LINEAR PROGRAMMING PROBLEM USING TEACHING-LEARNING BASED OPTIMIZATION IN THE SPHERICAL FUZZY ENVIRONMENT

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Declaration

I, hereby declare that the presented work in the thesis entitled "**LINEAR PROGRAMMING PROBLEM USING TEACHING-LEARNING BASED OPTIMIZATION IN THE SPHERICAL FUZZY ENVIRONMENT**" in fulfillment of degree of **Doctor of Philosophy (Ph.D.)** is outcome of research work carried out by me under the supervision **Dr.Rakesh Yadav**, working as Professor, in the Department of Mathematics, School of Chemical Engineering and Physical Sciences of Lovely Professional University, Punjab, India. In keeping with general practice of reporting scientific observations, due acknowledgments have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

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Certificate

This is to certify that the work reported in the Ph.D. thesis entitled "**LINEAR PROGRAMMING PROBLEM USING TEACHING-LEARNING BASED OPTIMIZATION IN THE SPHERICAL FUZZY ENVIRONMENT**" submitted in fulfillment of the requirement for the reward of degree of **Doctor of Philosophy (Ph.D.)** in the Department of Mathematics, School of Chemical Engineering and Physical Sciences, Lovely Professional University, Phagwara, Punjab, India is a research work carried out by **Ms. Manbir Kaur**, 42000296, is bonafide record of her original work carried out under my supervision and that no part of thesis has been submitted for any other degree, diploma or equivalent course.

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Abstract

In many fields, from industry to engineering to the social sector, decision makers are increasingly required to weigh several, conflicting objectives in their decision-making processes. Multi-objective optimization models can be used to describe real-world decision difficulties in this scenario.

Conventional optimization methods presume that all of an optimization model's parameters and goals are perfectly known. However, due to a lack of information, subjective viewpoints, insufficient description of objectives, and incapacity to evaluate the relative importance of the objectives, challenges arise in the building of an accurate mathematical model of the system. In this situation, fuzzy set theory is a suitable choice for modeling system imprecision and uncertainty since it may define imprecise information in a more logical and understandable way. Several authors have researched fuzzy multi-objective optimization issues in the literature.

Teaching-Learning Based Optimization (TLBO) approach is widely accepted in numerous engineering applications.Despite the similarity of this algorithm to other Meta-heuristic techniques in terms of employing a set of solution and stochastic nature, the inspiration of algorithm is unique. The effectiveness of this simple and robust paradigm in the fields of engineering and sciences is gaining popularity rapidly due to its less complex approach in tuning with the algorithm and other common parameters like population size, number of iterations and stopping criteria, etc. Furthermore, it is concluded that TLBO is a promising variant in the class of stochastic methods that can be extended to find solutions of challenging real-world problems.

In fuzzy optimization, we searching for the best possible solution that can be found in the face of incomplete, imprecise, or uncertainty data. The bulk of real-world optimization issues are fuzzy in nature, despite the fact that they are frequently considered to be crisp for the sake of solving them. Many of them are multi-objective optimization problems, in which multiple conflicting objectives must be satisfied while adhering to the same set of constraints.

Keywords: *Linear Programming Problem, Spherical Fuzzy Optimization, Teaching Learning Based Optimization, Pareto Optimal Solution, Mathematical Modelling, Municipal Solid Waste Management*

Thesis Layout

The thesis is divided into following chapters:

Chapter 1: Introduction This chapter includes the basic concepts related to the terms involved in our presented research work like different models of spherical fuzzy linear programming problem (single as well as multi objectives), membership functions for spherical fuzzy number, objective functions, and constraints, algorithm of proposed method, spherical fuzzy programming, municipal solid waste management in Indian cities. This is further followed by literature review, research gap and the defined research objectives of our work.

Chapter 2: Teaching-Learning Based Optimization: A review on its Background & Development

This chapter is based on one of the sub-objectives of our first research objective. It is about Teaching-Learning Based Optimization (TLBO) approach which is widely accepted in numerous engineering applications.Despite the similarity of this algorithm to other Meta-heuristic techniques in terms of employing a set of solution and stochastic nature, the inspiration of algorithm is unique. This paper aims at discussing the position of TLBO within the domain of Nature-Inspired Algorithms (NIA). The effectiveness of this simple and robust paradigm in the fields of engineering and sciences is gaining popularity rapidly due to its less complex approach in tuning with the algorithm and other common parameters like population size, number of iterations and stopping criteria, etc. The paper also highlights the development, mechanism, mathematical formulation and algorithm of TLBO. Furthermore, it is concluded that TLBO is a promising variant in the class of stochastic methods that can be extended to find solutions of challenging real-world problems. The review is expected to be useful for the freshers in research area of Optimization techniques.

Chapter 3: Assessment of Municipal Solid Waste Management of Dinanagar city of Punjab India

This chapter is also contributes as sub-objective of first objective. It is about the assessment of MSWM by MCD in the study area. Unscientific waste management is increasingly becoming a major reason for environmental issues in Indian cities. Unlike previous municipal solid waste management in cities of Punjab(India), this study analyzes the implementation of solid waste management. Also, examine the factors responsible for the dysfunction of the municipal corporation of Dinanagar(MCD) city of Punjab(India). To fulfill the research objectives, primary and secondary data are collected from various sources. However, some drawbacks and flaws were found in the existing practices of Municipal Solid Waste Management. The Strengths, Weaknesses, Opportunities, Threats(SWOT) analysis is performed to conclude the present scenario of MCD. This study is eventually concluded with some suggestions to waste managing authorities and researchers for contribution in improvement of current system.

Chapter 4: Implementation Analysis of Municipal Solid Waste Management of Dinanagar city of Punjab India

This chapter is also contributes as sub-objective of first and second objective.It is based on the study of implementation analysis of MSWM by MCD, the study area. Unscientific waste management is increasingly becoming a major reason for environmental issues in Indian cities. Unlike previous municipal solid waste management in cities of Punjab(India), this study analyzes the implementation of solid waste management. Also, examine the factors responsible for the dysfunction of the municipal corporation of Dinanagar(MCD) city of Punjab(India). To fulfill the research objectives, primary and secondary data are collected from various sources for qualitative and quantitative analysis. However, some drawbacks and flaws were found in the existing practices of Municipal Solid Waste Management. The internal consistency and validity are measured using Cronbach's alpha. The importance- performance analysis is performed to conclude the present scenario of MCD. This study is eventually concluded with some suggestions to waste managing authorities and researchers for contribution in improvement of current system.

Chapter 5: Prioritizing the indicators responsible for sustainable municipal solid waste management using SF-AHP and SF-TOPSIS

This chapter contributes to the completion of our one of the sub-objectives of second objective. It is about the study of various factors responsible for sustainable waste management in local study area and then prioritize them using spherical fuzzy concept. Municipal corporations of small cities in India are struggling with the challenges that emerge from the unstructured solid waste management system. For successful implementation of the Municipal solid waste management system, all the basic components of this system need to work effectively and efficiently. Identification of performance indicators is completed with the help of a literature review and experts information. Interviews were conducted with the experts working in the field of solid waste management and three decisions criteria i.e., importance, performance, and understandability were defined to evaluate the selected performance indicators. For uncertainties, criteria weights were established using the spherical fuzzy analytical hierarchy process approach and the combined pairwise comparison matrix was aggregated by applying the spherical fuzzy TOPSIS method. Eventually, the indicators responsible for sustainable waste management were selected and ranked. An assessment of the conceptual framework of MSWM is also proposed for practical implementation of the ranked indicators for the MSWM system in Dinanagar and other cities with similar situations.

Chapter 6: Spherical fuzzy programming approach to optimize the transportation problem

This chapter also contributes to the completion of our one of the sub-objectives of second objective. It is based on the single objective model of transportation problem involved in municipal solid waste management. In any real-life problem, decision-making plays a very important role. It is always observable that uncertainty, hesitation, vagueness, etc involves in real-life situations. The existence of such factors results in increases the difficulty for the decision maker(s) to decide the accurate/precise, crisp value of parameters involved in the specific problem. In the present study, the uncertainty included in the transportation problem deals with the proposed method based on the derived accuracy function. The function is derived with the centroid method and is further used to convert the fuzzy number into a crisp value in the proposed approach. The applicability and validity are presented with the numerical illustration. The superiority and comparative study are shown by applying it to the real-life transportation problem as a case study of Dinanagar city, India.

Chapter 7: Mathematical Modelling Of Municipal Solid Waste Management In Spherical Fuzzy Environment
This chapter is based on our last two defined objectives. It is based on the application of proposed integrated novel method to optimize the waste management. Waste management is increasingly becoming a global resource management issue. The health and environmental issues are the main concerns of this management. Municipal solid waste management systems in developing countries, such as India, are facing challenges as a result of long-term uncertainty. Despite the fact that numerous studies on waste management have been conducted in a variety of fuzzy environments around the world. This study uses a mathematical model to include all of the major components of municipal solid waste management. To deal with uncertainty, the mathematical model of municipal solid waste management is defined using the spherical fuzzy environment. The designed model is multi-objective so to find the pareto optimal solution teaching learning based optimization is used in the spherical fuzzy environment. The goal of this research is to determine the current state of waste management in the Dinanagar area of Punjab, India. Finally, the constructed mathematical model is in possession of long-term waste management in the study area, Dinanagar city in Punjab, India. The findings of comparing the suggested model to the current framework show that the new model provides better solutions in terms of sustainability.

Chapter 8: Conclusion & Future scope This chapter concludes the various approaches already discussed and introduces new topics for the future in terms of spherical fuzzy linear programming problems, numerical illustrations and applications. This would put forward future aspirations for the area of optimization in other fields. It summarizes the inputs and identifies the areas that need attention in the near future.

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Chapter 1

Introduction

In many fields, from industry to engineering to the social sector, decision makers are increasingly required to weigh several, conflicting objectives in their decision-making processes. Multi-objective optimization models can be used to describe real-world decision difficulties in this scenario.

The three main subjects discussed in this work are Municipal Solid Waste Management in India, teaching-learning based optimization, and spherical fuzzy mathematical programming. Traditional optimization techniques make the assumption that the parameters and objectives of an optimization model are fully understood. However, difficulties are encountered in creating a precise mathematical model of the system because of a lack of knowledge, subjective opinions, an inadequate description of the aims, and an inability to assess the relative importance of the objectives. Since it has the potential to define imprecise information in a more logical and intelligible way, fuzzy set theory is a good option in this case for describing system imprecision and uncertainty. Several authors have been studied multi-objective optimization problems under fuzziness.

In fuzzy optimization, we look for the best feasible outcome in the presence of insufficient, inaccurate, or uncertain data. Even though they are typically thought of as being crisp for the purpose of addressing them, the majority of optimization problems in the real world are actually fuzzy in nature. Numerous them involve balancing multiple conflicting objectives while keeping to the same set of constraints. These are known as multi-objective optimization issues.

The TLBO algorithm is divided into two primary phases because it imitates the learning process by using the class teacher's and other students' expertise. The nicest thing about TLBO is that it's simpler to learn and use than other optimization approaches because it only needs two algorithmic parameters. Learners, who make up the population in a classroom, are generated at random. In programming languages like MATLAB, MATHEMATICA, SCILAB, etc., the hybrid TLBO method in the SF environment can be simulated.

The basic concept of this study is introduced from [6]

In this study a novel hybrid method is developed by integrating linear programming problem(Single or multi objective LPP) under spherical fuzzy environment with a robust Teaching-Learning based optimization technique.

1.1 Basic terminology

There are many basic terms and concepts are used throughout this research work such as fuzzy number, extended versions of fuzzy number, hierarchy diagram of different fuzzy numbers, membership function etc. Some of the crucial definitions and concepts are given in the proceeding sections.

1.2 Different models of SFMOLPP

The extension of MOLPP by introducing SF concept named as SFMOLPP and can be expressed as:

Model-I: This model presents the SFMOLPP in which coefficients of objective functions are represented in spherical fuzzy number(SFN) but the variable's coefficients and r.h.s constants of constraints are in real numbers.

$$Maximize\ F_o(x) = (\widetilde{f}_1, \widetilde{f}_2, \widetilde{f}_3, ..., \widetilde{f}_o) \quad \forall o = 1, 2, ..., O_1$$
$$Minimize\ F_o(x) = (\widetilde{f}_1, \widetilde{f}_2, \widetilde{f}_3, ..., \widetilde{f}_o) \quad \forall o = O_1 + 1, O_2 + 2, ..., O$$

Subject to

$$\sum_{k=1}^{K} a_{ik} x_k \leq, =, \geq b_i, \quad \forall i = 1, 2, ..., I$$
$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

where each $\widetilde{f}_o = \sum_{k=1}^{K} \widetilde{c}_k x_k, \widetilde{c}_k$ denotes a SFN and a_{ik}, b_i are real numbers.

Model-II: This model presents the SFMOLPP in which objective function's coefficients are in real numbers but the variable's coefficients and r.h.s constants of constraints are in SF numbers.

$Maximize\ F_o(x) = (f_1, f_2, f_3, ..., f_o) \quad \forall o = 1, 2, ..., O_1$

$Minimize\ F_o(x) = (f_1, f_2, f_3, ..., f_o) \quad \forall o = O_1 + 1, O_2 + 2, ..., O$

Subject to

$$\sum_{k=1}^{K} \widetilde{a}_{ik} x_k \leq, =, \geq \widetilde{b}_i, \quad \forall i = 1, 2, ..., I$$

$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

where each $f_o = \sum_{k=1}^{K} c_k x_k$, c_k is a real number and $\widetilde{a}_{ik}$ and $\widetilde{b}_i$ are SF numbers.

Model-III: This model presents the SFMOLPP in which coefficients of objective functions, the variable's coefficients and r.h.s constants of constraints all are in SF numbers.

$Maximize\ F_o(x) = (\widetilde{f}_1, \widetilde{f}_2, \widetilde{f}_3, ..., \widetilde{f}_o) \quad \forall o = 1, 2, ..., O_1$

$Minimize\ F_o(x) = (\widetilde{f}_1, \widetilde{f}_2, \widetilde{f}_3, ..., \widetilde{f}_o) \quad \forall o = O_1 + 1, O_2 + 2, ..., O$

Subject to

$$\sum_{k=1}^{K} \widetilde{a}_{ik} x_k \leq, =, \geq \widetilde{b}_i, \quad \forall i = 1, 2, ..., I$$

$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

where each $\widetilde{f}_o = \sum_{k=1}^{K} \widetilde{c}_k x_k$, $\widetilde{c}_k$ denotes a SFN and $\widetilde{a}_{ik}$, $\widetilde{b}_i$ are also SF numbers.

1.3 Membership functions for SFN

Membership functions for crisp formulation of SFLPP :

The SFN $\widetilde{a} = (t, i, f)$ where each $t, i, f \in [0, 1]$.

The truthiness or positive membership function for SFN, $\widetilde{a}$ can be defined as follows:

$$t_{\widetilde{a}}(x) = \begin{cases} \frac{x - t_1}{t_2 - t_1}, \ t_1 \leq x \leq t_2 \\ \frac{t_2 - x}{t_3 - t_2}, \ t_2 \leq x \leq t_3 \\ 0 \qquad otherwise \end{cases} \tag{1.1}$$

The indeterminacy or neutral membership function for SFN, $\widetilde{a}$ can be defined as follows:

$$i_{\widetilde{a}}(x) = \begin{cases} \frac{x-i_1}{i_2-i_1}, & i_1 \leq x \leq i_2 \\ \frac{i_2-x}{i_3-i_2}, & i_2 \leq x \leq i_3 \\ 0 & otherwise \end{cases} \tag{1.2}$$

The falsity or negative membership function for SFN $\widetilde{a}$ can be defined as follows:

$$f_{\widetilde{a}}(x) = \begin{cases} \frac{x-f_1}{f_2-f_1}, & f_1 \leq x \leq f_2 \\ \frac{f_2-x}{f_3-f_2}, & f_2 \leq x \leq f_3 \\ 0 & otherwise \end{cases} \tag{1.3}$$

1.4 Membership functions for Objective function

Membership functions for objective function

Case-I: For **single** objective function

Firstly, find the upper and lower bound with respect to every membership functions as follows:

$$F_U^t = \max\{F(x_k)\} \quad \text{and } F_L^t = \min\{F(x_k)\} \quad \text{(Positive membership)} \tag{1.4}$$

$$F_U^i = F_L^t + p_0(F_U^t - F_L^t) \quad \text{and } F_L^i = F_L^t \quad \text{(Neutral membership)} \tag{1.5}$$

$$F_U^f = F_U^t \quad \text{and } F_L^f = F_L^t + q_0(F_U^t - F_L^t) \quad \text{(Negative membership)} \tag{1.6}$$

where p_o and $q_0 \in [0,1]$ real positive numbers which are the predetermined.

On the basis of eqs. (1.4) to (1.6), the objective function's membership functions can be as follows:

$$\mathbf{T}(F(x)) = \begin{cases} 1 & if\ f \geq f_U^t \\ \frac{f-f_L^t}{f_U^t-f_L^t} & if\ f_L^t \leq f \leq f_U^t \\ 0 & if\ f \leq f_L^t \end{cases} \tag{1.7}$$

$$\mathbf{I}(F(x)) = \begin{cases} 1 & if\ f \geq f_U^i \\ \frac{f-f_L^i}{f_U^i-f_L^i} & if\ f_L^i \leq f \leq f_U^i \\ 0 & if\ f \leq f_L^i \end{cases} \tag{1.8}$$

$$\mathbf{F}(F(x)) = \begin{cases} 1 & if\ f \geq f_U^{\mathrm{f}} \\ \frac{f-f_L^{\mathrm{f}}}{f_U^{\mathrm{f}}-f_L^{\mathrm{f}}} & if\ f_L^{\mathrm{f}} \leq f \leq f_U^{\mathrm{f}} \\ 0 & if\ f \leq f_L^{\mathrm{f}} \end{cases} \tag{1.9}$$

where $\mathbf{T}(F(x)), \mathbf{I}(F(x))$ and $\mathbf{F}(F(x))$ are the truthiness or positive, indeterminacy or neutral and falsity or negative membership functions for objective function respectively.

Case-II: For **multi** objective function

According to [7], the fuzzy decision set includes objectives and constraints under fuzziness. Hence, based on the theory of fuzzy decision set , the decision set of spherical fuzzy can be obtained under spherical fuzziness as follows:

$$\widetilde{D} = \left(\bigcap_{o=1}^{O} \widetilde{f}_o\right)\left(\bigcap_{i=1}^{I} \widetilde{C}_i\right) = (x, \mathbf{T}_D(x), \mathbf{I}_D(x), \mathbf{F}_D(x)) \tag{1.10}$$

where $\widetilde{f}_o, \widetilde{D},$ and $\widetilde{C}$ are the objective, decision set, and constraints under spherical fuzziness, respectively.

& $\mathbf{T}_{\widetilde{D}}(x), \mathbf{I}_{\widetilde{D}}(x)$ and $\mathbf{F}_{\widetilde{D}}(x)$ are the truthiness or positive, indeterminacy or neutral and falsity or negative membership function under $\widetilde{D}$, respectively.

For marginal calculation of each objective function, firstly determine the lower and upper bounds for every objective function by substituting the resulting values of decision variables say $X^1, X^2, X^3, ..., X^o$ in each objective function. The expressions for lower bounds L_o and upper U_o for O^{th} objective function can be as follows:

$$L_o = \text{min} F_o(X^o) \text{ and } U_o = \text{max} F_o(X^o) \quad \forall o = 1, 2, 3, ..., O \tag{1.11}$$

Now, the lower and upper bounds for truthiness or positive, indeterminacy or neutral and falsity or negative membership function of each objective function can be expressed as the following:

$$U_o^T = U_o \qquad \text{and } L_o^T = L_o \qquad \text{Positive membership} \tag{1.12}$$

$$U_o^I = L_o^T + p_0(U_o^T - L_o^T) \qquad \text{and } L_o^I = L_o^T \qquad \text{Neutral membership} \tag{1.13}$$

$$U_o^F = U_o^T \qquad \text{and } L_o^F = L_o^T + q_0(U_o^T - L_o^T) \quad \text{Negative membership} \tag{1.14}$$

where p_o and $q_0 \in (0,1)$ real positive numbers which are the predetermined or assigned by decision makers.

Subcase-I: Minimum objective function

The truthiness or positive, indeterminacy or neutral and falsity or negative membership function are:

$$\mathbf{T}_{\widetilde{D}}(x) = \begin{cases} 1 & if\ F_o(x) \leq L_o^T \\ 1 - \frac{F_o(x) - L_o^T}{U_o^T - L_o^T} & if\ L_o^T \leq F_o(x) \leq U_o^T \\ 0 & if\ F_o^T \geq U_o^T \end{cases} \tag{1.15}$$

$$\mathbf{I}_{\widetilde{D}}(x) = \begin{cases} 1 & if\ F_o(x) \leq L_o^I \\ 1 - \frac{F_o(x) - L_o^I}{U_o^I - L_o^I} & if\ L_o^I \leq F_o(x) \leq U_o^I \\ 0 & if\ F_o^I \geq U_o^I \end{cases} \tag{1.16}$$

$$\mathbf{F}_{\widetilde{D}}(x) = \begin{cases} 1 & if\ F_o(x) \leq L_o^F \\ 1 - \frac{U_o^F - F_o(x)}{U_o^F - L_o^F} & if\ L_o^F \leq F_o(x) \leq U_o^F \\ 0 & if\ F_o^F \geq U_o^F \end{cases} \tag{1.17}$$

Subcase-II: Maximum objective function

The truthiness or positive, indeterminacy or neutral and falsity or negative membership function are:

$$\mathbf{T}_{\widetilde{D}}(x) = \begin{cases} 1 & if\ F_o(x) \leq L_o^T \\ 1 - \frac{U_o^T - F_o(x)}{U_o^T - L_o^T} & if\ L_o^T \leq F_o(x) \leq U_o^T \\ 0 & if\ F_o^T \geq U_o^T \end{cases} \tag{1.18}$$

$$\mathbf{I}_{\widetilde{D}}(x) = \begin{cases} 1 & if\ F_o(x) \leq L_o^I \\ 1 - \frac{U_o^I - F_o(x)}{U_o^I - L_o^I} & if\ L_o^I \leq F_o(x) \leq U_o^I \\ 0 & if\ F_o^I \geq U_o^I \end{cases} \tag{1.19}$$

$$\mathbf{F}_{\widetilde{D}}(x) = \begin{cases} 1 & if\ F_o(x) \leq L_o^F \\ 1 - \frac{L_o^F - F_o(x)}{U_o^F - L_o^F} & if\ L_o^F \leq F_o(x) \leq U_o^F \\ 0 & if\ F_o^F \geq U_o^F \end{cases} \tag{1.20}$$

where $L_o \neq U_o$ must hold $\forall F_o$. If for any objective $L_o = U_o$ then there will be one membership value.

The decision making Spherical Fuzzy model based on [7] concept for MOLPP can be as follows:

$$Max \min_{o=1,2,....,O} \mathbf{T}_o(F_o(x))^2$$
$$Min \max_{o=1,2,....,O} \mathbf{I}_o(F_o(x))^2$$
$$Min \max_{o=1,2,....,O} \mathbf{F}_o(F_o(x))^2$$

subject to

$$\sum_{k=1}^{K} a_{ik}x_k \leq, =, \geq b_i, \quad \forall i = 1, 2, ..., I$$
$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

$$\mathbf{T}_o(F_o(x))^2 \geq \mathbf{I}_o(F_o(x))^2, \mathbf{T}_o(F_o(x))^2 \geq \mathbf{F}_o(F_o(x))^2$$
$$0 \leq \mathbf{T}_o(F_o(x))^2 + \mathbf{I}_o(F_o(x))^2 + \mathbf{F}_o(F_o(x))^2 \leq 1 \tag{1.21}$$

Using auxiliary variables, the eq. (1.21) can be formulated again as:

$$Max\ \alpha^2$$
$$Min\ \beta^2$$
$$Min\ \gamma^2$$

subject to

$$\mathbf{T}_o(F_o(x))^2 \geq \alpha^2, \mathbf{I}_o(F_o(x))^2 \leq \beta^2, \mathbf{F}_o(F_o(x))^2 \leq \gamma^2$$

$$\sum_{k=1}^{K} a_{ik}x_k \leq, =, \geq b_i, \quad \forall i = 1, 2, ..., I$$

$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

$$\alpha^2 \geq \beta^2, \ \alpha^2 \geq \gamma^2, \ 0 \leq \alpha^2 + \beta^2 + \gamma^2 \leq 1 \tag{1.22}$$

After simplifying the eq. (1.22), the following SFMO optimization model is obtained:

$$Max(\alpha^2 - \beta^2 - \gamma^2)$$

subject to

$$\mathbf{T}_o(F_o(x))^2 \geq \alpha^2, \mathbf{I}_o(F_o(x))^2 \leq \beta^2, \mathbf{F}_o(F_o(x))^2 \leq \gamma^2$$

$$\sum_{k=1}^{K} a_{ik}x_k \leq, =, \geq b_i, \quad \forall i = 1, 2, ..., I$$

$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

$$\alpha^2 \geq \beta^2, \ \alpha^2 \geq \gamma^2, \ 0 \leq \alpha^2 + \beta^2 + \gamma^2 \leq 1 \tag{1.23}$$

Hence, the optimum model eq. (1.23) gives the optimized solution under the environment of spherical fuzzy for the MOLPP .

1.5 Membership functions for constraints

Membership functions for constraints:

The truthiness or positive, indeterminacy or neutral and falsity or negative membership functions for constraints under SFN are as follows:

$$T(C_i(x)) = \begin{cases} 1 & if\ b_i \geq \sum\limits_{k=1}^{K} (a_{ik} + d_{ik})x_k \\ \frac{b_i - \sum_{k=1}^{K} a_{ik}x_k}{\sum_{k=1}^{K} d_{ik}x_k} & if\ \sum\limits_{k=1}^{K} a_{ik}x_k \leq b_i \leq \sum\limits_{k=1}^{K} (a_{ik} + d_{ik})x_k \\ 0 & if\ b_i \leq \sum\limits_{k=1}^{K} a_{ik}x_k \end{cases} \tag{1.24}$$

$$I(C_i(x)) = \begin{cases} 1 & if\ b_i \geq \sum_{k=1}^{K}(a_{ik}+d_{ik})x_k \\ \frac{b_i - \sum_{k=1}^{K} d_{ik}x_k}{\sum_{k=1}^{K} a_{ik}x_k} & if\ \sum_{k=1}^{K} a_{ik}x_k \leq b_i \leq \sum_{k=1}^{K}(a_{ik}+d_{ik})x_k \\ 0 & if\ b_i \leq \sum_{k=1}^{K} a_{ik}x_k \end{cases} \tag{1.25}$$

$$F(C_i(x)) = \begin{cases} 1 & if\ b_i \geq \sum_{k=1}^{K}(a_{ik}+d_{ik})x_k \\ \frac{\sum_{k=1}^{K}(a_{ik}+d_{ik})x_k - b_i}{\sum_{k=1}^{K} d_{ik}x_k} & if\ \sum_{k=1}^{K} a_{ik}x_k \leq b_i \leq \sum_{k=1}^{K}(a_{ik}+d_{ik})x_k \\ 0 & if\ b_i \leq \sum_{k=1}^{K} a_{ik}x_k \end{cases} \tag{1.26}$$

where $d_{ik} \in [0,1]$ is predetermined limit of tolerance for the i^{th} constraint.

1.6 Algorithm for Proposed method

The step wise algorithm to find the solution of the MOLPP under spherical fuzzy conditions using TLBO can be explained as:

- **Step-I** Construct the table table 1.1 as shown below, for pay-off matrix by finding the solution of each defined objective and define their the lower and upper bound..

Table 1.1: Pay-off matrix(for SFMOLPP only)

	Z_1	Z_2	...	Z_o
X^1	$Z_1(X^1)$	$Z_2(X^1)$	...	$Z_o(X^1)$
X^2	$Z_1(X^2)$	$Z_2(X^2)$	...	$Z_o(X^2)$
...	...	...	...	...
...	...	...	...	...
X^o	$Z_1(X^o)$	$Z_2(X^1)$	...	$Z_o(X^o)$

- **Step-II** Determine the lower and upper bound for truthiness or positive, indeterminacy or neutral and falsity or negative membership function by using respective lower and upper values.

- **Step-III** Formulate the positive, neutral and negative membership functions under environment of spherical fuzziness.

- **Step-IV** Define the mathematical programming problem under spherical fuzzy environment(known as SFMPP).
- **Step-V(a)** Compute the crisp version of SFSOLPP by using robust optimization techniques to obtain the optimal solution.
- **Step-V(b)** Compute the obtained optimization model with suitable robust optimization techniques like TLBO to obtain the pareto-optimal i.e., compromise solution of SFMOLPP.
- **Step-VI** Present some numerical illustration to show applicability and validity of proposed integrated method for SF single/multi objective LPP with TLBO.
- **Step-VII** Application of integrated novel method to solve mathematically modeled Municipal Solid Waste Management(MSWM).

1.7 Spherical Fuzzy Mathematical Programming

In the last few years, lots of different robust techniques for mathematical programming problems are being introduced by various authors. The real-life problems like transportation problems, assignment problems, and problems related to supplier selection chain etc expressed in the form of multi objective optimization problems. Multi-objective optimization techniques are most popular to encounter problems in real-life. In real-life problems having more than one objectives is to be optimize under well-defined set of constraints. But, it is not always possible to obtain a single solution satisfying each objective efficiently; however a pareto optimal solution can be obtained. The compromise solutions set for multi-objective problem is called Pareto optimization. In literature, various authors approaches for MOLPP under different conditions to handle different real-life problems involving uncertainties. In article [8] author proposed a fuzzy technique to compute multi objective LPP in a fuzzy set. In article [9] proposed the programming approach based on intuitionistic fuzzy(IF) for multi objective LPP, which considers the degree of non-membership and membership of the solution set into the IF set.

Based on different vague or uncertain sets, like fuzzy set, intuitionistic fuzzy set, neutrosophic set and etc. various different multi objective optimization techniques have been developed. The authors of [10] studied on nonlinear transportation problems in the neutrosophic conditions. The article [11], also presented the neutrosophic optimization methodology for the optimal shale gas water management system under uncertainty. Recently, [12] suggested modified version of neutrosophic optimization approach under fuzziness and successfully implemented the proposed method to supply chain planning problems.

The hybridization of fuzzy set, Pythagorean fuzzy set(PyFS), Intuitionistic fuzzy set(IFS), is introduced in [13] and named as spherical fuzzy sets (SFS) which is defined on the basis of three different membership degrees termed as truthiness or positive, indeterminacy or neutral and falsity or negative membership degrees of the object into the feasible set of solution. Hence [14], to deal with such situations involving uncertainty in parameters, the SFS is useful and powerful tool for decision making under the environment spherical fuzzy. Moreover it incorporate indeterminacy or neutral thoughts and allows the decision makers to include such factors in decision making processes.

1.8 Municipal Solid Waste Management in India

Municipal Solid Waste Management(MSWM) Municipal solid waste management decision making becomes more complex with the increase in urbanization so it requires to design supporting Modelling formulations. **Waste generation:**

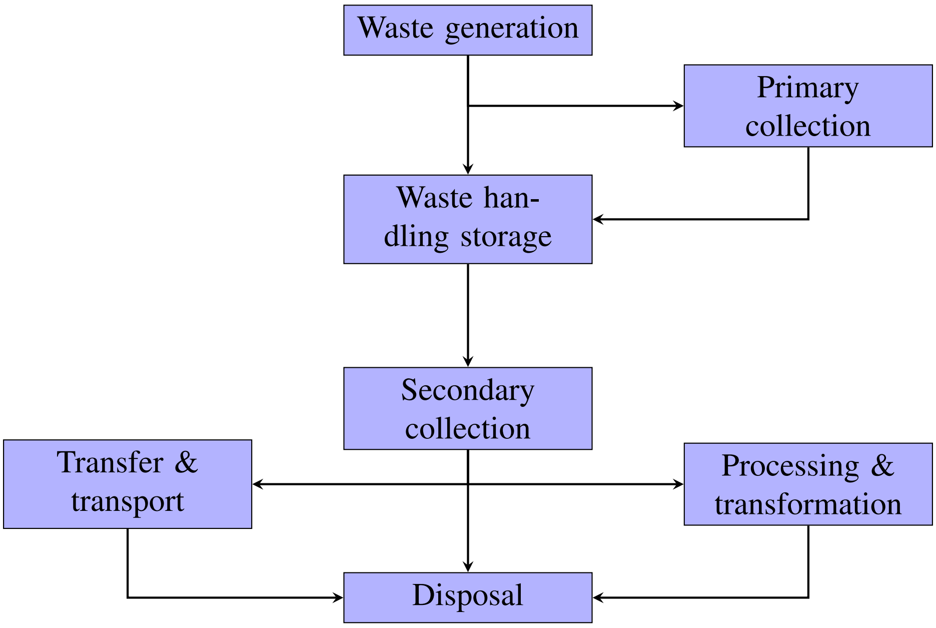
- Residential waste
- Commercial waste
- Industrial waste
- Agriculture waste

Functional elements of MSWM: shown in figure 1. The two major stages of waste management which requires research methodology are to minimize the cost of transportation and to maximize the revenue generated by transforming the waste into valuable material.Including these there are more other challenges also which will also be the part of this research.

1.8.1 Indian Government initiatives for MSWM [1]

In India's cities, it is getting harder to meet the incremental infrastructural needs of the expanding urban population. As per the 2011 census, population of India was 1.21 billion, having 31% of people living in the cities. By 2050, the population is expected to be half of it would live in city areas. With the increase in country's population, MSW management has become a very serious problem, not only due to aesthetic and environmental concerns, but also due to the massive amount of generated waste every day. As

Figure 1.1: Functional elements of MSWM



per the Central Pollution Control Board of India, the amount of waste generated was 1, 27,486 Tons per day of MSW during 2011-12, with an average generation rate of waste 0.11 kg per capita per day. Only 15,881 Tons per day that is 12.45% of the total waste generated was collected, and only 89,334 Tons per day that is 70% was treated or processed. Segregation of waste at the level of source, transportation, collection, processed or treatment, and scientific disposal of trash were all not satisfactory, as a result it leads to degradation of environment and a decline in living standards.

The reason behind that MSWM was becoming increasingly important first became apparent during 1990, when concerns about unsuitable MSWM resulted in number of Public Interest Litigations, prompting the Supreme Court of India to order the Ministry of Environment and Forests (MoEF), Government of India, to release Municipal Solid Waste Management Guidelines in 1996. The rules required all Urban Local Bodies, not just Class I cities and metro cities, to implement a proper system for waste management, with a deadline for the installation of waste collection, transport, processing, treatment and disposal facilities by the end of year 2003.The Indian government's Ministry of Urban Development (MoUD) prepared a solid waste management guidance manual for all Urban Local Bodies and released the same with the defined rules during the year 2000. All Urban Local Bodies, moreover, were not able to introduce a long-term municipal solid waste management system, having transportation, collection, processing, treatment and disposal of waste, until the year 2003. Despite positive accomplishments and pilots, most urban local bodies continue to face problems not only in transportation systems, advanced collection or selecting adequate and processing or treatment technologies and disposal techniques, but also in managing long-term financial requirement in MSWM system. Non-compliance with rules of MSW management is still a reason for issue of worry 14 years after the compliance of rules for municipal SW, 2000 were already published. Through the Ministry of Environment and Forests' draft amended municipal SWM Rules of year 2013, the Indian government continues to highlight these problems or issues and support states and urban local bodies in creating adequate and contemporary MSW management systems and the parallel revision of MSW management manual of year 2000 by the Ministry of Urban Development. It is based on learning from fourteen years' experience gained post the notification of the rules in year 2000. The Indian government has announced and sanctioned the 12th and 13th Finance Commission Grants and finances have been provided for development and improvement of MSW management under projects (such as UIDSSMT and JnNURM) from the year 2005 to give an importance to municipal solid waste management in Indian cities. State funds are also available for the proper implementation of solid waste management initiatives.

1.8.2 MSWM in Indian Cities

The study [15] contribute to evaluate the current status of MSWM in India and suggest some significant improvements for better functioning of waste management. The article [16] reflects a case study of city **Lucknow** ,which is one of the main metropolises in India, and struggles to a major problem of MSW management. Through this community participation study, a qualitative study using a SWOT analysis (strengths, weaknesses, opportunities and threats) has been effectively implemented. The SWOT analysis was found to be a useful tool for investigating the possibilities and strategies for starting and successfully conducting the MSWM programming.[17] The research analyses the cost of WM in **Mumbai**, India, and examines alternate methods for managing municipal solid waste (MSW). The first approach is community participation and the second is private sector participation. Moreover, there is a need to further analyze the role of PPP(Public Private Partnership) in waste management. [18]The MSWM, **Allahabad** data derived from ArcGIS maps is responsible for this work's information retrieval, updating, and essential information visualization. [19]the Master Plan (2005–2021) provides instructions for the removal and treatment of municipal solid waste for the entire state of **Delhi** after the Delhi government recognized the gravity of the matter. It is concluded that a three-way cooperation between the public sector, the commercial sector, and the general public is necessary to make any significant changes to the current situation. [20], By measuring the site-specific emission variables in conjunction with pertinent activity data and using the IPCC's 1996 procedures for CH4 inventory preparation, an inventory of **Chennai's** landfills' glasshouse gas emissions has been created. As intermediate phases of waste management also affect its quantity, it is crucial to examine MSW reaching landfills along with the generation of waste and its composition determination at source level for CH4 emission inventory. By separating garbage at the source and encouraging recycling or reuse of separated items, waste volume and the strain on landfills are reduced, and producers have access to raw materials. The author of [4] highlights the current practices of MSW in **Puducherry**, proper record of generated, collected, treated, and disposed waste is needed as per rules drafted by GoI. The physical & chemical composition is also a necessary aspect to be maintained by municipality authorities. [21], this article helps characterize the municipal solid trash that currently exists in **Jalandhar**, India, and assess its viability for different waste-processing methods. The author of [22] Guided with the specific objective of analyzing the implementation of various SWM programs in **Ludhiana** city, the study found political commitment, facilities provided to the staff, participation of public and commercial sector significantly affected the successful solid waste management implementation. The study also revealed some problems from administration that affected implementation at the initial stages like inadequate land for final disposal of untreated waste, inadequate resources, shortage of staff, public unawareness, lack of integrated solid waste management plan, etc. In article [23] the purpose of this paper is to add to the characterizations of the current municipal solid waste in Jalandhar, India, and to assess its suitability for different waste-processing methods.

1.8.3 Teaching Learning Based Optimization

The literature review of TLBO shows that this technique is very versatile to be applicable in solving the various optimization problems. The results in various research articles proved that different variants and hybridization of TLBO is also very interesting field to explore more about its applicability. The variety of different benchmark functions in this field of optimization gives so many gaps to study or introduce new hybrid or modified versions of optimization techniques. Metaheuristic methods perform differently in different problems. One may results better than the other in a particular problem and worse in other problems. In literature survey, it is concluded that TLBO is superior than other popular optimization techniques in obtaining the optimal solution. Besides previously developed metaheuristic methods, the investigations on metaheuristic techniques are still being done and new algorithms are being developed continually. For future

works, suggest developing a unified platform for analyzing, evaluating, and comparing different optimization techniques.
Some proposed variants are shown in table 1.2 and some areas of application are given in table 1.3.

Table 1.2: Hybrid/Modified variants of TLBO.

Proposed variant	**Abbreviation**	**Authors**
Teaching & peer-learning PSO	TPLPSO	[24]
Bidirectional TPLPSO	BTPLPSO	[25]
Hybrid with PSO	HTL-MOPSO	[26]
Hybrid with GA	G-TLBO	[27]
Hybrid with ABC	TLBO-ABC	[28]
Hybrid with GSA	TLBO-GSA	[29]
Hybrid with CS	TLCS	[30]
Hybrid with HS	HSTL	[31]
Elitist TLBO	ETLBO	[32]
Hybrid with Simulated Annealing	SAMCCTLBO	[33]
Hybrid with Grid based search	MO-ITLBO	[34]
Hybrid with NNA	TLNNA	[35]
Hybrid with JAYA & SVM	ITLBO-IPJAYA-SVM	[36]

In the last few years, lots of different robust techniques for mathematical programming problems are being introduced by various authors. The real-life problems like transportation problems, assignment problems, supplier selection problems etc expressed in the form of MO optimization problems. MO optimization techniques are most popular to encounter problems in real-life. In real-life problems having multiple objectives is to be optimize in well-defined constraints set. But, it is sometimes impossible to find a single solution satisfying every objective; moreover a pareto optimal solution can be obtained. The compromise solutions set for multi-objective problem is called Pareto optimization. In literature, various authors approaches for MOLPP under different conditions to handle different real-life problems involving uncertainties. The author of [8] proposed a fuzzy programming approach to find the solution of multi objective LPP in a fuzzy environment. In article [9] (IF) technique for MOLPP is introduced, involves the degree of non-membership and membership of the solution set into the intuitionistic fuzzy environment.

Based on different uncertain sets, such as fuzzy set, intuitionistic fuzzy set, neutrosophic set etc. different multi objective optimization approach have been introduced.

Table 1.3: Some more applications of TLBO.

Variants of TLBO	**Area of application**	**Reference**
ETLBO	Mechanical Engg.	[37]
TLBO + DE	Time series prediction	[38]
TLBO + DE	Power dispatch	[39]
TLBO + DE	Proton exchange membrane	[40]
GBTLBO	Thermal power system	[41]
HTLBO	Flow shop scheduling	[42]
TLBO	Two sided assembly lines	[43]
MO-ITLBO	Plate-fin heat ex-changer	[44]
TS-TLBO	Stirling heat engine	[45]
TLBO	Solar micro CCHP	[46]
TLBO + JAYA	structure optimization	[47]
ETLBO	Hybrid clustering	[48]
TLBO + NN	Bio-diesel	[49]
AutoTLBO	Micro-array datasets	[50]
TLBO + FUZZY	Geometric image segmentation	[51]
TLBO + FLANN	Learning from non-lin data	[52]
Modi-TLBO	Min surface roughness of alloy	[53]
TLBO+PST	Automated power system	[54]
ETLBO	Network design for resilience	[55]
TLBO	Surface roughness in EDM	[56]
TLBO	Energy storage system	[57]

Further, The authors of [10] developed a method on nonlinear transportation problems in the neutrosophic environment. The authors of [11] studied and developed the neutrosophic optimization approach for the optimum value of shale gas water management system in fuzziness . Recently, [12] proposed modified version of neutrosophic optimization in fuzziness and successfully solved supply chain management problems.

The generalization of Pythagorean fuzzy set(PyFS), fuzzy set, and Intuitionistic fuzzy set(IFS) is developed in [13] and named as spherical fuzzy sets (SFS) which is defined on the basis of three different membership degrees termed as truthiness or positive, indeterminacy or neutral and falsity or negative membership degrees of the object into the set of feasible solution. The limitations of fuzzy, IF, PyF sets can be explained by the following illustration:

Consider a decision making problem with multi attribute, in which 0.6, 0.7 and 0.8 are the acceptance degree, the rejection degree, and the indeterminacy or neutral degree corresponding to selected alternative then this condition is nt in the coverage of mentioned fuzzy sets. Hence, to tackle with such constraints, the SFS is useful and powerful tool for decision making in the spherical fuzzy environment [14]. It permits the decision takers to include neutral and indeterminacy findings in decision making problems.

1.9 Research Gap

Over the past few decades, numerous inexact optimization models have been developed for handling the uncertainties involved in MSWM systems. In article [58], the author has developed a GA aided with fuzzy chance constraint programming problem model(GAFCCP) for solving uncertainty in municipal solid waste (MSW) management. The developed model is a combination of the genetic algorithm and fuzzy chance constraints programming problem(FCCP) method which makes a unique contribution to enhancement of the applicability and feasibility of the optimization method. In research article [59] an interactive method is introduced for computing MOOPs. The proposed method can be used to obtain those Pareto-optimal solutions of the mathematical models of linear as well as nonlinear multi-objective optimization problems modeled in fuzzy or crisp environment which reasonably meet users aspirations. The authors of article [60] targeted to develop a mathematical model with multiple intermediate facilities and multiple depots to minimize variable and fixed waste collection cost. Because the amount of garbage produced every day is not deterministic in reality, a fuzzy optimization strategy suggested to deal with the problem of uncertainty in the generated waste. As per literature review it is observed that the characterization(physical and chemical) of waste varies according to the lifestyle of inhabitants, running industries, role of government or non-government local bodies, public awareness and so on. Focus on minimizing the generation of waste at local level which implies requirement of smaller landfill areas or open dump sites. Integration of waste/unwanted resources and energy generation system at city level. Finally, converge towards the concept of zero waste strategy.

1.10 Research Objectives

This study address a fuzzy mathematical (single or multi-objective) programming issue in which the functional relationships between decision variables and objective functions are unknown. It might be difficult to determine the exact functional link between objectives and choice factors in real-life decision situations due to uncertainty.

A fuzzy concept is supposed to be the information source from which some knowledge about the goal functions can be derived. Decision-making is difficult in such scenarios, and the inclusion of several objectives leads to a multi-objective optimization problem with fuzzy constraints. Appropriate fuzzy reasoning strategies are utilized to discover crisp functional relations between the objective functions and the choice variables to find the solution of the problem. The original fuzzy-based multi-objective optimization model is therefore transformed into a multi-objective optimization problem. A deterministic single-objective optimization problem is reformed with the help of fuzzy optimization techniques to solve the resultant problem.

Finally, the Teaching Learning-Based Optimization (TLBO) algorithm is used to solve the resulting linear programming model, and numerical examples are used to demonstrate the computing procedure.

The following are the main objectives of our study to achieve the defined aim or goal of our research work:

- To explore Teaching learning-based Optimization, spherical fuzzy environment, different optimization techniques are aided by variants of fuzzy sets.
- Teaching learning-based optimization algorithm integrated with spherical fuzzy sets.
- Design and model an interactive method for multi-objective linear programming problems in the spherical fuzzy environment using teaching-learning-based optimization.
- Application of the proposed integrated novel method to solve the mathematically modeled municipal solid waste management

Chapter 2

Teaching-learning based optimization-a review on background and development

2.1 Introduction

Survival is primary aim of all living beings. Every creature on the earth has different properties of the fittest survival. These variant properties are one of the reason behind the rapid increase in number of Nature-Inspired Algorithms(NIA) and second reason is NO FREE LUNCH theorem proved by [61] states the non-existence of universal algorithm. Most of NIA are based on the specific community group of species like PSO developed by simulating the behavior of swarms such as birds flocking, fish schooling etc.In-spite of this, there is competition of survival between the dominated one specie(s) over the other(s), here the balance in nature is required. The concept of fittest survival can be categorize in two ways:

- best fittest survival within the same species community.
- best fittest survival between the two or more different species community.

Till now, all NIA belongs to first category.

Artificial intelligence has many categories and one of the most specialized is Computational Intelligence(CI),some researchers considered it to be equivalent to Soft Computing(SC). The field of Computational Intelligence can be divided into further three sub fields: Evolutionary Computation(EC), Fuzzy Logic, and Neural Network. NIA investigates natural phenomenon define in computational programming to find optimal solution of well defined problem. Further this review covers the conventional applications in the field of Evolutionary computational algorithms, meta-heuristic techniques, and swarm intelligence. Finally, focus on the basic terminology, mechanism and applications of already developed technique called TLBO.

2.2 Nature Inspired Optimization Techniques(NIOT)

Fields of sciences and engineering involves different problems requires an optimal solution or near to optimal solutions. Optimization is a process to find global or nearest to global(local) optima of objective function defined in terms of decision variables and within the constraints(if applicable). The problem can be single or multiple objective depending upon application field. The presence of restrictions over the objective make it as constrained optimization problem otherwise unconstrained. The simulated NIOT is initialized with random population consist of members having different values bounded within the range of parameters. This population is updated over each generation after following the procedure of position update because it is a step where members learn from their environment and improve accordingly. The population belongs to DE(Differential Evolution), GA(Genetic Algorithm), ES(Evolutionary Strategy), EP(Evolutionary Programming), GP(Genetic programming) and ACO(Ant Colony Optimization), ABC(Artificial Bee Colony), PSO(Particle Swarm Optimization), SMO(Spider Monkey Optimization) [62] is said to be competitive(also named as first category) and cooperative(also named as second category), respectively. The brief introductions of some popular optimization techniques which are hybridized with TLBO to enhance its performance are as follows:

In article [63] developers introduced a Swarm Intelligence technique named as PSO inspired by the collective behavior of cooperative societal population to construct distributed plan for finding the optimal solution or near to the optimal solution of a problem. Bees swarming around their hive, Ants food foraging behavior,flocking of birds in the sky, fish schooling under water, immune system is swarm of cells, crowd of people is human swarm are some examples of swarm. Self organization, division of labor, simultaneous tasks performance by specialized individuals are some basic properties of swarms. PSO mimics the social behavior of birds flocking, each bird/particle associated with it's own position and velocity. Particles update their position and velocity by maintaining the balance between exploration and exploitation [64] *i.e.* local or global best. [65], in this paper the developers proposed a new modified PSO called as selectively-informed PSO which is based on the strategy of dense connections between the particles as it gets more and more information from its neighbor while less dense with few connections gives only a single best performed neighbor. The dense particles converge towards the correct direction to attain optimization.

DE belongs to [66], categorize as evolutionary algorithm introduced by Kenneth Price & Rainer Storn in 1997 is a simple and efficient heuristic technique for global optimisation over continuous region. The population in DE is stochastic and competitive,each member is chromosome consists of genes.Every chromosome undergoes mutation followed by recombination. Greedy selection strategy is performed to update the population. GA introduced by [67] is another meta-heuristic technique inspired by the principles of natural genetics and selection known as Darwin's principle of natural evolution. Like DE, in GA each member is chromosome undergone tournament selection to enter mating pool as parent produces offspring after crossover. These offspring mutated to obtain more offspring and constitute population-offspring.

Among these, candidate with best fittest value will survive. During the process of GA good solutions are retained and bad are eliminated. In Binary coded GA, real variables are encoded into binary variables *i.e.*0 and 1 which constitutes discrete search space. The increase in bit-length lead to higher precision but population becomes very large which is drawback of BGA. To overcome the drawbacks of BGA, Real coded GA is introduced, has real variable population instead of binary. Therefore, encoding of real to binary is not required which means decision variables can be directly used to compute the objective function value. [68], this paper concludes that the combination of two or more optimized methods can obtain the solution with high accuracy and will have good application prospect in other biological problems. The developers combines GA with ANN to optimize the plant tissue cultural conditions because plants have different properties when grows under different environments for example medicinal plants. This method is flexible to set experimental factors and levels to achieve success quickly.

The authors of [69] discovered ABC which is another swarm intelligence technique inspired from the cooperative population of honey bees. Working of bees distributed into three phases: Employed bees, Scout bees and Onlooker bees. Employed bees try to find out the better food source than the one associated with it and follow the greedy selection to update the solution. Scout bees phase may or may not encountered in generation. It occurs only when the trial counter of at least one solution is greater than limit. Limit is a user specified integer value. Onlooker bees select food source according to probability of nectar amount and accept new solution if it is better than current solution(*i.e.*greedy selection). Every solution is associated with an individual trial counter. There is possibility of elimination of best solution from population during second phase due to limit value, therefore it is required to memorize the best solution before performing the scout phase.

ACO is designed by [70], a very popular and efficient cooperative population based technique commonly used for solving vehicle routing problems. This technique simulates the behavior of real ants to search the shortest route which connects a food source to their nest. Ants are capable to search the shortest route using their feature of depositing trail of pheromones on the path followed by them. The amount of deposited pheromone is inversely proportional to the length of route followed by the ants towards food destination. Therefore, all other ants follow the route with more pheromone because it must be the shorter path. Slightly modified version is used to solve some bigger traveling salesman problems. To explore more about ACO applications go through paper [71].The article [72] based on investigating the behavior of arboreal turtle ant to find the alternatives for broken links in network and maintain the network. This proposed probabilistic algorithm uses less computational resources than other graph search algorithms, and thus it is more useful in other search spaces. In [73] paper, the author and co-authors proposed an effective hybrid gene selection method based on ReliefF algorithm and ACO algorithm called as RFACO-GS. The experimental results successfully proves the efficacy of proposed algorithm that it can decrease the conditionality of gene expression dataset, and concentrate on most relevant genes with high classification accuracy. Some recently proposed optimization techniques are Marine predators algorithm(MPA)[74], Arithmetic optimization algorithm(AOA)[75], Tree growth algorithm(TGA)[76], Equilibrium optimizer(EO)[77] and many more explore and develop.

2.2.1 Deterministic vs Stochastic Methods

Deterministic methods search entire feasible region which is a time consuming process and then gives exact solution. It can be applied to limited problems only and not much efficient to solve NP hard, combinatorial, non-linear continuous, complex constrained optimization problems. This inefficiency is alleviates by introducing stochastic methods and the respective algorithm is termed as search algorithm designed to locate desired solution from a given set of all possible solutions to optimize the single/multiple objective(s). The main challenge with this, search is exponentially increase proportional to the size of the problem and number of objectives. However this issue arises when solving problems having wide range.

Stochastic methods become more popular because of gradient-free nature and random population initialization, therefore applicable to non-convex, continuous or explicitly defined problems. Stochastic algorithms are single or multiple solution based algorithms. Hill Climbing(HC)[78], Simulated Annealing(SA)[79], Tabu Search(TS)[80], Iterated Local Search(ILS)[81] are some most popular single solution based algorithms and GA, PSO, ACO, GE are multiple solution based algorithms. Stochastic methods also have some demerits like sometimes get stuck at local optima and in some cases converges to global optima probabilistically.

2.2.2 Heuristic vs Meta-heuristic techniques

Heuristic methods are problem dependent whereas meta-heuristic(MH) techniques can work well with the black box problem. Due to this meta-heuristic methods are more popular in the field of optimization. A comprehensive review on MH can be collected from [82]. A general scheme of meta-heuristic method is given in the form of flowchart figure 2.1.

2.2.3 Benchmark functions

To study or to develop the optimization techniques, the benchmark functions(also known as test functions) plays a very important role. These are the pre-defined functions designed by well experienced scientists/researchers which have been used to compare and validate the optimization (heuristic or metaheuristic) algorithms. The algorithms that perform well on the different test functions are considered as effective for solving real world problems. In terms of applied mathematics, also known as artificial landscapes which are helpful to visualize or evaluate the characteristics, precision, robustness and convergence rate of optimization problems. These are categorized on the basis of similarities in their shapes and physical properties. These are also play a significant role in understanding the advantage or disadvantage of newly developed algorithm over the popular algorithms. There are some issues with test functions, one issue is that these are often well-defined and sufficiently smooth, but the real world problems are diverse in nature and different from these functions. Second issue is that these are typically unconstrained or with regular search space, but the real world applications many non-linear complex constraints and irregular search space. Therefore, the algorithm that works well with test functions may not works well with real world applications.

2.3 TLBO: Teaching-Learning based Optimization

In present era it is gaining the popularity of being less complex and only two algorithmic parameters based algorithm. Due to this it become flexible to inculcate with other optimization techniques in the form of hybridization or modification, so that standard

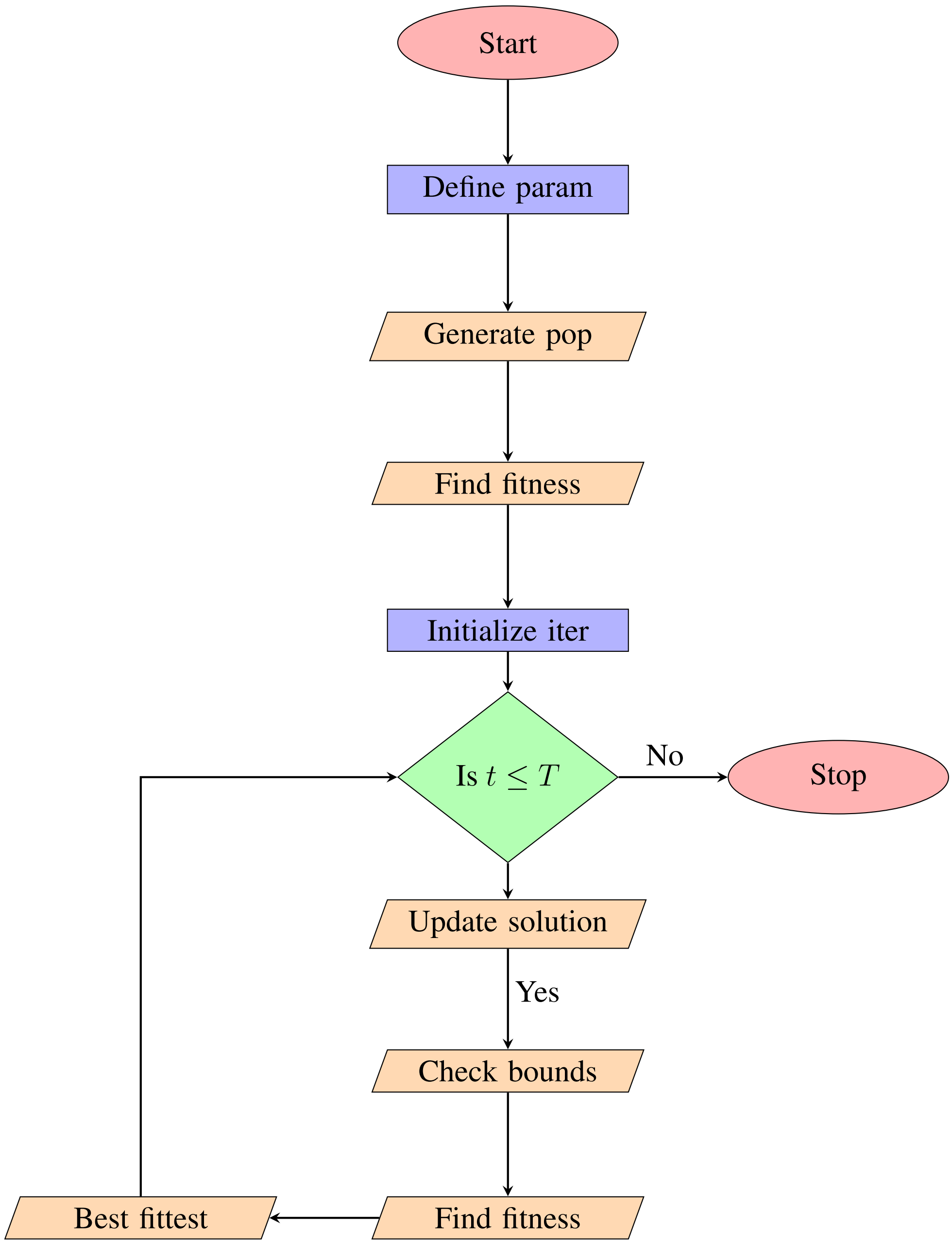


Figure 2.1: General scheme of MH.

TLBO can be enhanced to perform well with fast convergence towards the local/global optima as compare to other optimization algorithms. The more detailed discussion about its background, development and variety of applications is as follows:

2.3.1 Background and Development

TLBO is stochastic based population technique developed by Rao et.al. in 2011 [83], inspired from the transfer of knowledge between teacher-student and within students in class-room environment. In this paper proposed method is explained by the developers and tested its performance on various benchmark functions, mechanical engineering testing functions and other design problems which show the better results over other nature inspired optimization techniques. This novel method is applicable for the various engineering optimization designs. TLBO is efficient to find optimum solution for continuous non-linear large scale problems [84]. With some modifications in the method, it becomes capable to solve complex constrained optimization problems [32].

Author introduced the concept of elitism with TLBO and tested on 35 constrained benchmark functions to investigate elitist effect on its performance. This modified method can be easily optimize various industrial problems like job scheduling, pricing policies, supplier selection, order lot sizing, inventory control production planning and control, vehicle-routing etc.
Other papers [85] and [86] by Rao proved that TLBO can be successfully applicable for solving multi-objective problems also.

TLBO's dominance over existing NIAs steered focus of Matej Crepinsek [87] and reported to reveal quantitative and qualitative mistakes in TLBO. Furthermore, the authors had presented results and tried to invalidate the TLBO algorithm on the basis of only two papers of Rao [83], [84], without reviewed the latest research on TLBO. After that Waghmare [88] clarifies those doubts or misconceptions about algorithm in an objective way. In this paper author presented valid performance of TLBO on the problems having low fitness-distance correlations by proper setting of algorithm parameters.

One more paper by Rao et al. [89], proposed an elitist TLBO for solving unconstrained optimization problems is tested on 76 unconstrained benchmark functions(unimodal, multi-modal, separable, non-separable) and compared with other algorithms. The statistical results have proved that elitist TLBO is better than that not considered elitism.

2.3.2 Applications

The applications of TLBO are introduced in almost every field of engineering to solve various types of optimization problems. In [90] the author proposed an optimized method by combining the hybrid TLBO-DE with Elan neural network to estimate the proton exchange membrane fuel cell parameters and its performance is compared with popular techniques based on well-known benchmark functions. The authors of [91] proposed a new method of non-dominating sorting TLBO (NSTLBO) on the basis of group learning and experienced learners to find the solution of MOOPs. The algorithm named as NSGLTLBOLE is introduced based on the concept of micro teaching in which the learners of a class (called population) are divided into subgroups (called sub-populations) and algorithm is applied one by one for all the subgroups and after achieving some improvements collected together to form a diverse population. The algorithm find the optimal set of solutions for each subgroup using NSTLBO and crowding distance technique which are continued to the next iteration of whole population. Moreover, learner phase use the basis of learning from other experienced learners within the population. The concept of exploitation is incorporated within the group, whereas the exploration search completely inculcated in randomly regrouping of subgroups. Its efficiency over other algorithms is cross verified for various bi-objective benchmark functions which proved that NSGLTLBOLE has performed better. SCA–TLBO is proposed in [92] which is a novel hybrid optimization algorithm, for finding optimum solutions to the problems arises in the field of visual tracking. The convergence of TLBO with SCA to escape from local optima is faster than the individual SCA and TLBO. The performance of this algorithm is confirmed using several benchmark functions. Statistical methods are used to calculate the qualitative efficiency of the SCA–TLBO, and results prove its capability. This hybrid algorithm is applied for real visual tracking case study. The different parameters of visual tracking such as object root mean square error, tracking error, tracking detection rate, absolute error, and time cost are experimentally measured by using SCA-TLBO based tracking framework. The capability of hybrid SCA & TLBO based tracking framework over another, like alpha–beta filter, particle filter, extended Kalman filterand linear Kalman filter, particle swarm optimization, scale-invariant feature transform, and bat algorithm, is explained in the paper. One of the interesting application of TLBO is introduced in [93]. The main concern of author is optimize the aerodynamic shape of aircraft. The author proposes a novel TLBO mimetic algorithm (TLBO+MA) for attaining his main concern to search for optimal shape parameters which have significant role in characterizing aerodynamic vehicle. In the introduced TLBO+MA, an adaptive teaching factor, conservation of information inspired operator and multi-meme learning are incorporated to enhance the searching behavior of standard TLBO. Simulation based on well-known benchmarks and ASO for HTV-2 prototype demonstrates the efficiency of the proposed TLBO-MA. In [94] article, the authors obtained the multi objective optimal solution to power flow by implementing TLBO. A MO problem is formulated in terms of desired decision variables depending upon parameters. The results of TLBO are compared with mixed integer PSO which clearly proves that the new TLBO is better than the former in terms of convergence and searching globally. The developer of standard TLBO compiled applications inculcated to solve unconstrained and constrained optimization problems in [95]. One can go through this article to know more about applicability area of TLBO algorithm. The authors of [96] contributes to develop surrogate-based optimization technique using teaching learning optimization (TLBO) algorithm and modeling technique called high-dimensional model representation (HDMR). The aim of this techniques is to solve the problems of high-dimensional engineering optimization like optimize high-dimensional design space and meta-modeling. The difficulty of high-dimensional model is reduced by dividing it into low dimensional model with HDMR modeling technique. The authors verify the effectiveness of developed optimization method for high-dimensional problems using various pre defined functions. Moreover, The proposed optimization based on surrogate technique is successfully applied to find the solution of a typical engineering optimization problem: optimize parameters of high-dimensional aerodynamic shape of aircraft. It concludes that the performance of traditional surrogate-based optimization framework become improved with HDMR-based modeling technique and TLBO algorithm for high-dimensional engineering problems. In [97] paper, a new optimization technique i.e. teaching learning based optimization (TLBO) is presented to optimize power dispatch (CHPD) problem and combined heat having feasible & bounded space. The basic TLBO algorithm is hybridized with Opposition based learning (OBL) to achieve better results and fast convergence towards optima. The efficiency and accuracy of the hybrid OTLBO algorithms are proved better by comparing the results with other popular optimization techniques through the simulation in MATLAB programming. [98] introduces a two-stage method approach, using Finite Element Method (FEM) and two-dimensional Isogeometric Analysis (IGA) incorporate with TLBO, PSO, Bat algorithm(BA) to observe damage in beam-like structures. On the basis of laboratory experiments it is concluded that the introduced approach can be used to find out the location and severity damage in beam-like structures. To explore applications on various manufacturing process one can read the paper [99]. A new hybrid version of TLBO and the mutated fuzzy adaptive particle swarm optimization (PSO) algorithm is developed in [100] to diagnose the genes responsible for development of breast cancer. The mathematical formulation of problem is defined in terms of multi objective function which means minimization of the number of infected genes and maximization of the classification performance. The obtained results concludes that this hybrid technique is capable to attain the desired accuracy, sensitivity and the specificity in the breast cancer. The authors of [101] contributes to develop a novel hybrid method having ability to solve complex multi-objective optimization problem without entangling in local optima. An improved TLBO is proposed to achieve economic dispatch of power generation through distributed energy resources considering environmental constraints. In [102] article a multi-unit production planning based optimization strategy that involves a set of integer and continuous variables to overcome the challenges of formulation/strategies and able to find efficient production plannings. In addition, they have designed an effective strategy to manage the constraints. The proposed methods are verified on eight cases, which have been already discussed in literature to potentially guide the petrochemical industries, and increases the profit. They also discusses the computational performance of other popular optimization algorithms such as sanitized TLBO, differential evolution, binary PSO, ABC and the popular real coded GA. The exploration and exploitation of search space is improved in the basic MO-TLBO (multi objective teaching-learning-based optimization) by inculcating the strategy of tutorial training and self motivated learning. The multi-objective TS-TLBO approach can be adopted to find the non-dominated solutions maintained in an external archive. The various applications are presented to support the efficiency and accuracy of TS-TLBO algorithm.

2.4 Algorithm structure

TLBO algorithm constitutes of two main phases as it mimics the learning process via class teacher's and other student's knowledge within the class. The best feature of TLBO is, it requires only two algorithmic parameters which makes it easier to understand and less complex than other optimization techniques. Generate learners randomly which constitutes the population that is classroom environment. The population represents in the form of matrix say $P(i, j)$ where $1 \leq i \leq m$ and $1 \leq j \leq n$, m rows depicts as number of students and n columns as number of subjects(decision variables) in course.

2.4.1 Teacher Phase

This phase simulates the process of increase in knowledge of student through teacher and deviation of result of particular student from the subject-wise mean(M) result of whole class. The efforts of teacher to improve the knowledge of students leads towards the best objective function value. The evaluated fitness function value for each student act as their results and best learner among all plays the role of teacher. The obtained results of learners is to be considered as new solution if it lies within defined range of decision variables and obtain respective fitness function value. The updated learner's knowledge at t^{th} iteration through teacher's efforts is evaluated by using the relation given as:

$$X_{new,t} = X_{old,t} + r \times (M_{new} - T_f \times M_j) \tag{2.1}$$

where $r \in (0, 1)$ is random number.

$T_f \in 1, 2$ is known as teaching factor which is chosen by using the relation:

$T_f = round[1 + rand(0, 1)]$

2.4.2 Learner Phase

In this phase the learner gains knowledge from other students may have more or less information about the subjects of course. The improvement in learner's X_i knowledge at t^{th} iteration through mutual interaction with other student X_j, which is randomly and uniquely selected from classmates by the learner.

Evaluate the corresponding fitness function $f(X_i)$ and $f(X_j)$.

The updated solution is calculated by using one relation as follows accordingly:

$$X'_{new,t} = X_{new,t} + rand(X_i - X_j) \qquad if \qquad f(X_i) \leq f(X_j) \tag{2.2}$$

$$X'_{new,t} = X_{new,t} + rand(X_j - X_i) \qquad if \qquad f(X_i) \geq f(X_j) \tag{2.3}$$

The two essential elements in the TLBO Algorithm are the teacher and the students. This outlines two fundamental learning methods: instructor-led instruction (known as the teacher phase) and student-led interaction (known as the learner phase). A teacher is typically thought of as a highly educated somebody who instructs students to get better achievements in terms of their grades or marks. Additionally, students learn by their interactions with one another, which also enhances their performance. The TLBO approach is population-based. The results of the learners are equivalent to the fitness value of the optimization issue in this optimization algorithm, which considers a population of learners and various design factors as various subjects presented to the learners. The instructor is seen as the best answer across the board. The instructor phase and learner phase are the two primary working stages of the TLBO algorithm. The author of [99] concluded that in the engineering fields like chemical, electrical, production, thermal, and civil, TLBO proved to be the best. Moreover it excels in bio-medicine, maintenance, and process planning. This optimization approach can be applied to problems with several restrictions and multiple objectives. The TLBO algorithm also included duplicate solution removal and the elitism concept. TLBO is an algorithm-specific parameter that just needs standard control factors, such the size of the population, the number of generations, and the size of the elite.

The step by step working of standard TLBO is briefly explained in 1. By making the desired changes in 1, anyone can prepare the algorithm of any hybrid or modified version of TLBO. On the basis of algorithm, the problem which requires optimal solution can be simulated in programming codes like MATLAB, MATHEMATICA, SCILAB etc.

2.4.3 Pseudocode of TLBO

Algorithm 1 Pseudo-code of TLBO

Require: Problem settings: Fitness function, Domain of search space
Require: Algorithm parameters: Number of learners, Maximum Iterations
Ensure: Best Fitness function value

1: Generate random learners
2: Evaluate fitness function value of each Learner
3: **TEACHER PHASE:** Assign the best Learner as a Teacher
4: Obtain the mean of each subject of learners
5: Generate new solution in teacher phase using 2.1
6: Apply corner bound strategy if solution does not lies within domain of search space
7: Evaluate the fitness function value of new solution
8: Apply greedy selection strategy on new and old solution to choose the better solution
9: **LEARNER PHASE:** Choose any random solution as partner
10: Generate the new solution in learner phase using 2.2 or 2.3 accordingly
11: Apply corner bound strategy if solution does not lies within domain of search space
12: Evaluate the fitness function value of new solution
13: Apply greedy selection strategy on new and old solution to choose the better solution
14: Repeat the steps or vary the maximum number of iteration to get the desired results

List of Abbreviations

ABC	Artificial Bee Colony
ACO	Ant Colony Optimization
ANN	Artificial Neural Network
CI	Computational Intelligence
DE	Differential Evolution
EP	Evolutionary Programming
ES	Evolutionary Strategy
GA	Genetic Algorithm
GP	Genetic Programming
iter	iteration
MH	Meta-heuristic
NIA	Nature Inspired Algorithm
NIOT	Nature Inspired Optimization Techniques
param	parameters
pop	population
PSO	Particle Swarm Optimization
rand	random
SMO	Spider Monkey Optimization
sol	solution

T	Maximum Iterations
t	Current Iteration

2.5 Conclusion

This review article of TLBO shows that this technique is very versatile to be applicable in solving the various optimization problems. The results in various research articles proved that different variants and hybridization of TLBO is also very interesting field to explore more about its applicability. The variety of different benchmark functions in this field of optimization gives so many gaps to study or introduce new hybrid or modified versions of optimization techniques. Metaheuristic methods perform differently in different problems. One may results better than the other in a particular problem and worse in other problems. In literature survey, it is concluded that TLBO is superior than other popular optimization techniques in obtaining the optimal solution. Besides previously developed metaheuristic methods, the investigations on metaheuristic techniques are still being done and new algorithms are being developed continually. For future works, suggest developing a unified platform for analyzing, evaluating, and comparing different optimization techniques. In present era it is gaining the popularity of being less complex and only two algorithmic parameters based algorithm. Due to this it become flexible to inculcate with other optimization techniques in the form of hybridization or modification, so that standard TLBO can be enhanced to perform well with fast convergence towards the local/global optima as compare to other optimization algorithms.

Chapter 3

Assessment of Municipal Solid Waste Management in Dinanagar city of Punjab, India

3.1 Introduction

Indian government introduced and implemented the various rules, policies and schemes to handle & manage the different types of waste [103].With rapid increase in modernize population the challenges to manage the trash is encountered in developing countries. So, different methods and technologies such as Geographical Information System aided methods [104], MSWM using mathematical modeling [105], simulation optimization method [106] are developed by the researchers.The authors of [107] proposed Genetic Algorithm to solve the issue of transportation cost involved in collecting the generated waste from various source sites to transfer station and to other interlinked stations such as incineration, composting and landfills. In the article [108], author presented a new optimization-simulation approach based method on firefly algorithm & procedure is demonstrated using MSWM case study.The review article [109] is presented the challenges related to economic optimization of SWM. The classification of waste modeling methods on the basis of regions, time series interval, waste stream are reviewed in article [110]. According to literature one major reason behind poor WM is inhabitants perception, concern, behavior, participation and awareness. The typical characterization of waste in India is: Biodegradable, Non-biodegradable, hazardous. On the basis of characterization three main ways followed to manage and handle the waste are: centralized, decentralized, integrated SWM. Biodegradable waste is preferably handled at decentralize level as it reduces its value with delay in treatment. Whereas non-biodegradable and hazardous trash can be managed at centralized level as it generally requires to establish a long term plant investment, which are successfully run with at least 5 tons of waste as per drafts of SWM. The proper utilization of wastes leads to the advantages [111] such as reducing Greenhouse Gas emission, biogas generation, quality compost production, soil fertility improvements etc. The article is based on the assessments of 5.4current implication as per following aspects

- Sources of waste generation.
- Quantification & Characterization of waste.
- Citizen's awareness and participation.
- Willingness to pay for SW services.
- Suggestion to implement more effective & sustainable waste management plan.

It is observed that the some problems or challenges related to MSWM are different in different cities so there is need to monitor the WM practices at local level.

Table 3.1: Details of Study area-Dinanagar

Municipal Corporation Area	14.36 km sq.
Total No. of wards	15
Total Houses	4840
No. of Households	300(approx)in each ward
Total Population(2011)	23976
Present Population(2021)	25000(approx.)
Literacy rate	88.66%
Male Literacy	92.26%
Female Lieracy	84.73%
State avg. Literacy rate	75.84%

Source: Census of India and DMC

3.1.1 Sources & characteristics of MSW

The generators of MSW are broadly categorize into Residential and Non-residential waste. Further, the residential waste includes kitchen waste(left over food items), paper, cardboard, plastic, sanitary waste, inert and Non-residential waste includes bulk quantity of refused fruit, vegetables, packaging cardboard, plastic, construction & demolition waste, Industrial, restaurants & hotels waste etc. In Dinanagar, the average Municipal solid waste generation is about 6 metric tonnes(only of residential area, no

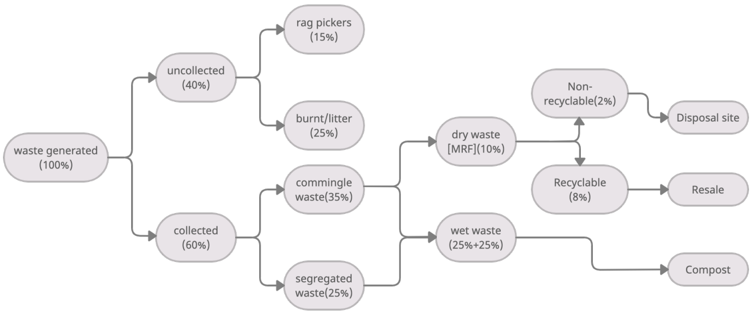


Figure 3.1: Life cycle of waste in Dinanagar city

record found for non-residential contribution in trash production), and typical physical composition of waste is mainly three types biodegradable, non-biodegradable, and hazardous. No any record is available for chemical composition of waste generated in city.

3.1.2 Problem identification

The Indian government establishes new targets to minimize the quantity of biodegradable waste in landfill or dumping sites. To achieve this target, the composting is primarily solution in small municipality like Dinanagar. Using this technique the waste volume is reduced by 50-65%. Composting can be done either manually or mechanically. Presently, 33 number of manual composting pits at different locations are successfully maintained by Municipal Corporation of Dinanagar(MCD). There is need to improve the collection rate of generated waste, treatment rate of collected waste, and disposal rate of untreated waste. The population of Dinanagar is 25376 inhabitants and 15 square km land area which is further distributed in 15 wards with average of 376 inhabitants each. The MSW generated is only about 0.2-0.25 kg per capita per day, of which 60% is wet waste, 40% is dry waste and only 50-60% of generated waste in the city is collected with the utilization of presently provided collection services. In particular, material recovery facility(MRF) is also adapted by the MCD to treat all metal, paper and plastic from the perspective of cost management. The need of effective and efficient MSWM is increasing as the poor management contributes adverse effects on economy, health, environment and one major threat is global warming which is due to increase in Green House Gas(GHG) emissions because of uncollected/untreated waste lying in open dump sites. The inefficient collection and treatment services are the reasons for gap existence in present practices of MCD (see table 3.2).

Parameters	Existing	Benchmark	Gap-existing
Waste collection	60% of waste generated	100%	40%
Waste treatment	65% of waste collected	100%	35%

Table 3.2: Gap analysis
Source: Municipal corporation of Dinanagar

3.1.3 Waste Management

Almost in every residential regions of 15 wards, the unpleasant view of openly dumped commingle waste is spreading odor, threat of health issues, causing environmental problems, stray animals(like dog, cow) are creating more mess with waste(like diapers, sanitary napkins and left over food). The blockage of sewage is also a very frequently faced problem especially in rainy season. All these issues or problems are directly or indirectly related to the biodegradable fraction present in the openly dumped commingle waste by the unaware or may be aware inhabitants. The only solution to all issue is not to dump such type of waste in open areas, it should be collected separately and treated in decentralized manual composting plant as early as possible to produce good quality compost. Because, the delay in treatment of biowaste leads to low quality of compost. Further, it can be used for gardening purpose and the rest can be convert into revenue by marketing which could be led to compensate the expenditure by some extent. The life cycle of waste in the city is shown in figure 3.1.

3.2 Common indicators influencing the proper functioning of MSWM

Citizen's participation(CP)

In the earlier plans of waste management it was a belief that public participation is the least most factor affecting the MSWM but in recent years the researchers and the experiences proved that it is the most primarily aspect to be focused for proper utilization of waste as resources. Without citizen's participation, it is impossible to meet the challenges emerges out from the waste quantification. So, it is important to think out off the box to handle and manage with the issues related with WM. It is necessity to inspire and aware each other about the impacts of poor waste management. The negligence towards it's bad effects will be finally ended up with the loss of inhabitants life.

Waste minimization(WM)

Now a days, to manage the waste production in the country, concept of waste minimization becomes more important to focus. This practice will led to converge towards the successful achievement of waste management objectives. It is the most effective way to reduce the quantity of waste, the cost associated with its handling and its environmental & aesthetic effects. Waste minimization strategies requires national or state level interventions such as extended producer responsibility can be established for waste like electronics, batteries, ban to use or sell certain types of products and packaging that cannot be reused, repaired, recycled or composted. Waste minimization usually requiring ULB support or action like promoting & developing at source level reduction program, awareness & education programs, banning the use of plastic products by replacing with reusable and recyclable material.

Five R's policy(FRP)

The technique of five R's is responsible for sustainable and zero waste objective of solid waste management. The definition of 5 R's are :

- **Refuse:** Say no to non-biodegradable material or products.
- **Reduce:** Replace the non-biodegradable with biodegradable material.
- **Reuse:** Do not use disposable products. Replace them with more sustainable alternatives.
- **Recycle:** Use the material which can be transform into another usable form.
- **Recover:** Convert the kitchen waste into compost.

By adopting these 5R's policy in daily routine life, the challenges of waste management can be achieved effectively and efficiently.

Segregation at source level(SAS)

Segregation of waste into different fractions at source level is an essential practice for effective and efficient WM. Unwanted material should be treated or stored by the source of trash until it is collected for treatment or disposal by the waste collectors. It should be separated by waste generators into three fractions: wet, dry and hazardous. Apart from these wastes horticulture waste, construction & demolition and sanitary waste should be collected separately and designated for treatment. Collection and transportation of segregated waste from source to destination is highly essential in SWM system.

Waste collection(WC)

Waste collection is next essential step after segregation at source level. Inefficient collection services of waste has a bad impact on and aesthetics of cities & public health. Waste collection is generally divided into primary collection which refers to the collection of waste directly from waste generators and secondary collection includes picking up waste from community bins, waste storage depots and transfer to treatment facilities or disposal sites. A proper synchronized primary collection and secondary collection and transportation system is necessary to avoid waste littering and container's overflow in the city.

Service charges(SC)

Households that produce the majority of solid waste and are directly affected by uncollected solid waste should be able to participate in improving SWM. As a result, the contribution of city dwellers to SWM service plays a significant role in improving SWM in the community. As result, service rates for SWM should be set at a level that discourages illegal dumping while still maximizing cost recovery. Before it's implementation, the proposed sanitation cost must be changed based on willingness and ability to pay. At present only 1500 households are paying for the waste management services. The residents are paying directly to waste collectors as an incentive. So, it is not a part of revenue for MCD. MCD expects to receive ₹197,295 every month in revenue from households. When correctly collected, this revenue covers 85% of the cost of providing 100% solid waste collection coverage in study area.

Training & awareness of WM staff(TAS)

The workers involved in collection and treatment process of Waste Management should be trained enough to run the system effectively. The number of safai sewaks, sewadars, laborers etc working with MCD are 72, which is more than enough but as per formal talk with municipal authorities " The maximum number of workers are females(not to be involved in field work), of old age or suffering from some disease, some(the most efficient workers) are deputed at ministers and their relatives residents for personal(cooking, cleaning, gardening etc) purposes ". Their work and duties are divided as sweeper, collector from primary and secondary source , separator at transfer station and manage and inspect working of composting plant etc. The overall working of workers is further supervised by a sanitary inspector. One person is appointed for maintaining the records of waste in terms of dry & wet waste and other information related to compost production, utilization and recycled, non-recycled waste at MRF. No record is available in terms of components of generated waste, physical-chemical composition of collected, treated or disposed waste. The lack of well qualified and experienced staff to manage finance, accounts, propose and suggest to generate the revenue from the collected resources. The present staff is not capable to maintain the proper records which is responsible for insufficient planning and processing of waste management. Other professionals like environmental engineers, medical officers, security and legal personnel's are also lacking in present WM practices.

Selection of appropriate technique(SAT)

The treatment of produced waste depends upon its quantity, sources, physical & chemical composition, collection services, disposal facility and perception & participation of inhabitants. The sources of waste are domestic, commercial, fruit and vegetable market, fish and meat market, green waste from parks, gardens, construction & demolition etc. The physical composition includes food, plastic, paper, metal, rubber, glass, sanitary pads, napkins, diapers, wood, textile, dust, stone and inert. The MSWM is responsible for maintaining the waste produced in household as well as commercial regions of the city. As per record available by MCD the maximum fraction of generated waste is biodegradable. In small cities, the most effective and efficient technique to manage the waste is manual decomposition of biowaste at decentralized level which includes kitchen waste(left overs of food and vegetables), wet paper, cardboard(non-recyclable). The rest waste is transfer to material recovery facility(MRF) for further segregation and treatment accordingly . At present both(manual composting and MRF) techniques are adapted by MCD. But the improper & manipulated record, delayed & incomplete waste collection, processing of whole generated waste, awareness & participation of residents, dedication of workers, political influence in the system are the major reasons behind inefficiency of MSWM in Dinanagar.

Marketing of resources(MR)

Marketing of resources obtained from city waste is also become more challenging after collection & segregation of recyclable material and compost prepared from biowaste. For complete utilization of resources, an effective marketing strategy has to be designed. The compost produced at different composting plants of MCD is distributed free of cost to the citizens. The wet waste collected at MRF is further sold via waste collectors. The rate list of recyclable or resalable material is given in table 3.4 as per local waste collector.

Environmental & economic effects(EEE)

Another main effects of poor management of solid waste are environmental and economic effects. The improper planning of SWM destroy the soil, water, air and habitat. Leachate released from the openly dumped waste contaminates the soil, air and water which is very harmful for plants, animals and human. Stray animals & scavengers litter the waste and invade the roadside garbage causing damage to the surroundings. As organic solid wastes decompose, release an unpleasant stench that pollutes the environment. Waste items such as plastic and rubber emit poisonous gases into the atmosphere. The clogging of drains by plastic trash causes water logging, which encourages mosquito breeding and the spread of diseases.

Political role(PR)

The role of politicians is very important and influential in local areas of small municipalities. Without the support of political leaders, the proper implementation of SWM is not possible. The study area is divided into 15 wards and each ward is associated with a municipal commissioner(MC). It should be MC's responsibility to inspect the working of MCD in the associated ward. They can promote the campaigning to aware and inspire the residents of ward. They should be more interactive with the public to resolve their problems. A responsive and dedicated leader can contribute in sustainable SWM. Both the national and local governments must make a clear commitment to waste management in order to improve the city's garbage condition. National strategies for proper waste management should be vigorously promoted. To attract more attention to this sector, the jobs available to both skilled and unskilled labor should be made more appealing. Public knowledge and education on many facets of waste management should accompany political commitment. Environmental education and awareness should be made a requirement in all schools.

National policies & initiatives(NP)

MSWM in urban areas has emerged as one of the biggest challenge encountered by developing countries like India, not only in terms of aesthetic impact and environmental but also the potential threat to public health, resulting from improper and non-scientific handling of municipal waste. Acknowledging the magnitude of this challenge, on 2^{nd} October 2014 the GoI launched Swachh Bharat Mission(SBM) with a goal to make India clean & free from open defecation. Under SBM 100% scientific management of MSW has been identified as one of the critical objectives. The GoI has taken several initiatives to achieve these objectives. As a part of these initiatives the government is facilitating assistance for market development in form of financial assistance of ₹1500 per ton on sale of compost to farmers to increase and encourage compost sale. The Central Electricity Regulatory Commission(CERC) has announced mandated 100% procurement of power generated from WtE plants and generic tariff for Waste-to-Energy(WtE) of ₹7.90 per unit of power. These initiatives will ensure financial viability of setting up of waste-to-Compost(WtC) and WtE plants.

In order to assist cities and states to understand and effectively implement SWM systems, the Ministry of Urban Development, in partnership with Government of Germany has published the revised MSWM manual 2016, in alignment with the SWM rules 2016.

Citizen's awareness(CA)

Inhabitants of the region are responsible for waste generation. Public awareness is critical to the successful implementation of SWM's strategic plans. The public's understanding and attitudes toward waste disposal have a direct impact on the SWM plan. Several academics concluded that it is a critical aspect in determining the fate of any area's waste management strategy. Local government units should stimulate citizen engagement by providing widespread information. Non-governmental organizations (NGOs) might be enlisted to help organise public awareness campaigns. It's also critical to encourage the usage of composts made from organic resources. The media has a significant role to play in public education.

Effective & efficient management(EEM)

The city's integrated SWM policy is a necessity for improvement. To prioritize the best waste management techniques, the MCD should use the waste hierarchy model. By developing a national strategy, it is feasible to gather the necessary resources, develop norms and processes, and establish other supporting organizations. Only a nationwide plan, which engages the entire country in developing a well-developed strategy, would provide an effective solution to the solid waste problem. SWM entails the planning and long- term implementation of all aspects of a waste management plan that are relevant to the region's waste management. The available waste management strategy must be adopted and implemented in such a way that local governments are prepared to face any issues that may arise. Innovations should also be integrated into the planning.

Table 3.3: Decomposition period of waste material [4]

S.No.	Waste material	Decomposition rate
1.	Kitchen waste	10-14 days
2.	Paper	3-4 weeks
3.	Cotton cloth	2-5 months
4.	Wood	10-15 years
5.	Metal	100-500 years
6.	Plastic	more than 1 million years

Table 3.4: Rates of recyclable material(in ₹(Rupee): symbol for Indian currency)

S.No.	Recyclable material	Resale rates(per kg)
1.	Copper	₹350
2.	Aluminum	₹100
3.	Steel	₹35
4.	Rigid iron	₹25
5.	Plastic bottles	₹22
6.	Mix plastic	₹20
7.	Rigid plastic	₹19
8.	Light iron	₹18
9.	Paper	₹12
10.	News paper	₹10
11.	Cardboard	₹10
12.	Glass	₹3

Waste record & accounting(WR)

The analysis of implementation of SWM is completely dependence upon the proper maintenance of waste record in terms of quantification, composition. On the basis of record, the better planning and decisions can be made for successful and sustainable SWM.

3.3 Methodology

The methodology adopted for preparing this article is mainly based on literature review and the data collected by means of questionnaire survey, interviews with citizens of different localities(low,medium and high income regions), local waste collectors, waste management staff of MCD, persons involved in commercial sector. The reason behind selecting this method is that there is no proper record maintained by MCD. All data is recorded manually by the data operator in the form of hard copies, and by observing

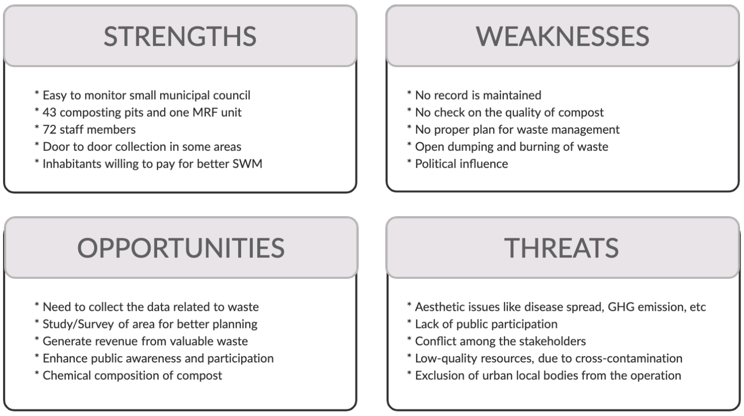


Figure 3.2: SWOT analysis of WM in MCD

the figure it was quite clear that the data is completely manipulated just to meet the guidelines issued by GoI. So, the assumed data is used as per standard waste generation value and the state wise data available at different online sites and research articles. The quantitative analysis is performed on numeric data collected from various primary and secondary locations. The qualitative analysis is further expressed and explained in narrative form on the basis of interview, small group discussions & responses collected in structured questionnaire survey from samples selected in the categories: low,middle and high income localities. The SWOT analysis of present practices of MCD is briefly described in figure 3.2.

3.3.1 Conclusion

The study, which had the explicit goal of analyzing the implementation of several SWM programs in Dinanagar city, discovered that improper record about quantification, collection, treatment and disposal of wastes, political commitment, personnel facilities, and public and commercial sector commitment all had a substantial impact on the success of solid waste management implementation. The study also revealed some administrative issues that hampered implementation in the early stages, such as a lack of resources, unscientific land for final waste disposal, a lack of an effective and efficient solid waste management plan, public awareness, staff training, and poor enforcement of laws and regulations.For successful and effective management of solid waste in the Punjab city of Dinanagar, encouraging waste reduction, adequate resources, political leadership support, providing monetary incentives for recyclable items, encouraging workers, developing a strong policy, composting, incorporating technological innovations, and recycling techniques have been suggested and recommended. Other areas which are linked and have a direct impact on the city's waste management but are understudied were observed. It includes like revenue from local sources, waste management finance and commingled waste analysis in the city, and adequate logistics and modern technologies for SWM to be used in the city.Future research will focus on urban growth and related concerns in order to gain a better knowledge of such difficulties and create solutions for long-term development and enhancing environmental sanitation in Dinanagar.

Chapter 4

Implementation Analysis of Municipal Solid Waste Management in Dinanagar city of Punjab, India

4.1 Introduction

4.1.1 Indian Government initiatives for MSWM [1]

In Indian cities, it is becoming more difficult to provide the growing urban population with the infrastructure demands. As per the 2011 census, population of India was 1.21 billion, having 31% of people living in the cities. By 2050, the population is expected to be half of it would live in city areas. With the increase in country's population, MSW management has become a very serious problem, not only due to aesthetic and environmental concerns, but also due to the massive amount of generated waste every day. As per the Central Pollution Control Board of India, the amount of waste generated was 1, 27,486 Tons per day of MSW during 2011-12, with an average generation rate of waste 0.11 kg per capita per day. Only 15,881 Tons per day that is 12.45% of the total waste generated was collected, and only 89,334 Tons per day that is 70% was treated or processed. Segregation of waste at the level of source, transportation, collection, processed or treatment, and scientific disposal of trash were all not satisfactory, as a result it leads to degradation of environment and a decline in living standards.

The reason behind that MSWM was becoming increasingly important first became apparent during 1990, when concerns about unsuitable MSWM resulted in number of Public Interest Litigations, prompting the Supreme Court of India to order the Ministry of Environment and Forests (MoEF), Government of India, to release Municipal

Solid Waste Management Guidelines in 1996. The rules required all Urban Local Bodies, not just Class I cities and metro cities, to implement a proper system for waste management, with a deadline for the installation of waste collection, transport, processing, treatment and disposal facilities by the end of year 2003.The Indian government's Ministry of Urban Development (MoUD) prepared a solid waste management guidance manual for all Urban Local Bodies and released the same with the defined rules during the year 2000. All Urban Local Bodies, moreover, were not able to introduce a long-term municipal solid waste management system, having transportation, collection, processing, treatment and disposal of waste, until the year 2003. Despite positive accomplishments and pilots, most urban local bodies continue to face problems not only in transportation systems, advanced collection or selecting adequate and processing or treatment technologies and disposal techniques, but also in managing long-term financial requirement in MSWM system. Non-compliance with rules of MSW management is still a reason for issue of worry 14 years after the compliance of rules for municipal SW, 2000 were already published. Through the Ministry of Environment and Forests' draft amended municipal SWM Rules of year 2013, the Indian government continues to highlight these problems or issues and support states and urban local bodies in creating adequate and contemporary MSW management systems and the parallel revision of MSW management manual of year 2000 by the Ministry of Urban Development. It is based on learning from fourteen years' experience gained post the notification of the rules in year 2000. The Indian government has announced and sanctioned the 12th and 13th Finance Commission Grants and finances have been provided for development and improvement of MSW management under projects (such as UIDSSMT and JnNURM) from the year 2005 to give an importance to municipal solid waste management in Indian cities. State funds are also available for the proper implementation of solid waste management initiatives.

4.1.2 MSWM in Indian Cities

The study [15] contribute to evaluate the current status of MSWM in India and suggest some significant improvements for better functioning of waste management. The article [16] reflects a case study of city **Lucknow** ,which is one of the main metropolises in India, and struggles to a major problem of MSW management. Through this community participation study, a qualitative study using a SWOT analysis (strengths, weaknesses, opportunities and threats) has been effectively implemented. The SWOT analysis was found to be a useful tool for investigating the possibilities and strategies for starting and successfully conducting the MSWM programme.[17] The research analyses the cost of

WM in **Mumbai**, India, and examines alternate methods for managing municipal solid waste (MSW). The first approach is community participation and the second is private sector participation. Moreover, there is a need to further analyze the role of PPP(Public Private Partnership) in waste management. [18]The MSWM, **Allahabad** data derived from ArcGIS maps is responsible for this work's information retrieval, updating, and essential information visualisation. [19]the Master Plan (2005–2021) provides instructions for the removal and treatment of municipal solid waste for the entire state of **Delhi** after the Delhi government recognised the gravity of the matter. It is concluded that a three-way cooperation between the public sector, the commercial sector, and the general public is necessary to make any significant changes to the current situation. [20], By measuring the site-specific emission variables in conjunction with pertinent activity data and using the IPCC's 1996 procedures for CH4 inventory preparation, an inventory of **Chennai's** landfills' glasshouse gas emissions has been created. As intermediate phases of waste management also affect its quantity, it is crucial to examine MSW reaching landfills along with the generation of waste and its composition determination at source level for CH4 emission inventory. By separating garbage at the source and encouraging recycling or reuse of separated items, waste volume and the strain on landfills are reduced, and producers have access to raw materials. The author of [4] highlights the current practices of MSW in **Puducherry**, proper record of generated, collected, treated, and disposed waste is needed as per rules drafted by GoI. The physical & chemical composition is also a necessary aspect to be maintained by municipality authorities. [21], this article helps characterize the municipal solid trash that currently exists in **Jalandhar**, India, and assess its viability for different waste-processing methods. The author of [22] Guided with the specific objective of analyzing the implementation of various SWM programs in **Ludhiana** city, the study found political commitment, facilities provided to the staff, participation of public and commercial sector significantly affected the successful solid waste management implementation. The study also revealed some problems from administration that affected implementation at the initial stages like inadequate land for final disposal of untreated waste, inadequate resources, shortage of staff, public unawareness, lack of integrated solid waste management plan, etc. In article [23] the purpose of this paper is to add to the characterizations of the current municipal solid waste in Jalandhar, India, and to assess its suitability for different waste-processing methods.

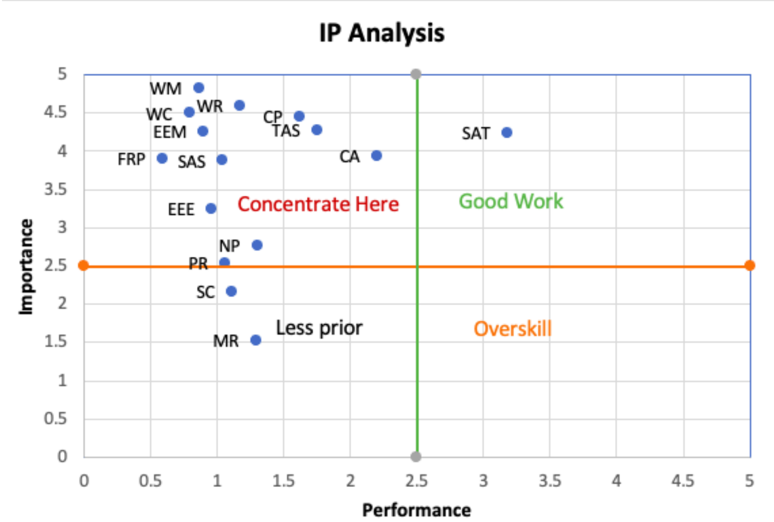


Figure 4.1: Quadrant chart-Importance Performance Analysis

4.2 Quantitative and Qualitative Analysis

Literature review is broadly used to prepare the questionnaire, which contains respondents personal information like age, gender, qualification, total family members, along with the knowledge, awareness, participation interest about SWM in city. The maximum responses are collected through Google form so as to follow the COVID-19 guidelines. The responses from the staff workers is collected via interview by following the measures of COVID-19 instructions that is wearing masks and maintain the precautionary distance. The questionnaire is broadly divided into three sections of indicators: 1. Inhabitants, 2. Waste management staff & Actors, 3. Policies & initiatives. The included questions are designed on the basis of importance and performance of existing practices. The respondent has to give rank between 0 to 5 as per the experience and knowledge with respect to the particular question. On the basis of collected responses the importance-performance analysis is completed using excel and the IP chart is shown in figure 4.1. In table 4.1 the performance and importance column are representing the mean of total responses w.r.t each indicator. The internal consistency analysis of collected data is performed using R / R studio software with cronbach's alpha.

4.3 Descriptive Analysis

Importance performance chart and scores are constituted on the basis of responses (shown in table 4.2) collected from the citizens of Dinanagar. The internal consistency & validity of collected responses is performed using coefficient alpha(also known

Table 4.1: Importance Performance score Analysis

S.No	Indicators	Performance	Importance
1.	CP	1.62	4.45
2.	WM	0.86	4.83
3.	FRP	0.59	3.9
4.	SAS	1.04	3.88
5.	WC	0.79	4.5
6.	SC	1.11	2.17
7.	TAS	1.75	4.28
8.	SAT	3.18	4.24
9.	MR	1.29	1.52
10.	EEE	0.96	3.25
11.	PR	1.06	2.54
12.	NP	1.3	2.77
13.	CA	2.2	3.94
14.	EEM	0.9	4.26
15.	WR	1.17	4.59

as cronbach's alpha). The range of alpha is 0.65-0.69 with respect to each indicator. as shown in table 4.3. The significance of defined indicators with effective & efficient management is also done using chi-square test shown in table 4.4. Importance performance analysis of 15 indicators is represented in figure 4.1 in the form of quadrant chart. The quadrant chart is divided into four sections: first, second, third and fourth quadrant divided as good work, concentrate here, less prior and possible over-skill respectively. The indicator defined as selection of appropriate technique is lying in first quadrant which means the treatment facilities presently used by MCD are satisfactory. Presently, MCD has adopted material recovery facility for other waste materials and composting of organic waste. IP analysis shows that the indicators SC(Service charges) and MR(Marketing resources) are lying in less prior category. Rest of the 12 indicators are required to focus for improved and better working of MSWM in Dinanagar city. The IP scores are given in table 4.1.

4.4 Results & Conclusion

The following are the suggested conclusions and recommendations to improve the present status of municipal waste management.

Table 4.2: Response frequency

Indicators	0	1	2	3	4	5
CP1P	0.17	0.27	0.33	0.23	0	0
CP1I	0	0	0.02	0.07	0.35	0.56
WM2P	0.36	0.42	0.22	0	0	0
WM2I	0	0	0	0	0.17	0.83
FRP3P	0.42	0.58	0	0	0	0
FRP3I	0	0	0.17	0.07	0.46	0.3
SAS4P	0.24	0.48	0.28	0	0	0
SAS4I	0	0	0.13	0.17	0.39	0.31
WC5P	0.44	0.33	0.23	0	0	0
WC5I	0	0	0	0	0.5	0.5
SC6P	0.31	0.27	0.42	0	0	0
SC6I	0	0.54	0.04	0.13	0.29	0
TAS7P	0.17	0.26	0.32	0.2	0	0.05
TAS7I	0	0	0	0.13	0.46	0.41
SAT8P	0	0	0.3	0.22	0.48	0
SAT8I	0	0	0.08	0.08	0.36	0.48
MR9P	0.49	0	0.24	0.27	0	0
MR9I	0	0.67	0.14	0.19	0	0
EEE10P	0.22	0.6	0.18	0	0	0
EEE10I	0	0	0.15	0.45	0.4	0
PR11P	0.21	0.52	0.27	0	0	0
PR11I	0	0	0.46	0.54	0	0
NP12P	0.19	0.32	0.49	0	0	0
NP12I	0	0.37	0	0.12	0.51	0
CA13P	0	0.27	0.41	0.17	0.15	0
CA13I	0	0.09	0.05	0.14	0.27	0.45
EEM14P	0.41	0.28	0.31	0	0	0
EEM14I	0	0	0	0.14	0.46	0.4
WR15P	0.22	0.39	0.39	0	0	0
WR15I	0	0	0	0.05	0.31	0.64

4.4.1 Proper Planning of SWM

The proper planning to manage the generated waste in city is a most essential part of municipal corporation. The first step is to maintain the record related to the waste like waste generation, sources of waste production, physical & chemical composition of waste. This data is necessary to decide the transportation services to collect the waste from primary and secondary sources. The selection of treatment facility is depending upon the production of waste composition in city. The data collection of waste by MCD from every aspect is very poor. But still the selection of appropriate technique for waste

Table 4.3: Internal consistency analysis
(-) indicates the reverse order of ranking

Indicators	alpha	correlation	mean	sd
CP1P-	0.67	0.254759	3.38	1.022771
CP1I-	0.67	0.233417	0.55	0.715979
WM2P	0.66	0.417917	0.86	0.75237
WM2I	0.68	0.117238	4.83	0.377525
FRP3P-	0.65	0.459812	4.42	0.496045
FRP3I-	0.69	0.04228	1.11	1.023906
SAS4P	0.67	0.301158	1.04	0.723627
SAS4I	0.67	0.227993	3.88	0.997775
WC5P	0.66	0.364242	0.79	0.795124
WC5I-	0.67	0.30323	0.5	0.502519
SC6P	0.64	0.590311	1.11	0.851558
SC6I	0.64	0.673897	2.17	1.348812
TAS7P	0.68	0.18003	1.75	1.242147
TAS7I	0.67	0.174783	4.28	0.682834
SAT8P	0.69	0.021649	3.18	0.868995
SAT8I-	0.65	0.525136	0.76	0.911431
MR9P	0.66	0.412223	1.29	1.320354
MR9I-	0.67	0.235712	3.48	0.797471
EEE10P-	0.67	0.195442	4.04	0.634369
EEE10I	0.68	0.136521	3.25	0.70173
PR11P	0.66	0.410833	1.06	0.693695
PR11I-	0.66	0.339648	2.46	0.500908
NP12P	0.66	0.389869	1.3	0.771984
NP12I	0.68	0.127879	2.77	1.398809
CA13P	0.68	0.176834	2.2	1.005038
CA13I-	0.65	0.53882	1.06	1.269853
EEM14P	0.68	0.149841	0.9	0.84686
EEM14I-	0.66	0.373542	0.74	0.690776
WR15P-	0.66	0.301907	3.83	0.76614
WR15I-	0.67	0.154316	0.41	0.587668

treatment is satisfactory. Mainly two waste treatment facilities are adopted by MCD, one is composting of biodegradable waste and second is material recovery facility. No doubt these two are sufficient for treatment of collected waste but only 60% of generated waste is collected by MCD. Moreover, 65% of collected waste is treated using available facilities because of commingled waste which cannot be segregate manually. Finally, the uncollected waste creating the several common issues like odor, choking of sewage etc and the untreated waste if dumped to open dump sites.

The district authorities are encouraging and motivating the concept of no bins and

Table 4.4: χ^2 test at 95% level of significance

EEM with other indicators	χ^2	df	p-value
CP	17.629	6	0.007229
WM	11.189	4	0.002452
FRP	1.021	2	0.6002
SAS	8.2861	4	0.08164
WC	9.5515	4	0.0487
SC	16.361	4	0.002571
TAS	32.202	8	8.57E-05
SAT	20.813	4	0.0003449
MR	12.913	4	0.01171
EEE	0.65504	4	0.9568
PR	3.5048	4	0.4771
NP	47.083	4	1.47E-09
CA	12.959	6	0.04369
WR	1.9134	4	0.7517

no landfill site by focusing on the zero waste approach. The issues of resource conservation cannot be fully addressed by striving for zero waste. Continued dependence on waste management strategies that have been shown to be ineffective in addressing the growing complexity of solid waste and a lack of data characterizing and quantifying waste generating patterns are the main causes of this [112].

SWM makes use of a variety of technologies to manage waste generation,collection, transportation, handling, processing, storage, treatment and disposal. Ineffective management of MSW cause human health problems, create long term environmental issues & degradation of valuable land resources. A sustainable and sufficient waste management approach is required to balance the quality of human life, the need for development, and clean environment. [112, 113].

4.4.2 Availability of equipment & funds

In-spite of various schemes or policies are framed for funds to improve WM services by GoI, the acceptance of grant proposal is not an easy task. The waste management problems related to small municipalities are not much important to focus on by the government. On the other end, any establishment which depends on government funding to manage waste related services will never be able to perform effectively and efficiently. The lack of instruments like brooms, brushes, dust pans & bins, choppers etc has led to improper cleanliness services. As per records provided by data operator of MCD, one separator machine is installed but not in functioning condition. One bailing machine is also installed which is in functioning condition. Moreover, the three E-rickshaws, three trolleys, six cycle rickshaws and four hand carts are used for collection and transportation purposes like transfer waste from secondary point to MRF, to compost pits in local nearby areas. The number of waste bags/bins provided for collection of segregated wet and dry waste from secondary point located near PSPCL(Punjab State Power corporation limited) is 60(30 for wet waste + 30 for dry waste). The total number of composting pits is 33, out of which 26 are lying filled at different location(near municipal council(6/7), sewa kendra(9/11), singowal road(11/15)) in the city. All the composting plants are managed scientifically and the manure is distributed free of cost to people as per the demand. One MRF is also developed at singowal road, to segregate the recyclable material from collected dry waste which is further sold via local waste collectors. An agreement is also executed on 22/02/2021 between ”THE SHAKTI PLASTIC INDUSTRIES, Mumbai” and MCD, district Gurdaspur for purchase/lift/process and disposal of mixed plastic waste under ”The Punjab Municipal Act, 1911” and ”Punjab Municipal Corporation Act, 1976”.

Chapter 5

Prioritizing the indicators responsible for sustainable municipal solid waste management using SF-AHP and SF-TOPSIS

5.1 Introduction

Unscientific and unplanned management of solid waste is becoming a major reason of environmental problems in the cities of developing countries. Solid waste management(SWM) by municipal committee is one of the major task creating many challenges for the Municipal Corporations, in Indian cities. The rise in urbanization and population also increases the waste generation quantity. As the life style of the population changes, the number of different kinds of discarded material also changes.

Waste management is one of the most important concerns in environmental protection and natural resource conservation. To manage solid waste, a variety of approaches can be used. MSWM is a complicated topic that is influenced by a number of specific aspects relating to the state of the country. For decision makers, choosing the optimal option for handling and managing MSW in terms of environmental quality and economic value is a critical priority. The use of spherical fuzzy environment with Multi Criteria Decision Making(MCDM) in MSWM is still limited. According to the findings of several research, this approach has the ability to handle dataset uncertainty, and decision makers can readily evaluate attributes using linguistic expressions.

Human beings are the worlds largest source of garbage, everyone has the power and obligation to prevent it. Waste management has become a key issue to address on a priority basis as a result of negligence. Waste is treated as a resource rather than trash or garbage all over the world. Research and Development is becoming increasingly popular for resource management. Researchers in all disciplines are continuously focusing on recreating valuable resources from the trash. Nowadays, the 6 R's strategy (Refuse, Reduce, Repair, Reuse, Recover, Recycle) could be adopted in tandem with modern technological inventions. In general developing countries produce more waste than that of under development. The government of every country initiated various projects to alter the different type of waste to resources.

This research will lead to better waste management planning and decision-making. In addition, waste will be managed self-sustainably to some extent through effective application of the recommended methodology.

5.2 Literature review

Intutionistic fuzzy sets (IFS), Neutrosophic fuzzy sets (NFS), and Intutionistic fuzzy sets of second type (IFS2) are the extensions of ordinary fuzzy sets with membership functions in three dimensions, aiming at defining the judgments of decision makers with a detailed description. The authors of [114] introduced spherical fuzzy sets (SFS) in terms of a generalized three dimensions, including some required differences from the already defined fuzzy sets.. [115] introduced index-based PFVIKOR method which includes vague information defined by PF values and solved problems like service quality, R&D project investment, internet stock performance evaluation, and internet stock problems. [116] introduced a approach of PFVIKOR for solving the problems related to site selection for charging stations of electric vehicles. [117] introduced an approach of MCDM whivh is stochastic and solving the problems related to warehouse location in the uncertain environment. In this method, the stochastic AHP technique is used to calculate the weights criteria. The ranking of alternatives was calculated by FVIKOR method. [118] introduced an integrated FVIKOR and ANP (analytical network process) method with the help of IT2FSs, solving problem related to supplier selection in supply chain management. [119] introduced modified and extended FVIKOR model for the construction of an automatic robotic system in the healthcare department or industry. [120] developed a new extension of VIKOR model for DM problems using Pythagorean hesitant fuzzy system. [121] developed a method based on VIKOR and AHP to solve the supply chain management and business analysis by formulating both long-term and short-term flexible decision making strategies for successfully implementing and managing the reverse logistics adoption in the supply chain systems. [122] introduced SFVIKOR to deal with the waste management problems. In [123] an

Table 5.1: Linguistic terms, SFN and score index for constructing PCM

Linguistic terms	Spherical fuzzy number	Score index
Absolutely strong important	$[0.9, 0.1, 0.0]$	9
Very strong important	$[0.8, 0.2, 0.1]$	7
Fairly strong important	$[0.7, 0.3, 0.2]$	5
Slightly strong important	$[0.6, 0.4, 0.3]$	3
equal important	$[0.5, 0.4, 0.4]$	1
Slightly low important	$[0.4, 0.6, 0.3]$	$1/3$
Fairly low important	$[0.3, 0.7, 0.2]$	$1/5$
Very low important	$[0.2, 0.8, 0.1]$	$1/7$
Absolutely low important	$[0.1, 0.9, 0.0]$	$1/9$

Table 5.2: Eleven point spherical fuzzy linguistic term scale.

Linguistic terms	Spherical fuzzy number
Extremely low	$[0.045, 0.955, 0.045]$
Very low	$[0.135, 0.865, 0.135]$
Low	$[0.255, 0.745, 0.255]$
Fair	$[0.335, 0.665, 0.335]$
Medium	$[0.410, 0.590, 0.410]$
Good	$[0.500, 0.500, 0.500]$
Very good	$[0.590, 0.410, 0.410]$
High	$[0.665, 0.335, 0.335]$
Very high	$[0.745, 0.255, 0.255]$
Exceptionally high	$[0.865, 0.135, 0.135]$
Excellent	$[0.955, 0.045, 0.045]$

efficient algorithm is presented by the authors with combination theory and combined fuzzy TOPSIS method to select the best suitable alternative out of all possible hybrid and single energy resources in Turkey. In [124] the authors developed an approach to calculate the light business jet aircraft provide less travel time, long-range, on-board lavatory facility, cozy seating arrangements, other aesthetic ambiance (audio systems, and light systems, and temperature-noise control) and appliances at affordable cost of flight.

Table 5.3: Random indices for consistency ratio [5]

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

5.3 Evaluation of selected methodology

This research is based on the article [125], a novel work is established by integrating TOPSIS and AHP with the spherical fuzzy set. Integrated AHP with SF is used to determine the spherical fuzzy criteria weights, while integrated TOPSIS with SF is used to calculate the final ranking of the alternatives. A new geometric mean formula for SF is developed for determining the spherical fuzzy weights of criteria. Also presented, a new 11-point spherical fuzzy linguistic term scale, which can be used by the decision experts to quantify the preferences. The step by step procedure is as follows:

5.3.1 Stage1: Spherical fuzzy analytical hierarchy process(SF-AHP)-To construct SF weights of criteria

This stage involves major three steps: Firstly, structure decision hierarchy by defining criteria for selected performance indicators through decision makers or experts. Secondly, construct the spherical fuzzy pairwise comparison matrix(SF-PCM) with the help of expert(s) to determine criteria weights on the basis of 9-pt linguistic scale given in table 5.1, using SF-AHP. In case of multiple experts the PCM has to be constructed by each expert and then the combined SF-PCM will be computed using spherical fuzzy geometric mean(SFGM). Let $\widetilde{S}_{GM_i}$ be the SFGM for criteria i is as follows:

$$\begin{aligned}\widetilde{S}_{GM_i} =&(\widetilde{S}_{i1} \times \widetilde{S}_{i2} \times ... \times \widetilde{S}_{in})^{\frac{1}{n}} \\ =& \left[\prod_{j=1}^{n}(\mu_{ij})^{\frac{1}{n}}, \sqrt{1-\prod_{j=1}^{n}(1-\nu_{ij}^2)^{\frac{1}{n}}}, \sqrt{\prod_{j=1}^{n}(1-\nu_{ij}^2)^{\frac{1}{n}} - \prod_{j=1}^{n}(1-\nu_{ij}^2-\pi_{ij}^2)^{\frac{1}{n}}}\right]\end{aligned} \tag{5.1}$$

Let $[\widetilde{P}_{ij}^1]_{n\times n}, [\widetilde{P}_{ij}^2]_{n\times n}, ..., [\widetilde{P}_{ij}^k]_{n\times n}$ be the SF-PCM for different $'k'$ number of experts. So, the combined SF-PCM will be calculated as:

$$[\widetilde{P}_{ij}^C]_{n\times n} = ([\widetilde{P}_{ij}^1]_{n\times n} \otimes [\widetilde{P}_{ij}^2]_{n\times n} \otimes ... \otimes [\widetilde{P}_{ij}^k]_{n\times n})^{\frac{1}{k}}.$$

where $[\widetilde{P}_{ij}^k]_{n\times n} = \begin{bmatrix} \widetilde{S}_{11} & \widetilde{S}_{12} & \dots & \widetilde{S}_{1n} \\ \widetilde{S}_{21} & \widetilde{S}_{22} & \dots & \widetilde{S}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{S}_{n1} & \widetilde{S}_{n2} & \dots & \widetilde{S}_{nn} \end{bmatrix}$ for all finite k and $\widetilde{S}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$ for $i, j = 1, 2, ..., n$.

Last and important step is to check the consistency ratio, $CR = \frac{CI}{RI}$ which should be less than 0.1 and is calculated by taking the corresponding score index number of values substituted in pairwise comparison matrix. Eventually, the second step is completed with the satisfying condition of consistency ratio. The score index is calculated by equation: $SI = (|100 \times ((\mu - \pi)^2 - (\nu - \pi)^2)|)^{\frac{1}{2}}$. Consider the same formula as $\frac{1}{SI}$ to find the inverse of score index corresponding to the spherical fuzzy number. Finalize, weights of criteria using formula (5.1) of SFGM, if the CR is satisfied otherwise reconstruct the SF-PCM.

5.3.2 Stage2: Spherical fuzzy technique for order preference by similarity to ideal solution(SF-TOPSIS)- To select the best indicator/alternative

At this stage, we apply SF-TOPSIS method proposed in the article [114] with 11-pt linguistic term scale of spherical fuzzy shown in table 5.2. At this stage six steps includes: **1.** Initially, construct the spherical fuzzy evaluation matrix using eleven point SF linguistic term scale. **2.** Compute the weighted SF evaluation matrix by multiplying the SF evaluation matrix with the weights of criteria. **3.** Determine the SF positive and SF negative ideal solutions of defuzzified weighted SF evaluation matrix. The formula (5.2) to defuzzify the SFN adopted from the article [126]. **4.** Now, find the Euclidean distance from SF-PIS and SF-NIS for each alternatives using the formula (5.3) and (5.4.) **5.** Calculate relative closeness to the SF positive and negative ideal solutions based on the formula (5.5). **6.** Finally, on the basis of relative closeness values assign the ranking to multiple alternatives such that the best one is ranked with higher value.

$$\sqrt{|100 \times [(3\mu - \frac{\pi}{2})^2 - (\frac{\nu}{2} - \pi)^2]|} \tag{5.2}$$

$$D^+ = \sqrt{\frac{1}{2n}\sum_{j=1}^{n}((\mu_j - \mu_j^+)^2 + (\nu_j - \nu_j^+)^2 + (\pi_j - \pi_j^+)^2)} \tag{5.3}$$

$$D^- = \sqrt{\frac{1}{2n}\sum_{j=1}^{n}((\mu_j - \mu_j^-)^2 + (\nu_j - \nu_j^-)^2 + (\pi_j - \pi_j^-)^2)} \tag{5.4}$$

$$C = \frac{D^-}{\min D^-} - \frac{D^+}{\max D^+} \quad \text{or} \quad C = \frac{D^-}{D^- + D^+} \tag{5.5}$$

5.4 Application to MSWM- A case study of Indian city, Dinanagar

5.4.1 Current situation in study area

The study area selected in this research is Dinanagar, which is a small city located in Punjab, India. The details of this area is shown in table 5.4. Waste management practices in Punjab is more challenging because of various reason such as human resources, financial and political constraints influencing the effectiveness of SWM process in the cities. The Municipal corporation/committee, Dinanagar city of Punjab(India) is facing a lot of hindrances due to insufficient funds for maintaining the services related waste management, non-supportive behavior of urban local bodies, unawareness and lack of interest of inhabitants. It is observed via thorough literature review over Indian cities that the problems or challenges related to MSWM are different in different cities so there is need to monitor the WM practices at local level. Municipal solid waste management(MSWM) is one of the major task creating many challenges [127] for the Municipal corporations, in India [128]. With the rise in population & urbanization the waste generation also increases. As the lifestyle of the population changes, the number of produced garbage kinds also increased. For MSWM, the collection of waste materials from primary, secondary or other source is the major problem. According to municipal corporations and in literature, it is clear that the expenditure over waste management is almost 70-80 percent of municipalities total budget. Instead of trash treatment, the major objective of this research is to concentrate on waste minimization by examining existing adapted strategies. This study will lead to better waste management planning and decision-making.The generators of MSW are broadly categorize into Residential and Non-residential waste. Further, the residential waste includes kitchen waste(left over food items), paper, cardboard, plastic, sanitary waste, inert and Non-residential waste includes bulk quantity of refused fruit, vegetables, packaging cardboard, plastic, construction & demolition waste, Industrial, restaurants & hotels waste etc. In Dinanagar, the average Municipal solid waste generation is about 6 metric tonnes(only of residential area, no record found for non-residential contribution in trash production), and typical physical composition of waste is mainly three types biodegradable, non-biodegradable, and hazardous. No any record is available for chemical composition of waste generated in city.

The Indian government establishes new targets to minimize the quantity of biodegradable waste in landfill or dumping sites. To achieve this target, the composting is primarily solution in small municipality like Dinanagar. Using this technique the waste volume is reduced by 50-65%. Composting can be done either manually or mechanically. Presently, 33 number of manual composting pits at different locations are successfully maintained by Municipal Corporation of Dinanagar(MCD). There is need to improve the collection rate, treatment rate and disposal rate of generated solid waste. The population of Dinanagar is 25376 inhabitants and 15 square km land area which is further distributed in 15 wards with average of 376 inhabitants each. The MSW generated is only about 0.2-0.25 kg per capita per day, of which 60% is wet waste, 40% is dry waste and only 50-60% of generated waste in the city is collected with the utilization of presently provided collection services.

In particular, material recovery facility(MRF) is also adapted by the MCD to treat all metal, paper and plastic from the perspective of cost management. The need of effective and efficient MSWM is increasing as the poor management contributes adverse effects on economy, health, environment and one major threat is increase in Green House Gas(GHG) emissions which further responsible for global warming due to uncollected/untreated waste lying in open dump sites. The inefficient collection and treatment services are the reasons for unsuccessful practices of MCD.

Parameters	**Numbers or %**
Municipal Corporation Area:	14.36 km sq.
Total No. of wards:	15
Total Houses:	5637
Population in each ward:	376(approx.)
Total Population(2011):	23976
Present Population(2021):	25376(approx.)
Literacy rate:	88.66%
Male Literacy:	92.26%
Female Literacy:	84.73%
State avg. Literacy rate:	75.84%

Table 5.4: Details of Study area-Dinanagar
Source: Census of India and MCD

5.4.2 Defining criteria and indicators

Three criteria were defined C1-importance, C2-performance and C3-understandability based on judgments by the experts. Furthermore, a communication considering present condition of MSWM was done with experts of waste management. It led to define different alternatives-performance indicators responsible for sustainable municipal solid waste management. Some performance indicators (PIs) were identified by thorough literature review and some were developed by the experts based on the actual situation of study area. Finally, the potential PIs were selected by the experts as per the limitations like data availability and other inadequacies(shown in table 5.5).

As per the experts in the field of MSWM, the criteria matrix & PSPA evaluation

Table 5.5: Selected performance indicators for sustainable MSWM

Performance indicators (PIs)	
Operational	**Economic**
-waste minimization -six Rs policy -composition of waste generated -waste collection & handling -segregation of waste at source level -composting facility -material recovery facility -landfill & disposal sites	-collection cost per ton of generated waste -transportation cost -sorting cost -cost of operating & maintaining composting facilities -recycling cost -disposal cost of rejected waste -losses due to inefficient MSWM -operators revenue
Waste management staff	**Public service, participation and awareness**
-employees per ton of daily waste -employees per 100 household served -employees for waste collection per 100 household served -employees working at MRF per ton of daily waste generated -staff training & awareness -on duty accidents -qualification of waste management staff -staff incentives	-public satisfaction with waste management -waste generation per capita per day -public participation in current practices -public acceptance for waste management plans & actions -performance of waste collection -incentives for sorting at source level -recycling practices -public awareness about importance of MSWM
Technology & innovation	**Environmental**
-equipment/instrument relevance -equipment/instrument efficiency -degree of innovation -employees reluctance -technological capacity for adaptability	-physicochemical quality parameters of Leachate -air quality parameters of leachate & landfill -application of environment standards at disposal sites -water saving -visual and odor impact

matrix with their respective score indices are shown in appendix, table 8.1, 8.2, 8.3, 8.4.

5.4.3 Results and Discussions

The reason behind using the SF-AHP and SF-TOPSIS methods in prioritizing the indicators responsible for sustainable MSWM is the occurrence of uncertainty at various stages involved in waste management. Initially, spherical fuzzy evaluation matrix of PSPA (Public service participation & awareness) is constructed (shown in table 5.6). Now, the weighted SF PSPA evaluation matrix is constructed(shown in table 5.7) by multiplying the SF-PSPA with weights of criteria, calculated by SF-AHP(refer to table 5.10).

The SFP and SFN ideal solutions of defuzzified values(obtained in table 5.8) are also calculated and shown below the table 5.7. The relative closeness is calculated using the euclidean distances and finally the ranking of indicators PSPA is shown in table 5.9. From this table, it is clear that PU2 ranked 1, which means for effective im-

Table 5.10: SF criteria weights using SFGM

	μ	ν	π
C1	0.643	0.334	0.298
C2	0.509	0.474	0.319
C3	0.268	0.730	0.213

Table 5.6: SF PSPA evaluation matrix

	C1			**C2**			**C3**		
	μ	ν	π	μ	ν	π	μ	ν	π
PU1	0.865	0.135	0.135	0.5	0.5	0.5	0.745	0.255	0.255
PU2	0.745	0.255	0.255	0.955	0.045	0.045	0.865	0.135	0.135
PU3	0.865	0.135	0.135	0.41	0.59	0.41	0.5	0.5	0.5
PU4	0.59	0.41	0.41	0.335	0.665	0.335	0.5	0.5	0.5
PU5	0.745	0.255	0.255	0.865	0.135	0.135	0.665	0.335	0.335
PU6	0.5	0.5	0.5	0.59	0.41	0.41	0.255	0.745	0.255
PU7	0.41	0.59	0.41	0.355	0.665	0.335	0.5	0.5	0.5
PU8	0.745	0.255	0.255	0.5	0.5	0.5	0.59	0.41	0.41
SF wts	**0.643**	**0.334**	**0.298**	**0.509**	**0.474**	**0.319**	**0.268**	**0.73**	**0.213**

Table 5.7: Weighted SF PSPA evaluation matrix

Criteria	C1			C2			C3		
Indicators	μ	ν	π	μ	ν	π	μ	ν	π
PU1	0.556	0.291	0.453	0.255	0.488	0.662	0.200	0.642	0.398
PU2	0.479	0.302	0.551	0.486	0.453	0.369	0.232	0.673	0.327
PU3	0.556	0.291	0.453	0.209	0.541	0.606	0.134	0.646	0.506
PU4	0.379	0.378	0.651	0.171	0.593	0.552	0.134	0.646	0.506
PU5	0.479	0.302	0.551	0.440	0.423	0.450	0.178	0.633	0.438
PU6	0.322	0.441	0.699	0.300	0.445	0.621	0.068	0.738	0.369
PU7	0.264	0.512	0.638	0.181	0.593	0.552	0.134	0.646	0.506
PU8	0.479	0.302	0.551	0.255	0.488	0.662	0.158	0.634	0.471
SFP	**0.556**	**0.291**	**0.453**	**0.486**	**0.453**	**0.369**	**0.232**	**0.673**	**0.327**
SFN	**0.264**	**0.512**	**0.638**	**0.181**	**0.593**	**0.552**	**0.068**	**0.738**	**0.369**

Table 5.8: Defuzzified values of table 5.7

Indicators	C1	C2	C3
PU1	14.087	1.088	3.923
PU2	10.905	12.657	5.320
PU3	14.087	0.892	1.063
PU4	6.690	1.002	1.063
PU5	10.905	10.694	2.915
PU6	3.864	4.361	0.204
PU7	2.770	0.726	1.063
PU8	10.905	1.088	1.828
Maximum	**14.087**	**12.657**	**5.320**
Minimum	**2.770**	**0.726**	**0.204**

Table 5.9: Euclidean distances and rank of indicators

D+	D-	C	Rank
0.157	0.193	2.024	3
0.051	0.215	2.751	1
0.175	0.185	1.840	4
0.213	0.103	0.541	6
0.084	0.197	2.369	2
0.212	0.094	0.427	7
0.244	0.073	0.000	8
0.175	0.161	1.504	5
0.244	**0.073**		

Table 5.11: Consistency check

	C1	C2	C3	Matrix weights	SFN weights	Ratios
C1	1.000	3.195	7.901	8.447	3.433	2.46
C2	0.319	1.000	4.742	3.107	1.087	2.858
C3	0.123	0.200	1.000	0.835	0.195	4.28
					$\lambda_{max} =$	**3.2**

plementation of MSWM the quantity of generated
waste per capita per day has to be as accurate or certain as possible. After this, PU5 i.e., performance of waste collection is ranked at 2, which concludes that proper and in time collection of generated waste from sources led to maximize the expected revenue. The achievement of PU2 & PU5, the indicator PU1 i.e., public satisfaction is ranked with number 3. As per $\lambda_{max} = 3.2$ after consistency check (see table 5.11) the SF criteria

Table 5.12: Ranking of indicators PSPA

Public service, participation and awareness	Short form	Ranking
Public satisfaction with waste management	PU1	3
Waste generation per capita per day	PU2	1
Public participation in current practices	PU3	4
Public acceptance for WM plans & actions	PU4	6
Performance of waste collection	PU5	2
Incentives for sorting at source level	PU6	7
Recycling practices	PU7	8
Public awareness about importance of MSWM	PU8	5

weights are acceptable and the obtained $CI = 0.1$ and $CR = 0.089 < 0.1$.

5.5 Conclusion

The majority of multi-criteria decision-making procedures in the literature use scoring methods, which are quite basic. These strategies contain physical and intangible criteria in real-world challenges, which makes assigning a single numerical value difficult. For cases like this, fuzzy sets can be helpful. Although a wider range of linguistic scales introduces subjectivity into decision-making, current article has applied the developed larger linguistic scale (moving from a five-point to a seven-point linguistic scale) that allows decision-makers to choose from a wider range of linguistic words. The framework is used to solve a problem of sustainable MSWM by prioritizing the performance indicators, and it is discovered that the ranks generated from spherical fuzzy AHP-TOPSIS are the best. This approach can also be used in variety of real world problems involving uncertainty. In future, the same method applied to rest of the defined PIs to find the global ranking.

Chapter 6

Spherical Fuzzy Programming Approach to Optimize the Transportation Problem

6.1 Introduction

The transportation problem is initially introduced by the author of [129]. The goal of [130] study is to handle a symmetric trapezoidal fuzzy number-related fuzzy linear programming issue. It is possible to solve fuzzy LPPs without first turning them into crisp version of LPPs according to several significant and intriguing findings.The authors of [131] provide a model of estimating uniformity and demonstrate the system's capabilities for monitoring and rescheduling. The resulting method can handle situations requiring poorly defined expertise, provide plans that are roughly consistent, and modify the execution of strategies to account for unexpected events. In [132] authors formulate the problem and utilize triangular intuitionistic fuzzy numbers to deal with uncertainty and hesitation. In [133] authors define a transportation problem in which costs are represented as triangular intuitionistic fuzzy numbers. The ideal solutions to the stated problem are determined using two methods in article [134], namely intuitionistic fuzzy programming and goal programming, and the optimal solutions are then contrasted. In study [135], a multi objective nonlinear transportation problem formulated in terms of fuzzy parameters.The method involves neutrosophic compromise programming approach, which is based on neutrosophic decision set has been investigated which contains the concept of indeterminacy mf or degree along with truthiness membership function and falsity membership function of different goals. The fuzzy programming approach of Zimmermann and the concept of neutrosophic sets serve as inspirations for the development of a new compromise method for the multi-objective transportation problem (MO-TP) in this [136] paper. In the paper [137], authors used and applied Interval-valued spherical fuzzy AHP approach to solve the public transportation problem. The obtained results are observed and analyzed & step-by-step explanation of the approach might useful to other applications. The authors of [138] introduced Fuzzy Analytic Hierarchy Process to the model with two extensions which are Intutionistic Fuzzy Sets and Spherical Fuzzy Sets to evaluate the solution set and also provided with a traditional AHP in order to check the robustness of the former methods. In [139] work, authors investigated the solution of the spherical fuzzy transportation problem (SFTP) and presented three different models of the spherical fuzzy transportation problem. In [123] an efficient algorithm is developed by the authors with the help of combination theory and combined fuzzy TOPSIS method to choose the best suitable alternative out of all possible single and hybrid energy resources in Turkey. In [124] the authors developed an approach to calculate the light business jet aircraft provide less travel time, long-range, on-board lavatory facility, cozy seating arrangements, other aesthetic ambiance (audio systems, and light systems, and temperature-noise control) and appliances at affordable cost of flight.

6.2 Preliminaries

6.2.1 Fuzzy set

Definition 6.2.1 *Ordinary Fuzzy set : [7] Let* $\mathbf{U}$ *be the universe of discourse then a fuzzy set* $\widetilde{X}_f$ *in* $\mathbf{U}$ *is defined as follows:*

$$\widetilde{X_f} = \{(x, t_{\widetilde{X_f}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_f}}(x) : \mathbf{U} \longrightarrow [0,1]$ *is the membership function and* $0 \leq t_{\widetilde{X_f}}(x) \leq 1 \; \forall x \in \mathbf{U}$*, represents the membership degree of each* $x \in \mathbf{U}$ *to* $\widetilde{X_f}$*.*

Definition 6.2.2 *Triangular fuzzy number: The ordered triplets* $\widetilde{X_f}(t_1, t_2, t_3)$ *denoting lower value, middle value & upper value of a m.f, is known as triangular fuzzy number if its m.f defined as:*

$$t_{\widetilde{X_f}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & if \; t_1 \leq x \leq t_2 \\ \dfrac{t_3 - x}{t_3 - t_2} & if \; t_2 \leq x \leq t_3 \\ 0 & otherwise \end{cases} \tag{6.1}$$

6.2.2 Defuzzification of triangular fuzzy number

In literature, there are various methods available to defuzzify the fuzzy number [140]. Among all, the centroid method is most widely used as it gives a value which is deterministic based on center of gravity of fuzzy numbers. In this article, the same method is used to obtain the defuzzified version of the triangular fuzzy number, defining as follows:

$$def(\widetilde{X}) = \frac{\int\limits_x x t_{\widetilde{X}}(x)dx}{\int\limits_x t_{\widetilde{X}}(x)dx} \tag{6.2}$$

where x is the output variable and $t_{\widetilde{X}}(x)$ is the m.f.

Hence, by calculating the integrals of (7.2), the defuzzified version of the TFN $\widetilde{X}(t_1, t_2, t_3)$ is:

$$def(\widetilde{X}) = \frac{(\frac{t_3 - t_1}{2})(\frac{3t_1 + t_2 + 3t_3}{3})}{(\frac{t_3 - t_1}{2})} = \frac{3t_1 + t_2 + 3t_3}{3} \tag{6.3}$$

Definition 6.2.3 *Pythagorean fuzzy set: [141] Let* **U** *be the universe of discourse then a fuzzy set* $\widetilde{X_p}$ *in* **U** *is defined as follows:*

$$\widetilde{X_p} = \{(x; t_{\widetilde{X_p}}(x), f_{\widetilde{X_p}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_p}}(x) : \mathbf{U} \longrightarrow [0, 1]$ *and* $f_{\widetilde{X_p}}(x) : \mathbf{U} \longrightarrow [0, 1]$ *are the truthiness m.f and falsity m.f respectively. Also* $0 \leq t^2_{\widetilde{X_p}}(x) + f^2_{\widetilde{X_p}}(x) \leq 1 \ \forall x \in \mathbf{U}$, *represents the membership degree with respect to every element* $x \in \mathbf{U}$ *to* $\widetilde{X_p}$.

Definition 6.2.4 *Spherical fuzzy sets: [6] Let* **U** *be the universal set then a fuzzy set* $\widetilde{X}$ *in* **U** *is defined as follows:*

$$\widetilde{X_s} = \{(x; t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x), f_{\widetilde{X_s}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0, 1]$,$i_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0, 1]$ *and* $f_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0, 1]$ *are the truthiness m.f , indeterminacy m.f and falsity m.f respectively. Also* $0 \leq t^2_{\widetilde{X_s}}(x) + i^2_{\widetilde{X_s}}(x) + f^2_{\widetilde{X_s}}(x) \leq 1 \ \forall x \in \mathbf{U}$, *represents the membership degree with respect to every element* $x \in \mathbf{U}$ *to* $\widetilde{X_s}$.

Definition 6.2.5 *Spherical triangular fuzzy number(STFN) The spherical triangular fuzzy number* $\widetilde{X_s} = (t, i, f) = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ *s.t* $t, i, f \in [0, 1]$

The m.f for t, i, and f can be defined by using (7.1):

$$t_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & if\ t_1 \le x \le t_2 \\ \dfrac{t_3 - x}{t_3 - t_2} & if\ t_2 \le x \le t_3 \\ 0 & otherwise \end{cases} \tag{6.4}$$

$$i_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x - i_1}{i_2 - i_1} & if\ i_1 \le x \le i_2 \\ \dfrac{i_3 - x}{i_3 - i_2} & if\ i_2 \le x \le i_3 \\ 0 & otherwise \end{cases} \tag{6.5}$$

$$f_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x - f_1}{f_2 - f_1} & if\ f_1 \le x \le f_2 \\ \dfrac{f_3 - x}{f_3 - f_2} & if\ f_2 \le x \le f_3 \\ 0 & otherwise \end{cases} \tag{6.6}$$

6.3 Ranking of STFN

In the literature, the [142] authors proposed ranking functions for ordering the SFNs but the procedure is not clear, so the existing ranking functions are not universal and cannot be used for ordering or defuzzify the SFNs. To overcome this situation, we develop a new score function using the centroid method (7.2) and used this to develop an algorithm to optimize the transportation problems.

Definition 6.3.1 *Score function & Accuracy function Let* $\widetilde{X_s} = (t, i, f) = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ *such that* $t, i, f \in [0, 1]$ *be a STFN. Score functions for the m.f* $t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x), and f_{\widetilde{X_s}}(x)$ *are denoted and defined respectively as follows:*

$$Sc(t_{\widetilde{X_s}}) = \frac{3t_1 + t_2 + 3t_3}{3}; Sc(i_{\widetilde{X_s}}) = \frac{3i_1 + i_2 + 3i_3}{3}; Sc(f_{\widetilde{X_s}}) = \frac{3f_1 + f_2 + 3f_3}{3} \tag{6.7}$$

Now, the accuracy function of $\widetilde{X_s}$ *is denoted and defined by:*

$$\begin{aligned} Acc(\widetilde{X_s}) &= \frac{Sc(t_{\widetilde{X_s}}) + Sc(i_{\widetilde{X_s}}) + Sc(f_{\widetilde{X_s}})}{3} \\ &= \frac{(3t_1 + t_2 + 3t_3) + (3i_1 + i_2 + 3i_3) + (3f_1 + f_2 + 3f_3)}{9} \end{aligned} \tag{6.8}$$

Example 6.3.2 *Let* $\widetilde{X_s} = (2.5, 3, 4.5; 2.4, 3, 4.8; 2.3, 3, 5)$ *and* $\widetilde{Y_s} = (4.5, 5, 6.3; 4.3, 5, 6.5; 4, 5, 6.7)$ *be the two STFNs, then their respective accuracy functions using proposed method (7.8) are 2.5000 and 3.9222.*

Theorem 6.3.3 *The score functions for truthiness, indeterminacy, & falsity are linear functions and accuracy function is the average of their score functions. The accuracy function is also a linear function.*

proof: *Let* $\tilde{X} = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ & $\tilde{Y} = (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)$ *are any two STFNs. Then for any scalar* a*, we have*

$$\begin{aligned}
Acc(a\tilde{X} + \tilde{Y}) =& Acc(a(t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3) \\
&+(t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)) \\
=& Acc((at_1, at_2, at_3; ai_1, ai_2, ai_3; af_1, af_2, af_3) \\
&+(t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)) \\
=& Acc(at_1 + t'_1, at_2 + t'_2, at_3 + t'_3; ai_1 + i'_1, ai_2 + i'_2, \\
& ai_3 + i'_3; af_1 + f'_1, af_2 + f'_2, af_3 + f'_3) \\
=& \{(3(at_1 + t'_1) + (at_2 + t'_2) + 3(at_3 + t'_3)) + (3(ai_1 + i'_1) \\
&+(ai_2 + i'_2) + 3(ai_3 + i'_3)) + (3(af_1 + f'_1) + (af_2 + f'_2) \\
&+3(af_3 + f'_3))\}/9 \\
=& a\frac{(3t_1 + t_2 + 3t_3) + (3i_1 + i_2 + 3i_3) + (3f_1 + f_2 + 3f_3)}{9} \\
&+\frac{(3t'_1 + t'_2 + 3t'_3) + (3i'_1 + i'_2 + 3i'_3) + (3f'_1 + f'_2 + 3f'_3)}{9} \\
=& aAcc(\tilde{X}) + Acc(\tilde{Y})
\end{aligned}$$

Hence, Acc() is linear function.

Definition 6.3.4 *Ordering of STFNs using accuracy function*

Let $\tilde{X} = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ & $\tilde{Y} = (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)$ *are any two STFNs. Then,*

1. *If* $Acc(\tilde{X}) \geq Acc(\tilde{Y})$ *then* $\tilde{X} \geq \tilde{Y}$
2. *If* $Acc(\tilde{X}) \leq Acc(\tilde{Y})$ *then* $\tilde{X} \leq \tilde{Y}$
3. *If* $Acc(\tilde{X}) = Acc(\tilde{Y})$ *then* $\tilde{X} = \tilde{Y}$
4. *If* $\tilde{X} \geq \tilde{Y}$ *then* $\max(\tilde{X}, \tilde{Y}) = \tilde{X}$
5. *If* $\tilde{X} \leq \tilde{Y}$ *then* $\min(\tilde{X}, \tilde{Y}) = \tilde{X}$

Definition 6.3.5 *General form of optimization problem The general form of optimization problem(OP) mainly consist of Objective function(s) which has to be optimize(maximize or minimize), constraints related to the problem, and non-negative restrictions in problem which are called as decision variables. The mathematical formulation is as follows:*

$$\begin{aligned} Optimize\ Z =& (Z_1, Z_2, ..., Z_k) \\ subject\ to \quad g_j(x) \leq b_j&, \quad j = 1, 2, 3, ..., m \\ x_i \geq 0&, \quad i = 1, 2, 3, ...n \end{aligned} \tag{6.9}$$

where $k, m,$ and, n are the number of objectives, constraints, and variables respectively.

6.4 Spherical fuzzy optimization problem(SFOP)

The extension of (6.9) by introducing SF concept named as SFOP and can be expressed as:

$$\begin{aligned} Optimize\ \tilde{Z} =& (\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_k) \\ \text{subject to} \quad \tilde{g}_j(x) \leq \tilde{b}_j&, \quad j = 1, 2, 3, ..., m \\ x_i \geq 0&, \quad i = 1, 2, 3, ...n \end{aligned} \tag{6.10}$$

where $k, m, \&, n$ represents the number of objectives, number of constraints, and number of variables respectively. **Model-I:Partial fuzzified OP-** When one or more but not all parameters involved in optimization problem can be considered as fuzzy.
Model-II:Full fuzzified OP- When all parameters involved in optimization problem can be considered as fuzzy.

6.5 Formulation of proposed SFTP

In general TP are concerned with the transporting of goods from different sources to different destinations to obtain the best(optimal) solution of defined goal(s). Consider, there are m sources with a_i where$i = 1, 2, ..., m$ units of items availability and items are to be transported between n destinations with b_j where$j = 1, 2, ..., n$ units of items requirement. The unit transportation cost of transporting items from source i to destination j is c_{ij}. Let x_{ij} as quantity (known as decision variables & $m \times n$ in number) which are to be transported from all the sources to all the destinations in such a way that the total transportation cost is minimum. The mathematical formulation is given as:

$$\begin{aligned}
\mathbf{TP_1}: \quad & MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1,2,...,m \\
& \sum_{i=1}^{m} x_{ij} \leq b_j, \quad j = 1,2,...,n \\
& x_{ij} \geq 0 \quad \forall\, i \,\&\, j
\end{aligned}$$

Further, $\sum_{i=1}^{m} a_i$ and $\sum_{j=1}^{n} b_j$ are the total availability and total demand, which may be either equal(called as balanced) or not(called as unbalanced). If balanced then it is standard TP otherwise nonstandard. We develop a method for solving standard TP so in case of unbalanced, must be converted to standard TP by using additional dummy source or destination.

In reality, some or all the parameters involved in TPs may be fuzzy in nature due to various uncontrollable factors such as fuel rates, traffic jams, inexactness of supply-demand, poor decision making, fluctuation in market prices, environmental conditions, consumer's behavior etc. In such situations the fuzzy parameters are more reliable to achieve the prescribed objective(s). $\mathbf{TP_1}$: a fuzzified version can be as follows:

$$\begin{aligned}
\mathbf{TP_2}: \quad & MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq \tilde{a}_i, \quad i = 1,2,...,m \\
& \sum_{i=1}^{m} x_{ij} \leq \tilde{b}_j, \quad j = 1,2,...,n \\
& x_{ij} \geq 0 \quad \forall\, i \,\&\, j
\end{aligned}$$

Further, $\sum_{i=1}^{m} \tilde{a}_i$ and $\sum_{j=1}^{n} \tilde{b}_j$ are the total availability and total demand which must be as per the standard TP(as discussed above in (TP_1)) The parameters involved in TP_2 are considered as STFNs and defuzzified by using equation 7.8 and TP_2 represented as below:

$$\mathbf{TP_3}: \quad MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n} def(\tilde{c}_{ij})x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \leq def(\tilde{a}_i), \quad i = 1, 2, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \leq def(\tilde{b}_j), \quad j = 1, 2, ..., n$$

$$x_{ij} \geq 0 \quad \forall\, i \,\&\, j$$

where $def(\tilde{c}_{ij}), def(\tilde{a}_i),$ and $def(\tilde{b}_j)$ are the defuzzified versions of $\tilde{c}_{ij}, \tilde{a}_i, \&\tilde{b}_j$ respectively. Further, $\sum_{i=1}^{m} def(\tilde{a}_i)$ and $\sum_{j=1}^{n} def(\tilde{b}_j)$ are the total availability and total demand which must be as per the standard TP(as discussed above in (TP_1))

Transportation table

Since the TP is a special case of general OP, the application of any optimization method would give an optimal solution. But whenever it is possible to represent the OP in the form of TP, it is simpler to express in the form of transportation table, which displays all the values c_{ij}, a_i, b_j associated with the problem

6.6 Algorithm of proposed SFTP

The step-wise procedure from problem formulation to final optimal solution is summarized below:
Step 1: Define the problem with the available data and information collected from the decision maker(s) after a through discussion. Formulate the problem in transportation table form for better visualization of numeric data.
Step 2: Check whether the problem is in standard form of TP or not. Two cases arises:
sub-step 2(a): If TP is balanced go to Step 3.
sub-step 2(b): If not balanced then convert it into balanced TP and go to next step
Step 3: Formulate the TP in mathematical form as in TP_1.
Step 4: Convert the given TP into SFTP by converting the uncertain parameters in terns of STFNs with through discussion.
Step 5: Compute the score function values for all the STFN parameters represent in tabular form.

Step 6: Convert the score function values into defuzzified values using proposed accuracy function & express in tabular form and as TP_2. Again check for standard TP(make it standard, in case not).
Step 7: Formulate the crisp version of SFTP in mathematical form as TP_3
Step 8: Solve both given TP and SFTP using any appropriate optimization technique or mathematical software for comparison of optimal solution.

6.7 Numerical illustration

The following TP is considered in tabular form (shown in table 6.1):

Table 6.1: Considered transportation problem

	M_1	M_2	M_3	M_4	Origin
W_1	2	2	2	1	$a(1) = 3$
W_2	10	8	5	4	$a(2) = 5$
W_3	7	6	6	8	$a(3) = 7$
Destination	$b(1) = 4$	$b(2) = 3$	$b(3) = 4$	$b(4) = 4$	

The STFN for the considered numerical example are presented in the table 6.2

The step-wise procedure & solution of defined transportation cost or route problem is explained as below:

Step 1: Mathematical model of given TP is represented (using the data available in table 6.1) below as TP_1:

$$\begin{aligned}
\mathbf{TP_1}: \quad & MinZ = 2x_{11} + 2x_{12} + 2x_{13} + x_{14} \\
& + 10x_{21} + 8x_{22} + 5x_{23} + 4x_{24} \\
& + 7x_{31} + 6x_{32} + 6x_{233} + 8x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq 3 \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq 5 \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq 7 \\
& x_{11} + x_{12} + x_{31} \leq 4 \\
& x_{12} + x_{22} + x_{32} \leq 3 \\
& x_{13} + x_{23} + x_{33} \leq 4 \\
& x_{14} + x_{24} + x_{34} \leq 4 \\
& x_{ij} \geq 0 \quad \forall\, i = 1,2,3 \,\&\, j = 1,2,3,4
\end{aligned}$$

Table 6.2: STFN for Numerical example

$\tilde{2}$	$W_1 \to M_1$	$(1, 2, 3; 0.5, 2, 2.5; 0.3, 2, 2.8)$
$\tilde{2}$	$W_1 \to M_2$	$(0.8, 2, 1.5; 0.5, 2, 2.5; 0.3, 2, 3)$
$\tilde{2}$	$W_1 \to M_3$	$(1, 2, 2.5; 0.7, 2, 2.8; 0.5, 2, 3)$
$\tilde{1}$	$W_1 \to M_4$	$(0.6, 1, 1.4; 0.5, 1, 1.5; 0.3, 1, 1.7)$
$\tilde{10}$	$W_2 \to M_1$	$(8, 10, 11; 7.5, 10, 11.5; 7, 10, 12)$
$\tilde{8}$	$W_2 \to M_2$	$(7, 8, 9; 6.5, 8, 8.2; 6, 8, 8.5)$
$\tilde{5}$	$W_2 \to M_3$	$(4, 5, 5.5; 3.8, 5, 5.8; 3.5, 5, 6)$
$\tilde{4}$	$W_2 \to M_4$	$(3, 4, 5; 2.7, 4, 5.3; 2.5, 4, 5.5)$
$\tilde{7}$	$W_3 \to M_1$	$(6.5, 7, 8; 6, 7, 8.2; 5.8, 7, 8.5)$
$\tilde{6}$	$W_3 \to M_2$	$(5.5, 6, 7; 5.1, 6, 7.2; 4.8, 6, 7.6)$
$\tilde{6}$	$W_3 \to M_3$	$(5.6, 6, 6.3; 5.4, 6, 6.6; 5, 6, 7)$
$\tilde{8}$	$W_3 \to M_4$	$(7.5, 8, 9; 7, 8, 9.3; 6.8, 8, 9.5)$
$\tilde{3}$	$a(1)$	$(2.5, 3, 4.5; 2.4, 3, 4.8; 2.3, 3, 5)$
$\tilde{5}$	$a(2)$	$(4.5, 5, 6.3; 4.3, 5, 6.5; 4, 5, 6.7)$
$\tilde{7}$	$a(3)$	$(6.8, 7, 7.2; 6.5, 7, 7.5; 6.6, 7, 8.7)$
$\tilde{4}$	$b(1)$	$(3.8, 4, 6; 4.5, 4, 6.2; 5.5, 4, 6.5)$
$\tilde{3}$	$b(2)$	$(2.8, 3, 3.2; 2.7, 3, 4; 2.5, 3, 4.4)$
$\tilde{4}$	$b(3)$	$(2.7, 4, 5; 2.8, 5, 5.3; 2.5, 4, 5.5)$
$\tilde{4}$	$b(4)$	$(3.2, 4, 4.5; 3, 4, 4.8; 2.7, 4, 5)$

Step 2: The formulation of given TP_1 in terms of STFN using table 6.2 as follows:

$$\begin{aligned}
\mathbf{TP_2}: \quad MinZ =\ & (1, 2, 3; 0.5, 2, 2.5; 0.3, 2, 2.8)x_{11} + (0.8, 2, 1.5; 0.5, 2, 2.5; 0.3, 2, 3)x_{12} \\
& + (1, 2, 2.5; 0.7, 2, 2.8; 0.5, 2, 3)x_{13} + (0.6, 1, 1.4; 0.5, 1, 1.5; 0.3, 1, 1.7)x_{14} \\
& + (8, 10, 11; 7.5, 10, 11.5; 7, 10, 12)x_{21} + (7, 8, 9; 6.5, 8, 8.2; 6, 8, 8.5)x_{22} \\
& + (4, 5, 5.5; 3.8, 5, 5.8; 3.5, 5, 6)x_{23} + (3, 4, 5; 2.7, 4, 5.3; 2.5, 4, 5.5)x_{24} \\
& + (6.5, 7, 8; 6, 7, 8.2; 5.8, 7, 8.5)x_{31} + (5.5, 6, 7; 5.1, 6, 7.2; 4.8, 6, 7.6)x_{32} \\
& + (5.6, 6, 6.3; 5.4, 6, 6.6; 5, 6, 7)x_{33} + (7.5, 8, 9; 7, 8, 9.3; 6.8, 8, 9.5)x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \le (2.5, 3, 4.5; 2.4, 3, 4.8; 2.3, 3, 5) \\
& x_{21} + x_{22} + x_{23} + x_{24} \le (4.5, 5, 6.3; 4.3, 5, 6.5; 4, 5, 6.7) \\
& x_{31} + x_{32} + x_{33} + x_{34} \le (6.8, 7, 7.2; 6.5, 7, 7.5; 6.6, 7, 8.7) \\
& x_{11} + x_{12} + x_{31} \le (3.8, 4, 6; 4.5, 4, 6.2; 5.5, 4, 6.5) \\
& x_{12} + x_{22} + x_{32} \le (2.8, 3, 3.2; 2.7, 3, 4; 2.5, 3, 4.4) \\
& x_{13} + x_{23} + x_{33} \le (2.7, 4, 5; 2.8, 5, 5.3; 2.5, 4, 5.5) \\
& x_{14} + x_{24} + x_{34} \le (3.2, 4, 4.5; 3, 4, 4.8; 2.7, 4, 5) \\
& x_{ij} \ge 0 \quad \forall\, i = 1, 2, 3 \;\&\; j = 1, 2, 3, 4
\end{aligned}$$

Table 6.3: Score function values of STFNs

	M_1	M_2	M_3	M_4	$a(i)$
W_1	(4.0000,3.0000,3.1000)	(2.3000,3.0000,3.3000)	(3.5000,3.5000,3.5000)	(1.6667,1.6667,1.6667)	(7.3333,7.5333,7.6333)
W_2	(21.6667,21.6667,21.6667)	(18.0000,16.7000,16.5000)	(10.5000,10.6000,10.5000)	(8.6667,8.6667,8.6667)	(11.8000,11.8000,11.7000)
W_3	(16.1667,15.8667,15.9667)	(13.8333,13.6333,13.7333)	(13.2333,13.3333,13.3333)	(18.5000,18.3000,18.3000)	(15.6667,15.6667,16.9667)
$b(j)$	(10.4667,11.3667,12.6667)	(6.3333,7.0333,7.2333)	(8.3667,8.7667,8.6667)	(8.3667,8.4667,8.3667)	

Table 6.4: Accuracy function values of score function values: Defuzzification of STFNs

	M_1	M_2	M_3	M_4	$a(i)$
W_1	1.1222	0.9556	1.1667	0.5556	2.5000
W_2	7.2222	5.6889	3.5111	2.8889	3.9222
W_3	5.3333	4.5778	4.4333	6.1222	5.3667
$b(j)$	3.8333	2.2889	2.8667	2.8000	

Step 3: For the Defuzzification of STFNs, using score function values table 6.3, the TP is as follows:

$$\begin{aligned}
MinZ = & (4.0000, 3.0000, 3.1000)x_{11} + (2.3000, 3.0000, 3.3000)x_{12} \\
& + (3.5000, 3.5000, 3.5000)x_{13} + (1.6667, 1.6667, 1.6667)x_{14} \\
& + (21.6667, 21.6667, 21.6667)x_{21} + (18.0000, 16.7000, 16.5000)x_{22} \\
& + (10.5000, 10.6000, 10.5000)x_{23} + (8.6667, 8.6667, 8.6667)x_{24} \\
& + (16.1667, 15.8667, 15.9667)x_{31} + (13.8333, 13.6333, 13.7333)x_{32} \\
& + (13.2333, 13.3333, 13.3333)x_{33} + (18.5000, 18.3000, 18.3000)x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq (7.3333, 7.5333, 7.6333) \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq (11.8000, 11.8000, 11.7000) \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq (15.6667, 15.6667, 16.9667) \\
& x_{11} + x_{12} + x_{31} \leq (10.4667, 11.3667, 12.6667) \\
& x_{12} + x_{22} + x_{32} \leq (6.3333, 7.0333, 7.2333) \\
& x_{13} + x_{23} + x_{33} \leq (8.3667, 8.7667, 8.6667) \\
& x_{14} + x_{24} + x_{34} \leq (8.3667, 8.4667, 8.3667) \\
& x_{ij} \geq 0 \quad \forall\, i = 1, 2, 3\; and\; j = 1, 2, 3, 4
\end{aligned}$$

Step 4:After checking the condition of standard TP in accuracy function values table 6.4. The balanced sum of last most column and lower most row of this table is 11.7889.

Step 5: the final representation of given TP_2 in crisp form is as below:

$$\begin{aligned}\textbf{TP}_3: \quad & MinZ = 1.1222x_{11} + 0.9556x_{12} + 1.1667x_{13} + 0.5556x_{14} \\ & \quad + 7.2222x_{21} + 5.6889x_{22} + 3.5111x_{23} + 2.8889x_{24} \\ & \quad + 5.3333x_{31} + 4.5778x_{32} + 4.4333x_{33} + 6.1222x_{34} \\ \text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq 2.5000 \\ & x_{21} + x_{22} + x_{23} + x_{24} \leq 3.9222 \\ & x_{31} + x_{32} + x_{33} + x_{34} \leq 5.3667 \\ & x_{11} + x_{12} + x_{31} \leq 3.8333 \\ & x_{12} + x_{22} + x_{32} \leq 2.2889 \\ & x_{13} + x_{23} + x_{33} \leq 2.8667 \\ & x_{14} + x_{24} + x_{34} \leq 2.8000 \\ & x_{ij} \geq 0 \quad \forall\, i = 1,2,3 \;\&\; j = 1,2,3,4\end{aligned}$$

Step 6: Solve the original TP using an appropriate method or optimizing s/w packages for optimal solution which is shown in table 6.5

Step 7: Similarly, Solve the crisp version of SFTP using same method or optimizing s/w packages for optimal solution which is shown in table 6.5

Table 6.5: Optimal solutions using optimizing method with online Matlab

Solu.	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	Min Z
TP	0	0	3	0	4	1	0	0	0	2	1	4	104
SFTP	0	0	2.5	0	3.8333	0.0889	0	0	0	2.2	0.3667	2.8	59.9464

6.8 Application

The data related to TP is represented in tabular form in table 6.6 collected from Municipal corporation of Dinanagar city, India which takes the responsibility of waste management in the city. There are three waste generation points denoted as W_1, W_2, W_3 (waste considered to be segregated at source level) from where the recoverable material is transfer to three MRF(Material Recovery Facility) stations denoted as M_1, M_2, M_3. Due to several uncertainties like variations in fuel rates, traffic jams, weather etc. the transporter is not sure about the transportation cost involved here. Also due to some unavoidable factors like uncertainty in amount of waste generation, waste characteristics as per the season etc., transportation cost involved is unpredictable. According to the previous experience of Decision maker(s) the transportation costs are estimated

Table 6.6: Application proposed method on transportation problem: Dinanagar city, India

	M_1	M_2	M_3	M_4	Generated waste
W_1	16	20	12	0	$a(1) = 200$
W_2	14	8	18	0	$a(2) = 160$
W_3	26	24	16	0	$a(3) = 90$
MRF capacity	$b(1) = 180$	$b(2) = 120$	$b(3) = 140$	$b(4) = 10$	

as STFNs shown in table 6.8 after a thorough discussion. The MC of Dinanagar city requires optimal solution so the TP cost is minimum. All the given parameters are spherical triangular fuzzy numbers and converted into crisp version by applying the proposed accuracy function. Formulate the given TP in terms of crisp values of STFNs i.e; $TP_3$and solve with the help of suitable method of optimization. The mathematical formulation of SFTP after Defuzzification is as follows:

$$\begin{aligned}
\mathbf{TP_3}: \quad & MinZ = 12.0556x_{11} + 15.3444x_{12} + 7.7778x_{13} - 0.2222x_{14} \\
& \quad + 10.7778x_{21} + 5.6889x_{22} + 13.8000x_{23} - 0.2222x_{24} \\
& \quad + 20.0111x_{31} + 18.4778x_{32} + 12.3000x_{33} - 0.2222x_{34} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} \leq 157.111 \\
& x_{21} + x_{22} + x_{23} + x_{24} \leq 123.0667 \\
& x_{31} + x_{32} + x_{33} + x_{34} \leq 66.6556 \\
& x_{11} + x_{12} + x_{31} \leq 138.4444 \\
& x_{12} + x_{22} + x_{32} \leq 91.1111 \\
& x_{13} + x_{23} + x_{33} \leq 109.5556 \\
& x_{14} + x_{24} + x_{34} \leq 7.7222 \\
& x_{ij} \geq 0 \quad \forall\, i = 1, 2, 3 \,\&\, j = 1, 2, 3, 4
\end{aligned}$$

The optimal solution with optimal transportation cost are obtained by proposed method with the help of online version of Matlab. Thus, the optimal solution and spherical fuzzy transportation cost are: $x_{11} = 66.0000, x_{12} = 91.1111, x_{21} = 5.7888, x_{23} = 109.5556, x_{31} = 66.6556$, rest of the decision variables are zero and $MinZ_{sftp} = 5101.8$.

Solution of TP without applying the proposed method is $x_{11} = 170, x_{12} = 30, x_{21} = 10, x_{23} = 140, x_{32} = 90$, rest of the decision variables are zero and $MinZ_{tp} = 8140$.

Table 6.7: STFNs for the application of TP

	t	i	f
		M_1	
W_1	(15,16,16.5)	(14.8,16,16.7)	(14.5,16,17)
W_2	(13.5,14,15)	(13.3,14,15.2)	(13,14,15)
W_3	(25.4,26,26.5)	(25.2,26,26.8)	(25.2,26,27)
		M_2	
W_1	(19,20,21)	(18.8,20,21.2)	(18.6,20,21.5)
W_2	(7,8,9)	(6.6,8,8.2)	(6,8,8.5)
W_3	(23.8,24,24.4)	(23.5,24,24.6)	(23.2,24,24.8)
		M_3	
W_1	(11,12,11.5)	(10.7,12,12.3)	(10.5,12,3)
W_2	(17.5,18,18.6)	(17.3,18,18.8)	(17,18,19)
W_3	(15.8,16,16.4)	(15.5,16,16.8)	(15.2,16,17)
		M_4	
W_1	0	0	0
W_2	0	0	0
W_3	0	0	0
		$a(i)$	
W_1	(198,200,205)	(195,200,210)	(193,200,215)
W_2	(155,160,162.75)	(152,160,164.35)	(150,160,165.5)
W_3	(75,90,90.5)	(75.7,90,90.7)	(80,90,100)
		$b(j)$	
M_1	(170,180,190)	(167,180,193)	(165,180,183)
M_2	(110,120,125)	(108,120,126)	(105,120,128)
M_3	(138,140,145)	(135,140,148)	(132,140,150)
M_4	(9.5,10,11)	(9.3,10,11.2)	(9,10,11.5)

Clearly, $MinZ_{sftp} < MinZ_{tp}$. Hence, the resulting solution using proposed approach is better than the existing methods.

6.9 Comparative study

The transportation problem involves uncertain parameters has been solved by proposed accuracy function derived with centroid method. The applied approach is based on the spherical fuzzy numbers which comprise with membership functions: truthiness m.f , indeterminacy m.f and falsity m.f. These m.f increases the flexibility for decision making by decision makers. To prove the effectiveness and efficiency of introduced method one illustration is explained step-wise and the same is applied to a real life problem related to TP exists in Dinanagar city India. The obtained optimal solutions are presented in 6.5 and application is discussed in section 8. The values of objective function of given transportation problem and spherical fuzzy transportation problem are 104 and 59.9464 respectively. Similarly, objective function values of real life application are 8140 and 5101.8 corresponding to given TP and SFTP. Hence, it is concluded that by applying the proposed accuracy function and method, the obtained objective values are optimal than the existing methods applied on original data. This confirms the superiority of SFTP.

6.10 Conclusions

In this research article, SFTP is proposed. Accuracy function is derived from the centroid method and is used to defuzzify the parameters defined in terms of fuzzy number. The method is applicable to solve any type of TP having partial or fully fuzzified parameters In future research, we are working on finding the solution of multi objective problem using SF optimization technique. The compromised solution will be obtained by hybridizing the SFOP with Teaching Learning Based Optimization technique.

Chapter 7

Mathematical Modelling Of Municipal Solid Waste Management In Spherical Fuzzy Environment

7.1 Introduction

Humans are frequently confronted with several decision-making challenges, including uncertainty or ambiguity, in their daily lives. The information in decision-making situations is not always easy to explain in terms of crisp numbers, thus fuzzy sets are a better option. The newest extensions of Intuitionistic fuzzy sets are the SF sets, having an independent indeterminate membership degree from the other parameters [6, 143–147]. Most academics are likely to prefer spherical fuzzy sets in the near future since their principles are strong enough to develop further. In case of spherical fuzzy optimization, we aim for the best solution which can be found in the face of inadequate, imprecise, or ambiguous data. Most real-world optimization issues have fuzzy mathematical models [58–60], despite the fact that they are frequently supposed to be crisp for the sake of solving them quickly. In such instances, one normally strives to find a solution that is as approachable as possible and feasible given the decision maker's (DM) requirements. To arrive at such a satisfactory result, fuzzy optimization in an interactive manner is required, with the DM being asked to first describe his or her preferences and expectations. The constraints of pythagorean, intuitionistic, and neutrosophic fuzzy sets can be explained using the illustration as follows: In DM problems with multi-attributes, if the truthiness m.f of an alternative is 0.7, the falsity m.f of the alternatives is 0.6, and the indeterminacy m.f or neutral degree of selecting the alternative is found to be 0.9, the scenario is out of the coverage of the above fuzzy sets. As a result, in case of decision-making the spherical fuzzy set provides a valid tool for decision making in a uncertain environment for such situations. The basic concept of this article is introduced from [2].

The studies [15, 128, 148–151] contribute to evaluating the current status of MSWM in India and suggest some significant improvements for better waste management. For articles on MSWM modeling, see [110] for a review of previously published models on generation of MSW and to suggest implementation guidelines that provide a compromise between economically efficient model development and environmental.The authors of [152, 153] also contribute their work to the mathematical modelling of MSWM. We investigate the LPP in a spherical fuzzy environment in this paper. The deterministic version is calculated using SFS theory after the parameters are converted to SF numbers. Many SF optimization models are proposed to find the best SFLPP solution. A numerical example of a real-life problem, MSWM is solved to demonstrate the applicability and validity of the SF optimization models. Based on the offered work and conclusions, the future scopes are also considered. In [123], an efficient algorithm is developed by the authors with the help of combination theory and the combined fuzzy TOPSIS method to choose the best suitable alternative out of all possible single and hybrid energy resources in Turkey. The latest articles based on MSWM in Indian cities [154–160] discussed about challenges, current status, sustainable waste management, life-cycle assessment and so on but the concept of mathematical modelling is not introduced by any author. Moreover, some authors worked on waste management issues at global level like [161–168]. In article [169] the authors worked on Forecasting of MSW generation using non-linear auto regressive neural models. The authors of article [170] introduced LandGEM mathematical model for energy production potential and quantification of landfill gas emissions in Tirupati MSW disposal. In paper [171] the authors used multi-criteria decision making under fuzzy environment for the evaluation of municipal solid waste management scenarios. In article [172] the authors write about detailed literature review on application of stochastic, deterministic, and fuzzy lpp models in SWM studies. In paper [173] the authors introduced method based on spherical fuzzy to optimize the transportation problem. One more article [174] based on spherical fuzzy is prepared to prioritize the indicators responsible for sustainable municipal solid waste management using SF-AHP and SF-TOPSIS after working and writing an article [175] over the implementation analysis of municipal solid waste management in study area.

7.2 Preliminaries

7.2.1 Fuzzy set

Definition: Ordinary Fuzzy set : [7] Let **U** be the universe of discourse then a fuzzy set $\widetilde{X_f}$ in **U** is defined below:

$$\widetilde{X_f} = \{(x, t_{\widetilde{X_f}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_f}}(x) : \mathbf{U} \longrightarrow [0,1]$ is the membership function and $0 \le t_{\widetilde{X_f}}(x) \le 1 \; \forall x \in \mathbf{U}$, represents the membership degree of each $x \in \mathbf{U}$ to $\widetilde{X_f}$.

Definition: Triangular fuzzy number: The ordered triplets $\widetilde{X_f}(t_1, t_2, t_3)$ denotes lower value, middle value & upper value of a m.f, is said to be triangular fuzzy number if its m.f is defined as:

$$t_{\widetilde{X_f}}(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & if\ t_1 \le x \le t_2 \\ \dfrac{t_3 - x}{t_3 - t_2} & if\ t_2 \le x \le t_3 \\ 0 & otherwise \end{cases} \tag{7.1}$$

Defuzzification of triangular fuzzy number

In literature, there are various methods available to defuzzify the fuzzy number [140]. Among all, the centroid method is most widely used as it gives a value which is deterministic based on center of gravity of fuzzy numbers. In this article, the same method is used to obtain the defuzzified version of the triangular fuzzy number which is defined as follows:

$$def(\widetilde{X}) = \frac{\int_x x t_{\widetilde{X}}(x)dx}{\int_x t_{\widetilde{X}}(x)dx} \tag{7.2}$$

where x is the output variable and $t_{\widetilde{X}}(x)$ is the m.f.

Hence, by calculating the integrals of (7.2), the defuzzified version of the TFN $\widetilde{X}(t_1, t_2, t_3)$ is:

$$def(\widetilde{X}) = \frac{(\frac{t_3 - t_1}{2})(\frac{3t_1 + t_2 + 3t_3}{3})}{(\frac{t_3 - t_1}{2})} = \frac{3t_1 + t_2 + 3t_3}{3} \tag{7.3}$$

Definition: Spherical fuzzy sets: [6] Let **U** be the universal set then a fuzzy set $\widetilde{X}$

in $\mathbf{U}$ is written and defined as:

$$\widetilde{X_s} = \{(x; t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x), f_{\widetilde{X_s}}(x)) \,|\, x \in \mathbf{U}\}$$

such that $t_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$,$i_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$ and $f_{\widetilde{X_s}}(x) : \mathbf{U} \longrightarrow [0,1]$ are the truthiness m.f , indeterminacy m.f and falsity m.f respectively. Also $0 \leq t^2_{\widetilde{X_s}}(x) + i^2_{\widetilde{X_s}}(x) + f^2_{\widetilde{X_s}}(x) \leq 1 \ \forall x \in \mathbf{U}$, represents the membership degree for every element $x \in \mathbf{U}$ to $\widetilde{X_s}$.

Definition: Spherical triangular fuzzy number(STFN) The spherical triangular fuzzy number $\widetilde{X_s} = (t,i,f) = (t_1,t_2,t_3; i_1,i_2,i_3; f_1,f_2,f_3)$ s.t $t,i,f \in [0,1]$

The m.f for t, i, and f can be defined by using (7.1):

$$t_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x-t_1}{t_2-t_1} & if\ t_1 \leq x \leq t_2 \\ \dfrac{t_3-x}{t_3-t_2} & if\ t_2 \leq x \leq t_3 \\ 0 & otherwise \end{cases} \tag{7.4}$$

$$i_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x-i_1}{i_2-i_1} & if\ i_1 \leq x \leq i_2 \\ \dfrac{i_3-x}{i_3-i_2} & if\ i_2 \leq x \leq i_3 \\ 0 & otherwise \end{cases} \tag{7.5}$$

$$f_{\widetilde{X_s}}(x) = \begin{cases} \dfrac{x-f_1}{f_2-f_1} & if\ f_1 \leq x \leq f_2 \\ \dfrac{f_3-x}{f_3-f_2} & if\ f_2 \leq x \leq f_3 \\ 0 & otherwise \end{cases} \tag{7.6}$$

7.3 Ranking of STFN

In the literature, the [142] authors proposed ranking functions for ordering the SFNs but the procedure is not clear, so the existing ranking functions are not universal and cannot be used for ordering or defuzzify the SFNs. To overcome this situation, we develop a new score function using the centroid method (7.2) and used this to develop an algorithm to optimize the transportation problems.

Definition: Score function & Accuracy function Let $\widetilde{X_s} = (t,i,f) = (t_1,t_2,t_3; i_1,i_2,i_3; f_1,f_2,f_3)$ such that $t,i,f \in [0,1]$ be a STFN. Score functions for the m.f $t_{\widetilde{X_s}}(x), i_{\widetilde{X_s}}(x)$, and$f_{\widetilde{X_s}}(x)$

are denoted and defined respectively as follows:

$$Sc(t_{\widetilde{X_s}}) = \frac{3t_1 + t_2 + 3t_3}{3}; Sc(i_{\widetilde{X_s}}) = \frac{3i_1 + i_2 + 3i_3}{3}; Sc(f_{\widetilde{X_s}}) = \frac{3f_1 + f_2 + 3f_3}{3} \tag{7.7}$$

Now, the accuracy function of $\widetilde{X_s}$ is denoted and defined by:

$$\begin{aligned} Acc(\widetilde{X_s}) =& \frac{Sc(t_{\widetilde{X_s}}) + Sc(i_{\widetilde{X_s}}) + Sc(f_{\widetilde{X_s}})}{3} \\ =& \frac{(3t_1 + t_2 + 3t_3) + (3i_1 + i_2 + 3i_3) + (3f_1 + f_2 + 3f_3)}{9} \end{aligned} \tag{7.8}$$

Example: Let $\widetilde{X_s} = (2.5, 3, 4.5; 2.4, 3, 4.8; 2.3, 3, 5)$ and $\widetilde{Y_s} = (4.5, 5, 6.3; 4.3, 5, 6.5; 4, 5, 6.7)$ be the two STFNs, then their respective accuracy functions using proposed method (7.8) are 2.5000 and 3.922

Theorem: Let the score functions for truthiness, indeterminacy, & falsity are linear functions and accuracy function is the average of their score functions then the accuracy function is also a linear function.

proof: Let $\tilde{X} = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ & $\tilde{Y} = (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)$ are any two STFNs. Then for any scalar a, we have

$$\begin{aligned} Acc(a\tilde{X} + \tilde{Y}) =& Acc(a(t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3) \\ &+ (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)) \\ =& Acc((at_1, at_2, at_3; ai_1, ai_2, ai_3; af_1, af_2, af_3) \\ &+ (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)) \\ =& Acc(at_1 + t'_1, at_2 + t'_2, at_3 + t'_3; ai_1 + i'_1, ai_2 + i'_2, \\ & ai_3 + i'_3; af_1 + f'_1, af_2 + f'_2, af_3 + f'_3) \\ =& \{(3(at_1 + t'_1) + (at_2 + t'_2) + 3(at_3 + t'_3)) + (3(ai_1 + i'_1) \\ &+ (ai_2 + i'_2) + 3(ai_3 + i'_3)) + (3(af_1 + f'_1) + (af_2 + f'_2) \\ &+ 3(af_3 + f'_3))\}/9 \\ =& a\frac{(3t_1 + t_2 + 3t_3) + (3i_1 + i_2 + 3i_3) + (3f_1 + f_2 + 3f_3)}{9} \\ &+ \frac{(3t'_1 + t'_2 + 3t'_3) + (3i'_1 + i'_2 + 3i'_3) + (3f'_1 + f'_2 + 3f'_3)}{9} \\ =& aAcc(\tilde{X}) + Acc(\tilde{Y}) \end{aligned}$$

Hence, Acc() is linear function.

Definition: Ordering of STFNs using accuracy function

Let $\tilde{X} = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3)$ & $\tilde{Y} = (t'_1, t'_2, t'_3; i'_1, i'_2, i'_3; f'_1, f'_2, f'_3)$ are any two STFNs. Then,

1. If $Acc(\tilde{X}) \geq Acc(\tilde{Y})$ then $\tilde{X} \geq \tilde{Y}$
2. If $Acc(\tilde{X}) \leq Acc(\tilde{Y})$ then $\tilde{X} \leq \tilde{Y}$
3. If $Acc(\tilde{X}) = Acc(\tilde{Y})$ then $\tilde{X} = \tilde{Y}$
4. If $\tilde{X} \geq \tilde{Y}$ then $\max(\tilde{X}, \tilde{Y}) = \tilde{X}$
5. If $\tilde{X} \leq \tilde{Y}$ then $\min(\tilde{X}, \tilde{Y}) = \tilde{X}$

7.4 Methodology

The linear model, sometimes known as the linear programming problem (LPP), is the most popular, most straightforward, and frequently used mathematical programming model. The LPP model is straightforward and may be used to a variety of real-world problems, including those involving transportation, supply chain management, job assignment, manufacturing and production planning, supplier selection, and other issues. Over several decades, traditional LPP has changed and expanded. Uncertainty is included in the LPP, which is often used by academics. A typical and frequently used mathematical programming issue is the linear programming problem. Numerous academicians have researched the numerous fuzzy environment extensions of the linear programming issue, including ordinary fuzzy, intuitionistic, Pythagorean, neutrosophic, and others.

7.4.1 Spherical fuzzy linear programming problem [2]

The introduction of a spherical fuzzy idea known as the spherical fuzzy linear programming problem (SFLPP) is presented as a further expansion of LPP.
The extension of LPP by introducing SF concept named as SFLPP and can be expressed as:
Model-I: This model presents the SFLPP in which coefficients of objective functions are represented in spherical fuzzy number(SFN) but right hand side constants of constraints and the coefficients of variables are represented in real numbers.

$$Optimize\ Z = \sum_{k=1}^{K} \widetilde{c}_k x_k$$

Subject to

$$\sum_{k=1}^{K} a_{ik}x_k \leq, =, \geq b_i, \quad \forall i = 1, 2, ..., I$$

$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

where $\widetilde{c}_k$ denotes a SFN and a_{ik}, b_i are real numbers.

Model-II: This model presents the SFLPP in which coefficients of objective functions are represented by real numbers but the coefficients of variables and right hand side constants of constraints are represented in SF numbers.

$$Optimize\ Z = \sum_{k=1}^{K} c_k x_k$$

Subject to

$$\sum_{k=1}^{K} \widetilde{a}_{ik}x_k \leq, =, \geq \widetilde{b}_i, \quad \forall i = 1, 2, ..., I$$

$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

where c_k is a real number and $\widetilde{a}_{ik}$ and $\widetilde{b}_i$ are SF numbers.

Model-III: This model presents the SFLPP in which coefficients of objective functions, the coefficients of variables and right hand side constants of constraints all are represented in SF numbers.

$$Optimize\ Z = \sum_{k=1}^{K} \widetilde{c}_k x_k$$

Subject to

$$\sum_{k=1}^{K} \widetilde{a}_{ik}x_k \leq, =, \geq \widetilde{b}_i, \quad \forall i = 1, 2, ..., I$$

$$x_k \geq 0, \quad \forall k = 1, 2, ..., K$$

where $\widetilde{c}_k$ denotes a SFN and $\widetilde{a}_{ik}, \widetilde{b}_i$ are also SF numbers.

The crisp version of different spherical fuzzy parameters are obtained by using Eqs. 7.4,7.5,7.6 for every membership degree decided by the decision makers. The obtained crisp form can be solved by using any suitable optimization method.

7.4.2 Algorithm

- **Step-I** Define the problem and examine the parameters which contains uncertainty.
- **Step-II** Formulate the mathematical model in terms of spherical fuzzy environment.
- **Step-III** Formulate the truthiness, indeterminacy and falsity membership functions under spherical fuzzy environment.
- **Step-IV** Decide the confirmation degree according to the experience of decision maker for the truthiness, indeterminacy and falsity memberships of spherical fuzzy numbers.
- **Step-V** Convert the defined SFLPP into the crisp version using equations 7.4,7.5,7.6.
- **Step-VI** Compute the defined optimization model by applying robust optimization method or suitable technique to get the desired solution of SFLPP.
- **Step-VII** Application of novel method to solve mathematically modeled Municipal Solid Waste Management(MSWM) problem.

7.4.3 Assessment of MSWM system in study area [3]

Waste management in Punjab, India: Data is taken from cpcb.nic.in In the state of Punjab, there are 167 ULBs(urban local bodies) in total that are in charge of managing MSW. There are 56 Nagar Panchayats, 13 Municipal Corporations, 26 Class-I, 47 Class-II, and 25 Class-III cities/towns in the State. Punjab produces a total of about 4338.37 TPD(tons per day) of solid waste, of which 4278.86 TPD is collected, 1894.04 TPD is processed, and 2384.82 TPD is landfilled. In 142 ULBs, house-to-house collection is used; in 113 ULBs, segregation is used; in 98 ULBs, storage facilities are available; and in 143 ULBs, covered conveyance is used. In the State, there are 1572 operational composting facilities, 1 vermicomposting facility (at Shamchaurassi), and 2 RDF/palletization facilities (at Bathinda and Ludhiana). For the management of solid waste, the Department of Local Government (DLG) has chosen a decentralised strategy. Until the end of 2020, the State will have 1572 processing sites (composting pits) put up for the processing of wet waste. Only inert garbage will be dumped in landfills, and recyclable waste is being channelled through 235 Material Recovery Facilities (MRFs). Although installed, the two Waste to Energy plants (in Bathinda and Ludhiana) are not yet operational. The ULBs in the State have set up a total of 143 solid waste dumping sites. Due to a number of variables, such as human resource, financial, and political constraints, which hinder the performance of SWM processes in cities, waste management techniques in Punjab are more challenging to implement.

Waste management in Dinanagar: Insufficient money for maintaining waste management services, non-supportive conduct of urban local bodies, unawareness, and lack of enthusiasm among residents are all obstacles that the Municipal Corporation/committee of Dinanagar(MCD) in Punjab, India is facing. The Indian government establishes new targets to minimize the quantity of biodegradable waste in landfill or dumping sites. To achieve this target, the composting is primarily solution in small municipality like Dinanagar. Using this technique the waste volume is reduced by 50-65%. Composting can be done either manually or mechanically. Presently, 33 number of manual composting pits at different locations are successfully maintained by Municipal Corporation of Dinanagar(MCD). There is need to improve the collection rate, treatment rate and disposal rate of generated solid waste. The population of Dinanagar is 25376 (table 7.1) inhabitants and 15 square km land area which is further distributed in 15 wards with average of 376 inhabitants each. The MSW generated is only about 0.2-0.25 kg per capita per day, of which 60% is wet waste, 40% is dry waste and only 50-60% of generated waste in the city is collected with the utilization of presently provided collection services. Material recovery facility(MRF) is also adapted by the MCD to treat all recoverable material from the perspective of cost management. The need of effective and efficient MSWM is increasing as the poor management contributes adverse effects on economy, health, environment and one major threat is increase in emission of Green House Gases which further accountable for the global warming due to uncollected/untreated waste lying in open dump sites. The inefficient collection and treatment services are the reasons for gap existence in present practices of MCD. The technique of five R's is responsible for sustainable and zero waste objective of solid waste management. The definition of 5 R's are: REFUSE, REDUCE, REUSE, RECYCLE, RECOVER.

- **Refuse:** Say no to non-biodegradable material or products.
- **Reduce:** Replace the non-biodegradable with biodegradable material.
- **Reuse:** Do not use disposable products. Replace them with more sustainable alternatives.
- **Recycle:** Use the material which can be transform into another usable form.

Ward No.	Total Houses*	Population	No. of houses paying for WM*	Waste generated (in kg/day)
1.	497	2236	120	457
2.	466	2097	125	429
3.	523	2354	150	481
4.	425	1912	130	391
5.	388	1746	165	357
6.	438	1971	125	403
7.	393	1768	50	362
8.	415	1868	120	382
9.	256	1152	100	236
10.	276	1242	75	254
11.	428	1926	80	394
12.	264	1188	50	243
13.	283	1274	70	260
14.	231	1040	90	213
15.	354	1593	50	326
Total	**5637**	**25376**	**1500**	**5188**

Table 7.1: Estimated ward wise population and waste generation
***Source:** Municipal corporation of Dinanagar

- **Recover:** Convert the organic waste into compost.

By adopting these 5R's policy in daily routine life, the challenges of waste management can be achieved effectively and efficiently.

7.5 Mathematical formulation of MSWM: Multi Objective Problem

7.5.1 Indices notation:

$i-$ Generators of solid waste, $i = 1$ to I
$j-$ Segregation stations(SS), $j = 1$ to J
$k-$ Material recovery stations(MRS), $k = 1$ to K
$l-$ Composting stations(CS), $l = 1$ to L
$m-$ Incineration stations(IS), $m = 1$ to M
$n-$ Anaerobic digestion stations(ADS), $n = 1$ to N
$o-$ Landfill sites(LF) , $o = 1$ to O

7.5.2 Decision variables & Binary variables:

x_{ij}^{seg}– Amount of waste transferred from waste generators to segregation stations per day
x_{jk}^{mrf}– Amount of waste transferred from segregation stations to MRS per day
x_{jl}^{comp}– Amount of waste transferred from segregation stations to composting stations per day
x_{jm}^{inc}– Amount of waste transferred from segregation stations to incineration stations per day
x_{jn}^{anb}– Amount of waste transferred from segregation stations to ADS per day
x_{jo}^{lf}– Amount of waste transferred from segregation stations to landfill sites per day
$x_j^{seg}, x_k^{mrf}, x_l^{comp}, x_m^{inc}, x_n^{anb}, x_o^{lf}$ are the **binary variables** , which takes value 1 if SS, MRS, CS, IS, ADS, LF facilities are provided in the area under study and 0 otherwise.

7.5.3 Input values/parameters:

Fixed costs such as maintenance etc at various stations of waste management are denoted as follows:

c_j^{seg}– Fixed cost related to segregation stations per unit weight
c_k^{mrf}– Fixed cost related to MRS per unit weight
c_l^{comp}– Fixed cost related to composting stations per unit weight
c_m^{inc}– Fixed cost related to incineration stations per unit weight
c_n^{anb}– Fixed cost related to ADS per unit weight
c_o^{lf}– Fixed cost related to landfill sites per unit weight

Capacity of various waste management stations to process the waste are denoted as follows:

p_j^{seg}– Waste processing capacity at segregation stations per day
p_k^{mrf}– Waste processing capacity at MRS per day
p_l^{comp}– Waste processing capacity at composting stations per day
p_m^{inc}– Waste processing capacity at incineration stations per day
p_n^{anb}– Waste processing capacity at ADS per day
p_o^{lf}– Waste processing capacity at landfill sites per day

Processing costs of waste at different stations of waste management are denoted as follows:

$\tilde{pc}_j^{seg}$– Waste processing cost at segregation stations per unit weight
$\tilde{pc}_k^{mrf}$– Waste processing cost at MRS per unit weight
$\tilde{pc}_l^{comp}$– Waste processing cost at composting stations per unit weight
$\tilde{pc}_m^{inc}$– Waste processing cost at incineration stations per unit weight
$\tilde{pc}_n^{anb}$– Waste processing cost at ADS per unit weight
$\tilde{pc}_o^{lf}$– Waste processing cost at landfill sites per unit weight

Transportation costs of waste from one station to another station are denoted as follows:

$\tilde{tc}_{ij}^{seg}$– Transportation cost of waste transferred from waste generators to segregation stations per day
$\tilde{tc}_{jk}^{mrf}$– Transportation cost of waste transferred from segregation stations to MRS per day
$\tilde{tc}_{jl}^{comp}$– Transportation cost of waste transferred from segregation stations to composting stations per unit weight
$\tilde{tc}_{jm}^{inc}$– Transportation cost of waste transferred from segregation stations to incineration stations per unit weight
$\tilde{tc}_{jn}^{anb}$– Transportation cost of waste transferred from segregation stations to ADS per unit weight
$\tilde{tc}_{jo}^{lf}$– Transportation cost of waste transferred from segregation stations to landfill sites per unit weight

Revenue produce from different stations of waste management are denoted as follows:

$\tilde{R}_k^{mrf}$– Revenue generated from MRS per unit weight
$\tilde{R}_l^{comp}$– Revenue generated from composting stations per unit weight
$\tilde{R}_m^{inc}$– Revenue generated from incineration stations per unit weight
$\tilde{R}_n^{anb}$– Revenue generated from ADS per unit weight
$\tilde{R}_o^{lf}$– Revenue generated from landfill sites per unit weight

Fractions of waste transferred from segregation station to different stations are denoted as below:

$\alpha^{mrf}-$ Fraction of recoverable waste transfer to MR stations.
$\alpha^{comp}-$ Fraction of compostable waste transfer to composting stations.
$\alpha^{inc}-$ Fraction of dry waste transfer to incineration stations.
$\alpha^{anb}-$ Fraction of waste transfer to anaerobic digestion stations.
$\alpha^{lf} = 1-(\alpha^{mrf}+\alpha^{comp}+\alpha^{inc}+\alpha^{anb})$- Fraction of untreated waste transfer to the landfill.

Emission coeff. for GHG effect

$coeff_j^{seg}-$ GHG emission coeff. from SS per unit weight per day
$coeff_k^{mrf}-$ GHG emission coeff. from MRS per unit weight per day
$coeff_l^{comp}-$ GHG emission coeff. from composting stations per unit weight per day
$coeff_m^{inc}-$ GHG emission coeff. from incineration stations per unit weight per day
$coeff_n^{anb}-$ GHG emission coeff. from ADS per unit weigh per dayt
$coeff_o^{lf}-$ GHG emission coeff. from landfill sites per unit weight per day
$\widetilde{\mathbf{W_i}}-$ Total generated waste at source i per day
Note: Some parameters denoted with (˜) tilde sign is to be considered as fuzzy in nature.

7.6 Defining Objective functions

The following three objective functions are defined to optimize:

- **OP1:**Minimization of total cost which includes transportation cost, maintenance cost, processing cost or other costs related to different stations of solid waste management system.
- **OP2:**Minimization of GHG emissions like carbon and methane from different waste management stations.
- **OP3:**Minimization of final waste disposal at landfill sites to approach the aim of zero waste.

7.6.1 Mathematical expressions for the defined objectives is as follows:

$$Min\ OP1 = Fixed\ cost + Processing\ cost + Transportation\ cost - Revenue$$

where

$$Fixed\ cost(FC) = \sum_{j=1}^{J} c_j^{seg} x_j^{seg} + \sum_{k=1}^{K} c_k^{mrf} x_j^{mrf} + \sum_{l=1}^{L} c_l^{comp} x_l^{comp} + \sum_{m=1}^{M} c_m^{inc} x_m^{inc} + \sum_{n=1}^{N} c_n^{anb} x_n^{anb} + \sum_{o=1}^{O} c_o^{lf} x_o^{lf}$$

$$Processing\ cost(PC) = pc_j^{seg} \sum_{j=1}^{J} \sum_{i=1}^{I} x_{ij}^{seg} + pc_k^{mrf} \sum_{k=1}^{K} \sum_{j=1}^{J} x_{ij}^{mrf} + pc_l^{comp} \sum_{l=1}^{L} \sum_{j=1}^{J} x_{ij}^{comp} + pc_m^{inc} \sum_{m=1}^{M} \sum_{j=1}^{J} x_{ij}^{inc} + pc_n^{anb} \sum_{n=1}^{N} \sum_{j=1}^{J} x_{ij}^{anb} + pc_o^{lf} \sum_{o=1}^{O} \sum_{j=1}^{J} x_{ij}^{lf}$$

$$Transportation\ cost(TC) = \sum_{i=1}^{I} \sum_{j=1}^{J} tc_{ij}^{seg} x_{ij}^{seg} + \sum_{j=1}^{J} \sum_{k=1}^{K} tc_{jk}^{mrf} x_{jk}^{mrf} + \sum_{j=1}^{J} \sum_{l=1}^{L} tc_{jl}^{comp} x_{jl}^{comp} + \sum_{j=1}^{J} \sum_{m=1}^{M} tc_{jm}^{inc} x_{jm}^{inc} + \sum_{j=1}^{J} \sum_{n=1}^{N} tc_{jn}^{anb} x_{jn}^{anb} + \sum_{j=1}^{J} \sum_{o=1}^{O} tc_{jo}^{lf} x_{jo}^{lf}$$

$$Revenue(R) = R_k^{mrf} \sum_{j=1}^{J} \sum_{k=1}^{K} x_{jk}^{mrf} + R_l^{comp} \sum_{j=1}^{J} \sum_{l=1}^{L} x_{jl}^{comp} + R_m^{inc} \sum_{j=1}^{J} \sum_{m=1}^{M} x_{jm}^{inc} + R_n^{anb} \sum_{j=1}^{J} \sum_{n=1}^{N} x_{jn}^{anb} + R_o^{lf} \sum_{j=1}^{J} \sum_{o=1}^{O} x_{jo}^{lf}$$

$$Min\ OP2 = coeff_j^{seg} \sum_{j=1}^{J} \sum_{i=1}^{I} x_{ij}^{seg} + coeff_k^{mrf} \sum_{k=1}^{K} \sum_{j=1}^{J} x_{jk}^{mrf} + coeff_l^{comp} \sum_{l=1}^{L} \sum_{j=1}^{J} x_{jl}^{comp} + coeff_m^{inc} \sum_{m=1}^{M} \sum_{j=1}^{J} x_{jm}^{inc} + coeff_n^{anb} \sum_{n=1}^{N} \sum_{j=1}^{J} x_{jn}^{anb} + coeff_o^{lf} \sum_{o=1}^{O} \sum_{j=1}^{J} x_{jo}^{lf}$$

$$Min\ OP3 = \sum_{j=1}^{J}\sum_{o=1}^{O} x_{jo}^{lf}$$

$$subject\,to\,the\,constraints:$$

$$\sum_{j=1}^{J} x_{ij}^{seg} = W_i, \quad i = 1 \text{ to } I$$

$$\sum_{k=1}^{K}\sum_{j=1}^{J} x_{jk}^{mrf} = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha^{mrf} x_{ij}^{seg}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} x_{jl}^{comp} = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha^{comp} x_{ij}^{seg}$$

$$\sum_{m=1}^{M}\sum_{j=1}^{J} x_{jm}^{inc} = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha^{inc} x_{ij}^{seg}$$

$$\sum_{n=1}^{N}\sum_{j=1}^{J} x_{jn}^{anb} = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha^{anb} x_{ij}^{seg}$$

$$\sum_{o=1}^{O}\sum_{j=1}^{J} x_{jo}^{lf} = \sum_{i=1}^{I}\sum_{j=1}^{J} \alpha^{lf} x_{ij}^{seg}$$

Note: $\alpha^{lf} = 1 - (\alpha^{mrf} + \alpha^{comp} + \alpha^{inc} + \alpha^{anb})$

$$\sum_{i=0}^{I} x_{ij}^{seg} \leq p_j x_j^{seg}, \quad j = 1\ to\ J$$

$$\sum_{j=0}^{J} x_{jk}^{mrf} \leq p_k x_k^{mrf}, \quad k = 1\ to\ K$$

$$\sum_{j=0}^{J} x_{jl}^{comp} \leq p_l x_l^{comp}, \quad l = 1\ to\ L$$

$$\sum_{j=0}^{J} x_{jm}^{inc} \leq p_m x_m^{inc}, \quad m = 1\ to\ M$$

$$\sum_{j=0}^{J} x_{jn}^{anb} \leq p_n x_n^{anb}, \quad n = 1\ to\ N$$

$$\sum_{j=0}^{J} x_{jo}^{lf} \leq p_o x_o^{lf}, \quad o = 1\ to\ O$$

$$where \quad x_{ij}^{seg} \geq 0, \; x_{jk}^{mrf} \geq 0, \; x_{jl}^{comp} \geq 0, \; x_{jm}^{inc} \geq 0, \; x_{jn}^{anb} \geq 0, \; x_{jo}^{lf} \geq 0.$$
$$and \quad x_{j}^{seg}, \; x_{k}^{mrf}, \; x_{l}^{comp}, \; x_{m}^{inc}, \; x_{n}^{anb}, \; x_{o}^{lf} \quad are \;\; 0 \;\; or \;\; 1.$$

7.7 Numerical Illustration

The following considerations are according to the present situation of Dinanagar, study area. After a detailed discussion with the municipal manager and other workers of municipal corporation of Dinanagar(MCD), the collected data is summarized in the tables 7.3,7.4,7.5 according to which the various parameters like different costs, GHG emission coefficient, and waste generation at sources and so on are decided. Total budget for MSWM is 50 lac per year(Source: MCD) and 10 % of total budget is reserved by MCD The objectives of this study are defined and shown in table 7.2.

Table 7.2: Objectives defined for optimization

Objectives	functions	Target	Current
Minimize OP1	Total cost	₹20000(with revenue)	4 lac per month
Minimize OP2	GHG emissions	Reduce to some extent	Not available
Minimize OP3	landfill disposal	500 kg per day	more than 2.5 ton per day

Consider the following:

$i-$ Generators of solid waste, $i = 1$ to $I, Take \; I = 3$

$j-$ Segregation stations(SS), $j = 1$ to $J, Take \; J = 1$

$k-$ Material recovery stations(MRS), $k = 1$ to $K, Take \; K = 3$

$l-$ Composting stations(CS), $l = 1$ to $L, Take \; L = 3$

$m-$ Incineration stations(IS), $m = 1$ to $M, Take \; M = 0$

$n-$ Anaerobic digestion stations(ADS), $n = 1$ to $N, Take \; N = 1$

$o-$ Landfill sites(LF) , $o = 1$ to $O, Take \; O = 1$

Table 7.3: Parametric values

Parameters	per capita/day
Cost of WM	₹7 - 8
GHG emissions	$3.16 Kg \; CO_2 - eq$
Waste generation	200 gm

Table 7.4: Amount of waste treated at different stations

Stations	% of waste	waste/day(in tons)
Compost	50 -60	2.5 - 3
MRF	20 -30	1 - 1.8
ANB	15 - 20	0.75 - 1.2
Landfill	5 - 6	0.25 - 0.3

Table 7.5: Cost (in ₹1000s) related to different stations for MSWM in Dinanagar

Stations	Fixed cost	Transportation	Processing
Segregation	18	27	27
MRF	40	60	60
Compost	20	30	30
ANB	10	15	15
LF	2	3	3

Table 7.6: Green House Gas emission values

Stations	GHG*(per ton)	waste treated(in tons)*	waste (in Kg)	GHG (per Kg)
MRF	4	60	1800	0.12
Compost	25700	40	2000	1285
ANB	46200	340	1000	135.68
Landfill	12900	600	200	4.3

Note: * [176] to estimate GHG (in $Kg\ CO_2 - eq$)

7.7.1 Mathematical formulation of MSWM under consideration is as follows:

$$
\begin{aligned}
Min\ \ OP1 =& FC + PC + TC - R \\
& \text{where} \\
FC =& c_1^{seg} x_1^{seg} + c_1^{mrf} x_1^{mrf} + c_2^{mrf} x_2^{mrf} + c_3^{mrf} x_3^{mrf} + c_1^{comp} x_1^{comp} \\
& + c_2^{comp} x_2^{comp} + c_3^{comp} x_3^{comp} + c_1^{anb} x_1^{anb} + c_1^{lf} x_1^{lf} \\
PC =& pc_1^{seg}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + pc_1^{mrf} x_{11}^{mrf} + pc_2^{mrf} x_{12}^{mrf} \\
& + pc_3^{mrf} x_{13}^{mrf} + pc_1^{comp} x_{11}^{comp} + pc_2^{comp} x_{12}^{comp} + pc_3^{comp} x_{13}^{comp} \\
& + pc_1^{anb} x_{11}^{anb} + pc_1^{lf} x_{11}^{lf} \\
TC =& tc_{11}^{seg} x_{11}^{seg} + tc_{21}^{seg} x_{21}^{seg} + tc_{31}^{seg} x_{31}^{seg} + tc_{11}^{mrf} x_{11}^{mrf} + tc_{12}^{mrf} x_{12}^{mrf} \\
& + tc_{13}^{mrf} x_{13}^{mrf} + tc_{11}^{comp} x_{11}^{comp} + tc_{12}^{comp} x_{12}^{comp} + \\
& tc_{13}^{comp} x_{13}^{comp} + tc_{11}^{anb} x_{11}^{anb} + tc_{11}^{lf} x_{11}^{lf} \\
R =& R_1^{mrf} x_{11}^{mrf} + R_2^{mrf} x_{12}^{mrf} + R_3^{mrf} x_{13}^{mrf} + R_1^{comp} x_{11}^{comp} \\
& + R_2^{comp} x_{12}^{comp} + R_3^{comp} x_{13}^{comp} + R_1^{anb} x_{11}^{anb} + R_1^{lf} x_{11}^{lf} \\
Min\ \ OP2 =& coeff_1^{seg}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + coeff_1^{mrf} x_{11}^{mrf} \\
& + coeff_2^{mrf} x_{12}^{mrf} + coeff_3^{mrf} x_{13}^{mrf} + coeff_1^{comp} x_{11}^{comp} \\
& + coeff_2^{comp} x_{12}^{comp} + coeff_3^{comp} x_{13}^{comp} + coeff_1^{anb} x_{11}^{anb} \\
& + coeff_1^{lf} x_{11}^{lf} \\
Min\ \ OP3 =& x_{11}
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
x_{11}^{seg} =& W_1;\ x_{21}^{seg} = W_2;\ x_{31}^{seg} = W_3 \\
x_{11}^{mrf} + x_{12}^{mrf} + x_{13}^{mrf} =& \alpha_{mrf}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{comp} + x_{12}^{comp} + x_{13}^{comp} =& \alpha_{comp}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{anb} =& \alpha_{anb}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{lf} =& \alpha_{lf}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg} \leq& p_1^{seg} x_1^{seg} \\
x_{11}^{mrf} \leq& p_1^{mrf} x_1^{mrf};\ x_{12}^{mrf} \leq p_2^{mrf} x_2^{mrf};\ x_{13}^{mrf} \leq p_3^{mrf} x_3^{mrf} \\
x_{11}^{comp} \leq& p_1^{comp} x_1^{comp};\ x_{12}^{comp} \leq p_2^{comp} x_2^{comp};\ x_{13}^{comp} \leq p_3^{comp} x_3^{comp} \\
x_{11}^{anb} \leq& p_1^{anb} x_1^{anb};\ x_{11}^{lf} \leq p_1^{lf} x_1^{lf}
\end{aligned}
$$

Table 7.7: Score function and Accuracy function values of fuzzy parameters

IV	FV	$t = FV * 0.9$	$i = FV * 0.4$	$f = FV * 0.2$	$t_1 = t - 0.2 * t$	$t_2 = t$	$t_3 = t + 0.1 * t$	$i_1 = i - 0.9 * i$	$i_2 = i$
pc_1^{seg}	0.17400	0.15660	0.06960	0.03480	0.12528	0.15660	0.17226	0.00696	0.06960
pc_1^{mrf}	1.21000	1.08900	0.48400	0.24200	0.87120	1.08900	1.19790	0.04840	0.48400
pc_2^{mrf}	0.96700	0.87030	0.38680	0.19340	0.69624	0.87030	0.95733	0.03868	0.38680
pc_3^{mrf}	1.80750	1.62675	0.72300	0.36150	1.30140	1.62675	1.78943	0.07230	0.72300
pc_1^{comp}	0.55200	0.49680	0.22080	0.11040	0.39744	0.49680	0.54648	0.02208	0.22080
pc_2^{comp}	0.46900	0.42210	0.18760	0.09380	0.33768	0.42210	0.46431	0.01876	0.18760
pc_3^{comp}	0.42900	0.38610	0.17160	0.08580	0.30888	0.38610	0.42471	0.01716	0.17160
pc_1^{anb}	0.48300	0.43470	0.19320	0.09660	0.34776	0.43470	0.47817	0.01932	0.19320
pc_1^{lf}	0.48500	0.43650	0.19400	0.09700	0.34920	0.43650	0.48015	0.01940	0.19400
tc_{11}^{seg}	0.15220	0.13698	0.06088	0.03044	0.10958	0.13698	0.15068	0.00609	0.06088
tc_{21}^{seg}	0.15760	0.14184	0.06304	0.03152	0.11347	0.14184	0.15602	0.00630	0.06304
tc_{31}^{seg}	0.20190	0.18171	0.08076	0.04038	0.14537	0.18171	0.19988	0.00808	0.08076
tc_{11}^{mrf}	0.96750	0.87075	0.38700	0.19350	0.69660	0.87075	0.95783	0.03870	0.38700
tc_{12}^{mrf}	0.96670	0.87003	0.38668	0.19334	0.69602	0.87003	0.95703	0.03867	0.38668
tc_{13}^{mrf}	1.20880	1.08792	0.48352	0.24176	0.87034	1.08792	1.19671	0.04835	0.48352
tc_{11}^{comp}	0.46140	0.41526	0.18456	0.09228	0.33221	0.41526	0.45679	0.01846	0.18456
tc_{12}^{mrf}	0.58540	0.52686	0.23416	0.11708	0.42149	0.52686	0.57955	0.02342	0.23416
tc_{13}^{mrf}	0.43060	0.38754	0.17224	0.08612	0.31003	0.38754	0.42629	0.01722	0.17224
tc_{11}^{anb}	0.48300	0.43470	0.19320	0.09660	0.34776	0.43470	0.47817	0.01932	0.19320
tc_{11}^{lf}	0.48500	0.43650	0.19400	0.09700	0.34920	0.43650	0.48015	0.01940	0.19400
R_1^{mrf}	7.50000	6.75000	3.00000	1.50000	5.40000	6.75000	7.42500	0.30000	3.00000
R_2^{mrf}	6.67000	6.00300	2.66800	1.33400	4.80240	6.00300	6.60330	0.26680	2.66800
R_3^{mrf}	7.50000	6.75000	3.00000	1.50000	5.40000	6.75000	7.42500	0.30000	3.00000
R_1^{anb}	7.00000	6.30000	2.80000	1.40000	5.04000	6.30000	6.93000	0.28000	2.80000
R_1^{lf}	5.00	4.50	2.00	1.00	3.60	4.50	4.95	0.20	2.00
W_1	2115.00	1903.50	846.00	423.00	1522.80	1903.50	2093.850	84.60	846.00
W_2	1637.00	1473.30	654.80	327.40	1178.64	1473.30	1620.63	65.48	654.80
W_3	1436.00	1292.4	574.4	287.2	1033.92	1292.4	1421.64	57.44	574.4

Note:IV-Input value, FV-Fuzzy value

continue on next page........

Table 7.8: Score function and Accuracy function values of fuzzy parameters(continuation of table7.7)

IV	FV	$i_3 = i + 0.1 * i$	$f_1 = f - 0.9 * f$	$f_2 = f$	$f_3 = f + 0.1 * f$	Sc(t)	Sc(i)	Sc(f)	Acc(FV)
pc_1^{seg}	0.07656	0.17400	0.00348	0.03480	0.03828	0.34974	0.10672	0.05336	0.16994
pc_1^{mrf}	0.53240	1.21000	0.02420	0.24200	0.26620	2.43210	0.74213	0.37107	1.18177
pc_2^{mrf}	0.42548	0.96700	0.01934	0.19340	0.21274	1.94367	0.59309	0.29655	0.94444
pc_3^{mrf}	0.79530	1.80750	0.03615	0.36150	0.39765	3.63308	1.10860	0.55430	1.76533
pc_1^{comp}	0.24288	0.55200	0.01104	0.11040	0.12144	1.10952	0.33856	0.16928	0.53912
pc_2^{comp}	0.20636	0.46900	0.00938	0.09380	0.10318	0.94269	0.28765	0.14383	0.45806
pc_3^{comp}	0.18876	0.42900	0.00858	0.08580	0.09438	0.86229	0.26312	0.13156	0.41899
pc_1^{anb}	0.21252	0.48300	0.00966	0.09660	0.10626	0.97083	0.29624	0.14812	0.47173
pc_1^{lf}	0.21340	0.48500	0.00970	0.09700	0.10670	0.97485	0.29747	0.14873	0.47368
tc_{11}^{seg}	0.06697	0.15220	0.00304	0.03044	0.03348	0.30592	0.09335	0.04667	0.14865
tc_{21}^{seg}	0.06934	0.15760	0.00315	0.03152	0.03467	0.31678	0.09666	0.04833	0.15392
tc_{31}^{seg}	0.08884	0.20190	0.00404	0.04038	0.04442	0.40582	0.12383	0.06192	0.19719
tc_{11}^{mrf}	0.42570	0.96750	0.01935	0.19350	0.21285	1.94468	0.59340	0.29670	0.94493
tc_{12}^{mrf}	0.42535	0.96670	0.01933	0.19334	0.21267	1.94307	0.59291	0.29645	0.94414
tc_{13}^{mrf}	0.53187	1.20880	0.02418	0.24176	0.26594	2.42969	0.74140	0.37070	1.18059
tc_{11}^{comp}	0.20302	0.46140	0.00923	0.09228	0.10151	0.92741	0.28299	0.14150	0.45063
tc_{12}^{comp}	0.25758	0.58540	0.01171	0.11708	0.12879	1.17665	0.35905	0.17952	0.57174
tc_{13}^{comp}	0.18946	0.43060	0.00861	0.08612	0.09473	0.86551	0.26410	0.13205	0.42055
tc_{11}^{anb}	0.21252	0.48300	0.00966	0.09660	0.10626	0.97083	0.29624	0.14812	0.47173
tc_{11}^{lf}	0.21340	0.48500	0.00970	0.09700	0.10670	0.97485	0.29747	0.14873	0.47368
R_1^{mrf}	3.30000	7.50000	0.15000	1.50000	1.65000	15.07500	4.60000	2.30000	7.32500
R_2^{mrf}	2.93480	6.67000	0.13340	1.33400	1.46740	13.40670	4.09093	2.04547	6.51437
R_3^{mrf}	3.30000	7.50000	0.15000	1.50000	1.65000	15.07500	4.60000	2.30000	7.32500
R_1^{anb}	3.08000	7.00000	0.14000	1.40000	1.54000	14.07000	4.29333	2.14667	6.83667
R_1^{lf}	2.20	5.00000	0.10000	1.00000	1.10000	10.05000	3.06667	1.53333	4.88333
W_1	930.60	2115.0000	42.30000	423.00000	465.30000	4251.15000	1297.20000	648.60000	2065.65000
W_2	720.28	1637.0000	32.74000	327.40000	360.14000	3290.37000	1004.02667	502.01333	1598.80333
W_3	631.84	1436.0000	28.72000	287.20000	315.92000	2886.36000	880.74667	440.37333	1402.49333

7.7.2 Mathematical model formulation using parametric values is as given below:

$$
\begin{aligned}
Min\ \ OP1 =& 0.166x_1^{seg} + 1.21x_1^{mrf} + 0.805x_2^{mrf} + 0.404x_3^{mrf} + 0.46x_1^{comp} \\
&+ 0.88x_2^{comp} + 0.644x_3^{comp} + 0.323x_1^{anb} + 0.325x_1^{lf} \\
&+ 0.174(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + 1.21x_{11}^{mrf} + 0.967x_{12}^{mrf} \\
&+ 1.0875x_{13}^{mrf} + 0.552x_{11}^{comp} + 0.469x_{12}^{comp} + 0.429x_{13}^{comp} \\
&+ 0.483x_{11}^{anb} + 0.485x_{11}^{lf} + 0.1522x_{11}^{seg} + 0.1576x_{21}^{seg} \\
&+ 0.2019x_{31}^{seg} + 0.9675x_{11}^{mrf} + 0.9667x_{12}^{mrf} + 1.2088x_{13}^{mrf} \\
&+ 0.464x_{11}^{comp} + 0.5854x_{12}^{comp} + 0.4306x_{13}^{comp} + 0.483x_{11}^{anb} \\
&+ 0.485x_{11}^{lf} - (7.5x_{11}^{mrf} + 6.67x_{12}^{mrf} + 7.5x_{13}^{mrf} + 0x_{11}^{comp} \\
&+ 0x_{12}^{comp} + 0x_{13}^{comp} + 7x_{11}^{anb} + 5x_{11}^{lf}) \\
Min\ \ OP2 =& 0(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + 0.0267x_{11}^{mrf} + 0.04x_{12}^{mrf} + 0.0533x_{13}^{mrf} \\
&+ 449.75x_{11}^{comp} + 353.375x_{12}^{comp} + 481.875x_{13}^{comp} + 135.88x_{11}^{anb} \\
&+ 4.3x_{11}^{lf} \\
Min\ \ OP3 =& x_{11}^{lf}
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
x_{11}^{seg} =& 2115;\ x_{21}^{seg} = 1637;\ x_{31}^{seg} = 1436 \\
x_{11}^{mrf} + x_{12}^{mrf} + x_{13}^{mrf} =& \frac{9}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{comp} + x_{12}^{comp} + x_{13}^{comp} =& \frac{1}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{anb} =& \frac{2}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{lf} =& \frac{1}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg} \leq& 5000x_1^{seg} \\
x_{11}^{mrf} \leq& 400x_1^{mrf};\ x_{12}^{mrf} \leq 600x_2^{mrf};\ x_{13}^{mrf} \leq 800x_3^{mrf} \\
x_{11}^{comp} \leq& 700x_1^{comp};\ x_{12}^{comp} \leq 550x_2^{comp};\ x_{13}^{comp} \leq 750x_3^{comp} \\
x_{11}^{anb} \leq& 1000x_1^{anb};\ x_{11}^{lf} \leq 200x_1^{lf}
\end{aligned}
$$

7.7.3 Mathematical formulation: defuzzified parametric values is:

The defuzzified values of parameters having fuzzy nature are shown in table 7.7 and 7.8. The confirmation degree on the basis of the previous knowledge of municipal manager and others is decided as (0.9, 0.4, 0.2) for truthiness, indeterminacy, & falsity memberships of spherical fuzzy numbers respectively.

$$\begin{aligned}
Min\ \ OP1 =& 0.166x_1^{seg} + 1.21x_1^{mrf} + 0.805x_2^{mrf} + 0.404x_3^{mrf} \\
&+ 0.46x_1^{comp} + 0.88x_2^{comp} + 0.644x_3^{comp} + 0.323x_1^{anb} \\
&+ 0.325x_1^{lf} + 0.16994(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
&+ 1.18177x_{11}^{mrf} + 0.94444x_{12}^{mrf} + 1.7653x_{13}^{mrf} \\
&+ 0.53912x_{11}^{comp} + 0.45806x_{12}^{comp} + 0.41899x_{13}^{comp} \\
&+ 0.47173x_{11}^{anb} + 0.47368x_{11}^{lf} + 0.14865x_{11}^{seg} \\
&+ 0.15392x_{21}^{seg} + 0.19719x_{31}^{seg} + 0.94493x_{11}^{mrf} \\
&+ 0.94414x_{12}^{mrf} + 1.18059x_{13}^{mrf} + 0.45063x_{11}^{comp} \\
&+ 0.57174x_{12}^{comp} + 0.42055x_{13}^{comp} + 0.47173x_{11}^{anb} \\
&+ 0.47368x_{11}^{lf} - (7.32500x_{11}^{mrf} + 6.51437x_{12}^{mrf} \\
&+ 7.32500x_{13}^{mrf} + 0x_{11}^{comp} + 0x_{12}^{comp} + 0x_{13}^{comp} \\
&+ 6.83667x_{11}^{anb} + 4.88333x_{11}^{lf}) \\
Min\ \ OP2 =& 0(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) + 0.0267x_{11}^{mrf} + 0.04x_{12}^{mrf} \\
&+ 0.0533x_{13}^{mrf} + 449.75x_{11}^{comp} + 353.375x_{12}^{comp} \\
&+ 481.875x_{13}^{comp} + 135.88x_{11}^{anb} + 4.3x_{11}^{lf} \\
Min\ \ OP3 =& x_{11}^{lf}
\end{aligned}$$

subject to the constraints

$$\begin{aligned}
x_{11}^{seg} =& 2065.65;\ x_{21}^{seg} = 1598.80333;\ x_{31}^{seg} = 1402.49333 \\
x_{11}^{mrf} + x_{12}^{mrf} + x_{13}^{mrf} =& \frac{9}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{comp} + x_{12}^{comp} + x_{13}^{comp} =& \frac{1}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{anb} =& \frac{2}{5}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{lf} =& \frac{1}{25}(x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg}) \\
x_{11}^{seg} + x_{21}^{seg} + x_{31}^{seg} \leq& 5000x_1^{seg} \\
x_{11}^{mrf} \leq& 400x_1^{mrf};\ x_{12}^{mrf} \leq 600x_2^{mrf};\ x_{13}^{mrf} \leq 800x_3^{mrf} \\
x_{11}^{comp} \leq& 700x_1^{comp};\ x_{12}^{comp} \leq 550x_2^{comp};\ x_{13}^{comp} \leq 750x_3^{comp} \\
x_{11}^{anb} \leq& 1000x_1^{anb};\ x_{11}^{lf} \leq 200x_1^{lf}
\end{aligned}$$

7.8 Results & Discussion

The optimal values of defined objectives are shown in table 7.9. As per records provided by data operator of MCD, the problem is defined. It is worth mentioning that some of the data is to be assumed due to unavailability of data and information.

Table 7.9: Objective functions value after optimization

Objectives(units)	functions	without fuzzy	with fuzzy	fuzzy+TLBO
Min OP1(in ₹)	Total cost	15382	14555	12538
Min OP2*	GHG emissions	1142000	1115300	1002600
Min OP3(in Kg/day)	landfill disposal	207.5200	202.6779	180.3245

Note: *(in $Kg\ CO_2 - eq$)

The study determines the present status of solid waste management in the study region, Dinanagar, Punjab, India. Finally, constructed and solved the mathematical model of SWM in the study area. The findings of comparing the suggested model to the current framework show that the new model provides better solutions in terms of sustainability.

7.9 Conclusion

In this study, a SFLPP model with TLBO was developed for supporting the municipal solid waste management under fuzzy environment. Spherical fuzzy set's ability to capture imprecise and contradictory information results in a substantial contribution to decision-making issues. Thus, in this paper we presented SFLPP which means LPP in the environment of spherical fuzzy, which entails maximum of truthiness and minimum of indeterminacy, and falsity degree or membership function. In present era TLBO is gaining the popularity of being less complex and only two algorithmic parameters based algorithm. Due to this it become flexible to inculcate with other optimization techniques in the form of hybridization or modification, so that standard TLBO can be enhanced to perform well with fast convergence towards the local/global optima as compare to other optimization algorithms. In addition, an illustration of MSWM is provided to represent the efficacy of the suggested SFLPP solution technique. The obtained results reflects that the proposed model has the capability to handle uncertainties involved at various stages of waste management. Comparison with current practices in study area demonstrated the advances of general solutions in the aspects of minimization of cost, GHG emission and landfill disposal. Some amendments and parameters estimation in this model could further increase the applicability to many other problems having fuzziness. The SFLPP can also used to solve other real-world problems, involving parameters contains uncertainty. The application of SFLPP to real-world problems, like inventory control, transportation, supply chain management, supplier selection, and portfolio management, is also an open door for academics.

Chapter 8

Conclusion & Future scope

Humans are frequently confronted with several decision-making challenges, including uncertainty or ambiguity, in their daily lives. The information in decision-making situations is not always easy to explain in terms of crisp numbers, thus fuzzy sets are a better option.

Initially, the explicit goal of study is to assess and analyzing the implementation of several SWM programs in Dinanagar city, discovered that improper record about quantification, collection, treatment and disposal of wastes, political commitment, personnel facilities, and public and commercial sector commitment all had a substantial impact on the success of solid waste management implementation. The study also revealed some administrative issues that hampered implementation in the early stages, such as a lack of resources, unscientific land for final waste disposal, a lack of an effective and efficient solid waste management plan, public awareness, staff training, and poor enforcement of laws and regulations.For successful and effective management of waste in Dinanagar city of Punjab, providing monetary incentives for recyclable items, developing a strong policy,adequate resources, political leadership support, encouraging workers, incorporating technological innovations, composting, encouraging waste reduction, and recycling techniques have been suggested and recommended. Other areas which are linked and have a direct effect on the city's SWM but are understudied were observed. The included topics; revenue from local resources, commingled waste analysis and waste management finance in the city and adequate logistics, modern technologies for SWM to be used in the city.Future research will focus on urban growth and related concerns in order to gain a better knowledge of difficulties and find solutions for long-term development and enhancing environmental sanitation in Dinanagar.

After assessment and implementation analysis of waste management at study area, our aim is to prioritize the factors responsible for sustainable municipal solid waste management. The majority of multi-criteria decision-making procedures in the literature use scoring methods, which are quite basic. These strategies contain physical and intangible criteria in real-world challenges, which makes assigning a single numerical value difficult. For cases like this, fuzzy sets can be helpful. Although a wider range of linguistic scales introduces subjectivity into decision-making, current article has applied the developed larger linguistic scale (moving from a five-point to a seven-point linguistic scale) that allows decision-makers to choose from a wider range of linguistic words. The framework is used to solve a problem of sustainable MSWM by prioritizing the performance indicators, and it is discovered that the ranks generated from spherical fuzzy AHP-TOPSIS are the best. This approach can also be used in variety of real world problems involving uncertainty. In future, the same method applied to rest of the defined PIs to find the global ranking.

During this work, SFTP is also proposed. Accuracy function is derived from the centroid method and is used to defuzzify the parameters defined in terms of fuzzy number. The method is applicable to solve any type of TP having partial or fully fuzzified parameters In future research, we are working on finding the solution of multi objective problem using SF optimization technique. The compromised solution will be obtained by hybridizing the SFOP with Teaching Learning Based Optimization technique.

Finally in this study, a SFLPP model with TLBO was proposed for supporting the fuzzy environment exists in municipal solid waste management. Spherical fuzzy set's ability to capture imprecise and contradictory information results in a substantial contribution to decision-making issues. Thus, in this work we developed SFLPP in a SF environment, which entails max of truthiness and min of indeterminacy and falsity degree or membership functions. In present era TLBO is gaining the popularity of being less complex and only two algorithmic parameters based algorithm. Due to this it become flexible to inculcate with other optimization techniques in the form of hybridization or modification, so that standard TLBO can be enhanced to perform well with fast convergence towards the local/global optima as compare to other optimization algorithms. Additionally, a illustration of MSWM is provided to represent the effectiveness and efficacy of the suggested SFLPP model. The obtained results reflects that the proposed model has the capability to handle uncertainties involved at various stages of waste management. Comparison with current practices in study area demonstrated the advances of general solutions in the aspects of minimization of cost, GHG emission and landfill disposal.

Some amendments and parameters estimation in this model could further increase the applicability to many other problems having fuzziness. The SFLPP can also used to solve other real-world problems, involving parameters contains uncertainty. The application of SFLPP to real-world problems, like supply chain management, transportation problem, supplier selection, portfolio management and inventory control is also an open door for academics.

Bibliography

[1] CPHEEO Ministry of Urban Development. Municipal solid waste management manual(draft). In *GoI MSWM*, pages 100–1. German International Cooperation, 2014.

[2] Cengiz Kahraman and Fatma Kutlu Gündoğdu. *Decision making with spherical fuzzy sets: theory and applications*, volume 392. Springer Nature, 2020.

[3] Manbir Kaur and Rakesh Yadav. Assessment of municipal solid waste management in dinanagar city of punjab india. *PIMT Journal of Research*, 14(1), 2021.

[4] Swati Pattnaik and M Vikram Reddy. Assessment of municipal solid waste management in puducherry (pondicherry), india. *Resources, Conservation and Recycling*, 54(8):512–20, 2010.

[5] Thomas L Saaty and Luis G Vargas. Hierarchical analysis of behavior in competition: Prediction in chess. *Behavioral science*, 25(3):180–191, 1980.

[6] Cengiz Kahraman and Fatma Kutlu Gundougdu. *Decision Making with Spherical Fuzzy Sets*, volume 392. Springer, 2021.

[7] Richard E Bellman and Lotfi Asker Zadeh. Decision-making in a fuzzy environment. *Management science*, 17(4):B–141, 1970.

[8] H-J Zimmermann. Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems*, 1(1):45–55, 1978.

[9] Plamen P Angelov. Optimization in an intuitionistic fuzzy environment. *Fuzzy sets and Systems*, 86(3):299–306, 1997.

[10] Firoz Ahmad and Ahmad Yusuf Adhami. Neutrosophic programming approach to multiobjective nonlinear transportation problem with fuzzy parameters. *International journal of management science and engineering management*, 14(3):218–29, 2019.

[11] Firoz Ahmad, Ahmad Yusuf Adhami, and Florentin Smarandache. Neutrosophic optimization model and computational algorithm for optimal shale gas water management under uncertainty. *Symmetry*, 11(4):544, 2019.

[12] Firoz Ahmad, Ahmad Yusuf Adhami, and Florentin Smarandache. Modified neutrosophic fuzzy optimization model for optimal closed-loop supply chain management under uncertainty. In *Optimization theory based on neutrosophic and plithogenic sets*, pages 343–403. Elsevier, 2020.

[13] Fatma Kutlu Gündoğdu, Cengiz Kahraman, and Ali Karaşan. Spherical fuzzy vikor method and its application to waste management. In *International Conference on Intelligent and Fuzzy Systems*, pages 997–1005. Springer, 2019.

[14] Muhammad Rafiq, Shahzaib Ashraf, Saleem Abdullah, Tahir Mahmood, and Shakoor Muhammad. The cosine similarity measures of spherical fuzzy sets and their applications in decision making. *Journal of Intelligent & Fuzzy Systems*, 36(6):6059–73, 2019.

[15] Mufeed Sharholy, Kafeel Ahmad, Gauhar Mahmood, and RC Trivedi. Municipal solid waste management in indian cities–a review. *Waste management*, 28(2):459–467, 2008.

[16] PK Srivastava, K Kulshreshtha, CS Mohanty, P Pushpangadan, and A Singh. Stakeholder-based swot analysis for successful municipal solid waste management in lucknow, india. *Waste management*, 25(5):531–37, 2005.

[17] Sarika Rathi. Alternative approaches for better municipal solid waste management in mumbai, india. *Waste management*, 26(10):1192–200, 2006.

[18] Mufeed Sharholy, Kafeel Ahmad, Rakesh Chandra Vaishya, and Rana Datta Gupta. Municipal solid waste characteristics and management in allahabad, india. *Waste management*, 27(4):490–96, 2007.

[19] Vikash Talyan, RP Dahiya, and TR Sreekrishnan. State of municipal solid waste management in delhi, the capital of india. *Waste management*, 28(7):1276–87, 2008.

[20] Arvind K Jha, C Sharma, Nahar Singh, R Ramesh, R Purvaja, and Prabhat K Gupta. Greenhouse gas emissions from municipal solid waste management in indian mega-cities: A case study of chennai landfill sites. *Chemosphere*, 71(4):750–58, 2008.

[21] Sapna Sethi, NC Kothiyal, Arvind K Nema, and MK Kaushik. Characterization of municipal solid waste in jalandhar city, punjab, india. *Journal of hazardous, toxic, and radioactive waste*, 17(2):97–106, 2013.

[22] Ishfaq Showket Mir, Puneet Pal Singh Cheema, and Sukhwinder Pal Singh. Implementation analysis of solid waste management in ludhiana city of punjab. *Environmental Challenges*, 2:100023, 2021.

[23] Ram Kumar Ganguly and Susanta Kumar Chakraborty. Integrated approach in municipal solid waste management in covid-19 pandemic: Perspectives of a developing country like india in a global scenario. *Case Studies in Chemical and Environmental Engineering*, 3:100087, 2021.

[24] Wei Hong Lim and Nor Ashidi Mat Isa. Teaching and peer-learning particle swarm optimization. *Applied Soft Computing*, 18:39–58, 2014.

[25] Wei Hong Lim and Nor Ashidi Mat Isa. Bidirectional teaching and peer-learning particle swarm optimization. *Information Sciences*, 280:111–134, 2014.

[26] Tingli Cheng, Minyou Chen, Peter J Fleming, Zhile Yang, and Shaojun Gan. A novel hybrid teaching learning based multi-objective particle swarm optimization. *Neurocomputing*, 222:11–25, 2017.

[27] Mehmet Güçyetmez and Ertuğrul Çam. A new hybrid algorithm with genetic-teaching learning optimization (g-tlbo) technique for optimizing of power flow in wind-thermal power systems. *electrical engineering*, 98(2):145–157, 2016.

[28] Xu Chen, Bin Xu, Congli Mei, Yuhan Ding, and Kangji Li. Teaching–learning–based artificial bee colony for solar photovoltaic parameter estimation. *Applied energy*, 212:1578–1588, 2018.

[29] Mehmet Fatih Tefek, Harun Uğuz, and Mehmet Güçyetmez. A new hybrid gravitational search–teaching–learning-based optimization method for energy demand estimation of turkey. *Neural Computing and Applications*, 31(7):2939–2954, 2019.

[30] Jida Huang, Liang Gao, and Xinyu Li. An effective teaching-learning-based cuckoo search algorithm for parameter optimization problems in structure designing and machining processes. *Applied Soft Computing*, 36:349–356, 2015.

[31] Shouheng Tuo, Longquan Yong, and Tao Zhou. An improved harmony search based on teaching-learning strategy for unconstrained optimization problems. *Mathematical Problems in Engineering*, 2013, 2013.

[32] R Venkata Rao and Vivek Patel. An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems. *International Journal of Industrial Engineering Computations*, 3(4):535–60, 2012.

[33] Debao Chen, Feng Zou, Jiangtao Wang, and Wujie Yuan. Samcctlbo: a multi-class cooperative teaching–learning-based optimization algorithm with simulated annealing. *Soft Computing*, 20(5):1921–1943, 2016.

[34] Vivek K Patel and Vimal J Savsani. A multi-objective improved teaching–learning based optimization algorithm (mo-itlbo). *Information Sciences*, 357:182–200, 2016.

[35] Yiying Zhang, Zhigang Jin, and Ye Chen. Hybrid teaching–learning-based optimization and neural network algorithm for engineering design optimization problems. *Knowledge-Based Systems*, 187:104836, 2020.

[36] Mohammad Aljanabi, Mohd Arfian Ismail, and Vitaly Mezhuyev. Improved tlbo-jaya algorithm for subset feature selection and parameter optimisation in intrusion detection system. *Complexity*, 2020, 2020.

[37] Sandeep Singh Chauhan and Prakash Kotecha. An efficient multi-unit production planning strategy based on continuous variables . *Applied Soft Computing Journal*, 2018.

[38] Lei Wang, Feng Zou, Xinhong Hei, Dongdong Yang, Debao Chen, Qiaoyong Jiang, and Zijian Cao. A hybridization of teaching–learning-based optimization and differential evolution for chaotic time series prediction. *Neural computing and applications*, 25(6):1407–1422, 2014.

[39] Mojtaba Ghasemi, Mohammad Mehdi Ghanbarian, Sahand Ghavidel, Shima Rahmani, and Esmaeil Mahboubi Moghaddam. Modified teaching learning algorithm and double differential evolution algorithm for optimal reactive power dispatch problem: a comparative study. *Information Sciences*, 278:231–249, 2014.

[40] Oguz Emrah Turgut and Mustafa Turhan Coban. Optimal proton exchange membrane fuel cell modelling based on hybrid teaching learning based optimization–

differential evolution algorithm. *Ain Shams Engineering Journal*, 7(1):347–360, 2016.

[41] Rasoul Azizipanah-Abarghooee, Taher Niknam, Farhad Bavafa, and Mohsen Zare. Short-term scheduling of thermal power systems using hybrid gradient based modified teaching–learning optimizer with black hole algorithm. *Electric power systems research*, 108:16–34, 2014.

[42] Zhanpeng Xie, Chaoyong Zhang, Xinyu Shao, Wenwen Lin, and Haiping Zhu. An effective hybrid teaching–learning-based optimization algorithm for permutation flow shop scheduling problem. *Advances in Engineering Software*, 77:35–47, 2014.

[43] Qiuhua Tang, Zixiang Li, Liping Zhang, CA Floudas, and Xiaojun Cao. Effective hybrid teaching-learning-based optimization algorithm for balancing two-sided assembly lines with multiple constraints. *Chinese Journal of Mechanical Engineering*, 28(5):1067–1079, 2015.

[44] Vivek Patel and Vimal Savsani. Optimization of a plate-fin heat exchanger design through an improved multi-objective teaching-learning based optimization (mo-itlbo) algorithm. *Chemical Engineering Research and Design*, 92(11):2371–2382, 2014.

[45] Faiza Dib and Ismail Boumhidi. Hybrid algorithm de–tlbo for optimal h inf and pid control for multi-machine power system. *International Journal of System Assurance Engineering and Management*, 8(2):925–936, 2017.

[46] Behzad Azizimehr, Ehsanolah Assareh, and Rahim Moltames. Thermoeconomic analysis and optimization of a solar micro cchp by using tlbo algorithm for domestic application. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, 42(14):1747–1761, 2020.

[47] Hasan Tahsin Öztürk, Tayfun Dede, and Emel Türker. Optimum design of reinforced concrete counterfort retaining walls using tlbo, jaya algorithm. In *Structures*, volume 25, pages 285–296. Elsevier, 2020.

[48] DP Kanungo, Janmenjoy Nayak, Bighnaraj Naik, and Himansu Sekhar Behera. Hybrid clustering using elitist teaching learning-based optimization: an improved hybrid approach of tlbo. *International Journal of Rough Sets and Data Analysis (IJRSDA)*, 3(1):1–19, 2016.

[49] Alireza Baghban, Mohammad Navid Kardani, and Amir H Mohammadi. Improved estimation of cetane number of fatty acid methyl esters (fames) based biodiesels using tlbo-nn and pso-nn models. *Fuel*, 232:620–631, 2018.

[50] Ramachandra Rao Kurada, K Karteeka Pavan, and Allam Appa Rao. Automatic teaching–learning-based optimization: A novel clustering method for gene functional enrichments. In *Computational Intelligence Techniques for Comparative Genomics*, pages 17–35. Springer, 2015.

[51] Haiyan Jin, Yaning Li, Bei Xing, and Lei Wang. A geometric image segmentation method based on a bi-convex, fuzzy, variational principle with teaching-learning optimization. *Journal of Intelligent & Fuzzy Systems*, 31(6):3075–3081, 2016.

[52] Bighnaraj Naik, Janmenjoy Nayak, and HS Behera. A tlbo based gradient descent learning-functional link higher order ann: An efficient model for learning from non-linear data. *Journal of King Saud University-Computer and Information Sciences*, 30(1):120–139, 2018.

[53] D Devarasiddappa, M Chandrasekaran, and R Arunachalam. Experimental investigation and parametric optimization for minimizing surface roughness during wedm of ti6al4v alloy using modified tlbo algorithm. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 42(3):1–18, 2020.

[54] Dillip Khamari, Rabindra Kumar Sahu, Tulasichandra Sekhar Gorripotu, and Sidhartha Panda. Automatic generation control of power system in deregulated environment using hybrid tlbo and pattern search technique. *Ain Shams Engineering Journal*, 11(3):553–73, 2020.

[55] R Rajesh. Network design for resilience in supply chains using novel crazy elitist tlbo. *Neural Computing and Applications*, 32(11):7421–37, 2020.

[56] Soni Kumari, Pankaj Sonia, Bharat Singh, Kumar Abhishek, and Kuldeep K Saxena. Optimization of surface roughness in edm of pure magnesium (mg) using tlbo. *Materials Today: Proceedings*, 26:2458–61, 2020.

[57] Preet Lata and Shelly Vadhera. Reliability improvement of radial distribution system by optimal placement and sizing of energy storage system using tlbo. *Journal of Energy Storage*, 30:101492, 2020.

[58] Ye Xu, Xin Liu, Xiaoguang Hu, Guohe Huang, and Na Meng. A genetic-algorithm-aided fuzzy chance-constrained programming model for municipal solid waste management. *Engineering Optimization*, 2019.

[59] Kusum Deep, Krishna Pratap Singh, Mitthan Lal Kansal, and C Mohan. An interactive method using genetic algorithm for multi-objective optimization problems modeled in fuzzy environment. *Expert Systems with Applications*, 38(3):1659–1667, 2011.

[60] Seyed Zeinab Aliahmadi, Farnaz Barzinpour, and Mir Saman Pishvaee. A fuzzy optimization approach to the capacitated node-routing problem for municipal solid waste collection with multiple tours: A case study. *Waste Management & Research*, 38(3):279–290, 2020.

[61] David H Wolpert, William G Macready, et al. No free lunch theorems for search. Technical report, Technical Report SFI-TR-95-02-010, Santa Fe Institute, 1995.

[62] Jagdish Chand Bansal, Harish Sharma, Shimpi Singh Jadon, and Maurice Clerc. Spider monkey optimization algorithm for numerical optimization. *Memetic computing*, 6(1):31–47, 2014.

[63] James Kennedy and Russell Eberhart. Particle swarm optimization. In *Proceedings of ICNN'95-International Conference on Neural Networks*, volume 4, pages 1942–1948. IEEE, 1995.

[64] Agoston E Eiben and Cornelis A Schippers. On evolutionary exploration and exploitation. *Fundamenta Informaticae*, 35(1-4):35–50, 1998.

[65] Yang Gao, Wenbo Du, and Gang Yan. Selectively-informed particle swarm optimization. *Scientific reports*, 5(1):1–7, 2015.

[66] Rainer Storn and Kenneth Price. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization*, 11(4):341–359, 1997.

[67] John H Holland. Genetic algorithms. *Scientific american*, 267(1):66–73, 1992.

[68] Qiang Zhang, Dandan Deng, Wenting Dai, Jixin Li, and Xinwen Jin. Optimization of culture conditions for differentiation of melon based on artificial neural network and genetic algorithm. *Scientific reports*, 10(1):1–8, 2020.

[69] Dervis Karaboga and Bahriye Basturk. A powerful and efficient algorithm for numerical function optimization: artificial bee colony (abc) algorithm. *Journal of global optimization*, 39(3):459–471, 2007.

[70] Marco Dorigo and Luca Maria Gambardella. Ant colonies for the travelling salesman problem. *biosystems*, 43(2):73–81, 1997.

[71] Marco Dorigo, Mauro Birattari, and Thomas Stutzle. Ant colony optimization. *IEEE computational intelligence magazine*, 1(4):28–39, 2006.

[72] Arjun Chandrasekhar, Deborah M Gordon, and Saket Navlakha. A distributed algorithm to maintain and repair the trail networks of arboreal ants. *Scientific reports*, 8(1):1–19, 2018.

[73] Lin Sun, Xianglin Kong, Jiucheng Xu, Ruibing Zhai, Shiguang Zhang, et al. A hybrid gene selection method based on relieff and ant colony optimization algorithm for tumor classification. *Scientific reports*, 9(1):1–14, 2019.

[74] Afshin Faramarzi, Mohammad Heidarinejad, Seyedali Mirjalili, and Amir H Gandomi. Marine predators algorithm: A nature-inspired metaheuristic. *Expert Systems with Applications*, 152:113377, 2020.

[75] Laith Abualigah, Ali Diabat, Seyedali Mirjalili, Mohamed Abd Elaziz, and Amir H Gandomi. The arithmetic optimization algorithm. *Computer methods in applied mechanics and engineering*, 376:113609, 2021.

[76] Armin Cheraghalipour, Mostafa Hajiaghaei-Keshteli, and Mohammad Mahdi Paydar. Tree growth algorithm (tga): A novel approach for solving optimization problems. *Engineering Applications of Artificial Intelligence*, 72:393–414, 2018.

[77] Afshin Faramarzi, Mohammad Heidarinejad, Brent Stephens, and Seyedali Mirjalili. Equilibrium optimizer: A novel optimization algorithm. *Knowledge-Based Systems*, 191:105190, 2020.

[78] Lawrence Davis. Bit-climbing, representational bias, and test suit design. In *Proc. Intl. Conf. Genetic Algorithm, 1991*, pages 18–23, 1991.

[79] Scott Kirkpatrick, C Daniel Gelatt, and Mario P Vecchi. Optimization by simulated annealing. *science*, 220(4598):671–680, 1983.

[80] Fred Glover. Tabu search—part i. *ORSA Journal on computing*, 1(3):190–206, 1989.

[81] Helena R Lourenço, Olivier C Martin, and Thomas Stützle. Iterated local search. In *Handbook of metaheuristics*, pages 320–353. Springer, 2003.

[82] Mohamed Abdel-Basset, Laila Abdel-Fatah, and Arun Kumar Sangaiah. Metaheuristic algorithms: A comprehensive review. *Computational intelligence for multimedia big data on the cloud with engineering applications*, pages 185–231, 2018.

[83] R V Rao, V J Savsani, and D P Vakharia. Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems. *CAD Computer Aided Design*, 43(3):303–315, 2011.

[84] R V Rao, V J Savsani, and D P Vakharia. Teaching – Learning-Based Optimization : An optimization method for continuous non-linear large scale problems. *Information Sciences*, 183(1):1–15, 2012.

[85] R Venkata Rao and Vivek Patel. Multi-objective optimization of two stage thermoelectric cooler using a modified teaching–learning-based optimization algorithm. *Engineering Applications of Artificial Intelligence*, 26(1):430–445, 2013.

[86] R Venkata Rao and Vivek Patel. Multi-objective optimization of heat exchangers using a modified teaching-learning-based optimization algorithm. *Applied Mathematical Modelling*, 37(3):1147–1162, 2013.

[87] Matej Črepinšek, Shih Hsi Liu, and Luka Mernik. A note on teaching-learning-based optimization algorithm. *Information Sciences*, 212:79–93, 2012.

[88] Gajanan Waghmare. Comments on ”a note on teaching-learning-based optimization algorithm”. *Information Sciences*, 229(December):159–169, 2013.

[89] R Venkata Rao and Vivek Patel. Comparative performance of an elitist teaching-learning-based optimization algorithm for solving unconstrained optimization problems. *International Journal of Industrial Engineering Computations*, 4(1):29–50, 2013.

[90] Chengjun Guo, Juncheng Lu, Zhong Tian, Wei Guo, and Aida Darvishan. Optimization of critical parameters of PEM fuel cell using TLBO-DE based on Elman neural network. *Energy Conversion and Management*, 183(December 2018):149–158, 2019.

[91] Jatinder Kaur, Surjeet Singh Chauhan, and Pavitdeep Singh. *NSGLTLBOLE : A Modified Non-dominated Sorting TLBO Technique Using Group Learning and Learning Experience of Others for Multi-objective Test Problems*. Springer Singapore, 2019.

[92] Hathiram Nenavath. Hybrid SCA – TLBO : a novel optimization algorithm for global optimization and visual tracking. *Neural Computing and Applications*, 6, 2018.

[93] Xinghua Qu, Ran Zhang, Bo Liu, and Huifeng Li. An improved tlbo based memetic algorithm for aerodynamic shape optimization. *Engineering Applications of Artificial Intelligence*, 57(October 2016):1–15, 2017.

[94] Subhrajyoti Sahu, Ajit Kumar Barisal, and Abhishek Kaudi. Multi-objective optimal power flow with dg placement using tlbo and mipso: A comparative study. *Energy Procedia*, 117:236–243, 2017.

[95] R. Venkata Rao. Review of applications of tlbo algorithm and a tutorial for beginners to solve the unconstrained and constrained optimization problems, 2016.

[96] Xiaojing Wu, Xuhao Peng, Weisheng Chen, and Weiwei Zhang. A developed surrogate-based optimization framework combining HDMR-based modeling technique and TLBO algorithm for high-dimensional engineering problems. *Structural and Multidisciplinary Optimization*, 60(2):663–680, 2019.

[97] Provas Kumar Roy, Chandan Paul, and Sneha Sultana. Oppositional teaching learning based optimization approach for combined heat and power dispatch. *International Journal of Electrical Power and Energy Systems*, 57:392–403, 2014.

[98] Samir Khatir, Magd Abdel Wahab, Djilali Boutchicha, and Tawfiq Khatir. Structural health monitoring using modal strain energy damage indicator coupled with teaching-learning-based optimization algorithm and isogoemetric analysis. *Journal of Sound and Vibration*, 448:230–246, 2019.

[99] Atul Tiwari and M K Pradhan. Applications of TLBO algorithm on various manufacturing processes: AReview. In *Materials Today: Proceedings*, volume 4, pages 1644–1652. Elsevier Ltd, 2017.

[100] Saleh Shahbeig, Mohammad Sadegh Helfroush, and Akbar Rahideh. A fuzzy multi-objective hybrid TLBO–PSO approach to select the associated genes with breast cancer. *Signal Processing*, 131:58–65, 2017.

[101] Prachi Mafidar Joshi and H K Verma. An improved TLBO based economic dispatch of power generation through distributed energy resources considering environmental constraints. *Sustainable Energy, Grids and Networks*, 18:100207, 2019.

[102] Sandeep Singh Chauhan, Chinta Sivadurgaprasad, Rajasekhar Kadambur, and Prakash Kotecha. A novel strategy for the combinatorial production planning problem using integer variables and performance evaluation of recent optimization algorithms. *Swarm and Evolutionary Computation*, 43(December 2017):225–243, 2018.

[103] Namita Gupta and Rajiv Gupta. Solid waste management and sustainable cities in india: the case of chandigarh. *Environment and urbanization*, 27(2):573–88, 2015.

[104] Debishree Khan and Sukha Ranjan Samadder. Municipal solid waste management using geographical information system aided methods: A mini review. *Waste management & research*, 32(11):1049–62, 2014.

[105] Nur Ayvaz-Cavdaroglu, Asli Coban, and Irem Firtina-Ertis. Municipal solid waste management via mathematical modeling: A case study in istanbul, turkey. *Journal of environmental management*, 244:362–369, 2019.

[106] Gordon H Huang, Jonathan D Linton, Julian Scott Yeomans, and Reena Yoogalingam. Policy planning under uncertainty: efficient starting populations for simulation-optimization methods applied to municipal solid waste management. *Journal of Environmental Management*, 77(1):22–34, 2005.

[107] Nikolaos V Karadimas, Katerina Papatzelou, and Vassili G Loumos. Genetic algorithms for municipal solid waste collection and routing optimization. In *IFIP International Conference on Artificial Intelligence Applications and Innovations*, pages 223–31. Springer, 2007.

[108] Julian Scott Yeomans and Xin-She Yang. Municipal waste management optimisation using a firefly algorithm-driven simulation-optimisation approach. *International Journal of Process Management and Benchmarking*, 4(4):363–375, 2014.

[109] Nina Juul, Marie Münster, Hans Ravn, and M Ljunggren Söderman. Challenges when performing economic optimization of waste treatment: a review. *Waste Management*, 33(9):1918–25, 2013.

[110] Peter Beigl, Sandra Lebersorger, and Stefan Salhofer. Modelling municipal solid waste generation: A review. *Waste management*, 28(1):200–214, 2008.

[111] Antonio Gallardo, Francisco J Colomer-Mendoza, Mar Carlos-Alberola, Cristóbal Badenes, Natalia Edo-Alcón, and Joan Esteban-Altabella. Efficiency of a pilot scheme for the separate collection of the biowaste from municipal solid waste in spain. *Scientific Reports*, 11(1):1–13, 2021.

[112] Sinnott Murphy and Stephanie Pincetl. ”zero waste in los angeles: Is the emperor wearing any clothes?”. *Resources, Conservation and Recycling*, 81:40–51, (2013).

[113] Sie Ting Tan, Chew Tin Lee, Haslenda Hashim, Wai Shin Ho, and Jeng Shiun Lim. ”optimal process network for municipal solid waste management in iskandar malaysia”. *Journal of Cleaner Production*, 71:48–58, (2014).

[114] Fatma Kutlu Gündoğdu and Cengiz Kahraman. Spherical fuzzy sets and spherical fuzzy topsis method. *Journal of intelligent & fuzzy systems*, 36(1):337–52, 2019(a).

[115] Ting-Yu Chen. Remoteness index-based pythagorean fuzzy vikor methods with a generalized distance measure for multiple criteria decision analysis. *Information Fusion*, 41:129–150, 2018.

[116] Feng-Bao Cui, Xiao-Yue You, Hua Shi, and Hu-Chen Liu. Optimal siting of electric vehicle charging stations using pythagorean fuzzy vikor approach. *Mathematical Problems in Engineering*, 2018, 2018.

[117] Şeyma Emeç and Gökay Akkaya. Stochastic ahp and fuzzy vikor approach for warehouse location selection problem. *Journal of Enterprise Information Management*, 2018.

[118] Kuan Liu, Yanwu Liu, and Jindong Qin. An integrated anp-vikor methodology for sustainable supplier selection with interval type-2 fuzzy sets. *Granular Computing*, 3(3):193–208, 2018.

[119] Fuli Zhou, Xu Wang, and Mark Goh. Fuzzy extended vikor-based mobile robot selection model for hospital pharmacy. *International Journal of Advanced Robotic Systems*, 15(4):1729881418787315, 2018.

[120] Muhammad Sajjad Ali Khan, Saleem Abdullah, Asad Ali, and Fazli Amin. An extension of vikor method for multi-attribute decision-making under pythagorean hesitant fuzzy setting. *Granular Computing*, 4(3):421–434, 2019.

[121] Patchara Phochanikorn, Chunqiao Tan, and Wen Chen. Barriers analysis for reverse logistics in thailand's palm oil industry using fuzzy multi-criteria decision-making method for prioritizing the solutions. *Granular Computing*, 5(4):419–436, 2020.

[122] Fatma Kutlu Gündoğdu, Cengiz Kahraman, and Ali Karaşan. Spherical fuzzy vikor method and its application to waste management. In *International Conference on Intelligent and Fuzzy Systems*, pages 997–1005. Springer, 2019(b).

[123] Gurpreet Kaur, Arunava Majumder, and Rakesh Yadav. An efficient generalized fuzzy topsis algorithm for the selection of the hybrid energy resources: A comparative study between single and hybrid energy plant installation in turkey. *RAIRO-Operations Research*, 56(3):1877–1899, 2022.

[124] Aishwarya Dhara, Gurpreet Kaur, Pon Maa Kishan, Arunava Majumder, and Rakesh Yadav. An efficient decision support system for selecting very light business jet using critic-topsis method. *Aircraft Engineering and Aerospace Technology*, 2021.

[125] Manoj Mathew, Ripon K Chakrabortty, and Michael J Ryan. A novel approach integrating ahp and topsis under spherical fuzzy sets for advanced manufacturing system selection. *Engineering Applications of Artificial Intelligence*, 96:103988, 2020.

[126] Fatma Kutlu Gündoğdu and Cengiz Kahraman. A novel spherical fuzzy qfd method and its application to the linear delta robot technology development. *Engineering Applications of Artificial Intelligence*, 87:103348, 2020.

[127] Lal Chand Malav, Krishna Kumar Yadav, Neha Gupta, Sandeep Kumar, Gulshan Kumar Sharma, Santhana Krishnan, Shahabaldin Rezania, Hesam Kamyab, Quoc Bao Pham, Shalini Yadav, et al. A review on municipal solid waste as a renewable source for waste-to-energy project in india: current practices, challenges, and future opportunities. *Journal of Cleaner Production*, 277:123227, 2020.

[128] Kurian Joseph. Municipal solid waste management in india. In *Municipal solid waste management in Asia and the Pacific Islands*, pages 113–138. Springer, 2014.

[129] Frank L Hitchcock. The distribution of a product from several sources to numerous localities. *Journal of mathematics and physics*, 20(1-4):224–230, 1941.

[130] K Ganesan and P Veeramani. Fuzzy linear programs with trapezoidal fuzzy numbers. *Annals of operations research*, 143(1):305–315, 2006.

[131] Marc Asunción, Luis Castillo, Juan Fernández-Olivares, Oscar García-Pérez, Antonio González, and Francisco Palao. Handling fuzzy temporal constraints in a planning environment. *Annals of Operations Research*, 1(155):391–415, 2007.

[132] Sujeet Kumar Singh and Shiv Prasad Yadav. Efficient approach for solving type-1 intuitionistic fuzzy transportation problem. *International journal of system assurance engineering and management*, 6(3):259–267, 2015.

[133] Sujeet Kumar Singh and Shiv Prasad Yadav. A new approach for solving intuitionistic fuzzy transportation problem of type-2. *Annals of Operations Research*, 243(1):349–363, 2016.

[134] Sankar Kumar Roy, Ali Ebrahimnejad, José Luis Verdegay, and Sukumar Das. New approach for solving intuitionistic fuzzy multi-objective transportation problem. *Sādhanā*, 43(1):1–12, 2018.

[135] Firoz Ahmad and Ahmad Yusuf Adhami. Neutrosophic programming approach to multiobjective nonlinear transportation problem with fuzzy parameters. *International journal of management science and engineering management*, 14(3):218–229, 2019.

[136] Rizk M Rizk-Allah, Aboul Ella Hassanien, and Mohamed Elhoseny. A multi-objective transportation model under neutrosophic environment. *Computers & Electrical Engineering*, 69:705–719, 2018.

[137] Szabolcs Duleba, Fatma Kutlu Gündoğdu, and Sarbast Moslem. Interval-valued spherical fuzzy analytic hierarchy process method to evaluate public transportation development. *Informatica*, 32(4):661–686, 2021.

[138] Büşra Buran and Mehmet Erçek. Public transportation business model evaluation with spherical and intuitionistic fuzzy ahp and sensitivity analysis. *Expert Systems with Applications*, page 117519, 2022.

[139] Vineet Kumar, Anjana Gupta, and HC Taneja. Solution of transportation problem under spherical fuzzy set. In *2021 IEEE 6th International Conference on Computing, Communication and Automation (ICCCA)*, pages 444–448. IEEE, 2021.

[140] Timothy J Ross. *Fuzzy logic with engineering applications*. John Wiley & Sons, 2005.

[141] Ronald R Yager. Pythagorean fuzzy subsets. In *2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)*, pages 57–61. IEEE, 2013.

[142] Firoz Ahmad and Ahmad Yusuf Adhami. Spherical fuzzy linear programming problem. In *Decision Making with Spherical Fuzzy Sets*, pages 455–472. Springer, 2021.

[143] Fatma Kutlu Gündoğdu. A spherical fuzzy extension of multimoora method. *Journal of Intelligent & Fuzzy Systems*, 38(1):963–978, 2020(a).

[144] Fatma Kutlu Gündoğdu and Cengiz Kahraman. A novel spherical fuzzy qfd method and its application to the linear delta robot technology development. *Engineering Applications of Artificial Intelligence*, 87:103348, 2020(b).

[145] Fatma Kutlu Gündoğdu and Cengiz Kahraman. Extension of codas with spherical fuzzy sets. *Journal of Multiple-Valued Logic & Soft Computing*, 33, 2019(c).

[146] Fatma Kutlu Gündoğdu and Cengiz Kahraman. Spherical fuzzy sets and decision making applications. In *International Conference on Intelligent and Fuzzy Systems*, pages 979–987. Springer, 2019(a).

[147] Fatma Kutlu Gündoğdu and Cengiz Kahraman. Spherical fuzzy analytic hierarchy process (ahp) and its application to industrial robot selection. In *International Conference on Intelligent and Fuzzy Systems*, pages 988–996. Springer, 2019(b).

[148] Neha Gupta, Krishna Kumar Yadav, and Vinit Kumar. A review on current status of municipal solid waste management in india. *Journal of environmental sciences*, 37:206–217, 2015.

[149] Tapas Kumar Ghatak. Municipal solid waste management in india: A few unaddressed issues. *Procedia Environmental Sciences*, 35:169–175, 2016.

[150] Yash Pujara, Pankaj Pathak, Archana Sharma, and Janki Govani. Review on indian municipal solid waste management practices for reduction of environmental impacts to achieve sustainable development goals. *Journal of environmental management*, 248:109238, 2019.

[151] Sonil Nanda and Franco Berruti. Municipal solid waste management and landfilling technologies: a review. *Environmental Chemistry Letters*, pages 1–24, 2020.

[152] Mohsen Akbarpour Shirazi, Reza Samieifard, Mohammad Ali Abduli, and Babak Omidvar. Mathematical modeling in municipal solid waste management: case study of tehran. *Journal of Environmental Health Science and Engineering*, 14(1):1–12, 2016.

[153] C Manjusha and B Sajeen Beevi. Mathematical modeling and simulation of anaerobic digestion of solid waste. *Procedia Technology*, 24:654–660, 2016.

[154] Ashish Soni, Pankaj Kumar Das, and Prabhat Kumar. A review on the municipal solid waste management status, challenges and potential for the future indian cities. *Environment, Development and Sustainability*, pages 1–49, 2022.

[155] Abhishek Dixit, Deepesh Singh, and Sanjay Kumar Shukla. Changing scenario of municipal solid waste management in kanpur city, india. *Journal of Material Cycles and Waste Management*, 24(5):1648–1662, 2022.

[156] Ashootosh Mandpe, Ayushman Bhattacharya, Sonam Paliya, Vinay Pratap, Athar Hussain, and Sunil Kumar. Life-cycle assessment approach for municipal solid waste management system of delhi city. *Environmental Research*, 212:113424, 2022.

[157] Abhishek Singhal, Anil Kumar Gupta, Brajesh Dubey, and Makrand M Ghangrekar. Seasonal characterization of municipal solid waste for selecting feasible waste treatment technology for guwahati city, india. *Journal of the Air & Waste Management Association*, 72(2):147–160, 2022.

[158] Gaurav Sharma and Baerbel Sinha. Future emissions of greenhouse gases, particulate matter and volatile organic compounds from municipal solid waste burning in india. *Science of The Total Environment*, 858:159708, 2023.

[159] Anant V Suryavanshi, M Mansoor Ahammed, and Irshad N Shaikh. Energy, economic, and environmental analysis of waste-to-energy technologies for municipal solid waste treatment: A case study of surat, india. *Journal of Hazardous, Toxic, and Radioactive Waste*, 27(2):04023005, 2023.

[160] Ajaz Ahmad Mir, Jasir Mushtaq, Abdul Qayoom Dar, and Mahesh Patel. A quantitative investigation of methane gas and solid waste management in mountainous srinagar city-a case study. *Journal of Material Cycles and Waste Management*, 25(1):535–549, 2023.

[161] Shijun Ma, Chuanbin Zhou, Jingjin Pan, Guang Yang, Chuanlian Sun, Yijie Liu, Xinchuang Chen, and Zhilan Zhao. Leachate from municipal solid waste landfills in a global perspective: Characteristics, influential factors and environmental risks. *Journal of Cleaner Production*, 333:130234, 2022.

[162] Afzal Husain Khan, Eduardo Alberto López-Maldonado, Shah Saud Alam, Nadeem A Khan, Juan Ramon López López, Perla Fabiola Méndez Herrera, Ahmed Abutaleb, Sirajuddin Ahmed, and Lakhveer Singh. Municipal solid waste generation and the current state of waste-to-energy potential: State of art review. *Energy Conversion and Management*, 267:115905, 2022.

[163] Kunsen Lin, Youcai Zhao, Jia-Hong Kuo, Hao Deng, Feifei Cui, Zilong Zhang, Meilan Zhang, Chunlong Zhao, Xiaofeng Gao, Tao Zhou, et al. Toward smarter management and recovery of municipal solid waste: A critical review on deep learning approaches. *Journal of Cleaner Production*, page 130943, 2022.

[164] Qing Ye, Qasim Umer, Rongting Zhou, Amna Asmi, and Fahad Asmi. How publications and patents are contributing to the development of municipal solid waste management: Viewing the un sustainable development goals as ground zero. *Journal of Environmental Management*, 325:116496, 2023.

[165] Costas A Velis, David C Wilson, Yoni Gavish, Sue M Grimes, and Andrew Whiteman. Socio-economic development drives solid waste management performance in cities: A global analysis using machine learning. *Science of The Total Environment*, 872:161913, 2023.

[166] Weiping Huang and Mohammad Marefati. Development, exergoeconomic assessment and optimization of a novel municipal solid waste-incineration and solar thermal energy based integrated power plant: An effort to improve the performance of the power plant. *Process Safety and Environmental Protection*, 2023.

[167] Arunodaya Raj Mishra, Pratibha Rani, Dragan Pamucar, Ibrahim M Hezam, and Abhijit Saha. Entropy and discrimination measures based q-rung orthopair fuzzy multimoora framework for selecting solid waste disposal method. *Environmental Science and Pollution Research*, 30(5):12988–13011, 2023.

[168] Elham Abdollahi Saadatlu, Farnaz Barzinpour, and Saeed Yaghoubi. A sustainable municipal solid waste system under leachate treatment impact along with leakage control and source separation. *Process Safety and Environmental Protection*, 169:982–998, 2023.

[169] Sunil Kumar, Rakesh Kumar, et al. Forecasting of municipal solid waste generation using non-linear autoregressive (nar) neural models. *Waste Management*, 121:206–214, 2021.

[170] C Ramprasad, Hari Charan Teja, Vunnam Gowtham, and Varadam Vikas. Quantification of landfill gas emissions and energy production potential in tirupati municipal solid waste disposal site by landgem mathematical model. *MethodsX*, 9:101869, 2022.

[171] Aakash Gaur, Harshita Prakash, Kuwar Anand, Girish Kumar, and Athar Hussain. Evaluation of municipal solid waste management scenarios using multi-criteria decision making under fuzzy environment. *Process Integration and Optimization for Sustainability*, 6(2):307–321, 2022.

[172] V Vivekanand and GS Prakash. Application of deterministic, stochastic and fuzzy linear programming models in solid waste management studies: literature review. *The Journal of Solid Waste Technology and Management*, 45(1):68–75, 2019.

[173] Rakesh Yadav and Manbir Kaur. Spherical fuzzy programming approach to optimize the transportation problem. *Mathematical Statistician and Engineering Applications*, 71(4):10216–31, 2022.

[174] Manbir Kaur and Rakesh Yadav. Prioritizing the indicators responsible for sustaintable municipal solid waste management using sf-ahp and sf-topsis. *NeuroQuantology*, 20(17):247–54, 2022.

[175] Manbir Kaur and Rakesh Yadav. Implementation analysis of municipal solid waste management in dinanagar city of punjab india. *Seyboid Report*, 17(11):1832–49, 2022.

[176] Gabriel Andari Kristanto and William Koven. Estimating greenhouse gas emissions from municipal solid waste management in depok, indonesia. *City and environment interactions*, 4:100027, 2019.

[177] Franck Michaud, Patrick Castéra, Christophe Fernandez, and Amadou Ndiaye. Meta-heuristic methods applied to the design of wood—plastic composites, with some attention to environmental aspects. *Journal of composite materials*, 43(5):533–548, 2009.

[178] Shyamala Mani and Satpal Singh. Sustainable municipal solid waste management in india: A policy agenda. *Procedia Environmental Sciences*, 35:150–157, 2016.

[179] Rajkumar Joshi and Sirajuddin Ahmed. Status and challenges of municipal solid waste management in india: A review. *Cogent Environmental Science*, 2(1):1139434, 2016.

[180] Harshit Khandelwal, Arun Kumar Thalla, Sunil Kumar, and Rakesh Kumar. Life cycle assessment of municipal solid waste management options for india. *Bioresource technology*, 288:121515, 2019.

[181] Rishi Rana, Rajiv Ganguly, and Ashok Kumar Gupta. Physico-chemical characterization of municipal solid waste from tricity region of northern india: a case study. *Journal of Material Cycles and Waste Management*, 20(1):678–689, 2018.

[182] Rishi Rana, Rajiv Ganguly, and Ashok Kumar Gupta. Life-cycle assessment of municipal solid-waste management strategies in tricity region of india. *Journal of Material Cycles and Waste Management*, 21(3):606–623, 2019.

[183] Chander Kumar Singh, Anand Kumar, and Soumendu Shekhar Roy. Quantitative analysis of the methane gas emissions from municipal solid waste in india. *Scientific reports*, 8(1):1–8, 2018.

[184] Manoj Mathew, Ripon K Chakrabortty, and Michael J Ryan. Selection of an optimal maintenance strategy under uncertain conditions: An interval type-2 fuzzy ahp-topsis method. *IEEE Transactions on Engineering Management*, 2020.

[185] Xunchang Fei, Hongping He, Xiaoqing Pi, Xuhong Lu, Qinqin Chen, Jun Ma, Yao Wang, Mingliang Fang, Chuangzhou Wu, and Shijin Feng. The distribution, behavior, and release of macro-and micro-size plastic wastes in solid waste disposal sites. *Critical Reviews in Environmental Science and Technology*, 53(3):366–389, 2023.

Appendices

Table 8.1: Criteria matrix in linguistic terms by five experts using table 5.1

Experts	Criteria	C1	C2	C3
E1	C1	E	SS	AS
	C2	SL	E	FS
	C3	AL	FL	E
E2	C1	E	FS	VS
	C2	FL	E	VS
	C3	VL	VL	E
E3	C1	E	SS	AS
	C2	SL	E	FS
	C3	AL	FL	E
E4	C1	E	SS	AS
	C2	SL	E	SS
	C3	AL	SL	E
E5	C1	E	SS	AS
	C2	SL	E	FS
	C3	AL	FL	E

Table 8.2: PSPA evaluation matrix using 11 point scale given in table 5.2

Indicators	C1	C2	C3
PU1	EH	G	VH
PU2	VH	EX	EH
PU3	EH	M	G
PU4	VG	F	G
PU5	VH	EH	H
PU6	G	VG	L
PU7	M	F	G
PU8	VH	G	VG

Table 8.3: Score index of criteria for table 8.1

Experts	Criteria	C1	C2	C3
E1	C1	1.000	3.000	9.000
	C2	0.333	1.000	5.000
	C3	0.111	0.200	1.000
E2	C1	1.000	5.000	7.000
	C2	0.200	1.000	7.000
	C3	0.143	0.143	1.000
E3	C1	1.000	3.000	9.000
	C2	0.333	1.000	5.000
	C3	0.111	0.200	1.000
E4	C1	1.000	3.000	7.000
	C2	0.333	1.000	3.000
	C3	0.143	0.333	1.000
E5	C1	1.000	3.000	9.000
	C2	0.333	1.000	5.000
	C3	0.111	0.200	1.000

Table 8.4: SF-PCM from five experts

	C1			C2			C3		
Experts	μ	ν	π	μ	ν	π	μ	ν	π
E1	0.500	0.400	0.400	0.600	0.400	0.300	0.900	0.100	0.000
	0.400	0.600	0.300	0.500	0.400	0.400	0.700	0.300	0.200
	0.100	0.900	0.000	0.300	0.700	0.200	0.500	0.400	0.400
E2	0.500	0.400	0.400	0.700	0.300	0.200	0.800	0.200	0.100
	0.300	0.700	0.300	0.500	0.400	0.400	0.800	0.200	0.100
	0.200	0.800	0.100	0.200	0.800	0.100	0.500	0.400	0.400
E3	0.500	0.400	0.400	0.600	0.400	0.300	0.900	0.100	0.000
	0.400	0.600	0.300	0.500	0.400	0.400	0.700	0.300	0.200
	0.100	0.900	0.000	0.300	0.700	0.200	0.500	0.400	0.400
E4	0.500	0.400	0.400	0.600	0.400	0.300	0.800	0.200	0.100
	0.400	0.600	0.300	0.500	0.400	0.400	0.600	0.400	0.300
	0.200	0.800	0.100	0.400	0.600	0.300	0.500	0.400	0.400
E5	0.500	0.400	0.400	0.600	0.400	0.300	0.900	0.100	0.000
	0.400	0.600	0.300	0.500	0.400	0.400	0.700	0.300	0.200
	0.100	0.900	0.000	0.300	0.700	0.200	0.500	0.400	0.400

Table 8.5: Combined SF-PCM using SFGM for SF weight calculation

	C1			C2			C3		
	μ	ν	π	μ	ν	π	μ	ν	π
C1	0.500	0.400	0.400	0.619	0.383	0.285	0.859	0.149	0.064
C2	0.378	0.623	0.301	0.500	0.400	0.400	0.697	0.308	0.214
C3	0.132	0.869	0.052	0.293	0.709	0.201	0.500	0.400	0.400

Table 8.6: SFN of Combined SF-PCM to check the consistency

Criteria	C1	C2	C3
C1	1.000	3.195	7.901
C2	0.319	1.000	4.742
C3	0.123	0.200	1.000

Research Publications:

The contribution of authors in research publications of this thesis are mentioned below:

1. Y. Rakesh and K. Manbir, " Teaching learning based optimization-a review on background and development ". *AIP conference proceedings* **2986(030173)** 1-12(2024). https://doi.org/10.1063/5.0197786

2. K. Manbir and Y. Rakesh, " Assesment of Municipal Solid Waste Management in Dinanagar city of Punjab, India ". *PIMT journal of Research* **14(1)** 27-31(2021). https://pimtjr.in/wp-content/uploads/2022/02/Abstract-Vol.14-No.-1-Oct-Dec-2021-issue-1.pdf **[UGC journal]**

3. K. Manbir and Y. Rakesh, " Implementation analysis of Municipal Solid Waste Management in Dinanagar city of Punjab, India ". *The Seybold Report* **17(11)** 1832-49(2022).https://seyboldreport.org/article_overview?id=MTIyMDIyMDMwMTU2MDc4MzE2 **[SJR-0.104]**

4. K. Manbir and Y. Rakesh, " Prioritizing The Indicators Responsible For Sustainable Municipal Solid Waste Management Using SF-AHP And SF-TOPSIS ". *NeuroQuantology: An Interdisciplinary Journal* **20(17)** 247-54(2022).https://www.neuroquantology.com/article.php?id=10668 **[SJR-0.285]**

5. K. Manbir and Y. Rakesh, " Spherical fuzzy programming approach to optimize the transportation problem ". *Mathematical Statistician and Engineering Applications* **71(4)** 10216-31(2021). https://www.philstat.org/index.php/MSEA/article/view/1849 **[SJR-0.105]**

6. K. Manbir and Y. Rakesh, " Mathematical Modelling Of Municipal Solid Waste Management In Spherical Fuzzy Environment ". *Advances in Nonlinear Variational Inequalities* **26(4)** 47-64(2023). https://doi.org/10.52783/anvi.v26.i4.308 **[SJR-0.12]**

Conferences Attended:

1. 3^{rd} International conference on *" Recent Advances in Fundamental and Advanced Sciences (RAFAS 2021)"* is attended on 25-26 June, 2021 organized by School of Chemical Engineering & Physical Sciences, Lovely Professional University, Phagwara Punjab(India) 144411 and a paper entitled " Teaching learning based optimization-a review on background and development " is presented in the form of poster.

2. 5^{th} International conference on *" Recent Advances in Mathematical Sciences with Application in Engineering & Technology (IC-RA-MSA-ET 2022)"* is attended on 16-18 June, 2022 organized by School of Computational & Integrative Sciences, Jawaharlal Nehru University, New Delhi(India) 110067, sponsored by Department of Biotechnology Ministry of Science & Technology GoI and a paper entitled " Mathematical Modelling Of Municipal Solid Waste Management In Spherical Fuzzy Environment: A case study " is presented.