Theoretical Investigation on Stimulated Brillouin Scattering of Self Focused Laser Beams in Underdense Plasma Targets

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DECLARATION

I hereby declared that the presented work in the thesis entitled **"Theoretical Investigation on Stimulated Brillouin Scattering of Self Focused Laser Beams in Underdense Plasma Targets"** in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy (Ph.D.) in Physics** is the outcome of original research work carried out by me under the supervision of **Dr. Naveen Gupta**, Assistant Professor, Department of Physics, Lovely Professional University, Phagwara, Punjab, India and co-supervision of **Dr. Shri Bhagwan Bhardwaj**, working as Assistant Professor in the Department of Physics, S.U.S. Government College, Matak Majri, Indri, Karnal, Haryana, India. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

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CERTIFICATE

This is to certify that the work reported in the Ph.D. thesis entitled **"Theoretical Investigation on Stimulated Brillouin Scattering of Self Focused Laser Beams in Underdense Plasma Targets"** submitted in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy (Ph.D.)** in **Physics** is a research work carried out by **Suman Choudhry, Registration No. 42000183**, is bonafide record of her original work carried out under my supervision and that no part of thesis has been submitted for any other degree, diploma or equivalent course.

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Abstract

This thesis focuses on nonlinear dynamics of different laser beams (q-Gaussian and Quadruple-Gaussian (Q.G.) laser beams) in different nonlinear media such as plasmas. Further, the impacts of self action effects and stimulated Brillouin scattering (SBS) of laser beams in plasmas have been studied theoretically. In the midst of different existing semi-analytical approaches, variational theory approach has been used to investigate the propagation of q-Gaussian and Quadruple-Gaussian (Q.G.) laser beams in nonlinear medium such as underdense plasmas under nonlinear regime.

Theoretical investigation has been conducted to study the impact of relativistic self-focusing of q-Gaussian laser beams on Stimulated Brillouin Scattering (SBS) in underdense plasma targets characterized by axially-increasing plasma density. Through our observations, we have noted that the amplitude distribution across the cross-sectional area of a beam significantly influences the propagation dynamics within nonlinear media. The laser beams, whose amplitude structure is deviated from ideal Gaussian profile, possess more self-focusing in plasmas and get scattered more by preexisting IAWs through the SBS phenomenon. For instance, as compared to an ideal Gaussian beam, a laser beam with deviation parameter $q = 3$ gets 2.75 times more self-focused as well as, for the same propagation distance, the power of scattered wave for the beam with $q = 3$ is four times greater as compared to that for the ideal Gaussian beam. Therefore, by controlling the deviation parameter q, one can optimize the Stimulated Brillouin Scattering (SBS) of the laser light beam. The power carried by scattered beam can also be controlled by changing the initial intensity of the laser beam or by changing the density ramp slope.

Theoretical investigations have been conducted to explore the stimulated Brillouin scattering of elliptical q-Gaussian laser beams in axially inhomogeneous plasmas, considering the impact of self-focusing of the laser light beam on the power of the scattered wave. Observations have indicated that as the irradiance across the cross-section of the laser beam approaches an ideal Gaussian profile, there is a substantial reduction in the power of the scattered wave. Hence, to mitigate stimulated Brillouin scattering of laser beams in laser-

plasma interaction applications, it is advisable to strive for an irradiance profile that closely approximates the ideal Gaussian profile.

The nonlinear characteristics of Quadruple Gaussian (Q.G.) laser beam have been observed in relativistic plasma. The comparisons between linear and nonlinear propagation of Q.G. and Gaussian laser beam have been made. It has been investigated that the extent of self focusing is enhanced at $\frac{x_0}{r_0} = 1.5$ for Q.G. laser beam as compare to the Gaussian laser beam.

Dedication

"Every challenging work needs self-efforts as well as Guidance, support and motivation of

Some special peoples"

I dedicate my humble efforts to special persons of my life who were very close to my heart

During this research work journey

***My Mother** (Mrs. Kamlesh Choudhry) and **Father** (Mr. Yashpal Choudhry)*

And

***My Husband** (Mr. Dinesh) and **Lovely Son** (Priyanshu Choudhary)*

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Preface

The main focus of this thesis especially on the Theoretical Investigation on Stimulated Brillouin Scattering of Self Focused Laser Beams (q-Gaussian and Quadruple Gaussian profile) in Underdense Plasma Targets. The present research work has been divided into total eight chapters. The nonlinear Schrodinger wave equation for the evolution of beam envelope is solved numerically by using variational theory. In particular,

Chapter- 1 incorporates the introduction part which involves the scattering of light, origin of scattering, scalar and tensor scattering, Brillouin scattering of light, stimulated Brillouin scattering (SBS), application of SBS (optical phase conjugation, production of phase conjugation by SBS, phonon maser).

Chapter- 2 includes the literature review of different research papers which are based on self-action effects, Brillouin scattering and variational theory approach.

Chapter- 3 involves the description of the amplitude structures of q-Gaussian and Quadruple Gaussian Laser Beams laser profiles and their effective beam width with different laser-plasma parameters in detail, variational theory approach and research objectives.

In Chapter- 4, study has delved into how the self-focusing of the laser beam influences the excitation of ion acoustic waves in axially inhomogeneous plasmas. The incorporation of beam ellipticity and deviations from the ideal Gaussian profile in the amplitude structure of the beam of laser light has been considered in this study. Based on the findings of this investigation, it can be inferred that as the amplitude distribution of the beam of laser light approaches the ideal Gaussian profile, there is a notable reduction in the power of the excited Ion Acoustic Wave (IAW).

In Chapter- 5, the emphasis is placed on studying the impact of relativistic self-focusing of q-Gaussian laser beams on Stimulated Brillouin Scattering (SBS) in underdense plasma targets characterized by axially-increasing plasma density. It has been noted that the amplitude distribution across the cross-sectional area of a beam significantly influences the propagation dynamics within nonlinear media.

In Chapter- 6, the focus is on exploring the stimulated Brillouin scattering of elliptical q-Gaussian laser beams in axially inhomogeneous plasmas, considering the impact of self-focusing of the laser light beam on the power of the scattered wave. Observations have indicated that as the irradiance across the cross-section of the laser beam approaches an ideal

Gaussian profile, there is a substantial reduction in the power of the scattered wave. Hence, to mitigate stimulated Brillouin scattering of laser beams in laser-plasma interaction applications, it is advisable to strive for an irradiance profile that closely approximates the ideal Gaussian profile.

Chapter- 7 explores potential well dynamics of self focusing of Quadruple Gaussian Laser Beams in Thermal Quantum Plasma. This chapter also includes self-channeling of laser beam.

Chapter- 8 includes conclusions and future scope.

List of Research Paper Publications

1. Naveen Gupta, **Suman Choudhry**, S. B. Bhardwaj, Sanjeev Kumar and Sandeep Kumar, "Relativistic effects on Stimulated Brillouin Scattering of self-focused q-Gaussian laser beams in plasmas with axial density ramp", Journal of Russian Laser Research 42, 418-429 (Published online: 5 July 2021).
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13. Naveen Gupta, A. K. Alex, Rohit Johari, **Suman Choudhry**, Sanjeev Kumar, Aatif Ahmad & S. B. Bhardwaj, "Formation of elliptical q-Gaussian breather solitons in diffraction managed nonlinear optical media: effect of cubic quintic nonlinearity", Journal of Optics (Published online: 18 August 2023).
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15. Naveen Gupta, Rohit Johari, A. K. Alex, **Suman Choudhry**, "Beat wave excitation of electron plasma wave by cross-focused q-Gaussian laser beams in thermal quantum plasmas", Journal of Optics (Published online: 04 November 2023).
16. Naveen Gupta, **Suman Choudhry**, Rohit Johari, A. K. Alex, Sanjeev Kumar, S. B. Bhardwaj, "Quadruple Gaussian laser beams in thermal quantum plasma: self-focusing, self-trapping and self-phase modulation", Journal of Optics (Published online: 21 November 2023).
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19. Naveen Gupta, A. K. Alex, Rohit Johari, **Suman Choudhry**, Sanjeev Kumar, S. B. Bhardwaj, "Excitation of electron plasma wave by self-focused Laguerre-Gaussian laser beams in axially inhomogeneous plasma: effect of orbital angular momentum of photons", Journal of Optics (Published online: 23 December 2023).

20. Naveen Gupta, Sanjeev Kumar, **Suman Choudhry**, A. K. Alex, Rohit Johari, S. B. Bhardwaj, "Quadruple Gaussian laser beam in cubic-quintic nonlinear media: effect of nonlinear absorption", Journal of Optics (Published online: 02 January 2024).

Conferences/Seminars

1. Presented a paper titled "Ion Acoustic Decay Instability of Elliptical q-Gaussian Laser Beams in Plasma with Axial Density Ramp: effect of self focusing" in International Conference on "Frontiers in Physics, Materials Science and Nanotechnology" organised by Department of Physics, Chaudhary Devi Lal University, Sirsa on 25-26 March, 2022.
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3. Presented paper titled "Stimulated Brillouin Scattering of Self Focused q-Gaussian Laser Beams in Underdense Plasma Targets: Effect of Density Ramp" in International Symposium on Recent Trends in Optical Materials and Photonic Devices organised by Department of Pure & Applied Physics, Guru Ghasidas Vishwavidyalaya, Bilaspur, INDIA on 7 December, 2022.
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Chapter- 1

Stimulated Scattering of Laser Light

1.1 Introduction

One of the most significant milestones in the history of light is marked by the introduction of laser, attributed by Maiman[1] in 1960. The invention represents a remarkable leap in optical science and technology, paving the way for a new era. For the very first time, humanity has succeeded in creating a remarkable and inventive tool that enables the direct generation and control of coherent light. In optics, lasers brought the same revolution as transistors did in electronics, and cyclotrons brought revolution to realm of nuclear physics. Due to the coherence properties of the laser, the light beam has precisely defined optical phase in both space and temporal domains[2]. Laser light exhibits a narrow frequency spectrum due to this well defined phase, which limits the optical spatial wavelength and frequency of laser light. In addition to its directionality, laser light is capable of propagating over long distances with minimal spreading, and conventional optical elements can be easily employed to manipulate it. With the phase coherence and directionality of lasers, extremely large optical powers can be created, pushing optical systems from linear to nonlinear[3].

Coherent frequency mixing effects encompassing difference-frequency, third-harmonic generation, and optical sum-frequency were observed sequentially soon after laser coherent frequency mixing was invented[4-6] (fig. 1.1).

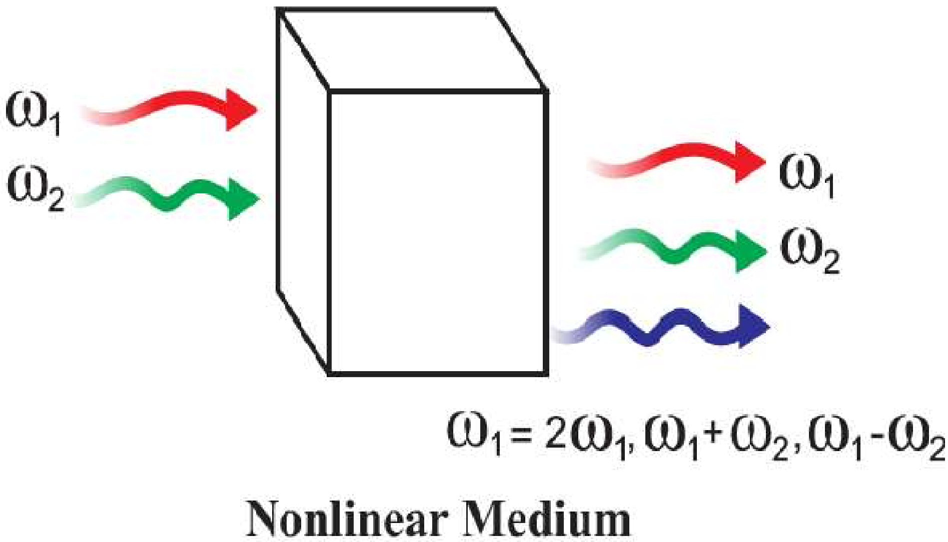

Fig.1.1: Frequency mixing in nonlinear media

The researcher promptly recognized that all these new optical phenomena could be explained if the polarization of the medium under a laser beam were expanded in power series of electric fields as described below:

$$P = \epsilon_0(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \dots\dots\dots\dots) \quad (1.1)$$

In this context $\chi^{(1)}$ represents the linear susceptibility and $\chi^{(2)}, \chi^{(3)}$ represent nonlinear susceptibilities. The above equation is valid for one dimensional case i.e., polarized electrons undergo oscillations aligned with the direction of the propagating light wave. When an electron is moved from its equilibrium position within the dielectric lattice, it will experience restoring forces as well as forces from the neighboring molecule. (fig. 1.2).

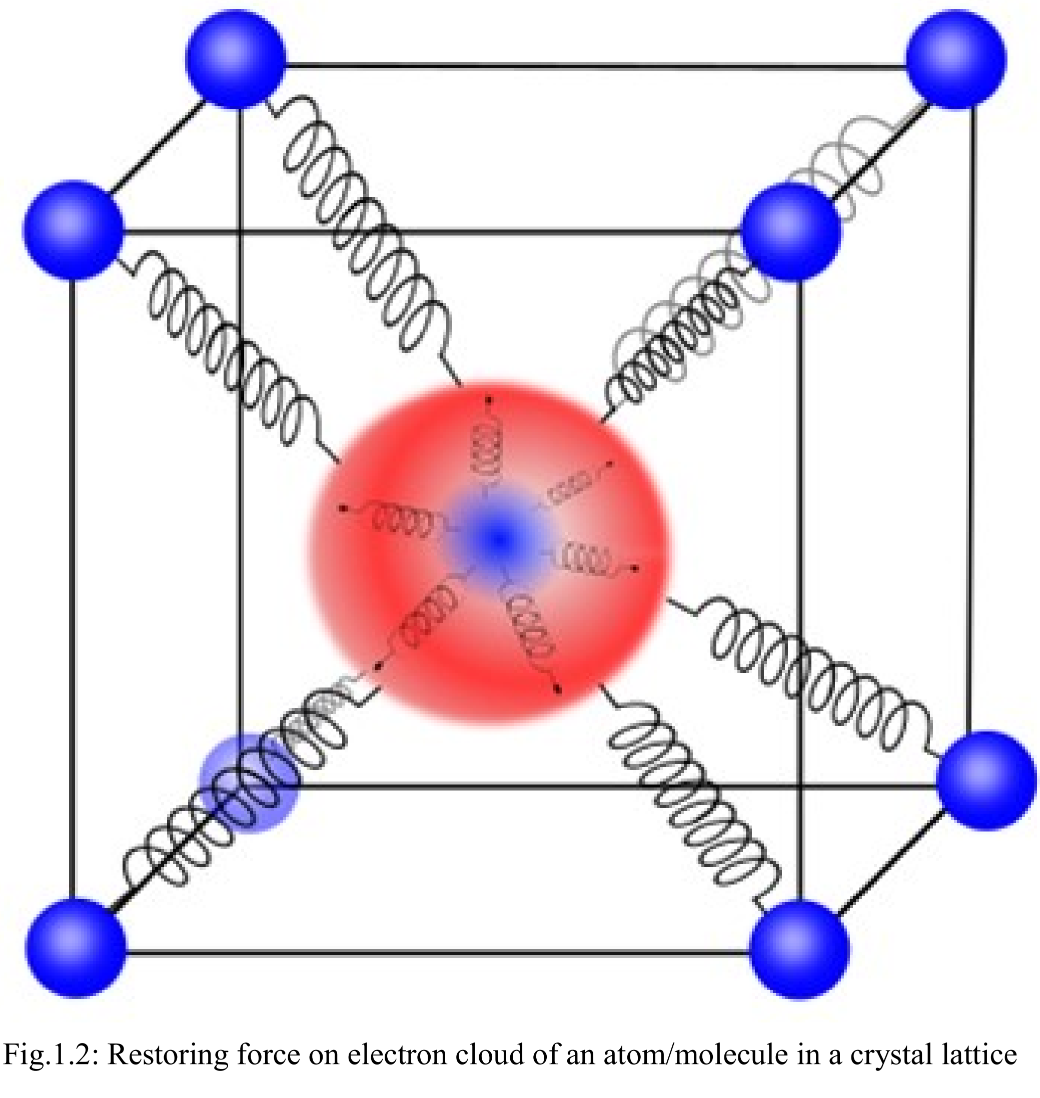

Fig.1.2: Restoring force on electron cloud of an atom/molecule in a crystal lattice

When a field is applied in the x direction, an electron may also move in y and z directions. (fig. 1.3).

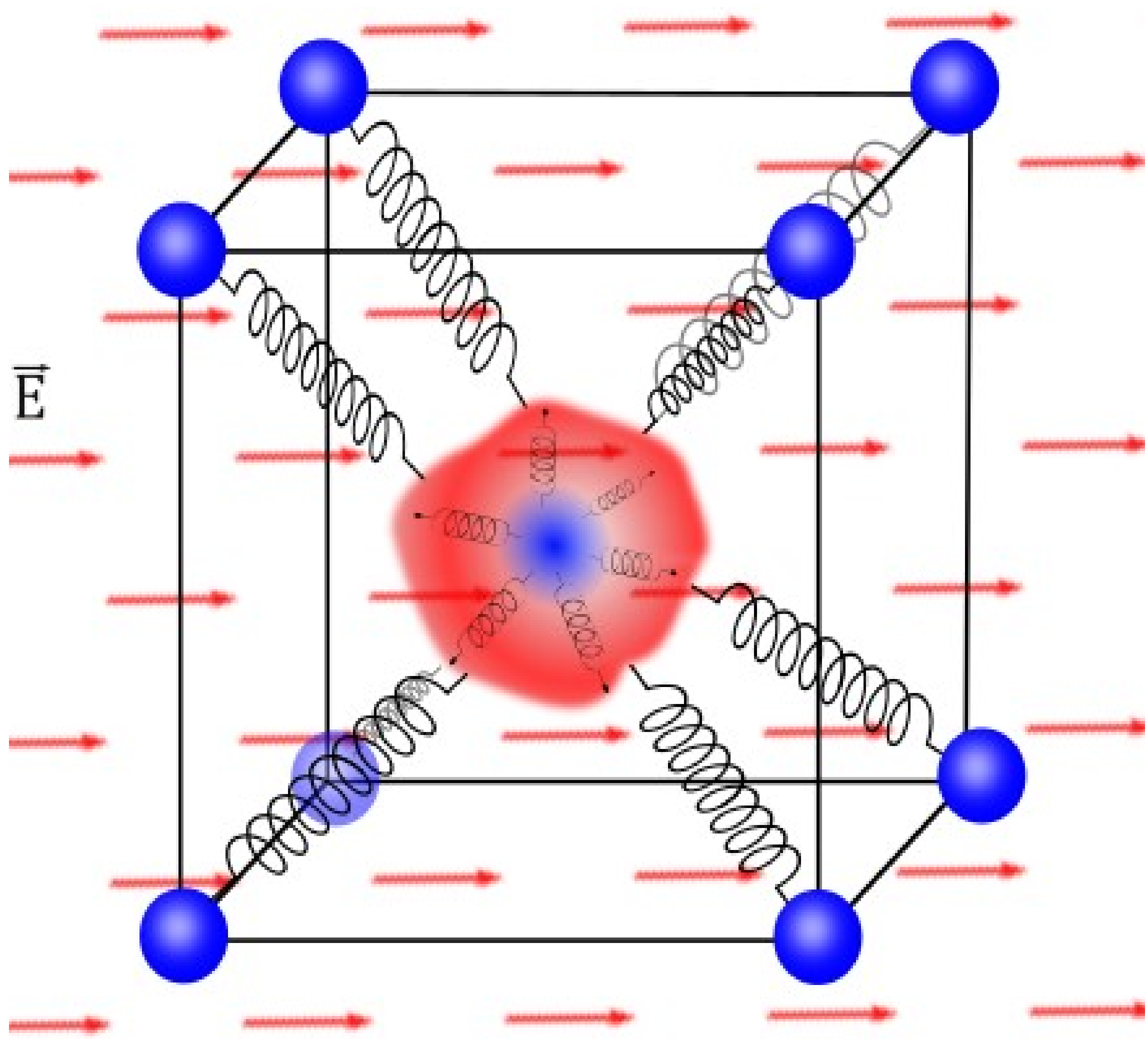

Fig.1.3: Tensor nature of nonlinear susceptibility

Crystal structure strongly influences the strength and direction of these forces. Thus, the susceptibility of an anisotropic medium is in general a tensor[7] and hence Eq. (1.1) can be written in more generalized way as

$$\mathbf{P} = \epsilon_0 \sum_{i=1}^{3} \boldsymbol{x_i} \sum_j \chi_{ij}^{(1)} E_j + \sum_{jk} \chi_{ijk}^{(2)} E_j E_k + \sum_{jkl} \chi_{ijkl}^{(3)} E_j E_k E_l + \dots \dots \dots \quad (1.2)$$

$\chi^{(n)}$=Tensor of rank $n + 1$ having 3^{n+1} terms. Thus,

$\chi^{(1)}$ ≡9 terms

$\chi^{(2)}$ ≡27 terms

$\chi^{(3)}$ ≡81 terms

$\chi^{(2)} \equiv \chi_{ijk}$ relates each of the three components of the polarization $P = (P_x, P_y, P_z)$ to the nine products of the two applied light fields, for a total 27 terms.

By substituting Eq. (1.2) into Maxwell's equations, a system of nonlinear differential equations emerges incorporating electric field strength of high-order. These terms play a crucial role in facilitating coherent optical frequency mixing. Based on Eq. (1.2), it can also be derived that upon the action of laser radiation, the index of refraction of a medium is no longer a constant even at a given wavelength, instead there will be an induced refractive-index change, depending on the intensity (in a third-order nonlinear medium) or the amplitude (in a second order nonlinear medium) of the incident laser beam(s). This nonlinear assumption can be used to well explain the observed self-focusing effect[8-10] of a laser beam and some other related nonlinear optical effects.

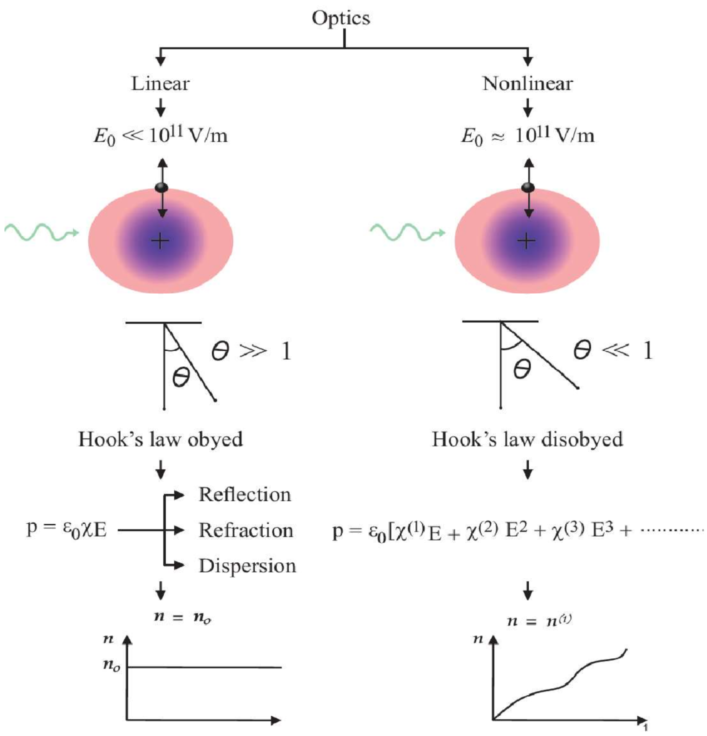

Fig.1.4: Linear vs. nonlinear optics

The light and matter that interact in a nonlinear manner result in myriad optical effects. This span a gamut from excitation of frequency overtones of laser beams to several stimulated scattering phenomenon such as Stimulated Brillouin Scattering (SBS), Stimulated Raman Scattering (SRS) and others.

1.2 Scattering

Scattering of light refers deflection in its direction of propagation to random directions when it encounters a material object[11-14]. It takes place due to variations in the refractive properties of the medium or due to interaction of light with bosonic excitations of the medium. Like emission of light due to electronic transitions can be spontaneous or stimulated depending upon whether it occurs by its own or is triggered by external photon, scattering of light can also be spontaneous or stimulated[15].

Fig.1.5: Scattering of light

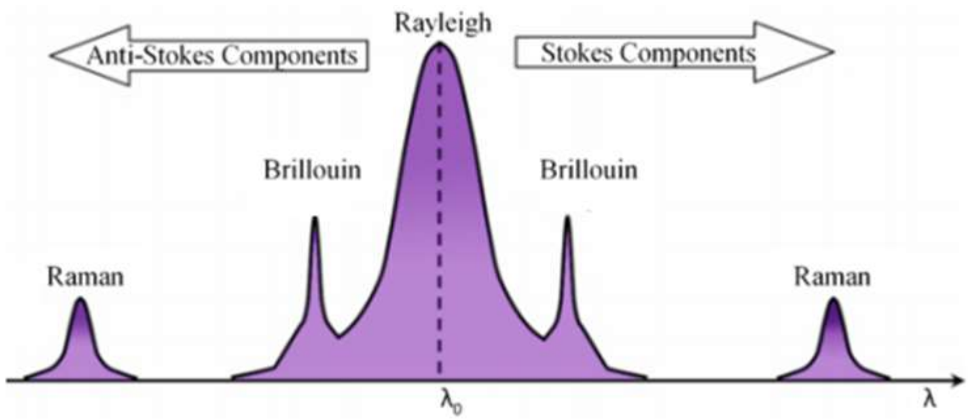

Fig.1.6: Spectrum of scattering

Scattering of light is said to be spontaneous if the medium's optical properties remain unaffected during the propagation of light i.e., the bosonic excitations of the medium are neither damped nor amplified due to their interaction with light. In case of stimulated scattering, the medium's optical properties get modified due to presence of light. In other words in the stimulated version of scattering the bosonic excitations of the medium get amplified. Stimulated scattering differs from spontaneous scattering in several other aspects like:

1. It is observed at high intensity i.e., there exists a certain threshold intensity below which scattering will be spontaneous.
2. It occurs for radiations with very narrow spectral width i.e., for quasi monochromatic radiations.
3. It is highly coherent process in contrast to spontaneous scattering which is totally non coherent.
4. It does not involve any anti stoke's component.

Depending on the kind of bosonic excitation due to which scattering is occurring, there are number of different scattering processes like Rayleigh scattering, Rayleigh wing scattering, Raman scattering, Brillouin scattering etc.

1. Raman Scattering[16]: Scattering due to molecular vibrations or optical phonons.
2. Brillouin Scattering[17]: Scattering due to lattice vibrations or acoustical phonons.
3. Rayleigh Scattering[18]: Scattering due to propagating density fluctuations or entropy fluctuations.
4. Rayleigh Wing Scattering[19]: A scattering due to an anisotropic molecule's fluctuating orientation.

1.3 Origin of scattering

Scattering of light occurs as result of variations in the optical properties (index of refraction, dielectric function, susceptibility). In purely homogeneous media scattering of light cannot occur. This fact can be explained as follows:

Consider a purely homogeneous medium which is illuminated by a plane wave. Let the volume element dV_1 scatters light into the direction θ. In homogeneous media, one can always locate a nearby volume element dV_2 whose scattered light interferes destructively with the scattered light from volume element dV_1. We can conclude that for completely homogeneous media, light will not scatter since this argument applies to any volume element in the medium. Hence, scattering of light occurs due to variation in the optical properties of medium. E.g. If the density of the medium is not uniform then there can be a possibility for the total number of molecules in a volume element dV_1 not to equal the total number of molecules in dV_2, which will result in net scattered light because the destructive interference between the fields scattered by these two elements cannot exactly cancel each other.

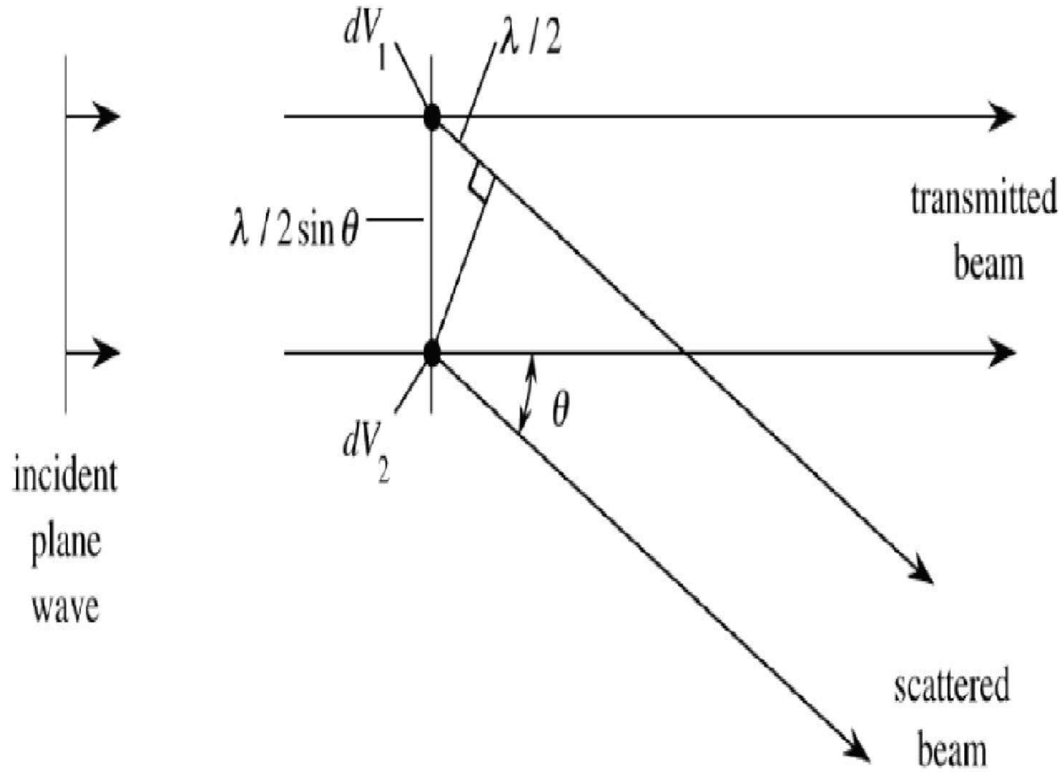

Fig.1.7: Origin of scattering

1.4 Scalar and tensor scatterings

As the scattering of light results from changes in the optical properties of material medium, the dielectric tensor of scattering medium can be written as

$$\epsilon_{ik} = \bar{\epsilon}\ \delta_{ik} + \Delta\epsilon_{ik}$$

where, $\bar{\epsilon}$ represents the average dielectric constant of the medium, $\Delta\epsilon_{ik}$ is the spatial or temporal variation in the dielectric tensor.

It is convenient to decompose fluctuations $\Delta\epsilon_{ik}$ into scalar and tensor contributions as

$\Delta\epsilon_{ik}$=$\Delta\epsilon\ \delta_{ik} + \Delta\epsilon_{ik}^{\mathrm{t}}$

1.4.1 Scalar Scattering

The scalar contribution $\Delta\epsilon$ results due to variation in thermodynamic quantities such as density or temperature, pressure, entropy. Scattering of light that results from $\Delta\epsilon$ is called scalar scattering e.g., Brillouin scattering, Rayleigh scattering.

1.4.2 Tensor Scattering

Scattering that result from $\Delta\epsilon_{ik}^{t}$ is called tensor light scattering. It is further convenient to express $\Delta\epsilon_{ik}^{t}$ as sum of symmetric and anti symmetric parts as

$$\Delta\epsilon_{ik}^{t} = \Delta\epsilon_{ik}^{s} + \Delta\epsilon_{ik}^{a}$$

Where $\Delta\epsilon_{ik}^{s} = \Delta\epsilon_{ki}^{s}$ ⟶Rayleigh Wing scattering and

$\Delta\epsilon_{ik}^{a} = -\Delta\epsilon_{ki}^{a}$ → Raman scattering

1.5 Brillouin scattering of light

In 1915, Louis Brillouin suggested that light travelling through a material medium must undergo slight changes in wavelength as the result of encounter with the high frequency sound waves that arise from the ordinary thermal vibrations of atoms in the material. The sound waves from the vibrating atoms fan out in all directions, and since the vibrations may vary considerably in frequency, the waves have a wide range of frequencies up into the infrared. Like other sound waves, they consist of alternate compressions and expansions in the direction of the wave propagation. Since the velocity of such a wave is much slower than that of a light wave, it follows that the sound wave is much shorter than an electromagnetic wave of the same frequency. Hence a wave of visible light passing through a liquid in the same direction as a sound wave of the same frequency will encounter regularly spaced maximal compressions and expansions of the sound wave. A part of the light will be reflected from each sound wave crest, and if the spacing between the crests is just half the wavelength of the light, the reflections will add in phase to produce an appreciable amount of light. Brillouin scattering is analogous to Doppler Effect. The frequency of the scattered light is shifted up or down depending on whether the sound wave is approaching towards the incident light wave or propagating in the same direction as that of the light wave.

The formula for Brillouin scattering makes it possible to measure the velocity of sound at various wavelengths in any liquid and thus to study important properties of materials. Observation of scattering is so difficult with ordinary light sources; however, that very little work was done along this line before the laser became available. With a laser beam thanks to monochromaticity and directionality, it is now relatively easy to observe Brillouin scattering.

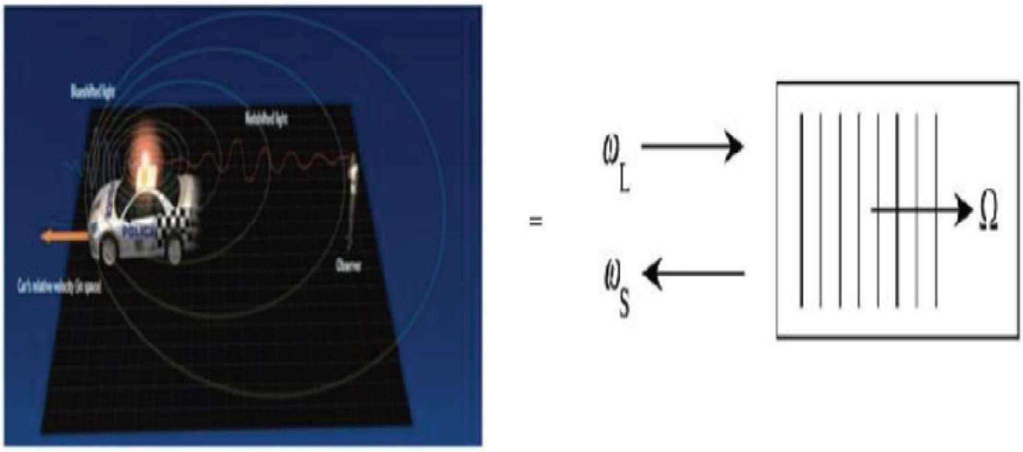

Fig.1.8: Equivalence of Doppler Effect and Brillouin scattering

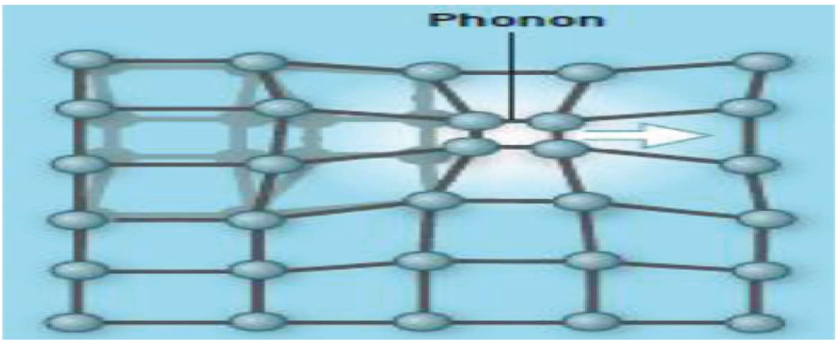

Fig.1.9: Acoustical phonon

By bringing together the incident laser beam and scattered light on the cathode of a phototube, it is possible to measure extremely minute frequency shifts in the scattered light. The phototube current shows a beat signal at the difference frequency, even when the shift amounts to only a few cycles per second.

From the perspective of quantum mechanics, Brillouin scattering refers to the scattering of light caused by acoustical phonons.

1.6 Stimulated Brillouin Scattering

In stimulated Brillouin scattering, the material is not subjected to an external sound wave or pressure–density variation. Instead, the stimulation occurs internally through a pair of light waves propagating in opposite directions. Any medium traversed by light is subjected to strong mechanical forces caused by the high electric fields created by a high-power laser. These forces are known as electrostrictive forces and exhibit proportionality to the square of the electric field. The electrostrictive force causes compression in a medium that exhibits isotropic properties and another medium appears to have a nearly compressive electrostrictive force. When electric field reverses, it remains unchanged in sign as it is proportional to the square of the electric field. In other words there is consistent compressive force exerted on the medium which is proportional to the intensity of the light. Approximately one dyne per kilowatt is estimated to be the magnitude of this force. Consequently, a light flux of 10^8 watts per square centimeter leads to an electrostrictive pressure of approximately $10^5 \mathrm{dyne/cm^2}$, which is equivalent to around $100\,\mathrm{g/cm^2}$ in magnitude. A change in light intensity is accompanied by a change in electrostrictive pressure.

The generation of beats between two distinct light waves is achieved by using the time coherence property of laser. The light frequency is exceedingly high, approximately 10^{14} cycles per second so that when the various frequencies of light waves differ by a very small fraction, the absolute frequency difference is large. Thus, beats will be produced by two light signals of frequencies ν_1 and ν_2 propagating through the medium simultaneously. The instantaneous intensity at a given point will fluctuate; exhibiting rises and falls at the difference frequency ν_1 - ν_2. At this difference frequency, there will be an electrostrictive force on the medium, and it can set up vibration in the medium.

When the difference ν_1 - ν_2 is of the same order of magnitude as the frequencies at which sound waves can propagate through the medium, electrostrictive forces can give rise to intense sound waves. The production of sound waves is feasible at any frequency within the range where sound can propagate including above microwave frequencies. During propagation of an intense laser beam of frequency ν_1 through a medium, it can be scattered by a preexisting thermally generated sound wave of frequency ν_s. As a result, another frequency $\nu_2 = \nu_1 - \nu_s$ of light will be created. The interaction of highly intense incident light results in combination of scattered and incident light, generating an electrostrictive force at the difference frequency $\nu_s = \nu_1 - \nu_2$. As a result the existing sound wave within the medium undergoes amplification, and this leads to further scattering of light. Such a scattering of light is known as Stimulated Brillouin Scattering (SBS). In solids, the accompanying sound waves can be so large that the crystal is broken. This provides a mean of inducing and examining the behavior of high frequency sound waves in practically any light transmitting material. The experimental arrangement for SBS is shown below:

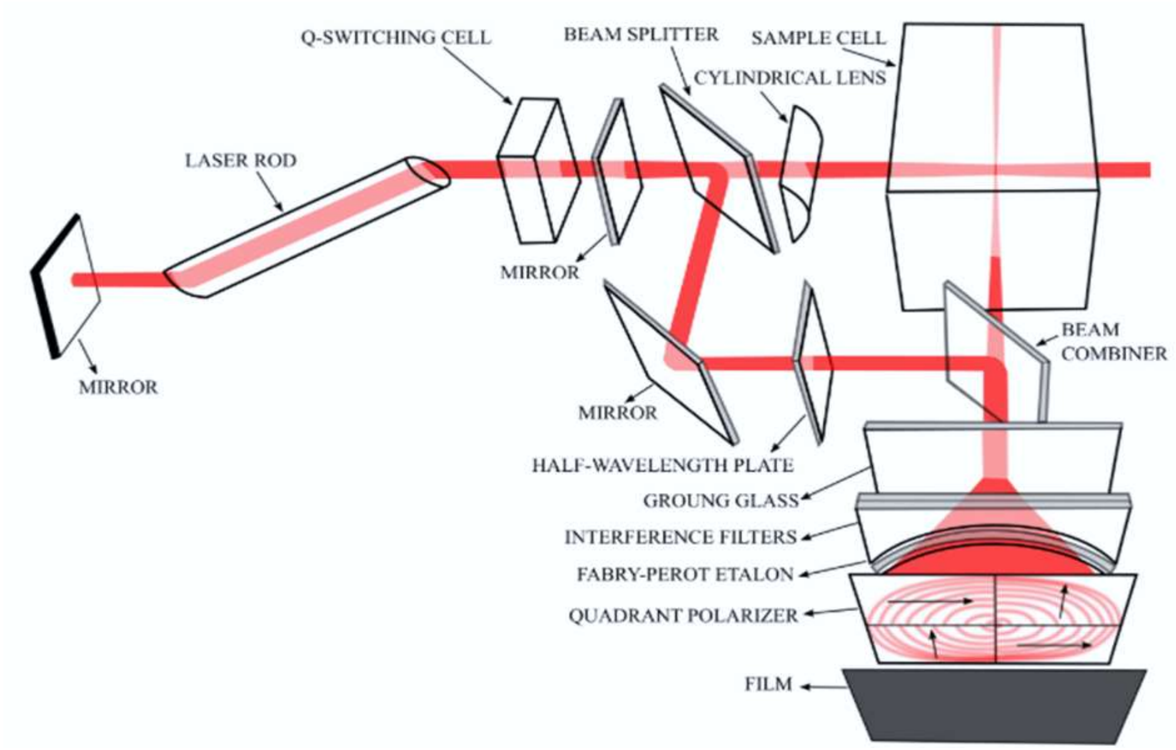

Fig.1.10: Experimental arrangement for SBS

1.7 Applications of Stimulated Brillouin Scattering

1.7.1 Optical Phase Conjugation

Imagine an athlete standing on a diving board preparing for the high jump. Run fast and fly. However, a slight technical error results in the body entering the water an incorrect angle, creating a large splash and spreading waves from the point of contact. How cool would it be if you could go back in time, fix your mistakes, and score high? The jet of water converges, the waves return to the point of contact, and the diver is pushed out of the water onto the platform, leaving the surface as smooth as it was before the dive. Unfortunately, although such scenarios can easily be viewed using a movie projector. The observations of time reversal disprove our common everyday perception. It contradicts the second law of thermodynamics, which states that systems naturally move towards maximum entropy.

Nonetheless if the actor is a light wave or any other type of electromagnetic radiation, the scenario can be executed successfully. Such phenomena are made possible by a striking and well-known property of light rays: the reversibility of light. For each individual light beam characterized by specific arrangement of rays, there is a existence of a corresponding “time-reversed" beam in which the rays follows the identical trajectory however in the opposite direction, analogous to rewinding film. The achievement of wave motion reversal is attributed to an exceptionally simplified treatment of the problem. There is no need to reverse the quantum mechanical or thermal motion of atoms or electrons that emit or refract light. For practical purposes, the inverse of the temporal behavior of macroscopic parameters that describe the average motion of a substantial number of particles is sufficient.

The existence of reversed beams has significant implications. For example, it is evident that an ideal beam, or one that is free of distortion and has minimum divergence, can be degraded by transmitting it through inhomogeneities, such as a glass plate of non uniform thickness. The property of reversibility means it is possible to create an "anti-distorted" beam that becomes ideal after being transmitted backward through the inhomogeneities. The technology by which such beams are created and manipulated is called optical phase conjugation. The waves making up such a beam are called phase-conjugate waves.

Optical phase conjugation is employed to eliminate aberrations effects from specific types of optical systems. The characteristics of phase conjugation process can be understood by comparing the reflection of an optical beam from an ordinary metallic mirror with that from Phase Conjugated Mirror (PCM) (fig. 1.11). It can be seen that in case of ordinary metallic mirror the most advanced section of the incident wavefront is reflected as most advanced section. However, in case of reflection from PCM the most advanced section of the incident wavefront is reflected as most retarded section.

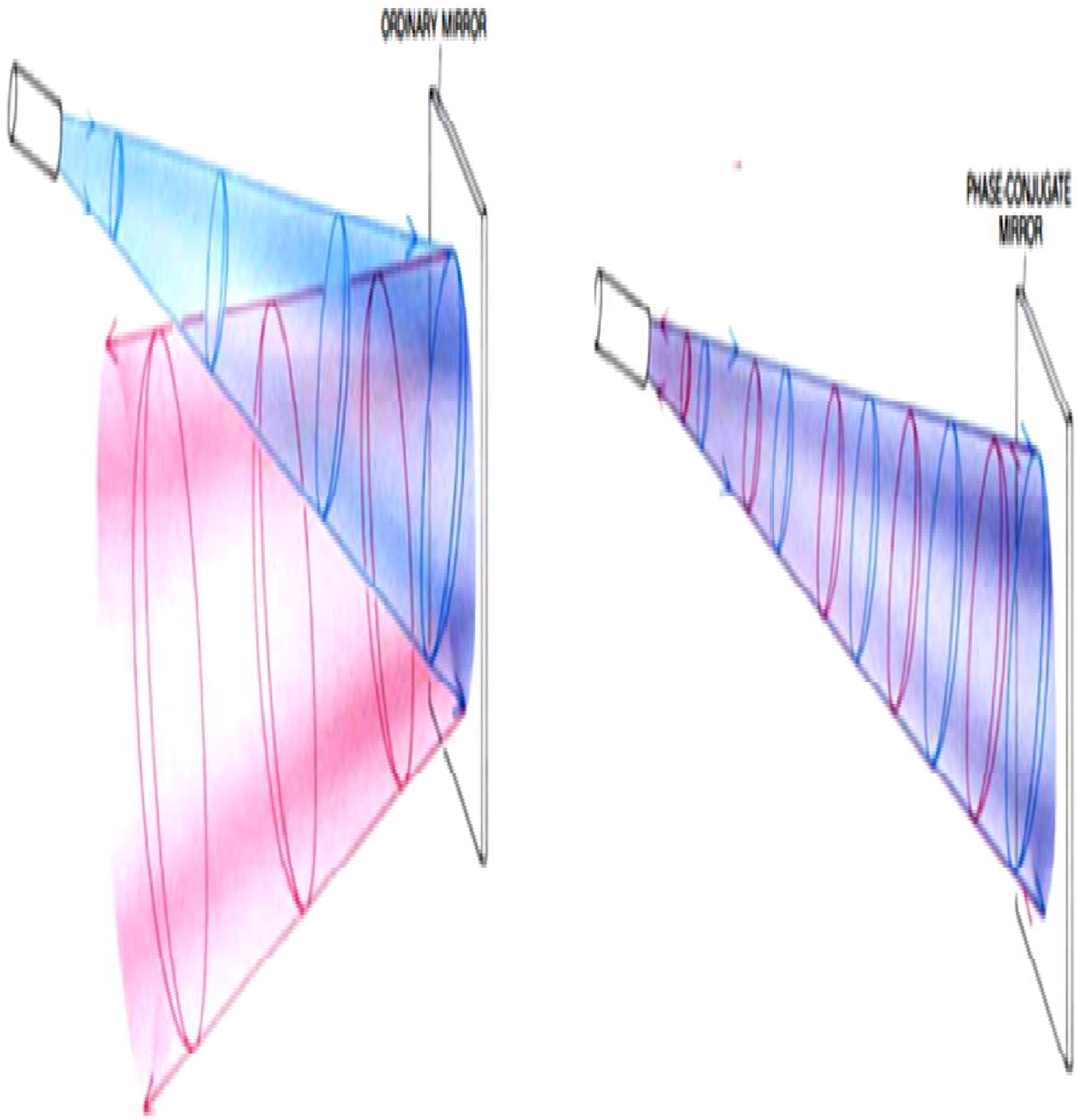

Fig.1.11: Ordinary mirror vs. phase conjugate mirror

1.7.2 Production of Phase Conjugation by SBS

It is easy to generate the phase conjugation of a plane wave. For this one simply need to mount a plane mirror so that the wave is reflected precisely backward. It is not much more difficult to conjugate a wave that has a spherical wavefront. A concave mirror in the shape of a section of a sphere would be mounted so that the center of the mirror corresponds to the source of the wave. Then at every point on the mirror the rays would be incident perpendicularly to the surface and would be reflected precisely backward.

To conjugate a beam that has an arbitrary wavefront, position a mirror whose profile coincides with that of the wavefront. Unfortunately such a method is difficult to realize in practice due to following reasons:

1. One would have to make a new mirror for each particular incident beam.
2. Even the shape of a wavefront of a laser beam can change during a brief pulse; one would therefore have to change the shape of the mirror continuously to match the changing shape of the wave.
3. The precision required for prepare and position such a mirror would be extremely high.

To produce a phase conjugate wave, a medium or surface is required whose properties are affected by the characteristics of waves that are incident on it. That dependence allows the medium or surface to adjust itself with such delicate correspondence to the structure of the incoming beam that a reflected phase conjugate beam is produced under certain conditions. Fortunately such materials do exist. They are termed as optically nonlinear media.

Two widely used methods of optical phase conjugation that rely on such nonlinear optical media are:

1. Four wave mixing
2. Stimulated Brillouin scattering

The main trick that allowed phase conjugation to take place was the use of a special glass plate that had been made non uniform by etching with hydrofluoric acid. A beam of red light from a pulsed ruby laser was distorted by being passed through the plate. The distorted beam was directed into a pipe one meter long, four millimeters wide and four millimeters high that had been filled with gaseous methane at a pressure of 140 atmospheres. Stimulated

Brillouin scattering occurred in the pipe, and the reflected beam, when it was passed backward through the same hedplate etc., emerged undistorted. That is to say, its structure was identical with that of the incident beam.

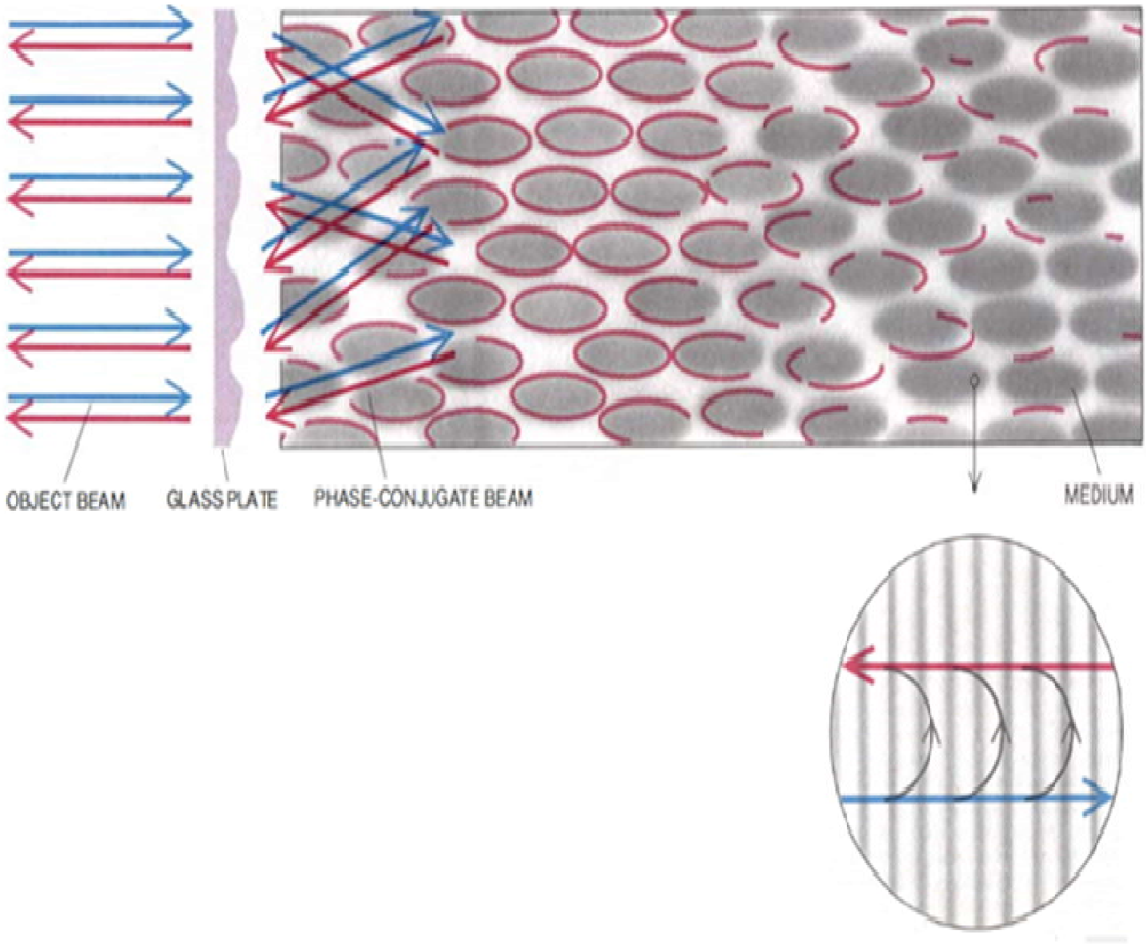

Fig.1.12: Phase conjugation by SBS

Phase conjugation by stimulated scattering has now been realized in a large number of scattering mediums and many types of lasers. The main advantage of the technique is that it only requires a cell filled with an appropriate gas, liquid or solid.

1.7.3 The Phonon Maser

Acoustic waves adhere to equations that share a similar general form as those governing light waves, thereby demonstrating various analogous phenomena. An acoustic wave has the capability to induce excitation of atoms or molecules and can obtain energy from it through either spontaneous or stimulated emission. Therefore, if molecules are suitably coupled to an acoustic field and the excitation meets the threshold condition, masers can effectively stimulate the generation of acoustic waves. The initial proposed systems in this context included spin states inversion of impurities within a crystal, employing methods analogous to those utilized in solid-state electromagnetic masers. It seems that Brillouin scattering and its closely related counterpart, Raman scattering appears to be more generally applicable techniques that rely on phase correlation instead of population inversion to generate amplification. Parametric amplification can also be regarded as an alternative perspective on this process.

Like a grating, an acoustic wave can scatter light by its train of crests and troughs. A Doppler shift occurs because the wave is moving. Brillouin was first to analyze, scattered light frequency is shifted from the original beam frequency ν_0 by certain amount

$$\nu = 2\,\nu_0 \frac{v}{c} \sin\frac{\theta}{2} \tag{1.3}$$

Here c and v denote the phase velocities of light and sound, respectively within the medium while θ represents angle of scattering. The energy $h\nu$ that is lost is transferred to acoustic wave involved in scattering, which posses a frequency of ν. In the event that the light possesses ample intensity, it has the capability to impart energy to the acoustic field at a rate that exceeds its rate of loss. This enables the fulfillment of a threshold condition, resulting in the steady accumulation of acoustic energy.

Typically, achieving significant amplification using regular light is challenging due to the substantial losses incurred at the high acoustic frequencies (10^9 to 10^{10}cy/sec) described by Eq. (1.3), especially when θ is not significantly small. But, it is possible to produce highly intense acoustic wave by SBS that can shatter glass or quartz by laser beams with power intensities of range of hundreds of megawatts per square centimeter. As a result, extremely high-frequency acoustic waves can be generated and studied almost in any material where there is light transmission, a possibility not previously possible.

It has been studied for some time that Brillouin scattering is caused by spontaneous emission. With the advent of highly intense and monochromatic laser light, this technique can now be employed with enhanced precision. Consequently, it provides valuable insights into the propagation of hypersonic waves in materials, yielding interesting information in the process. Despite Eq.1.3 that put a kind of limit, for $\theta = \pi$, of $2\,\nu_0 \frac{v}{c}$, stimulated emission has no fixed limit on the acoustic frequencies it can produce. However, within the optical branch of acoustic waves, the phase velocity ν can attain exceptionally high values. The Raman maser, or stimulated Raman scattering, is an optical branch of an acoustic spectrum that generates coherent molecular oscillations. It is thus possible to generate and investigate a significant portion of both the acoustic spectrum and electromagnetic spectrum using quantum-electronic techniques.

1.8 SBS in Plasmas

Plasma as a whole maintains quasineutrality; however, due to the separation of electrons and positively charged ions, disturbances can give rise to regions with net negative and net positive charges, resembling the plates of a charged parallel plate capacitor. The uneven distribution of charges in such cases set up an electric field from the positive regions to the negative regions. The presence of this electric field exerts equal forces that attract the electrons and ions towards each other. As a result of this attractive force electrons and ions start moving towards each other. While the electrons and ions approach each other, their velocity and momentum gradually increase, analogous to a pendulum moving from an extreme position towards its mean position. The increase in momentum causes the electrons and ions to surpass their equilibrium positions, leading to a reversal in the direction of the electric field. With the electric field now reversed, it acts in opposition to the motions of electrons and ions, causing them to decelerate and eventually be pulled back in the opposite direction. The process continues to repeat itself in a cyclic manner, creating an electron-ion oscillator. When considering the influence of thermal velocity, these electron-ion oscillators give rise to a longitudinal wave that alternately compresses and rarifies the density of the plasma (fig. 1.14).

These density fluctuations associated with IAW act like acoustical phonons and thus scatter the incident laser beam through the process of SBS.

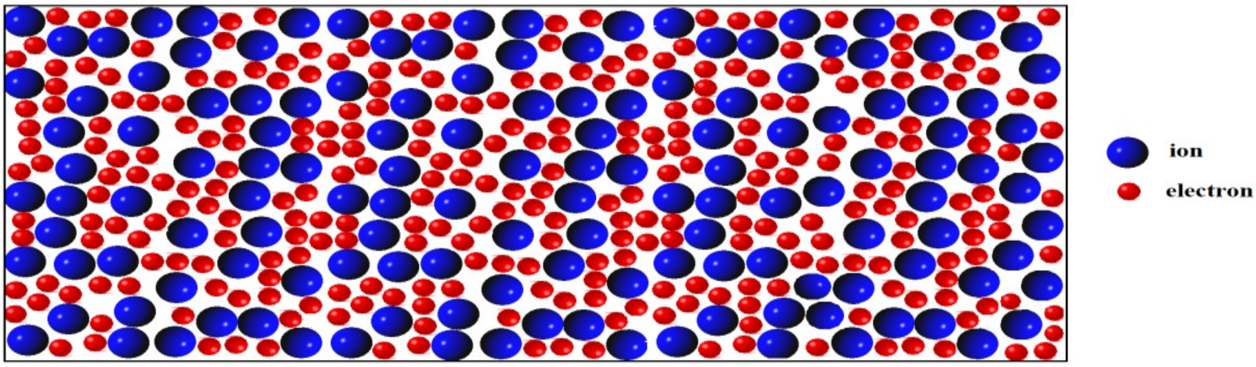

Fig.1.13: Plasma

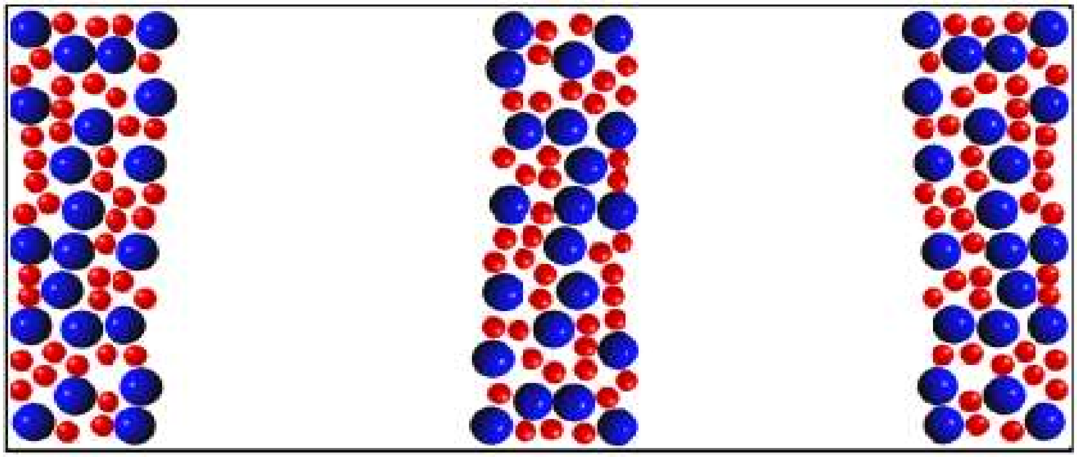

Fig.1.14: Ion acoustic wave in plasma

1.9 Objectives

1. To study excitation of ion acoustic waves self focused q-Gaussian laser beams in relativistic plasmas with axial density ramp.
2. To study Stimulated Brillouin scattering of q-Gaussian laser beams in axially inhomogeneous plasmas.
3. Self action effects of quadruple Gaussian laser beams in thermal quantum plasmas.
4. Stimulated Brillouin scattering of self focused quadruple Gaussian laser beams in thermal quantum plasmas

Chapter- 2
Literature Review

2.1 Introduction

This chapter unravels the contribution of different researchers in the field of laser-matter interactions.

2.2 Review of Literature

1. **Y. P. Nayyar et al. (1977)[20]** investigated that in the extra-paraxial region, beam of laser is self-focused and defocused. In this paper, they have studied that in the extra paraxial region, transverse Gaussian (TEM_{00} mode) and TEM_{10} laser beams are self-focused/defocused. Gaussian mode shows self-focusing for axial points while for off-axial point, self defocusing takes place. For the TEM_{10} mode, the reverse effect has been observed.
2. **V. S. Soni et al. (1980)[21]** presented that in a collisionless magnetoplasma, elliptical beam of laser is self-trapped and self-focused. It has been observed that for the extraordinary mode, the critical power is half the critical power of self-trapping for ordinary mode.
3. **R. N. Agarwal et al. (1996)[22]** studied that self guided high power beam of laser while transmitting through plasma creates channel of low density. The author studied phenomenon of Stimulated Brillouin Scattering (SBS) of laser in plasma medium. The growth rate is decreased because of geometrical factor according to nonlocal theory of SBS.
4. **W. Jinsong (1996)[23]** studied that density fluctuation in absorption media causes stimulated brillouin scattering of thermally generated acoustic wave. The author predicted the threshold for the phenomenon of SBS generation and the line width of stokes radiation by utilizing the Fourier transform.
5. **A. V. Maximov et al. (2004)[24]** investigated inertial confinement fusion targets with direct-drive inertial confinement that have been modeled as nonlinear laser beam propagation at the critical density surface, smoothed by spatial and temporal bandwidth.

6. **J. Myatt et al. (2004)[25]** developed the scheme for simulating LPI using the LPI codepF3D, in the underdense corona of direct drive targets where critical density surface is always present and the plasma is extremely inhomogeneous.
7. **L. Deng et al. (2005)[26]** used Fresnel-Kirchhoff diffraction theory to study the emergence of far-field patterns of Gaussian profile of laser propagating through medium exhibiting the phenomenon of self-focusing/defocusing. Pattern formation was studied using various Gaussian profile and various Kerr media.
8. **B. Eliasson et al. (2005)[27]** purposed that in plasmas, whistlers are linearly self-focused. The analytical results of the wave model are compared with the direct simulation of the whistler wave equation. Numerical results describe the frequency-modulated whistlers and their rapid localization within plasmas.
9. **P. Jha et al. (2006)[28]** studied the dynamics of laser in underdense, cold, transversely magnetized plasma. The author represented that perpendicular magnetization of plasma increases the self-focusing of laser beam inside plasma and calculated the critical power needed for self focusing of laser.
10. **Amrita et al. (2006)[29]** studied progression of Gaussian laser beam inside the collisional plasma causes non uniform heating of electrons that causes self-focusing of beam of laser**.** For the axial points, temperature is maximum and decreases rapidly for the off axial points.
11. **S. P. Singh et al. (2007)[30]** studied the nonlinear effects in fiber optics. Nonlinearity in optical fiber arises due to dependence of intensity on refractive index and inelastic scattering phenomenon. The author discussed self-phase modulation, cross-phase modulation and four-wave mixing nonlinear effects in optical fiber.
12. **J. Parashar (2009)[31]** studied that propagation of highly intense Gaussian laser beam through gas containing atomic clusters generate nano plasma medium that assist self focusing of beam of laser. The impact of self focusing on laser third harmonic generation has been studied. Due to nonlinear response of electrons, laser beam generate third harmonic.
13. **K. I. Hassoon et al. (2010)[32]** investigated the self-focusing of high intense laser by relativistic effects guiding in a plasma perpendicular to the ambient magnetic field using the slab model.

14. **R. Singh et al. (2010)[33]** investigated ponderomotive acceleration of electrons caused by short self focused laser by relativistic means. Recurrent self-focusing of laser occurs due to saturation in nonlinear plasma permittivity. The periodic lengths vary across different axial segments of the pulse, leading to distortion in the pulse shape.
15. **D. Tripathi et al. (2010)[34]** explained relaxing ponderomotive nonlinearity to study the nonstationary self-focusing of a Gaussian laser in plasma. The wave equation was work out by paraxial ray approximation method.
16. **N. Kant et al. (2011)[35]** analyzed the effect of ponderomotive force on self-focusing of laser in underdense plasma with upward ramp of density. During propagation of laser inside the plasma, the spot size show oscillatory behavior. The study showed defocusing can be minimized with upward ramp of density in plasma.
17. **N. Kant et al. (2012)[36]** studied how hermite–Gaussian beams of laser are self-focussed in plasma medium having upward density ramp transition. The spot size varies harmonically with propagation distance. Paraxial ray approximation was used to find the equations for beam width of laser beam.
18. **S.V. Chekalin et al. (2013)[37]** studied in general terms, similarities and dissimilarities between the phenomena of laser beam self-focusing and laser pulse filamentation. Their studies describe the current status of nonlinear optics and laser physics.
19. **V. Nanda et al. (2013)[38]** described how Hermite-cosh Gaussian beam of laser is self focused in a magnetoplasma with density ramp profile. Hermite-cosh-Gaussian beam of laser is early and strongly self-focussed in magnetoplasma with density ramp.
20. **P. Sprangle et al. (2014)[39]** studied interaction and propagation of high intense and high power laser. This paper gives overview of the number of applications and processes associated with high-intensity, high-power lasers, and their interactions.
21. **M. Aggarwal et al. (2014)[40]** studied the propagation of a cosh Gaussian beam of laser in plasma having density ripple using the paraxial approximation. The variation of refractive index because of relativistic interaction of laser-plasma and self focusing of beam of laser by ponderomotive force has been considered.
22. **V. Nanda et al. (2014)[41]** studied that in plasma with density transition, enhanced self-focusing by relativistic means occurs for Hermite-cosh-Gaussian beam of laser. Using

WKB approximation and paraxial ray approach, the equation of beam width parameter have been derived.

23. **P. Sati et al. (2014)[42]** studied frequency broadening and self-focusing pulse of laser in water. An analytical model is developed for the avalanche breakdown of water by finite spot size, intense short pulse of laser.

24. **M. Aggarwal et al. (2015)[43]** studied the dynamics of Quadruple Gaussian in inhomogeneous and non-linear plasmas using paraxial approximation and Wentzel-Kramers-Brillouin approximation. The impact of magnetic field is studied on self-focusing of laser beam in plasma.

25. **N. Kant et al. (2015)[44]** purposed dynamics of cosh-Gaussian beam of laser in plasma with linear absorption and density transition ramp by paraxial approximation method. The density ramp of plasma and non-linear absorption play significant role in self focusing of laser beam inside plasma.

26. **R. P. Sharma et al. (2015)[45]** proposes numerical simulation to explain the phenomenon of self focusing of laser inside plasma. The coupled equations describing the propagation of ion acoustic wave and laser in collisionless plasma are presented by numerical simulation. The effect of perturbation number is studied on nonlinear propagation of laser.

27. **A. Singh et al. (2015)[46]** investigated that in preformed parabolic channel of plasma, second harmonic is generated by relativistically self-focused q-Gaussian beam of laser. Moment theory approach was used to obtain differential equation for evolution of spot size of laser with respect to propagation distance.

28. **A. Singh et al. (2015)[47]** investigated in collisionless plasma; second harmonic is generated by self focusing of cosh-Gaussian beam of laser. The differential equation governing the evolution of spot size of ChG beam of laser in collisionless plasma has been derived using moment theory.

29. **R. S. Craxton et al. (2015)[48]** presented a review paper on direct-drive inertial confinement fusion. The author identified problems demonstrated and solutions implemented by target-physics experiments that have evolved scientific understanding in many areas.

30. **C. Riconda et al. (2016)[49]** studied that for laser-plasma interactions relevant to inertial confinement fusion, Raman–Brillouin interplay is important. Based on extensive one-dimensional (1D) particle-in-cell (PIC) simulations across a variety of parameters, the present paper extends and complements previous work.
31. **M. A. Wani et al. (2016)[50]** studied that in linear absorbing collisional plasma; a chirped Gaussian laser beam undergoes self-focusing and defocusing. The chirp parameters and laser parameters greatly affect the spot size of laser beam. The chirp parameters minimize the divergence of laser beam and assist in self-focusing of laser beam inside plasma.
32. **E. Garmire (2017)[51]** studied the important development that have led to today's non linear optics field of stimulated Brillouin scattering are described in this perspectives about the earliest years of physics of stimulated Brillouin scattering. Controlling stimulated Brillouin scattering in fibers and exploiting its phase conjugation properties in both bulk media and fibers has enabled a wide range of applications.
33. **L. Devi et al. (2017)[52]** investigated the validity of paraxial theory for propagation of super-Gaussian laser in plasma. Formation of plasma takes place during the propagation of high intense laser through gas. Super-Gaussian beam is most intense on axis which causes plasma density and hence velocity of laser to be maximum on axis which in turns lead to defocusing of laser beam.
34. **E. S. Zyryanova (2018)[53]** studied the phenomenon of stimulated Brillouin scattering in optical fiber communication system. Phenomenon of stimulated Brillouin scattering occurs both in single and multimode and threshold value does not depends on modes.
35. **E. Garmire (2018)[54]** explained stimulated Brillouin scattering is ubiquitously used in optical systems today, including sensors, high-power lasers, scientific instruments microwave signal processors and opto mechanical systems. SBS is useful and practical technology in a range of applications for fifty years, beginning with its conceptual discovery 50 years ago.
36. **Z. Bai et al. (2018)[55]** reviewed materials for stimulated Brillouin scattering experimental design and uses. SBS materials, SBS application, experimental design and parameter optimization method are discussed in detail. In this article, this is expected to provide reference and guidance to SBS related experiments.

37. **S. A. Kozlov et al. (2018)[56]** studied that pulses with short periods do not self-focus. Author describes surprises in nonlinear optics for few-period waves, such as the vanishing self-focusing phenomenon of high-power radiation.
38. **M. Habibi et al. (2018)[57]** studied the propagation of laser beam inside plasma medium. During the propagation of high intense laser in plasma, the spot size of laser depends on plasma parameters and laser. Nonlinear phenomenon that occur during laser plasma interaction are discussed. Various types of phenomenon of self-focusing of laser are discussed.
39. **V. Nanda et al. (2018)[58]** compares the self-focusing of beam of laser in classical relativistic and relativistic cold quantum case for decentered parameter $b = 0.9$. Self-focusing is boosted with penetration of laser in relativistic cold quantum plasma.
40. **R. Borghi (2018)[59]** studied that in quantum mechanics, variational methods are typically presented as useful strategies for obtaining rough estimates of ground state energies. A brief list of well-known potential distributions in both 1D and 3D is provided in this paper.
41. **N. Kant et al. (2019)[60]** studied the dynamics of hermite cosh Gausssian in magnetoplasma having exponential slope of plasma density. The exponential slope of density of plasma causes stronger convergence of laser in comparison to tangential plasma density slope. Self focusing is stronger in high dense plasma with strong magnetic field.
42. **V. Thakur et al. (2019)[61]** studied that in a collisionless plasma with exponential slope of density, Hermite-cosine-Gaussian (HChG) is self-focussed by relativistic means. For different values of laser intensity, plasma density, and decentered parameters, self-trapping and self-focusing of Hermite-cosine-Gaussian (HChG) laser is investigated.
43. **K. Walia (2020)[62]** studied stimulated Brillouin scattering in unmagnetized plasma under the effect of ponderomotive force and relativistic motion of electrons. The interaction of high intense laser causes variation in mass of electrons that produces nonlinearity in plasma. Nonlinearity is also produced due to ponderomotive force. The equation governing the spot size of laser is solved using 4^{th} order RK method.
44. **H. K. Malik et al. (2020)[63]** studied the phenomenon of self-focusing by relativistic nonlinearity and shift in frequency of super-Gaussian beam of laser in plasma medium.

Interaction of high intense laser beam causes variation in mass of electron that in turn leads to self focusing of laser beam. The author investigated that frequency shift is increased due to self-focusing of laser beam.

45. **L. Devi et al. (2020)[64]** investigated the effect of magnetic field on lasers having same spot size. Super-Gaussian lasers are strongly self-focused under the action of stronger magnetic field. Laser beam with larger spot size is much better self-focused. Due to pinching effect of quasi-stationary magnetic field, earlier self focusing of high intense laser occurs.
46. **K. Virk (2021)[65]** reviewed a paper on self focusing of laser beam during its propagation inside plasma. Interaction of high intense laser in plasma causes change in its optical properties that produces non linearity in plasma that results in self focusing of laser inside plasma. Mechanisms of self focusing and second harmonic generation are discussed.
47. **R. J. J. Rioboo et al. (2021)[66]** studied Brillouin spectroscopy in the biomedical field to study mechanical properties in biology. Brillouin imaging is governing the viscoelastic behavior of biological samples. This technique has been used to analyze animal tissues, sub cellular components, cells and human samples.
48. **Bashan et al. (2021)[67]** studied Forward Stimulated Brillouin Scattering(FSBS) within fiber optics and photonic integrated circuits, delving into its implications for optomechanical non-reciprocity in standard polarization maintaining fibres.
49. **D. Asgharnejad et al. (2021)[68]** studied the phenomenon of self-focusing of Gaussian laser under ponderomotive and relativistic nonlinearity in magnetized, collisionless, warm quantum plasma. The study shows relativistic and ponderomotive nonlinearity boost self focusing of laser in warm quantum plasma in contrast to cold quantum plasma.
50. **N. Gupta et al. (2021)[69]** studied the propagation dynamics of elliptical q-Gaussian beam of laser inside plasma that generate the electron plasma wave inside plasma. As irradiance profile converges to Gaussian profile, the self focusing is reduced which decreases the amplitude of electron plasma wave in plasma.
51. **N. Gupta et al. (2022)[70]** investigated the pulse width, beam widths and axial phase shift of laser beam by considering the elliptical profile of laser beam inside plasma. Gaussian profile increases self focusing of laser beam but it slows down the focusing.

The laser pulse self-focuses less in the transverse direction, which is initially more elliptical.

52. **P. P. Nikam et al. (2022)[71]** studied the impact of magnetization parameter on self focusing of Gaussian laser beam within magnetized collisionless plasma. In comparison to unmagnetized case, self focusing is increased by forward magnetic field and vice versa.
53. **S. Kojima (2022)[72]** reviewed Brillouin scattering studies on proteins, ferroelectric materials and glasses in materials science. The ferroelectric phase transition with the sharp elastic anomaly and critical slowing down is demonstrated. Protein polymorphism and denaturation is confirmed by change in qausi-elastic and elastic properties.
54. **M. Merklein et al. (2022)[73]** reviewed of 100 years of stimulated Brillouin scattering together with its history and future perspects. Applications of SBS are discussed in fiber optics and medical sciences.
55. **X. Zeng et al. (2022)[74]** studied that in chiral photonic crystal fiber, phenomenon of SBS occurs. SBS has many applications in signal processing, microwave photonics, sensing.
56. **K. Tian et al. (2022)[75]** investigated self focusing and self defocusing of Hermite-sinh-Gaussian (HsHG) in underdense non uniform plasma medium. The impacts of beam intensity, plasma parameters on phenomenon of self focusing/defocusing are discussed. Increase in hermite mode index decrease self focusing and increase self defocusing.
57. **N. Gupta et al. (2022)[76]** studied the self action effects of high intense laser during its interaction with plasma. An equation set has been derived to describe the changes in beam width and axial phase shift of a laser beam. During the propagation of laser inside plasma, irradiance profile of laser beam play important role. Gaussian profile possesses fast focusing character.
58. **L. Devi et al. (2023)[77]** studied the phenomenon of self focusing and defocusing of super Gaussian laser beam in plasma with various density profiles. Moment theory is used to explain self focusing of super Gaussian profile. Results of self focusing and defocusing phenomenon are being compared for linear, tangent and exponential density profiles.

59. **A. Butt et al. (2023)[78]** studied the effect of plasma density ramp on self-focusing of q-Gaussian laser inside plasma. Laser spot size shows oscillatory behavior inside plasma medium. Increase in q value decreases self focusing of laser but higher value of q possesses fastest focusing character.

Chapter- 3
Amplitude structure of laser beam

3.1 Introduction

To characterize a laser beam, its spatial amplitude profile is measured at points that are perpendicular to the propagation direction. The spatial amplitude profile refers to the change in intensity with distance from the center of beam, within a plane that is perpendicular to the propagation direction. Distribution of intensity over the beam of laser is determined by the shape of its laser cavity. Laser beams with different spatial amplitude profiles demonstrate varying behavior within nonlinear media. Based on review of literature it has been noted that previous theoretical studies on SBS laser beams in plasma primarily focused on the idealized Gaussian profile of the laser beam[45,46,63]. However, the investigations carried out to measure the amplitude structures of laser beams suggests that even when the laser system operates in its fundamental TEM_{00} mode, the amplitude structure across the cross section of the beam does not exhibit an ideal Gaussian profile. Additionally, it's been discovered that a significant amount of laser energy exists beyond the full width half maximum (FWHM) region. This means that the wings of the intensity profile are expanded compared to the Gaussian profile. Nakatsutsumi et al.[79] have proposed that by incorporating experimental data, Tsallis' q-Gaussian distribution can be employed to model the true amplitude structure across the beam. Hence, in the present investigation to get more realistic results, the amplitude structure across the cross section of the laser beam has been considered as q-Gaussian distribution.

In contrast to ideal Gaussian and q-Gaussian laser beams[4-6], now a days, researchers are showing notable interest in a novel variety of laser beams termed as flat top laser beams. These beams exhibit a consistent irradiance throughout their cross-sectional and thus possess more power and lesser divergence compared to the Gaussian and q-Gaussian beams. Mathematically the spatial variation of amplitude across the cross section of such beams is described by super Gaussian function. But again super Gaussian approximation represents an idealized concept. The laser beams characterized by flat top irradiance profile produced in laboratory, have a uniform irradiance only up to some finite extent, after which the irradiance starts decreasing radially, as in case of Gaussian or q-Gaussian irradiance.

The amplitude structure of such beams can be modeled by Cosh-Gaussian (ChG) function or Quadruple-Gaussian (Q.G) functions.

Thus, in the present investigation two types of amplitude profiles for the laser beams have been investigated[18]:

1. q-Gaussian
2. Quadruple-Gaussian (Q.G)

Following sections describe the physical characteristics (amplitude profile, effective beam width, spectral width etc.) of these laser beams.

3.2 q-Gaussian Laser Beams

The expression that describes the spatial distribution of the transverse amplitude of a q-Gaussian laser beam across its cross section at the plane $(z = 0)$ where it enters the medium is as follows[76]:

$$A_0(r, z) = \frac{E_{00}}{f}\left(1 + \frac{r^2}{q r_0^2 f^2}\right)^{-q/2} \tag{3.1}$$

where $'q'$ is linked to the extent by which the irradiance distribution of the beam deviates from an ideal Gaussian profile across its cross sectional area. Consequently, it is commonly referred to as the deviation parameter (DP). When $f(z)$ is multiplied with the initial beam width r_0, it yields the instantaneous spot size of the laser beam, representing the size at a specific location within the medium. As a result, the function $f(z)$ is commonly mentioned as a dimensionless beam-width parameter (BWP). With an increase in DP $'q'$, the intensity profile of the beam gradually converges toward a Gaussian profile, and it changes to an exact Gaussian distribution when $q = \infty$ i.e.

$$\lim_{q\to\infty} A_0(r, z) = \frac{E_{00}}{f}\exp\left(-\frac{r^2}{2 r_0^2 f^2}\right) \tag{3.2}$$

Thus, $q = \infty$ corresponds to an ideal Gaussian beam.

To examine the impact of the DP $'q'$ on the distribution of intensity over the beam, we have illustrated in fig. 3.1 the relationship between normalized intensity and radial coordinate for various values of q.

It is evident that the DP $'q'$ has no impact on the illumination within the axial region of the beam. Nevertheless, it does influence the irradiance in the off-axial regions, as lower values of q result in expanded wings. As the value of q increases, illuminance in the peripheral region of the cross-sectional area of beam diminishes more rapidly in comparison to beams having lower q values. Indeed, an increase in q leads to a higher intensity gradient in the peripheral region of the beam, making the off-axial part weaker compared to the central region.

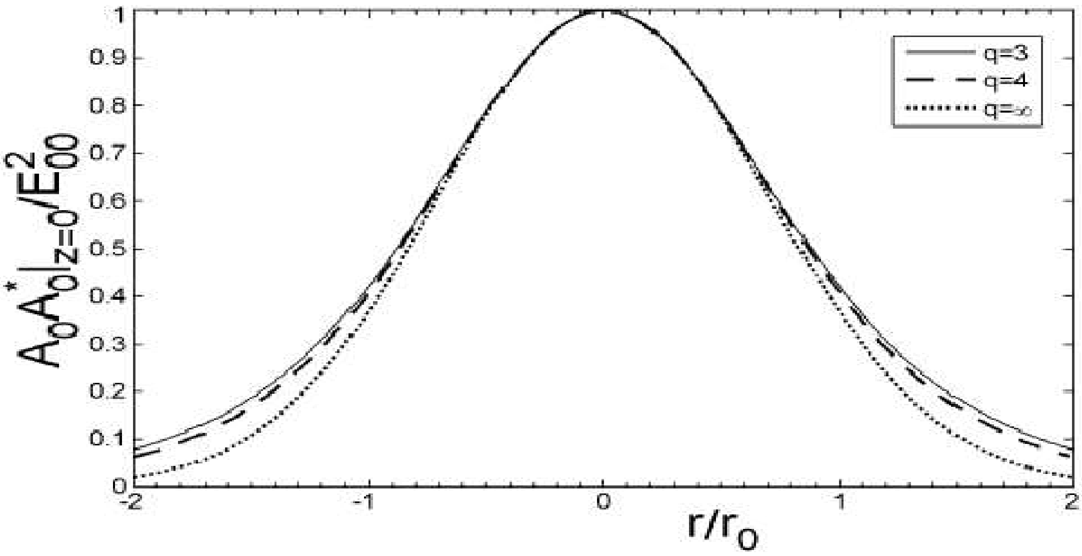

Fig.3.1: Effect of q on distribution of intensity over the beam

To investigate the influence of the parameter q on the initial width of the laser beam, the effective radius of the beam is defined as:

$$< a^2 >= \frac{1}{I_0}\iint A_0 r^2 A_0^* d^2 r \tag{3.3}$$

with $$I_0 = \iint A_0 A_0^* d^2 r \tag{3.4}$$

and $$d^2 r = r dr d\theta \tag{3.5}$$

Therefore, for the intensity profile described by Eq. (3.1), the effective width of the laser beam is determined to be

$$< a^2 >= r_0^2 f^2 \left(1 - \frac{2}{q}\right)^{-1} \tag{3.6}$$

The plot (fig. 3.2) illustrates the relationship between the normalized beam width of the q-Gaussian laser beam and the DP $'q'$ at the plane of incidence ($z = 0$). From the plot it can be observed that as the DP $'q'$ increases, the effective beam width of the beam decreases. This occurs because, as the value of q increases, the intensity of the beam shifts close to the off-axial parts. Therefore, it is evident that ideal Gaussian laser beams have the smallest beam width.

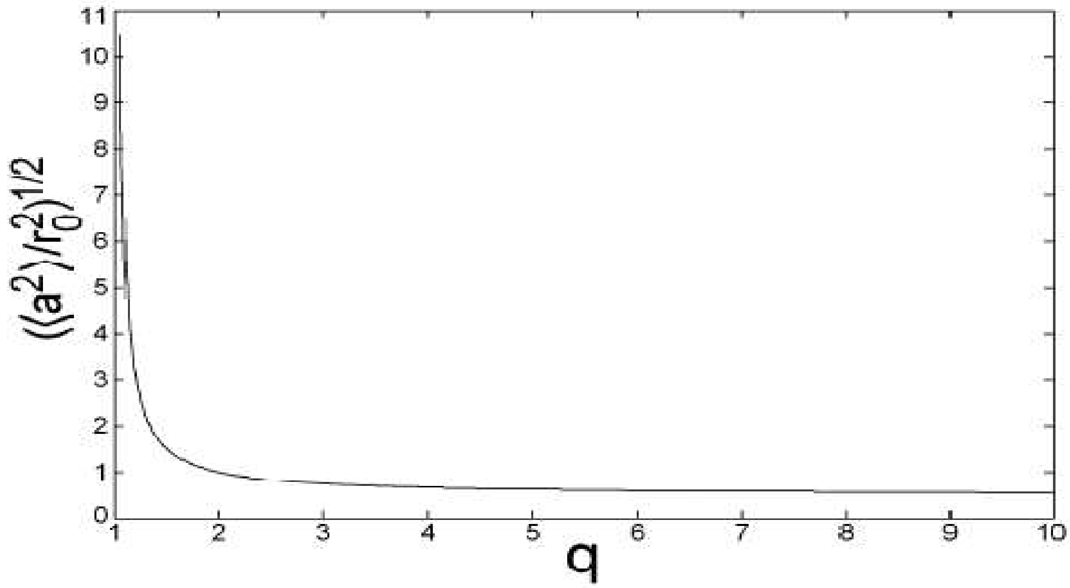

Fig.3.2: Effect of DP $'q'$ on beam width

3.3 Quadruple Gaussian Laser Beams

The transverse amplitude structure $A(x,y)$ of the laser beam is represented by[77]

$$A(x,y) = \frac{E_{00}}{f}\left[e^{\frac{-(x-x_0f)^2+y^2}{2r_0^2f^2}} + e^{\frac{-(x+x_0f)^2+y^2}{2r_0^2f^2}} + e^{\frac{-(y-x_0f)^2+x^2}{2r_0^2f^2}} + e^{\frac{-(x+x_0f)^2+y^2}{2r_0^2f^2}}\right] \quad (3.7)$$

Here, E_{00} represents the axial amplitude and r_0 represents equilibrium radius of the laser beam.

Eq. (3.7) reveals Q.G laser beams can be generated by the constructive interference of four identical laser beams, each exhibiting a Gaussian intensity profile. The coordinates of their intensity maxima are located at $(-x_0, 0)$, $(x_0, 0)$, $(0, x_0)$, $(0, -x_0)$.

Fig. 3.3 portray the initial profile of beam for various values of $\frac{x_0}{r_0}$. The plots presented in fig. 3.3 demonstrate that laser beams with $\frac{x_0}{r_0}$ values within the range $0 \leq \frac{x_0}{r_0} \leq 1.5$ exhibit a wider region of uniform irradiance across its cross section. Moreover, the size of this uniformly illuminated portion grows with an increase in the value of $\frac{x_0}{r_0}$ .When $\frac{x_0}{r_0} > 1.5$ the intensity profile peaks are observed in regions of the cross section that are significantly distant from the axis of the beam of laser. In the axial region of the laser beam, a valley of intensity emerges, accompanied by four off-axial peaks.

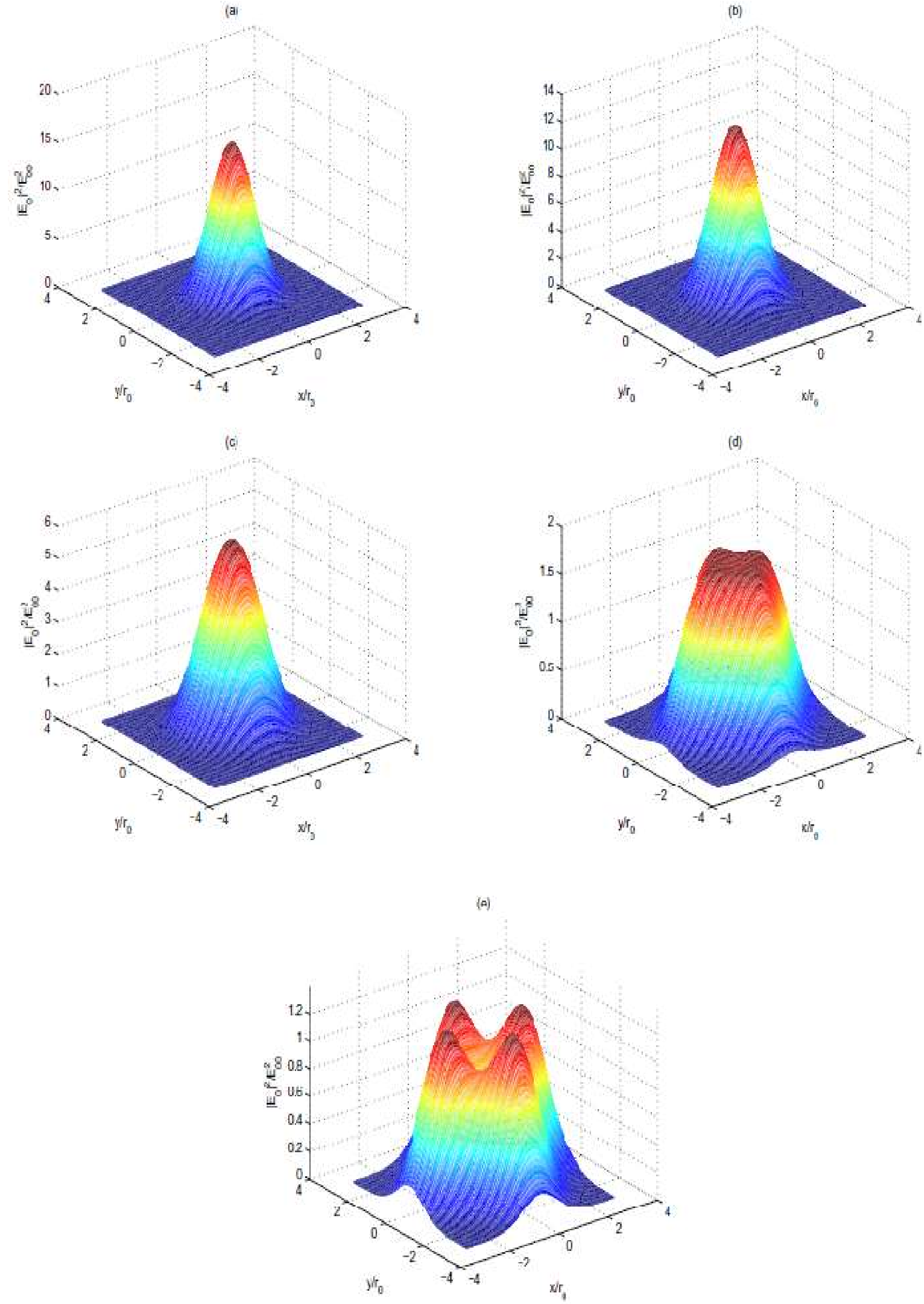

Fig.3.3: Surface plots for intensity of Q.G laser beam for different values of $\frac{x_0}{r_0}$

The effective beam width of the beam is defined as follows:

$$\sigma^2 = <x^2> + <y^2> \tag{3.8}$$

where,

$$<x^2> = \frac{1}{I_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 A_0 A_0^* \, dxdy \tag{3.9}$$

$$<y^2> = \frac{1}{I_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 A_0 A_0^* \, dxdy \tag{3.10}$$

Using Eqs. (3.8) to (3.10), the root mean square (RMS) radius of a quadruple-Gaussian laser beam can be obtained in the following manner

$$\sigma_{Q.G} = \frac{\sqrt{2} r_0 f}{\sqrt{3}\left(1+3e^{-\frac{x_0^2}{2r_0^2}}\right)^{\frac{1}{2}}}\left\{\left(2+2\frac{x_0^2}{r_0^2}\right)+\left(2+\frac{x_0^2}{r_0^2}\right)e^{-\frac{x_0^2}{2r_0^2}} + 2e^{-\frac{x_0^2}{2r_0^2}}\right\}^{\frac{1}{2}} \tag{3.11}$$

The RMS radius of the corresponding Gaussian beam can be determined as $\sigma_G = r_0 f$. The ratio $\sum$ of the root mean square (RMS) beam widths between the quadruple-Gaussian beam $\sigma_{Q.G}$ and the Gaussian beam σ_G is given by:

$$\sum = \frac{\sigma_{Q.G}}{\sigma_G} = \frac{\sqrt{2}}{\sqrt{3}\left(1+3e^{-\frac{x_0^2}{2r_0^2}}\right)^{\frac{1}{2}}}\left\{\left(2+2\frac{x_0^2}{r_0^2}\right)+\left(2+\frac{x_0^2}{r_0^2}\right)e^{-\frac{x_0^2}{2r_0^2}} + 2e^{-\frac{x_0^2}{2r_0^2}}\right\}^{1/2} \tag{3.12}$$

Fig. 3.4 shows the relationship between normalized beam widths of the quadruple-Gaussian with $\frac{x_0}{r_0}$. Effective beam width of the laser beam increases as the value of $\frac{x_0}{r_0}$ increases. Thus, for a fixed geometrical radius r_0, a quadruple-Gaussian laser beam with a higher value of $\frac{x_0}{r_0}$ will have a larger effective beam width. This is attributed to the transfer of laser intensity from central or axial to the peripheral or off-axial portion of the wavefronts as the value of $\frac{x_0}{r_0}$ increases. From Eq. (3.12) one can deduce or infer that quadruple-Gaussian beams exhibit a smaller diffraction divergence.

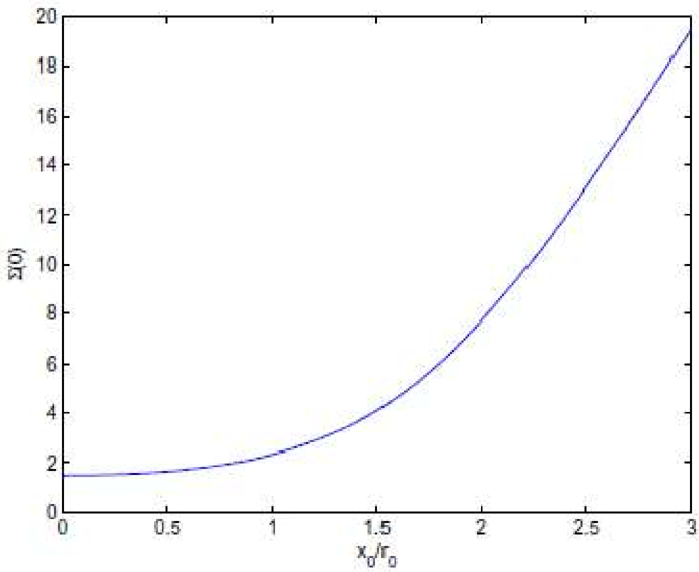

Fig.3.4: Effect of $\frac{x_0}{r_0}$ on effective radius of Q.G laser beam

Due to the finite effective width of the laser beam, it exhibit behavior similar to that of traversing through a thin aperture or opening. Consequently, it encounters a broadening within its transverse momenta, which can be explained by position-momentum uncertainty $\Delta x \Delta p_x$ = constant. Since the momentum of photon is linked to its propagation constant as $(p = \hbar k)$, the broadening within transverse momentum transform the mean value of the propagation constant. The root mean square (RMS) value of spectral width σ_k of beam of laser is expressed in the following manner:

$$\sigma_{k,Q.G} = \sqrt{< k_x^2 > + < k_y^2 >} \tag{3.13}$$

$$< k_x^2 > = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_x^2 S(k_x,k_y) dk_x dk_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(k_x,k_y) dk_x dk_y} \tag{3.14}$$

$$< k_y^2 > = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_y^2 S(k_x,k_y) dk_x dk_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(k_x,k_y) dk_x dk_y} \tag{3.15}$$

And,

$$S(k_x, k_y) = \left| \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_0(x,y)|_{z=0}\, e^{-i(k_x x + k_y y)} dx dy \right|^2 \tag{3.16}$$

The ratio of $\sum_k$ of spectral widths of two otherwise identical beams with Q.G and Gaussian irradiance profiles can be expressed as

$$\sum_k = \frac{\sigma_{k,Q.G}}{\sigma_{k,G}} = \left\{ \frac{1+e^{-\frac{x_0^2}{2r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right)}{1+e^{-\frac{x_0^2}{2r_0^2}}} \right\}^{\frac{1}{2}} \tag{3.17}$$

Fig 3.5 illustrates the variation of $\sum_k$ with $\frac{x_0}{r_0}$. It is noticeable that as a function of $\frac{x_0}{r_0}$, this ratio $\sum_k$ initially decreases and subsequently begins to increase, with a transition point observed at $\frac{x_0}{r_0} = 1.50$.

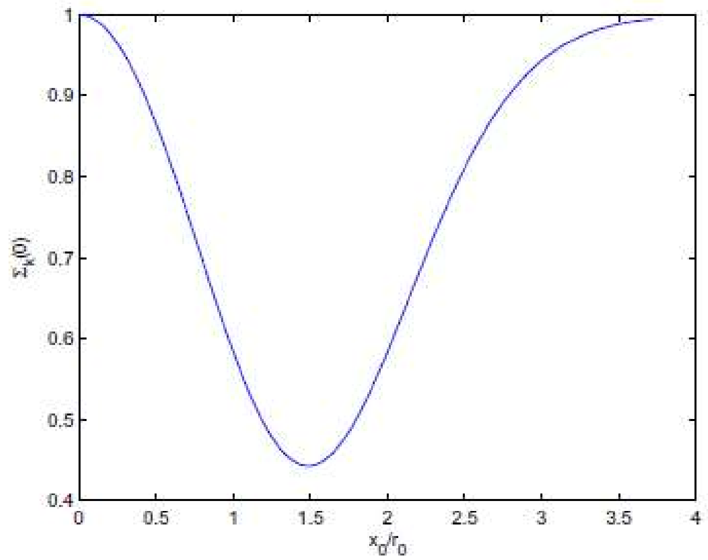

Fig.3.5: Effect of $\frac{x_0}{r_0}$ on spectral width of Q.G laser beam

This occurs due to the inverse correlation between the spectral width of the laser beam in the (k_x, k_y) space and the effective area across its cross-section (i.e., in the (x, y) plane), where the majority of its intensity is concentrated. It is demonstrated from the surface plots of the laser beam for various values of $\frac{x_0}{r_0}$ (fig. 3.3) that the effective area where the most of the laser intensity is concentrated increases with $\frac{x_0}{r_0}$ in the range $0 \leq \frac{x_0}{r_0} \leq 1.50$ and the same decreases with $\frac{x_0}{r_0}$ for $\frac{x_0}{r_0} > 1.50$. Therefore, a transitional dip is observed in the spectral width of the laser beam in the (k_x, k_y) space at $\frac{x_0}{r_0} = 1.50$.

3.4 Variational Theory

Literature review reveals that almost the base of the nonlinear dynamics of laser beams have built on the paraxial theory. Paraxial theory only takes into account the axial portion of the laser beam means it does not involve the effect of off-axial part of laser beam. This approach is not properly suitable when we need to deal with situations related to non-Gaussian beams such as q-Gaussian, Cosh-Gaussian, super-Gaussian, Quadruple beams in which off-axial fields play a crucial role in propagation dynamics. In present work, I have proposed the application of variational theory which removes the shortcomings of paraxial theory. This methodology is obtained by examining some integral relations derived from the nonlinear Schrodinger equation (NLSE). This method is more acceptable from the elementary point of view, since the whole wave front of the beam is examined in the interaction procedure.

Variational theory[70] is based on variational calculus. It is a semi analytical technique to obtain approximate solutions of partial differential equations which cannot be solved analytically. This technique replaces a set of partial differential equations by a set of coupled ordinary differential equations those can be solved either analytically or by using simple numerical techniques like Runge Kutta fourth-order method etc. It can be used successfully to have physical insight into number of nonlinear systems like propagation of waves in various nonlinear media, super conductors, Bose Einstein condensates etc. In present investigation, variational technique has been used to investigate nonlinear interactions of intense laser beams with plasmas and narrow band gap semiconductors. A detailed description of this technique is as follows:

$$\nabla_{\perp}^{2} E_0 + \frac{\omega_0^2}{c^2}\Phi(E_0 E_0^{*})E_0 - 2ik_0\frac{\partial E_0}{\partial z} = 0 \qquad (3.18)$$

Where, $\nabla_{\perp}^{2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ is the transverse Laplacian and $\Phi(E_0 E_0^{*}) = \varepsilon(E_0 E_0^{*}) - \varepsilon_0$ is the intensity dependent nonlinear dielectric function of the medium.

Because of non linearity, Eq. (3.18) does not adhere to the superposition principle. Consequently, combining two solutions linearly does not yield another solution to this equation. Hence, traditional techniques of solving partial differential equations are not applicable for this equation. Although there exist a number of numerical methods to solve this equation, but these methods suffer from major difficulties due to dependence of accuracy and speed on the functional form and number of the nonlinear terms incorporated in Eq.

(3.18). Achieving the desired accuracy requires careful consideration when selecting step size. All these difficulties can pose serious convergence problems for beam propagation algorithm. In contrast to this, explicit analytical or semi-analytical solutions to Eq. (3.18) are more advantageous instead than numerical methods, since they give comprehensive understanding of the behavior of the system.

There are different semi analytical techniques for solving Eq. (3.18) among which variational approaches is one which provide both qualitative and quantitative results and are very close to numerical results. The method relies on Ritz's optimization technique and finds broad utilization in the investigation of NLSE, which arises not only in nonlinear optics but also in numerous other physical problems. It transforms the infinite-dimensional problem of solving partial differential equations into a Newton-like ordinary differential equation of second-order for the parameters that describe the solution. The validity of variational results, however, can be qualitative in some cases. Variational techniques are most likely to produce results in good agreement with the actual solutions when the trial function closely resembles the shape of the actual solution. Nevertheless, in certain situations, the method may prove ineffective or yield imprecise results.

We first write the Nonlinear Schrodinger Wave Equation in the form:

$$\hat{F}[\Psi] = 0$$

We can subsequently establish a Lagrangian density £ $[\Psi, \Psi^*]$, in a manner such that:

$$\frac{\partial £}{\partial \Psi^*} = \hat{F}[\Psi]$$

In accordance with variational Method, we are required to solve the following set of extended Euler-Lagrange equation for the variational parameter $g_i(z)$ with $i = 1, 2, 3,4 \ldots . N$:

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial q_i}{\partial z}\right)}\right) - \frac{dL}{dz} = 0 \tag{3.19}$$

Here, L represents the average Lagrangian obtained by integrating £ over the transverse coordinates x and y.

$$L = \iint £ \; dxdy$$

For our particular problem, the Lagrangian density corresponding to eq. (3.18) is

$$£ = \mathrm{i}k_0\left(\Psi\frac{\partial\Psi^*}{\partial z} - \Psi^*\frac{\partial\Psi}{\partial z}\right) + \left|\frac{\partial\Psi}{\partial x}\right|^2 + \left|\frac{\partial\Psi}{\partial y}\right|^2 - \frac{\omega_0^2}{c^2}\int_0^{E_0E_0^*}\Phi(E_0E_0^*)d(E_0E_0^*) \qquad (3.20)$$

The variational method investigates the dynamic characteristics of the laser beam through medium under nonlinear regime by using different trial functions for laser beam.

3.5 Research Objectives

The Objectives of this present research is to theoretically investigate the phenomenon of stimulated Brillouin scattering of intense laser beams in underdense plasma targets through the excitation of ion acoustic waves.

Following problems are analyzed in order to achieve the objectives:

1. Excitation of ion acoustic waves self focused q-Gaussian laser beams in relativistic plasmas with axial density ramp.
2. Stimulated Brillouin scattering of q-Gaussian laser beams in axially inhomogeneous Plasmas.
3. Self action effects of quadruple Gaussian laser beams in thermal quantum plasmas.
4. Stimulated Brillouin scattering of self focused quadruple Gaussian laser beams in thermal quantum plasmas.

Chapter- 4

Excitation of Ion Acoustic Waves by Self Focused q -Gaussian Laser Beam in Plasma with Axial Density Ramp

4.1 Introduction

Plasma consists of particles carrying both positive and negative charges moving about so energetically that they do not readily combine. Plasmas exist ubiquitously throughout the universe. They form the extremely heated gas subjected to high pressure found in the sun and other stars, as well as the rarefied gas in interstellar space and in the ionospheric envelope surrounding the earth. Plasmas also exist closer to hand. They exist within the flames of burning fuel and within gas-discharge devices like neon signs. Plasmas exhibit such an enormous variety of physical effects that physicist have studied their properties for about 200 years. Researchers have discovered the electron and elucidated the structure of atoms through research on plasmas, particularly gas discharges.

The current interest in plasmas reflects two principal motives. The first one is technological. It is crucial to understand plasma behavior in order to release thermo nuclear energy safely, the attempt to reproduce in man-made plasma the kind of nuclear reaction found in the sun. Another technical goal is the design of magnetohydrodynamic generators, where electrical power is produced by the movement of gas plasma jets through magnetic fields. The second broad motive for the study of plasmas is the importance of plasma phenomena in space and in astrophysics. When plasma is subjected to electromagnetic fields, the motion of the particles is no longer completely random. One important consequence of this imposed order is that plasmas can transmit certain kinds of waves that are related to electromagnetic waves but that have unique and curious properties. These waves include high frequency electron plasma waves (EPWs) and low frequency ion acoustic waves (IAWs).

Due to their notable characteristics of quasineutrality and collective behavior, IAWs can be induced in plasmas. Plasma represents a state of matter characterized by a high temperature that causes atoms losing their distinct identities. Within plasma, the negatively charged electrons continue to experience the pull of positively charged nuclei, even though their binding differs from that found in solid or liquid states of matter. Plasma possesses unique characteristics that set it apart from ordinary states of matter such as solids, liquids, and gases. Unlike these forms, plasma exhibits distinct behavior due to the strong influence of electric and magnetic fields on its electrons and ions moving independently within the medium. Plasma as a whole maintains quasineutrality; however, due to the separation of electrons and positively charged ions, disturbances can give rise to regions with net negative and positive charges, resembling the charged plates in a parallel plate capacitor. The non uniform arrangement of charges in such cases set up an electric field that spans from the regions of positive charge to those of negative charge. The presence of this electric field exerts equal forces that attract the electrons and ions towards each other. As a result of this attractive force electrons and ions start moving towards each other. While the electrons and ions approach each other, their velocity and momentum gradually increase, analogous to a pendulum moving from an extreme position towards its mean position. The increase in momentum causes the electrons and ions to surpass their equilibrium positions, leading to a reversal of electric field direction. With the electric field now reversed, it acts in opposition to the motions of electrons and ions, causing them to decelerate and eventually be pulled back in the opposite direction. The process continues to repeat itself in a cyclic manner, creating an electron-ion oscillator. When considering the influence of thermal velocity, these oscillations between electrons and ions result in the generation of a longitudinal wave that alternately compresses and rarifies the density of the plasma. This wave propagates through the plasma and is known as an ion acoustic wave. (fig. 4.1)

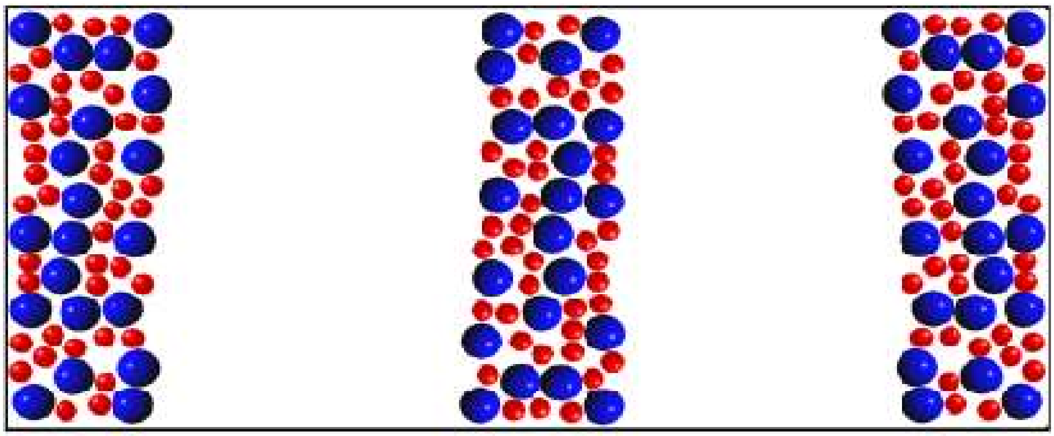

Fig.4.1: Ion acoustic wave

Indeed, ion acoustic waves (IAWs) share similarities with sound waves in the sense that they travel in the form of compressions and rarefactions of particles, these align with the direction of motion of the wave. Like sound waves in a medium, IAWs cause variations in the density of plasma as they propagate through it. The main distinction between sound waves and plasma waves lies in their dispersion characteristics. Plasma waves exhibit dispersion, meaning their propagation characteristics depend on the frequency or wavelength of the wave. In contrast, sound waves in ordinary matter generally do not exhibit significant dispersion and propagate with a constant speed determined by the properties of the medium. In case of inertial confinement fusion, these IAWs leads to reflection of a substantial amount of laser energy through a phenomenon of stimulated Brillouin scattering.

The interaction between incident laser beams and ion acoustic waves (IAWs) can also be utilized as a diagnostic tool to obtain information about the physical properties of plasma like temperature, ion density, conductivity etc. This is achieved by assessing the frequency variation between the driving wave and the wave that scatters as a result of ion acoustic waves (IAW). One inherent advantage of this diagnostic method is that it does not require the insertion of any physical probe into the plasma. This will be specifically useful in case of space plasmas where probing is challenging and expensive.

The review of existing literature indicated that a significant portion of theoretical investigations concerning the excitation of ion acoustic waves (IAWs) through plasmas has primarily centered around lasers featuring a circular cross-section. In practice, the cross-sectional shape of laser beam is not perfectly circular. The experimentally produced laser beams have small ellipticity that significantly affects their propagation characteristics through nonlinear media. Till date, to the best of our knowledge, no theoretical investigations have been reported regarding the excitation of ion acoustic waves (IAWs) propagating in plasmas with varying densities using elliptical beams of laser light. Hence, the aim of this chapter is to present the pioneering theoretical study on the on the excitation of IAWs using relativistically focused elliptical q-Gaussian laser beams in plasmas featuring an axial density ramp.

4.2 Relativistic Nonlinearity in Plasma

The dielectric response of a plasma with an increasing equilibrium electron density with longitudinal distance as $n(z) = n_0(1 + \tan(dz))$ is given by[52]

$$\varepsilon = 1 - \frac{4\pi e^2 n_0}{m_e \omega_0^2}(1+\tan(dz)) \quad (4.1)$$

Where $'e'$ represents the electronic charge and $'m_e'$ denotes the electronic mass. The constant $'d'$ characterizes the electron density gradient and is commonly referred to as the gradient of the density ramp. The term $'n_0'$ represents the equilibrium density of the plasma electrons at the reference point $'z = 0'$. The symbol $'\omega_0'$ represents the angular frequency of the incident beam of laser light. When a laser beam with an electric field vector

$$E(r, z, t) = A_0(x, y, z)e^{i(k_0 z - \omega_0 t)} e_X$$

transmit through the plasma target, it induces oscillations in the velocity of plasma electrons [47]

$$v = -i\frac{e}{\omega_0 m_e}E$$

If the incident laser beam possesses enough intensity to induce plasma electrons to oscillate at velocities comparable to the speed of light then m_e i.e. the effective mass of plasma electrons in Eq. (4.1) would then be substituted by $m_e = m_0\gamma$. Here, m_0 represents the (rest) mass of electron and γ denotes the relativistic Lorentz factor which is associated with laser intensity as

$$\gamma = (1 + \beta A_0 A_0^*)^{\frac{1}{2}}$$

Where β represents a constant defined as $\beta = \frac{e^2}{m_0^2 c^2 \omega_0^2}$. It is related to magnitude of relativistic nonlinearity and is often referred as "relativistic nonlinearity coefficient". Consequently, when a laser beam is present, Eq. (4.1) is modified as

$$\varepsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}(1 + \beta A_0 A_0^*)^{-\frac{1}{2}}(1 + \tan(dz)) \quad (4.2)$$

where $\omega_{p0} = \sqrt{\frac{4\pi e^2 n_0}{m_0}}$ is the plasma frequency without a laser beam. Based on Eq. (4.2) it is evident that the dependence of electron mass on laser intensity makes the plasma an optically nonlinear medium in a similar way as Kerr effect makes ordinary dielectrics an optically nonlinear medium.

Expressing Eq. (4.2) as

$$\varepsilon = \varepsilon_0 + \phi(A_0A_0^*) \tag{4.3}$$

we get

$$\varepsilon_0 = 1 - \omega_{p0}^2/\omega_0^2 \tag{4.4}$$

and

$$\phi(A_0A_0^*) = \frac{\omega_{p0}^2}{\omega_0^2}[1 - \frac{1}{(1+\beta A_0A_0^*)^{1/2}}][1 + \tan(dz)] \tag{4.5}$$

Equation (4.5) provides the dielectric function of plasma incorporating relativistic nonlinear effects.

4.3 Evolution of Beam Envelope

The behavior of an optical beam as it passes through a medium, described by a dielectric function represented as $\phi(A_0A_0^*)$ is elucidated through a wave equation[70,71]

$$2ik_0\frac{\partial A_0}{\partial z} = \nabla_\perp^2 A_0 + \frac{\omega_0^2}{c^2}\phi(A_0A_0^*)A_0 \tag{4.6}$$

Eq. (4.6) is commonly referred to as the NSWE owing to its notable mathematical similarity to the Schrödinger wave equation encountered in the realm of quantum mechanics. The existence of nonlinear term represented by $\phi(A_0A_0^*)$ in Eq. (4.6) prevents it from having an exact analytic solution. To gain a deeper understanding of the dynamics of laser beam propagation, the solution to Eq. (4.6) can be obtained either through numerical methods or semi-analytic methods. When dealing with partial differential equations, numerical methods can be tedious and prone to convergence issues. Therefore, in the current analysis, a semi-analytical technique called the variational method has been employed.

The variational method enables the approximation of a solution for Eq. (4.6). The core principle of the variational method revolves around discovering solutions for distinct category of functions, denoted as $A_0(r,\sigma)$, where the parameters $\sigma = \left(f_x(z), f_y(z)\right)$ depend on the evolution variable and derived from the solutions of the corresponding set of ordinary differential equations. Following this methodology, Eq. (4.6) can be cast into a variational framework by employing the action principle that relies on the Lagrangian density[71]

$$\mathcal{L} = i\left(A_0\frac{\partial A_0^*}{\partial z} - A_0^*\frac{\partial A_0}{\partial z}\right) + |\nabla_\perp A_0|^2 - \frac{\omega_0^2}{c^2}\int^{A_0A_0^*}\phi(A_0A_0^*)d(A_0A_0^*) \tag{4.7}$$

In the current study, we have employed a trial function in the following form[77]:

$$A_0(x,y,z) = \frac{E_{00}}{\sqrt{f_x f_y}}\left\{1+\frac{1}{q}\left(\frac{x^2}{a^2 f_x^2}+\frac{y^2}{b^2 f_y^2}\right)\right\}^{-\frac{q}{2}} \tag{4.8}$$

where, E_{00} represents the laser beam's axial amplitude, and af_x and bf_y are the laser beam's widths along the x and y directions, respectively. The parameters a and b represent the beam widths at equilibrium at the plane of entrance ($z = 0$), making f_x and f_y the dimensionless beam width parameters. Putting the trial function provided in Eq. (4.8) into Lagrangian density and performing integration across the cross-section of the beam results in a reduced Lagrangian as $L = \int_0^\infty \mathcal{L} r dr$. The associated Euler-Lagrange equations

$$\frac{d}{dz}\left(\frac{\partial L}{\partial(\frac{\partial f_{x,y}}{\partial z})}\right) - \frac{\partial L}{\partial f_{x,y}} = 0 \tag{4.9}$$

give

$$\frac{d^2 f_x}{dz^2} = \frac{1}{2k_0^2 a^4}\frac{1}{f_x^3}\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left[\left(1+\frac{1}{q}\right)^{-1} + \left(\frac{<L_1>}{E_{00}^2} f_x f_y + \frac{2E_{00}^2}{f_x^2 f_y}\frac{\partial <L_1>}{\partial f_x}\right)\right] \tag{4.10}$$

$$\frac{d^2 f_y}{dz^2} = \frac{1}{2k_0^2 b^4}\frac{1}{f_y^3}\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left[\left(1+\frac{1}{q}\right)^{-1} + \left(\frac{<L_1>}{E_{00}^2} f_x f_y + \frac{2E_{00}^2}{f_x f_y^2}\frac{\partial <L_1>}{\partial f_y}\right)\right] \tag{4.11}$$

where,

$$< L_1 > = \frac{\omega_0^2}{c^2}\int (\int^{A_0 A_0^*} \phi(A_0\, A_0^*) d(A_0 A_0^*))\, d^2 r$$

Equation (4.10) and (4.11) can be written as

$$\frac{d^2 f_x}{dz^2} = \frac{1}{2k_0^2 a^4 f_x^3}\frac{\left(1-\frac{1}{q}\right)(1-\frac{2}{q})}{(1+\frac{1}{q})} + \frac{1}{2}\frac{(1-\frac{2}{q})}{a^2 \epsilon_0 I_0}\int x A_0 A_0^* \frac{\partial \phi}{\partial x} d^2 r \tag{4.12}$$

$$\frac{d^2 f_y}{dz^2} = \frac{1}{2k_0^2 b^4 f_y^3}\frac{\left(1-\frac{1}{q}\right)(1-\frac{2}{q})}{(1+\frac{1}{q})} + \frac{1}{2}\frac{(1-\frac{2}{q})}{b^2 \epsilon_0 I_0}\int y A_0 A_0^* \frac{\partial \phi}{\partial y} d^2 r \tag{4.13}$$

Using eqs. (4.5) and (4.8) in eqs. (4.12) and (4.13) we get

$$\frac{d^2 f_x}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f_x^3} - \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_x^2 f_y} I \tag{4.14}$$

$$\frac{d^2 f_y}{d\xi^2} = \left(\frac{a}{b}\right)^4 \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f_y^3} - \left(\frac{a}{b}\right)^2\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_x f_y^2} I \tag{4.15}$$

where

$$d' = dk_0a^2$$

$$\xi = \frac{z}{k_0a^2}$$

$$I = \int_0^\infty t\left(1+\frac{t}{q}\right)^{-2q-1} \{1+\frac{\beta E_{00}^2}{f_xf_y}\left(1+\frac{t}{q}\right)^{-q}\}^{-3/2} dt$$

Equations (4.14) and (4.15) depict the interconnected nonlinear differential equations that dictate the evolution of beam widths for elliptical q-Gaussian beams of laser light along both the x-axis and y-axis as they propagate longitudinally. In present investigation, the Runge Kutta RK4 method was employed for solving the equations for a specific collection of laser-plasma parameters:

$$\omega_0 = 1.78\times10^{15}\text{rad/sec}, \qquad a = 10\mu\text{m},$$

$$\beta E_{00}^2 = 3(\text{corrresponding to laser intensity } 4\times10^{16}\text{W/cm}^2, \frac{\omega_{p0}^2a^2}{c^2} =$$

$$9\ (\text{corresponding to electron density} = 5\times10^{16}\text{electrons per cm}^3)$$

and for various values of $q, d'$and $\frac{a}{b}$ viz.,

$$q = (3,4,\infty), d' = (0.25,0.35,0.45) \text{ and } \frac{a}{b} = (1,1.1,1.2)$$

In this study, the boundary condition adopted is that the laser beam displays a plane wavefront at the plane of incidence.

Expressed in mathematical terms, this condition signifies that at $\xi = 0$:

$$f_{x,y} = 1$$

$$\frac{df_{x,y}}{d\xi} = 0$$

Figure 4.2 depicts the change in widths of the beam of laser light as it evolves. The plots in fig. 4.2 exhibit the oscillatory characteristics of the beam widths of laser light in both perpendicular directions with respect to its longitudinal axis. By studying the functions and origins of the different terms present in the Eqs. (4.14) and (4.15) that govern the beam widths, we can comprehend the observed response of the beam of laser light. The first terms on the RHS of these equations depict the spatial dispersive factors, showcasing an inversely proportional relationship with the cube of their corresponding beam widths. (i.e. as $f_{x,y}^{-3}$). These terms capture the spreading of the beam of laser light in the along $x-$ and $y-$axes, resulting from diffraction divergence. It is obvious that smaller the beam width in any perpendicular direction more is the magnitude of diffraction divergence along that direction. Indeed, this observation is a consequence of the position-momentum uncertainty principle of photons. The diffraction of a laser beam arises fundamentally from the inherent uncertainty in the position and momentum of the photons. In order to keep the products $\Delta x \Delta p_x$ and $\Delta y \Delta p_y$ to be constant, the reduction in beam width along any of the transverse direction leads to gain in additional momentum along that direction. The gain in this transverse momentum is the actual cause of the diffraction divergence.

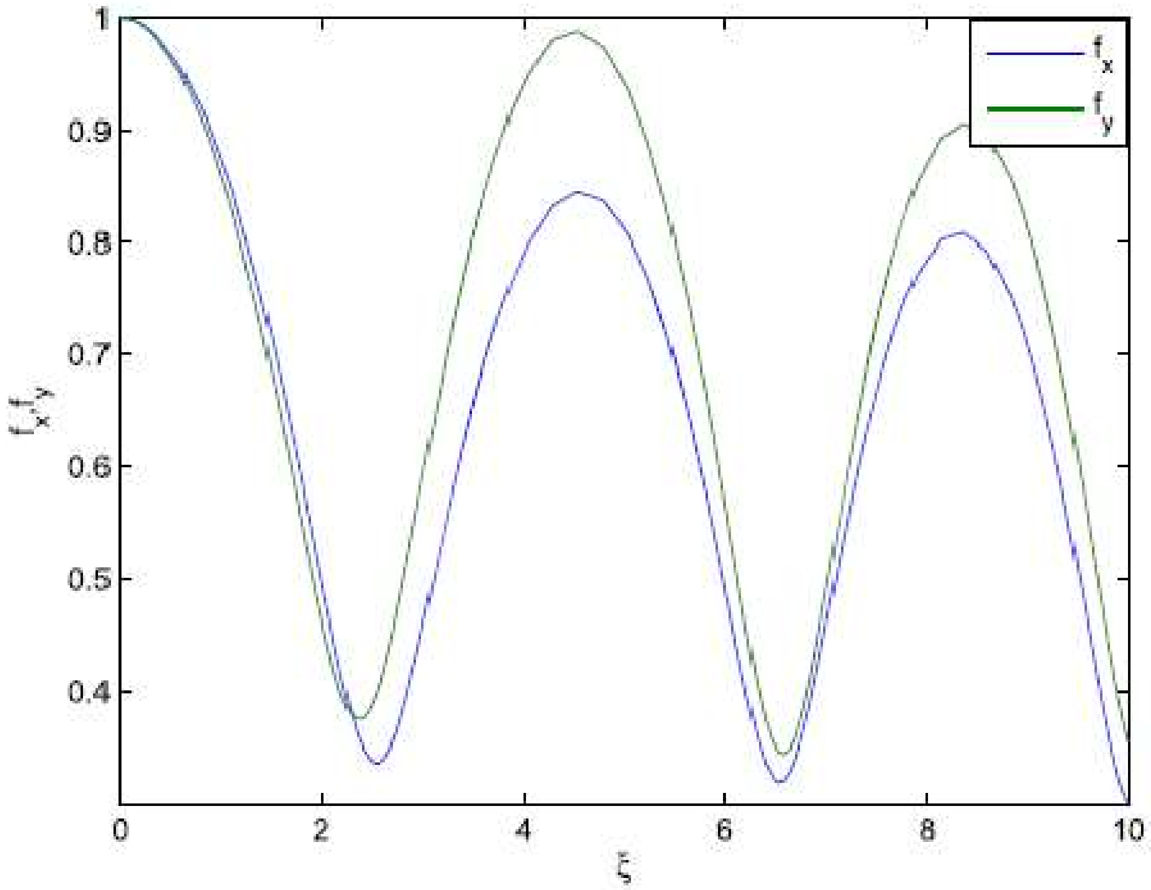

Fig.4.2: Variation in the laser beam spot size while traversing within the plasma for

$q = 3$, $d' = 0.25$ and $\frac{a}{b} = 1.1$

The emergence of the second terms on RHS of these equations can be attributed to the complex influence of the beam widths $f_{x,y}$ .These terms arise due to relativistic effects caused by the interaction between the laser beam and the plasma electrons. These terms describe both the nonlinear refraction of the beam of laser light and the nonlinear interaction that takes place between the beam widths in different perpendicular directions. The nonlinear refraction of the beam of laser light caused by the plasma's nonlinearity counteracts the diffraction effects occurring in both perpendicular directions. While a beam of laser light propagates within plasma, a competition arises between the phenomena of diffraction and nonlinear refraction. The outcome of this competition between diffraction and nonlinear refraction determines the final behavior of the beam of laser light, if it converges or diverges. Above a certain critical value of beam intensity, achieved by equating the two terms on the right-hand side of Eqs. (4.14) and (4.15), the laser beam will converge in both perpendicular directions. The intensity of the beam correspondingly increases when the cross-sectional area of the beam of laser light decreases or contracts. Once the intensity of the beam of laser light reaches a certain high level, the mass of plasma electron within the illuminated area becomes saturated. Therefore the optical nonlinear behavior of the plasma diminishes or vanishes. Consequently, after reaching a minimum value, the laser's beam width rebound or revert back to their initial values. As the laser beam extends in both orthogonal directions, the interplay of diffraction broadening and nonlinear refraction resumes once more time. The interplay of diffraction broadening and nonlinear refraction continues until the quantities $f_{x,y}$ reach their peak values. These processes occur in a repetitive manner, leading to an oscillatory behavior in the beam widths of the beam of laser light in the both perpendicular directions.

Furthermore, it is observed that following each focal spot, both the highest and lowest values of the beam width experience a downward shift. This phenomenon occurs because the electron density at equilibrium increases with longitudinal distance. As a result, the plasma's refractive index continues to decrease as the laser beam achieves greater penetration into the plasma. This effect is attributed to the behavior observed at relativistic intensities. With an increase in plasma density, a beam containing a higher number of relativistic electrons travels alongside the laser beam. This generates a higher current and, in turn, produces an extremely strong quasi-stationary magnetic field. Consequently, the pinching effect depicted in fig. 4.3 becomes more pronounced, amplifying the self-focusing of the beam of laser light.

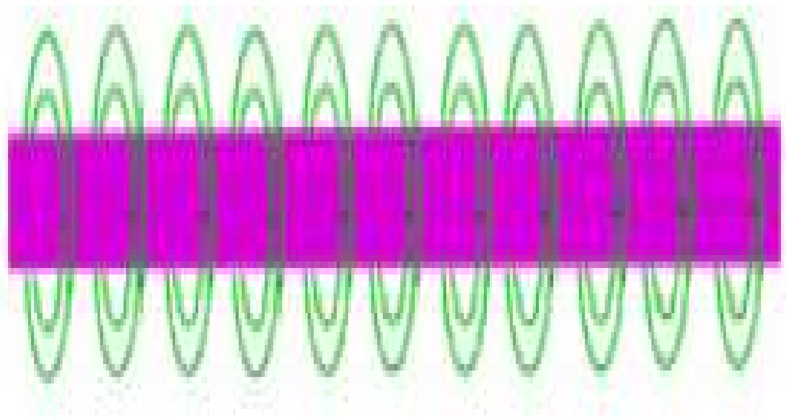

Fig.4.3: Pinching effect in plasma

As a result, the self-focusing effect is amplified, leading to a further downward shift of both the highest and lowest value of the width of laser at each focal spot. Additionally, it has been noted that the oscillation frequency of the beam intensifies as it propagates over distance. This can be attributed to the physics behind it: the denser the plasma, the greater the phase velocity of the laser beam as it traverses it. Consequently, in plasma with higher density, the beam of laser light undergoes faster self-focusing, which results in an upsurge in the oscillation frequency.

Furthermore, it is noted that the laser beam exhibits a noticeable difference in self-focusing between x and y directions. This discrepancy arises from the original beam width, where the width in the x- direction is greater than that in the y- direction ($\frac{a}{b} = 1.1$). As a consequence, the resistance caused by diffraction against the nonlinear refraction is pronounced in the y-direction to the greater extent. Consequently, the focusing ability of the beam of laser light diminishes in the y direction.

The figures presented in fig. 4.4 illustrate that as the deviation parameter (DP) $'q'$ increases; there is a corresponding reduction in the degree of self-focusing in both transverse directions. This behavior can be attributed to beams of laser with larger q, where the majority of the energy of laser beam is localized and concentrated within narrow area near the axis of beam. Consequently, these beams obtain minimal aid from off-axis rays towards the nonlinear refraction phenomenon. As self-focusing stems from the equilibrium between the optical nonlinearity of the medium and the nonlinear refraction of the optical beam, an elevation in the q value diminishes the level of self-focusing demonstrated by the laser beam.

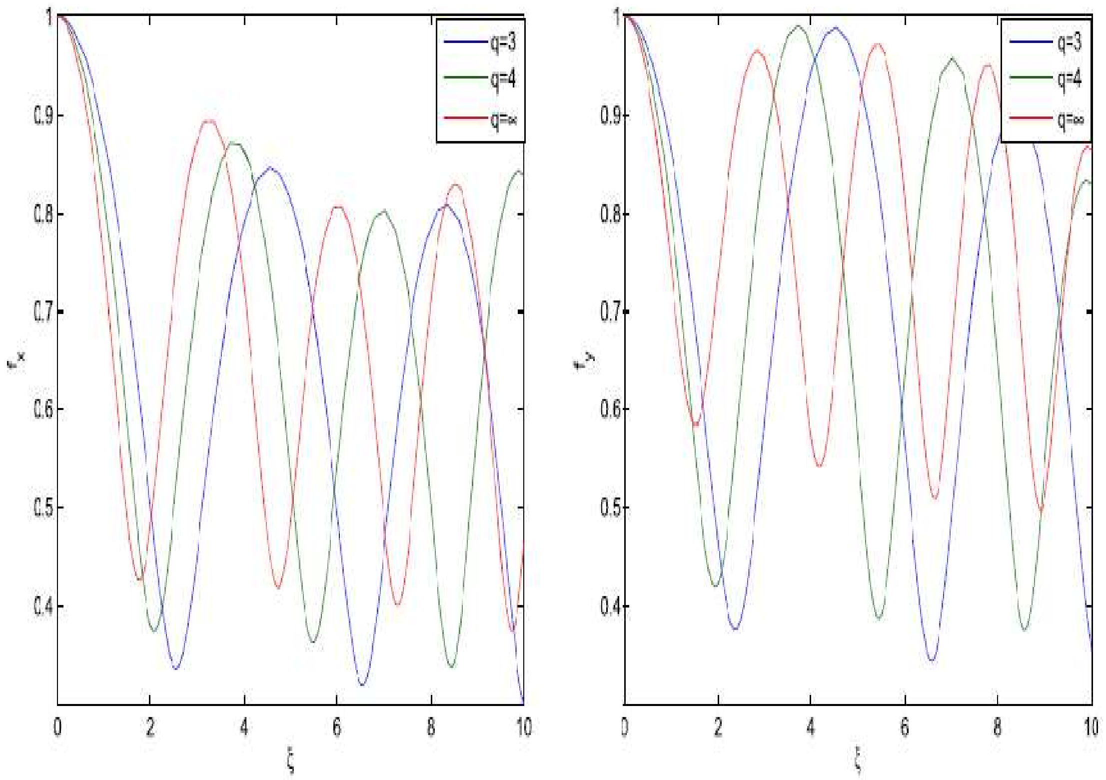

Fig.4.4: Impact of the deviation parameter q on the progression of the laser beam's spot size within the plasma, while maintaining a constant ramp slope and beam ellipticity

$$(d' = 0.25, \ \frac{a}{b} = 1.1)$$

Fig. 4.5 demonstrates the impact of beam ellipticity in the y direction on the self-focusing of a beam of laser light. The plot reveals that as ellipticity of beam increases in the y direction, a corresponding decrease in the degree of self-focusing is present along the same direction. This effect originates due to decrease in the initial width of the beam in the y direction with an increase in beam ellipticity (i.e.,$\frac{a}{b}$). Consequently, an increased beam ellipticity strengthens the diffraction effect in the y direction. Consequently the laser beam exhibits decreased focusing in the y direction due to the enhanced influence of diffraction caused by the increased beam ellipticity.

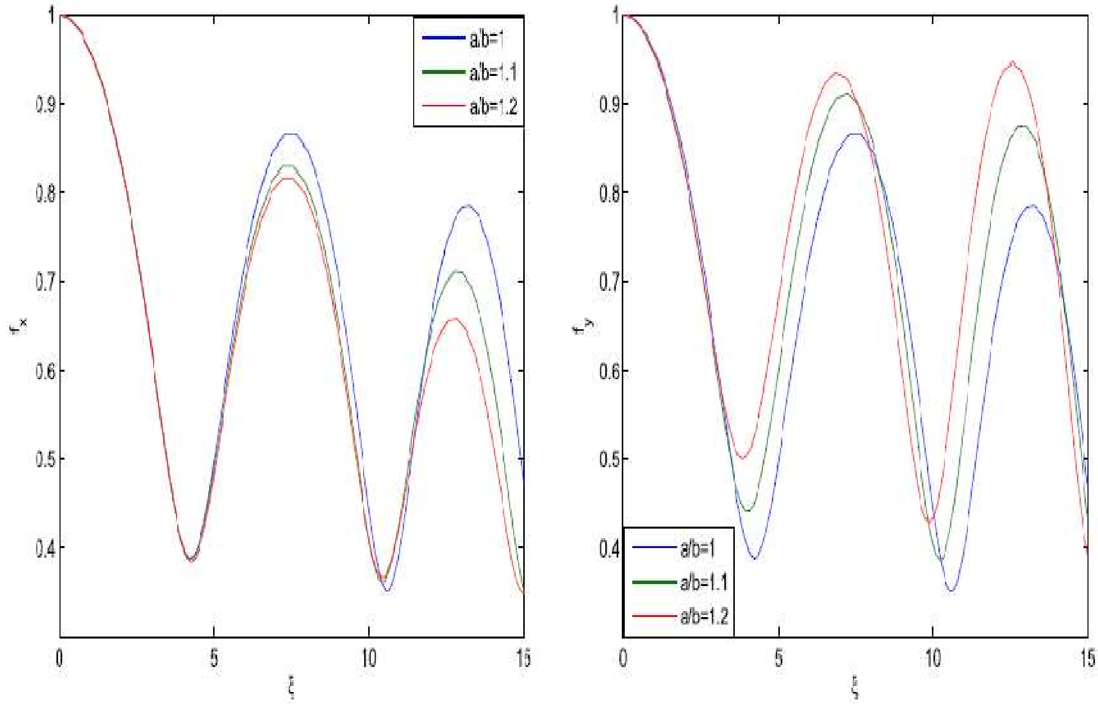

Fig.4.5: Impact of ellipticity of the beam of laser light on progression of its spot size as it propagates through plasma keeping deviation parameter and slope of the ramp fixed i.e., $q = 3$ and $d' = 0.25$.

Furthermore, it is apparent that at the initial stage, the increase in beam ellipticity has minimal impact on the self-focusing of the beam in the x direction. Nonetheless, as the beam continues to propagate through the plasma, the focusing of beam in the x direction progressively diminishes. That occurrence can be attributed to the strengthening nonlinear coupling between the two beam widths as the beam delves further into the plasma medium.

Fig. 4.6 depicts the influence of the gradient of the density ramp on the self-focusing of a beam of laser in both perpendicular directions. As the gradient of the density ramp increases, the degree of self-focusing exhibited by the beam of laser in two orthogonal directions becomes more pronounced, as observed in fig. 4.6. An increased gradient of the density ramp results in a larger the count of electrons influencing the relativistic nonlinearity in the propagation direction, which accounts for this behavior. Consequently, there is an enhanced transverse and longitudinal gradient in the refractive index within the plasma. This increased gradient ultimately amplifies the degree of self-focusing of the beam of laser in two perpendicular directions.

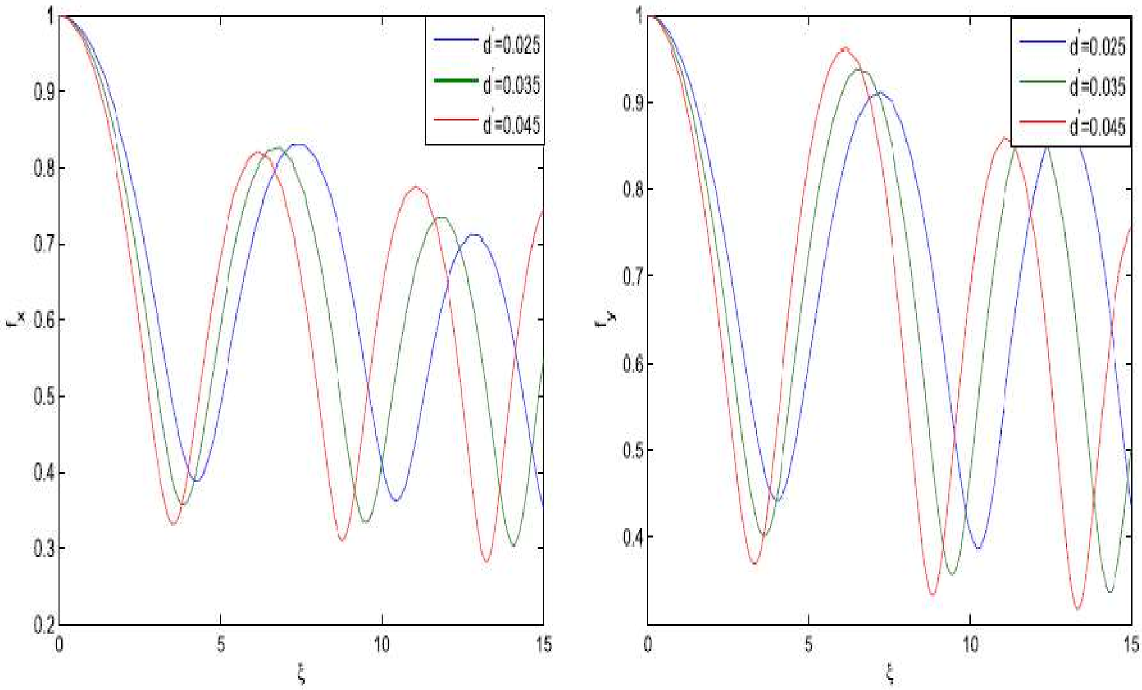

Fig.4.6: Impact of slope of density ramp on evolution of spot size of the laser beam through plasma keeping deviation parameter and ellipticity fixed i.e.,

$$q = 3 \text{ and } \frac{a}{b} = 1.1$$

4.4. Excitation of Ion Acoustic Wave

Laser beams can excite IAWs through primarily two techniques. The first technique involved in the excitation of IAWs by laser beams is known as resonance absorption. When a laser beam interacts with plasma, a significant portion of its oscillatory electric field energy is transferred to the neighboring electrons and ions. In the vicinity of the critical-density surface, an interesting phenomenon occurs where the natural oscillating frequency of IAW matches the frequency of the laser beam. At this specific depth within the plasma, the energy carried by the laser beam can resonantly drive IAWs to reach significant amplitudes. This phenomenon can be likened to a child on a swing who increases their height by exerting force at the precise timing that matches the natural motion of the swing. The second mechanism involved in the excitation of IAWs is known as three-wave mixing. In this process, an incident laser beam undergoes a decay process, resulting in the generation of two daughter waves. Typically, the strength of the mixing process is most pronounced when the waves that interact have large amplitudes. In the context of three-wave mixing, the frequency of the pump wave is equal to the combined frequencies of the two daughter waves.

There are two mechanisms by which three-wave mixing results in the excitation of IAWs. The first mechanism involved in the excitation of IAWs through three-wave mixing is referred to as decay instability. In the decay instability mechanism of three-wave mixing, the pump beam undergoes a splitting, resulting in the generation of an IAW and an EPW. (fig. 4.7).

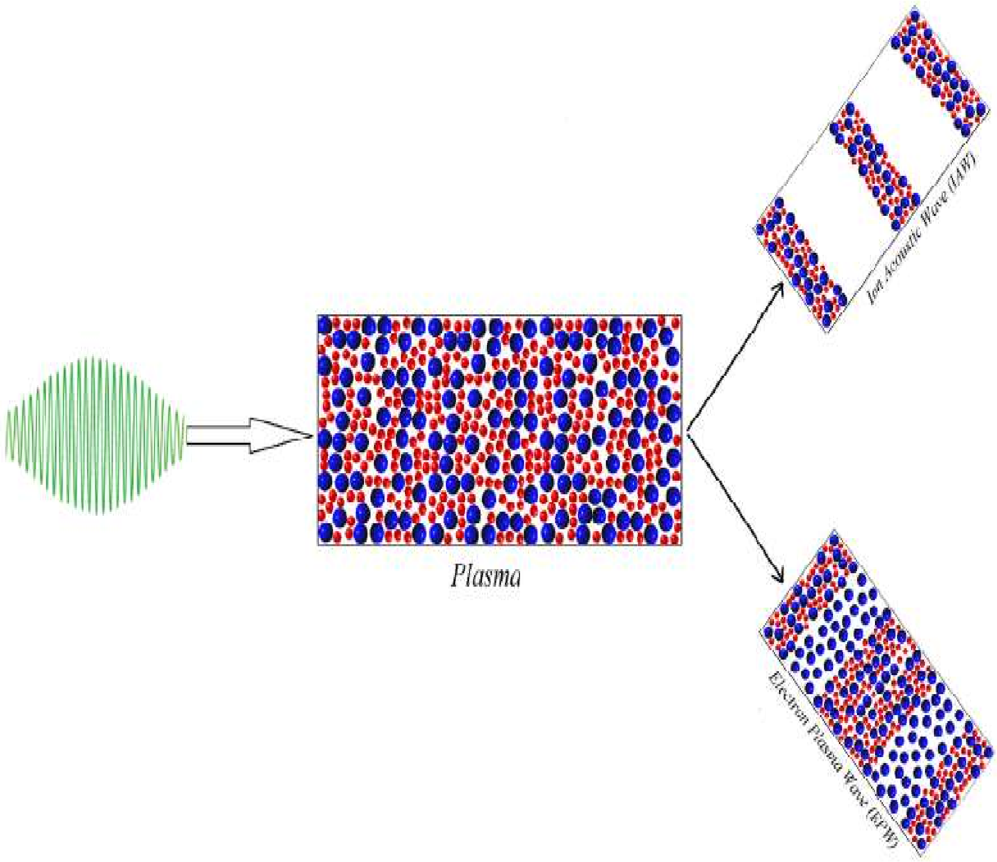

Fig.4.7: Excitation of IAW by decay instability

The second mechanism for the excitation of IAWs is known as Brillouin scattering. In Brillouin scattering, the daughter waves generated are an IAW and a reflected light wave (fig. 4.8). In this current study, the main mechanism considered for the excitation of IAWs is resonance absorption.

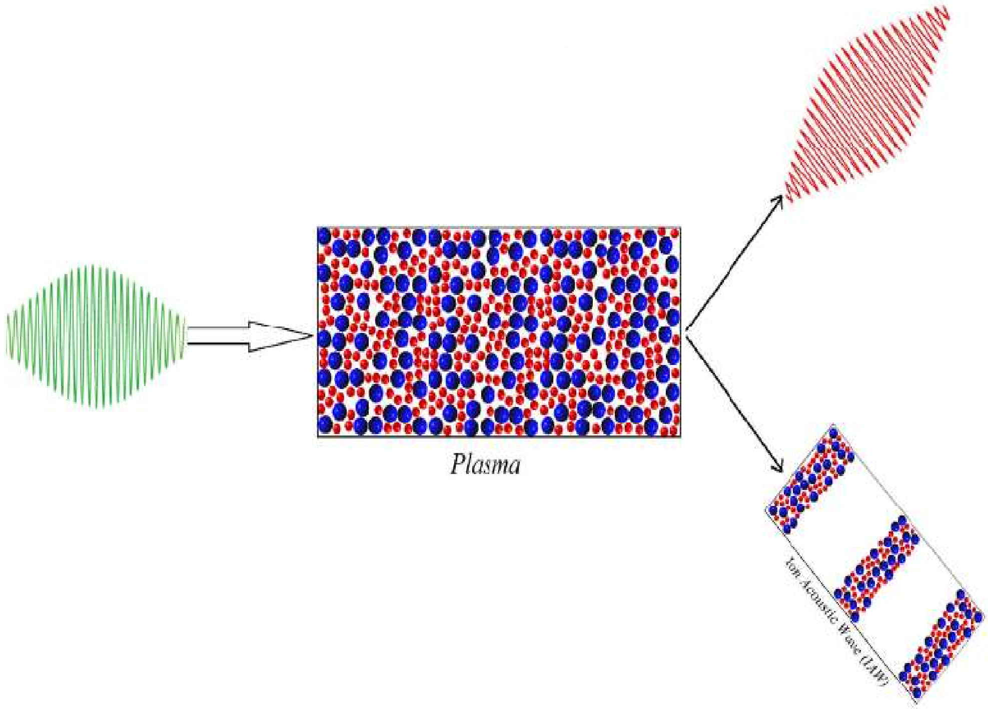

Fig.4.8: Excitation of IAW by SBS

The evolution of ion acoustic wave through plasma is determined by wave equation

$$2ik_{ia}\frac{\partial n_{ia}}{\partial z} = \nabla_{\perp}^{2} n_{ia} + \frac{\omega_{ia}^{2}}{v_{th}^{2}}(1+\tan(dz))\left(1-\frac{1}{(1+\beta A_0 A_0^{*})^{\frac{1}{2}}}\right)n_{ia} \tag{4.16}$$

where

$$v_{th} = \sqrt{\frac{2KT_i}{M_i}}$$

is the thermal velocity of ions.

Given the Gaussian ansatz for the IAW as

$$n_{ia} = \frac{n_{00}}{\sqrt{f_{xi} f_{yi}}} e^{-\left(\frac{x^2}{2a^2 f_{xi}^2}+\frac{y^2}{2b^2 f_{yi}^2}\right)} \tag{4.17}$$

and using the procedure of "Evolution of beam envelope" we obtain the equation that describes beam width evolution of IAW as

$$\frac{d^2 f_{xi}}{d\xi^2} = \frac{1}{f_{xi}^3} - \left(\frac{\omega_{ia}^2 a^2}{v_{th}^2}\right)^2 (1+\tan(d'\xi))\frac{1}{f_{xi}^2 f_{yi}} T \tag{4.18}$$

$$\frac{d^2 f_{yi}}{d\xi} = \left(\frac{a}{b}\right)^4 \frac{1}{f_{yi}^3} - \left(\frac{a}{b}\right)^2 \left(\frac{\omega_{ia}^2 a^2}{v_{th}^2}\right)^2 (1 + \tan(d'\xi)) \frac{1}{f_{yi}^2 f_{xi}} T \tag{4.19}$$

Where

$$T = \int_0^{2\pi} \int_0^{\infty} e^{-x\left(\frac{f_x^2}{f_{xi}^2}\cos(\theta) + \frac{f_y^2}{f_{yi}^2}\sin(\theta)\right)} \left(1 + \frac{x}{q}\right)^{-q-1} \left\{1 + \frac{\beta E_{00}^2}{f_x f_y}\left(1 + \frac{x}{q}\right)^{-q}\right\}^{-3/2} x dx d\theta$$

For the initially plane wavefront eqs. (4.18) and (4.19) are subject to boundary equations $f_{xi,yi} = 1$ and $\frac{df_{xi,yi}}{d\xi} = 0$ at $\xi = 0$.

Eqs. (4.18) and (4.19) illustrate the coupling between the IAW and the pump beam, which in this case is a q-Gaussian laser beam. These equations demonstrate that the density perturbations linked to the IAW are highly dependent on the self-focusing behavior of the beam of laser light. In other words, any changes in the self-focusing characteristics of the beam of laser light can significantly impact the density perturbations linked with the IAW. By employing Poisson's equation, it is possible to determine the electrostatic field generated by the excited Ion Acoustic Wave (IAW) given by

$$E_{ia} = E_{ia} e^{-i(k_{ia} z - \omega_{ia} t)} e_z \tag{4.20}$$

$$E_{ia} = \frac{im\omega_{ia}^2}{ek_{ia}\sqrt{f_{xi} f_{yi}}} e^{-\left(\frac{x^2}{2a^2 f_{xi}^2} + \frac{y^2}{2b^2 f_{yi}^2}\right)} \tag{4.21}$$

Eq. (4.21) provides the expression for the magnitude of the electrostatic field connected to the generated Ion Acoustic Wave (IAW). Eqs. (4.18) and (4.19) have been solved in conjunction with Eqs. (4.14) and (4.15) for $\omega_{ia} = \omega_0$ to obtain the field strength of excited IAW. The variations of the field strength of the Ion Acoustic Wave (IAW) as it propagates with distance have been illustrated in fig. 4.9, 4.10, and 4.11.

Evidently, the magnitude of the electric field linked to the generated Ion Acoustic Wave (IAW) exhibits oscillatory behavior as a function dependent on the propagation distance. Notably, the maximum field strength occurs precisely at the specified locations corresponding to the focal spots of the beam of laser light. The reason behind this behavior is that the amplitude of the Ion Acoustic Wave (IAW) is greatly affected by the degree of self-focusing of the laser light beam. When the pump beam undergoes self-focusing, its intensity grows, resulting in increased oscillation amplitude of the plasma electrons. Consequently, the increased amplitude of the plasma electrons causes a proportional rise in the amplitude of the IAW. The oscillatory evolution of the pump beam's widths in the two perpendicular directions is reflected in a similar oscillatory behavior of the magnitude of the generated Ion Acoustic Wave. Notably, the maximum field strength of the IAW occurs at the location where the pump beam exhibits the minimum beam width.

The graphical representations shown in fig. 4.9 demonstrate that the strength of the excited IAW decreases as the DP $'q'$ increases. This can be attributed to two factors:

(1) The focusing of the laser light beam is reduced as the DP $'q'$ of the pump beam increases.

(2) Convergence of intensity towards the beam axis: The intensity of the laser beam becomes more concentrated around the beam axis with increase in DP $'q'$. Consequently, the impact of the off-axial portion of the beam on the plasma electrons diminishes, leading to a decrease in the strength of the excited IAW.

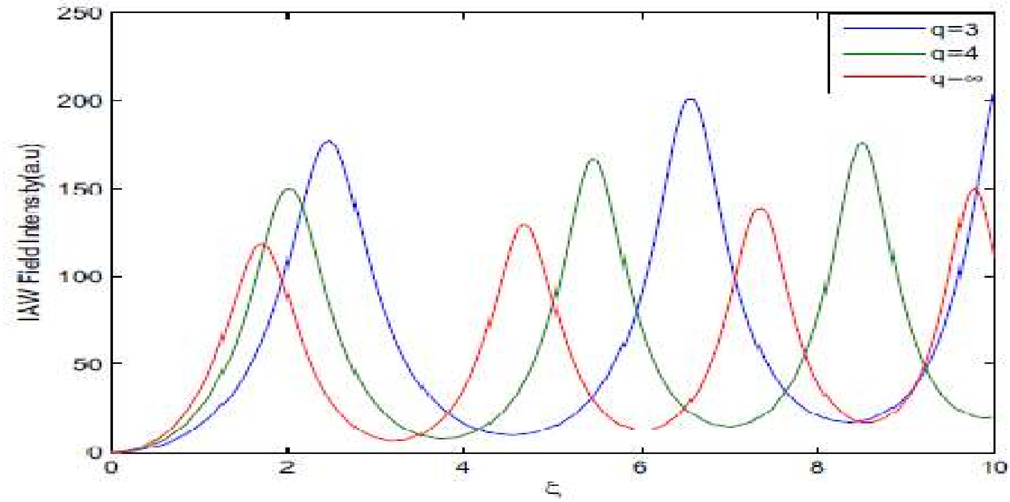

Fig.4.9: Impact of DP on evolution of field strength of excited IAW through plasma keeping slope of the ramp and ellipticity of the beam fixed i.e., $d' = 0.25$ and $\frac{a}{b} = 1.1$

Fig. 4.10 illustrates that as the ellipticity of the beam of laser increases, a notable reduction occurs in the magnitude of the electric field connected with the generated Ion Acoustic Wave (IAW). This reduction can be attributed to the overall decrease in the degree of self-focusing exhibited by the pump beam as its ellipticity increases.

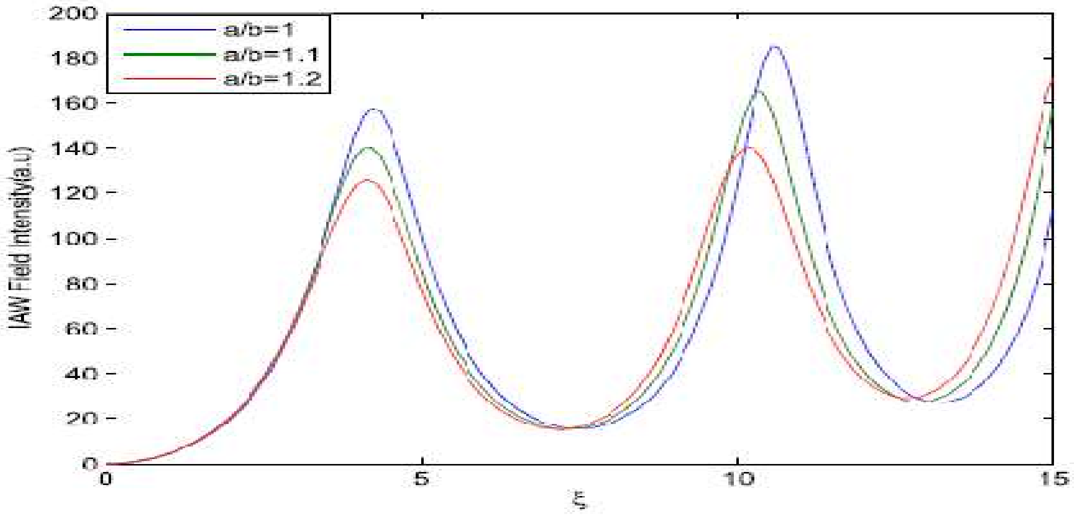

Fig.4.10: Impact of ellipticity of the laser beam on evolution of field strength of excited IAW through plasma keeping deviation parameter and slope of the ramp fixed i.e. $q = 3$ and $d' = 0.25$

Fig. 4.11 demonstrates the impact of the slope of the plasma density ramp on the field strength of the excited IAW. The plot reveals that the slope of the plasma density ramp significantly contributes to augmenting the intensity of the IAW. This is because the plasma density ramp significantly contributes to the enhancement of the IAW strength.

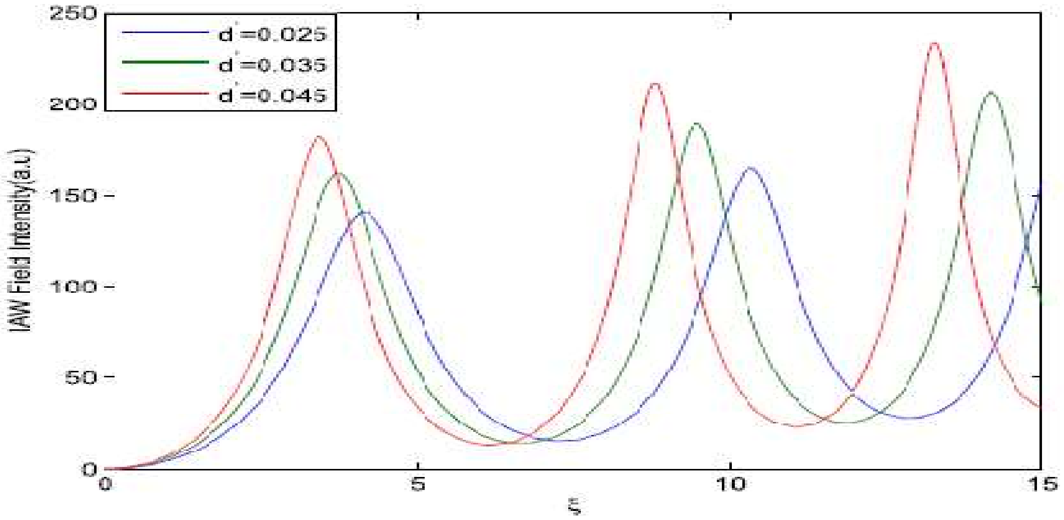

Fig.4.11: Impact of slope of the density ramp on evolution of field strength of excited IAW through plasma keeping deviation parameter and ellipticity fixed, i.e., $q = 3$ and $\frac{a}{b} = 1.1$

Chapter- 5

Relativistic effects on Stimulated Brillouin Scattering of self-focused q-Gaussian laser beams in plasmas with axial density ramp

5.1. Introduction

Ever since the scheme to initiate nuclear fusion using high-intensity laser beams (ICF) for viable energy production without producing any harm to global climate, there was a considerable interest in the context of laser beams of high intensity interacting with plasmas in a nonlinear manner. In the realm of laser-driven fusion, the objective is to accurately target laser energy onto a specific plasma density. This enables fuel pellet to be compressed and subsequently heated. With adequate compression, the pellet has the potential to undergo fusion, resulting in the release of a considerable energy. Nevertheless, the laser-plasma interaction may occur at a density different from the intended value, resulting in various undesirable effects that hinder the efficient implosion of the target.

Owing to their notable characteristics of quasineutrality and collective behavior, plasmas exhibit a range of inherent oscillation patterns. This encompasses high-frequency EPWs and low-frequency IAWs. The IAWs, similar to acoustical phonons in solids, exhibit characteristics akin to propagating sound waves. Consequently, it is logical to explore the possibility of Stimulated Brillouin Scattering (SBS) occurring in plasmas, just as it does in solids. Stimulated scattering processes involve the nonlinear interactions[67], where an incident wave interacts with the bosonic excitations of the medium, resulting in the conversion of the wave into a scattered wave that is either upshifted or downshifted in frequency. In the context of plasmas, the function of bosonic excitations is fulfilled by the traversing plasma oscillations, namely, EPWs or ion acoustic waves IAWs. The nonlinear medium either supplies or absorbs the energy difference between the incoming and scattered photon waves. SBS involves the interaction of light with sound waves in solids, liquids, and gases or with IAWs in plasmas.

Brillouin scattering is the scattering of light from propagating sound waves. In the case of stimulated Brillouin scattering, the sound wave or pressure-density variation is not externally imposed on the material; instead, it is internally stimulated by a pair of counter-propagating light waves. Similar to how sound can be conceptualized as a pressure-density wave, light can be considered as a traveling electric field.

An electric field can induce compression in materials a phenomenon known as the electrostrictive effect. If an electric field pattern moves at the speed of sound through a material, it can therefore give rise to a sound wave. Such an electric field pattern can be generated by the interference of two optical beams traveling in opposite directions, if the frequency difference between the two beams is equal to the frequency of the sound. In the case of stimulated Brillouin scattering, one of the beams is the incident light beam.

The other light beam arises from the scattering of the incident beam by small, statistically-distributed density fluctuations in the medium (i.e., thermally-fluctuating sound waves). When the frequency and direction of a scattered wave are just right, the wave will interfere with the incident beam and amplify the pressure-density variations in the material. The variations subsequently lead to the reflection of a minute portion of the incident beam. The reflected portion interferes, in turn, with the incident beam generating more pressure-density variations. The pressure-density variations lead to more reflections of the incident beam. The reflections build exponentially as distance increases, until a reflected beam emerges from the material. Although the amplification depends on the intensity of the incident beam, nevertheless, a prerequisite for producing the reflected beam is that the power of the incident beam must exceed a certain threshold. In the context of plasmas, the function of the sound wave is fulfilled by the IAW.

The discovery of SBS was initially noted by Chio et al. in 1964, shortly after Maiman's invention of the laser in 1960[1]. Thereafter, numerous studies examining both the theoretical and experimental facts of SBS have been conducted by various scholars.

Gaussian irradiance profiles have been predominantly considered, assuming that the laser operates in the TEM_{00} mode. However, experimental investigations of laser beam irradiance across its cross section have revealed that though the laser may be functioning in the TEM_{00} mode, the irradiance distribution is not perfectly Gaussian. A substantial part of the beam energy distribution is observed to deviate from the axis of the beam, resulting in expanded intensity wings. Based on experimental findings there have been suggestions that that the irradiance distribution can be effectively simulated using a group of distribution functions called Tsallis q −Gaussian distributions. To date, no theoretical exploration of SBS in plasmas featuring density ramps when subjected to q-Gaussian light emitted by laser. The purpose of this article is to introduce the initial theoretical exploration of Stimulated

Brillouin Scattering (SBS) for relativistically-focused q-Gaussian laser light in plasmas with a varying density along the axial direction.

5.2 Relativistic Dielectric Function of Plasma

The plasma produced during ICF by the irradiance of fuel pellet by intense laser beams expands radially outwards. Hence, as seen from the side of observer, the plasma density tends to increase as distance increases. One possible representation for this plasma density is a gradually rising ramp as $n(z) = n_0(1 + \tan(az))$. The dielectric function for this type of plasma is defined in the following manner[62]:

$$\varepsilon = 1 - \frac{4\pi e^2 n_0}{m_e \omega_0^2}[1+\tan(az)], \tag{5.1}$$

In this equation, e represents the charge of an electron, while m_e corresponds to the mass of an electron. The constant 'a' quantifies the rate at which the electron density changes as a function of distance and it is frequently denoted as the gradient of the density ramp. At $z = 0$, n_0 represents the equilibrium density of the plasma electrons, ω_0 represents the angular frequency of the incident beam of laser. During the transmission of intense laser light having an electric field vector

$$E(r, z, t) = A_0(r, z)e^{i(k_0 z - \omega_0 t)}e_X \tag{5.2}$$

into a plasma target, it induces oscillations in the plasma electrons, resulting in their motion with velocity

$$v = -i\frac{e}{\omega_0 m_e}E \tag{5.3}$$

Here, $A_0(r, z)$ represents the amplitude field of the laser beam and r is the radial distance measured from the axis of the laser light. When magnitude of the electric field strength in the laser light beam reaches a certain threshold then quiver velocity of plasma electrons reaches a magnitude similar to the speed of light in a vacuum. Therefore, when high intense laser light interacts with plasma, the mass of plasma electrons undergoes relativistic growth. Thus, in Eq. (5.1), the effective mass m_e of plasma electrons must be substituted with $m_e = \gamma m_0$. The parameter γ is related to field strength as[62]:

$$\gamma = (1 + \beta A_0 A_0^*)^{1/2} \tag{5.4}$$

Where β represents a constant defined as $\beta = \frac{e^2}{m_0^2 c^2 \omega_0^2}$. It is related to magnitude of relativistic nonlinearity and is often referred as "relativistic nonlinearity coefficient". Therefore, when a laser beam is present, Eq. (5.1) undergoes modification and can be expressed as follows:

$$\varepsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}(1 + \beta A_0 A_0^*)^{-\frac{1}{2}}(1 + \tan(az)) \quad (5.5)$$

where $\omega_{p0} = \sqrt{\frac{4\pi e^2 n_0}{m_0}}$ represents plasma frequency when there is no laser beam present. Writing Eq. (5.5) as

$$\varepsilon = \varepsilon_0 + \phi(A_0 A_0^*)$$

we arrive at

$$\varepsilon_0 = 1 - \omega_{p0}^2/\omega_0^2$$

and

$$\phi(A_0 A_0^*) = \frac{\omega_{p0}^2}{\omega_0^2}[1 - \frac{1}{(1+\beta A_0 A_0^*)^{1/2}}][1 + \tan(az)] \quad (5.6)$$

5.3 Dynamics of Pump Beam

The mathematical representation of the propagation behavior of a laser beam in a nonlinear medium is characterized by the wave equation, obtainable from the Maxwell equations

$$2ik_0 \frac{\partial A_0}{\partial z} = \nabla_\perp^2 A_0 + \frac{\omega_0^2}{c^2} \phi(A_0 A_0^*) A_0 \quad (5.7)$$

Lagrangian density relating to eq. (5.7) is:

$$\mathcal{L} = i\left(A_0 \frac{\partial A_0^*}{\partial z} - A_0^* \frac{\partial A_0}{\partial z}\right) + |\nabla_\perp A_0|^2 - \frac{\omega_0^2}{c^2}\int^{A_0 A_0^*} \phi(A_0 A_0^*) d(A_0 A_0^*) \quad (5.8)$$

In this study, the functional form of $A_0(r, z)$ is taken as

$$A_0(r, z) = \frac{E_{00}}{f}\left(1 + \frac{r^2}{q r_0^2 f^2}\right)^{-q/2} \quad (5.9)$$

Through the substitution of the trial function into the Lagrangian density and subsequent integration over the variable $'r'$, the reduced Lagrangian can be obtained as $L = \int_0^\infty \mathcal{L} r dr$. The resulting Euler Lagrange equation is then derived as

$$\frac{d}{dz}\left(\frac{\partial L}{\partial(\frac{\partial f}{\partial z})}\right) - \frac{\partial L}{\partial f} = 0 \quad (5.10)$$

yields the subsequent ordinary differential equation governing the laser beam width:

$$\frac{d^2f}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f^3} - \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2 {r_0}^2}{c^2}\right)\left(1+\tan(d\xi)\right)\frac{\beta E_{00}^2}{f^3}T \qquad (5.11)$$

where

$$T = \int_0^\infty x\left(1+\frac{x}{q}\right)^{-2q-1}\left(1+\frac{\beta E_{00}^2}{f^2}\left(1+\frac{x}{q}\right)^{-q}\right)^{-3/2} dx, \quad x = \frac{r^2}{r_0^2 f^2}, \; d = ak_0 r_0^2, \; \xi = \frac{z}{k_0 r_0^2}.$$

In this study, we solved Eq. (5.11) numerically using Runge-Kutta fourth-order method. The analysis assumes that as the beam enters the plasma medium, it is initially collimated and possesses a planar wavefront. These conditions impose the following boundary conditions $f = 1$ and $\frac{df}{d\xi}$=0 at $\xi = 0$.

Fig. 5.1 illustrates the change in normalized density $\frac{n(\xi)}{n_0}$ as a function of longitudinal distance ξ for varying values of d. The influence of factor d is evident in the fig. 5.1, as it amplifies gradient of the ramp, resulting in a higher rate of plasma density growth with increasing distance.

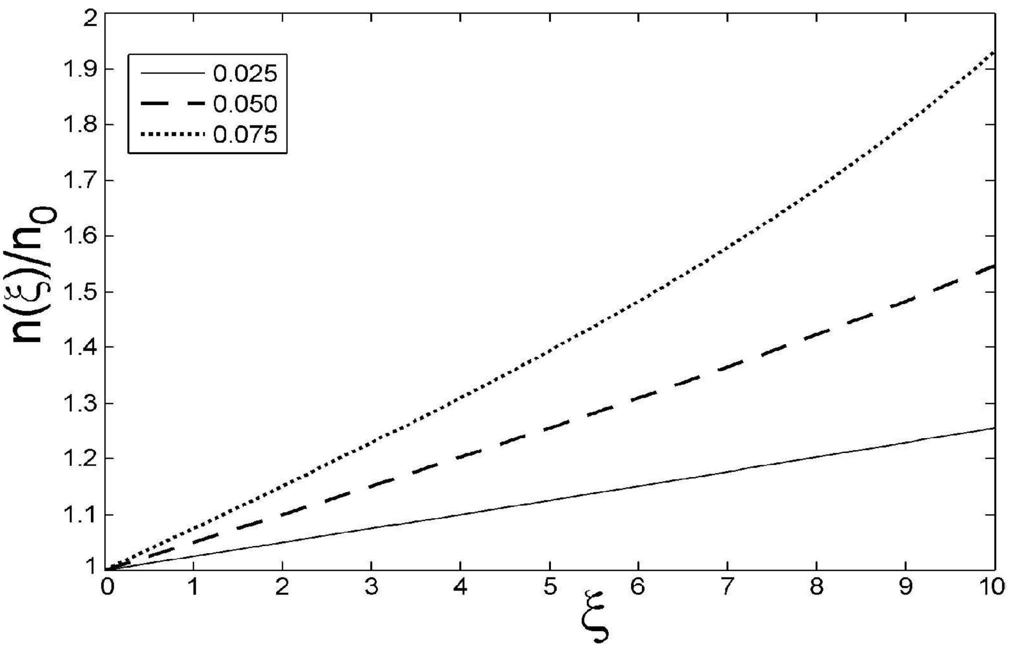

Fig.5.1: Change in normalized density $n(\xi)/n_0$ with longitudinal distance ξ for different values of d (the solid curve for $d = 0.025$, the dashed curve for $d = 0.050$, and the dotted curve for $d = 0.075$

To observe the linear propagation of a q-Gaussian laser beam, Eq. (5.11) needs to be solved in a vacuum (absence of a plasma medium). The resulting fluctuations are depicted in fig. 5.2.

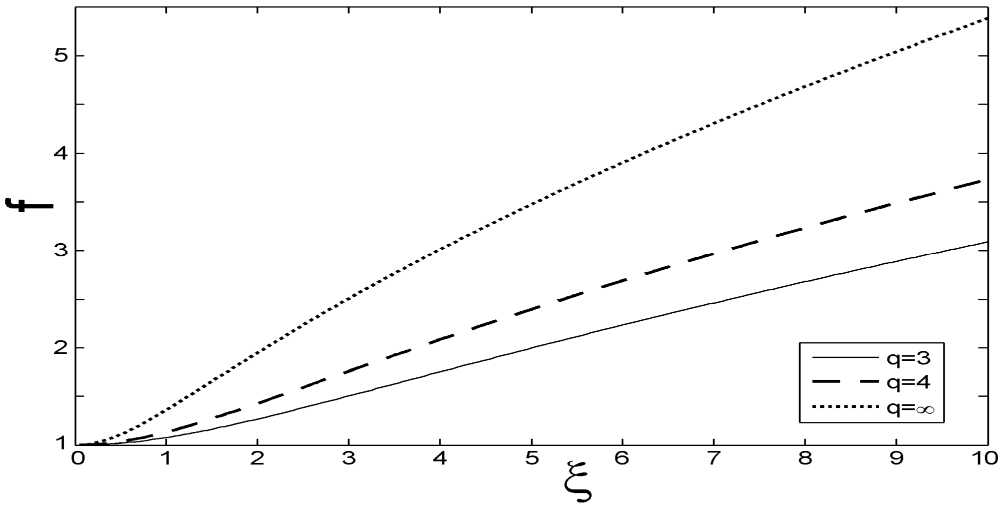

Fig.5.2: Effect of q (the solid curve for $q = 3$, the dashed curve for $q = 4$ and the dotted curve for $q = \infty$) on vacuum diffraction of laser beam

It is clear that the laser beam will diffract in a vacuum irrespective of the value of q. This behavior arises from the fundamental wave property of light, which naturally leads to diffraction phenomena. A noteworthy finding is that when the beam profile approaches a Gaussian distribution, the laser beam experiences enhanced diffraction. This phenomenon can be comprehended by analyzing the initial beam profile shown in fig. 3.1 or considering the influence of q on the initial beam width depicted in fig. 3.2. In these figures, it is evident that laser beams with intensity distributions closer to a Gaussian profile have smaller effective beam widths. The diffraction-induced broadening of an optical beam exhibits an inverse relationship with its beam width. Therefore, an augmentation in the q value enhances the diffraction of the beam of laser light, making it more pronounced or stronger.

To observe the progression of the beam profile as it propagates within the plasma, we solve Eq. (5.11) for

$$\omega_0 = 1.78 \times 10^{14} \text{ rad}/sec, \quad r_0 = 10\mu m, \quad \frac{\omega_{p0}^2 r_0^2}{c^2} = 12.$$

Fig. 5.3 demonstrates the associated evolution of the beam width as it travels within the plasma. It is apparent from the fig. 5.3 that width of laser light undergoes harmonic variations with distance during its propagation within the plasma.

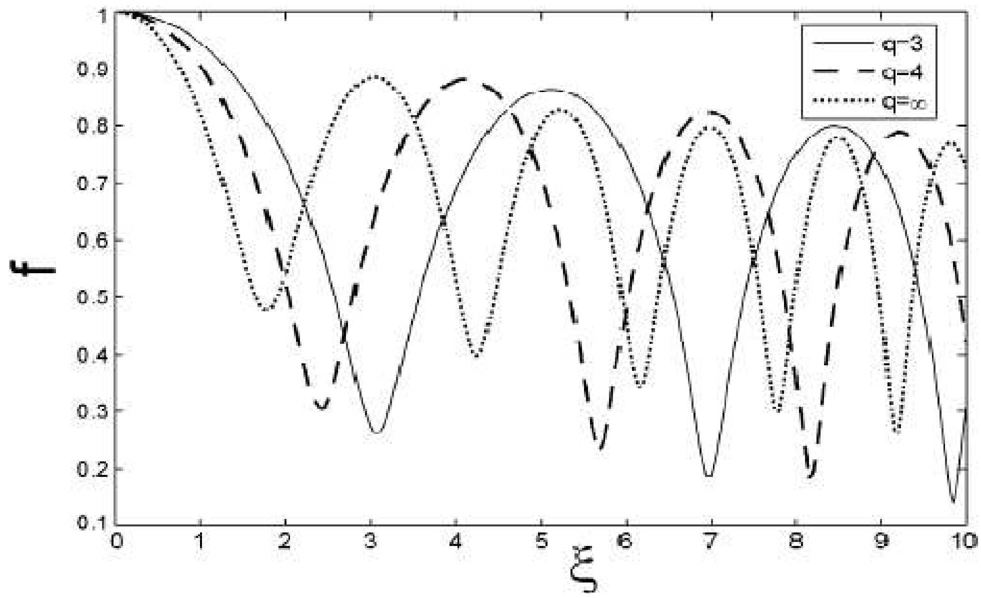

Fig.5.3: Effect of q (the solid curve for $q = 3$, the dashed curve for $q = 4$, and the dotted curve for $q = \infty$ on the beam width evolution in plasma with the longitudinal distance ξ

Fig. 5.3 also exhibits the diminished focusing of laser beams at higher q values where we take $q = \infty$ to compare the results of present study with that for ideal Gaussian beams. One can see that the laser beam with $q = 3$ obtains a minimum beam width of 2μm at a distance of $2.8z_R$, where $z_R = k_0 r_o^2/2$ is Rayleigh length. Compared to this laser beam with $q = \infty$ (i.e., ideal Gaussian beam), we obtain a minimum beam width of 2.3 μm and 5.5μm at distances of $2z_R$ and $1.6z_R$, respectively. Consequently, it can be concluded that as the deviation parameter rises, while maintaining the other laser-plasma parameters constant, the self-focusing of the beam of laser light diminishes.

The impact of the initial beam intensity on the self-focusing of the beam of laser light has been depicted in fig. 5.4; here, $\beta E_{00}^2 = 1.5, 2$ and 2.5 correspond to laser intensities of $1.2.10^6, 1.6.10^6$ and $2.10^{16} W/cm^2$, respectively. One can see that, as the initial intensity of the laser beam rises, a substantial improvement is observed in both the magnitude and rapidity of self-focusing. This occurs due to the strengthening of the relativistic effect when the intensity of the pump beam increases. Consequently, this leads to an increased transverse gradient of the refractive index within the exposed section of the plasma. Consequently, the degree of focusing of laser light increases by amplifying the beam's intensity.

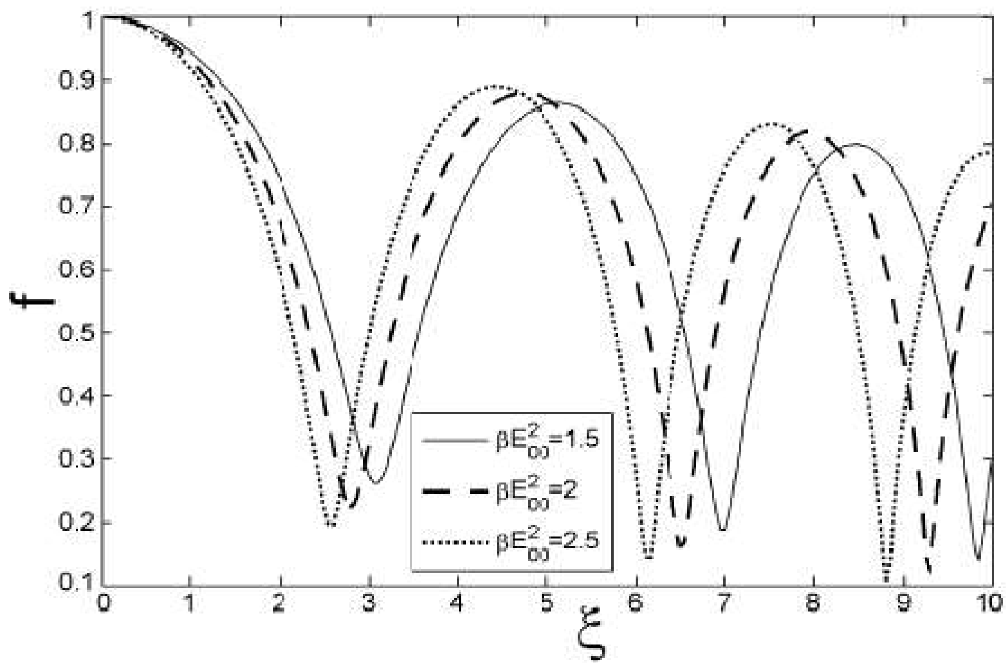

Fig.5.4: Impact of the initial beam intensity on the evolution on the laser beam width; here, $\beta E_{00}^2 = 1.5$ (the solid curve), 2 (the dashed curve), and 2.5 (the dotted curve) correspond to laser intensities of $1.2 .10^{16}$, $1.6 .10^{16}$, and 2.10^{16} W/cm^2, respectively

In fig. 5.5, we illustrate the impact of the density ramp slope on the laser-beam self-focusing. This figure demonstrates that an increase in the density ramp slope results in an enhanced degree of self-focusing in the laser beam. This happens because of the reduction in the refractive index of the plasma in deeper regions, associated with a rise in the gradient of the density ramp. As a result, the laser beam encounter a lower refractive index, leading to enhanced self-focusing. Consequently, amplification in the gradient of the density ramp amplifies the degree of focusing

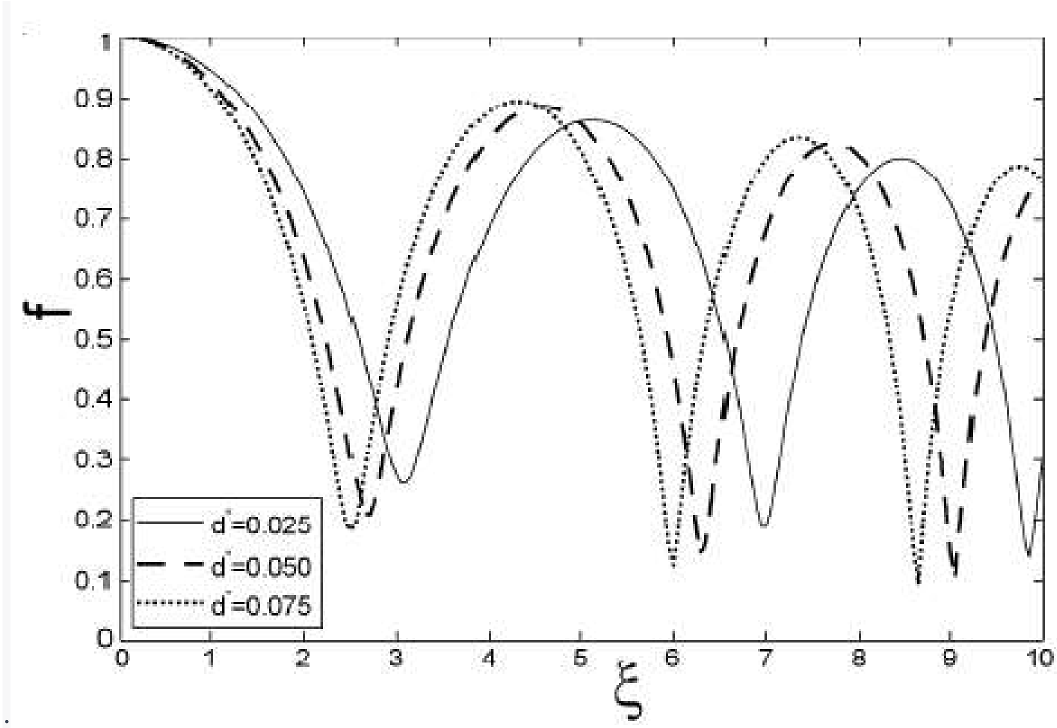

Fig.5.5: Impact of the density ramp slope (the solid curve for d' = 0.025, the dashed curve for d' = 0.050, and the dotted curve for d' = 0.075) on the beam width evolution

5.4 The Evolution of Ion Acoustic Wave

Plasma as a whole maintains quasineutrality; however, due to the separation of electrons and positively charged ions, disturbances can give rise to regions with net negative and net positive charges, resembling the charged plates in a parallel plate capacitor. The non uniform arrangement of charges in such cases set up an electric field that spans from the regions of positive charge to those of negative charge. The presence of this electric field exerts equal forces that attract the electrons and ions towards each other. As a result of this attractive force electrons and ions start moving towards each other. While the electrons and ions approach each other, their velocity and momentum gradually increase, analogous to a pendulum moving from an extreme position towards its mean position. The increase in momentum causes the electrons and ions to surpass their equilibrium positions, leading to a reversal in the electric field direction. With the electric field now reversed, it acts in opposition to the motions of electrons and ions, causing them to decelerate and eventually be pulled back in the opposite direction. The process continues to repeat itself in a cyclic manner, creating an electron-ion oscillator. When considering the influence of thermal velocity, these oscillations between electrons and ions result in the generation of a longitudinal wave that alternately compresses and rarifies the density of the plasma. This wave propagates through the plasma and is known as an ion acoustic wave.

The spatiotemporal evolution of the density perturbation n_{ia} related with IAW in plasma with the refractive index specified by Eq. (5.6) is given by the wave equation

$$2ik_{ia}\frac{\partial n_{ia}}{\partial z}=\nabla_{\perp}^{2}n_{ia}+\frac{\omega_{ia}^{2}}{v_{th}^{2}}[1+\tan(az)](1-\frac{1}{(1+\beta A_0 A_0^*)^{1/2}})\, n_{ia} \tag{5.12}$$

where ω_{ia} is the angular frequency of IAW and $v_{th}=\sqrt{\frac{2KT_i}{M_i}}$

By considering the Gaussian ansatz for the Ion Acoustic Wave (IAW) as follows:

$$n_{ia}=\frac{n_{00}}{f_i}\exp\left(-\frac{r^2}{2r_0^2 f_i^2}\right) \tag{5.13}$$

By employing the Gaussian ansatz for the IAW and applying the procedure outlined in section 5.3, we derive the equation for the beam's evolution in the presence of the IAW as follows:

$$\frac{d^2 f_i}{d\xi^2}=\left(\frac{\omega_0}{\omega_{ia}}\right)^2[\frac{1}{f_i^3}-\left(\frac{\omega_{ia}r_0}{v_{th}^2}\right)^2[1+\tan(d\xi)]\frac{1}{f_i^3}I \tag{5.14}$$

where

$$I=\int xe^{-f^2 t/f_i^2}\left(1+\frac{x}{q}\right)^{-(q+1)}\left\{1+\frac{\beta E_{00}^2}{f^2}\left(1+\frac{x}{q}\right)^{-q}\right\}^{3/2}dx$$

For the initially plane wavefront, Eq. (5.14) is subject to the boundary conditions $f_i=1$ and $\frac{df_i}{d\xi}=0$ at $\xi=0$.

5.5 The Evolution of Scattered Wave

The nonlinear interaction between the pump beam and the IAW gives rise to a nonlinear current density J_{NL} at frequency $\omega_s=\omega_0-\omega_{ia}$ given by

$$J_{NL}=\frac{e^2 n_0}{m_0\omega_s}\frac{n_{ia}}{n_0}e^{i(\omega_s t-k_s z)}A_0(r,z) \tag{5.15}$$

A scattered wave emerges as a consequence of the nonlinear current density and its evolution is given by the wave equation.

$$\nabla^2 E_s = \frac{1}{c^2}\frac{\partial^2 E_s}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial J_{NL}}{\partial t} \tag{5.16}$$

The above equation determines the magnitude of the electric field for the scattered radiation, which can be expressed as follows:

$$E_s = i\frac{\omega_{p0}^2/c^2}{(\omega_s^2/c^2)-k_s^2}\frac{n_{ia}}{n_0}A_0(r,z)rdr \tag{5.17}$$

Through the definition of normalized power for scattered radiation, we can express it as follows:

$$P = \frac{\int E_s E_s^* rdr}{\int A_0 A_0^* rdr} \tag{5.18}$$

We arrive at

$$P = \frac{(\omega_{po}^2/c^2)^2}{\left[\left(\frac{\omega_s^2}{c^2}\right)-k_s^2\right]^2}\frac{\int (n_{ia}/n_0)^2 A_0 A_0^* rdr}{\int A_0 A_0^* rdr} \tag{5.19}$$

Eq. (5.19) describes the variation in the power of the scattered wave as it propagates over distance. We numerically solved Eq. (5.19) along with Eqs. (5.11) and (5.14) for the ion temperature $T_i = 10^6 K$, $\omega_{ia} = 10^{12}$rad/s, $\frac{n_{00}}{n_0} = 0.001$ to gain insight into how the power of a scattered wave changes as it propagates longitudinally through the plasma. Fig. 5.6 illustrates the corresponding variation of the power (*P*) as a function of the longitudinal distance (ξ). It is noticeable that the power of the scattered radiation exhibits a step-like behavior and monotonically increases in relation to the propagation distance. Each step in the power of the scattered radiation occurs precisely at the position where the beam width reaches its minimum value. This occurs due to the self-focusing of the pump beam, leading to an amplified intensity. As a consequence, the amplitude of the plasma species, including electrons and ions, oscillations likewise experiences an increase, subsequently amplifying the amplitude of the generated IAW. Indeed, the density perturbations linked with the IAW result from the scattering of the pump wave. As a consequence, there is a continuous and monotonic increase in the power of the scattered wave as the distance of propagation increases.

The step-like behavior observed in the power of the scattered wave coinciding with positions of the minimum width of the laser pump beam arises due to the correlation with the highest intensity in these regions. Consequently, in these regions, the maximum current density is observed for scattered radiation.

Therefore, following its peak value at the initial focal point, the scattered radiation encounters a sudden rise at the subsequent focal point. These abrupt spikes in the scattered radiation at the pump beam's focal points contribute to the observed step-like pattern in the scattered wave's power.

Furthermore, it is noted that as the longitudinal distance increases, the steps in the power of the scattered wave become steeper while the size of each step diminishes. The reason behind the increasing steepness of the steps is that it is directly related to the degree of self-focusing. On the other hand, the decrease in step size is attributed to the inverse proportionality between step size and the oscillation frequency of the beam width. It has been noted that as the distance progresses, both the degree and frequency of self-focusing increase. Therefore, the steps become steeper as observed in the P vs. ξ curves, while simultaneously causing a decrease in the size of each step.

One can see that at a distance of $10z_R$ through the plasma, the power of scattered wave for laser beam with $q = 3$ is $0.44.10^{-4}$ P_0 and, for ideal Gaussian beam, it is $1.1.10^{-4}$ P_0 where P_0 is the incident beam power. The plots presented in Fig. 5.6(a) also illustrate the decrease in the rate of amplification of scattered radiation as the deviation parameter (DP) $'q'$ increases in value. The fundamental physics that explain this observation is the direct correlation between the power of the scattered wave and the degree of self-focusing exhibited by the pump beam. As q increases, causing a decrease in the degree of focusing within the pump beam, a corresponding decline in the scattered radiation power is observed. Thus, one can conclude that SBS of Gaussian beam is minimum. Indeed, this effect arises from the minimum focusing of the laser light beam in the plasma medium. Consequently, the SBS of a laser light can be controlled by fine-tuning the magnitude of its DP $'q'$. Figures 5.6(b) and (c) illustrate that the amplification of SBS can be achieved by raising either the initial intensity of the pump beam or the gradient of the density ramp. This effect arises as a consequence of the increased degree of self-focusing in the pump beam, which occurs when its initial intensity or the density ramp slope is increased.

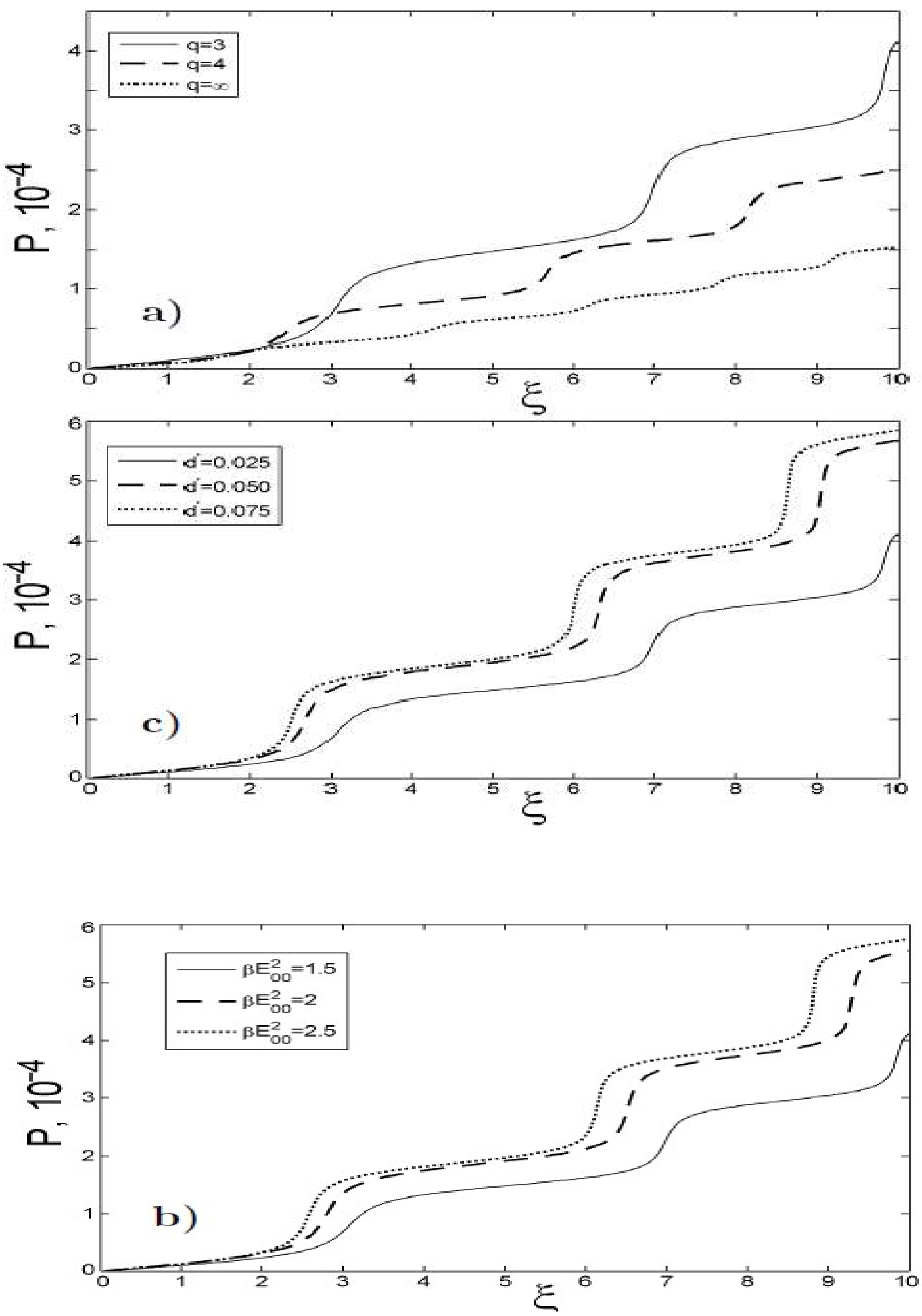

Fig.5.6: The evolution of the scattered wave power with the longitudinal distance within the plasma; here, effect of q (a) (the solid curve for $q = 3$, the dashed curvefor $q = 4$, and the dotted curve for $q = \infty$), the initial intensity (b) (the solid curve for $\beta E_{00}^2 = 1.5$,, the dashed curve for $\beta E_{00}^2 = 2$, and the dotted curve for $\beta E_{00}^2 = 2.5$), and the slope of the density ramp (c)(the solid curve for d' = 0.025, the dashed curve for d' = 0.050, and the dotted curve for d'= 0.075)

Chapter- 6

Self-Focusing effected Stimulated Brillouin Scattering in Density Ramped Plasma

6.1. Introduction

Powering the world without harming the global climate is a mounting concern. Thus, in the past few decades, developed countries have added substantial amounts of solar, geothermal, wind, and biomass power to decarbonize electrical power production. Despite the presence of renewable power sources, they are insufficient to satisfy humanity's insatiable demand for energy. Regarding this matter, the suggestion to commence nuclear fusion through the use of powerful laser beams (known as inertial confinement fusion or ICFs) for viable energy production has been at the vanguard of research since the successful realization of the thermonuclear fusion – the hydrogen bomb. The objective in laser-driven fusion is to focus laser energy onto a specific plasma density, inducing compression and then heating of the fuel pellet. With adequate compression, the pellet has the potential to undergo fusion, resulting in the release of a significant amount of energy. It can be likened to having a miniature piece of the sun on Earth.

The fascination and obstacles of fusion emanate from the inherent properties of the fusion process by itself. Abundant and affordable, fusion fuel is readily available. The notable benefits include the ample availability of fuel, with deuterium and tritium being the most readily accessible and exploitable fuels. Deuterium is naturally present in all water sources, particularly seawater.

However, tritium is not naturally abundant and can be produced within the fusion reactor by bombarding lithium with abundant neutrons, which are also readily available in nature. Fusion, known as the cleanest energy source, does not directly generate nuclear waste. It is worth noting that tritium possesses mild radioactivity. Furthermore, the activation of the reactor chamber by neutrons influences the selection of valuable structural materials, aiming to minimize the disposal of components during maintenance or the decommissioning of the entire reactor assembly at the end of its operational life.

In contrast, fission reactions take place at average temperatures, while fusion reactions occur only at extremely high temperatures, similar to those found in stars. The fuel with the lowest ignition temperature is a combination of deuterium and tritium, which ignites at temperatures approximately around 50 keV (i.e., 50×10^6K). At such high temperatures, the fuel undergoes complete ionization, transforming into a plasma state. This is why plasma physics holds a vital significance within the realm of fusion research.

In the case of inertial confinement fusion (ICF), lasers can sometimes interact with the plasma at densities that differ from the intended conditions. This could lead to undesired effects and hinder the efficient implosion of the target. This encompasses a wide range of phenomena, including the self-action effects of the laser beam, as well as various stimulated scattering processes like stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS). Stimulated scatterings are the processes in which an incident electromagnetic beam interacts nonlinearly with bosonic excitations of a medium and hence transformed into a scattered wave with a frequency either higher (up-shifted) or lower (down-shifted). The nonlinear medium supplies the difference in the photon energy between the incident and scattered waves. Within the context of plasmas, the propagating plasma waves, commonly referred to as ion acoustic waves (IAWs), fulfill the role of bosonic excitations. SBS involves the interaction of light with sound waves in solids, liquids, gases or with IAWs in plasmas. In the case of ICF, the crests of these ion acoustic waves (IAWs) act as partially reflecting mirrors for the incident laser beam. Consequently a considerable amount of laser energy is reflected by these waves, leading to a decrease in the effectiveness of laser-plasma interaction. Thus, in the context of ICF it becomes vital to investigate different aspects of the phenomenon of SBS.

It is not SBS that always plays the role of villain. By serving as a diagnostic tool in experiments on laser– plasma interactions, it also plays the hero role. By measuring the frequency shift between the pump wave and the scattered wave, one can determine the different parameters of plasma-like electron and ion temperatures, ion density, conductivity, refractive index of plasma, etc. The significant advantage of using SBS as a diagnostic tool is that it does not require any physical probe to be inserted into the plasma. Another vital application of SBS is optical phase conjugation, which is known as time-reversal or wavefront reversal. Optical phase conjugation is a method that involves generating an optical beam with wavefront or phase variations that are reversed compared to a reference beam. The time-reversed process contradicts our everyday experience, and for good reason: the sequences of events disobey the second law of thermodynamics (the law that states that systems tend toward maximum entropy). However, the scenario can be played successfully if the actor is the wave motion of light or some other electromagnetic radiation. Such a phenomenon is possible because of a remarkable property of light rays that has been long recognized namely the reversibility of their propagation. For every light beam with an arbitrary structure of rays, there exists a possible "time-reversed" beam whose rays run along the same trajectories but in the opposite direction, similar to when a film is run backward. The phase conjugate mirrors reflect light radically differently than conventional mirrors. Let's consider a beam of light interacting with two types of mirrors: a traditional mirror and a phase conjugator. When light rays hit a conventional mirror, they are characterized by a wave vector k aligned with the direction of propagation. Upon reflection, only the component $k_{\perp}$, which is normal to the mirror surface, is inverted. As a result, the orientation of the conventional mirror can be adjusted to arbitrarily redirect the light beam. In contrast, a phase conjugator acts differently. It inverts all components of the wave vector k, resulting in a total reversal of sign and direction. Regardless of the orientation of the phase conjugator, the reflected beam always retraces the path of the incident beam.

In chapter 5, cross sectional shape of the laser beam was considered to be exactly circular. However, practically the shape of the laser beam is slightly elliptical and that too affect significantly the progression of the laser beam. Thus in this chapter SBS of elliptical q-Gaussian laser beam has been investigated.

6.2 Evolution of pump beam

Let's examine the propagation of a beam of laser light characterized by electric field vector

$$\vec{E}(r,t) = A_0\,(x,y,z)e^{i(k_0 z-\omega_0 t)}e_x \tag{6.1}$$

Through plasma having equilibrium electron density increases monotonically with longitudinal coordinate z . Such an electron density profile of plasma can be modeled as:

$$n(z) = n_0(1+\tan(dz)) \tag{6.2}$$

Here n_0 represents electron density of plasma at the initial plane where the laser beam enters i.e., $z = 0$. The constant $'d'$ signifies the rate at which the electron density of the plasma increases. Therefore, $'d'$ is commonly called the gradient of the density ramp.

The dielectric function of plasma, with electron density given by eq. (6.2) can be written as

$$\varepsilon = 1-\frac{4\pi e^2 n_0}{m_e\omega_0^2}(1+\tan(dz)) \tag{6.3}$$

Where e represents the electronic charge and m_e denotes the electron mass. The electric field of the laser beam imparts oscillatory velocity

$$v = -i\frac{e}{\omega_0 m_e}E \tag{6.4}$$

to the plasma electrons. In the case where the laser light beam is adequately intense enough to cause the plasma electrons the electrons in plasma undergo oscillations at a velocity similar to speed of light in a vacuum, it is necessary to replace the effective mass m_e of plasma electrons in Eq. (6.3) with $m_e = m_0\gamma$. Here, m_0 corresponds to the rest mass of electron, and γ denotes the relativistic Lorentz factor. The Lorentz factor γ is linked with the laser intensity in the following manner:

$$\gamma = (1 + \beta A_0 A_0^*)^{\frac{1}{2}}$$

Where β represents a constant defined as $\beta = \frac{e^2}{m_0^2 c^2 \omega_0^2}$. It is related to magnitude of relativistic nonlinearity and is often referred as "relativistic nonlinearity coefficient". Therefore, under the influence of a laser beam, Eq. (6.3) undergoes modification as follows:

$$\varepsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2}(1 + \beta A_0 A_0^*)^{-\frac{1}{2}}(1 + \tan(dz)) \tag{6.5}$$

Where $\omega_{p0} = \sqrt{\frac{4\pi e^2 n_0}{m_0}}$ is the plasma frequency when there is no laser beam present. By observing Eq. (6.5), one can deduce that the dependence of electron mass on laser intensity makes the plasma an optically nonlinear medium in a similar way to the Kerr effect makes ordinary dielectrics an optically nonlinear medium. Expressing the effective dielectric function of the plasma as the combination of linear and nonlinear components:

$$\varepsilon = \varepsilon_0 + \phi(A_0 A_0^*) \tag{6.6}$$

we get

$$\varepsilon_0 = 1 - \omega_{p0}^2/\omega_0^2 \tag{6.7}$$

and

$$\phi(A_0 A_0^*) = \frac{\omega_{p0}^2}{\omega_0^2}[1 - \frac{1}{(1+\beta A_0 A_0^*)^{1/2}}][1 + \tan(dz)] \tag{6.8}$$

Eq. (6.8) provides the relation for the relativistic nonlinear dielectric function of the plasma The wave equation governs the behavior of an optical beam as it propagates within a nonlinear medium with a nonlinear dielectric function represented by $\phi(A_0 A_0^*)$ is given as

$$2ik_0\frac{\partial A_0}{\partial z}= \nabla_\perp^2 A_0+\frac{\omega_0^2}{c^2}\phi(A_0A_0^*)A_0 \tag{6.9}$$

Lagrangian density for Eq. (6.9) is:

$$\mathcal{L}=i\left(A_0\frac{\partial A_0^*}{\partial z}-A_0^*\frac{\partial A_0}{\partial z}\right)+|\nabla_\perp A_0|^2-\frac{\omega_0^2}{c^2}\int^{A_0A_0^*}\phi(A_0A_0^*)d(A_0A_0^*) \tag{6.10}$$

In this present study, we have utilized a trial function with the following structure:

$$A_0(x,y,z)=\frac{E_{00}}{\sqrt{f_xf_y}}\left\{1+\frac{1}{q}\left(\frac{x^2}{a^2f_x^2}+\frac{y^2}{b^2f_y^2}\right)\right\}^{-\frac{q}{2}} \tag{6.11}$$

Upon inserting the trial function as defined in eq. (6.11) into the Lagrangian density and performing integration over the entire cross-sectional area of the laser light beam, we derive the reduced Lagrangian given by $L=\int_0^\infty \mathcal{L}rdr$. The corresponding Euler–Lagrange equations

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial f_{x,y}}{\partial z}\right)}\right)-\frac{\partial L}{\partial f_{x,y}}=0 \tag{6.12}$$

give

$$\frac{d^2f_x}{dz^2}=\frac{1}{2k_0^2a^4}\frac{1}{f_x^3}\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left[\left(1+\frac{1}{q}\right)^{-1}+\left(\frac{<L_1>}{E_{00}^2}f_xf_y+\frac{2E_{00}^2}{f_x^2f_y}\frac{\partial<L_1>}{\partial f_x}\right)\right] \tag{6.13}$$

$$\frac{d^2f_y}{dz^2}=\frac{1}{2k_0^2b^4}\frac{1}{f_y^3}\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left[\left(1+\frac{1}{q}\right)^{-1}+\left(\frac{<L_1>}{E_{00}^2}f_xf_y+\frac{2E_{00}^2}{f_xf_y^2}\frac{\partial<L_1>}{\partial f_y}\right)\right] \tag{6.14}$$

where,

$$<L_1>=\frac{\omega_0^2}{c^2}\int\left(\int^{A_0A_0^*}\phi(A_0\,A_0^*)d(A_0A_0^*)\right)d^2r$$

Equations (6.13) and (6.14) can be written as

$$\frac{d^2f_x}{dz^2}=\frac{1}{2k_0^2a^4f_x^3}\frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}+\frac{1}{2}\frac{\left(1-\frac{2}{q}\right)}{a^2\epsilon_0I_0}\int xA_0A_0^*\frac{\partial\phi}{\partial x}d^2r \tag{6.15}$$

$$\frac{d^2f_y}{dz^2}=\frac{1}{2k_0^2a^4f_y^3}\frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}+\frac{1}{2}\frac{\left(1-\frac{2}{q}\right)}{a^2\epsilon_0I_0}\int yA_0A_0^*\frac{\partial\phi}{\partial y}d^2r \tag{6.16}$$

Using Eqs. (6.8) and (6.11) in Eqs. (6.15) and (6.16) we get

$$\frac{d^2f_x}{d\xi^2}=\frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f_x^3}-\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_x^2f_y}I \tag{6.17}$$

$$\frac{d^2f_y}{d\xi^2}=\left(\frac{a}{b}\right)^4\frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f_y^3}-\left(\frac{a}{b}\right)^2\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)\left(\frac{\omega_{p0}^2a^2}{c^2}\right)\left(1+\tan(d'\xi)\right)\frac{\beta E_{00}^2}{f_xf_y^2}I \tag{6.18}$$

where

$$d' = dk_0 a^2$$

$$\xi = \frac{z}{k_0 a^2}$$

$$I = \int_0^\infty t\left(1+\frac{t}{q}\right)^{-2q-1} \{1+\frac{\beta E_{00}^2}{f_x f_y}\left(1+\frac{t}{q}\right)^{-q}\}^{-3/2} dt$$

The beam width evolution of elliptical q-Gaussian laser beams in x and y directions with respect to longitudinal distance of propagation is given by a system of nonlinearly coupled differential equations, specifically denoted as Eqs. (6.17) and (6.18). For an initial plane wavefront of the pump beam, Eqs. (6.17) and (6.18) are governed by specific boundary conditions.

$$f_{x,y}(\xi = 0) = 1$$

$$\frac{df_{x,y}}{d\xi} = 0 \text{ at } \xi = 0$$

Equations (6.17) and (6.18) are identical to the equation describing the motion of driven oscillators with a unit mass where the beam width parameters $f_{x,y}$ assume the role of displacement and the longitudinal distance ξ play that of time. Thus, from these equations one can infer that while the beam of laser light propagates within the plasma, its beam widths will undergo harmonic oscillations over the propagation distance. From a physical perspective, the harmonic fluctuations in beam widths arise as a result of the interplay between two phenomena. The first is the self-focusing of the laser light beam, which is driven by the plasma's optical nonlinearity. The second is the inherent wave nature of light, leading to diffraction. The laser beam's beam widths exhibit similar behavior in previous investigations for ponderomotive optical nonlinearity of plasma.

6.3 Evolution of ion acoustic wave

The perturbation in plasma density, denoted as n_{ia} linked with the IAW, undergoes evolution governed by the wave equation.

$$2ik_{ia}\frac{\partial n_{ia}}{\partial z} = \nabla_\perp^2 n_{ia} - \frac{\omega_{ia}^2}{v_{th}^2}[1+\tan(dz)](1-\frac{1}{(1+\beta A_0 A_0^*)^{1/2}})\, n_{ia} \quad (6.19)$$

where $v_{th} = \sqrt{\frac{2KT_i}{M_i}}$ is the thermal velocity of ions.

Considering the Gaussian ansatz for the IAW as

$$n_{ia} = \frac{n_{00}}{\sqrt{f_{xi}f_{yi}}} e^{-\left(\frac{x^2}{2a^2 f_{xi}^2} + \frac{y^2}{2b^2 f_{yi}^2}\right)} \quad (6.20)$$

By following the aforementioned procedure, we can derive the equation describing the evolution of the beam width in the presence of the Ion Acoustic Wave (IAW):

$$\frac{d^2 f_{xi}}{d\xi^2} = \frac{1}{f_{xi}^3} - \left(\frac{\omega_{ia}^2 \; a^2}{v_{th}^2}\right) (1 + \tan(d'\xi)) \frac{1}{f_{xi}^2 f_{yi}} \; T \quad (6.21)$$

$$\frac{d^2 f_{yi}}{d\xi^2} = \left(\frac{a}{b}\right)^4 \frac{1}{f_{yi}^3} - \left(\frac{a}{b}\right)^2 \left(\frac{\omega_{ia}^2 \; a^2}{v_{th}^2}\right) (1 + \tan(d'\xi)) \frac{1}{f_{yi}^2 f_{xi}} \; T \quad (6.22)$$

where

$$T = \int_0^{2\pi} \int_0^{\infty} exp[-u(\frac{f_x^2}{f_{xi}^2}\cos(\theta) + \frac{f_y^2}{f_{yi}^2}\sin(\theta)) \; \left(1 + \frac{u}{q}\right)^{(-q-1)} \left\{1 + \frac{\beta E_{00}^2}{f_x f_y}\left(1 + \frac{u}{q}\right)^{-q}\right\}^{-3/2} u du d\theta$$

For the initial plane wavefront, Eqs. (6.21) and (6.22) are governed by boundary equations $f_{xi,yi} = 1$ and $\frac{df_{xi,yi}}{d\xi} = 0 \; at \; \xi = 0$. Eqs. (6.21) and (6.22) show the interaction between the IAW and the pump beam. The density perturbations associated with the IAW exhibit a high sensitivity to the self-focusing of the laser light beam. By employing Poisson's equation, it becomes possible to determine the electrostatic field generated by the excited Ion Acoustic Wave (IAW) as follows:

$$E_{ia} = E_{ia} e^{-i(k_{ia} z - \omega_{ia} t)} e_z \quad (6.23)$$

$$E_{ia} = \frac{i m_0 \omega_{ia}^2}{e k_{ia} \sqrt{f_{xi} f_{yi}}} e^{-\left(\frac{x^2}{2a^2 f_{xi}^2} + \frac{y^2}{2b^2 f_{yi}^2}\right)} \quad (6\,.24)$$

The Eq. (6.24) provides the magnitude of the field corresponding to the excited IAW.

6.4 Evolution of scattered wave

SBS, one of the most beautiful effects in nonlinear optics, involves directing a beam of light into a transparent medium such as liquid, compressed gas, glass, or crystal. When light of low intensity passes through such a sample, no attenuation is observed. The behavior of a high-intensity light beam, however, is astonishing. The beam is reflected backward almost wholly when beginning at a threshold power of roughly a million watts. Although such power is relatively high, it is easily attainable with a laboratory pulsed laser.

The reflected beam is a consequence of events that produce Brillouin scattering of light – the scattering of light due to its encounter with high-frequency acoustic waves. These are present in all materials because of thermal agitation and cover a range of frequencies extending from zero up through microwaves into the infrared region. Consider, in particular, a thermal acoustic wave that travels in the same direction as a light wave and has the same wavelength. Of course, its frequency is much lower than that of a light wave, corresponding to the much lower sound velocity. This sound wave has a maximum pressure for every wavelength, so the light wave also sees a regularly repeated density fluctuation slowly moving through the liquid. To fix the density fluctuation, it would be a three-dimensional grating, like a crystal is for X-rays, and would reflect some of the light with the original frequency. This reflection phenomenon is reminiscent of the experience of observing a thin oil film on water, where one can observe its surface displaying a vibrant range of rainbow colors. At every location on the film, a specific color is reflected more effectively than others, and this particular color corresponds to a wavelength that is half the thickness of the film layer. Due to variation in the thickness of the layer, different points on the film reflect distinct colors and because the wave is in fact moving, the frequency of the reflected light is reduced by a Doppler shift of twice v/c, where v is sound velocity and c is light velocity in a vacuum. As thermal waves are moving in all directions in the liquid, they can scatter light in all directions with slightly different frequencies.

However, when an intense light wave, such as that from a giant pulse laser, traverses the medium, it is not only reflected by thermal vibrations, it combines with them to drive these vibrations to a much greater intensity. The process is that the direct light and the reflected light, especially in the backward direction, combine to produce maxima and minima of the resultant intensity. At the places where the light intensity is most significant, where the two waves are in phase, the material experiences an electrostrictive force that compresses it. This force, although negligible with ordinary light waves, can be fairly large with lasers, and it has just the suitable periodicity to apply pressure and drive the sound waves to higher intensities. The stronger sound waves reflect more light, so the process goes. The reflections thus build exponentially as distance increases until a reflected beam emerges from the material. Indeed, some liquids may reflect as much as 80 or 90% of the light with a slight change in frequency by the SBS. In solids, the intense vibrations may be one cause of fracture.

As the amplification relies on the intensity of the incident beam, a prerequisite for producing the reflected beam is that the power of the incident beam must exceed a certain threshold. In the context of plasma, the role analogous to a sound wave is fulfilled by an Ion Acoustic Wave (IAW). IAWs, similar to propagating sound waves in solids, are associated with acoustic phonons. Consequently, it is reasonable to explore the possibility of Stimulated Brillouin Scattering (SBS) occurring in plasmas, just as it does in solid materials. Nevertheless, elementary considerations have demonstrated that nonlinear effects in plasmas are comparatively weaker than their counterparts observed in liquids or solids by a factor of 10^7. In applications involving interactions between lasers and plasmas, laser beams with intensities more than 10^{16} W/cm^2 are being used. At these ultrahigh intensity plasmas become prone to SBS.

The nonlinear interaction of the pump beam and IAW leads to generation of a nonlinear current density, denoted as J_{NL} at a frequency $\omega_S = \omega_0 - \omega_{ia}$. The expression for this nonlinear current density is given by:

$$J_{NL} = \left(\frac{en_0}{m_0\omega_s}\right)\frac{n_{ia}}{n_0}e^{i(\omega_s t - k_s z)}A_0(x, y, z) \tag{6.25}$$

The presence of this nonlinear current density gives rise to a scattered wave, which follows the evolution dictated by the wave equation.

$$\nabla^2 E_s = \frac{1}{c^2}\frac{\partial^2 E_s}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial J_{NL}}{\partial t} \tag{6.26}$$

The equation provides the measure of the electric field associated with the scattered radiation as:

$$E_s = i\frac{\left(\frac{\omega_{P0}^2}{c^2}\right)}{\left(\frac{\omega_S^2}{c^2} - k_S^2\right)}\frac{n_{ia}}{n_0}\,A_0(x, y, z) \tag{6.27}$$

By denoting the normalized power of scattered radiation as

$$P_S = \frac{\int E_S E_S^* d^2r}{\int A_0 A_0^* d^2r} \tag{6.28}$$

we get

$$P_S = \frac{\left(\frac{\omega_{p0}^2}{c^2}\right)^2}{\left(\frac{\omega_S^2}{c^2} - k_S^2\right)^2}\frac{\int\left(\frac{n_{ia}}{n_0}\right)^2 A_0 A_0^* d^2r}{\int A_0 A_0^* d^2r} \tag{6.29}$$

In the current study, Equation (6.29) has been solved in association with Eqs. (6.17), (6.18), (6.21), and (6.22) using the following collection of laser-plasma parameters: $\omega_0 = 1.78 \times$

10^{15} rad/s $\omega_{ia} = 10^{12}$rad/s, $a = 10$ μm, $\beta E_{00}^2 = 3$, $\frac{\omega_{po}^2 a^2}{c^2} = 9$, $T = 10^6\ K$, $\frac{n_{00}}{n_0} = 0.001$ and for various values of q, d' and $\frac{a}{b}$ viz; $q = (3,\ 4, \infty)$, $d' = (0.25, 0.35, 0.45)$ and $\frac{a}{b} = (1, 1.1, 1.2)$, in order to visualize the progression of the power of scattered waves with longitudinal distance as they propagate within the plasma. The change in the scattered power $P_S$with the distance ξ is illustrated in figs. 6.1–6.3. It is noticeable that the power of the scattered radiation exhibits a step-like behavior and monotonically increases with the propagation distance.

Furthermore, upon observation, it is noticed that as the longitudinal distance increases, the steps in the power of the scattered wave become steeper while the size of each step diminishes. The reason behind the increasing steepness of the steps is that it is directly related to the degree of self-focusing. On the other hand, the decrease in step size is attributed to the inverse proportionality between step size and the frequency of oscillations of the beam width. It has been noted that as the distance progresses, both the extent and frequency of self-focusing increases. Consequently, the observed P vs. ξ curves display an escalation in step steepness alongside a reduction in individual step size.

Figure 6.1 clearly demonstrate a decrease in the rate at which scattered radiation is amplified when deviation parameter q increases. This observation can be attributed to the direct connection between the power of the scattered wave and the degree of self-focusing displayed by the pump beam.

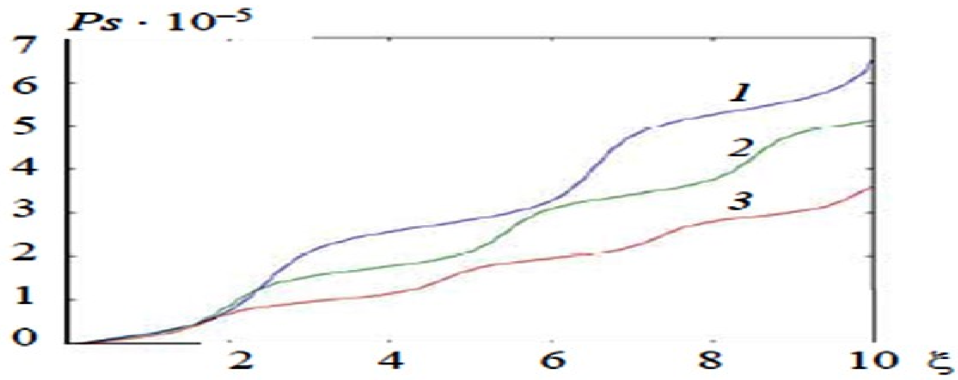

Fig. 6.1: Variation of the power of scattered waves with distance in plasma for $q = 3(1),\ 4(2)$ and ∞ (3); $d' = 0.25$ and $\frac{a}{b} = 1.1$

When the q value grows, the focusing of the pump beam becomes diminishes, leading to a reduction of the scattered radiation power. Figure 6.2 demonstrate that Stimulated Brillouin Scattering (SBS) diminishes as the ellipticity of the laser beam increases. This reduction can be attributed to the diminished overall self-focusing of the laser light beam that occurs with an enhancement in its ellipticity.

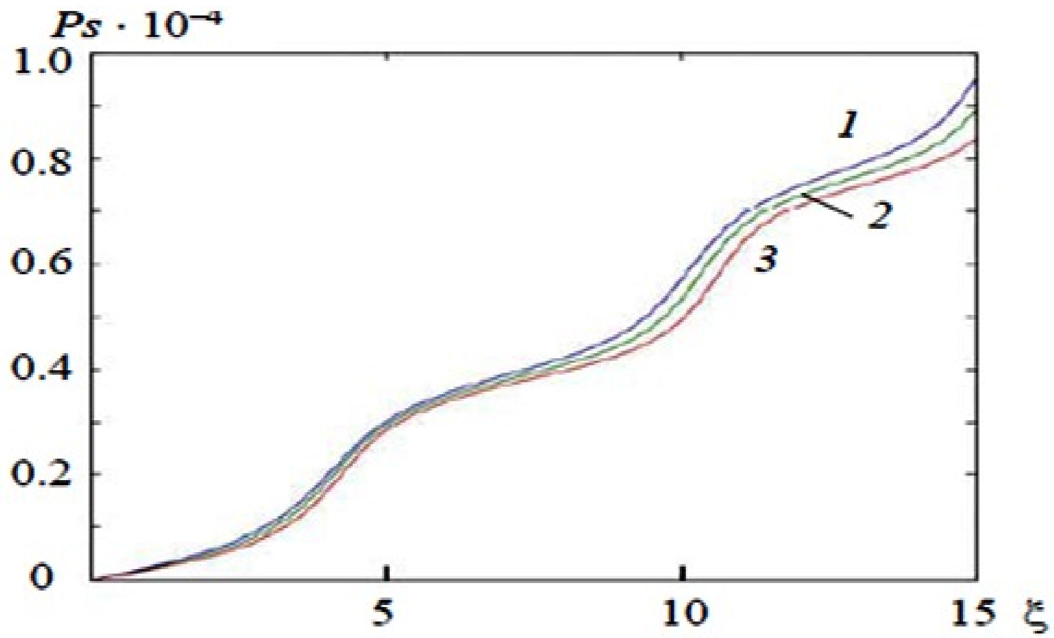

Fig.6.2: Variation of the power of the scattered waves as the function of propagation distance in plasma for $\frac{a}{b} = 1(1), 1.1(2),$ and 1.2 (3); $q = 3$ and $d' = 0.25$

Figure 6.3 illustrates how the slope of the density ramp affects the power of the scattered wave. It's clear that augmenting the slope of the density ramp intensifies the amplification of the scattered wave. Once more, this observation is connected to the amplified self-focusing of the laser light beam as the slope of the density ramp increases. An increase in the slope of the density ramp, leads to higher reflectivity of the focal spots that amplify the scattered wave.

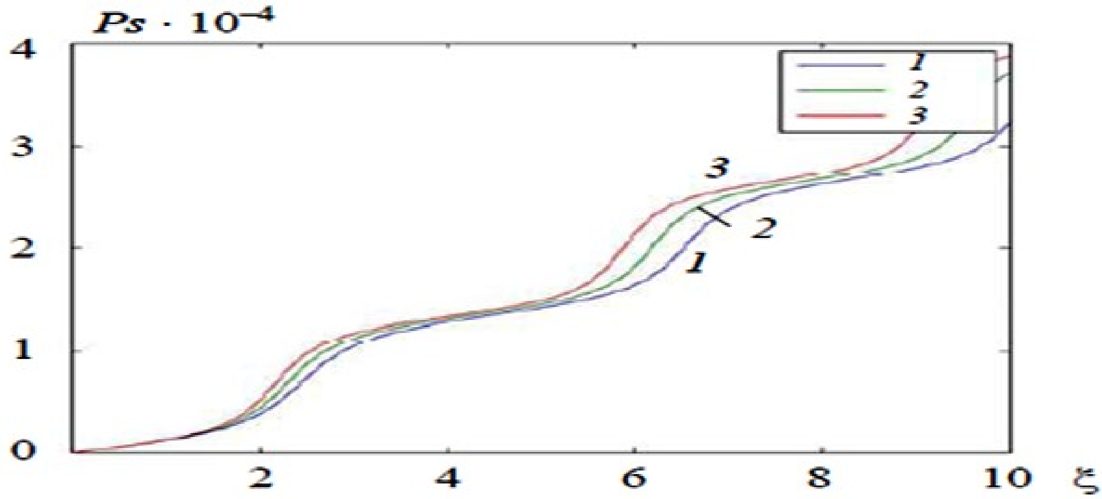

Fig.6.3: Variation of power of scattered waves with distance in plasma for $d' = 0.25\ (1),\ 0.35\ (2),$ and $0.45\ (3)$; $\frac{a}{b} = 1.1$ and $q = 3$

Chapter- 7

Self-Focused Quadruple Gaussian Laser Beams in Thermal Quantum Plasma

7.1 Introduction

Exploring self-action effects and their impact on the stimulated Brillouin scattering (SBS) of intense laser beams in plasmas has been at the forefront of research for several decades. Thus, this chapter aims to give theoretical study on self focusing, self trapping and SBS of QG laser beams in thermal quantum plasmas.

7.2 Relativistic Nonlinearity of Fusion Plasma

In case of inertial confinement fusion (ICF), the generated plasma is so dense that the wave functions of the electrons start overlapping with each other. In such a case the dielectric characteristics of plasma correspond to thermal quantum plasma (TQP). Let us take the interaction of a circularly polarized laser beam having electric field vector

$$E(r,z,t) = A(r,z)e^{i(k_0 z-\omega_0 t)}\left(e_x + ie_y\right) \tag{7.1}$$

with TQP of equilibrium electron density $n_0(z)$. Upon irradiation of fuel pellet during ICF the generated plasma expands radially outwards. Hence, as seen from the side of laser beam, the plasma density varies as an increasing function of distance. Such a density profile of plasma has been modeled by Gupta *et al.*[11] as $n_0(z) = n_0(0)(1+\tan(dz))$. The impact of quantum effects within plasmas is dictated by the spatial dimensions of the particles wave packets. If the average size of electron wave packets is greater than the average separation between them resulting in the merging of wave functions, quantum effects assume a substantial role. When considering the impact of quantum mechanical diffraction, it becomes possible to formulate the dielectric function of TQP as[62]

$$\epsilon = 1 - \frac{\omega_p^2}{(\omega_0^2 - \alpha k_0^4 - k_0^2 v_F^2)} \tag{7.2}$$

where, $\omega_p^2 = \frac{4\pi e^2}{m_e} n_0(z)$ is the frequency of plasma, the constant α that can be expressed as $\alpha = \frac{\hbar^2}{4m_e^2}$ is associated with the effect of diffraction of electron wave function resulting from quantum correction of density fluctuations, e and m_e are electronic charge and mass, respectively and $v_F = \left(\frac{2K_B T_F}{m_e}\right)^{1/2}$ is the electron's fermi speed. For $T_F = 0$, Eq. (7.2) represents the dielectric function of cold quantum plasma (CQP). Also, along with $T_F = 0$ and the condition $\alpha = 0$, Eq. (7.2) represents the dielectric function of classical plasma. When subjected to the circularly polarized field of the laser beam, the unbound charge carriers within the plasma experience movement along circular paths, characterized by oscillations around the perimeter. When subjected to the effects of an ultra-intense laser, the plasma experiences an increase in the quiver velocity of its electrons, causing them to approach the speed of light. In Eq. (7.2), the effective mass $m_e$of plasma electrons must be substituted by $m_0\gamma$, where γ represents the relativistic Lorentz factor that is connected to the intensity of laser by the following expression[62]

$$\gamma = (1 + \beta AA^*)^{1/2} \tag{7.3}$$

Where β represents a constant defined as $\beta = \frac{e^2}{m_0^2 c^2 \omega_0^2}$. It is related to the magnitude of relativistic nonlinearity and is often referred as "relativistic nonlinearity coefficient". Relativistic nonlinearity lacks transient behavior as it does not involve the modification of electron density of plasma. It immediately comes into existence as the laser power exceeds a certain threshold. Therefore, when the laser beam is present, the altered dielectric function of TQP can be expressed as follows:

$$\epsilon = 1 - \frac{\omega_{p0}^2(z)\gamma^{-1}}{(\omega_0^2 - \alpha_0\gamma^{-2}k_0^4 - \gamma^{-1}k_0^2 v_{F0}^2)} \tag{7.4}$$

Where $\omega_{p0}^2(z) = \frac{4\pi e^2}{m_0} n_0(z)$ is the equilibrium plasma frequency, $\alpha_0 = \frac{\hbar^2}{4m_0^2}$ and $v_{F0} = \left(\frac{2K_B T_F}{m_0}\right)^{1/2}$. Writing eq. (7.4) as a linear combination of linear and nonlinear parts as

$$\epsilon = \epsilon_0 + \phi(AA^*) \tag{7.5}$$

We get

$$\epsilon_0 = 1 - \frac{\omega_{p0}^2(z)}{\omega_0^2} \tag{7.6}$$

$$\phi(AA^*) = \frac{\omega_{p0}^2(z)}{\omega_0^2}\{1 - \frac{\gamma^{-1}}{1 - \frac{\gamma^{-2}\alpha_0 k_0^4}{\omega_0^2} - \frac{k_0^2 v_{F0}^2 \gamma^{-1}}{\omega_0^2}}\} \tag{7.7}$$

Eq. (7.7) gives the relativistic nonlinearity of TQP. Eq. (7.7) also models the dielectric function of plasmas occurring in solids like narrow band gap semiconductors e.g., InSb. In semiconductors with narrow band gaps, due to the non parabolicity of the conduction band the motion of conduction electrons becomes relativistic even at moderate laser intensities.

Thus, their nonlinear dielectric response is governed by equation similar to that of Eq. (7.7). Thus, the results of present study are also applicable qualitatively to that for the interaction of high-intensity laser light beams with narrow band gap semiconductors or plasma occurring in solids.

7.3 Quadruple Gaussian Laser Beam

The representation of transverse amplitude structure, of the beam of laser light denoted as $A(x, y)$, is described by:

$$A(x,y) = \frac{E_{00}}{f}[e^{\frac{-(x-x_0 f)^2+y^2}{2r_0^2 f^2}} + e^{\frac{-(x+x_0 f)^2+y^2}{2r_0^2 f^2}} + e^{\frac{-(y-x_0 f)^2+x^2}{2r_0^2 f^2}} + e^{\frac{-(x+x_0 f)^2+y^2}{2r_0^2 f^2}}] \quad (7.8)$$

here, E_{00} represents the axial amplitude and r_0 represents equilibrium radius of the beam of laser light. The dimensionless beam width parameter, denoted as $f(z)$, describes a function that represents the instantaneous spot size when multiplied by the initial beam width. Furthermore, when divided by the amplitude of laser, it acts as an indicator of the intensity along the axis. Eq. (7.8) reveals Q.G. laser beams can be generated by the constructive interference of four identical beams of laser light, each exhibiting a Gaussian intensity profile. The coordinates of their intensity maxima are located at $(-x_0, 0)$,$(x_0, 0)$,$(0, x_0)$,$(0, -x_0)$.

7.4 Evolution of beam envelope

The wave equation employed to model the propagation of laser beams within nonlinear media is given by

$$i\frac{\partial A}{\partial z} = \frac{1}{2k_0}\nabla_{\perp}^2 A_0 + \frac{k_0}{2\varepsilon_0}\phi(AA^*)A \quad (7.9)$$

The Lagrangian density associated to Eq. (7.9) can be expressed as follows:

$$\mathcal{L} = i\left(A_0\frac{\partial A_0^*}{\partial z} - A_0^*\frac{\partial A_0}{\partial z}\right) + |\nabla_{\perp}A_0|^2 - \frac{\omega_0^2}{c^2}\int^{A_0A_0^*}\phi(A_0A_0^*)d(A_0A_0^*) \quad (7.10)$$

By putting the trial function (Eq. 7.8) into the Lagrangian density (Eq. 7.10) and conducting integration with respect to the x and y, the outcome is the reduced Lagrangian as follows:

$$L = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathcal{L}r dx dy \quad (7.11)$$

Hence, by treating $(f, \frac{df}{dz})$ as generalized coordinates, the associated Lagrange's equation of motion can be formulated as follows:

$$\frac{d}{dz}\left(\frac{\partial L}{\partial(\frac{\partial f}{\partial z})}\right)-\frac{\partial L}{\partial f}=0 \tag{7.12}$$

$$\frac{d^2f}{d\xi^2}=\frac{1}{4f^3}\left[\frac{1+e^{-\frac{x_0^2}{r_0^2}}\left(1-\frac{x_0^2}{r_0^2}\right)+\ e^{-\frac{x_0^2}{r_0^2}}\left(2-\frac{x_0^2}{r_0^2}\right)}{\left(2+2\frac{x_0^2}{r_0^2}\right)+\left(2+2\frac{x_0^2}{r_0^2}\right)e^{-\frac{x_0^2}{2r_0^2}}+2e^{-\frac{x_0^2}{r_0^2}}}\right]-\frac{1}{2\pi}\left(\frac{\omega_{p0}(0)r_0}{c}\right)^2\left(1+\tan(d'\xi)\right)$$

$$\frac{\beta E_{00}^2}{f^3}\frac{1}{\left(2+2\frac{x_0^2}{r_0^2}\right)+\left(2+2\frac{x_0^2}{r_0^2}\right)e^{-\frac{x_0^2}{2r_0^2}}+2e^{-\frac{x_0^2}{r_0^2}}}(K_1+K_2+K_3+K_4) \tag{7.13}$$

$$K_1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}t_1\left(t_1-\frac{x_0^2}{r_0^2}\right)e^{-\frac{\left(t_1-\frac{x_0^2}{r_0^2}\right)^2+t_2^2}{2}}G_1^3(t_1,t_2)G_2dt_1dt_2$$

$$K_2=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}t_1\left(t_1+\frac{x_0^2}{r_0^2}\right)e^{-\frac{\left(t_1+\frac{x_0^2}{r_0^2}\right)^2+t_2^2}{2}}G_1^3(t_1,t_2)G_2dt_1dt_2$$

$$K_3=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}t_1^2e^{-\frac{t_1^2+\left(t_2-\frac{x_0^2}{r_0^2}\right)^2}{2}}G_1^3(t_1,t_2)G_2dt_1dt_2$$

$$K_4=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}t_1^2e^{-\frac{t_1^2+\left(t_2+\frac{x_0^2}{r_0^2}\right)^2}{2}}G_1^3(t_1,t_2)G_2dt_1dt_2$$

$$G_1(t_1,t_2)=e^{-\frac{\left(t_1-\frac{x_0^2}{r_0}\right)^2+t_2^2}{2}}+e^{-\frac{\left(t_1+\frac{x_0^2}{r_0}\right)^2+t_2^2}{2}}+e^{-\frac{t_1^2+\left(t_2-\frac{x_0^2}{r_0^2}\right)^2}{2}}+e^{-\frac{t_1^2+\left(t_2+\frac{x_0^2}{r_0^2}\right)^2}{2}}$$

$$G_2=\frac{\left(1-F^{-2}\frac{\alpha_0k_0^4}{\omega_0^2}-F^{-1}\frac{k_0^2v_{F0}^2}{\omega_0^2}\right)F^{-3}+F^{-2}(2F^{-3}\frac{\alpha_0k_0^4}{\omega_0^2}+F^2\frac{k_0^2v_{F0}^2}{\omega_0^2})}{\left(1-F^{-2}\frac{\alpha_0k_0^4}{\omega_0^2}-F^{-1}\frac{k_0^2v_{F0}^2}{\omega_0^2}\right)^2}$$

$$F=\left(1+\frac{\beta E_{00}^2}{f^2}G_1^2(t_1,t_2)\right)^{\frac{1}{2}}$$

$$t_1=\frac{x}{r_0f}$$

$$t_2=\frac{y}{r_0f}$$

$$\xi = \frac{z}{k_0 r_0^2}$$

$$d' = k_0 r_0^2 d$$

Hence, Eq. (7.13) derived from the variational theory presents a mathematical expression in the form of an ordinary differential equation. The equation serves as a guiding principle for comprehending the laser beam's beam width evolution while it propagates through a nonlinear medium, particularly in the context of thermal quantum plasma in our case. Although the reduced equation cannot be directly integrated to yield a closed-form solution, it is possible to obtain an approximate solution using the Runge-Kutta fourth-order method with ease. In the current study, Eq. (7.13) was obtained using the given set of parameters:

$\omega_0 = 1.78 \times 10^{20}$rad sec^{-1}, $r_0 = 20\mu$m, $T_f = 10^9$K, $n_0 = 4 \times 10^{19}$cm^{-3}, $\beta E_{00}^2 = 3$

and for various values of $\frac{x_0}{r_0}$ under the initial conditions $f = 1$ and $\frac{df}{d\xi} = 0$ at $\xi = 0$. The significance of these conditions is as follows:

- $f = 1$ at $\xi = 0$ implies that at the entrance in the plasma the radius of the laser light beam is r_0.
- $\frac{df}{d\xi} = 0$ at $\xi = 0$ implies that the beam of laser light is initially collimated.

In order to comprehend the alterations in the linear propagation of the laser light beam corresponding to its beam profile, we solved Eq. (7.13) for varying $\frac{x_0}{r_0}$ values while excluding the presence of plasma. Fig. 7.1 illustrates the progression of the beam width with increasing distance. It is evident from the results that regardless of the value of $\frac{x_0}{r_0}$, the beam width of Q.G beams of laser light consistently diverges in a monotonically increasing manner during its propagation in vacuum. This is an obvious result as any optical beam propagating in vacuum or in linear medium (the medium with an index of refraction that remains unaffected by changes in intensity) undergoes self broadening due to diffraction. The interesting result is that, the rate of divergence reduces by increasing $\frac{x_0}{r_0}$. Thus, one can have the control diffraction a laser beam of given radius by choosing suitable value of $\frac{x_0}{r_0}$.

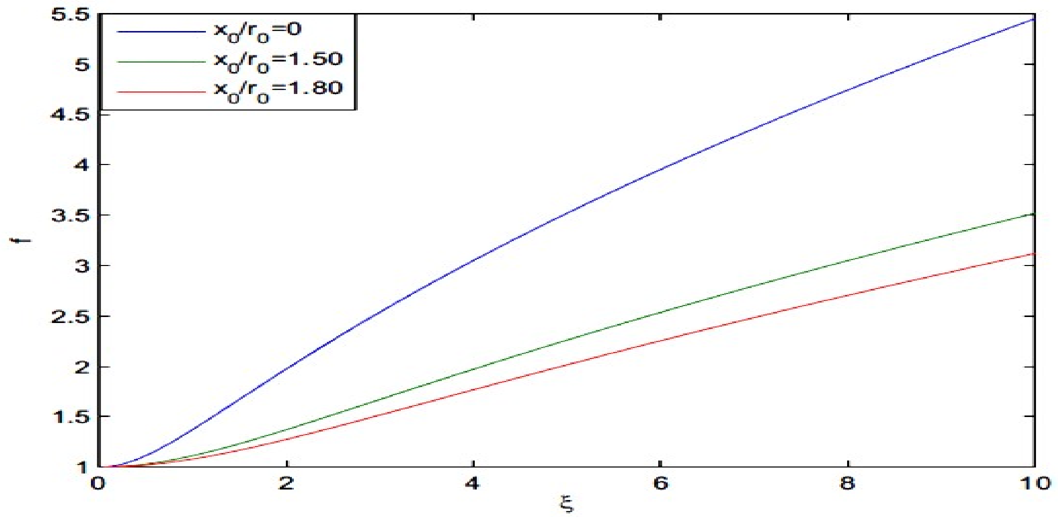

Fig.7.1: Impact of $\frac{x_0}{r_0}$ on vacuum diffraction of Q.G laser beam

Figs. 7.2(a) and 7.2(b) illustrate how the parameter $\frac{x_0}{r_0}$ influences the transmission of the laser light beam. These graphs imply that the beam width of the laser undergoes harmonic fluctuations when it propagates through the plasma. This phenomenon can be attributed to the beam's traversal through a self-induced oscillatory waveguide, which comprises a sequence of convex lenses.

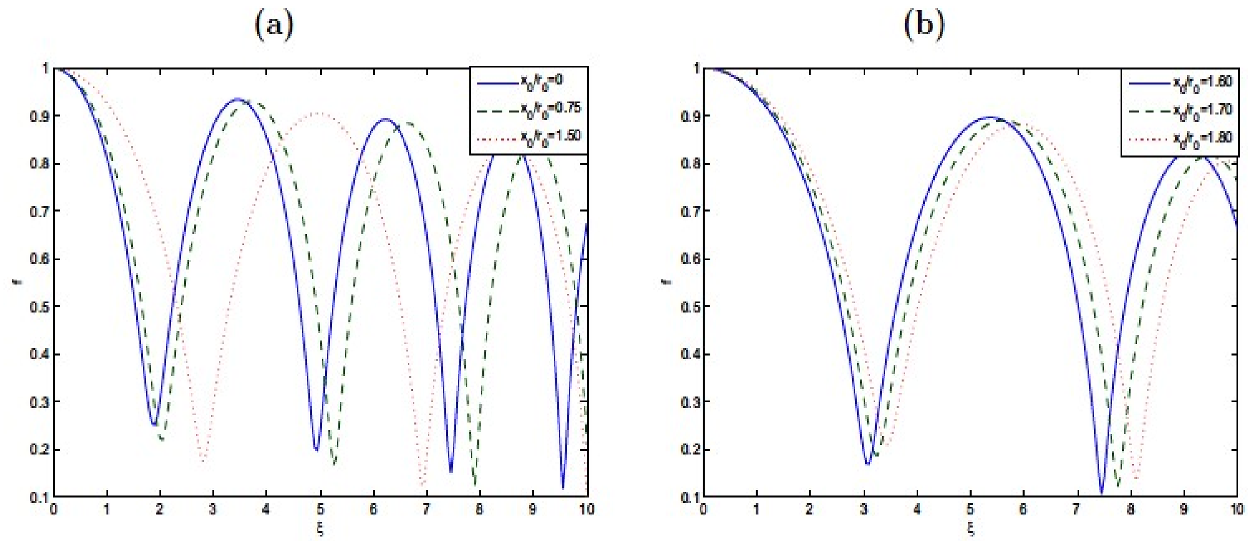

Fig.7.2: Impact of $\frac{x_0}{r_0}$on width of laser light beam at fixed values of

$$\beta E_{00}^2 = 3\ d' = 0.025 \text{ and } \left(\frac{\omega_{p0} r_0}{c}\right)^2 = 9$$

Furthermore, an increase in the value of $\frac{x_0}{r_0}$ within the range of $0 \leq \frac{x_0}{r_0} < 1.5$ enhances the focusing of the laser light beam. Conversely, beyond $\frac{x_0}{r_0} = 1.5$, an increase in $\frac{x_0}{r_0}$ value results in a reduction in the extent of self-focusing exhibited by the laser light beam. This happens because, for $0 \leq \frac{x_0}{r_0} < 1.5$, as $\frac{x_0}{r_0}$ grows, the intensity distribution across the cross-sectional area of the laser light beam becomes increasingly uniform. Consequently, the laser light beam receives a balanced contribution from the off-axial regions similar to the contribution provided by the axial portion, in terms of nonlinear refraction. As self-focusing of the laser light beam emerges due to nonlinear refraction so when the value of $\frac{x_0}{r_0}$ falls within the range of $0 \leq \frac{x_0}{r_0} < 1.5$, self-focusing effect experiences an increase in its value. In the present investigation the plasma's refractive index has been considered as a whole. Thus, the present investigation is valid for all the values of $\frac{x_0}{r_0}$ those are achievable experimentally.

The reduction in the degree of self-focusing of the laser light beam when $\frac{x_0}{r_0}$ exceeds 1.5 can be attributed to the following reason: for $\frac{x_0}{r_0} > 1.5$ the intensity peaks of the individual Gaussian laser that make up the Q.G (Quadruple-Gaussian) laser light beams are significantly separated from one another. As a result, when these beams are superimposed, the peaks of intensity in the resultant beam are observed in the off-axial regions, as illustrated in fig. 3.3. Consequently, the axial part of the laser light beam contributes significantly less to nonlinear refraction due to its weaker intensity in comparison to the off-axial part. Consequently, the laser light beam experiences minimal contribution from the axial region in terms of nonlinear refraction. As a consequence, the diminished contribution from the axial part of the laser light beam results in a reduction in the overall focusing capability of the beam.

To investigate the impact of the initial intensity of the laser light beam on its self-focusing, we have solved Eq. (7.13) for various values of the initial beam intensity βE_{00}^2 while maintaining other laser and plasma parameters constant. Fig. 7.3 demonstrates characteristics of the beam width. It is evident that laser beams with larger intensity produce larger relativistic nonlinearity in plasma. Consequently, this amplifies the self-focusing of the laser beam.

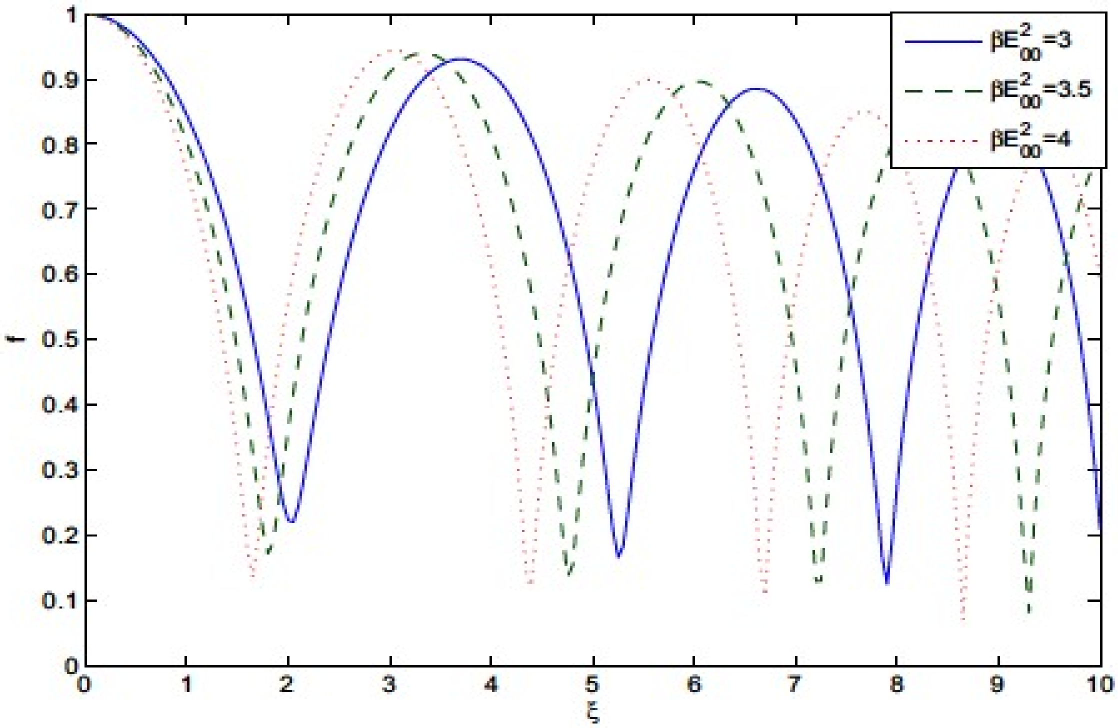

Fig.7.3: Impact of laser intensity on beam width at fixed values of

$\frac{x_0}{r_0} = 0.75, d' = 0.025$ and $\left(\frac{\omega_{po} r_0}{c}\right)^2 = 9$

To compare the behavior of the laser beam in thermal quantum plasma (TQP), cold quantum plasma (CQP) and relativistic classical plasma (RCP) we have solved Eq. (7.13) for $T_F = 0\,K,\ \alpha_0 = 0$ (RCP); $T_F \neq 0\,K, \alpha_0 = 0$ (CQP) and $T_F \neq 0\,K, \alpha_0 \neq 0$ (TQP). From the observations in fig. 7.4, it becomes apparent that for specific laser parameters, the degree of focusing of laser light can be arranged in the following order: TQP > CQP > RCP i.e., laser beam possesses highest focusing in TQP and minimum focusing in RCP. This is because in TQP the finite values of electron fermi temperature and quantum diffraction effects make the motion of plasma electrons relativistic at significantly lower values of laser intensities. As a result of the increased relativistic mass nonlinearity, the focusing of the laser light beam becomes most pronounced in TQP. As no such effect is present in RCP, the focusing effect is minimum in this case.

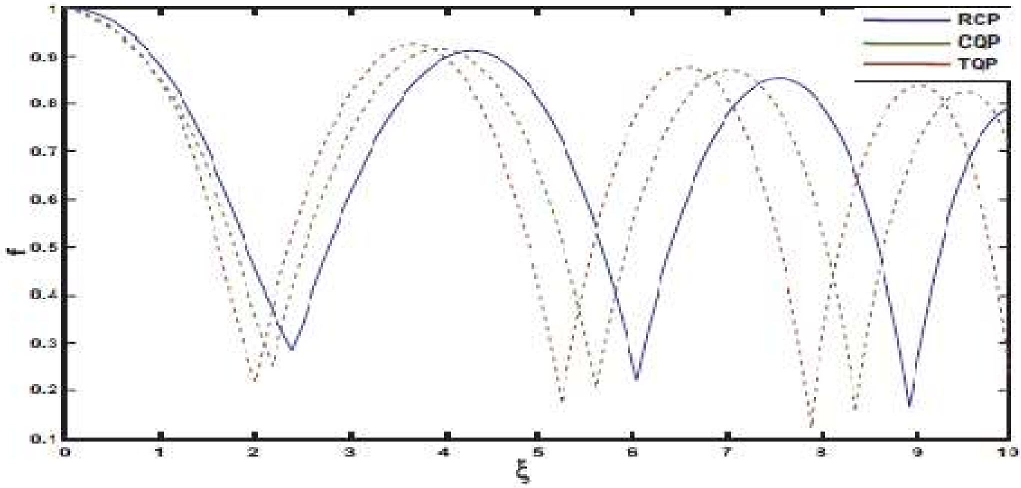

Fig.7.4: Comparison of self focusing of the laser light beam in different regimes of plasma i.e., RCP, CQP and TQP

Fig. 7.5 shows the influence of gradient of the density ramp on self focusing of laser light. The plots in fig. 7.5 illustrate that self focusing of the beam of laser light can also be enhanced by increasing the slope of the density ramp. This happens because when the slope of the density ramp is increased, the beam of laser light encounters a lower refractive index in regions within the plasma at greater depths. As a result, an increased slope in the density ramp increases the focusing of the laser light.

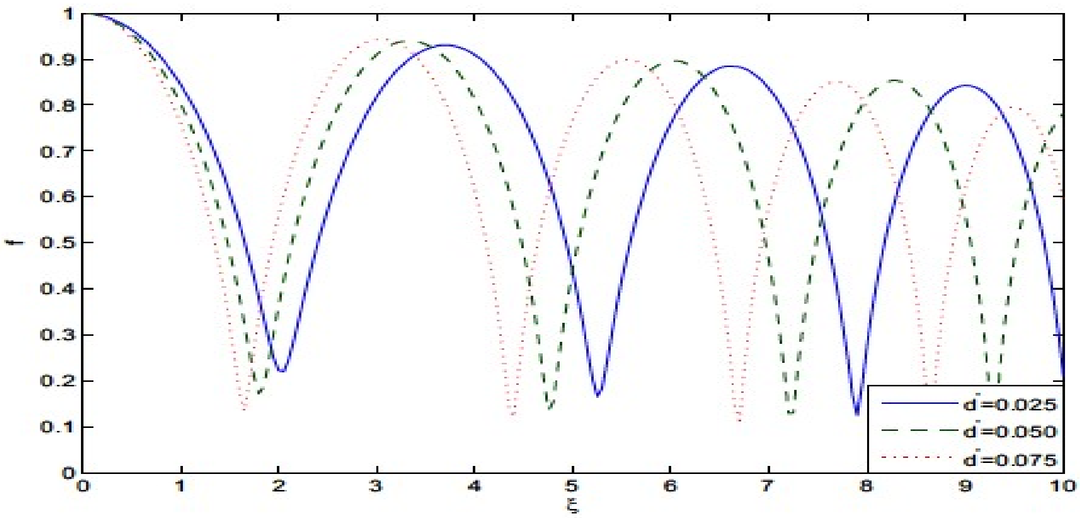

Fig.7.5: Impact of slope of density ramp on beam width of laser beam at fixed values of

$\frac{x_0}{r_0} = 0.75$, $\beta E_{00}^2 = 3$ and $\left(\frac{\omega_{po} r_0}{c}\right)^2 = 9$

7.5 Potential well dynamics

Eq. (7.13) bears resemblance to the equation of motion of a forced harmonic oscillator with unit mass and is expressed as follows:

$$\frac{d^2 f}{d\xi^2} + \frac{\partial V(f)}{\partial f} = 0 \tag{7.14}$$

where,

$$V(f) = -\int \big(D(f) - R(f)\big) df \tag{7.15}$$

Using Eqs. (7.13), (7.14) in (7.15) we get

$$V(f) = \frac{1}{8f^2}\left[\frac{1 + e^{-\frac{x_0^2}{r_0^2}}\left(1 - \frac{x_0^2}{r_0^2}\right) + e^{-\frac{x_0^2}{r_0^2}}\left(2 - \frac{x_0^2}{r_0^2}\right)}{\left(2 + 2\frac{x_0^2}{r_0^2}\right) + \left(2 + 2\frac{x_0^2}{r_0^2}\right)e^{-\frac{x_0^2}{2r_0^2}} + 2e^{-\frac{x_0^2}{r_0^2}}}\right] - \frac{1}{4\pi}\left(\frac{\omega_{p0}(0) r_0}{c}\right)^2$$

$$\big(1 + \tan(d'\xi)\big)\frac{\beta E_{00}^2}{f^2}\frac{1}{\left(2 + 2\frac{x_0^2}{r_0^2}\right) + \left(2 + 2\frac{x_0^2}{r_0^2}\right)e^{-\frac{x_0^2}{2r_0^2}} + 2e^{-\frac{x_0^2}{2r_0^2}}}(K_1 + K_2 + K_3 + K_4) \tag{7.16}$$

From Eqs. (7.13) and (7.16) it is noted that the application of variational theory has effectively simplified the problem of nonlinear wave propagation, transforming it into a straightforward mechanical problem. Specifically, it can be likened to the motion of a particle with unit mass oscillating within a central potential represented by $V(f)$. In this transformed problem, the role of the phase space coordinates $(r, v)$for the particle is now being played by $(f, \frac{df}{d\xi})$.

Figure 7.6 illustrate the effect of $\frac{x_0}{r_0}$ on potential function $V(f)$. It is observed that$V(f) \longrightarrow \infty$ for $f \longrightarrow 0$. The reason for this is that diffraction effects dominate for $f \longrightarrow 0$. For $f \longrightarrow \infty$; $V(f) \longrightarrow 0$ because laser beams possessing exceedingly large spot dimensions exhibit negligible diffraction phenomena. Additionally, they fail to incite any non linear interactions within the medium.

Another noteworthy observation is that as $\frac{x_0}{r_0}$ grows within the range of $0 \le \frac{x_0}{r_0} < 1.5$, the lower point of the potential well moves in an upward direction. Conversely, when the value of $\frac{x_0}{r_0}$ increases beyond 1.5, the lower point of the potential well moves in a downward direction. This indicates that for $0 \le \frac{x_0}{r_0} < 1.5$, Q.G beams of laser light with larger value of

$\frac{x_0}{r_0}$ require lower intensity to achieve self focusing. However, for $\frac{x_0}{r_0} > 1.5$, beams of laser light with increased values of $\frac{x_0}{r_0}$ necessitate higher intensity levels for achieving self-focusing.

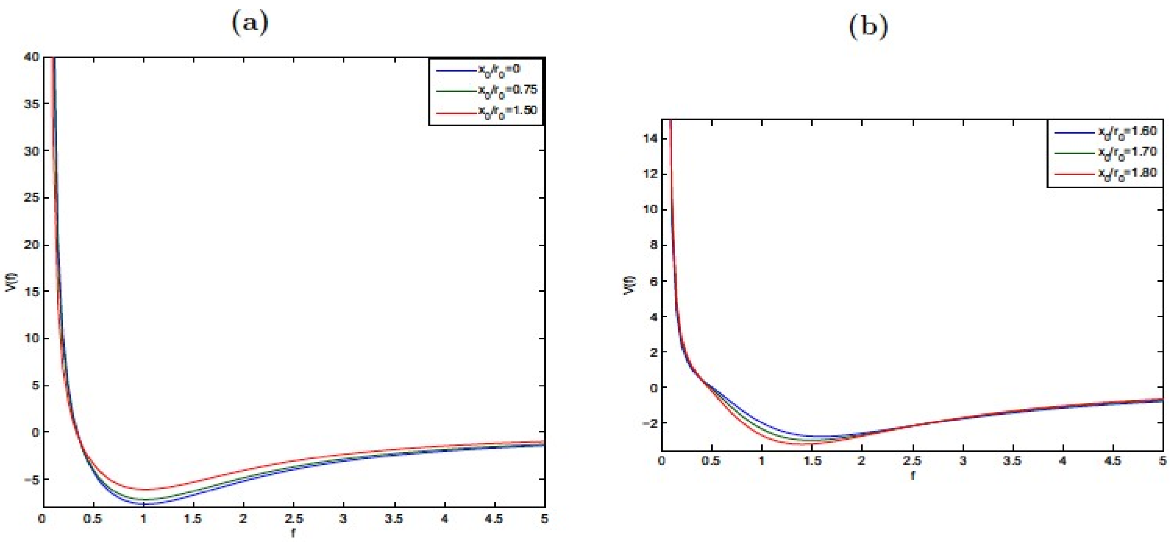

Fig.7. 6: Effect of $\frac{x_0}{r_0}$ on potential well for self-focusing

As $V(f) \longrightarrow \infty$ for $f \longrightarrow 0$ and $V(f) \longrightarrow 0$ for $f \longrightarrow \infty$, it can be interpreted that the potential function $V(f)$ for self-focusing is identical to the potential function experienced by a particle moving under the influence of a central force field. Therefore, f as regarded as radial coordinate within the polar coordinate system (f, θ_f). The Lagrangian L describing particle with unit mass within the plane (f, θ_f)can be defined as follows.

$$L' = \frac{1}{2}\left[\left(\frac{df}{d\xi}\right)^2 + f^2\left(\frac{d\theta_f}{d\xi}\right)^2\right] - U(f) \tag{7.17}$$

Multiplying Eq. (7.17) by $\frac{df}{d\xi}$ and performing integration on both sides, we get

$$\frac{1}{2}\left(\frac{df}{d\xi}\right)^2 + V(f) = E \tag{7.18}$$

Here, E represents the integration constant, representing the energy function. The canonical momentum linked with the polar angle θ_f can be expressed as follows.

$$p_{\theta_f} = \frac{dL'}{d\dot{\theta_f}} = f^2\dot{\theta}_f \tag{7.19}$$

where, $\dot{\theta}_f = \frac{d\theta_f}{d\xi}$. The corresponding Lagrange equation

$$\frac{d}{d\xi}\left(\frac{\partial L'}{\partial \dot{\theta}_f}\right) - \frac{\partial L'}{\partial \theta_f} = 0$$

gives

$$\frac{d}{d\xi}\left(f^2 \dot{\theta}_f\right) = 0$$

which implies that

$$f^2 \dot{\theta}_f = 1 \tag{7.20}$$

This shows that the angular momentum associated with polar coordinate θ_f remain conserved. The Lagrange equation for radial coordinate f is given by

$$\frac{d}{d\xi}\left(\frac{\partial L'}{\partial \dot{f}}\right) - \frac{\partial L'}{\partial f} = 0$$

Using Eq. (7.19) we get

$$\frac{d^2 f}{d\xi^2} = -\frac{d}{df}(U(f) + \frac{l^2}{2f^2})$$

After multiplying the equation with $\frac{df}{d\xi}$, and integrating, we obtain the following result

$$\frac{1}{2}\left(\dot{f}^2 + \frac{l^2}{f^2}\right) + U(f) = E \tag{7.21}$$

Comparing Eq. (7.21) with (7.18)

$$V(f) = U(f) + \frac{l^2}{2f^2} \tag{7.22}$$

By comparing Kepler's potential with the potential function described by Eq. (7.22) for self-focusing, we can establish a relationship

$$l = \frac{1}{4}\left[\frac{1 + e^{-\frac{x_0^2}{r_0^2}}\left(1 - \frac{x_0^2}{r_0^2}\right) + e^{-\frac{x_0^2}{2r_0^2}}\left(2 - \frac{x_0^2}{r_0^2}\right)}{\left(2 + 2\frac{x_0^2}{r_0^2}\right) + \left(2 + 2\frac{x_0^2}{r_0^2}\right)e^{-\frac{x_0^2}{2r_0^2}} + 2e^{-\frac{x_0^2}{2r_0^2}}}\right] \tag{7.23}$$

This is an important result which shows how the divergence of a laser beam is influenced by $\frac{x_0}{r_0}$, since the angular momentum of the beam in (f, θ_f) plane relies on $\frac{x_0}{r_0}$.

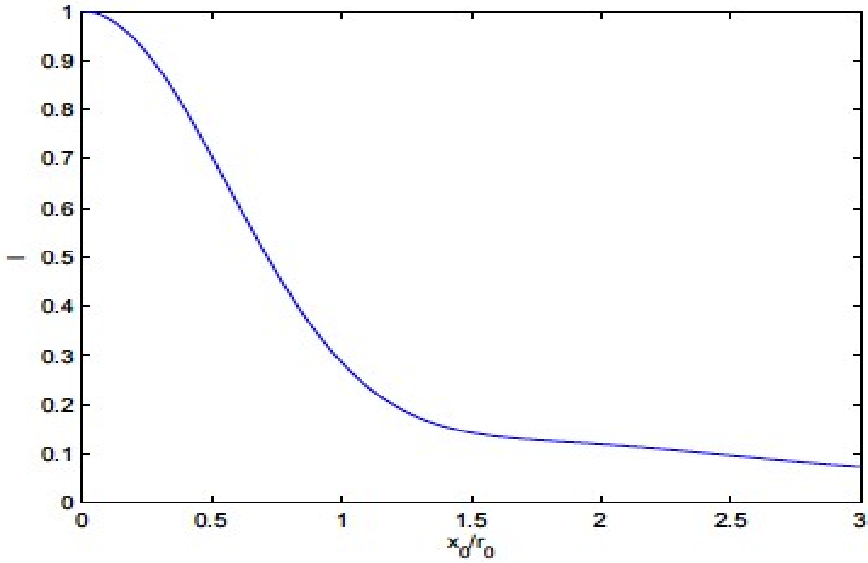

Fig.7.7: Fluctuations in angular momentum of Q.G with $\frac{x_0}{r_0}$ in (f, θ_f) plane

7.6 Self-channeling of laser beam

Since, $\frac{df}{d\xi}$ represents the curvature of the wavefront of the laser light beam, therefore, if rate of change of curvature i.e., $\frac{d^2f}{d\xi^2}$ vanishes at the entrance into the plasma then $\frac{df}{d\xi}$ will remain constant for subsequent values of ξ i.e., it will stay vanished throughout the complete passage of the laser beam as it passes through the plasma. As a result this will cause f to preserve its initial value. Hence, along with the condition $f = 1$ at $\xi = 0$, the condition $\frac{df}{d\xi} = 0$ at $\xi = 0$ ensures that the laser beam maintains its shape and size intact throughout its propagation. As a result, the beam is referred to as a spatial optical soliton. Thus by taking $f = 1$ and $\frac{df}{d\xi} = \frac{d^2f}{d\xi^2}$=0 in Eq. (7.13) we get the relation between dimensionless beam width $\left(\frac{\omega_{p0}(0)r_0}{c}\right)$ and the critical beam intensity βE_{00}^2 as

$$r_e^2 = \frac{\pi}{2\beta E_{00}^2} \frac{1 + e^{-\frac{x_0^2}{r_0^2}}\left(1 - \frac{x_0^2}{r_0^2}\right) + e^{-\frac{x_0^2}{r_0^2}}\left(2 - \frac{x_0^2}{r_0^2}\right)}{K_1' + K_2' + K_3' + K_4'} \tag{7.24}$$

Where,

$$K_i' = K_i|_{f=1};\ \text{i=1-4}$$

$$r_e = \frac{\omega_{p0}(0) r_0}{c}$$

For the laser beams satisfying Eq. (7.24), $\frac{d^2 f}{d\xi^2}$will vanish at $\xi = 0$. Thus, as the beam of laser light traverses the plasma, the wavefront curvature will remain unchanged. In other words, the value of $\frac{df}{d\xi}$ will remain constant throughout the journey, and this constant value will be identical to the initially presumed value, which is zero. Indeed, if, $\frac{d^2 f}{d\xi^2} = \frac{df}{d\xi} = 0 \ at\ \xi = 0$, it indicates that $\frac{df}{d\xi} = 0$ for $\xi > 0$ as well. Physically, this implies that beam maintains its shape and size throughout its propagation. In such a case the laser beam is said to be self trapped. Thus, the points lying over the critical curve correspond to self trapped mode for which the laser beam itself creates a waveguide inside the plasma for its stable propagation.

The points $(\beta E_{00}^2, r_e)$ situated above the critical curve correspond to the positive value of $\frac{d^2 f}{d\xi^2}$ at $\xi = 0$. Such beams will broaden monotonically with distance. This is because in such a case the optical nonlinearity lacks the required strength to compensate the natural diffraction of the laser beam. However, it will still oppose natural diffraction and hence the extent of broadening will be lesser compared to that in vacuum.

When the point $(\beta E_{00}^2, r_e)$ is located below the critical curve, the initial value of $\frac{d^2 f}{d\xi^2}$will be negative. This indicates that the beam will converge as it propagates, resulting in self-focusing

The graphs depicted in figs. 7.8 (a) and 7.8 (b) demonstrate that at lower value of laser intensities$(\beta E_{00}^2 \ll 1)$, the equilibrium beam width $r_e$decreases steeply. Nonetheless, under exceedingly high laser intensities$(\beta E_{00}^2 \gg 1)$, the equilibrium beam width remains unaffected by the laser intensity. The noticed behavior can be attributed by the fact that at extremely high laser intensities, the plasma region exposed to laser experiences significant depletion of electron density. As a result, the dielectric function of the plasma becomes unrelated to the laser intensity. Consequently, under high intensities, the laser beam width becomes unaffected by the intensity. Furthermore, it is noted that laser beams with extremely narrow widths cannot undergo self-channeling. The reason for this is the wider diffraction angles exhibited by narrower beams. As a result, to achieve self-guidance, narrower beams necessitate larger differences in refractive index.

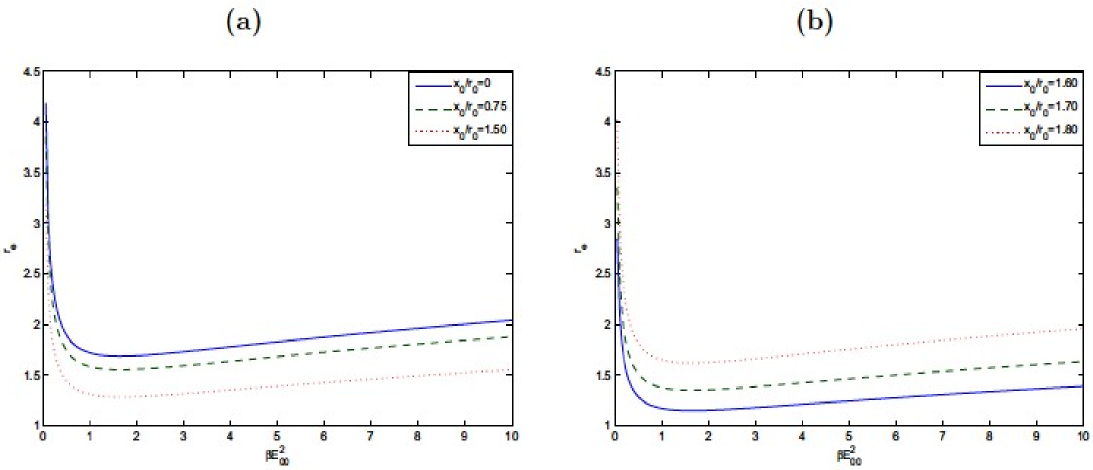

Fig.7.8: Critical curves of the laser beam in TQP for various values of $\frac{x_0}{r_0}$

(a) $\frac{x_0}{r_0}$ =0, 0.75, 1.50; (b) $\frac{x_0}{r_0}$=1.60, 1.70, 1.80

It becomes evident that as the value of $\frac{x_0}{r_0}$ increases within the range of $0 \leq \frac{x_0}{r_0} < 1.5$, the critical curves undergo a downward shift. Physically this implies that the laser light beams characterized by larger values of $\frac{x_0}{r_0}$ in the range $0 \leq \frac{x_0}{r_0} < 1.5$, have the ability to self guide at comparatively reduced power. This is because beam of laser light with larger values of $\frac{x_0}{r_0}$ receive significant contributions from off-axis rays, which lead to nonlinear refraction effects.

7.7 SBS of QG Laser Beam

The progression of laser inside plasma medium excites an IAW whose beam width evolves according to

$$\frac{d^2 f_{ia}}{d\xi^2} = \frac{1}{f_{ia}^3} - \frac{1}{2\pi}\left(\frac{\omega_{ia} r_0}{c}\right)^2 \left(1 + \tan(d'\xi)\right) \frac{\beta E_{00}^2}{f_{ia}^3} \frac{1}{\left(2 + 2\frac{x_0^2}{r_0^2}\right) + \left(2 + 2\frac{x_0^2}{r_0^2}\right) e^{-\frac{x_0^2}{2r_0^2}} + 2e^{-\frac{x_0^2}{r_0^2}}} (K_1 + K_2 + K_3 + K_4) \quad (7.25)$$

Now the power of SBS scattered wave can be obtained as obtained in chapter 5 as:

$$P = \frac{\int E_s E_s^* r dr}{\int A_0 A_0^* r dr}$$

Figs. 7.9 and 7.10 show the effect of $\frac{x_0}{r_0}$ on power of scattered wave. It can be seen that in the range $0 \le \frac{x_0}{r_0} \le 1.5$ the SBS scattering increases and then it starts decreasing for $\frac{x_0}{r_0} > 1.5$

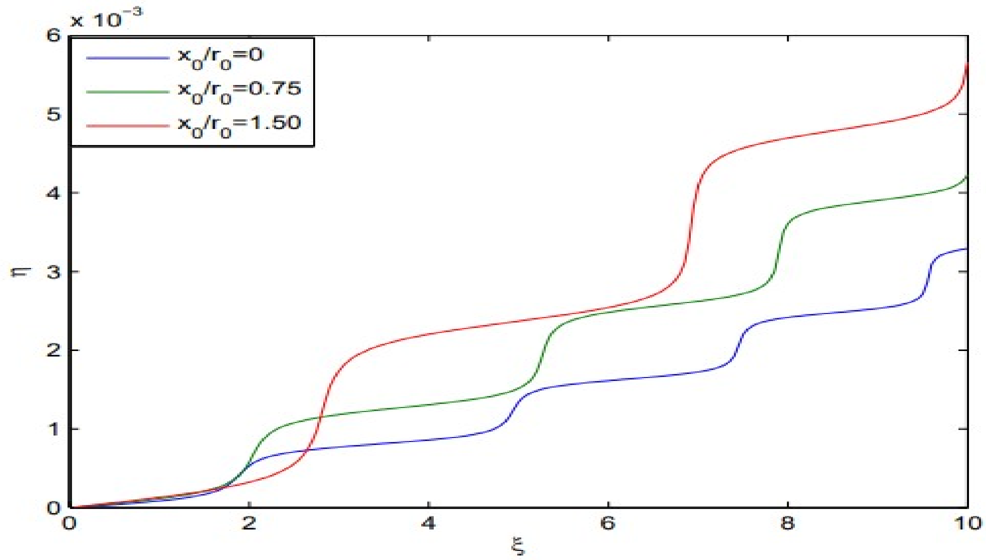

Fig.7.9: Effect of $\frac{x_0}{r_0}$ on SBS in the range $0 \le \frac{x_0}{r_0} \le 1.5$

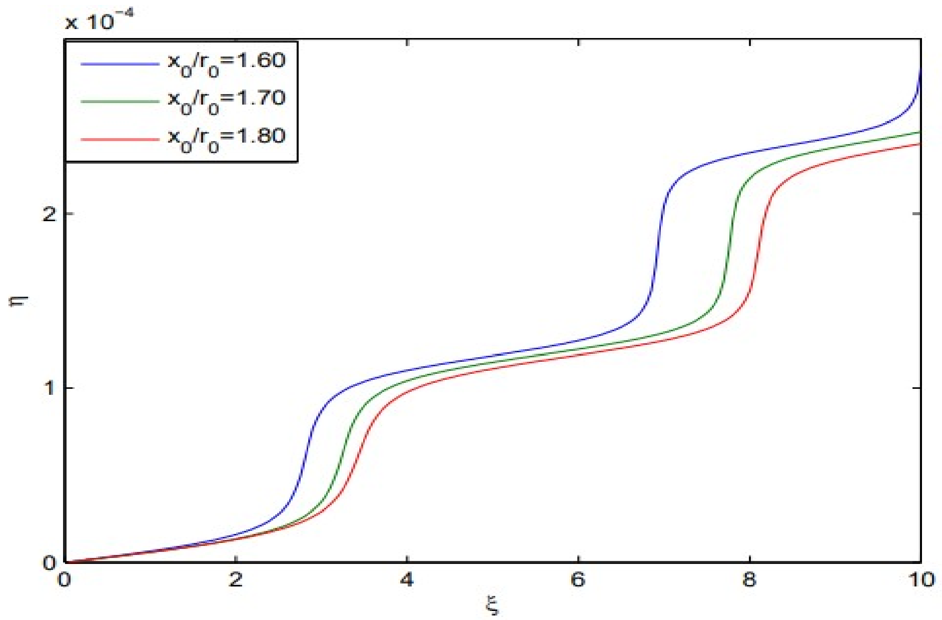

Fig.7.10: Effect of $\frac{x_0}{r_0}$ on SBS in the range $\frac{x_0}{r_0} > 1.5$

Chapter- 8

Conclusions and Future Scope

8.1 Conclusion

In present work, self-action effects of non Gaussian laser beams (q-Gaussian and Quadruple Gaussian) have been investigated semi analytically by using variational theory. The study is then extended to see the effect of self-focusing of the laser beam on excitation of IAWs and then further its effect on SBS. The major conclusions of present study are as follows:

1. The impact of relativistic self-focusing of q-Gaussian laser beams on Stimulated Brillouin Scattering (SBS) in underdense plasma targets characterized by axially-increasing plasma density has been studied. Through our observations, we have noted that the amplitude distribution across the cross-sectional area of a beam significantly influences the propagation dynamics within nonlinear media. The laser beams, whose amplitude structure is deviated from ideal Gaussian profile, possess more self-focusing in plasmas and get scattered more by preexisting IAWs through the SBS phenomenon. For instance, as compared to an ideal Gaussian beam, a laser beam with deviation parameter $q = 3$ gets 2.75 times more self-focused as well as, for the same propagation distance, the power of scattered wave for the beam with $q = 3$ is four times greater as compared to that for the ideal Gaussian beam. Therefore, by controlling the deviation parameter q, one can optimize the Stimulated Brillouin Scattering (SBS) of the laser light beam. The power carried by scattered beam can also be controlled by changing the initial intensity of the laser beam or by changing the density ramp slope.
2. For a fixed geometrical radius r_0, a quadruple-Gaussian laser beam with a larger value of $\frac{x_0}{r_0}$ will have a larger effective beam width.This is attributed to the transfer of laser intensity from central or axial to the peripheral or off-axial portion of the wavefronts as the value of $\frac{x_0}{r_0}$ increases. One can deduce or infer that quadruple-Gaussian beams exhibit a smaller diffraction divergence.
3. The self-focusing of the laser beam influences the excitation of (IAWS) in axially inhomogeneous plasmas. The incorporation of beam ellipticity and deviations from

the ideal Gaussian profile in the amplitude structure of the beam of laser light has been studied. It is inferred that as the amplitude distribution of the beam of laser light approaches the ideal Gaussian profile, there is a notable reduction in the power of the excited Ion Acoustic Wave (IAW). Thus in order to mitigate the excitation of IAWs in ICF, the laser beams should have nearly ideal Gaussian intensity across their cross sectional areas.

8.2 Future Scope

Based on laser plasma interactions, various projects and investigations on inertial confinement fusion, particle acceleration, THz generation etc. are going on worldwide. The breath of all these applications is ultimately dependent on the efficiency of laser plasma coupling where SBS is a major nuisance. Despite being a nuisance, SBS is having important applications as well. In lasers, the major issue is to have desired wavelengths as laser action is possible only for specific wavelengths. SBS gives the possibility to have tunable lasers with broad bandwidths. Thus in future, I will investigate SBS of laser beams with other beam profiles like Hermite Gaussian, Super Gaussian, Airy Gaussian etc. If the study is successful, it will be extended to see the effect of hot spots on the beam wavefronts on SBS in plasmas.

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RELATIVISTIC EFFECTS ON STIMULATED BRILLOUIN SCATTERING OF SELF-FOCUSED q-GAUSSIAN LASER BEAMS IN PLASMAS WITH AXIAL DENSITY RAMP

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Abstract

We investigate the phenomenon of stimulated Brillouin scattering (SBS) of q-Gaussian laser beams nonlinearly-interacting with underdense plasmas. When an intense laser beam with frequency ω_0 propagates through plasma, due to relativistic mass nonlinearity of plasma electrons, it gets coupled to a preexisting-ion acoustic wave (IAW) at frequency ω_{ia}. The nonlinear interaction of pump beam with IAW produces a back-scattered wave at frequency $\omega_s = \omega_0 - \omega_{ia}$. In view of the variational theory, we obtain semi-analytical solutions of the coupled nonlinear wave equations for the three waves (pump, IAW, and scattered wave) under the Wentzel-Kramers-Brillouin (WKB) approximation. We show that the scattered-wave power is significantly affected by the self-focusing effect of the pump beam.

Keywords: q-Gaussian beam, density pump, relativistic plasma, stimulated Brillouin scattering, self-focusing.

1. Introduction

Ever since the proposal of initiating nuclear fusion by intense laser beams (ICF) for viable energy production [1] without producing any harm to global climate, there was a considerable interest in the nonlinear interaction of intense laser beams with plasmas. In laser-driven fusion, the goal is to deposit the laser energy at a particular density in the plasma in order to derive the compression and subsequent heating of the fuel pellet. If the pellet is sufficiently compressed, it may undergo fusion, with the release of a large amount of energy. However, the laser may interact with the plasma at a density different to that which is intended, leading to myriad undesirable effects [2–5] and preventing the effective implosion of the target.

Due to their remarkable properties of quasineutrality and collective behavior, plasmas possess a number of natural modes of oscillations [6–8]. This includes high-frequency electron plasma waves (EPWs) and low-frequency ion acoustic waves (IAWs); the latter ones correspond to acoustical phonons, as do the

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RESEARCH ARTICLE

Self-focusing of cosh-Gaussian laser beam in collisional plasma: effect of nonlinear absorption

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Abstract Self-focusing phenomenon of intense laser beams in underdense plasmas has been investigated theoretically. The mechanism of optical nonlinearity of plasma has been modeled by Ohmic heating of the plasma electrons resulting from their collisions with other species. The effect of nonlinear absorption of laser energy in plasma also has been incorporated. Formulation is based on finding a semi-analytical solution of the nonlinear wave equation for the slowly varying beam envelope. For this purpose, moment theory in W. K. B approximation has been invoked that converts nonlinear wave equation to an ordinary differential equation governing the evolution of spot size of the laser beam. The differential equation so obtained has been solved numerically to envision the effect of laser–plasma parameters on self-focusing of the laser beam

Keywords Self focusing · Cosh Gaussian · Plasma · Moment theory

Introduction

Light has always fascinated man and investigation of interaction of light with matter is as old as human civilization. Ancient people used glass-made lenses to focus light to burn pieces of papers. However, with the debut of laser [1] in 1960, the twentieth century witnessed a dramatic shift in our perception and understanding of light. Due to its extraordinary properties of coherence, high intensity and monochromaticity, laser light revealed true beauty of light matter interactions. When laser was born, little did its inventors and aficionados realize that it would not only sweep that era of scientists off its feet, but would continue to challenge and mesmerize generations to come. With that high expectation as a benchmark, the laser has proved to be nothing short of a miracle. The laser has become ubiquitous in the almost every field of modern age science and technology. Even routine life applications are abound and still too many applications are in pipeline and are waiting for their turn.

Amelioration in laser technology fueled by the advent of chirp pulse amplification [2] (CPA) technique has led to a resurgence in the field of light matter interactions by giving birth to two entirely new areas of science, i.e., nonlinear optics and laser–plasma interactions. Interactions of intense coherent beams of light produced by modern laser systems with plasmas are rich in copious nonlinear phenomena those were not possible before the invention of laser. This includes a gamut from optical self-action effects like[3, 4] (self-focusing, self-guiding, self-phase modulation, etc.) to several frequency mixing processes[5, 6] like sum frequency generation, difference frequency generation, second harmonic generation (SHG), etc. Being extremely complex but rich in physics, these nonlinear effects have the potential to keep researchers busy for several upcoming years. Over past few years, veteran physicists are attempting to improve on the understanding of laser–plasma interactions by carrying out experimental as well as theoretical investigations. The major impetus behind these investigations on laser–plasma interactions was built by the proposal of initiating controlled nuclear fusion reaction by using ultra-intense laser beams [7]. Fusion is considered to be the cleanest source of energy that bears the promise to

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RESEARCH ARTICLE

Nonlinear interaction of quadruple Gaussian laser beams with narrow band gap semiconductors

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Abstract This paper presents an investigation on nonlinear propagation of quadruple Gaussian (Q.G) laser beam in narrowband semiconductor (e.g., n-type InSb) plasmas. In the presence of laser beam, the electron fluid in the conduction band becomes relativistic that makes the medium highly nonlinear. As a result the laser beam gets self-focused. Following variational theory approach in W.K.B approximation the numerical solution of the nonlinear Schrodinger wave equation (NSWE) for the field of incident laser beam has been obtained. Particular emphasis is put on dynamical variations of beam spot size and longitudinal phase (Gouy phase). Self-trapping of the laser beam resulting from the dynamical balance between diffraction broadening and nonlinear refraction also has been investigated.

Introduction

The advent of laser [1] in the early 1960s set in motion a train of events that led to a renaissance in the field of light–matter interactions. The past few years have seen two important advances. One was the proposal of initiating fusion reactions [2] for viable energy production that would quench humanity's endless thirst for energy without worsening the global climate change. Another noteworthy advance was the laser-driven particle accelerators [3]. Particle acceleration by laser-driven plasma wave is an extremely interesting and far-reaching idea that can bring huge particle accelerators to bench top. The efforts to translate these concepts into reality, however, have to surmount two serious problems: (1) The creation of relativistic plasmas requires ultrahigh laser intensities in the excess of 10^{18}-10^{20} W/cm^2, and (2) the plasmas have to be extremely homogeneous. These rather daunting requirements have made it difficult even to carry out exploratory experiments to test the proposed ideas.

Therefore, there have been ongoing efforts to find alternatives to standard plasma experiments, where these severe constraints could be mitigated. One could then validate the theoretical frameworks and shed light on the eventual feasibility of these ideas. Fortunately, such an alternative exists; it is provided by certain special plasmas found in the narrow-band semiconductors [4, 5] (Fig. 1). Plasmas contain negative and positive carriers under conditions in which they do not combine. In Fig. 1 a red dot is an electron, or negative charge, a blue dot containing a plus sign is a positive charge, and neutral atoms are shown green. In a gas there are two kinds of charge carrier: electrons and positive ions (atoms lacking electrons). In a simple metal the only mobile carriers are electrons; positive ions are locked in the crystal lattice. A semiconductor has two kinds of mobile carrier: electrons and positive "holes" or missing electrons. All three plasmas can transmit waves.

Interaction of intense laser beams with semiconductor plasmas is rich in copious nonlinear effects. This spans a gamut from parametric instabilities to several self-action effects like self-focusing, self-trapping self-phase modulation, etc. All these nonlinear effects are extremely complex but rich in physics to provide a necessary test bed for

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Scattering of Laser Light in Dielectrics and Plasmas: A Review

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A review on some nonlinear phenomena associated with light matter interactions has been presented. Emphasis is put on explaining the basic physics of the phenomenon while minimizing the mathematics. Particularly the phenomena of Rayleigh scattering, stimulated Raman and Brillouin scattering have been discussed in detail. As a special case scattering of intense laser beams with fourth state of matter i.e., plasma also has been discussed.

Keywords: Scattering of light, raman scattering, rayleigh scattering, brillouin scattering

1 INTRODUCTION

Laser[1] is one of the most successful pieces of apparatus gifted by 20th century science. When laser made its debut in 1960 people considered it to be solution which is searching for its problem. Since its invention the imact of laser on our lives has changed with time and still is changing. Now laser is ubiquitous in every aspect of life: from super matket barcode scanners, security checkpoints, CD writers to high end applications like medical diagnisis and surgery[2][3], inertial confinement fusion[4][5]. The extent of diversity in the applications of laser can be estimated from

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RESEARCH ARTICLE

Optical guiding of *q*-Gaussian laser beams in radial density plasma channel created by two prepulses: ignitor and heater

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Abstract Self-action effects (self focusing) and self phase modulation of *q*-Gaussian laser beam in plasma channel created by ignitor heater technique have been investigated theoretically. The ignitor beam causes tunnel ionization of air. The heater beam heats the plasma electrons and establishes a parabolic channel and also prolongs the channel life by delaying the electron ion recombination. The third beam (*q*-Gaussian beam) is guided in the plasma channel under the combined effects of density nonuniformity of the parabolic channel and relativistic mass nonlinearity of the plasma electrons. Formulation is based on finding the numerical solution of nonlinear Schrodinger wave equation (NSWE) for the fields of incident laser beams with the help of moment theory approach. Particular emphasis is put on dynamical variations of the spot size of the laser beams and longitudinal phase-shift of the guided beam with distance of propagation.

Keywords Self-Focusing · Self-Trapping · Phase Modulation · Bessel Gauss Lasers · Ponderomotive Force

Introduction

After the transistor, lasers [1] are considered to be one of the most successful inventions of 20th century science. When laser made its debut in 1960, some people called it solution in search of a problem. Today lasers have reserved their place in almost every aspect of life: consumer technologies like CD players, super market checkout scanners to higher end technologies. With the advent of chirped pulse amplification (CPA) technique [2], the turn of last century has witnessed a giant leap in laser technology leading to a renaissance in the field of light-matter interactions. This amelioration in laser technology has given birth to an entirely new field of science known as laser–plasma interactions. An agglomeration of nonlinear phenomena viz., parametric instabilities, [3, 4] higher harmonic generation, [5, 6] Self-focusing, [7] self-phase modulation [8], etc is ubiquitous in these laser plasma interactions.

A major impetus behind the investigations on laser plasma interactions was provided by the proposal of initiating fusion reactions [9, 10] for viable energy production by using intense laser beams. Fusion is considered to be the cleanest source of energy as there will be no emission of radioactive end products and green house gases. Thus, it bears the promise to quench humanity's endless thirst for energy without making any harm to global climate. Along the way the field of laser–plasma interactions has branched into a number of potential applications like laser-driven accelerators, [11, 12] X-ray lasers, [13, 14] higher harmonic generation [5, 6], etc. The ultimate breath of most of these applications depends on stable guiding of intense laser beams over longer distances, without significant energy loss. However, due to lights natural wave property of diffraction, a light beam traveling in vacuum or in a medium always broadens in the absence of an optical guiding mechanism. Diffraction broadening of the laser beam is thus the fundamental phenomenon that jeopardizes the feature of aforesaid applications by negating the efficiency of laser–plasma coupling. Hence, there is surging interest to explore the methods that may aid to increase the

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Potential Well Dynamics of Self Focusing of Quadruple Gaussian Laser Beams in Thermal Quantum Plasma

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This paper presents theoretical study on self-action effects of intense laser beams interacting with fusion plasmas. Particularly the phenomena associated with the nonlinear refraction of the laser beam have been investigated in detail. In order to see the effect of uniformity of the illumination over the beam phase fronts on its propagation characteristics the irradiance profile of the beam has been modeled by quadruple Gaussian (Q.G) profile. Following Variational theory approach, the nonlinear partial differential equation (PDE) for the beam envelope has been reduced to a set of coupled ordinary differential equations for the evolution of beam width and axial phase. The equations so obtained have been solved numerically to envision the effect of laser as well as medium parameters on the propagation characteristics of the laser beam.

1. INTRODUCTION

The quest to initiate nuclear fusion by employing intense laser beams [1-3] to quench endless thirst of human for energy without harming the global climate is at the vanguard of research since past few years. This will be similar to

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RESEARCH ARTICLE

Excitation of ion acoustic waves by self-focused *q*-Gaussian laser beam in plasma with axial density ramp

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Abstract Dynamics of the laser-driven ion acoustic waves (IAWs) in plasmas with axial density ramp has been investigated theoretically. The effect of self-focusing of the laser beam on the power of laser excited IAW has been incorporated. During its propagation through the plasma, the laser beam excites an IAW at frequency ω_{ia} that due to the optical nonlinearity of plasma gets nonlinearly coupled to the laser beam. Using variational theory, semianalytical solutions of the coupled nonlinear wave equations for the pump wave and IAW have been obtained under W.K.B approximation technique. It has been observed that power of the IAW is significantly affected by the self-focusing effect of pump beam.

Keywords *q*-Gaussian · Density ramp · Relativistic plasma · Self-focusing · Ion acoustic waves

Introduction

Plasma is a collection of positively and negatively charged particles moving about so energetically that they do not readily combine. Plasmas are everywhere in the universe [1]. They form the intensely hot gas under high pressure in the sun and the stars, as well as the rarefied gas in interstellar space and in the ionospheric envelope surrounding the earth. Plasmas also exist closer to hand. They are present in the flames of burning fuel and in gas-discharge devices such as neon signs. Plasmas exhibit such an enormous variety of physical effects that physicists have studied their properties for about 200 years. Past research on plasmas, particularly on gas discharges, led to the discovery of the electron and to the elucidation of atomic structure [2].

The current interest in plasmas reflects two principal motives. The first one is technological. An understanding of plasma behavior is crucial to the controlled release of thermonuclear energy [3–6], the attempt to reproduce in a man-made plasma the kind of nuclear reaction found in the sun. Another technical goal is the design of magnetohydrodynamic generators [7], in which electric power is generated by jets of gas plasma traversing magnetic fields. The second broad motive for the study of plasmas is the importance of plasma phenomena in space and in astrophysics. When a plasma is subjected to electromagnetic fields, the motion of the particles is no longer completely random. One important consequence of this imposed order is that a plasmas can transmit certain kinds of waves that are related to electromagnetic waves but that have unique and curious properties. These waves include high-frequency electron plasma waves [8, 9] (EPWs) and low frequency one called ion acoustic waves [10, 11] (IAWs).

IAWs can be excited in plasmas due to their remarkable properties of quasineutrality and collective behavior. Plasma is a state of matter that contains enough heat that atoms lose their individuality. The negatively charged electrons are still attracted by positively charged nuclei, but they are not bound together. This gives a plasma some unusual properties unlike most kind of ordinary matter—solids, liquids and gases—the free floating electrons and ions of a plasma are strongly affected by electric and magnetic fields. Plasma as a whole is quasineutral, but as

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RESEARCH ARTICLE

Electron plasma wave excitation by self-focused cosh gaussian laser beams in axially inhomogeneous plasma: effect of density ramp

Naveen Gupta[1] · **Sanjeev Kumar**[1,2] · **S. B. Bhardwaj**[3] · **Rohit Johari**[1] · **Suman Choudhry**[1]

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Abstract Dynamics of the laser driven electron plasma waves (EPWs) in plasmas with axial density ramp has been investigated theoretically. The effect of self focusing of the laser beam on the power of laser excited EPW has been incorporated. During its propagation through the plasma, the laser beam excites an EPW at frequency ω_{ep} that due to the optical nonlinearity of plasma gets nonlinearly coupled to the laser beam due to the ponderomotive nonlinearity of plasma electrons. Using variational theory semi analytical solutions of the coupled nonlinear wave equations for the pump wave and EPW have been obtained under W.K.B approximation technique. It has been observed that power of the EPW is significantly affected by the self-focusing effect of pump beam.

Keywords Self-Focusing · Electron plasma wave · Cosh-gaussian · Ponderomotive force · Clean energy

Introduction

Investigations on coupling of intense laser beams with plasmas are at the vanguard of research since past few decades due to its importance in many potential applications including laser fusion [1–3], plasma wake field accelerators [4, 5], X-ray lasers [6, 7], terahertz generation [8], etc. The ultimate breath of these applications depends on the efficiency of laser plasma coupling which is further decided by many different nonlinear processes [9–11]. These processes range from collisional absorption to excitation of copious laser driven instabilities [12–15]. These instabilities can be represented as the resonant coupling of the incident laser beam into two daughter waves. In the absence of external magnetic field these daughter waves can be electron plasma waves, ion acoustic waves along with a scattered electromagnetic wave.

EPWs can be excited in plasmas due to their remarkable properties of quasi neutrality and collective behaviour. Plasma is a state of matter that contains enough heat that atoms lose their individuality. The negatively charged electrons are still attracted by positively charged nuclei, but they are not bound together. This gives a plasma some unusual properties unlike most kind of ordinary matter-solids, liquids and gases-the free-floating electrons and ions of a plasma are strongly affected by electric and magnetic fields. Plasma as a whole is quasi neutral, but as the electrons and positively charged ions are separated, a disturbance can create regions of net negative and net positive charges acting like the plates of a charged parallel plate capacitor. Such an uneven distribution of charges results in an electric field running from positive to negative regions. This electric field pulls the electrons and ions towards each other with equal forces. Due to their large mass ions are lazy and thus remain at rest and the electrons move towards the ions. As the electrons move towards the ions, they steadily gain velocity and momentum like a pendulum moving towards its mean position from an extreme position. Due to this gain in momentum the electrons overshoot their equilibrium positions resulting in reversing the direction of electric field. Now the reversed electric field opposes the electron motion slow them down and then pulling them back again. The process repeats itself, establishing an electron oscillator. In the presence of thermal velocity these electron oscillations lead to a longitudinal wave compression and rarefaction regions of electrons

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RESEARCH ARTICLE

Self-focusing of laser-driven ion acoustic waves in plasma with axial density ramp

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Received: 11 June 2022 / Accepted: 20 August 2022

Abstract In the present work, dynamics of the laser-driven ion acoustic waves (IAWs) in plasmas with axial density ramp has been investigated theoretically. The effect of propagation characteristics of the laser beam on self-focusing of IAW has been investigated in detail. During its propagation through the plasma, the laser beam excites an IAW at frequency ω_{ia} due to the optical nonlinearity of plasma and gets nonlinearly coupled to the laser beam. Semi-analytical solutions of the coupled nonlinear wave equations for the pump wave and IAW have been obtained under W.K.B approximation technique using variational theory. It has been observed that propagation of the IAW is significantly affected by the self-focusing effect of laser beam.

Keywords q-Gaussian · Density ramp · Relativistic plasma · Self-focusing · Ion acoustic wave · Clean energy

Introduction

The fundamental currency of our universe is energy. It lights up our homes, grows our food and powers our computers. There is no end to world's energy appetite. The need of energy is so great and growing so rapidly around the world that alternate sources of energy to quench humanity's endless thirst of energy without doing any harm to global climate are required. In this regard, the quest to tap the energy of nuclear fusion [1] by employing intense laser beams to confine an ultra-hot plasma and generate electric power has been in progress since past few decades. The fusion power plants will be fueled by a form of heavy hydrogen found in ordinary sea water and will produce no harmful emissions—no sooty pollutants, no nuclear waste and no greenhouse gases. In laser-driven fusion, the goal is to deposit laser energy at a particular density in the plasma in order to derive the compression and subsequent heating of the fuel pellet. If the pellet is compressed sufficiently, it may undergo fusion, leaving to the release of a large amount of energy. It is as if there is a tiny hunk of the sun on Earth.

Both the allure and the challenges of fusion arise from the nature of the fusion process itself. Fusion fuel is abundant and cheap. The major advantages are: (1) The abundance of fuel—the most easily exploitable fuels are deuterium and tritium. Deuterium occurs naturally in all sources of water specially sea water. Tritium, however, is not readily available naturally, it can easily be manufactured inside the fusion reactor by the bombardment of lithium with neutrons, which also abundant in nature. (2) Cleanest source of energy—fusion does not produce nuclear waste directly. Although tritium is mildly radioactive and neutron activation of the reactor chamber dictates which structural materials are most useful to minimize waste disposal of components discarded in maintenance or the entire reactor assembly at the end of its life.

However, fusion only happens at the extremely high temperatures that are typical characteristics of stars, whereas fission only happens at normal temperatures. The fuel with the lowest kindling point is a mixture of deuterium and tritium that ignites at temperatures around 50 keV (i.e., 50

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RESEARCH ARTICLE

Second-harmonic generation of two cross-focused *q*-Gaussian laser beams by nonlinear frequency mixing in plasmas

Naveen Gupta[1] · S. B. Bhardwaj[2] · Sanjeev Kumar[1] · Suman Choudhry[1] · Rishabh Khatri[1] · Siddhanth Shishodia[1] · Rohit Johari[1]

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Abstract A scheme for second-harmonic generation (SHG) of a pair of q-Gaussian laser beams interacting nonlinearly with underdense plasma has been proposed. Due to the relativistic increase in electron mass under the intense fields of the laser beam, the resulting optical nonlinearity of plasma leads cross-focusing of the laser beams. The resulting nonlinear coupling between the two laser beams makes the oscillations of plasma electrons to contain a frequency component equal to the sum of frequencies of the pump beams. This results in a nonlinear current density at frequency equal to the sum of frequencies of the pump beams. If the frequencies of the pump beams are equal, then the resulting nonlinear current generates a new radiation at frequency twice the frequencies of the pump beams— a phenomenon known as SHG. Starting from nonlinear Schrodinger wave equation a set of coupled differential equations governing the evolution of beam widths of the laser beams and power of generated second-harmonic radiation with longitudinal distance has been obtained with the help of variational theory. The equations so obtained have been solved numerically to envision the effect of laser as well as plasma parameters on the power of generated second-harmonic radiation.

Keywords Cosh Gaussian · Self-focusing · Nonlinear optics · Second-harmonic generation · Clean energy

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Introduction

The invention of the laser [1] is the most towering achievement in the long history of light. It brought an extraordinary technological leap, which has since paved the way for a startling new era in optical science and technology. For the first time, man got a remarkable tool for direct generation and manipulation of coherent light. Laser brought same revolution to optics that transistor brought to electronics and cyclotron brought to nuclear physics. The distinctive qualities of laser derive from its coherence properties, which result in a beam of light with a well-defined optical phase both in space and time. This prescribed phase generally confines the wavelength and frequency of the laser light to a restricted range, so that the beam exhibits a narrow frequency spectrum. Another unique property of laser light is its directionality, which means that the beam can propagate over great distances without significant spreading and can be readily manipulated using conventional optical elements. The phase coherence and directionality of the laser make it possible to create extremely large optical powers and focused intensities that cannot be obtained from incoherent light emitters. These characteristics also allow accurate transfer of information [2], precise calibration of time [3, 4], and measurements of many physical constants [5, 6], among numerous other applications, using laser light. Lasers are now standard components of such commonplace objects as compact-disk players and printers. The everyday presence of lasers does not mean, however, that they have been reduced to performing only pedestrian tasks. Higher-end applications like laser surgery [7], laser-driven particle accelerators [8, 9], inertial confinement fusion [10], etc., are also abound.

Success is never without limitations, and laser is also not an exception. By the virtue of its unique coherence properties, laser light contains only a confined band of frequencies

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STIMULATED BRILLOUIN SCATTERING OF ELLIPTICAL *q*-GAUSSIAN LASER BEAMS IN PLASMAS WITH AXIAL DENSITY RAMP: EFFECT OF SELF-FOCUSING

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UDC 533.9;535.36

Theoretical investigation on stimulated Brillouin scattering of intense q-Gaussian laser beams interacting with axially nonuniform plasmas has been presented. A ramp-shaped density profile has modelled the axial inhomogeneity of the plasma and the optical nonlinearity of the plasma has been considered to be originating owing to the dependence of the relativistic mass of the plasma electrons on laser intensity. An intense laser beam with frequency ω_0 propagating through plasma gets coupled with a preexisting ion acoustic wave (IAW) at frequency ω_{ia} and produces a back-scattered wave at frequency $\omega_s = \omega_0 - \omega_{ia}$. Using variational theory semi-analytical solution of the set of coupled wave equations for the pump, an IAW and a scattered wave have been obtained under W.K.B. approximation. It has been noted that the power of the scattered wave is significantly affected by the self-focusing effect of the pump beam.

Keywords: *q-Gaussian, density ramp, relativistic plasma, self-focusing, stimulated Brillouin scattering.*

Introduction. Powering the world without harming the global climate is a mounting concern. Thus, in the past few decades, developed countries have added substantial amounts of solar, geothermal, wind, and biomass power to decarbonize electrical power production. But these forms of renewable power cannot quench humanity's endless thirst for energy. In this regard, the proposal of initiating nuclear fusion by intense laser beams (inertial confinement fusion, ICFs) for viable energy production [1] has been at the vanguard of research since the successful realization of the thermonuclear fusion — the hydrogen bomb. In laser-driven fusion, the goal is to deposit laser energy at a particular density in the plasma to derive the compression and subsequent heating of the fuel pellet. If the pellet is compressed sufficiently, it may undergo fusion, releasing a large amount of energy. It is as if there is a tiny chunk of the sun on Earth.

Both the allure and the challenges of fusion arise from the nature of the fusion process itself. Fusion fuel is abundant and cheap. The significant advantages are: the abundance of fuel — the most easily exploitable fuels are deuterium and tritium. Deuterium occurs naturally in all sources of water, especially seawater. Tritium, however, is not readily available naturally; it can easily be manufactured inside the fusion reactor by the bombardment of lithium with neutrons, which are also abundant in nature. The cleanest source of energy — fusion — does not produce nuclear waste directly. Although tritium is mildly radioactive and neutron activation of the reactor chamber dictates which structural materials are most valuable to minimize waste disposal of components discarded in maintenance or the entire reactor assembly at the end of its life.

However, fission occurs at average temperatures, and fusion occurs only at extreme temperatures, which are characteristic of stars. The fuel with the lowest kindling point is a mixture of deuterium and tritium that ignites at temperatures around 50 keV (i.e., 50×10^6 K). At such high temperatures, the fuel becomes a fully ionized gas or plasma, hence the prominence of plasma physics in fusion research.

In ICF, lasers may interact with the plasma at a density different than that which is intended, leading to undesirable effects [2–5] and preventing the effective implosion of the target. This spans a gamut from self-action effects of the laser beam to several stimulated scattering processes such as stimulated Raman scattering (SRS) [6, 7] and stimulated Brillouin scattering (SBS) [8–12]. Stimulated scatterings are the processes in which an incident electromagnetic beam interacts nonlinearly with bosonic excitations of a medium and hence becomes converted to a frequency up- or down-shifted scattered

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RESEARCH ARTICLE

Self-focusing of rippled q-Gaussian laser beams in plasmas: effect of relativistic nonlinearity

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Abstract Theoretical investigation on self-focusing of q-Gaussian laser beam propagating through underdense plasma has been presented. The optical nonlinearity of plasma has been modeled by the relativistic mass nonlinearity of plasma electrons in the field of laser beam. Using variational theory approach, semi-analytical solutions of the wave equations for the fields of main beam and that of ripple have been obtained. Emphasis has been put on the evolutions of the intensities of main beam and that of ripple.

Keywords q-Gaussian · Laser ripple · Variational theory · Clean energy · Self-focusing

Introduction

Since the 1930s, when scientists began to realize that the sun and other stars are powered by nuclear fusion, their thoughts turned toward recreating this process in the laboratory for the viable energy production. Because fusion can use atoms present in ordinary water as a fuel, harnessing the process could assure future generations of adequate electric power [1]. The ultimate stakes are so high, fusion will produce no harmful emissions—no sooty pollutants, no nuclear waste and no greenhouse gases. All the stars and the sun use their strong gravitational pull to compress nuclei to high densities. In addition, temperatures in the sun are extremely high, so that the positively charged nuclei have enough kinetic energy to overcome their mutual electrostatic repulsion and draw near enough to fuse. However, such resources are not readily available on the earth. The particles that fuse most easily are the nuclei of deuterium and tritium. To fuse even deuterium and tritium, hydrogen gas has to be heated intensely and also has to be confined long enough that the particle density multiplied by the confinement time exceeds 10^{14} seconds per cubic centimeter. Since the 1950s, fusion research has focused on two ways of achieving this number: inertial confinement and magnetic confinement.

The strategy of inertial confinement fusion (ICF) is to shine a symmetrical array of powerful laser beams onto a spherical capsule containing a D-T mixture. The laser beams vaporize the surface of the pellet that explodes outward. To conserve momentum, the inner sphere of fuel simultaneously shoots inward just like the recoil of a gun when the bullet is fired. Although the fuel is compressed for only a brief moment (about 10^{-10} second), extremely high densities of almost 10^{25} particles per cubic centimeter can be obtained [2, 3].

For the successful realization of ICF, it is highly necessary that the fuel pellet should be heated uniformly. However, due to the nonuniform irradiance (intensity ripples) over the cross sections of the laser beams, the pellet is not heated uniformly that derives an instability known as Rayleigh–Taylor (RT) instability [4–6]: Whenever a not-very-dense fluid (like air) pushes on a denser fluid (like water), the situation is inherently unstable. If the interface between the two fluids is having any imperfection like bumps or divots, then these imperfections will immediately grow with time.

This fundamental instability can even be observed in everyday kitchen scenarios. It may be difficult to imagine an instability in the kitchen, but consider the following question: Why does not the water stay in a glass when you invert it? At first glance, the answer may seem obvious:

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RESEARCH ARTICLE

Formation of elliptical *q*-Gaussian breather solitons in diffraction managed nonlinear optical media: effect of cubic quintic nonlinearity

Naveen Gupta[1] · A. K. Alex[1] · Rohit Johari[1] · Suman Choudhry[1] · Sanjeev Kumar[1] · Aatif Ahmad[1] · S. B. Bhardwaj[2]

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Abstract This paper presents theoretical investigation on the formation of elliptical q-Gaussian breather solitons in diffraction managed optical media. The optical nonlinearity of the medium has been modeled by cubic–quintic nonlinearity. To obtain the physical insight into the propagation dynamics of the laser beam, semi-analytical solution of the wave equation for the laser beam has been obtained by using variational theory approach in W.K.B approximation. Emphasis is put on investigating evolutions of transverse dimensions and axial phase of the optical beam.

Keywords Soliton · Clean Energy · Self Focusing · Variational Theory · Breather

Introduction

Since the debut of quantum mechanics in the 1920s, the two different aspects of physical quantities, i.e., waves and particles, have been intimately related in physical theories. Although both the aspects appear to be physically different, there are a number of experimental evidences that show correlation among both. In the past few years, solutions of certain wave equations have revealed another correlation between waves and particles. The surprising fact is that these wave equations are not the part of quantum mechanics but instead have been derived from classical physics [1]. Solutions to these equations describe waves those neither spread in space (i.e., those do not diffract) nor disperse in time.

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Diffraction and dispersion (Fig. 1) are the inherent properties of all kind of waves whether it is electromagnetic wave, mechanical wave (sound wave) or even matter wave.

However, these new kinds of waves retain their size and shape indefinitely (Fig. 2). These waves can be regarded as a quantity of energy localized permanently to a definite region of space. It can be set in motion but it cannot dissipate by spreading out. When two such waves collide, each comes away from the encounter with its identity intact (Fig. 3). If a wave meets an “anti-wave,” both can be annihilated. This kind of behavior is extraordinary in waves, but it is familiar in another context, i.e., with particles. Thus, such waves can be considered as particles and are termed as “solitons.”

The first recorded observation of a soliton was made almost 200 years ago by Russell [2], an engineer and naval architect. He reported to the British Association for the Advancement of Science: “I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses. When the boat suddenly stopped-not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation then suddenly leaving it behind rolled forward with great velocity assuming the form of a large solitary elevation a rounded smooth and well-defined heap of water which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback and overtook it still rolling on at a rate of some eight or nine miles per hour preserving its original figure some 30 feet long and a foot to a foot and a half in height. Its height gradually diminished and after a chase of one or two miles I lost it in the windings of the channel.”

Our topic of investigation, i.e., spatial optical solitons, arises due to dynamical balance of diffraction with induced focusing of the optical beam in a nonlinear medium. By nonlinear medium, it is meant by a medium whose index

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RESEARCH ARTICLE

Excitation of upper hybrid wave by cross focused *q*-Gaussian laser beams in graded index plasma channel

Naveen Gupta[1] · Rohit Johari[1] · Sanjeev Kumar[1] · Suman Choudhry[1] · S. B. Bhardwaj[2] · A. K. Alex[1]

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Abstract In this paper, a method is presented for exciting an upper hybrid wave (UHW) in a preformed parabolic plasma channel. The plasma channel is magnetized perpendicular to the propagation direction of laser beams. The UHW is generated through the interaction of two *q*-Gaussian laser beams with frequencies ω_1 and ω_2, employing the ponderomotive nonlinearity. The evolution of the laser beam spot sizes along the propagation distance is described by a set of coupled differential equations derived using the moment theory approach in the W.K.B approximation. The ponderomotive nonlinearity depends on the intensities of both laser beams, resulting in a mutual influence between the two beams, leading to cross-focusing. Numerical simulations are conducted to examine the impact of laser and channel parameters on the cross-focusing of laser beams and its effect on the power of the generated UHW. The results indicate that the intensity profiles of the laser beams, channel depth, and strength of the static magnetic field significantly affect the power of the generated UHW.

Keywords q-Gaussian · Plasma Waves · Moment Theory · Clean Energy · Self Focusing

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Introduction

At the turn of the last century, the introduction of lasers [1] sparked a significant surge in research within the field of plasma physics. The study of plasmas began in the nineteenth century, when Michael Faraday investigated electrical discharges through gases. Modern plasma research dates from 1957 and 1958. During those years, Soviet Sputnik and American Explorer spacecrafts discovered that space near the earth is filled with plasma. At the same time, till then secret research on controlled thermonuclear fusion conducted by the USA, Soviet Union and Europe was revealed at the Atoms for Peace Conference in Geneva, greatly increasing the freely available information on plasmas. Fusion research focuses on producing extremely hot plasmas and confining them in magnetic "bottles," to create the conditions necessary for energy-producing nuclear reactions to occur.

Extensive studies, incorporating both theoretical and experimental approaches, have been conducted to enhance our understanding of this subject. These collective efforts have given rise to various potential applications, such as laser-driven particle accelerators [2–5], inertial confinement fusion [6, 7], X-ray lasers [8–10], laser plasma channeling [11, 12], and supercontinuum generation [13]. The successful realization of these applications relies heavily on the efficient coupling of laser energy with plasmas. Unfortunately, the interaction length between lasers and plasmas is inherently limited by diffraction divergence, restricting it to approximately a Rayleigh length in the absence of an optical guiding mechanism. Diffraction broadening, therefore, represents a fundamental phenomenon that hampers the efficiency of laser–plasma coupling. Consequently, there has been a renewed interest in extending the propagation distance of laser beams through

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RESEARCH ARTICLE

Beat wave excitation of electron plasma wave by cross-focused *q*-Gaussian laser beams in thermal quantum plasmas

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Abstract This paper presents a theoretical investigation on excitation of an electron plasma wave by beating two *q*-Gaussian laser beams in thermal quantum plasmas. Moment theory in W.K.B. approximation has been invoked to find numerical solution of nonlinear Schrodinger equation for Drude model of dielectric function of plasma. Optical nonlinearity of plasma has been modeled by relativistic mass nonlinearity of plasma electrons. The laser-induced nonlinearity in the dielectric properties of plasma depends not only on the intensity of the first beam but also on that of second beam. Therefore, propagation characteristics of one laser beam affects that of second beam, and hence, cross-focusing of the two laser beams takes place. Due to nonuniform intensity distribution along the wavefronts of the laser beams, the background electron concentration gets modified. The amplitude of EPW, that depends on the background electron concentration, thus gets nonlinearly coupled with the laser beams. Numerical simulations have been carried out to investigate the effects of laser-plasma parameters on cross-focusing of the laser beams and further its effect on power of EPW.

Keywords *q*-Gaussian · Beat wave · Quantum plasma · Clean energy · Self-focusing

Introduction

The introduction of the laser by Maiman [1] in 1960 revolutionized the field of plasma physics, giving rise to a new area of study known as laser-plasma interactions. This field is highly complex yet filled with physics that will continue to engage researchers for years to come. Nowadays, lasers play a pivotal role in plasma physics, driven in part by the proposal of using high-power laser beams to initiate fusion reactions [2–4] for practical energy production. However, laser-plasma interactions have branched out into numerous potential applications. Some of these applications, such as microwave guiding [5], lightning protection [6], and the manipulation of rain and snowfall [7], have direct real-world impacts. Other applications, like the generation of terahertz radiations [8–10], electron acceleration [11–14], and X-ray lasers [15, 16], are more subtle but equally significant for the scientific community and technological advancements.

Most of these applications rely on efficient coupling between lasers and plasmas. Unfortunately, when a light beam travels through a medium or vacuum, it naturally broadens due to the wave property of diffraction. This diffraction broadening fundamentally undermines the efficiency of laser-plasma coupling, thereby jeopardizing the effectiveness of the aforementioned applications. Consequently, there is a growing interest in exploring methods or processes that can enhance the efficiency of laser-plasma coupling by increasing the effective interaction length between laser beams and plasmas. The such nonlinear phenomenon that addresses this challenge is self-focusing, which occurs due to the material medium's nonlinear response to incident laser beams. Under certain conditions, the medium behaves like a convex lens [17], altering its refractive properties when exposed to an intense laser beam in a plasma. This variation in the light's velocity across the beam's wavefront causes the beam to bend and converge spherically, leading to self-focusing of the laser beam.

Recently, the high-density and low-temperature plasmas known as quantum plasmas are encountered in modern

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RESEARCH ARTICLE

Quadruple Gaussian laser beams in thermal quantum plasma: self-focusing, self-trapping and self-phase modulation

Naveen Gupta[1] · Suman Choudhry[1,2] · Rohit Johari[1] · A. K. Alex[1] · Sanjeev Kumar[1,2] · S. B. Bhardwaj[3]

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Abstract This paper presents a theoretical investigation into the self-action effects of intense laser beams interacting with fusion plasmas. The focus of the study is on the phenomena related to the nonlinear refraction of the laser beam, which are examined in detail. To analyze the impact of illumination uniformity across the beam's phase fronts on its propagation characteristics, the irradiance profile of the beam is modeled using a quadruple Gaussian (Q.G) profile. By employing the variational theory approach, the nonlinear partial differential equation (PDE) governing the beam envelope is transformed into a system of coupled ordinary differential equations that describe the evolution of the beam width and axial phase. These equations are then solved numerically to investigate the influence of both laser and medium parameters on the propagation characteristics of the laser beam.

Keywords Self focusing · Self trapping · Self phase modulation · Moment theory · Quadruple Gaussian laser · Clean energy

Introduction

The pursuit of harnessing intense laser beams [1–3] to achieve nuclear fusion and meet humanity's ever-growing energy needs while mitigating the impact on the global climate has been a prominent area of research in recent years. This endeavor can be likened to creating a miniature sun on Earth to power industrial machinery. Inertial confinement fusion (ICF) serves as the underlying principle for this approach, where an intense laser beam is divided into multiple smaller beams of equal intensity. These divided beams are then amplified in energy and subsequently recombined using a system of mirrors and lenses. Through this process, the beams converge on a small region from different directions. To facilitate the fusion reaction, a fuel pellet containing deuterium and tritium is encapsulated within a spherical shell measuring a few millimeters in diameter. The shell itself can be composed of materials such as plastic, glass, or other suitable substances. The fuel pellet is precisely positioned at the intersection point of the converging laser beams, ensuring uniform illumination of the pellet [4–6].

Upon interaction with the fuel pellet, the intense laser beam swiftly ionizes the atoms situated in the outermost layer. However, the material within a critical radius acts as an obstacle to the laser energy, rendering it opaque. Consequently, the incident energy becomes absorbed by a dense plasma layer enveloping the deuterium tritium fuel. As a result, this heated plasma layer expands and ablates, essentially tearing away explosively from the rest of the pellet. The ablated plasma attains velocities of approximately 1000 kms per second. In accordance with Newton's third law, an equal and opposite force propels the material inside the ablation layer inward, akin to a rocket being propelled by the plasma escaping from all sides. In essence, this system can be likened to a spherical rocket powered by lasers, with its payload being the rapidly contracting fuel pellet. However, the energy efficiency of this implosion process is relatively low, measuring less than 10 percent, mainly due to the exhaust velocity of the ablated material being significantly higher than the velocity of the imploding pellet. Nonetheless, the concentric implosive force exerted is still sufficient to accelerate the remaining shell to velocities of several hundred

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RESEARCH ARTICLE

Spatial frequency chirping of *q*-Gaussian laser beams in graded index plasma channel with ponderomotive nonlinearity

Naveen Gupta[1] · Rohit Johari[1] · A. K. Alex[1] · Suman Choudhry[1] · Sanjeev Kumar[1] · S. B. Bhardwaj[2]

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Abstract Chirping of spatial frequency of *q*-Gaussian laser beams interacting nonlinearly with plasmas with radially inhomogeneous electron density has been investigated theoretically. Due to the radial nonuniformity of the electron density, the index of refraction of the plasma channel resembles to that of a graded index fiber. Chirping or modulation of spatial frequency also known as phase anomaly occurs due to position momentum uncertainty of photons. Due to intensity gradient over the laser cross section, the transverse component of ponderomotive force becomes finite. This results in redistribution of carriers in the irradiated potion of the channel. This results in the enhancement of the radial gradient of the density profile that stimulates the laser beam to get self-focused. The reduction in transverse dimensions of the laser beam in turn leads to spread in transverse momentum of its photons. This transverse momentum spread then modifies the axial phase of the laser beam. Following Virial theory, equations of motion for radius and spatial frequency of the laser beam have been obtained. The equations so obtained have been solved numerically to envision the effect of various laser and plasma parameters on the evolution of beam envelope. Manifestation of axial phase to Berry phase has also been explained.

Keywords *q*-Gaussian · Virial theory · Self-focusing · Gouy phase

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Introduction

The additional accumulation of axial phase of a converging beam in comparison with an infinite plane wave is known as modulation of spatial frequency or Gouy phase shift [1]. In his experiment, Gouy reflected a beam of light emerging from a pin hole, from both a curved and flat mirror. The focusing beam over lapped with the nonfocusing beam in a region near the focus and created a circular diffraction pattern. Gouy then looked at the circular diffraction pattern at several different locations, both before and after the focus. He observed that the central region of the diffraction pattern changed from bright to dark, indicating an axial phase jump of the focusing beam—the Gouy phase shift.

Observing Gouy phase shift is relatively easy, but explaining it is not. Since its discovery, the Gouy phase shift has remained a matter of debate. Curiosity about its origin and physical meaning is still at the vanguard of investigations. Various theories [2, 3] (ranging from classical to quantum) have been used to explain its origin. Classically the phase shift of an optical beam arises due to the contribution of an additional phase in the neighborhood of the beam focal spot arising from the second-order derivative of field amplitude with respect to transverse coordinates. However, in quantum mechanical terms the Gouy phase shift is considered to be originating as a consequence of modification of its transverse dimensions. Converging beams going through focus have finite spatial extent in the transverse plane. The uncertainty relation then induces some distribution over the transverse and consequently longitudinal wave vectors. The net effect of this distribution over wave vectors is an overall phase shift.

Since the discovery of lasers [4], the anomalous behavior of the spatial frequency also known as axial phase or wave number of the optical beams has been drawing attention of

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RESEARCH ARTICLE

Third harmonic generation of self-focused *q*-Gaussian laser beams in nonlinear media: effect of cubic quintic nonlinearity

Naveen Gupta[1] · Sanjeev Kumar[1,2] · S. B. Bhardwaj[3] · Suman Choudhry[1,2] · A. K. Alex[1] · Rohit Johari[1] · Amit Bindra[1]

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Abstract This paper presents theoretical investigation on third harmonic generation (THG) in nonlinear media. Variational method has been adopted to find the semi analytical solution of the wave equation governing the evolution of slowly varying beam envelop in nonlinear medium. Emphasis is put on investigating the evolution of beam width of the laser beam with distance through the medium. When laser beam with frequency ω_0 propagates through a nonlinear medium dominated by $\chi^{(3)}$ nonlinearity, oscillations of the electrons of the medium contain a frequency component $\omega_3 = 3\omega_0$ and thus produce third harmonic of the pump beam. An equation governing the conversion efficiency of third harmonic has been derived. Deviation of intensity profile of the incident laser beam from ideal Gaussian profile has also been incorporated through *q*-Gaussian profile.

Keywords Self focusing · Cubic quintic · *q*-Gaussian · Harmonic generation

Introduction

The quest for short wavelength coherent radiation sources for medical diagnostics and treatment, homeland security, plasma diagnostics, etc. has a long history. For past few decades only two main approaches, namely free electron lasers and synchrotron had been considered for this purpose. However, involvement of large infrastructure, accelerators, beam lines and massive gantries of more than 100 tons, makes these techniques are quite expensive. As a result, access to these facilities is quite limited, specifically for less affluent institutes like universities and hospitals these facilities are not affordable and therefore the research related to them is not growing at a faster pace.

By bringing coherent short wavelength sources off the shelf, the process of laser HHG can reduce the cost of coherent radiation sources. The reduction in cost is not only due to the replacement of accelerator but also due to the fact that there will be no requirement of large building footprints and massive gantries. To understand the generation of higher harmonics in nonlinear media let us see what happens when an intense laser beam passes through a transparent optical medium. The focused light from certain lasers has an electric field as strong as 10^7 V cm^{-1}. Such optical fields are as intense as the cohesive local electric fields in the crystal. Consequently when intense laser beams enter a transparent crystal, they cause a massive redistribution of the electrons and the resulting polarization is no longer proportional to the optical electric field. In fact, at optical fields of 10^7 V cm^{-1} and higher many materials break down completely.

Figure 1 illustrates the characteristic response when an intense optical electric field travels through a nonlinear, or ionic, material. It shows that an intense field in the "right" direction is more effective in polarizing the material than a field in the "left" direction. Such a situation can occur only in a crystal that has a "one-wayness" in its structure, or, to be more precise, one that has no center of symmetry. Such crystals are called as noncentro symmetric crystals. Of the crystals found in nature only about 10% fall in this class, and they usually exhibit the phenomenon called piezoelectricity. When a piezoelectric crystal is subjected to mechanical pressure, its asymmetry leads

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RESEARCH ARTICLE

Excitation of electron plasma wave by self-focused Laguerre–Gaussian laser beams in axially inhomogeneous plasma: effect of orbital angular momentum of photons

Naveen Gupta[1] · **A. K. Alex**[1] · **Rohit Johari**[1] · **Suman Choudhry**[1,2] · **Sanjeev Kumarand**[1,2] · **S. B. Bhardwaj**[3]

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Abstract Theoretical investigation has been conducted into the dynamics of electron plasma waves (EPWs) driven by lasers in plasmas featuring axial density ramps. To explore the impact of the laser beams' orbital angular momentum on both the propagation dynamics and the strength of the excited EPWs, the irradiance across the laser beam's cross section has been represented using a Laguerre–Gaussian beam profile. As the laser beam travels through the plasma, it stimulates an EPW at the pump frequency. This EPW becomes nonlinearly coupled to the laser beam due to the optical nonlinearity of the plasma. By employing variational theory and the W.K.B. approximation, semi-analytical solutions for the coupled nonlinear wave equations governing the pump wave and EPW have been derived. Notably, it has been observed that the power of the EPW is significantly influenced by the self-focusing effect of the pump beam.

Keywords Self-focusing · Electron plasma wave · Clean energy · Laguerre–Gaussian · Ponderomotive force

Introduction

The introduction of the laser in 1960 initiated a sequence of events that triggered a resurgence in the study of interactions between light and matter. This revolutionary development gave rise to an entirely novel realm of research termed as laser–plasma interactions. Over the recent years, this field has garnered remarkable attention from researchers, owing to its paramount significance for numerous potential applications, including nuclear fusion [1–3], plasma wakefield accelerators [4, 5], and coherent radiation sources [6–8]. The efficacy of these applications hinges upon the efficiency of the coupling between lasers and plasmas, a factor influenced by various nonlinear processes [9–11]. These processes encompass an array of phenomena, ranging from the self-focusing of laser beams to the excitation of diverse modes of wave propagation within plasmas. In scenarios where external magnetic fields are absent, these modes of wave propagation primarily involve electron plasma waves (EPWs) and ion acoustic waves (IAWs).

In the context of laser-driven nuclear fusion, the activation of electron plasma waves (EPWs) holds paramount importance. However, the EPWs excited by the pump beam have a dual nature. On one side, they reflect a substantial portion of the laser energy out of the fusion pellet through a phenomenon known as stimulated Raman scattering. Conversely, they generate superthermal electrons, which in turn cause premature heating of the pellet. Specifically, electrons that synchronize their motion with the excited plasma wave and roughly match its speed become entrapped within the plasma wave structure. Leveraging the potent electric field of the EPW, these trapped electrons undergo acceleration to attain superthermal velocities (as illustrated in Fig. 1). Possessing remarkable penetration capabilities, these superthermal electrons contribute to the untimely heating of the pellet before it reaches the critical density required for fusion ignition. This phenomenon is termed as 'preheating.' Furthermore, the appealing electrostatic attraction between these superthermal electrons and the ions in the surrounding corona can lead to the ejection of energetic ions from the ablation layer. These outward-propelled energetic ions

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RESEARCH ARTICLE

Quadruple Gaussian laser beam in cubic-quintic nonlinear media: effect of nonlinear absorption

Naveen Gupta[1] · **Sanjeev Kumar**[1] · **Suman Choudhry**[1] · **A. K. Alex**[1] · **Rohit Johari**[1] · **S. B. Bhardwaj**[2]

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Abstract An investigation on nonlinear propagation of Quadruple Gaussian (Q.G) laser beams propagating through dissipative media possessing cubic-quintic nonlinearity has been presented. The formulation is based on finding the numerical solution of Ginzburg–Landau equation for the field of incident laser beam followed by moment theory approach in W.K.B approximation. In particular, dynamical variations of beam spot size and longitudinal phase of the laser beam have been investigated in detail. Self-trapping of the laser beam resulting from the balance between diffraction broadening and nonlinear refraction has been also investigated.

Keywords Self focusing · Moment theory · Axial phase · Quadruple Gaussian · Clean energy

Introduction

Lasers [1] are one of the most successful pieces of apparatus born from twentieth century science. Rapid progress in laser technology fueled by the advent of chirped pulse amplification [2] (CPA) technique has enabled the investigation of highly nonlinear interactions of light with matter. Exotic nonlinear phenomena such as higher harmonic generation [3–5] (HHG), self-focusing [6], self-phase modulation [7, 8], and formation of spatial solitons [9, 10] are ubiquitous in laser-matter interactions. These nonlinear phenomena are extremely complex but are bursting with enough physics to keep researchers busy for years to come. Over last five decades, physicists are attempting to improve upon the understanding of these phenomena in order to have deep insight into laser-matter interaction physics. Major impetus was provided by the relevance of laser-matter interactions to potential applications like optical communication, optical device fabrication, etc. Spatial solitons, i.e., optical beams that do not disperse or dissipate, rather maintain their size and shape indefinitely, have attracted considerable interest recently, due to their applications in optical communication, optical device fabrication, ultrafast signal routing systems, etc. Spatial solitons are also familiar with mathematics, mechanical engineering, fluid dynamics, thermodynamics, biology, etc. Even spatial solitons are ubiquitous in nature and can be found in a variety of systems: from matter waves and EM waves [11]. Solitary waves were first observed by Scottish civil engineer John Scott Russel in 1834 when he noticed a curious occurrence during his ride along side a canal. When a horse-drawn barge suddenly stopped, it generated a single wave that continued to wave along the canal for kilometers without any change in the form or speed [12]. Our topic of interest, i.e., optical spatial solitons, arises due to counter balance of self-diffraction with nonlinearity induced self-focusing in conservative media. When very narrow optical beams propagate without affecting the properties of a medium, they undergo natural diffraction and broaden with distance. The narrower the initial beam is, faster it diverges (diffracts [13]). In nonlinear materials, the presence of light modifies their properties (refractive index, absorption, or conversion to other frequencies). The refractive index change resembles the intensity profile of the laser beam, forming an optical lens that increases the index in the beam's center while leaving it unchanged in the beam's tail. This induced lens focuses the beam, a phenomenon called self-focusing [6] that is a precursor of spatial solitons. When

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This is to certify that **Suman Choudhry, Department of Physics, Lovely Professional University, Phagwara, Jalandhar, India** has participated in FPMSN-2022 held on March 25-26, 2022 at Chaudhary Devi Lal University, Sirsa and presented a research paper (poster) **Ion Acoustic Decay Instability of Elliptical q-Gaussian Laser Beams in Plasma with Axial Density Ramp: Effect of Self Focusing.**

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