STUDY OF SOME PLAIN STRAIN PROBLEMS IN THE MICROPOLAR ELASTICITY

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by

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Declaration of Authorship

I, **Rafiya Nazir**, hereby declare that this thesis, entitled **"Study of some plain strain problems in the micropolar elasticity,"** and the work presented in it are my own. I confirm that:

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- Where any part of this thesis has not previously been submitted for a degree or any other qualification at this university or any other institution, this has been clearly stated.
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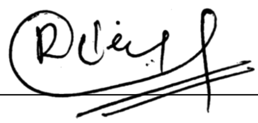
Signed: 

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CERTIFICATE

This is to certify that the thesis entitled **"Study of some plain strain problems in the micropolar elasticity"** submitted in fulfillment of the requirement for the reward of the degree of Doctor of Philosophy (Ph.D.) in the Department of mathematics, School of Physical Sciences and Chemical Engineering, is a research work carried out by **Rafiya Nazir**, Registration No. 11916851, is a bonafide record of her original work carried out under my supervision, and that no part of the thesis has been submitted for any other degree, diploma, or equivalent course.

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Abstract

The mathematical theory of elasticity is an endeavour to lessen the work involved in determining stress-strain, or relative displacement, within a solid body which is subjected to an equilibrating system of forces or might be in a state of slight internal relative motion. It aims to derive results which shall be essentially vital in application to engineering, architecture, and all other useful areas in which the material of construction is used. Classical theory of elasticity is one of the most important branches of continuum mechanics, which deals with the stresses and deformations in elastic materials generated due to the action of external forces or a change in temperature. The classical theory of elasticity serves as an excellent model for studying the mechanical behaviour of a wide variety of solid materials and is used extensively in civil, mechanical, and aeronautical engineering design. This is the oldest established theory governing the behaviour of deformable solid materials, which was founded in the early nineteenth century. However, the classical theory of elasticity was unable to analyze materials possessing microstructre, and as such, researchers started to focus on a new theory known as the micropolar theory of elasticity, where the microstructure of the materials plays a significant role. Micropolar theory assumes materials to be made up of small dumbwell-like interconnected molecules, which can undergo rotational motion independently in addition to translational motion.

The thesis consists of six chapters where in chapter 1, a general overview on the micropolar theory of elasticity is given. The development of theory of elasticity from classical to micropolar elasticity via several generalizations is also discussed, along with the memory-dependent derivative. In second Chapter, the elastodynamic responses of magneto micropolar isotropic media is studied under the gravitational influence. The Matlab software has also been used in order to illustrate the obtained results graphically. In Chapter 3, we have investigated and studied the 2D mathematical model in the framework of the Green-Lindsay model in the presence and absence of the micropolar effect by using a memory-dependent derivative. In Chapter 4, the memory response of a rotating micropolar elastic media under the thermo-mechanical effect has been investigated. The potential displacement approach, along with the normal mode analysis, has been used for solving systems of differential equations. In Chapter 5, a 2D model has

been developed in micropolar theory of elasticity, subjected to magnetic and thermal effects in the context of memory-dependent derivatives. In this chapter, the interaction of magnetic, thermal, and rotational fields has been studied using a memory-dependent derivative. In Chapter 6, a novel mathematical model in the micropolar theory of generalized thermoelasticity is established under the framework of photothermal theory. The resulting differential equations have been solved by using integral transforms along with the potential displacement approach.

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List of Symbols and Abbreviations

$\overrightarrow{H_0}$	Initial magnetic field
$\overrightarrow{h}$	induced magnetic field vector
$\overrightarrow{E}$	induced electric field vector
$\overrightarrow{J}$	current density vector
$\overrightarrow{u}$	displacement vector
$\overrightarrow{\Omega}$	rotation vector
$\overrightarrow{\phi}$	microrotation vector
$\overrightarrow{F}$	Lorentz force
$\overrightarrow{G}$	gravitational force
μ_0 and ε_0	magnetic and electric permeabilities
δ_{ij}	kronecker's delta
λ, μ	Lames constants
$\kappa, \alpha, \beta, \gamma$	micropolar elastic constants
T	thermodynamic temperature
φ	conductive temperature
T_0	reference temperature
K	thermal conductivity
K^*	material characteristic
C_e	specific heat
α_t	linear thermal expansion
τ_0, τ_1	relaxation times
ρ	material density
j	microrotation

$a^* > 0$	temperature discrepancy
D_e	coefficient of carrier diffusion
n_0	equilibrium carrier concentration
τ_p	photogenerated carrier lifetime
τ	time-delay
b	complex constant
a	wave number along x-axis
δ	coupling parameter of thermal activation
t_p	characteristic time of pulse heat flux
s_0	surface recombination speed
q_0	a constant
H	Heaviside unit step function
σ_{ij}	stress tensor
m_{ij}	couple stress tensor
ε_{ijk}	alternate tensor
E_g	semiconducting energy gap
d_n	electronic deformation coefficient
ν	$= (3\lambda + 2\mu + \kappa)\alpha_t$
N	$= n - n_0$
i	$= \sqrt{-1}$
γ'	$= (3\lambda + 2\mu)\alpha_t$
δ	$= \frac{\partial n_0}{\partial \theta}$
γ_n	$= (3\lambda + 2\mu)d_n$
CST	couple stress theory
DPL model	dual-phase-lag model
2TT	two-temperature theory
MDD	memory-dependent derivative
FD	Fractional derivative
TPL model	three-phase-lag model
LS model	Lord-Shulman model
GL model	Green-Lindsay model
GN model	Green-Nagdhi model
NMA	normal mode analysis

PDA	potential displacement approach
EVA	Eigen value approach
BC	boundary conditions

Chapter 1

Introduction

In this chapter, a brief and concise introduction to the micropolar theory of elasticity is given, along with several generalizations. In order to study the processes reflecting memory effect, the notion of memory-dependent derivative in elasticity theory is also discussed. The main theme of the thesis is presented, along with a brief history and an extensive survey of the literature. The research gap is identified based on the survey of the literature, and as a result, the objectives of the current work are proposed accordingly.

1.1 Classical elasticity

The mathematical elasticity theory is a sublime and attention-grabbing subject that studies the stresses and distortions created in elastic media due to some external force or due to temperature change. An adequate outcomes has been produced by classical elasticity in so many engineering problems involving various structural materials. The material in classical elasticity is treated as a continuum, and hence the molecular structure of materials in such a continuum is completely ignored. In this theory, the displacement vector is used to characterize the distortion of the body, and a force, namely the stress vector, is used to determine the transmission of loads across a surface element. Therefore, the symmetric tensors of stress and strain are used to describe the distortion of the body. The materials used in construction, for instance, aluminum, steel, and concrete, are effectively defined by classical elasticity, provided the elastic limit is not crossed by stresses. However, the materials possessing microstructure e.g., soil, bone, composites, polymers etc and those materials with huge stress gradients are neglected to convey agreeable results by classical elasticity. But after comparing the experimental results and the results that were acquired using classical elasticity, significant discrepancies were discovered in some of the materials, for instance, fibrous, polymers, asphalts, etc. The reason behind these discrepancies is the material's atomic structures, which is overlooked in classical elasticity. The premise of this theory is shaped by Hook's law [1], which was found in 1660 and revealed in 1678. In general, Hooke's law (constitutive relations) can be expressed as

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl},$$

and equation of motion as

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i,$$

where, c_{ijkl} is a fourth order tensor having 81 components which depend upon the nature of medium.

Actually, Galileo was the first mathematician to study the resistance of solids to rupture by treating solids as inelastic objects. His investigations laid the foundation of a field that was later investigated by many researchers. Two major breakthroughs in the history of elasticity initiated by Galileo's observations were the discovery of Hooke's law in 1660 by British mathematician Robert Hooke and the formulation of the general equations by French engineer Navier in 1821. Scientific thinking has been shaped by Hooke's law for a long period of time, and its outcomes generally matched experimental findings aswell.

Any solid is said to be elastic if it has the ability to deform when subjected to a load and then revert back to its primitive form after the removal of deforming forces. An elastic solid is said to be linear, if an infinitesimal deformation is experienced by that body and for which the governing material law is linear. About any given point, if the body's elastic properties are the same in all directions, then the body is said to be isotropic. If it happens that the body's elastic properties are independent of the positions of the points, then the body is said to be homogeneous. For such materials Hooke's law reduces to

$$\sigma_{ij} = \lambda\delta_{ij}\upsilon + 2\mu e_{ij},$$

where, $\upsilon = e_{11} + e_{22} + e_{33}$; λ and μ are material constants known as Lame's constants, σ_{ij} represents force stresses, and δ_{ij} represents the kronecker delta.

In 1887, the concept of couple stress was introduced in classical elasticity for the first time by Viogt [2] in addition to the force stresses. It was introduced in order to address the flaws of classical elasticity and hence lead to another theory known as "couple stress theory". In 1909, Cosserat and Cosserat [3] further extended the couple stress theory (CST) and hence introduced the complete theory of asymmetric elasticity, which was non-linear initially. In this theory, it was assumed that in a 3D continuum, each material particle is associated with a 'rigid triad'. So in addition to the displacement, the material particles can rotate independently during the distortion process. Therefore, in this way, the concept of rotation was incorporated into the continuum, which leads to additional degrees of freedom, and these additional degrees of freedom in turn lead to the asymmetry of stress and strain tensors. Hence, an excellent continuum modelization was provided by the Cosserat brothers idea for molecular lattices. However, the Cosserat brothers work did not pique anyone's interest and thus remained dormant during their lifetime. It may be due to the non-linear nature of this theory and its presentation as a unified theory including electrodynamics, optics, and mechanics. Then, around 50 years of break, so many researchers were fascinated by this theory, and various Cosserat-type theories were established by them. For instance, Gunther [4], Grioli [5], Rajagopal [6], Aero and Kuvshinskii [7], Toupin [9], Mindlin and Tiersten [8], Eringen [10], Koiter [11], Palmov [12], etc., established different Cosserat-type theories. In each of the above mentioned theories, the kinematic variable has been taken in account, corresponding to rotation of a material point. However, the theories developed by these researchers were called by names, for instance, the theory developed by Toupin, was referred to as "Cosserat theory with constrained motion", theory developed by Koiter was referred to as "Couple stress theory", theory developed by Eringen was referred to as "Indeterminate couple stress theory" and theory developed by Nowacki, was referred to as "Cosserat pseudo-continuum theory" etc. Furthermore, the theories developed by these researchers were completely identical to the Cosserats theory. The micro-rotation vector ϕ in Nowackis theory, is described in terms of displacement vector u by the formula $\phi = \frac{1}{2}\nabla \times u$. Later on, following Eringen [13], the general Cosserat continuum theory was renamed "micropolar continuum theory," where the micro-rotation vector is not defined in terms of a displacement vector. A non-linear theory for micro-elasticity was devised in [14, 15], and in this theory, the intrinsic motions of the microelements are taken into consideration. This theory is actually an extension of "Indeterminate couple stress theory" and "Cosserat theory" because in this, the skew-symmetric part of the stress tensor, the symmetric part of the couple stress tensor, and the spin inertia are entirely covered.

1.1.1 Thermoelasticity and magneto-elasticity

The thermoelasticity theory, which is a broadened version of the classical theories of elasticity and thermal conductivity, deals with the effect of thermomechanical disturbances on an elastic body. The heating of a body leads to temperature change and deformation of its structure, which causes thermal deformation. The term "thermal deformation" simply means that a material expands when its thermal energy (and temperature) rises, causing its atoms (or molecules) to vibrate more often. This increased vibration in turn results stretching of the molecular bonds. Also, if the material's thermal energy is reduced, then accordingly material will contract. Hence, it can be concluded that the theory of thermoelasticity is actually based on temperature changes. Interaction between elastic and temperature fields leads to coupling between deformation and temperature distribution, so Hooke's law gets replaced by the Duhamel-Neumann [16, 17] equation

$$\sigma_{ij} = \lambda\delta_{ij}\nu + 2\mu e_{ij} + \beta_{ij}T,$$

where, β_{ij} are thermal moduli. For homogeneous isotropic material

$$\beta_{ij} = \beta = -(3\lambda + 2\mu)\alpha_t,$$

where, α_t is the coefficient of thermal expansion.

The thermoelasticity theory is classified into three different forms: uncoupled, coupled, and generalized thermoelasticity. The uncoupled theory of thermoelasticity (classical thermoelasticity) was introduced by Duhamel and Neumann [16, 17], but this theory had two shortcomings, namely, the heat conduction equation is free of elastic terms, and another shortcoming of this theory is the heat equations parabolic nature, anticipating infinite velocities of thermal signals, so opposing the actual physical phenomena and therefore doesn't present exact outcomes. A century later, Biot [18] in 1956 removed the first paradox of the classical uncoupled thermoelasticity theory, that the temperature remains unaffected by elastic changes, and hence established a new theory known as the coupled theory of thermoelasticity. However, the second shortcoming of the above two theories is the same. Therefore, in order to remove the latter shortcoming of the classical coupled thermoelasticity, various generalizations were established. The first and foremost generalization was given by Lord and Shulman [19] (LS theory) in 1967, with one relaxation time, and another generalization was given by Green and Lindsay [20] (GL theory) in 1972, with two relaxation times. In the above theories, namely the LS and GL theories, the heat conduction equation is of the hyperbolic type, which therefore removes the shortcoming of infinite velocities of thermal signals. Later on, three additional models were introduced by Green and Nagdhi ([21], [22], and [23]) (GN theory), viz., GN types I, II, and III.

Magnetoelasticity is another generalization of elasticity theory, and it came into existence when magnetic effects were introduced in the theory of elasticity. Due to its innumerable applications in different fields, it has grabbed the attention of a number of researchers. Knopoff [24] and Chadwick [25] were the first ones to establish the basis of magnetoelasticity, and later on, Kaliski and Petykiewicz [26] further developed it.

1.2 Micropolar theory of elasticity

So far, classical elasticity was not completely successful in elucidating the behaviour of those materials possessing microstructure structure. Therefore, in order to study such types of materials, A.C. Eringen [13] established in 1966 a new theory called as the *"Micropolar theory of elasticity"*, which successfully studies the deformation of such materials or any material whose microstructure plays a crucial part in their macroscopic reactions. The micropolar elasticity contemplates the granular character of the medium, and the deformation is described by microrotation and displacement. The granular character of the medium is proposed to be imposed to such type of materials, where the ordinary classical elasticity theory is ineffective in analyzing their behavior. The motion of materials in micropolar elasticity is described by displacement and rotation vectors, though in classical elasticity, only displacement vector is utilized to analyze the motion of material points. Therefore, the motion of a material point (particle) in the micropolar theory of elasticity is described by six degrees of freedom. Usually in micropolar elasticity, in addition to the displacement components (u_1, u_2, u_3), the microrotational angles (ϕ_1, ϕ_2, ϕ_3) are also utilized to describe the rotation of the microstructure. The interaction taking place between two parts of a body is transmitted by a torque vector along with a force vector, which results in assymetrical force aswell as couple stresses. The entire class of materials is represented by this medium, which are basically formed by dumbbell-like molecules or dipole atoms. In micropolar theory, an additional object, namely the *director*, is assigned in order to define the orientation of material particles, and hence the microrotation of the material particles is examined by it.

Now, in isotropic and homogeneous micropolar media, the stress-strain relation, as defined in [13], is given by

$$\sigma_{ij} = \lambda u_{r,r} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \varepsilon_{ijr}\phi_r),$$
$$\mu_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{ij} + \gamma\phi_j.$$

where, λ, μ represents Lame's constants, $\alpha, \beta, \gamma, \kappa$ represents micropolar elastic constants, σ_{ij} represents force stress, and μ_{ij} represents couple stress.

The micropolar elasticity is diverse in scope because of its applications in acoustics, optics, geophysics, etc. In both isotropic and anisotropic media, plenty of research work has been done, and at present, hundreds of papers are present in this area and related areas. Moreover, an intensive study is going on in this field. A cubic crystal that is subjected to a mechanical source is studied by Kumar and Ailawalia [27] by utilizing the eigenvalue approach. Ramezani and Naghdabadi [28] established the concept of energy pairs in the micropolar continuum. Kumar and Choudary [29] studied the axi-symmetric problem in a micropolar medium by using integral transforms. So much research has been done in this field that one can refer to [30], [31], [32], etc for more review.

1.2.1 Micropolar thermoelasticity

In 1966, thermal effects were introduced into the micropolar elasticity theory by Nowacki [33], which gave rise to a new theory of elasticity called the *"Micropolar theory of Thermoelasticity"*. Thus, micropolar thermoelasticity theory is the generalization of micropolar theory and hence is comprised of not only heat equation but also of stress strains that in turn are produced because of heat flow. Therefore, the temperature distribution and the stresses which are produced by temperature fields can be calculated easily. Taking Eringen [34], Lord and Shulman [19], and Green and Lindsay [20] under consideration, the mathematical model for computing stress-strain along with modified Fourier's heat conduction law for homogeneous and isotropic micropolar generalized thermoelastic solids is as follows:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \varepsilon_{ijr}\phi_r) - \upsilon\left(T + \tau_1\frac{\partial T}{\partial t}\right)\delta_{ij},$$

$$\mu_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i},$$

$$K\nabla^2 T = \rho C_e\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \nu T_0\left(\frac{\partial}{\partial t} + \Xi\tau_0\frac{\partial^2}{\partial t^2}\right)u_{i,i}.$$

where, σ_{ij} represents force stress, μ_{ij} represents couple stress, λ, μ represents Lame's constants, $\alpha, \beta, \gamma, \kappa$ represents micropolar elastic constants, δ_{ij} represents Kronecker's delta, K represents thermal conductivity, C_e represents specific heat, T represents thermodynamic temperature, T_0 represents reference temperature, $\nu = (3\lambda + 2\mu + \kappa)\alpha_t$, and τ_0, τ_1 represents relaxation times.

So, because of its innumerable applications, a lot of research has been done in this field till now. Therefore, in 1973, the generalized micropolar thermoelasticity theory was extended by Boschi and Iesan [35]. In 1978, Dost and Taborrok [36] introduced generalized thermoelasticity by making use of Green and Lindsay theory. The micropolar thermoelasticity theory was established by Ciarletta [37] without dissipation of energy, and the thermal signals were allowed to propagate at finite speeds.

The micropolar thermoelasticity with stretch that was established by Nowacki [38] and Eringen ([13], [39]) in the framework of Lord-Shulman [19] and Green-Lindsay [20] theories was further generalized by Kumar and Singh [40]. In micropolar generalized thermoelastic solids, the plane waves were studied by Singh and Kumar [41] and Singh [42]. A 2D model of a generalized thermo-microstretch elastic solid is studied by Kumar and Singh [43], in which the elastic solid is exposed to impulsive force. The resulting non-dimensional coupled equations were solved by utilizing the technique of integral transforms along with the eigenvalue approach, and then the obtained results were illustrated graphically. In micropolar thermoelasticity, the response of impedance parameters was studied by Kumar et al. [44] in the context of modified GL theory. The impact of rotation was investigated by Othman and Abbas [45] in micropolar thermoelasticity in the context of TPL theory.

1.2.2 Magneto Micropolar elasticity

The magneto-micropolar elasticty theory is the generalization of the micropolar elasticity, and this theory deals with the distortion of a solid body placed in an external magnetic field. The two fields, namely magnetic and elastic, which are present in this theory contribute to the total deformation of the body. Moreover, the governing laws of these two fields get changed due to the interaction of these fields, and hence the elastic field enters into the governing equations of electromagnetism, i.e., Maxwell's equations, by modifying Ohm's law, and in turn the elastic field is affected by electro-magnetic field by inclusion of Lorentz's pondermotive force in Hook's law.

Taking [106] into consideration, the mathematical model for this type of media along with Lorentz force is as

$$\sigma_{ij} = \lambda u_{k,k} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \epsilon_{ijk}\phi_k),$$
$$\mu_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j},$$
$$(\mu + k)u_{i,jj} + (\lambda + \mu)u_{j,ji} + k\epsilon_{ijk}\phi_{k,j} + \epsilon_{ijk}J_jB_k = \rho\ddot{u}_i,$$
$$k\epsilon_{ijk}u_{k,j} - 2k\phi_i + (\alpha + \beta)\phi_{j,ji} + \gamma\phi_{i,jj} = \rho j\ddot{\phi}_i.$$

1.2.3 Magneto Micropolar thermoelasticity

In this theory, not only elastic and electromagnetic fields are present, but it also includes thermal fields, and the total distortion of the body is contributed while these fields interact with each other. The magnetomicropolar thermoelasticity is one of the delightful branch of micropolar theory, and because of its applications in different fields, it has grabbed the attention of thousands of researchers, due to which a lot of research has been done in this field and in related fields as well. A 2D model was constructed in the electromagnetic theory of micropolar elasticity by Kumar and Rupender [47], and the interaction was inquired between the mechanical and electromagnetic fields. Integral transforms have also been employed to figure out the problem's required solution. For two distinct theories, Lord-Shulman and Green-Lindsay, the behaviour of obtained physical quantities has been visualised graphically by the authors. A plain strain problem where the half-space was imposed to distributed thermal and mechanical sources was explored by Kumar [48]. To figure out the problem's required solution, Integral transforms have been utilized. For generalized theories of thermoelasticity, a considerable impact of the magnetic field has been observed graphically by authors on different physical quantities. In a generalized thermo-microstretch elastic medium, a general solution of field equations is found by Singh and Kumar [49] by making use of the eigenvalue approach along with Laplace and Hankel transformations for an axisymmetric problem, and the obtained results were depicted graphically. [50] introduced a simple mathematical way of obtaining the solution of a boundary value problem exposed to a mechanical source in magneto-micropolar infinite space in the presence of a transverse magnetic field. Laplace and Hankel transforms have been utilized to address the non-dimensional coupled equations. A 2D problem is explored by Kumar et al. [51] in magneto-micropolar thermoelastic half-space by taking hall current and rotational effects into account with a fractional order derivative. Furthermore, integral transforms along with a potential displacement approach are utilized in order to find out the solution of the required problem. The mechanical force and the transverse magnetic field were applied to 2D generalized magneto-micropolar thermoelastic infinite space by Singh and Kumar [52] and the technique of integral transforms was utilized for finding the solution of the required problem. A thermomechanical interactions was studied by Lata and Kaur [53] in homogeneous magneto thermoelastic medium (which is transversely isotropic aswell) in the framework of heat transfer (fractional order) and hall current. A new mathematical model was examined in [54] for a homogeneous magneto-thermoelastic (which is also transversely isotropic) medium in the context of fractional order theory. The impact of Hall current was studied by Lata and Kaur [55] in a homogeneous magnetothermoelastic rotating medium by taking fractional order theory into consideration. In 2022, a 2D problem was addressed by Abouelregal et al. [56] in generalized micropolar thermoelasticity and hence established a novel heat transfer model. A 2D model was established in magneto-micropolar thermoelasticity by Abouelregal et al. [57] and the higher order DPL model along with two-temperature theory (2TT) has been utilized. The different physical quantities has been obtained by adapting the technique of normal mode analysis.

A mathematical model [58] for an isotropic thermoelastic homogeneous solid placed in the externally applied magnetic field is usually taken as,

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \varepsilon_{ijk}\phi_k) - \nu T\delta_{ij},$$
$$\mu_{ij} = \alpha\phi_k\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i},$$
$$K\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\rho C_e T + T_0\nu e) + \pi_0 J_{i,i},$$
$$(\mu + k)u_{i,jj} + (\lambda + \mu)u_{j,ji} + k\varepsilon_{ijk}\phi_{k,j} + \varepsilon_{ijk}J_jB_k - \nu T_{,i} = \rho\ddot{u}_i,$$
$$k\varepsilon_{ijk}u_{k,j} - 2k\phi_i + (\alpha + \beta)\phi_{j,ji} + \gamma\phi_{i,jj} = \rho j\ddot{\phi}_i.$$

Again, because of its applications in various fields, it has captured the attention of thousands of researchers, resulting in a significant amount of research in this field and related areas.

1.3 Memory dependent derivative

The introduction of fractional calculus to the world was made possible by a letter exchange between Leibniz and de lHospital [59] in 1695. Due to its countless applications in various fields, fractional calculus has a very long history and has been utilised for decades. Since both fractional derivatives (FDs) and memory-dependent derivatives (MDDs) are employed to reflect processes with memory, it has been found that MDDs are significantly more useful than FDs when both derivatives are taken into account. One of the most notable differences between MDD and FD is that MDD has only one form, which is the most important feature, whereas FDs have several. In the context of MDDs definition, it has been observed that the physical meaning of MDDs is much clearer than that of FDs, which is their second distinguishing feature. Furthermore, MDDs can be represented by integer order differentials and integrals, making them easier to use, particularly when numerical calculations are required. In FDs, the kernel remains fixed, and we know that different processes need different kernels, which requires freedom in the kernel selection depending upon the nature of the problem. This requirement is also fulfilled by MDDs, in which the kernels and time delays can be randomly chosen. As a result, memory dependent derivatives are more effective in analysing material memory responses.

The α-order fractional derivative for a function $f(t)$ (Caputo [60] derivative) is taken as:

$$D_a^\alpha f(t) = \int_a^t K_\alpha(t-r)f^m(r)dr, \tag{1.1}$$

with kernel function,

$$K_\alpha(t-r) = \frac{(t-r)^{m-\alpha-1}}{\Gamma(m-\alpha)}. \tag{1.2}$$

In the equations (1.1) and (1.2), the kernel function, which remains constant for a real number α (which is given), is represented by $K_\alpha(t-r)$ and f^m represents the m^{th} order derivative.

Since one of the most important property of fractional calculus is that it can be used to study processes possessing memory with a fixed kernel. However, due to the fixed nature of kernel, it becomes quite difficult to study the processes possessing memory because different processes require different kernels. As such the kernels should be chosen freely. So, to address this drawback, a novel derivative was proposed by Wang and Li [61] in 2011, known as a *memory-dependent derivative*, and in this derivative the kernel can be selected freely according to the problem's nature. In this, the first order derivative of a function f has been described in an integral form of a common derivative with a kernel function $K(t-r)$ on an interval $[t-\tau, t]$ in the following form

$$D_\tau f(t) = \frac{1}{\tau}\int_{t-\tau}^{t} K(t-r)f^{'}(r)dr.$$

here, $K(t-r)$ denotes the kernal function and $\tau(>0)$ denotes time delay.

In general, the weight required by memory effect is $0 \leq K(t-r) \leq 1$ for $r\epsilon[t-r,t]$ such that the MDD's magnitude $D_\tau f(t)$ is typically lesser than that of $f^{'}(t)$, which is the common derivative. The kernel function $K(t-r)$ can be taken randomly, for instance 1, $r-t+1$, $\left[\frac{r-t}{\tau+1}\right]^p$, where $p = 0.25, 1, 2$ etc, which may be more practical. If, $K(t-r) = 1$, then

$$D_\tau f(t) = \frac{1}{\tau}\int_{t-\tau}^{t} f^{'}(r)dr = \frac{f(t)-f(t-\tau)}{\tau} \to f^{'}(t),$$

This makes it clear that $\frac{d}{dt}$ (which is a common derivative) is the limit of D_τ as τ approaches to 0.

Because of the countless and fascinating applications of fractional calculus in different fields, it has captured a lot of interest over the last decade. A mathematical model in thermoelasticity theory was introduced by Youssef [62] in the context of a heat conduction equation with fractional order. Sherief and Latief [63] employed a fractional order thermoelastic model for a spherical cavity which is subjected to a thermal shock. In [64]-[72], there have been certain important developments that are specifically connected to fractional calculus.

To explore the notion of MDD, the following fascinating and remarkable research works ([73], [74], [75], and [76]) can be reviewed. In an infinite space, a 2D problem was investigated by Purkait et al. [77] by using the Green-Naghdi model III with MDD. In addition to this, the authors have recognized the graphical analysis of different physical quantities w.r.t the time parameter. In elastic solid with voids, the MDD model was established by Sur and Kanoria [78] on thermal wave propagation. The Laplace transform along with the EVA have been adapted so as to get the desired solution of the problem. In a generalized thermoelastic medium, a 2D model was examined by Othman and Mondal [79] in the framework of the LordShulman model with MDD. A new model with higher-order MDD was examined by Abouelregal et al. [80] in generalized thermoelasticity theory. Sun et al. [81] developed a new generalized thermoelastopiezoelectric model in the context of MDD. In 2021, a quality factor of a microbeam was studied by Kumar [82] by using three-phase-lag thermoelasticity with MDD. In 2022, the quality factor was analyzed by Kumar et al. [83] of a micro-beam resonator in the framework of MDD. The impact of kernel function and delay time was also analyzed on the micro-beam resonator parameters. In addition to this, the normal mode technique was utilized to figure out the problem's required solution. In 2022, a model of thermoelasticity was investigated by Abouelregal [84] with higher order MDDs and with two delay-time parameters. For the solution of the problem, the Laplace transform technique has been utilized. The TPL model with MDD was used by Bayones et al. [85] to study the thermoelastic interaction in magneto-thermoelasticity. Laplace transform has been utilized by the authors for finding the solution of the required problem. In 2023, the thermoelastic behavior was investigated by Abouelregal et al. [86] of rotating size-dependent nanobeams in the context of MDD. Moreover, the impact of memory-dependent parameters (kernel function and delay time) has also been recognized graphically for different physical quantities.

1.4 Research Gap and Objectives

While undergoing a literature survey, it has been observed that thermoelasticity and generalized thermoelasticity involving memory-dependent derivatives are active area of the research. So, in the proposed research work, the main objective has been to study the plain-strain problems of micropolar elasticity by employing memory-dependent derivatives. Moreover, most of the work is available in the classical elasticity theory, thus leaving scope for the study of plain-strain problems in the field of micropolar elasticity. Furthermore, the response of micropolar materials under magnetic and thermal fields can be studied, which can be useful for modern-day engineering. We have studied the impact of different effects such as thermal, rotational, or electromagnetic effects as well as memory-dependent derivatives in problems of micropolar/classical theory of elasticity in order to explore new results. Based on these research gaps, the objectives of the thesis were framed as follows:

1. To formulate a mathematical model for problems on micropolar theory of elasticity using memory dependent derivative.
2. Analyse the response of material by including and excluding micropolar effect.
3. To study the response of micropolar elastic material when subjected to thermal, electromagnetic or rotation effect.

Chapter 2

Elastodynamic responses of magneto micropolar isotropic media under the gravitational influence

2.1 Introduction

Since each object possessing mass exerts a gravitational force on other objects with mass, the magnitude of this pull (force) is determined by the masses of the objects involved. Moreover, it's because of this gravity that keeps the moon in orbit around the earth and other planets around the sun. Thus, gravity can be defined as the type of force that attracts an object (with mass) towards the earth's centre or towards any other object (with mass). Keeping gravity under consideration, it has been observed that insufficient attention has been paid to the classical elasticity theory with gravitational effects. So, in order to study the impact of gravity in elasticity, Bromwich [87] was the first to study its impact on wave propagation in elastic solids, in 1898. Then in 1974, De and Sengupta [88] examined the gravitational impact on wave propagation in an elastic layer. In 2010, Ailawalia et al. [89] established a new mathematical model of rotating generalized thermoelasticity under hydrostatic initial stress and gravity. The authors have also analyzed the rotational and gravitational impacts on the obtained outcomes graphically. For three different theories, a problem was developed by Othman et al. [90] for the generalized thermoelastic medium under the gravitational effect. They have also compared the outcomes of the three different theories in the absence and presence of temperature dependence. Othman and Hilal [91] explored a 2D problem of a thermoelastic rotating media with voids while taking the gravitational field into account. For the required solution of the problem, the normal mode technique along with the Helmholtz potentials has been utilized. On the plane waves, the impact of gravity and magnetic fields were explored by Othman and Hilal [92] imposed to laser pulse heating. A significant impact of gravitational and magnetic fields were also recognized graphically by authors on thermoelastic material (porous). Othman and Elaziz [93] used the DPL model in order to examine the impact of gravitational and rotational fields on the micropolar magneto-thermoelastic solid. Taking LS-theory (Lord-Shuman model) and the DPL model under consideration, the authors have discussed and compared the obtained outcomes graphically with and without magnetic, roational, and gravitational fields. Othman et al. [94] used the II and III types of the Green-Naghdi theory to examine the effects of gravity on plane waves in a micropolar thermoelastic medium. The obtained outcomes were compared not only with and without gravitational effects but also for distinct values of inclination angle. On a rotating magneto-micropolar thermoelastic medium, the effect of gravitation and magnetic fields were studied by Hilal [95] with temperature dependency. For the desired solution of the problem, the authors have utilized normal mode technique along with the Helmholtz potentials. A 2D model was developed to study the gravitational impact in micropolar thermoelasticity by Kumar et al. [96] by using Dual-phase-lag theory.

This chapter investigates a 2D mathematical model of magneto-micropolar thermoelasticity with two temperatures under the impact of gravity. For deriving the solution of the required problem, the normal mode analysis (NMA) along with the potential displacement approach (PDA) are utilized. Finally, for various values of gravity acquired outside the mesosphere, the components of stress, strain, and temperature distribution have been compared and illustrated graphically.

2.2 Basic equations

Following [97], the equations of electromagnetism for a perfectly conducting, homogeneous, and slowly moving elastic medium, along with the equations of motion and constitutive relations in micropolar generalized thermoelasticity, in the context of Lorentz and gravitational forces, are as follows:

$$\nabla \times \overrightarrow{h} = \overrightarrow{J} + \varepsilon_0 \frac{\partial \overrightarrow{E}}{\partial t}, \tag{2.1}$$

$$\nabla \times \overrightarrow{E} = -\mu_0 \frac{\partial \overrightarrow{h}}{\partial t}, \tag{2.2}$$

$$\overrightarrow{E} = -\mu_0 \left(\frac{\partial \overrightarrow{u}}{\partial t} \times \overrightarrow{H_0} \right), \tag{2.3}$$

$$\nabla . \overrightarrow{h} = 0, \tag{2.4}$$

$$(\lambda + \mu)\nabla(\nabla . \overrightarrow{u}) + (\mu + \kappa)\nabla^2 \overrightarrow{u} + \kappa(\nabla \times \overrightarrow{\phi}) - \nu \nabla T + \overrightarrow{F} + \overrightarrow{G} = \rho \frac{\partial^2 \overrightarrow{u}}{\partial t^2}, \tag{2.5}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla . \overrightarrow{\phi}) - \gamma \nabla \times (\nabla \times \overrightarrow{\phi}) + \kappa(\nabla \times \overrightarrow{u}) - 2\kappa \overrightarrow{\phi} = \rho j \frac{\partial^2 \overrightarrow{\phi}}{\partial t^2}, \tag{2.6}$$

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa(u_{j,i} - \varepsilon_{ijr}\phi_r) - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{2.7}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}. \tag{2.8}$$

Heat conduction equation is

$$K\nabla^2\varphi = \rho C_e\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \nu T_0\left(\frac{\partial}{\partial t} + \tau_0 n_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\overrightarrow{u}), \tag{2.9}$$

where,

$$\varphi - T = a^*\nabla^2\varphi. \tag{2.10}$$

Moreover, $\overrightarrow{F}$ and $\overrightarrow{G}$ has been defined as follows

$$\begin{aligned} \overrightarrow{F} &= \mu_0(\overrightarrow{J} \times \overrightarrow{H_0}), \\ \overrightarrow{G} &= \rho g(w_x, 0, -u_x). \end{aligned} \tag{2.11}$$

The equations of motion (2.5)-(2.6) along with heat equation (2.9) in Cartesian coordinates (x, y, z) in component form can be written as

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) + (\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \kappa\left(\frac{\partial\phi_3}{\partial y} - \frac{\partial\phi_2}{\partial z}\right) -$$
$$\nu\frac{\partial T}{\partial x} + (J_2H_3 - J_3H_2) + \rho g\frac{\partial w}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.12}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial y\partial x} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y\partial z}\right) + (\mu+\kappa)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \kappa\left(\frac{\partial\phi_1}{\partial z} - \frac{\partial\phi_3}{\partial x}\right) -$$
$$\nu\frac{\partial T}{\partial y} + (J_3H_1 - J_1H_3) = \rho\frac{\partial^2 v}{\partial t^2}, \tag{2.13}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial z\partial x} + \frac{\partial^2 v}{\partial z\partial y} + \frac{\partial^2 w}{\partial z^2}\right) + (\mu+\kappa)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \kappa\left(\frac{\partial\phi_2}{\partial x} - \frac{\partial\phi_1}{\partial y}\right) -$$
$$\nu\frac{\partial T}{\partial z} + (J_1H_2 - J_2H_1) - \frac{\partial u}{\partial x} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{2.14}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_2}{\partial x\partial y} + \frac{\partial^2\phi_3}{\partial x\partial z}\right) + \gamma\left(\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} + \frac{\partial^2\phi_1}{\partial z^2}\right) + \kappa\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - 2\kappa\phi_1$$
$$= \rho j\frac{\partial^2\phi_1}{\partial t^2}, \tag{2.15}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial y\partial x} + \frac{\partial^2\phi_2}{\partial y^2} + \frac{\partial^2\phi_3}{\partial y\partial z}\right) + \gamma\left(\frac{\partial^2\phi_2}{\partial x^2} + \frac{\partial^2\phi_2}{\partial y^2} + \frac{\partial^2\phi_2}{\partial z^2}\right) + \kappa\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - 2\kappa\phi_2$$
$$= \rho j\frac{\partial^2\phi_2}{\partial t^2}, \tag{2.16}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial z\partial x}+\frac{\partial^2\phi_2}{\partial z\partial y}+\frac{\partial^2\phi_3}{\partial z^2}\right)+\gamma\left(\frac{\partial^2\phi_3}{\partial x^2}+\frac{\partial^2\phi_3}{\partial y^2}+\frac{\partial^2\phi_3}{\partial z^2}\right)+\kappa\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)-2\kappa\phi_3$$
$$=\rho j\frac{\partial^2\phi_3}{\partial t^2}, \tag{2.17}$$
$$K\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right)=\rho C_e\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+\upsilon T_0\left(\frac{\partial}{\partial t}+\tau_0 n_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right), \tag{2.18}$$

where,

$$\varphi-T=a^*\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right). \tag{2.19}$$

where, (u,v,w), (ϕ_1,ϕ_2,ϕ_3), (J_1,J_2,J_3) and (H_1,H_2,H_3) are the components of displacement vector $\overrightarrow{u}$, microrotation vector $\overrightarrow{\phi}$, current density vector $\overrightarrow{J}$ and magnetic field vector $\overrightarrow{H}$ respectively.

2.3 Formulation and Solution of the problem

A generalized micropolar thermoelastic medium with gravity is considered. In addition to this, the considered medium is isotropic, homogeneous, and permeated by $\overrightarrow{H_0}$ which acts along the y-axis. The origin of a rectangular cartesian co-ordinate system (x, y, z) is taken at any point on the plane surface of half-space z = 0, as shown in figure 2.1. Moreover, $\overrightarrow{u}$ and $\overrightarrow{\phi}$ for the considered plane strain problem are defined as

$$\overrightarrow{u}=(u,0,w), \quad \overrightarrow{\phi}=(0,\phi_2,0), \quad u(x,z,t), \quad and \quad w(x,z,t). \tag{2.20}$$

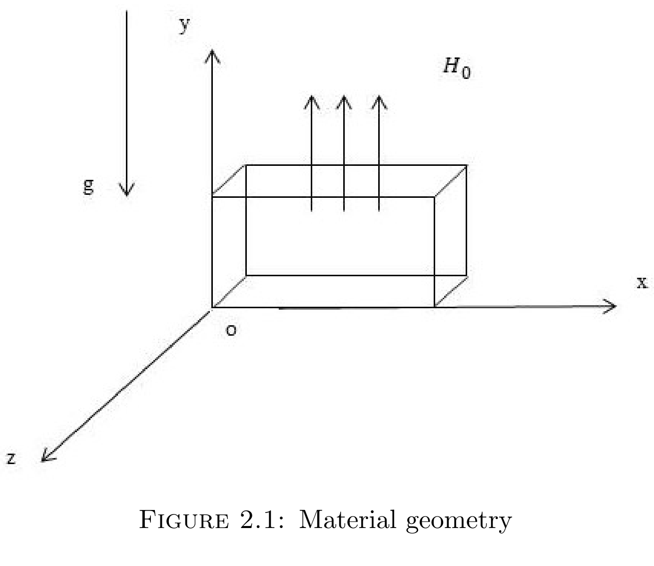

FIGURE 2.1: Material geometry

After plugging equation (2.20) in equations (2.1)-(2.3), we get

$$\overrightarrow{E} = \mu_0 H_0(\dot{w}, 0, -\dot{u}), \tag{2.21}$$

$$\overrightarrow{h} = -H_0(0, e, 0), \tag{2.22}$$

$$\overrightarrow{J} = ((H_0 e_{,z} - \mu_0 H_0 \epsilon_0 \ddot{w}), 0, (-H_0 e_{,x} + \mu_0 H_0 \epsilon_0 \ddot{u})), \tag{2.23}$$

where,

$$e = u_x + w_z. \tag{2.24}$$

represents the cubical dilatation.

After some simplification, equation (2.11) turns into

$$\overrightarrow{F} = (\mu_0 H_0^2(e_{,x} - \epsilon_0\mu_0\ddot{u}), 0, \mu_0 H_0^2(e_{,z} - \epsilon_0\mu_0\ddot{w})). \tag{2.25}$$

Using equation (2.20) and under the influence of Lorentz and gravitational forces, the above equations (2.12)-(2.19), along with equations (2.7)-(2.8), can be expressed as

$$(\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + (\mu+\kappa)\nabla^2 u - \kappa\frac{\partial \phi_2}{\partial z} - \nu\frac{\partial T}{\partial x} - \mu_0 H_0 J_3 + \rho g\frac{\partial w}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.26}$$

$$(\lambda+\mu)\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + (\mu+\kappa)\nabla^2 w + \kappa\frac{\partial \phi_2}{\partial x} - \nu\frac{\partial T}{\partial z} + \mu_0 H_0 J_1 - \rho g\frac{\partial u}{\partial x} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{2.27}$$

$$\gamma\nabla^2\phi_2 + \kappa\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - 2\kappa\phi_2 = \rho j\frac{\partial^2 \phi_2}{\partial t^2}, \tag{2.28}$$

$$K\nabla^2\varphi = \rho C_e\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \nu T_0\left(\frac{\partial}{\partial t} + \tau_0 n_0\frac{\partial^2}{\partial t^2}\right)e, \tag{2.29}$$

$$\varphi - T = a^*\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2}\right), \tag{2.30}$$

$$\sigma_{xx} = (\lambda+2\mu+\kappa)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{2.31}$$

$$\sigma_{zz} = (\lambda+2\mu+\kappa)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{2.32}$$

$$\sigma_{xz} = (\mu+\kappa)\frac{\partial w}{\partial z} + \mu\frac{\partial u}{\partial z} + \kappa\phi_2, \tag{2.33}$$

$$\sigma_{zx} = (\mu+\kappa)\frac{\partial u}{\partial z} + \mu\frac{\partial w}{\partial x} - \kappa\phi_2, \tag{2.34}$$

$$m_{xy} = \gamma\frac{\partial\phi_2}{\partial x}, \tag{2.35}$$

$$m_{zy} = \gamma\frac{\partial\phi_2}{\partial z}. \tag{2.36}$$

Now, the non-dimensional quantities are defined below as

$$(x', z') = \frac{\omega*}{c_1}(x, z),\ (u', w') = \frac{\rho c_1 \omega^*}{\nu T_0}(u, w),\ \sigma'_{i,j} = \frac{\sigma_{ij}}{\nu T_0},\ (t^{'}, \tau_0^{'}, \tau_1^{'}) = \omega^*(t, \tau_0, \tau_1),\ T^{'} = \frac{T}{T_0},$$
$$\phi_2{}' = \frac{\rho c_1^2}{\nu T_0}\phi_2,\ m_{ij}^{'} = \frac{\omega^*}{c_1 \nu T_0} m_{ij},\ g^{'} = \frac{g}{c_1 \omega^*},\ \varphi^{'} = \frac{\varphi}{T_0}, \tag{2.37}$$

where, $\omega^* = \frac{\rho C_e {c_1}^2}{K}$, $c_1^2 = \frac{\lambda + 2\mu + \kappa}{\rho}$.

After using equation (2.37), equations (2.26)-(2.30) turn into (dropping the dashes for convenience)

$$a_1 \nabla^2 u + a_2 \frac{\partial e}{\partial x} - a_3 \frac{\partial \phi_2}{\partial z} - a_4 \frac{\partial T}{\partial x} + g a_5 \frac{\partial w}{\partial x} = a_6 \frac{\partial^2 u}{\partial t^2}, \tag{2.38}$$
$$a_1 \nabla^2 w + a_2 \frac{\partial e}{\partial z} + a_3 \frac{\partial \phi_2}{\partial x} - a_4 \frac{\partial T}{\partial z} - g a_5 \frac{\partial u}{\partial x} = a_6 \frac{\partial^2 w}{\partial t^2}, \tag{2.39}$$
$$a_7 \nabla^2 \phi_2 + a_8 \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) - a_9 \phi_2 = a_{10} \frac{\partial^2 \phi_2}{\partial t^2}, \tag{2.40}$$
$$a_{12} \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + a_{13} \left(\frac{\partial}{\partial t} + \tau_0 n_0 \frac{\partial^2}{\partial t^2} \right) e = a_{11} \nabla^2 \varphi, \tag{2.41}$$
$$\varphi - T = \eta_1 \nabla^2 \varphi, \tag{2.42}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$, $a_1 = \frac{(\mu+\kappa)\omega^*}{\rho c_1^2}$, $a_2 = \frac{(\lambda+\mu)\omega^* + \mu_0 H_0^2 \omega^*}{\rho c_1^2}$, $a_3 = \frac{\kappa \omega^*}{\rho c_1^2}$, $a_4 = \omega^*$, $a_5 = \frac{1}{c_1}$, $a_6 = \frac{\mu_0^2 H_0^2 \varepsilon_0 \omega^*}{\rho} + \omega^*$, $a_7 = \frac{\gamma \omega^{*2}}{\rho {c_1}^2}$, $a_8 = \frac{\kappa}{\rho}$, $a_9 = \frac{2\kappa}{\rho}$, $a_{10} = j\omega^{*2}$, $a_{11} = \frac{K\omega^*}{c_1^2}$, $a_{12} = \rho c^*$, $a_{13} = \frac{\gamma \nu T_0}{\rho c_1^2}$, $\eta_1 = \frac{a^* \omega^{*2}}{c_1^2}$.

Now, for obtaining solution of the required problem, the potential displacements $q(x, z, t)$ and $\psi(x, z, t)$ which are defined below, are introduced as

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z},\ w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{2.43}$$

Using equation (2.43) in equations (2.38), (2.40), (2.41), we obtain

$$\left[(a_1 + a_2)\nabla^2 - a_6 \frac{\partial^2}{\partial t^2} \right] q - a_4 T - a_5 g \frac{\partial \psi}{\partial x} = 0, \tag{2.44}$$
$$\left[a_1 \nabla^2 - a_6 \frac{\partial^2}{\partial t^2} \right] \psi - a_3 \phi_2 + g a_5 \frac{\partial q}{\partial x} = 0, \tag{2.45}$$
$$\left[a_7 \nabla^2 - a_9 - a_{10} \frac{\partial^2}{\partial t^2} \right] \phi_2 + a_8 \nabla^2 \psi = 0, \tag{2.46}$$
$$a_{12} \left[\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right] T + a_{13} \left[\frac{\partial}{\partial t} + \tau_0 n_0 \frac{\partial^2}{\partial t^2} \right] \nabla^2 q = a_{11} \nabla^2 \varphi. \tag{2.47}$$

Using equation (2.42) in equations (2.44) and (2.47), we obtain

$$\left[(a_1+a_2)\nabla^2 - a_6\frac{\partial^2}{\partial t^2}\right]q - a_4(1-\eta_1\nabla^2)\varphi - a_5 g\frac{\partial\psi}{\partial x} = 0, \tag{2.48}$$

$$a_{12}\left[\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right](1-\eta_1\nabla^2)\varphi + a_{13}\left[\frac{\partial}{\partial t} + \tau_0 n_0\frac{\partial^2}{\partial t^2}\right]\nabla^2 q = a_{11}\nabla^2\varphi. \tag{2.49}$$

2.4 Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal modes as follows

$$[u, w, q, \psi, \phi_2, m_{ij}, \sigma_{ij}, \varphi](x, z, t) = [\bar{u}, \bar{w}, \bar{q}, \bar{\psi}, \bar{\phi}_2, \bar{m}_{ij}, \bar{\sigma}_{ij}, \bar{\varphi}](z)exp(bt + iax). \tag{2.50}$$

After using equation (2.50), equations (2.45), (2.46), (2.48) and (2.49) turns into

$$[D^2 - A_1]\bar{\psi} - A_2\bar{\phi}_2 + A_3\bar{q} = 0, \tag{2.51}$$

$$[D^2 - A_6]\bar{q} - A_7[1-\eta_1(D^2-a^2)]\bar{\varphi} - A_8\bar{\psi} = 0, \tag{2.52}$$

$$[D^2 - A_6]\bar{q} - A_7[1-\eta_1(D^2-a^2)]\bar{\varphi} - A_8\bar{\psi} = 0, \tag{2.53}$$

$$[A_9(D^2-a^2) - A_{10}]\bar{\varphi} - [A_{11}(D^2-a^2)]\bar{q} = 0, \tag{2.54}$$

where,
$D = \frac{\partial}{\partial z}$, $A_1 = \frac{a_1a^2+a_6b^2}{a_1}$, $A_2 = \frac{a_3}{a_1}$, $A_3 = \frac{ga_5ia}{a_1}$, $A_4 = \frac{a_7a^2+a_9+a_{10}b^2}{a_7}$, $A_5 = \frac{a_8}{a_7}$, $A_6 = \frac{(a_1+a_2)a^2+a_6b^2}{(a_1+a_2)}$, $A_7 = \frac{a_4}{a_1+a_2}$, $A_8 = \frac{a_5gia}{a_1+a_2}$ $A_9 = a_{11} + \eta_1 a_{12}b(1+\tau_0 b)$, $A_{10} = a_{12}b(1+\tau_0 b)$, $A_{11} = a_{13}b(1+\tau_0 n_0 b)$.

After some simplification, equations (2.51)-(2.54) can be expressed as

$$[D^4 + A'D^2 + B']\bar{\psi} + [C'D^2 - D']\bar{q} = 0, \tag{2.55}$$

$$[A''D^4 - B''D^2 + C'']\bar{q} - [D''D^2 - E']\bar{\psi} = 0, \tag{2.56}$$

where,
$A' = A_2A_5 - A_4 - A_1$, $B' = A_4A_1 - A_2A_5a^2$, $C' = A_3$, $D' = A_3A_4$, $A'' = A_9 + A_7A_{11}\eta_1$, $B'' = A_9A_6 + A_9a^2 + A_{10}$, $C'' = A_9A_6a^2 + A_{10}A_6 - A_7 - \eta_1A_7A_{11}a^4$, $D'' = A_9A_8$, $E' = A_9A_8a^2 + A_{10}A_8$.

Eliminating $\bar{\psi}$ from equations (2.55) and (2.56), we get

$$[D^8 + AD^6 + BD^4 + CD^2 + F]\bar{q}(z) = 0,$$

where, $A = \frac{A'A''-B''}{A''}$, $B = \frac{D''C'+C''-A'B''+B'A''}{A''}$, $C = \frac{A'C''-D''D'-E'C'-B'B''}{A''}$, $F = \frac{E'D'+B'C''}{A''}$.

Similarly,

$$[D^8 + AD^6 + BD^4 + CD^2 + F][\bar{\phi_2}(z), \bar{\psi}(z), \bar{q}(z), \bar{\varphi}(z)] = 0, \tag{2.57}$$

Rewriting equation (2.57), in the factored form as

$$[(D^2 - {k_1}^2)(D^2 - {k_2}^2)(D^2 - {k_3}^2)(D^2 - {k_4}^2)][\bar{\phi_2}, \bar{\psi}, \bar{q}, \bar{\varphi}] = 0, \tag{2.58}$$

here, ${k_n}^2 (n = 1, 2, 3, 4)$ represents characteristic roots of the equation (2.57). The general solution of equation (2.58), has the form

$$\bar{\phi_2}(z) = \sum_{n=1}^{4} M_n e^{-k_n z}, \tag{2.59}$$

$$\bar{\psi}(z) = \sum_{n=1}^{4} {M_n}' e^{-k_n z}, \tag{2.60}$$

$$\bar{q}(z) = \sum_{n=1}^{4} {M_n}'' e^{-k_n z}, \tag{2.61}$$

$$\bar{\varphi}(z) = \sum_{n=1}^{4} {M_n}''' e^{-k_n z}, \tag{2.62}$$

Here, M_n, ${M_n}'$, ${M_n}''$, ${M_n}'''$ are parameters that depends on a and b.

Using equations (2.59) - (2.62) in equations (2.51) - (2.54), we get

$$\bar{\psi}(z) = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z}, \tag{2.63}$$

$$\bar{q}(z) = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z}, \tag{2.64}$$

$$\bar{\varphi}(z) = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z}, \tag{2.65}$$

where, $H_{1n} = -\frac{({k_n}^2 - A_4)}{A_5({k_n}^2 - a^2)}$, $H_{2n} = \left[\frac{-A_2 A_5 (k_n^2 - a^2)}{A_3(k_n^2 - A_4)} - \frac{({k_n}^2 - A_1)}{A_3}\right] H_{1n}$, $H_{3n} = \left[\frac{A_{11}({k_n}^2 - a^2) H_{2n}}{A_9({k_n}^2 - a^2) - A_{10}}\right]$.

In general, equations (2.59)-(2.62), can be written as

$$(\bar{\phi_2}, \bar{\psi}, \bar{q}, \bar{\varphi})(z) = \sum_{n=1}^{4} (1, H_{1n}, H_{2n}, H_{3n}) M_n exp(-k_n z). \tag{2.66}$$

Now, using equations (2.37) and (2.43) in equations (2.32), (2.34), (2.36), we get

$$\sigma_{zz} = \left(a_{14}\frac{\partial^2}{\partial z^2} + a_{15}\frac{\partial^2}{\partial x^2}\right) q + (a_{15} - a_{14})\frac{\partial^2 \psi}{\partial x \partial z} - \left[1 + \tau_1 \frac{\partial}{\partial t}\right](1 - \eta_1 \nabla^2)\varphi, \tag{2.67}$$

$$\sigma_{zx} = (a_{16} + a_{17})\frac{\partial^2 q}{\partial x \partial z} - a_{17}\frac{\partial^2 \psi}{\partial x^2} + a_{16}\frac{\partial^2 \psi}{\partial z^2} - a_{18}\phi_2, \tag{2.68}$$

$$m_{zy} = a_{19}\frac{\partial \phi_2}{\partial z}, \tag{2.69}$$

where,

$a_{14} = \frac{\lambda+2\mu+\kappa}{\rho c_1^2}$, $a_{15} = \frac{\lambda}{\rho {c_1}^2}$, $a_{16} = \frac{\mu+\kappa}{\rho c_1^2}$, $a_{17} = \frac{\mu}{\rho c_1^2}$, $a_{18} = \frac{\kappa}{\rho c_1^2}$, $a_{19} = \frac{\gamma \omega^{*2}}{\rho c_1^4}$.

Applying equation (2.50) in equations (2.67) - (2.69), we get

$$\bar{\sigma}_{zz} = [D^2 - A_{12}a^2]\bar{q} + iaA_{13}D\bar{\psi} + A_{14}[\eta_1(D^2 - a^2) - 1]\bar{\varphi}, \tag{2.70}$$

$$\bar{\sigma}_{zx} = iaA_{15}D\bar{q} + [A_{16}a^2 + A_{17}D^2]\bar{\psi} - A_{18}\bar{\phi}_2, \tag{2.71}$$

$$\bar{m}_{zy} = a_{19}D\bar{\phi}_2, \tag{2.72}$$

where,

$A_{12} = \frac{a_{15}}{a_{14}}$, $A_{13} = \frac{a_{15}-a_{14}}{a_{14}}$, $A_{14} = \frac{1+\tau_1 b}{a_{14}}$, $A_{15} = (a_{16} + a_{17})$, $A_{16} = a_{17}$, $A_{17} = a_{16}$, $A_{18} = a_{18}$.

Using equation (2.90) in equations (2.43), (2.70) - (2.72), we get

$$\bar{u} = \sum_{n=1}^{4} M_n H_{4n} e^{-k_n z}, \tag{2.73}$$

$$\bar{w} = \sum_{n=1}^{4} M_n H_{5n} e^{-k_n z}, \tag{2.74}$$

$$\bar{\sigma}_{zz} = \sum_{n=1}^{4} M_n H_{6n} e^{-k_n z}, \tag{2.75}$$

$$\bar{\sigma}_{zx} = \sum_{n=1}^{4} M_n H_{7n} e^{-k_n z}, \tag{2.76}$$

$$\bar{m}_{zy} = \sum_{n=1}^{4} M_n H_{8n} e^{-k_n z}, \tag{2.77}$$

where,

$$H_{4n} = iaH_{2n} - H_{1n}k_n,$$
$$H_{5n} = -H_{2n}k_n - iaH_{1n},$$
$$H_{6n} = (k_n^2 - a^2 A_{12})H_{2n} - iaA_{13}k_nH_{1n} + A_{14}(\eta_1 k_n^2 - \eta_1 a^2 - 1)H_{3n},$$
$$H_{7n} = -iaA_{15}H_{2n}k_n + (A_{16}a^2 + A_{17}k_n)H_{1n} - A_{18},$$
$$H_{8n} = -a_{19}k_n.$$

2.5 Boundary conditions

The following boundary conditions have been taken at $z = 0$ to figure out the M_n, (where, $n = 1, 2, 3, 4$) parameters. The thermal boundary condition is taken as

$$\varphi = B_1 e^{bt+iax}, \tag{2.78}$$

and the mechanical boundary conditions are taken such that the bounding plane z = 0 is traction-free i.e.,

$$\sigma_{zz} = 0, \tag{2.79}$$
$$\sigma_{zx} = 0, \tag{2.80}$$
$$m_{zy} = 0. \tag{2.81}$$

here, B_1 represents a constant.

After using equation (2.50) and after some simplification, we get

$$\bar{\varphi} = B_1, \tag{2.82}$$
$$\bar{\sigma}_{zz} = 0, \tag{2.83}$$
$$\bar{\sigma}_{zx} = 0, \tag{2.84}$$
$$\bar{m}_{zy} = 0, \tag{2.85}$$

Using equations (2.82)-(2.85) along with (2.62), (2.75)-(2.77), and after some simplification, we obtain

$$\sum_{n=1}^{4} H_{3n} M_n = B_1, \tag{2.86}$$

$$\sum_{n=1}^{4} H_{6n} M_n = 0, \tag{2.87}$$

$$\sum_{n=1}^{4} H_{7n} M_n = 0, \tag{2.88}$$

$$\sum_{n=1}^{4} H_{8n} M_n = 0. \tag{2.89}$$

The equations (2.86)-(2.89) have been solved for M_n, $(n = 1, 2, 3, 4)$ to figure out the solution of the required problem and by making use of Inverse matrix method, which is as:

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} H_{31} & H_{32} & H_{33} & H_{34} \\ H_{61} & H_{62} & H_{63} & H_{64} \\ H_{71} & H_{72} & H_{73} & H_{74} \\ H_{81} & H_{82} & H_{83} & H_{84} \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

2.6 Validity of the Problem

When the gravitational effect is ignored, we obtain the following results

$$(\bar{q}, \bar{\varphi}, \bar{\psi}, \bar{\phi}_2)(z) = \sum_{n=1}^{4} (1, \mathbb{H}_{1n}, \mathbb{H}_{2n}, \mathbb{H}_{3n}) M_n exp(-k_n z). \tag{2.90}$$

where,

$\mathbb{H}_{1n} = \frac{A_{11}(k_n^2 - a^2)}{A_9(k_n^2 - a^2)}$, $\mathbb{H}_{2n} = \frac{(k_n^2 - A_6)}{A_8} - \frac{A_7[1 - \eta_1(k_n^2 - a^2)] A_{11}(k_n^2 - a^2)}{A_8 A_9 (k_n^2 - a^2)}$, $\mathbb{H}_{3n} = \frac{(k_n^2 - A_1^{'})}{A_2^{'}} \mathbb{H}_{2n}$, $A_1^{'} = \left(a^2 + \frac{a_6 b^2}{a_1}\right)$, $A_2^{'} = a_3$.

which are in sync with the results explained in the study [98] considered in the context of magneto-micropolar generalized thermoelasticity.

2.7 Numerical results and Discussion

The numerical computations were done for distinct values of gravity, g particularly for g = 4, 6, and 8, acquired outside the earth's mesosphere. Then the graphical representation of different physical quantities, such as the components of displacement, force stresses, and the conductive temperature distribution, is done, as shown in figures 2.2-2.6. Moreover, while performing numerical computations, the following material properties of magnesium [106] are considered:

$$\rho = 1.74 \times 10^3 kg\, m^{-3},\ j = 0.2 \times 10^{-19} m^2,\ \gamma = 0.779 \times 10^{-9} kg\, ms^{-2},$$
$$\lambda = 9.4 \times 10^{10} kg\, m^{-1} s^{-2},\ \kappa = 1.0 \times 10^{10} kg\, m^{-1} s^{-2}\ \mu = 4.0 \times 10^{10} kg\, m^{-1} s^{-2}$$
$$K = 1.7 \times 10^2 Jm^{-1} s^{-1} deg^{-1},\ a^* = 0.074 \times 10^{-15} m^2,\ C_e = 1.04 \times 10^3 Jkg^{-1} deg^{-1},$$
$$T_0 = 298K,\ \alpha_t = 7.403 \times 10^{-7} K^{-1},\ \tau_1 = 1s,\ \tau_0 = 0.02s,\ B_1 = 1,\ n_0 = 0.$$

Figure 2.2 clearly demonstrates that the displacement component u reduces when the value of gravity is increased, and the maximum displacement distribution is observed, which is attained at g = 4, and it reduces when the value of gravity is increased, i.e., at g = 6. At g = 8, we observe the least displacement distribution. In other words, the amplitude of displacement component u reduces when the value of gravity is increased.

From Figure 2.3, it is clear that the displacement component w is less sensitive to the changes in value of g. Moreover, figure 2.3 shows that the displacement component w is least at g = 8 in terms of amplitude, and then it increases when g is reduced. Figure 2.4 demonstrates that the stress component σ_{zx} is maximum at g = 6, then the stress distribution is reduced when gravity is increased, i.e., at g = 8, and the least normal stress is observed at g = 4.

From figure 2.5, we observe that the Normal stress σ_{zz} shows the same variation as that of tangential stress σ_{zx}. Figure 2.6, demonstrates the impact of gravity on temperature distribution φ and describes the variation of conductive temperature w.r.t z. We notice that the temperature distribution is highest at g = 4, and it reduces when the value of gravity is increased, i.e., at g = 6, and we observe that it is least when the value of g is further increased, i.e., at g = 8. In other words, temperature distribution φ decreases when gravity is increased.

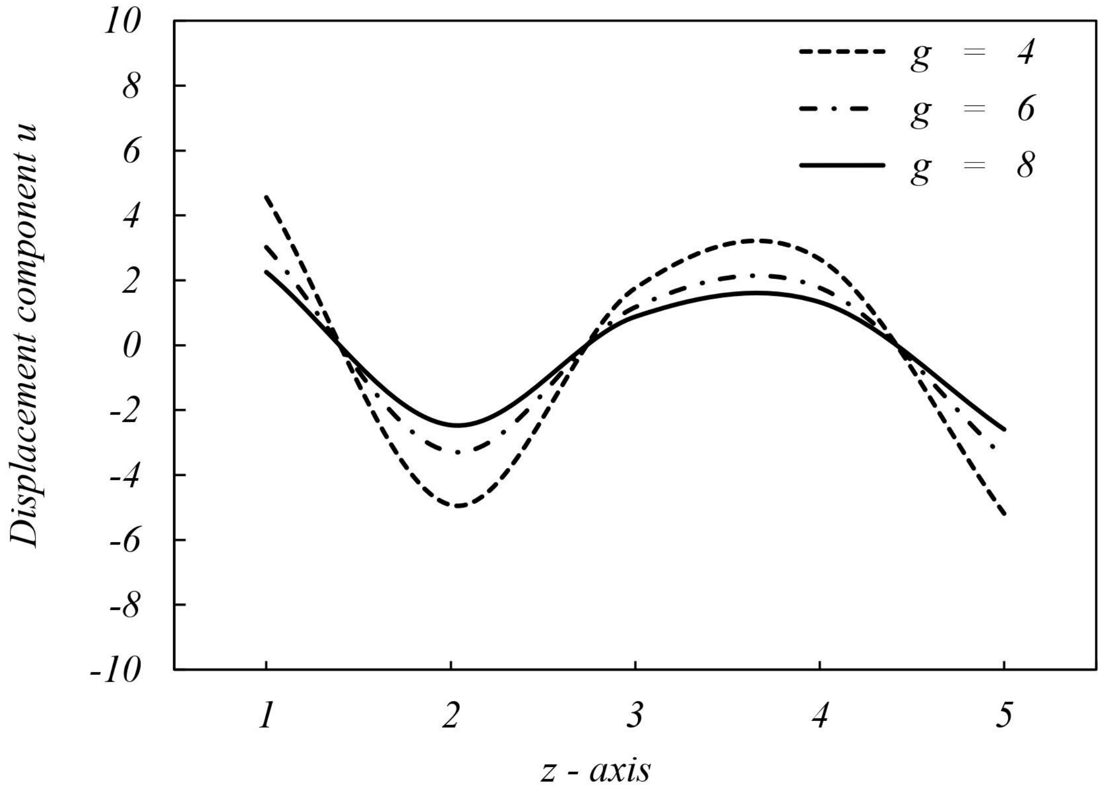

FIGURE 2.2: Variation of u at distinct values of gravity g

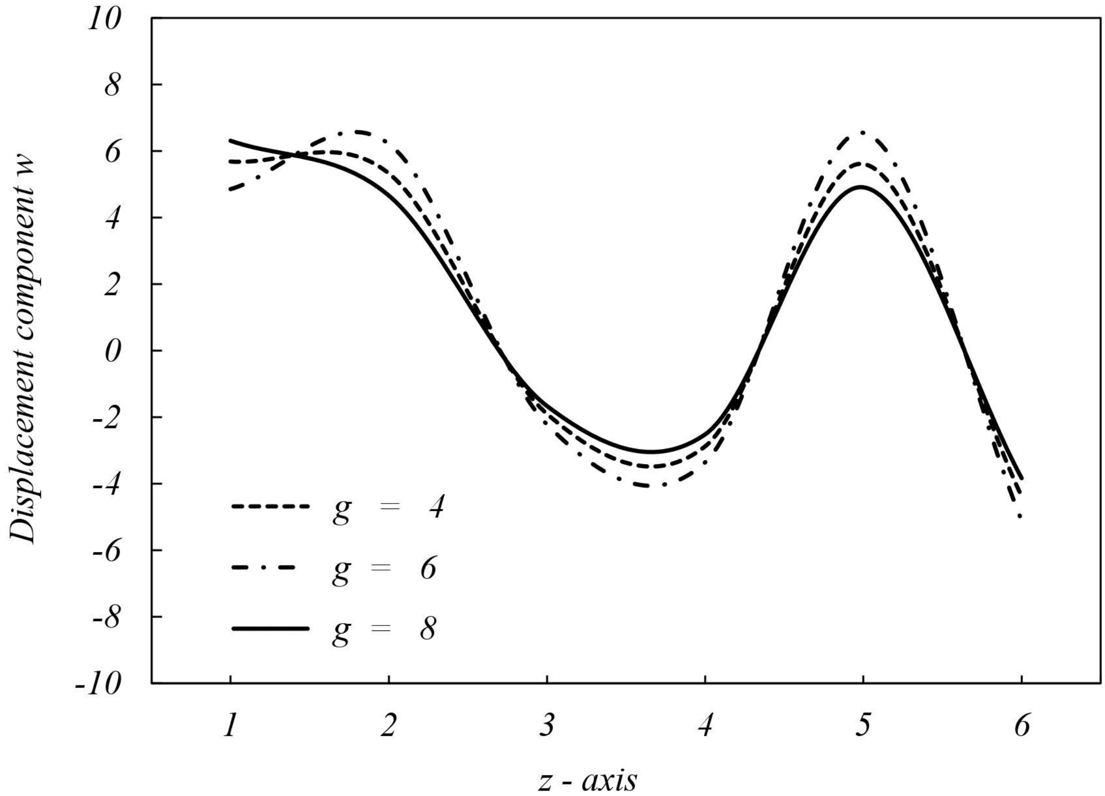

FIGURE 2.3: Variation of w at distinct values of gravity g

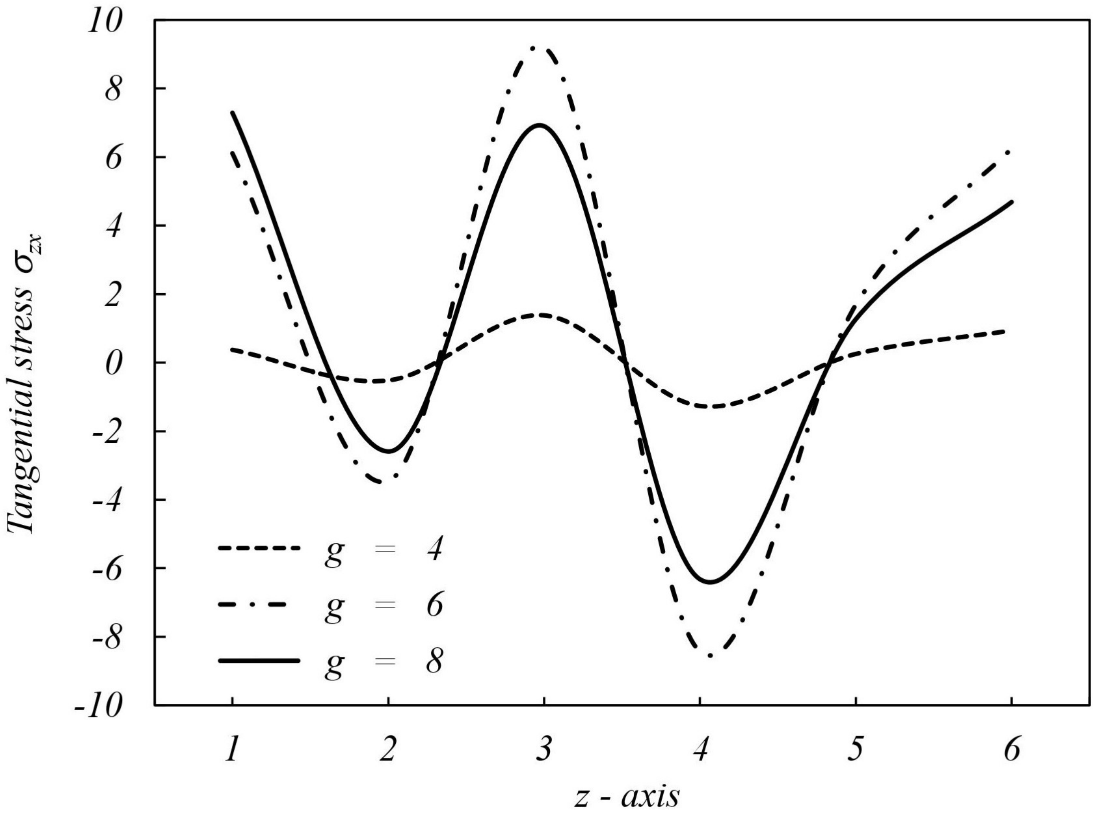

FIGURE 2.4: Variation of tangential stress σ_{zx} at distinct values of gravity g

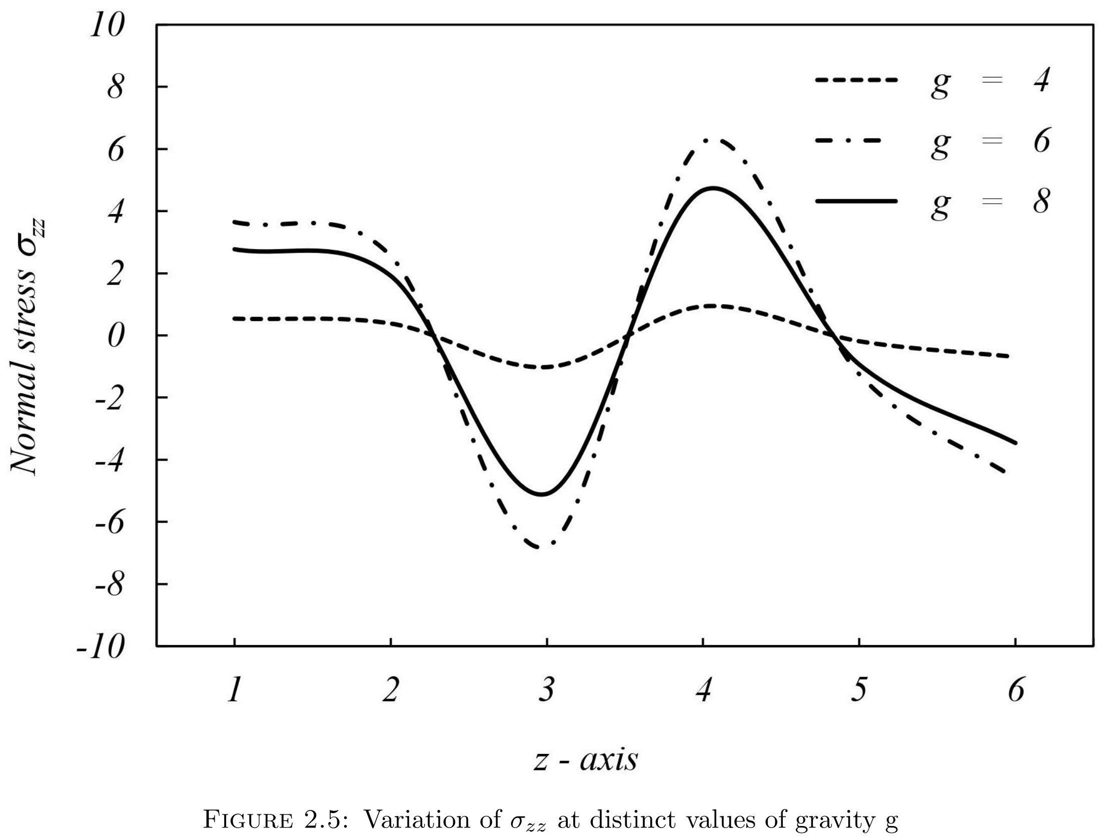

FIGURE 2.5: Variation of σ_{zz} at distinct values of gravity g

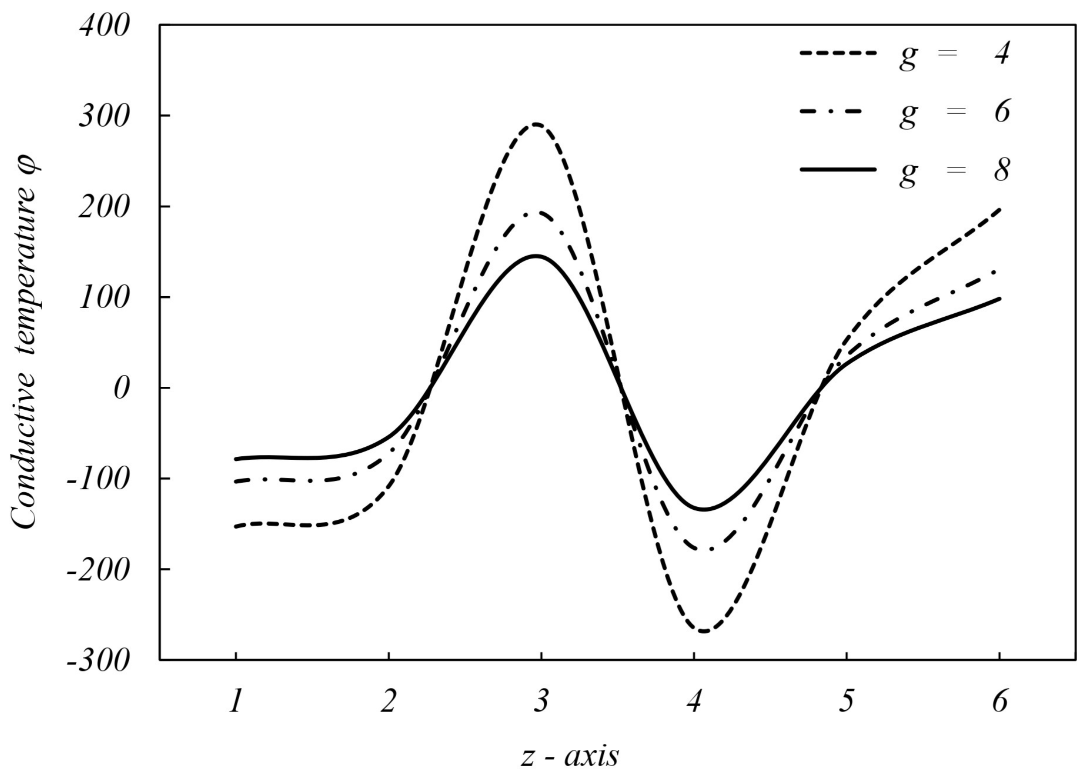

FIGURE 2.6: Variation of φ at distinct values of gravity g

2.8 Conclusion

In this study, the response of micropolar elastic material has been investigated by including magnetic and thermal effects. The solution of the required problem has been derived by using the Normal mode technique. Matlab software along with MS-Excel were used for numerical calculations and for graphical representations of physical quantities such as force stresses, temperature distributions, and displacement components. The major highlights of the study may be pointed out as follows:

1. The response of different physical quantities, namely, force stresses, couple stresses, temperature distributions, and displacement components, has been investigated for distinct values of "g", obtained outside the earth's mesosphere.

2. It has been observed that both the displacement components "u" and tangential stress "σ_{zx}" exhibit oscillatory behaviour. Moreover, it has been seen that the displacement component "u" has a negative correlation with the values of "g".

Chapter 3

Study of memory response in the presence and absence of micropolar effect under the framework of Green-Lindsay model

3.1 Introduction

In this chapter, a 2D model has been investigated in the presence and absence of micropolar effect in order to investigate the processes which possess memory with the help of a memory-dependent derivative (MDD) for a rotating medium. In generalised thermoelasticity, innumerable interesting results have been examined so far in the light of the memory-dependent derivatives ([73], [74], [75], [76]). The phase-lag models were utilized by Othman and Mondal [100] in order to study the impact of MDD and laser pulse on wave propagation of micropolar thermoelastic medium. In this investigation, different thermoelasticity theories, viz., LS, DPL, and TPL, have also been compared by the authors. In addition to this, they have also recognized an observable effect on different physical quantities with and without MDD. In generalized thermoelasticity, Biswas [101] studied a 2D problem under the framework of MDD. The integral transforms have been utilized for obtaining the desired solution of the problem, along with the eigenvalue approach. In magneto-micropolar thermoelastic media, the impact of MDD was studied by Said [102] by using the dual-phase lag (DPL) model. In a generalized thermoelastic medium, which is assumed to be orthotropic, the impact of a laser pulse and magnetic field for distinct values of delay time and for kernel functions was studied by Singh and Pal [103] in the context of Green-Naghdi theory with MDD. Taking MDD into consideration, the transient response of a half space was studied by Li and He [104] in generalized thermoelasticity. They have also investigated and discussed the behaviour of the obtained outcomes graphically for distinct values of delay time, kernel function, and gradient parameter. In this study, the authors have discussed and analyzed the obtained outcomes graphically for distinct values of delay time and for a fixed kernel function. Horgan and Murphy [105] investigated the classic deformation within the framework of nonlinear elasticity for isotropic, incompressible hyperelastic materials.

In the current study, a novel problem has been undertaken in which a two dimensional model has been developed to investigate the elastic response of an elastic media in the presence and absence of micropolar effect under the context of heat conduction equation possessing memory-dependent derivative. Moreover, the Helmholtz potential's along with the normal mode analysis (NMA) were utilized for finding the analytical solution of the required problem. Finally, Matlab software has been utilized in order to graphically demonstrate the components of displacement, force stresses, couple stresses, as well as the temperature distribution.

3.2 Basic equations

For a homogenous (which is perfectly conducting) elastic solid, the linearized equations of electrodynamic medium, which is moving slowly defined in [106], are taken as:

$$\nabla \times \overrightarrow{h} = \overrightarrow{J} + \varepsilon_0 \frac{\partial \overrightarrow{E}}{\partial t}, \tag{3.1}$$

$$\nabla \times \overrightarrow{E} = -\mu_0 \frac{\partial \overrightarrow{h}}{\partial t}, \tag{3.2}$$

$$\overrightarrow{E} = -\mu_0 \left(\frac{\partial \overrightarrow{u}}{\partial t} \times \overrightarrow{H_0} \right), \tag{3.3}$$

$$\nabla . \overrightarrow{h} = 0. \tag{3.4}$$

The field equations of motion and constitutive relations are added in equations (3.1)-(3.4), in micropolar theory of generalized thermoelasticity, by taking Lorentz force under consideration,

$$(\lambda + 2\mu + \kappa)\nabla(\nabla.\overrightarrow{u}) - (\mu + \kappa)\nabla \times (\nabla \times \overrightarrow{u}) + \kappa(\nabla \times \overrightarrow{\phi}) + \overrightarrow{F} - v\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\nabla T$$
$$= \rho \left[\frac{\partial^2 \overrightarrow{u}}{\partial t^2} + \overrightarrow{\Omega} \times (\overrightarrow{\Omega} \times \overrightarrow{u}) + 2\left(\overrightarrow{\Omega} \times \frac{\partial \overrightarrow{u}}{\partial t}\right)\right], \tag{3.5}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla.\overrightarrow{\phi}) - \gamma\nabla \times (\nabla \times \overrightarrow{\phi}) + \kappa(\nabla \times \overrightarrow{u}) - 2\kappa\overrightarrow{\phi} = \rho j \left(\frac{\partial^2 \overrightarrow{\phi}}{\partial t^2} + \overrightarrow{\Omega} \times \frac{\partial \overrightarrow{\phi}}{\partial t} \right). \tag{3.6}$$

The constitutive relations are

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa(u_{j,i} - \varepsilon_{ijr}\phi_r) - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{3.7}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}. \tag{3.8}$$

The heat conduction equation under Green-Lindsay model with MDD as defined in [99]

$$K\nabla^2 T = \rho C_e(1 + \tau D_\tau)\dot{T} + \gamma' T_0\dot{e} \tag{3.9}$$

The Lorentz force $\overrightarrow{F}$ is defined as

$$\overrightarrow{F} = \mu_0(\overrightarrow{J} \times \overrightarrow{H_0}), \tag{3.10}$$

The equations of motion (3.5)-(3.6) along with heat equation (3.9) in Cartesian coordinates (x, y, z) in component form can be written as

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right) + (\mu + \kappa)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \kappa\left(\frac{\partial\phi_3}{\partial y} - \frac{\partial\phi_2}{\partial z}\right) +$$
$$(J_2H_3 - J_3H_2) - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho\left(\frac{\partial^2 u}{\partial t^2} + (\Omega_2\Omega_1 v - \Omega_2^2 u - \Omega_3^2 u + \Omega_3\Omega_1 w) + 2(\Omega_2\dot{w} - \Omega_3\dot{v})\right), \tag{3.11}$$

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial y\partial x} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y\partial z}\right) + (\mu + \kappa)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \kappa\left(\frac{\partial\phi_1}{\partial z} - \frac{\partial\phi_3}{\partial x}\right) +$$
$$(J_3H_1 - J_1H_3) - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y} = \rho\left(\frac{\partial^2 v}{\partial t^2} + (\Omega_1\Omega_2 u + \Omega_3\Omega_2 w - \Omega_1^2 v - \Omega_3^2 v) + 2(\Omega_3\dot{u} - \Omega_1\dot{w})\right), \tag{3.12}$$

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial z\partial x} + \frac{\partial^2 v}{\partial z\partial y} + \frac{\partial^2 w}{\partial z^2}\right) + (\mu + \kappa)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \kappa\left(\frac{\partial\phi_2}{\partial x} - \frac{\partial\phi_1}{\partial y}\right) +$$
$$(J_1H_2 - J_2H_1) - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \rho\left(\frac{\partial^2 w}{\partial t^2} + (\Omega_1\Omega_3 u - \Omega_1^2 w - \Omega_2^2 w + \Omega_2\Omega_3 v) + 2(\Omega_1\dot{v} - \Omega_2\dot{u})\right), \tag{3.13}$$

$$(\alpha + \beta)\left(\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_2}{\partial x\partial y} + \frac{\partial^2\phi_3}{\partial x\partial z}\right) + \gamma\left(\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} + \frac{\partial^2\phi_1}{\partial z^2}\right) + \kappa\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - 2\kappa\phi_1$$
$$= \rho j\left(\frac{\partial^2\phi_1}{\partial t^2} + (\Omega_2\dot{\phi}_3 - \Omega_3\dot{\phi}_2)\right), \tag{3.14}$$

$$(\alpha + \beta)\left(\frac{\partial^2\phi_1}{\partial y\partial x} + \frac{\partial^2\phi_2}{\partial y^2} + \frac{\partial^2\phi_3}{\partial y\partial z}\right) + \gamma\left(\frac{\partial^2\phi_2}{\partial x^2} + \frac{\partial^2\phi_2}{\partial y^2} + \frac{\partial^2\phi_2}{\partial z^2}\right) + \kappa\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - 2\kappa\phi_2$$
$$= \rho j\left(\frac{\partial^2\phi_2}{\partial t^2} + (\Omega_3\dot{\phi}_1 - \Omega_1\dot{\phi}_3)\right), \tag{3.15}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial z\partial x}+\frac{\partial^2\phi_2}{\partial z\partial y}+\frac{\partial^2\phi_3}{\partial z^2}\right)+\gamma\left(\frac{\partial^2\phi_3}{\partial x^2}+\frac{\partial^2\phi_3}{\partial y^2}+\frac{\partial^2\phi_3}{\partial z^2}\right)+\kappa\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)-2\kappa\phi_3$$

$$=\rho j\left(\frac{\partial^2\phi_3}{\partial t^2}+(\Omega_1\dot{\phi_2}-\Omega_2\dot{\phi_1})\right), \tag{3.16}$$

$$K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho C_e(1+\tau D_\tau)\dot{T}+\gamma' T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right). \tag{3.17}$$

where, (u,v,w), (ϕ_1,ϕ_2,ϕ_3), (J_1,J_2,J_3), (H_1,H_2,H_3) and $(\Omega_1,\Omega_2,\Omega_3)$ are the components of displacement vector $\overrightarrow{u}$, microrotation vector $\overrightarrow{\phi}$, current density vector $\overrightarrow{J}$, magnetic field vector $\overrightarrow{H}$ and rotation vector, respectively.

3.3 Formulation and solution of the problem

A micropolar generalized thermoelastic medium, which is homogenous, isotropic and perfectly conducting, is taken into account and in which the initial magnetic field $\overrightarrow{H_0}$ is permeated along the y-axis. The origin of a rectangular cartesian co-ordinate system (x, y, z) is taken at any point on the plane surface of half-space z = 0, as shown in figure 3.1. For the 2D problem, we have taken displacement, rotation, and microrotation vectors as

$$\overrightarrow{u}=(u,0,w),\quad \overrightarrow{\Omega}=(0,\Omega,0),\quad \overrightarrow{\phi}=(0,\phi_2,0),\quad u(x,z,t),\quad and\quad w(x,z,t). \tag{3.18}$$

Using equation (3.18) in equations (3.1) - (3.3), we get

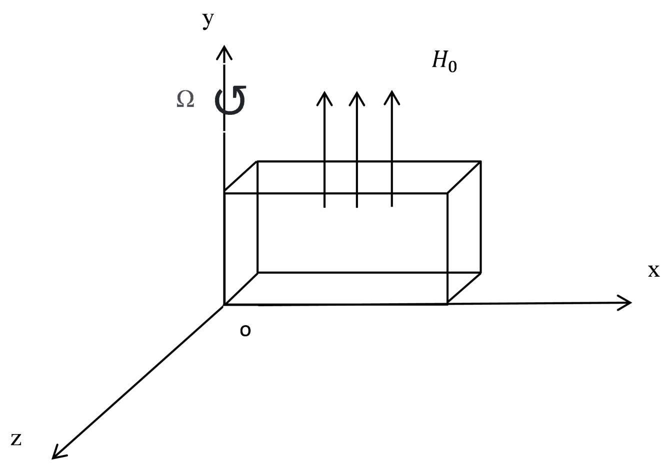

FIGURE 3.1: Rotating material geometry

$$\overrightarrow{E}=\mu_0H_0(\dot{w},0,-\dot{u}), \tag{3.19}$$

$$\overrightarrow{h}=-H_0(0,e,0), \tag{3.20}$$

$$\overrightarrow{J}=((H_0e_{,z}-\varepsilon_0\mu_0H_0\ddot{w}),0,(-H_0e_{,x}+\varepsilon_0\mu_0H_0\ddot{u})), \tag{3.21}$$

where e is cubical dilatation, defined as

$$e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}.$$

Thus from equation (3.10), we obtain

$$\overrightarrow{F} = (\mu_0 H_0^2(e_{,x} - \varepsilon_0\mu_0\ddot{u}), 0, \mu_0 H_0^2(e_{,z} - \varepsilon_0\mu_0\ddot{w})). \tag{3.22}$$

From equations (3.19)–(3.22), it is clear that the components of the induced electric field, induced magnetic field, and the Lorentz force are functions of the displacement components and the externally applied constant magnetic field.
After using equation (3.18) in equations (3.11)-(3.17) and in equations (3.7)-(3.8), we obtain the following components

$$(\lambda + 2\mu + \kappa)\frac{\partial^2 u}{\partial x^2} + (\mu + \kappa)\frac{\partial^2 u}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 w}{\partial x \partial z} - \kappa\frac{\partial \phi_2}{\partial z} + \mu_0 H_o^2(e_x - \varepsilon_0\mu_0\ddot{u}) - \nu$$
$$\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho(\ddot{u} - \Omega^2 u + 2\Omega\dot{w}), \tag{3.23}$$

$$(\lambda + \mu)\frac{\partial^2 u}{\partial x \partial z} + (\lambda + 2\mu + \kappa)\frac{\partial^2 w}{\partial z^2} + (\mu + \kappa)\frac{\partial^2 w}{\partial x^2} + \kappa\frac{\partial \phi_2}{\partial x} + \mu_0 H_0^2(e_z - \varepsilon_0\mu_0\ddot{w}) - \nu$$
$$\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \rho(\ddot{w} - \Omega^2 w - 2\Omega\dot{u}), \tag{3.24}$$

$$\gamma\nabla^2\phi_2 + \kappa\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - 2\kappa\phi_2 = \rho j\ddot{\phi}_2. \tag{3.25}$$

$$K\nabla^2 T = \rho C_e(1 + \tau D_\tau)\frac{\partial T}{\partial t} + \gamma' T_0\frac{\partial e}{\partial t}. \tag{3.26}$$

$$\sigma_{xx} = (\lambda + 2\mu + \kappa)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{3.27}$$

$$\sigma_{yy} = \lambda e - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{3.28}$$

$$\sigma_{zz} = \lambda\frac{\partial u}{\partial x} + (\lambda + 2\mu + \kappa)\frac{\partial w}{\partial z} - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{3.29}$$

$$\sigma_{xz} = \mu\frac{\partial u}{\partial z} + (\mu + \kappa)\frac{\partial w}{\partial x} + \kappa\phi_2, \tag{3.30}$$

$$\sigma_{zx} = \mu\frac{\partial w}{\partial x} + (\mu + \kappa)\frac{\partial u}{\partial z} - \kappa\phi_2, \tag{3.31}$$

$$m_{xy} = \gamma\frac{\partial \phi_2}{\partial x}, \tag{3.32}$$

$$m_{zy} = \gamma\frac{\partial \phi_2}{\partial z}. \tag{3.33}$$

where, $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ and $e = \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)$.

Introducing non-dimensional quantities as mentioned below in equations (3.23)-(3.33),

$$(x',z')=\frac{\bar{\omega}}{c_0}(x,z),\ (u',w')=\frac{\rho c_0\bar{\omega}}{\nu T_0}(u,w),\ \sigma'_{i,j}=\frac{\sigma_{ij}}{\nu T_0},\ (\tau_1^{'},\tau',t^{'})=\bar{\omega}(\tau_1,\tau,t),\ T'=\frac{T}{T_0},$$
$$\phi_2{}'=\frac{\rho c_0^2}{\nu T_0}\phi_2,\ \Omega^{'}=\frac{\Omega}{\bar{\omega}},\ m_{ij}^{'}=\frac{\bar{\omega}}{c_0\nu T_0}m_{ij}, \tag{3.34}$$

we get

$$\nabla^2 u+a_1\frac{\partial e}{\partial x}-a_2\frac{\partial \phi_2}{\partial z}-a_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}=a_4\frac{\partial^2 u}{\partial t^2}-a_5 u+a_6\frac{\partial w}{\partial t}, \tag{3.35}$$
$$\nabla^2 w+a_1\frac{\partial e}{\partial z}+a_2\frac{\partial \phi_2}{\partial x}-a_3\frac{\partial T}{\partial z}\left(1+\tau_1\frac{\partial}{\partial t}\right)=a_4\frac{\partial^2 w}{\partial t^2}-a_5 w-a_6\frac{\partial u}{\partial t}, \tag{3.36}$$
$$\nabla^2\phi_2+a_7\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2a_7\phi_2=a_8\frac{\partial^2\phi_2}{\partial t^2}, \tag{3.37}$$
$$\sigma_{xx}=\frac{\partial u}{\partial x}+a_9\frac{\partial w}{\partial z}-\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{3.38}$$
$$\sigma_{yy}=a_9 e-\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{3.39}$$
$$\sigma_{zz}=a_9\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}-\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{3.40}$$
$$\sigma_{xz}=a_{10}\frac{\partial u}{\partial z}+a_{11}\frac{\partial w}{\partial x}+a_{12}\phi_2, \tag{3.41}$$
$$\sigma_{zx}=a_{11}\frac{\partial u}{\partial z}+a_{10}\frac{\partial w}{\partial z}+a_{12}\phi_2, \tag{3.42}$$
$$m_{xy}=a_{13}\frac{\partial\phi_2}{\partial x}, \tag{3.43}$$
$$m_{zy}=a_{14}\frac{\partial\phi_2}{\partial z}, \tag{3.44}$$
$$a_{14}\nabla^2 T=a_{15}(\bar{\omega}+\tau D_\tau)\frac{\partial T}{\partial t}+a_{16}\frac{\partial e}{\partial t}. \tag{3.45}$$

where, $\bar{\omega}=\frac{\rho C_e {c_0}^2}{K}$, $c_0^2=\frac{\lambda+2\mu+\kappa}{\rho}$, $a_1=\frac{\lambda+\mu+\mu_0 H_0^2}{\mu+\kappa}$, $a_2=\frac{\kappa}{\mu+\kappa}$, $a_3=\frac{\lambda+2\mu+\kappa}{\mu+\kappa}$, $a_4=a_3\left(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)$, $a_5=a_3\Omega^2$, $a_6=2a_3\Omega$, $a_7=\frac{\kappa c_0^2}{\gamma\bar{\omega}^2}$, $a_8=\frac{\rho j c_0^2}{\gamma}$, $a_9=\frac{\lambda}{\lambda+2\mu+\kappa}$, $a_{10}=\frac{\mu}{\rho c_0^2}$, $a_{11}=\frac{\mu+\kappa}{\rho c_0^2}$, $a_{12}=\frac{\kappa}{\rho c_0^2}$, $a_{13}=\frac{\gamma^2\bar{\omega}^2}{\nu\rho c_0^4}$, $a_{14}=\frac{K\bar{\omega}}{c_0^2}$, $a_{15}=\frac{\rho c_e}{\bar{\omega}}$ and $a_{16}=\frac{\gamma^{'} T_0\nu}{\rho c_0^2}$.

Now, for obtaining the desired solution of the required problem, the potential displacements $q(x,z,t)$ and $\psi(x,z,t)$, which are defined below, are introduced as

$$u=\frac{\partial q}{\partial x}+\frac{\partial\psi}{\partial z},\ w=\frac{\partial q}{\partial z}-\frac{\partial\psi}{\partial x}. \tag{3.46}$$

which lead equations (3.35), (3.37) and (3.45) to the following form

$$\left((a_1+1)\nabla^2 - a_4\frac{\partial^2}{\partial t^2} + a_5\right) q - a_3\left(1+\tau_1\frac{\partial}{\partial t}\right) T + a_6\frac{\partial \psi}{\partial t} = 0, \tag{3.47}$$

$$\left(\nabla^2 - a_4\frac{\partial^2}{\partial t^2} + a_5\right)\psi - a_2\phi_2 - a_6\frac{\partial q}{\partial t} = 0, \tag{3.48}$$

$$\left(\nabla^2 - 2a_7 - a_8\frac{\partial^2}{\partial t^2}\right)\phi_2 + a_7\nabla^2\psi = 0, \tag{3.49}$$

$$a_{14}\nabla^2 T = a_{15}\left(\bar{\omega} + \tau D_\tau\right)\frac{\partial T}{\partial t} + a_{16}\frac{\partial}{\partial t}\nabla^2 q. \tag{3.50}$$

3.4 Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal mode as follows

$$[u, w, T, \sigma_{ij}, q, \psi, \phi_2, m_{ij}](x, z, t) = [\bar{u}, \bar{w}, \bar{T}, \bar{\sigma}_{ij}, \bar{q}, \bar{\psi}, \bar{\phi}_2, \bar{m}_{ij}](z)exp(bt + iax). \tag{3.51}$$

Use of equation (3.51) transforms equations (3.47)-(3.50) to

$$[D^2 - A_1]\bar{q} - A_2\bar{T} + A_3\bar{\psi} = 0, \tag{3.52}$$

$$[D^2 - A_4]\bar{\psi} - A_5\bar{\phi}_2 - A_6\bar{q} = 0, \tag{3.53}$$

$$[D^2 - A_7]\bar{\phi}_2 + [A_8 D^2 - A_9]\bar{\psi} = 0, \tag{3.54}$$

$$[D^2 - A_{10}]\bar{T} - [A_{11}D^2 - A_{12}]\bar{q} = 0. \tag{3.55}$$

where, $D = \frac{\partial}{\partial z}$, $A_1 = \frac{(a_1+1)a^2 + a_4 b^2 - a_5}{a_1+1}$, $A_2 = \frac{a_3(1+\tau_1 b)}{a_1+1}$, $A_3 = \frac{a_6 b}{a_1+1}$, $A_4 = a^2 + a_4 b^2 - a_5$, $A_5 = a_2$, $A_6 = a_6 b$, $A_7 = a^2 + 2a_7 + a_8 b^2$, $A_8 = a_7$, $A_9 = a_7 a^2$, $A_{10} = \frac{a_{14}a^2 + a_{15}b(\bar{\omega} + \tau G(\tau, b))}{a_{14}}$, $A_{11} = \frac{a_{16}b}{a_{14}}$, $A_{12} = \frac{a_{16}a^2 b}{a_{14}}$,

$G(\tau, b) = \frac{-(b^2(m^2-2n+1)\tau^2 + 2b\tau(m^2-n) + 2m^2)exp[b(t-\tau)] + (b^2\tau^2 - 2bn\tau + 2m^2)exp(bt)}{b^2\tau^2}$ and the kernel function $K(t-r)$ is defined as [61]

$$K(t-r) = 1 - \frac{2n}{\tau}(t-r) + \frac{m^2}{\tau^2}(t-r^2) = \begin{cases} 1; if, m = n = 0 \\ 1 - \frac{(t-r)}{\tau}; if, m = 0, n = \frac{1}{2} \\ 1 - (t-r); if, m = 0, n = \frac{\tau}{2} \\ \left(1 - \frac{t-r}{\tau}\right)^2; if, m = n = 1. \end{cases}$$

After some simplification, equations (3.52)-(3.55) become

$$[D^4 - B_1 D^2 + B_2]\bar{\psi} - [B_3 D^2 - B_4]\bar{q} = 0, \tag{3.56}$$

$$[D^4 - B_5 D^2 + B_6]\bar{q} + [B_7 D^2 - B_8]\bar{\psi} = 0. \tag{3.57}$$

where, $B_1 = A_7 + A_4 - A_8 A_5$, $B_2 = A_4 A_7 - A_9 A_5$, $B_3 = A_6$, $B_4 = A_6 A_7$, $B_5 = A_{10} + A_1 + A_{11} A_2$, $B_6 = A_1 A_{10} + A_2 A_{12}$, $B_7 = A_3$, $B_8 = A_3 A_{10}$.

Simplifying equations (3.56) and (3.57), we obtain,

$$[D^8 - AD^6 + BD^4 - CD^2 + F]\bar{\psi}(z) = 0, \tag{3.58}$$

Similarly,

$$[D^8 - AD^6 + BD^4 - CD^2 + F][\bar{q}(z), \bar{\phi}_2(z), \bar{T}(z)] = 0, \tag{3.59}$$

where, $A = B_5 + B_1$, $B = B_6 + B_1 B_5 + B_2 + B_3 B_7$, $C = B_1 B_6 + B_2 B_5 + B_3 B_8 + B_4 B_7$, $F = B_2 B_6 + B_4 B_8$.

In factored form equations (3.58) and (3.59) can be written as

$$[(D^2 - {k_1}^2)(D^2 - {k_2}^2)(D^2 - {k_3}^2)(D^2 - {k_4}^2)][\bar{\psi}(z), \bar{q}(z), \bar{\phi}_2(z), \bar{T}(z)] = 0, \tag{3.60}$$

where, $k_n (n = 1, 2, 3, 4)$ represents the characteristic roots of the equation (3.60). The general solution of equation (3.60), has the form

$$\bar{\psi}(z) = \sum_{n=1}^{4} M_n e^{-k_n z}, \tag{3.61}$$

$$\bar{\phi}_2(z) = \sum_{n=1}^{4} {M_n}' e^{-k_n z}, \tag{3.62}$$

$$\bar{q}(z) = \sum_{n=1}^{4} {M_n}'' e^{-k_n z}, \tag{3.63}$$

$$\bar{T}(z) = \sum_{n=1}^{4} {M_n}''' e^{-k_n z}, \tag{3.64}$$

Here, M_n, ${M_n}'$, ${M_n}''$, ${M_n}'''$ represents parameters that depends on a and b.

Using equations (3.61)-(3.64) in equations (3.52)-(3.55), we get

$$(\bar{\phi}_2(z), \bar{q}(z), \bar{T}(z)) = \sum_{n=1}^{4} M_n (H_{1n}, H_{2n}, H_{3n}) e^{-k_n z}, \tag{3.65}$$

where, $H_{1n} = \frac{-(A_8 k_n^2 - A_9)}{k_n^2 - A_7}$, $H_{2n} = -\frac{(k_n^2 - A_4)(k_n^2 - A_7) - A_5(A_8 k_n^2 - A_9)}{(A_8 k_n^2 - A_9)A_6} H_{1n}$ and $H_{3n} = \frac{A_{11}k_n^2 - A_{12}}{k_n^2 - A_{10}} H_{2n}$.

In general, equations (3.61) and (3.65), can be re-written as

$$(\bar{\psi}, \bar{\phi}_2, \bar{q}, \bar{T})(z) = \sum_{n=1}^{4} (1, H_{1n}, H_{2n}, H_{3n}) M_n exp(-k_n z). \tag{3.66}$$

Using equation (3.51) in equation (3.46) and after some simplification, the displacement components are obtained as

$$(\bar{u}, \bar{w})(z) = \sum_{n=1}^{4} M_n (H_{4n}, H_{5n}) e^{-k_n z}, \tag{3.67}$$

where, $H_{4n} = H_{2n} ia - k_n$ and $H_{5n} = -H_{2n} k_n - ia$.

Using equations (3.46) and (3.51) in equation (3.38) and in equations (3.40)-(3.44), we obtain the force and couple stress components

$$(\bar{\sigma}_{xx}, \bar{\sigma}_{xz}, \bar{\sigma}_{zz}, \bar{\sigma}_{zx}, \bar{m}_{xy}, \bar{m}_{zy})(z) = \sum_{n=1}^{4} M_n (H_{6n}, H_{7n}, H_{8n}, H_{9n}, H_{10n}, H_{11n}) e^{-k_n z}. \tag{3.68}$$

where, $H_{6n} = -H_{2n}a^2 - iak_n + a_9(H_{2n}k_n^2 + iak_n) - (1+\tau_1 b)H_{3n}$, $H_{7n} = [a_{10}(-iaH_{2n}k_n + k_n^2) + a_{11}(-H_{2n}iak_n + a^2)]$, $H_{8n} = a_9(-H_{2n}a^2 - iak_n) + (H_{2n}k_n^2 + iak_n) - (1+\tau_1 b)H_{3n}$, $H_{9n} = a_{11}(-k_n iaH_{2n} + k_n^2) + a_{10}(H_{2n}k_n^2 + iak_n) + a_{12}H_{1n}$, $H_{10n} = a_{13}H_{1n}ia$ and $H_{11n} = -H_{1n}k_n$.

3.5 Boundary conditions

To find out the M_n (where $n = 1, 2, 3, 4$) parameters, the boundary conditions has been imposed at $z = 0$, and are as

$$\sigma_{zz} = \sigma_{zx} = m_{xy} = 0, \tag{3.69}$$

(i.e., the bounding plane z = 0 is traction-free)

and the thermal boundary condition is

$$T = f(x, t), \tag{3.70}$$

where $f(x,t) = \theta_0 e^{bt+iax}$ and θ_0 represents the amplitude of the function $f(x,t)$.

Using equation (3.51) in equations (3.69) and (3.70), and after some simplification, we obtain

$$\sum_{n=1}^{4} H_{3n} M_n = \theta_0, \tag{3.71}$$

$$\sum_{n=1}^{4} H_{8n} M_n = 0, \tag{3.72}$$

$$\sum_{n=1}^{4} H_{9n} M_n = 0, \tag{3.73}$$

$$\sum_{n=1}^{4} H_{10} M_n = 0. \tag{3.74}$$

Now, solving equation (3.71)-(3.74) for M_n where $n = 1, 2, 3, 4$ by making use of Inverse matrix method which is given below:

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} H_{31} & H_{32} & H_{33} & H_{34} \\ H_{81} & H_{82} & H_{83} & H_{84} \\ H_{91} & H_{92} & H_{93} & H_{94} \\ H_{101} & H_{102} & H_{103} & H_{104} \end{bmatrix}^{-1} \begin{bmatrix} \theta_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

3.6 Particular case

The field equations can be obtained from the above mentioned cases for the generalized micropolar thermoelasticity medium without micropolar constants, by taking:

$$\kappa = \alpha = \beta = \gamma = j = 0, \tag{3.75}$$

After plugging equation (3.75) into equations (3.5)-(3.9) and using equations (3.34), (3.46) and (3.51), we get

$$[D^2 - A^{'}]\bar{q} - A^{''}\bar{T} + A^{'''}\bar{\psi} = 0, \tag{3.76}$$

$$[D^2 - B^{'}]\bar{\psi} - B^{''}\bar{q} = 0, \tag{3.77}$$

$$[D^2 - A_{10}]\bar{T} - [A_{11}D^2 - A_{12}]\bar{q} = 0, \tag{3.78}$$

where, $A^{'} = \frac{(1+c_1)a^2+c_3b^2-c_4}{1+c_1}$, $A^{''} = \frac{c_2(1+\tau_1 b)}{1+c_1}$, $A^{'''} = \frac{c_5 b}{1+c_1}$, $B^{'} = a^2 + c_3b^2 - c_4$, $B^{''} = c_5 b$, $c_1 = \frac{\lambda+\mu+\mu_0 H_0^2}{\mu}$, $c_2 = \frac{\lambda+2\mu}{\mu}$, $c_3 = c_2\left(\frac{\rho+\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)$, $c_4 = c_2\Omega^2$ and $c_5 = 2c_2\Omega$.

Eliminating $\bar{T}$ and $\bar{\psi}$ from above equations (3.76)-(3.78), we get

$$[D^6 - \alpha_1 D^4 + \alpha_2 D^2 - \alpha_3]\bar{q}(z) = 0, \tag{3.79}$$

Similarly, for different physical quantities, we have

$$[D^6 - \alpha_1 D^4 + \alpha_2 D^2 - \alpha_3](\bar{\psi}(z), \bar{T}(z)) = 0, \tag{3.80}$$

$\alpha_1 = B' + A' + A_{10} + A_{11}A''$, $\alpha_2 = B'(A_{10} + A' + A_{11}A'') + A'A_{10} + A''A_{12} + B''A'''$ and $\alpha_3 = B'(A'A_{10} + A''A_{12}) - B''A'''A_{12}$.

Thus equations (3.79) and (3.80) in factored form can be written as

$$[(D^2 - l_1^2)(D^2 - l_2^2)(D^2 - l_3^2)][\bar{q}(z), \bar{\psi}(z), \bar{T}(z)] = 0. \tag{3.81}$$

where, $l_\imath(\imath = 1, 2, 3)$ represents the characteristic roots of the equation (3.81).

The solution of above equation (3.81) has the form

$$(\bar{\psi}(z), \bar{q}(z), \bar{T}(z)) = \sum_{\imath=1}^{3}(1, L_{1\imath}, L_{2\imath})R_\imath exp(-l_n z). \tag{3.82}$$

where, $L_{1\imath} = \frac{l_\imath^2 - B'}{B''}$, $L_{2\imath} = \left(\frac{A_{11}l_\imath^2 - A_{12}}{l_\imath^2 - A_{10}}\right)L_{1\imath}$ and $R_\imath, \imath = 1, 2, 3$ are some parameters depending on a and b.

Now, using equations (3.75) and (3.51) in equation (3.46), and equations (3.75), (3.34), (3.46) and (3.51) in equations (3.27)-(3.33), and after some simplification, we get

$$(\bar{u}, \bar{w}, \bar{\sigma}_{xx}, \bar{\sigma}_{zz}, \bar{\sigma}_{zx}) = \sum_{\imath=1}^{3} R_\imath(L_{3\imath}, L_{4\imath}, L_{5\imath}, L_{6\imath}, L_{7\imath})e^{-l_\imath z}, \tag{3.83}$$

where, $L_{3\imath} = L_{1\imath}ia - l_\imath$, $L_{4\imath} = -L_{1\imath}l_\imath - ia$, $L_{5\imath} = -L_{1\imath}a^2 + c_6(L_{1\imath}l_\imath^2 + l_\imath ia) - ial_\imath - L_{2\imath}(1 + \tau_1 b)$, $L_{6\imath} = -c_6(L_{1\imath}a^2 + ial_\imath) + L_{1\imath}l_\imath^2 + l_\imath ia - L_{2\imath}(1 + \tau_1 b)$, $L_{7\imath} = c_7(-iaL_{1\imath}l_\imath + l_\imath^2(1 + L_{1\imath}) + l_\imath ia)$, $c_6 = \frac{\lambda}{\lambda + 2\mu}$ and $c_7 = \frac{\mu}{\rho c_0^2}$.

Now using equations (3.75) and (3.51) in equations (3.69) and (3.70) and after some simplification, we obtain

$$\sum_{\imath=1}^{3} L_{2\imath} R_\imath = \theta_0, \tag{3.84}$$

$$\sum_{\imath=1}^{3} L_{6\imath} R_\imath = 0, \tag{3.85}$$

$$\sum_{\imath=1}^{3} L_{7\imath} R_\imath = 0, \tag{3.86}$$

Now, solving equation (3.84)-(3.86) for $R_\imath$ where $\imath = 1, 2, 3$ by making use of Inverse matrix method which is given below:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} L_{21} & L_{22} & L_{23} \\ L_{61} & L_{62} & L_{63} \\ L_{71} & L_{72} & L_{73} \end{bmatrix}^{-1} \begin{bmatrix} \theta_0 \\ 0 \\ 0 \end{bmatrix}.$$

3.7 Validity of the Problem

When the memory effect is ignored, we obtain the following results

$$(\bar{\psi}, \bar{\phi}_2, \bar{q}, \bar{T})(z) = \sum_{n=1}^{4} (1, H_{1n}, H_{2n}, H_{3n}) M_n exp(-k_n z).$$

where,

$H_{1n} = \frac{-(A_8 k_n^2 - A_9)}{k_n^2 - A_7}$, $H_{2n} = -\frac{(k_n^2 - A_4)(k_n^2 - A_7) - A_5(A_8 k_n^2 - A_9)}{(A_8 k_n^2 - A_9) A_6} H_{1n}$ and $H_{3n} = \frac{A_{11} k_n^2 - A_{12}}{k_n^2 - \frac{a_{14} a^2 + a_{15} b\bar{\omega}}{a_{14}}} H_{2n}$.
which are in sync with the results explained in the study [106] considered in the context of magneto-micropolar generalized thermoelasticity.

3.8 Numerical Results and Discussion

The material properties of magnesium [106] are taken in order to demonstrate the numerical computations, which are as follows:

$\lambda = 9.4 \times 10^{10}\, kgm^{-1}s^{-2}$, $\kappa = 1.0 \times 10^{10} kg\, m^{-1}s^{-2}$, $\mu = 4.0 \times 10^{10} kg\, m^{-1}s^{-2}$, $j = 0.2 \times 10^{-19} m^2$, $\rho = 1.74 \times 10^3 kg\, m^{-3}$, $\gamma = 0.779 \times 10^{-9} kg\, ms^{-2}$, $K^* = 1.7 \times 10^2 Jm^{-1}s^{-1}deg^{-1}$, $C_e = 1.04 \times 10^3 Jkg^{-1}deg^{-1}$, $T_0 = 298K$, $\alpha_t = 7.403 \times 10^{-7} K^{-1}$, $\tau_1 = 1s$, $\tau = 0.1s$, $\theta_0 = 1$, $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$, $\varepsilon_0 = 1/36\pi \times 10^{-9} Fm^{-1}$.

A graphic analysis has been done for the components of displacement, force stresses, couple stress components, and the temperature distribution. Results have been compared with and without micropolar effect for distinct values of time, $t = 0.2,\ 0.4, 0.6$ and for the fixed kernel, $K(t-r) = \left(1 - \frac{t-r}{\tau}\right)^2$.

Figures 3.2-3.7 illustrate the graphical results for magneto micropolar thermoelasticity (MMT), and figures 3.8-3.12 illustrate the graphical results for (WMMT) (without magneto micropolar thermoelasticity).

Figure 3.2 and 3.3 demonstrate the effect of time, t, on the displacement components u and w with respect to distance z. Figure 3.2, clearly demonstrates that the displacement component u reduces when the value of time is decreased and the maximum displacement distribution is attained at $t = 0.6$. In other words, the displacement component u reduces when the value of time is decreased, in terms of magnitude.

From figure 3.3, it has been observed that the displacement component w shows the same variation as that of u. Figure 3.4 shows that the stress component σ_{zz} is maximum at $t = 0.2$, then it is reduced when time is increased, i.e., at $t = 0.4$, and least normal stress is observed at $t = 0.6$. In nutshell, σ_{zz} reduces when the value of time is increased.

From figure 3.5, we observe that the tangential stress σ_{zx} shows the same variation as that of σ_{zz}. Figure 3.6 depicts the variation of couple stress m_{zx} with respect to distance z, and the maximum couple stress is attained for $t = 0.6$. Figure 3.7 demonstrates the impact of time on temperature distribution T and describes the variation of temperature w.r.t z. It is evident from the figure that the temperature distribution T is highest at $t = 0.2$, and it decreases when the value of time is increased, i.e., at $t = 0.4$, and we observe the least temperature distribution at $t = 0.6$.

Figure 3.8 demonstrates the variation of u in the case of WMMT. Comparing the variation of u in the cases of MMT and WMMT, it is clearly noticed that the amplitude of displacement component u is larger when the micropolar influence is excluded (i.e., in case of WMMT). From figure 3.9, we observe that the variation of the displacement component w w.r.t z, and it is apparent that w decreases with an increase in time t.

Figures 3.10 and 3.11, demonstrate the effect of time on stress components σ_{zz} and σ_{zx} and describe the variation of stress components w.r.t distance z. We noticed that the stress component σ_{zz} is maximum for $t = 0.2$, then the stress distribution is reduced when time is increased, i.e., for $t = 0.4$, and

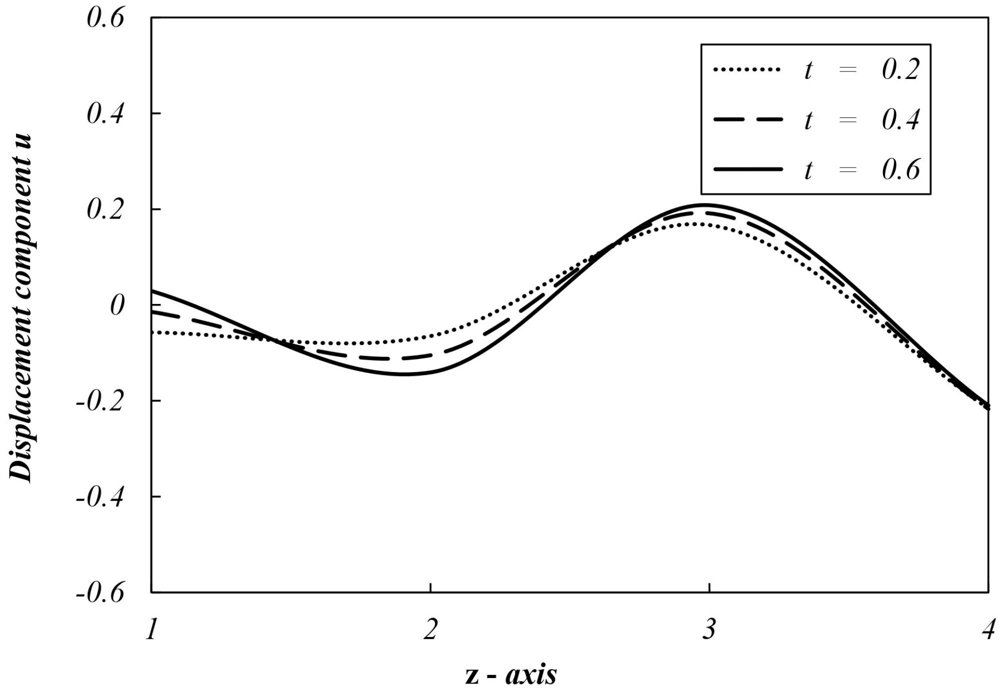

FIGURE 3.2: Variation of u at distinct values of time t in the presence of micropolar effect

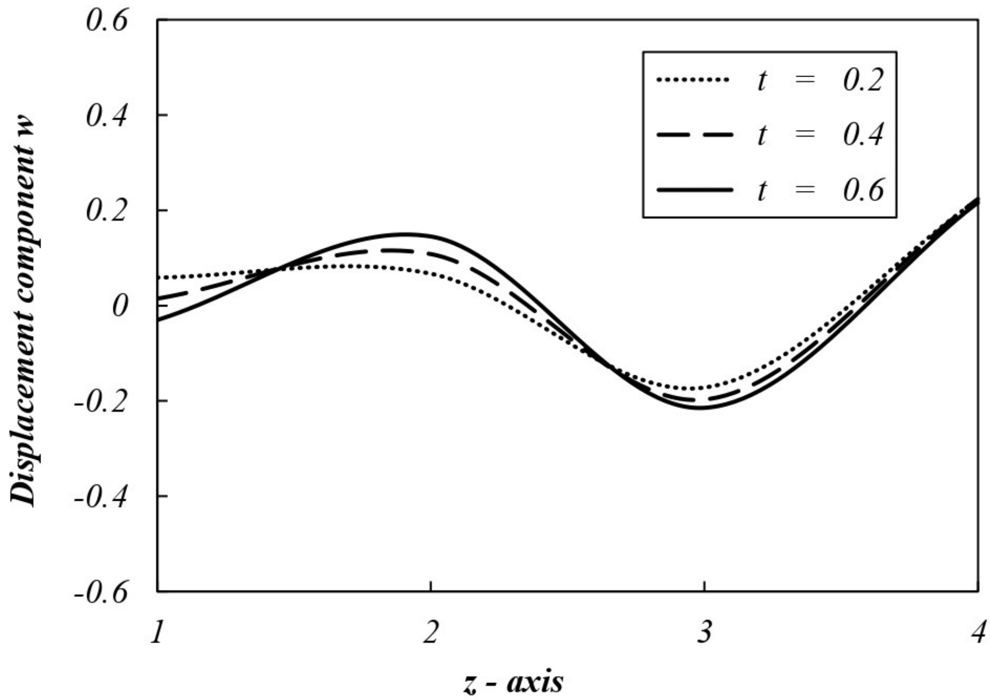

FIGURE 3.3: Variation of w at distinct values of time t in the presence of micropolar effect

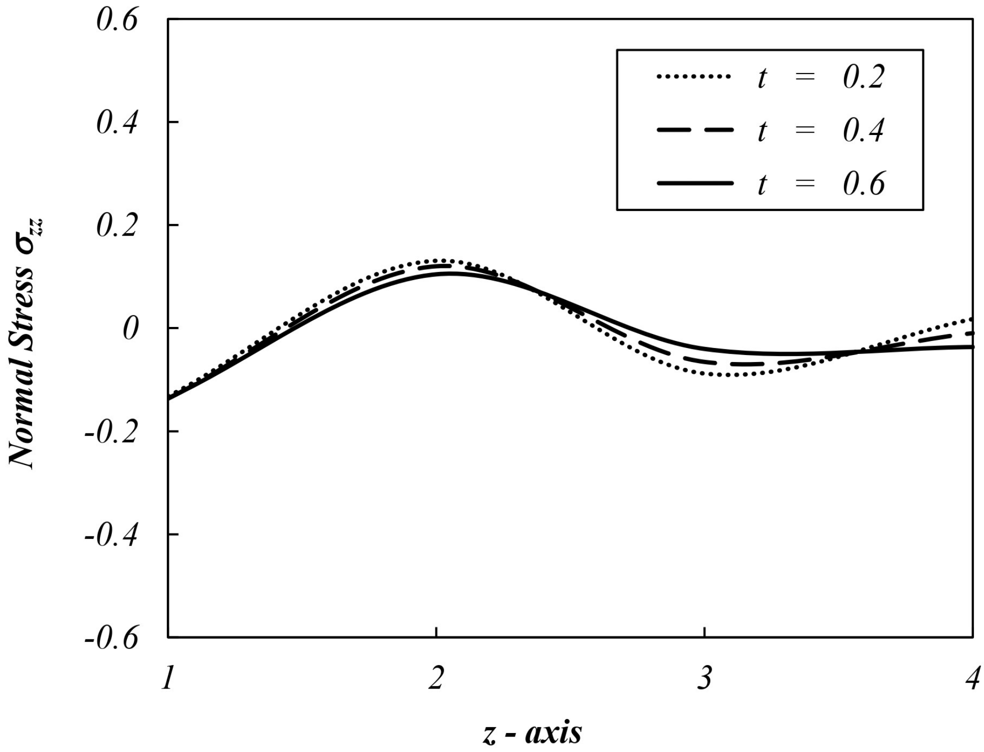

FIGURE 3.4: Variation of σ_{zz} at distinct values of time t in the presence of micropolar effect

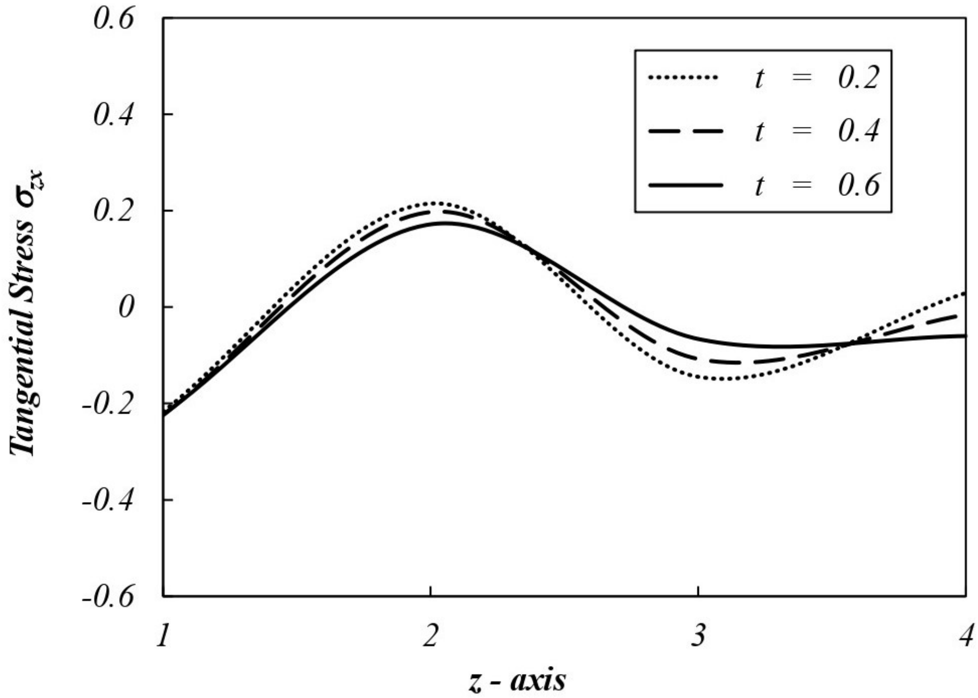

FIGURE 3.5: Variation of σ_{zx} at distinct values of time t in the presence of micropolar effect

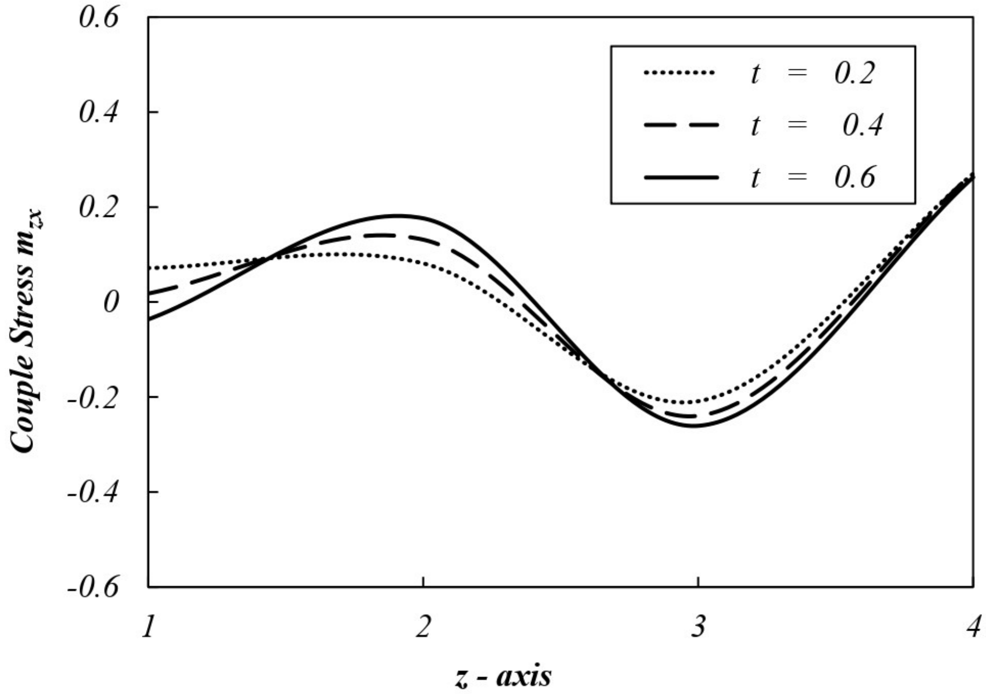

FIGURE 3.6: Variation of m_{zx} at distinct values of time t in the presence of micropolar effect

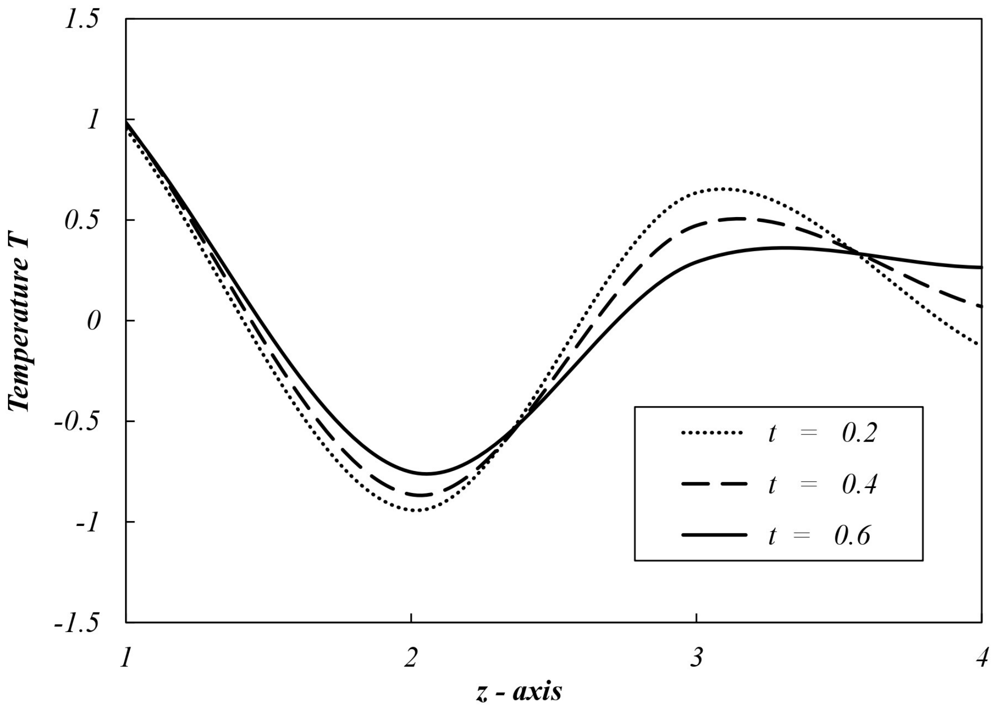

FIGURE 3.7: Variation of T at distinct values of time t in the presence of micropolar effect

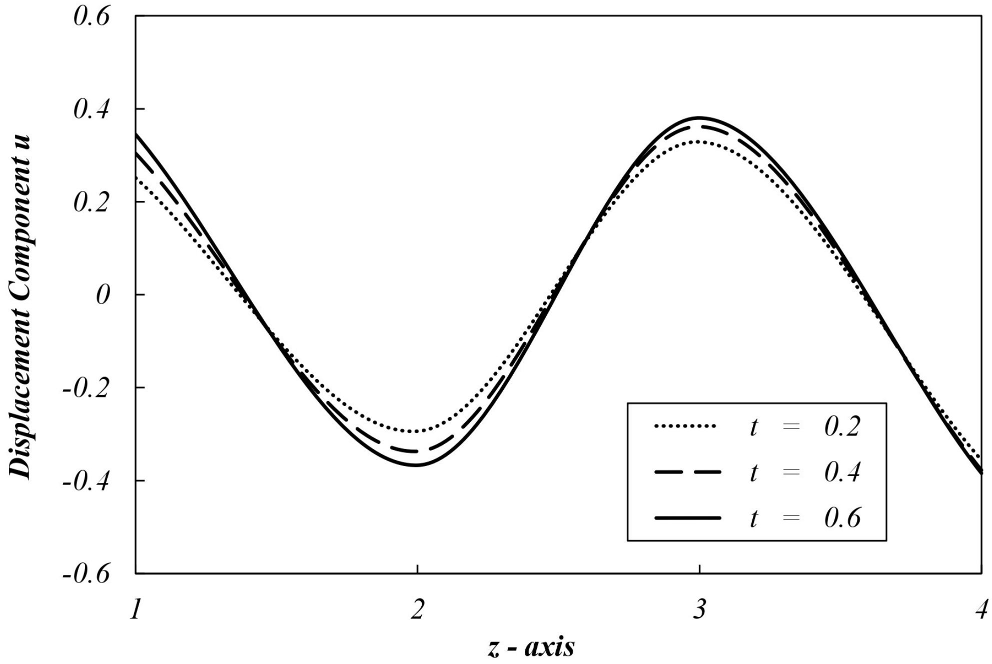

FIGURE 3.8: Variation of u at distinct values of time t in the absence of micropolar effect

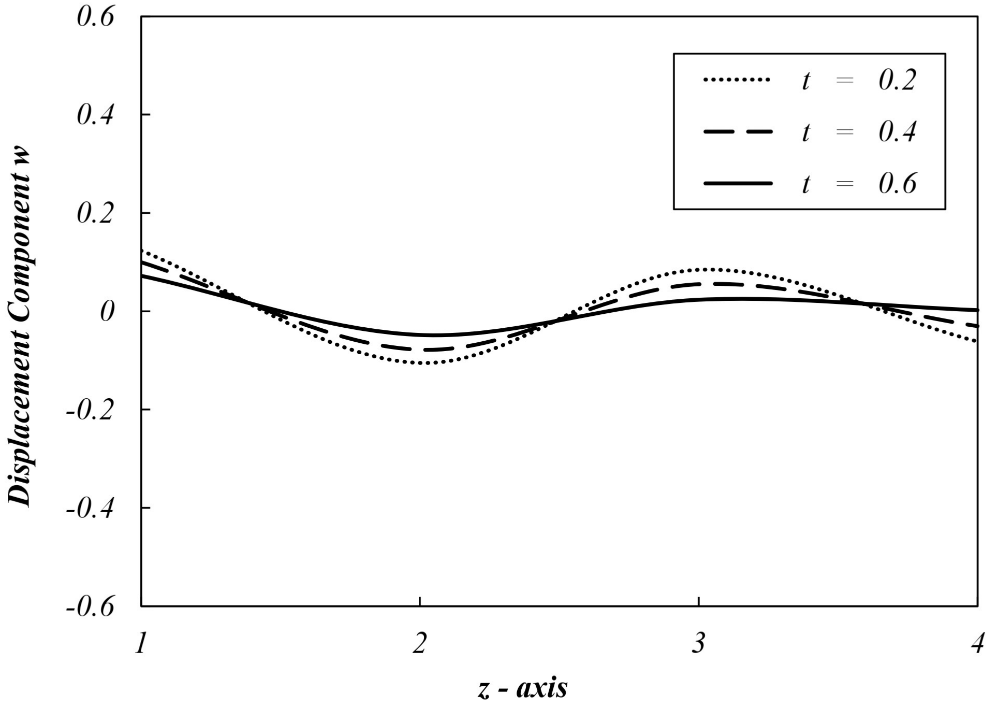

FIGURE 3.9: Variation of w at distinct values of time t in the absence of micropolar effect

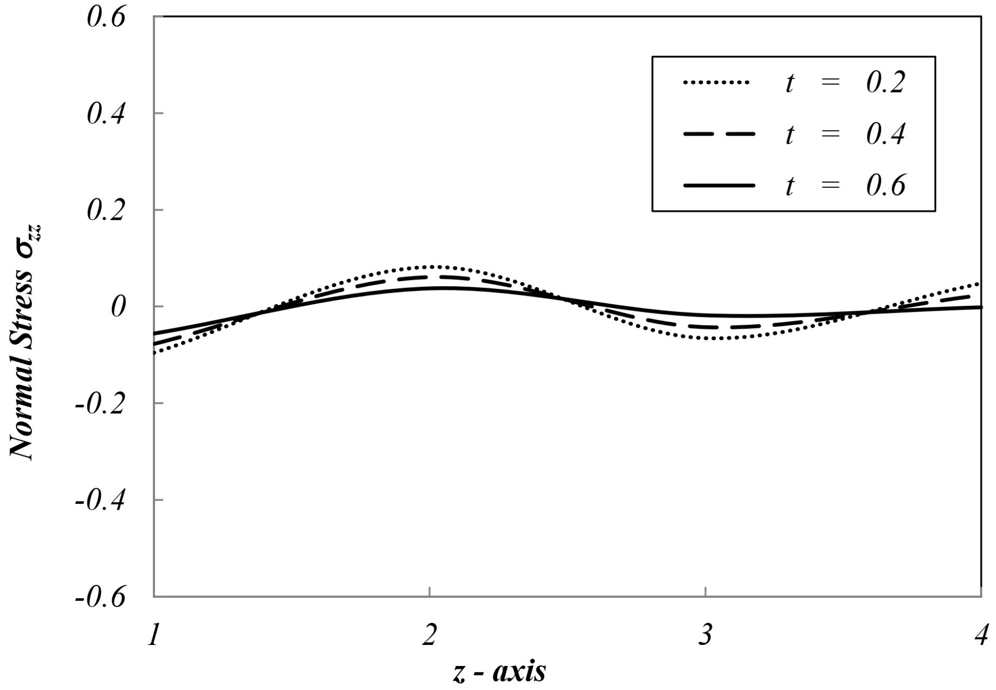

FIGURE 3.10: Variation of σ_{zz} at distinct values of time t in the absence of micropolar effect

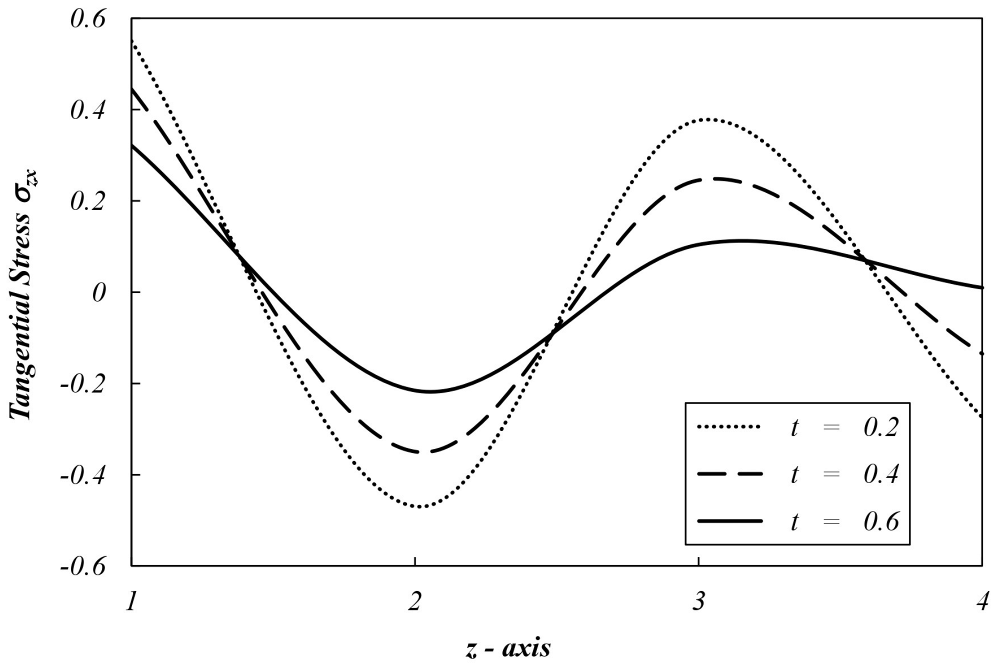

FIGURE 3.11: Variation of σ_{zx} at distinct values of time t in the absence of micropolar effect

when time is further increased, we observe the least normal stress, i.e., for $t = 0.6$. From figure 3.11 shows that the amplitude of σ_{zx} is much higher in the absence of micropolarity.

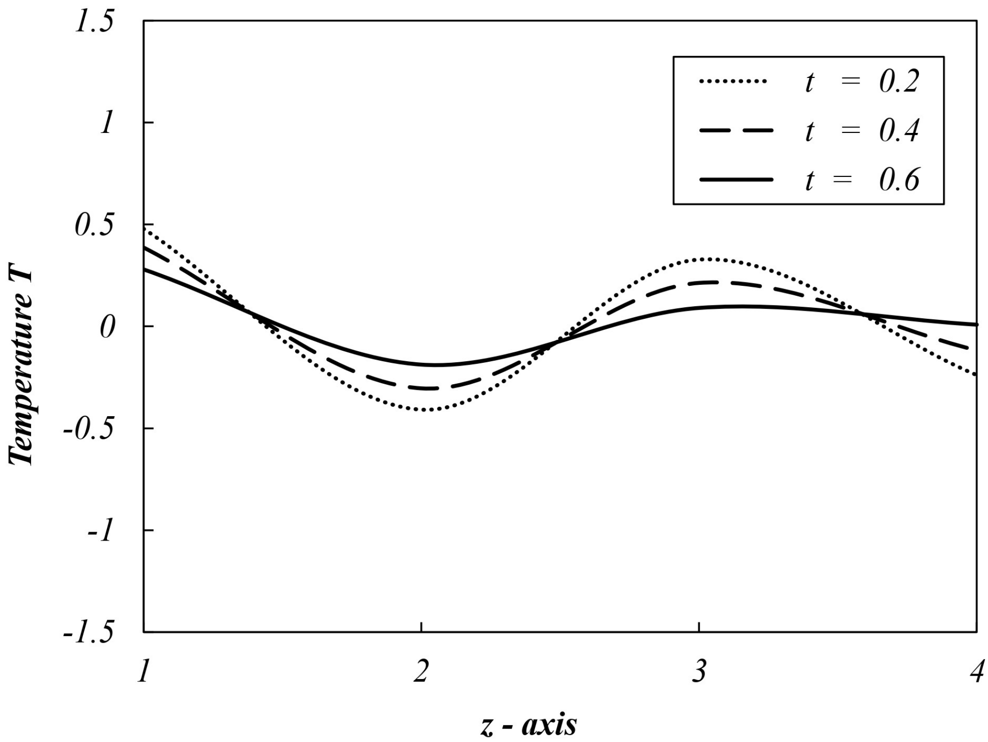

FIGURE 3.12: Variation of temperature T at distinct values of time t in the absence of micropolar effect

Figure 3.12 shows that the variation of temperature distribution T in case of WMMT, and it has been observed that the micropolarity has a significant effect on T in case of MMT, as it enhances the magnitude of T in the case of MMT.

If we compare Figure 3.2 with Figure 3.8, we learn that the addition of the micropolar effect leads to a significant decrease in the amplitude of displacement component u. It can be seen by comparing Figures 3.3 and 3.9 that when the micropolar effect is absent, the displacement component's (w) amplitude sharply declines. It is apparent from Figures 3.4 and 3.10 that when the micropolar effect is not considered, the amplitude of normal stress σ_{zz} drastically decreases. When the micropolar effect is present, Figures 3.7 and 3.12 reveals a significant change in the temperature amplitude.

3.9 Conclusion

A mathematical model for magneto-micropolar thermoelasticity problem has been developed in the framework of the Green-Lindsay model with MDD to examine the elastic behaviour of materials in the presence and absence of the micropolar effect. Moreover, the components of displacement, force stresses, couple stresses, and temperature distribution are illustrated graphically by making use of Matlab software.

Following the above numerical discussion, it can be concluded that,

1. It has been observed that the time parameter is causing significant effect on the elastic response of the considered material.

2. Micropolar nature plays a significant role in the elastic response of the material.

Chapter 4

A study of thermo-mechanical interactions in the rotating micropolar elastic solid with two temperatures using memory-dependent derivative

4.1 Introduction

A new approach to two-temperature micropolar thermoelasticity has been studied in the framework of the Green-Nagdhi theory (type III) with memory-dependent derivative for a rotating medium. The two - temperature thermoelasticity theory for the first time was proposed by Chen and Gurtin [108] and Chen et al. [109]-[110] in deformable bodies. This theory relies upon the conductive φ and thermodynamic temperature θ and is used to figure out the phonon and electron temperature distributions in ultra-short laser processing of metals. The difference between the above two temperatures, viz., φ and θ for time-independent situations, is proportional to heat supply, and both parameters can be treated as identical when heat supply is excluded [109]. In 2006, a generalized theory of thermoelasticity was established by Youssef [112] by using heat conduction theory in deformable bodies. Youssef [113] in 2008, examined a 2D model of generalized two-temperature thermoelastic half space. For obtaining the desired solution, the author has adapted the integral transform technique. For distinct values of the ramp parameter, the author has shown the variation of different physical quantities graphically. Abbas and Youssef [114] studied a transient phenomena in thermoelastic solids. The authors have also utilized copper material for numerical purposes. Another 2D problem in generalized magneto-micropolar thermoelastic medium was examined by Singh and Kumar [97] with a rotating effect. Radaev [115] studied a factorization of the main hyperbolic differential operator of the micropolar elasticity theory. The effect of three theories of thermoelasticity was studied in [116] on the propagation of a set of two coupled transverse waves and a set of two coupled longitudinal waves.

In this chapter, the memory response of a rotating micropolar elastic media under the thermo-mechanical effect has been investigated. The Helmholtz potential's along with the normal mode technique, is utilized for finding the desired solution of the required problem. Additionally, the Matlab software is used for numerical computations. The behaviour of the field quantities is studied for a fixed kernel $\tilde{K}(t-r)$ and for distinct values of time, t. Finally, the material properties of magnesium are considered to demonstrate the components of displacement, force stress, couple stress, thermodynamic temperature, as well as the conductive temperature distribution graphically.

4.2 Basic equations

The system of governing equations of a linear micropolar thermoelastic medium (which is rotating) with two temperatures and without body forces has been taken [107] as

$$(\lambda+\mu)\nabla(\nabla.\vec{u})+(\mu+\kappa)\nabla^2\vec{u}+\kappa(\nabla\times\vec{\phi})-\nu\nabla T$$
$$=\rho\left[\frac{\partial^2\vec{u}}{\partial t^2}+\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2\left(\vec{\Omega}\times\frac{\partial\vec{u}}{\partial t}\right)\right], \tag{4.1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+\kappa(\nabla\times\vec{u})-2\kappa\vec{\phi}=\rho j\left[\frac{\partial^2\vec{\phi}}{\partial t^2}+\vec{\Omega}\times\frac{\partial\vec{\phi}}{\partial t}\right]. \tag{4.2}$$

The constitutive relations are

$$\sigma_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+\kappa(u_{j,i}-\varepsilon_{ijr}\phi_r)-\nu T\delta_{ij}, \tag{4.3}$$
$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}. \tag{4.4}$$

Under Green-Naghdi theory [22], the heat conduction equation free from heat sources is given by

$$K^*\nabla^2\varphi+K\nabla^2\dot{\varphi}=\rho C_e\ddot{T}+\gamma' T_0\ddot{e}. \tag{4.5}$$

Furthermore, superposed dot represents the time derivative and e is cubical dilatation, defined as

$$e=u_x+w_z.$$

In the context of MDD, equation (4.5) of G-N type III is written as

$$K^*\nabla^2\varphi + K\nabla^2\dot{\varphi} = D_\tau\left[\rho C_e\frac{\partial T}{\partial t} + \gamma^{'}T_0\frac{\partial e}{\partial t}\right], \tag{4.6}$$

Also,

$$T = (1 - a^*\nabla^2)\varphi. \tag{4.7}$$

where, T and φ represents thermodynamic and conductive temperatures.

The equations of motion (4.1)-(4.2) along with the equations (4.6) and (4.7) in Cartesian coordinates (x, y, z) in component form can be written as

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}\right)+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\kappa\left(\frac{\partial\phi_3}{\partial y}-\frac{\partial\phi_2}{\partial z}\right)-$$

$$\nu\frac{\partial T}{\partial x}=\rho\left(\frac{\partial^2 u}{\partial t^2}+(\Omega_2\Omega_1 v-\Omega_2^2 u-\Omega_3^2 u+\Omega_3\Omega_1 w)+2(\Omega_2\dot{w}-\Omega_3\dot{v})\right), \tag{4.8}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)+(\mu+\kappa)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\kappa\left(\frac{\partial\phi_1}{\partial z}-\frac{\partial\phi_3}{\partial x}\right)-$$

$$\nu\frac{\partial T}{\partial y}=\rho\left(\frac{\partial^2 v}{\partial t^2}+(\Omega_1\Omega_2 u+\Omega_3\Omega_2 w-\Omega_1^2 v-\Omega_3^2 v)+2(\Omega_3\dot{u}-\Omega_1\dot{w})\right), \tag{4.9}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial z\partial x}+\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial z^2}\right)+(\mu+\kappa)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\kappa\left(\frac{\partial\phi_2}{\partial x}-\frac{\partial\phi_1}{\partial y}\right)-$$

$$\nu\frac{\partial T}{\partial z}=\rho\left(\frac{\partial^2 w}{\partial t^2}+(\Omega_1\Omega_3 u-\Omega_1^2 w-\Omega_2^2 w+\Omega_2\Omega_3 v)+2(\Omega_1\dot{v}-\Omega_2\dot{u})\right), \tag{4.10}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial x^2}+\frac{\partial^2\phi_2}{\partial x\partial y}+\frac{\partial^2\phi_3}{\partial x\partial z}\right)+\gamma\left(\frac{\partial^2\phi_1}{\partial x^2}+\frac{\partial^2\phi_1}{\partial y^2}+\frac{\partial^2\phi_1}{\partial z^2}\right)+\kappa\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)-2\kappa\phi_1$$

$$=\rho j\left(\frac{\partial^2\phi_1}{\partial t^2}+(\Omega_2\dot{\phi_3}-\Omega_3\dot{\phi_2})\right), \tag{4.11}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial y\partial x}+\frac{\partial^2\phi_2}{\partial y^2}+\frac{\partial^2\phi_3}{\partial y\partial z}\right)+\gamma\left(\frac{\partial^2\phi_2}{\partial x^2}+\frac{\partial^2\phi_2}{\partial y^2}+\frac{\partial^2\phi_2}{\partial z^2}\right)+\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2\kappa\phi_2$$

$$=\rho j\left(\frac{\partial^2\phi_2}{\partial t^2}+(\Omega_3\dot{\phi_1}-\Omega_1\dot{\phi_3})\right), \tag{4.12}$$

$$(\alpha+\beta)\left(\frac{\partial^2\phi_1}{\partial z\partial x}+\frac{\partial^2\phi_2}{\partial z\partial y}+\frac{\partial^2\phi_3}{\partial z^2}\right)+\gamma\left(\frac{\partial^2\phi_3}{\partial x^2}+\frac{\partial^2\phi_3}{\partial y^2}+\frac{\partial^2\phi_3}{\partial z^2}\right)+\kappa\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)-2\kappa\phi_3$$

$$=\rho j\left(\frac{\partial^2\phi_3}{\partial t^2}+(\Omega_1\dot{\phi_2}-\Omega_2\dot{\phi_1})\right), \tag{4.13}$$

$$K^*\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right)+K\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right)\dot{\varphi}=D_\tau\left[\rho C_e\frac{\partial T}{\partial t}+\gamma^{'}T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right], \tag{4.14}$$

$$T=\left(1-a^*\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\right)\varphi. \tag{4.15}$$

where, (u, v, w), (ϕ_1, ϕ_2, ϕ_3), and $(\Omega_1, \Omega_2, \Omega_3)$ are the components of displacement vector $\overrightarrow{u}$, microrotation vector $\overrightarrow{\phi}$ and rotation vector $\overrightarrow{\Omega}$, respectively.

4.3 Formulation and solution of the problem

A homogeneous micropolar thermoelastic rotating half-space with two temperatures is taken into account and rotation acts along the y-axis. Also, the origin of a rectangular cartesian coordinate system (x, y, z) is taken at any point on the plane surface of half-space z = 0, as shown in figure 4.1.

The considered medium rotates with a uniform angular velocity $\Omega = \Omega n$, here n represents a unit vector which denotes the direction of rotation axis. The dynamic displacement vector $\overrightarrow{u}$, rotation vector $\overrightarrow{\Omega}$ and microrotation vector $\overrightarrow{\phi}$ for the considered 2D problem are taken as

$$\overrightarrow{u} = (u, 0, w), \quad \overrightarrow{\Omega} = (0, \Omega, 0), \quad \overrightarrow{\phi} = (0, \phi_2, 0), \quad u(x, z, t), \quad and \quad w(x, z, t). \tag{4.16}$$

Using equation (4.16) in equations (4.8)-(4.15) and in equations (4.3)-(4.4), we get the

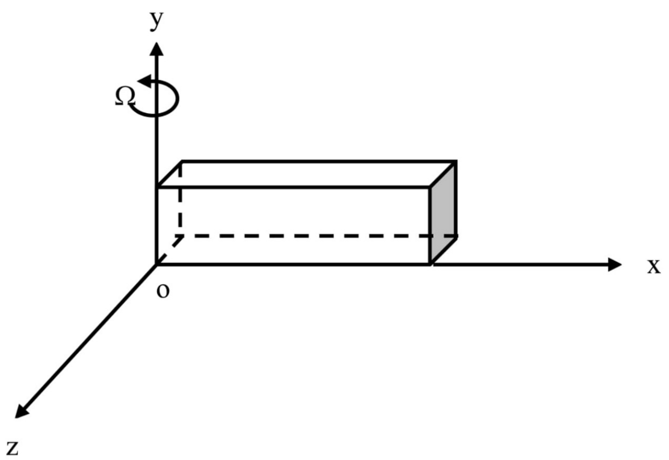

FIGURE 4.1: Rotating material geometry

following components

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x\partial z}\right)+(\mu+\kappa)\nabla^2 u-\kappa\frac{\partial \phi_2}{\partial z}-\nu(1-a^*\nabla^2)\frac{\partial \varphi}{\partial x}=\rho[\ddot{u}-\Omega^2 u+2\Omega\dot{w}] \tag{4.17}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial z\partial x}+\frac{\partial^2 w}{\partial z^2}\right)+(\mu+\kappa)\nabla^2 w+\kappa\frac{\partial \phi_2}{\partial x}-\nu(1-a^*\nabla^2)\frac{\partial \varphi}{\partial z}=\rho[\ddot{w}-\Omega^2 w-2\Omega\dot{u}] \tag{4.18}$$

$$\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)+\gamma\nabla^2\phi_2-2\kappa\phi_2=j\rho\ddot{\phi_2}. \tag{4.19}$$

$$K^*\left(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial z^2}\right)+K\left(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial z^2}\right)\dot{\varphi}=D_\tau\left[\rho C_e\frac{\partial T}{\partial t}+\gamma' T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)\right], \tag{4.20}$$

$$T=\left(1-a^*\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right)\varphi, \tag{4.21}$$

$$\sigma_{xx}=\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+(2\mu+\kappa)\frac{\partial u}{\partial x}-\nu T, \tag{4.22}$$

$$\sigma_{yy}=\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)-\nu T, \tag{4.23}$$

$$\sigma_{zz}=\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+(2\mu+\kappa)\frac{\partial w}{\partial z}-\nu T, \tag{4.24}$$

$$\sigma_{xz}=\mu\frac{\partial u}{\partial z}+(\mu+\kappa)\frac{\partial w}{\partial x}+\kappa\phi_2, \tag{4.25}$$

$$\sigma_{zx}=(\mu+\kappa)\frac{\partial u}{\partial z}+\mu\frac{\partial w}{\partial x}-\kappa\phi_2, \tag{4.26}$$

$$m_{xy}=\gamma\frac{\partial \phi_2}{\partial x}, \tag{4.27}$$

$$m_{zy}=\gamma\frac{\partial \phi_2}{\partial z}. \tag{4.28}$$

Introducing the below mentioned non-dimensional quantities in equations (4.17)-(4.21),

$$(x',z')=\frac{\eta_0}{c_0}(x,z),\ (u',w')=\frac{\rho\eta_0 c_0}{\gamma_1 T_0}(u,w),\ {\sigma'}_{i,j}=\frac{\sigma_{ij}}{\nu T_0},\ (\tau',t')=\eta_0(\tau,t),\varphi'=\frac{\varphi}{T_0},$$
$${\phi_2}'=\frac{\rho c_0^2}{\nu T_0}\phi_2,\ \Omega'=\frac{\Omega}{\eta_0},\ m'_{ij}=\frac{\eta_0}{c_0\nu T_0}m_{ij},\ T'=\frac{T}{T_0}, \tag{4.29}$$

where, $\eta_0 = \frac{\rho C_e {c_0}^2}{K^*}$, $c_0^2 = \frac{\lambda+2\mu+\kappa}{\rho}$.
we get

$$\frac{(\lambda+\mu)}{\rho c_0^2}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x \partial z}\right)+\left(\frac{\mu+\kappa}{\rho c_0^2}\right)\nabla^2 u-\frac{\kappa}{\rho c_0^2}\frac{\partial \phi_2}{\partial z}-(1-a_1\nabla^2)\frac{\partial \varphi}{\partial x}=\ddot{u}-\Omega^2 u+2\Omega\dot{w}, \tag{4.30}$$

$$\frac{(\lambda+\mu)}{\rho c_0^2}\left(\frac{\partial^2 u}{\partial x \partial z}+\frac{\partial^2 w}{\partial z^2}\right)+\left(\frac{\mu+\kappa}{\rho c_0^2}\right)\nabla^2 w+\frac{\kappa}{\rho c_0^2}\frac{\partial \phi_2}{\partial x}-(1-a_1\nabla^2)\frac{\partial \varphi}{\partial z}=\ddot{w}-\Omega^2 w-2\Omega\dot{u}, \tag{4.31}$$

$$\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)+\frac{\gamma\eta_0^2}{c_0^2}\nabla^2\phi_2-2\kappa\phi_2=j\rho\eta_0^2\frac{\partial^2\phi_2}{\partial t^2}, \tag{4.32}$$

$$D_\tau\left((1-a_1\nabla^2)\frac{\partial\varphi}{\partial t}+a_4\frac{\partial e}{\partial t}\right)=a_2\nabla^2\varphi+a_3\nabla^2\dot{\varphi}, \tag{4.33}$$

$$T=(1-a_1\nabla^2)\varphi, \tag{4.34}$$

where, $a_1 = \frac{a^*\eta_0^2}{c_0^2}$, $a_2 = \frac{K^*\eta_0}{\rho C_e c_0^2}$, $a_3 = \frac{K\eta_0^2}{\rho C_e c_0^2}$, and $a_4 = \frac{\gamma' \nu T_0}{\rho^2 C_e c_0^2}$.

Now, to obtain the solution, the displacement potentials $q(x,z,t)$ and $\psi(x,z,t)$ are introduced as

$$u=\frac{\partial q}{\partial x}+\frac{\partial\psi}{\partial z},\ w=\frac{\partial q}{\partial z}-\frac{\partial\psi}{\partial x}. \tag{4.35}$$

which leads equations (4.30), (4.32), (4.33) to

$$\left(\nabla^2-\frac{\partial^2}{\partial t^2}+\Omega^2\right)q+2\Omega\frac{\partial\psi}{\partial t}-(1-a_1\nabla^2)\varphi=0, \tag{4.36}$$

$$\left(\left(\frac{\mu+\kappa}{\rho c_0^2}\right)\nabla^2+\Omega^2-\frac{\partial^2}{\partial t^2}\right)\psi-\frac{\kappa}{\rho c_0^2}\phi_2-2\Omega\frac{\partial q}{\partial t}=0, \tag{4.37}$$

$$\left(\frac{\gamma\eta_0^2}{c_0^2}\nabla^2-j\rho\eta_0^2\frac{\partial^2}{\partial t^2}-2\kappa\right)\phi_2+\kappa\nabla^2\psi=0, \tag{4.38}$$

$$D_\tau\left((1-a_1\nabla^2)\frac{\partial\varphi}{\partial t}+a_4\frac{\partial}{\partial t}\nabla^2 q\right)=a_2\nabla^2\varphi+a_3\nabla^2\dot{\varphi}, \tag{4.39}$$

4.4 Normal mode analysis

In term of normal modes, the solution of the considered physical variables can be decomposed as

$$[u,w,q,\psi,T,\phi_2,m_{ij},\sigma_{ij},\varphi](x,z,t)=[\bar{u},\bar{w},\bar{q},\bar{\psi},\bar{T},\bar{\phi_2},\bar{m}_{ij},\bar{\sigma}_{ij},\bar{\varphi}](z)exp(bt+iax). \tag{4.40}$$

Use of equation (4.40) transforms equations (4.36)-(4.39) to

$$[D^2 - A_1]\bar{q} + A_2\bar{\psi} + [a_1 D^2 - A_3]\bar{\varphi} = 0, \tag{4.41}$$

$$[A_4 D^2 - A_5]\bar{\psi} - A_6\bar{\phi_2} - A_2\bar{q} = 0, \tag{4.42}$$

$$[A_7 D^2 - A_8]\bar{\phi_2} + \kappa[D^2 - a^2]\bar{\psi} = 0, \tag{4.43}$$

$$[A_9 D^2 - A_{10}]\bar{\varphi} - [A_{11} D^2 - A_{12}]\bar{q} = 0, \tag{4.44}$$

where, $D = \frac{\partial}{\partial z}$, $A_1 = a^2 + b^2 - \Omega^2$, $A_2 = 2\Omega b$, $A_3 = a_1 a^2 + 1$, $A_4 = \frac{\mu+\kappa}{\rho c_0^2}$, $A_5 = A_4 a^2 - \Omega^2 + b^2$, $A_6 = \frac{\kappa}{\rho c_0^2}$, $A_7 = \frac{\gamma\eta_0^2}{c_0^2}$, $A_8 = A_7 a^2 + j\rho\eta_0^2 b^2 + 2\kappa$, $A_9 = a_2 + a_3 b + a_1 b G(\tau, b)$, $A_{10} = a_2 a^2 + a_3 a^2 b + G(\tau, b)(1 + a_1 a^2)b$, $A_{11} = G(\tau, b) a_4 b$, $A_{12} = a_4 b G(\tau, b) a^2$, $G(\tau, b) = \frac{-(b^2(m^2-2n+1)\tau^2 + 2b\tau(m^2-n) + 2m^2)exp[b(t-\tau)] + (b^2\tau^2 - 2bn\tau + 2m^2)exp(bt)}{b^2\tau^2}$.

After some simplification, equations (4.41)-(4.44) can be reduced to

$$[A_{13} D^4 - A_{14} D^2 + A_{15}]\bar{\psi} - [A_{16} D^2 - A_{17}]\bar{q} = 0, \tag{4.45}$$

$$[A_{18} D^4 - A_{19} D^2 + A_{20}]\bar{q} + [A_{21} D^2 - A_{22}]\bar{\psi} = 0, \tag{4.46}$$

where, $A_{13} = A_4 A_7$, $A_{14} = A_4 A_8 + A_5 A_7 - \kappa A_6$, $A_{15} = A_5 A_8 - \kappa A_6 a^2$, $A_{16} = A_2 A_7$, $A_{17} = A_2 A_8$, $A_{18} = A_9 + A_{11} a_1$, $A_{19} = A_9 A_1 + A_{10} + A_{11} A_3 + A_{12} a_1$, $A_{20} = A_{10} A_1 + A_{12} A_3$, $A_{21} = A_2 A_9$, $A_{22} = A_2 A_{10}$.

Simplifying equations (4.45) and (4.46), we obtain,

$$[D^8 - AD^6 + BD^4 - CD^2 + F]\bar{\psi}(z) = 0, \tag{4.47}$$

Similarly,

$$[D^8 - AD^6 + BD^4 - CD^2 + F][\bar{q}(z), \bar{\phi_2}(z), \bar{\varphi}(z)] = 0, \tag{4.48}$$

where, $A = \frac{A_{13}A_{19} + A_{14}A_{18}}{A_{13}A_{18}}$, $B = \frac{A_{13}A_{20} + A_{14}A_{19} + A_{15}A_{18} + A_{21}A_{16}}{A_{13}A_{18}}$, $C = \frac{A_{14}A_{20} + A_{15}A_{19} + A_{21}A_{17} + A_{22}A_{16}}{A_{13}A_{18}}$, $F = \frac{A_{15}A_{20} + A_{22}A_{17}}{A_{13}A_{18}}$.

Rewriting equation (4.47) and (4.48), in the factored form as

$$[(D^2 - {k_1}^2)(D^2 - {k_2}^2)(D^2 - {k_3}^2)(D^2 - {k_4}^2)][\bar{\psi}(z), \bar{q}(z), \bar{\phi_2}(z), \bar{\varphi}(z)] = 0, \tag{4.49}$$

here, ${k_n}^2 (n = 1, 2, 3, 4)$ represents the characteristic roots of the equation (4.47) and (4.48).

The general solution of equation (4.49), has the form

$$\bar{\phi_2}(z) = \sum_{n=1}^{4} M_n e^{-k_n z}, \tag{4.50}$$

$$\bar{\psi}(z) = \sum_{n=1}^{4} {M_n}' e^{-k_n z}, \tag{4.51}$$

$$\bar{q}(z) = \sum_{n=1}^{4} {M_n}'' e^{-k_n z}, \tag{4.52}$$

$$\bar{\varphi}(z) = \sum_{n=1}^{4} {M_n}''' e^{-k_n z}, \tag{4.53}$$

Here, M_n, ${M_n}'$, ${M_n}''$, ${M_n}'''$ are parameters that depends on a and b.

Using equations (4.50)-(4.53) in equations (4.41)-(4.44), we get

$$\bar{\psi}(z) = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z}, \tag{4.54}$$

$$\bar{q}(z) = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z}, \tag{4.55}$$

$$\bar{\varphi}(z) = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z}, \tag{4.56}$$

where, $H_{1n} = -\frac{(A_7 {k_n}^2 - A_8)}{\kappa({k_n}^2 - a^2)}$, $H_{2n} = \left[\frac{(A_4 {k_n}^2 - A_5)}{A_2} + \frac{A_6 \kappa (k_n^2 - a^2)}{(A_7 k_n^2 - A_8) A_2}\right] H_{1n}$, $H_{3n} = \left[\frac{A_{11} {k_n}^2 - A_{12}}{A_9 {k_n}^2 - A_{10}}\right] H_{2n}$.

In general, equations (4.50) and (4.54)-(4.56), can be written as

$$(\bar{\phi_2}, \bar{\psi}, \bar{q}, \bar{\varphi})(z) = \sum_{n=1}^{4} (1, H_{1n}, H_{2n}, H_{3n}) M_n exp(-k_n z). \tag{4.57}$$

By making use of equations (4.29) and (4.40), equations (4.22)-(4.28), (4.34) and (4.35) yield to

$$\bar{u} = \sum_{n=1}^{4} M_n H_{4n} e^{-k_n z}, \tag{4.58}$$

$$\bar{w} = \sum_{n=1}^{4} M_n H_{5n} e^{-k_n z}, \tag{4.59}$$

$$\bar{T} = \sum_{n=1}^{4} M_n H_{6n} e^{-k_n z}, \tag{4.60}$$

$$\bar{\sigma}_{xx} = \sum_{n=1}^{4} M_n H_{7n} e^{-k_n z}, \tag{4.61}$$

$$\bar{\sigma}_{yy} = \sum_{n=1}^{4} M_n H_{8n} e^{-k_n z}, \tag{4.62}$$

$$\bar{\sigma}_{zz} = \sum_{n=1}^{4} M_n H_{9n} e^{-k_n z}, \tag{4.63}$$

$$\bar{\sigma}_{xz} = \sum_{n=1}^{4} M_n H_{10n} e^{-k_n z}, \tag{4.64}$$

$$\bar{\sigma}_{zx} = \sum_{n=1}^{4} M_n H_{11n} e^{-k_n z}, \tag{4.65}$$

$$\bar{m}_{zy} = \sum_{n=1}^{4} M_n H_{12n} e^{-k_n z}, \tag{4.66}$$

where,

$$H_{4n} = iaH_{2n} - H_{1n}k_n,$$
$$H_{5n} = -H_{2n}k_n - iaH_{1n},$$
$$H_{6n} = (1 - a_1 k_n^2 + a_1 a^2) H_{3n}$$
$$H_{7n} = (a_5 k_n^2 - a^2) H_{2n} - (1 - a_5) iaH_{1n}k_n - H_{6n},$$
$$H_{8n} = (a_5 k_n^2 - a_5 a^2) H_{2n} - H_{6n},$$
$$H_{9n} = (k_n^2 - a_5 a^2) H_{2n} - ia(a_5 - 1) H_{1n}k_n - H_{6n},$$
$$H_{10n} = \frac{1}{\rho c_0^2}\left[-(2\mu + \kappa) iaH_{2n}k_n + \mu H_{1n}k_n^2 + (\mu + \kappa) a^2 H_{1n} + \kappa\right],$$

$$H_{11n} = \frac{1}{\rho c_0^2}\left[-(2\mu + \kappa) iaH_{2n}k_n + (\mu + \kappa) H_{1n}k_n^2 + \mu a^2 H_{1n} - \kappa\right],$$
$$H_{12n} = -\frac{\eta_0^2 \gamma k_n}{\rho c_0^4},$$

Also, here $a_5 = \frac{\lambda}{\rho c_0^2}$.

4.5 Boundary conditions

To find out the M_n parameters (where $n = 1, 2, 3, 4$), the boundary conditions are taken as

Mechanical boundary conditions.

The normal and tangential stress conditions (which are mechanically stressed by constant forces R_1 and R_2) for the plane $z = 0$ can be taken as

$$\sigma_{zz} = -R_1\psi_1(x,t) \tag{4.67}$$

$$\sigma_{zx} = -R_2\psi_1(x,t) \tag{4.68}$$

$$m_{zy} = 0, \tag{4.69}$$

Thermal boundary conditions.

Since, the plane $z = 0$ is taken to be isothermal, therefore the thermal boundary condition is taken as

$$\varphi = 0, \tag{4.70}$$

where, $\psi_1(x,t) = e^{bt+iax}$, R_1 and R_2 are constants.

Now using equation (4.40) in the above equations, (4.67)-(4.70), and after some simplification, we get

$$\sum_{n=1}^{4} H_{9n}M_n = -R_1, \tag{4.71}$$

$$\sum_{n=1}^{4} H_{11n}M_n = -R_2, \tag{4.72}$$

$$\sum_{n=1}^{4} H_{12n}M_n = 0, \tag{4.73}$$

$$\sum_{n=1}^{4} H_{3n}M_n = 0. \tag{4.74}$$

Now, to solve the system of equations (4.71)-(4.74), matrix Inversion method is deployed as

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} H_{31} & H_{32} & H_{33} & H_{34} \\ H_{91} & H_{92} & H_{93} & H_{94} \\ H_{111} & H_{112} & H_{113} & H_{114} \\ H_{121} & H_{122} & H_{123} & H_{124} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -R_1 \\ -R_2 \\ 0 \end{bmatrix}.$$

For inversion matrix method matlab software has been utilized.

4.6 Validity of the Problem

When the memory effect is ignored, we obtain the following results

$$(\bar{\phi_2}, \bar{\psi}, \bar{q})(z) = \sum_{n=1}^{4}(1, H_{1n}, H_{2n})M_n exp(-k_n z).$$

where, $H_{1n} = -\frac{(A_7 {k_n}^2 - A_8)}{\kappa({k_n}^2 - a^2)}$, $H_{2n} = \left[\frac{(A_4 {k_n}^2 - A_5)}{A_2} + \frac{A_6\kappa(k_n^2 - a^2)}{(A_7 k_n^2 - A_8)A_2}\right] H_{1n}$

which are in sync with the results explained in the study [107] considered in the context of micropolar generalized (two-temperature) thermoelasticity.

4.7 Numerical results and Discussion

The material properties of magnesium [106] has been considered for numerical simulations, which are as:

$\rho = 1.74 \times 10^3 kg\, m^{-3},\ j = 0.2 \times 10^{-19} m^2,\ \gamma = 0.779 \times 10^{-9} kg\, m^{-2},$

$\kappa = 1.0 \times 10^{10} kg\, m^{-1} s^{-2},\ \lambda = 9.4 \times 10^{10} kg\, m^{-1} s^{-2}\ \mu = 4.0 \times 10^{10} kg\, m^{-1} s^{-2}$

$K^* = 1.7 \times 10^2 J m^{-1} s^{-1} deg^{-1},\ K = 0.1 W m^{-1} K^{-1},\ a^* = 0.15 \times 10^{-14} m^2,$

$C_e = 1.04 \times 10^3 J kg^{-1} deg^{-1},\ T_0 = 298K,\ \alpha_t = 7.403 \times 10^{-7} K^{-1},\ \tau = 1s.$

The numerical computations were accomplished for a fixed kernel, $K(t-r) = 1 - \frac{t-r}{\tau}$ (as defined in [61]) and for a time-delay, $\tau = 1s$ as well as for different values of time t, i.e., for $t = 0.2,\ 0.3,\ 0.4$. The graphical analysis of components of displacement, force stress, couple stress, thermodynamic temperature as well as conductive temperature distributions has been done, as shown in figures 4.2-4.8.

Figure 4.2 shows that u is significantly influenced by the time t and it decreases with z. The least displacement distribution is observed for $t = 0.4$, and it increases when time is reduced. Figure 4.3 shows the variation of w w.r.t z, and the maximum displacement

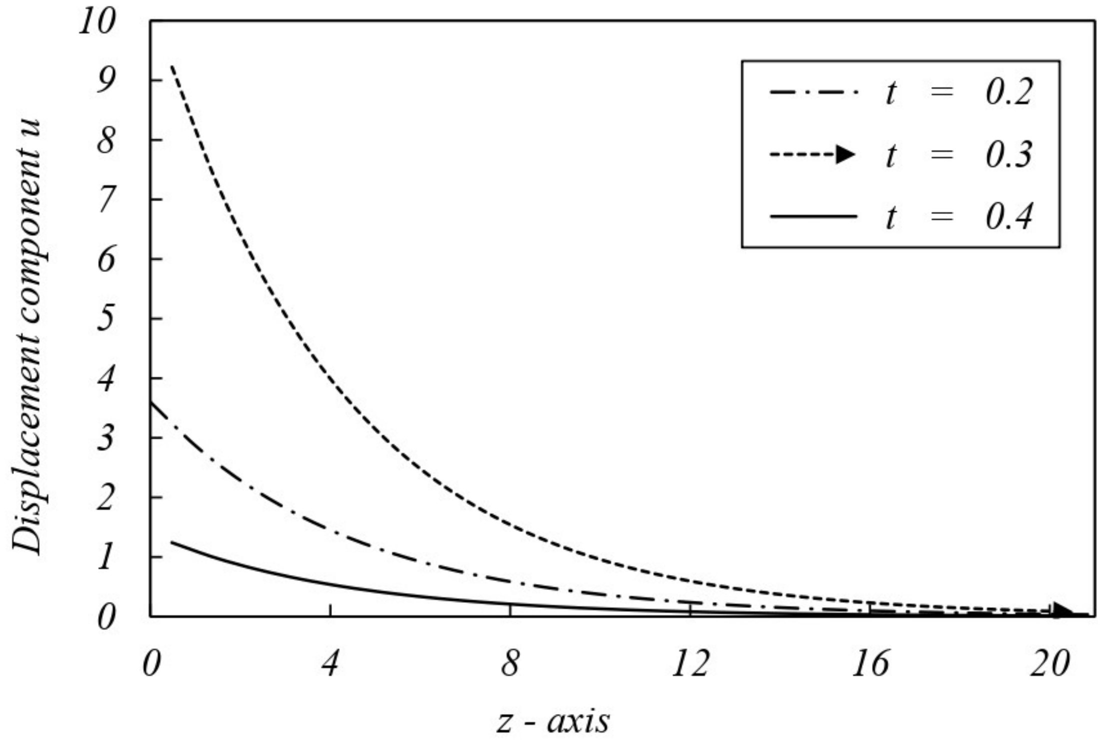

FIGURE 4.2: Variation of u at distinct values of time t

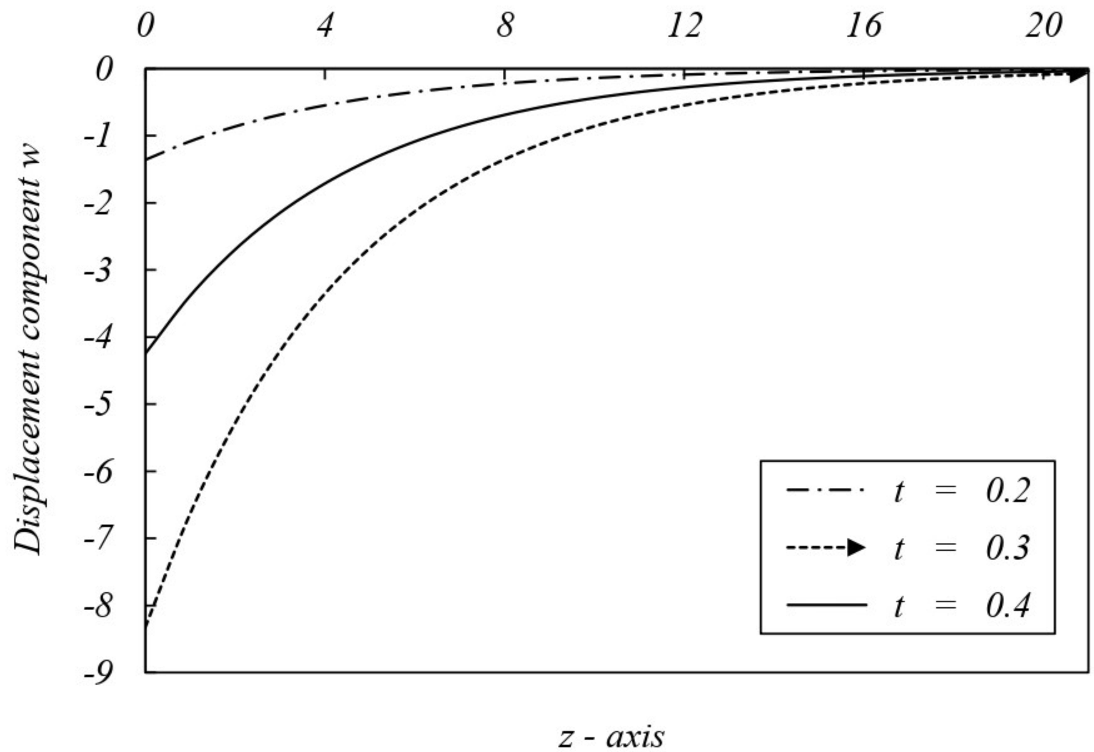

FIGURE 4.3: Variation of w at distinct values of time t

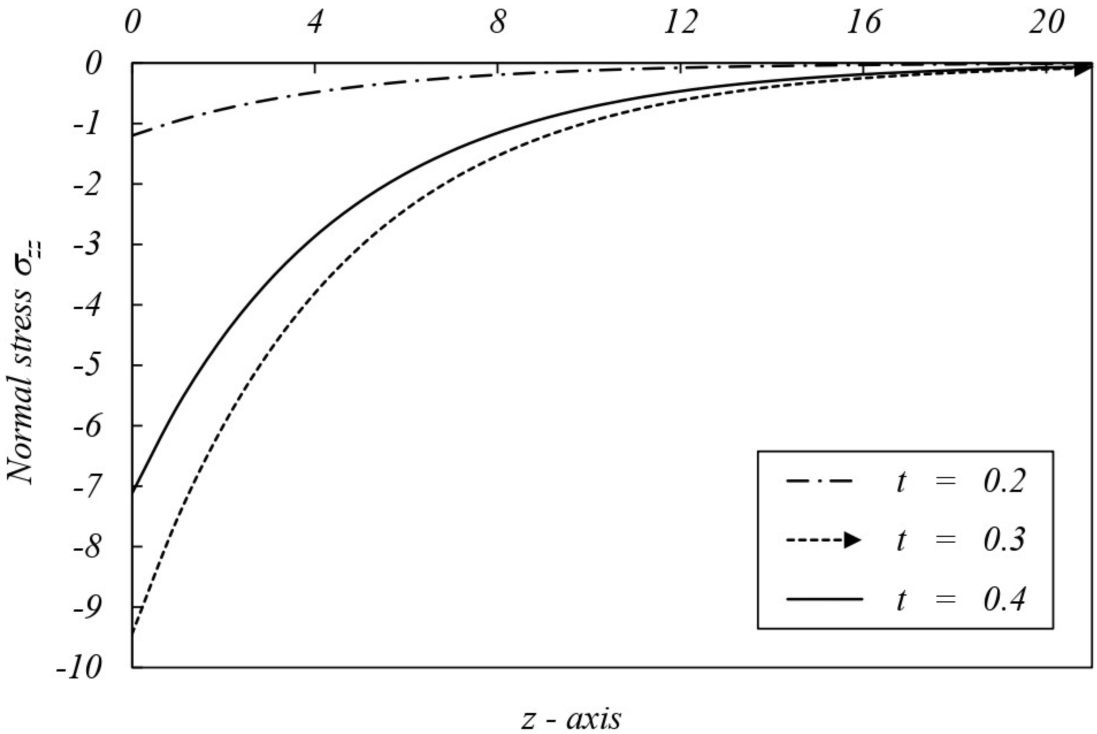

FIGURE 4.4: Variation of σ_{zz} at distinct values of time t

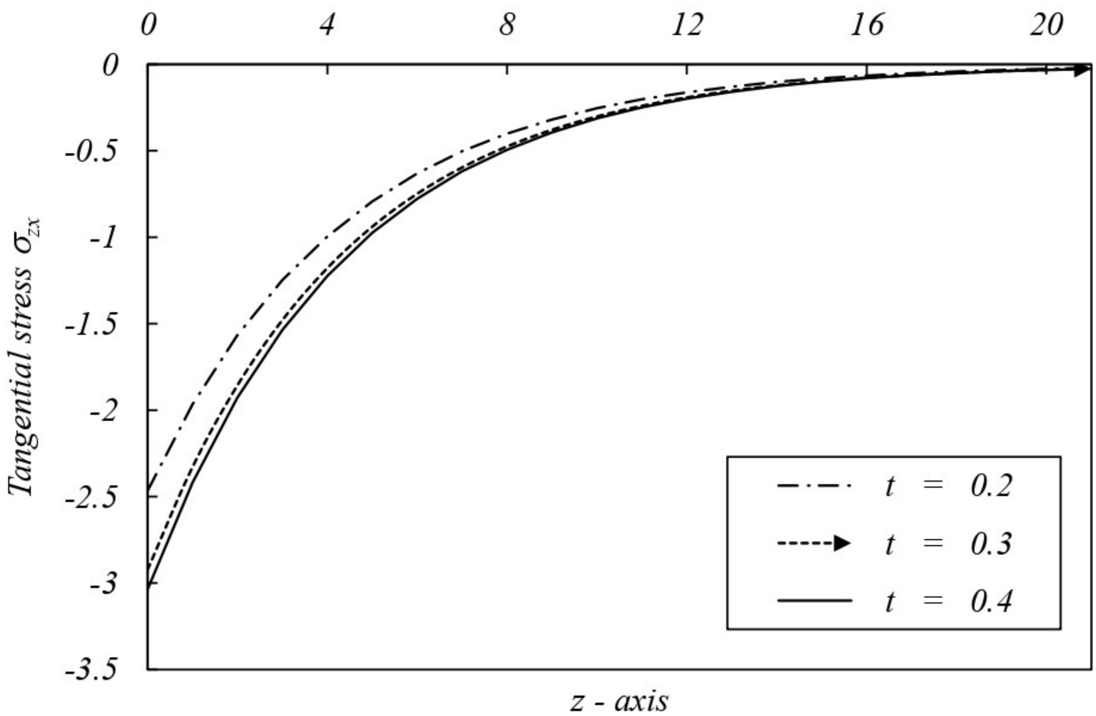

FIGURE 4.5: Variation of σ_{zx} at distinct values of time t

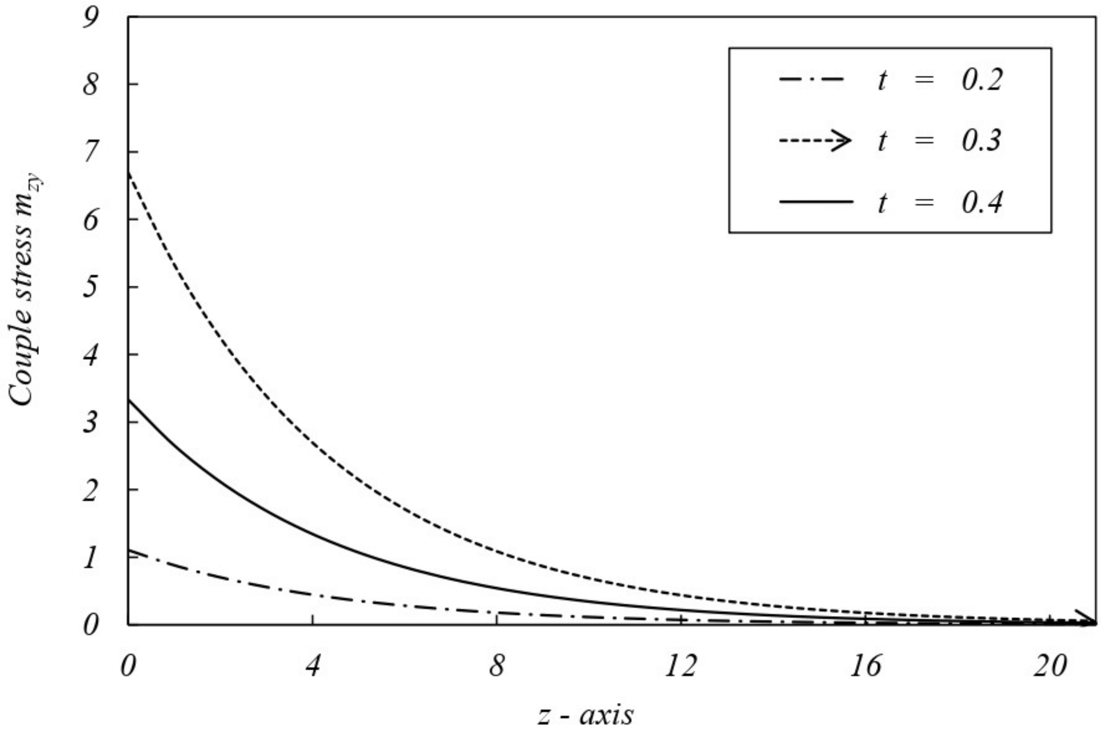

FIGURE 4.6: Variation of m_{zy} at distinct values of time t

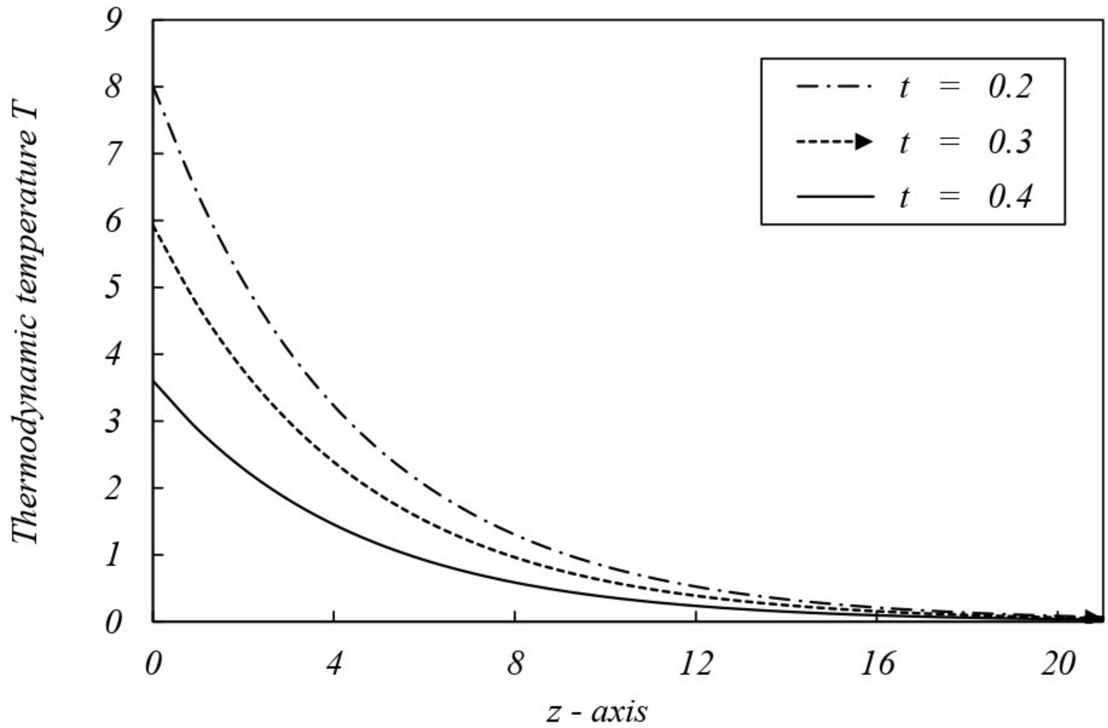

FIGURE 4.7: Variation of T at distinct values of time t

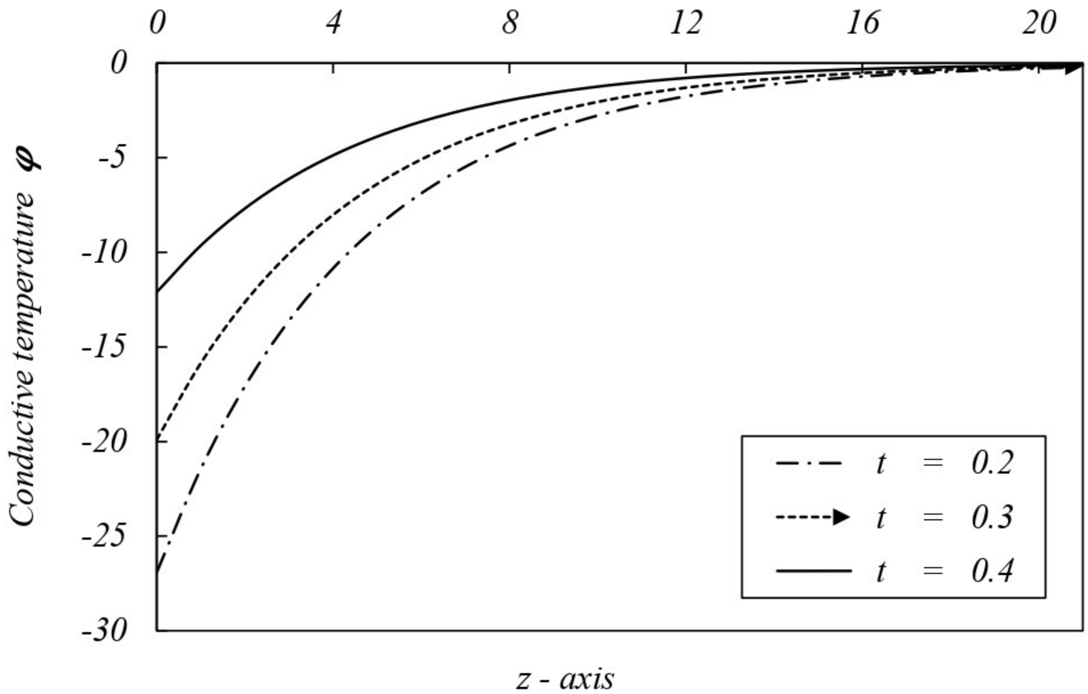

FIGURE 4.8: Variation of φ at distinct values of time t

is attained for $t = 0.3$, and it reduces when time is increased, i.e., for $t = 0.4$, and for $t = 0.2$, we observe the least displacement distribution. It is observed that all curves begin with distinct negative values, then increase, and eventually converge to zero. Figure 4.4 demonstrates that the similar variation is shown by the normal stress σ_{zz} and by the displacement component w.

Figure 4.5 clearly shows that the σ_{zx} increases when time t is increased and hence tangential stress σ_{zx} is maximum for $t = 0.4$. It has also been noticed that all curves begin with distinct values and then eventually converge to zero for distinct values of t. Figure 4.6 shows that when $t = 0.2$, the couple stress, m_{zy}, is the lowest.

Figure 4.7 clearly shows that T (thermodynamic temperature) is maximum for $t = 0.2$ and then it reduces with increase in time i.e, for $t = 0.3$. For $t = 0.4$, we observe the least temperature distribution. In other words, its clear that T reduces with increasing time t. From figure 4.8, it has been noticed that φ (conductive temperature) is highest for $t = 0.2$.

4.8 Conclusion

A new mathematical model of micropolar thermoelasticity has been established in the context of Green-Naghdi theory using a memory-dependent derivative. The thermo-mechanical interactions using two-temperature theory (2TT) with memory-dependent derivative have been demonstrated in the micropolar media. Using magnesium, the impact of time t on the components of displacement, force stress, couple stress, thermodynamic temperature, and conductive temperature distribution has been graphically explained. Following observations has been made in the following study:

1. Significant variation is shown by different physical quantities such as force stresses, couple stresses, displacement components and temperature distribution for different values of time “t”.

2. All the physical quantities converge to zero as the distance z increases.

Chapter 5

Interactions of magneto micropolar thermoelastic rotating medium with Memory dependent derivative

5.1 Introduction

A 2D mathematical model has been established in micropolar elasticity theory, subjected to magnetic and thermal effects in the context of memory-dependent derivatives. On a nanoscale thermoelastic micropolar material, the impact of the memory effect has been studied by Abouelregal et al. [131] under varying pulsed heating flow. By making use of dual-phase-lag model with memory-dependent derivative, a 2D problem has been studied by Kumar et al. [132] for micropolar elastic plate. The impact of rotation has been studied by authors in [133] on a micro-stretch medium under dual-phase-lag model model with memory-dependent derivative. The three-phase lag model has been explored by Jojare and Gaikwad [134] in isotropic semiconductors in order to study the memory effect. The MDD concept has been utilized by Bhattacharya and Kanoria [135] in order to examine the generalized magneto-thermo-diffusion relations in an isotropic medium. The laplace transforms along with the finite element method has been utilized for finding the required solution of the considered problem. The fractional heat conduction theory has been adopted by Xue et al. [136] to study the transient thermoelastic response in a porous half-space in the context of memory-dependent derivative. In order to solve the problem analytically, the authors have adopted Integral transforms.

The current chapter is concerned with the effect of the time parameter on a magneto-micropolar thermoelastic solid with a memory-dependent derivative. Furthermore, the medium is analyzed using the Lord-Shulman's model with memory-dependent derivative. The normal mode technique has been adopted to analytically solve the problem. The variation of various physical quantities, such as temperature distribution, displacement, force stress, and couple stress components, has been discussed w.r.t the time 't'.In addition to this, the potential displacement approach along with the normal analysis technique has been used to figure out the desired solution of the problem. For numerical computations, Matlab software along with the MS-excel has been utilized.

5.2 Basic equations

Following [137], the equations of electromagnetism for a perfectly conducting, homogeneous, slowly moving elastic medium along with the motion equations, the constitutive relations in micropolar generalized thermoelasticity in the context of Lorentz force are taken as

$$\nabla \times \overrightarrow{h} = \overrightarrow{J} + \varepsilon_0 \frac{\partial \overrightarrow{E}}{\partial t}, \tag{5.1}$$

$$\nabla \times \overrightarrow{E} = -\mu_0 \frac{\partial \overrightarrow{h}}{\partial t}, \tag{5.2}$$

$$\overrightarrow{E} = -\mu_0 \left(\frac{\partial \overrightarrow{u}}{\partial t} \times \overrightarrow{H_0} \right), \tag{5.3}$$

$$\nabla . \overrightarrow{h} = 0. \tag{5.4}$$

$$(\lambda + 2\mu + \kappa)\nabla(\nabla . \overrightarrow{u}) - (\mu + \kappa)\nabla \times (\nabla \times \overrightarrow{u}) + \kappa(\nabla \times \overrightarrow{\phi}) + \overrightarrow{F} - \upsilon \nabla T$$
$$= \rho \left[\frac{\partial^2 \overrightarrow{u}}{\partial t^2} + \overrightarrow{\Omega} \times (\overrightarrow{\Omega} \times \overrightarrow{u}) + 2(\overrightarrow{\Omega} \times \frac{\partial \overrightarrow{u}}{\partial t}) \right], \tag{5.5}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla . \overrightarrow{\phi}) - \gamma \nabla \times (\nabla \times \overrightarrow{\phi}) + \kappa(\nabla \times \overrightarrow{u}) - 2\kappa \overrightarrow{\phi} = \rho j \left(\frac{\partial^2 \overrightarrow{\phi}}{\partial t^2} + \overrightarrow{\Omega} \times \frac{\partial \overrightarrow{\phi}}{\partial t} \right), \tag{5.6}$$

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa(u_{j,i} - \varepsilon_{ijr}\phi_r) - \nu T \delta_{ij}, \tag{5.7}$$

$$m_{ij} = \alpha \phi_{r,r}\delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i}. \tag{5.8}$$

and

$$\overrightarrow{F} = \mu_0(\overrightarrow{J} \times \overrightarrow{H_0}). \tag{5.9}$$

The heat conduction equation with MDD [138] has been taken as

$$K\nabla^2 T = (1 + \tau D_\tau)(\rho C_e \dot{T} + \gamma^{'} T_0 \dot{e}). \tag{5.10}$$

The equations of motion (5.5)-(5.6) along with the equations (5.10) in Cartesian coordinates (x, y, z) in component form can be written as

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}\right)+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\kappa\left(\frac{\partial \phi_3}{\partial y}-\frac{\partial \phi_2}{\partial z}\right)+$$
$$(J_2H_3-J_3H_2)-\nu\frac{\partial T}{\partial x}=\rho\left(\frac{\partial^2 u}{\partial t^2}+(\Omega_2\Omega_1 v-\Omega_2^2 u-\Omega_3^2 u+\Omega_3\Omega_1 w)+2(\Omega_2\dot{w}-\Omega_3\dot{v})\right), \tag{5.11}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)+(\mu+\kappa)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\kappa\left(\frac{\partial \phi_1}{\partial z}-\frac{\partial \phi_3}{\partial x}\right)+$$
$$(J_3H_1-J_1H_3)-\nu\frac{\partial T}{\partial y}=\rho\left(\frac{\partial^2 v}{\partial t^2}+(\Omega_1\Omega_2 u+\Omega_3\Omega_2 w-\Omega_1^2 v-\Omega_3^2 v)+2(\Omega_3\dot{u}-\Omega_1\dot{w})\right), \tag{5.12}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial z\partial x}+\frac{\partial^2 v}{\partial z\partial y}+\frac{\partial^2 w}{\partial z^2}\right)+(\mu+\kappa)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\kappa\left(\frac{\partial \phi_2}{\partial x}-\frac{\partial \phi_1}{\partial y}\right)+$$
$$(J_1H_2-J_2H_1)-\nu\frac{\partial T}{\partial z}=\rho\left(\frac{\partial^2 w}{\partial t^2}+(\Omega_1\Omega_3 u-\Omega_1^2 w-\Omega_2^2 w+\Omega_2\Omega_3 v)+2(\Omega_1\dot{v}-\Omega_2\dot{u})\right), \tag{5.13}$$

$$(\alpha+\beta)\left(\frac{\partial^2 \phi_1}{\partial x^2}+\frac{\partial^2 \phi_2}{\partial x\partial y}+\frac{\partial^2 \phi_3}{\partial x\partial z}\right)+\gamma\left(\frac{\partial^2 \phi_1}{\partial x^2}+\frac{\partial^2 \phi_1}{\partial y^2}+\frac{\partial^2 \phi_1}{\partial z^2}\right)+\kappa\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)-2\kappa\phi_1$$
$$=\rho j\left(\frac{\partial^2 \phi_1}{\partial t^2}+(\Omega_2\dot{\phi_3}-\Omega_3\dot{\phi_2})\right), \tag{5.14}$$

$$(\alpha+\beta)\left(\frac{\partial^2 \phi_1}{\partial y\partial x}+\frac{\partial^2 \phi_2}{\partial y^2}+\frac{\partial^2 \phi_3}{\partial y\partial z}\right)+\gamma\left(\frac{\partial^2 \phi_2}{\partial x^2}+\frac{\partial^2 \phi_2}{\partial y^2}+\frac{\partial^2 \phi_2}{\partial z^2}\right)+\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2\kappa\phi_2$$
$$=\rho j\left(\frac{\partial^2 \phi_2}{\partial t^2}+(\Omega_3\dot{\phi_1}-\Omega_1\dot{\phi_3})\right), \tag{5.15}$$

$$(\alpha+\beta)\left(\frac{\partial^2 \phi_1}{\partial z\partial x}+\frac{\partial^2 \phi_2}{\partial z\partial y}+\frac{\partial^2 \phi_3}{\partial z^2}\right)+\gamma\left(\frac{\partial^2 \phi_3}{\partial x^2}+\frac{\partial^2 \phi_3}{\partial y^2}+\frac{\partial^2 \phi_3}{\partial z^2}\right)+\kappa\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)-2\kappa\phi_3$$
$$=\rho j\left(\frac{\partial^2 \phi_3}{\partial t^2}+(\Omega_1\dot{\phi_2}-\Omega_2\dot{\phi_1})\right), \tag{5.16}$$

$$K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=(1+\tau D_\tau)\left(\rho C_e\dot{T}+\gamma' T_0\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right) \tag{5.17}$$

where, (u, v, w), (ϕ_1, ϕ_2, ϕ_3), (J_1, J_2, J_3), (H_1, H_2, H_3) and $(\Omega_1, \Omega_2, \Omega_3)$ are the components of displacement vector $\overrightarrow{u}$, microrotation vector $\overrightarrow{\phi}$, current density vector $\overrightarrow{J}$, magnetic field vector $\overrightarrow{H}$ and rotation vector, respectively.

5.3 Formulation and Solution of the problem

A generalized micropolar thermoelastic medium is considered, which is perfectly conducting, homogeneous, and isotropic. In addition to this, $\overrightarrow{H_0}$ is also permeated along the y-axis. The origin of a rectangular cartesian co-ordinate system (x, y, z) is taken at any point on the plane surface of half-space z = 0, as shown in Fig 5.1. As we are considering a 2D plane strain problem, $\overrightarrow{u}$, $\overrightarrow{\Omega}$, and $\overrightarrow{\phi}$ are taken as

$$\overrightarrow{u} = (u, 0, w), \quad \overrightarrow{\Omega} = (0, \Omega, 0), \quad \overrightarrow{\phi} = (0, \phi_2, 0), \quad u(x, z, t), \quad and \quad w(x, z, t). \tag{5.18}$$

Using equation (5.18) in equations (5.1)-(5.3), we get

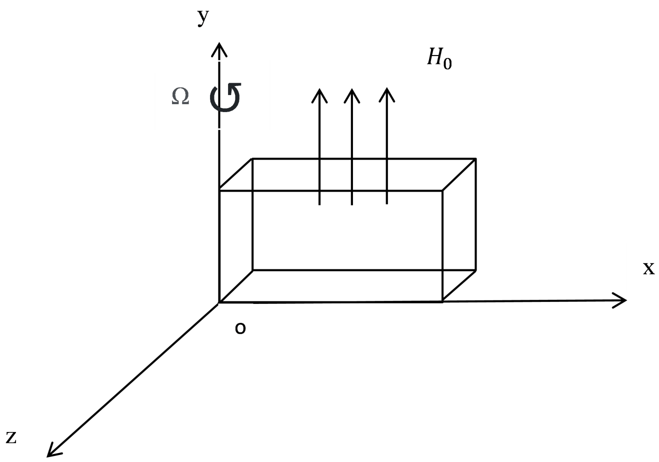

FIGURE 5.1: Rotating material geometry

$$\overrightarrow{E} = \mu_0 H_0(\dot{w}, 0, -\dot{u}), \tag{5.19}$$
$$\overrightarrow{h} = -H_0(0, e, 0), \tag{5.20}$$
$$\overrightarrow{J} = ((H_0 e_{,z} - \varepsilon_0\mu_0 H_0\ddot{w}), 0, (-H_0 e_{,x} + \varepsilon_0\mu_0 H_0 \ddot{u})), \tag{5.21}$$

where

$$e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}.$$

represents the cubical dilatation.

Now, simplifying equation (5.9), we get

$$\overrightarrow{F} = (\mu_0 H_0^2(e_{,x} - \varepsilon_0\mu_0\ddot{u}), 0, \mu_0 H_0^2(e_{,z} - \varepsilon_0\mu_0\ddot{w})). \tag{5.22}$$

The following components are obtained after using the expression (5.18) in (5.11)-(5.17) and (5.7)-(5.8),

$$(\lambda + 2\mu + \kappa)\frac{\partial^2 u}{\partial x^2} + (\mu + \kappa)\frac{\partial^2 u}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 w}{\partial x \partial z} - \kappa\frac{\partial \phi_2}{\partial z} + \mu_0 H_o^2(e_x - \varepsilon_0\mu_0\ddot{u}) - \nu\frac{\partial T}{\partial x}$$
$$= \rho(\ddot{u} - \Omega^2 u + 2\Omega\dot{w}), \tag{5.23}$$

$$(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial z}+(\lambda+2\mu+\kappa)\frac{\partial^2 w}{\partial z^2}+(\mu+\kappa)\frac{\partial^2 w}{\partial x^2}+\kappa\frac{\partial \phi_2}{\partial x}+\mu_0 H_0^2(e_z-\varepsilon_0\mu_0\ddot{w})-\nu\frac{\partial T}{\partial z}$$
$$=\rho(\ddot{w}-\Omega^2 w-2\Omega\dot{u}), \tag{5.24}$$
$$\gamma\nabla^2\phi_2+\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2\kappa\phi_2=\rho j\ddot{\phi_2}. \tag{5.25}$$
$$K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=(1+\tau D_\tau)\left(\rho C_e\dot{T}+\gamma^{'}T_0\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)\right) \tag{5.26}$$
$$\sigma_{xx}=(\lambda+2\mu+\kappa)\frac{\partial u}{\partial x}+\lambda\frac{\partial w}{\partial z}-\nu T, \tag{5.27}$$
$$\sigma_{yy}=\lambda e-\nu T, \tag{5.28}$$
$$\sigma_{zz}=\lambda\frac{\partial u}{\partial x}+(\lambda+2\mu+\kappa)\frac{\partial w}{\partial z}-\nu T, \tag{5.29}$$
$$\sigma_{xz}=\mu\frac{\partial u}{\partial z}+(\mu+\kappa)\frac{\partial w}{\partial x}+\kappa\phi_2, \tag{5.30}$$
$$\sigma_{zx}=\mu\frac{\partial w}{\partial x}+(\mu+\kappa)\frac{\partial u}{\partial z}-\kappa\phi_2, \tag{5.31}$$
$$m_{xy}=\gamma\frac{\partial \phi_2}{\partial x}, \tag{5.32}$$
$$m_{zy}=\gamma\frac{\partial \phi_2}{\partial z}. \tag{5.33}$$

The below non-dimensional quantities are introduced in equations (5.23)-(5.33),

$$(x',z')=\frac{\bar{\omega}}{c_1}(x,z),\ (u',w')=\frac{\rho c_1\bar{\omega}}{\nu T_0}(u,w),\ {\sigma'}_{i,j}=\frac{\sigma_{ij}}{\nu T_0},\ (\tau',t^{'})=\bar{\omega}(\tau,t),\ T'=\frac{T}{T_0},$$
$${\phi_2}'=\frac{\rho c_1^2}{\nu T_0}\phi_2,\ \Omega^{'}=\frac{\Omega}{\bar{\omega}},\ m_{ij}^{'}=\frac{\bar{\omega}}{c_1\nu T_0}m_{ij}, \tag{5.34}$$

we obtain (dropping the dashes for convenience)

$$\nabla^2 u+a_1\frac{\partial e}{\partial x}-a_2\frac{\partial \phi_2}{\partial z}-a_3\frac{\partial T}{\partial x}=a_4\frac{\partial^2 u}{\partial t^2}-a_5 u+a_6\frac{\partial w}{\partial t}, \tag{5.35}$$
$$\nabla^2 w+a_1\frac{\partial e}{\partial z}+a_2\frac{\partial \phi_2}{\partial x}-a_3\frac{\partial T}{\partial z}=a_4\frac{\partial^2 w}{\partial t^2}-a_5 w-a_6\frac{\partial u}{\partial t}, \tag{5.36}$$
$$\nabla^2\phi_2+a_7\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2a_7\phi_2=a_8\frac{\partial^2 \phi_2}{\partial t^2}, \tag{5.37}$$
$$\sigma_{xx}=\frac{1}{\rho c_1^2}\left(\lambda e+(2\mu+\kappa)\frac{\partial u}{\partial x}\right)-T, \tag{5.38}$$
$$\sigma_{yy}=\frac{\lambda}{\rho c_1^2}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)-T, \tag{5.39}$$
$$\sigma_{zz}=\frac{1}{\rho c_1^2}\left[\lambda e+(2\mu+\kappa)\frac{\partial w}{\partial z}\right]-T, \tag{5.40}$$
$$\sigma_{zx}=\frac{1}{\rho c_1^2}\left((\mu+\kappa)\frac{\partial u}{\partial z}+\mu\frac{\partial w}{\partial x}-\kappa\phi_2\right), \tag{5.41}$$
$$\sigma_{xz}=\frac{1}{\rho c_1^2}\left(\mu e+\kappa\frac{\partial w}{\partial x}+\kappa\phi_2\right) \tag{5.42}$$

$$m_{xy} = \alpha_6 \frac{\partial \phi_2}{\partial x}, \tag{5.43}$$

$$\alpha_3 \nabla^2 T = (\bar{\omega} + \tau D_\tau)\left(\alpha_4 \frac{\partial T}{\partial t} + \alpha_5 \frac{\partial e}{\partial t}\right) \tag{5.44}$$

where, $a_1 = \frac{\lambda+\mu}{\mu+\kappa} + \alpha_1$, $a_2 = \frac{\kappa}{\mu+\kappa}$, $a_3 = \frac{\lambda+2\mu+\kappa}{\mu+\kappa}$, $a_4 = \frac{\lambda+2\mu+\kappa}{\mu+\kappa}(1+\alpha_2)$, $a_5 = \Omega^2 a_3$, $a_6 = 2\Omega a_3$, $a_7 = \frac{\kappa {c_1}^2}{\gamma \bar{\omega}^2}$, $a_8 = \frac{\rho i j {c_1}^2}{\gamma}$, $\alpha_1 = \frac{\mu_0 {H_0}^2}{\mu+\kappa}$, $\alpha_2 = \frac{\epsilon_0 {\mu_0}^2 {H_0}^2}{\rho}$, $\alpha_3 = \frac{K \bar{\omega}^2 T_0}{{c_1}^2}$, $\alpha_4 = \rho C_e T_0$, $\alpha_5 = \frac{{T_0}^2 \nu \gamma'}{\rho {c_1}^2}$.

Now, to obtain the solution, we now introduce the displacement potentials $q(x,z,t)$ and $\psi(x,z,t)$, as described in [139]

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \; w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{5.45}$$

Using equation (5.45) in equations (5.35), (5.37), (5.44), we obtain

$$\left((a_1+1)\nabla^2 - a_4 \frac{\partial^2}{\partial t^2} + a_5\right) q - a_3 T + a_6 \frac{\partial \psi}{\partial t} = 0, \tag{5.46}$$

$$\left(\nabla^2 - a_4 \frac{\partial^2}{\partial t^2} + a_5\right) \psi - a_2 \phi_2 - a_6 \frac{\partial q}{\partial t} = 0, \tag{5.47}$$

$$\left(\nabla^2 - 2a_7 - a_8 \frac{\partial^2}{\partial t^2}\right) \phi_2 - a_7 \nabla^2 \psi = 0, \tag{5.48}$$

$$\alpha_3 \nabla^2 T = (\bar{\omega} + \tau D_\tau)\left(\alpha_4 \frac{\partial T}{\partial t} + \alpha_5 \nabla^2 \frac{\partial q}{\partial t}\right) \tag{5.49}$$

5.4 Normal mode analysis

In term of normal modes, the solution of the considered physical variables can be decomposed as

$$[u, w, T, \sigma_{ij}, q, \psi, \phi_2, m_{ij}](x,z,t) = [\bar{u}, \bar{w}, \bar{T}, \bar{\sigma}_{ij}, \bar{q}, \bar{\psi}, \bar{\phi}_2, \bar{m}_{ij}](z) exp(bt + iax). \tag{5.50}$$

Use of equation (5.50) transforms equations (5.46)-(5.49) to

$$[A_1 D^2 - A_2]\bar{q} + A_3 \bar{\psi} - A_4 \bar{T} = 0, \tag{5.51}$$

$$[D^2 - A_5]\bar{\psi} - A_6 \bar{q} - A_7 \bar{\phi}_2 = 0, \tag{5.52}$$

$$[D^2 - A_8]\bar{\phi}_2 - [A_9 D^2 - A_{10}]\bar{\psi} = 0, \tag{5.53}$$

$$[A_{11} D^2 - A_{12}]\bar{T} - [A_{13} D^2 - A_{14}]\bar{q} = 0, \tag{5.54}$$

where, $D = \frac{\partial}{\partial z}$, $A_1 = 1 + a_1$, $A_2 = a^2 + a_1 a^2 + a_4 b^2 - a_5$, $A_3 = a_6 b$, $A_4 = a_3$, $A_5 = a^2 + a_4 b^2 - a_5$, $A_6 = a_6 b$, $A_7 = a_2$, $A_8 = a^2 + 2a_7 + a_8 b^2$, $A_9 = a_7$, $A_{10} = A_9 a^2$, $A_{11} = \alpha_3$, $A_{12} = \alpha_3 a^2 + \alpha_4 b(\bar{\omega} + \tau G(\tau, b))$, $A_{13} = \alpha_5 b(\bar{\omega} + \tau G(\tau, b))$, $A_{14} = \alpha_5 b a^2(\bar{\omega} + \tau G(\tau, b))$.

and $G(\tau,b)=\frac{-(b^2(m^2-2n+1)\tau^2+2b\tau(m^2-n)+2m^2)exp[b(t-\tau)]+(b^2\tau^2-2bn\tau+2m^2)exp(bt)}{b^2\tau^2}$,

and the kernel function $K(t-r)$ is defined as [61]

$$K(t-r)=1-\frac{2n}{\tau}(t-r)+\frac{m^2}{\tau^2}(t-r^2)=\begin{cases}1; if, m=n=0\\ 1-\frac{(t-r)}{\tau}; if, m=0, n=\frac{1}{2}\\ 1-(t-r); if, m=0, n=\frac{\tau}{2}\\ \left(1-\frac{t-r}{\tau}\right)^2; if, m=n=1.\end{cases}$$

After some simplification, equations (5.51)-(5.54) become

$$[D^4-A^{'}D^2+A^{''}]\bar{\psi}+[A^{'''}D^2-A^{''''}]\bar{q}=0, \tag{5.55}$$

$$[B^{'}D^4-B^{''}D^2+B^{'''}]\bar{q}+[B^{''''}D^2-B^{'''''}]\bar{\psi}=0, \tag{5.56}$$

where, $A^{'}=(A_8+A_5-A_7A_9)$, $A^{''}=A_5A_8+A_7A_{10}$, $A^{'''}=A_6$, $A^{''''}=A_8A_6$, $B^{'}=A_{11}A_1$, $B^{''}=A_{11}A_2+A_{12}A_1+A_4A_3$, $B^{'''}=A_{12}A_2-A_4A_{14}$, $B^{''''}=A_{11}A_3$, $B^{'''''}=A_{12}A_3$.

Simplifying equations (5.55) and (5.56), we obtain,

$$[D^8-AD^6+BD^4-CD^2+F]\bar{\psi}(z)=0, \tag{5.57}$$

Similarly,

$$[D^8-AD^6+BD^4-CD^2+F][\bar{\psi}(z),\bar{q}(z),\bar{\phi}_2(z),\bar{T}(z)]=0, \tag{5.58}$$

where, $A=\frac{B^{''}+A^{'}B^{'}}{B^{'}}$, $B=\frac{B^{'''}+A^{'}B^{''}+A^{''}B^{'}+A^{'''}B^{''''}}{B^{'}}$, $C=\frac{A^{'}B^{'''}+A^{''}B^{''}+A^{'''}B^{'''''}+A^{''''}B^{''''}}{B^{'}}$, $F=\frac{A^{''}B^{'''}+A^{''''}B^{'''''}}{B^{'}}$.

Rewriting equation (5.58), in the factored form as

$$[(D^2-{k_1}^2)(D^2-{k_2}^2)(D^2-{k_3}^2)(D^2-{k_4}^2)][\bar{\psi}(z),\bar{q}(z),\bar{\phi}_2(z),\bar{T}(z)]=0, \tag{5.59}$$

where, ${k_n}^2(n=1,2,3,4)$ represents the characteristic roots of the equation (5.59).

The general solution of equation (5.59), has the form

$$\bar{\psi}(z)=\sum_{n=1}^{4}P_ne^{-k_nz}, \tag{5.60}$$

$$\bar{\phi}_2(z)=\sum_{n=1}^{4}{P_n}'e^{-k_nyz}, \tag{5.61}$$

$$\bar{q}(z) = \sum_{n=1}^{4} P_n{}'' e^{-k_n z}, \tag{5.62}$$

$$\bar{T}(z) = \sum_{n=1}^{4} P_n{}''' e^{-k_n z}, \tag{5.63}$$

Here, P_n, $P_n{}'$, $P_n{}''$, $P_n{}'''$ are parameters that depends on a and b.
Using equations (5.60)-(5.63) in equations (5.51)-(5.54), we get

$$\bar{\phi}_2(z) = \sum_{n=1}^{4} L_{1n} P_n e^{-k_n z}, \tag{5.64}$$

$$\bar{q}(z) = \sum_{n=1}^{4} L_{2n} P_n e^{-k_n z}, \tag{5.65}$$

$$\bar{T}(z) = \sum_{n=1}^{4} L_{3n} P_n e^{-k_n z}, \tag{5.66}$$

where, $L_{1n} = \frac{A_9 {k_n}^2 - A_{10}}{{k_n}^2 - A_8}$, $L_{2n} = \left[\frac{(k_n^2 - A_5)(k_n^2 - A_8)}{A_6(A_9 k_n^2 - A_{10})} - \frac{A_7}{A_6}\right] L_{1n}$, $L_{3n} = \left[\frac{A_{13} k_n^2 - A_{14}}{A_{11} k_n^2 - A_{12}}\right] L_{2n}$.

In general, equation (5.60) along with equations (5.64)-(5.66), can be written as

$$(\bar{\psi}, \bar{\phi}_2, \bar{q}, \bar{T})(z) = \sum_{n=1}^{4} (1, L_{1n}, L_{2n}, L_{3n}) P_n exp(-k_n z). \tag{5.67}$$

Using equations (5.50) and (5.67) in equation (5.45), we obtain the components of displacement as

$$\bar{u} = \sum_{n=1}^{4} P_n L_{4n} e^{-k_n z}, \tag{5.68}$$

$$\bar{w} = \sum_{n=1}^{4} P_n L_{5n} e^{-k_n z}, \tag{5.69}$$

where, $L_{4n} = [iaL_{2n} - k_n]$ and $L_{5n} = [-L_{2n} k_n - ia]$.
Using equations (5.50) and (5.67) into equations (5.38)-(5.43), we obtain the force stress components

$$\bar{\sigma}_{xx} = \sum_{n=1}^{4} P_n L_{6n} e^{-k_n z}, \tag{5.70}$$

$$\bar{\sigma}_{zz} = \sum_{n=1}^{4} P_n L_{7n} e^{-k_n z}, \tag{5.71}$$

$$\bar{\sigma}_{xz} = \sum_{n=1}^{4} P_n L_{8n} e^{-k_n z}, \tag{5.72}$$

$$\bar{m}_{xy} = \sum_{n=1}^{4} P_n L_{9n} e^{-k_n z}, \tag{5.73}$$

where,

$$L_{6n} = \frac{1}{\rho c_1^2}\left[\lambda(L_{4n} ia - L_{5n} k_n) + (2\mu + \kappa) ia L_{4n}\right] - L_{3n},$$
$$L_{7n} = \frac{1}{\rho c_1^2}[(\lambda ia L_{4n} - k_n L_{5n}) - (2\mu + \kappa) k_n L_{5n}] - L_{3n},$$
$$L_{8n} = \frac{1}{\rho c_1^2}[\mu(L_{4n} ia - k_n L_{5n}) + \kappa(L_{5n} ia + L_{1n})],$$
$$L_{9n} = \alpha_6 ia L_{1n}$$

5.5 Boundary conditions

The following boundary conditions have been taken at $z = 0$ to figure out the P_n $(n = 1, 2, 3, 4)$ parameters

Thermal boundary conditions:

A thermal shock has been employed to the plane's isothermal boundary $z = 0$, which is given as

$$T = B_1 e^{bt+iax}, \tag{5.74}$$

Mechanical boundary conditions:

The normal stress condition (which is mechanically stressed by constant force B_2), is taken as

$$\sigma_{zz} = -B_2 e^{bt+iax}, \tag{5.75}$$

The tangential stress σ_{xz} condition (which is stress free) is taken as

$$\sigma_{xz} = 0, \tag{5.76}$$

The couple stress condition (the couple stress is constant in x-direction) is taken as

$$m_{xy} = 0. \tag{5.77}$$

Now, using equation (5.50) along with equations (5.67), (5.71), (5.72) in equation (5.74)-(5.77), we get

$$\sum_{n=1}^{4} L_{3n}P_n = B_1, \tag{5.78}$$

$$\sum_{n=1}^{4} L_{7n}P_n = -B_1, \tag{5.79}$$

$$\sum_{n=1}^{4} L_{8n}P_n = 0, \tag{5.80}$$

$$\sum_{n=1}^{4} L_{9n}P_n = 0. \tag{5.81}$$

The equations (5.78)-(5.81) have been solved for P_n, $(n = 1, 2, 3, 4)$ to figure out the solution of the required problem and by making use of Inverse matrix method, which is as follows:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} L_{31} & L_{32} & L_{33} & L_{34} \\ L_{71} & L_{72} & L_{73} & L_{74} \\ L_{81} & L_{82} & L_{83} & L_{84} \\ L_{91} & L_{92} & L_{93} & L_{94} \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\ -B_2 \\ 0 \\ 0 \end{bmatrix}.$$

5.6 Validity of the Problem

When the micropolar and magnetic effects are ignored, we obtain the following results

$$(\bar{\psi}, \bar{q}, \bar{T})(z) = \sum_{n=1}^{4} (1, V_{1n}, V_{2n}) T_n exp(-k_n z).$$

where, $V_{1n} = -\frac{-(k_n^2 - X_4)}{X_5}$, $V_{2n} = 1 - \frac{(k_n^2-1)(k_n^2-X_4)}{X_3 X_5}$,

$X_3 = \frac{a_6' b}{a_1' + 1}$, $X_5 = a_6' b$, $a_1' = \frac{\lambda+\mu}{\mu} + \alpha_1$, $a_6' = 2\Omega\frac{\lambda+2\mu}{\mu}$

which are in sync with the results explained in the study [79] considered in the context of the generalized thermoelasticity with memory effect.

5.7 Numerical results and Discussion

Analysis has been conducted for aluminium epoxy, using [140]. A graphical analysis of variation in displacement components, stress components, couple stress components,

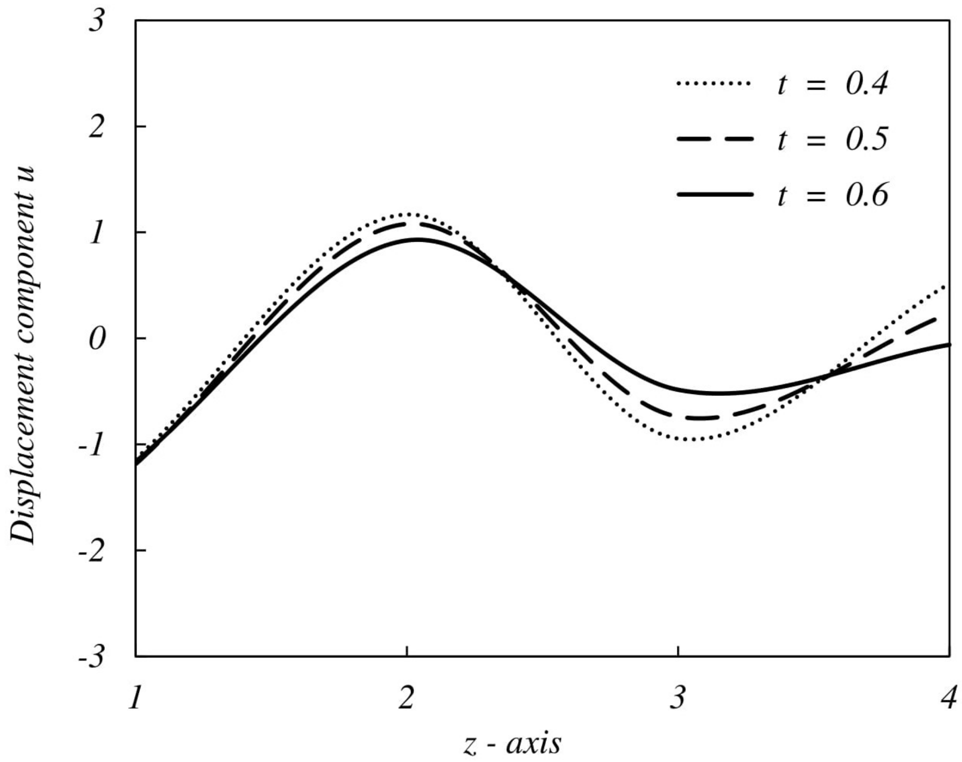

FIGURE 5.2: Variation of u at distinct values of t and for $K(t-r) = 1 - \frac{t-r}{\tau}$

and temperature distribution has been done. Results have been compared for three distinct values of t viz., $t = 0.4$, $t = 0.5$, and $t = 0.6$ w.r.t distance z, as shown in below figures.

From the figure 5.2, it is clear that the displacement component u is influenced by the time t. The maximum displacement distribution is observed and is attained for $t = 0.4$. It reduces when the time is increased to $t = 0.5$. For $t = 0.6$, we observe the least displacement. From the figure 5.3, it is clearly observed that the variation of w is same as that of u for different values of time t. From figure 5.4, it is observed that the stress component σ_{xz} is maximum for $t = 0.6$, and it reduces with decrease in time. As such, the least tangential stress is observed for $t = 0.4$.

From figure 5.5, we observe that the normal stress σ_{zz} is maximum for $t = 0.6$. Then it reduces when the value of t is reduced, as shown in figure. Figure 5.6 demonstrates that the maximum couple stress m_{xy} is attained for $t = 0.4$.

From figure 5.7, we noticed that the temperature distribution T is highest for $t = 0.6$ and it reduces when time is reduced, i.e., for $t = 0.5$. Also it can be seen that T is smallest for $t = 0.4$.

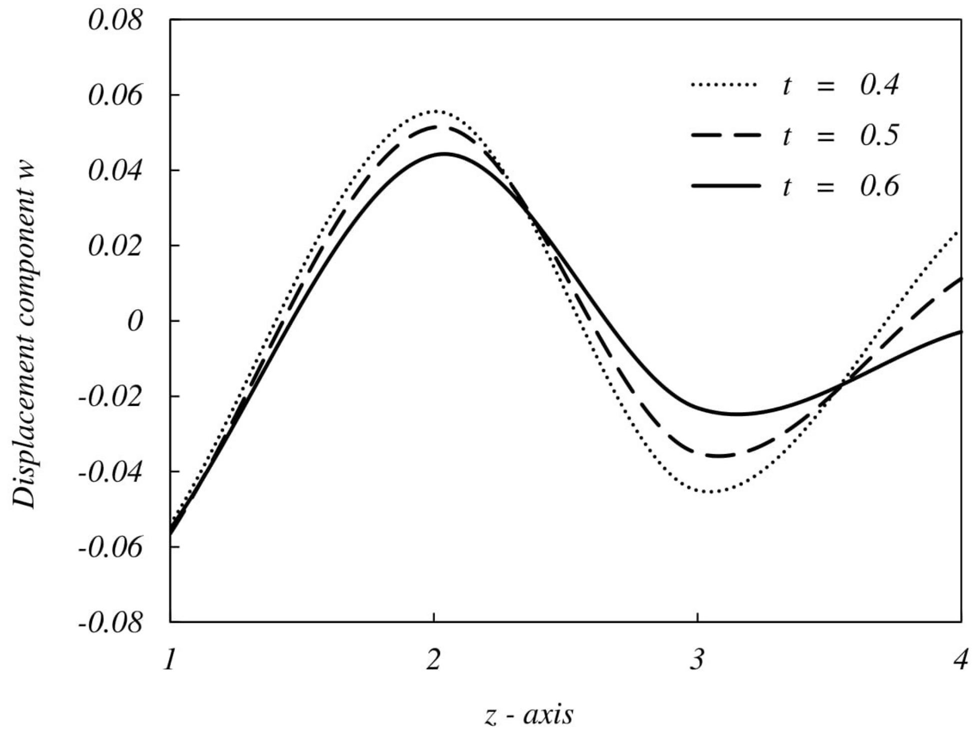

FIGURE 5.3: Variation of w at distinct values of t and for $K(t-r) = 1 - \frac{t-r}{\tau}$

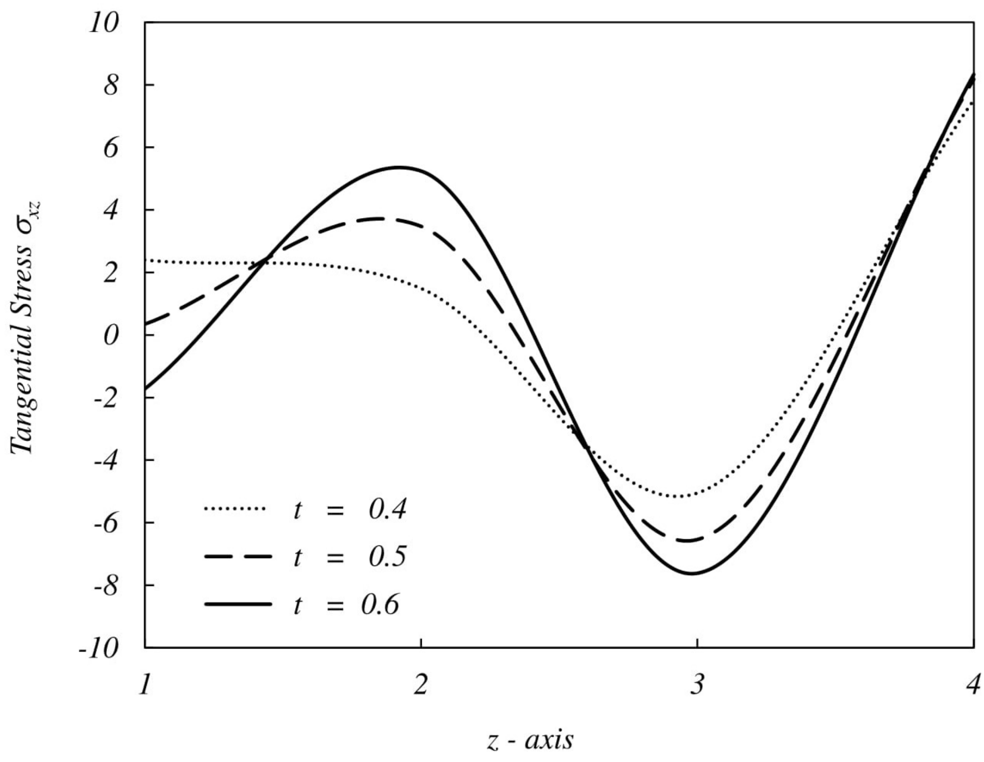

FIGURE 5.4: Variation of σ_{xz} at distinct values of t and for $K(t-r) = 1 - \frac{t-r}{\tau}$

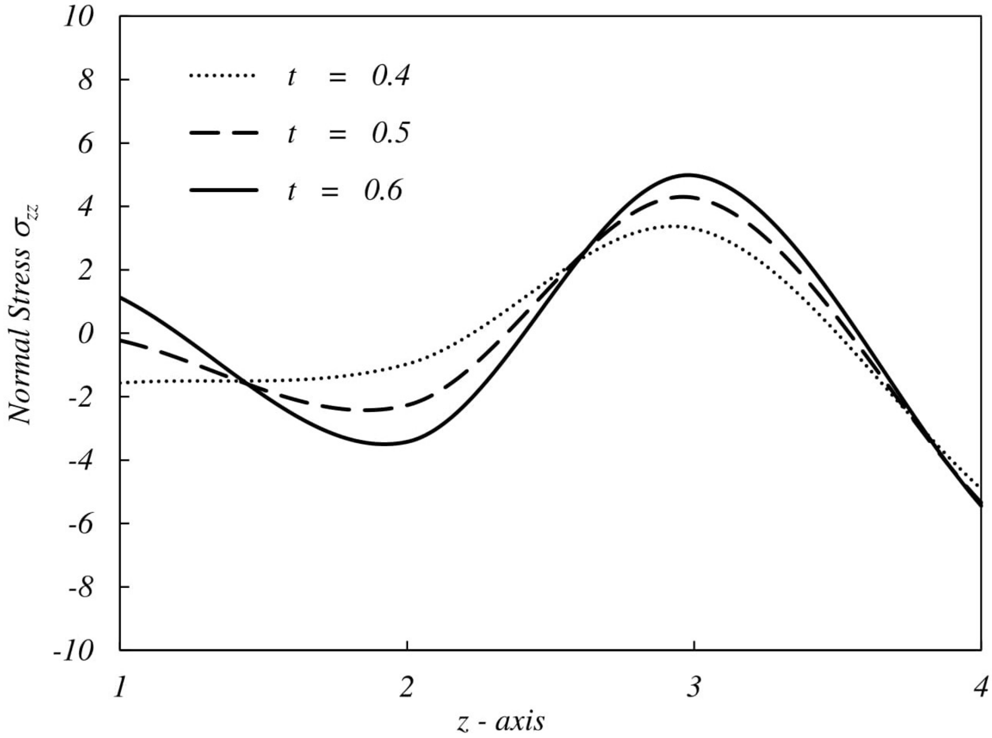

FIGURE 5.5: Variation of σ_{zz} at distinct values of t and for $K(t-r) = 1 - \frac{t-r}{\tau}$

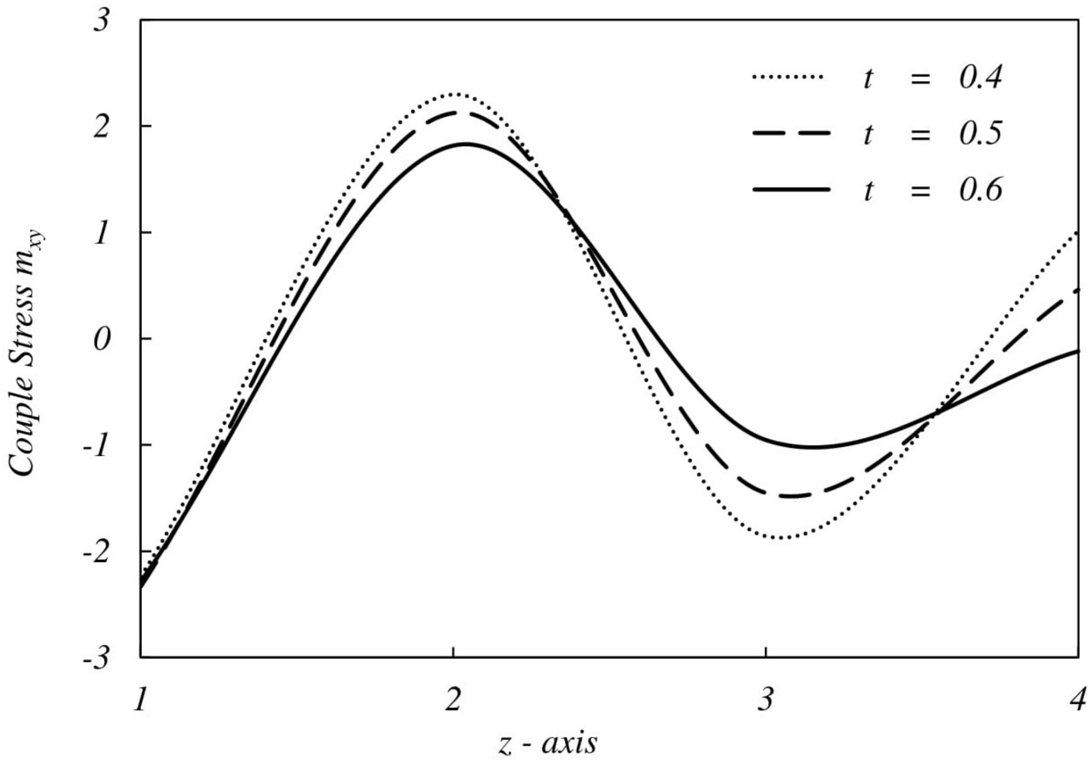

FIGURE 5.6: Variation of m_{xy} at distinct values of t and for $K(t-r) = 1 - \frac{t-r}{\tau}$

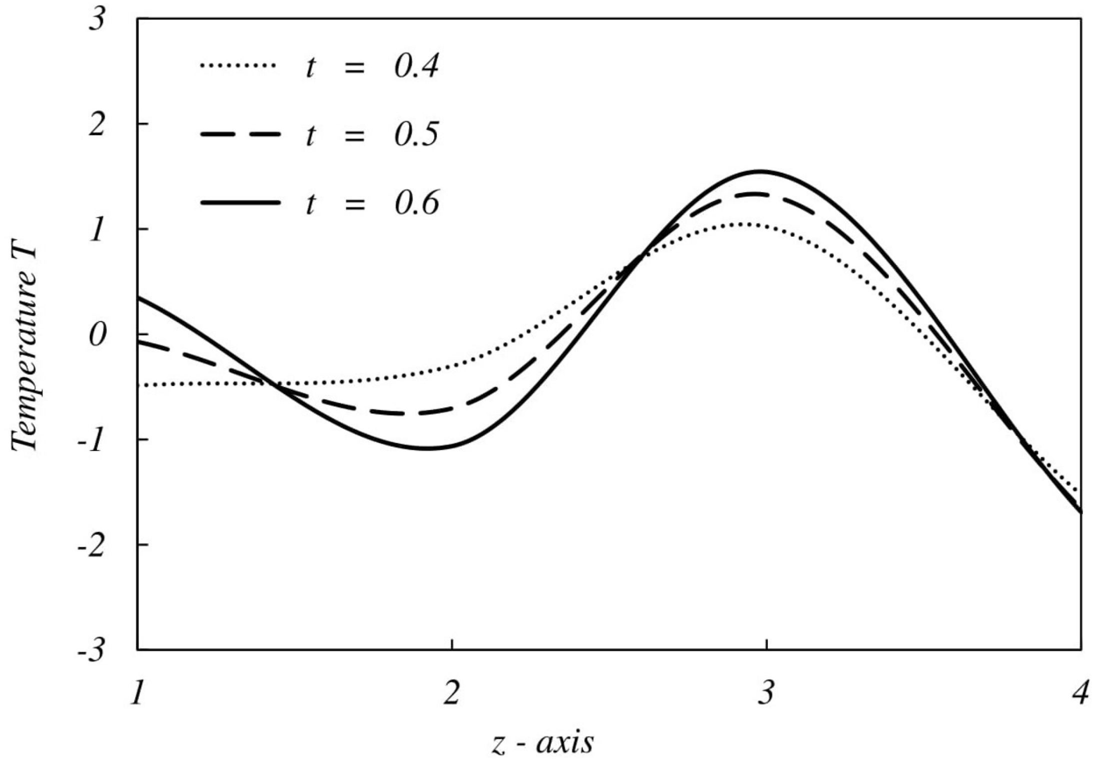

FIGURE 5.7: Variation of T at distinct values of t and for $K(t-r) = 1 - \frac{t-r}{\tau}$

5.8 Conclusion

Potential displacement approach along with the normal mode technique has been utilized for obtaining the solution of the required 2D problem in magneto micropolar thermoelasticity using memory-dependent derivatives. Significant variation is shown by different physical quantities, viz., displacement components, stress components, couple stress components, and temperature distribution for distinct values of time. Matlab software and MS-Excel have been used for graphical representation.

Following observations has been made in this study:

1. A new mathematical model has been developed in magneto-micropolar thermoelasticity in the framework of a heat conduction equation having a memory-dependent derivative for a rotating medium.
2. The time parameter has a strong impact on the elastic response of all physical quantities except u.

Chapter 6

Photo-thermo-elastic interactions in micropolar generalized thermoelasticity theory in the framework of Green-Naghdi theory

6.1 Introduction

Semiconducting materials serve an important role in modern engineering, and the electrical conductivity of these materials lies between conductors and insulators. One of the most important characteristics of semiconducting materials is that they possess optical properties. Therefore, when they are subjected to sunlight or a laser beam, some of the energy will be soaked up, whereas some of the energy will be liberated in the form of heat or thermal energy. This phenomenon is known as the photothermal effect. This effect is often utilized to measure thermal properties of materials, especially in the semiconductor industry. Furthermore, when a laser beam or sunlight is imposed on an elastic medium, a free charge carrier emerges on the surface, which creates the plasma waves. In other words, plasma waves are developed due to the excited electrons which move randomly on the surface of semiconducting material. The photothermal method was first established by Gordon et al. [117] when an intracavity spherical semiconducting material sample was subjected to a beam of laser. Employing spectroscopy analysis, Kreuzer [118] investigated photoacoustic waves generated by a laser source of light. The equations of plasmaelastic and thermoelastic waves were investigated by Todorovic [119] in a semiconducting plate. A thermoelastic interaction was considered by Abo-Dahab and Lotfy [120] in order to investigate the photothermal effects on a semiconductor structure. The photothermal interactions were investigated by Hobiny and Abbas [121] in a 2D semiconducting half-space by employing Green and Naghdi theory. The photo-thermal waves were studied by Mabrouk et al. [122] in a magneto-rotating semiconducting elastic medium by making use of dual-phase-lag model. A novel mathematical model of a rotating elastic semiconducting medium is developed by Lotfy [123] in the context of photothermal excitation. Another mathematical model was examined by Raddadi et al. [124] in the microstretch photo-thermoelasticity theory.

The current field of study requires more work due to its numerous applications in the scientific field. At present, a very few researchers are working in this field. So in the current study, a new mathematical model has been developed in the micropolar theory of thermoelasticity which includes the photothermal effect. The components of displacements, force and couple stresses, carrier density, as well as the temperature distribution are obtained by utilizing the technique of Laplace and Fourier transforms along with the potential displacement approach.

6.2 Basic equations

The field equations of motion and constitutive relations for the micropolar theory of generalized thermoelasticity in the context of photothermal theory is given by

$$(\lambda+\mu)\nabla(\nabla.\overrightarrow{u})+(\mu+\kappa)\nabla^2\overrightarrow{u}+\kappa(\nabla\times\overrightarrow{\phi})-\nu\nabla T-\gamma_n\nabla\overrightarrow{N}=\rho\ddot{\overrightarrow{u}}, \tag{6.1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\overrightarrow{\phi})-\gamma\nabla\times(\nabla\times\overrightarrow{\phi})+\kappa\nabla\times\overrightarrow{u}-2\kappa\overrightarrow{\phi}=\rho j\frac{\partial^2\overrightarrow{\phi}}{\partial t^2}, \tag{6.2}$$

$$\sigma_{ij}=(\lambda u_{r,r}-\nu T-\gamma_n N)\delta_{ij}+\mu(u_{i,j}+u_{j,i})+\kappa(u_{j,i}-\varepsilon_{ijr}\phi_r), \tag{6.3}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}. \tag{6.4}$$

The heat conduction equation in the presence of photothermal theory under Green-Naghdi theory as defined in [121], is given by

$$\left(K^*+K\frac{\partial}{\partial t}\right)\nabla^2 T+\frac{E_g}{\tau_p}N-\rho C_e\frac{\partial^2 T}{\partial t^2}-\gamma^{'}T_0\frac{\partial^2 e}{\partial t^2}=0. \tag{6.5}$$

The coupling between plasma and thermoelastic waves, as defined in [121], can be written as

$$D_e\nabla^2 N-\frac{\partial N}{\partial t}-\frac{N}{\tau_p}+\frac{\delta T}{\tau_p}=0, \tag{6.6}$$

The equations of motion (6.1)-(6.2) along with the equations (6.5) and (6.6) in Cartesian coordinates (x, y, z) in component form can be written as

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x \partial y}+\frac{\partial^2 w}{\partial x \partial z}\right)+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\kappa\left(\frac{\partial \phi_3}{\partial y}-\frac{\partial \phi_2}{\partial z}\right)-$$
$$\nu\frac{\partial T}{\partial x}-\gamma_n\frac{\partial N}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{6.7}$$
$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial y \partial x}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y \partial z}\right)+(\mu+\kappa)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\kappa\left(\frac{\partial \phi_1}{\partial z}-\frac{\partial \phi_3}{\partial x}\right)-$$
$$\nu\frac{\partial T}{\partial y}-\gamma_n\frac{\partial N}{\partial y}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{6.8}$$
$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial z \partial x}+\frac{\partial^2 v}{\partial z \partial y}+\frac{\partial^2 w}{\partial z^2}\right)+(\mu+\kappa)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\kappa\left(\frac{\partial \phi_2}{\partial x}-\frac{\partial \phi_1}{\partial y}\right)-$$
$$\nu\frac{\partial T}{\partial z}-\gamma_n\frac{\partial N}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}, \tag{6.9}$$
$$(\alpha+\beta)\left(\frac{\partial^2 \phi_1}{\partial x^2}+\frac{\partial^2 \phi_2}{\partial x \partial y}+\frac{\partial^2 \phi_3}{\partial x \partial z}\right)+\gamma\left(\frac{\partial^2 \phi_1}{\partial x^2}+\frac{\partial^2 \phi_1}{\partial y^2}+\frac{\partial^2 \phi_1}{\partial z^2}\right)+\kappa\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)-2\kappa\phi_1$$
$$=\rho j\frac{\partial^2 \phi_1}{\partial t^2}, \tag{6.10}$$
$$(\alpha+\beta)\left(\frac{\partial^2 \phi_1}{\partial y \partial x}+\frac{\partial^2 \phi_2}{\partial y^2}+\frac{\partial^2 \phi_3}{\partial y \partial z}\right)+\gamma\left(\frac{\partial^2 \phi_2}{\partial x^2}+\frac{\partial^2 \phi_2}{\partial y^2}+\frac{\partial^2 \phi_2}{\partial z^2}\right)+\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2\kappa\phi_2$$
$$=\rho j\frac{\partial^2 \phi_2}{\partial t^2}, \tag{6.11}$$
$$(\alpha+\beta)\left(\frac{\partial^2 \phi_1}{\partial z \partial x}+\frac{\partial^2 \phi_2}{\partial z \partial y}+\frac{\partial^2 \phi_3}{\partial z^2}\right)+\gamma\left(\frac{\partial^2 \phi_3}{\partial x^2}+\frac{\partial^2 \phi_3}{\partial y^2}+\frac{\partial^2 \phi_3}{\partial z^2}\right)+\kappa\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)-2\kappa\phi_3$$
$$=\rho j\frac{\partial^2 \phi_3}{\partial t^2}, \tag{6.12}$$
$$\left(K^*+K\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)T+\frac{E_g}{\tau_p}N-\rho C_e\frac{\partial^2 T}{\partial t^2}-\gamma' T_0\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)$$
$$=0, \tag{6.13}$$
$$D_e\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)N-\frac{\partial N}{\partial t}-\frac{N}{\tau_p}+\frac{\delta T}{\tau_p}=0. \tag{6.14}$$

where, (u, v, w) and (ϕ_1, ϕ_2, ϕ_3) are the components of displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$, respectively.

6.3 Formulation and solution of the problem

A homogeneous, isotropic, elastic semiconducting material in micropolar generalized thermoelasticity is taken into consideration under the Green-Naghdi theory. The origin of a rectangular cartesian co-ordinate system (x, y, z) is taken at any point on the plane surface of half-space z = 0, as shown in Figure 6.1. For the 2D problem, the displacement vector $\overrightarrow{u}$ and microrotation vector $\overrightarrow{\phi}$ is given by

$$\overrightarrow{u} = (u, 0, w), \quad \overrightarrow{\phi} = (0, \phi_2, 0), \quad u = u(x, z, t), \quad w = w(x, z, t), \quad N = N(x, z, t)$$
$$\text{and} \quad T = T(x, z, t). \tag{6.15}$$

Using equation (6.15) in equations (6.7)-(6.14) and in (6.3)-(6.4), the following compo-

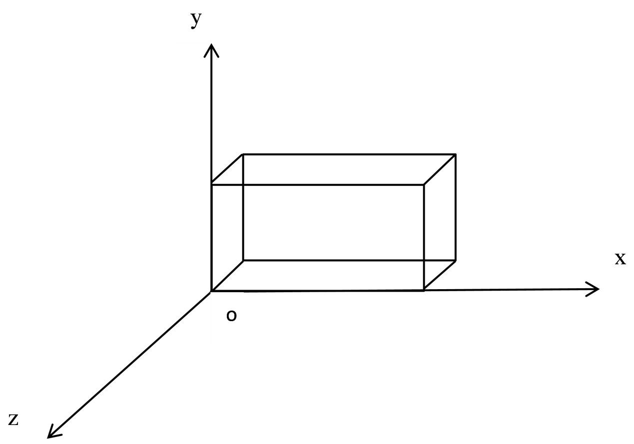

FIGURE 6.1: Material geometry

nents are obtained

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x \partial z}\right)+(\mu+\kappa)\nabla^2 u-\kappa\frac{\partial \phi_2}{\partial z}-\nu\frac{\partial T}{\partial x}-\gamma_n\frac{\partial N}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{6.16}$$

$$(\lambda+\mu)\left(\frac{\partial^2 u}{\partial z \partial x}+\frac{\partial^2 w}{\partial z^2}\right)+(\mu+\kappa)\nabla^2 w+\kappa\frac{\partial \phi_2}{\partial x}-\nu\frac{\partial T}{\partial z}-\gamma_n\frac{\partial N}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}, \tag{6.17}$$

$$\gamma\nabla^2\phi_2+\kappa\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2\kappa\phi_2=\rho j\frac{\partial^2 \phi_2}{\partial t^2}, \tag{6.18}$$

$$\left(K^*+K\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=-\frac{E_g}{\tau_p}N+\frac{\partial^2}{\partial t^2}\left(\rho C_e T+\gamma' T_0\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)\right), \tag{6.19}$$

$$D_e\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial z^2}\right)-\frac{\partial N}{\partial t}-\frac{N}{\tau_p}+\delta\frac{T}{\tau_p}=0. \tag{6.20}$$

$$\sigma_{xx}=(\lambda+2\mu+\kappa)\frac{\partial u}{\partial x}+\lambda\frac{\partial w}{\partial z}-\nu T-\gamma_n N, \tag{6.21}$$

$$\sigma_{zz}=(\lambda+2\mu+\kappa)\frac{\partial w}{\partial z}+\lambda\frac{\partial u}{\partial x}-\nu T-\gamma_n N, \tag{6.22}$$

$$\sigma_{xz}=\mu\frac{\partial u}{\partial z}+(\mu+\kappa)\frac{\partial w}{\partial x}+\kappa\phi_2, \tag{6.23}$$

$$\sigma_{zx}=\mu\frac{\partial w}{\partial x}+(\mu+\kappa)\frac{\partial u}{\partial z}-\kappa\phi_2, \tag{6.24}$$

$$m_{zy}=\gamma\frac{\partial \phi_2}{\partial z}, \tag{6.25}$$

$$m_{xy}=\gamma\frac{\partial \phi_2}{\partial x}, \tag{6.26}$$

Initial and Boundary conditions The initial conditions are as

$$u(x,z,0)=w(x,z,0)=T(x,z,0)=N(x,z,0)=0, \frac{\partial N(x,z,0)}{\partial t}=\frac{\partial T(x,z,0)}{\partial t}=$$
$$\frac{\partial u(x,z,0)}{\partial t}=\frac{\partial w(x,z,0)}{\partial t}=0 \tag{6.27}$$

Also, the boundary conditions on $x=0$, are defined as

$$\begin{cases} (i)\sigma_{xx}=\sigma_{xz}=m_{xy}=0, \text{(i.e., the bounding plane z = 0 is traction-free)}\\ (ii)-K\frac{\partial T(x,z,t)}{\partial x}=q_0\frac{t^2e^{-\frac{t}{t_p}}}{16t_p^2}H(\hat{a}-|z|),\\ \text{(i.e., the surface } x=0 \text{ is caused by the heating flux with the exponentially decaying pulses [125])}\\ (iii)D_e\frac{\partial N}{\partial x}-s_0N=0,\\ \text{(which is the boundary conditions of the carrier density)} \end{cases} \tag{6.28}$$

Now, the non-dimensional quantities are defined as follows

$$(x^{'},z^{'})=\frac{\omega^*}{c_1}(x,z),\quad (u^{'},w^{'})=\frac{\rho\omega^*c_1}{T_0\nu}(u,w),\quad T^{'}=\frac{T}{T_0},\quad \sigma_{ij}^{'}=\frac{\sigma_{ij}}{T_0\nu},\quad \phi_2^{'}=\frac{\rho c_1^2}{T_0\nu}\phi_2,$$
$$m_{ij}^{'}=\frac{\omega^*}{c_1T_0\nu}m_{ij},\quad (t^{'},\tau_p^{'},t_p^{'})=\omega^*(t,\tau_p,t_p),\quad N^{'}=\frac{N}{n_0},\quad q_0^{'}=\frac{q_0c_1}{\omega^*T_0K} \tag{6.29}$$

where, $\omega^*=\frac{\rho c_1^2c_e}{K},\quad c_1^2=\frac{\lambda+2\mu+\kappa}{\rho}$.
Using equation (6.29) in equations (6.16)-(6.26), we get (dropping the dashes for convenience)

$$\nabla^2u+a_1\frac{\partial e}{\partial x}-a_2\frac{\partial\phi_2}{\partial z}-a_3\frac{\partial T}{\partial x}-a_4\frac{\partial N}{\partial x}=a_3\frac{\partial^2u}{\partial t^2}, \tag{6.30}$$
$$\nabla^2w+a_1\frac{\partial e}{\partial z}+a_2\frac{\partial\phi_2}{\partial x}-a_3\frac{\partial T}{\partial z}-a_4\frac{\partial N}{\partial z}=a_3\frac{\partial^2w}{\partial t^2}, \tag{6.31}$$
$$\nabla^2\phi_2+a_5\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}-2\phi_2\right)=a_6\frac{\partial^2\phi_2}{\partial t^2}, \tag{6.32}$$
$$\sigma_{xx}=\frac{\partial u}{\partial x}+a_7\frac{\partial w}{\partial z}-T-a_8N, \tag{6.33}$$
$$\sigma_{zz}=\frac{\partial w}{\partial z}+a_7\frac{\partial u}{\partial x}-T-a_8N, \tag{6.34}$$
$$\sigma_{xz}=a_9\frac{\partial u}{\partial z}+a_{10}\frac{\partial w}{\partial x}+a_{11}\phi_2, \tag{6.35}$$
$$\sigma_{zx}=a_9\frac{\partial w}{\partial x}+a_{10}\frac{\partial u}{\partial z}-a_{11}\phi_2, \tag{6.36}$$
$$m_{zy}=a_{12}\frac{\partial\phi_2}{\partial z}, \tag{6.37}$$
$$m_{xy}=a_{12}\frac{\partial\phi_2}{\partial x}, \tag{6.38}$$

$$\left(a_{13}+\frac{\partial}{\partial t}\right)\nabla^2 T = a_{14}\frac{\partial^2 T}{\partial t^2} + a_{15}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) - a_{16}\frac{N}{\tau_p}, \tag{6.39}$$

$$\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial z^2}\right) = a_{17}\left(\frac{\partial N}{\partial t}+\frac{N}{\tau_p}\right) - a_{18}\frac{T}{\tau_p}. \tag{6.40}$$

Using equation (6.29) in equation (6.28), we get

$$\frac{\partial T}{\partial x} = -q_0\frac{t^2 e^{-\frac{t}{t_p}}}{16 t_p^2} H(\hat{a}-|z|) \quad ,$$

$$\frac{\partial N}{\partial x} - a_{19}N = 0,$$

$$\sigma_{xx} = \sigma_{xz} = 0, \quad m_{xy} = 0. \tag{6.41}$$

where, $a_1 = \frac{\lambda+\mu}{\mu+\kappa}$, $a_2 = \frac{\kappa}{\mu+\kappa}$, $a_3 = \frac{\rho c_1^2}{\mu+\kappa}$, $a_4 = \frac{\rho c_1^2 \gamma_n n_0}{T_0\nu(\mu+\kappa)}$, $a_5 = \frac{\kappa c_1^2}{\omega^{*2}\gamma}$, $a_6 = \frac{\rho j c_1^2}{\gamma}$, $a_7 = \frac{\lambda}{\rho c_1^2}$, $a_8 = \frac{\gamma_n n_0}{T_0\nu}$, $a_9 = \frac{\mu}{\rho c_1^2}$, $a_{10} = \frac{\mu+\kappa}{\rho c_1^2}$, $a_{11} = \frac{\kappa}{\rho c_1^2}$, $a_{12} = \frac{\gamma\omega^{*2}}{\rho c_1^4}$, $a_{13} = \frac{K^*}{K\omega^*}$, $a_{14} = \frac{\rho c_e c_1^2}{K\omega^{*2}}$, $a_{15} = \frac{\gamma' T_0\nu}{K\rho c_1\omega^{*2}}$, $a_{16} = \frac{E_g n_0 c_1^2}{T_0 K\omega^{*2}}$, $a_{17} = \frac{n_0 c_1^2}{D_e\omega^* n_0}$, $a_{18} = \frac{\delta T_0 c_1^2}{D_e\omega^* n_0}$, $a_{19} = \frac{s_0 c_1}{D_e\omega^*}$.

Now, for obtaining the desired solution of the required problem, the potential displacements $q(x,z,t)$ and $\psi(x,z,t)$ are introduced as

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \; w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{6.42}$$

Plugging equation (6.42) in equations (6.30), (6.32) and (6.39), we obtain

$$\nabla^2(a_1+1)q - a_3 T - a_4 N - a_3\frac{\partial^2 q}{\partial t^2} = 0, \tag{6.43}$$

$$\left(\nabla^2 - a_3\frac{\partial^2}{\partial t^2}\right)\psi - a_2\phi_2 = 0, \tag{6.44}$$

$$\left(\nabla^2 - 2a_5 - a_6\frac{\partial^2}{\partial t^2}\right)\phi_2 + a_5\nabla^2\psi = 0, \tag{6.45}$$

$$\left(\left(a_{13}+\frac{\partial}{\partial t}\right)\nabla^2 - a_{14}\frac{\partial^2}{\partial t^2}\right)T - a_{15}\nabla^2\frac{\partial^2 q}{\partial t^2} + a_{16}\frac{N}{\tau_p} = 0, \tag{6.46}$$

Laplaces transforms

For any function $f(x,z,t)$, the Laplace transform has been described as

$$L[f(x,z,t)] = \int_0^\infty f(x,z,t)e^{-st}dt = \bar{f}(x,z,s), \tag{6.47}$$

where, s represents laplace transforms parameter.

Using equation (6.47) in equations (6.43)-(6.46) and in equation (6.40), we get

$$[\nabla^2(a_1+1) - a_3 s^2]\bar{q} - a_3\bar{T} - a_4\bar{N} = 0, \tag{6.48}$$

$$[\nabla^2 - a_3 s^2]\bar{\psi} - a_2\bar{\phi}_2 = 0, \tag{6.49}$$

$$[\nabla^2 - 2a_5 - a_6 s^2]\bar{\phi}_2 + a_5\nabla^2\bar{\psi} = 0, \tag{6.50}$$

$$[(a_{13}+s)\nabla^2 - a_{14}s^2]\bar{T} - a_{15}\nabla^2 s^2\bar{q} + a_{16}\frac{\bar{N}}{\tau_p} = 0, \tag{6.51}$$

$$\left[\nabla^2 - a_{17}\left(s + \frac{1}{\tau_p}\right)\right]\bar{N} + a_{18}\frac{\bar{T}}{\tau_p} = 0. \tag{6.52}$$

Fouriers transforms

For any function $\bar{f}(x,z,s)$, the Fourier transform has been described as

$$F[\bar{f}(x,z,s)] = \int_{-\infty}^{\infty} f(x,\zeta,s)e^{-i\zeta z}dz = \bar{f}^*(x,\zeta,s). \tag{6.53}$$

Using equation (6.53) in equations (6.48)-(6.52), we get

$$[(D^2-\zeta^2)(a_1+1) - a_3 s^2]\bar{q}^* - a_3\bar{T}^* - a_4\bar{N}^* = 0, \tag{6.54}$$

$$[(D^2-\zeta^2) - a_3 s^2]\bar{\psi}^* - a_2\bar{\phi}_2^* = 0, \tag{6.55}$$

$$[(D^2-\zeta^2) - 2a_5 - a_6 s^2]\bar{\phi}_2^* + a_5[D^2-\zeta^2]\bar{\psi}^* = 0, \tag{6.56}$$

$$[(a_{13}+s)(D^2-\zeta^2) - a_{14}s^2]\bar{T}^* - a_{15}[D^2-\zeta^2]s^2\bar{q}^* + \frac{a_{16}}{\tau_p}\bar{N}^* = 0, \tag{6.57}$$

$$\left[(D^2-\zeta^2) - a_{17}\left(s + \frac{1}{\tau_p}\right)\right]\bar{N}^* + \frac{a_{18}}{\tau_p}\bar{T}^* = 0. \tag{6.58}$$

The 4^{th} order ordinary differential equation (ODE) is obtained after removing $\bar{\phi}_2^*$ and $\bar{\psi}^*$ between (6.55)-(6.56), which in turn is satisfied by $\bar{\phi}_2^*$ and $\bar{\psi}^*$, is as

$$[D^4 - AD^2 + B][\bar{\psi}^*(x), \bar{\phi}_2^*(x)] = 0, \tag{6.59}$$

The other quantities $\bar{N}^*$, $\bar{q}^*$, $\bar{T}^*$ can be eliminated between equations (6.54), (6.57) and (6.58), the following 6^{th} order ODE (satisfied by $\bar{N}^*$, $\bar{q}^*$ and $\bar{T}^*$) can be obtained as follows

$$[D^6 - CD^4 + ED^2 - F][\bar{N}^*(x), \bar{q}^*(x), \bar{T}^*(x)] = 0, \tag{6.60}$$

where, $D = \frac{\partial}{\partial x}$, $A = 2\zeta^2 + (a_3+a_6)s^2 + a_5(2+a_2)$, $B = (\zeta^2 + 2a_5 + a_6 s^2)(\zeta^2 + a_3 s^2) - a_2 a_5\zeta^2$, $C = \frac{-a_3 a_{15} a_{18} s^2 + a_{18}(a_1+1)\left[(a_{13}+s)\zeta^2 + a_{14}s^2 + \left(\zeta^2 + a_{17}\left(s+\frac{1}{\tau_p}\right)\right)(a_{13}+s)\right] + [a_{18}(\zeta^2(a_1+1) + a_3 s^2)](a_{13}+s)}{a_{18}(a_1+1)(a_{13}+s)}$,
$E = \frac{-\alpha_1\alpha_2 - \alpha_3 - \alpha_4\alpha_5 + \alpha_6\alpha_7}{\alpha_8}$, $F = \frac{-\alpha_1\alpha_2\zeta^2 - \alpha_6\alpha_5}{\alpha_8}$
$\alpha_1 = \frac{a_{15}a_{18}s^2}{\tau_p}$, $\alpha_2 = \frac{1}{\tau_p}(a_4 a_{18} + a_3(\zeta^2\tau_p + a_{17}s\tau_p + a_{17}))$, $\alpha_3 = \frac{a_3 a_{15} a_{18}\zeta^2 s^2}{\tau_p}$, $\alpha_4 =$

$\frac{a_{18}(a_1+1)}{\tau_p}$,
$\alpha_5 = \frac{a_{16}a_{18}}{\tau_p^2} - \left[\zeta^2 + a_{17}\left(s + \frac{1}{\tau_p}\right)\right][(a_{13}+s)\zeta^2 + a_{14}s^2]$, $\alpha_6 = \frac{a_{18}\zeta^2(a_1+1)+a_{18}a_3 s^2}{\tau_p}$,
$\alpha_7 = (a_{13}+s)\zeta^2 + a_{14}s^2 + \left[\zeta^2 + a_{17}\left(s + \frac{1}{\tau_p}\right)\right][a_{13}+s]$, $\alpha_8 = \frac{a_{18}(a_1+1)(a_{13}+s)}{\tau_p}$.
In factored form equations (6.59) and (6.60) can be written as

$$[(D^2-h_1^2)(D^2-h_2^2)][\bar{\psi}^*(x), \bar{\phi}_2^*(x)] = 0, \tag{6.61}$$
$$[(D^2-k_1^2)(D^2-k_2^2)(D^2-k_3^2)][\bar{N}^*(x), \bar{q}^*(x), \bar{T}^*(x)] = 0, \tag{6.62}$$

here, ${h_\jmath}^2(\jmath = 1,2)$ and ${k_n}^2(n = 1,2,3)$ represents characteristic roots of the equations (6.61) and (6.62).

The general solution of equations (6.61) and (6.62), has the form

$$\bar{\psi}^*(x) = \sum_{\jmath=1}^{2} M_\jmath e^{-h_\jmath x}, \tag{6.63}$$
$$\bar{\phi_2}^*(x) = \sum_{\jmath=1}^{2} {M_\jmath}' e^{-h_\jmath x}, \tag{6.64}$$
$$\bar{N}^*(x) = \sum_{n=1}^{3} H_n e^{-k_n x}, \tag{6.65}$$
$$\bar{T}^*(x) = \sum_{n=1}^{3} {H_n}' e^{-k_n x}, \tag{6.66}$$
$$\bar{q}^*(x) = \sum_{n=1}^{3} {H_n}'' e^{-k_n x}, \tag{6.67}$$

Here, $M_\jmath$, $M_\jmath^{'}$, H_n, ${H_n}'$ and ${H_n}''$ are some parameters.

Using equations (6.63)-(6.67) in (6.54)-(6.58), we get

$$\bar{\psi}^*(x) = \sum_{\jmath=1}^{2} M_\jmath e^{-h_\jmath x}, \tag{6.68}$$
$$\bar{\phi_2}^*(x) = \sum_{\jmath=1}^{2} L_{1\jmath} M_\jmath e^{-h_\jmath x}, \tag{6.69}$$
$$\bar{N}^*(x) = \sum_{n=1}^{3} H_n e^{-k_n x}, \tag{6.70}$$
$$\bar{T}^*(x) = \sum_{n=1}^{3} P_{1n} H_n e^{-k_n x}, \tag{6.71}$$
$$\bar{q}^*(x) = \sum_{n=1}^{3} P_{2n} H_n e^{-k_n x}, \tag{6.72}$$

where, $L_{1j} = \frac{h_j^2-(\zeta^2+a_3 s^2)}{a_2}$, $\quad P_{1n} = \frac{-\tau_p\left[k_n^2-\zeta^2-a_{17}\left(s+\frac{1}{\tau_p}\right)\right]}{a_{18}}$, $\quad P_{2n} = \frac{a_3 P_{1n}+a_4}{(a_1+1)[k_n^2-\zeta^2]-a_3 s^2}$.

Using equations (6.47) and (6.53) in equations (6.42), (6.33)-(6.41) and (6.33) we get

$$\bar{u}^* = \frac{d\bar{q}^*}{dx} + i\zeta\bar{\psi}^*, \tag{6.73}$$

$$\bar{w}^* = i\zeta\bar{q}^* - \frac{d\bar{\psi}^*}{dx} \tag{6.74}$$

$$\bar{\sigma}_{xx}^* = \frac{d\bar{u}^*}{dx} + a_7 i\zeta\bar{w}^* - \bar{T}^* - a_8\bar{N}^*, \tag{6.75}$$

$$\bar{\sigma}_{zz}^* = i\zeta\bar{w}^* + a_7\frac{d\bar{u}^*}{dx} - \bar{T}^* - a_8\bar{N}^*, \tag{6.76}$$

$$\bar{\sigma}_{xz}^* = a_9 i\zeta\bar{u}^* + a_{10}\frac{d\bar{w}^*}{dx} + a_{11}\bar{\phi}_2^*, \tag{6.77}$$

$$\bar{\sigma}_{zx}^* = a_9\frac{d\bar{w}^*}{dx} + a_{10}i\zeta\bar{u}^* - a_{11}\bar{\phi}_2^*, \tag{6.78}$$

$$\bar{m}_{zy}^* = a_{12}i\zeta\bar{\phi}_2^*, \tag{6.79}$$

$$\bar{m}_{xy}^* = a_{12}\frac{d\bar{\phi}_2^*}{dx}, \tag{6.80}$$

$$\begin{cases} \frac{d\bar{T}^*}{dx} = \frac{-q_0 t_p}{8(1+st_p)^3}\frac{2sin\zeta\hat{a}}{\zeta}, \\ \frac{d\bar{N}^*}{dx} = a_{19}\bar{N}^*, \\ \bar{\sigma}_{xx}^* = \bar{\sigma}_{xz}^* = \bar{m}_{xy}^* = 0, \end{cases} \tag{6.81}$$

Using equations (6.68)-(6.72) in equations (6.73)-(6.80), we get

$$\bar{u}^* = -\sum_{n=1}^{3} P_{2n}k_n H_n e^{-k_n x} + i\zeta\sum_{j=1}^{2} M_j e^{-h_j x}, \tag{6.82}$$

$$\bar{w}^* = i\zeta\sum_{n=1}^{3} P_{2n}H_n e^{-k_n x} + \sum_{j=1}^{2} h_j M_j e^{-h_j x}, \tag{6.83}$$

$$\bar{\sigma}_{xx}^* = \sum_{n=1}^{3}\{(k_n^2 - a_7\zeta^2)P_{2n} - (P_{1n}+a_8)\}H_n e^{-k_n x} - i\zeta\sum_{j=1}^{2} h_j\{1-a_7\}M_j e^{-h_j x}, \tag{6.84}$$

$$\bar{\sigma}_{zz}^* = \sum_{n=1}^{3}\{-\zeta^2 P_{2n} + a_7 P_{2n}k_n^2 - (P_{1n}+a_8)\}H_n e^{-k_n x} + i\zeta\sum_{j=1}^{2}\{1+h_j\}M_j e^{-h_j x}, \tag{6.85}$$

$$\bar{\sigma}_{xz}^* = \sum_{n=1}^{3}\{-(a_9+a_{10})i\zeta P_{2n}k_n\}H_n e^{-k_n x} - \sum_{j=1}^{2}\{(a_9\zeta^2 - a_{11}L_{1j} - h_j)\}M_j e^{-h_j x}, \tag{6.86}$$

$$\bar{\sigma}_{zx}^* = \sum_{n=1}^{3}\{-(a_9+a_{10})i\zeta P_{2n}k_n\}H_n e^{-k_n x} - \sum_{j=1}^{2}\{a_9 h_j^2 + a_{10}\zeta^2 + a_{11}L_{1j}\}M_j e^{-h_j x}, \tag{6.87}$$

$$\bar{m}^*_{zy} = a_{12} i\zeta \sum_{\jmath=1}^{2} L_{1\jmath} M_\jmath e^{-h_\jmath x}, \tag{6.88}$$

$$\bar{m}^*_{xy} = -a_{12} \sum_{\jmath=1}^{2} L_{1\jmath} h_\jmath M_\jmath e^{-h_\jmath x}. \tag{6.89}$$

Using equations (6.71), (6.70), (6.84), (6.86) and (6.89) in equation (6.81) for $x = 0$, we obtain

$$\sum_{n=1}^{3} P_{1n} k_n H_n = A_1, \tag{6.90}$$

$$\sum_{n=1}^{3} P_{3n} H_n = 0, \tag{6.91}$$

$$\sum_{n=1}^{3} P_{4n} H_n - \sum_{\jmath=1}^{2} L_{2\jmath} M_\jmath = 0, \tag{6.92}$$

$$\sum_{n=1}^{3} P_{5n} H_n - \sum_{\jmath=1}^{2} L_{3\jmath} M_\jmath = 0, \tag{6.93}$$

$$\sum_{\jmath=1}^{2} L_{4\jmath} M_\jmath = 0 \tag{6.94}$$

where $A_1 = \frac{q_0 t_p}{8(1+st_p)^3} \frac{2sin\zeta\hat{a}}{\zeta}$, $P_{3n} = -k_n - a_{19}$, $P_{4n} = (k_n^2 - a_7\zeta^2)P_{2n} - (P_{1n} + a_8)$, $L_{2\jmath} = i\zeta(1-a_7)h_\jmath$, $P_{5n} = -(a_9 + a_{10})i\zeta P_{2n} k_n$, $L_{3\jmath} = a_9\zeta^2 - h_\jmath - a_{11} L_{1\jmath}$, $L_{4\jmath} = -a_{12} L_{1\jmath} h_\jmath$.

Now, solving equations (6.90)-(6.94) for H_n and $M_\jmath$ (where, $n = 1, 2, 3$ and $\jmath = 1, 2$) by making use of the Inverse matrix method, which is given below:

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} P_{11}k_1 & P_{12}k_2 & P_{13}k_3 & 0 & 0 \\ P_{31} & P_{32} & P_{33} & 0 & 0 \\ P_{41} & P_{42} & P_{43} & -L_{21} & -L_{22} \\ P_{51} & P_{52} & P_{53} & -L_{31} & -L_{32} \\ L_{41} & L_{42} & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} A_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Inversion of the Transformation

For any function $\bar{f}^*(x, \zeta, s)$, the inversion of the Fourier transform can be defined as follows

$$F^{-1}[\bar{f}^*(x, \zeta, s)] = \bar{f}(x, z, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}^*(x, \zeta, s) e^{i\zeta z} d\zeta. \tag{6.95}$$

Consequently, with respect to x, z and s, the solutions of field variables can be obtained by using equation (6.95) in equations (6.82)-(6.89), (6.70) and (6.71), as follows

$$\bar{u}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\sum_{n=1}^{3}P_{2n}k_nH_ne^{-k_nx} + i\zeta\sum_{j=1}^{2}M_je^{-h_jx}\right]e^{i\zeta z}d\zeta, \tag{6.96}$$

$$\bar{w}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[i\zeta\sum_{n=1}^{3}P_{2n}H_ne^{-k_nx} + \sum_{j=1}^{2}h_jM_je^{-h_jx}\right]e^{i\zeta z}d\zeta, \tag{6.97}$$

$$\bar{\sigma}_{xx}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_{n=1}^{3}\{(k_n^2 - a_7\zeta^2)P_{2n} - (P_{1n} + a_8)\}H_ne^{-k_nx} - i\zeta\sum_{j=1}^{2}h_j\{1 - a_7\}M_je^{-h_jx}\right]$$
$$e^{i\zeta z}d\zeta, \tag{6.98}$$

$$\bar{\sigma}_{zz}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_{n=1}^{3}\{-\zeta^2P_{2n} + a_7P_{2n}k_n^2 - (P_{1n} + a_8)\}H_ne^{-k_nx} + i\zeta\sum_{j=1}^{2}\{1 + h_j\}M_je^{-h_jx}\right]$$
$$e^{i\zeta z}d\zeta, \tag{6.99}$$

$$\bar{\sigma}_{xz}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_{n=1}^{3}\{-(a_9 + a_{10})i\zeta P_{2n}k_n\}H_ne^{-k_nx} - \sum_{j=1}^{2}\{(a_9\zeta^2 - a_{11}L_{1j} - h_j)\}M_je^{-h_jx}\right]$$
$$e^{i\zeta z}d\zeta, \tag{6.100}$$

$$\bar{\sigma}_{zx}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_{n=1}^{3}\{-(a_9 + a_{10})i\zeta P_{2n}k_n\}H_ne^{-k_nx} - \sum_{j=1}^{2}\{a_9h_j^2 + a_{10}q^2 + a_{11}L_{1j}\}M_je^{-h_jx}\right]$$
$$e^{i\zeta z}d\zeta, \tag{6.101}$$

$$\bar{m}_{zy}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[a_{12}i\zeta\sum_{j=1}^{2}L_{1j}M_je^{-h_jx}\right]e^{i\zeta z}d\zeta, \tag{6.102}$$

$$\bar{m}_{xy}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-a_{12}\sum_{j=1}^{2}L_{1j}h_jM_je^{-h_jx}\right]e^{i\zeta z}d\zeta, \tag{6.103}$$

$$\bar{N}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_{n=1}^{3}H_ne^{-k_nx}\right]e^{i\zeta z}d\zeta, \tag{6.104}$$

$$\bar{T}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_{n=1}^{3}P_{1n}H_ne^{-k_nx}\right]e^{i\zeta z}d\zeta. \tag{6.105}$$

Following Honig and Hirdes [126], the Laplace transform function $\bar{f}(x,z,s)$ can be inverted to $f(x,z,t)$ by

$$f(x,z,t) = \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty}\bar{f}(x,z,s)e^{-st}ds. \tag{6.106}$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(x, z, s)$.

The last step is to calculate the integral in equation (6.106). The method for evaluating this integral is described by Press [127].

6.4 Validity of the Problem

When the plasma wave (which is illustrated by carrier density $\overrightarrow{N}(r, t)$) is ignored, we obtain the following results

$$\bar{\psi}^*(x) = \sum_{j=1}^{2} M_j e^{-h_j x},$$

$$\bar{\phi_2}^*(x) = \sum_{j=1}^{2} L_{1j} M_j e^{-h_j x},$$

$$\bar{T}^*(x) = \sum_{n=1}^{3} P_{1n} H_n e^{-k_n x},$$

$$\bar{q}^*(x) = \sum_{n=1}^{3} P_{2n} H_n e^{-k_n x},$$

where, $L_{1j} = \frac{h_j^2 - (\zeta^2 + a_3 s^2)}{a_2}$, $\quad P_{1n} = \frac{-\tau_p \left[k_n^2 - \zeta^2 - a_{17}\left(s + \frac{1}{\tau_p}\right)\right]}{a_{18}}$, $\quad P_{2n} = \frac{a_3 P_{1n} + a_4}{(a_1+1)[k_n^2 - \zeta^2] - a_3 s^2}$.

which are in sync with the results explained in the study [128] considered in the context of micropolar generalized thermoelasticity.

6.5 Numerical results and Discussion

The material properties of Silicon as in [129, 130] are taken under consideration for numerical simulations, which are as:

$$\lambda = 3.64 \times 10^{10} Nm^{-2},\ \mu = 5.46 \times 10^{10} Nm^{-2},\ \kappa = 10^{10} Nm^{-2},\ \rho = 2330\ kg\ m^{-3}$$

$$\gamma = 0.779 \times 10^{-9} N,\ j = 0.2 \times 10^{-19} m^2,\ K = 150 Wm^{-1}K^{-1},\ K^* = 3 Wm^{-1}K^{-1},$$

$$C_e = 695 Jkg^{-1}K^{-1},\ T_0 = 800K,\ \tau_p = 5 \times 10^{-5}s,\ E_g = 1.11eV,\ D_e = 2.5 \times 10^{-3} m^2 s^{-1},$$

$$d_n = -9 \times 10^{-31} m^3,\ s_0 = 2ms^{-1},\ n_0 = 10^{20} m^{-3},\ \alpha_t = 3 \times 10^{-6} K^{-1},\ \hat{a} = 0.25,\ q_0 = 10.$$

The impact of characteristic time of pulse heat flux t_p is illustrated on the behavior of the field quantities such as the components of displacement, force stresses, couple stress components, temperature distribution, and carrier density. The graphical analysis has been done for different values of t_p, i.e., for $t_p = 0.3,\ 0.4,\ 0.5$ w.r.t x, as shown in the below figures.

The following analysis depicts that, except the displacement component w which is less sensitive towards the changes in characteristic time t_p, rest of the parameters namely displacement component u, normal stress σ_{xx}, tangential stress σ_{xz}, couple stress m_{xy}, temperature distribution T, and carrier density N are highly sensitive towards changes in t_p.

Figures 6.2 and 6.3 demonstrate the influence of characteristic time t_p on the displacement components u and w with respect to distance x. Figure 6.2 clearly reveals that the displacement component u exhibits oscillatory behavior in the region $0 \leq x \leq 8$ and is greatly influenced by t_p. Due to the photothermal effect, the elastic waves (described by displacement component u) on the surface are generated with a positive amplitude, which starts reducing when moving away from the source. After that, the elastic waves start showing periodic nature. It is clearly evident that the amplitude of the displacement component is maximum for $t_p = 0.5$, and it reduces when the value of t_p is reduced, i.e., for $t_p = 0.4$. For $t_p = 0.3$, we observe the least amplitude of the displacement distribution. In other words, the displacement component u reduces when the value of t_p is reduced in terms of amplitude.

From figure 6.3, we observe that the displacement component w exhibits oscillatory behavior in the region $0 \leq x \leq 8$ and is moderately influenced by the characteristic time t_p. In terms of amplitude, the displacement component w attains its maximum for $t_p = 0.5$. Then it reduces when the value of characteristic time is reduced.

Figures 6.4 and 6.5 demonstrate the impact of t_p on stress components σ_{xx} and σ_{xz} w.r.t. distance x. It is noticed that both the normal and tangential stresses start from zero, and hence the boundary conditions are satisfied. In figure 6.4, the normal stress σ_{xx} describes the mechanical wave inside the semiconductor medium. From the graph, it is clearly observed that the mechanical waves are highly sensitive towards the characteristic time t_p.

Figures 6.5 and 6.6 illustrates that similar observations are exhibited by the tangential stress σ_{xz} and couple stress m_{xy} with a higher amplitude. Moreover, it is also clear from figure 5 that the couple stress m_{xy} starts from zero, and hence satisfies the boundary condition.

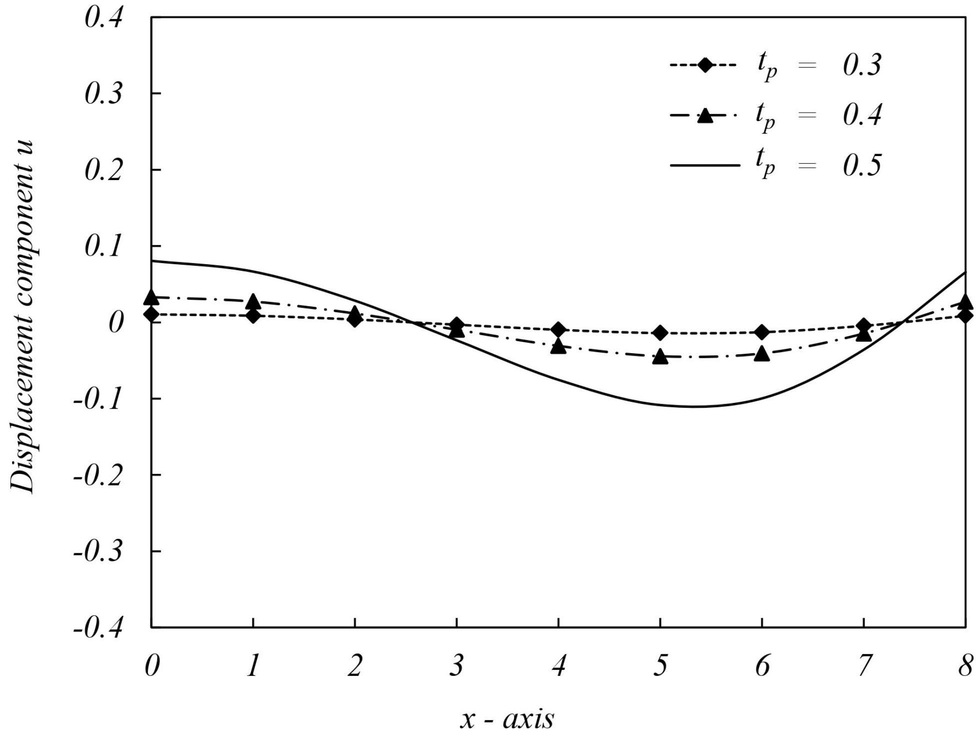

FIGURE 6.2: Variation of displacement component u at distinct values of t_p

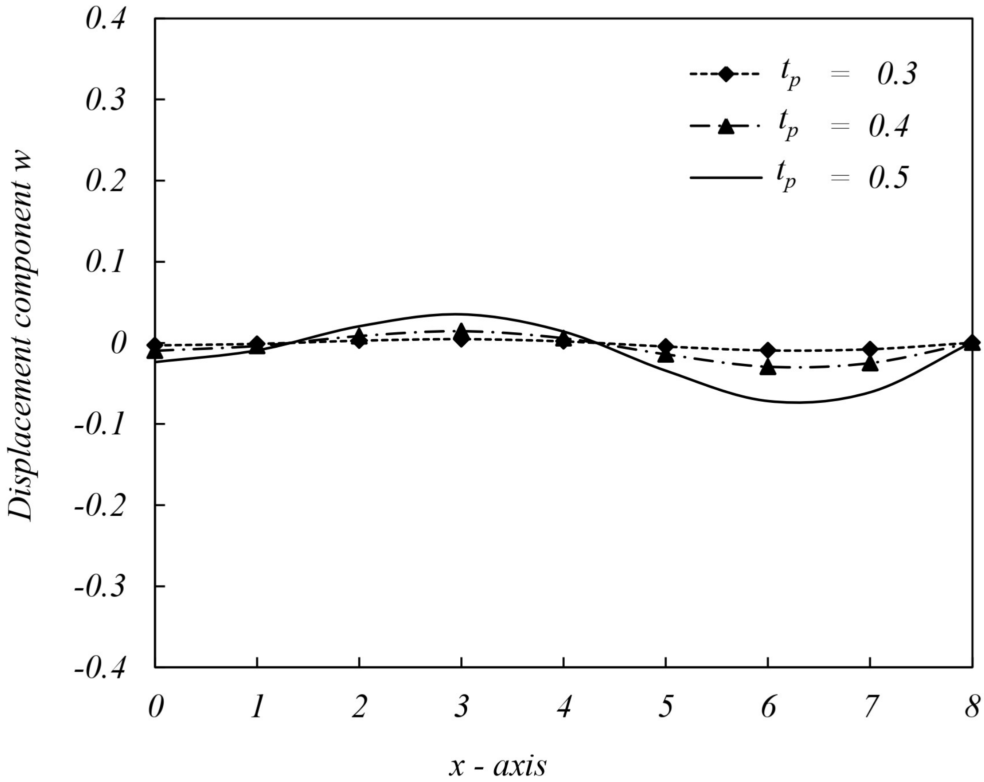

FIGURE 6.3: Variation of displacement component w at distinct values of t_p

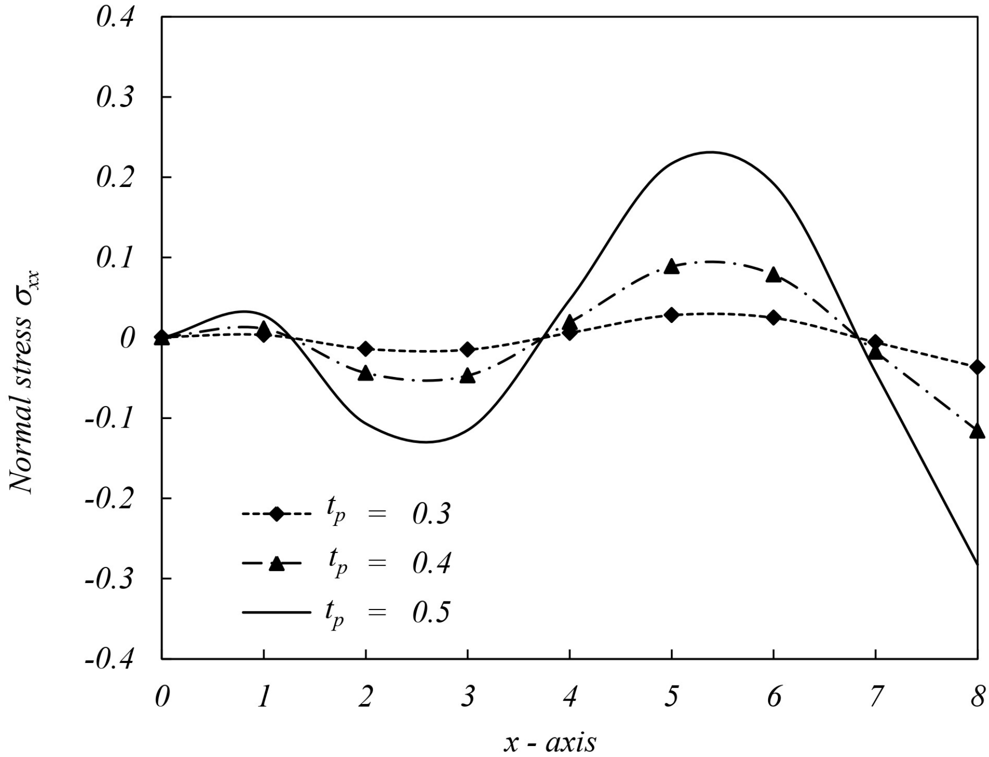

Figure 6.4: Variation of normal stress σ_{xx} at distinct values of t_p

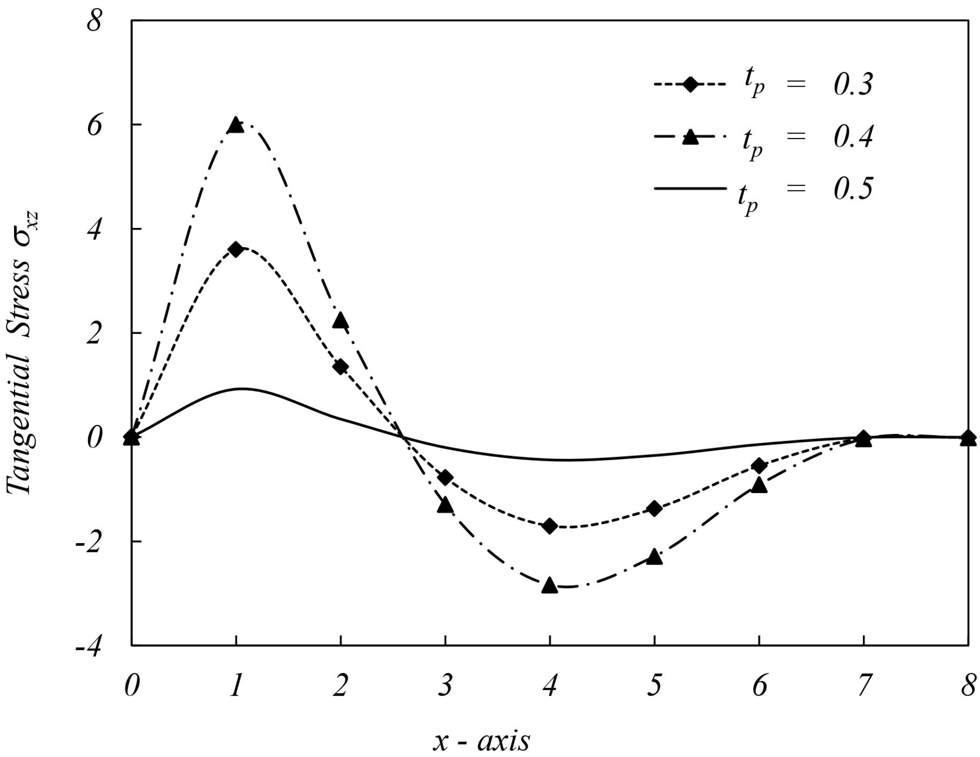

Figure 6.5: Variation of tangential stress σ_{xz} at distinct values of t_p

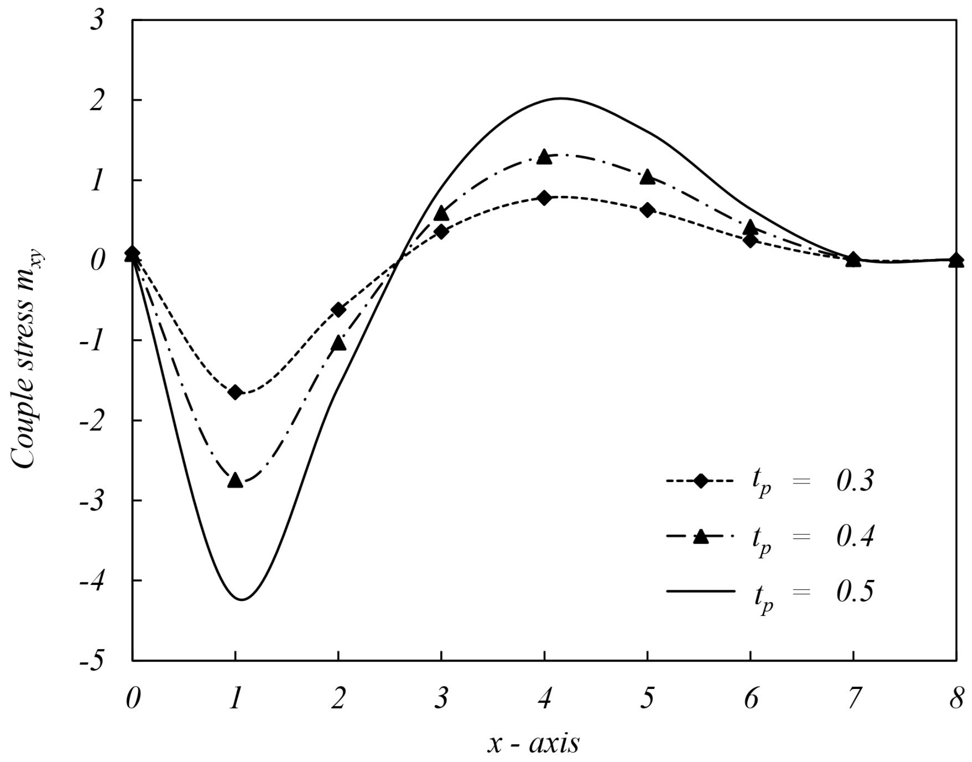

FIGURE 6.6: Variation of couple stress m_{xy} at distinct values of t_p

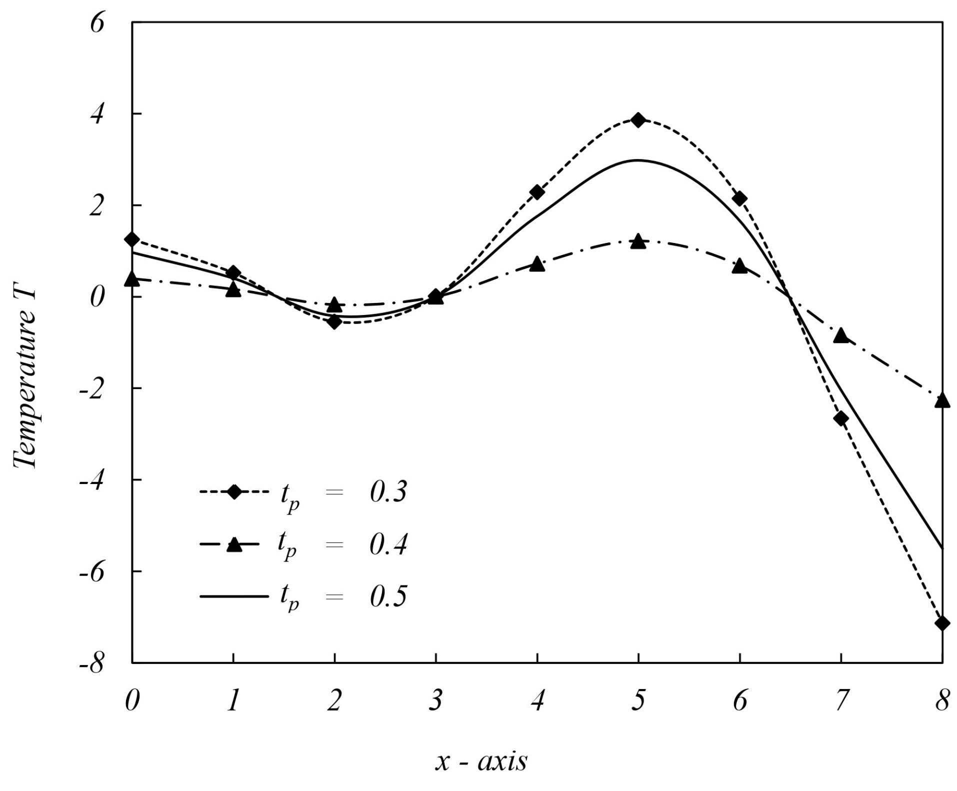

FIGURE 6.7: Variation of temperature T at distinct values of t_p

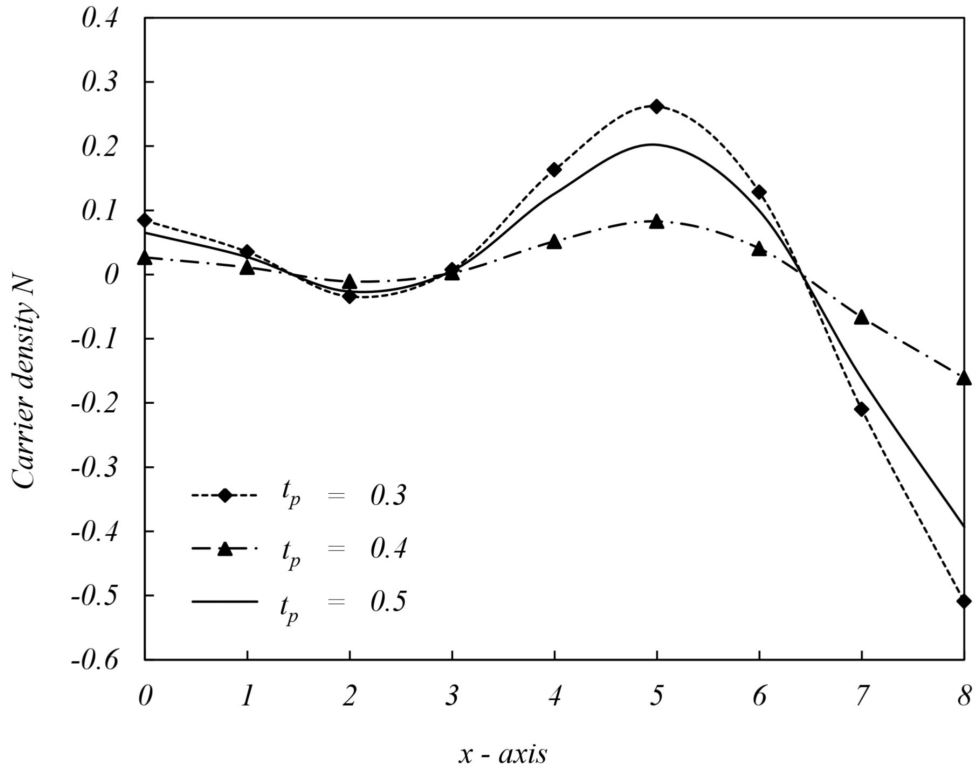

FIGURE 6.8: Variation of Carrier density N at distinct values of t_p

Figure 6.7 and 6.8 demonstrates the impact of the characteristic time t_p on temperature distribution T and carrier density N. The temperature distributions expresses the thermal waves w.r.t the distance x. The thermal waves starts from positive values near the source of application, thus satisfying the thermal boundary conditions.

The amplitude of thermal waves is observed to be maximum for $t_p = 0.3$, and it decreases when the value of characteristic time t_p is increased, i.e., for $t_p = 0.5$. Both the physical quantities viz. T and N show similar sensitivity towards t_p. Starting from a positive value, then showing an oscillatory nature with increasing amplitude as x keeps increasing. For higher values of x, graphs depict that for $t_p = 0.4$, amplitude shows significantly lower variations. Thus, it can be said that for higher values of x, we can find the same values of t_p, which can keep the amplitude in a controlled range.

6.6 Conclusion

In this study, a homogeneous, isotropic, semiconducting material (Silicon) has been taken into consideration, and a new mathematical model suitable for the micropolar thermoelastic media which includes the photothermal effect, is developed. The effect of coupling of thermal, plasma, and elastic waves has been studied under the Green-Naghdi theory. The study reveals a strong influence of the characteristic time of pulse heat flux t_p on different physical quantities, which has been discussed numerically and depicted graphically.

From the above study, we can conclude that

1. In the context of the Green-Naghdi theory with photothermal effect, a new mathematical model for the micropolar theory of generalized thermoelasticity has been constructed.
2. The boundary conditions are satisfied by all the field variables, therefore the deformation of an elastic body depends both on the type of boundary conditions as well as on the type of applied force.
3. The physical quantities namely, temperature T and carrier density N, have been found to exhibit a similar sensitivity towards t_p.
4. The analytical solutions are obtained by using normal mode technique along with Integral transforms.

Conclusion and Future work

In this thesis, mathematical models for plain-strain problems in micropolar elasticity have been studied. Using the potential displacement approach in conjunction with the normal mode technique and/or integral transforms, each problem has been analytically addressed. Matlab software and Microsoft Excel were used to create graphical demonstrations of the results obtained in each problem. Taking into account both thermal and magnetic forces, Chapter 2 explores the behavior of micropolar elastic materials. Furthermore, the fluctuation of several physical quantities was demonstrated for various gravity values. The Green-Lindsay model with MDD has been used in Chapter 3rd to build a mathematical model for the magneto-micropolar thermoelasticity problem, and a considerable impact on the elastic response of the material has been found to be caused by the time parameter. In the 4th chapter of the thesis, a 2D mathematical model has been established for the micropolar thermoelasticity in the context of MDD. In order to determine the appropriate solution to the required problem, the Helmholtz potentials are used in conjunction with the normal mode technique. The subject of the fifth chapter is to study the impact of the time parameter on a magneto-micropolar thermoelastic solid (with MDD). Furthermore, the problem has been solved analytically using the normal mode technique. In chapter 6, a novel mathematical model, incorporating the photothermal effect, is devised for micropolar thermoelastic media. The required problem has been solved analytically and depicted graphically.

As the photo-thermo-elastic interactions in micropolar generalized thermoelasticity were covered in chapter 6. More research is required because this field of study has numerous applications in the scientific domain. Thus, in subsequent research, the focus will be on expanding the breadth of this topic to include the fields of classical elasticity and micropolar elasticity under the photothermal theory, which are not covered in the thesis. These models will be helpful in analyzing the stress-strain bodies having microstructure subjected to mechanical, thermal or electromagnetic forces.

Paper Publications, Presentations, Seminar and Webinar

Published papers

- Kumar, V. and Nazir, R., 2022. Elastodynamic Responses of Magneto Micropolar Isotropic Media under the Gravitational Influence. Mechanics of Solids, 57(5), pp.949-959.
- Kumar, V., Nazir, R. and Lotfy, K., 2022. Interactions of magneto-micropolar thermoelastic rotating medium with memory-dependent derivative. Indian Journal of Physics, 96(13), pp.3809-3816.
- Kumar, V., Nazir, R. A Study of Thermo-Mechanical Interactions in the Rotating Micropolar Elastic Solid with Two Temperatures Using Memory-Dependent Derivative. Mech. Solids (2023).
- Nazir, R. and Kumar, V., 2024. Photo-thermo-elastic interactions in micropolar generalized thermoelasticity theory in the framework of Green-Naghdi theory. Journal of Thermal Stresses, pp.1-16.

Communicated papers

- Rafiya Nazir and Varun kumar, 2023. Study of memory response in the presence and absence of micropolar effect under the framework of Green-Lindsay model.
- Rafiya Nazir and Varun kumar, 2023. A review on the development of micropolar theory of elasticity.

Papers presented in conferences

- Presented paper in International conference on "Recent advances in fundamental and applied sciences (RAFAS)" held on June 25-26, 2021 at LPU.

- Presented paper in International conference on “Materials for Emerging Technologies (ICMET-21)” held on February 18-19, 2022, organized by Lovely Professional University, Punjab.

- Presented paper in International conference on “Advance trends in Computational mathematics, statistics and Operations research (ICCMSO-2022)” held on April 2-3, 2022, organized by Department of Applied Sciences, The NorthCap University, Gurugram Haryana.

Seminar and Webinar attended so far

- Participated in three days International E-Seminar on “Advances in Mathematical Sciences (ISAMS-2022)” during March 14-16, 2022, organized by Central University of Karnataka.

- Participated in International Seminar on “Indian Mathematicians and their Contributions (ISIMC-2022)” during December 21-22, 2022, Organized by Department of Mathematics, School of Physical Sciences, Central University of Karnataka, India.

- Participated in International webinar on “Recent trends in Mathematical sciences” organized by Hans Raj Mahila Maha Vidhyalaya on 28th July, 2020.

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