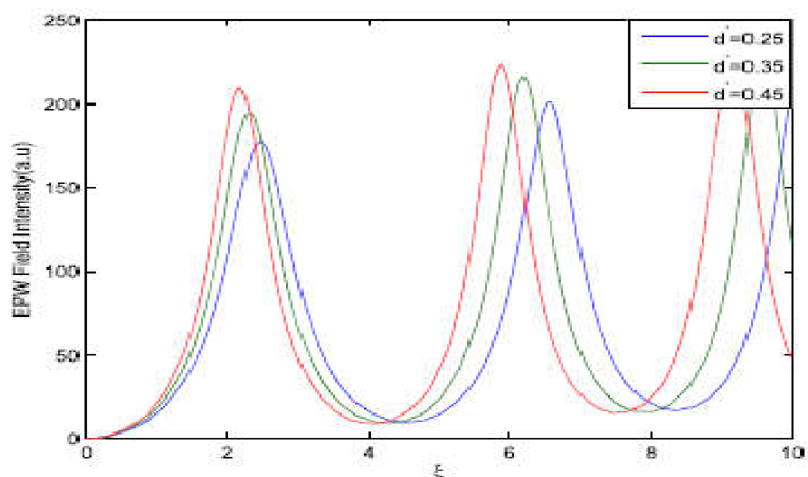



Fig. 6.6: Effect of density ramp on strength of EPW. The parameters used are

$$q = 3, \;\; d' = (0.25, 0.35, 0.45) \; and \; \frac{a}{b} = 1.1.$$

Effect of slope of the ramp on power of EPW is shown in fig. 6.6. It can be seen that power of the excited EPW can be enhanced by making axial density of the plasma more and more steeper.

**6.4 Evolution of scattered wave**

The nonlinear coupling of the pump with EPW produces a current density $J_{NL}$ at frequency $\omega_s = \omega_0 - \omega_{ep}$ given by

$$J_{NL} = \left(\frac{e^2 n_0}{m_0 \omega_s}\right) \frac{n_{ep}}{n_0} e^{i(\omega_s t - k_s z)} A_0(r,z) \quad 6.13)$$

The presence of a non-linear current density gives rise to a wave that is scattered, and its behavior follows the governing wave equation:

$$\nabla^2 E_s = \frac{1}{c^2}\frac{\partial^2 E_s}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial J_{NL}}{\partial t} \quad (6.14)$$

The following equation provides the magnitude of the electric field of scattered radiation as

$$E_s = i\frac{\left(\frac{\omega_{P0}^2}{c^2}\right)}{\left(\frac{\omega_s^2}{c^2} - k_s^2\right)} \frac{n_{ep}}{n_0} A_0(r,z) \quad (6.15)$$

Normalizing the power of scattered wave with that of pump

$$P_S = \frac{\int E_s E_s^* d^2 r}{\int A_0 A_0^* d^2 r} \quad (6.16)$$

One can get

$$P_S = \frac{\left(\frac{\omega_{p0}^2}{c^2}\right)^2}{\left(\frac{\omega_s^2}{c^2} - k_s^2\right)^2} \frac{\int \left(\frac{n_{ep}}{n_0}\right)^2 A_0 A_0^* d^2 r}{\int A_0 A_0^* d^2 r} \quad (6.17)$$

Eq. (6.17) governs the evolution of the power of scattered wave. This equation has been solved in accordance with eqs. (6.5), (6.6) and (6.9), (6.10) for electron temperature $T_0 = 10^6$ K, $\omega_{ep} = 10^{12} rad/\sec\, and\ \frac{n_{00}}{n_0} = 0.001$. The corresponding variation of $P_S$ with $\xi$ has been depicted in figs. (6.7, 6.8 and 6.9). The phenomenon of scattered radiation exhibits a consistent pattern of growth with increasing distance of propagation, resembling a series of step-like increments. Each step coincides with the location where the beam width reaches its minimum value.

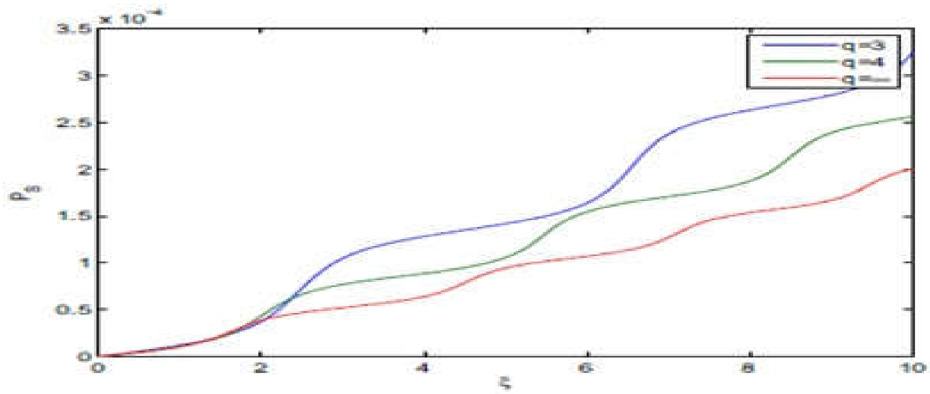



Fig.6.7: Effect of DP $q$ on power of SRS scattered wave. The parameters used are

$$q = (3, 4, \infty), \quad d' = 0.25 \; and \; \frac{a}{b} = 1.1.$$

The plots in fig. (6.7) reveal that as the DP $q$ increases, there is a decrease in the rate at which scattered radiation amplifies. This phenomenon can be attributed to the relationship between the power of the scattered wave and the degree of self-focusing of the pump. Specifically, when the value of $q$ increases, the focusing of the pump is diminished, then leading to a decrease in the power of the scattered radiation.

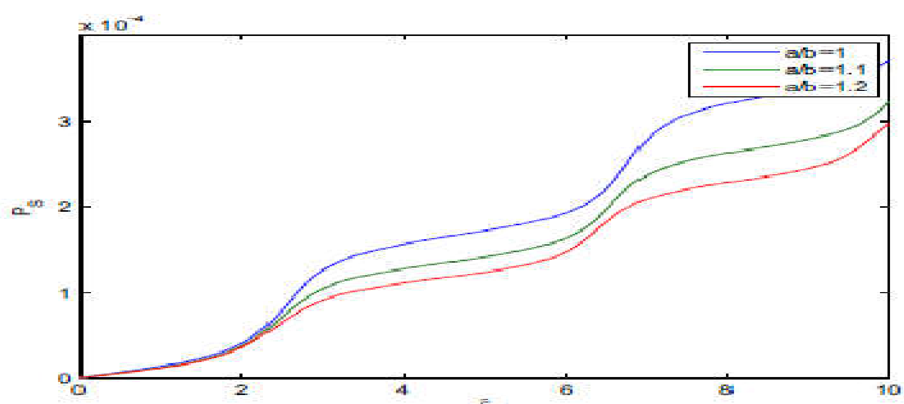



Fig. 6.8: Effect of beam ellipticity on power of SRS scattered wave. The parameters used are $q = 3, \; d' = 0.25 \; and \; \frac{a}{b} = (1, 1.1, 1.2)$.

The plots in fig.(6.8) indicate that the SRS decreases with increase in the ellipticity of the pump. The reason is that with increase in the ellipticity of the pump its overall self-focusing gets reduced.

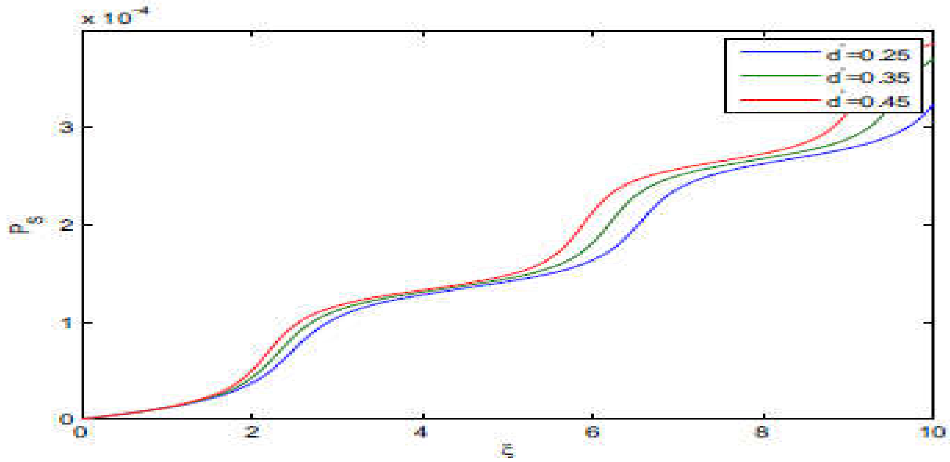



Fig. 6.9: Effect of density ramp on power of SRS scattered wave. The parameters used are $q = 3,\ d' = (0.25, 0.35, 0.45)\ and\ \frac{a}{b} = 1.1$.

The plots in fig. 6.9 illustrates that as the slope of the ramp increases, there is a corresponding increase in the power of the scattered wave. This occurs because the self-focusing is enhanced as the slope of the density ramp increases.

## Chapter- 7

## Self Focusing of Cosh Gaussian Laser Beam in Collisionless Plasma and Its Effect on Excitation of Electron Plasma Wave

### 7.1 Introduction

Plasma is a collection of positively and negatively charged particles moving about so energetically that they do not readily combine. Plasmas are everywhere in the universe. They form the intensely hot gas under high pressure in the sun and the stars, as well as the rarefied gas in interstellar space and in the ionospheric envelope surrounding the earth. Plasmas also exist closer to hand. They are present in the flames of burning fuel and in gas-discharge devices such as neon signs. Plasmas exhibit such an enormous variety of physical effects that physicists have studied their properties for about 200 years. Past research on plasmas, particularly on gas discharges, led to the discovery of the electron and to the elucidation of atomic structure.

The current interest in plasmas reflects two principal motives. The first one is technological. An understanding of plasma behaviour is crucial to the controlled release of thermonuclear energy, the attempt to reproduce in a man-made plasma, the kind of nuclear reaction found in the sun. Another technical goal is the design of magneto-hydrodynamic generators, in which electric power is generated by jets of gas plasma traversing magnetic fields. The second broad motive for the study of plasmas is the importance of plasma phenomena in space and in astrophysics. When plasma is subjected to electromagnetic fields, the motion of the particles is no longer, completely random. One important consequence of this imposed order is that a plasma can transmit certain kinds of waves that are related to electromagnetic waves but they have unique and curious properties. These waves include high frequency EPWs and low frequency one called IAWs. In ICF, the EPWs are of main concern as the excited EPWs can reflect a significant amount of laser energy by SRS.

The aim of present study is to investigate excitation of EPWs by ChG laser beams in axially inhomogeneous plasma.

## 7.2 Evolution of Beam Envelope

The propagation of an optical beam through an axially inhomogeneous plasma whose equilibrium electron density is modeled as $n_0(z) = n_0(1 + \tan(dz))$, is governed by wave equation

$$2ik_0 \frac{\partial A_0}{\partial z} = \nabla_\perp^2 A_0 + \frac{\omega_0^2}{c^2} \phi(A_0 A_0^*) A_0 \tag{7.1}$$

where, the symbols have their usual meaning as given in chapter-5 and

$$\phi(A_0 A_0^*) = \frac{\omega_{p0}^2}{\omega_0^2} \left\{ 1 - \big(1 + tan(dz)\big) e^{-\left(\frac{e^2}{8m\omega_0^2 T_0 K_0} A_0 A_0^*\right)} \right\} \tag{7.2}$$

Eq.(7.2) has been obtained under the assumption that the plasma is collisionless and thus the optical nonlinearity of plasma is dominated by ponderomotive force.

In present study, the irradiance over the pump is modeled by ChG profile which is given by eq.(3.8). The Lagrangian density corresponding to wave eq.(3.1) is

$$\mathcal{L} = \iota \left( E_0 \frac{\partial E_0^\star}{\partial z} - E_0^\star \frac{\partial E_0}{\partial z} \right) + |\nabla_\perp \mathrm{E}_0|^2 - \frac{\omega_0^2}{c^2} \int \phi(E_0 E_0^\star) d(E_0 E_0^\star) \tag{7.3}$$

Substituting eq. (3.8) in eq. (7.3), we get

$$L = \int \mathcal{L}(E_0, E_0^\star, \phi) d^2 r$$

The corresponding Euler Lagrange equation i.e., eq.(3.21) by considering BWP $'f'$ as a variational parameter, we obtained following ordinary differential equation for the evolution of beam waist size.

$$\frac{d^2 f}{d\xi^2} = \left( \frac{1 + e^{-b^2}(1 - b^2)}{2(1 + b^2)} \right) \frac{1}{f^3} - \left( \frac{e^{-b^2}}{1 + b^2} \right) \left( \frac{\omega_{p0}^2 c^2}{c^2} \right) \frac{\beta E_{00}^2}{f^3} (1 + \tan(d'\xi))(T_1 - bT_2) \tag{7.4}$$

where,

$$T_1 = \int_0^\infty x^3 e^{-2x^2} \cosh^4(bx) e^{-\frac{\beta E_{00}^2}{f^2} e^{-x^2} \cosh^2(bx)} dx$$

$$T_2 = \int_0^\infty x^2 e^{-2x^2} \cosh^3(bx) \sinh(bx) e^{-\frac{\beta E_{00}^2}{f^2} e^{-x^2} \cosh^2(bx)} dx$$

$$x = \frac{r}{r_0 f}$$

$$\xi = \frac{z}{k_0 r_0^2}$$

Eq. (7.4) governs the evolution of beam width of ChG laser beam. For initially collimated laser beam (i.e., laser beam having a plane wave front), eq.(7.4) is subjected to initial conditions:

$$f(0) = 1$$

and

$$\frac{df}{d\xi}|_{\xi=0} = 0$$

**7.3 Excitation of Electron Plasma Wave**

The propagation of excited EPW is governed by the wave equation:

$$2\iota k_{ep}\frac{\partial n_{ep}}{\partial z} = \nabla_\perp^2 \mathrm{n_{ep}} + \frac{\omega_{\mathrm{ep}}^2}{v_{th}^2}(1+\tan(dz))\left\{1 - e^{-\frac{e^2}{8m\omega_0^2 T_0 K_0}E_0 E_0^\star}\right\} n_{ep} \qquad (7.5)$$

where, $v_{th} = \sqrt{\frac{2KT_0}{m}}$ is the thermal velocity of plasma electrons.

Considering the Gaussian ansatz for the EPW as

$$n_{ep} = \frac{n_{00}}{f_{ep}} e^{-\frac{r}{2r_0^2 f_{ep}^2}} \qquad (7.6)$$

And using the procedure of section 7.2, we get the equations for the evolution of beam widths of the EPW as

$$\frac{d^2 f_{ep}}{d\xi^2} = \frac{1}{f_{ep}^3} - \left(\frac{\omega_{ep}^2 r_0^2}{v_{th}^2}\right)\frac{1}{f_{ep}^3}(1+\tan(d'\xi))(T_3 - bT_4) \qquad (7.7)$$

$$T_3 = \int_0^\infty x^3 e^{-x^2} e^{-\frac{f^2}{f_{ep}^2}x} \cosh^4(bx) e^{--\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)} dx$$

$$T_4 = \int_0^\infty x^2 e^{-x^2} e^{-\frac{f^2}{f_{ep}^2}x} \cosh^3(bx)\sinh(bx)\, e^{--\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)} dx$$

Eq. (7.7) shows the coupling of EPW with pump beam. Using Poisson's equation, the electric field of the EPW has been obtained as

$$\boldsymbol{E_{ep}} = E_{ep} e^{\iota(k_{ep} z - \omega_{ep} t)} \hat{\boldsymbol{z}}$$

$$E_{ep} = \frac{\iota m \omega_{ep}^2}{c k_{ep} f_{ep}} e^{-\frac{r}{2 r_0^2 f_{ep}^2}} \tag{7.8}$$

Eq. (7.8) gives the field strength of the EPW. Eqs. (7.7) and (7.8) have been solved in accordance with eq.(7.4) by taking following parameters:

$$\omega_0 = 1.78 \times \frac{10^{15}\mathrm{rad}}{\mathrm{sec}}; \; \mathrm{r_0} = 15\mu\mathrm{m}; \beta \mathrm{E}_{00}^2 = 3; \left(\frac{\omega_{p0} r_0}{c}\right)^2 = 9;$$

$$\mathrm{T_0} = 10^6 \mathrm{K}; d' = 0.25$$

$$\omega_{ep} = 10^{15}\mathrm{rad/sec}$$

and for different values of cosh factor

$$b = (0, 0.5, 1, 1.1, 1.2, 1.3)$$

The corresponding behaviour of the field strength of the EPW for different values of cosh factor has been shown in fig.7.1.

It can be seen that the EPW excited by ChG beams exhibit similar behaviour depicted by EPW excited by $q$ -Gaussian beam (Chapter 6). The behaviour of EPW for different values of $b$ again depict that the amplitude of EPW is dictated by the extent of self-focusing of the pump.

As a comparative analysis for different values of Dc.P $b$ on the amplitude of EPW, it can be seen that for adopted laser plasma parameters for $b$=0, the intensity of excited EPW is 99 au, however for $b = 0.5$ it is 110 au and for $b = 1$ it is 125 au that indicates that amplitude of EPW can be enhanced by increasing Dc.P $b$ in the range $0 \leq b \leq 1$. However beyond $b > 1$, there is abrupt decrease in amplitude of EPW as for $b = 1.1$, amplitude of EPW is 74 au, for $b = 1.2$, it is 64 au and for $b = 1.3$, it is 49 au. Thus, Dc.P $b$ act as a control parameter for the amplitude of EPW.

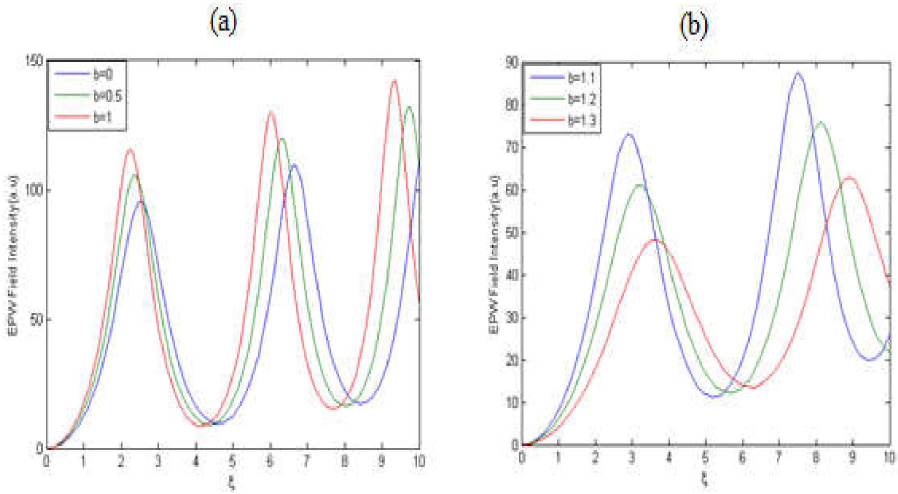



Fig. 7.1: Effect of cosh factor on strength of EPW. The parameters used are

$b = (0, 0.5, 1, 1.1, 1.2, 1.3)\ and\ d' = 0.025.$

## Chapter- 8

## Stimulated Raman Scattering of ChG Laser Beams in Axially Inhomogeneous Plasma

### 8.1 Introduction

In this chapter, the study of previous chapter is extended to investigate SRS of ChG laser beam by nonlinearly excited EPW.

### 8.2 Evolution of Beam Envelope

Following the procedure of chapter-7, the equation of motion of beam widths of laser beam and EPW have been obtained as

$$\frac{d^2 f}{d\xi^2} = \left(\frac{1+e^{-b^2}(1-b^2)}{2(1+b^2)}\right)\frac{1}{f^3} - \left(\frac{e^{-b^2}}{1+b^2}\right)\left(\frac{\omega_{p0}^2 c^2}{c^2}\right)\frac{\beta E_{00}^2}{f^3}(1+\tan(d'\xi))(T_1 - bT_2) \quad (8.1)$$

$$\frac{d^2 f_{ep}}{d\xi^2} = \frac{1}{f_{ep}^3} - \left(\frac{\omega_{ep}^2 r_0^2}{v_{th}^2}\right)\frac{1}{f_{ep}^3}(1+\tan(d'\xi))(T_3 - bT_4) \quad (8.2)$$

where, $\beta = \frac{e^2}{8m\omega_0^2 T_0 K_0}$ is the coefficient of ponderomotive nonlinearity and

$$T_1 = \int_0^\infty x^3 e^{-2x^2}\cosh^4(bx) e^{-\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)}dx$$

$$T_2 = \int_0^\infty x^2 e^{-2x^2}\cosh^3(bx)\sinh(bx) e^{-\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)}dx$$

$$T_3 = \int_0^\infty x^3 e^{-x^2} e^{-\frac{f^2}{f_{ep}^2}x}\cosh^4(bx) e^{-\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)}dx$$

$$T_4 = \int_0^\infty x^2 e^{-x^2} e^{-\frac{f^2}{f_{ep}^2}x}\cosh^3(bx)\sinh(bx) e^{-\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)}dx$$

$$x = \frac{r}{r_0 f}$$

$$\xi = \frac{z}{k_0 r_0^2}$$

$$d' = dk_0 r_0^2$$

Eqs.(8.1) and (8.2) govern the evolution of beam width of ChG laser beam and EPW along the distance of propagation through plasma. For initially collimated beams (i.e., for planar plane wave fronts), eqs.(8.1) and (8.2) are subjected to initial conditions:

$$f(0) = f_{ep} = 1$$

and

$$\frac{df}{d\xi}|_{\xi=0} = \frac{df_{ep}}{d\xi}|_{\xi=0} = 0$$

In the present investigation, eqs.(8.1) and (8.2) have been solved numerically by using Runge Kutta fourth order method for the following set of laser and plasma parameters

$$\omega_0 = 1.78 \times 10^{15}\text{rad/sec}; \;\; \mathrm{r}_0 = 15\mu\mathrm{m}; \beta\mathrm{E}_{00}^2 = 3; \left(\frac{\omega_{\mathrm{p}0}\mathrm{r}_0}{c}\right)^2 = 9; \mathrm{T}_0 = 10^6\mathrm{K}$$

and for different values of Cosh factor and the slope of density ramp i.e.,

$$b = (0, 0.5, 1, 1.1, 1.2, 1.3) \text{ and } d' = (0.025, 0.035, 0.045)$$

**8.3 Evolution of Scattered Beam**

Under the proper phase matching conditions

$$k_0 = k_{ep} + k_s$$

$$\omega_0 = \omega_{ep} + \omega_s$$

The coupling of pump with EPW generates a current density $J_{NL}$ at frequency $\omega_s = \omega_0 - \omega_{ep}$ given by

$$J_{NL} = \left(\frac{e^2 n_0}{m_0 \omega_s}\right)\frac{n_{ep}}{n_0} e^{\iota(\omega_s t - k_s z)} E_0(r, z) \quad (8.3)$$

This nonlinear current density is the source of scattered wave that evolves according to

$$\nabla^2 E_s = \frac{1}{c^2}\frac{\partial^2 E_s}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial J_{NL}}{\partial t} \quad (8.4)$$

From this equation, field strength of scattered wave has been obtained as

$$E_s = \iota \frac{\left(\frac{\omega_{p0}^2}{c^2}\right)}{\left(\frac{\omega_S^2}{c^2} - k_S^2\right)} \frac{n_{ep}}{n_0^0} E_0(r, z) \tag{8.5}$$

Defining the normalized reflectivity of plasma due to SRS as

$$R = \frac{\int E_s E_S^\star d^2 r}{\int E_0 E_0^\star d^2 r} \tag{8.6}$$

we get

$$R = \frac{\left(\frac{\omega_{p0}^2}{c^2}\right)^2}{\left(\frac{\omega_S^2}{c^2} - k_S^2\right)^2} \frac{\int \left(\frac{n_{ep}}{n_0^0}\right)^2 E_0 E_0^\star d^2 r}{\int E_0 E_0^\star d^2 r} \tag{8.7}$$

Eq.(8.7) gives the reflectivity of plasma due to SRS. This equation has been solved numerically for $\omega_{ep} = 1.48 \times 10^{14}$ rad/sec and $\frac{n_{ep}}{n_0^0} = 0.0001$ in association with eqs.(8.1) and (8.2), in order to envision how the SRS reflectivity of plasma is affected by the propagation characteristics of the pump beam. The effect of various laser and plasma parameters on SRS reflectivity is depicted in figs. (8.1) and (8.2).

The reflectivity of plasma, as observed, exhibits a monotonically increasing pattern as the propagation distance increases. This behavior resembles steps, with each step aligning with the minimum beam width. This phenomenon occurs because the self-focusing of the pump leads to an increase in its intensity, thereby amplifying the oscillation amplitude of the plasma electrons. As a result, the amplitude of the generated EPW also increases.

The peaks of this EPW exhibit characteristics similar to partially reflective mirrors, causing an electromagnetic beam to reflect. When the amplitude of the EPW increases, the reflectivity of these partially induced mirrors also increases. As a result, the amplitude of the scattered wave continues to grow as it propagates longitudinally, resulting in the reflectivity of SRS becoming a function that steadily increases with distance.

The plots in fig. 8.1 indicate that with increase in cosh factor for $0 \le b \le 1$, the SRS reflectivity of plasma also increases. But for $b > 1$, with increase in the value of cosh factor, the SRS reflectivity starts decreasing. This is due to enhanced self-focusing of the ChG laser beam with increase in the cosh factor in the range for $0 \leq b \leq 1$. But as the extent of self-focusing of ChG laser beam decreases with increase in cosh factor for $b > 1$, the SRS reflectivity shows a similar behaviour i.e., it also gets reduced.

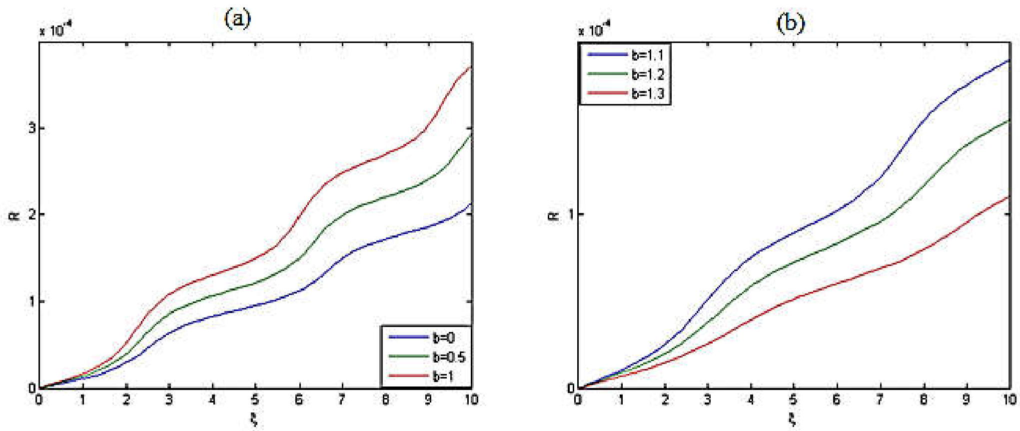



Fig. 8.1: Effect of cosh factor power of SRS scattered wave. The parameters used are $b = (0, 0.5, 1, 1.1, 1.2, 1.3)$ and $d' = 0.025$.

The plots in fig.8.2 depict that slope of density ramp plays a significant role in enhancing the SRS reflectivity of plasma i.e., with increase in slope of density ramp, the SRS reflectivity of plasma increases. This is due to enhancement of self focusing of the beam with increasing slope of density ramp.

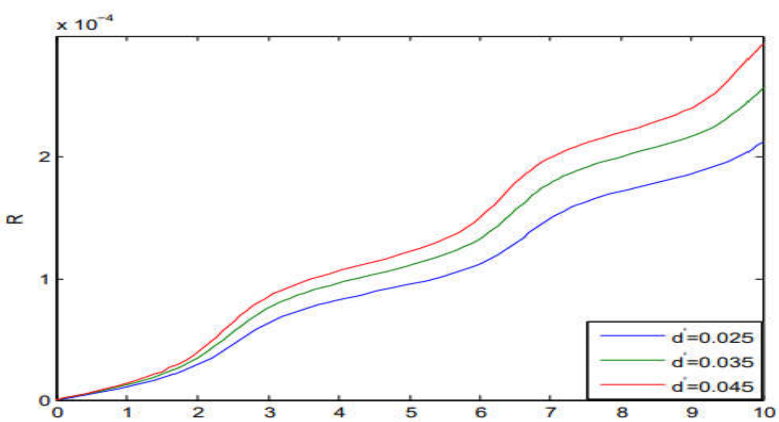



Fig.8.2: Effect density ramp on power of SRS scattered wave. The parameters used are $d' = (0.025, 0.035, 0.045)$ and $b = 0.5$.

## Chapter- 9

## Conclusions and Future Scope

### 9.1 Conclusion

The present study delves into the intricate dynamics of non-Gaussian laser beams, namely $q$-Gaussian and Cosh Gaussian beams, employing a semi-analytical approach grounded in variational theory. This investigation holds paramount significance across numerous disciplines, particularly in the realm of laser-plasma interactions and nonlinear optics. By scrutinizing the self-action effects of these beams, the research unravels pivotal insights into their behavior when interacting with plasma environments, illuminating pathways for optimizing laser performance in various applications. Notably, the findings unveil that self-focusing of laser beams can be notably enhanced by integrating the influence of off-axial intensity and engaging with ramped density plasma, opening avenues for advancements in laser beam manipulation and control. Furthermore, in the realm of $q$-Gaussian beams, the research identifies an optimal parameter configuration—specifically, setting DP $q$ to its lowest value of $q$ = 3 to maximize self-focusing while acknowledging the intricacies of Variational theory's limitations. This comprehension of optimal parameters stands to revolutionize laser systems design, promising heightened efficiency and precision in energy delivery for tasks like laser material processing and particle acceleration. Similarly, in the domain of Cosh Gaussian beams, the study elucidates the role of the decentered parameter (b) as a pivotal control factor in regulating self-focusing, affording versatility in beam profiles that can be tailored for myriad applications, from optical tweezing in biological research to beam shaping in laser-based manufacturing. In essence, the amalgamation of theoretical insights and practical implications from this research not only enriches our understanding of laser-plasma interactions but also paves the way for transformative innovations across diverse fields, spanning laser physics, plasma dynamics, and advanced materials processing.

The primary objective of this study was to explore how the profiles of laser beams influence their Stimulated Raman Scattering (SRS) interactions within plasmas. It has been deduced that the distribution of intensity across the wavefronts of the laser beam plays a pivotal role in shaping its SRS behavior within the plasma medium. Notably, as the intensity profile of the laser beam deviates from its central axis, the SRS reflectivity of the plasma undergoes significant changes. For instance, in the case of $q$-Gaussian laser beams, the SRS reflectivity experiences a notable increase when the value of the parameter $q$ is positioned towards the lower end of its spectrum. Similarly, for Cosh Gaussian laser beams, there exists a critical range of the decentered parameter (b) between 0 and 1, within which the SRS reflectivity escalates. However, beyond this range, the reflectivity diminishes. These findings shed light on the intricate interplay between laser beam profiles and SRS phenomena in plasma environments, offering valuable insights for optimizing laser-plasma interactions in diverse applications ranging from fusion research to laser-based material processing.

### 9.2 Future Scope

Based on laser plasma interactions, various projects and investigations on inertial confinement fusion, particle acceleration, THz generation etc. are going on worldwide. The breath of all these applications is ultimately dependent on the efficiency of laser plasma coupling where SRS is a major nuisance. Along with being a nuisance SRS is having important applications as well. In lasers, the major issue is to have desired wavelengths as laser action is possible only for specific wavelengths. However, SRS gives the possibility to have tunable lasers with broad bandwidths known as Raman lasers. Thus in future, I will try to investigate SRS of laser beams with other beam profiles like Hermite Gaussian, Super Gaussian, Airy Gaussian etc. If the study will be successful, then it will be extended to see the effect of hot spots on the beam wavefronts on SRS in plasmas.

## References


**[1]** G. A. Mourou and D. Umstadter, "Extreme light", Sci. Am. 286, 5 (2002).

**[2]** T. H. Maiman, "Stimulated optical radiation in ruby", Nature 187, 493 (1960).

**[3]** J. A. Giordmaine, "The Interaction of Light with Light", Sci. Am. 210, 38 (1964).

**[4]** A. L. Schawlow, "Lasers", Science 149, 13 (1965).

**[5]** C. H. Townes, "Production of Coherent Radiation by Atoms and Molecules", Science 149, 831 (1965).

**[6]** D. Strickland, "Nobel Lecture: Generating high-intensity ultrashort optical pulses", Rev. Mod. Phys. 91, 030502 (2019).

**[7]** J. A. Giordmaine, "Nonlinear optics", Phys. Today 22, 38 (1969).

**[8]** E. Garmire, "Special issue: Nonlinear optics", Phys. Today 47, 23 (1994).

**[9]** N. Gupta, N. Singhand A. Singh, "Second harmonic generation of $q$-Gaussian laser beam in preformed collisional plasma channel with nonlinear absorption", Phys. Plasma, 22, 113106 (2015).

**[10]** A. Singh and N. Gupta, "Second harmonic generation by self focused $q$-Gaussian laser beam in preformed collisional parabolic plasma channel", Optik 127, 2432 (2016).

**[11]** J. E. Bjorkholm, "Optical Second-Harmonic Generation Using a Focused Gaussian Laser Beam", Phys. Rev. 142, 126 (1966).

**[12]** N. Gupta and S. Kumar, "Generation of Second Harmonics of $q$-Gaussian Laser Beams in Collisionless Plasma with Axial Density Ramp", NLOQO 54, 45 (2021).

**[13]** N. Gupta and S. Kumar, "Self-focusing of multi-Gaussian laser beams in nonlinear optical media as a Kepler's central force problem", Opt. and Quant. Electronics 52, 178 (2020).

**[14]** N. Gupta, "Stimulated Raman scattering: A Review", NLOQO 56, 123 (2021).

**[15]** N. Gupta, S. Choudhry, S. Kumar, S. B. Bhardwaj, S. Kumar, Gnaneshwaran A., S. Shishodia and Kishore B**., "**Scattering of Laser Light in Dielectrics and Plasmas: A Review", NLOQO 55, 1 (2021).

**[16]** R. W. Wood, "Scattered light and the Raman effect", Sci. Am. 143, 452 (1930).

**[17]** A. F. Gibson, "Non-linear optics", Sc. Progress 56, 479 (1968).

**[18]** G. Eckhardt, R. W. Hellwarth, F. J. McClung, S. E. Schwarz, D. Weiner, and E. J. Woodbury, "Stimulated Raman Scattering From Organic Liquids", Phys. Rev. Lett. 9, 455 (1962).

**[19]** N. Gupta and S. B. Bhardwaj, "Relativistic effects on electron acceleration by elliptical $q$-gaussian laser beam driven electron plasma wave", Opt. and Quant. Electronics 53, 700 (2021).

**[20]** N. Gupta and S. Kumar, "Generation of second harmonics of self-focused quadruple -Gaussian laser beams in collisional plasmas with density ramp", J. Opt. 49, 455 (2020).

**[21]** G. G. Comisar, "Theory of the Stimulated Raman Effect in Plasmas", Phys. Rev. 141, 200 (1966).

**[22]** N. Gupta, "Hot electron generation by self focused quadruple Gaussian laser beams during inertial confinement fusion", NLOQO 55, 63 (2022).

**[23]** N. Gupta, N. Singh and A. Singh, "Second harmonic generation of $q-$Gaussian laser beam in preformed collisional plasma channel with nonlinear absorption", Phys. Plasma 22, 113106 (2015).

**[24]** A. Singh and N. Gupta, "Beat wave excitation of electron plasma wave by relativistic cross focusing of cosh-Gaussian laser beams in plasma", Phys. Plasma 22, 062115 (2015).

**[25]** K. Walia, "Self-focusing of high power beam in unmagnetized plasma and its effect on Stimulated Raman scattering process", Optik 225, 165592 (2021).

**[26]** A. Singh and N. Gupta, "Higher harmonic generation by self-focused $q$-Gaussian laser beam in preformed collisionless plasma channel", Laser Particle Beams 32, 621 (2014).

**[27]** A. Singh and N. Gupta, "Electron plasma wave excitation by beating of two $q$-Gaussian laser beams in collisionless plasma", Laser Particle Beams 34, 230 (2016).

**[28]** A. Singh and N. Gupta, "Beat wave excitation of electron plasma wave by coaxial cosh-Gaussian laser beams in collisional plasma", Laser Particle Beams 33, 621 (2015).

**[29]** N. Singh, N. Gupta and A. Singh, "Second harmonic generation of Cosh-Gaussian laser beam in collisional plasma with nonlinear absorption", Optics Communications 381, 180 (2016).

**[30]** A. Singh and N. Gupta, "Second-harmonic generation by relativistic self-focusing of cosh-Gaussian laser beam in underdense plasma", Laser Particle Beams 34, 1 (2016).

**[31]** N. Gupta and A. Singh, "Dynamics of quadruple laser pulses in under-dense plasmas", Contributions to Plasma Physics 57, 258 (2017).

**[32]** V. P. Nayyar and N. K. Verma, "Self-focusing and self-defocusing of laser beams in the extra-paraxial region: geometrical optics approximation", J. Phys. D: Appl. Phys. 10, 2363 (1977).

**[33]** V. S. Soni and V. P. Nayyar, "Self-trapping and self-focusing of an elliptical laser beam in a collisionless magneto plasma", J. Phys. D: Appl. Phys. 13, 361 (1980).

**[34]** J. S. Bakos, I. B. Földes, P. N. Ignácz and Z.Sorlei, "Self-focusing of laser beam crossing a laser plasma", Optics Communications 46, 209 (1983).

**[35]** A. M. Rubenchik and S. K. Turitsyn, "On self-focusing of laser beams in plasma", Laser Particle Beams 5, 3 (1987).

**[36]** P. Vyas and M. P. Srivastava, "Study of Dynamics of Plasma Produced due to Self-Focussed Laser Beam on a Cylindrical Target", Contrib. Plasma Phys. 30, 799 (1990).

**[37]** B. I. Cohen, B. F. Lasinski, A. B. Langdon and J. C. Cummings, "Dynamics of ponderomotive self focusing in plasmas", Phys. Fluids B: Plasma Phys 3, 766 (1991).

**[38]** X. L. Chen and R. N. Sudan, "Two dimensional self focusing of short intense laser pulse in underdense plasma", Phys. Fluids B: Plasma Phys 5, 1336 (1993).

**[39]** G. Bonnaud, H. S. Brandi, C. Manus, G. Mainfray and T. Lehner, "Relativistic and ponderomotive self focusing of a laser beam in a radially inhomogeneous plasma. II. Beyond the paraxial approximation", Phys. Plasmas 1, 968 (1994).

**[40]** P. Chessa, P. Mora, and T. M. Antonsen, "Numerical simulation of short laser pulse relativistic self-focusing in underdense plasma", Phys. Plasmas 5, 3451 (1998).

**[41]** R. Bharuthram, J. Parashar, and V. K. Tripathi, "Transient and time periodic self focusing of a laser beam in a plasma", Phys. Plasmas 6, 1611 (1999).

**[42]** V. Malka, N. R. L. Galloudec, S. Hüller, D. Pesme, F. Amiranoff, S. D. Baton, A. Modena, Ph. Mounaix, C. Rousseaux and M. Salvati, "Strong self-focusing in quasi-stationary laser plasmas", Phys. Plasmas 7, 4259 (2000).

**[43]** L. M. Chen and H. Lin, "Anisotropic self-focusing of intense lasers in under critical plasma", Phys. Plasmas 8, 2974 (2001).

**[44]** R. W. Boyd, "Self-Action Effects in Non-linear Optics", Portal de la CienciaVenezolana 1, 100 (2002).

**[45]** J. Faure, V. Malka, J. R. Marquès, P. G. David, F. Amiranoff, K. Ta Phuoc and A. Rousse, "Effects of pulse duration on self-focusing of ultra-short lasers in underdense plasmas", Phys. Plasmas 9, 756 (2002).

**[46]** B. Eliasson and P. K. Shukla, "Linear self-focusing of whistlers in plasmas", New J. Phys. 7 95 (2005).

**[47]** P. Jha, R. K. Mishra, A. K. Upadhyaya, and G. Raj, "Self-focusing of intense laser beam in magnetized plasma", Phys. Plasmas 13, 103102 (2006).

**[48]** Amrita and A. K. Sharma, "Thermal self-focusing of a laser in collisional plasma: beyond the near axis approximation", Phys. Scr. 74, 128 (2006).

**[49]** A. M. Rubenchik, M. P. Fedoruk, and S. K. Turitsyn, "Laser Beam Self-Focusing in the Atmosphere", PRL 102, 233902 (2009).

**[50]** J. Parashar, "Effect of self-focusing on laser third harmonic generation in a clustered gas", Phys. Scr. 79, 015501 (2009).

**[51]** R. Singh and A. K. Sharma, "Ponderomotive acceleration of electrons by a self focused laser pulse", Phys. Plasmas 17, 123109 (2010).

**[52]** D. N. Gupta and H. Suk, "Enhanced thermal self-focusing of a Gaussian laser beam in a collisionless plasma", Phys. Plasmas 18, 124501 (2011).

**[53]** P. Sati, A. Sharma, and V. K. Tripathi, "Self focusing of a quadruple Gaussian laser beam in a plasma", Phys. Plasmas 19, 092117 (2012).

**[54]** V. V. Semak and M. N. Shneider, “Effect of power losses on self-focusing of high-intensity laser beam in gases”, J. Phys. D: Appl. Phys. 46 185502 (2013).

**[55]** D. N. Gupta, M. R. Islam, D. G. Jang, H. Suk and D. A. Jaroszynski, “Self-focusing of a high-intensity laser in a collisional plasma under weak relativistic- ponderomotive nonlinearity”, Phys. Plasmas 20, 123103 (2013).

**[56]** B. Bokaei and A. R. Niknam, “Increasing the upper-limit intensity and temperature range for thermal self-focusing of a laser beam by using plasma density ramp-up”, Phys. Plasmas 21, 032309 (2014).

**[57]** M. E. Abari, M. Sedaghat and B. Shokri, “Self-focusing and defocusing of Gaussian laser beams in collisional underdense magnetized plasmas with considering the nonlinear ohmic heating and ponderomotive force effects”, Phys. Plasmas 22, 103112 (2015).

**[58]** G. Purohit, P. Rawat and R. Gauniyal, “Second harmonic generation by self-focusing of intense hollow Gaussian laser beam in collisionless plasma”, Phys. Plasmas 23, 013103 (2016).

**[59]** A. A. Balakin, A. G. Litvak, V. A. Mironov and S. A. Skobelev, “Self-compression of soliton-like laser pulses in the process of self-focusing”, J. Opt.19, 095503 (2017).

**[60]** S. A. Kozlov, A. A. Drozdov, M. A. Kniazev, D. A. Kislin, S. Choudhary and R. W. Boyd, “Self focusing does not occur for few-cycle pulses”, J. Phys.: Conf. Ser. 1092, 012066 (2018).

**[61]** C. Hazarika, A. Das and S. Hazarika, “Investigation of paraxial and nonparaxial self-focusing of Gaussian beam in chalcogenide glass medium using NLSE”, J. Phys.: Conf. Ser. 1330, 012020 (2019).

**[62]** K. Virk, “A review on self focusing in laser plasma interaction”, Materials Today: Proceeding 37, 2786 (2021).

**[63]** P. P. Nikam, V. S. Pawar, S. D. Patil and M. V. Takale, “Self-Focusing of Gaussian Laser Beam in an Axially Magnetized Plasma”, J. Phys.: Conf. Ser. 2267, 012001 (2022).

**[64]** N. Bloembergen, “The Stimulated Raman Effect”, American Journal of Physics 35, 989 (1967).

**[65]** R. A. Smith, "Review Lecture: Lasers and Light Scattering", Proc. Roy. Soc. Lond. A. 323, 305-320 (1971).

**[66]** A. L. Schawlow, "Lasers and physics: a pretty good hint", Physics Today 35, 46 (1982).

**[67]** C. J. McKinstrie and M. Yu, "The role of ion momentum in stimulated Raman scattering", Physics of Fluids B: Plasma Physics 3, 3041 (1991).

**[68]** H. C. Barr, T. J. M. Boyd, and A. P. Mackwood, "Stimulated Raman scattering in inhomogeneous collisional laser produced plasmas", Phys. Plasmas 1, 993 (1994).

**[69]** V. Y. Bychenkov, W. Rozmus, and V. T. Tikhonchuk, "Stimulated Raman scattering in non-Maxwellian plasmas", Phys. Plasmas 4, 1481 (1997).

**[70]** K. L. Baker, R. P. Drake, B. S. Bauer, K. G. Estabrook, A. M. Rubenchik, C. Labaune, H. A. Baldis, N. Renard, S. D. Baton, E. Schifano, A. Michard, W. Seka, and R. E. Bahr, "Observation of the Langmuir decay instability driven by stimulated Raman scattering", Phys. Plasmas 4, 3012 (1997).

**[71]** N. Bloembergen, "From nanosecond to femtosecond science", Rev. Mod. Phys. 71, S283 Centenary (1999).

**[72]** J. C. Fernández, J. A. Cobble, D. S. Montgomery, M. D. Wilke and B. B. Afeyan, "Observed insensitivity of stimulated Raman scattering on electron density", Phys. Plasmas 7, 3743 (2000).

**[73]** S. T. Mahmoud and R. P. Sharma, "Relativistic self-focusing and its effect on stimulated Raman and stimulated Brillouin scattering in laser plasma interaction", Phys. Plasmas 8, 3419 (2001).

**[74]** E. S. Dodd and D. Umstadter, "Coherent control of stimulated Raman scattering using chirped laser pulses", Phys. Plasmas 8, 3531 (2001).

**[75]** H. A. Salih, V. K. Tripathi and M. Rafat, "Stimulated Raman scattering of the beat wave of counter-propagating lasers", Phys. Plasmas 11, 747 (2004).

**[76]** H. A. Salih, S. T. Mahmoud, R. P. Sharma, and M. Rafat, "Stimulated Raman scattering of relativistic laser beam in plasmas", Phys. Plasmas 12, 042302 (2005)

**[77]** J. Parashar, "Stimulated Raman scattering of laser from periodically spaced nanoparticles", Phys. Plasmas 16, 093106 (2009).

**[78]** R. Singh and V. K. Tripathi, “Stimulated Raman scattering coupled to decay instability in a plasma channel”, Phys. Plasmas 19, 012109 (2012).

**[79]** Z. J. Liu, B. Li, X. Y. Hu, J. Xiang, C. Y. Zheng, L. H. Cao and L. Hao, “The light diffraction effect on stimulated Raman scattering”, Phys. Plasmas 23, 022705 (2016).

**[80]** Y. X. Wang, Q. S. Feng, H. C. Zhang, Q. Wang, C. Y. Zheng, Z. J. Liu and X. T. He, “Transition of backward stimulated Raman scattering from absolute to convective instability via density modulation”, Phys. Plasmas 24, 103122 (2017).

**[81]** A. P. Misra and D. Chatterjee, “Stimulated scattering instability in a relativistic plasma”, Phys. Plasmas 25, 062116 (2018).

**[82]** K. Walia, Y. Tyagi, D. Tripathi, A. M. Alshehri and N. Ahmad, “Stimulated Raman Scattering of high power beam in thermal quantum plasma”, Optik195, 163166 (2019).

**[83]** A. Bierstedt, C. Warschat, Y. You, K.Rurack and J. Riedel, “Stimulated Raman scattering by intra cavity mixing of nanosecond laser excitation and fluorescence in acoustically levitated droplets”, Anal. Methods 12, 5046 (2020).

**[84]** Q. Feng, L. Cao, Z. Liu, C. Zheng and X. He, “Stimulated Brillouin scattering of backward stimulated Raman scattering”, Scientific Reports 10, 3492 (2020).

**[85]** Y. Ji, C. W. Lian, R. Yan, C. Ren, D. Yang, Z. H. Wan, B. Zhao, C. Wang, Z. H. Fang, and J. Zheng, “Convective amplification of stimulated Raman rescattering in a picosecond laser plasma interaction regime”, Matter Radiat. Extremes 6, 015901 (2021).

**[86]** O. Kamboj, A. H. Ali and N. Kant, “Suppression of stimulated backward Raman scattering in a magnetized density rippled plasma”, Optical and Quantum Electronics 54, 539 (2022).

**[87]** K. Brzozowski, E. Matuszyk, A. Pieczara, J. Firlej, A. M. Nowakowska and M. Baranska, “Stimulated Raman scattering microscopy in chemistry and life science –

Development, innovation, perspectives", Biotechnology Advances 60, 108003 (2022).

**[88]** F. Cooper, C. Lucheroni and H. Shepard, "Variational method for studying self-focusing in a class of nonlinear Schrodinger equations", Physics Letters A 184 (1992).

**[89]** D. Anderson, M. Lisak and A. Berntson, "A variational approach to nonlinear evolution equations in optics", Pramana – J. Phys. 57, 917 (2001).

**[90]** G. Bouchitte and I. Fragala (2002) "Variational theory of weak geometric structures: The measure method and its applications", Progress in Nonlinear Differential Equations and Their Applications 51, 19 (2002).

**[91]** E. Arevalo and A. Becker, "Variationsal analysis of self-focusing of intense ultrashort pulses in gases", Physical Review E 72, 026605 (2005).

**[92]** J. Zhang, J. Y. Yu and N. Pan, "Variational principles for nonlinear fiber optics", Chaos, Solitons and Fractals 24, 309 (2005).

**[93]** C. P. Jisha, V. C. Kuriakose and K. Porsezian, "Spatio-temporal solitons in bulk cubic–quintic media stabilized by self-induced multiphoton ionization", Physics Letters A 352, 496 (2006).

**[94]** C. P. Jisha, V. C. Kuriakose and K. Porsezian, "Variational method in soliton theory", Eur. Phys. J. Special Topics 173, 341 (2009).

**[95]** M. Syafwan, Efendi and N. Arifin, "Variational approximations for twisted solitons in a parametrically driven discrete nonlinear Schrödinger equation", J. Phys.: Conf. Ser. 983, 012145 (2018).

**[96]** E. I. Duque, S. L. Aguayo and B. A. Malomed, "Numerical realization of the variational method for generating self-trapped beams", Optics Express 26, 7451 (2018).

**[97]** N. Rivera, J. Flick and P. Narang, "Variational theory of non-relativistic quantum electrodynamics", Phys. Rev. Lett. 122, 193603 (2019).

**[98]** M. F. Ferreira and S. C. Latas, "A Variational Approach to Dissipative Optical Solitons", J. Phys.: Conf. Ser. 1730, 012019 (2021).

**[99]** A. Djazet, S. I. Fewo, C. B. Tabi and T. C. Kofane, "Dynamics of moving cavity solitons in two-level laser system from symmetric Gaussian input:

vectorial cubic-quintic complex Ginzburg–Landau equation", Applied Physics B 127:151 (2021).

**[100]** Y. Tian, "Variational principle for some nonlinear problems", GEM - International Journal on Geomathematics 13, 4 (2022).

**[101]** Y. Li, J. Dai, J. Liu, Z. Weng, W. Hu and D. Lu, "Evolution of pure-quartic solitons studied by the variational approach", Optics Communications 524, 128790 (2022).

*Journal of Russian Laser Research, Volume 42, Number 4, July, 2021*

# RELATIVISTIC EFFECTS ON STIMULATED BRILLOUIN SCATTERING OF SELF-FOCUSED $q$-GAUSSIAN LASER BEAMS IN PLASMAS WITH AXIAL DENSITY RAMP

**Naveen Gupta,**[1*] **Suman Choudhry,**[1,2] **S. B. Bhardwaj,**[3]
**Sanjeev Kumar,**[1,2] **and Sandeep Kumar**[1]

[1] *Lovely Professional University*
*Phagwara, India*
[2] *Government College for Women*
*Karnal, India*
[3] *Pt. C. L. S. Government College*
*Karnal, India*
*Corresponding author e-mail: naveens222@rediffmail.com

**Abstract**

We investigate the phenomenon of stimulated Brillouin scattering (SBS) of $q$-Gaussian laser beams nonlinearly-interacting with underdense plasmas. When an intense laser beam with frequency $\omega_0$ propagates through plasma, due to relativistic mass nonlinearity of plasma electrons, it gets coupled to a preexisting-ion acoustic wave (IAW) at frequency $\omega_{ia}$. The nonlinear interaction of pump beam with IAW produces a back-scattered wave at frequency $\omega_s = \omega_0 - \omega_{ia}$. In view of the variational theory, we obtain semi-analytical solutions of the coupled nonlinear wave equations for the three waves (pump, IAW, and scattered wave) under the Wentzel-Kramers-Brillouin (WKB) approximation. We show that the scattered-wave power is significantly affected by the self-focusing effect of the pump beam.

**Keywords:** $q$-Gaussian beam, density pump, relativistic plasma, stimulated Brillouin scattering, self-focusing.

## 1. Introduction

Ever since the proposal of initiating nuclear fusion by intense laser beams (ICF) for viable energy production [1] without producing any harm to global climate, there was a considerable interest in the nonlinear interaction of intense laser beams with plasmas. In laser-driven fusion, the goal is to deposit the laser energy at a particular density in the plasma in order to derive the compression and subsequent heating of the fuel pellet. If the pellet is sufficiently compressed, it may undergo fusion, with the release of a large amount of energy. However, the laser may interact with the plasma at a density different to that which is intended, leading to myriad undesirable effects [2–5] and preventing the effective implosion of the target.

Due to their remarkable properties of quasineutrality and collective behavior, plasmas possess a number of natural modes of oscillations [6–8]. This includes high-frequency electron plasma waves (EPWs) and low-frequency ion acoustic waves (IAWs); the latter ones correspond to acoustical phonons, as do the

Manuscript submitted by the authors in English on March 23, 2021.
 

J Opt
https://doi.org/10.1007/s12596-021-00738-3

RESEARCH ARTICLE

# Self-focusing of cosh-Gaussian laser beam in collisional plasma: effect of nonlinear absorption

**Naveen Gupta**[1] · **Sandeep Kumar**[1] · **A Gnaneshwaran**[1] · **Sanjeev Kumar**[1,2] · **Suman Choudhry**[1,2]

Received: 10 May 2021 / Accepted: 2 July 2021


**Abstract** Self-focusing phenomenon of intense laser beams in underdense plasmas has been investigated theoretically. The mechanism of optical nonlinearity of plasma has been modeled by Ohmic heating of the plasma electrons resulting from their collisions with other species. The effect of nonlinear absorption of laser energy in plasma also has been incorporated. Formulation is based on finding a semi-analytical solution of the nonlinear wave equation for the slowly varying beam envelope. For this purpose, moment theory in W. K. B approximation has been invoked that converts nonlinear wave equation to an ordinary differential equation governing the evolution of spot size of the laser beam. The differential equation so obtained has been solved numerically to envision the effect of laser–plasma parameters on self-focusing of the laser beam

**Keywords** Self focusing · Cosh Gaussian · Plasma · Moment theory

## Introduction

Light has always fascinated man and investigation of interaction of light with matter is as old as human civilization. Ancient people used glass-made lenses to focus light to burn pieces of papers. However, with the debut of laser [1] in 1960, the twentieth century witnessed a dramatic shift in our perception and understanding of light. Due to its extraordinary properties of coherence, high intensity and monochromaticity, laser light revealed true beauty of light matter interactions. When laser was born, little did its inventors and aficionados realize that it would not only sweep that era of scientists off its feet, but would continue to challenge and mesmerize generations to come. With that high expectation as a benchmark, the laser has proved to be nothing short of a miracle. The laser has become ubiquitous in the almost every field of modern age science and technology. Even routine life applications are abound and still too many applications are in pipeline and are waiting for their turn.

Amelioration in laser technology fueled by the advent of chirp pulse amplification [2] (CPA) technique has led to a resurgence in the field of light matter interactions by giving birth to two entirely new areas of science, i.e., nonlinear optics and laser–plasma interactions. Interactions of intense coherent beams of light produced by modern laser systems with plasmas are rich in copious nonlinear phenomena those were not possible before the invention of laser. This includes a gamut from optical self-action effects like[3, 4] (self-focusing, self-guiding, self-phase modulation, etc.) to several frequency mixing processes[5, 6] like sum frequency generation, difference frequency generation, second harmonic generation (SHG), etc. Being extremely complex but rich in physics, these nonlinear effects have the potential to keep researchers busy for several upcoming years. Over past few years, veteran physicists are attempting to improve on the understanding of laser–plasma interactions by carrying out experimental as well as theoretical investigations. The major impetus behind these investigations on laser–plasma interactions was built by the proposal of initiating controlled nuclear fusion reaction by using ultra-intense laser beams [7]. Fusion is considered to be the cleanest source of energy that bears the promise to

✉ Naveen Gupta
naveens222@rediffmail.com

1 Lovely Professional University Phagwara, Phagwara, India

2 Government College for Women, Karnal, India

Published online: 11 August 2021

*Nonlinear Optics, Quantum Optics,* Vol. 0, pp. 1–44
Reprints available directly from the publisher
Photocopying permitted by license only



# Scattering of Laser Light in Dielectrics and Plasmas: A Review

NAVEEN GUPTA[1,*], SUMAN CHOUDHRY[1,2], SANJEEV KUMAR[1,2], S. B. BHARDWAJ[3], SANDEEP KUMAR[1], GYANESH[1], SIDDHANTH SHISHODIA AND KISHORE B

[1]*Lovely Professional University, Phagwara, India*
[2]*Government College for Women, Karnal, India*
[3]*Pt. C. L. S. Government College, Karnal, India*

*Received: January 4, 2021. Accepted: April 27, 2021.*

A review on some nonlinear phenomena associated with light matter interactions has been presented. Emphasis is put on explaining the basic physics of the phenomenon while minimizing the mathematics. Particularly the phenomena of Rayleigh scattering, stimulated Raman and Brillouin scattering have been discussed in detail. As a special case scattering of intense laser beams with fourth state of matter i.e., plasma also has been discussed.

*Keywords: Scattering of light, raman scattering, rayleigh scattering, brillouin scattering*

## 1 INTRODUCTION

Laser[1] is one of the most successful pieces of apparatus gifted by 20$^{th}$ century science. When laser made its debut in 1960 people considered it to be solution which is searching for its problem. Since its invention the imact of laser on our lives has changed with time and still is changing. Now laser is ubiquitous in every aspect of life: from super matket barcode scanners, security checkpoints, CD writers to high end applications like medical diagnisis and surgery[2][3], inertial confinement fusion[4][5]. The extent of diversity in the applications of laser can be estimated from

Corresponding author's e-mail: naveens222@rediffmail.com

JOURNAL OF ELECTROMAGNETIC WAVES AND APPLICATIONS
2022, VOL. 36, NO. 6, 767–786
https://doi.org/10.1080/09205071.2021.1983877

Taylor & Francis
Taylor & Francis Group

Check for updates

# Stimulated Raman scattering of self focused elliptical *q*-Gaussian laser beam in plasma with axial temperture ramp: effect of ponderomotive force

Naveen Gupta[a], Sanjeev Kumar[a,b] and S. B. Bhardwaj[c]

[a]Lovely Professional University, Phagwara, India; [b]Government College for Women, Karnal, India; [c]SUS Govt College, Matak-Majri, Indri, Karnal, INDIA

**ABSTRACT**

The phenomenon of stimulated Raman scattering (SRS) of elliptical *q*-Gaussian laser beams interacting nonlinearly with underdense plasmas has been investigated theoretically. Using variational theory semi analytical solutions of the coupled nonlinear wave equations for the three waves (pump, EPW and scattered) have been obtained under W.K.B approximation technique. The equations so obtained have been solved numerically to envision the effects of laser as well as plasma parameters on the dynamics of pump beam and further its effect on the power of scattered wave. It has been observed that power of the scattered wave is significantly affected by the self focusing effect of pump beam.

**ARTICLE HISTORY**
Received 7 February 2021
Accepted 17 September 2021

**KEYWORDS**
Self focusing; q-Gaussian; temperature ramp; stimulated Raman scattering

## 1. Introduction

Laser [1] is one of the most successful pieces of apparatus gifted by twentieth century science. When laser made its debut in 1960 people considered it to be solution which is searching for its problem. Since its invention the imact of laser on our lives has changed with time and still is changing. Now laser is ubiquitous in every aspect of life: from super market barcode scanners, security checkpoints, CD writers to high end applications like medical diagnosis and surgery, inertial confinement fusion. The extent of diversity in the applications of laser can be estimated from the fact that same instrument is being used to produce highest [2] as well as lowest temperature [3] on earth i.e. the instrument can heat as well as can cool down.

Laser has played an important role in revealing true beauty of interaction of light with matter through the appearance of several new phenomena. This includes a gamut from parametric instabilities [4–6] to several self action effects like self-focusing [7], self-channeling [8], self-phase modulation [9] etc. Being extremely complex and rich in physics, these nonlinear phenomena have potential to engage researchers for several upcoming years. Therefore, for the better under-standing of light matter interactions several researchers are making conscious efforts to improve upon the understanding of these

**CONTACT** Naveen Gupta naveens222@rediffmail.com

J Opt
https://doi.org/10.1007/s12596-021-00776-x

RESEARCH ARTICLE

# Nonlinear interaction of quadruple Gaussian laser beams with narrow band gap semiconductors

Naveen Gupta[1] · Sandeep Kumar[1] · Gnaneshwaran A[1] · S. B. Bhardwaj[2] · Sanjeev Kumar[1,3] · Suman Choudhry[1,3]

Received: 1 June 2021 / Accepted: 4 September 2021


**Abstract** This paper presents an investigation on nonlinear propagation of quadruple Gaussian (Q.G) laser beam in narrowband semiconductor (e.g., n-type InSb) plasmas. In the presence of laser beam, the electron fluid in the conduction band becomes relativistic that makes the medium highly nonlinear. As a result the laser beam gets self-focused. Following variational theory approach in W.K.B approximation the numerical solution of the nonlinear Schrodinger wave equation (NSWE) for the field of incident laser beam has been obtained. Particular emphasis is put on dynamical variations of beam spot size and longitudinal phase (Gouy phase). Self-trapping of the laser beam resulting from the dynamical balance between diffraction broadening and nonlinear refraction also has been investigated.

## Introduction

The advent of laser [1] in the early 1960s set in motion a train of events that led to a renaissance in the field of light–matter interactions. The past few years have seen two important advances. One was the proposal of initiating fusion reactions [2] for viable energy production that would quench humanity's endless thirst for energy without worsening the global climate change. Another noteworthy advance was the laser-driven particle accelerators [3]. Particle acceleration by laser-driven plasma wave is an extremely interesting and far-reaching idea that can bring huge particle accelerators to bench top. The efforts to translate these concepts into reality, however, have to surmount two serious problems: (1) The creation of relativistic plasmas requires ultrahigh laser intensities in the excess of $10^{18}$-$10^{20}$ $W/cm^2$, and (2) the plasmas have to be extremely homogeneous. These rather daunting requirements have made it difficult even to carry out exploratory experiments to test the proposed ideas.

Therefore, there have been ongoing efforts to find alternatives to standard plasma experiments, where these severe constraints could be mitigated. One could then validate the theoretical frameworks and shed light on the eventual feasibility of these ideas. Fortunately, such an alternative exists; it is provided by certain special plasmas found in the narrow-band semiconductors [4, 5] (Fig. 1). Plasmas contain negative and positive carriers under conditions in which they do not combine. In Fig. 1 a red dot is an electron, or negative charge, a blue dot containing a plus sign is a positive charge, and neutral atoms are shown green. In a gas there are two kinds of charge carrier: electrons and positive ions (atoms lacking electrons). In a simple metal the only mobile carriers are electrons; positive ions are locked in the crystal lattice. A semiconductor has two kinds of mobile carrier: electrons and positive "holes" or missing electrons. All three plasmas can transmit waves.

Interaction of intense laser beams with semiconductor plasmas is rich in copious nonlinear effects. This spans a gamut from parametric instabilities to several self-action effects like self-focusing, self-trapping self-phase modulation, etc. All these nonlinear effects are extremely complex but rich in physics to provide a necessary test bed for

✉ Naveen Gupta
naveens222@rediffmail.com

1 Lovely Professional University, Phagwara, India

2 SUS Govt College, Matak-Majri, Indri, Karnal, India

3 Government College for Women, Karnal, India

Published online: 07 October 2021

J Opt
https://doi.org/10.1007/s12596-021-00822-8

RESEARCH ARTICLE

# Stimulated Raman scattering of self-focused elliptical *q*-Gaussian laser beam in plasma with axial density ramp: effect of ponderomotive force

**Naveen Gupta[1] · Sanjeev Kumar[1,2] · S. B. Bhardwaj[3]**

Received: 8 September 2021 / Accepted: 29 December 2021


**Abstract** The phenomenon of stimulated Raman scattering (SRS) of elliptical $q$-Gaussian laser beams interacting nonlinearly with underdense plasmas has been investigated theoretically. When an intense laser beam with frequency $\omega_0$ propagates through plasma, due to the nonuniform irradiance over its cross-section d.c component of ponderomotive force becomes active. Due to this ponderomotive nonlinearity of plasma, the laser beam gets coupled with a preexisting electron plasma wave (EPW) at frequency $\omega_{ep}$. The nonlinear interaction of pump beam with EPW produces a back scattered wave at frequency $\omega_s = \omega_0 - \omega_{ep}$. Using variational theory semi-analytical solution of the set of coupled nonlinear wave equations for the three waves (pump, EPW and scattered) has been obtained under W.K.B approximation technique. It has been observed that power of the scattered wave is significantly affected by the self-focusing effect of pump beam.

## Introduction

After transistor, laser [1] is considered to be the most successful pieces of apparatus born from 20th century science. When laser made its debut in 1960 people considered it to be solution which is searching for its problem. Since its invention the impact of laser on our lives has changed with time, and still is changing. Now, laser is ubiquitous in every aspect of life: from super market barcode scanners, security checkpoints, CD writers to high end applications like medical diagnosis and surgery, inertial confinement fusion. The extent of diversity in the applications of laser can be estimated from the fact that same instrument is being used to produce highest [2] as well as lowest temperature [3] on earth i.e., the instrument can heat as well as can cool down. Laser has played an important role in revealing true beauty of interaction of light with matter through the appearance of several new phenomena. This spans a gamut from parametric instabilities [4–6] to several self action effects like self-focusing [7], self-channelling [8], self-phase modulation [9], etc. Being extremely complex and rich in physics, these nonlinear phenomena have potential to engage researchers for several upcoming years. Therefore, for the better under-standing of light matter interactions several researchers are making conscious efforts to improve upon the understanding of these nonlinear phenomena. These efforts have collectively laid the foundation of an entirely new branch of science known as laser plasma interactions.

Raman scattering occurs due to interaction of light with optical phonons. Equivalently, we can see that it is the scattering of light due to quantized molecular vibrations of the medium (Fig. 1). Basically, Raman scattering is an inelastic scattering in which an incident photon with energy $h\nu_L$ produces a scattered photon with energy $h\nu_S$ while the remaining energy $h(\nu_L - \nu_S)$=$h\Omega$ results in the vibrational excitation of the molecule. Thus, Stokes component of the Raman scattering corresponds to creation of an optical phonon. The frequency $\nu_S$ corresponding to the scattered photon is called Stokes frequency and is smaller than the incident light frequency by an amount equal to that of generated phonon.

✉ Naveen Gupta
naveens222@rediffmail.com

1 Lovely Professional University, Phagwara, India

2 Government College for Women, Karnal, India

3 SUS Govt College, Matak-Majri, Indri, Karnal, India

Published online: 29 January 2022

J Opt
https://doi.org/10.1007/s12596-021-00804-w

RESEARCH ARTICLE

# Optical guiding of *q*-Gaussian laser beams in radial density plasma channel created by two prepulses: ignitor and heater

**Naveen Gupta[1] · Rohit Johari[1] · Sanjeev Kumar[1,2] · S. B. Bhardwaj[3] · Suman Choudhry[1,2]**

Received: 4 September 2021 / Accepted: 16 November 2021


**Abstract** Self-action effects (self focusing) and self phase modulation of *q*-Gaussian laser beam in plasma channel created by ignitor heater technique have been investigated theoretically. The ignitor beam causes tunnel ionization of air. The heater beam heats the plasma electrons and establishes a parabolic channel and also prolongs the channel life by delaying the electron ion recombination. The third beam (*q*-Gaussian beam) is guided in the plasma channel under the combined effects of density nonuniformity of the parabolic channel and relativistic mass nonlinearity of the plasma electrons. Formulation is based on finding the numerical solution of nonlinear Schrodinger wave equation (NSWE) for the fields of incident laser beams with the help of moment theory approach. Particular emphasis is put on dynamical variations of the spot size of the laser beams and longitudinal phase-shift of the guided beam with distance of propagation.

**Keywords** Self-Focusing · Self-Trapping · Phase Modulation · Bessel Gauss Lasers · Ponderomotive Force

## Introduction

After the transistor, lasers [1] are considered to be one of the most successful inventions of 20th century science. When laser made its debut in 1960, some people called it solution in search of a problem. Today lasers have reserved their place in almost every aspect of life: consumer technologies like CD players, super market checkout scanners to higher end technologies. With the advent of chirped pulse amplification (CPA) technique [2], the turn of last century has witnessed a giant leap in laser technology leading to a renaissance in the field of light-matter interactions. This amelioration in laser technology has given birth to an entirely new field of science known as laser–plasma interactions. An agglomeration of nonlinear phenomena viz., parametric instabilities, [3, 4] higher harmonic generation, [5, 6] Self-focusing, [7] self-phase modulation [8], etc is ubiquitous in these laser plasma interactions.

A major impetus behind the investigations on laser plasma interactions was provided by the proposal of initiating fusion reactions [9, 10] for viable energy production by using intense laser beams. Fusion is considered to be the cleanest source of energy as there will be no emission of radioactive end products and green house gases. Thus, it bears the promise to quench humanity's endless thirst for energy without making any harm to global climate. Along the way the field of laser–plasma interactions has branched into a number of potential applications like laser-driven accelerators, [11, 12] X-ray lasers, [13, 14] higher harmonic generation [5, 6], etc. The ultimate breath of most of these applications depends on stable guiding of intense laser beams over longer distances, without significant energy loss. However, due to lights natural wave property of diffraction, a light beam traveling in vacuum or in a medium always broadens in the absence of an optical guiding mechanism. Diffraction broadening of the laser beam is thus the fundamental phenomenon that jeopardizes the feature of aforesaid applications by negating the efficiency of laser–plasma coupling. Hence, there is surging interest to explore the methods that may aid to increase the

✉ Naveen Gupta

1 Lovely Professional University, Phagwara, India
2 Government College for Women, Karnal, India
3 SUS Govt College, Matak-Majri, Indri, Karnal, India

Published online: 06 March 2022

*Nonlinear Optics, Quantum Optics*, Vol. 00, pp. 1–28
Reprints available directly from the publisher
Photocopying permitted by license only



# Potential Well Dynamics of Self Focusing of Quadruple Gaussian Laser Beams in Thermal Quantum Plasma

NAVEEN GUPTA[1,*], SANJEEV KUMAR[1,2], SUMAN CHOUDHRY[1,2],
S. B. BHARDWAJ[3] AND SANDEEP KUMAR[1]

[1]*Lovely Professional University, Phagwara*
[2]*Government College for Women, Karnal*
[3]*Pt. C. L. S. Government College, Karnal*

*Received: August 14, 2020. Accepted: December 17, 2020.*

This paper presents theoretical study on self-action effects of intense laser beams interacting with fusion plasmas. Particularly the phenomena associated with the nonlinear refraction of the laser beam have been investigated in detail. In order to see the effect of uniformity of the illumination over the beam phase fronts on its propagation characteristics the irradiance profile of the beam has been modeled by quadruple Gaussian (Q.G) profile. Following Variational theory approach, the nonlinear partial differential equation (PDE) for the beam envelope has been reduced to a set of coupled ordinary differential equations for the evolution of beam width and axial phase. The equations so obtained have been solved numerically to envision the effect of laser as well as medium parameters on the propagation characteristics of the laser beam.

## 1. INTRODUCTION

The quest to initiate nuclear fusion by employing intense laser beams [1-3] to quench endless thirst of human for energy without harming the global climate is at the vanguard of research since past few years. This will be similar to

* Contact Author: e-mail: naveens222@rediffmail.com

J Opt
https://doi.org/10.1007/s12596-022-00873-5

RESEARCH ARTICLE

# Excitation of ion acoustic waves by self-focused $q$-Gaussian laser beam in plasma with axial density ramp

**Naveen Gupta[1] · Suman Choudhry [1,2] · S. B. Bhardwaj [3] · Sanjeev Kumar[1,2]**

Received: 31 January 2022 / Accepted: 6 April 2022


**Abstract** Dynamics of the laser-driven ion acoustic waves (IAWs) in plasmas with axial density ramp has been investigated theoretically. The effect of self-focusing of the laser beam on the power of laser excited IAW has been incorporated. During its propagation through the plasma, the laser beam excites an IAW at frequency $\omega_{ia}$ that due to the optical nonlinearity of plasma gets nonlinearly coupled to the laser beam. Using variational theory, semianalytical solutions of the coupled nonlinear wave equations for the pump wave and IAW have been obtained under W.K.B approximation technique. It has been observed that power of the IAW is significantly affected by the self-focusing effect of pump beam.

**Keywords** $q$-Gaussian · Density ramp · Relativistic plasma · Self-focusing · Ion acoustic waves

## Introduction

Plasma is a collection of positively and negatively charged particles moving about so energetically that they do not readily combine. Plasmas are everywhere in the universe [1]. They form the intensely hot gas under high pressure in the sun and the stars, as well as the rarefied gas in interstellar space and in the ionospheric envelope surrounding the earth. Plasmas also exist closer to hand. They are present in the flames of burning fuel and in gas-discharge devices such as neon signs. Plasmas exhibit such an enormous variety of physical effects that physicists have studied their properties for about 200 years. Past research on plasmas, particularly on gas discharges, led to the discovery of the electron and to the elucidation of atomic structure [2].

The current interest in plasmas reflects two principal motives. The first one is technological. An understanding of plasma behavior is crucial to the controlled release of thermonuclear energy [3–6], the attempt to reproduce in a man-made plasma the kind of nuclear reaction found in the sun. Another technical goal is the design of magnetohydrodynamic generators [7], in which electric power is generated by jets of gas plasma traversing magnetic fields. The second broad motive for the study of plasmas is the importance of plasma phenomena in space and in astrophysics. When a plasma is subjected to electromagnetic fields, the motion of the particles is no longer completely random. One important consequence of this imposed order is that a plasmas can transmit certain kinds of waves that are related to electromagnetic waves but that have unique and curious properties. These waves include high-frequency electron plasma waves [8, 9] (EPWs) and low frequency one called ion acoustic waves [10, 11] (IAWs).

IAWs can be excited in plasmas due to their remarkable properties of quasineutrality and collective behavior. Plasma is a state of matter that contains enough heat that atoms lose their individuality. The negatively charged electrons are still attracted by positively charged nuclei, but they are not bound together. This gives a plasma some unusual properties unlike most kind of ordinary matter—solids, liquids and gases—the free floating electrons and ions of a plasma are strongly affected by electric and magnetic fields. Plasma as a whole is quasineutral, but as

✉ Naveen Gupta
naveens222@rediffmail.com

[1] Lovely Professional University, Phagwara, India
[2] Government College for Women, Karnal, India
[3] SUS Govt College, Matak-Majri, Indri, Karnal, India

Published online: 07 May 2022

J Opt
https://doi.org/10.1007/s12596-022-00932-x

RESEARCH ARTICLE

# Electron plasma wave excitation by self-focused cosh gaussian laser beams in axially inhomogeneous plasma: effect of density ramp

**Naveen Gupta[1] · Sanjeev Kumar[1,2] · S. B. Bhardwaj[3] · Rohit Johari[1] · Suman Choudhry[1]**

Received: 19 May 2022 / Accepted: 9 July 2022


**Abstract** Dynamics of the laser driven electron plasma waves (EPWs) in plasmas with axial density ramp has been investigated theoretically. The effect of self focusing of the laser beam on the power of laser excited EPW has been incorporated. During its propagation through the plasma, the laser beam excites an EPW at frequency $\omega_{ep}$ that due to the optical nonlinearity of plasma gets nonlinearly coupled to the laser beam due to the ponderomotive nonlinearity of plasma electrons. Using variational theory semi analytical solutions of the coupled nonlinear wave equations for the pump wave and EPW have been obtained under W.K.B approximation technique. It has been observed that power of the EPW is significantly affected by the self-focusing effect of pump beam.

**Keywords** Self-Focusing · Electron plasma wave · Cosh-gaussian · Ponderomotive force · Clean energy

## Introduction

Investigations on coupling of intense laser beams with plasmas are at the vanguard of research since past few decades due to its importance in many potential applications including laser fusion [1–3], plasma wake field accelerators [4, 5], X-ray lasers [6, 7], terahertz generation [8], etc. The ultimate breath of these applications depends on the efficiency of laser plasma coupling which is further decided by many different nonlinear processes [9–11]. These processes range from collisional absorption to excitation of copious laser driven instabilities [12–15]. These instabilities can be represented as the resonant coupling of the incident laser beam into two daughter waves. In the absence of external magnetic field these daughter waves can be electron plasma waves, ion acoustic waves along with a scattered electromagnetic wave.

EPWs can be excited in plasmas due to their remarkable properties of quasi neutrality and collective behaviour. Plasma is a state of matter that contains enough heat that atoms lose their individuality. The negatively charged electrons are still attracted by positively charged nuclei, but they are not bound together. This gives a plasma some unusual properties unlike most kind of ordinary matter-solids, liquids and gases-the free-floating electrons and ions of a plasma are strongly affected by electric and magnetic fields. Plasma as a whole is quasi neutral, but as the electrons and positively charged ions are separated, a disturbance can create regions of net negative and net positive charges acting like the plates of a charged parallel plate capacitor. Such an uneven distribution of charges results in an electric field running from positive to negative regions. This electric field pulls the electrons and ions towards each other with equal forces. Due to their large mass ions are lazy and thus remain at rest and the electrons move towards the ions. As the electrons move towards the ions, they steadily gain velocity and momentum like a pendulum moving towards its mean position from an extreme position. Due to this gain in momentum the electrons overshoot their equilibrium positions resulting in reversing the direction of electric field. Now the reversed electric field opposes the electron motion slow them down and then pulling them back again. The process repeats itself, establishing an electron oscillator. In the presence of thermal velocity these electron oscillations lead to a longitudinal wave compression and rarefaction regions of electrons

✉ Naveen Gupta
naveens222@rediffmail.com

1 Lovely Professional University, Phagwara, India

2 Government College for Women, Karnal, India

3 SUS Government College, Matak-Majri, Indri, Karnal, India

Published online: 28 July 2022

J Opt
https://doi.org/10.1007/s12596-022-00955-4

RESEARCH ARTICLE

# Self-focusing of laser-driven ion acoustic waves in plasma with axial density ramp

**Naveen Gupta[1] · Suman Choudhry[1,2] · Sanjeev Kumar[1,2] · S. B. Bhardwaj[3]**

Received: 11 June 2022 / Accepted: 20 August 2022


**Abstract** In the present work, dynamics of the laser-driven ion acoustic waves (IAWs) in plasmas with axial density ramp has been investigated theoretically. The effect of propagation characteristics of the laser beam on self-focusing of IAW has been investigated in detail. During its propagation through the plasma, the laser beam excites an IAW at frequency $\omega_{ia}$ due to the optical nonlinearity of plasma and gets nonlinearly coupled to the laser beam. Semi-analytical solutions of the coupled nonlinear wave equations for the pump wave and IAW have been obtained under W.K.B approximation technique using variational theory. It has been observed that propagation of the IAW is significantly affected by the self-focusing effect of laser beam.

**Keywords** *q*-Gaussian · Density ramp · Relativistic plasma · Self-focusing · Ion acoustic wave · Clean energy

## Introduction

The fundamental currency of our universe is energy. It lights up our homes, grows our food and powers our computers. There is no end to world's energy appetite. The need of energy is so great and growing so rapidly around the world that alternate sources of energy to quench humanity's endless thirst of energy without doing any harm to global climate are required. In this regard, the quest to tap the energy of nuclear fusion [1] by employing intense laser beams to confine an ultra-hot plasma and generate electric power has been in progress since past few decades. The fusion power plants will be fueled by a form of heavy hydrogen found in ordinary sea water and will produce no harmful emissions—no sooty pollutants, no nuclear waste and no greenhouse gases. In laser-driven fusion, the goal is to deposit laser energy at a particular density in the plasma in order to derive the compression and subsequent heating of the fuel pellet. If the pellet is compressed sufficiently, it may undergo fusion, leaving to the release of a large amount of energy. It is as if there is a tiny hunk of the sun on Earth.

Both the allure and the challenges of fusion arise from the nature of the fusion process itself. Fusion fuel is abundant and cheap. The major advantages are: (1) The abundance of fuel—the most easily exploitable fuels are deuterium and tritium. Deuterium occurs naturally in all sources of water specially sea water. Tritium, however, is not readily available naturally, it can easily be manufactured inside the fusion reactor by the bombardment of lithium with neutrons, which also abundant in nature. (2) Cleanest source of energy—fusion does not produce nuclear waste directly. Although tritium is mildly radioactive and neutron activation of the reactor chamber dictates which structural materials are most useful to minimize waste disposal of components discarded in maintenance or the entire reactor assembly at the end of its life.

However, fusion only happens at the extremely high temperatures that are typical characteristics of stars, whereas fission only happens at normal temperatures. The fuel with the lowest kindling point is a mixture of deuterium and tritium that ignites at temperatures around 50 keV (i.e., 50

✉ S. B. Bhardwaj
sbbbhardwaj09@gmail.com

Naveen Gupta
naveens222@rediffmail.com

1 Department of Physics, Lovely Professional University, Phagwara 144402, India

2 Department of Physics, Government College for Women, Karnal 132001, India

3 Department of Physics, SUS Govt College, Matak-Majri, Karnal 132041, India

Published online: 03 November 2022

J Opt
https://doi.org/10.1007/s12596-022-00995-w

RESEARCH ARTICLE

# Second-harmonic generation of two cross-focused *q*-Gaussian laser beams by nonlinear frequency mixing in plasmas

**Naveen Gupta[1] · S. B. Bhardwaj[2] · Sanjeev Kumar[1] · Suman Choudhry[1] · Rishabh Khatri[1] · Siddhanth Shishodia[1] · Rohit Johari[1]**

Received: 11 July 2022 / Accepted: 18 October 2022


**Abstract** A scheme for second-harmonic generation (SHG) of a pair of *q*-Gaussian laser beams interacting nonlinearly with underdense plasma has been proposed. Due to the relativistic increase in electron mass under the intense fields of the laser beam, the resulting optical nonlinearity of plasma leads cross-focusing of the laser beams. The resulting nonlinear coupling between the two laser beams makes the oscillations of plasma electrons to contain a frequency component equal to the sum of frequencies of the pump beams. This results in a nonlinear current density at frequency equal to the sum of frequencies of the pump beams. If the frequencies of the pump beams are equal, then the resulting nonlinear current generates a new radiation at frequency twice the frequencies of the pump beams— a phenomenon known as SHG. Starting from nonlinear Schrodinger wave equation a set of coupled differential equations governing the evolution of beam widths of the laser beams and power of generated second-harmonic radiation with longitudinal distance has been obtained with the help of variational theory. The equations so obtained have been solved numerically to envision the effect of laser as well as plasma parameters on the power of generated second-harmonic radiation.

**Keywords** Cosh Gaussian · Self-focusing · Nonlinear optics · Second-harmonic generation · Clean energy

✉ Naveen Gupta
naveens222@rediffmail.com

[1] Lovely Professional University, Phagwara, India

[2] SUS Govt College, Matak-Majri, Indri, Karnal, India

## Introduction

The invention of the laser [1] is the most towering achievement in the long history of light. It brought an extraordinary technological leap, which has since paved the way for a startling new era in optical science and technology. For the first time, man got a remarkable tool for direct generation and manipulation of coherent light. Laser brought same revolution to optics that transistor brought to electronics and cyclotron brought to nuclear physics. The distinctive qualities of laser derive from its coherence properties, which result in a beam of light with a well-defined optical phase both in space and time. This prescribed phase generally confines the wavelength and frequency of the laser light to a restricted range, so that the beam exhibits a narrow frequency spectrum. Another unique property of laser light is its directionality, which means that the beam can propagate over great distances without significant spreading and can be readily manipulated using conventional optical elements. The phase coherence and directionality of the laser make it possible to create extremely large optical powers and focused intensities that cannot be obtained from incoherent light emitters. These characteristics also allow accurate transfer of information [2], precise calibration of time [3, 4], and measurements of many physical constants [5, 6], among numerous other applications, using laser light. Lasers are now standard components of such commonplace objects as compact-disk players and printers. The everyday presence of lasers does not mean, however, that they have been reduced to performing only pedestrian tasks. Higher-end applications like laser surgery [7], laser-driven particle accelerators [8, 9], inertial confinement fusion [10], etc., are also abound.

Success is never without limitations, and laser is also not an exception. By the virtue of its unique coherence properties, laser light contains only a confined band of frequencies

Published online: 05 November 2022

DOI 10.1007/s10946-022-10093-8
*Journal of Russian Laser Research, Volume 43, Number 6, November, 2022*

# STIMULATED RAMAN SCATTERING OF COSH-GAUSSIAN LASER BEAMS IN PLASMA WITH AXIAL DENSITY RAMP: EFFECT OF SELF-FOCUSING

**Naveen Gupta,**[1*] **Sanjeev Kumar,**[1,2] **and S. B. Bhardwaj**[3]

[1]*Lovely Professional University*
*Phagwara, Punjab, India*

[2]*Government College for Women*
*Karnal, Haryana, India*

[3]*SUS Goverment College*
*Matak-Majri, Indri, Karnal, Haryana, India*

*Corresponding author e-mail: naveens222@rediffmail.com

**Abstract**

We present theoretical study of stimulated Raman scattering (SRS) of intense cosh-Gaussian (ChG) laser beams interacting with axially non-uniform plasmas. The axial inhomogeneity of the plasma is modeled by a ramp-shaped density profile and the optical nonlinearity of the plasma is considered to be due to the ponderomotive force acting on the plasma electrons. An intense laser beam with frequency $\omega_0$ propagating through plasma gets coupled with a pre-existing electron plasma wave (EPW) with frequency $\omega_{ep}$ and produces a back-scattered wave at frequency $\omega_s = \omega_0 - \omega_{ep}$. Using the variational theory, we obtain a semi-analytical solution of the set of coupled wave equations for the pump wave, EPW, and scattered wave under the WKB approximation. We observe that the SRS reflectivity of plasma is significantly affected by self-focusing of the pump beam.

**Keywords:** self-focusing, stimulated Raman scattering, cosh-Gaussian beam, ponderomotive force.

## 1. Introduction

Investigations on coupling of intense laser beams with plasmas have been at the vanguard of research for the past few decades due to their importance in many potential applications including laser fusion [1–3], plasma wake-field accelerators [4, 5], $X$-ray lasers [6, 7], THz generation [8], etc. The ultimate breath of these applications depends on the efficiency of laser– plasma coupling, which is governed by many different nonlinear processes [9–11]. These processes range from collisional absorption to excitation of several laser-driven instabilities [12–15]. These instabilities can be represented as resonant coupling of the incident laser beam with two daughter waves. In the absence of an external magnetic field, these daughter waves can be electron plasma waves and ion acoustic waves, as well as a scattered electromagnetic wave. In addition to the above-mentioned instabilities, there is one more possibility where the amplitude structure over the cross-section of the laser beam produces density modulation of plasma which, in turn, leads to self-focusing or filamentation of the laser beam.

Manuscript submitted by the authors in English on September 22, 2022.

DOI 10.1007/s10812-023-01518-6
*Journal of Applied Spectroscopy, Vol. 90, No. 1, March, 2023 (Russian Original Vol. 90, No. 1, January–February, 2023)*

# EXCITATION OF ELECTRON PLASMA WAVE BY SELF FOCUSED COSH-GAUSSIAN LASER BEAMS IN COLLISIONLESS PLASMAS: EFFECT OF DENSITY RAMP

**Naveen Gupta,[a] Sanjeev Kumar,[b] and S. B. Bhardwaj[c,*]** UDC 535.375.5;621.375.826

*Dynamics of the laser-driven electron plasma waves (EPWs) in plasmas with axial density ramp has been investigated theoretically. The effect of self-focusing of the laser beam on the power of laser-excited EPW has been incorporated. During its propagation through the plasma, the laser beam excites an EPW at frequency $\omega_{ep}$ that due to the optical nonlinearity of plasma gets nonlinearly coupled to the laser beam. Using variational theory semi analytical solutions of the coupled nonlinear wave equations for the pump wave and EPW have been obtained under W.K.B. approximation technique. It has been observed that power of the EPW is significantly affected by the self-focusing effect of pump beam.*

***Keywords:*** *self-focusing, electron plasma wave, cosh-Gaussian, ponderomotive force.*

**Introduction.** Investigations on coupling of intense laser beams with plasmas is at the vanguard of research since past few decades due to its importance in many potential applications including laser fusion [1–3], plasma wake field accelerators [4, 5], X-ray lasers [6, 7], terahertz generation [8] etc. The ultimate breath of these applications depends on the efficiency of laser plasma coupling which is further decided by many different nonlinear processes [9–11]. These processes range from collisional absorption to excitation of copious laser driven instabilities [12–15]. These instabilities can be represented as the resonant coupling of the incident laser beam into two daughter waves. In the absence of external magnetic field these daughter waves can be electron plasma waves, ion acoustic waves along with a scattered electromagnetic wave.

Electron plasma waves (EPWs) can be excited in plasmas due to their remarkable properties of quasi neutrality and collective behavior. Plasma is a state of matter that contains enough heat that atoms lose their individuality. The negatively charged electrons are still attracted by positively charged nuclei, but they are not bound together. This gives a plasma some unusual properties unlike most kind of ordinary matter — solids, liquids, and gases — the free-floating electrons and ions of a plasma are strongly affected by electric and magnetic fields. Plasma as a whole is quasi neutral, but as the electrons and positively charged ions are separated, a disturbance can create regions of net negative and net positive charges acting like the plates of a charged parallel plate capacitor. Such an uneven distribution of charges results in an electric field running from positive to negative regions. This electric field pulls the electrons and ions towards each other with equal forces. Due to their large mass ions are lazy and thus remain at rest and the electrons move towards the ions. As the electrons move towards the ions, they steadily gain velocity and momentum like a pendulum moving towards its mean position from an extreme position. Due to this gain in momentum the electrons overshoot their equilibrium positions resulting in reversing the direction of electric field. Now the reversed electric field opposes the electron motion slow them down and then pulling them back again. The process repeats itself, establishing an electron oscillator. In the presence of thermal velocity these electron oscillations lead to a longitudinal wave compression and rarefaction regions of electrons propagating through the plasma known as EPW.

The excited EPW through SRS is having very high phase velocity and therefore can lead to the generation of superthermal electrons in inertial confinement fusion (ICF). These penetrating electrons can preheat the fuel and prevent the efficient compression required for high gain [16]. When a superthermal electron escapes from the pellet, it leaves a

*To whom correspondence should be addressed.

[a]Department of Physics, Lovely Professional University, Phagwara, India; [b]Department of Physics, Government College for Women, Karnal, India; [c]Department of Physics, SUS Govt. College, Matak-Majri, Karnal, India; email: naveens222@rediffmail.com. Abstract of article is published in Zhurnal Prikladnoi Spektroskopii, Vol. 90, No. 1, p. 120, January–February, 2023.

J Opt
https://doi.org/10.1007/s12596-023-01201-1

RESEARCH ARTICLE

# Stimulated Raman scattering of self-focused Laguerre–Gaussian laser beams in axially inhomogeneous plasma

**Naveen Gupta[1] · Alex AK[1] · Rohit Johari[1] · Sanjeev Kumar[1,2] · S. B. Bhardwaj[3] · A. Saini[4]**

Received: 12 February 2023 / Accepted: 23 April 2023


**Abstract** This paper presents a theoretical investigation on stimulated Raman scattering (SRS) of intense Laguerre–Gaussian (LG) laser beams propagating through plasma with axial density ramp. The optical nonlinearity of the plasma has been considered to be originating due the ponderomotive force acting on the plasma electrons due to intensity gradient over the cross section of laser beam. An intense laser beam with frequency $\omega_0$ propagating through plasma gets coupled with a preexisting electron plasma wave (EPW) at frequency $\omega_{ep}$ and produces a back scattered wave at frequency $\omega_s = \omega_0 - \omega_{ep}$. Using variational theory semi-analytical solution of the set of coupled wave equations for the pump, EPW and scattered wave has been obtained under W.K.B approximation. It has been observed that power of the scattered wave is significantly affected by the self-focusing effect of pump beam.

**Keywords** Self-focusing · Stimulated Raman scattering · Laguerre–Gaussian · Clean energy · Ponderomotive force

## Introduction

The invention of the laser led to a renaissance in the field of light matter interactions by giving birth to an entirely new area of research known as laser plasma interactions. Since the past few decades, this new field is at vanguard of research due to its importance in many potent applications [1–8]. The impetus was built by the proposal of initiating thermonuclear fusion [1, 3] for viable energy production by using intense laser beams.

Both the allure and the challenges of fusion arise from the nature of the fusion process itself. Fusion fuel is abundant and cheap. The major advantages are: (1). The abundance of fuel—the most easily exploitable fuels are deuterium and tritium. Deuterium occurs naturally in all sources of water specially sea water. Tritium, however, is not readily available naturally, it can easily be manufactured inside the fusion reactor by the bombardment of neutrons with lithium, which also abundant in nature. (2). Cleanest source of energy—Fusion does not produce nuclear waste directly. In laser driven fusion, the goal is to deposit laser energy at a particular density in the plasma in order to derive the compression and subsequent heating of the fuel pellet. If the pellet is compressed sufficiently, it may undergo fusion, leaving to the release of a large amount of energy. It's as if there is a tiny hunk of the sun on Earth.

The successful implosion of the fuel pellet depends on the efficiency of laser plasma coupling which is decided by several nonlinear processes [9–11] ranging from collisional absorption to excitation of several laser driven instabilities [12–15]. In laser plasma instabilities, the pump beam (incident laser beam) splits into two daughter waves. If there is no external magnetic field, these daughter waves will be a scattered electromagnetic wave along with an electron plasma wave (EPW) or ion acoustic waves (IAW).

For laser driven fusion, these instabilities are of serious concern because if they are operative to a significant extent, they make it difficult to achieve high gain. In this context, SRS [16, 17], in which the incident laser beam decays into an EPW and a scattered electromagnetic wave, is of more

✉ Naveen Gupta
naveens222@rediffmail.com

[1] Lovely Professional University, Phagwara, Punjab, India
[2] Government College for Women, Karnal, Haryana, India
[3] SUS Govt College, Matak-Majri, Indri, Karnal, Haryana, India
[4] M. D. Govt. Girls College, Dadupur Roran, Karnal, India

Published online: 08 May 2023

J Opt
https://doi.org/10.1007/s12596-023-01349-w

RESEARCH ARTICLE

# Formation of elliptical *q*-Gaussian breather solitons in diffraction managed nonlinear optical media: effect of cubic quintic nonlinearity

**Naveen Gupta[1] · A. K. Alex[1] · Rohit Johari[1] · Suman Choudhry[1] · Sanjeev Kumar[1] · Aatif Ahmad[1] · S. B. Bhardwaj[2]**

Received: 3 June 2023 / Accepted: 30 July 2023


**Abstract** This paper presents theoretical investigation on the formation of elliptical $q$-Gaussian breather solitons in diffraction managed optical media. The optical nonlinearity of the medium has been modeled by cubic–quintic nonlinearity. To obtain the physical insight into the propagation dynamics of the laser beam, semi-analytical solution of the wave equation for the laser beam has been obtained by using variational theory approach in W.K.B approximation. Emphasis is put on investigating evolutions of transverse dimensions and axial phase of the optical beam.

**Keywords** Soliton · Clean Energy · Self Focusing · Variational Theory · Breather

## Introduction

Since the debut of quantum mechanics in the 1920s, the two different aspects of physical quantities, i.e., waves and particles, have been intimately related in physical theories. Although both the aspects appear to be physically different, there are a number of experimental evidences that show correlation among both. In the past few years, solutions of certain wave equations have revealed another correlation between waves and particles. The surprising fact is that these wave equations are not the part of quantum mechanics but instead have been derived from classical physics [1]. Solutions to these equations describe waves those neither spread in space (i.e., those do not diffract) nor disperse in time. Diffraction and dispersion (Fig. 1) are the inherent properties of all kind of waves whether it is electromagnetic wave, mechanical wave (sound wave) or even matter wave.

However, these new kinds of waves retain their size and shape indefinitely (Fig. 2). These waves can be regarded as a quantity of energy localized permanently to a definite region of space. It can be set in motion but it cannot dissipate by spreading out. When two such waves collide, each comes away from the encounter with its identity intact (Fig. 3). If a wave meets an "anti-wave," both can be annihilated. This kind of behavior is extraordinary in waves, but it is familiar in another context, i.e., with particles. Thus, such waves can be considered as particles and are termed as "solitons."

The first recorded observation of a soliton was made almost 200 years ago by Russell [2], an engineer and naval architect. He reported to the British Association for the Advancement of Science: "I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses. When the boat suddenly stopped-not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation then suddenly leaving it behind rolled forward with great velocity assuming the form of a large solitary elevation a rounded smooth and well-defined heap of water which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback and overtook it still rolling on at a rate of some eight or nine miles per hour preserving its original figure some 30 feet long and a foot to a foot and a half in height. Its height gradually diminished and after a chase of one or two miles I lost it in the windings of the channel."

Our topic of investigation, i.e., spatial optical solitons, arises due to dynamical balance of diffraction with induced focusing of the optical beam in a nonlinear medium. By nonlinear medium, it is meant by a medium whose index

✉ Naveen Gupta
naveens222@rediffmail.com

[1] Lovely Professional University, Phagwara, India

[2] SUS Govt College, Matak-Majri, Indri, Karnal, India

Published online: 18 August 2023

*Nonlinear Optics, Quantum Optics*, Vol. 58, pp. 99–126
Reprints available directly from the publisher
Photocopying permitted by license only



# Effect of Self Focusing on Stimulated Raman Scattering of Elliptical $q$-Gaussian Laser Beam in Underdense Plasma with Axial Density Ramp

NAVEEN GUPTA[1], SANJEEV KUMAR[1,2] AND S. B. BHARDWAJ[3]

[1]*Lovely Professional University, Phagwara, India*
*naveens222@rediffmail.com*
[2]*Government College for Women, Karnal, India*
[3]*SUS Govt College, Matak-Majri, Indri, Karnal, India*

*Received: September 5, 2021. Accepted: February 17, 2022.*

Stimulated Raman scattering (SRS) of elliptical $q$-Gaussian laser beams interacting with axially inhomogeneous plasmas has been investigated theoretically. An intense laser beam with frequency $\omega_0$ propagating through plasma gets coupled with a preexisting electron plasma wave (EPW) at frequency $\omega_{ep}$, due to ponderomotive nonlinearity on plasma electrons exerted by the laser beam. This nonlinear interaction of pump beam with EPW produces a back scattered wave at frequency $\omega_s = \omega_0 - \omega_{ep}$. Using variational theory semi analytical solutions of the coupled nonlinear wave equations for the three waves (pump, EPW and scattered) have been obtained under W.K.B approximation technique. It has been observed that self focusing of the laser beam significantly affects the power of back scattered wave.

## 1. INTRODUCTION

Laser [1] is one of the most successful pieces of apparatus gifted by 20th century science. When laser made its debut in 1960 people considered it to be solution which is searching for its problem. Since its invention the imact of laser on our lives has changed with time and still is changing. Now laser is ubiquitous in every aspect of life: from super market barcode scanners, security checkpoints, CD writers to high end applications like medical diagnosis

J Opt
https://doi.org/10.1007/s12596-023-01390-9

RESEARCH ARTICLE

# Excitation of upper hybrid wave by cross focused *q*-Gaussian laser beams in graded index plasma channel

**Naveen Gupta[1] · Rohit Johari[1] · Sanjeev Kumar[1] · Suman Choudhry[1] · S. B. Bhardwaj[2] · A. K. Alex[1]**

Received: 6 July 2023 / Accepted: 26 August 2023


**Abstract** In this paper, a method is presented for exciting an upper hybrid wave (UHW) in a preformed parabolic plasma channel. The plasma channel is magnetized perpendicular to the propagation direction of laser beams. The UHW is generated through the interaction of two $q$-Gaussian laser beams with frequencies $\omega_1$ and $\omega_2$, employing the ponderomotive nonlinearity. The evolution of the laser beam spot sizes along the propagation distance is described by a set of coupled differential equations derived using the moment theory approach in the W.K.B approximation. The ponderomotive nonlinearity depends on the intensities of both laser beams, resulting in a mutual influence between the two beams, leading to cross-focusing. Numerical simulations are conducted to examine the impact of laser and channel parameters on the cross-focusing of laser beams and its effect on the power of the generated UHW. The results indicate that the intensity profiles of the laser beams, channel depth, and strength of the static magnetic field significantly affect the power of the generated UHW.

**Keywords** q-Gaussian · Plasma Waves · Moment Theory · Clean Energy · Self Focusing

✉ Sanjeev Kumar
skpanwar82@gmail.com

Naveen Gupta
naveens222@rediffmail.com

1 Lovely Professional University, Phagwara, India

2 SUS Govt College, Matak-Majri, Indri, Karnal, India

## Introduction

At the turn of the last century, the introduction of lasers [1] sparked a significant surge in research within the field of plasma physics. The study of plasmas began in the nineteenth century, when Michael Faraday investigated electrical discharges through gases. Modern plasma research dates from 1957 and 1958. During those years, Soviet Sputnik and American Explorer spacecrafts discovered that space near the earth is filled with plasma. At the same time, till then secret research on controlled thermonuclear fusion conducted by the USA, Soviet Union and Europe was revealed at the Atoms for Peace Conference in Geneva, greatly increasing the freely available information on plasmas. Fusion research focuses on producing extremely hot plasmas and confining them in magnetic "bottles," to create the conditions necessary for energy-producing nuclear reactions to occur.

Extensive studies, incorporating both theoretical and experimental approaches, have been conducted to enhance our understanding of this subject. These collective efforts have given rise to various potential applications, such as laser-driven particle accelerators [2–5], inertial confinement fusion [6, 7], X-ray lasers [8–10], laser plasma channeling [11, 12], and supercontinuum generation [13]. The successful realization of these applications relies heavily on the efficient coupling of laser energy with plasmas. Unfortunately, the interaction length between lasers and plasmas is inherently limited by diffraction divergence, restricting it to approximately a Rayleigh length in the absence of an optical guiding mechanism. Diffraction broadening, therefore, represents a fundamental phenomenon that hampers the efficiency of laser–plasma coupling. Consequently, there has been a renewed interest in extending the propagation distance of laser beams through

Published online: 23 September 2023



Conferences/Seminars

## INTERNATIONAL CONFERENCE
ON
# FRONTIERS IN PHYSICS, MATERIALS SCIENCE & NANOTECHNOLOGY
(FPMSN-2022)

MARCH 25-26, 2022

Organised by

DEPARTMENT OF PHYSICS

CHAUDHARY DEVI LAL UNIVERSITY, SIRSA-125055, HARYANA

## Certificate

This is to certify that **Sanjeev Kumar, Department of Physics, Lovely Professional University, Phagwara, Punjab, India** has participated in FPMSN-2022 held on March 25-26, 2022 at Chaudhary Devi Lal University, Sirsa and presented a research paper (oral) **Excitation of Electron Plasma Wave by Cosh-Gaussian Laser Beams in Underdense Plasma Targets: Effects of Self Focusing and Axial Density Ramp.**

| Dr. Ram Mehar S. Dixit | Prof. Sushil Kumar | Dr. Dharamvir S. Ahlawat |
|---|---|---|
| Organizing Secretary | Patron, FPMSN-2022 | Convener, FPMSN-2022 |
| Associate Professor | Dean, Faculty of Physical Sciences | Chairman, Dept. of Physics |

आज़ादी का अमृत महोत्सव

## International Symposium on Recent Trends in Optical Materials and Photonic Devices

(RTOMPD-2022)
December 07, 2022

## Certificate

This certificate is awarded to

**Sanjeev Kumar**

**Lovely Professional University, Phagwara, Punjab**

for Participating / Presenting Paper / Invited Speaker on topic **Stimulated Raman Scattering of Intense Cosh-Gaussian Laser Beams Interacting with Axially Non-Uniform Underdense Plasmas: Effect of Density Ramp and Self Focusing** in the International Symposium on Recent Trends in Optical Materials and Photonic Devices (RTOMPD-2022) organized by Department of Pure & Applied Physics, **Guru Ghasidas Vishwavidyalaya, Bilaspur** on December 07, 2022 in Hybrid Mode.

*Guru Ghasidas Vishwavidyalaya wishes him/her a bright and self-reliant future.*

Department of Pure & Applied Physics

| Prof. Parmendra Kumar Bajpai | Dr. Rajesh Sharma |
|---|---|
| Chairman, Organizing Committee | Convenor |
| Guru Ghasidas Vishwavidyalaya, Bilaspur | Guru Ghasidas Vishwavidyalaya, Bilaspur |

**CHAIR OF OPTICS OF T. SHEVCHENKO UNIVERSITY**

**Education in domain of optics and laser physics**

**FACULTY OF PHYSICS**

**23rd International Young Scientists Conference Optics and High Technology Material Science - SPO 2022**

# Certificate of Attendance

THIS CERTIFICATE IS AWARDED TO

*Mr. Sanjeev Kumar*

**with an oral presentation entitled**
"*Stimulated Raman Scattering of Self Focused Cosh-Gaussian Laser Beams in Underdense Plasma Targets: Effect of Density Ramp*" **in the "23rd International Young Scientists Conference Optics and High Technology Material Science - SPO 2022" .**

*24-26 November, 2022*

**Prof. Leonid V. Poperenko**
Head of Chair of Optics
Faculty of Physics
Taras Shevchenko National
University of Kyiv

**optics.univ.kiev.ua** **fb.com/knuoptics/**

# CERTIFICATE

This certificate is presented to

**Sanjeev Kumar**

for participating in the International Conference of Students and Young Researchers in Theoretical and Experimental Physics "**HEUREKA-2023**"
with oral presentation
**"STIMULATED RAMAN SCATTERING OF SELF FOCUSED ELLIPTICAL q-GAUSSIAN LASER BEAM IN PLASMA: EFFECT OF DENSITY RAMP"**

**Chairman of the Program Committee** ____________ **Prof. V. Stadnyk**

May 16-18, 2023. Lviv (Online), Ukraine