STUDY OF SOME ELASTODYNAMIC PROBLEMS IN NON-LOCAL MICROPOLAR MEDIA

Thesis Submitted for the Award of the Degree of

DOCTOR OF PHILOSOPHY
in

Mathematics

By

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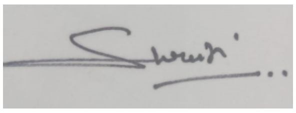
2024

DECLARATION

I, hereby declared that the presented work in the thesis entitled "**Study of some elastodynamic problems in non-local micropolar media**" in fulfilment of degree of **Doctor of Philosophy (Ph. D.)** is outcome of research work carried out by me under the supervision of **Dr. Kulwinder Singh**, working as Associate Professor ,in the Department of Mathematics/ School of chemical engineering and physical sciences of Lovely Professional University, Punjab, India. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.



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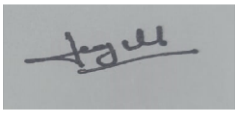
CERTIFICATE

This is to certify that the work reported in the Ph. D. thesis entitled “**study of some elastodynamic problems in non-local micropolar media**” submitted in fulfillment of the requirement for the award of degree of **Doctor of Philosophy (Ph.D.)** in the Department of Mathematics/ School of chemical engineering and physical sciences, is a research work carried out by **Ms. Shruti** , Registration no. 41800940, is bonafide record of her original work carried out under my supervision and that no part of thesis has been submitted for any other degree, diploma or equivalent course.



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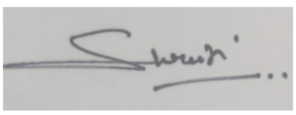

SHRUTI

ABSTRACT

In the thesis entitled "Study of some Elastodynamic problems in Non-local Micropolar media", we have studied the wave propagation problems in non-local Micropolar elastic material using Eringen's non local Micropolar theory. These problems investigated the effects of various parameters on propagation of surface waves in non-local Micropolar half space alone as well as in structured consisting of Non-local Micropolar half space and piezoelectric layer. The parameters whose effects are investigated in details are non-locality of the material, piezoelectricity, initial stress, interface imperfection, thermal effects, impedance and memory dependent derivatives etc. The thesis has been divided into 5 chapters, details of each chapter has been given below.

Chapter 1 contains the basics and literature review of classical theory of elasticity, Micropolar theory, non-local theory of elasticity and non-local Micropolar theory of elasticity and surface waves in elastic media. Based upon the literature review the research gap and then objectives of the study are given in this chapter.

Chapter 2 includes the study of propagation of shear waves in piezoelectric layered non-local Micropolar half space composite structure. The general equation of shear waves in the coupled structure is obtained analytically in the closed form. In the particular case the result obtained is in accordance with the classical Love wave equation. The effects of key factor like non-locality, characteristic length, piezoelectric and elastic constants on the phase velocity of shear waves has been investigated and the results are depicted graphically. The theoretical results obtained shows that the phase velocity of shear wave is significantly affected due the presence of non-locality and size effects on small length scale in Micropolar elastic material.

Chapter 3 deals with the study of the effect of initial stress on the shear waves in non-local Micropolar half space and non-local piezoelectric layer bounded

imperfectly. Imposing the initial stress and a condition of imperfect surface between the layered composite structures could lead to new insights in design. Taking the non-local and microstructure effects into consideration, the propagation of shear waves has been investigated in an initially stressed piezoelectric layer imperfectly bounded to a Micropolar half-space under the non-local theory. The general phase velocity equation for shear waves has been obtained analytically in closed form. The phase velocity equation is in agreement with the classical Love wave equation in a particular case. The effects of key factors such as non-locality, interfacial imperfection, initial stress and thickness of the layer on the phase velocity have been evaluated. Graphical analysis has been performed and the results obtained indicate that shear wave propagation is significantly affected by various parameters considered in the study and are useful for designing high performance surface acoustic devices and sensors.

Chapter 4 deals with the analysis of propagation of Rayleigh waves in a non-local Micropolar thermoelastic half space with impedance boundary conditions. Dispersion equation of Rayleigh wave propagation with impedance boundary conditions is obtained and the effect of impedance and non-local parameters are studied. Dispersion equation of Rayleigh waves for a Micropolar thermoelastic half space with impedance boundary as well as traction free half space is obtained in the particular case. The non-dimensional speed of Rayleigh wave is computed as function of impedance parameters and presented graphically for aluminum epoxy material. It is observed that non-local and impedance parameter has significant effects on Rayleigh wave speed.

Chapter 5 studied the problem of Rayleigh waves propagation in a non-local Micropolar thermoelastic material within the framework of memory-dependent heat conduction model. The secular equation of Rayleigh waves, describing the dependence of Rayleigh wave speed on the memory dependent parameter and non-local parameter, is obtained analytically under stress-free and thermally insulated/isothermal boundary conditions. In the particular case, the secular equation obtained is in agreement with previously published results. Numerical computations have been performed to investigate the effects of key factors such as time delay parameter, non-local parameter, and kernel functions on propagation of Rayleigh waves in the aluminum-epoxy composite material. The numerical and graphical analysis validate that the speed of Rayleigh waves is influenced significantly by time delay heat transfer, selection of kernel and non-local characteristic of the material.

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LIST OF SYMBOLS

Symbol	Meaning
u_1^n, u_2^n, u_3^n	displacement components in non-local micropolar elastic half space
u_1, u_2, u_3	displacement components in piezoelectric layer
$\sigma_{k\ell}$	stress tensor in local Micropolar half space
$\in_{ij}$	infinitesimal strain tensor
$\sigma_{k\ell}^{nl}$	stress tensor in non-local Micropolar half space
f_ℓ	body force density
$m_{k\ell}$	couple stress tensor
$m_{k\ell}^{nl}$	Non-local couple stress tensor
l_ℓ	body couple density
j	micro inertia density

ϕ_ℓ	micro-rotation vector
ρ	mass density
u_ℓ	displacement vector
$\in_{\ell \mathrm{mn}}$	alternating symbol
$\varepsilon(= e_0 l)$	non-local parameter for Micropolar material
l	characteristic length
$\alpha, \beta, \gamma, \kappa$	micro polar material constants
δ_{ij}	Kronecker delta
k	wave-number
ω	phase velocity of wave
τ_{ij}	stress tensor for piezoelectric material
τ_{ij}^{nl}	Non-local stress tensor for piezoelectric material
D_j	electric displacement for piezoelectric material
D_j^{nl}	Non-local electric displacement for piezoelectric material

$\varepsilon' = e_0 a$	non-local parameter for piezoelectric material
e_{kij}	Piezoelectric constants
ϵ_{jk}	Dielectric constants
c_{ijkl}	elastic constants
ρ'	Density of piezoelectric material
u	mechanical displacement
S_{kl}	strain tensor
E_k	electric field intensity
φ	electric potential
χ	Degree of imperfectness of the interface.

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CHAPTER – 1

INTRODUCTION

1.1 PRELIMINARIES

Mathematical modeling is necessary for comprehending both the theoretical foundations of solid mechanics as well as its practical applications. The use of mathematical modeling opens up the possibility of discovering answers to arduous mechanical problems. Theory of elasticity developed various mathematical models to study the deformation in elastic medium using laws of mechanics.

The application of an external force onto a material induces a displacement of the constituent particles leading to a consequential deformation of the material. Elastic and plastic deformations are distinct modes of deformation that can be discerned based on their characteristic behavior. Elastic deformation pertains to the reversible alteration in the shape and dimensions of a physical entity, which is contingent upon the discontinuation of external forces, thereby leading to the entity's reinstatement of its original configuration. Elastic deformation is a phenomenon that can manifest in a diverse range of materials. Material exhibits the elastic behavior until the external material forces are applied to a certain limit known as elastic limit. Plastic deformation is the result of the application of forces that exceed the elastic limit of a body. This phenomenon results in a permanent alteration in the body's configuration, persisting even in the absence of the impelling forces.

1.2 THEORY OF CLASSICAL ELASTICITY

Classical elasticity theory is based upon on an idealized model of elastic and continuous body such that the material is perfect and spread out evenly throughout its whole volume without any defects. Irrespective of the positions of the particles, they possess the same properties throughout the material. This assumption serves as the fundamental principle upon which the theory is constructed. Materials like concrete, steel, and aluminium behave according to the "Theory of classical elasticity" when subjected to stresses that do not exceed their elastic limits.

1.2.1 STRESS AND STRAIN COMPONENTS

The responses of elastic materials under the act of external forces have been studied by using “theory of classical elasticity” which is comprised of some fundamental equations and constitutive relations. These fundamental equations are developed using universal principles of physics including the conservation of mass, energy and momentum etc. The medium is said to be strained when the relative position of the particle is altered in a continuous medium. Material deformation occurs when there is a simultaneous change in the relative position of material points and the distance between them. Measure of deformation is known as strain and its study is referred as strain analysis.

Let us consider a continuous elastic medium having surface S and volume V. After the deformation, let us assume that the particle in continuous medium changes its position from $P\ (x_1, x_2, x_3)$ to $P'(x_1', x_2', x_3')$ as per the fig. 1.1. Displacement from old to new point has been represented by $u_i = x_i' - x_i$. Here $u_i, i(= 1,2,3)$ represents the displacement components.

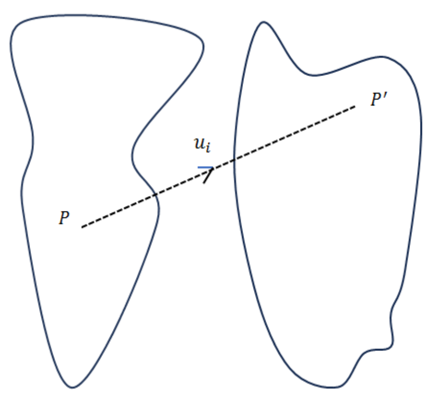

Fig. 1.1 Deformation of an elastic body

The infinitesimal strain tensor $\in_{ij}$ is symmetrical

and is represented as $\in_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$. Here, $\in_{11}, \in_{22}, \in_{33}$ are normal strain components and $\in_{12} = \in_{21}, \in_{13} = \in_{31}, \in_{23} = \in_{32}$ are shear strain component.

The internal resistance developed within the material due to external forces is known as stress. It is quantified as force extended per unit area. Considering an arbitrary element with area ΔA in the continuum within normal n_i and stress vector on elementary area is $T_i^n \Delta A$. Here, T_i^n force traction vector and symmetric stress tensor σ_{ij} are related as $T_i^n = \sigma_{ij} n_j, i, j = 1,2,3$. Components $\sigma_{11}, \sigma_{22}, \sigma_{33}$ are normal stress components and $\sigma_{12} = \sigma_{21}, \sigma_{13} = \sigma_{31}, \sigma_{23} = \sigma_{32}$ represents shear stresses as represented in fig. 1.2.

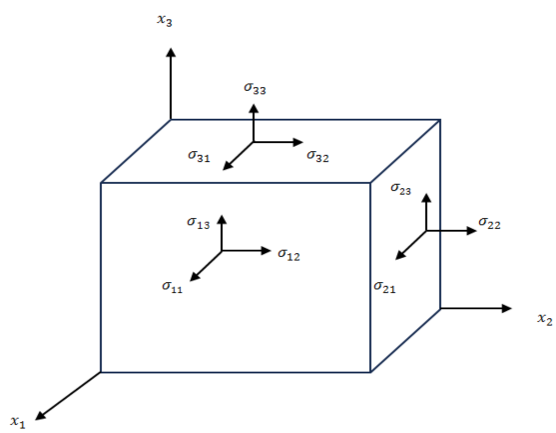

Fig. 1.2 Stress components

Relationship between strain and stress for an elastic medium could be defined as per the Hooke's law. The simplest form of Hooke's law $(F = -kx)$ is valid for ideal linear elastic material within the elastic limit of the material. For materials with complex behavior and structure under varying conditions the generalized Hooke's law is used which in tensor form can be written $\sigma_{ij} = C_{ijkl} \in_{kl}, i, j, k, l = 1,2,3$. where $\in_{kl}, \sigma_{ij}$ are infinitesimal strain and stress components and C_{ijkl} are elastic constants. The count of C_{ijkl} are 81 which reduces to 36 by imposing the symmetry condition of stress and strain.

Further these constants reduce to 21 by the condition $C_{ijkl} = C_{klij}$ which arises due to isothermal and adiabatic system in nature. Now, by assuming symmetry of axis and planes, and considering the material to be isotropic in nature, the elastic constant reduces to only 2. Hence the Hooke's law for homogeneous, isotropic material becomes

$$\sigma_{ij} = \lambda \, \Delta \, \delta_{ij} + 2 \, \mu \; \epsilon_{ij} \, . \tag{1.1}$$

Here, λ and μ are Lame's contents, $\Delta = \epsilon_{ii} = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$ is volume dilation and δ_{ij} is the Kronecker delta represented as

$$\delta_{ij} = \begin{cases} 1 \, , if \; i = j \\ 0 \, , if \; i \neq j \, . \end{cases}$$

The condition of equilibrium could be written as

$$\sigma_{ij,j} = -F_i \, . \tag{1.2}$$

The above equation represents the equation of motion in classical mechanics. Here, F_i are force components and, $\sigma_{ij,j} = \frac{\partial \sigma_{ij}}{\partial x_j}$.

Using equation (1.1) in equation (1.2), the equation of motion of an isotropic elastic medium can be written as given below

$$(\lambda + \mu) u_{j,ji} + \mu \, u_{i,jj} + F_i = 0 \, . \tag{1.3}$$

Equation of motion is derived from the equilibrium condition by adding components of inertial force per unit volume. Hence, the equation of motion in an isotropic elastic medium could be written as

$$(\lambda + \mu) u_{j,ji} + \mu \, u_{i,jj} + F_i = \rho \, \frac{\partial^2 u_i}{\partial t^2} \, . \tag{1.4}$$

Where u_i is the displacement component, $i (= 1,2,3)$. F_i represents the force components.

1.3 MICROPOLAR THEORY OF ELASTICITY

Micropolar theory addresses the limitations of classical elasticity by considering the effects of material microstructures, including particle rotations and couple stresses. It enables more precise modeling of materials like composites and foams that exhibit size-dependent behaviors. By accounting for internal micro-rotations, it provides better analysis for materials with complex microstructures, which classical elasticity cannot handle effectively. When attempting to characterize the properties of materials in which the microstructure does not play any part in the material's mechanical responses, the "theory of classical elasticity" can be of great assistance. But certain discrepancies have been perceived between the conclusions of classical theory and the results of empirical experimentation in many materials indicating that the microstructure of the material may have an impact on its mechanical response. The aforementioned inconsistencies serve to underscore the potential influence of the material's microstructure on its mechanical properties. Such discrepancies serve as proof that the material's microstructure may have an impact on its mechanical properties. Consequently, the examination of the material's mechanical properties necessitates a significant focus on the role played by the microstructures embedded within it. Therefore one must take into consideration the material's microstructure in order to make sense of the findings of an examination into the mechanical properties of such materials. Only then will the findings make sense. In an effort to find a solution to these discrepancies in classical theory of elasticity and identify a way to reconcile them, a number of authors have put out a wide variety of different theories relating to the continuum of micromechanics. Voigt (1887) tried to explain the discrepancies in classical theory and attempted to establish a theory of the micromechanics continuum while working under an array of assumptions. He came to the conclusion that the interaction that takes place between two particles of a substance is conveyed by the force vector along with the moment vector, that results in couple-stress in elasticity. The next stage of this process will include the beginning of an investigation into the microstructure of the substance that we are now studying.

After this, Cosserat and Cosserat (1909) proceeded to develop the fundamental "Theory of Micro-Mechanics." The proposed theory is reliant not solely on "The Theory of Linear Displacement," but also on an autonomous rotational motion exhibited by each material particle during the course of deformation. The theory under consideration is commonly known as the "Cosserat Theory of Elasticity," which was named as such by its originator. These hypotheses have demonstrated both the existence of couple-stress within the medium and the correlation between stress and strain. The observed outcome led to significant revisions in the constitutive equations, as well as various other aspects of classical elasticity.

Eringen and Suhubi (1964a, 1964b) formulated a comprehensive theory for nonlinear microelastic solids. They achieved this by augmenting the balancing laws of continuum mechanics with additional laws that consider the inherent motions of the microelements present within the macro volume. A microcontinuum is a continuous medium where each point is linked to a sub-continuum, allowing translation, rotation, and deformation. Eringen (1966) extended the asymmetric theory of elasticity and introduced micropolar theory of elasticity with body microinertia effects. The micropolar medium is a specific type of microcontinuum, wherein the associated sub-continuum can solely undergo translation and rotation, while the entire medium has the capacity for translation, rotation, and deformation. In other words, a micropolar elastic body is comprised of interconnected atoms or particles shaped as small rigid bodies capable of simultaneous translational and rotational motion. These particles have the autonomy to rotate independently. In the context to micropolar elasticity theory, the stress and strain tensors exhibit asymmetry that is, it no longer exhibit the property of symmetry anymore. This unique feature in micropolar continuum theory enables the incorporation of internal long-range cohesive forces while staying within the limits of the basic continuum framework. Fiber glass, solid propellant granules, and polymeric compounds are all examples of this category of material. It's possible that micro-cracks and micro-fractures can be explained by this notion as well.

The theory of micropolar elastic plates was conceived of and developed by Eringen (1967). Parfitt and Eringen (1969) conducted a study on plane wave propagation within a micropolar elastic half space, as well as their subsequent reflections on a stress-free flat surface. Furthermore, an examination was conducted on the propagation of plane waves as they traversed the given spatial domain. Based on the outcomes of several conducted studies, it has been observed that a micropolar elastic solid has the potential to facilitate the coexistence of four distinct waves, each characterized by its unique velocity. Below a critical frequency, two waves undergo a phenomenon wherein they cease to propagate due to their interaction within the medium they traverse. The current investigation presents a set of successive equations that pertain to non-linear theory. It is noteworthy that the linear theory can be obtained as a particular instance by imposing specific constraints related to the stability of local materials.

In the field of micropolar elasticity, Nowacki (1970, 1971) conducted a study on the axially symmetric problem and the second plane problem. Eringen (1999) studied the field of solid mechanics for the evaluation of the deformation of materials with microstructures. This theory takes into account 6 degrees of freedom, 3 translational elements, and 3 micro-rotational components in order to describe motion. Additionally, there are six micro rotational components.

As per the Eringen‘s (1966)“theory of micropolar elasticity”, the field equations for homogenous isotropic solids are given by

$$\sigma_{k\ell,k} + \rho\,(f_\ell - \ddot{u}_\ell) = 0, \tag{1.5}$$

$$m_{k\ell,k} + \in_{\ell mn} \sigma_{mn} + \rho\left(l_\ell - j\,\ddot{\phi}_\ell\right) = 0. \tag{1.6}$$

The stress tensor can be represented as

$$\sigma_{k\ell} = \lambda e_{rr}\delta_{k\ell} + (\mu+\kappa)e_{k\ell} + \mu e_{\ell k}\,, \tag{1.7}$$

$$m_{k\ell} = \alpha\,\gamma_{rr}\delta_{k\ell} + \beta\gamma_{k\ell} + \gamma\gamma_{k\ell}\,, \tag{1.8}$$

$$e_{k\ell} = (u_{\ell,k} - \epsilon_{k\ell m}\,\phi_{m,\ell}),\ \gamma_{k\ell} = \phi_{k,\ell}\,. \tag{1.9}$$

Using the above equation, the field equations can be written as

$$(\lambda + \mu)u_{k,k\ell} + (\mu + \kappa)u_{\ell,kk} + \kappa \in_{\ell mn} \phi_{n,m} + \rho(f_\ell - \ddot{u}_\ell) = 0, \tag{1.10}$$

$$(\alpha+\beta)\,\phi_{k,k\ell}+\gamma\,\phi_{\ell,kk}\;+\;\kappa\;\in_{\ell mn}u_{m,m}-2\,\kappa\,\phi_\ell+\rho\big(l_\ell-\;j\,\ddot{\phi}_\ell\big)=0. \tag{1.11}$$

Here $\alpha,\beta,\gamma,\kappa$ are the micro polar material constants. δ_{kl} is the Kronecker delta. $\sigma_{k\ell}$is micropolar stress and $m_{k\ell}$ represents couple stress tenser.j is the micro inertia density. ϕ_ℓ represents micro-rotation vector. ρ represents mass density, u_ℓ is the displacement vector. $\in_{\ell\mathrm{mn}}$ is the alternating symbol. Index after comma denotes the partial derivative.

Few of the latest studies on micropolar materials are as given here, Sharma and Kumar(2009), Sharma et al. (2009), Kaur et al. (2016), Kaur et al. (2017), Zhang et al. (2016), Kaur et al. (2016) , Barak and Kaliraman (2019), Singh et al. (2019), Khurana and Tomar (2022), Abo-Dahab et al. (2022), Singh and Kashyap (2023), Kumar et al. (2023), Sahu et al. (2023), Somaiah and Kumar (2023), Kumar and Pratap (2023).

1.3.1 MICROPOLAR THEORY OF THERMOELASTICITY

The fundamental study conducted by Nowacki (1966) investigated the impact of temperature on various phenomena, thereby contributing significantly to the advancement of the micropolar continuum concept and introduced the linear theory of micropolar thermoelasticity. Extensive advancements are covered by thermoelasticity. It is composed out of the theories of strains, stresses and heat transfer arising from the coupling of temperature and strain fields when subjected to heat flow. Thermoelasticity allows one to compute the temperature scattering resulting from the action of time-varying internal forces, as well as the stresses induced by the temperature field. Theory of classical thermoelasticity is unable to describe the material’s response to a quick transient loading and at low temperatures. Due to these limitations, numerous researchers have developed generalized theories of thermoelasticity. Then models featuring single or dual relaxation times, models centered on two temperatures, models without energy dissipation, dual-phase-lag theories, and explanation for anomalous heat conduction through fractional calculus have also been put forwarded. Biot (1956) developed the heat conduction equation based on the thermodynamics of irreversible processes that includes the dilatation term. It includes two equations:

a) Hyperbolic equation of motion.
b) Parabolic equation of heat conduction.

Anomalies that have been detected in the classical theory of thermoelasticity have been removed by Lord and Shulman (1967). Instead of using the Fourier law, they proposed a novel law of heat conduction, and this theory includes a new constant that serves as the relaxation period. The remaining field equations in this theory are the same as they are in coupled and uncoupled theories of thermoelasticity, but the heat equation is of wave type (hyperbolic). The theory of thermoelasticity with two relaxation times is another generalization of the linked theory. Green and Laws (1971) were able to generalize this inequality. Eringen (1970) made significant contributions to the field by expanding the existing theory and introducing the linear theory of micropolar thermal elasticity. Many researchers have studied the thermoelastic property in Micropolar materials and few of them are listed below: Tauchert and Claus (1968) , Dost and Tabarrok (1978) , Chandrasekhariah (1986), Sharma et al. (2007), Kumar et al. (2011), sharma et al. (2011), Zakaria (2012) , Kumar and Abbas (2013) , Kumar et al. (2016), Marin et al. (2019) , Othman et al. (2020), Lianngenga and Singh (2020) , Tarun (2022), Abouelregal et al. (2022), Sharma and Kumar (2023).

1.4 NON-LOCAL THEORY OF ELASTICITY:

The nonlocal elasticity theories were initially developed by Edelen and Laws (1971). The distinguishing factor among these various theories lies in the existence of nonlocality residuals within the fields. In accordance with the nonlocal theory of elasticity, it is posited that the stress experienced at any given point situated in a continuous body is dependent not solely upon the strain observed at that point, but also on the strains exhibited at all other sites within the body. Due to this phenomenon, it can be observed that the nonlocal stress forces functions as remote action forces. At extended wavelengths, the non-local theory converges to the classical local theory, while at abbreviated wavelengths it converges to the atomic lattice's dynamics. Eringen (1983) studied the differential equations of nonlocal elasticity and proposed few results for surface waves and screw dislocation.

The utilization of nonlocal elasticity theory has been widely implemented in the analysis of the flexural, vibrational, and buckling characteristics of 1-D nanostructure. Several researchers have incorporated nonlocal elasticity theory in their investigations of the diverse applications of micro and nanostructures. The nonlocal elasticity theory has been extensively employed by numerous scholars who recognize the significance of its application to structures at a micro-level.

In classical field theories, it is commonly observed that the ratio between the external characteristic length, represented by L (referred to as crack length, sample size or wavelength) and the internal characteristic length (generally referred as lattice parameter or size of the grain) denoted as l, tends to be significantly large. The aforementioned ratio is commonly denoted as the L/l ratio. The present findings provide evidence supporting the notion that accurate calculations can be achieved under the condition that the value of L exceeds that of l. Despite this, researchers are compelled to resort to non-classical theories when their local classical field theories (denoted as ($L \approx l$) are proven to be incorrect, necessitating the adoption of non-classical theories.
In the same way, in a dynamic problems if t_e and t_i denotes external and internal characteristic time respectively then $\left(\frac{t_e}{t_i}\right)$ is called as pertinent ratio. Where t_e is the period of variation. Consider, for instance, the amount of time that the external loads are applied. Here, t_i denotes travel time of the signal between the molecules. For example, the relaxation time. For $\frac{t_e}{t_i} \approx 1$, the classical theory fails to explain many physical phenomena. So there is a need to incorporate the non-local theories. Three possible kinds of non-locality are: mixed spatial-temporal, temporal, and spatial. Following Povstenko (1999), any physical quantity (referred to as effect-"s") at a given reference point $\boldsymbol{x}$ and

time t is dependent locally on a different physical quantity (referred to as cause-“q”) at same position and time could be represented by the following relation:

$$s(x_1, x_2, x_3, t) = s(q\,(x_1, x_2, x_3, t)) \tag{1.12}$$

Spatial non-locality could be defined as “effect” ($\bar{s}$) at time t and point $x$is dependent upon the “causes” from all other points x' at the same instant of time t.Therefore the nonlocal average or the spatial non-locality of a local field $s(x_1, x_2, x_3)$ within the domain V at the same time t could be written as:

$$\bar{s}(x_1, x_2, x_3, t) = \int_V \alpha_1\,(|x - x'|, \zeta)\; s(q(x_1', x_2', x_3', t))\; dx_1' dx_2'\; dx_3'. \tag{1.13}$$

Here, α_1 is the positive continuous function of the *spatial non-local kernels*. ζ is proportional to the characteristic length ratio $\left(\frac{L}{l}\right)$. In case of constitutive equation, the “effect” is referred as “stress” and the “cause” is referred as “strain”. The non-local consecutive equation (1.13) could be written as:

$$\sigma_{ij}^{nl}(x_1, x_2, x_3, t) = \int_V \alpha_1(|x - x^{'}|, \zeta) C_{ijkl}\epsilon_{kl}(x_1^{'}, x_2^{'}, x_3^{'}, t)\; dx_1^{'} dx_2^{'}\; dx_3^{'}. \tag{1.14}$$

Where σ_{ij}^{nl}denotes the non-local stress components. For long rage interactions, we use spatial non-locality.

In case of temporal non-locality (with time dependent memory), “effect” $\bar{s}$ at point x and time t relies on the history of “causes” at the same point x and at all proceeding times to the time t.

Mathematically,

$$\bar{s}(x_1, x_2, x_3, t) = \int_0^t \beta_1\,(t - t', \eta)\; s\left(q(x_1', x_2', x_3', t')\right) dt'. \tag{1.15}$$

Here β_1 represents the time dependent non-locality kernel. The parameter η is proportional to the characteristic time ratio $\left(\frac{t_e}{t_i}\right)$. So the time dependent constitutive equation could be represented as

$$\sigma_{ij}^{nl}(x_1, x_2, x_3, t) = \int_0^t \beta_1(t - t^{'}, \eta)\, C_{ijkl}\epsilon_{kl}(x_1, x_2, x_3, t^{'}) dt^{'}. \tag{1.16}$$

The above constitutive equation implies that the stress depends upon the past deformation also. When both spatial and temporal non-locality effects are accompanied, then it is known as mixed spatial-temporal non-locality. In this case the effect $\bar{s}$ at a position $\boldsymbol{x}$ and time t depends on the “causes” at all other positions $\boldsymbol{x}'$ and at all the proceeding time t'**.**

Mathematically,

$$\bar{s}(x_1, x_2, x_3, t) = \int_0^t \int_V \gamma_1 \, (|x - x'| , \quad t - t' , \zeta , \eta) \;\; s\left(q(x_1', x_2', x_3' , t')\right) dx_1' dx_2' \;\; dx_3' dt' . \tag{1.17}$$

Here γ' is the space-time non-locality kernel. Therefore the constitutive equation for mixed spatial-temporal non-locality could be expressed as:

$$\sigma_{ij}^{nl}(x_1, x_2, x_3 , t) = \int_0^t \int_V \gamma_1(|x - x'| \, t - t' , \zeta , \eta) \, C_{ijkl} \epsilon_{kl}(x_1', x_2', x_3' , t') \, dx_1' dx_2' \;\; dx_3' dt' . \tag{1.18}$$

As, $\eta \rightarrow 0$, the spatial and temporal non-locality effect disappears.

PROPERTIES OF SPATIAL NON-LOCAL KERNEL:

- $\alpha_1 \, (|x - x'|, \zeta)$ has a maximum at $|\boldsymbol{x} - \boldsymbol{x}'|$.
- $\alpha_1(|x - x'|, \zeta) \rightarrow 0$ with increase in $|x - x'|$.
- $\alpha_1(|x - x'|, \zeta)$ is continuous function of $|x - x'|$.
- $\alpha_1(|x - x'|, \zeta)$ represents a delta-sequence which tends to direct $\delta -$ function as $\zeta \rightarrow 0$ i.e. $\lim_{\zeta \rightarrow 0} \alpha_1(|x - x'|, \zeta) = \delta \, (\, |x - x'|)$
- $\int_V \alpha_1(|x - x'|, \zeta) \, dV(x') = 1$.

The properties of other kernels are similar.

The theory of nonlocal elasticity resolves the shortcomings of classical elasticity by incorporating long-range interactions between material points. This provides more accurate modeling of small-scale materials, such as nanomaterials, where size effects are important. It improves the understanding of phenomena like stress concentration and size-dependent mechanical behaviors that classical elasticity cannot capture.

1.4.1 THEORYOF NON-LOCAL MICROPOLAR ELASTICITY:

Eringen (1984) extended the "theory of non-local elasticity" to the Micropolar materials. He developed the dispersion relations in linear, non-local micropolar elastic solids for transverse plane waves. Non-local micropolar theory takes micropolar theory a step further by considering non-local interactions between different points within the material. Unlike "classical continuum mechanics", at a given point the deformation is influenced solely by its immediate neighborhood whereas non-local micropolar theory incorporates the influence of deformation at all other points within the material. However, the influence decreases with increasing distance. This non-local interaction effect enables a more accurate representation of materials with long-range interactions or small-scale features. At the sub molecular or atomic level, it is observed that materials exhibiting elastic properties invariably exhibit a discernible internal structure. The traditional classical theory is deemed inaccurate whenever the internal and external scales are situated within a context that can be likened to the contrast between them. Non-local micropolar theory finds applications in various areas, including modeling granular materials, biological tissues, and heterogeneous materials. It provides a comprehensive framework for analyzing the behavior of these materials, considering their microstructural characteristics and non-local interactions.

In conclusion, non-local micropolar theory is a valuable approach for studying the material's mechanical behavior with complex microstructures and long-range interactions. By incorporating these effects, engineers and researchers can gain deeper insights into the behavior of such materials, leading to improved designs and a better understanding of their mechanical properties. So, Non-local micropolar theory enhances traditional micropolar theory by considering interactions over a finite distance between material points, rather than just local interactions. This allows for a more precise representation of materials with microstructures, capturing size effects and long-range influences. As a result, it provides better modeling of wave dispersion and stress distribution, particularly in materials where microstructural effects are significant.

By using Eringen's non-local formulation (1972, 1973, 1976), "the theory of non-local Micropolar elasticity" has been stated by Eringen (1984). For homogeneous and isotropic solids, the basic equations may be represented as

$$\sigma^{nl}_{k\ell,k} + \rho\,(f_\ell - \ddot{u}^n_\ell) = 0, \tag{1.19}$$

$$m^{nl}_{k\ell,k} + \epsilon_{\ell mn}\,\sigma_{mn} + \rho\big(l_\ell - j\,\ddot{\phi}_\ell\big) = 0, \tag{1.20}$$

$$\sigma^{nl}_{k\ell} = \int[\lambda'\, e'_{rr}\delta_{k\ell} + (\mu' + \kappa')e'_{k\ell} + \mu' e'_{\ell k}]\,dv', \tag{1.21}$$

$$m^{nl}_{k\ell} = \int[\alpha'\,\gamma'_{rr}\delta_{k\ell} + \beta'\,\gamma'_{kl} + \gamma'\gamma'_{k\ell}]\,dv', \tag{1.22}$$

$$e'_{k\ell} = \frac{\partial u^n_\ell(\boldsymbol{x}',t)}{\partial x'_k} - \epsilon_{k\ell m}\,\frac{\partial \phi_m(\boldsymbol{x}',t)}{\partial x'_l}, \tag{1.23}$$

$$\gamma_{k\ell} = \frac{\partial \phi_k(\boldsymbol{x}',t)}{\partial x'_l}. \tag{1.24}$$

where, $\sigma^{nl}_{k\ell}$ = non-local stress tensor, u^n_ℓ = displacement vector, ρ = mass density, f_ℓ = body force, ϕ_ℓ = microinertia vector, j = microinertia density, $m^{nl}_{k\ell}$ = non-local couple stress tensor, l_ℓ = body couple density. λ', μ', α', β' and γ' represents the non-local material moduli which is dependent on the distance $|x - x'|$. $\delta_{k\ell}$ is the Kronecker delta. $\in_{\ell mn}$ is the alternating symbol. Index after comma denotes the partial derivative and dot (·) represents the partial derivative w.r.t time. The sole distinction between the local and non-local theories lies in the equations (1.21) and (1.22). These equations specify the stress and couple stress at a point x within a body are contingent upon the strain measures $e_{k\ell}$ and $\gamma_{k\ell}$ at all points x' of the body. Eringen (1984) used the special case of nonlocality could be considered for the simplicity of mathematical problems

$$\{\lambda', \mu', \kappa', \alpha', \beta', \gamma'\} = \alpha_1(|x - x'|)\{\lambda, \mu, \kappa, \alpha, \beta, \gamma\} \tag{1.25}$$

where, $\kappa, \alpha, \beta, \gamma$	=	local Micropolar material constants
λ, μ	=	Classical Lame's constant.
$\alpha_1(|x - x'|)$	=	Attenuation function

Here, the attenuation function is calculated as per the distance of the material point $\boldsymbol{x}'$ and $\boldsymbol{x}$. The greater the distance the lesser is the effect. This function shows its maximum value when $\boldsymbol{x}' = \boldsymbol{x}$. Also, it must be a Dirac-Delta sequence which depends upon the internal characteristic length a. From the equation (1.21) and (1.22), it is could be written that,

$$\lim_{a \to 0} \alpha_1 (|x - x'|, a) = \delta |x - x'|. \tag{1.26}$$

This study worked on nonlocal elasticity and predicted the wave dispersion and other phenomena which are in accordance with the atomic theories. In two dimensions, the representation could be given as:

$$\alpha_1(|x - x'|, a) = (2\pi \epsilon^2)^{-1} \kappa_0[|x - x'|/\varepsilon] \tag{1.27}$$

Here, κ_0= modified Bessel's function, and $\varepsilon = e_0 l$ (1.28)

Here, l= internal characteristic length, e_0= material constant.

For infinite plane, Eringen (1984) proposed a special case using Green's function which can be written as follows

$$(1 - \in^2 \nabla^2)\alpha_1 = \delta |x - x'|. \tag{1.29}$$

So applying the operator $(1 - \in^2 \nabla^2)$ on equation (1.21) and (1.22), we get,

$$(1 - \varepsilon^2 \nabla^2) \sigma_{k\ell}^{nl} = \sigma_{k\ell} = \lambda e_{rr} \delta_{k\ell} + (\mu + \kappa) e_{k\ell} + \mu e_{\ell k}, \tag{1.30}$$

$$(1 - \varepsilon^2 \nabla^2)\, m_{k\ell}^{nl} = m_{k\ell} = \alpha\, \gamma_{rr} \delta_{k\ell} + \beta \gamma_{k\ell} + \gamma \gamma_{k\ell}, \tag{1.31}$$

$$e_{k\ell} = (u_{\ell,k}^n - \epsilon_{k\ell m}\, \phi_{m,\ell}), \gamma_{k\ell} = \phi_{k,\ell}. \tag{1.32}$$

Using the above equation, the field equations can be written as

$$(\lambda + \mu)u^n_{k,k\ell} + (\mu + \kappa)u^n_{\ell,kk} + \kappa \in_{\ell mn} \phi_{n,m} + \rho(1 - \varepsilon^2\nabla^2)(f_\ell - \ddot{u}^n_\ell) = 0, \quad (1.33)$$

$$(\alpha + \beta)\, \phi_{k,k\ell} + \gamma\, \phi_{\ell,kk} + \kappa \in_{\ell mn} u^n_{n,m} - 2\,\kappa\, \phi_\ell + \rho(1 - \varepsilon^2\nabla^2)\big(l_\ell - j\, \ddot{\phi}_\ell\big) = 0. \quad (1.34)$$

Where ε is a non-local parameter $(\varepsilon = e_0 l)$, l characteristic length, $\alpha, \beta, \gamma, \kappa$ are the micro polar material constants. δ_{kl} is the Kronecker delta. $\sigma_{k\ell}$, $m_{k\ell}$ are local micropolar stress and couple stress tenser. $\sigma^{nl}_{k\ell}$, $m^{nl}_{k\ell}$ represents the non-local micropolar stress and non-local couple stress tenser. j is the micro inertia density. ϕ_ℓ is the micro-rotation vector. ρ is the mass density, u_ℓ is the displacement vector. $\in_{\ell \mathrm{mn}}$ is the alternating symbol. Index after comma denotes the partial derivative. Numerous authors have made significant contributions to the advancement of the non-local theory. The development of this theory was undertaken by a diverse group of authors. Few of the researches are mentioned here. The study conducted by Nowinski (1993) explored the investigation of Eringen's theory pertaining to micromorphic bodies. In doing so, Nowinski successfully derived the equilibrium equation for linear isotropic micropolar and microstretch bodies featuring nonlocal cohesion. Following papers could be referred for elastodynamical problems in non-local Micropolar media. Tien-min (1980), Hsieh (1982), Kaliski et al. (1992), Jun and Dhaliwal (1993),Trovalusci, and Masiani (2003), Acharya (2004), Huang et al. (2005), Lazar and Kirchner (2006), Chakraborty (2007), Khurana and Tomar (2013, 2017), Ding et al.(2016), Mondal et al. (2019), Kalkal et al. (2020), Kumar and Tomar (2020),Sahrawa et al. (2020), Deswal et al. (2021), Poonam et al. (2021), Ceballes et al.(2021), Sheoram et al. (2022), Shorkin et al. (2023).

1.5 THEORY OF PIEZOELECTRICITY

Piezoelectricity is the effect through which electrical energy is generated from mechanical energy in certain materials. There are some natural and manmade materials which exhibit such phenomena such as Quartz, Rochelle salt and ceramics like lead zirconate etc. Piezoelectric materials have been used in many appliances such as powerful sonar, microphones and sonobuoys etc. The study of composite structures containing a piezoelectric material is in demand nowadays due to their remarkably different physical and chemical properties and applications in sensor technology. By using ideal combination of control elements in piezoelectric structures, we can achieve the effective control of electromechanical coupling. The wave propagation in composite materials has been used extensively in sensor technology and nondestructive testing techniques to determine strength of materials. In the study by Qian et al. (2004), the constitutive relations and equations of motion for a homogeneous transversely isotropic piezoelectric medium are formulated to account for the material's specific anisotropy and symmetry properties. When explaining piezoelectricity in such materials, it's important to note that anisotropy refers to the directional dependence of properties, and in the case of transversely isotropic piezoelectric materials (such as crystal class 6mm and certain piezoelectric ceramics), the x_3-axis is typically the axis of symmetry. This symmetry plays a crucial role in determining how the material responds to mechanical and electrical fields, influencing the form of the constitutive equations. These equations reflect the coupling between mechanical stresses and electric fields, and are expressed in terms of specific components that respect the symmetry of the material.

The constitutive relations and equation of motion for a homogeneous transversely isotropic piezoelectric medium given by Qian et al. (2004)are given as given

$$\begin{cases}\tau_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \\ D_j = e_{jkl}S_{kl} + \epsilon_{jk}E_k.\end{cases} \tag{1.35}$$

$$\begin{cases}\tau_{ij,j} = \rho'\ddot{u_i}, \\ D_{i,i} = 0.\end{cases} \tag{1.36}$$

Here τ_{ij} represents stress tensor, D_j is the electric displacement, e_{kij} , ϵ_{jk} and c_{ijkl} represents piezoelectric, dielectric and elastic constants respectively. Density of piezoelectric material is represented by ρ'. u is the mechanical displacement. S_{kl} and E_k are the strain tensor and electric field intensity and can be denoted in terms of displacement u and electric potential φ as

$$S_{ij} = \frac{u_{i,j} + u_{j,i}}{2}, \tag{1.37}$$

$$E_i = -\partial\varphi/\partial x_i. \tag{1.38}$$

Following papers can be referred for various types of problems in piezoelectric materials. Curtis and Redwood (1973), Li et al. (2001) , Sharma et al. (2004), Jin et al. (2005), Sharma et al. (2005), Sharma et al. (2008), Walia et al. (2009), Sharma et al. (2010), Liu and He (2010a), Liu and He (2010b), Sharma et al. (2011), sharma et al. (2011) , Huang and Li (2011), Abd-alla et al. (2012), Cui et al. (2013), Liu et al (2013), Li and Jin (2015), Yanping Kong and Nie (2015), Lee et al. (2016), Goyal (2020) ,Goyal et al. (2020), Sharma and Kumar (2022), Sharma et al.(2023).

1.6 WAVES IN ELASTIC MEDIA

1.6.1 BODY WAVES:

Body waves travel through the medium's interior. It exhibits shorter wavelengths and smaller amplitudes compared to surface waves. It also possesses a higher velocity of propagation. The waves mentioned can be categorized into two distinct types: Longitudinal waves and Transverse waves.

a) LONGITUDINAL WAVES:

Longitudinal waves, also called as dilatational/irrational or compressional waves. These are such type of waves that travels through the medium so they are associated with the refraction and compression of the particles. These waves are the fastest waves and are first to appear on the seismograms. These waves are also known as primary waves or P-waves in seismology.

b) TRANSVERSE WAVES :

Transverse waves, also known as equivoluminal/shear/rotational waves and are characterized by the rotational and shearing motion of particles as the wave propagates through a medium, while maintaining a constant volume. Within these waves, particles undergo vibrations that occur in a direction ⊥ to the wave's propagation. These waves are also known as S- waves or secondary waves in seismology. These particular waves are categorized as SH-waves and SV-waves. When the particles' motion exhibits polarization exclusively in the horizontal plane, it is referred to as shear horizontal (SH) waves. When vibrational motion is confined to the vertical plane of the particles, it designates the waves as vertically polarized shear (SV) waves. Both transverse and longitudinal waves have the ability to propagate through a solid medium.

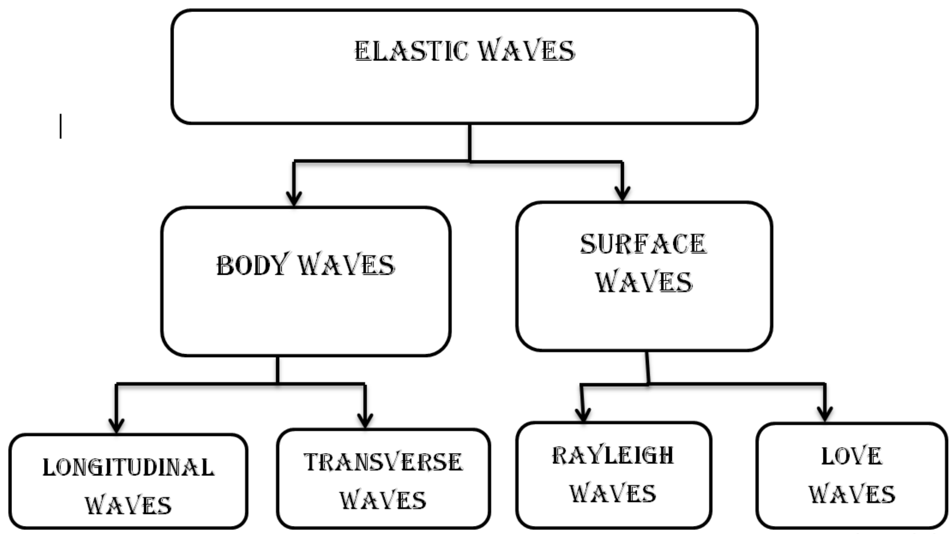

Fig. 1.3: Types of elastic waves

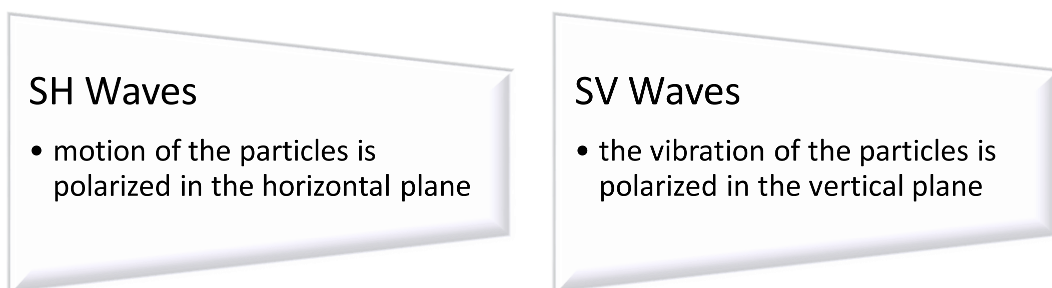

Fig. 1.4 SH- waves, SV - waves

1.6.2 SURFACE WAVES:

Surface waves are waves that are able to travel along the free surface of a bounded medium and are given their own category. Since surface waves travel at a velocity that is lower than that of body waves, when seismic activity occurs, surface waves are recognized on the seismogram after body waves have been recorded. Love and Rayleigh waves are two types of surface waves that can be encountered.

a) LOVE WAVES:

Love waves have the ability to propagate through layered structures that are composed of a distinct layer with a finite thickness that is firmly attached to a substrate. The Love wave's propagation is contingent upon condition that the velocity of the layer is lower than the velocity of the half-space. In the context of these waves, it is observed that the particles exhibit transverse vibrations that are parallel to the surface. The investigation of Love waves was conducted by A.E.H Love (1911). Love waves, a type of surface wave, have found utility in different applications such as sensors and non-destructive testing techniques. These waves have been extensively studied by researchers including Tamarin et al. (2003). Midya (2004) studied the love type waves in homogeneous micropolar isotropic elastic media consisting of layer over a semi-infinite medium. Sharma et al. (2020) studies the love waves in layered media. The dispersion relation for Love-waves in a piezomagnetic layered pair stress substrate under viscous liquid loading was developed by Sharma and Kumar (2021). Following studies could also be referred. Papadakis et al. (2009), Kuznetsov (2010), Zhu et al. (2014), Qingzeng et al. (2014), Saha et al. (2015), Kaur et al. (2019), Goya and Kumar (2019), Nobili and Volpini (2021), Manna et al. (2022), Sharma and Kumar (2022), Singh et al. (2023), Hrytsyna et al. (2023).

b) RAYLEIGH WAVES:

Lord Rayleigh (1887) studied about the waves that travel along the homogeneous half-space in free-surface. Due to the fact that surface waves have a tendency to cause damage during earthquakes, seismology places a particularly great emphasis on their study. Rayleigh waves are a type of surface wave that have been the subject of investigation by a number of different researchers. Since Rayleigh waves in micropolar elastic materials have potential practical applications in a variety of fields, including seismology, acoustics, aerospace, and undersea structures, they have been the subject of investigation by a large number of researchers. Deresiewicz (1961) analyzed the Rayleigh waves for thermoelastic solid. Chimenti et al. (1982) studied the Rayleigh waves on a layered half space. Smith and Dahlen (1973) studied the Rayleigh waves in an anisotropic medium. Tolipov (2002) investigated the Rayleigh waves in an elastic wedge. Vashishth and

Khurana (2005) studied the Rayleigh waves in anisotropic, heterogeneous poroelastic layer. Vinh (2009) investigated the Rayleigh waves in elastic medium influenced by initial stress. Zhang et al. (2014) analyzed Rayleigh wave's propagation in magneto elastic half space. Ozisik (2021) investigated the Rayleigh waves in the pre-stressed layers under complete contact. Mrithymjayo and Reddy (1993), Kumar and Singh (1996), Tomar (2005), Kumar and Deswal (2006), Kumar and Pratap (2006), Sharma and Kumar (2009), Zhang et al. (2015), Singh et al. (2016) can be referred to understand the Rayleigh wave's propagation in micropolar elastic material.

The choice of Love and Rayleigh waves for studying non-local micropolar materials is both practical and innovative. Love waves are vital for applications in geophysics and earthquake engineering, as they help analyze surface vibrations and subsurface structures. Rayleigh waves are significant for surface inspection and non-destructive testing, providing insights into material responses to surface stresses. By applying non-local micropolar theory, the research can better account for the effects of microstructures and size-dependent behaviors that traditional theories might overlook. This approach promises to improve the accuracy of wave propagation models and advance the understanding of material behavior in engineering and materials science.

1.7 RESEARCH GAP:

Classical elasticity theory struggles to account for size-dependent material responses, prompting the development of microcontinuum theories with additional parameters. While micropolar elasticity has been extensively studied in contexts like wave propagation and various effects (thermal, magnetic, etc.), the non-local version of this theory is less explored. Non-local elasticity has shown promise in solving fracture mechanics problems, but its application to micropolar materials remains under-researched. This study aims to address this gap by examining elastodynamic issues within the non-local micropolar elasticity framework. Utilizing mathematical approaches, we will explore wave propagation problems in non-local micropolar elastic materials.

1.8 OBJECTIVES OF THE WORK:

1. Comprehensive and in-depth study of the literature to acquire the knowledge to construct mathematical models in non-local micropolar theory of elasticity.
2. To formulate the problem related to wave propagation using non-local micropolar theory and to solve them by using suitable mathematical methods.
3. To study the effects of inner microstructures and other additional material parameters in two dimensional problems in non-local micropolar elasticity under different types of boundary conditions.

CHAPTER- 2

SHEAR WAVES PROPAGATION IN LAYERED STRUCTURE HAVING NON-LOCAL MICROPOLAR/PIEZOELECTRIC MATERIALS[1]

2.1. INTRODUCTION:

Many researchers studied the phenomenon of wave propagation in composite materials consisting of piezoelectric and elastic materials. Tiersten (1963) investigated the pure piezoelectric materials and studied about the thickness vibrations in these materials. Piezoelectric materials in purely elastic homogeneous material has been investigated by Bleustein (1968) and found that that there exists a new type surface waves in the considered structure. Liu et al. (1973) studied the dispersive behavior of shear waves in the piezoelectric layer and elastic half space with an imperfect interface between them. Qian et al. (2004) worked on the dispersion relation of SH-waves propagating in piezoelectric composite layered structure. Many more studies has been done in the field of piezoelectricity, few of them has been mentioned here. Mindlin (1952), Wang and Zhao (2013), Wang and Jin (2016), Chenlin et al., Kumar et al. (2018), Kumar et al. (2019), Sharma and Kumar (2021), Yang et al. (2021) , Shatalov et al. (2021) etc.

Studying anti-plane waves in layered structures is important for understanding how waves interact with different materials stacked together. Layered materials are frequently used in applications such as advanced composites, coatings, and electronic devices. By examining how anti-plane waves propagate through these structures, we can optimize their design for better performance and functionality. Additionally, this research supports

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the development of precise models that improve material design and engineering, and it advances theoretical knowledge by exploring how materials with unique properties, such as non-local micropolar or piezoelectric materials, affect wave propagation.

In the present chapter, a composite structure consisting of homogeneous, isotropic non-local micropolar elastic half space in perfect contact with a piezoelectric layer has been considered to investigate surface wave propagation. An analysis has been performed to explore how various parameters influence the phase-velocity of shear waves. The dispersion relation of shear waves in closed form is obtained analytically. Numerical computation for phase-velocity is carried out and the results are illustrated graphically.

2.2 FORMULATION OF THE PROBLEM AND ITS SOLUTION

In this problem, a composite structure consisting of layer of piezoelectric material of thickness "h" lying in perfect contact over a non-local micropolar elastic half space. The origin "O" of the Cartesian coordinate system is considered to be at the joining surface of piezoelectric layer and the substrate as shown in the fig.-2.1. The piezoelectric layer is polarized along x_3-axis direction perpendicular to $x_1 - x_2$ plane. The x_2-axis is taken positive in the vertically downward direction and shear waves are considered as propagating in x_1 direction so the displacement components will be free from x_3 coordinates.

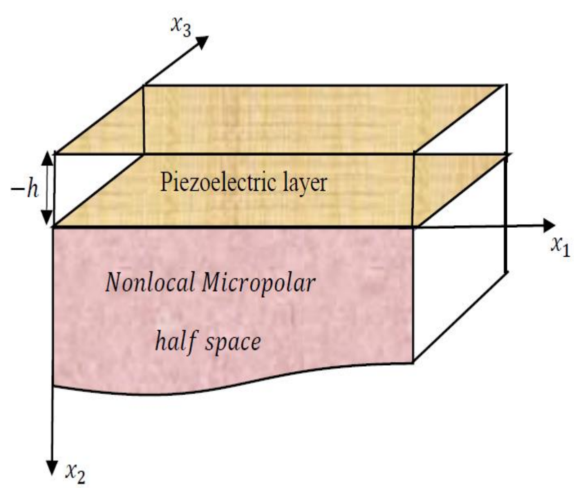

Fig.2.1 Geometry of the problem

Let $\boldsymbol{u^n} = (u_1^n, u_2^n, u_3^n)$ and $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ denotes the displacement and micro-rotation vector components in non-local micropolar elastic half space respectively. Also, $\boldsymbol{u} = (u_1, u_2, u_3)$ and φ represents displacement components and electric field potential respectively in the upper piezoelectric layer.

As shear wave propagates in x_1-direction, causing displacement in x_3 -direction so the components of displacement and microrotation for non-local Micropolar material can be expressed as:

$$u_1^n = u_2^n = 0, u_3^n = u_3^n(x_1, x_2, t), \phi_1 = \phi_1(x_1, x_2, t),\ \phi_2 = \phi_2(x_1, x_2, t),\ \phi_3 = 0. \tag{2.1}$$

The displacement component for piezoelectric layer can be taken as:

$$u_1 = u_2 = 0, u_3 = u_3(x_1, x_2, t), \tag{2.2}$$

2.2.1. SOLUTION OF NON-LOCAL MICROPOLAR ELASTIC HALF SPACE

As per the Eringen‘s (1984) “theory of non-local micropolar elasticity”, the field equations for homogenous, isotropic solids are given by

$$\sigma^{\mathrm{nl}}_{\mathrm{k}\ell,\mathrm{k}} + \rho\,(\mathrm{f}_\ell - \ddot{\mathrm{u}}^{\mathrm{n}}_\ell) = 0, \tag{2.3}$$

$$m^{nl}_{k\ell,k} + \in_{\ell mn} \sigma_{mn} + \rho\big(l_\ell - j\,\ddot{\phi}_\ell\big) = 0. \tag{2.4}$$

Here, $\sigma^{nl}_{\mathrm{k}\ell}$ is the force stress tensor, f_ℓ is the body force density, $\mathrm{m}^{nl}_{\mathrm{k}\ell}$ is the couple stress tensor, l_ℓ is the body couple density. j is the micro inertia density. ϕ_ℓ is the micro-rotation vector. ρ is the mass density, u^n_ℓ is the displacement vector. $\in_{\ell\mathrm{mn}}$ is the alternating symbol. Index after comma denotes the partial derivative.

By using Eringen’s non-local formulation as given explained in section (1.4.1) in chapter 1, the non-local stress tensor can be represented in terms of local stress tensor as

$$(1 - \varepsilon^2 \nabla^2)\sigma^{nl}_{k\ell} = \sigma_{k\ell} = \lambda e_{rr}\delta_{k\ell} + (\mu + \kappa)e_{k\ell} + \mu e_{\ell\mathrm{k}}, \tag{2.5}$$

$$(1 - \varepsilon^2 \nabla^2)m^{nl}_{k\ell} = m_{k\ell} = \alpha\,\gamma_{rr}\delta_{k\ell} + \beta\gamma_{k\ell} + \gamma\gamma_{k\ell}, \tag{2.6}$$

$$e_{k\ell} = (u^n_{\ell,k} - \epsilon_{k\ell m}\,\phi_{m,\ell}), \gamma_{k\ell} = \phi_{k,\ell}. \tag{2.7}$$

Where the symbols have their usual meanings as explained in the section (1.4.1) in chapter 1.

Using equations (2.5)-(2.7) in equations (2.3)-(2.4) the field equations can be written as

$$(\lambda + \mu)u^n_{k,k\ell} + (\mu + \kappa)u^n_{\ell,kk} + \kappa \in_{\ell mn} \phi_{n,m} + \rho(1-\varepsilon^2\nabla^2)(f_\ell - \ddot{u}_\ell) = 0, \tag{2.8}$$

$$(\alpha+\beta)\,\phi_{k,k\ell} + \gamma\phi_{\ell,kk} + \kappa \in_{\ell mn} u^{\mathrm{n}}_{\mathrm{n},m} - 2\,\kappa\phi_\ell + \rho(1-\varepsilon^2\nabla^2)\left(l_\ell - j\ddot{\phi}_\ell\right) = 0. \tag{2.9}$$

It can be noted here that in the absence of body forces and the nonlocal Micropolar parameterε, the system reduced to those of local micropolar theory.

Hence, equations (2.8) and (2.9) reduce to following equations without applied body force and body couple densities using equation (2.1) and (2.2).

$$(\mu + \kappa)\nabla^2 u^n_3 + \kappa\left(\frac{\partial\phi_2}{\partial x_1} - \frac{\partial\phi_1}{\partial x_2}\right) = \rho(1-\varepsilon^2\nabla^2)\frac{\partial^2 u^n_3}{\partial t^2}, \tag{2.10}$$

$$(\alpha+\beta)\,\frac{\partial}{\partial x_1}\left(\frac{\partial\phi_1}{\partial x_1} + \frac{\partial\phi_2}{\partial x_2}\right) + \gamma\nabla^2\phi_1 - 2\kappa\phi_1 + \kappa\frac{\partial u^n_3}{\partial x_2} = \rho j(1-\varepsilon^2\nabla^2)\frac{\partial^2\phi_1}{\partial t^2}, \tag{2.11}$$

$$(\alpha+\beta)\,\frac{\partial}{\partial x_2}\left(\frac{\partial\phi_1}{\partial x_1} + \frac{\partial\phi_2}{\partial x_2}\right) + \gamma\nabla^2\phi_2 - 2\kappa\phi_2 - \kappa\frac{\partial u^n_3}{\partial x_1} = \rho j(1-\varepsilon^2\nabla^2)\frac{\partial^2\phi_2}{\partial t^2}. \tag{2.12}$$

To decouple the equation (2.10)-(2.12) introducing the potential functions ψ and ξ as

$$\phi_1 = \frac{\partial\psi}{\partial x_1} + \frac{\partial\xi}{\partial x_2} \;\; and \;\; \phi_2 = \frac{\partial\psi}{\partial x_2} - \frac{\partial\xi}{\partial x_1}. \tag{2.13}$$

On substituting equation (2.13) in (2.10)-(2.12), we obtained

$$\nabla^2 u^n_3 - c_1\nabla^2\xi = (1-\varepsilon^2\nabla^2)\frac{1}{c_2^2}\frac{\partial^2 u^n_3}{\partial t^2}, \tag{2.14}$$

$$\nabla^2\psi - \frac{2c_5^2}{c_3^2 + c_4^2}\,\psi = (1-\varepsilon^2\nabla^2)\frac{1}{c_3^2+c_4^2}\frac{\partial^2\psi}{\partial t^2}, \tag{2.15}$$

$$\nabla^2\xi - \frac{2c_5^2}{c_3^2}\xi + \frac{c_5^2}{c_3^2}u^n_3 = (1-\varepsilon^2\nabla^2)\frac{1}{c_3^2}\frac{\partial^2\xi}{\partial t^2}. \tag{2.16}$$

where$c_1 = \frac{\kappa}{\mu+\kappa}$,$c_2 = \sqrt{\frac{\mu+\kappa}{\rho}}$, $c_3 = \sqrt{\frac{\gamma}{\rho j}}$, $c_4 = \sqrt{\frac{\alpha+\beta}{\rho j}}$ and $c_5 = \sqrt{\frac{\kappa}{\rho j}}$.

The wave solution of equation (2.14)-(2.16) can be expressed as

$$(\psi,\xi,u^n_3) = (\psi,\xi,u^n_3)(x_2)e^{\iota(kx_1-\omega t)}. \tag{2.17}$$

In this context, ω is expressed as $\omega = k$c, where c signifies the phase-velocity of the wave and k represents the wave number.

Using the equation (2.17) in equations (2.14)-(2.16), we obtained,

$$(D^2 - r^2)\psi(x_2) = 0. \tag{2.18}$$

$$(D^4 - PD^2 + Q)(u_3^n, \xi)(x_2) = 0. \tag{2.19}$$

where $r^2 = \dfrac{k^2 + \frac{2c_5{}^2}{c_3{}^2 + c_4{}^2} - \frac{\omega^2}{c_3{}^2 + c_4{}^2} - \frac{\omega^2\varepsilon^2 k^2}{c_3{}^2 + c_4{}^2}}{\left(1 - \frac{\omega^2\varepsilon^2}{c_3{}^2 + c_4{}^2}\right)}$,

$$P = \frac{\left(k^2 - \frac{\omega^2}{c_2{}^2}\right) + \left(k^2 - \frac{\omega^2}{c_3{}^2}\right) + \frac{c_5{}^2}{c_3{}^2}(2 - c_1) - \frac{2\omega^2\varepsilon^2k^2}{c_3{}^2} - \frac{2\omega^2\varepsilon^2c_5{}^2}{c_2{}^2c_3{}^2} + \frac{2\omega^4\varepsilon^2}{c_2{}^2c_3{}^2} + \frac{2\omega^4\varepsilon^4k^2}{c_2{}^2{c_3}^2} + \frac{\omega^4\varepsilon^2k^2}{c_2{}^2{c_3}^2}}{\left(1 - \frac{\varepsilon^2\omega^2}{c_3{}^2} + \frac{\varepsilon^4\omega^4}{c_2{}^2c_3{}^2} - \frac{\varepsilon^2\omega^2}{c_2{}^2}\right)},$$

$$Q = \frac{k^2 - \frac{\omega^2}{c_3{}^2} + \frac{2c_5{}^2}{c_3{}^2} - \frac{c_1c_5{}^2k^2}{c_3{}^2} - \frac{\omega^2\varepsilon^2k^4}{c_3{}^2} + \frac{2\omega^4\varepsilon^2k^2}{c_2{}^2{c_3}^2} - \frac{\omega^2\varepsilon^2k^4}{c_2{}^2} - \frac{2\omega^2\varepsilon^2kc_5{}^2}{c_2{}^2c_3{}^2} + \frac{\omega^4\varepsilon^4k^4}{c_2{}^2{c_3}^2}}{\left(1 - \frac{\varepsilon^2\omega^2}{c_3{}^2} + \frac{\varepsilon^4\omega^4}{c_2{}^2c_3{}^2} - \frac{\varepsilon^2\omega^2}{c_2{}^2}\right)}.$$

We consider the above calculation under the following assumption,

$$\frac{\omega^2\varepsilon^2}{c_3{}^2 + c_4{}^2} \neq 1, \frac{\varepsilon^2\omega^2}{c_3{}^2} + \frac{\varepsilon^4\omega^4}{c_2{}^2c_3{}^2} - \frac{\varepsilon^2\omega^2}{c_2{}^2} \neq 1, \frac{\varepsilon^2\omega^2}{c_3{}^2} + \frac{\varepsilon^4\omega^4}{c_2{}^2c_3{}^2} - \frac{\varepsilon^2\omega^2}{c_2{}^2} \neq 1.$$

Using radiation condition $(u_3^n, \xi, \psi)(x_2) \longrightarrow 0 \; as \; x_2 \longrightarrow \infty$, the general solution of equations (2.18)-(2.19) can be written as

$$\psi = (S\, e^{-r\, x_2})e^{\iota(kx_1 - \omega t)}, \tag{2.20}$$

$$\xi = (Me^{-p\, x_2} + Ne^{-q\, x_2})e^{\iota(kx_1 - \omega t)}, \tag{2.21}$$

$$u_3^n = (Mq_1e^{-p\, x_2} + Nq_2e^{-q\, x_2})e^{\iota(kx_1 - \omega t)}. \tag{2.22}$$

Where $p, q = \sqrt{\frac{P \pm \sqrt{P^2 - 4Q}}{2}}$, $p^2 + q^2 = P, p^2q^2 = Q$,

$$q_1 = \frac{c_3{}^2}{c_5{}^2}\left(k^2 - \frac{\omega^2}{c_3{}^2} + 2\frac{c_5{}^2}{c_3{}^2} - p^2\right), q_2 = \frac{c_3{}^2}{c_5{}^2}\left(k^2 - \frac{\omega^2}{c_3{}^2} + 2\frac{c_5{}^2}{c_3{}^2} - q^2\right).$$

and S, M and N are arbitrary constants. The components of microrotation are obtained from equation (2.12) by using equations (2.19)-(2.20) as

$$\phi_1 = (\iota kSe^{-rx_2} - p\,Me^{-p\,x_2} - q\,Ne^{-q\,x_2})e^{\iota(kx_1-\omega t)}, \tag{2.23}$$

$$\phi_2 = (-rSe^{-Rx_2} - \iota k(Me^{-p\,x_2} + Ne^{-q\,x_2})e^{\iota(kx_1-\omega t)}. \tag{2.24}$$

The expressions (2.20) - (2.24) are obtained under the assumption that $c < c_2$ and $c < c_3$. The wave corresponding to $c > c_2$ is represents the refracted wave which lose their energy very quickly and hence not significant.

2.2.2 SOLUTION OF PIEZOELECTRIC LAYER

The constitutive relations and equation of motion for a homogeneous transversely isotropic piezoelectric medium as given by Qian et al. (2004) are

$$\begin{cases}\tau_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \\ D_j = e_{jkl}S_{kl} + \epsilon_{jk}E_k.\end{cases} \tag{2.25}$$

$$\begin{cases}\tau_{ij,j} = \rho' \ddot{u_i}, \\ D_{i,i} = 0.\end{cases} \tag{2.26}$$

Here τ_{ij} represents stress tensor, D_j is the electric displacement, $e_{kij}, \epsilon_{jk}\ and c_{ijkl}$ represents piezoelectric, dielectric and elastic constants respectively. Density of piezoelectric material is represented by ρ'. u is the mechanical displacement. S_{kl} and E_k are the strain tensor and electric field intensity and can be shown in terms of displacement u and electric potential φ as

$$S_{ij} = \frac{u_{i,j} + u_{j,i}}{2}, \tag{2.27}$$

$$E_i = -\,\partial\varphi/\partial x_i. \tag{2.28}$$

Using equations (2.27) -(2.28) and (2.2) in equations (2.25)-(2.26), we obtained the following equations

$$c_{44}\left[\frac{\partial^2 u_3}{\partial x_1{}^2} + \frac{\partial^2 u_3}{\partial x_2{}^2}\right] + e_{15}\left[\frac{\partial^2 \varphi}{\partial x_1{}^2} + \frac{\partial^2 \varphi}{\partial x_2{}^2}\right] = \rho' \frac{\partial^2 u_3}{\partial t^2}, \tag{2.29}$$

$$e_{15}\left[\frac{\partial^2 u_3}{\partial {x_1}^2}+\frac{\partial^2 u_3}{\partial {x_2}^2}\right]-\epsilon_{11}\left[\frac{\partial^2 \varphi}{\partial {x_1}^2}+\frac{\partial^2 \varphi}{\partial {x_2}^2}\right]=0, \tag{2.30}$$

$$\begin{cases}\tau_{11}=\tau_{22}=\tau_{33}=\tau_{12}=0,\\ \tau_{23}=c_{44}\frac{\partial u_3}{\partial x_2}+e_{15}\frac{\partial \varphi}{\partial x_2},\\ \tau_{31}=c_{44}\frac{\partial u_3}{\partial x_1}+e_{15}\frac{\partial \varphi}{\partial x_1}.\end{cases} \tag{2.31}$$

$$\begin{cases}D_1=e_{15}\frac{\partial u_3}{\partial x_1}-\epsilon_{11}\frac{\partial \varphi}{\partial x_1},\\ D_2=e_{15}\frac{\partial u_3}{\partial x_2}-\epsilon_{11}\frac{\partial \varphi}{\partial x_2},\\ D_3=0.\end{cases} \tag{2.32}$$

where the constants c_{44}, ϵ_{11} and e_{15} represents elastic coefficient, dielectric coefficients and piezoelectric coefficients respectively.

The solution of the differential equations (2.29)-(2.30) is assumed as

$$u_3(x_1,x_2,t)=u_3(x_2)e^{\iota(kx_1-\omega t)}, \tag{2.33}$$

$$\varphi(x_1,x_2,t)=\varphi(x_2)e^{\iota(kx_1-\omega t)}. \tag{2.34}$$

Using equations (2.33) and (2.34) in (2.29) - (2.30), we get

$$u_3=(E_1\cos\lambda_1 x_2+E_2\sin\lambda_1 x_2)e^{\iota(kx_1-\omega t)}, \tag{2.35}$$

$$\varphi=\left[F_1e^{kx_2}+F_2e^{-kx_2}+\frac{e_{15}}{\epsilon_{11}}(E_1\cos\lambda_1 x_2+E_2\sin\lambda_1 x_2)\right]e^{\iota(kx_1-\omega t)}. \tag{2.36}$$

Here, $\lambda_1=k\sqrt{\frac{(c^2-c_{sh}^2)}{c_{sh}^2}}$, $c_{sh}=\sqrt{\frac{c_{44}\,\epsilon_{11}+e_{15}^2}{\rho\,\epsilon_{11}}}$ represents bulk shear wave velocity in piezoelectric layer. E_1, E_2, F_1, F_2 are the arbitrary constants. The expressions (2.35)-(2.36) are obtained under the assumption $c>c_{sh}$.

2.3. BOUNDARY CONDITIONS

The below mentioned boundary conditions should be satisfied for the shear wave propagation in the considered model.

a) The top layer of the piezoelectric layer is taken to be electrically open and mechanically free i.e. at $x_2=-h$

$$\tau_{32}=0\,,\ D_2=0, \tag{2.37}$$

b) As the piezoelectric layer is in perfect contact with non-local micropolar half space so continuity conditions at the common interface $x_2 = 0$ can be written as,

Displacement components should be continuous: $u_3 = u_3^n$,

Stress components should be continuous: $\sigma_{32}^{nl} = \tau_{32}$.

Apply the operator$(1 - \varepsilon^2 \nabla^2)$ on the stress components and using equation (2.5), we obtained

$$(1 - \varepsilon^2 \nabla^2)\sigma_{32}^{nl} = (1 - \varepsilon^2 \nabla^2)\tau_{32},$$

$$\sigma_{32} = (1 - \varepsilon^2 \nabla^2)\tau_{32} . \tag{2.38}$$

Here σ_{32} is the stress component of local micropolar half space.

c) The non-local couple stress vanishes at $x_2 = 0$, as the piezoelectric layer does not exhibit micropolar property. $m_{21}^{nl} = 0 , m_{22}^{nl} = 0.$

Apply the operator $(1 - \varepsilon^2 \nabla^2)$ on both sides of the equation and using equation (2.6), we get

$$m_{21} = 0 , m_{22} = 0. \tag{2.39}$$

Where m_{21} and m_{22} are the local micropolar couple stress components.

d) Electric potential should vanish at the common interface $x_2 = 0$.

$$\varphi = 0. \tag{2.40}$$

2.4. DERIVATION OF DISPERSION RELATION

On substituting the values of displacement and stress components form equations (2.5) - (2.6), (2.20)-(2.24), (2.31)-(2.32) and (2.35)-(2.36) in the boundary conditions (2.37)-(2.40), we obtained following equations in seven unknowns coefficients E_1 , E_2 , F_1 , F_2 , S , M , N.

$$\bar{c}_{44}\lambda_1(E_1 \sin\lambda_1 h + E_2 \cos\lambda_1 h) + e_{15}\, k\, (F_1 e^{-kh} - F_2 e^{kh}) = 0,$$

$$F_1 e^{-kh} - F_2 e^{kh} = 0,$$

$$E_1 = M\, q_1 + N\, q_2,$$

$$\bar{c}_{44}\lambda_1[1+\epsilon^2(\lambda_1^2+k^2)]\,E_2+\ e_{15}\,k\,(F_1-\ F_2)=\ q_3\ \ M+\ q_4\ N+\iota\, q_5 S, \tag{2.41}$$

$$\frac{e_{15}}{\varepsilon_{11}}E_1+\ F_1\ +\ F_2=0,$$

$$\iota q_6\ S+q_7\,M+\ q_8\ N=0,$$

$$q_9 S+\iota q_{10}\ M+\iota\, q_{11}\ N=0.$$

Where $\bar{c}_{44}=\ c_{44}+\frac{{e_{15}}^2}{\varepsilon_{11}}$, $q_1\ =\ \delta-\ \frac{\gamma p^2}{\kappa}$, $q_2\ =\ \delta-\frac{\gamma q^2}{\kappa}$, $q_3\ =\ p[\mu\, q_1+\kappa(1-q_1)]$, $q_4\ =q\ (\ \mu\, q_2\ +\kappa(1-\ q_2\ )$,$q_5\ =\kappa k$, $q_6\ =(\beta+\ \gamma)kr$, $q_7\ =\ \beta\ k^2+\ \gamma p^2$, $q_8\ =\beta\ k^2+\gamma q^2$, $q_9\ =-\alpha k^2+(\alpha+\beta+\gamma)r^2$, $q_{10}=(\beta+\gamma)k\ p$, $q_{11}=(\beta+\gamma)kq$, $\delta=\frac{{c_3}^2k^2-\omega^2+2\ {c_5}^2}{{c_5}^2}$.

In the system of equations (2.41) for non-trivial solution, the determinant of the 7×7 coefficient matrix must be zero which will results in the subsequent dispersion relation of shear waves in the considered composite structure.

$$\frac{{e_{15}}^2k\,tanh(kh)W_1-\varepsilon_{11}W_2}{\varepsilon_{11}W_1}+\ \bar{c}_{44}\lambda_1[1+\epsilon^2(\lambda_1^2+k^2)]tan(\lambda_1 h)=0, \tag{2.42}$$

where, $W_1\ =\ q_1\ q_6\ q_{11}\ -\ q_2\ q_6\ q_{10}\ -\ q_1\ q_8 q_9\ +\ q_2\ q_7 q_9$,

$$W_2\ =\ q_5\ q_7\ q_{11}\ -\ q_5\ q_8\ q_{10}\ +\ q_3\ q_6 q_{11}\ +\ q_4\ q_6 q_{10}-\ q_3\ q_8 q_9\ -\ q_4\ q_7 q_9.$$

2.5. PARTICULAR CASES

CASE I:

In order to obtained the dispersion equation in piezoelectric/local micropolar half space, we assume that the nonlocality parameter is absent that is $\epsilon=0$, then the dispersion equation (2.42) reduces to

$$\frac{e_{15}^2 k\, tanh(kh) W_1 - \varepsilon_{11} W_2}{\varepsilon_{11} W_1} = \bar{c}_{44} \lambda_1 tan(\lambda_1 h). \tag{2.43}$$

Also the values of r, P and Q in the differential equations (2.18)-(2.19) are reduces to

$$r^2 = k^2 + \frac{2{c_5}^2}{{c_3}^2 + {c_4}^2} - \frac{\omega^2}{{c_3}^2 + {c_4}^2}, P = \left(k^2 - \frac{\omega^2}{c_2^2}\right) + \left(k^2 - \frac{\omega^2}{c_3^2}\right) + \frac{c_5^2}{c_3^2}(2 - c_1),$$

$$Q = \left(k^2 - \frac{\omega^2}{c_3^2} + \frac{2c_5^2}{c_3^2}\right) - \frac{c_1 c_5^2 k^2}{c_3^2}. \tag{2.44}$$

The dispersion relation (2.43) and values in equations (2.44) are in agreement with already published results given by Kumar et al. (2019)

CASE II:

By neglecting the micropolar parameters i.e., in the limiting case when $, \beta, \gamma, j\kappa \to 0$, we get the new value of c_2 as $c_2 = \sqrt{\frac{\mu}{\rho}}$ and $\frac{W_2}{W_1} \to -\mu k \sqrt{1 - \frac{c^2}{c_2^2}}$ and then by substituting these values in equation (2.43), we obtained

$$\frac{e_{15}^2 tahh(kh)}{\varepsilon_{11}} + \mu\sqrt{1 - \frac{c^2}{c_2^2}} = \overline{c_{44}}\sqrt{\frac{c^2}{c_{sh}^2} - 1}\, tan\left(kh\sqrt{\frac{c^2}{c_{sh}^2} - 1}\right). \tag{2.45}$$

Further in the absence of piezoelectric parametere_{15}, the relation $\bar{c}_{44} = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}$ and $c_{sh} = \sqrt{\frac{c_{44}\,\epsilon_{11} + e_{15}^2}{\rho'\epsilon_{11}}}$, reduced to $\bar{c}_{44} = c_{44}$ and $c_{sh} = \sqrt{\frac{c_{44}}{\rho'}} = c_1'$. On substituting the values of $e_{15}, \bar{c}_{44}$ and c_{sh} in equation (2.45), we get

$$\mu\sqrt{1 - \frac{c^2}{c_2^2}} = c_{44}\sqrt{\frac{c^2}{c_1'^2} - 1}\, tan\left(kh\sqrt{\frac{c^2}{c_1'^2} - 1}\right) \tag{2.46}$$

Equation (2.46) is the well-known classical equation given by Love (1911).

2.6. NUMERICAL ANALYSIS AND DISCUSSION

For the purpose of verification and illustration of the theoretical results obtained in the preceding section numerical computations have been performed and the results are depicted graphically. The variations in non dimensional phase-velocity$\left(\frac{c}{c_{sh}}\right)$w.r.t non dimensional wave number kh under the effects of key parameters like non-local, piezoelectric and elastic stiffness constants have been exhibited in figures (2.2-2.6). The fundamental mode together with some higher modes of dispersion curves are depicted in these figures to demonstrate the influence of various parameters. The thickness of the piezoelectric layer is taken as $h = 0.001m$ and $e_0 = 0.39$. For numerical calculations we took aluminum epoxy material and piezoelectric material (PZT-4) for which the values of relevant parameters are given in table-2.1

Table 2.1 Material parameters Gauthier (1982) and Liu et al. (2010)

Aluminum epoxy material	Piezoelectric layer PZT-4
$\rho = 2.19 \times 10^3 kg/m^3$,	$\rho' = 7.5 \times 10^3 kg/m^3$
$\lambda = 7.59 \times 10^{10} N/m^2$	$C_{44} = 2.56 \times 10^{10} N/m^2$
$\mu = 1.89 \times 10^{10} N/m^2$	$e_{15} = 12.7\ C/m^2$
$\kappa = 0.0149 \times 10^{10} N/m^2$	$\epsilon_{11} = 6.46 \times 10^{-9} C^2/Nm^2$
$\alpha = 0.01 \times 10^6 N$	
$\beta = 0.015 \times 10^6 N$	
$\gamma = 0.268 \times 10^6 N$	
$j = 0.196 \times 10^4 m^2$	

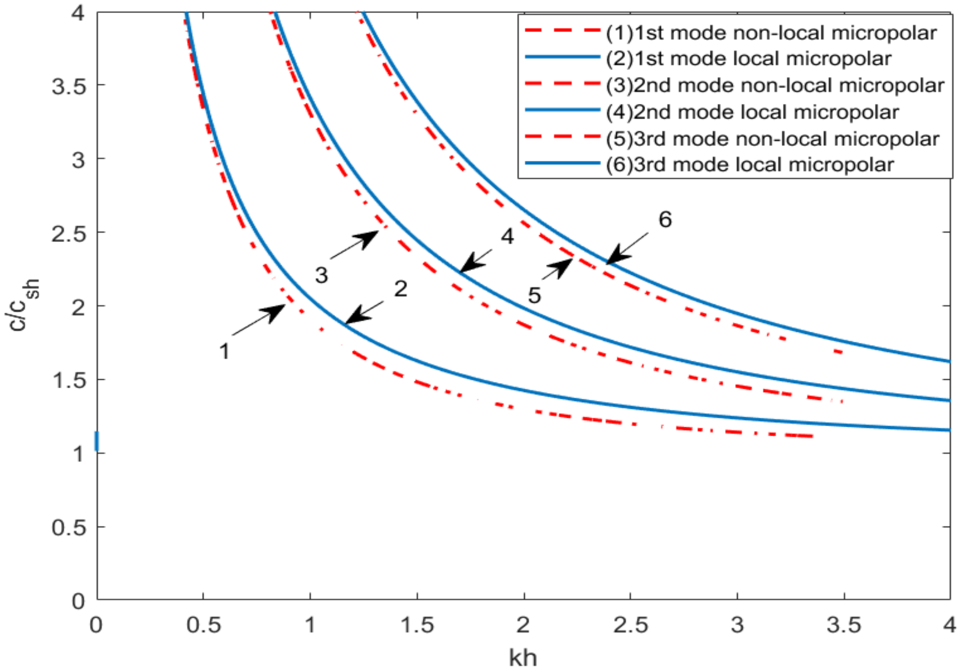

Fig.2.2 Dispersion curves for piezoelectric/local micropolar elastic and piezoelectric/ non-local micropolar elastic structure.

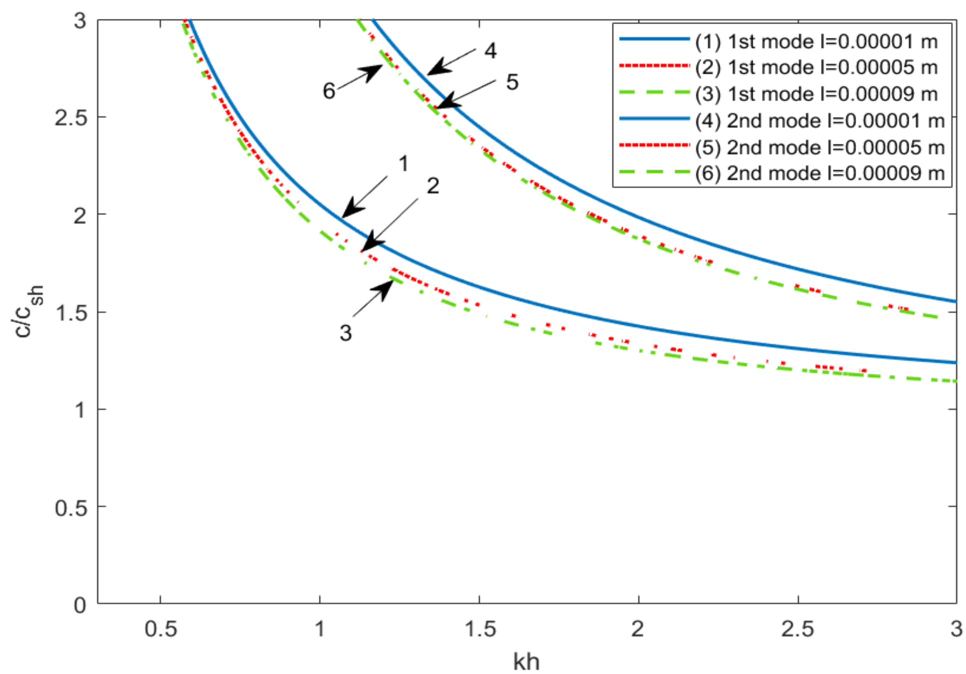

Fig.2.3 Effect of characteristic length of non-local micropolar elastic half space on the phase-velocity of shear waves.

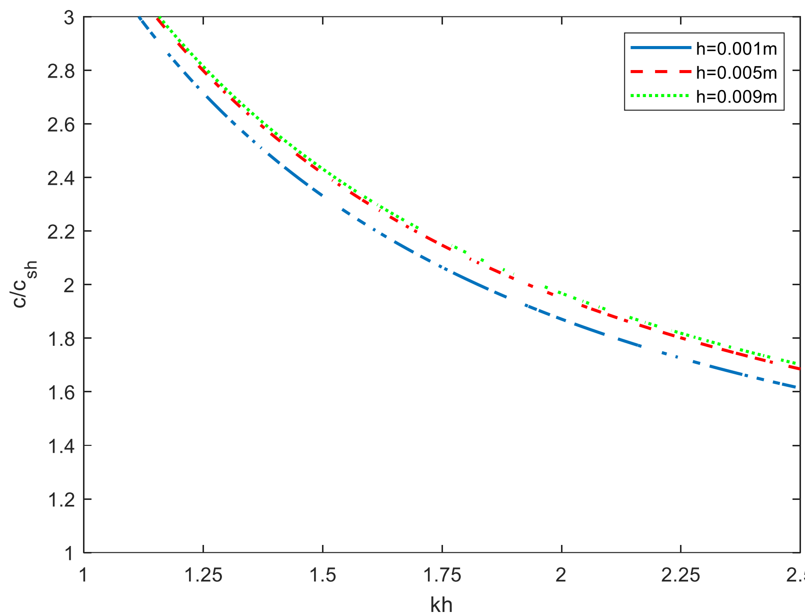

Fig.2.4 Effect of thickness of piezoelectric layer on the phase-velocity of shear waves.

Figure-2.2 compares the variations of non dimensionalphase-velocity w.r.t. non dimensional wave number in a piezoelectric/non-local micropolar half space and piezoelectric/micropolar half space. The dispersion curves for first three modes have been drawn for both local micropolar and non-local micropolar half space. It is noticeable here that the phase-velocityminimized gradually in both the structures with increase in wave number. With same value of wave number, the phase-velocity is greater in case of local micropolar half space as compared to non-local micropolar half space. For higher wave number values, it is noticed that dispersion curves vanished for non-local micropolar half space which is due to decreasing effects of neighboring points on the reference point. This suggests that local interactions among particles enhance wave propagation, while in non-local micropolar materials, the diminishing influence of surrounding points leads to reduced phase velocity at higher wave numbers.

Figure-2.3 illustrates how the internal characteristic length affects the phase-velocity of shear waves, and as visible from the graph this parameter has significant effect. First two modes have been plotted to study the behavior on different modes. We consider three distinct values of internal characteristic length and as shown in graph, with rise in internal characteristic length the phase-velocity falls in all modes. The dispersion curves for two modes are of same type but the distance between them corresponding to different characteristic length decreases in second modes. The increment of internal characteristic length results in increased wave attenuation and a reduction in phase velocity, as the waves encounter greater resistance during propagation.

Figure 2.4 demonstrates the influence of the thickness of piezoelectric layer on phase velocity of the shear waves. The layer thickness has a significant effect on the phase velocity of shear waves. It is observed that, as the thickness increases, the phase velocity also increases for the selected values of this parameter.

2.7. CONCLUSIONS

This chapter delves into the transmission of shear waves within a composite structure comprising a piezoelectric layer atop a non-local micropolar elastic half space. An overarching wave dispersion equation for the propagation of shear waves in such a piezoelectric and non-local micropolar structure has been analytically derived. In specific instances, this relationship aligns well with the classical Love wave equation. The following points encapsulate the findings of the current investigation:

- **Non-local Micropolar Effects:** The non-local characteristics of the micropolar elastic material significantly reduce the phase velocity of shear waves in the composite structure.
- **Characteristic Length Influence:** An increase in the characteristic length parameter of the non-local micropolar material decreases the phase velocity for a fixed wave number.
- **Phase Velocity and Wave Number**: The shear waves exhibit dispersive behavior, characterized by a decrease in phase velocity as the wave number increases.
- **Dispersion Curve Analysis**: The dispersion curves for various modes show similar shapes, but lower modes are more affected by the parameters compared to higher modes.

- **Thickness of piezoelectric layer**: The analysis reveals that the thickness parameter (h) has a substantial impact on the phase velocity of shear waves in layered structures. As the thickness increases, the phase velocity rises, indicating that thicker layers enhance wave propagation speed. This relationship between thickness and phase velocity highlights the importance of layer geometry in determining wave behavior in non-local composite materials.

In conclusion, the study highlights the critical role of non-local and piezoelectric effects on shear wave propagation in layered structures. The results provide valuable insights into how these materials influence wave behavior, which is essential for designing advanced composite materials and understanding their dynamic properties.

Understanding shear wave propagation in layered structures with non-local micropolar and piezoelectric materials can have applications in many fields such as in aerospace engineering, it can improve the design and performance of composite materials used in aircraft by offering better predictions of wave interactions under various conditions. In civil engineering, the insights can enhance the development of earthquake-resistant structures by improving the understanding of seismic wave behavior in layered composites. For electronics, this research is valuable for optimizing sensors and actuators that rely on piezoelectric materials. Overall, accurately predicting and controlling shear wave behavior in these advanced materials can lead to significant advancements in material design and structural integrity.

CHAPTER 3

INFLUENCE OF INITIAL STRESS AND SURFACE IMPERFECTION ON SHEAR WAVES IN NONLOCAL COMPOSITE MATERIAL[2]

3.1. INTRODUCTION:

The utilization of wave propagation in composite materials is a common practice in sensor technology and nondestructive testing procedures for the purpose of assessing material strength. The surface wave's propagation in composite materials comprising of piezoelectric and elastic materials has been studied by many researchers. While most research assumes a perfectly bounded surface but in practicality the materials possess the imperfect surfaces due many reasons like manufacturing defects. Furthermore, the multilayer structure is typically pre-stressed throughout the manufacturing process to avoid brittle fracture. As a result, more accurate findings can be obtained by taking into account the impacts of uneven surfaces and beginning tension.

The literature has a large number of research publications that examine the impact of initial stress and imperfect interfaces in many types of composite materials. For instance, Chen et al.(2004) investigated how waves move across two distinct mediums with poor bonding. Their results revealed that the presence of interface flaws can greatly impact how waves propagate through a material. Qian et al. (2004) examined how Love waves moved through a piezoelectric multilayer device under initial stress. Chen et al. (2008) analyzed the shear horizontal piezoelectric wave's propagation through a plate composed of polarized piezoceramics with an imperfect interface between the two surfaces. Kurt

[2]The content of this chapter is published in journal "International Journal of Applied and Computational Mathematics" entitlled "Effects of nonlocal characteristics of composite material on shear waves propagation with an imperfect interface", vol. 9 and isuue 5 page-113. (SCOPUS , SJR-0.37)

(2016) studied the influence of initial stress for the Lamb wave in a material comprising of piezoelectric and pre stressed elastic layers. Singh et al. (2017) explored the transmission characteristics of Love-waves in a composite structure. The structure consisted of bounded piezoelectric layer at top and lower fiber-reinforced half-space, taking into account a rectangular-shaped irregularity at the common interface. Kumar et al. (2019) derive dispersion relation for shear wave's propagation in a piezoelectric device in contact with a micropolar elastic half space under initial load. Qing-tian and Song-nan (2018) established dispersion equation in piezoelectric circular curved rods in an orthogonal curvilinear coordinate system. Qing et al. (2019) investigated the application of piezoelectric materials used as transducers in structural health monitoring which is also widely examined by the aerospace industry for improving the safety and reliability of aircraft structures. Bharti et al. (2021) have used shear waves to study the impact of soft and rigid mountain surfaces on the phase-velocity in a composite structure consisting of fluid saturated porous medium and orthotropic half space. Dhabal et al. (2022) studied the interaction of shear waves has been studied in an infinite magnetoelastic orthotropic medium. Liu et al. (2022) described effects of piezoelectricity on the guided waves in nonlocal piezoelectric nano plates. Liu et al. (2022) investigated the shear wave propagation in functionally graded small scaled plates. The piezoelectric effects have been studied on the same. Sharma & Kumar (2022) investigated the Bleustein–Gulyaev wave's propagation in non-local piezoelectric film on the non-local piezoelectric half space.

The motivation for exploring layered structures in the study of shear waves, particularly concerning initial stress and surface imperfections in non-local composite materials, lies in the desire to understand how these factors influence wave propagation. Layered composites present intricate interactions due to their diverse mechanical properties and stress distributions, which are crucial for analyzing shear wave behavior under various conditions. This approach helps to illuminate the complexities involved in the use of layered composites. The current chapter pertains to an intricate configuration comprising a non-local micropolar elastic half-space that is homogeneous and isotropic in nature, along with a pre-stressed non-local piezoelectric layer. The two components are separated by an imperfect interface. Our study delves into the examination of the influence of diverse parameters, including non-locality, imperfect interface and initial stress on the phase-velocity of shear waves. We derive closed-form analytical expression for the dispersion relation of shear waves and conduct numerical computations to determine the phase-velocity. Subsequently, the outcomes are depicted in a graphical format.

3.2. BASIC EQUATIONS

Using basic equation applicable to a homogeneous, isotropic, non-local micropolar elastic material as given in section (1.4.1) of chapter 1.

$$\sigma^{nl}_{k\ell,k} + \rho\,(f_\ell - \ddot{u}^n_\ell) = 0, \tag{3.1}$$

$$m^{nl}_{k\ell,k} + \in_{\ell mn} \sigma_{mn} + \rho\big(l_\ell - j\,\ddot{\phi}_\ell\big) = 0, \tag{3.2}$$

$$(1 - \varepsilon^2\nabla^2)\sigma^{nl}_{k\ell} = \sigma_{k\ell} = \lambda e_{rr}\delta_{k\ell} + (\mu + \kappa)e_{k\ell} + \mu e_{\ell k}, \tag{3.3}$$

$$(1 - \varepsilon^2\nabla^2)m^{nl}_{k\ell} = m_{k\ell} = \alpha\,\gamma_{rr}\delta_{k\ell} + \beta\gamma_{k\ell} + \gamma\gamma_{k\ell}, \tag{3.4}$$

$$e_{k\ell} = (u^n_{\ell,k} - \epsilon_{k\ell m}\phi_{m,\ell}),\ \gamma_{k\ell} = \phi_{k,\ell}. \tag{3.5}$$

The physical meaning of symbols has already been defined in chapter 1.

Using above equations, the field equations can be written as:

$$(\lambda + \mu)u^n_{k,k\ell} + (\mu + \kappa)u^n_{\ell,kk} + \kappa \in_{\ell mn} \phi_{n,m} + \rho(1 - \varepsilon^2\nabla^2)(f_\ell - \ddot{u}^n_\ell) = 0, \tag{3.6}$$

$$(\alpha + \beta)\,\phi_{k,k\ell} + \gamma\,\phi_{\ell,kk} + \kappa \in_{\ell mn} u^n_{n,m} - 2\,\kappa\,\phi_\ell + \rho(1 - \varepsilon^2\nabla^2)\big(l_\ell - j\,\ddot{\phi}_\ell\big) = 0. \tag{3.7}$$

Now, the constitutive relations and equation of motion for a homogeneous, transversely isotropic non-local piezoelectric medium can be expressed as in Sharma and Kumar (2022)

$$\begin{cases} \big(1 - \varepsilon'^2\nabla^2\big)\tau^{nl}_{ij} = \tau_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \\ \big(1 - \varepsilon'^2\nabla^2\big)D^{nl}_j = e_{jkl}S_{kl} + \epsilon_{jk}E_k. \end{cases} \tag{3.8}$$

$$\begin{cases}\tau_{ij,j}^{nl} + (u_{i,j}\tau_{kj}^{0})_{,j} = (1-\varepsilon'^{2}\nabla^{2})\rho'\ddot{u_i}\,, \\ D_{i,i}^{nl} + (u_{i,k}D_{j}^{0})_{,i} = 0.\end{cases} \tag{3.9}$$

Where

$$S_{ij} = \frac{u_{i,j} + u_{j,i}}{2}, \tag{3.10}$$

$$E_i = -\,\partial\varphi/\partial x_i\,. \tag{3.11}$$

Here τ_{ij}^{nl} represents non-local stress tensor andτ_{ij}represents local stress tensor. D_j^{nl} is the electric displacement, $\varepsilon' = e_0 a$ is the non-local parameter which denotes small scale effect. $e_{kij}\,, \epsilon_{jk}\; and\; c_{ijkl}$ represents piezoelectric, dielectric and elastic constants respectively. Density of piezoelectric material is represented by ρ'. $u$is the mechanical displacement. It could be observed that in absence of non-local constraint the above equation will represent the one for local piezoelectric medium. S_{kl} and E_k are the strain tensor and electric field intensity.

3.3. FORMULATION AND PROBLEM'S SOLUTION:

As illustrated in figure 3.1, we consider a composite structure with a pre-stressed layer of non-local piezoelectric material of thickness h on top of a non-local micropolar elastic half space. The starting point of the chosen Cartesian coordinate system is established at the location marked as point O, as illustrated in Figure 3.1.The non-local piezoelectric layer polarized along x_3-axis direction perpendicular to $x_1 - x_2$ plane is considered. We assume that shear waves propagates in x_1 direction and x_2-axis is assumed to be positive in perpendicularly downward direction so the displacement components will be free from x_3 coordinates.

Thus, $(u_1, u_2\,, u_3)$ and $(u_1^n, u_2^n\,, u_3^n)$ stand for the displacement components resulting from shear wave propagation in a non-local piezoelectric layer and a NLME half space, respectively.

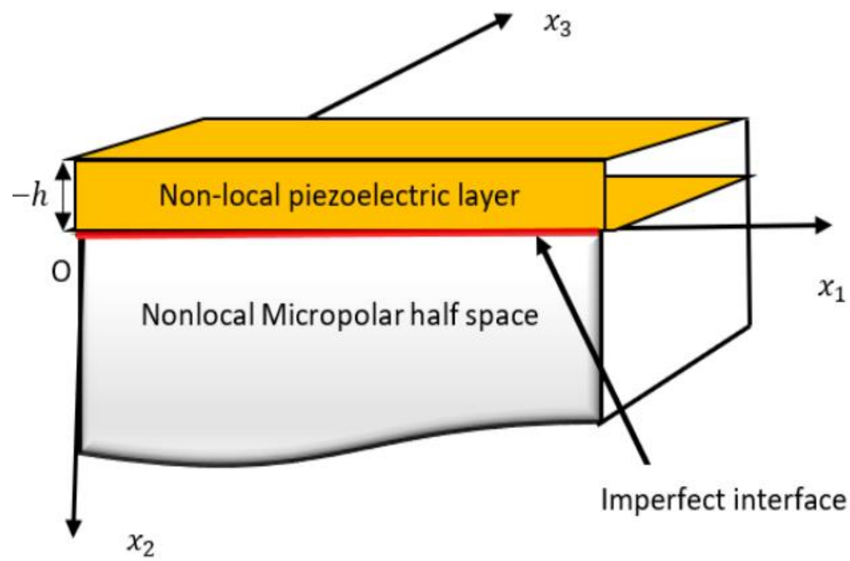

Fig.3.1 Geometry of the problem

The components of displacement, micro rotation and electric field potential can be represented as

$$u_1^n = u_2^n = 0,\ u_3^n = u_3^n(x_1, x_2, t), \phi_1 = \phi_1(x_1, x_2, t),\ \phi_2 = \phi_2(x_1, x_2, t), ,$$
$$\phi_3 = 0,\ \ u_1 = u_2 = 0, u_3 = u_3(x_1, x_2, t), \varphi = \varphi(x_1, x_2, t). \tag{3.12}$$

3.3.1 SOLUTION OF NON-LOCAL MICROPOLAR HALF SPACE:

In the absence of body force and body couple densities, equations (3.6) and (3.7) can be simplified as follows

$$(\mu + \kappa)\nabla^2 u_3^n + \kappa\left(\frac{\partial \phi_2}{\partial x_1} - \frac{\partial \phi_1}{\partial x_2}\right) = \rho(1 - \varepsilon^2\nabla^2)\frac{\partial^2 u_3^n}{\partial t^2}, \tag{3.13}$$

$$(\alpha + \beta)\ \frac{\partial}{\partial x_1}\left(\frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2}\right) + \gamma\nabla^2\phi_1 - 2\kappa\phi_1 + \kappa\frac{\partial u_3^n}{\partial x_2} = \rho \mathrm{j}(1 - \varepsilon^2\nabla^2)\frac{\partial^2 \phi_1}{\partial t^2}, \tag{3.14}$$

$$(\alpha + \beta)\ \frac{\partial}{\partial x_2}\left(\frac{\partial \phi_1}{\partial x_1} + \frac{\partial \phi_2}{\partial x_2}\right) + \gamma\nabla^2\phi_2 - 2\kappa\phi_2 - \kappa\frac{\partial u_3^n}{\partial x_1} = \rho \mathrm{j}(1 - \varepsilon^2\nabla^2)\frac{\partial^2 \phi_2}{\partial t^2}. \tag{3.15}$$

To decouple the equation (3.13)-(3.15) introducing the potential functions ψ and ξ as

$$\phi_1 = \frac{\partial\psi}{\partial x_1} + \frac{\partial\xi}{\partial x_2} \;\; and \;\; \phi_2 = \frac{\partial\psi}{\partial x_2} - \frac{\partial\xi}{\partial x_1}. \tag{3.16}$$

On substituting equation (3.16) in (3.13)-(3.15), we obtained

$$\nabla^2 u_3^n - c_1\nabla^2 \xi = (1-\varepsilon^2\nabla^2)\frac{1}{c_2^2}\frac{\partial^2 u_3^n}{\partial t^2}, \tag{3.17}$$

$$\nabla^2 \psi - \frac{2c_5^2}{c_3^2 + c_4^2}\psi = (1-\varepsilon^2\nabla^2)\frac{1}{c_3^2 + c_4^2}\frac{\partial^2 \psi}{\partial t^2}, \tag{3.18}$$

$$\nabla^2 \xi - \frac{2c_5^2}{c_3^2}\xi + \frac{c_5^2}{c_3^2}u_3^n = (1-\varepsilon^2\nabla^2)\frac{1}{c_3^2}\frac{\partial^2 \xi}{\partial t^2}. \tag{3.19}$$

where $c_1 = \frac{\kappa}{\mu+\kappa}$, $c_2 = \sqrt{\frac{\mu+\kappa}{\rho}}$, $c_3 = \sqrt{\frac{\gamma}{\rho j}}$, $c_4 = \sqrt{\frac{\alpha+\beta}{\rho j}}$ and $c_5 = \sqrt{\frac{\kappa}{\rho j}}$.

The solution for waves described in equations (3.17)-(3.19) can be represented as

$$(\psi, \xi, u_3^n) = (\psi, \xi, u_3^n)(x_2)e^{\iota(kx_1-\omega t)}. \tag{3.20}$$

where k and ω are as explained in equation (2.17) in chapter 2.

Using the equation (3.20) in equations (3.17)-(3.19), we obtained,

$$(D^2 - R^2)\psi(x_2) = 0. \tag{3.21}$$

$$(D^4 - PD^2 + Q)(u_3^n, \xi)(x_2) = 0. \tag{3.22}$$

where $$R^2 = \frac{k^2 + \frac{2c_5^2}{c_3^2+c_4^2} - \frac{\omega^2}{c_3^2+c_4^2} - \frac{\omega^2\varepsilon^2k^2}{c_3^2+c_4^2}}{\left(1-\frac{\omega^2\varepsilon^2}{c_3^2+c_4^2}\right)},$$

$$P = \frac{\left(k^2 - \frac{\omega^2}{c_2^2}\right) + \left(k^2 - \frac{\omega^2}{c_3^2}\right) + \frac{c_5^2}{c_3^2}(2-c_1) - \frac{2\omega^2\varepsilon^2k^2}{c_3^2} - \frac{2\omega^2\varepsilon^2c_5^2}{c_2^2c_3^2} + \frac{2\omega^4\varepsilon^2}{c_2^2c_3^2} + \frac{2\omega^4\varepsilon^4k^2}{c_2^2c_3^2} + \frac{\omega^4\varepsilon^2k^2}{c_2^2c_3^2}}{\left[1 - \varepsilon^2\omega^2\left(\frac{1}{c_2^2} + \frac{1}{c_3^2}\right) + \frac{\varepsilon^4\omega^4}{c_2^2c_3^2}\right]},$$

$$Q = \frac{\left(k^2 - \frac{\omega^2}{c_2^2}\right)\left(k^2 - \frac{\omega^2}{c_3^2} + \frac{2c_5^2}{c_3^2}\right) - \frac{c_1c_5^2k^2}{c_3^2} - \frac{\omega^2\varepsilon^2k^4}{c_3^2} + \frac{2\omega^4\varepsilon^2k^2}{c_2^2c_3^2} - \frac{\omega^2\varepsilon^2k^4}{c_2^2} - \frac{2\omega^2\varepsilon^2kc_5^2}{c_2^2c_3^2} + \frac{\omega^4\varepsilon^4k^4}{c_2^2c_3^2}}{\left[1 - \varepsilon^2\omega^2\left(\frac{1}{c_2^2} + \frac{1}{c_3^2}\right) + \frac{\varepsilon^4\omega^4}{c_2^2c_3^2}\right]}.$$

Using radiation condition $(u_3^n, \xi, \psi)(x_2) \longrightarrow 0 \; as \; x_2 \longrightarrow \infty$, the general solution of equations (3.21)-(3.22) is taken as:

$$\psi = (S\, e^{-r\, x_2}) e^{\iota(kx_1 - \omega t)}, \tag{3.23}$$

$$\xi = (M e^{-p\, x_2} + N e^{-q\, x_2}) e^{\iota(kx_1 - \omega t)}, \tag{3.24}$$

$$u_3^n = (M r_1 e^{-p\, x_2} + N r_2 e^{-q\, x_2}) e^{\iota(kx_1 - \omega t)}. \tag{3.25}$$

Where $p, q = \sqrt{\frac{P \pm \sqrt{P^2 - 4Q}}{2}}$, $p^2 + q^2 = P$, $p^2 q^2 = Q$, S, M and N are arbitrary constants. The components of microrotation are obtained from equation (3.16) by using equations (3.23) - (3.24) as

$$\phi_1 = (\iota k S e^{-r x_2} - p\, M e^{-p\, x_2} - q\, N e^{-q\, x_2}) e^{\iota(kx_1 - \omega t)}, \tag{3.26}$$

$$\phi_2 = (-r S e^{-R x_2} - \iota k (M e^{-p\, x_2} + N e^{-q\, x_2}) e^{\iota(kx_1 - \omega t)}. \tag{3.27}$$

The expressions (3.23)–(3.27) are obtained under the assumption that c is less than both c_2 and c_3. The wave corresponding to $c > c_2$ is represents the refracted wave which lose their energy very quickly and hence not significant.

3.3.2 SOLUTION OF NON-LOCAL PIEZOELECTRIC LAYER

Now for non-local piezoelectric layer, using equations (3.10)-(3.12) in equations (3.8)-(3.9) we obtained the following equations

$$(c_{44} + \tau_{11}^0) \frac{\partial^2 u_3}{\partial {x_1}^2} + c_{44} \frac{\partial^2 u_3}{\partial {x_2}^2} + e_{15} \left[\frac{\partial^2 \varphi}{\partial {x_1}^2} + \frac{\partial^2 \varphi}{\partial {x_2}^2} \right] = \left(1 - \varepsilon'^2 \nabla^2\right) \rho' \frac{\partial^2 u_3}{\partial t^2}, \tag{3.28}$$

$$\mathrm{e}_{15} \left[\frac{\partial^2 \mathrm{u}_3}{\partial {\mathrm{x}_1}^2} + \frac{\partial^2 \mathrm{u}_3}{\partial {\mathrm{x}_2}^2} \right] - \epsilon_{11} \left[\frac{\partial^2 \varphi}{\partial {\mathrm{x}_1}^2} + \frac{\partial^2 \varphi}{\partial {\mathrm{x}_2}^2} \right] = 0. \tag{3.29}$$

c_{44} , ϵ_{11} and e_{15} represents the terms as explained in section 2.2.2 in chapter 2.

Solution of eq. (3.28)-(3.29) are assumed as

$$u_3(x_1, x_2, t) = u_3(x_2) e^{\iota k (x_1 - ct)}, \tag{3.30}$$

$$\varphi(x_1, x_2, t) = \varphi(x_2) \mathrm{e}^{\iota \mathrm{k}(\mathrm{x}_1 - \mathrm{ct})}. \tag{3.31}$$

Where, the symbols have their usual meaning as discussed in section 2.2.1 in chapter 2. Substituting (3.30) and (3.31) in (3.28)-(3.29), we get

$$u_3 = (E_1 \cos \lambda_1 x_2 + E_2 \sin \lambda_1 x_2) \mathrm{e}^{\iota \mathrm{k}(\mathrm{x}_1 - \mathrm{ct})}, \tag{3.32}$$

$$\varphi = \left[F_1 e^{kx_2} + F_2 e^{-kx_2} + \frac{e_{15}}{\epsilon_{11}}(E_1 \cos\lambda_1 x_2 + E_2 \sin\lambda_1 x_2)\right] e^{\iota k(x_1 - ct)}. \tag{3.33}$$

Here, $\lambda_1 = k\sqrt{\frac{c^2\left(1+k^2\epsilon'^2\right) - \frac{\tau_{11}^0}{\rho'} - c_{sh}^2}{c_{sh}^2 + \omega^2 \epsilon'^2}}$, $c_{sh} = \sqrt{\frac{c_{44}\,\epsilon_{11} + e_{15}^2}{\rho\epsilon_{11}}}$ represents the bulk shear wave velocity in the piezoelectric layer. E_1, E_2, F_1, F_2 are the arbitrary constants. The expressions (3.32) - (3.33) are obtained under the assumption $c > c_{sh}$.

3.4. BOUNDARY CONDITIONS

For the propagation of shear waves in the model being considered, it is necessary to meet the specified boundary conditions.

a) The upper layer of non-local piezoelectric layer is taken to be electrically open and mechanically free i.e. at $x_2 = -h$

$$\tau_{32}^{nl} = 0\,,\; D_2^{nl} = 0\,. \tag{3.34}$$

b) As we are considering imperfect interface between the layer and the half space so to characterize the interfacial imperfection the linear spring model (2004) and has applied which can be expressed as below.

At $x_2 = 0$, $\chi\, \sigma_{32}^{nl} = ({u_3}^n - u_3)$ where χindicates the degree of imperfectness of the interface.

Apply the operator $(1 - \varepsilon^2 \nabla^2)$ on both sides, we get

$$\chi\, \sigma_{32} = (1 - \varepsilon^2 \nabla^2)({u_3}^n - u_3)\,, \tag{3.35}$$

where σ_{23} is the local micropolar stress component.

c) Stress components should be continuous: $\sigma_{32}^{nl} = \tau_{32}^{nl}$.

Apply the operator $(1 - \varepsilon^2 \nabla^2)\left(1 - \varepsilon'^2 \nabla^2\right)$ on both sides of the equation, we get

$$(1 - \varepsilon^2 \nabla^2)\left(1 - \varepsilon'^2 \nabla^2\right) \sigma_{32}^{nl} = (1 - \varepsilon^2 \nabla^2)\left(1 - \varepsilon'^2 \nabla^2\right)\tau_{32}^{nl}.$$

$$\text{or } \left(1 - \varepsilon'^2 \nabla^2\right) \sigma_{32} = (1 - \varepsilon^2 \nabla^2)\tau_{32}. \tag{3.36}$$

where τ_{32} is the local piezoelectric stress component.

d) At $x_2 = 0$, the non-local couple stress vanishes as the piezoelectric layer does not does not exhibit micropolar property.

$$m_{21}^{nl} = 0\,, \quad m_{22}^{nl} = 0.$$

Apply the operator $(1 - \varepsilon^2 \nabla^2)$ on both sides of the equation, we get

$$(1 - \varepsilon^2 \nabla^2) m_{21}^{nl} = 0\,, (1 - \varepsilon^2 \nabla^2) m_{22}^{nl} = 0.$$

$$\text{or } m_{21} = 0\,, \quad m_{22} = 0. \tag{3.37}$$

where m_{21} and m_{22} are the local micropolar couple stress component.

e) Electric potential must be zero at the shared interface. i.e. at $x_2 = 0$.

$$\varphi = 0 \tag{3.38}$$

3.5. DERIVATION OF DISPERSION RELATION

By plugging in the values of displacement and stress components from equations (3.3)-(3.4), (3.23)-(3.27) and (3.32)-(3.33) into equations (3.34)-(3.38), we obtain

$$\bar{c}_{44}\lambda_1(E_1 \sin\lambda_1 h + E_2 \cos\lambda_1 h) + e_{15}\,k\,(F_1 e^{-kh} - F_2 e^{kh}) = 0,$$
$$F_1 e^{-kh} - F_2 e^{kh} = 0,$$
$$E_1[1 + k^2(\epsilon^2 + \lambda_1^2)] = M\{[1 + \epsilon^2(k^2 - p^2)]r_1 + \chi r_3\} + N\{[1 + \epsilon^2(k^2 - q^2)]r_2 + \chi r_4\} - \chi\, \iota r_5\, S,$$
$$\bar{c}_{44}\lambda_1(1 + \epsilon^2(k^2 + \lambda_1^2)E_2 + e_{15}\,k\,(F_1 - F_2) \qquad (3.39$$
$$= -r_3\,[1 + \varepsilon'^2(k^2 - p^2)]M - r_4\,[1 + \varepsilon'^2(k^2 - q^2)]N + \iota\, r_5 S[1 + \varepsilon'^2(k^2 - r^2)],$$
$$\frac{e_{15}}{\varepsilon_{11}}E_1 + F_1 + F_2 = 0,$$
$$\iota r_6\, \mathrm{S} + r_7\, M + r_8\, N = 0,$$
$$r_9 S + \iota r_{10}\, M + \iota\, r_{11}\, N = 0.$$

Where $E_1, E_2, F_1, F_2, S, M, N$ are unknown coefficients.

$\bar{c}_{44} = c_{44} + \frac{{e_{15}}^2}{\varepsilon_{11}}, r_1 = \delta - \frac{\gamma p^2}{\kappa}, r_2 = \delta - \frac{\gamma q^2}{\kappa}, r_3 =$ p $(\mu r_1 + \kappa)$,

$r_4 = q\,(\mu r_2 + \kappa)$, $r_5 = \kappa k, r_6 = v r, r_7 = \beta k^2 + \gamma p^2$, $r_8 = \beta k^2 + \gamma q^2$, $r_9 = -\alpha k^2 + (\alpha + \beta + \gamma) r^2,\ r_{10} = v\,p,\ r_{11} = v k q,\ \ \delta = \frac{{c_3}^2 k^2 - \omega^2 + 2\,{c_5}^2}{{c_5}^2},\ v = (\beta + \gamma)k.$

The condition of non-trivial solution of equations (3.39) leads to the following dispersion relation for shear waves in the composite structure,

$$\frac{{e_{15}}^2 k \tanh(kh)}{\varepsilon_{11}} + \bar{c}_{44}\lambda_1 L_1 tan(\lambda_1 h) = \frac{L_1[L_4 L_7 - L_5 L_8 + L_6 L_9]}{[(L_2 L_{10} + L_3 L_{11} + \chi(L_7 + L_8 - L_9)}, \tag{3.40}$$

where, $L_1 = 1 + \epsilon^2(\lambda_1^2 + k^2)$, $L_2 = 1 + \epsilon^2(k^2 - p^2), L_3 = 1 + \epsilon^2(k^2 - q^2), L_4 = 1 + \epsilon'^2(k^2 - p^2)$, $L_5 = 1 + \epsilon'^2(k^2 - q^2)$, $L_6 = 1 + \epsilon'^2(k^2 - r^2)$, $L_7 = r_3 r_6 r_{11} - r_3 r_8 r_9$, $L_8 = r_4 r_7 r_9 - r_4 r_6 r_{10}$, $L_9 = r_5 r_7 r_{11} - r_5 r_8 r_{10}$, $L_{10} = r_1 r_6 r_{11} - r_1 r_8 r_9$, $L_{11} = r_2 r_7 r_9 - r_2 r_6 r_{10}$.

3.6. PARTICULAR CASES

CASE I:

In order to obtained the dispersion equation in local piezoelectric/non-local micropolar half space, we assume that the non-locality parameter of piezoelectric material is absent that is $\epsilon' = 0$, then the dispersion equation (3.42) reduces to

Here, $L_4 = L_5 = L_6 = 1$ for $\epsilon' = 0$.

$$\frac{{e_{15}}^2 k \tanh(kh)}{\varepsilon_{11}} + \bar{c}_{44}\lambda_1 L_1 tan(\lambda_1 h) = \frac{L_1[L_7 - L_8 + L_9]}{[(L_2 L_{10} + L_3 L_{11} + \chi(L_7 + L_8 - L_9)}. \tag{3.41}$$

CASE II:

In order to obtained the dispersion equation in local piezoelectric/ local micropolar half space, we assume that the non-locality parameter of piezoelectric material and micropolar material are absent that is $\epsilon = 0$, $\epsilon' = 0$, then the dispersion equation (3.43) reduces to

$$\frac{{e_{15}}^2 k\tanh(kh)}{\varepsilon_{11}} + \bar{c}_{44}\lambda_1 tan(\lambda_1 h) = \frac{[L_7 - L_8 + L_9]}{[L_{10} + L_{11} + \chi(L_7 + L_8 - L_9)]}\,. \tag{3.42}$$

Here , $L_1 = L_2 = L_3 = L_4 = L_5 = L_6 = 1$ for $\epsilon' = 0\,, \epsilon = 0\,.$

Also the values r, P and Q in the differential equations (3.21)-(3.22) reduced to

$$r^2 = k^2 + \frac{2{c_5}^2}{{c_3}^2 + {c_4}^2} - \frac{\omega^2}{{c_3}^2 + {c_4}^2},$$
$$P = \left(k^2 - \frac{\omega^2}{c_2^2}\right) + \left(k^2 - \frac{\omega^2}{c_3^2}\right) + \frac{c_5^2}{c_3^2}(2 - c_1), \tag{3.43}$$
$$Q = \left(k^2 - \frac{\omega^2}{{c_2}^2}\right)\left(k^2 - \frac{\omega^2}{c_3^2} + \frac{2c_5^2}{c_3^2}\right) - \frac{c_1 c_5^2 k^2}{c_3^2}.$$

The dispersion relation (3.41) and values in equations (3.42) are in agreement with already published results Kumar et al. (2019).

CASE-III:

On substituting, χ = 0, τ_{11}^0 in the equation (3.43) we obtain,

$$\frac{{e_{15}}^2 k\tanh(kh)}{\varepsilon_{11}} + \bar{c}_{44}\lambda_1 tan(\lambda_1 h) = \frac{L_7 - L_8 + L_9}{L_{10} + L_{11}}, \tag{3.44}$$

where, $\lambda_1 = k\sqrt{\frac{c^2 - c_{sh}^2}{c_{sh}^2}}\,, c_{sh} = \sqrt{\frac{c_{44}\,\epsilon_{11} + e_{15}^2}{\rho\epsilon_{11}}}.$

Which is same as the result of obtained under case I in equation (2.43) in chapter 2.

CASE-IV:

By neglecting the piezoelectric and micropolar parameters i.e., under limiting case when $\alpha, \beta, \gamma, j, \kappa, e_{15}, \rightarrow 0$,

We have, $c_2 = \sqrt{\frac{\mu}{\rho}}$ and , $\bar{c}_{44} = c_{44}$ and $c_{sh} = \sqrt{\frac{c_{44}}{\rho}} = c_1',\ \frac{L_7 - L_8 + L_9}{L_{10} + L_{11}} \rightarrow \mu k\sqrt{1 - \frac{c^2}{c_2^2}}.$

On substituting the values of $e_{15}, \bar{c}_{44}$ and c_{sh} in equation (3.44), we get

$$\mu\sqrt{1-\frac{c^2}{c_2^2}}=c_{44}\sqrt{\frac{c^2}{c_1'^2}-1}\,tan\left(kh\sqrt{\frac{c^2}{c_{sh}^2}-1}\right). \tag{3.45}$$

Which is same as result obtained under case II in equation (2.45) in chapter 2.

3.7. NUMERICAL ANALYSIS AND DISCUSSION

To study the behavior of propagating shear wave in the considered structure numerical computations has been performed by considering a combination of piezoelectric layer PZT-4 and aluminum epoxy half space. The numerically computed results are presented graphically in figures (3.2-3.7). The variations in non dimensional phase-velocity w.r.t. non dimensional wave number has been investigated under the effects of key parameters like non-local piezoelectric, non-local micropolar, thickness parameter, initial stress and imperfect parameter. The half space is considered of aluminum epoxy material for which the values of relevant parameters are as given in table 2.1 of chapter 2.

Figure–3.2 Describes the behavior of phase-velocity w.r.t. wave number in non-local piezoelectric layer imperfectly bonded over non-local micropolar half space. Dispersion curves of various mode of shear waves are shown and it is noticed that the phase-velocity decreases with an increase in wave number values which indicates the dispersive wave behavior in the composite structure. This suggests that wave speed varies with frequency. Figure-3.3 shows the impact of non-local parameter of piezoelectric material (ε') over the variations of phase-velocity w.r.t. wave number. To investigate the effect of(ε'), we considered different values of ε' (0 nm, 0.2 nm, 0.25 nm), while the non-local parameter of micropolar material (ε) is assumed to be fixed ($i.e.\,\varepsilon$ =0.30 nm). As visible in the graph the phase-velocity decreases with increase in non-local parameter of piezoelectric material. It is also seen that the phase-velocity is on higher side in case of local piezoelectric half space ($i.e.\,\varepsilon'$ =0) as compared to non-local piezoelectric half space. This reduction is due to the size-dependent effects introduced by the non-local characteristic, which results in lower wave speeds compared to a local piezoelectric material. Figure-3.4 presents the influence of non-local parameter of micropolar material

(ε) on the variation phase-velocity for fixed value of ε'=0.3nm. It could be seen from the graph, dispersion curves vanishes after certain wave number values for different values of ε . With increase in this parameter the dispersion curve can be seen for longer wave number values. So this parameter also has significant effects on phase-velocity of shear waves. Figure-3.5 describes the effects of initial stress on the variation of phase-velocity w.r.t. wave number. It is noticed that the initial stress has little effect on the dispersion relation for $|\tau_{11}^{0}| < 10^{8}$ and significant variations are observed for $|\tau_{11}^{0}| > 10^{8}$ as shown in graph. With increase in initial stress the phase-velocity increases for same wave number value.

Figure-3.6 describes the impact of an imperfect surface on the change in phase-velocity w.r.t. wave number. It has been demonstrated that at the imperfect interface, the phase-velocity decreases compared to a perfectly bounded surface. Figure-3.7 illustrates the impact of thickness parameter (h) on the variation in non dimensional phase-velocity and non dimensional wave number. The layer's significantly influences the phase-velocity of shear waves. For considered values of this parameter it is noticed that phase-velocity increases with increase in thickness.

These numerical results provide a comprehensive understanding of how various parameters such as non-local effects, initial stress, interface conditions, and layer thickness affect shear wave propagation in layered structures.

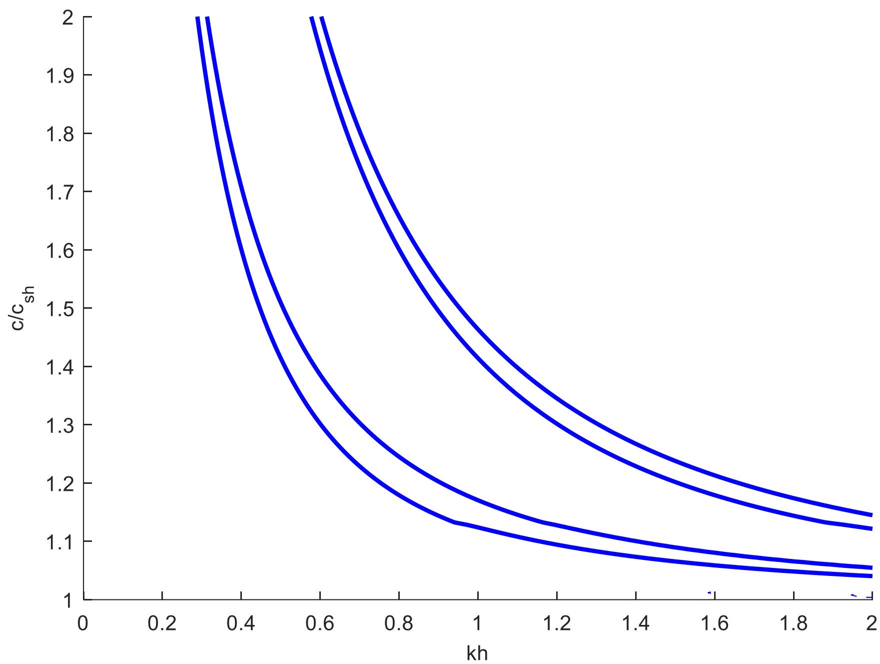

Fig. 3.2 Dispersion curve for shear waves for the considered model.

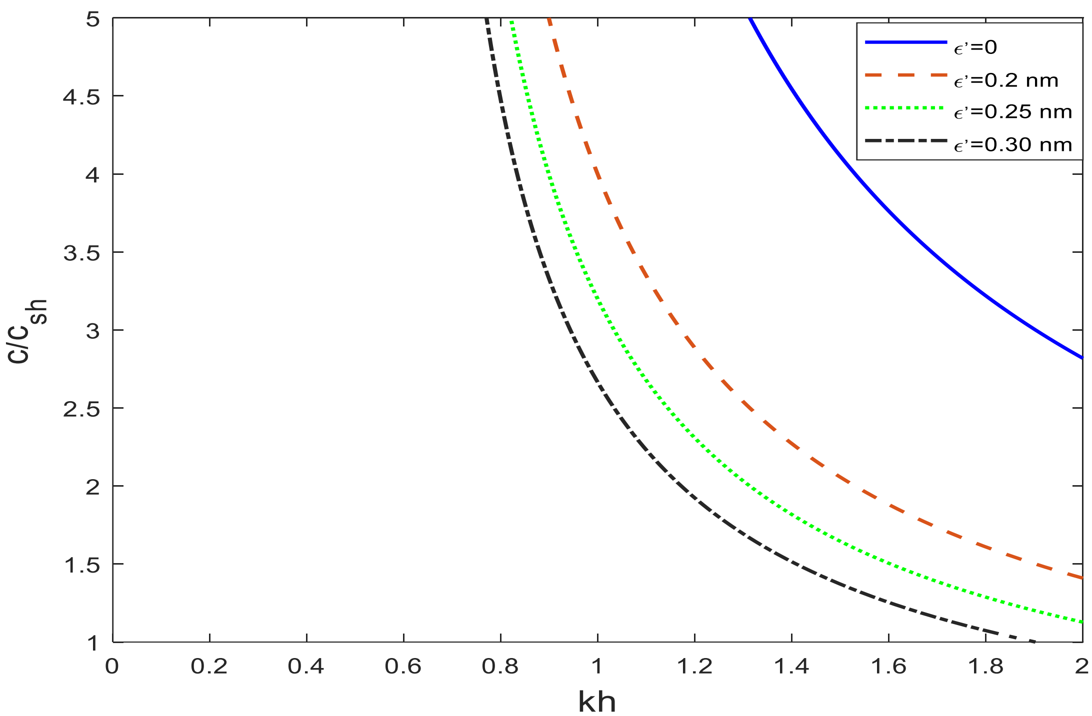

Fig. 3.3 Influence of non-local piezoelectric parameter.

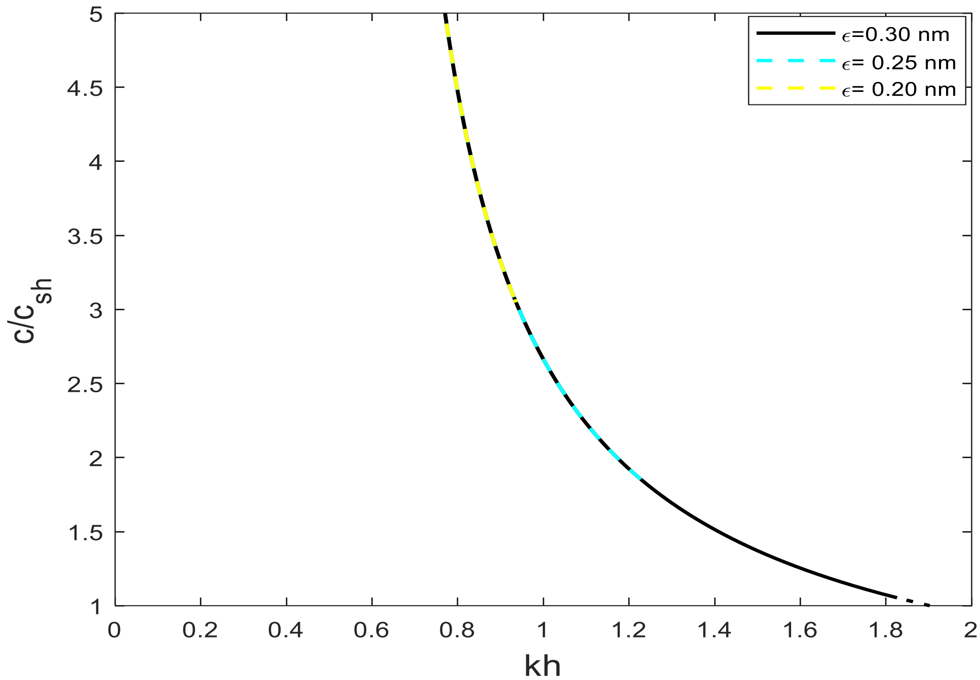

Fig. 3.4 Influence of non-local micropolar parameter.

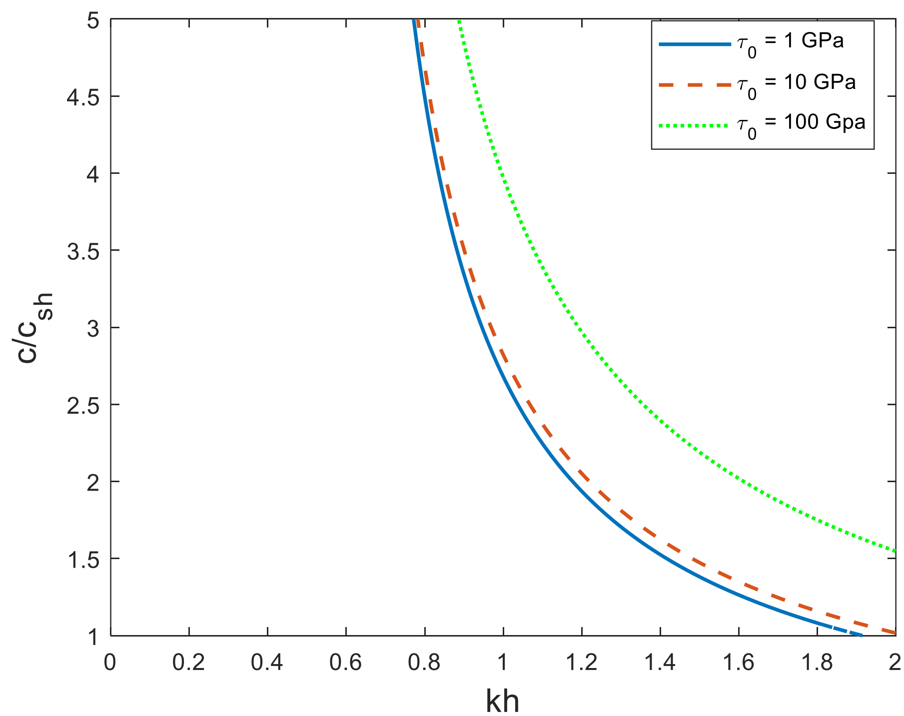

Fig. 3.5 Influence of initial stress parameter.

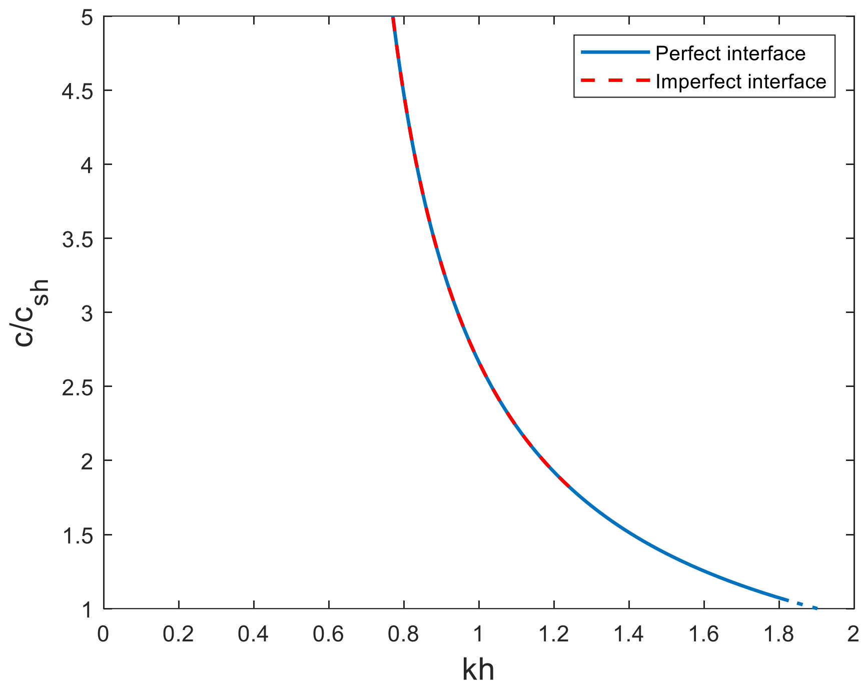

Fig. 3.6 Effect of imperfect parameter.

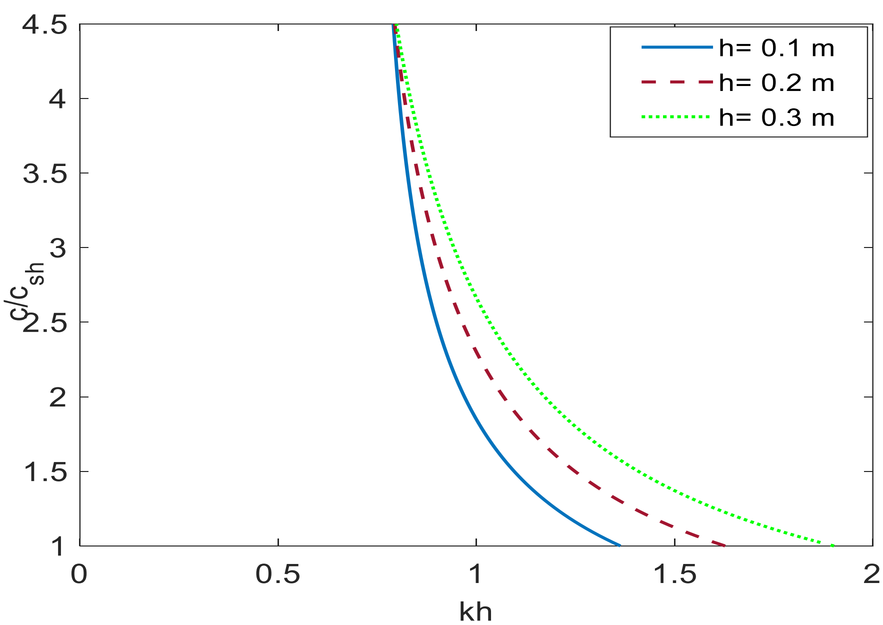

Figure 3.7 Variation due to thickness of layer.

3.8. CONCLUSION

The focus of this research pertains to the transmission of shear waves within a composite construction composed of a nonlocal piezoelectric layer that is imperfectly connected to a nonlocal micropolar elastic half space, which has an effect on the initial stress. A general wave dispersion equation for the shear wave propagation in considered structure has been derived analytically. Following points are concluded out of the present study:

- **Dispersion Relations**: The wave dispersion relations are affected by the non-local properties of both piezoelectric and micropolar materials.
- **Phase Velocity and Wave Number**: An increase in wave number leads to a reduction in phase velocity, illustrating the dispersive nature of wave propagation in this composite structure.
- **Effect of Non-locality**: Non-local characteristics of both piezoelectric and micropolar materials significantly lower the phase velocity of shear waves. This indicates that size-dependent and non-local effects play a crucial role in modifying wave propagation behavior.
- **Impact of Initial Stress**: Initial stress has a significant effect on the phase velocity of shear waves, with increase in initial stress the phase velocity also increases.
- **Layer Thickness**: The thickness of the piezoelectric layer is a vital factor in designing wave propagation within the composite structure. Changes in layer thickness can influence the dispersion and phase velocity of shear waves, making it a key consideration for material optimization.

In this chapter, considering the non-local effect in a piezoelectric material bonded imperfectly over NLME half space that will undoubtedly support investigators those are functioning in non-local piezoelectric materials. The study of shear wave propagation in layered structures with non-local micropolar and piezoelectric materials has significant practical applications. The insights gained can enhance the design of sensors by improving their sensitivity and performance, making them more effective in various environments. Additionally, this research is valuable for structural health monitoring, providing a deeper understanding of wave behavior in composite materials used in aerospace and civil engineering. It also contributes to better vibration control solutions for machinery and infrastructure, enhancing safety and reducing wear. Furthermore, the findings can guide the development of advanced composite materials with tailored properties for specific applications, leveraging the unique characteristics of non-local micropolar and piezoelectric materials. Overall, these applications highlight the study's potential to advance both theoretical knowledge and practical engineering solutions.

CHAPTER 4

RAYLEIGH WAVES IN NON-LOCAL MICROPOLAR THERMOELASTIC MATERIALS UNDER IMPEDANCE BOUNDARY CONDITIONS[3]

4.1 INTRODUCTION:

Rayleigh waves are important to investigate in materials due to their unique surface-propagation characteristics. These waves travel along the surface of a solid and provide valuable insights into how energy disperses in complex materials. The inclusion of non-local micropolar effects, which account for rotational motions and size-dependent behavior, allows for a deeper understanding of wave dispersion and energy transmission beyond classical models. Rayleigh waves are highly sensitive to surface properties, making them important for studying surface behavior, stress distribution, and the effects of material imperfections. Rayleigh waves in the layered model has several applications, including non-destructive testing techniques for detecting faults and weaknesses in materials, SAW devices, and geotechnical and geophysics engineering to investigate the earth's internal structure. Eringen (1968) determined the dispersion relation for Rayleigh surface waves in micropolar elastic half space under stress-free boundary. In the majority of scenarios involving Rayleigh waves, the standard practice is to consider boundary conditions where the surface is treated as traction-free. The exploration of alternative boundary conditions such as impedance boundary (IB) conditions is seldom undertaken in seismology. Nevertheless, in other branches of physics such as acoustics and electromagnetism, the use of IB conditions is prevalent. The IB conditions refer to a linear combination of unspecified functions and their derivatives that are defined along a boundary. The research articles given by Godoy et al. (2012), Vinh and Hue (2014a),

[3]The content of this chapter is published in “Journal of Physics: Conference Series” in the research paper entitled “Rayleigh waves with impedance boundary conditions in a non-local micropolar thermoelastic material”, VOL.1531 , p. 012048 (scopus indexed, 0.183).

Vinh and Hue (2014b), Singh (2015), Kaur and Singh (2015). Vinh and Xuan (2017), Singh (2016), Giang and Vinh (2021), Singh (2017), Anh et al. (2023) used IB conditions to study Rayleigh wave propagation in elastic materials.

This chapter focuses on examining the Rayleigh wave's propagation within a non-local micropolar thermoelastic half space characterized by presence of IB conditions. The dispersion equation governing the propagation of Rayleigh waves under IB conditions is derived, and the impact of both impedance and non-local parameters is investigated. Some earlier published results have been derived as particular cases from this study. The Rayleigh waves speed is calculated based on impedance parameters and then visually illustrated for an aluminum epoxy material. It is observed that the non-local and impedance parameter have significant effect on the Rayleigh waves speed.

4.2 BASIC EQUATIONS:

Using Eringen's (1984) non-local theory as explained in chapter 1, the equation of motion for homogenous, isotropic non-local micropolar elastic solids are given by

$$\sigma^{\mathrm{nl}}_{k\ell,k} + \rho\,(f_\ell - \ddot{u}^{n}_\ell) = 0, \tag{4.1}$$

$$m^{\mathrm{nl}}_{k\ell,k} + \in_{\ell mn} \sigma_{mn} + \rho\left(l_\ell - j\ddot{\phi}_\ell\right) = 0. \tag{4.2}$$

where, f_ℓ is the body force density, l_ℓ is the body couple density, j denotes micro inertia density. ϕ_ℓ is the micro-rotation vector. ρ is the mass density, u_ℓ represents displacement vector. $\in_{\ell mn}$ is the alternating symbol. Index after comma signifies the partial derivative.

The non-local stress tensor ($\sigma^{nl}_{k\ell}$) and couple stress tensor ($m^{nl}_{k\ell}$) can be represented in terms of local stress tensor ($\sigma_{k\ell}$) and local couple stress tensor ($m_{k\ell}$) as

$$(1-\varepsilon^2\nabla^2)\,\sigma^{\mathrm{nl}}_{k\ell} = \sigma_{k\ell} \tag{4.3}$$

$$(1-\varepsilon^2\nabla^2)m^{\mathrm{nl}}_{k\ell} = m_{k\ell}. \tag{4.4}$$

where ε is a non-local parameter ($\varepsilon = e_0 l$) , l is characteristic length $e_0$is the non-local constant. The local stress tensor $\sigma_{k\ell}$ and local couple stress tensor $m_{k\ell}$ for an isotropic, homogeneous micropolar thermoelastic material are given by Eringen (1970)

$$\begin{cases} \sigma_{k\ell} = \lambda e_{rr}\delta_{k\ell} + (\mu+\kappa)e_{k\ell} + \mu e_{\ell k} - \nu T\delta_{k\ell} , \\ m_{k\ell} = \alpha\gamma_{rr}\delta_{k\ell} + \beta\gamma_{k\ell} + \gamma\gamma_{k\ell} , \\ e_{k\ell} = (u^{n}_{\ell,k} - \epsilon_{k\ell m}\phi_{m,\ell}) , \gamma_{k\ell} = \phi_{k,\ell} . \end{cases} \tag{4.5}$$

where, T is the change in temperature from the reference temperature T_o of the body and $\nu = (3\lambda + 2\mu + \kappa)\alpha_t$. Here, α_t denotes coefficient of thermal linear expansion.

Using equations (4.3)-(4.5) in equations (4.1)-(4.2), the field equations of a non-local micropolar thermoelastic material can be written as

$$(\lambda + \mu)u^{n}_{k,k\ell} + (\mu + \kappa)u^{n}_{\ell,kk} + \kappa \in_{\ell mn} \phi_{n,m} - \nu\nabla T + \rho(1 - \varepsilon^2\nabla^2)(f_\ell - \ddot{u}^{n}_{\ell}) = 0, \tag{4.6}$$

$$(\alpha + \beta)\,\phi_{k,k\ell} + \gamma\phi_{\ell,kk} + \kappa \in_{\ell mn} u^{n}_{m,m} - 2\,\kappa\phi_\ell + \rho(1 - \varepsilon^2\nabla^2)\big(l_\ell - j\ddot{\phi}_\ell\big) = 0. \tag{4.7}$$

Equations (4.3)-(4.4) and (4.6)-(4.7) reduced to those of local micropolar theory in the static case and in the absence of body forces and couples.

The heat conduction equation as per the Lord Shulman (1967) is given as,

$$K^*\nabla^2 T = \rho\, C^*\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right) T + \nu\, T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\nabla.\boldsymbol{u^n} \tag{4.8}$$

where, K^*= coefficient of thermal conductivity,

C^*= specific heat at constant strain

τ_0= thermal relaxation time.

4.3 PROBLEM’S FORMULATION

In the initial state, a non-local thermoelastic half space, which is homogeneous and isotropic, is considered to be at a uniform temperature T_0. The geometry of the problem is as shown in the figure 4.1. Wave propagation direction is assumed along the x_1- axis, ensuring the particles vibrating on a line parallel to the x_3-axis experiences equal displacement. Consequently, all field quantities are presumed to be independent of x_3-axis.

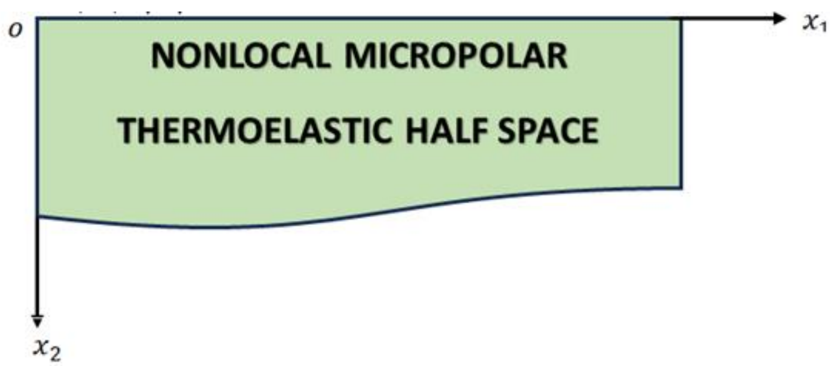

Fig. 4.1 Geometry of the problem

For the two- dimensional problem in $x_1 - x_2$plane, we have

$$\boldsymbol{u^n} = (u_1^n\,, u_2^n, 0), \qquad \boldsymbol{\phi} = (0, 0, \phi_3\,) \tag{4.9}$$

Using (4.9), equation (4.6) and (4.7) can written as :

$$(\lambda + 2\mu + K)\frac{\partial^2 u_1^n}{\partial x_1^2} + (\mu + K)\frac{\partial^2 u_1^n}{\partial x_2^2} + (\lambda + \mu)\frac{\partial^2 u_2^n}{\partial x_1 \partial x_2} + K\frac{\partial \phi_3}{\partial x_2} - \nu\frac{\partial T}{\partial x_1} \;=\; \rho(1 - \varepsilon^2 \nabla^2\,)\frac{\partial^2\, u_1^n}{\partial t^2}, \tag{4.10}$$

$$(\lambda + 2\mu + K)\frac{\partial^2 u_2^n}{\partial x_2^2} + (\mu + K)\frac{\partial^2 u_2^n}{\partial x_1^2} + (\lambda + \mu)\frac{\partial^2 u_1^n}{\partial x_1 \partial x_2} - K\frac{\partial \phi_3}{\partial x_1} - \nu\frac{\partial T}{\partial x_2} \;=\; \rho$$

$$(1 - \varepsilon^2 \nabla^2\,)\frac{\partial^2\, u_2^n}{\partial t^2}, \tag{4.11}$$

$$\gamma\left(\frac{\partial^2 \phi_3}{\partial x_1^2} \;+\; \frac{\partial^2 \phi_3}{\partial x_2^2}\right) + K\left(\frac{\partial u_2^n}{\partial x_1} - \;\frac{\partial u_1^n}{\partial x_2}\right) - 2K\phi_3 \;=\; \rho \mathrm{j}(1 - \varepsilon^2 \nabla^2\,)\frac{\partial^2\, \phi_3}{\partial t^2}. \tag{4.12}$$

By employing Helmholtz's decomposition, u_1^n and u_2^n can be expressed in terms of potential function Φ and Ψ as

$$u_1^n \;=\; \frac{\partial \Phi}{\partial x_1} + \;\frac{\partial \Psi}{\partial x_2} \;\;, u_2^n \;=\; \frac{\partial \Phi}{\partial x_1} - \;\frac{\partial \Psi}{\partial x_2}. \tag{4.13}$$

Substituting (4.13) in equation (4.8) and (4.10) to (4.12) , we get

$$(\lambda + 2\mu + K)\nabla^2\Phi - \nu T = \rho(1 - \varepsilon^2\nabla^2)\frac{\partial^2 \Phi}{\partial t^2}, \tag{4.14}$$

$$(\mu + K)\nabla^2\Psi + K\,\phi_3 = \rho(1 - \varepsilon^2\nabla^2)\frac{\partial^2 \Psi}{\partial t^2}, \tag{4.15}$$

$$\gamma\nabla^2\phi_3 - 2K\,\phi_3 - K\,\nabla^2\Psi = \rho \mathrm{j}(1 - \varepsilon^2\nabla^2)\frac{\partial^2 \phi_3}{\partial t^2}, \tag{4.16}$$

$$K^*\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\rho\, C^* T + \nu\, T_0\nabla^2\Phi). \tag{4.17}$$

4.4. PROBLEM'S SOLUTION

The surface wave solutions of the equation (4.14) to (4.17) is taken as

$$\{\Phi, \Psi, T, \phi_3\} = \{\bar{\Phi}(x_2), \bar{\Psi}(x_2), \bar{T}(x_2), \overline{\phi_3}(x_2)\}e^{ik(x_1 - ct)} \tag{4.18}$$

where $\omega = kc$. c, k are as explained in chapter 2. Here, the assumption is made that Rayleigh surface waves exhibit temporal damping and propagation side $x_1 - axis$ with wave speed $Re\,(c) = V > 0$and $Im\,(c) \le 0$.

The equations (4.14) to (4.17) by using (4.18), we obtain

$$[D^4 - AD^2 + B]\left(\bar{\Phi}(x_2), \bar{T}(x_2)\right) = 0 \tag{4.19}$$

$$[D^4 - A'D^2 + B']\left(\bar{\Psi}(x_2), \overline{\phi_3}(x_2)\right) = 0 \tag{4.20}$$

Where,

$$D = \frac{d}{dx_2}, A = k^2\left[2 - \frac{c^2\left(1 + A_2 + \frac{A_1}{c_1^2}\right)}{A_1}\right] + \frac{k^4\varepsilon^2 c^2}{c_1^2}\left(\frac{c^2}{A_1} - 2\right)$$

$$B = k^4\left[\frac{A_1 - c^2\left(1 + A_2 + \frac{A_1}{c_1^2}\right) + \frac{c^4}{c_1^2}}{A_1}\right] + k^6\varepsilon^2 c^2\left(\frac{c^2}{A_1} - 1\right)$$

$$A' = k^2\left(1 - \frac{c^2}{c_2^2}\right) + k^2 - \frac{k^2 c^2 \rho \mathrm{j}}{\gamma} + \frac{2K}{\gamma} - \frac{K^2}{\gamma\,(\mu + K)} + \frac{2k^2c^2\rho\,\mathrm{j}\,\varepsilon^2}{\gamma\, c_2^2}$$

$$(c_2^2 + k^2c^2 + k^4\varepsilon^4c^4) - \frac{2k^2c^2\varepsilon^2}{c_2^2}\left(1 + \frac{2K}{\gamma}\right)$$

$$B' = k^2\left(k^2 - \frac{k^2c^2\rho j}{\gamma} + \frac{2K}{\gamma}\right)\left(1 - \frac{c^2}{{c_2}^2}\right) - \frac{k^2K^2}{\gamma\,(\mu+K\,)} - \frac{2k^2\,K\,c^2\varepsilon^2}{\gamma\,{c_2}^2} + \frac{k^6\varepsilon^2c^2}{{c_2}^2}\left(\frac{k^2c^2\rho\,j\,\varepsilon^2}{\gamma} + \frac{2\,\rho\,j}{\gamma} - \frac{\rho\,j\,{c_2}^2}{\gamma} - 1\right),$$

$$ {c_1}^2 = \frac{\lambda + 2\mu + K}{\rho}\,, {c_2}^2 = \frac{\mu + K}{\rho}\,, \tau^* = \tau_0 + \frac{i}{\omega}\,,$$

$$A_1 = \frac{K^*}{\rho\,C^*\tau^*}\,, A_2 = \frac{v^2T_0}{\rho^2{c_1}^2C^*} \tag{4.21}$$

Using the radiation conditions $\bar{\Phi}(x_2),\ \bar{\Psi}(x_2), \bar{T}(x_2), \overline{\phi_3}(x_2) \to\ 0\ as\ x_2\ \to \infty$ on the general solution of the equations (4.19) and (4.20) using (4.18) , we obtain

$$\Phi = (B_1e^{-b_1\,y} + B_2e^{-b_2\,y})e^{ik(x_1-ct)}, \tag{4.22}$$

$$\Psi = (B_3e^{-b_3\,y} + B_4e^{-b_4\,y})e^{ik(x_1-ct)}, \tag{4.23}$$

$$T = (r_1B_1e^{-b_1\,y} + r_2B_2e^{-b_2\,y})e^{ik(x_1-ct)}, \tag{4.24}$$

$$\phi_3 = (r_3\,B_3e^{-b_3\,y} + r_4\,B_4e^{-b_4\,y})e^{ik(x_1-ct)}. \tag{4.25}$$

Where

$$b_1^2 + b_2^2 = A\,, b_1^2b_2^2 = B\,, b_3^2 + b_4^2 = A'\,, b_3^2b_4^2 = B'. \tag{4.26}$$

$$\begin{cases} r_i = \frac{k^2}{v}\left[(\lambda+2\mu+K)\left(\frac{b_i}{k^2}-1\right)+\rho c^2\left(1+\varepsilon^2(k^2-1)\right)\right], & (i=1,2) \\ r_j = \dfrac{k^2(\mu+K)}{K}\left[1 - \dfrac{c^2}{{c_2}^2} - \dfrac{b_j}{k^2} + \dfrac{\varepsilon^2c^2}{{c_2}^2}(b_3^2 - k^2)\right], & (j = 3,4) \end{cases} \tag{4.27}$$

here, B_1 , B_2 , B_3 , B_4 are arbitrary constants.

4.5. BOUNDARY CONDITIONS

Following Godoy and Nedelec (2012), the IB conditions at the surface $x_2 = 0$ can be written as:

$$\sigma_{2i} + \omega\,Z_iu_i = 0.$$

Therefore, the IB conditions for non-local Micropolar thermoelastic half space can be formulated as:

$$(1 - \varepsilon^2\nabla^2)(\sigma_{21}^{nl} + \omega\,Z_1u_1) = 0,$$

$$(1-\varepsilon^2\nabla^2)\left(\sigma_{22}^{nl}+\ \omega\, Z_2 u_2\right)=0,$$
$$(1-\varepsilon^2\nabla^2)\left(m_{23}^{nl}+\ \omega\, Z_3 \phi_3\right)=0.$$

Or

$$\begin{aligned}\sigma_{21}+\ (1-\varepsilon^2\nabla^2)\omega\, Z_1 u_1 &= 0,\\ \sigma_{22}+\ (1-\varepsilon^2\nabla^2)\,\omega\, Z_2 u_2 &= 0,\\ m_{23}+(1-\varepsilon^2\nabla^2)\,\omega\, Z_3\phi_3 &= 0.\end{aligned} \tag{4.28}$$

We consider thermal boundary condition as $,\frac{\partial T}{\partial u_2}+hT=0.$

Where h approaches to ∞ corresponds to isothermal surface and h approaches to 0 corresponds to thermally insulated surface.

Using condition (4.28) on the surface x_2= 0, The resulting secular equation for the velocity of Rayleigh wave propagation is as follows

$$M_1[T_1(L_2N_4-N_2L_4)-T_2(L_1N_4-N_1L_4)]=M_2[T_1(L_2N_3-N_2L_3)T_2(L_1N_3-N_1L_3)] \tag{4.29}$$

Where,

$$L_1=\left[k\,V_1Z_1^*-b_1-\left(1+\frac{K}{\mu}\right)b_1\right],$$

$$L_2=\left[k\,V_1Z_1^*-b_2-\left(1+\frac{K}{\mu}\right)b_2\right],$$

$$L_3=\ V_1Z_1^*b_3-k-\ k\ \left(1+\frac{K}{\mu}\right)\left(1-\frac{c^2}{c_2^2}-\frac{\varepsilon^2k^2c^2}{c_2^2}+\frac{\varepsilon^2b_3^2c^2}{c_2^2}\right),$$

$$L_4=\ V_1Z_1^*b_4-k-\ k\ \left(1+\frac{K}{\mu}\right)\left(1-\frac{c^2}{c_2^2}-\frac{\varepsilon^2k^2c^2}{c_2^2}+\frac{\varepsilon^2b_4^2c^2}{c_2^2}\right),$$

$$N_1=2+\frac{K}{\mu}-V_1^2+(k^2-1)\varepsilon^2V_1^2-V_1Z_2^*\,b_1\ ,$$

$$N_2=2+\frac{K}{\mu}-V_1^2+(k^2-1)\varepsilon^2V_1^2-V_1Z_2^*\,b_2\ ,$$

$$N_3=\left[\left(2+\frac{K}{\mu}\right)b_3-k\,V_1Z_2^*\right],$$

$$N_3=\left[\left(2+\frac{K}{\mu}\right)b_4-k\,V_1Z_2^*\right],$$

$$M_1 = (k\,\mu\,V_1 Z_3^* - \gamma\, b_3)\left(1 - \frac{c^2}{c_2^2} - \frac{b_3^2}{k^2} - \frac{\varepsilon^2 k^2 c^2}{c_2^2} + \frac{\varepsilon^2 b_3^2 c^2}{c_2^2}\right),$$

$$M_2 = (k\,\mu\,V_1 Z_3^* - \gamma\, b_4)\left(1 - \frac{c^2}{c_2^2} - \frac{b_4^2}{k^2} - \frac{\varepsilon^2 k^2 c^2}{c_2^2} + \frac{\varepsilon^2 b_4^2 c^2}{c_2^2}\right),$$

$$V_1 = \sqrt{\frac{\rho\, c^2}{\mu}},$$

$$Z_1^* = \frac{z_i}{\sqrt{\rho\mu}}.$$

For isothermally insulated surface:

$$T_1 = b_1\left[\left(2 + \frac{\lambda+K}{\mu}\right)\left(\frac{b_1^2}{k^2} - 1\right) + V_1^2\left(1 + \varepsilon^2(k^2 - 1)\right)\right],$$

$$T_2 = b_2\left[\left(2 + \frac{\lambda+K}{\mu}\right)\left(\frac{b_2^2}{k^2} - 1\right) + V_1^2\left(1 + \varepsilon^2(k^2 - 1)\right)\right].$$

For isothermal surface:

$$T_1 = \left[\left(2 + \frac{\lambda+K}{\mu}\right)\left(\frac{b_1^2}{k^2} - 1\right) + V_1^2\left(1 + \varepsilon^2(k^2 - 1)\right)\right],$$

$$T_2 = \left[\left(2 + \frac{\lambda+K}{\mu}\right)\left(\frac{b_2^2}{k^2} - 1\right) + V_1^2\left(1 + \varepsilon^2(k^2 - 1)\right)\right].$$

4.6. PARTICULAR CASES

Case I:

When the non-local parameter is absent, equation (4.29) simplifies to the secular equation of Rayleigh waves in a micropolar thermoelastic half-space under IB conditions.

Case II:

On overlooking the non-local and Micropolar effects ($\varepsilon = K = j = 0$) in the equation (4.29), we obtain the following equation

$$L_3(N_1T_2 - N_2T_1) - N_3(L_1T_2 - L_2T_1) = 0. \qquad (4.30)$$

Equation (4.30) aligns with the secular equation derived by the author Kumar et al. (2018) for Rayleigh waves in a thermoelastic half-space featuring IB conditions.

Case III:

If we neglect non-local, impedance, thermal and Micropolar effects from the model i.e.

$\varepsilon = K = j = Z_1^* = Z_2^* = Z_3^* = \nu = 0,$

Then equation (4.30) becomes,

$$\left(2-\frac{c^2}{c_2^2}\right)^2 = 4\sqrt{1-\frac{c^2}{c_1^2}}\sqrt{1-\frac{c^2}{c_2^2}}. \tag{4.31}$$

where $c_1^2 = \frac{\lambda+2\mu}{\rho}$, $c_2^2 = \frac{\mu}{\rho}$

This is widely recognized dispersion equation for the Rayleigh wave's phase-velocity in classical elastic half-space.

4.7 NUMERICAL RESULTS AND DISCUSSIONS

We have considered aluminum epoxy material for numerical results. The values of relevant physical parameters are mentioned in the table 2.1 in chapter 2. The values of additional physical parameters are mentioned here.

Table 4.1 material parameters

Aluminum epoxy material	
$\rho = 2.19 \times 10^3 kg/m^3$,	$j = 0.196 \times 10^4 m^2$
$\lambda = 7.59 \times 10^{10} N/m^2$	K^*=$0.492 \times 10^2 W/m$
$\mu = 1.89 \times 10^{10} N/m^2$	$C^* = 1.89 \times 10^{10} j/kg$
$\alpha = 0.01 \times 10^6 N$	$\tau_0 = 0.5 \times 10^{-10} s$
$\beta = 0.015 \times 10^6 N$	T_0=$298K$
$\gamma = 0.268 \times 10^6 N$	α_t=$2.36 \times 10^{-6} K^{-1}$
$j = 0.196 \times 10^4 m^2$	K=$0.0149 \times 10^{10}\ N/m^2$

The phase velocity c is considered as a complex constant with $Re\,(c) = V \geq 0$. The Rayleigh wave speed $V_1 = \sqrt{\frac{\rho V^2}{\mu}}$ is taken in non-dimensional form and calculated by using the equation (4.29). The influence of non-local and impedance parameters on the

Rayleigh-wave speed w.r.t non dimensional wave number (ka) , where a is the internal characteristic length, under thermally insulated boundary surface have been discussed in figure 4.2 to 4.6.

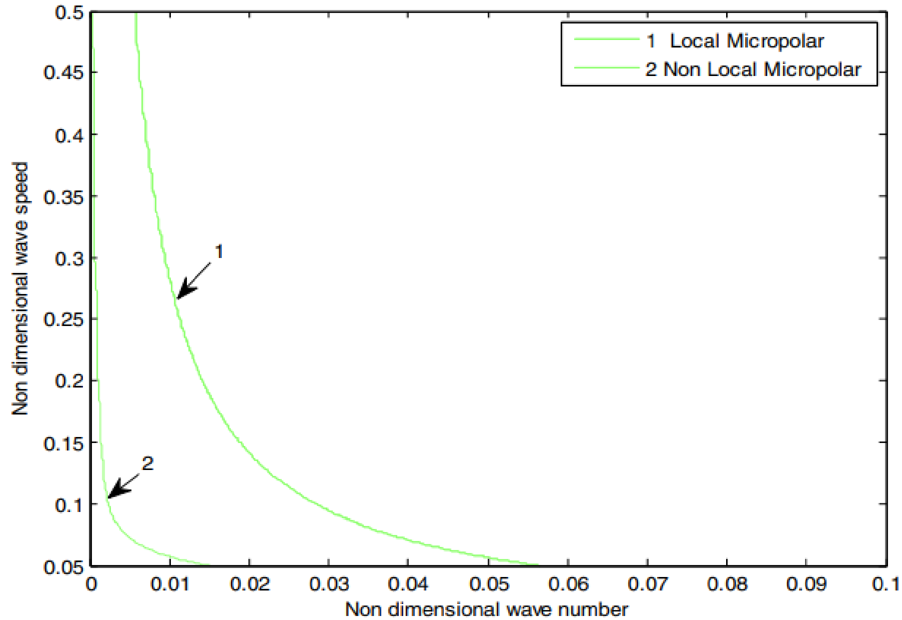

Fig.4.2 Non dimensional Rayleigh wave speed behavior w.r.t. non dimensional wave number in local and non-local Micropolar thermoelastic half spaces.

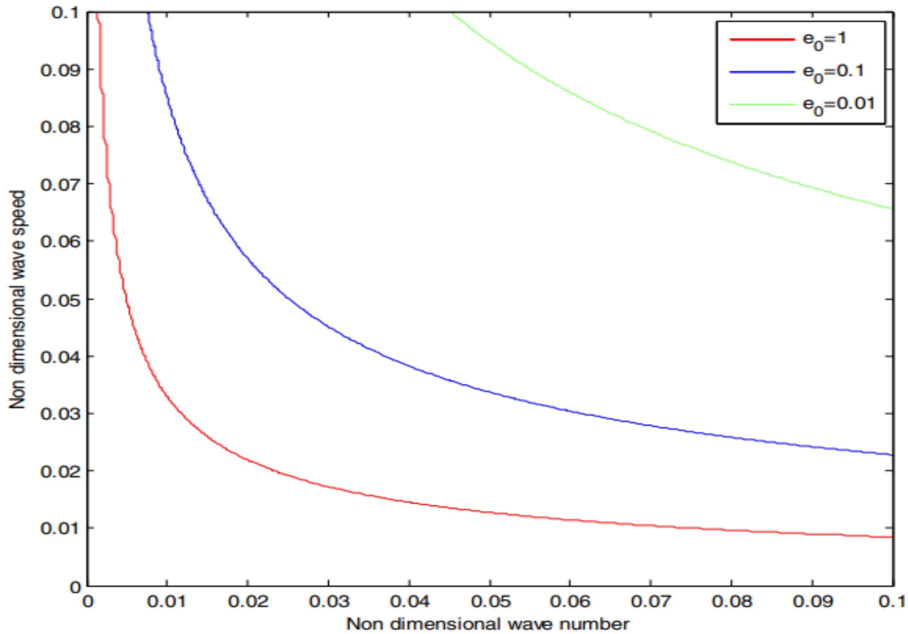

Fig 4.3 Effects of non-local constants on the variations of non dimensional wave speed w.r.t. non dimensional wave number

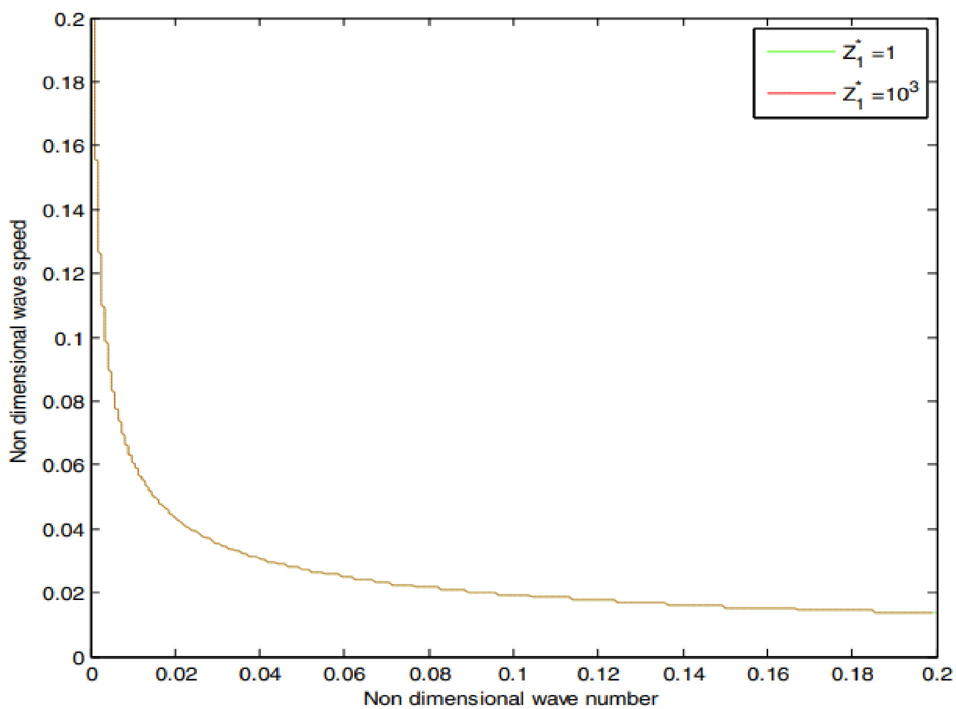

Fig. 4.4 Effects of impedance parameter Z_1^* on Rayleigh wave speed

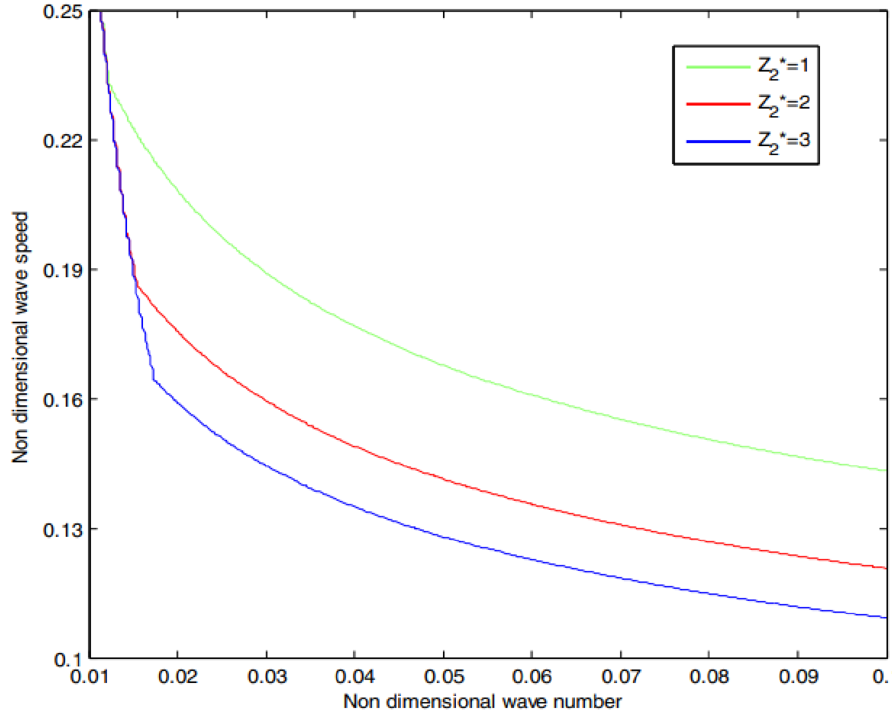

Fig4.5 Effects of impedance parameter Z_2^* on Rayleigh wave speed

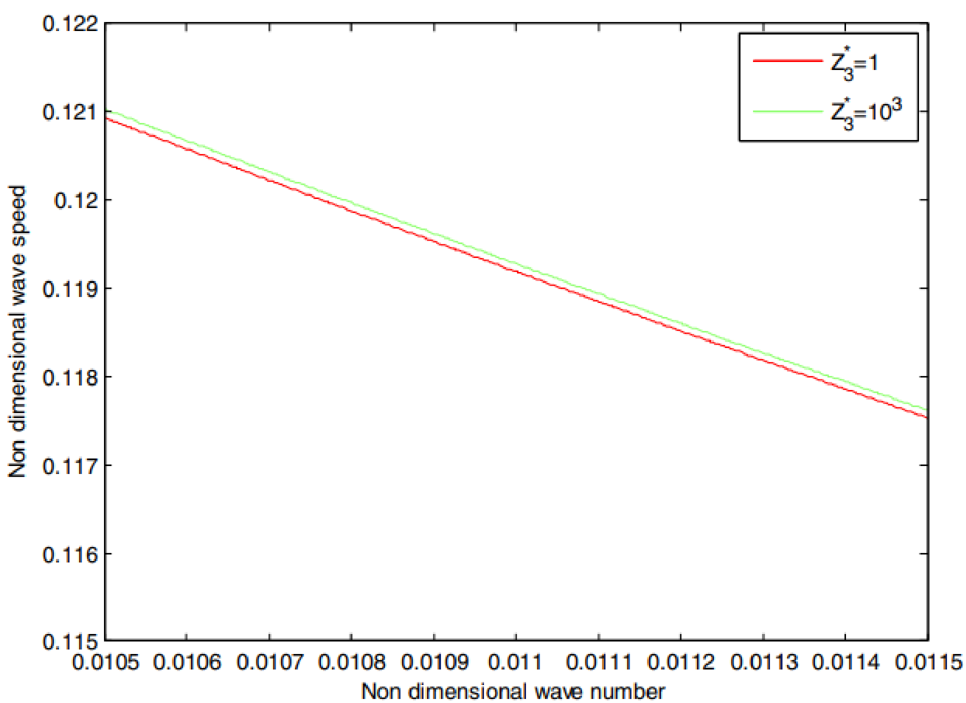

Fig. 4 . 6 Effects of impedance parameter Z_3^* on Rayleigh wave speed

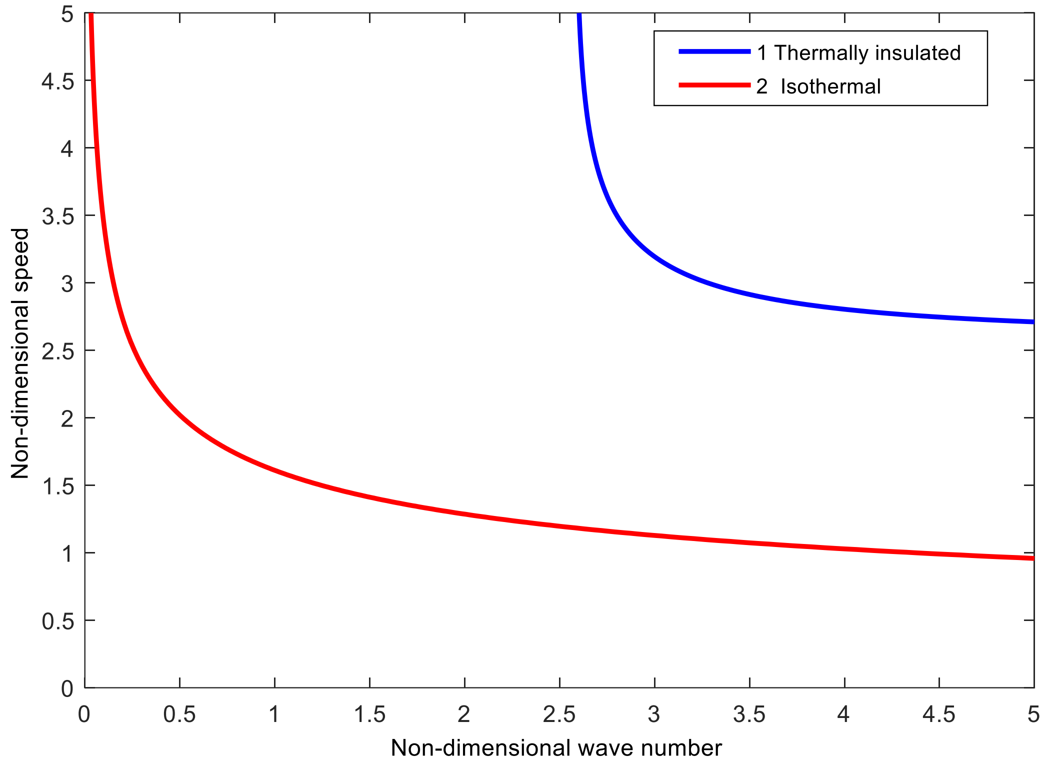

Fig. 4.7 Effects of isothermal and insulated boundary conditions on the variation of non dimensional wave speed with non dimensional wave number.

Figure.4.2 compares the wave speed in local and non-local Micropolar thermoelastic half space under fixed values of impedance parameters ($Z_1^* = 1,\ Z_2^* = 0,\ Z_3^* = 0$) and internal characteristic length $a = 10^{-9}m$. As visible, the graph shows that speed of wave vanishes quickly with wave number in case of non-local Micropolar thermoelastic material. This rapid decline suggests that as wave number rises, the impact of the material's microstructural characteristics becomes more significant, leading to increased energy loss and diminished efficiency of wave propagation.

Figure 4.3 shows the effect of non-local constant. It is noticed that as the value of non-local constant decreases the wave speed increases. This relationship indicates a more effective coupling between microstructural dynamics and macroscopic behavior, resulting in better wave transmission at reduced non-local constants. Figure (4.4 – 4.6) describes the effect of impedance parameters on the wave speed in a non-local Micropolar thermoelastic half space. The impedance parameters Z_2^* and Z_3^* has significant effects on Rayleigh wave speed rather it is has minimal effect by Z_1^*. These insights emphasize the importance of carefully selecting material properties to achieve desired wave behavior in practical applications.

Figure 4.7 compares the non-dimensional wave speed for thermally insulated and isothermal boundary conditions as a function of the non-dimensional wave number. The isothermal condition exhibits a consistently lower wave speed across all wave numbers compared to the thermally insulated case, which shows a higher wave speed, especially for smaller wave numbers. As the wave number increases, both curves tend to stabilize, indicating a reduced effect of thermal boundary conditions on wave speed at higher wave numbers.

Overall, these results shed light on the intricate factors influencing wave propagation in micropolar thermoelastic materials, revealing the essential contributions of microstructural features and impedance parameters.

4.8. CONCLUSION

In this chapter, we investigate Rayleigh waves in a non-local micropolar thermoelastic half-space under IB conditions for both thermally insulated and isothermal surfaces. We derive the explicit form of the secular equation that satisfies the IB conditions. Specific instances reveal that when the non-local parameter is eliminated, the resulting secular equation agrees with the earlier published results. From numerical discussions, we can conclude that

- **Effect of Non-Local Parameter on Wave Speed**: The non-local parameter has significant effects on Rayleigh waves speed. The wave decreases in case of non-local Micropolar material as compared to the local Micropolar material.

- **Impact of Increasing Non-Local Parameter**: With increase in non-local parameter value the wave speed decreases.
- **Role of Impedance and Thermal Parameters**: The Rayleigh wave speed depends upon impedance and thermal parameters and affected significantly by these parameters. The wave speed is highly or low dispersive according to the range of impedance parameters and wave number.

The study of Rayleigh waves in non-local micropolar materials has several practical applications. These waves are widely used in non-destructive testing to detect surface flaws, such as cracks or delaminations, in materials. In the field of geophysics, Rayleigh waves help in seismic surveys, providing insights into the Earth's subsurface properties. They are also crucial in the development of microelectromechanical systems (MEMS), where they enhance the sensitivity of devices such as sensors. Additionally, they play a key role in the design of surface acoustic wave (SAW) devices, used in telecommunications for filtering and signal processing. So the results obtained such as effects of impedance and non –local on the wave speed and the secular equation of Rayleigh wave in non-local Micropolar material may be useful for the researcher working in material design and solid mechanics.

Chapter -5

RAYLEIGH WAVES IN NON-LOCAL MICROPOLAR MATERIAL UNDER MEMORY DEPENDENT HEAT TRANSFER[4]

5.1 INTRODUCTION

The memory dependent derivatives now days have been applied to number of problems related to heat conduction equation as in certain materials, where the heat transfer rate depends not only on the local temperature gradient but also on the past history of temperature changes in the material. The memory dependent derivative (MDD) has been used in some interesting problems such as Ezzat et al. (2015) applied the concept of MDD in magneto-thermoelasticity to solve a one-dimensional problem of elastic half space. Ezzat et al. (2016) introduced MDD in generalized thermoelasticity theory and derived a new heat conduction equation. Othman and Mondal (2019) studied the effects of MDD under phase-lag modals on the wave propagation in generalized micropolar thermoelasticity using laser beam thermal shock conditions. Kumar and Pratap (2022) used MDD to analyzed wave propagation in micropolar generalized thermoelastic plate. Ahmed et al. (2018) developed a model based on MDD to study damping in different types of oscillatory systems. Kant and Mukhopadhyay (2019) investigated thermoelastic interaction using MDD within a thick plate. Sarkar and Mondal (2019) solved a two-temperature problem of generalized thermoelasticity using memory dependent derivative. Mondal and Othman (2021) obtained the general solution for the propagation of plane waves in generalized piezo-thermoelastic medium using memory dependent derivatives. Purkait et al. (2021) studied the elasto thermo-diffusion in a spherical shell using MDD for two temperature theory. Mondal and Sur (2023) applied MDD to investigate the

[4]Communicated in SCI journal.

interaction in a functionally graded thermoelastic rod and explored the influence of memory effects, magnetic field, and non-homogeneity of the material.

The motivation for examining Rayleigh-type waves in non-local micropolar materials with memory-dependent heat transfer is rooted in the complex interplay between microstructural characteristics and thermal effects. Traditional models often fail to account for the significant impact of non-local behavior and memory effects, which can

result in more precise predictions of wave propagation in advanced materials. By investigating these interactions, this study aims to enhance our understanding of wave dynamics, which is vital for optimizing material performance in various applications. This research addresses a significant gap in the existing literature, offering novel insights that are essential for developing more comprehensive models of wave behavior in intricate material systems.

This chapter addressed the propagation of Rayleigh waves in a non-local micropolar thermoelastic material under the memory-dependent heat conduction model. The secular equation is obtained analytically. Numerical computations have been performed to investigate impact of time delay parameter, non-local parameter, and kernel functions on propagation of Rayleigh waves in the aluminum-epoxy composite material. The numerical and graphical analysis validate that the speed of Rayleigh waves is influenced significantly by time delay heat transfer, selection of kernel and non-local characteristic of the material.

5.2 BASIC EQUATIONS

As discussed in section 4.2 of chapter 4, the basic equations of motion for homogenous, isotropic non-local micropolar elastic solids are given by

$$\sigma^{nl}_{k\ell,k} + \rho\,(f_\ell - \ddot{u}_\ell) = 0, \tag{5.1}$$

$$m^{nl}_{k\ell,k} + \epsilon_{\ell mn}\,\sigma_{mn} + \rho\left(l_\ell - j\ddot{\phi}_\ell\right) = 0. \tag{5.2}$$

The non-local stress tensor ($\sigma_{k\ell}^{nl}$) and couple stress tensor ($m_{k\ell}^{nl}$) can be represented in terms of local stress tensor ($\sigma_{k\ell}$) and local couple stress tensor ($m_{k\ell}$) as

$$(1-\varepsilon^2\nabla^2)\,\sigma_{k\ell}^{\mathrm{nl}} = \sigma_{k\ell} \tag{5.3}$$

$$(1-\varepsilon^2\nabla^2)m_{k\ell}^{\mathrm{nl}} = m_{k\ell}. \tag{5.4}$$

$$\begin{cases} \sigma_{k\ell} = \lambda e_{rr}\delta_{k\ell} + (\mu+\kappa)e_{k\ell} + \mu e_{\ell \mathrm{k}} - \nu T\delta_{k\ell}\,, \\ m'_{k\ell} = \alpha\gamma_{rr}\delta_{k\ell} + \beta\gamma_{k\ell} + \gamma\gamma_{k\ell}\,, \\ e_{k\ell} = (u_{\ell,k}^{\mathrm{n}} - \epsilon_{k\ell m}\phi_{m,\ell})\,, \gamma_{k\ell} = \phi_{k,\ell}\,. \end{cases} \tag{5.5}$$

where, symbols are having the usual meaning as explained in chapter - 4.

Using equations (5.3)-(5.5) in equations (5.1)-(5.2), the field equations of a non-local micropolar thermoelastic material can be written as

$$(\lambda+\mu)u_{k,k\ell}^{\mathrm{n}} + (\mu+\kappa)u_{\ell,kk}^{\mathrm{n}} + \kappa \in_{\ell mn} \phi_{n,m} - \nu\nabla T + \rho(1-\varepsilon^2\nabla^2)(f_\ell - \ddot{u}_\ell^n) = 0, \tag{5.6}$$

$$(\alpha+\beta)\,\phi_{k,k\ell} + \gamma\phi_{\ell,kk} + \kappa \in_{\ell mn} u_{n,m}^{\mathrm{n}} - 2\,\kappa\phi_\ell + \rho(1-\varepsilon^2\nabla^2)\big(l_\ell - j\ddot{\phi}_\ell\big) = 0. \tag{5.7}$$

As stated by Wang and Li (2011), the memory dependent derivative of a function f(t) is given as

$$D_\xi\big(f(t)\big) = \frac{1}{\xi}\int_{t-\xi}^{t} K(t-s)f'(s)ds \tag{5.8}$$

Where $K(t-s)$ is the kernel function and ξ is the time delay parameter, which can be adjusted based on the system requirement, and its units are same as that of time. The kernel function may be thought of as the strength of the impact that the past on the present and it is freely selectable such as 1, $\left[1+\frac{s-t}{\xi}\right]^2$ and $\left[1+\frac{s-t}{\xi}\right]^{\frac{1}{4}}$ according to the system. In general, the inequality $0 \le K(t-s) < 1$ should be satisfied by the function $K(t-s)\ fors \in [t-\xi, t]$ from the perspective of applications.

In case of $K(t-s) = 1$ the memory dependent derivative D_ξ can be expressed as follows

$D_\xi\big(f(t)\big) = \frac{1}{\xi}\int_{t-\xi}^{t} f'(s)\,ds = \frac{f(t)-f(t-\xi)}{\xi} \to f'(t)$ when $\xi \to 0$.

Therefore, the limit of a memory-dependent derivative D_ξ can be considered as the common derivative f' as $\xi \to 0$. Also, second order memory dependent derivative can be written as $D_\xi^2 = DD_\xi$ i.e. derivative of memory dependent derivative.

Based upon memory dependent derivative a new energy equation having the time delay parameter ξ is defined as (2016)

$$K^*\nabla^2 T = \left(\rho C^*\frac{\partial}{\partial t}T + \nu T_0\frac{\partial}{\partial t}\nabla.\boldsymbol{u^n}\right) + \frac{\tau_0}{\xi}\left[\int_{t-\xi}^{t} K(t-s)\left(\rho C^*\frac{\partial^2 T}{\partial s^2} + \nu T_0\frac{\partial\nabla.\boldsymbol{u^n}}{\partial s^2}\right)ds\right] \quad (5.9)$$

The symbols K^*, C^*, τ_o, T, T_o are as mentioned in chapter 4 in section 4.1.

The kernel function $K(t - s)$ is consider to be in the following forms, Ezzat et al. (2014)

$$K(t-s) = 1 - \frac{2n}{\xi}(t-s) + \frac{m^2}{\xi^2}(t-s)^2, \quad (5.10)$$

$$K(t-s) = \begin{cases} 1 & m = 0, n = 0 \\ 1 - \left(\frac{t-s}{\xi}\right) & m = 0, n = \frac{1}{2} \\ \left(1 - \frac{t-s}{\xi}\right)^2, & m = 1, n = 1. \end{cases}$$

5.3 PROBLEM FORMULATION

Considering a non-local Micropolar thermoelastic half space which is isotropic and homogeneous, the configuration assumes absence of body forces with coordinate origin located on surface of the plane and x_2-axis is oriented vertically downward. The half space is considered to be undeformed at constant temperature T_0. It is considered that Rayleigh wave propagates along the x_1-axis and all the particles vibrating on a line parallel to the x_3-axis are equally spaced. So all field variables will be independent of x_3-Coordinates. In $x_1 - x_2$ plane, the components of displacement vector $\boldsymbol{u^n}$ and microrotation vector $\boldsymbol{\phi}$ are taken as

$$\boldsymbol{u^n} = (\mathrm{u}_1^n, \mathrm{u}_2^n, 0) \;, \boldsymbol{\phi} = \left(0,0,\phi_3\right). \tag{5.11}$$

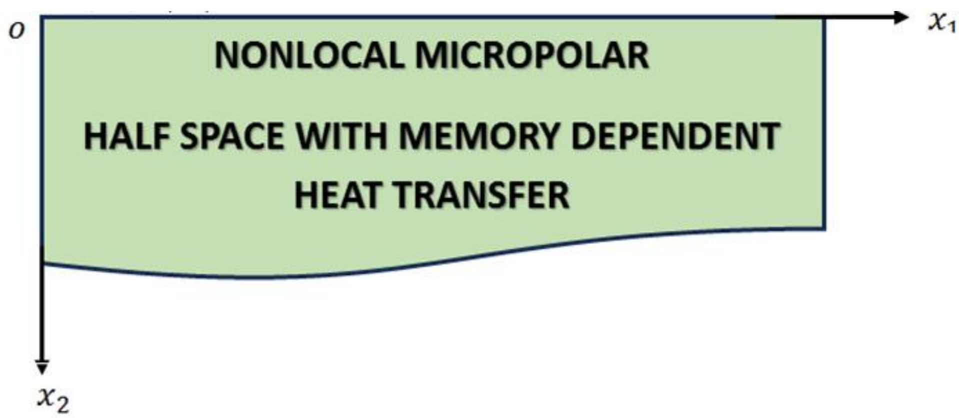

Fig. 5.1 Geometry of the problem

By using equation (5.11), equation (5.6) & (5.7) can be written as

$$(\lambda + 2\mu + \kappa)\frac{\partial^2 u_1^n}{\partial x_1^2} + (\mu + \kappa)\frac{\partial^2 u_1^n}{\partial x_2^2} + (\lambda + \mu)\frac{\partial^2 u_2^n}{\partial x_1 \partial x_2} + \kappa\frac{\partial \phi_3}{\partial x_2} - \nu\frac{\partial T}{\partial x_1} = (1 - \epsilon^2\nabla^2)\rho\frac{\partial^2 u_1^n}{\partial t^2}, \tag{5.12}$$

$$(\lambda + 2\mu + \kappa)\frac{\partial^2 u_2^n}{\partial x_2^2} + (\mu + \kappa)\frac{\partial^2 u_2^n}{\partial x_1^2} + (\lambda + \mu)\frac{\partial^2 u_1^n}{\partial x_1 \partial x_2} + \kappa\frac{\partial \phi_3}{\partial x_1} - \nu\frac{\partial T}{\partial x_2} = (1 - \epsilon^2\nabla^2)\rho j\frac{\partial^2 u_2}{\partial t^2}, \tag{5.13}$$

$$\gamma\nabla^2\phi_3 + \kappa\left(\frac{\partial u_2^n}{\partial x_1} - \frac{\partial u_1^n}{\partial x_2}\right) - 2\kappa\phi_3 = (1 - \epsilon^2\nabla^2)\rho j\frac{\partial^2\phi_3}{\partial t^2}\,. \tag{5.14}$$

By using the Helmholtz's decomposition, we have,

$$u_1^n = \frac{\partial\Phi}{\partial x_1} + \frac{\partial\Psi}{\partial x_2} \;, u_2^n = \frac{\partial\Phi}{\partial x_1} - \frac{\partial\Psi}{\partial x_2} \;. \tag{5.15}$$

On substituting (5.15) in equations (5.9), (5.12), (5.13), (5.14), we obtained

$$(\lambda + 2\mu + \kappa)\nabla^2\Phi - \nu T = (1 - \epsilon^2\nabla^2)\rho\frac{\partial^2\Phi}{\partial t^2}, \tag{5.16}$$

$$(\mu+\kappa)\nabla^2\Psi+\kappa\phi_3=(1-\epsilon^2\nabla^2)\rho\frac{\partial^2\Psi}{\partial t^2}\,, \tag{5.17}$$

$$\gamma\nabla^2\phi_3-\kappa\nabla^2\Psi-2\kappa\phi_3=(1-\epsilon^2\nabla^2)\rho_j\frac{\partial^2\phi_3}{\partial t^2}\,, \tag{5.18}$$

$$K^*\nabla^2T=\left(\rho C^*\frac{\partial}{\partial t}T+\nu T_0\frac{\partial}{\partial t}\nabla.\boldsymbol{u^n}\right)+\frac{\tau_0}{\xi}\left[\int_{t-\xi}^{t}K(t-s)\left(\rho C^*\frac{\partial^2T}{\partial s^2}+\nu T_0\frac{\partial\nabla^2\Phi}{\partial s^2}\right)ds\right]. \tag{5.19}$$

5.4 PROBLEM'S SOLUTION

The surface wave equations (5.11)-(5.13) is taken as

$$\{\phi_1,\psi_1,T,\phi\}=\{\overline{\phi_1}(x_2),\overline{\psi_1}(x_2),\bar{T}(x_2),\bar{\phi}(x_2)\}e^{\iota k(x_1-ct)}\,. \tag{5.20}$$

where $\omega=kc$ is the circular frequency, c is the phase-velocity and k is the wave number. For the propagation of Rayleigh wave, we assumed that c is a complex and $Re(c)>0$ gives the Rayleigh wave speed and $\exp(k\,im(c)t)$ is giving the damping in time, therefore it is assumed that $Im(c)\leq 0$.

By using (5.20) in equations (5.16)-(5.19), we get

$$[D^4-A_mD^2+B_m](\overline{\phi_1}(x_2),\bar{T}(x_2))=0\,, \tag{5.21}$$

$$[D^4-A^*D^2+B^*]\left(\overline{\psi_1}(x_2),\bar{\phi}(x_2)\right)=0. \tag{5.22}$$

Where,

$$D=\frac{d}{dx_2},\ A_m=2k^2-\frac{\iota\omega}{A_1\tau^*}a_m-\frac{\omega^2}{(C_1^2-\epsilon^2\omega^2)}-\frac{\iota\omega C_1^2A_2a_m}{\tau^*A_1(C_1^2-\epsilon^2\omega^2)}\,,m=1,2,3,4,$$

$$B_m=k^4-\frac{\iota k^3ca_m}{\tau^*A_1}-\frac{k^4c^2}{[c_1^2-\epsilon^2\omega^2]}+\frac{\iota\omega^3a_m}{A_1\tau^*[c_1^2-\epsilon^2\omega^2]}-\frac{A_2c_1^2k^3a_m\iota c}{A_1\tau^*[c_1^2-\epsilon^2\omega^2]},\ m=1,2,3,4,$$

$$A^*=k^2\left(1-\frac{c^2}{c_2^2}\right)+k^2-\frac{k^2c^2\rho j}{\gamma}+\frac{2\kappa}{\gamma}-\frac{\kappa^2}{\gamma(\mu+\kappa)}+\frac{2k^2c^2\rho j\epsilon^2}{\gamma c_2^2}(c_2^2+k^2c^2+k^4c^4\epsilon^4)-\frac{2k^2c^2\epsilon^2}{c_2^2}\left(1+\frac{2\kappa}{\gamma}\right),$$

$$B^* = k^2\left(k^2 - \frac{k^2c^2\rho j}{\gamma} + \frac{2\kappa}{\gamma}\right)\left(1 - \frac{c^2}{c_2^2}\right) - \frac{k^2\kappa^2}{\gamma(\mu+\kappa)} - \frac{2k^2\kappa\epsilon^2c^2}{\gamma c_2^2} + \frac{k^6\epsilon^2c^2}{c_2^2}\left(\frac{k^2\epsilon^2c^2\rho j}{\gamma} + \frac{2\rho j}{\gamma} - \frac{\rho j c_2^2}{\gamma} - 1\right),$$

$$c_1^2 = \frac{\lambda+2\mu+\kappa}{\rho}, \quad c_2^2 = \frac{\mu+\kappa}{\rho}, \mathrm{A}_1 = \frac{K^*}{\rho c^*\tau^*}, A_2 = \frac{\nu^2T_0}{\rho^2c_1^2c^*}, \tau^* = \tau_0 + \frac{\iota}{\omega},$$

$$a_m = \begin{cases} a_1 = 1 + \frac{\tau_0}{\xi}\left(1 - e^{\iota kc\xi}\right), & K(t-s) = 1 \\ a_3 = 1 + \frac{\tau_0}{\xi} + \frac{\tau_0}{\xi^2(ikc)}\left(1 - e^{\iota kc\xi}\right), & K(t-s) = 1 - \frac{t-s}{\xi} \\ a_4 = 1 + \frac{\tau_0}{\xi} - \frac{2\tau_0}{\xi^2(kc)}\left(\iota + \frac{(1-e^{\iota kc\xi})}{\xi(kc)}\right), & K(t-s) = \left[1 - \frac{t-s}{\xi}\right]^2 \end{cases}$$

The general solution of equation (5.21) and (5.22) by utilizing the radiation conditions i.e., $\overline{\phi_1}(x_2), \overline{\psi_1}(x_2), \bar{T}(x_2), \bar{\phi}(x_2) \rightarrow 0, x_2 \rightarrow \infty$, can be written as

$$\phi_1 = (D_1e^{-b_1x_2} + D_2e^{-b_2x_2})e^{\iota k(x_1-ct)}, \tag{5.23}$$

$$T = (r_1 D_1e^{-b_1x_2} + r_2D_2e^{-b_2x_2})e^{\iota k(x_1-ct),} \tag{5.24}$$

$$\psi_1 = (D_3e^{-b_3x_2} + D_4e^{-b_4x_2})e^{\iota k(x_1-ct)}, \tag{5.25}$$

$$\phi = (r_3 D_3e^{-b_3x_2} + r_4D_4e^{-b_4x_2})e^{\iota k(x_1-ct)}. \tag{5.26}$$

Where, D_1, D_2, D_3 and D_4 are arbitrary constants. b_1, b_2 and b_3, b_4 are the roots of the equations (5.21) and (5.22) respectively whose real parts are positive to ensure the decay conditions. These roots satisfy the following equations.

$$b_1^2 + b_2^2 = A_m, \; b_1^2b_2^2 = B_m, \; b_3^2 + b_4^2 = A^*, \; b_3^2b_4^2 = B^*, \tag{5.27}$$

$$r_i = \frac{k^2}{\nu}\left[(\lambda+2\mu+\kappa)\left(\frac{b_i^2}{k^2} - 1\right) + \rho c^2(1 + \epsilon^2(k^2-1))\right], \; (i = 1,2), \tag{5.28}$$

$$r_j = \frac{k^2(\mu+\kappa)}{\kappa}\left[1 - \frac{c^2}{c_2^2} - \frac{b_j^2}{k^2} + \frac{\epsilon^2 c^2}{c_2^2}(b_j^2 - k^2)\right], (j = 3,4).$$

5.5 BOUNDARY CONDITIONS AND SECULAR EQUATIONS

The boundary condition at the surface $x_2 = 0$ are taken as:

a) $$\sigma_{21}^{nl} = 0, \sigma_{22} = 0 \ , m_{23}^{nl} = 0. \tag{5.29}$$

Using equation (5.3) and (5.4), we have, $\sigma_{21} = 0$, $\sigma_{22} = 0$, $m_{23} = 0$, which are the components of stresses in local micropolar elastic material.

b) $$\frac{\partial T}{\partial x_2} + hT = 0. \tag{5.30}$$

Boundary changes to thermally insulated surface when h $\rightarrow 0$ and isothermal when h$\rightarrow \infty$.

Using equations (5.3)-(5.4), (5.15) and (5.23)-(5.26), the boundary conditions (5.29)-(5.30) are imposed at the surface $x_2 = 0$, we obtained a system of four homogeneous equations in the unknown D_1, D_2, D_3 and D_4. For non-trivial solution we have equations for which the determinant of the coefficients matrix must vanish which gives,

$$p_3[m_4(q_1 m_2 - q_2 m_1) + n_4(q_2 T_1 - q_1 T_2)] = p_4[m_3(q_1 m_2 - q_2 m_1) + n_3(q_2 T_1 - q_1 T_2)]. \tag{5.31}$$

Eq. (5.31) represents the secular equation of Rayleigh waves.

$$m_i = k b_i \left(2 + \frac{\kappa}{\mu}\right), \qquad (i = 1,2,3,4)$$

$$T_i = \left(2 + \frac{\lambda+\kappa}{\mu}\right) b_i^2 - \frac{\lambda}{\mu} k^2 - \frac{\nu}{\mu} r_i \ , \qquad (i = 1,2)$$

$$n_i = \left[k^2 + \left(1 + \frac{K}{\mu}\right)\left(2b_j^2 - k^2\left(1 - \frac{c^2}{c_2^2} + + \frac{\epsilon^2 c^2}{c_2^2}(b_j^2 - k^2)\right)\right)\right] \ , \quad (j = 1,2)$$

$$p_i = b_j\left(1+\frac{\kappa}{\mu}\right)\left[1-\frac{c^2}{c_2^2}-\frac{b_j^2}{k^2}+\frac{\epsilon^2 c^2}{c_2^2}\left(b_j^2-k^2\right)\right], \qquad (j=3,4)$$

$q_i = b_i\left[\left(2+\frac{\lambda+\kappa}{\mu}\right)\left(1-\frac{b_i^2}{k^2}\right)+\frac{\rho c^2}{\mu}\left(1+\epsilon^2(k^2-1)\right)\right]$, $(i=1,2)$, for thermally insulated surface.

$q_i = \left[\left(2+\frac{\lambda+\kappa}{\mu}\right)\left(1-\frac{b_i^2}{k^2}\right)+\frac{\rho c^2}{\mu}\left(1+\epsilon^2(k^2-1)\right)\right]$, $(i=1,2)$, for isothermal surface.

5.6 PARTICULAR CASES

(a) When non-local parameter $\epsilon \to 0$ then we obtained the following changed values of some parameters.

$$A_m = 2k^2-\frac{\iota\omega}{A_1\tau^*}a_m-\frac{\omega^2}{(c_1^2)}-\frac{i\omega c_1^2 A_2 a_m}{\tau^* A_1 c_1^2},$$

$$B_m = k^4-\frac{ik^3 c a_m}{\tau^* A_1}-\frac{k^4 c^2}{c_1^2}+\frac{i\omega^3 a_m}{A_1\tau^* c_1^2}-\frac{A_2 c_1^2 k^3 a_m ic}{A_1\tau^* c_1^2},$$

$$A^* = k^2\left(1-\frac{c^2}{c_2^2}\right)+k^2-\frac{k^2c^2\rho j}{\gamma}+\frac{2\kappa}{\gamma}-\frac{\kappa^2}{\gamma(\mu+\kappa)},$$

$$B^* = k^2\left(k^2-\frac{k^2c^2\rho j}{\gamma}+\frac{2\kappa}{\gamma}\right)\left(1-\frac{c^2}{c_2^2}\right)-\frac{k^2\kappa^2}{\gamma(\mu+\kappa)}.$$

The equation (5.31) with above parameters is the secular equation of Rayleigh wave in micropolar thermoelastic material with MDD which is in agreement with the earlier results published by Singh and Kashyap (2023)

(b) When the kernel $K(t-s)=1$ and the memory dependent parameter $\xi \to 0$, the memory dependent tends to the ordinary derivative i.e.

$$D_\xi\big(f(t)\big)=\frac{1}{\xi}\int_{t-\xi}^{t} f'(s)\,ds=\frac{f(t)-f(t-\xi)}{\xi} \to f'(t)\ as\ \xi \to 0.$$

From this we obtained the following changed values of A_m and B_m

$$A_m = k^2\left[2-c^2\left(\frac{1}{A_1}-\frac{1}{c_1^2}-\frac{A_2}{A_1}\right)\right], \quad B_m=\frac{k^4}{A_1}\left[\frac{c^4}{c_1^2}+A_1-c^2\left(1+A_2+\frac{A_1}{c_1^2}\right)\right]$$

The equation (5.31) with these changed values of A_m and B_m is the secular equation of Rayleigh wave in micropolar thermoelastic material without MDD. These equations coincide with the solution of the problem derived by Kumar et al.(2018)

(c) Further in the absence of micropolar effects ($\kappa = j = 0$), we obtained the following changed values

$$c_1^2 = \frac{\lambda+2\mu}{\rho},\ c_2^2 = \frac{\mu}{\rho},\ A^* = k^2\left(1-\frac{c^2}{c_2^2}\right)+k^2, B^* = k^4\left(1-\frac{c^2}{c_2^2}\right),$$

$$p_3 = b_3\left(1-\frac{c^2}{c_2^2}-\frac{b_3^2}{k^2}\right), p_4 = b_4\left(1-\frac{c^2}{c_2^2}-\frac{b_4^2}{k^2}\right),$$

Using condition (5.27), in the particular case we have $b_3^2 = k^2\left(1-\frac{c^2}{c_2^2}\right)$, $b_4^2 = k^2$which implies that $p_3 = 0$ and p_4 will be a non-zero term. Using these values in the equation (5.31), we obtained

$$m_3(q_1m_2 - q_2m_1) + n_3(q_2T_1 - q_1T_2) = 0. \qquad (5.32)$$

where,

$$m_i = 2kb_i, (i = 1,2,3),$$

$$n_3 = k^2\left(2-\frac{c^2}{c_2^2}\right),$$

$$T_1 = T_2 = k^2\left(2-\frac{c^2}{c_2^2}\right),$$

q_i=$b_i\left[\left(2+\frac{\lambda}{\mu}\right)\left(1-\frac{b_i^2}{k^2}\right)+\frac{\rho c^2}{\mu}\right]$, $(i = 1,2)$, For thermally insulated boundary,

q_i=$\left[\left(2+\frac{\lambda}{\mu}\right)\left(1-\frac{b_i^2}{k^2}\right)+\frac{\rho c^2}{\mu}\right]$, $(i = 1,2)$, For isothermal boundary.

Equation (5.32) is the secular equation of Rayleigh wave in thermoelastic half-space.

(d) On neglecting the thermal effects in the equation (5.30) it reduces to

$$\left(2-\frac{c^2}{c_2^2}\right)^2 = 4\sqrt{1-\frac{c^2}{c_1^2}}\sqrt{1-\frac{c^2}{c_2^2}}. \qquad (5.33)$$

Equation (5.33) is the well-known dispersion equation of Rayleigh waves in the elastic half space.

5.7 NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical computations have been executed to illustrate the theoretical findings, and the outcomes are visually presented through graphical representations. For numerical calculation the relevant parameters of aluminum epoxy material taken from Gauthier (1982) are given below in table-5.1

Table 5.1. Material parameters

Aluminum epoxy material	
$\rho = 2.19 \times 10^3 kg/m^3$,	$j = 0.196 \times 10^4 m^2$
$\lambda = 7.59 \times 10^{10} N/m^2$	K^*=$0.492 \times 10^2 W/m$
$\mu = 1.89 \times 10^{10} N/m^2$	$C^* = 1.89 \times 10^{10} j/kg$
$\alpha = 0.01 \times 10^6 N$	$\tau_0 = 0.5 \times 10^{-10} s$
$\beta = 0.015 \times 10^6 N$	T_0=$298K$
$\gamma = 0.268 \times 10^6 N$	α_t=$2.36 \times 10^{-6} K^{-1}$
$j = 0.196 \times 10^4 m^2$	K=$0.0149 \times 10^{10}\ N/m^2$

Assuming that c is the complex constant with $Re(c) = V \geq 0$, the non dimensional Rayleigh wave speed $V_1 = \sqrt{\frac{\rho V^2}{\mu}}$ has been calculated and its variations has been shown graphical with wave number. The impact of key factors under consideration such as nonlocality, time delay heat transfer, kernels and insulated and thermal boundary conditions on the non dimensional Rayleigh wave speed have been analyzed and visualized graphically in figures (5.2-5.8).

Figure (5.2) exhibits the effects of time delayed heat transfer on the non dimensional Rayleigh wave speed w.r.t. wave number for kernels $K(t-s) = 1$. The wave speed has been calculated at constant non-local parameter value $\epsilon = 0.003mm$ and three different time delay parameter values ($\xi = 0.1\ s, 0.2s\ and\ 0.3s$). As shown in the graph the wave speed decreases with increasing values of the wave number initially and then becomes almost constant at higher wave number values. The wave speed is significantly affected by different values of the time delay parameter. As clear from the magnified image, the wave speed decreases with increment in time delay parameter.

Figure (5.3-5.4) demonstrate the variations of non dimensional wave speed for the kernel $K(t-s) = \left(1-\frac{t-s}{\xi}\right)^2$ and $K(t-s) = 1-\frac{t-s}{\xi}$ with same parameter values as consider for figure (5.2). It is noticed from a magnified image of a small portion of the dispersion curve of figure (5.3) and (5.4) that the non dimensional Rayleigh wave speed increases with increasing values of time delay parameter. It is noticed that the variations of Rayleigh wave speed depend upon the combination of time delay parameter, kernel as well as wave number values.

Figures (5.5-5.7) depict the impact of non-local parameter on the non dimensional Rayleigh wave speed w.r.t. wave number and different kernels. The non dimensional wave speed $V_1 = \sqrt{\frac{\rho V^2}{\mu}}$ is computed and visualized graphically at fixed time delay parameter ($\xi = 0.1s$) and for three non-local parameter values ($\epsilon = 0.001\text{mm}, 0.002\text{mm}, 0.003\text{mm}$). Figure (5.5) display the effects of non-local parameter on the wave speed for the kernel $K(t-s) = 1$. As noticed from the figure, the non dimensional wave speed decreases with an increment in the non-local parameter. Figures (5.6-5.7) also display the same type of effects for the kernel $K(t-s) = \left(1-\frac{t-s}{\xi}\right)^2$ and $K(t-s) = 1-\frac{t-s}{\xi}$.

Figure (5.8) shows the effects of the isothermal and thermally insulated boundary on the Rayleigh wave speed with a time delay heat transfer ($\xi = 0.1s$) and non-local parameter ($\epsilon = 0.001\text{mm}$). It could be witnessed that Rayleigh wave speed is on the higher side in the case of an isothermal boundary as compared to thermally insulated boundary.

Figure (5.9-5.10) shows effect of thermal relaxation parameter. Initially, the wave speed increases for all values of thermal relaxation parameter then drops down smoothly. As thermal relaxation parameter increases, the wave speed decreases more rapidly with increasing wave number, indicating slower heat transfer and thus slower wave propagation. Smaller thermal relaxation parameter allow for faster wave speeds, as the material responds more quickly to thermal changes. Overall, higher thermal relaxation times lead to more significant damping of wave speed, especially at higher wave numbers.

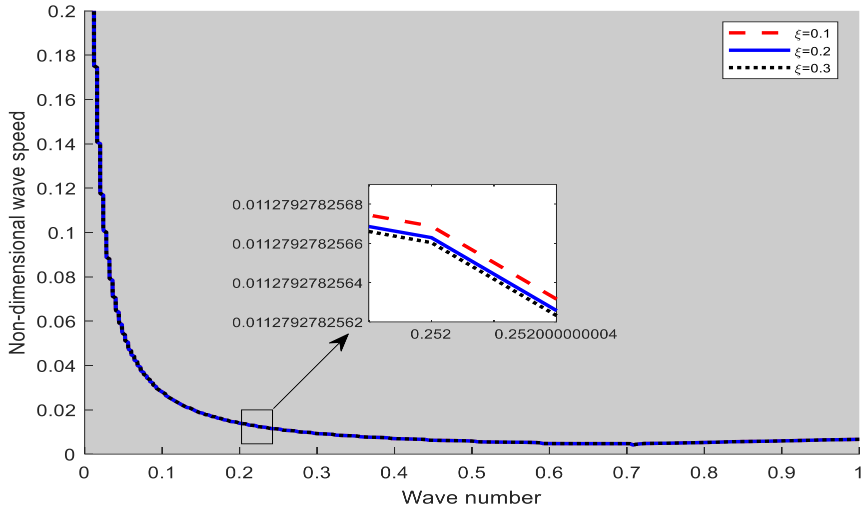

Fig. 5.2 Effects of time delay parameter on non dimensional wave speed w.r.t. wave number for the kernel $K(t-s) = 1$

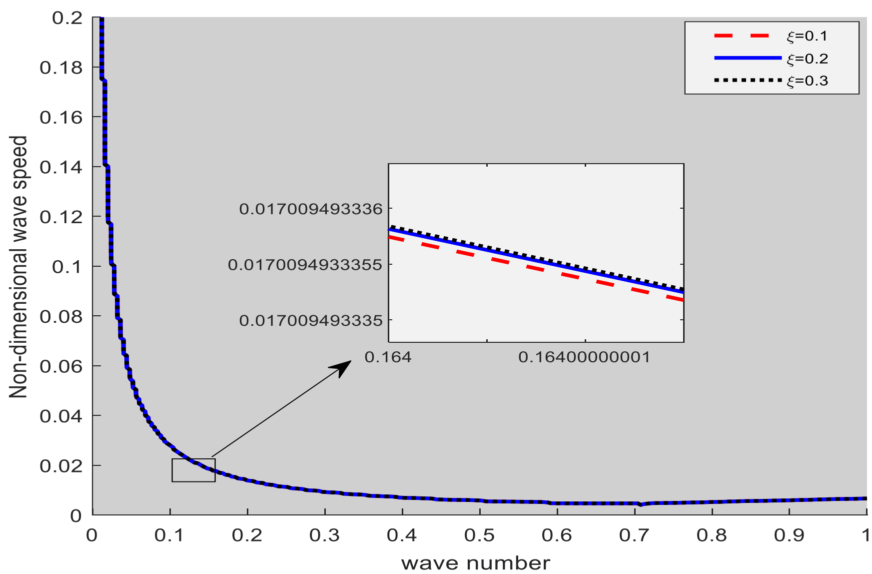

Fig. 5.3 Effects of time delay parameter on non dimensional wave speed w.r.t. wave number for the kernel $K(t-s) = \left(1 - \frac{t-s}{\xi}\right)^2$

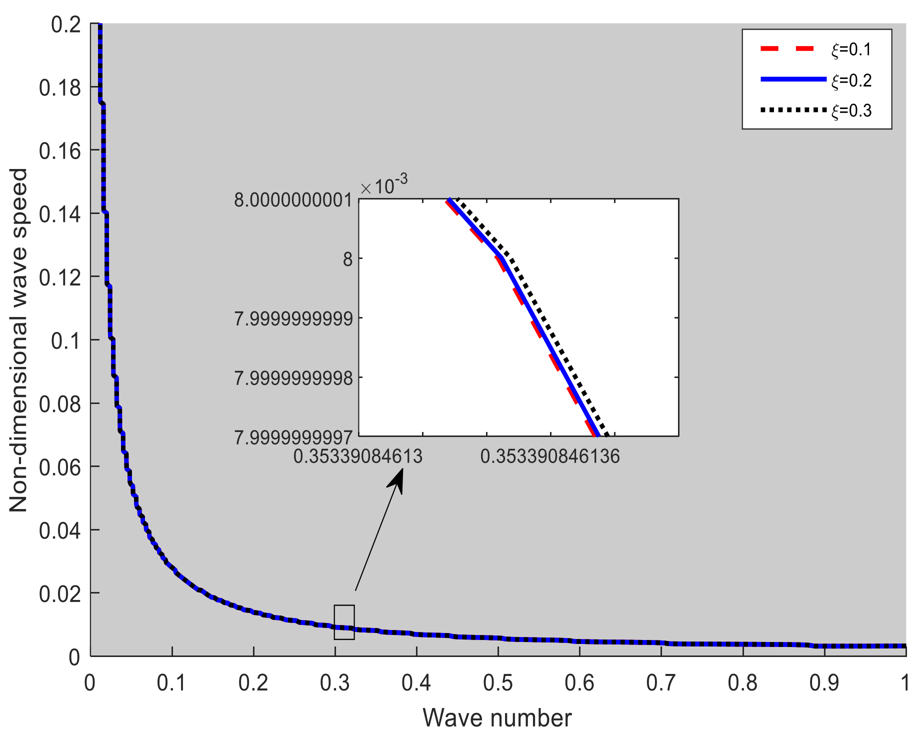

Fig. 5.4 Effects of time delay parameter on non dimensional wave speed w.r.t. wave number for the kernel $K(t-s) = 1 - \frac{t-s}{\xi}$

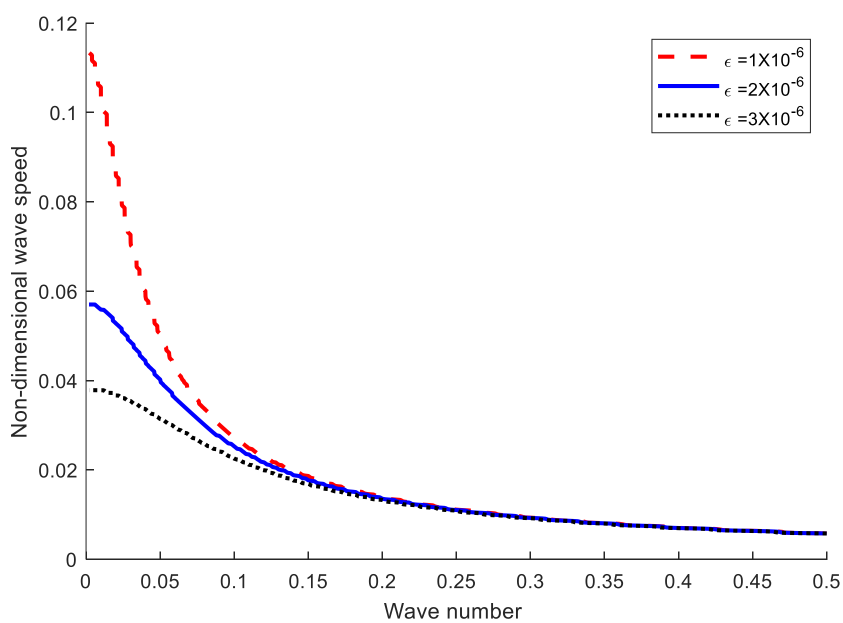

Fig. 5.5 Effects of non-local parameter on non dimensional Rayleigh wave speed w.r.t. wave number for the kernel $K(t-s) = 1$

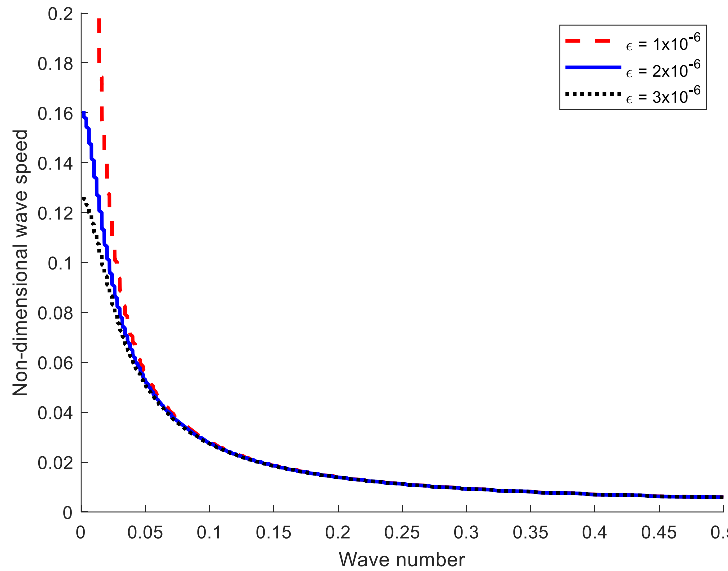

Fig. 5.6 Effects of non-local parameter on non dimensional Rayleigh wave speed w.r.t. non-dimensional wave number for the kernel $K(t-s) = \left(1 - \frac{t-s}{\xi}\right)^2$

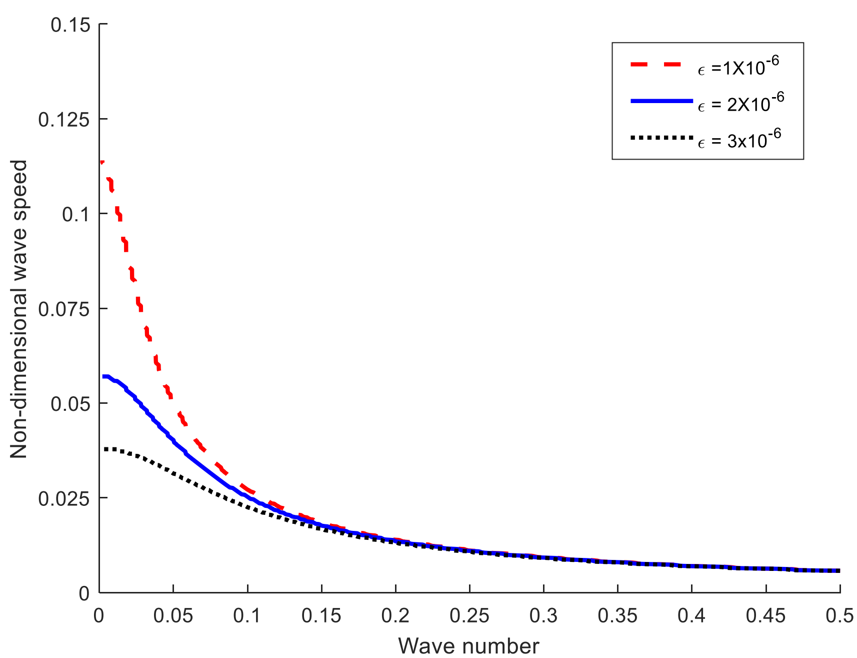

Fig. 5.7 Effects of non-local parameter on non dimensional Rayleigh wave speed w.r.t. wave number for the kernel $K(t-s) = 1 - \frac{t-s}{\xi}$

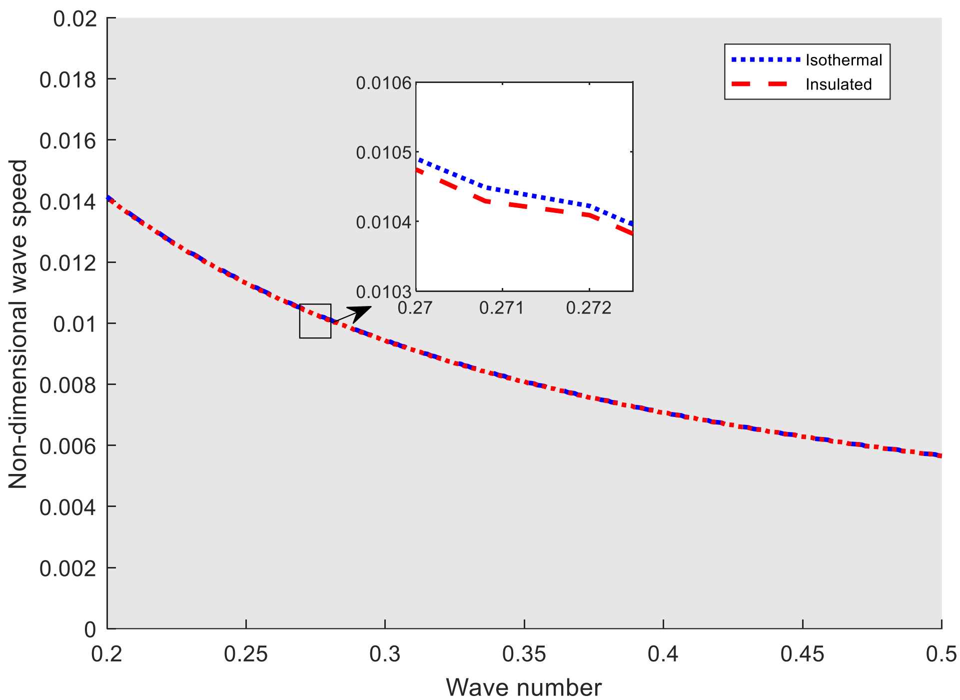

Fig. 5.8 Effects of isothermal and insulated boundary conditions on the non dimensional wave speed with time delay ($\xi = 0.1s$) and nonlocal parameter ($\epsilon = 0.001mm$).

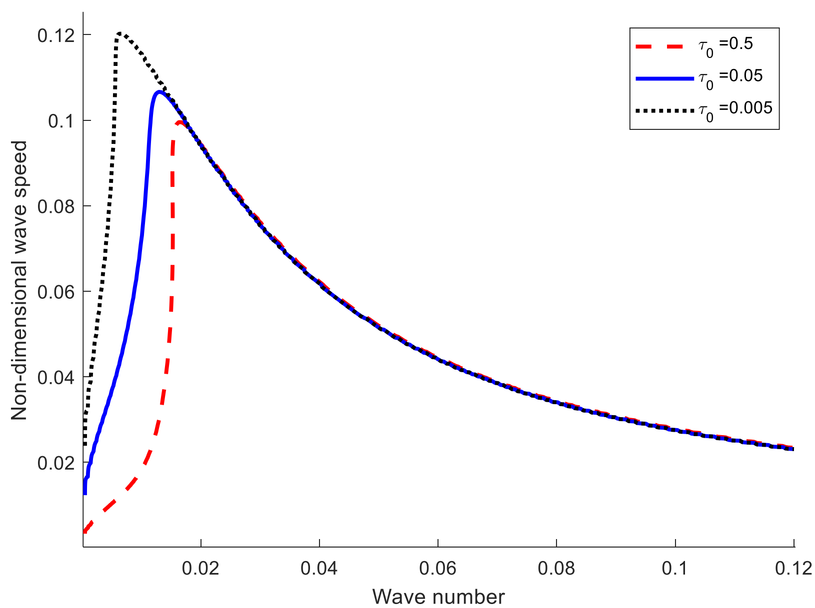

Fig. 5.9 Effects of thermal relaxation parameterime on non dimensional Rayleigh wave speed w.r.t. wave number for the kernel $K(t-s) = 1$

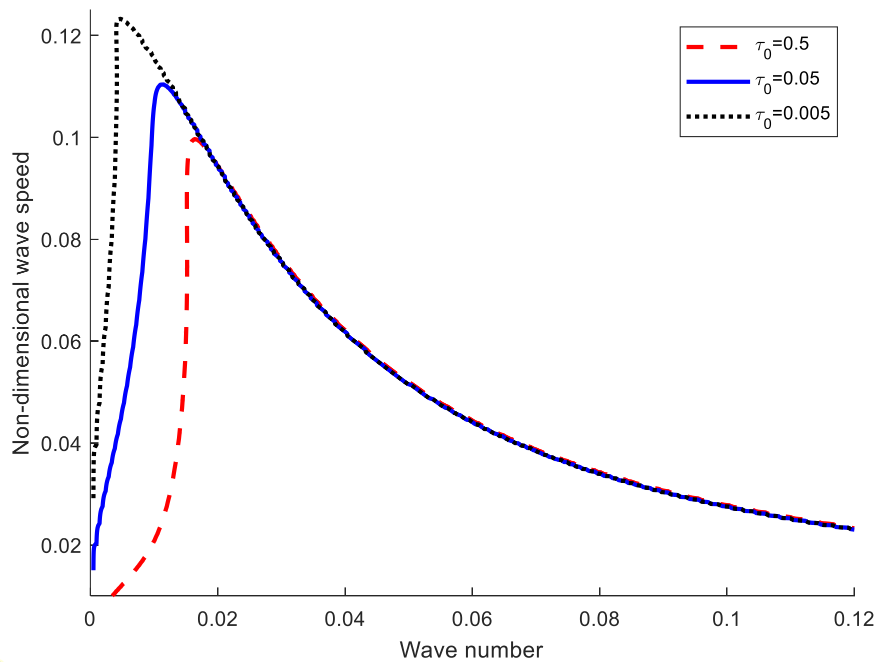

Fig. 5.10 Effects of thermal relaxation on non dimensional Rayleigh wave speed w.r.t. wave number for the kernel $K(t-s) = 1 - \frac{t-s}{\xi}$

5.8 CONCLUSIONS

The core findings of the present study can be summarized in the following points:

- **Existence of Rayleigh Waves:** The secular equation shows that there exist Rayleigh waves in non-local micropolar thermoelastic half space under memory dependent heat transfer.
- **Effect of Non-local Parameter:** The non-local parameter has substantial effects on the Rayleigh wave speed. The increase in non-local parameter values decreases the non dimensional Rayleigh wave speed.

- **Role of Memory Effects (Kernels):** Different types of kernels describing the memory effects have significant effects on the Rayleigh wave speed and hence kernels may be used according the problem.
- **Dispersive Nature of Rayleigh Waves:** The Rayleigh waves are dispersive in nature and the non dimensional phase-velocity decreases with increase in wave number.
- The presence of a time delay parameter in the heat equation influenced the Rayleigh wave speed significantly.
- **Impact of Thermal Boundary Conditions:** It is found that the Rayleigh wave speed is influenced by the thermal boundary conditions. For small wave number values, the Rayleigh wave speed is on the higher side for isothermal boundary as compared to a thermally insulated surface.
- **Thermal Relaxation Parameter Influence:** As seen in the previous trends, the thermal relaxation parameter plays a pivotal role, affecting the wave speed across different wave numbers. Higher thermal relaxation leads to pronounced changes at lower wave numbers.

The current study presents a theoretical model to analyze the Rayleigh wave propagation in non-local micropolar thermoelastic material with time delay heat transfer. The study may provide valuable information to the researcher and engineer working in seismology and related fields. The study of Rayleigh waves in non-local micropolar materials with memory-dependent heat transfer holds significant potential in various applied fields. In seismology and earthquake engineering, the insights gained can lead to more accurate predictions of seismic wave behavior through complex, heterogeneous earth materials, aiding in the design of earthquake-resistant infrastructure. In nanotechnology and materials science, understanding wave propagation in non-local micropolar materials is essential for developing advanced materials for applications in micro-electromechanical systems and nano-electromechanical systems where size effects and thermal management are critical. Additionally, in geophysical surveys, Rayleigh waves help in subsurface explorations, and the inclusion of thermal effects makes this study highly relevant for geothermal energy and oil exploration. The thermal aspects also apply to aerospace and nuclear reactors, where managing thermal stresses is crucial for the integrity and performance of engineering structures. Overall, this research enhances our ability to model and apply wave phenomena in technologically critical areas, leading to improvements in material design and diagnostic techniques.

FUTURE SCOPE

The following suggestions have been made for the future work as the extension of present work done in this thesis:

- The propagation of shear waves has been studied in the layered structure of non-local Micropolar elastic half space attached along with piezoelectric/non-local piezoelectric layer in perfect and imperfect contact. Effect of initial stress has also been studied on the proposed structure. Now as extension of this work, The behavior of non-local Micropolar martial could be studied under the different effects like hall effect, magnetic effect or gravity.
- We have studied the propagation of Rayleigh waves in the non-local Micropolar thermoelastic material under the impedance boundary condition in this thesis. Further as the futuristic scope of this work, love waves could be studied under the impedance boundary conditions in the layered structure as well.
- Wave propagation in non-local micropolar half space with double layer structure or sandwich structure can be studied.

BIBLIOGRAPHY

- Abd-alla, A.E.N.N., Al-sheikh, F.A. and Al-Hossain, A.Y., 2012. The reflection phenomena of quasi-vertical transverse waves in piezoelectric medium under initial stresses. Meccanica, 47, 731-744.
- Abo-Dahab, S.M., Abd-Alla, A.M., Alsharif, A. and Alotaibi, H., 2022. On generalized waves reflection in a micropolar thermodiffusion elastic half-space under initial stress and electromagnetic field. Mechanics Based Design of Structures and Machines, 50(8), 2670-2687.
- Abouelregal, A.E., Marin, M. and Alsharari, F., 2022. Thermoelastic plane waves in materials with a microstructure based on micropolar thermoelasticity with two temperature and higher order time derivatives. Mathematics, 10(9), 1552.
- Acharya, D.P. and Mondal, A., 2004. Effect of rotation on Rayleigh surface waves under the linear theory of non-local elasticity. Indian Journal of Physics, 52, 81-89.
- Al-Jamel, A., Al-Jamal, M.F. and El-Karamany, A., 2018. A memory-dependent derivative model for damping in oscillatory systems. Journal of Vibration and Control, 24(11), 2221-2229.
- Anh, V.T.N., Vinh, P.C., Tuan, T.T. and Hue, L.T., 2023. Weakly nonlocal Rayleigh waves with impedance boundary conditions. Continuum Mechanics and Thermodynamics, 35(5), 2081-2094.
- Barak, M.S. and Kaliraman, V., 2019. Reflection and transmission of elastic waves from an imperfect boundary between micropolar elastic solid half space and fluid saturated porous solid half space. Mechanics of Advanced Materials and Structures, 26(14), 1226-1233.

- Bharti, U., Vaishnav, P.K., Abo-Dahab, S.M., Bouslimi, J. and Mahmoud, K.H., 2021. Analysis of phase-velocity of love waves in rigid and soft mountain surfaces: Exponential law model. Complexity, 2021(2), 1-12.
- Biot, M.A., 1956. Thermoelasticity and irreversible thermodynamics. Journal of applied physics, 27(3), 240-253.
- Bleustein, J.L., 1968. A new surface wave in piezoelectric materials. Applied Physics Letters, 13(12), pp.412-413.
- Ceballes, S., Larkin, K., Rojas, E., Ghaffari, S.S. and Abdelkefi, A., 2021. Nonlocal elasticity and boundary condition paradoxes: a review. Journal of Nanoparticle Research, 23, 1-27.
- Chakraborty, A., 2007. Wave propagation in anisotropic media with non-local elasticity. International journal of solids and structures, 44(17), 5723-5741.
- Chandrasekharaiah, D.S., 1986. Heat-flux dependent micropolar thermoelasticity. International Journal of Engineering Science, 24(8), pp.1389-1395.
- Chen, W.Q., Cai, J.B., Ye, G.R. and Wang, Y.F., 2004. Exact three-dimensional solutions of laminated orthotropic piezoelectric rectangular plates featuring interlaminar bonding imperfections modeled by a general spring layer. International Journal of Solids and Structures, 41(18-19), 5247-5263.
- Chen, Z.G., Hu, Y.T. and Yang, J.S., 2008. Shear horizontal piezoelectric waves in a piezoceramic plate imperfectly bonded to two piezoceramic half-spaces. Journal of Mechanics, 24(3), 229-239.
- Cosserat, E. and Cosserat, F., 1913. Théorie des corps déformables. Bull. Am. Math. Soc., 19(1913), 242-246.
- Cui, J., Du, J., Wang, J., 2013. Study on SH waves in piezoelectric structure with an imperfectly bonded viscoelastic layer. IEEE International Ultrasonics Symposium (IUS), 1017-1020.

- Curtis, R.G. and Redwood, M., 1973. Transverse surface waves on a piezoelectric material carrying a metal layer of finite thickness. Journal of Applied Physics, 44(5), 2002-2007.
- Deresiewicz, H., 1961. A note on thermoelastic Rayleigh waves. Journal of the Mechanics and Physics of Solids, 9(3), 191-195.
- Chimenti, D. E., Nayfeh, A. H., & Butler, D. L. (1982). Leaky Rayleigh waves on a layered halfspace. Journal of Applied Physics, 53(1), 170-176.
- Deswal, S., Punia, B.S., Gunghas, A. and Kalkal, K.K., 2021. Nonlocal and thermal phase-lag effects on an exponentially graded micropolar elastic material with rotation and gravity. Waves in Random and Complex Media, 1-25.
- Dhabal, U., Panja, S.K. and Mandal, S.C., 2022. Shear Wave Interaction of Two Collinear Finite Cracks in an Infinite Magnetoelastic Orthotropic Media. International Journal of Applied and Computational Mathematics, 8(5), 243.
- Ding, N., Xu, X. and Zheng, Z., 2016. A size-dependent nonlinear microbeam model based on the micropolar elasticity theory. Acta Mechanica, 227, 3497-3515.
- Dost, S. and Tabarrok, B., 1978. Generalized micropolar thermoelasticity. International Journal of Engineering Science, 16(3), 173-183.
- Edelen, D. G. B., Green, A. E., & Laws, N. 1971. Continuum nonlocal mechanics. Archive for Rational Mechanics and Analysis, 43, 36-44.
- Edelen, D.G. and Laws, N., 1971. On the thermodynamics of systems with nonlocality. Archive for Rational Mechanics and Analysis, 43, 24-35.
- Eringen A. C., 1968.Theory of micropolar elasticity. In: H. Leibowitz (Ed.) Chapter–7, Fracture, vol. II, Academic Press, New York, 621–729,
- Eringen, A.C. and Edelen, D., 1972. On nonlocal elasticity. International journal of engineering science, 10(3), 233-248.
- Eringen, A.C. and Eringen, A.C., 1999. Theory of micropolar elasticity Springer New York, 101-248.

- Eringen, A.C. and Suhubi, E.S., 1964a. Nonlinear theory of simple micro-elastic solids—I. International Journal of Engineering Science, 2(2), 189-203.
- Eringen, A.C. and Suhubi, E.S., 1964b. Non-linear theory of simple micro-elastic solids II, International Journal of Engineering Science, 2, 389-404.
- Eringen, A.C., 1966. Linear theory of micropolar elasticity. Journal of Mathematics and Mechanics, 909-923.
- Eringen, A.C., 1967. Theory of micropolar plates. Zeitschrift für angewandte Mathematik und Physik ZAMP, 18, 12-30.
- Eringen, A.C., 1970. Foundations of micropolar thermoelasticity , Berlin: Springer, 23.
- Eringen, A.C., 1972. Nonlocal polar elastic continua. International journal of engineering science, 10(1), 1-16.
- Eringen, A.C., 1983. On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves. Journal of applied physics, 54(9), 4703-4710.
- Eringen, A.C., 1984. Plane waves in nonlocal micropolar elasticity. International Journal of Engineering Science, 22(8-10), 1113-1121.
- Ezzat, M.A., El-Karamany, A.S. and El-Bary, A.A., 2014. Generalized thermo-viscoelasticity with memory-dependent derivatives. International Journal of Mechanical Sciences, 89, pp.470-475.
- Ezzat, M.A., El-Karamany, A.S. and El-Bary, A.A., 2015. A novel magneto-thermoelasticity theory with memory-dependent derivative. Journal of Electromagnetic Waves and Applications, 29(8), 1018-1031.
- Ezzat, M.A., El-Karamany, A.S. and El-Bary, A.A., 2016. Modeling of memory-dependent derivative in generalized thermoelasticity. The European Physical Journal Plus, 131, 1-12.
- Gauthier, R.D., 1982. Experimental investigations on micropolar media. Mechanics of micropolar media. 395-463.

- Giang, P.T.H. and Vinh, P.C., 2021. Existence and uniqueness of Rayleigh waves with normal impedance boundary conditions and formula for the wave velocity. Journal of Engineering Mathematics, 130, 1-14.
- Godoy, E., Durán, M. and Nédélec, J.C., 2012. On the existence of surface waves in an elastic half-space with impedance boundary conditions. Wave Motion, 49(6), 585-594.
- Goyal, R. and Kumar, S., 2019. Dispersion of Love waves in size-dependent substrate containing finite piezoelectric and viscoelastic layers. International Journal of Mechanics and Materials in Design, 15, 767-790.
- Goyal, R., Kumar, S. and Sharma, V., 2020. A size-dependent micropolar-piezoelectric layered structure for the analysis of love wave. Waves in Random and Complex Media, 30(3), 544-561.
- Green, A.E. and Lindsay, K., 1972. Thermoelasticity. Journal of elasticity, 2(1), 1-7.
- He, S., Wei, E.I., Jiang, L., Choy, W.C., Chew, W.C. and Nie, Z., 2011. Finite-element-based generalized impedance boundary condition for modeling plasmonic nanostructures. IEEE transactions on nanotechnology, 11(2), 336-345.
- Hrytsyna, O., Sladek, J., Sladek, V. and Hrytsyna, M., 2023. Love waves propagation in layered waveguide structures including flexomagneticity/flexoelectricity and micro-inertia effects. Mechanics of Advanced Materials and Structures, 30(23), 4933-4951.
- Huang, W., Hjiaj, M. and Sloan, S.W., 2005. Bifurcation analysis for shear localization in non-polar and micro-polar hypoplastic continua. Journal of engineering mathematics, 52, 167-184.
- Huang, Y. and Li, X.F., 2011. Interfacial waves in dissimilar piezoelectric cubic crystals with an imperfect bonding. IEEE transactions on ultrasonics, ferroelectrics, and frequency control, 58(6), 1261-1265.
- Jin, F., Qian, Z., Wang, Z. and Kishimoto, K., 2005. Propagation behavior of Love waves in a piezoelectric layered structure with inhomogeneous initial stress. Smart materials and structures, 14(4), 515-523.

- Jun, W. and Dhaliwal, R.S., 1993. On some theorems in the nonlocal theory of micropolar elasticity. International journal of solids and structures, 30(10), 1331-1338.
- Kaliski, S., Rymarz, C., Sobczyk, K. and Wlodarczyk, E., 1992. Surface waves in nonlocal media and in media with a microstructure. Studies in Applied Mechanics-B: Waves, 30, 261-270.
- Kalkal, K.K., Sheoran, D. and Deswal, S., 2020. Reflection of plane waves in a nonlocal micropolar thermoelastic medium under the effect of rotation. Acta Mechanica, 231, 2849-2866.
- Kamar, R., Singh, K., and Pathania, D., 2018. Propagation of Rayleigh waves in a micropolar thermelastic half space with impedance boundary conditions. Material. Physics and. Mechanics, 35, 115-125.
- Kansal, T., 2022. The theory of generalized micropolar thermoelastic diffusion with double porosity. Theoretical and Applied Mechanics, 49(1), 85-109.
- Kant, S. and Mukhopadhyay, S., 2019. An investigation on responses of thermoelastic interactions in a generalized thermoelasticity with memory-dependent derivatives inside a thick plate. Mathematics and Mechanics of Solids, 24(8), 2392-2409.
- Kaur, B. and Singh, B., 2021. Rayleigh waves on the impedance boundary of a rotating monoclinic half-space. Acta Mechanica, 232(6), 2479-2491.
- Kaur, G., Singh, D. and Tomar, S.K., 2019. Love waves in a nonlocal elastic media with voids. Journal of vibration and control, 25(8), 1470-1483.
- Kaur, I. and Singh, K., 2021. Effect of memory dependent derivative and variable thermal conductivity in cantilever nano-beam with forced transverse vibrations. Forces in Mechanics, 5, 100043.
- Kaur, T., Sharma, S.K. and Singh, A.K., 2016. Influence of imperfectly bonded micropolar elastic half-space with non-homogeneous viscoelastic layer on

propagation behavior of shear wave. Waves in Random and Complex Media, 26(4), 650-670.

- Kaur, T., Sharma, S.K. and Singh, A.K., 2017. Shear wave propagation in vertically heterogeneous viscoelastic layer over a micropolar elastic half-space. Mechanics of Advanced Materials and Structures, 24(2), 149-156.
- Khurana, A. and Tomar, S., Torsional waves propagation in micropolar elastic half-space. Journal of Theoretical and Applied Mechanics, Sofia, 52 (2022), 50-63
- Khurana, A. and Tomar, S.K., 2013. Reflection of plane longitudinal waves from the stress-free boundary of a nonlocal, micropolar solid half-space. Journal of mechanics of materials and structures, 8(1), 95-107.
- Khurana, A. and Tomar, S.K., 2017. Rayleigh-type waves in nonlocal micropolar solid half-space. Ultrasonics, 73, 162-168.
- Kong, Y., Liu, J. and Nie, G., 2015. Propagation characteristics of SH wave in an mm2 piezoelectric layer on an elastic substrate. AIP Advances, 5(9), 095135.
- Kumar, R. and Abbas, I.A., 2013. Deformation due to thermal source in micropolar thermoelastic media with thermal and conductive temperatures. Journal of Computational and Theoretical Nanoscience, 10(9), 2241-2247.
- Kumar, R. and Deswal, S., 2006. Some problems of wave propagation in a micropolar elastic medium with voids. Journal of Vibration and Control, 12(8), 849-879.
- Kumar, R. and Kaur, M., 2017. Reflection and transmission of plane waves at micropolar piezothermoelastic solids. Journal of Solid Mechanics, 9(3), 508-526.
- Kumar, R. and Partap, G., 2006. Rayleigh Lamb waves in micropolar isotropic elastic plate. Applied Mathematics and Mechanics, 27, 1049-1059.
- Kumar, R. and Partap, G., 2006. Rayleigh Lamb waves in micropolar isotropic elastic plate. Applied Mathematics and Mechanics, 27, 1049-1059.

- Kumar, R. and Singh, B., 1996. Wave propagation in a micropolar generalized thermoelastic body with stretch. In Proceedings of the Indian Academy of Sciences-Mathematical Sciences,106, 183-199.
- Kumar, R., Singh, K. and Pathania D.S., 2019. Shear waves propagation in an initially stressed piezoelectric layer imperfectly bonded over a micropolar elastic half space. Structural Engineering and Mechanics, An Int'l Journal, 69(2), 121-129.
- Kumar, R., Singh, K. and Pathania, D., 2016. Interactions due to hall current and rotation in a magneto-micropolar fractional order thermoelastic half-space subjected to ramp-type heating. Multidiscipline Modeling in Materials and Structures, 12(1), 133-150.
- Kumar, R., Singh, K. and Pathania, D.S., 2018. Propagation of rayleigh waves in a micropolar thermoelastic half-space with impedance boundary conditions. Materials Physics & Mechanics, 35(1), 115-125.
- Kumar, S. and Partap, G., 2023. Analysis of waves in micropolar generalized thermoelastic plate with memory dependent derivatives. ZAMM-Journal of Applied Mathematics and Mechanics, 103(2), e202200244.
- Kumar, S. and Tomar, S.K., 2020. Plane waves in nonlocal micropolar thermoelastic material with voids. Journal of Thermal Stresses, 43(11), 1355-1378.
- Kumar, S., Nagar, H. and Kumari, S., 2023. Plane wave reflection in micropolar hygro-thermoelastic half-space. Journal of Thermal Stresses, 1-19.
- Kumar, S., Sharma, J.N. and Sharma, Y.D., 2011. Generalized thermoelastic waves in microstretch plates loaded with fluid of varying temperature. International Journal of Applied Mechanics, 3(03), 563-586.
- Kurt, I., Akbarov, S.D. and Sezer, S., 2016. The influence of the initial stresses on Lamb wave dispersion in pre-stressed PZT/Metal/PZT sandwich plates. Structural Engineering and Mechanics, 58(2), 347-378.
- Kuznetsov, S.V., 2010. Love waves in nondestructive diagnostics of layered composites. Survey. Acoustical Physics, 56(6), 877-892.

- Lazar, M. and Kirchner, H., 2006. The Eshelby tensor in nonlocal elasticity and in nonlocal micropolar elasticity. Journal of Mechanics of Materials and Structures, 1(2), 325-337.
- Lee, J.C., Shin, S.W., Kim, W.J. and Lee, C.J., 2016. Electro-mechanical impedance based monitoring for the setting of cement paste using piezoelectricity sensor. Smart structures and systems, 17(1), 123-134.
- Li, C., Guo, H. and Tian, X., 2019. Size-dependent effect on thermo-electro-mechanical responses of heated nano-sized piezoelectric plate. Waves in Random and Complex Media, 29(3), 477-495.
- Li, P. and Jin, F., 2015. Excitation and propagation of shear horizontal waves in a piezoelectric layer imperfectly bonded to a metal or elastic substrate. Acta Mechanica, 226, 267-284.
- Lianngenga, R. and Singh, S.S., 2020. Reflection of coupled dilatational and shear waves in the generalized micropolar thermoelastic materials. Journal of Vibration and Control, 26(21-22), 1948-1955.
- Liu H., Wang Z.K., Wang T.J., 2001. Effect of initial stress on the propagation behavior of Love waves in a layered piezoelectric structure. International Journal of Solids and Structures, 38(1), 37–51.
- Liu, C., Yu, J., Zhang, B., Wang, X., Zhang, X. and Zhang, H., 2022. Complete guided wave in piezoelectric nanoplates: A nonlocal stress expansion polynomial method. European Journal of Mechanics-A/Solids, 94, 104588.
- Liu, C., Yu, J., Zhang, B., Zhang, X. and Elmaimouni, L., 2022. Size-dependent and piezoelectric effects on SH wave propagation in functionally graded plates. Mechanics Research Communications, 124, 103965.
- Liu, J. and He, S., 2010a. Theoretical analysis on Love waves in a layered structure with a piezoelectric substrate and multiple elastic layers. Journal of Applied Physics, 107(7).

- Liu, J. and He, S., 2010b. Properties of Love waves in layered piezoelectric structures. International Journal of Solids and Structures, 47(2), 169-174.
- Liu, J., Wang, L., Lu, Y. and He, S., 2013. Properties of Love waves in a piezoelectric layered structure with a viscoelastic guiding layer. Smart materials and structures, 22(12), 125034.
- Liu, J., Wang, Y. and Wang, B., 2010. Propagation of shear horizontal surface waves in a layered piezoelectric half-space with an imperfect interface. IEEE transactions on ultrasonics, ferroelectrics, and frequency control, 57(8), 1875-1879.
- Lord, H.W. and Shulman, Y., 1967. A generalized dynamical theory of thermoelasticity. Journal of the Mechanics and Physics of Solids, 15(5), 299-309.
- Love, A. E. H.,1944. A treatise on the mathematical theory of elasticity. Cambridge University Press, Cambridge. Dover Publications, New York, First American Printing.
- Love, A.E.H., 1911. Some Problems of Geodynamics: Being an Essay to which the Adams Prize in the University of Cambridge was Adjudged in 1911. University Press.
- Ma, Q., Jiao, J., Hu, P., Zhong, X., Wu, B. and He, C., 2014. Excitation and detection of shear horizontal waves with electromagnetic acoustic transducers for nondestructive testing of plates. Chinese Journal of Mechanical Engineering, 27(2), 428-436.
- Manna, S. and Bhat, M., 2022. Love wave fields in a non-local elastic model with reinforced and inhomogeneous media. Soil Dynamics and Earthquake Engineering, 161, 107388.
- Manna, S., Halder, T. and Althobaiti, S.N., 2022. Dispersion of Love-type wave and its limitation in a nonlocal elastic model of nonhomogeneous layer upon an orthotropic extended medium. Soil Dynamics and Earthquake Engineering, 153, 107117.
- Marin, M., Baleanu, D. and Vlase, S., 2017. Effect of microtemperatures for micropolar thermoelastic bodies. Structural Engineering and Mechanics, 61(3), 381-387.

- Marin, M., Chirila, A., Öchsner, A. and Vlase, S., 2019. About finite energy solutions in thermoelasticity of micropolar bodies with voids. Boundary Value Problems, 2019, 1-14.
- Midya, G., 2004. On Love-type surface waves in homogeneous micropolar elastic media. International Journal of Engineering Science, 42(11-12), 1275-1288.
- Mindlin, R.D., 1952. Forced thickness-shear and flexural vibrations of piezoelectric crystal plates. Journal of Applied Physics, 23(1), 83-88.
- Mondal, S. and Othman, M.I., 2021. Memory dependent derivative effect on generalized piezo-thermoelastic medium under three theories. Waves in Random and Complex Media, 31(6), 2150-2167.
- Mondal, S. and Sur, A., 2023. Field equations and memory effects in a functionally graded magneto-thermoelastic rod. Mechanics Based Design of Structures and Machines, 51(3), 1408-1430.
- Mondal, S., Sarkar, N. and Sarkar, N., 2019. Waves in dual-phase-lag thermoelastic materials with voids based on Eringen's nonlocal elasticity. Journal of Thermal Stresses, 42(8), 1035-1050.
- Mrithyumjaya Rao, K. and Reddy, M.P., 1993. Rayleigh-type wave propagation on a micropolar cylindrical surface, 857-865.
- Nobili, A. and Volpini, V., 2021. Microstructured induced band pattern in Love wave propagation for novel nondestructive testing (NDT) procedures. International Journal of Engineering Science, 168, 103545.
- Nowacki, W., 1966. Couple-stresses in the theory of thermoelasticity. III(Constitutive equations derived based on thermodynamics of irreversible processes and coupled thermoelasticity, formulating variational and reciprocity theorems). Academie polonaise des sciences, bulletin, serie des sciences techniques, 14(8), 801-809.
- Nowacki, W., 1968. Couple-stresses in the theory of thermoelasticity. In Irreversible Aspects of Continuum Mechanics and Transfer of Physical Characteristics in Moving Fluids: Symposia Vienna, Vienna: Springer Vienna, 259-278.

- Nowacki, W., 1970. The second plane problem of micropolar elasticity. Bulletin of the Polish Academy of Sciences Technical Sciences, 18(11), 899-906.
- Nowacki, W., 1971. Axial-symmetric problems in micropolar elasticity(Micropolar elastic theory axial symmetric problems, deriving differential equations for elastic potential and half space). Academie Polonaise des Sciences, Bulletin, Serie des Sciences Techniques, 19(7), 317.
- Nowinski, J.L., 1993. On the surface waves in an elastic micropolar and microstretch medium with nonlocal cohesion. Acta mechanica, 96, 97-108.
- Othman, M.I. and Mondal, S., 2020. Memory-dependent derivative effect on wave propagation of micropolar thermoelastic medium under pulsed laser heating with three theories. International Journal of Numerical Methods for Heat & Fluid Flow, 30(3), 1025-1046.
- Othman, M.I., Abd-alla, A.E.N.N. and Abd-Elaziz, E.M., 2020. Effect of heat laser pulse on wave propagation of generalized thermoelastic micropolar medium with energy dissipation. Indian Journal of Physics, 94, 309-317.
- Ozisik, M., 2021. Dispersion of generalized Rayleigh waves in the half-plane covered with pre-stretched two layers under complete contact. Thermal Science, 25(2), 247-253.
- Papadakis, G., Tsortos, A. and Gizeli, E., 2009. Triple-helix DNA structural studies using a Love wave acoustic biosensor. Biosensors and Bioelectronics, 25(4), 702-707.
- Parfitt, V. R. and Eringen, A. C., 1969. Reflection of plane waves from the flat boundary of a micropolar elastic half-space. The Journal of the Acoustical Society of America, 45(5), 1258-1272.
- Poonam, Sahrawat, R.K. and Kumar, K., 2021. Plane wave propagation and fundamental solution in non-local couple stress micropolar thermoelastic solid medium with voids. Waves in Random and Complex Media, 1-36.

- Povstenko, Y.Z., 1999. The nonlocal theory of elasticity and its applications to the description of defects in solid bodies. Journal of Mathematical Sciences, 97, 3840-3845.
- Purkait, P., Sur, A. and Kanoria, M., 2021. Elasto-thermodiffusive response in a spherical shell subjected to memory-dependent heat transfer. Waves in Random and Complex Media, 31(3), 515-537.
- Qian, Z., Jin, F., Wang, Z. and Kishimoto, K., 2004. Dispersion relations for SH-wave propagation in periodic piezoelectric composite layered structures. International Journal of Engineering Science, 42(7), 673-689.
- Qian, Z., Jin, F., Wang, Z. and Kishimoto, K., 2004. Love waves propagation in a piezoelectric layered structure with initial stresses. Acta Mechanica, 171, 41-57.
- Qing, X., Li, W., Wang, Y. and Sun, H., 2019. Piezoelectric transducer-based structural health monitoring for aircraft applications. Sensors, 19(3), 545.
- Qing-tian, D. and Song-nan, L., 2018. Wave propagation in piezoelectric circular curved rods. Iranian Journal of Science and Technology, Transactions A: Science, 42, 155-166.
- R.K.T., 1982. Volume defects in nonlocal micropolar elasticity. International Journal of Engineering Science, 20(2), 261-270.
- Rajneesh, K., Nidhi, S., Parveen, L. and Marin, M., 2018. Reflection of plane waves at micropolar piezothermoelastic half-space. CMST, 24(2), 113-124.
- Rayleigh, L., 1885. On waves propagated along the plane surface of an elastic solid. Proceedings of the London mathematical Society, 1(1), 4-11.
- Rayleigh, L., 1885. On waves propagated along the plane surface of an elastic solid. Proceedings of the London mathematical Society, 1(1), 4-11.
- Saha, A.N.U.P., Kundu, S., Gupta, S. and Vaishnav, P.K., 2015. Love waves in a heterogeneous orthotropic layer under initial stress overlying a gravitating porous half-space. Indian National Science Academy, 81(5), 1193-1205.

- Sahrawat, R.K., Poonam and Kumar, K., 2020. Plane wave and fundamental solution in non-local couple stress micropolar thermoelastic solid without energy dissipation. Journal of Thermal Stresses, 44(3), 295-314.
- Sahu, S.A., Mondal, S. and Nirwal, S., 2023. Mathematical Analysis of Rayleigh Waves at the Nonplanner Boundary between Orthotropic and Micropolar Media. International Journal of Geomechanics, 23(3), 04022313.
- Sarkar, N. and Mondal, S., 2019. Transient responses in a two-temperature thermoelastic infinite medium having cylindrical cavity due to moving heat source with memory-dependent derivative. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 99(6), e201800343.
- Sharma, J.N. and Kumar, S., 2009. Lamb waves in micropolar thermoelastic solid plates immersed in liquid with varying temperature. Meccanica, 44, 305-319.

- Sharma, J.N. and Kumar, S., 2009. Lamb waves in micropolar thermoelastic solid plates immersed in liquid with varying temperature. Meccanica, 44, 305-319.

- Sharma, J.N., Kumar, S. and Sharma, Y.D., 2007. Propagation of Rayleigh surface waves in microstretch thermoelastic continua under inviscid fluid loadings. Journal of Thermal Stresses, 31(1), 18-39.
- Sharma, J.N., Kumar, S. and Sharma, Y.D., 2009. Effect of micropolarity, microstretch and relaxation times on Rayleigh surface waves in thermoelastic solids. Int. J. Appl. Math. Mech, 5(2), 17-38.
- Sharma, J.N., Pal, M. and Chand, D., 2004. Thermoelastic Lamb waves in electrically shorted transversely isotropic piezoelectric plate. Journal of Thermal Stresses, 27(1), pp.33-58.
- Sharma, J.N., Pal, M. and Chand, D., 2005. Propagation characteristics of Rayleigh waves in transversely isotropic piezothermoelastic materials. Journal of Sound and Vibration, 284(1-2), 227-248.

- Sharma, J.N., Sharma, K.K. and Kumar, A., 2010. Surface waves in a piezoelectric–semiconductor composite structure. International Journal of Solids and Structures, 47(6), 816-826.
- Sharma, J.N., Sharma, K.K. and Kumar, A., 2011. Acousto-diffusive waves in a piezoelectric-semiconductor-piezoelectric sandwich structure, 5.
- Sharma, J.N., Sharma, K.K. and Kumar, A., 2011. Modelling of acoustodiffusive surface waves in piezoelectric-semiconductor composite structures. Journal of Mechanics of Materials and Structures, 6(6), 791-812.
- Sharma, J.N., Singh, H. and Sharma, Y.D., 2011. Modeling of thermoelastic damping and frequency shift of vibrations in a transversely isotropic solid cylinder. Multidiscipline Modeling in Materials and Structures, 7(3), 245-265.
- Sharma, J.N., Walia, V. and Gupta, S.K., 2008. Effect of rotation and thermal relaxation on Rayleigh waves in piezothermoelastic half space. International Journal of Mechanical Sciences, 50(3), 433-444.
- Sharma, S.K., Kumar, S. and Kumar, R., 2023. Parametric analysis of hybrid tribo-piezoelectric energy harvester. Mechanics Based Design of Structures and Machines, 51(11), 6360-6373.
- Sharma, V. and Kumar, S., 2021. Microstructural and viscous liquid loading effects on the propagation of love waves in a piezomagnetic layered structure. Mechanics of Advanced Materials and Structures, 28(16), 1703-1713.
- Sharma, V. and Kumar, S., 2021. Microstructural and viscous liquid loading effects on the propagation of love waves in a piezomagnetic layered structure. Mechanics of Advanced Materials and Structures, 28(16), 1703-1713.
- Sharma, V. and Kumar, S., 2022. Bleustein–Gulyaev wave in a nonlocal piezoelectric layered structure. Mechanics of Advanced Materials and Structures, 29(15), 2197-2207.
- Sharma, V. and Kumar, S., 2022. Comparative study of micro-scale size effects on mechanical coupling factors and SH-wave propagation in functionally graded

piezoelectric/piezomagnetic structures. Waves in Random and Complex Media, 32(5), 2332-2367.

- Sharma, V. and Kumar, S., 2022. Nonlocal and magneto effects on dispersion characteristics of Love-type waves in piezomagnetic media. Waves in Random and Complex Media, 1-19.
- Sharma, V. and Kumar, S., 2023. A Study of Plane and Rayleigh Waves in a Microstructural Medium: the Role of Size Dependency and Thermal Effects. Mechanics of Solids, 58(4), 1335-1350.
- Sharma, V., Goyal, R. and Kumar, S., 2020. Love waves in a layer with void pores over a microstructural couple stress substrate with corrugated boundary surfaces. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 42, 1-16.
- Shatalov, M., Murashkin, E.V., Mahamood, R.M., Skhosana, P. and Mkolesia, A., 2021. Axisymmetric wave propagation in transversely isotropic piezoelectric functionally grade cylinder. Mechanics of Solids, 56, 1091-1102.
- Sheoran, D., Kumar, R., Punia, B.S. and Kalkal, K.K., 2022. Propagation of waves at an interface between a nonlocal micropolar thermoelastic rotating half-space and a nonlocal thermoelastic rotating half-space. Waves in Random and Complex Media, 1-22.
- Shorkin, V.S., Vilchevskaya, E.N. and Altenbach, H., 2023. Linear theory of micropolar media with internal nonlocal potential interactions. ZAMM-Journal of Applied Mathematics and Mechanics, e202300099.
- Singh, A.K., Das, A., Mistri, K.C., Nimishe, S. and Koley, S., 2017. Effect of corrugation on the dispersion of Love-type wave in a layer with monoclinic symmetry, overlying an initially stressed transversely isotropic half-space. Multidiscipline Modeling in Materials and Structures, 13(2), 308-325.
- Singh, A.K., Ray, A. and Kumari, R., 2023. A new dispersive wave with Love-type waves in a microstructure due to an impulsive point source. Waves in Random and Complex Media, 33(4), 876-898.

- Singh, B., 2015. Rayleigh waves in an incompressible fibre-reinforced elastic solid with impedance boundary conditions. Journal of the Mechanical Behavior of Materials, 24(5-6), 183-186.
- Singh, B., 2016. Rayleigh wave in a thermoelastic solid half-space with impedance boundary conditions. Meccanica, 51(5), 1135-1139.
- Singh, B., 2017. Rayleigh Wave in a Micropolar Elastic Medium with Impedance Boundary Conditions. Geosci. Res, 2(1), 6-13.
- Singh, B., Sindhu, R. and Singh, J., 2016. Rayleigh wave in a micropolar thermoelastic medium without energy dissipation. Engineering Solid Mechanics, 4(1), 11-16.
- Singh, B., Yadav, A.K. and Gupta, D., 2019. Reflection of plane waves from a micropolar thermoelastic solid half-space with impedance boundary conditions. Journal of Ocean Engineering and Science, 4(2), 122-131.
- Singh, K. and Kashyap, M., 2023. Memory Effects on Rayleigh Waves Propagation in a Micropolar Thermoelastic Half Space. Mechanics of Solids, 58(4), 1228-1238.
- Smith, M.L. and Dahlen, F.A., 1973. The azimuthal dependence of Love and Rayleigh wave propagation in a slightly anisotropic medium. Journal of Geophysical Research, 78(17), 3321-3333.
- Sokolnikoff, I.S. and Specht, R.D., 1946. Mathematical theory of elasticity.
- Somaiah, K. and Kumar, A.R., 2023. Rayleigh Wave Propagation at Viscous Liquid/Micropolar Micro-stretch Elastic Solid. Communications in Mathematics and Applications, 14(1), 89.
- Tamarin, O., Comeau, S., Dejous, C., Moynet, D., Rebiere, D., Bezian, J. and Pistre, J., 2003. Real time device for biosensing: design of a bacteriophage model using love acoustic waves. Biosensors and Bioelectronics, 18(5-6), 755-763.
- Tauchert, T.R., Claus Jr, W.D. and Ariman, T., 1968. The linear theory of micropolar thermoelasticity. International Journal of Engineering Science, 6(1), 37-47.

- Tien-min, T., 1980. Various reciprocal theorems and variational principles in the theories of nonlocal micropolar linear elastic mediums. Applied Mathematics and Mechanics, 1(1), 91-111.
- Tiersten, H.F., 1963. Thickness vibrations of piezoelectric plates. The Journal of the Acoustical Society of America, 35(1), 53-58.
- Tolipov, K.B., 2002. Propagation of Rayleigh waves in an elastic wedge. Russian journal of nondestructive testing, 38(7), 493-497.
- Vashishth, A. K., & Khurana, P., 2005. Rayleigh modes in anisotropic, heterogeneous poroelastic layers. Journal of seismology, 9(4), 431-448.
- Tomar, S.K. and Gogna, M.L., 1992. Reflection and refraction of a longitudinal microrotational wave at an interface between two micropolar elastic solids in welded contact. International Journal of Engineering Science, 30(11), 1637-1646.
- Tomar, S.K. and Gogna, M.L., 1995. Reflection and refraction of coupled transverse and micro-rotational waves at an interface between two different micropolar elastic media in welded contact. International Journal of Engineering Science, 33(4), 485-496.
- Tomar, S.K. and Gogna, M.L., 1995. Reflection and refraction of longitudinal wave at an interface between two micropolar elastic solids in welded contact. The Journal of the Acoustical Society of America, 97(2), 822-830.
- Tomar, S.K., 2005. Wave propagation in a micropolar elastic plate with voids. Journal of Vibration and Control, 11(6), 849-863.
- Trovalusci, P. and Masiani, R., 2003. Non-linear micropolar and classical continua for anisotropic discontinuous materials. International Journal of Solids and Structures, 40(5), 1281-1297.
- Vinh, P.C. and Hue, T.T.T., 2014a. Rayleigh waves with impedance boundary conditions in anisotropic solids. Wave Motion, 51(7), pp.1082-1092.
- Vinh, P.C. and Hue, T.T.T., 2014b. Rayleigh waves with impedance boundary conditions in incompressible anisotropic half-spaces. International Journal of Engineering Science, 85, 175-185.

- Vinh, P.C. and Xuan, N.Q., 2017. Rayleigh waves with impedance boundary condition: Formula for the velocity, existence and uniqueness. European Journal of Mechanics-A/Solids, 61, 180-185.
- Vinh, P.C., 2009. Explicit secular equations of Rayleigh waves in elastic media under the influence of gravity and initial stress. Applied Mathematics and Computation, 215(1), 395-404.
- Voigt, W., 1887. Theoretical studies of the elastic behaviour of crystals. Presented at the session of the Royal Society of Science, 3-52.
- Walia, V., Sharma, J.N. and Sharma, P.K., 2009. Propagation characteristics of thermoelastic waves in piezoelectric (6 mm class) rotating plate. European Journal of Mechanics-A/Solids, 28(3), 569-581.
- Wang, H.M. and Zhao, Z.C., 2013. Love waves in a two-layered piezoelectric/elastic composite plate with an imperfect interface. Archive of Applied Mechanics, 83, 43-51.
- Wang, J.L. and Li, H.F., 2011. Surpassing the fractional derivative: Concept of the memory-dependent derivative. Computers & Mathematics with Applications, 62(3), 1562-1567.
- Wang, X. and Jin, F., 2016, October. Dispersion relations for SH-wave propagation in nanoscale periodic piezoelectric composite layered structures. In 2016 Symposium on Piezoelectricity, Acoustic Waves, and Device Applications (SPAWDA) IEEE, 438-442.
- Yang, J., Sun, G. and Fu, G., 2021. Bifurcation and chaos of functionally graded carbon nanotube reinforced composite cylindrical shell with piezoelectric layer. Mechanics of Solids, 56, 856-872.
- Zakaria, M., 2012. Effects of Hall current and rotation on magneto-micropolar generalized thermoelasticity due to ramp-type heating. International Journal of Electromagnetics and Applications, 2(3), 24-32.

- Zhang, P., Wei, P. and Li, Y., 2016. Wave propagation through a micropolar slab sandwiched by two elastic half-spaces. Journal of vibration and acoustics, 138(4), 041008.
- Zhang, P., Wei, P. and Tang, Q., 2015. Reflection of micropolar elastic waves at the non-free surface of a micropolar elastic half-space. Acta Mechanica, 226(9), 2925-2937.
- Zhang, R., Pang, Y. and Feng, W., 2014. Propagation of Rayleigh waves in a magneto-electro-elastic half-space with initial stress. Mechanics of Advanced Materials and Structures, 21(7), 538-543.
- Zhu, H., Zhang, L., Han, J. and Zhang, Y., 2014. Love wave in an isotropic homogeneous elastic half-space with a functionally graded cap layer. Applied Mathematics and Computation, 231, 93-99.

LIST OF PUBLISHED /COMMUNICATED PAPERS

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2. Effects of nonlocal characteristics of composite material on shear waves propagation with an imperfect interface, Journal - International Journal of Applied and Computational Mathematics , vol. 9 &isuue 5, SJR : 0.37 , (scopus)
3. Rayleigh waves with impedance boundary conditions in a non-local micropolar thermoelastic material, Jouranal- Journal of Physics: Conference Series, VOL.1531 & Issue 17426588, (scopus)
4. Propagation of SH-waves in two layered piezoelectric/fiber-reinforced composite plate, Jouranal- Journal of Physics: Conference Series , vol.2267, (scopus)
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Non-Locality Effects on the Propagation of Shear Waves in Piezoelectric/Non-local Micropolar layered structure

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Abstract—Considering the non-local effects in micropolar elastic material this paper studied the propagation of shear waves in a piezoelectric layered non-local micropolar half space composite structure. The general dispersion equation of shear waves in the coupled structure is obtained analytically in the closed form. In the particular case the result obtained is in accordance with the classical Love wave equation. The effects of key factors like non-locality, characteristic length, piezoelectric and elastic constants on the phase velocity of shear waves have been investigated and results are depicted graphically. The theoretical results obtained shows that the phase velocity of shear wave is significantly affected due the presence of non-locality and size effects on small length scale in micropolar elastic material.

Keywords: Shear waves, piezoelectric, non-local micropolar, phase velocity, characteristic length
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1. INTRODUCTION

The non-local theory proposed by Edelen and Laws [1, 2] and Eringen and Edelen [3] are well known due to its close agreement with atomic theory. It has been used to calculate the strength of materials having microstructure and nanostructure properties. The shortcomings of classical theory to explain the physical phenomenon at microstructure level led to the invention of non-local theories. This theory is characterized by the presence of nonlocality residuals of fields. In this theory, the stress at any reference point within a continuous body depends upon the strain at that point along with the strain at all other points from the certain neighborhood of that point. In long wavelength limits the non-local theory reduces to local theory (classical) and in short wavelength limit it reduces to atomic lattice dynamics. The non-local models are used to find the solution of elastodynamic problems such as material damage, deformation of nanobeams, wave propagation in solids [4–6]. Eringen [7, 8] extended the concept of non-locality to various fields including micropolar elasticity [9]. The micropolar theory of elasticity has wide applications in the treatment of mechanics of granular materials, composite fibrous materials, polymers, bones and materials with micro-cracks and micro-fracture. Many research articles on the applications of micropolar theory in different fields are available in the literature.

Recently, the non-local micropolar theory of elasticity has find its application in the various fields like dislocation, high frequency vibrations and wave propagation in materials with microstructure. Eringen [10] obtained the dispersion relation for transverse plane waves in non-local micropolar elastic solid. The results obtained are in well agreement with Born- Kármán theory of lattice dynamics in the entire Brillouin zone. Jun and Dhaliwal [11] studied the linear non local theory of elasticity and established work-energy and uniqueness theorem. Khurana and Tomar [12, 13] investigated reflection of plane waves and propagation of Rayleigh waves in non-local micropolar solids and established the dispersive characteristic of waves in this media. Khurana and Tomar [14] considered two different non-local micropolar materials to study effects of non-locality on the reflection and transmission of plane waves. Kalkal et al. [15] explored the effect of rotation on the reflection of plane waves on the free surface of non-local micropolar

Propagation of SH-waves in two layered piezoelectric/fiber-reinforced composite plate

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Abstract. This article deals with the study of propagation of shears waves in a composite structure consisting of a piezoelectric layer superimposed on a layer of fiber-reinforced material. A mathematical model is framed and the dispersion relation of shear wave is derived analytically. By suitable substitutions the dispersion relation obtained is reduced to the well-known classical Love wave equation. The effects of various parameters such as wave number, thickness ratio and the direction of fiber-reinforced parameters on the shears wave velocity has been investigated and depicted graphically.

1. Introduction

These guidelines, Piezoelectric materials are commonly used in modern smart materials devices like sensors, actuators and transducers due to their high sensitivity and enhanced electromechanical property. The micro-sensors constituted of piezoelectric composite based upon surface acoustic waves (SAW) have numerous applications in many fields. The effectiveness of piezoelectric material is measured by electromechanical coupling factor with which it converts electrical energy into mechanical energy or vice versa. High electromechanical coupling factor indicates the efficient energy conversion and to attain it, sensors are manufactured by considering an assembly of materials with a layered structure. In the layered structures a thin layer of particular material is deposited on a substrate or on another layer of different material. Many researchers [1]-[7] have investigated the phenomenon wave propagation in layered structures comprising of piezoelectric smart material layers

Fiber-reinforced materials are available naturally inside the earth and there are artificial materials also. These materials are commonly used to strengthen the other materials to minimize damages due to vibration. As a result, the study of wave propagation in fiber-reinforced materials plays an importance role in different fields likes geomechanics and engineering. The research articles [8]-[11] studied the phenomenon of wave propagation in fiber reinforced materials.

The presents study deals with shear wave propagation in a coupled plate consisting of a piezoelectric layer in perfect contact with another layer of fiber-reinforced material. The velocity equation is derived analytically and it is in agreement with the result of earlier works. The effects of wave number, thickness ratio and reinforced direction on the phase velocity has been investigated and depicted graphically.

Rayleigh waves with impedance boundary conditions in a non local micropolar thermoelastic material

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Abstract. The present paper analyzes the propagation of Rayleigh waves in a non-local micropolar thermoelastic half space with impedance boundary conditions. Dispersion equation of Rayleigh wave propagation with impedance boundary conditions is obtained and the effect of impedance and non local parameters are studied. Dispersion equation of Rayleigh waves for a micropolar thermoelastic half space with impedance boundary as well as traction free half-space is obtained as a particular case. The non-dimensional speed of Rayleigh wave is computed as a function of impedance parameters and presented graphically for a aluminum epoxy material. It is observed that non local and impedance parameters has significant effects on Rayleigh wave speed.

Keywords: Non local micropolar thermoelasticity, Impedance boundary conditions, Rayleigh wavesThe first section in your paper

1. Introduction

The micropolar theory of elasticity established by Eringen's [1] is a familiar theory of solid mechanics and being used in analyze the deformation of microstructure materials such as cellular solids, composite fibrous, granular material, polymers, bones etc. This theory takes into account the intrinsic rotational motion along with linear displacement in the materials. The motion in this theory is governed by six degrees of freedom, three of microrotation and three of translation. Nowacki [2] and Eringen [3] extended the micropolar theory by including thermal effects and presented linear theory of micropolar thermoelasticity. The micropolar theory along with generalized theory of thermoelasticity proposed by Lord and Shulman [4] and Green and Lindsay [5] can be seen in many research findings.

Surface waves are of particular significance in seismology due to their destructive nature during earthquake. Rayleigh waves are one of the surface type waves and have been explored by several researchers. Rayleigh waves in micropolar elastic materials has been explored by many researchers due to the practical applicability in the various fields such as, seismology, acoustics, aerospace and submarine structures. Eringen [6] obtained the frequency equation of Rayleigh surface waves in micropolar elastic half space along a stress free boundary. The research articles [7-9] can be seen on various problems related to propagation of Rayleigh waves in micropolar elastic material.

The wave propagation in non local micropolar elastic half space is of special interest as the non local theories is considered to cover microscopic phenomena, for example crack tip problems, high frequency vibrations and dislocations etc. Eringen [10] derived the dispersion relation for transverse plane waves in non local micropolar elastic solid. Kaliski et al [11] discussed various properties of

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ORIGINAL PAPER

Effects of Nonlocal Characteristics of Composite Material on Shear Waves Propagation with an Imperfect Interface

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Abstract
Imposing the non-local effect on a layered composite structure could lead to new insights in design. Taking the non-local and microstructure effects into consideration, the propagation of shear waves has been investigated in an initially stressed piezoelectric layer imperfectly bonded to a micropolar half-space under the non-local theory. The general phase velocity equation for shear waves has been obtained analytically in closed form. The phase velocity equation is in agreement with the classical Love wave equation in a particular case. The effects of key factors such as non-locality, interfacial imperfection, initial stress and thickness of the layer on the phase velocity have been evaluated. Graphical analysis has been performed and the results obtained indicate that shear wave propagation is significantly affected by various parameters considered in the study and are useful for designing high performance surface acoustic devices and sensors.

Keywords Non-local piezoelectric · Non-local micropolar · Phase velocity · Imperfect interface · Initial stress

Introduction

The insufficiency of classical theory in explicating physical phenomena at the microstructural level has necessitated the inclusion of non-local theories. The non-local theory of elasticity proposed by Eringen and Edelen elucidates the complex material behavior at the micro scale [12]. The aforementioned theory has garnered significant traction owing to its close correlation with the atomic theory, which is employed to gauge the mechanical strength of materials by analyzing their micro and nano structural characteristics. The theory in question is distinguished by the presence of non-local residual fields, whereby the stress experienced at a given reference point is contingent upon not only the strain present at that location, but also the strain present at all other proximal points. The utilization of non-local theory enables us to prognosticate and elucidate the conduct of materials at the micro-scale by means of the non-local parameter. ($\epsilon = e_0 a$). Here, e_0 is material constant and a being internal characteristic

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