INVENTORY MODELS WITH TRADE CREDIT FOR FAST MOVING CONSUMER GOODS (FMCG) IN A FINITE PLANNING HORIZON

Thesis Submitted for the Award of the Degree of

DOCTOR OF PHILOSOPHY

In

Mathematics

By

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2024

DECLARATION OF AUTHORSHIP

I, hereby declared that the presented work in the thesis entitled **"Inventory models with trade credit for fast moving consumer goods (FMCG) in a finite planning horizon"** in fulfillment of degree of **Doctor of Philosophy (Ph. D.)** is outcome of research work carried out by me under the supervision **Dr. Nitin K. Mishra**, working as Professor, in the School of Chemical Engineering and Physical Sciences, of Lovely Professional University, Punjab, India. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of another investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

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CERTIFICATE

This is to certify that the work reported in the Ph. D. thesis entitled **“Inventory Models with Trade Credit for Fast Moving Consumer Goods (FMCG) in a Finite Planning Horizon”** submitted in fulfillment of the requirement for the reward of degree of **Doctor of Philosophy (Ph.D.)** in the School of Chemical Engineering and Chemical Sciences, Lovely Professional University, Punjab, is a research work carried out by Sakshi Sharma, 11919681, is a bonafide record of his/her original work carried out under my supervision and that no part of thesis has been submitted for any other degree, diploma or equivalent course.

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Dedicated to God

Late Bhagwan Singh Sir

And

My Mother Late Mrs. Shashi Sharma

Abstract

The inventory or stocks are always influenced by various factors – deterioration, seasonal demand, inflation, shortages etc. This research work focuses on analysing the effect of – deteriorating inventory, trade credit, inflationary market conditions, seasonal ramp demand, fuzzified defective products on the costs of the system, the retailer, retailers and the supplier for FMCG (Fast-Moving Consumer Goods) industry in finite planning horizon.

The coordination between supplier – retailer is an important aspect of calculating trade credit's effect on the cost and profits of all the players involved in this FMCG market. Greater the coordination, more easily the negative effects of increased cost on both ends can be reduced. Considering this in mind, coordination along with trade credit is used effectively in reducing cost of system in chapter 3 and chapter 4. Chapter 5 takes coordination to another level, by involving multiple retailers along with novel concept of equipoise cost of the retailer.

Focus is on reducing costs and maximizing profits in the presence of deteriorating inventory along with variable demand of products in changing seasons. Ramp seasonal demand and its nature (that how it controls the demand of the product) are discussed which is regularly seen in FMCG oriented market in present scenario. Inflation and its inflationary effects are considered in every replenishment cycle, rather than taking inflation as a yearly scenario. Fuzzification of deterioration rate and defective products shows how the cost is reduced in the favour of the retailer.

Due to a competitive market, lead time is kept zero. Algorithms were framed and numerical examples were discussed in each chapter of the research work to numerically prove the effectiveness of the proposed models.

The results show maximization of the profits by reducing costs and benefitting every player individually and then to the system. The study and its research results will benefit FMCG industry along with enhancing coordination among suppliers and retailers and the benefit in the form of reduced cost of the final

product will ultimately travel to the end user (the consumer) of the FMCG industry.

A compendious of chapters are discussed in the next few pages to get an overview of this research work.

Acknowledgment

I thank my supervisor Dr. Nitin K. Mishra, Professor, Department of Mathematics, School of Chemical Engineering and Chemical Sciences, Lovely Professional University, Phagwara, Punjab, (India) for his unconditional support and guidance throughout this work. I can complete this research work based on his valuable advice and moral support.

I want to dedicate my degree to Late Mr. Bhagwan Singh, Mr. Anil Sharma, and Mr. Rajeev Sharma, my mathematics teachers, and mentors, who teaches me and laid a strong foundation on which this degree is being received.

I thank all my family members, friends, and relatives for their direct and indirect support and help. I would like to express my deepest gratitude to my husband Mr. Neeraj Kumar, for supporting me to start this work and helping me financially against the will of all others, for his immovable belief in me that I can complete this work. Finally, I thank my two kids (Neeshi and Neeraksh), whose time I have given to this research work. I didn't even remember, how many times I had failed as a mother in the whole process, but still, they never complained.

Words cannot express my gratitude for the Almighty, the creator God for the constant blessings bestowed on me for the completion of work.

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List of Abbreviations

FMCG – Fast-Moving Consumer Goods

CPG – Consumer Packaged Goods

O. R. – Operations Research

EOQ – Economic Order Quantity

EPQ – Economic Production Quantity

FST – Fuzzy Set Theory

List of Notations

Z = Planning horizon (in months) and is a non-negative integer.

IC = inventory cost and it includes both capital cost and holding cost

I_{i+1} = inventory level of deteriorating inventory during $(i+1)^{th}$ cycle at the time t_i

Y = Ordering cost per unit.

v = constant demand rate.

M = credit period rate or permissible delay in payment.

$T_{i+1} = t_{i+1} - t_i$ is the replenishment cycle's length of $(i+1)^{th}$ cycle.

E_{i+1} = interest earned by the retailer in $(i+1)^{th}$ replenishment cycle.

W = wholesale cost per unit (for the retailer and W > c).

c = purchasing cost per unit (for the supplier).

h = holding cost per unit.

s = selling price per unit.

E = interest earned per year.

E_1 = retailer's interest earned under coordination for subcase 1

E_2 = retailer's interest earned under coordination for subcase 2

E_3 = retailer's interest earned under coordination for subcase 3

E' = interest charged per year.

E_1' = supplier's interest charged under coordination for subcase 1

E_2' = supplier's interest charged under coordination for subcase 2

E_3' = supplier's interest charged under coordination for subcase 3

D_{i+1} = number of deteriorated units in $(i+1)^{th}$ cycle

D = cost of deteriorated units.

θ = Deterioration rate

RET_{TC} = Retailer's total cost during planning horizon Z including the interest earned by him.

n = number of replenishment cycles without coordination.

m = number of replenishment cycles with coordination.

R(t) = Ramp type demand which is a function of time t and

$R(t) = D_0\{t-(t-\mu)H(t-\mu)\} > 0$, moreover $D_0 > 0$

where $H(t-\mu)$ is the Heaviside's function which further follows the below identity:

$$H(t-\mu) = \begin{cases} 1, t \geq \mu \\ 0. t < \mu \end{cases}$$

Thus,

$$R(t) = \begin{cases} D_0 t, t_i \leq t \leq \mu \\ D_0 \mu, \mu \leq t \leq t_{i+1} \end{cases}$$

S_{i+1} = inventory level during (i + 1)th cycle at the time t_i without coordination.

S'_{j+1} = inventory level during (j + 1)th cycle at the time t_j with coordination.

M = the trade credit period given by the supplier to the retailer.

R_I = the retailer's total cost without coordination.

S_I = the supplier's total cost without coordination.

R_{J1} = the retailer's total cost under coordination for subcase 1.

R_{J2} = the retailer's total cost under coordination for subcase 2.

R_{J3} = the retailer's total cost under coordination for subcase 3.

S_{J1}= the Supplier's total cost under coordination for subcase 1.

S_{J2} = the Supplier's total cost under coordination for subcase 2.

S_{J3}= the Supplier's total cost under coordination for subcase 3.

k = constant, inventory dependent demand rate

b_1 = increasing demand rate per year

b_2 = the scaling for inflation function

α_i = variable rate of inflation

$\alpha_{i+1} = \alpha_i + 0.1$ (linear rise in inflation)

LC = Labour Cost

MC = Machinery cost

S_I = Supplier's total cost

M_i = the tenable layout for case 2, where $i = 1,2,3,4, \dots$....

$C_{R_i}, i = 1,2,3, \dots$.. the retailer who makes a proffer to the other retailer in the nabe as per Case 2

$C_i, i = 1,2,3, \ldots ..$ is the inventory cost for 10 units of the product for each retailer (used to attain Equipoise in the market)

R_{I1} = first retailer's total cost

R_{I2} = second retailer's total cost

R_{I3} = third retailer's total cost

R_{I4} = fourth retailer's total cost

Q_I = optimal order quantity of the system

K = Defective percentage

θ_i = Fuzzy deterioration rate where i=1,2,3,4

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Compendious of Chapters

The thesis starts with an introduction chapter 1 about Operations Research and its definition. The chapter moves ahead with introduction of inventory, different types of inventories and also need of inventories. Costs related to inventory and various other parameters (Inflation, deterioration, finite planning horizon, fuzzification, trade credit) effecting inventory are discussed for the smooth functioning of FMCG industry.

Literature review forms the part of Chapter 2. A detailed review is conducted in inventory, gaps are identified in the literature existed in this field. The research objectives are framed and further we discussed methodology used in the upcoming chapters.

Chapter 3 is for Supplier – retailer inventory coordination with trade credit for deteriorating items including FMCG. Deteriorating inventory and perishable items are considered. Thus, inventory finishes up due to demand and decaying. A period was given for credit payment by the supplier to the retailer and its greater than the replenishment cycle length. This results in larger period of interest rate which eventually reduces the cost of the retailer. The theoretical model is accompanied along with a mathematical model. The model was solved by using Mathematica 12.0. Sensitivity was performed with various parameters. The chapter is concluded by mathematically solving non-linear differential equations and proving results mathematically that greater is the interest rate earned by the retailer, lower will be the cost of the retailer.

Chapter 4 is for supplier – retailer inventory coordination with trade credit for seasonal demand in FMCG sector. Seasonal ramp demand is taken to represent seasonal FMCG products under finite planning horizon of 120 days with uneven replenishment cycle lengths. Coordination and no coordination cases are discussed separately along with trade credit period, interest rate charged by the supplier and the rate of interest earned by the retailer. All the related cases and subcases are discussed in the article under this chapter. Algorithm shows simplified steps to compute costs. Mathematica 12.0 was used to deduce replenishment lengths of inventory cycles and numerical examples justify the results in favour of coordination case.

Focusing on the next objective in Chapter 5 for studying the effect of inflation in supplier – retailer inventory coordination with trade credit in FMCG for inventory-dependent and time-dependent demand, some novel concepts of equipoise of the inventory cost of the retailer is attained. Inflation and its everchanging nature is the main focus of this chapter. Considering such nature of inflation a near to reality model was discussed. A linear and variable rise in inflation is taken to consider variable nature of inflation. A single supplier – multiple retailer coordination was discussed. Algorithm 5,1 was used to calculate the cost of different retailers and algorithm 5.2 was used to calculate the equipoise cost of all retailers. This equipoise cost will help to maximize profits of the retailers and will ultimately benefit the supply chain at every user's end.

The next objective of Fuzzification of supplier – retailer coordination under Finite planning horizon for deteriorating items and seasonal demand is dealt in chapter 6. For a time-sensitive seasonal demand and FMCG industry, seasonal ramp type demand is considered along with a deteriorating inventory. Deterioration rate and defective products within the total quantity are fuzzified to calculate the cost of the retailer. The chapter justify result in the favour of fuzzified retailer's cost. Lead time is zero keeping in view dynamic nature of FMCG industry.

Chapter 7 concludes the thesis with two major parts providing conclusion of the work done and the futuristic insights for the chain managers and for the players of this rapidly expanding FMCG industry.

Chapter 1

Introduction

1.1 Introduction to Operations Research (O. R.)

1.1.1 A Brief History of Operations Research

No science was born on a particular day. Each science emerges from the increased interest in some sort of problem, developing deductive methods, different procedures, and tools which are appropriate for solving problems. Operations Research (O.R.) is no exception. Its roots are as old as science and management functions, but its use started at a great level during World War II (1939 – 1945). During World War, both side countries were trying to use their resources optimally and hence the use of Operation Research started firstly in the military context. Fundamentally, the scientists of the United Kingdom adopted new techniques to adequately use their resources followed by other countries. This is where the origin of O.R. can be traced. Over time, O.R. was applied in many other fields and hence it took the multi-dimensional framework.

The centre of attention of Operations Research is to recommend the best course of action in an 'Operational' problem. Mc Closky and Trefthen in the United Kingdom, first coined the term "OPERATIONS RESEARCH" in 1940. O.R. club (new name is Operational Research Society of U.K.) was formed in England in 1948. Its journal by the name of O.R. quarterly first appeared in 1950. Additionally, The Operations Research Society of America (ORSA) was established in 1952.

In India, Operations Research came into existence in 1949 with the establishment of the Regional Research Laboratory, Hyderabad, and planning and research reached the next level. The O.R. Society of India (ORSI) was founded in 1957. India joined the International Federation of Operational Research Societies (IFORS) in 1960 [1]. Taking a step forward, ORSI published its 'OPSEARCH' journal for the first time in 1963.

1.1.2 Definition of Operations Research

Many scientists and researchers gave definitions of O.R. from time to time. According to *Morse and Kimball* (1946), "Operations Research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control." *(Morse is the father of Operations Research).* Hence over time, the O.R. which was only used in its earlier days in the context of the military and the navy started to sneak peek into the civilian world as well. OR started to be used widely in various fields like business, society, and even industry. Thus, its scope widens and by now it had covered almost all spheres be it governmental departments, finance, or logistics. Therefore, the need arises for developing mathematical models in OR.

1.2 Inventory

1.2.1 Introduction

It literally, means **stocks**. For a business, it is raw materials available in their warehouse, work in progress in production, and ready for sell products, packaging, and even spare parts. Academically it means stocks of goods that have demand and supply in the market, and which can be easily converted into cash within one year. For this reason, inventory is mentioned in current assets on balance sheets. In current times, anything which is usable even idle resources and even humans form the inventory. When inventory is materialistic it is termed as stocks or simply inventory. Different types of organisations maintain different kinds of inventory, in blood banks, blood forms the inventory, in hospitals, beds, medicines, trained staff is their inventory, in case of manufacturing firms, raw material, finished goods, semi-finished goods, spare parts all are the part of their stocks.

Inventory is vital for the world economy and from a business point of view inventories amount to an investment; capital is needed to store materials at every stage of completion. For the smooth functioning of every business house, some kind of inventory is needed and hence inventory control and inventory management system are also required. The most important work is to have a balance between the advantages of maintaining an inventory and the cost of having inventory so that the optimal level of inventory can be determined. [2]

1.2.2 Types of inventories [3]

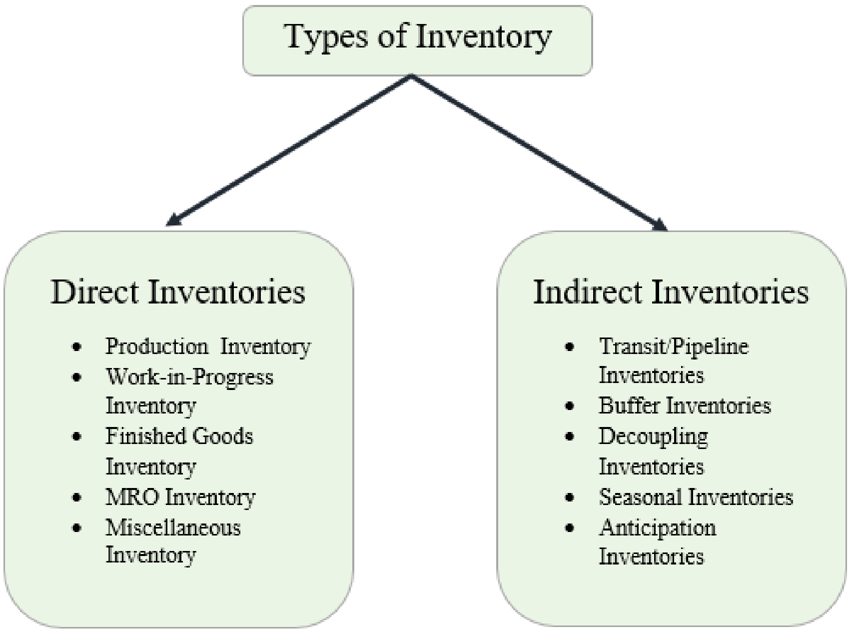


Figure 1.1: Different types of Direct and Indirect Inventories.

1.2.2A. Direct Inventories

- ***Production Inventory:*** These are also called *Raw Material Inventories.* The material which is used to produce finished goods is called raw materials. A warehouse of a firm doesn't only stock finished goods but also has a subsequent stock of raw materials as well. Consequently, the business houses need to have an inventory of raw materials so that machines keep on moving and finished goods are produced to meet the need of their customers.

- ***Work in progress inventory:*** It is those semi-finished stocks accumulated between two operations. The main purpose behind maintaining such inventories is to serve the purpose of replacement of wastages, maintain a uniform production rate even when sales are variable, and to cater different types of products.
- ***Finished Goods Inventory:*** These are finished goods, and such types of inventories are maintained to tackle different types of customers' demand, and even promote sales, it stabilizes the production level as well.
- ***MRO Inventory:*** It stands for *Maintenance, Repair,* and *Operating*. All such inventories which are very much essential for the production and manufacturing of final goods but don't form the part of the final product, fall under this category of inventories. Examples: petrol, coal, lubricants, oil, grease, etc.
- ***Miscellaneous Inventory:*** all other items like scrap or fragment pieces or outdated stocks *(maybe due to obsolescence or change of fashion or change of technology)* or stocks that are not sold, other items which are used in offices or various departments, and items included in stationery forms the part of this type of inventory.

1.2.2.B. Indirect Inventories

- ***Transit Inventories:*** these are *also termed Movement or Pipeline inventories.* Such inventories occur due to the shipment of various inventories from the production units to different distribution centres. Thus, to maintain smooth accomplishment of demand during the transit time, extra or transit inventories are maintained
- ***Buffer Inventories:*** it is also known as *Safety Stocks.* Companies many times prepare their safety inventory to cover various types of fluctuations in demand.
- ***Decoupling Inventories:*** These are internal inventories, which means it is maintained by the companies only to counter any unforeseen shutdowns or failure of machines. In today's manufacturing environment a single machine doesn't produce a whole product. A single product must pass through many stages to finally take the shape of the final product. Hence if at any stage

- machinery failed, it can lead to a shutdown of all the remaining machines, thus a firm, to tackle such scenarios, keep the stocks of all the products which a particular machine produces at every stage. It’s a sort of buffer stock being maintained by companies to keep running their machines at different stages of producing a product in case a machine stops working so that all other stages can proceed without a hold and the demand of customers were met on time.
- ***Seasonal Inventories:*** Some items have seasonal demand as cold drinks, iced tea, and juices are in high demand in summers. In winter sale of products like Chyawanprash, Honey, Tea, and Coffee increases, while in the rainy season, mosquito repellents are in high demand. Thus, the businessman must know when to stock for what products and for how much quantity stocks should be arranged so that neither they have to face lost sales nor stocks should be more than the required demand.
- ***Anticipation Inventories:*** These are kept as the future demand of some products can be predicted (or anticipated). Specialized items like rakhi before the festival of Raksha Bandhan or crackers before Diwali, air coolers, refrigerators, and air conditioners before the onset of summers are produced, describes the seasonal or anticipated inventories. The reason behind this is the smooth running of machinery all year round rather than overloading and working overtime in times of high demand.

1.2.3 Need for inventories

The foremost reason to hold inventories is to accomplish customers' demands on time else the customer can search for another buyer which ultimately leads to losses in a business (Lost Sale). Moreover, inventory is needed as every unit needs to have something to work with. There is a saying that defines inventory as a necessary evil. The reason behind this is due to inventory the capital is blocked, and the supplier had to forgo his opportunity cost of capital, but it is necessary as stocks allow you to fulfill the customers' demand, and having a strong customer base is a backbone of every business [4]. Some business processes also necessitate that there should be inventories, like

- ***Seasonal demand:*** There are many such items whose demand fluctuates due to changes in season, so the owner needs the stock to fulfill such changes in demand.
- ***High demand items:*** Many such items which are high in demand to be held as inventory as replenishment of such items may take time, as a consequence to be on the safer side its stockpiling is necessary.
- ***Lead Time:*** Overall models take lead time (the time between an order for inventory is placed and the order of inventory replenished in actual in the warehouse) as zero, but in practicality during this lead time to keep the machines moving, a business house needs inventory.
- ***Safety Stocks:*** Companies many times prepare their safety inventory to cover various types of fluctuations in demand.
- ***Miscellaneous:*** Apart from the above-mentioned reasons for maintaining inventory there are other factors also like *multi-stage production* as these days different parts of a product are manufactured at different production units and later transported to assemble them into a final product, thus it gives rise to compulsory maintenance of inventories. Other factors like bulk purchasing due to quantity discounts, deteriorating items, imperfect quality items, and the decay of goods all point towards having good inventory stocks, and thus there arise the need for an efficient inventory management system.

1.2.4 Different types of costs related to inventory

The inventory costs of every organization are very much affected by an increase or decrease in inventory levels. The policy framework of an organization always pays attention to efficiently maintaining its inventories. Inventory costs play a vital role in accurately controlling the inventories of a company. According to a general review, approximately 70 % capital of a company is invested in its different types of inventories. Thus, to maintain effectively this 70 % of its capital a company appoints the best brains as inventory managers and every such manager must know all the costs related to inventory. [5]

- ***Set up Cost:*** It is the cost of setting up machines. It includes even the cost of labour required to set up machines in their respective place. This is usually independent of units produced or ordered.
- ***Ordering Cost:*** It is the cost that must be borne by the forms whenever they make a new order. It includes various other costs like cost of advertisement, cost of placing an order, clerical cost, small administrative costs, travel expenditure, postal stamp and duty, and courier costs.
- ***Carrying Cost/ Holding Cost of Inventories:*** It is also called a *stock holding or storage cost.* Such costs come into the frame while maintaining inventories, it even includes interest on the capital invested in inventories. This type of cost is dependent directly on the number of stocks as well as the time for which the inventories are kept. It has various other components as follows:

a. *Capital tied up in inventories: or cost of money.* It is seen in a such way that how much a company or a firm has earned if it had invested the capital (which was tied up in inventories) in some other project from where they have earned revenue. Moreover, the company is not only losing its potential revenue but also has to pay interest if money is borrowed from a bank. This cost is rather the most important one under the heading of carrying or holding cost. It makes up around 15% to 20% of the total stock value.

b. *Depreciation cost:* The items like perfumes, chemicals, blood in the blood bank, food items, glassware or fragile items, fashion goods, the stocks for all such items deteriorate or depreciate with time. Such costs end up between 0.2 % to 1% of the total value of stocks.

c. *Storage space cost:* It is also called warehousing cost. It is the rent given for the storage space or a warehouse. It will include some overhead expenses such as the cost of lighting, electricity bill, and temperature control inside the warehouse to lower the depreciation cost. It varies between 1 to 3 % generally.

d. *Cost of obsolescence:* the word obsolescence in the case of inventory refers to all such stocks which become outdated or go out of fashion. This cost needs to be considered in the present scenario as fashion goods (clothes, accessories changes very soon), electronic goods, software, etc. these types of inventory are always sold on FIFO (First In First Out) basis, and if some stocks are left behind, mostly they can be given as freebies with other items or can be sold as stock clearance sales. It is generally taken as *5% of the total stock value.*

e. *Pilferage cost:* The word pilferage means theft or robbery. There are some items in inventory that need to be kept under security and some heavy items under inventory may be kept open and still, no one can misappropriate them. Its approximately taken to be around *1% of the total stock value.*

f. *Taxes and cost of insurance:* Maximum firms these days go for an additional cost of insurance, as it provides safety against many types of mishappening. Moreover, with the changed governmental norms, taxes are also to be filed on time and these too add an extra cost to the firm. Taxes and insurance costs may be taken as *1% to 2% of the total value of stocks.*

g. *Handling cost:* Movement of stocks, cost of labour, gantries, cranes all such types of costs are part of handling cost.

h. *Other administrative costs:* This even includes the cost of handling and keeping records, as inventories can be so large in number that to properly manage them a separate fund is to be allocated.

- ***Stockout Cost/ Shortages Cost:*** Shortages or stock out means unable to fulfill the demand of customers. It can be seen as the loss of potential profit and even loss of goodwill (customers can be lost forever, as they can fulfill their demands from another supplier. It can be seen as running out of stocks. It generally arises when demand is greater than supply but it's almost unable to predict exact demand as these days customer and their choices are uncertain and changes very quickly.
- ***Selling/Salvage Cost:*** it’s the total cost we will obtain after subtracting the depreciation. In other words, it’s the total amount that an organization will receive after selling the stocks after deducting deterioration.

Cost of money 15% - 20%	Deterioration 0.1% - 1%
Storage space cost 1% - 3%	Obsolescence 5% - 7%
Pilferage cost 1% - 2%	Taxes and Insurance 0% - 1%
Handling cost 2% - 5%	Other administrative cost 3% - 6%

Figure 1.2: Approximate value of different types of costs related to inventory.

1.2.5 Inflation in inventory

Inflation simply means a rise in prices. Economists have a separate point of view on inflation. According to economists, inflation is the result of a greater supply of money than its demand. When there is a general rise in prices of products (goods and services) over a while in an economy, it is defined as inflation. As a result, a person can buy fewer products with every single penny, implying that there is a fall in purchasing power. The effect of inflation on an economy can be both positive and negative. It is positive when it leads to a rise in the consumption power of the consumer and the demand of consumers is increased. It happens generally when the economy is running slow, thus in that scenario, inflation helps in raising the production levels.

Meanwhile, in higher inflation condition, the consumer will start holding more goods than required (hoarding) thinking that the price will further rise in the future. As a result, companies are encouraged to keep their level of inventories high [6], [7].

Generally, to dispose of inventories, FIFO, LIFO, and Weighted Average methods are used by the companies. But during inflation, the LIFO method is preferred because, under this, the cost of goods sold is based on the last material purchased, which will primarily reflect current costs. As a result, it will give proper estimates of sold inventories. This method is also desirable as it will lead to tax benefits during the inflation period. It will lead to a rise in income as well (due to tax benefits).

1.2.6 Planning Horizon in Inventory

Generally, inventory models can be broadly classified into two categories based on time horizon – infinite planning horizon and finite planning horizon [5]. A finite planning horizon means that the replenishment cycle did not repeat itself and that too not in the same manner. In other words, we can say that every time replenishment time of inventories varies. Thus, every replenishment cycle has a different optimal solution in a finite planning horizon and the main aim behind the whole process is to minimize the total cost within [0, T]. In the case of an infinite planning horizon, the cost of an entire period can go up to infinity and the managers are not able to compare costs for different replenishment cycles. But this does not mean that an infinite planning horizon never works. Both finite and infinite horizon solve their respective purpose but under different scenarios. Thus, the managers need to make appropriate decisions for developing inventory models that appropriately reduce the cost and maximize the profits. [8]

In today's time we have neither fixed inflation rate nor replenishment rate. The replenishment rate will change when there is any volatility in the market. That being the case, it can never be fixed in the Inventory Management System, and that is why the role of the Finite Planning Horizon increases here and that too with different cycle lengths. Replenishment rate depends on many things like, demand of product at particular time in the market, inflation in country, also on some external scenarios, effect of season also we cannot ignore here, how fast the product is selling and the platform through which it is being sold, these are all things that can boost or diminish the replenishment rate, and with this the timing of when the supplier or retailer will refill their inventory will also change.

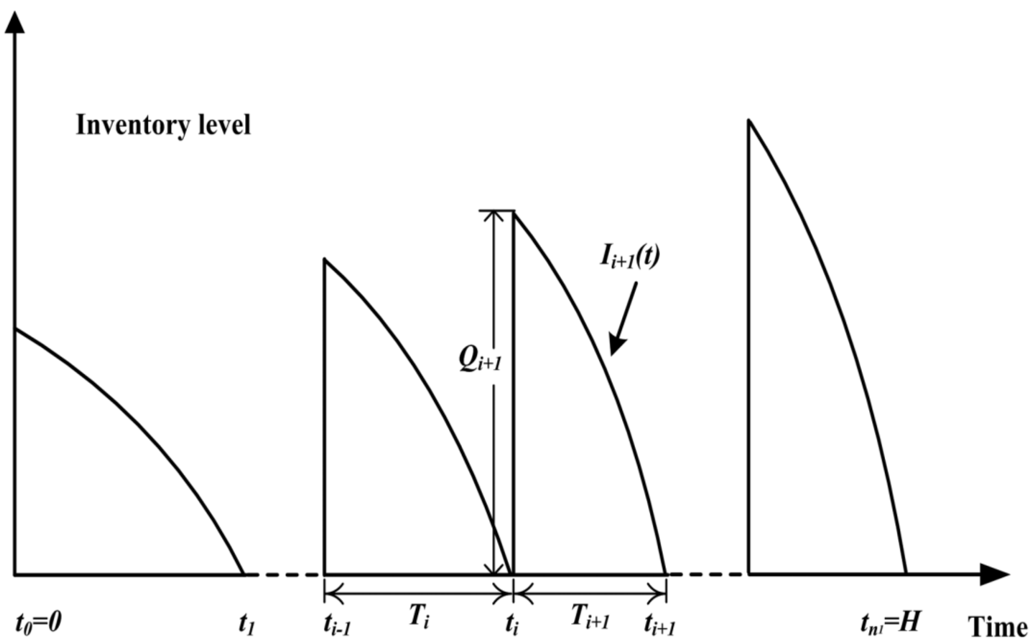


Figure 1.3: Finite Planning Horizon Model with unequal cycle lengths.

*The above **figure 1.3** shows a finite planning horizon model in which unequal cycle lengths are taken. It means the replenishment cycle is not replenished at equal intervals say every week. It can be replenished as and when there is a requirement. These type of planning horizons has an advantage over others as, the inventories can be renewed earlier also when there is a rise in demand, and even the stocks can be delayed depending upon a fall in sales due to various reasons, maybe change of season, festive effect, obsolescence, etc.*

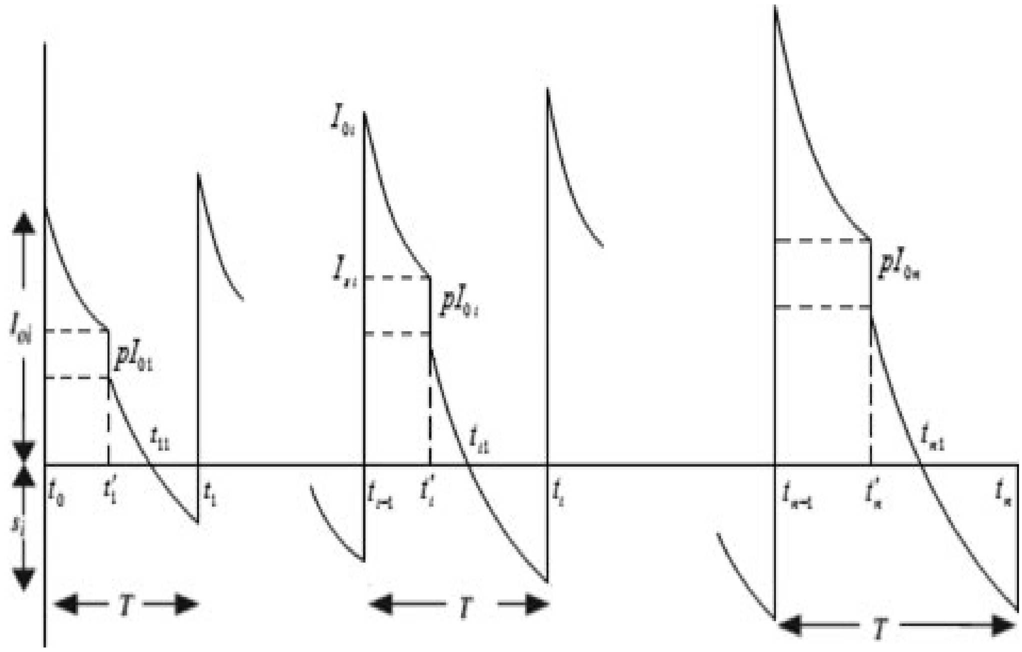


Figure 1.4: Finite Planning Horizon Model with equal cycle lengths.

*The above **figure 1.4** shows a finite planning horizon model in which equal cycle lengths are taken (as we can see every time, the time interval taken is T).it means inventories will be renewed after time T only. This type of planning horizon gives us a futuristic implication that the time to renew inventories is approaching. Moreover, these are successful, if the demand for a product is constant.*

1.3 FMCG (Fast Moving Consumer Goods)

The acronym ***FMCG*** stands for ***Fast Moving Consumer* Goods*.*** As the name says about itself this includes all those items or products which are consumed immediately and moves very fast because of high demand. Many FMCG products are short-lived as their shelf life is very less or are perishable like soft drinks, edible items, baked and dairy products. Many a time they are also termed as *consumer-packaged goods* [9]. FMCG is a fast-emerging sector. The people who are involved in this sector deal with all such items, which reach the consumer very quickly or get used. These items are not held for long. By keeping it for a long time, either these things will get blemishes, or their quality will be affected. Everything in this sector is or will be found, which is included in Inventory Management System.

1.3.1 Interesting points to know about FMCG

- It is a recession-proof industry.
- It is most stable in the current scenario.
- It is evolving constantly and getting bigger than its size of yesterday.
- It is truly a global industry including everything from rural to urban, from major to megacities, from national level giants to international sharks. [10]

1.3.2 FMCG in India

It is the 4^{th} largest sector of the Indian economy [11]. In India, this sector includes packaged food products, pharmaceuticals, personal care products, hygiene-related products, health products, beauty and fashion products, and consumer electronic items [12]. Rural India and urban India account for 40% and 60% respectively in FMCG products. Rural India goes for one-time use products, or we can say a sachet revolution [13], [14], [15]. Seeing such potential of FMCG in India the Indian Government has permitted 100% FDI in Single Brand Retail Trade (SBRT) and the minimum capitalization of foreign FMCG companies in India is $100 million. The government will also benefit from GST implementation in FMCG products as the maximum products that are sold in India under this category are soaps, hair oil, and toothpaste which falls under the 18% tax regime of GST [15], [16], [17].

1.3.3 Major FMCG Companies in India

1. Hindustan Unilever Ltd. (HUL)

2. ITC (Indian Tobacco Company)

3. Nestle India

4. Dabur India

5. Britannia Industries

6. GCMMF (AMUL)

7. Asian Paints

8.Cadbury India

9. Procter & Gamble Hygiene and Health Care

10. Marico Industries

*Based on a study conducted by AC Nielson [18].

1.4 Trade Credit

The simple way is we buy a product, and we pay for that. But in practicality in maximum businesses, there is an option called Trade Credit [19]. In this the supplier allows the retailer a particular period, when the purchases are made, to pay for that. It is within a permissible limit so that no losses are incurred by the supplier. Recurrently, to counter the risk of default, the supplier offers a discount to encourage earlier payments. For example: suppose the trade credit period was 60 days (as most common are 3 i.e. 30 days, 60 days, and 90 days trade credit). Then the supplier can say, he will offer a 2% discount on the final amount if payment is made within 7 days or 1 % discount if the same was made in 14 days. In accounting terms, it will be written as 2/7 net 60 or 1/14 net 60 [19].

Every supplier has its criteria by which they select that how much credit period should be allowed to any retailer (called creditworthiness or goodwill of the retailer). Some agencies provide credit scores to business houses like CIBIL, CRISIL.

1.5 Decay/Deteriorating Items

Deterioration means decay or to become worse or to fall from higher to lower level in quality. This is a common problem seen in almost all types of inventories, may it be food grains including vegetables, fruits, meat, or dairy products, the blood in the blood bank, medicinal and biological compounds, chemicals, perfumes, electrical and electronic items, i.e. loss in value of a product at any given time is deterioration. Thus, managing inventories of items that lose their value and weight with time is an important inventory problem in Fast Moving Consumer Goods [20], [21].

1.6 Fuzzification in Inventory Models

Fuzzy set theory has gained momentum in the present competitive and rapidly changing business world for almost the last fifty-odd years. The classical models of inventory management (EOQ and EPQ) assumed that all inventory-related scenarios occur in a predetermined and certain (crisp/ it is opposite of fuzzy) environment. But the fact is many scenarios are uncertain (fuzzy). Thus, the concept of Fuzzy Set Theory (FST) was introduced in inventory management. Application of fuzzy techniques can facilitate dealing of more real-world cases and that too in a properly managed way.

1.7 Emerging Trends in Inventory Management

1. **Impact of Inflation on Inventory Methods**
   - Preference for Last-In, First-Out (LIFO) method during inflationary periods.
   - LIFO reflects current costs, providing accurate inventory valuation.
   - Tax benefits associated with LIFO improve financial efficiency.
2. **Technological Advancements in Inventory Tracking**
   - Artificial Intelligence (AI) and Internet of Things (IoT) for real-time inventory monitoring.

- Automation and predictive analytics optimize inventory control and demand forecasting.
- Just-in-Time (JIT) strategies minimize holding costs and reduce excess stock accumulation.

3. **FMCG Sector and Inventory Dynamics**
   - FMCG, being a recession-proof and rapidly evolving sector, demands agile inventory management.
   - High turnover rates require efficient stock replenishment and demand anticipation models.
   - Digital transformation in inventory systems enhances operational efficiency.
4. **Role of Trade Credit in Inventory Management**
   - Credit-based purchasing systems (e.g., trade credit) help retailers manage liquidity.
   - Discount-based early payment incentives improve working capital cycles.
   - Creditworthiness assessment using agencies like CIBIL and CRISIL ensures risk mitigation.
5. **Mathematical Optimization in Inventory Management**
   - Stochastic models and linear programming used for inventory cost minimization.
   - Differential equations and Markov chains applied for demand prediction and supply chain efficiency.
   - Game theory and queuing models optimize resource allocation and warehouse logistics.
6. **Global and Policy Influences**
   - Government initiatives like 100% FDI in Single Brand Retail Trade (SBRT) impact inventory strategies.
   - GST implementation in FMCG affects pricing structures and tax efficiencies.
   - International trade policies and digitalization shaping modern inventory management framework.

Chapter 2
Literature Review

2.1 Literature Review

To talk about inventory management Herman Hollerith (an American inventor) first time designed a machine to record inventory information using punch cards [22]. In the late 1940s barcode was invented by Norman Woodland to track inventory [22]. Systematic analysis of inventory problems was started in 1913 when Ford W. Harris developed an Economic Order Quantity (EOQ) Model [23]. Later in 1918, E. W. Taft developed an Economic Production Quantity (EPQ) Model. This was an extension of the EOQ Model [24]. After this, many researchers made many changes in these models, developed new models, extended these models by adding new parameters; like in 1985 Goyal [25] developed an EOQ model with permissible delay in payments. Later Aggarwal and Jaggi, in 1995 [26], extended the work done by Goyal for deteriorating items with an added assumption that sales revenue can also be accumulated after the delay period.

Moreover, apart from all the above parameters, the relationship between supplier and retailer is also important. Thus, many researchers worked in this area. Banerjee [27] (1986), Liao et al. [6] (2000), Yang and Wee [28] (2000), Sarker et al. [29] (2000), Chung [30] (2000) all have one similarity that they prepared their models for deterioration in a two-level supply chain coordination (two-level supply chain implies to minimize inventory for all members and also to meet all the requirements of customers). In 1991, Raafat [31] prepared his model for deteriorating items and gave a review for the deteriorating inventory. He defined deterioration as decay, spoilage, and physical depletion. His work was extended by Goyal and Giri [32] (2001) for the permissible delay in payment, price discount, and price increase. In 2012, Bakker et al. [33] presented a detailed and up-to-date review of all the advancements done in the field of inventory control since 2001 (many classifications of Goyal and Giri (2001) were considered).

The classical EOQ model assumes that the retailer makes the full payment as soon as the products are received by him, but in practicality, this is one of the special cases. Generally, every supplier offers a credit period to retailers to settle the whole amount. Further to encourage faster payments, the supplier may also take the help of discounts, which means he can offer certain discounts if a retailer makes full payment before the credit period gets over. Wu and Zhao [34] (2014) describe that the relationship between supplier and retailer is becoming more stable over time. To strengthen the bond between the two, suppliers rely on trade credit, it can be said that it is a step to say that retailer is being trusted by the supplier, though the trade credit period itself depends on various other factors like the creditworthiness of the retailer, deterioration of items, the kind of the product purchased, and the demand of the products.

In the present world, inventory is not only affected by the above-mentioned parameters but is very much affected by demand also. Demand can be of various types. In the case of FMCG also, the inventory is affected by demand. Based on demand, inventory can be categorized into two types, and it is the reason for which the inventories are maintained. Two types of demand are Independent Demand for inventories and Dependent Demand for inventories. The earlier includes those items whose demand is not dependant on any other item/s. for example Mobile phones. And the latter includes those products whose demand is dependent on other items as well. For example, cover for the mobile phone or screen guard for the mobile [35]. Independent Demand for inventories is based on the customers' confirmed orders, prices of the item, the current scenario, past data, future forecasts, and estimates. On the other hand, the Dependent demand for inventories is generally based on the final product's sale. Apart from the above-mentioned two demands, the inventories for FMCG are also encountered with Seasonal demand or Festive demand. Thus, there arises the need for demand planning [36]. Because of its structure, the FMCG companies rely heavily on forecasted demand figures, and this includes all the downward processes like inventory management, production planning, buying, transportation – in short, the whole supply chain management and ultimately delivering to the customers (Basson et al. 2013) [36].

When the demand is greater than the supply, it leads to inflationary conditions in an economy. J. A. Buzzacott [37] (1975) was the first researcher who wrote something about inflation in inventory models. According to him, if the change in prices is independent of the replenishment cycle, then the inventory charge is low, and inflation also doesn't affect the inventory. He also formulated a minimum cost model for a single item in inventory and studies the effect of inflation on it. Guria et al. [7] (2013) framed an inventory policy for an item with inflation and demand depending on the selling price. The author added a new parameter of immediate part payments to the wholesaler.

From the days the EOQ and EPQ models were developed, almost all organizations and firms used these models to manage their inventories and the related issues. Over the period researchers continued to add more parameters like deterioration, inflation, demand, trade credit, partial back ordering, discount, coordination. All are above scenarios are based on one important assumption that all these parameters occur under a certain and deterministic environment. But this fact can't be always true in the current business world. Thus, an effective way to manage these discrepancies is to use fuzzy set theory, which can transform all the ill-defined information into valid mathematical expressions [38]. From the past fifty to sixty years, fuzzy set theory is gaining the necessary momentum and by now it's well applied in the field of inventory management. It has been extended to numerous parameters used in inventory models. According to reviewed literature, fuzzy set theory is applicable through given procedures.

Step 1: target an inventory model.

Step 2: identifying the ambiguous attributes.

Step 3: transforming the crisp model into the fuzzified one.

Step 4: finding an appropriate method for defuzzification.

Step 5: validating the Model.

Step 6: analysing the results.

2.2 Research Gap Identification

Enough research work has been done in the field of inventory management, be it preparing models, or adding more practically applicable parameters into the earlier derived models. But to the best of my knowledge, the work of inventory management is not extended to Fast Moving Consumer Goods (FMCG) area. Thus, there is a dire need of studying inventory theory which includes FMCG. As FMCG is the fastest growing and most promising business in the present scenario, thus this sector has a lot of potential, and many gaps are there in the inventory management system of FMCG which needs an address.

This sector faces maximum lost sales [39]. The main reason behind this being the increased use of e-commerce websites. Due to these websites, customers don't want to wait if there are shortages with one supplier; they immediately shift to another supplier where there is an availability of stocks. Studies point out that, out of every two items sold on e-commerce platforms one is from the FMCG sector. Thus its 50% share is still in a shadow and waiting for the covers to be uncovered by the new generation of research scholars.

FMCG sector has a lot of potential in rural areas as well. Thus, one can take up a project showing how inventory is different for the urban area and rural area customers [40] and what are the challenges in this field, may it be deteriorating items, seasonal demand, different demand patterns, shortages, increased transportation cost, or may it be a long payment period provided by the supplier to the rural area dealers.

Inflation is another parameter for inventories in the FMCG sector, as these goods are consumed very rapidly, thus if inflation stops, there is a lot of loss which the retailer and supplier need to bear. Thus, there is a need to work on the supplier – retailer coordination in managing inventories for the FMCG sector under inflationary conditions.

Since the sector of Fast-Moving Consumer Goods (FMCG) is very fuzzy thus there is a scope for Fuzzification of the supplier – retailer coordination.

1913	EOQ Model (Ford W. Harris)
1918	EPQ Model (E. W. Taft)
1940s	Barcode System (Norman Woodland)
1975	Inflation Effect in Inventory (Buzzacott)
1985	EOQ with Trade Credit (Goyal)
1986	Two-Level Supply Chain Model (Banerjee)
1991	Review of Deteriorating Inventory Models (Raafat)
1995	EOQ with Deterioration & Trade Credit (Aggarwal & Jaggi)
2000	Supply Chain Models for Deterioration (Liao, Yang, Wee, Sarker, Chung)
2001	Extended Deterioration Models (Goyal & Giri)
2012	Review of Inventory Advancements (Bakker et al.)
2013	Demand Planning for FMCG (Basson et al.)
2013	Inflation & Price Dependency in Inventory (Guria et al.)
2014	Trade Credit & Supplier Trust (Wu & Zhao)
Recent	Fuzzy Set Theory in Inventory (Various Researchers)

Table 2.1 Timeline of Key Inventory Management Developments

Year	**Key Contributions & Researchers**
Pre-20th Century	- Herman Hollerith (Punch card-based inventory recording)
1913	- Ford W. Harris: Economic Order Quantity (EOQ) Model
1918	- E.W. Taft: Economic Production Quantity (EPQ) Model
1940s	- Norman Woodland: Barcode for inventory tracking
1985	- Goyal: EOQ model with payment delays

Year	Key Contributions & Researchers
1986	- Banerjee: Two-level supply chain model with deterioration
1991	- Raafat: Deteriorating inventory review
1995	- Aggarwal & Jaggi: EOQ for deteriorating items with sales revenue accumulation
2000	- Liao et al., Yang & Wee, Sarker et al., Chung: Supplier-retailer coordination in inventory models
2001	- Goyal & Giri: Delay in payments, price discounting, and price increase
2009	- Mirzazadeh et al.: Inflationary inventory model under finite production rate
2011	- Tripathi: Pricing and ordering policy under inflation with permissible delay in payments
2013	-- Shastri et al.: Manufacturer-supplier-retailer inventory model under inflation
2013	- Guria et al.: Inflation-sensitive inventory model with part payments
2014	- Wu & Zhao: Trade credit and stable supplier-retailer relationships
2015	- Kiniwa et al.: Deflation in price stabilization process
2015	- Kumar & Rajput: Inflation & credit policy-based demand models
2017	- Singh et al.: EOQ model in finite planning horizon with and without delay in payments
2017	- Zhong et al.: Integrated location & two-way inventory networking under trade credit
2018	- Singh et al.: Inflation-based inventory model with trade credit
2019	- Singh et al.: Centralized & decentralized planning horizon with trade credit policy

Year	Key Contributions & Researchers
2019	- Saha & Sen: Selling price & time-dependent demand model under inflation
2020	- Esmaeili & Nasrabadi: Single supplier-multiple retailer model under inflation & trade credit
2020	- Taghizadeh-Yazdi et al.: Supplier-manufacturer-distributor coordination (15% profit increase)
2020	- Li et al.: Optimized inventory via replenishing & salvaging in e-commerce FMCG
2021	- Mahato et al.: Two-step supply chain under trade credit with hybrid pricing
2021	- Barman et al.: Variable & fuzzy demand models with inflation
2021	- Mandal et al.: Advertisement & stock level-dependent inventory model under trade credit policy
2022	- Khan et al.: Supply chain model with non-linear stock-dependent demand rate

Table 2.2 Summary of Literature Review on Inventory Management

2.2.A. Key Findings:

1. **Evolution of Inventory Models:** From punch cards and barcodes to advanced EOQ and EPQ models incorporating multiple parameters.
2. **Supplier-Retailer Coordination:** Trade credit, deterioration, and two-level supply chain integration play crucial roles.
3. **Impact of Inflation:** Various models examine price fluctuations, part payments, and inflation-sensitive demand.
4. **Demand Types & Effects:** Independent, dependent, seasonal, and inflation-induced demand patterns significantly affect inventory models.

5. **FMCG Industry Considerations:** Fast-moving consumer goods require dynamic inventory models with demand forecasting and e-commerce integration.
6. **Emergence of Fuzzy Set Theory:** Modern inventory models incorporate uncertainty through fuzzy logic for better real-world applicability.

2.3 Scope of Study

The models for inventory management have worked on many real-life problems of the supplier and the retailer. In the present model as well, we tried to establish a close relation between inflation and cost of the supplier and the retailer. The is much need that such a model is formed which consider the discount per cycle as well. Till now, this part has still not been worked upon. Using such a real-life model, the costs of the supplier, the retailer, the system can be reduced and hence raising, the profits of all. Other type of costs like transportation cost, mechanical costs, fuel efficiency, carbon points all can be clubbed into the exiting model under finite planning horizon.

2.4 Research Objectives

Based on the above research gap identifications, I have framed the following 4 objectives for my research work.

1. To analyse supplier – retailer coordination for managing inventory levels with trade credit for deteriorating items including Fast Moving Consumer Goods (FMCG).

2. To examine supplier – retailer inventory coordination with trade credit for seasonal demand in the Fast-Moving Consumer Goods (FMCG) sector.

3. To study the effect of inflation in supplier – retailer inventory coordination with trade credit in FMCG for inventory-dependent and time-dependent demand.

4. To fuzzify supplier – retailer coordination under Finite planning horizon for deteriorating items and seasonal demand.

2.5 Proposed Methodology

In solving inventory problems generally, three types of cost are taken into consideration, and that is carrying cost, cost of shortages, and replenishment cost. All three of them are closely related to each other and the main work of a researcher is to get the lowest cost (means the sum of three should be lowest). Inventory control is a tough challenge and to have a solution to these, many techniques and concepts were given from time to time. While considering inventory control two questions need to be addressed.

(a). when should the inventory be replenished by the retailer (means the time when the order should be placed to renew the stocks) for different cycles in a planning horizon?

(b). how much inventory should be replenished (i.e. how much stocks should be ordered so that even the shortages and backlogs of the previous cycle and the current demand all should be covered) in a planning horizon?

Finally, an inventory problem is to find such value of the variables that minimize the cost, and it can be done when there is a balance between the carrying cost and the replenishment cost.

For analysing an inventory system, the following steps has been considered:

(a). Determined the properties of the system

(b). Formulation of an inventory problem

(c). Developing a model of the system

(d). Deriving a solution of the system

To derive a solution, standard techniques, numerical analysis, numerical methods, Iterative or approximation methods, whatever is suitable depending on the problem has been taken up.

Methodology followed in the research work:

Moreover, a real-life problem by contacting a firm or a company that works in Fast Moving Consumer Goods has been taken up.

In the starting phase of my research work, I visited a local firm that deals in FMCG and works in Haryana and Delhi region, to have a piece of knowledge about working of FMCG firms.

What information and knowledge is gathered from there is listed below in some points:

- Inventory is an inseparable part of FMCG.
- Inventory is very much affected by seasonal demands.
- Inventory can't be separated from inflation even in these FMCG firms.
- Inventory for rural areas and urban areas do differ i.e. choices of goods change with the region also.
- Inventory also goes through decay or deterioration.

Based on collected data and values, a problem has been identified, a model was prepared, the solution has been derived by taking into consideration real-time problems under different chapter headings.

The help of software Mathematica 12.0 has been taken to solve the set of equations.

2.6 Conclusion

The main highlight of the work is that it all works in a finite planning horizon. Chapter 3, clearly classify the importance of coordination between the supplier and the retailer for the purpose of managing the supply chains in FMCG industry. It mainly focuses on the interest earned by the retailer when he replenishment cycle's length is smaller than the credit period and hence under this scenario the total cost of the retailer is obtained.

In chapter 4, the central point is the seasonal products and the seasonal demand in FMCG sector. The product under seasonal category follows a ramp type of demand under which in initial phase the demands rise, reached peak and then stabilizes at constant. The cases under coordination and no coordination cases were discussed among the suppliers and the retailers. The next portion focuses on the trade credit policy with both interest earned, and interest charged variables. The end part is concluded with the supplier's joint cost optimisation in a finite planning horizon.

Chapter 5 defines a role of variable inflation, whenever the inventory is replenished in a supply chain for FMCG products and FMCG market. An innovative idea of equipoise is instigated, and equipoise cost of the retailers is attained. The main advantage of this equipoise cost is the reduction in the final cost of the supply chain and in addition to this, individual costs of all the retailers in the supply chain and the supplier is reduced. Algorithms were discussed in detail as well.

Chapter 6 studies the effect of fuzzified variables on the cost. The results of the chapter shows that the system in which fuzzified variables are not used the cost of that system is higher and when the deterioration rate and defective products variables are fuzzified, the cost of the system is reduced. The graph in the chapter shows the two different costs.

To wind up the work, it's been deduced that in a finite planning horizon for a FMCG sector some novel concepts were inferred which helps in reducing the total cost of the retailer, the supplier and of the supply chain.

Chapter 3
Supplier-retailer coordination with trade credit for deteriorating items for FMCG

Abstract:

In the prevailing market conditions, an integrated approach of the supplier and the retailer is required to successfully manage the supply chains. The present time also focuses on the requirement of trade credit as well, as it's been proved till now that it is for the profit of both the supplier and the retailer. The inventories deteriorate with time, especially chemicals in the laboratory, thin films when exposed to different atmospheric forces, items having a shorter shelf life, items like fruits, food products, perishable products, consumer goods, and fashionable items (car accessories, mobile phones, clothes), pieces of machinery are also exposed to wear and tear. The research work focuses on a mathematical model with constant demand, deteriorating inventory, supplier–retailer coordination for the permissible delay in payments, and the retailer's interest earned when the replenishment cycle is shorter than the credit period rate, for various types of inventories related to different mechanical and engineering industries and even for fast-moving consumer goods or consumer packaged goods. Thus, it is attempted to calculate the total cost of the retailer with different parameters. A numerical example is also discussed, and sensitivity analysis is also done.

3.1 Introduction

The classical EOQ and EPQ models are extended by many researchers from time to time by adding new parameters accommodating more real-world scenarios, like deterioration, trade credit, permissible delay in payments, inflation, different types of demand, fuzzy set theory, single and multi-echelon supply chain and so on. The effect of deterioration on the inventory was firstly bought under the scope of study by Ghare and Schrader [41]. The model developed by two of them was simple and later many researchers extended their work, as in 1973 Covert and Philip [42] developed an EOQ model in which they had considered the variable rate of deterioration, and Weibull distribution was used along with other conditions in which no shortages were allowed, also constant demand was taken with instantaneous supply.

Generally, the norms of the market say that the simple way is, we buy a product, and we pay for that. But in practicality in maximum businesses, there is an option called Trade Credit. In this the supplier allows the retailer for a particular period, when the purchases are made, to pay for that. It is within a permissible limit so that no losses are incurred by the supplier. Many times, to counter the risk of default, the supplier offers a discount to encourage earlier payments. In 1973, Haley and Higgins [43], by considering the classical lot size model, introduced the relation between trade credit and inventory management. Their work focuses on deducing some conditions which can efficiently reduce a general solution into an optimal solution. In 1981, Ferris [44], deduced a theory (transaction theory) using trade credit for optimizing the cost of inventory exchange. After this, trade credit and deterioration were considered parallel by many researchers. In 1985, Goyal [25] formulated an EOQ model with permissible delays in payments. Later in 1995, Aggarwal and Jaggi [26], extended the work done by Goyal for deteriorating items with an added assumption that sales revenue can also be accumulated after the delay period. They further proved that no interest can be earned on these deteriorating items and also an increase in deterioration rate leads to shorter cycle lengths and a decrease in order quantity resulting in an overall increase in cost. This implies that the greater the deterioration rate more frequently inventory is to be replaced which will enhance the overall cost of the supplier. Researchers like Raafat, Goyal, Giri, Bakker, and Jaggi had contributed a lot to this field with their classic review projects. In 1991, Raafat [31] in his review paper, defines the deterioration as Decay, Spoilage, and Physical Depletion. He considers deterioration as the function of on-hand inventory. Further, he classified the different works done by other authors into different categories based on the type of deterioration. In 2001, Goyal and Giri, [32], reviewed all the advances done in deteriorating inventory literature after the survey of Raafat (1991). They defined inventoried goods into three major categories: Obsolescence, Deterioration, and No Obsolescence/ No Deterioration. In 2012, Bakker *et. al.* [33], extended the work done by Goyal and Giri in 2001and further made additions by including price discounts, lost sales, single and multiple items, payments delay, and one or two warehouses. In 2007, H. Soni *et.al.* [45] gave a review by taking trade credit as the basis of their research paper. Their work focused on finding optimal lot size solutions when the retailer is given trade credit by the supplier.

From the days the EOQ and EPQ models were developed, almost all organizations and firms used these models to manage their inventories and the related issues. Over the period researchers continued to add more parameters like deterioration, inflation, demand, trade credit, partial back ordering, discount, coordination. In 1995, Z. K. Weng [46] focused on inventory coordination among suppliers and buyers by taking selling price and order quantity into account. Lyu e*t. al.* [47], in 2010 focused on the need for coordination between retailer and supplier. His research proves that planning and coordination between the two can reduce costs borne by the two, deterioration or obsolescence of products can be controlled, demand patterns can be followed positively reducing shortages and lost sales as well. Wu and Zhao [34] (2014) describe that the relationship between supplier and retailer is becoming more stable over time. To strengthen the bond between the two, suppliers rely on trade credit, it can be said that it is a step to say that retailer is being trusted by the supplier, though the trade credit period itself depends on various other factors as well.

Generally, inventory models can be broadly classified into two categories based on time horizon – infinite and finite. A finite planning horizon means that the replenishment cycle did not repeat itself and that too not in the same manner. In other words, we can say that every time replenishment time of inventories varies. Thus, every replenishment cycle has a different optimal solution in a finite planning horizon. Wu and Zhao [34] have worked on finite planning horizons with unequal cycle length along with coordination for inventory management between the supplier and the retailer when a credit period is given by the supplier to the retailer. In 2017, Singh *et. al.* [48] presented an Economic

Order Quantity model in a finite planning horizon along with deteriorating items for two cases – with and without delay in payments. In 2018 Singh *et. al* [49], forms an inventory model in a finite planning horizon for deteriorating items under inflationary conditions including trade credit. In 2019, Singh *et. al* [50], under centralized and decentralized planning horizon with deteriorating items discussed the re-manufacturing of inventory. The paper incorporated trade credit policy as well.

Fast Moving Consumer Goods as the name says about itself, includes non-durable goods like food & drinks which are consumed immediately, and its inventory moves very rapidly through the producer, distributor, retailer, and consumer chain. Because of its structure, the FMCG companies rely heavily on forecasted demand figures, and this includes all the downward processes like inventory management, production planning, buying, transportation – in short, the whole supply chain management and ultimately delivering to the customers (Basson et al.) [36]. Nemtajela and Mbohwa (2017) [51], proved that there is a direct relation between managing inventory levels and uncertainty of demand i.e. more the uncertainty levels of demand, the inventory managers need to deal with more cumbersome inventory management problems for FMCG. Jaggi *et. al*. [52] (2018), explained that consumer goods or the products with displayed stocks need to have a centralized policy for achieving maximum joint profit by both the supplier and the retailer, though the profits are biased for the retailer. In 2020, Li *et. al.* [53], in their work explained that how the inventory can be optimized for fast-moving consumer products by replenishing and salvaging the stocks at the start of each cycle by focusing on the e-commerce platform for promoting these consumer-packaged goods.

After reviewing all the work done by the above-mentioned researchers, in this paper, we have focused to calculate the total cost of the retailer in different replenishment cycles by varying different parameters for fast-moving consumer products and for all such products whose inventory is maintained at some point and which deteriorates at some point of its life cycle, while taking into consideration constant demand along with the deteriorating inventory. Moreover, the supplier is giving the retailer trade credit to pay off the full amount of the purchased stocks, where credit period length depends upon many factors. A single retailer and a single supplier are taken while taking a single FMCG product into account.

The highlight of the paper is taking the rate of interest earned by the retailer during the credit period, which will affect its total cost. The coordination between the supplier and the retailer is taken into consideration but only for deciding the trade credit time period. Further, we assume that the time given to pay back all amounts is greater than the length of the replenishment cycle, so that the retailer gets a fair chance to earn the rate of interest, and hence its overall effect is studied in the article.

The paper proceeds with section 2, defining the problem, section 3, gives an account of assumptions and notations, section 4, is the mathematical formulation of the model, section 5 illustrates the model with a numerical example and section 6 gives the sensitivity analysis of different parameters summarised in the form of different tables along with the graphical representation of the tables in section 7, and section 8 gives the concluding words to the work done.

3.2 Defining the problem

In this paper, deterioration depending upon inventory level is considered along with constant demand. The coordination between supplier-retailer is used to decide the trade credit period given to the retailer by the supplier to pay for his purchases and the terms will be decided depending on various factors like creditworthiness, his records, quantity ordered, and the number of times ordered. The equation of the total cost of the retailer is deduced and its relationship with different parameters is shown. This research work particularly aims at interest earned by the retailer in the period of trade credit after the supplier–retailer coordination which was not taken up by the earlier researchers.

3.3 Assumptions and notations

The paper is based on the below-mentioned assumptions. The notations used in the paper are also listed after the assumptions in the form of separate subsections.

3.3.1 Assumptions

- The planning horizon considered is finite with equal lengths of replenishment cycle.
- The production rate is infinite which implies that the supplier himself doesn't maintain any inventory
- Discrete order quantity policy is followed by the supplier for the replenishment.
- The coordination between supplier- retailer is used only for deciding the trade credit time.
- The trade-credit period is greater than the length of the replenishment cycle.
- Shortages are not considered as for FMCG it's assumed that the consumer doesn't like to wait.
- The demand rate is considered constant.
- Inventory is replenished instantly and thus no lead time is taken into account.
- A single product of FMCG is taken into frame.
- One supplier and one retailer is taken for the framing of the model.
- Replenishment of stocks/inventory is instant or instantaneous.
- Inventory deteriorates at a constant rate $\boldsymbol{\theta}$.

3.3.2 Notations

c = Purchasing cost/unit (in Rupees/unit) and P < Wholesale Price/unit.

Z = Planning horizon (in months) and is a non-negative integer.

v = constant demand rate.

IC = inventory cost and it includes both capital cost and holding cost

I_{i+1} = inventory level of deteriorating inventory during $(i+1)^{th}$ cycle at the time t_i

S_{i+1} = order quantity or level of inventory in $(i+1)^{th}$ cycle in time t_i.

Y = ordering cost / order (rupees / order).

W = Wholesale price / unit.

Θ = Deterioration rate of inventory.

RET_{TC} = Total cost of the retailer during planning horizon Z including the interest earned by him.

D_{i+1} = number of deteriorated units in $(i+1)^{th}$ cycle

D = cost of deteriorated units.

n = number of replenishment cycles during the planning horizon Z.

3.3.3 Decision-making variables

t_i = replenishment time where t_0=0 and t_n= Z

M = credit period rate or permissible delay in payment.

T_{i+1} = t_{i+1} -t_i is the replenishment cycle's length of $(i+1)^{th}$ cycle.

E_{i+1} = interest earned by the retailer in $(i+1)^{th}$ replenishment cycle.

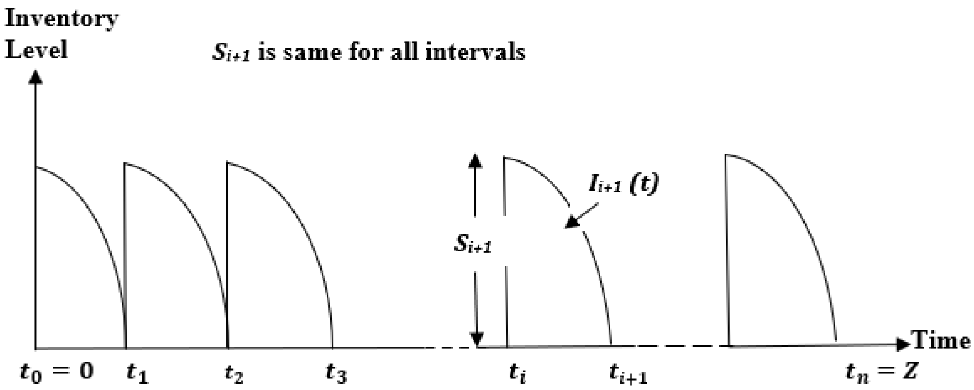


Figure 3.1. Graphical representation of the system discussed in the model.

3.4 The mathematical formulation of the model

The retailer orders the consumer products from the supplier before the earlier inventory level reaches zero, (shortages and lost sales – considered zero). The supplier will instantly replenish his order, and a same number of items are ordered.

The change in the inventory level during $(i+1)^{th}$cycle will be given by the following differential equation:

$$\frac{dI_{i+1}(t)}{dt} = -v - \theta I_{i+1}(t) \quad \text{where } t_i \leq t \leq t_{i+1} \tag{3.1}$$

Boundary conditions are I_{i+1} (t_{i+1}) = 0 and I_{i+1} (t_i) = Q_{i+1}

Solving the equation (3.1) we will get:

$$I_{i+1}(t) = e^{-\theta t} \int_t^{t_{i+1}} v\, e^{\theta u}\, du \quad \text{where } t_i \leq t \leq t_{i+1} \tag{3.2}$$

For details of the solution refer to Appendix A.3.1.

Now order quantity for $(i+1)^{th}$cycle will be

$$S_{i+1} = I(t_i) = e^{-\theta t_i} \int_{t_i}^{t_{i+1}} v\, e^{\theta t}\, dt \tag{3.3}$$

Now, the retailer's cost during Z = ordering cost + inventory cost + purchasing cost

$$RET_{TC} = n * Y + \sum_{i=0}^{n-1} IC \int_{t_i}^{t_{i+1}} I_{i+1}(t)\, dt + \sum_{i=0}^{n-1} W * S_{(i+1)} \tag{3.4}$$

Solving equation (3.4) we will get:

$$RET_{TC} = n * Y + \sum_{i=0}^{n-1} (\frac{IC}{\theta} + W) \int_{t_i}^{t_{i+1}} v\, e^{\theta(t - t_i)}\, dt - \frac{IC}{\theta} vZ \tag{3.5}$$

For details of the solution refer to Appendix A.3.2.

Here $t_0 = 0$ and $t_n = Z$.

When the retailer asks for trade credit from the supplier, it will be given depending upon various factors like the order quantity (larger the order, more will be the credit period length), the creditworthiness of the retailer, his previous records, and supplier-retailer coordination.

Now, we assume that $M \geq T_{i+1}$ i.e. when the trade credit time period is greater than or equal to the inventory cycle length. We will prove for $(i+1)^{th}$ cycle and generalize for rest. Under this scenario, the retailer get interest on the sales revenue up to the length of credit period and no interest is paid on the items already stocked up with the retailer. Moreover, the retailer can earn interest throughout the cycle length.

Therefore, the interest earned up to T_{i+1} for $(i+1)^{th}$ cycle will be:

$$W * E_{i+1} \int_0^{T_{i+1}} vtdt = W * E_{i+1} v \frac{T_{i+1}^2}{2}$$

Now, the interest will be earned beyond this cycle length also (i.e. during $\eta - T_{i+1}$) which will be:

$W * E_{i+1} v * T_{i+1}(M - T_{i+1})$

Thus, the total interest earned is

$W * E_{i+1} v \frac{T_{i+1}^2}{2} + W * E_{i+1} v * T_{i+1}(M - T_{i+1})$

$$= W * E_{i+1} T_{i+1} v * (M - \frac{T_{i+1}}{2}) \qquad (3.6)$$

Now the cost of deteriorated units = D = $A. d_{i+1}$ where $d_{i+1} = S_{(i+1)} - vT_{i+1}$

$$D = W \frac{e^{-\theta t_i}}{\theta} v \left(e^{\theta(t_{i+1})} - e^{\theta t_i}\right) - v * T_{i+1} \text{ where } T_{i+1} = t_{i+1} - t_i \qquad (3.7)$$

Thus, the final total cost of the retailer (including interest and deterioration cost) is

RET_{TC} = ordering cost + inventory cost + purchasing cost + cost of deteriorated items – interest earned.

$$RET_{TC} = nY + \left(\frac{IC}{\theta} + W\right)\left(\sum_{i=0}^{n-1} \frac{v}{\theta}\left(e^{\theta(t_{i+1}-t_i)} - 1\right)\right) - \frac{ICvZ}{\theta} + \frac{Wv}{\theta}\left(\sum_{i=0}^{n-1}\left(e^{\theta(t_{i+1}-t_i)} - 1\right)\right) - v\left(\sum_{i=0}^{n-1}(t_{i+1} - t_i)\right) - Wv\left(\sum_{i=0}^{n-1} E_{i+1} * T_{i+1}\left(\eta - \frac{T_{i+1}}{2}\right)\right) \qquad (3.8)$$

This equation (3.8) of the total cost of the retailer will be used further in the numerical example and to carry out a sensitivity analysis in further sections of this chapter.

3.5 Numerical example

Example: Given n = 2, Y = 1.2, IC = 0.4, W = 3, Z = 1, M = 7 months (0.58), E_1 = 0.13, E_2 = 0.13, θ = 0.01, t_0 =0, t_1 = 0.5, t_1 = 1, v =100, 500, 1500 and 2500 units/year respectively, the following table gives the result for total cost of the retailer which shows that as the value of demand (v) rises, the RET_{TC} also rises while the other parameters values are kept unchanged.

Constant variables	v	RET_{TC}
n = 2, W = 3 θ =0.01, Z = 1 t_0 = 0, t_1 = 0.5, t_2 = 1 E_1=0.13, E_2=0.13 M= 0.58 IC = 0.4 Y = 1.2	100	501.049
	500	2495.65
	1500	4782.14
	2500	12468.6

Table 3.1 The total cost of the retailer.

3.6 Sensitivity analysis

The next four tables in this section will give sensitivity analysis of various parameters and at the last of the tables, all the results are compiled together in a subsection.

n	θ	v	t_0	t_1	t_2	t_3	t_4	M	E_i = 0.13, i=1,2,3	IC	Y	W	Z	RET_{TC}
1	0.01	1500	0	1	-	-	-	366/365	0.13	0.4	1.2	3	1	7553.25
	0.02													7599.71
	0.03													7646.48
	0.04													7693.56
2	0.01	1500	0	0.5	1	-	-	0.58		0.4	1.2	3	1	7482.14
	0.02													7505.00
	0.03													7527.94
	0.04													7550.96
3	0.01	1500	0	0.33	0.67	1	-	0.416		0.4	1.2	3	1	7472.91
	0.02													7488.08
	0.03													7503.27
	0.04													7518.51

Table 3.2 Sensitivity analysis for θ (deterioration rate of inventories).

N	θ	v	t_0	t_1	t_2	t_3	M	E_i = 0.13, i=1,2,3	IC	Y	W	Z	RET_{TC}
1	0.03	100	0	1	-	-	366/365	0.13	0.4	1.2	3	1	510.88
		500											2549.63
		1800											9175.54
		2500											12743.3
2	0.03	100	0	0.5	1	-	0.58		0.4	1.2	3	1	504.10
		500											2510.91
		1800											9033.05
		2500											12545.0
3	0.03	100	0	0.33	0.67	1	0.416		0.4	1.2	3	1	503.58
		500											2503.49
		1800											9003.21
		2500											12503.10

Table 3.3 Sensitivity analysis for v (constant demand in a cycle)

Constant variables	n	M	t_1	t_2	t_3	t_4	RET_{TC}
θ = 0.02, W = 3, t_0 = 0, Z = 1, v = 1800, E_1=0.013, E_2=0.026, E_3=0.039, E_4=0.052, IC = 0.1, Y = 1.2	1	366/365	1	-	-	-	9437.04
	2	366/365	0.5	1	-	-	9202.43
	3	0.416	0.33	0.67	1	-	9124.98
	4	0.33	0.25	0.5	0.75	1	9086.02

Table 3.4 Sensitivity analysis for E_i (the rate of interest earned by the retailer in every cycle).

Constant variables	t_0[a]	t_1[a]	t_2[a]	t_3[a]	t_4[a]	M[b]	E_i = 0.13 ,i=1 ,2,3	RET_{TC}
θ = 0.01, W = 3, Z = 1, v = 1000, IC = 0.4, Y = 1.2, n = 4	0	1	2	3	4	5	0.13	133908
	0	1	2	3	4	6		132348
	0	1	2	3	4	7		130788
	0	1	2	3	4	8		129228

Table 3.5 Sensitivity analysis for M (the credit period).

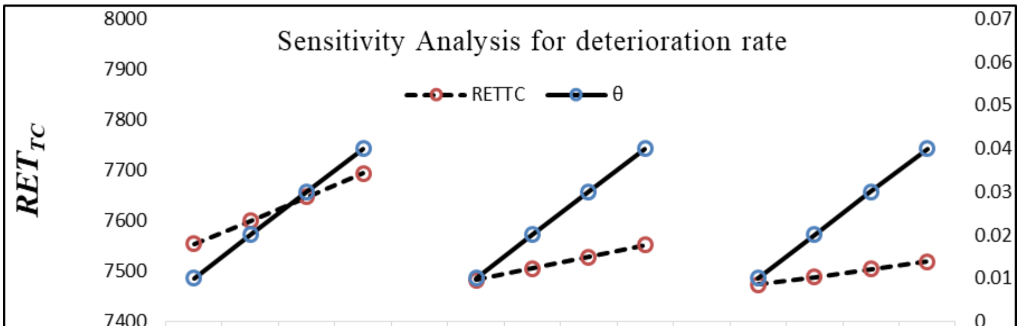
[a] time is taken in months

[b] credit period is taken in months

3.6.1 Sensitivity analysis takeaways

1. As per table 3.2, if there is an increase in the value of θ (deterioration rate of the inventory), the total cost of the retailer starts increasing. But, with the increase in the replenishment cycles, the total cost decreases which imply that if the inventory is replenished more frequently, the deterioration rate is less and so is the total cost. Thus, θ is directly related to total cost and indirectly with the number of replenishment cycles.
2. Table 3.3 shows that if the demand increases then the cost of the replenishment cycle also increases as more demand will lead to greater order quantities and hence final increase in the total cost of the retailer will be there. But, when there is an increase in the replenishment cycles per year, the cost will start decreasing showing an inverse relationship between the two, which implies that if the cycles are more, i.e. the replenishment occurs often, then for a particular value of v (demand), the cost starts decreasing, which will ultimately benefit the retailer.
3. The analysis in table 3.4 reveals that as the retailer's interest rate is increased in every replenishment cycle, the total cost of the retailer decreases, giving an inverse relationship between the two parameters.
4. Table 3.5 gives the detail of the relationship between M (trade credit period) and finite planning horizon (Z). As the credit period is increased, the retailer will get more time to settle the full payment, thus this will increase the interest earned period also, and as a result, the earnings of the retailer increase, finally resulting in a decrease in total cost.

3.7 Graphical representation of sensitivity analysis



	1	1	1	1	2	2	2	2	3	3	3	3
RETTC	7553	7600	7646	7694	7482	7505	7528	7551	7473	7488	7503	7519
θ	0.01	0.02	0.03	0.04	0.01	0.02	0.03	0.04	0.01	0.02	0.03	0.04

Figure 3.2 Graphical representation for table 3.2.

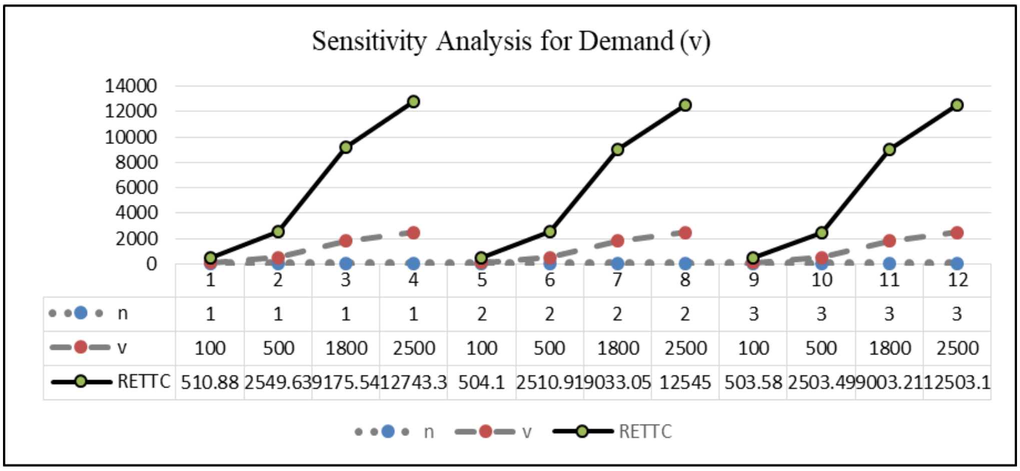


Figure 3.3. Graphical representation for table 3.3.

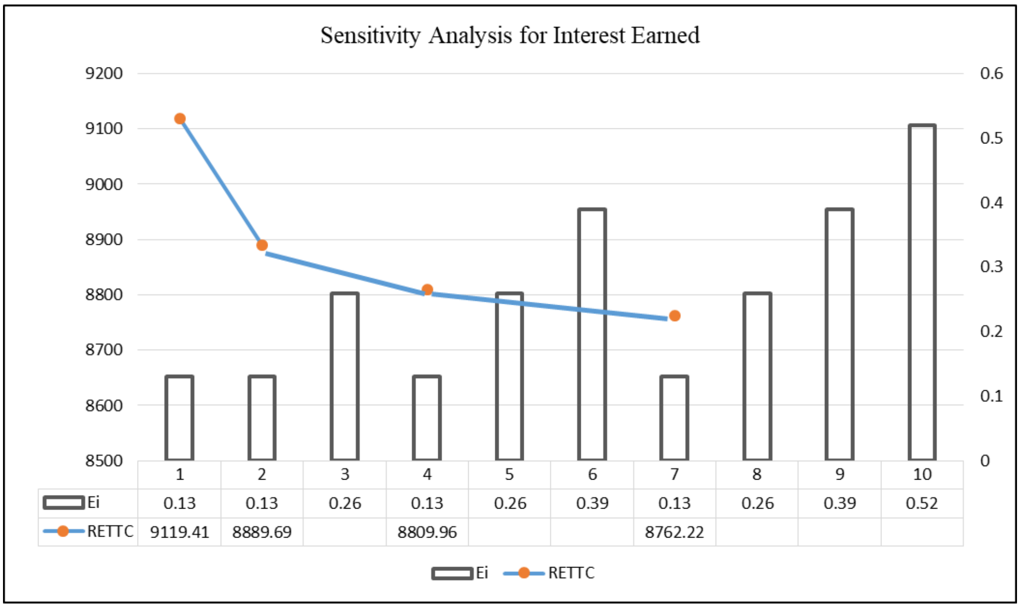


Figure 3.4 Graphical representation for table 3.4.

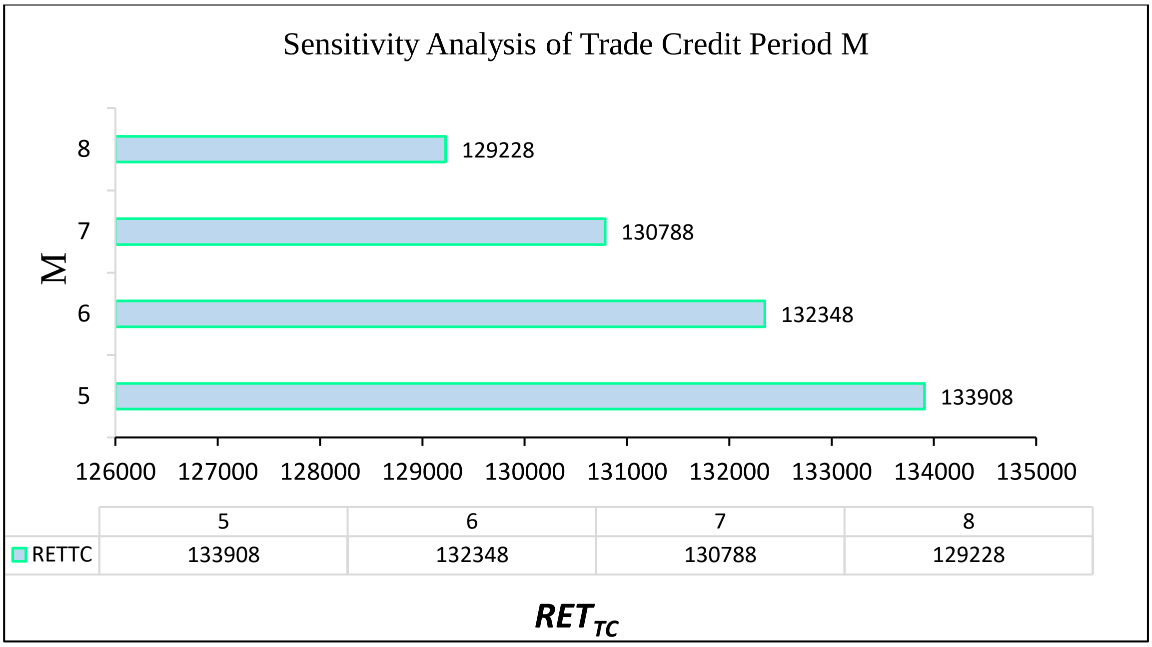


Figure 3.5. Graphical representation for table 3.5.

3.8 Practical Implications and Conclusion

3.8.1 The key practical implications are as follows:

Optimization of Trade Credit for Cost Reduction

- The analysis demonstrates that an extended trade credit period can significantly reduce the retailer's total cost by allowing interest accumulation on deferred payments.
- The supplier-retailer coordination mechanism is modeled to determine the optimal trade credit duration based on factors such as order frequency, volume, and creditworthiness.

Financial Implications of Interest Accumulation

- The study introduces a parameter EiE_i representing the interest earned in the ii-th cycle, which was previously unexplored in classical inventory models.
- Sensitivity analysis confirms that under favorable conditions, retailers can leverage the credit period to generate additional income, which can effectively offset inventory holding costs.

Deterioration and Its Economic Consequences

- The research incorporates deterioration effects, considering both direct decay (as in perishable goods) and obsolescence (as seen in technology and consumer-packaged goods).

3.8.2. Conclusion

The findings emphasize that optimal inventory replenishment policies must account for the rate of deterioration to prevent excessive holding costs and financial losses.

The article gives an insight into all those products which undergo deterioration at some point in their shelf life, may it be direct decay (as in the case of eatables), or physical spoilage (like mechanical goods, engineering items, radioactive substances, chemicals, or liquids) as every type of inventory deteriorate at some point any due to any reason. These days, obsolescence is also taken to be part of deterioration, which is more frequently seen in consumer packaged or fast-moving consumer goods. The article emphasizes constant demand, with supplier–retailer coordination when the interest is earned by the retailer during his credit period. For this a parameter E_i (interest earned in an i^{th} cycle) is introduced, the sensitivity analysis for the new parameter is done in section 6 of the research article. Further, it is shown that the total cost of the retailer is affected by varying different parameters (for this, a complete sensitivity analysis is carried out). Analysing the outcomes, it can be said that if the trade credit time is prolonged it allows the retailer to earn an extra income in the form of rate of interest, though it is not possible every time (if the retailer doesn’t deposit the money in the bank, no interest is earned), if it’s there, it can reduce the cost of the retailer. These days interest is earned by even lending money to other small retailers for a short period. In short, we can say that there can be many ways to earn extra in the form of interest, and every time the outcome is only one, i.e. reduced total cost.

The present paper can be extended by incorporating stock-dependent or time-varying demand functions, which would enhance its applicability across a broader range of industries. Additional complexities such as shortages, lost sales, and inflationary factors can be introduced to reflect real-world inventory management challenges more comprehensively. The study underscores the significance of trade credit as a strategic financial tool in inventory management. By systematically analysing the interplay between replenishment cycles, deterioration, and financial incentives, this research offers a robust decision-making framework for retailers. The insights presented in this work hold substantial relevance for industries dealing with perishable and rapidly depreciating goods, where cost optimization is paramount.

Chapter 4

Supplier – retailer inventory coordination with trade credit for seasonal demand in FMCG sector

Abstract: Seasonal products are vital for the Fast-Moving Consumer Goods (FMCG) industry. The FMCG sector is a promising sector and broad areas under this include electronics, electrical, computer accessories, fashion industry, eatables, generally those items which are perishable or become obsolete easily. These products follow the ramp type demand and the same is considered in this article focuses. Considering, FMCG sector, the planning horizon is kept finite. Supplier – retailer coordination is used to decide the trade credit period between the two. Also, its different cases and subcases are discussed. In this article, the total cost of the retailer and the supplier in no coordination and coordination cases are derived. Trade credit policy is discussed in detail, which includes both the interest earned and interest charged parameters. The optimality of the supplier's joint cost is also obtained. Numerical examples are discussed to support the theoretical outcomes. The same is solved by using Mathematica 12.0 software. A new algorithm has been framed. The sensitivity analysis and concluding remarks mark the end of the work for the supplier–retailer coordination under seasonal products with ramp type demand for all the consumer-packaged goods or FMCG industry.

4.1.Introduction

In today’s world, one alone can’t survive, same condition is applicable on the market also. Alone player (be it supplier or retailer/seller or buyer) can’t survive. Moreover, if they have coordination before stocking up their goods by understanding their local market very well, their profits can be increased manifolds, the losses can be reduced, customers’ grievances can be met successfully, lost sales can be reduced, seasonal demand can be taken care of properly, and even substitutable markets can be searched if there is a two – way coordination between the supplier and the retailer. The coordination between the parties can ease the position of market and even the financial constraints among the players of the market can also be taken care of. This coordination can lead to a very important aspect of present-day market and that is trade credit.

The general trend says that whenever we make a purchase, we pay to the seller, but in big businesses where there is involvement of millions and even more, a trust factor plays a vital role and this gave rise to a term called trade credit, in which a seller allows the buyer to pay for their purchased items in a span of time. This given period is such that no loss is borne by the seller or supplier of goods. The main important point here to be kept in mind is, this credit period is beneficial for both the parties though at one go it seems to be beneficial or biased for the buyer. Sellers can enhance their sales, their problem of storing inventory for a long can be taken care of, money will not be blocked at one place and from buyers point of view, they will never be out of stocks (if inventory coordination is proper), no need to worry about losing their customers as they will be having multiple substitutes for a single product, display counters will be fully loaded and even flow of cash is guaranteed as customers are many in the market. One more important benefit from buyers or retailers' point of view is, the interest earned by them during the trade credit period, this will be a source of extra income which will reduce their costs and multiple their gains.

The variation in season not only have its impact on temperature or weather conditions of an area, but this variation can be seen in market as well. Their impact is firstly visible in type of inventory that supplier – retailer holds, different types of products are being demanded by the consumers and as a result the whole market undergoes a major turmoil. Thus, here comes a study of seasonal demand in the picture. The seasonal demand is not only linked to seasons (summers, winters, autumn or spring) but a careful study of market reveals that the demand of products and items can see a variability not only according to seasons or changing weather conditions, but seasonality of demand can be an important case for study on the basis of months and even on weekly basis. Yearly demand may be encountered when a session of schools or colleges began in the form of raised demand in stationary items, or onset of a particular season can give rise to related products' demand (summers call for sunscreens, cold drinks; winters has a callout for warm clothes and so on). The other type of seasonal demand is monthly, like in the stating of the month when professionals receive their payments, they make purchases (in many countries it can be bimonthly as well, as they offer salaries fortnightly). Another type of demand can be categorised on weekly basis, as some product can be in high demand in some particular day of a week (‘Prasad’ (devotional offering) on Tuesdays and Thursdays generally all over India) and sometimes reverse can even happens like, a product can have nearly zero demand on a particular day of a week (meat in case of India).

The market of consumer packaged goods is so much diverse that it has a scope to cover almost all forces which co-exist in the market at a point of time, may it be – the supplier – retailer coordination, trade credit, lost sales, shortages, finite/infinite replenishment cycles, seasonal demand, inventory management, inflation, fuzzy or crisp environment, and many more to be named. These fast-moving consumer goods are having short shelf life, are consumed very swiftly, are very popular on e-commerce platform, even cover obsolescence and are the need of the market. It includes large packaged which is for sale in metropolitans and even have sachet revolution in its basket which relates particularly to the villages and target lower strata within the society. The suppliers and the retailers that deals with FMCG need to have a strong coordination among them so that inventory is not kept on hold for long (so that holding cost can be reduced), need to stack up those products which are in high demand in a particular area in a specific season with a new avenue to move the remaining stock to an alternative market of the product with the change of the season and thus to have a trade credit within the permissible limit.

4.2.Literature Review

The inventory management concept was first born around early 1940s and then in late 1960s, the modern-day barcode was invented to efficiently manage the inventories with the efforts of some retailers and then till date many upgradations has been done in this field.

Both the supplier and the retailer are important pillars of market. The coordination between the two can help both and in turn the customers in many diverse ways. This coordination can be the basis of decreased costs (holding, ordering, manufacturing, set up etc.), increased profits, permissible delay in payments/ trade credit period, reduced shortages, improved preservation technologies, reduced or very low/ negligible deterioration to name a few. With the passage of time, the trade credit period has become an inseparable part of inventory management, and this process also gives a peek in to the relation between a supplier and a retailer. Many times, it has seen in practicality that the supplier offers a long credit period to those retailers who has good credibility in the market and shorter payable period to the ones with low credibility as described by Teng (2009) [55]. In 1973, Haley and Higgins [43], gave the relation between inventory management and trade credit. In 1981, Ferris [44], deduced a transaction theory using trade credit for optimizing the cost of inventory exchange. In 1985, Goyal [25] gave an EOQ model with permissible delays in payments. Later in 1995, Aggarwal and Jaggi [26], extended their work and gave a relation between sales revenue and delay period along with rate of interest. In mid 90s, Rajan and Zingales [56] presented a detailed study that in G-7 nations their businesses are running considering trade credit as an important parameter. Further, in 2002, Wilson and Summers [57] presented a survey on trade credit conditions of small firms. They clearly stated that the size of the firm is an important aspect to decide the trade credit period and this credit period is also used to strengthen the relationship among the supplier, the retailer and the customer. Mateut *et. al.* [58], in 2006, with a survey of 16,000 manufacturing firms, they have proved that when monetary policy tightens, trade credit expands. In 2003, Huang and Chung [59], extended Goyal's model by including cash discounts to counter more real world problems of suppliers, retailers and customers that is by considering two level trade credit, first when the supplier gave the retailer a permissible time limit to pay the full amount and second, when the retailer in turn provides a time to the customer to pay back the whole amount. In their work, the time given by supplier to retailer is greater than that of the time provided by the retailer to its customer. In 2004, C-T Chang [60], proposed an inventory model in which bulk quantities if ordered by the retailer then they will get credit period to pay off their purchases. Sana and Chaudhary [61], in 2008, correlated the relation between the retailer's selling price and the supplier's offer of trade credit period. In 2008 itself, Jaggi *et. al.* [62], presented a more realistic model which proves that the demand of a product is linked to the credit period offered by the retailer to its customers. In 2009, Teng and Chang [63], extended the work done by Y-F Huang (2007) [64]. In their work, Huang's model in which only one case is considered (when credit time given by the supplier to the retailer is greater than the credit time given by the retailer to the customer) was analysed the other way round also.

In 2009, Skouri *et. al.* [65] explained that for seasonal products, demand will never be steady, and it will be explained by three successive time periods. In first phase it increases with time, followed by steady demand and in the final phase it decreases, i.e. the demand is ramp type. Though different researchers consider many different types of demands but for many products which are categorised in the category of Fast-Moving Consumer Goods (FMCG)/ Consumer Packaged Goods/ Seasonal Goods/ fashionable items, all these goods and their demand patterns can be very well explained time dependant ramp type demand. The concept of ramp type demand was firstly explained by Hill in 1995 [66]. Since then, different researchers extended Hill's model by adding more and more parameters. In 2011, Skouri *et. al.* [67], along with ramp type demand added deterioration and delay in payments within permissible limits. In 2012, Lin *et. al.* [68] studied ramp type demand under stock dependent consumption rate. In their work they had simplified the work done by Deng *et. al.* [69]. Deng *et. al.* [69] research work is the solution of all the questionable results of Mandal and Pal (1998) [70] and Wu and Ouyang (2000) [71]. Panda *et. al.* [72], in 2013, moved a step ahead by working on perishable items with ramp type demand which is both price and time dependent. In 2014, Saha [73], in 2014 took quadratic ramp type demand and included trade credit policy. Pal *et. al.* [74], [75], [76], in their three works done in 2014a, 2014b and in 2015, added many market-oriented parameters. In 2014a, Pal *et. al.* [74] they added three-layer supply chain trade credit i.e. when the supplier gives fixed period to pay off for their purchases to manufacturer, manufacturer does the same with the retailer and then in turn retailer gives credit period to each of its customers. In 2014b, Pal *et. al.* [75], ramp type demand was taken along with weibull deterioration under the environment of inflation, their model has no shortages and even the crisp and fuzziness of the model is discussed. Pal *et. al.* [76], in 2015 added shortages along with inflation in ramp type demand and fuzzy environment. In 2017, Chandra [77], presented a model of ramp type demand with holding cost dependable on time and where discounts were offered at backorders. Demand was ramp type and the linear function of time. In 2018, Singh *et. al.* [78], in their ramp type demand model, considered time dependent deteriorating items along with shortages which were completely backlogged.

The seasonal products (air conditioners, coolers, room heaters, cold drinks, juices), the fashionable items (like clothes, hair products etc.), FMC goods, a newly launched product, electronic items (like laptops, mobile phones, washing machines, printers, etc.) all these products follow ramp type demand pattern, in which in the beginning of the season the demand rises immediately, then becomes almost stable in a phase and towards the end of the season starts falling. FMCG industry has occupied a big part of the market. Skouri *et. al.* [67], has also described that thee products follow ramp type demand and added a permissible delay of payments. Wu and Zhao [34], in 2014, emphasised on supplier- retailer coordination along with trade credit period rate in a finite planning horizon. Their model is even applicable for consumer-packaged goods or FMCG. In 2017, Singh *et. al.* [48] presented an EOQ model in a finite planning horizon for two cases – with and without delay in payments. In 2018 again, Singh *et. al* [49], gives an inventory model in a finite planning horizon including trade credit. In 2019, Singh *et. al* [50], under centralized and decentralized planning horizon incorporated trade credit policy as well. Shaikh *et. al.* [79], in 2020, took up ramp type demand for deteriorating inventory with preservation facility and trade credit facility.

4.3.Defining the problem

In this article, a ramp type demand is taken into consideration. The main aim behind this to analyse properly the market scenario of seasonal goods. The seasonal goods functioned in the similar manner as of newly launched products, fashion items, clothes and fast-moving consumer goods. Their demand increases first, then slowly becomes steady and then starts falling. The supplier – retailer coordination is considered. Single way trade credit period is considered (period given by the supplier to its retailers to pay off the full amount for the purchases made). The retailer in turn takes full advantage of this period and earns interest.

When the permissible time is over, interest is charged by the supplier if the retailer fails to pay back within the decided period.

4.4.Assumptions and notations

The article is based on the below-mentioned assumptions. The notations used in the paper are also listed after the assumptions in the form of separate subsection.

4.4.1 Assumptions

- Ramp type demand rate is taken.
- Lead time is taken to be zero, this means instant replenishment of inventory is done.
- No shortages are taken as in case of FMCG, customers have lot of choices and mostly goes for substitutes and those who wait for the product are negligible.
- No deterioration rate is taken as shelf life of FMC products are very less.
- The planning horizon considered is finite with unequal lengths of replenishment cycle.
- Two cases of trade-credit time period is considered (a) when credit period is greater than the length of the replenishment cycle and (b) when credit period is less than the length of the replenishment cycle.
- Seasonal products of FMCG are taken into frame.
- One supplier and one retailer is taken for the framing of the model.
- Replenishment of stocks/inventory is instant or instantaneous.

4.4.2 Notations

Y = Ordering cost per unit.

W = wholesale cost per unit (for the retailer and W > c).

c = purchasing cost per unit (for the supplier).

h = holding cost per unit.

s = selling price per unit.

E = interest earned per year.

E' = interest charged per year.

n = number of replenishment cycles without coordination.

m = number of replenishment cycles with coordination.

R(t) = Ramp type demand which is a function of time t and

$R(t) = D_0\{t-(t-\mu)H(t-\mu)\} > 0$, moreover $D_0 > 0$

where $H(t-\mu)$ is the Heaviside's function which further follows the below identity:

$$H(t-\mu) = \begin{cases} 1, t \geq \mu \\ 0. t < \mu \end{cases}$$

Thus,

$$R(t) = \begin{cases} D_0 t, t_i \leq t \leq \mu \\ D_0 \mu, \mu \leq t \leq t_{i+1} \end{cases}$$

S_{i+1} = inventory level during (i + 1)th cycle at the time t_i without coordination.

S'_{j+1} = inventory level during (j + 1)th cycle at the time t_j with coordination.

M = the trade credit period given by the supplier to the retailer.

R_I = Retailer's total cost without coordination.

S_I = Supplier's total cost without coordination.

R_{J1} = Retailer's total cost under coordination for subcase 1.

R_{J2} = Retailer's total cost under coordination for subcase 2.

R_{J3} = Retailer's total cost under coordination for subcase 3.

S_{J1}= Supplier's total cost under coordination for subcase 1.

S_{J2} = Supplier's total cost under coordination for subcase 2.

S_{J3}= Supplier's total cost under coordination for subcase 3.

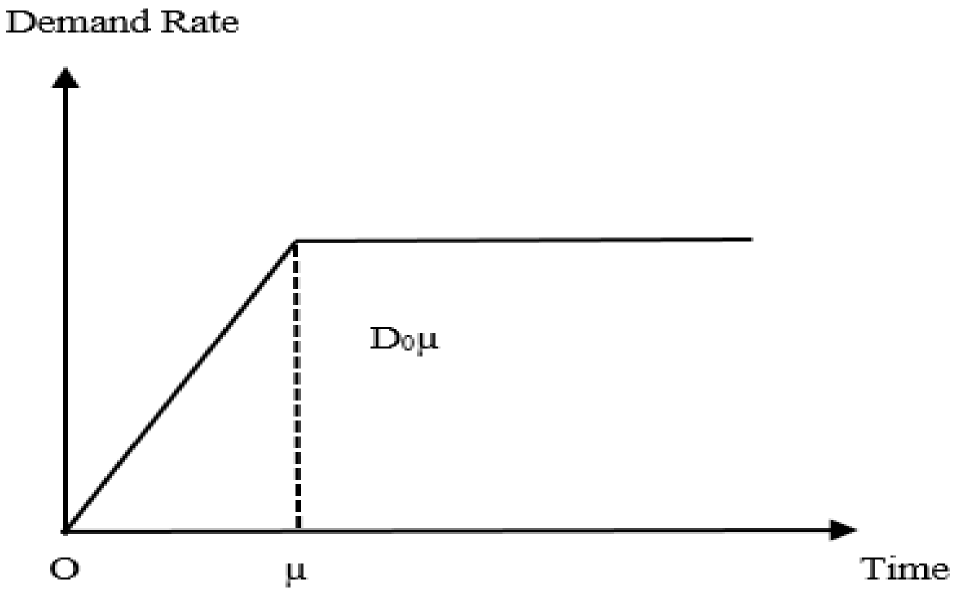


Figure 4.1. Ramp Type Demand, where demand increases from O to μ and then becomes constant.

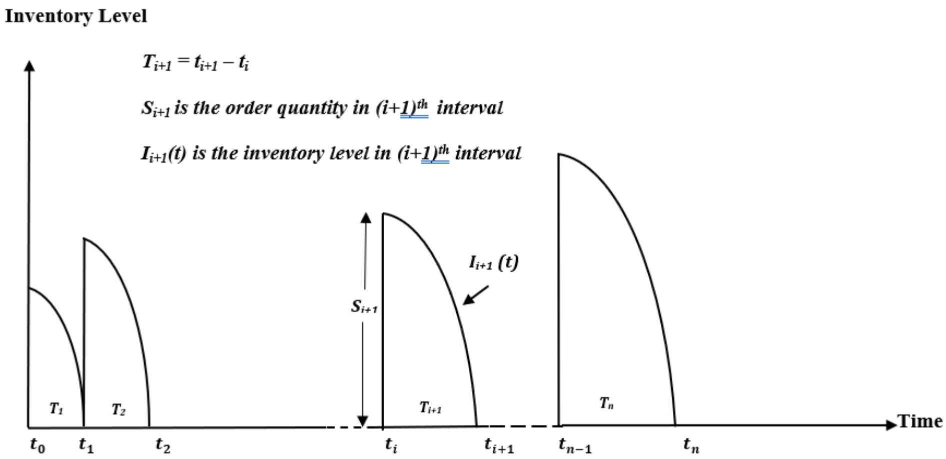


Figure 4.2. Graphical representation of the system discussed in the model.

4.5 The mathematical formulation of the model

The retailer orders a specific quantity of inventory from the supplier in the beginning of the cycle, thus in first replenishment cycle at t_0, the level of inventory is S_1. With the passage of time, the inventory level reaches zero at t_1. After this, the new replenishment cycle starts, and the supplier will take order of new inventory and will fulfill it instantly as no shortages are considered. Deterioration of inventory is not considered in our model as we are working on seasonal products of FMCG industry and with shorter time duration, the deterioration rate is highly negligible.

Case 1: When there is no coordination between the supplier and the retailer.

Here

$$R(t) = \begin{cases} D_0 t, t_i \le t \le \mu \\ D_0 \mu, \mu \le t \le t_{i+1} \end{cases}$$

The differential equation of the inventory level in (i+1)th cycle will be given as follows:

$$\frac{dI_{i+1}(t)}{dt} = -R(t), \; t_i \le t \le t_{i+1} \qquad (4.1)$$

Boundary conditions are $I_i(t_{i+1}) = 0$ and $I_i(t_i) = S_i$

The above equation (4.1) from demand function R(t) can be further reduced to

$$\frac{dI_{i+1}(t)}{dt} = -D_0 t, \qquad t_i \le t \le \mu \qquad (4.2)$$

$$\frac{dI_{i+1}(t)}{dt} = -D_o \mu, \qquad \mu \le t \le t_i \qquad (4.3)$$

Solving the equation (4.2) we will get:

$$I_{i+1}(t) = S_{i+1} - \frac{D_0(t^2 - t_i^2)}{2}, \qquad t_i \le t \le \mu \qquad (4.4)$$

For details of the solution refer to Appendix A.4.1.

Solving the equation (4.3) we will get:

$$I_{i+1}(t) = D_0 \mu (t_{i+1} - t), \qquad u \le t \le t_{i+1} \qquad (4.5)$$

For details of the solution refer to Appendix A.4.2.

From (4.4) and (4.5)

$S_{i+1} = D_o \mu (t_{i+1} - t) + \frac{D_0}{2}(t^2 - t_i^2)$ and

when $t = \mu$, $S_{i+1} = D_o \mu (t_{i+1} - \mu) + \frac{D_0}{2}(\mu^2 - t_i^2)$

Now,

(a) The ordering cost/ cycle = Y

(b) The purchase cost/ cycle = cS_{i+1} (for Supplier) and $W * S_{i+1}$ (for Retailer)

(c) the holding cost/cycle is $h\int_{t_i}^{t_{i+1}} I_{i+1}(t)dt$

$$= h\{S_{i+1}(\mu - t_i) - \frac{D_0}{6}(\mu^3 - 3t_i^2\mu - 2t_i^3) + \frac{D_0\mu}{2}(t_{i+1} - t_i)^2\} \quad (4.6)$$

Now, the retailer's cost during the finite planning horizon = ordering cost + purchasing cost + inventory holding cost

$$R_I = n * Y + W * \sum_{i=0}^{n-1} S_{i+1} + h * \sum_{i=0}^{n-1}\{S_{i+1}(\mu - t_i) - \frac{D_0}{6}(\mu^3 - 3t_i^2\mu - 2t_i^3) + \frac{D_0\mu}{2}(t_{i+1} - t_i)^2\} \quad (4.7)$$

Under this situation, the supplier's cost will be directly related to the behaviour of replenishment policy of the retailer and thus, the supplier's cost will be sum of manufacturing cost and set up cost. The set-up is further sum of labour cost (LC) and machinery set up cost (MC).

$$S_I = n * (LC + MC) + c * \sum_{i=0}^{n-1} S_{i+1} \quad (4.8)$$

Also, $S_{i+1} = \sum_{i=0}^{n-1} S_{i+1}$ is the order quantity during the finite planning horizon. (4.9)

Case 2: When there is coordination between the supplier and the retailer.

In this case, the supplier and retailer both agrees on some conditions of coordination. The main aim behind coordination is to reduce the total cost of both the retailer and the supplier or to maximise their profits. With the coordination, the replenishment cycles will be reduced which ultimately reduces the inventory holding cost of the retailer and from the supplier's point of view, his set up cost will be reduced significantly. Moving a step forward, in our model we will be using the coordination between the supplier and the retailer for deciding trade credit period as well.

Now the retailer's cost after coordination is

$$R_J = n * Y + W * \sum_{j=0}^{m-1} S'_{j+1} + h\sum_{j=0}^{m-1}\{(S'_{j+1}(\mu - t_j) - \frac{D_0}{6}(\mu^3 - 3t_j^2\mu - 2t_j^3) + \frac{D_0\mu}{2}(t_{j+1} - t_j)^2\} \quad (4.10)$$

The increased cost of the retailer will be absorbed by the supplier. Thus, the total cost of the supplier under the new case of coordination under the finite planning horizon will be

$$S_J = m(LC + MC) + c * \sum_{j=0}^{m-1} S'_{j+1} \quad (4.11)$$

Also, $S_{j+1} = \sum_{j=0}^{m-1} S'_{j+1}$ is the order quantity in the finite planning horizon. (4.12)

Now, at this point of time, the concept of trade credit plays a vital role. The supplier tends to reimburse the retailer for her/his losses (in the form of increased cost) by allowing him/her a particular period to pay back the whole amount for his/her purchases and this is termed as trade credit period.

If the supplier offers M credit period to its retailers, then the following subcases arises:

Subcase 1: $0 < M < \mu$,

Subcase 2: $\mu < M < t_{j+1}$,

Subcase 3: $M > t_{j+1}$

Subcase 1: 0 < M < μ

Interest Earned (E_1) = $sE \int_{t_j}^{\mu} \int_{t_j}^{t} D d\mu dt$

$$= sE\{\frac{D_0}{6}(\mu^3 + 2t_j^3) - \frac{D_0}{2}\mu t_j^2\} \quad (4.13)$$

Interest Charged (E'_1) = $cE' \int_{M}^{\mu} I_{j+1}(t)dt + cE' \int_{\mu}^{t_{j+1}} I_{j+1}(t)dt$

$$= cE'\{S_{j+1}(\mu - M) + \frac{D_0\mu}{2}(t_{j+1} - \mu)^2 - \frac{D_0}{6}\{\mu(\mu^2 - 3t_j^2) - M(M^2 - 3t_j^2)\}\} \quad (4.14)$$

Subcase 2: μ < M < t_{j+1}

Interest Earned (E_2) = $sE \int_{\mu}^{t_{j+1}} \int_{t_j}^{\mu} D d\mu dt$

$$= sE\frac{D_0}{2}(t_{j+1}^2 - \mu^2) * (\mu - t_j) \quad (4.15)$$

Interest Charged (E'_2) = $cE' \int_{M}^{t_{j+1}} I_{j+1}(t)dt$

$$= cE'\frac{D_0\mu}{2}(t_{j+1} - M)^2 \quad (4.16)$$

Subcase 3: M > t_{j+1}

Interest Earned (E_3) = = $sE\int_{\mu}^{t_{j+1}}\int_{t_j}^{\mu} Dd\mu dt$

$$=sE\frac{D_0}{2}(t_{j+1}^2-\mu^2)*(\mu-t_j) \quad (4.17)$$

Interest Charged (E_3') = 0

Under these subcases, the total cost of the retailer and the supplier will be different for all the cases and will be calculated differently.

Now the new R_J and S_J for the subcase 1 will be:

$$R_{J1} = n*Y + W*\sum_{j=0}^{m-1} S_{j+1}' + h\sum_{j=0}^{m-1}\{(S_{j+1}'(\mu-t_j) - \frac{D_0}{6}(\mu^3 - 3t_j^2\mu - 2t_j^3) + \frac{D_0\mu}{2}(t_{j+1}-t_j)^2\} - sE\sum_{j=0}^{m-1}\{\frac{D_0}{6}(\mu^3+2t_j^3) - \frac{D_0}{2}\mu t_j^2\} + cE'\sum_{j=0}^{m-1}\{S_{j+1}(\mu-M) + \frac{D_0\mu}{2}(t_{j+1}-\mu)^2 - \frac{D_0}{6}\{\mu(\mu^2-3t_j^2) - M(M^2-3t_j^2)\}\} \quad (4.18)$$

$$S_{J1} = m(LC+MC) + c*\sum_{j=0}^{m-1} S_{j+1}' - cE'\sum_{j=0}^{m-1}\{S_{j+1}(\mu-M) + \frac{D_0\mu}{2}(t_{j+1}-\mu)^2 - \frac{D_0}{6}\{\mu(\mu^2-3t_j^2) - M(M^2-3t_j^2)\}\} \quad (4.19)$$

In case of the supplier, the interest charged by the supplier from the retailer is his/her interest earned.

Also, the new R_{J2} and S_{J2} in subcase 2 will be given as:

$$R_{J2} = n*Y + W*\sum_{j=0}^{m-1} S_{j+1}' + h\sum_{j=0}^{m-1}\{(S_{j+1}'(\mu-t_j) - \frac{D_0}{6}(\mu^3 - 3t_j^2\mu - 2t_j^3) + \frac{D_0\mu}{2}(t_{j+1}-t_j)^2\} - sE\frac{D_0}{2}(t_{j+1}^2-\mu^2)*(\mu-t_j) + cE'\frac{D_0\mu}{2}(t_{j+1}-M)^2 \quad (4.20)$$

$$S_{J2} = m(LC+MC) + c*\sum_{j=0}^{m-1} S_{j+1}' - cE'\frac{D_0\mu}{2}(t_{j+1}-M)^2 \quad (4.21)$$

Similarly, the new R_{J3} and S_{J3} in subcase 3 will be given as:

$$R_{J3} = n * Y + W * \sum_{j=0}^{m-1} S'_{j+1} + h \sum_{j=0}^{m-1} \{(S'_{j+1}(\mu - t_j) - \frac{D_0}{6}(\mu^3 - 3t_j^2\mu - 2t_j^3) + \frac{D_0\mu}{2}(t_{j+1} - t_j)^2\} - sE\frac{D_0}{2}(t_{j+1}^2 - \mu^2) * (\mu - t_j) \quad (4.22)$$

$$S_{J3} = m(LC + MC) + c * \sum_{j=0}^{m-1} S'_{j+1} \quad (4.23)$$

To calculate the t_i for the total cost of the retailer and the supplier without coordination, we need to have, $\frac{\partial R_I}{\partial t_i} = 0$

Now, $$\frac{\partial R_I}{\partial t_i} = D_0 t_i(2h - W) + D_0\mu(W - 2(t_{i+1} + t_{i-1})) + h\frac{D_0}{2}(5t_i^2 + 3\mu^2) \quad (4.24)$$

Putting $\frac{\partial R_I}{\partial t_i} = 0$, we will be able to calculate the values of different $t_i's$ in days

t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
0	0.56	1.62	3.30	6.13	12.11	29.86	120

Table 4.1: Values of $t_i's$ for calculating the cost of the retailer and the supplier without coordination

R_I	2443.47	3382.40	4294.92	5199.54	6101.2	7001.49
S_I	68.24	126.43	185.98	245.79	305.62	365.40

Table 4.2: Values of R_I and S_I

From the table 4.2, it can be clearly seen that the cost of the retailer is higher than that of the supplier. Thus, to counter this problem, the supplier and the retailer will have a coordination. The main aim behind the coordination is to lower the cost and to maximize the profits of both the retailer and the supplier. The supplier will offer a trade credit to the retailer and based on the ramp type demand in case of seasonal goods, there will be 3 subcases: $0 < M < \mu$, $\mu < M < t_{j+1}$, $M > t_{j+1}$.

In all the subcases R_J will be different and so we will calculate $\frac{\partial R_{J1}}{\partial t_j}, \frac{\partial R_{J2}}{\partial t_j}, \frac{\partial R_{J3}}{\partial t_j}$

and

After this will put $\frac{\partial R_{J1}}{\partial t_j} = 0, \frac{\partial R_{J2}}{\partial t_j} = 0, \frac{\partial R_{J3}}{\partial t_j} = 0$

This will give $t_j{}'s$ for R_{J1}, R_{J2} and R_{J3}

$$\frac{\partial R_{J1}}{\partial t_j} = D_0 t_j(2h - W) + D_0\mu(W - 2(t_{j+1} + t_{j-1})) + h\frac{D_0}{2}(5t_j^2 + 3\mu^2) + cE_1'(D_0(M(t_j - \mu)) + ut_j) + 6Mt_j) - sE_1(D_0 t_j(t_j - \mu)) \quad (4.25)$$

$$\frac{\partial R_{J2}}{\partial t_j} = D_0 t_j(2h - W) + D_0\mu(W - 2(t_{j+1} + t_{j-1})) + h\frac{D_0}{2}(5t_j^2 + 3\mu^2) + cE_2'(D_0\mu(t_j - M)) - sE_2\frac{D_0}{2}(\mu - t_{j+1} - t_{j-1} + \mu^2) \quad (4.26)$$

$$\frac{\partial R_{J3}}{\partial t_j} = D_0 t_j(2h - W) + D_0\mu(W - 2(t_{j+1} + t_{j-1})) + h\frac{D_0}{2}(5t_j^2 + 3\mu^2) - sE_3\frac{D_0}{2}(\mu - t_{j+1} - t_{j-1} + \mu^2) \quad (4.27)$$

t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
0	0.58	1.64	3.33	6.20	12.29	30.23	120

Table 4.3: Values of $t_j{}'s$ for calculating the cost of the retailer and the supplier under coordination for subcase1: 0 < M < μ

$\boldsymbol{R_{J1}}$	2154.46	2787.37	3619.36	4492.25	5387.11	6269.09
$\boldsymbol{S_{J1}}$	757.48	241.93	127.67	***115.43***	138.81	176.94

Table 4.4: Values of R_{J1} and S_{J1}

The **bold** and *Italic* value of S_{J1} shows the optimal cost of the supplier under coordination situation in subcase 1: 0 < M < μ.

t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
0	1.90	3.20	4.49	6.68	11.96	29.07	120

Table 4.5: Values of $t_j{}'s$ for calculating the cost of the retailer and the supplier under coordination for subcase2: μ < M < t_{j+1}

$\boldsymbol{R_{J2}}$	2172.29	2984.49	3868.70	4764.89	5664.72	6565.96
$\boldsymbol{S_{J2}}$	392.74	***146.10***	178.87	234.42	293.37	353.00

Table 4.6: Values of R_{J2} and S_{J2}

The **bold** and *Italic* value of S_{J2} shows the optimal cost of the supplier under coordination situation in subcase 2: $\mu < M < t_{j+1}$

4.6 Algorithm

Step 1: For finding the total cost of the retailer and the supplier in Case 1: no coordination situation.

Step 1.a. Setting n = 2.

Step 1.b. Solving equation (24) by using Mathematica 12.0 to get the values of $t_i's$.

Step 1.c. Calculating R_I and S_I by using equations (4.10) and (4.11).

Step 1.d. Repeat Step 1.a. to Step 1.c. for n = 3 to n = 7.

Step 2. For finding the total cost of the retailer and the supplier in Case 2 – Subcase 1: under the coordination situation and $0 < M < \mu$.

Step 2.a. Setting n = 2.

Step 2.b. Solving equation (4.25) by using Mathematica 12.0 to get the values of $t_j's$.

Step 2.c. Calculating R_{J1} and S_{J1} by using equations (4.18) and (4.19).

Step 2.d. Repeat Step 2.a. to Step 2.c. for n = 3 to n = 7.

Step 3. For finding the total cost of the retailer and the supplier in Case 2 – Subcase 2: under the coordination situation and $\mu < M < t_{j+1}$. (Follow same steps as of step 2 by using equations (4.26), (4.20) and (4.21). From here will be get the values of R_{J2} and S_{J2}.

Step 3.d. Repeat Step 3.a. to Step 3.c. for n = 3 to n = 7.

Step 4. For finding the total cost of the retailer and the supplier in Case 2 – Subcase 3: under the coordination situation and $M > t_{j+1}$. (Follow same steps as of step 2 by using equations (4.27), (4.22) and (4.23). From here will be get the values of R_{J3} and S_{J3}.

4.7 Numerical Examples

Example 4.1: Given W = 0.05, μ = 0.15, Y = 900, h = 0.3, D_0 = 600, LC = 30, MC = 30, c = 0.049, the total cost of the retailer without coordination (R_I), the total cost of the supplier without coordination (S_I) are shown in the following table. The equations (4.7) and (4.8) will be used to calculate the values of R_I and S_I.

Constant Variables	$\boldsymbol{R_I}$ ***and*** $\boldsymbol{S_I}$
W=0.05 Y = 900 h = 0.3 LC = MC = 30 c = 0.049 μ = 0.15 D_0 = 600	R_I = 2443.66 S_I = 365.393

Table 4.7: The values of R_I and S_I for Example 4.1

Example 4.2: Given W = 0.05, μ = $(t_{i+1}-t_i)/2$, Y = 900, h = 0.3, D_0 = 600, c = 0.049, M = $(t_{i+1}-t_i)/3$, the total cost of the retailer with coordination (R_{J1}) and the total cost of the supplier (S_{J1}.) under coordination, is shown in the following table. The equation (4.18) and (4.19) will be used to calculate the value of R_{J1} and S_{J1}. In this case, 0 < M < μ

Constant Variables	$\boldsymbol{R_I}$ ***and*** $\boldsymbol{S_I}$
W = 0.05, Y = 900 h = 0.3, c = 0.049 E = 0.10, E' = 0.12 s = 0.3, D_0 = 600 μ = $(t_{i+1}-t_i)/2$ M = $(t_{i+1}-t_i)/3$	R_I = 2154.46 S_I = 757.48

Table 4.8: The values of R_{J1} and S_{J1} for Example 4.2

Example 4.3: Given W = 0.05, μ = (t_{i+1}-t_i)/3, Y = 900, h = 0.3, D_0 = 600, c = 0.049, M = (t_{i+1}-t_i)/2, the total cost of the retailer with coordination (R_{J2}) and the total cost of the supplier (S_{J2}) under coordination, is shown in the following table. The equation (4.20) and (4.21) will be used to calculate the value of R_{J2} and S_{J2}, is shown in the following table. In this case, μ < M < t_{j+1}

Constant Variables	R_I and S_I
W = 0.05, Y = 900 h = 0.3, c = 0.049 E = 0.10, E' = 0.12 s = 0.3, D_0 = 600 μ = (t_{i+1}-t_i)/3 M = (t_{i+1}-t_i)/2	R_I = 2172.29 S_I = 392.74

Table 4.9: The values of R_{J2} and S_{J2} for Example 4.3

4.8 Sensitivity analysis

The next four tables in this section will give sensitivity analysis of various parameters and at the last of the tables, all the results are compiled together in a subsection.

Parameters	%age change in parameters	%age change in R_I	Value of R_I	%age change in S_I	Value of S_I
D_0	20	11.5	2915.74	2.35	69.6013
	-20	-13.02	2243.82	-2.35	66.4009
A	20	6.97	2759.78	-	-
	-20	-6.97	2399.78	-	-
LC	20	-	-	8.82	74.0011
	-20	-	-	-8.82	62.0011
MC	20	-	-	8.82	74.0011
	-20	-	-	-8.82	62.0011
W	20	0.15	2456.59	-0.14	67.906
	-20	-0.15	2449.2	0.14	68.0951

Table 4.10: Sensitivity Analysis for Case 1: when there is no coordination between the supplier and the retailer.

Parameters	%age change in parameters	%age change in R_{J1}	Value of R_{J1}	Percentage change in S_{J1}	Value of S_{J1}
D_0	20	14.97	4122.15	18.39	882.8
	-20	-14.97	3048.11	-18.39	608.53
A	20	5.02	3765.13	-	-
	-20	-5.02	3405.13	-	-
LC	20	-	-	0.80	751.665
	-20	-	-	-0.80	739.665
MC	20	-	-	0.80	751.665
	-20	-	-	-0.80	739.665
c	20	-	-	2.97	-
	-20	-	-	-2.97	-
s	20	-0.27	3575.31	-	-
	-20	0.27	3594.93	-	-
E_1	20	-0.13	3580.12	-0.63	740.934
	-20	0.13	3590.12	0.63	750.388
E'_1	20	0.15	3590.42	-	-
	-20	-0.15	3579.84	-	-
h	20	8.23	3880.33	-	-
	-20	-8.25	3289.1	-	-

Table 4.11: Sensitivity Analysis for Case 2- Subcase 1: when there is coordination between the supplier and the retailer.

Parameters	%age change in parameters	Percentage change in R_{J2}	Value of R_{J2}	%age change in S_{J2}	Value of S_{J2}
D_0	20	8.17	3293.22	2.87	144.156
	-20	-8.17	2795.48	-2.87	136.104
A	20	8.05	2413.65	-	-
	-20	-8.05	2053.65	-	-
LC	20	-	-	10.48	189.705
	-20	-	-	-10.48	153.705
MC	20	-	-	10.48	189.705
	-20	-	-	-10.48	153.705
s	20	-0.13	2230.72	-	-
	-20	0.11	2236.31	-	-
E_2	20	-0.01	2244.33	-0.009	392.530
	-20	0.01	2244.80	0.009	392.602
E'_2	20	0.10	2236.03	-	-
	-20	-0.10	2231.28	-	-
h	20	10.98	2478.99	-	-
	-20	-10.98	2233.65	-	-

Table 4.12: Sensitivity Analysis for Case 2- Subcase 2: when there is coordination between the supplier and the retailer.

Parameter	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	n	R_I	S_I
W = 0.05 Y = 900, h = 0.3, μ = 0.15, LC = 30, MC = 30, D_0 =600, c = 0.049	0	28.3	120	-	-	-	-	-	2	2452.8	68.00
	0	10.6	29.9	120	-	-	-	-	3	3393.1	126.09
	0	4.96	12.0	30.1	120	-	-	-	4	4306.2	185.61
	0	2.48	5.96	12.3	30.2	120	-	-	5	5201.9	245.41
	0	1.12	3.07	6.19	12.39	30.21	120	-	6	6112.3	305.22
	0	0.18	1.25	3.13	6.22	12.39	30.21	120	7	7011.9	364.93

Table 4.13: Values of R_I and S_I for different replenishment cycles (in days).

Parameter	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	n	R_{J1}	S_{J1}
W =0.05 Y =900, h = 0.3, μ = 0.15, LC = 30, MC = 30, D_0 =600, c = 0.049, E_1 =0.10, E_1'=0.12, s = 0.3	0	28.3	120	-	-	-	-	-	2	2154.4	757.48
	0	10.5	29.9	120	-	-	-	-	3	2787.3	241.93
	0	4.90	11.9	30.1	120	-	-	-	4	3619.3	127.66
	0	2.51	5.89	12.2	30.21	120	-	-	5	4492.2	115.42
	0	1.30	3.16	6.14	12.27	30.23	120	-	6	5378.1	138.80
	0	0.57	1.64	3.33	6.20	12.28	30.23	120	7	6269.0	176.93

Table 4.14: Values of R_{J1} and S_{J1} for different replenishment cycles (in days).

Parameter	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	n	R_{J2}	S_{J2}
W =0.05 Y =900, h = 0.3, μ = 0.15, LC = 30, MC = 30, D_0 =600, c = 0.049, E_2 =0.10, E_2' =0.12	0	27.3	120	-	-	-	-	-	2	2172.29	392.739
	0	10.2	28.82	120	-	-	-	-	3	2984.49	146.102
	0	5.07	11.55	29.0	120	-	-	-	4	3868.70	178.867
	0	3.12	6.21	11.83	29.05	120	-	-	5	4764.89	234.462
	0	2.29	4.11	6.55	11.92	29.06	120	-	6	5664.72	293.366
	0	1.90	3.20	4.49	6.68	11.9	29.0	120	7	6565.96	353.006

Table 4.15: Values of R_{J2} and S_{J2} for different replenishment cycles (in days).

4.8.1 Sensitivity analysis takeaways

1. As per table 4.10, if there is an increase in the value of D_0 (demand), A (ordering cost) and W (the wholesale price), the cost of the retailer increases and when there is fall in all the above mentioned parameters, the cost of the retailer falls, which implies that D_0, A and W all are directly proportional to the cost of the retailer (R_J).
2. Table 4.10 also shows that the relation between D_0 and the total cost of the supplier is also directly proportional and same is applicable in case of LC (Labour cost) and MC (Manufacturing cost). In case of W, the cost of the supplier shows indirect relation, which is obvious in practical situations as well, because if the wholesale prices of a unit increase the profits of the supplier increases and as a result cost will decrease.
3. The analysis in table 4.11 reveals that the retailer's total cost (R_{J1}) is directly proportional to the D_0 (demand), A (ordering cost), s (the selling price), and h (the holding cost). The analysis in the table also clearly implies that R_{J1} is inversely proportional to the selling price (s), the interest earned E, and the interest charged (E_1').
4. Table 4.11 also indicates the relation between the supplier's total cost (S_{J1}) and various other parameters. S_{J1} is directly proportional to D_0, LC, MC, and c (the

purchasing cost of the supplier). With E$_1$, E_1', S_{J1} has inversely proportional relation.

5. Table 4.12 clearly defines the direct relation between R_{J2} and the parameters like, D$_0$, A, s and h, that i.e. as the D$_0$, A, s and h increases so the total cost of the retailer. Moreover, the other two parameters E$_2$ and E_2' have inversely proportional relation with the total cost of the retailer (R_{J2}).
6. Table 4.12 also says about the relation between S_{J2} and the parameters. S_{J2} is directly proportional to D$_0$, LC, and MC. With E$_2$ and E_2', S_{J2} has inversely proportional relation.
7. Table 4.13, along with all the parameters and values of $t_i's$ gives the value of R_I and S_I with different replenishment cycles.
8. Table 4.14 shows the values of R_{J1} and S_{J1} along with all parameters and values of $t_j's$ in different replenishment cycles.
9. Table 4.15 defines the values of R_{J2} and S_{J2} in subcase 2 of case 2 with the values of all parameters and $t_j's$ in different replenishment cycles.

4.9 Graphical representation of sensitivity analysis

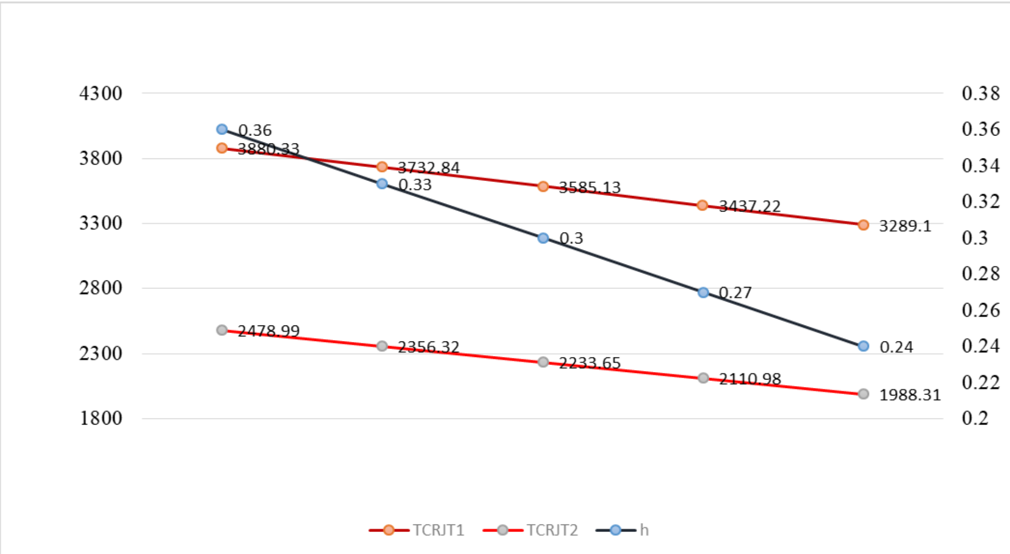
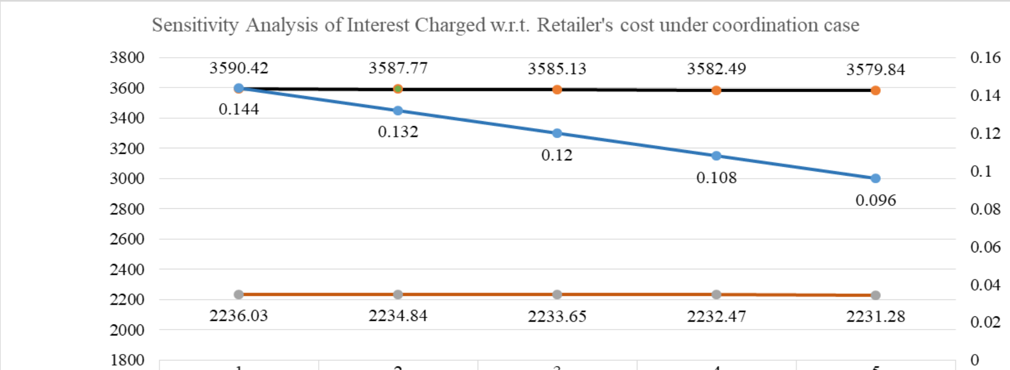


Figure 4.3: Sensitivity Analysis of Holding Cost (h) w.r.t. Retailer's cost (R_{J1}(TCRJT1) and R_{J2}(TCRJT2) under coordination.



	1	2	3	4	5
TCRJT1	3590.42	3587.77	3585.13	3582.49	3579.84
TCRJT2	2236.03	2234.84	2233.65	2232.47	2231.28
In. Charged	0.144	0.132	0.12	0.108	0.096

Figure 4.4: Sensitivity Analysis of Interest Charged w.r.t. Retailer's cost (R_{J1}(TCRJT1) and R_{J2}(TCRJT2) under coordination

4.10 Challenges of Implementing Inventory Coordination in Real-World Scenarios

1. **Complex Demand Patterns**

a. Seasonal items exhibit a ramp-like demand that varies over time, complicating inventory coordination efforts.

b. Demand fluctuations driven by market dynamics necessitate ongoing adjustments to replenishment schedules.

2. **Trade Credit and Financial Constraints**

a. Identifying the ideal trade credit duration poses difficulties, as longer periods may favor retailers but heighten financial risks for suppliers.

b. Retailers can lower expenses through interest accrued during the credit period; however, delayed payments may incur penalties, adding to financial unpredictability.

3. **Coordination Between Supplier and Retailer**

a. Formulating effective coordination agreements necessitates a foundation of trust and transparency in the exchange of information.

b. Suppliers seek to minimize setup expenses, while retailers prioritize reducing holding costs, resulting in potentially conflicting objectives.

4. **Inventory Holding and Replenishment Issues**

a. Disparate replenishment cycles within a limited planning timeframe complicate inventory oversight.

b. Assumptions of immediate replenishment may not consistently apply in real-world supply chains due to logistical challenges.

5. Market and Product Variability

a. Items with seasonal demand, fashion trends, and fast-moving consumer goods possess limited shelf lives, making accurate coordination essential.

b. Changes in the market may compel retailers to investigate new geographic areas, disrupting previously established coordination frameworks.

6. Cost Optimization Challenges

a. While coordination can lead to reduced overall costs for both suppliers and retailers, poor implementation may result in inefficiencies.

b. The sensitivity of cost factors necessitates ongoing monitoring to achieve cost reduction.

7. Technology and Analytical Limitations

a. The execution of real-time coordination strategies demands sophisticated software tools, such as Mathematica, which may not be available to all businesses.

8. Extension Possibilities and Future Considerations

a. The present model assumes a finite planning horizon with unequal cycle lengths. Future research can extend the model to an infinite planning horizon with dynamic replenishment strategies.

b. Additional considerations such as inventory deterioration, preservation technologies, and quadratic ramp-type demand functions can enhance model accuracy and applicability.

c. The trade credit framework can be further expanded to a multi-level credit system, wherein both retailers and end customers operate on credit, introducing new financial dimensions to the inventory coordination problem.

By addressing these challenges, future research can refine supplier-retailer coordination mechanisms, enhance inventory control methodologies, and contribute to the broader field of inventory optimization in applied mathematics.

4.11 Conclusion

The article is a work to get a clear view of the market conditions of the products that are seasonal in nature and follows a ramp type demand. The demand for such products follows a rise at the starting of the season, then towards the middle the demand for the product becomes steady and towards the end the demand decreases. These days, a different market can also be searched for the seasonal products so that their demand is there throughout the year, but whenever a market is changed, the product in its new market as well follows a ramp type demand only. So, we concluded that this type of demand for seasonal products or for the products which are perishable in nature or those that have shorter shelf life, like electronics, techno gadgets, fashionable items, or those items which are sold on e-commerce platforms all can be categorized in such type of demand.

We tried to categorize the article according to the parameter μ and thus the cases are discussed accordingly. The article calculated the cost of the retailer and the supplier in all the cases and subcases. A sensitivity analysis is done to see the behavior of all the variables and their effects on the total costs. In the due process, we reach the minimum cost of the supplier as well.

The interest earned by the retailer following the trade credit policy was also discussed. The greater the period, the higher interest will be earned, and this will reduce the cost of the retailer. The interest charged by the supplier is also taken up and thus, this part will raise the cost of the retailer but reduces that of the supplier as it will be the income source for the supplier. To tackle these scenarios, subcases were added under coordination case. The cost of the retailer and supplier after the coordination reduces and this means a successful coordination was done between the two. Numerical examples have been discussed as well to support the equations and results in the form of numerical values can be seen in the examples and in the tables constructed in the article.

The algorithm is created in the article, and it explains an easier way to calculate all the costs under no coordination and coordination situations. The practical applications of the case taken up are many. In day-to-day transactions, between the supplier and the retailer, the maximum runs on a credit basis. This situation is taken a step forward by including interest earned and charged subcases based on the seasonal products and ramp type demand. The model is constructed in a finite planning horizon with unequal cycle lengths. The replenishment time period for the quantity was calculated by using Mathematica 12.0 software which will help both the supplier and the retailer to have better coordination and a reduced cost. Our numerical examples and tables justify the coordination case. Our model can also be a good insight for the inventory managers. They can work on coordination schemes to reduce their costs taking into consideration the credit period length, the interest earned, and the interest charged in the process. This will help in maximizing their profits, to increase their customer base as well.

Our model can be extended in several ways. To point out some deterioration can be included, along with shortages. Preservation technology to further reduce the deterioration can be worked on. Quadratic ramp type demand can be added to give the model a nearer to reality approach. An inventory-dependent demand or lost sales can also be the extensions in our model. The trade credit policy can be two-fold as well, when the supplier gives trade credit to the retailer and the retailer in turn gives it to their customers. Thus, all the above-mentioned parameters and their effects can be studied in future research.

Chapter 5

Effect of Variable Inflation on Supply Chain Management in FMCG sector with Single Supplier – Multiple Retailers and trade Credit

Abstract: The main contribution of this article is to inspect the role of variable inflation rate in every replenishment cycle in a supply chain management. An inventory, time and inflation dependent demand is contemplated. Some completely novel concepts are introduced, inflation and its effect are studied in every single replenishment cycle, and furthermore equipoise is attained under single supplier – multiple retailers' coordination in a highly dynamic and competitive free markets. Moreover, equipoise cost of retailer is used to ultimately reduce costs of both the supplier and retailers. Algorithm is discussed in detail to understand the working of our new model, which is in close proximity to real world-based problems encountered in inventory management. Numerical examples are discussed in support of theoretical aspects mentioned with the help of Mathematica software 12.0.

5.1 Introduction

Inflation is the result of a greater supply of money than its demand. When the demand is greater than the supply, it leads to inflationary conditions in an economy. J. A. Buzzacott [37] (1975) was the first researcher who wrote something about inflation in inventory models. According to him, if the change in prices is independent of the replenishment cycle, then the inventory charge is low, and inflation also doesn't affect the inventory. He also formulated a minimum cost model for a single item in inventory and studies the effect of inflation on it. Guria et al. [7] (2013) framed an inventory policy for an item with inflation and demand depending on the selling price. The author added a new parameter of immediate part payments to the wholesaler.

In 1985, Goyal [25] gave an EOQ model with permissible delays in payments. Later in 1995, Aggarwal and Jaggi [26], extended their work and gave a relation between sales revenue and delay period along with rate of interest.

Supplier – Retailer coordination/ set up is taken up by innumerable researchers. M. Esmaeili & M. Nasrabadi [80], in 2020 talked about single vendor – multiple retailers set up.

Thusly, the main contribution of this article is to examine the role of variable inflation rate in inventory management. A supplier with multiple retailers' setup is considered. Equipoise of total inventory cost for multiple retailers in each competitive environment is calculated. The supplier and multiple retailer trade credit is considered.

5.2 Literature Review

Well, there is lot to discuss in literature review, here we will consider those research articles only which are related to the scope of our research. Till now, much has been talked about factors that impact cost and profit of a supplier and a retailer. Demand patterns, deterioration, trade credit, shortages, availability of raw material, transportation cot, set up cost, holding cost, purchase cost, ordering cost, lost sales, opportunity cost, labour cost, machinery cost, inflation, seasonal patterns of a product and many more are all pillars that need to be studied well to make a model in inventory management that reduces cost or maximizes the profit of a supplier or retailer or of the whole system. Inventory led and time led demand are such, which can't be overlooked to tackle inventory management paraphernalia. The inventory managers can't neglect the impact and after – effects of inflation, and the dynamic nature of inflation is the most cumbersome thing to be taken care of. In the market where there are high inflationary conditions and dynamic forces are most active, coordination among the players of the market can highly affect some important costs related to inventory system. In the futuristic and promising industry of FMCG, everything is or will be found, which is included in Inventory Management System.

In 2009, Mirzazadeh et.al [81], in their inventory model, under finite production rate talked about uncertain inflationary set up. In 2011, R. P. Tripathi [82], discussed important parameters of pricing and ordering policy under inflation with permissible delay in payments. In 2013, Shastri et.al [83], added manufacturer along with supplier and retailer for deteriorating items in inflationary situations. Weibull Deterioration is added by Singh and Panda [84] in 2015 along with above taken parameters. In 2015 itself, Kiniwa et.al. [85] added deflation in price stabilization process. In 2020, Geetha and Udayakumar [86], wrote about deteriorating items along with inflation induced demand which was time dependent as well in infinite horizon. In 2020, itself R. Sundararajan et. al. (a) [87] studied impact of delay in payments along with shortages and inflation. In 2021, R. Sundararajan et. al. (b) [88] gave price determination along with deterioration of items under inflation and delay in payments. Alterations are always perceived in inflation, under the impact of numerous dynamic constituents, thusly we can't take inflation in inventory as constant or same throughout the horizon.

So far, researchers have differently presently coordination between the supplier and the retailer. Guria et al. [7] (2013), Control system for inventory is developed in which immediate part payment is allowed and permissible delay in payments is also allowed for the rest of the period by the wholesaler for an item over a finite planning horizon or random planning horizon. Moreover, demand taken is inflation and selling price dependent. Chung et. al. [89], presented inventory models and discuss the case that at what time the retailer will prefer cash discounts and when they will go for trade credits. Zhong et. al. [90], in 2017 itself proposed an integrated location and two-way inventory networking model in consideration with trade credit. Mahato et. al. [91], in 2021, gave two step supply chain under trade credit considering hybrid price and stock depending on demand, and concluded that centralized policy will give more profits. In 2021, Mandal et. al. [92], gave advertisement and stock level dependent inventory model under trade credit policy with deterioration rate as constant function of on hand inventory.

Wu and Zhao [34], in 2014, emphasised on supplier- retailer coordination along with trade credit period rate in a finite planning horizon. Their model is even applicable for consumer-packaged goods or FMCG. In 2017, Singh *et. al.* [48] presented an EOQ model in a finite planning horizon for two cases – with and without delay in payments. In 2018 again, Singh *et. al* [49], gives an inflation-based inventory model in a finite planning horizon including trade credit. In 2019, Singh *et. al* [50], under centralized and decentralized planning horizon incorporated trade credit policy as well. In 2020, M. Esmaeili & M. Nasrabadi [80] takes the coordination between a supplier and a retailer a step forward by giving a single supplier and multiple retailer model under impact of inflation and trade credit. Taghizadeh-Yazdi et.al. [93], gave an integrated inventory model, considering coordination among supplier – manufacturer – distributor, and proves that profits raise by 15%.

Thus far, dissimilar demands were considered for developing profuse inventory models. To name few, inventory dependent, time led demand, selling price demand, inflation induced demand, and stock dependent demand etc. has been an inseparable part of supply chain management associated research from years on end. Liao et. al. [6] in 2000, gave inventory model stock dependent consumption rate under permissible delay in payments. In 2014 model by Wu and Zhao [34], took inventory and time dependent demand. In 2015, Kumar and Rajput [94], worked on time and stock dependent demand along with inflation and credit policy. Saha and Sen [95], 2019, added selling price dependent demand along with time dependency in their inflation based optimal inventory model. Shaikh et. al. [96], added partial backlogging and deteriorating items along with stock led demand in EOQ model with the facility of price discount. Barman et. al. [97], 2021, added variable and cloud fuzzy demand rate with inflation in their inventory model. Khan et. al. [98], 2022, gave supply chain model with non-linear stock dependent demand rate.

The FMCG sector is fast emerging sector. These Consumer-Packaged Goods (CPG) are inexpensive merchandise which sell quickly. This industry is mainly responsible for producing, distributing and even for marketing of these CPG. Nemtajela and Mbohwa (2017) [51], proved that there is a direct relation between managing inventory levels and uncertainty of demand i.e. more the uncertainty levels of demand, the inventory managers need to deal with more cumbersome inventory management problems for FMCG. Jaggi *et. al*. [52]

(2018), explained that consumer goods or the products with displayed stocks need to have a centralized policy for achieving maximum joint profit by both the supplier and the retailer, though the profits are biased for the retailer. In 2020, Li *et. al.* [53], in their work explained that how the inventory can be optimized for fast-moving consumer products by replenishing and salvaging the stocks at the start of each cycle by focusing on the e-commerce platform for promoting these consumer-packaged goods.

Authors (Name & Year)	**Inflation**	**Horizon**	**Coordination**	**Trade Credit**	**FMCG**
Mirzazadeh (2009)	Stochastic	Finite	S – R	No T.C	No
R. P. Tripathi (2011)	Constant	Infinite	S – R	IE – IC	No
Shastri et.al. (2013)	Constant	Infinite	S – M – R	No T.C	No
Singh, Panda (2015)	Constant	Finite	S – R	IE – IC	No
Singh *et. al* (2017)	Constant	Finite	S – R	Avg. (min, max.)	No
Singh *et. al* (2019)	No	Finite	S – R	Profit sharing	No
M. Esmaeili & M. Nasrabadi (2020)	Discrete-Time Markov Chain	Fixed	S – Multiple retailers	IE – IC	No
R. Sundararajan et. al. (a) (2020)	Constant	Infinite	S – R	IE – IC (single level)	No
R. Sundararajan et. al. (b) (2021)	Constant	Infinite	S – R	IE – IC (single level)	No
Geetha, Udayakumar (2020)	Constant	Infinite	S – R	No T.C	No
Present Paper	Exponential & Variable in each replenishment cycle	Finite	S – Multiple retailers	Equipoise	Yes

Table 5.1: A comparison of the inventory models

S – Supplier, R – Retailer, M – Manufacturer, IE – Interest Earned, IC – Interest Charged, T.C. – Trade Credit

Table 5.1 specifies that inflation is considered constant for quite a lot of times. Coordination between supplier and retailer is maximum time based on trade credit policy or credit period policy based on interest earned and interest charged. Though M. Esmaeili & M. Nasrabadi [5] (2020) moved out of the box and consider Discrete-Time Markov Chain inflation rate, they discussed two cases of inflation rate, one with Discrete-Time Markov Chain and other without Discrete-Time Markov Chain. They even consider single vendor – multiple retailers set up, but coming on to credit period, once again they had gone back to traditional interest charged and interest earned concept. The last column is for the models which are applicable of FMCG.

Some noticeable considerations of our research article are:

(i) variable inflation is considered in all replenishment cycles.

(ii) The finite planning horizon is taken.

(iii) Single supplier – multiple retailers set up is undertaken.

(iv) Equipoise is discussed for total cost of each retailer in coordination with the supplier and then the credit period is offered based on equipoise cost.

(v) The model introduced in this article is applicable to all the products encompassed in FMCG industry. All the novel concepts introduced are necessary to run the machinery of FMCG sector smoothly.

5.3 Defining the Problem

In this piece of writing, the effect of inflation is studied at every step. The main reason behind doing all this is to see how inflation affects demand that too when inflation rises every time inventory replenishes. We have considered linear rise in inflation which no one has taken before this, making our work a novel piece. The coordination between the supplier and the retailer has been taken to a different level. Using the co-ordination between the two, the retailer's cost equipoise has been worked out. The biggest advantage to the market (more precisely to the consumer) from this would be that the price of a product would remain the same with every retailer, that too when the deep analysis and effect of inflation has been considered. Retailer’s cost equipoise is an unconventional concept in inventory management.

5.4 Assumptions and Notations

5.4.1 Assumptions

- The planning horizon – finite, unequal lengths of replenishment cycle and foreseeing FMCG industry it is taken as 120 days (4 months).
- Shortages are not considered as for FMCG it's assumed that the retailers are many in the competitive environment and if the product is not available with one retailer, consumer immediately buy from the one who has the product.
- No lead time is considered as inventory is replenished instantly.
- Replenishment of stocks/inventory is instant or instantaneous.
- Inflation rate varies in every replenishment cycle.
- Linear rise in inflation is taken.
- No heed is paid to deterioration of the items as in FMCG products selling is much faster and thus deterioration is highly negligible when it kept as stock.

5.4.2 Notations

k = constant, inventory dependent demand rate

b_1 = increasing demand rate per year

b_2 = the scaling for inflation function

α_i = variable rate of inflation

$\alpha_{i+1} = \alpha_i + 0.1$ (linear rise in inflation)

Y = ordering cost per cycle

h = holding cost per cycle

W = wholesale price per cycle

I_{i+1} = inventory level during $(i+1)^{th}$ cycle at the time t_i and $t_i \leq t \leq t_{i+1}$

S_{i+1} = order quantity in $(i+1)^{th}$ cycle in time t_i without coordination where i = 0, 1, 2,….., n-1

LC = Labour Cost

MC = Machinery cost

c = purchasing cost per unit (for the supplier)

S_I = Supplier's total cost

M_i = the tenable layout for case 2, where $i = 1,2,3,4, \dots \dots$

$C_{R_i}, i = 1,2,3, \dots ..$ the retailer who makes a proffer to the other retailer in the nabe as per Case 2

$C_i, i = 1,2,3, \dots ..$ is the inventory cost for 10 units of the product for each retailer (used to attain Equipoise in the market)

R_{I1} = total cost of first retailer

R_{I2} = total cost of second retailer

R_{I3} = total cost of third retailer

R_{I4} = total cost of fourth retailer

Q_I = optimal order quantity of the system

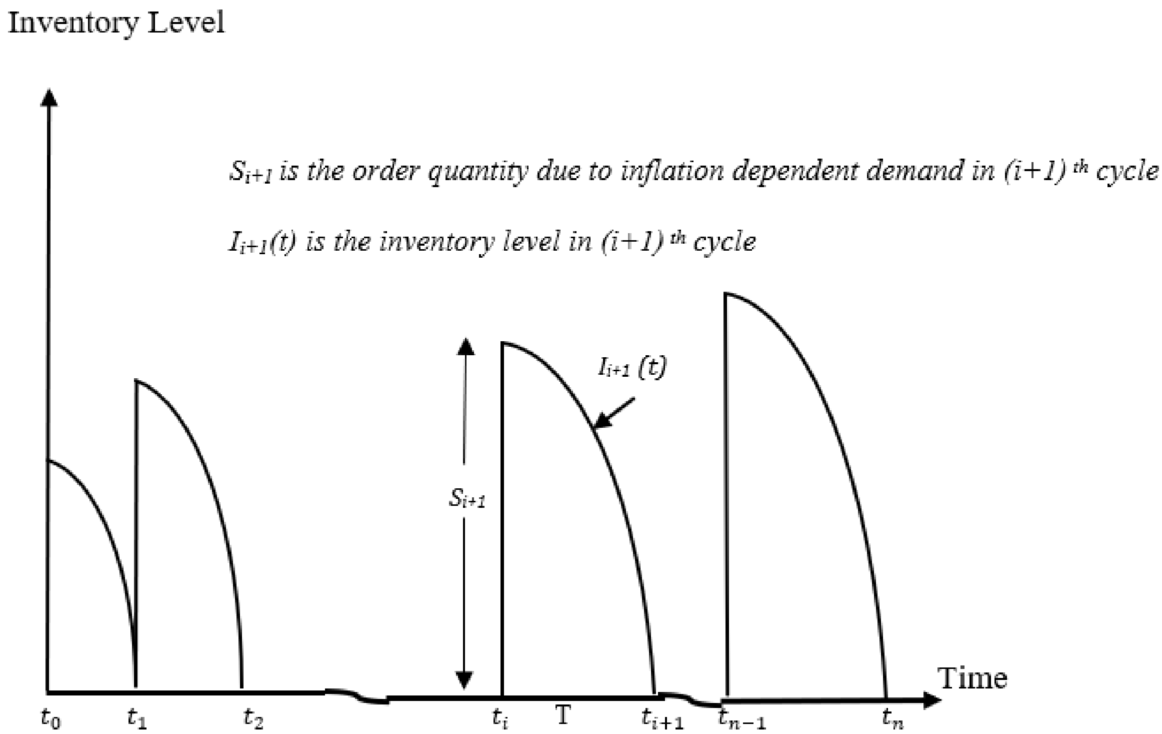


Figure 5.1: Graphical representation of the model.

5.5 The Mathematical Formulation of the model

The supplier takes an order from retailer at time $t_0 = 0$, the inventory level reaches zero at t_1 and then the inventory is refilled instantaneously. No shortages or deterioration rate is considered as targeted field is FMCG. Inventory and time dependent demand along with inflation-based demand is considered. The model and its graphical representation are shown in Figure 1, where t_0 is zero and the horizon is finite. $I_{i+1}(t)$ depicts the inventory level in (i+1)th cycle.

Case 1: When there is no coordination among the supplier and the retailers.

The differential equation of the inventory level in (i+1)th cycle will be given as follows:

$$\frac{dI_{i+1}(t)}{dt} + kI_{i+1}(t) = -(b_1 t + b_2 e^{\alpha_i t}),\ t_i \le t \le t_{i+1} \quad (5.1)$$

Where boundary conditions are $I_{i+1}(t_{i+1}) = 0$ and $I_i(t_i) = Q_{i+1}$. Therefore, solution of equation (5.1) is

$$I_{i+1}(t) = e^{-kt} \int_t^{t_{i+1}} e^{ku}(b_1 u + b_2 e^{\alpha_i u})\, du, t_i \le t \le t_{i+1} \quad (5.2)$$

Refer Appendix A.5.1 for the solution of Equation (5.1)

The order quantity for $(i+1)th$ cycle is

$$S_{i+1} = I_{i+1}(t_i) = e^{-kt_i} \int_{t_i}^{t_{i+1}} e^{kt}(b_1 t + b_2 e^{\alpha_i t})\, dt, \quad (5.3)$$

such that, $u \in [t, t_{i+1}]\ and\ t_i \le t \le u \le t_{i+1}$

Ordering cost of the retailer is $Y(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)})))$ (5.4)

Refer Appendix A.5.2 for the solution of Equation (5.4)

Holding cost of the retailer is

$$(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * \frac{1}{2k^3} b_1(2 - 2e^{k(-t_j+t_{1+j})} - 2kt_j + k^2 t_j^2 + 2e^{k(-t_j+t_{1+j})} kt_{1+j} - k^2 t_{1+j}^2) + \frac{b_2(\frac{e^{-kt_j+t_{1+j}\alpha_{1+j}}(-e^{kt_j}+e^{kt_{1+j}})}{k} + \frac{e^{t_j\alpha_{1+j}} - e^{t_{1+j}\alpha_{1+j}}}{\alpha_{1+j}})}{k+\alpha_{1+j}}) \quad (5.5)$$

Refer Appendix A.5.3 for the solution of Equation (5.5)

Purchasing cost of the retailer is

$$W(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)})*(b_1(\frac{e^{k(t_{j+1}-t_j)}*t_{j+1}}{k}-\frac{e^{k(t_{j+1}-t_j)}}{k^2}-\frac{t_j}{k}+\frac{1}{k^2})+ b_2(\frac{e^{\alpha_{j+1}*t_{j+1}}*e^{k(t_{j+1}-t_j)}-e^{t_j*\alpha_{j+1}}}{k+\alpha_{j+1}})))) \quad (5.6)$$

Refer Appendix A.5.4 for the solution of Equation (5.6)

The retailer's total cost will be sum of ordering cost, holding cost and purchasing cost

Adding Equation (5.4), (5.5) and (5.6), we get:

$$R_I = W(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)})*(b_1(\frac{e^{k(t_{j+1}-t_j)}*t_{j+1}}{k}-\frac{e^{k(t_{j+1}-t_j)}}{k^2}-\frac{t_j}{k}+\frac{1}{k^2})+b_2(\frac{e^{\alpha_{j+1}*t_{j+1}}*e^{k(t_{j+1}-t_j)}-e^{t_j*\alpha_{j+1}}}{k+\alpha_{j+1}}))))+Y(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)})))+ h(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)})*\frac{1}{2k^3}b_1(2-2e^{k(-t_j+t_{1+j})}-2kt_j+k^2t_j^2+ 2e^{k(-t_j+t_{1+j})}kt_{1+j}-k^2t_{1+j}^2)+\frac{b_2(\frac{e^{-kt_j+t_{1+j}\alpha_{1+j}}(-e^{kt_j}+e^{kt_{1+j}})}{k}+\frac{e^{t_j\alpha_{1+j}}-e^{t_{1+j}\alpha_{1+j}}}{\alpha_{1+j}})}{k+\alpha_{1+j}}) \quad (5.7)$$

Let $n^*, t_1^*, t_2^*, \dots . t_{n-1}^*$ be the optimal solution of Minimum of total cost of retailer (R_I). Under this first case, of no coordination supplier's cost will be affected by retailer's replenishment rate. Supplier's total cost will be sum of set up cost and manufacturing cost.

Set up cost is further sum of Labour Cost (LC) and Machinery Cost (MC).

$$S_I(n^*, t_0, t_1^*, t_2^*, \dots . t_{n-1}^*) = (n^*-1)(LC+MC)+\sum_{j=0}^{n^*-1} c*Q_{j+1}^* \quad (5.8)$$

The total order quantity during the planning horizon is

$$Q_I = \sum_{j=0}^{n^*-1} Q_{j+1}^* \quad (5.9)$$

5.6 Algorithm for Case 1:

Step 1: in case of no coordination, the optimization of order quantity schedule of the retailer will be deduced.

Step 2: Assign n = 2, 3, 4 and so on.

Step 3: Solve the Equation (5.10) for n = 2, 3, 4 and so on.

Step 4: Derived values of t_0, t_1, t_2, and so on will be used to calculate the value of total cost of the retailer from Equation (5.7).

Step 5: Find the optimal replenishment cycle.

Step 6: Use the optimal replenishment cycle value derived in previous step to calculate the optimal cost of the supplier from Equation (5.8) and total quantity from Equation (5.9).

Case 2: When there is coordination among the supplier and multiple retailers'

Credulous Arrangement for Competitive Market Model

Our system for Case 2 constitutes a set of retailers with different cost of the product. The intent behind fabricating this arrangement is to each an **Equipoise,** in inventory cost of retailers under the assistance of the supplier (at two phases, to achieve Equipoise and thenceforth deciding credit period).

Let $M_i, i = 1,2,3,4, \dots ....$ be the set of nabe retailers with dissimilar inventory costs at initial step. Here we presume, that each retailer has goods and in initial phase have different inventory costs. Under the defined array, a pair of retailers will be undertaken that have maximum and minimum inventory cost of the product. The inventory cost of each retailer will be designated as $C_{R_i}, i = 1,2,3, \dots ..$. to calculate the inventory cost of each retailer, we follow below mentioned procedure.

The inventory cost of the retailers' is calculated from equation (5.7). The unit cost of the product is given by dividing the solutions obtained from equation (5.7) with the solutions of Equation (5.9). This unit cost of the product will be different for all the retailers in the market (keeping in mind variable holding cost, ordering cost, purchasing cost of the retailer initially). The supplier will contact the retailers and will give them trade credits for bringing their cost up to equipoise in the competitive free market.

Proffer Design

In this section, we consider an arrangement most appropriate for FMCG competitive market. Under this, unabridged procedure, **RETProffer** is initiated, in which each retailer will always proffer another nabe retailer with the lowest cost of the product, where cost is always an integer.

RETProffer

Each retailer $C_{R_i}, i = 1,2,3, \ldots ..$ proffer

$C_{R_{ij}} = c_j + \left(\frac{C_i - C_j}{z}\right)$ $where\ z \geq 1,\ i, j\ \in\ M_i,$

to another retailer $C_{R_i}, i = 1,2,3, \ldots .. \in\ M_i$ by using the formula defined by above mentioned formula

For calculating Equipoise of Inventory Cost for Multiple Retailers in each Competitive Environment the following algorithm will be followed:

5.7 Algorithm for Case 2:

Step 1: Under coordination case, use the value derived in step 1 (c) (from Algorithm 5.6) of total cost of the retailer and the value of total quantity from Step 1 (e) (from Algorithm 5.6) divide total cost of retailer with quantity of the product to evaluate the per unit cost of the product for one retailer.

Step 2: Repeat Step 1 for 4 retailers.

Step 3: Consider the values of per unit cost of the product for Retailers as C_1, C_2, C_3, C_4.

Step 3 (a): Find the largest among the four values of C_1, C_2, C_3, C_4.

Step 3 (b): Find the smallest among the four values of C_1, C_2, C_3, C_4.

Step 3 (c): When C_1 is largest and C_2 is smallest, replace C_2 with S, and calculate $S + (C_1 - S)/z$ and replace $C_1 = C_2 = S$.

Step 3 (d): Repeat Step 3 (c) when C_1 is largest and C_3 is smallest by replacing C_2 with C_3.

Step 3 (e): Repeat Step 3 (c) when C_1 is largest and C_4 is smallest by replacing C_2 with C_4.

Step 3 (f): When C_2 is largest and C_1 is smallest, Calculate $VC_1C_2 = C_1 + ((C_2 - C_1)/z)$ and assign

$C_1 = C_2 = VC_1C_2$.

Step 3 (g): Repeat Step 3 (e) when C_2 is largest and C_3 is smallest by replacing C_1 with C_3.

Step 3 (h): When C_3 is largest and C_1 is smallest, Calculate $VC_1C_3 = C_1 + ((C_3 - C_1)/z)$

and replace $C_1 = C_3 = VC_1C_3$.

Step 3 (i): Repeat Step 3 (h) when C_3 is largest and C_2 is smallest by replacing C_1 with C_2.

Step 3 (j): If $C_1 = C_2 = C_3 = C_4$ then the equipoise is achieved and the program ends, if no then go to Step 3 (a).

Once the Equipoise is attained, it meant that all retailers now have the same inventory cost for a product, no matter how many different costs a retailer had at preliminary stage. The retailer whose cost was greater than the equipoise earlier, this meant that he would be given compensation in terms of interest through trade credit facility offered by the supplier whereas the retailer whose cost was prior less than the equipoise, would automatically benefit from the increased equipoise cost.

Definition 5.1: (**Tenable Layout**). *A layout is tenable if the cost of the product for different retailers is same.*

Definition 5.2: (**CostPact**). *A pact between two retailers during the execution of RETProffer.*

Theorem 5.1: The layout **RETProffer** is without any stalemate point. This implies that there prevails some inventory cost in M_i considering the layout to be tenable.

Proof: Refer Appendix A.5.5

Theorem 5.2: The layout in due course will reach Equipoise.

Proof: Refer Appendix A.5.6.

To determine optimality of the replenishment schedule

To resolve the optimality of replenishment schedule, from the equation (5.7) of total cost of the retailer we will calculate $\frac{\partial R_I}{\partial t_i}$ and then put $\frac{\partial R_I}{\partial t_i} = 0 \; where \; i = 1, 2, 3, \ldots \ldots n-1$. Since we our working on multiple retailers set up, the optimality will be for all the retailers individually. We will be proving for R_{I2} and will be same for all the other retailers.

$$\frac{\partial R_{I2}}{\partial t_i} = h\{(\alpha_i - \alpha_{1+i}) * (e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * (\frac{1}{2k^3} b_1(2 - 2e^{k(-t_j+t_{1+j})} -$$
$$2kt_j + k^2 t_j^2 + 2e^{k(-t_j+t_{1+j})} k t_{1+j} - k^2 t_{1+j}^2) +$$
$$\frac{b_2(\frac{e^{-kt_j+t_{1+j}\alpha_{1+j}}(-e^{kt_j}+e^{kt_{1+j}})}{k} + \frac{e^{t_j\alpha_{1+j}} - e^{t_{1+j}\alpha_{1+j}}}{\alpha_{1+j}})}{k+\alpha_{1+j}}) + (e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) *$$
$$\frac{(-e^{k(-t_{-1+i}+t_i)}k + e^{k(-t_i+t_{1+i})}k)b_1}{k^3} +$$
$$\frac{b_1(-1+e^{k(-t_{-1+i}+t_i)} + e^{k(-t_{-1+i}+t_i)}kt_i - e^{k(-t_i+t_{1+i})}kt_{1+i})}{k^2} + \frac{e^{k(-t_{-1+i}+t_i)+t_i\alpha_i}b_2}{k+\alpha_i} +$$
$$\frac{e^{t_i\alpha_i}b_2(-\frac{1}{k} + \frac{e^{k(-t_{-1+i}+t_i)}}{k} - \frac{1}{\alpha_i})\alpha_i}{k+\alpha_i} + \frac{e^{t_i\alpha_{1+i}}b_2}{k+\alpha_{1+i}} - \frac{e^{k(-t_i+t_{1+i})+t_{1+i}\alpha_{1+i}}b_2}{k+\alpha_{1+i}}\} +$$
$$Y\{e^{\sum_{i=0}^{i-1} \alpha_{i+1}(t_{i+1}-t_i)} * (\alpha_i - \alpha_{i+1})\} + e^{\sum_{i=0}^{i} \alpha_{i+1}(t_{i+1}-t_i)} * (\alpha_i - \alpha_{i+1}) *$$
$$(b_1(\frac{e^{k(t_{i+1}-t_i)} * t_{i+1}}{k} - \frac{e^{k(t_{i+1}-t_i)}}{k^2} - \frac{t_i}{k} + \frac{1}{k^2}) + b_2(\frac{e^{\alpha_{i+1} * t_{i+1}} * e^{k(t_{i+1}-t_i)} - e^{t_i * \alpha_{i+1}}}{k+\alpha_{i+1}})) +$$
$$e^{\sum_{i=0}^{i} \alpha_{i+1}(t_{i+1}-t_i)} * (b_1(-\frac{1}{k} + \frac{e^{k(-t_i+t_{1+i})}}{k} - e^{k(-t_i+t_{1+i})} * t_{1+i}) +$$
$$\frac{b_2(-e^{k(-t_i+t_{1+i})+t_{1+i}*\alpha_{1+i}}k - e^{t_i*\alpha_{1+i}}\alpha_{1+i})}{k+\alpha_{1+i}}))\} = 0 \qquad (5.10)$$

By using Equation (5.10) the value of t_i's is obtained and finally we calculate total cost of the retailers.

Next, we prove, Optimality of the replenishment cycle in Theorem 5.3

Theorem 5.3: The unique solution to nonlinear system of equations (5.10) derived by obtaining first derivative of $R_{I2}$is the optimal replenishment cycle length for given n.

Proof: Refer Appendix A.5.8. for proof.

5.9 Numerical Examples

To illustrate our model, following example is considered taking into account the above-mentioned assumptions under Assumptions and Notations section.

Example: For four retailers: Given W = 3, k = 1.1, b_1 = 40, b_2 = 20, Y = 500, $\alpha_{1+i} = \alpha_i + 0.01$, i = 1, 2, 3, ….,8, LC = 30, MC = 30, c = 2, h_1 = 0.0375, h_2 = 0.05, h_3 = 0.0625, h_4 = 0.10

By applying algorithm 5.6, the values of the total cost of the retailer is obtained and is shown in the table 5.2 below:

Retailers	**Holding cost**	**n = 2**	**n = 3**	**n = 4**	**n = 5**	**n = 6**	**n = 7**
$\boldsymbol{R_{I1}}$	h_1 = 0.0375	6762.30	328.28	2835.13	3680.48	4594.77	5590.03
$\boldsymbol{R_{I2}}$	h_2 = 0.05	6761.65	327.89	2834.61	3679.49	4593.08	5587.36
$\boldsymbol{R_{I3}}$	h_3 = 0.0625	6760.34	327.10	2833.56	3677.51	4589.72	5582.04
$\boldsymbol{R_{I4}}$	h_4 = 0.10	6763.43	328.81	2836.87	3682.10	4595.56	5589.03

Table 5.2: Total cost of all four retailers with different holding cost

Table 5.2 depicts the total cost of four different retailers having variable holding cost. The cost is minimum when n = 3, that is the optimal replenishment interval is 3. Optimal cost is achieved in 3rd cycle and now using these values we will calculate the value of total cost of supplier (S_I) with respect to all the four retailers which is tabled under Table 5.3 below.

$\boldsymbol{SR_{I1}}$	$\boldsymbol{SR_{I2}}$	$\boldsymbol{SR_{I3}}$	$\boldsymbol{SR_{I4}}$
62397.9	62380	62344	62309.9

Table 5.3: Total cost of Supplier with respect to retailers.

Calculation of Equipoise Inventory Cost

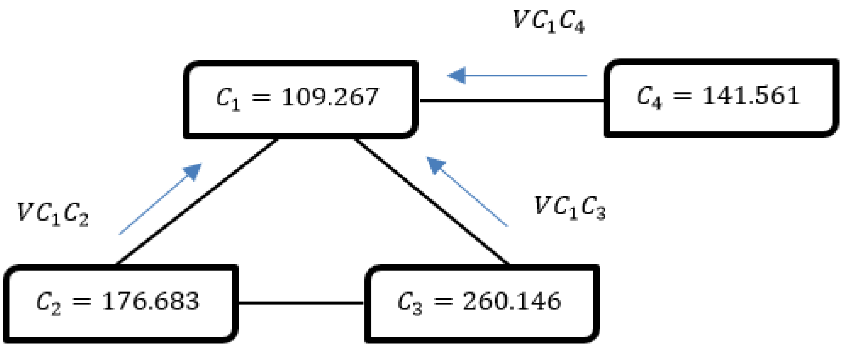


Figure 5.2: A specimen of RETProffer (Preliminary stage)

In accordance with Figure 5.2, VC_1C_2 means the proffer is made by C_2 to C_1. VC_1C_3 means the proffer is made by C_3 to C_1 and VC_1C_4 means the proffer is made by C_4 to C_1. C_1 is the smallest in the described system, thus all the nabe retailers are making proffer to C_1.

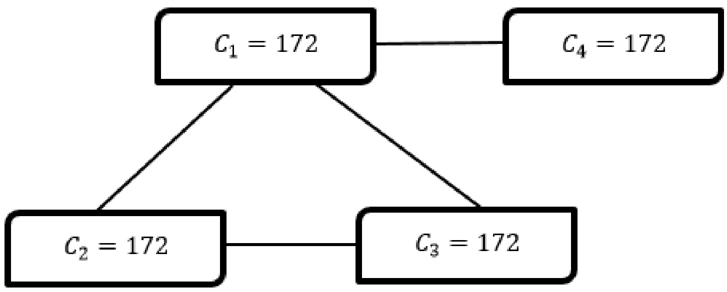


Figure 5.3: A specimen of RETProffer (Equipoise stage), when z = 2.

Since in Figure 5.3, $C_1 = C_2 = C_3 = C_4$, this implies that the equipoise has been achieved and the equipoise inventory cost when z = 2 is 172, which is greater than C_1 and C_4 (implying these two retailers has benefitted from the equipoise cost). The other two retailers, C_2 and C_3 have their equipoise lesser than their initial inventory cost, thus the supplier will now give credit period facility to these two retailers only.

Retailers	Total Quantity	Cost of retailer for 10 units of product	Equipoise Cost
$\boldsymbol{R_{I1}}$	259.625	$C_1 = 109.267$	172 (z = 2) 153 (z = 3) 146 (z = 4)
$\boldsymbol{R_{I2}}$	259.770	$C_2 = 176.683$	
$\boldsymbol{R_{I3}}$	259.917	$C_3 = 260.146$	
$\boldsymbol{R_{I4}}$	259.991	$C_4 = 141.561$	

Table 5.4: Calculation of Equipoise cost as explained above in Algorithm2

The last column of Table 4 shows the Equipoise inventory cost when z = 2, 3 and 4. As the value of z increases so the equipoise value is reducing, which implies inverse relation between the two.

Column 2 gives the total quantity of each of the retailers, Unit cost is calculated in column 3 by dividing values of total cost obtained in table 2 by dividing it with values of total quantity (column 2 of table 5.4).

5.10 Sensitivity Analysis

For sensitivity analysis out of four retailers, we are randomly choosing R_{I2} (as for all the other retailers process will be same). In this section Table and graphs will help to acknowledge relation among different parameters and the total cost of the retailer.

Parameters	%age change in parameters	%age change in R_I	Value of R_I	%age change in S_I	Value of S_I
Y	50	9.48	2948.41	-	-
	-50	-9.48	2437.37	-	-
	10	1.89	2743.99	-	-
	-10	-1.89	2641.79	-	-
W	50	40.63	3787.18	-	-
	-50	-40.63	1598.60	-	-
	10	8.12	2911.75	-	-
	-10	-8.12	2474.03	-	-
MC	50	-	-	25	77975.00
	-50	-	-	-25	46785.00
	10	-	-	5	65499.00
	-10	-	-	-5	59261.00
b_1	50	31.23	3533.90	30.21	833.17
	-50	-31.29	1850.21	-30.15	446.886
	10	6.24	2861.16	6.04	678.482
	-10	-6.25	2524.56	-6.03	601.199
b_2	50	9.19	2940.58	6.59	66493.20
	-50	-9.24	2443.91	-6.62	58245.10
	10	1.84	2742.52	1.32	63204.20
	-10	-1.84	2643.21	-1.32	61555.00
LC	50	-	-	25	77975.00
	-50	-	-	-25	46785.00
	10	-	-	5	65499.00
	-10	-	-	-5	59261.00
c	50	-	-	40.62	899.75
	-50	-	-	-40.62	379.917
	10	-	-	8.12	691.817
	-10	-	-	8.12	587.85

Table 5.5: Sensitivity Analysis of different parameters

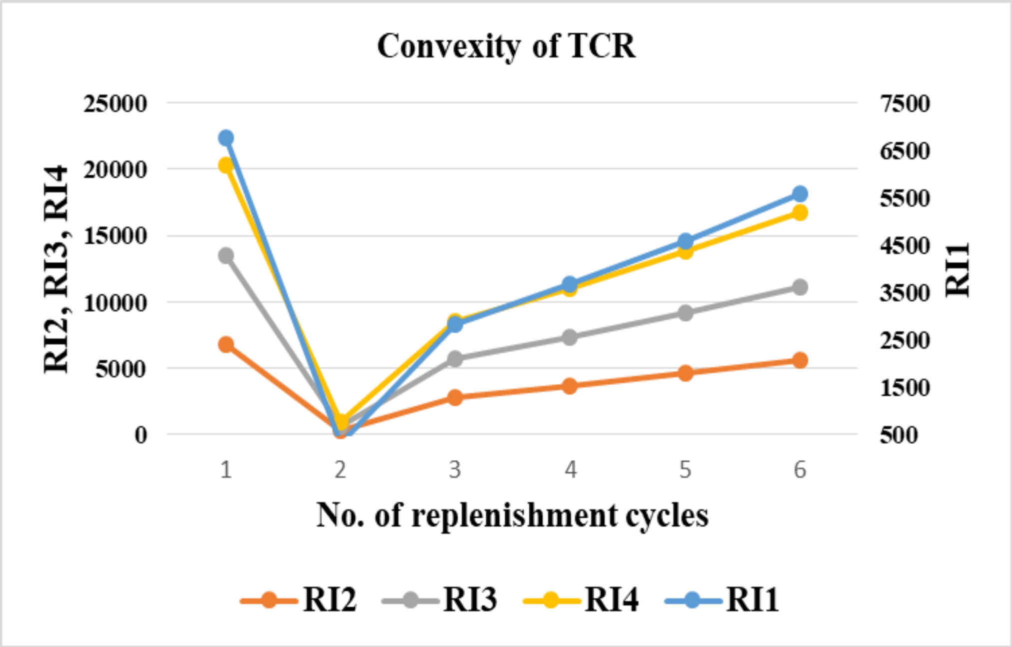


Figure 5.4. Graphical representation of Table 5.2

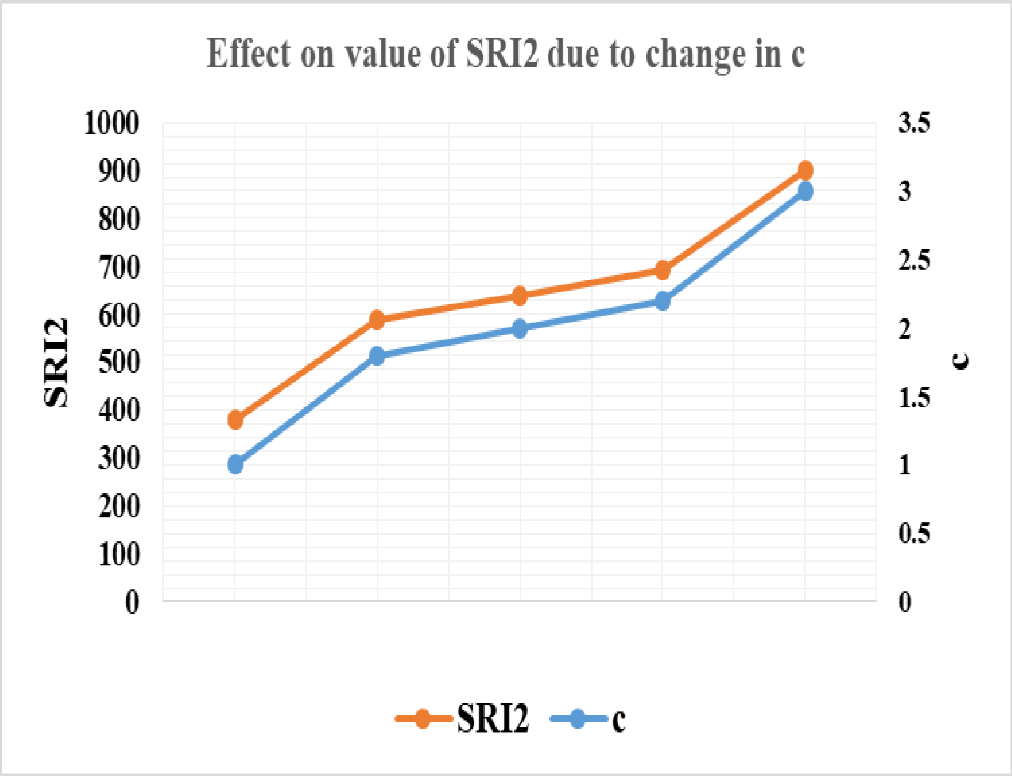


Figure 5.5. Effect on value of SR_{I2} due to change in c

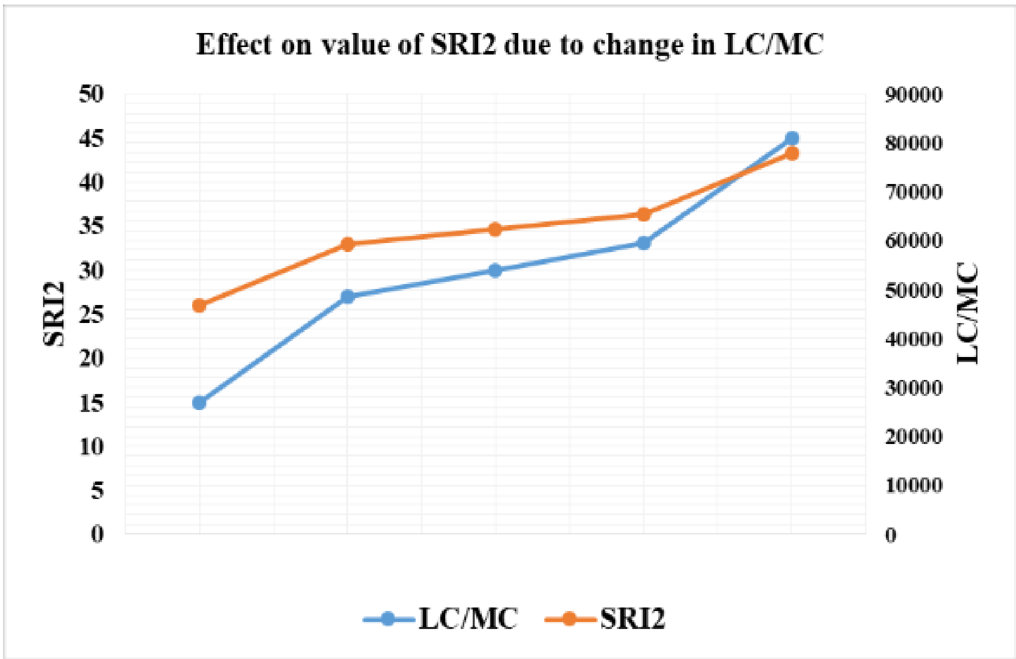


Figure 5.6. Effect on value of SR_{I2} due to change in LC/MC

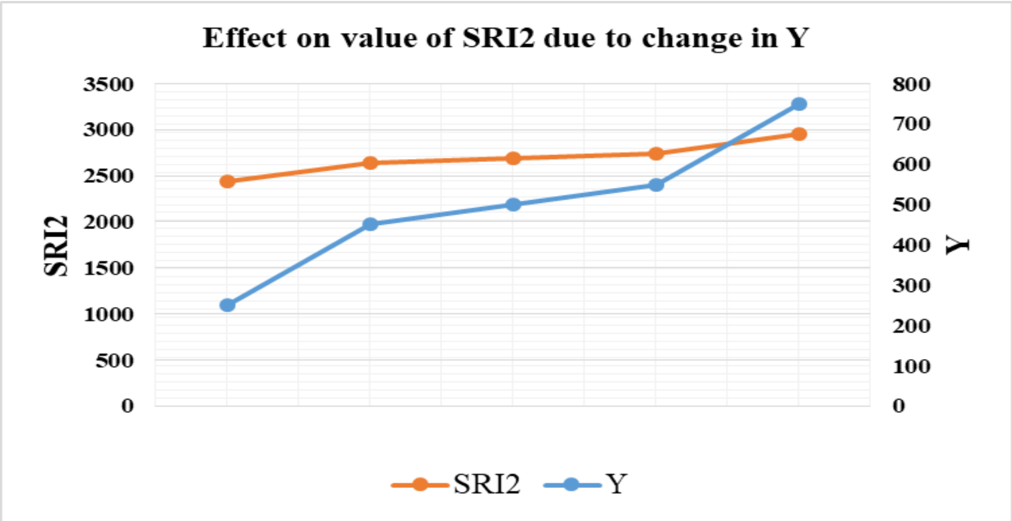


Figure 5.7. Effect on value of R_{I2} due to change in Y

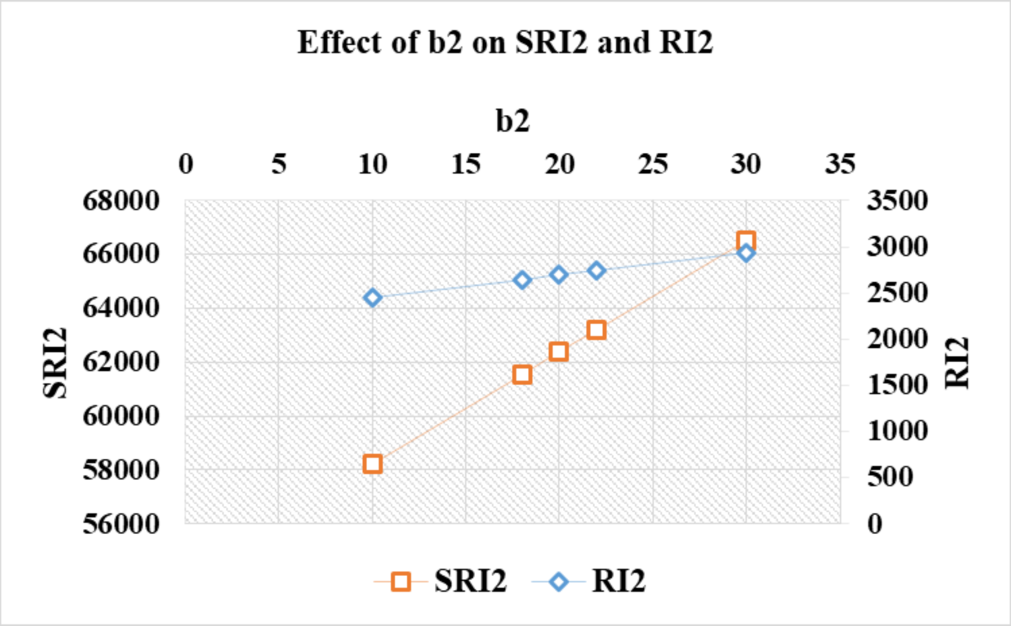


Figure 5.8. Effect on value of R_{I2} and SR_{I2} due to change in b_2

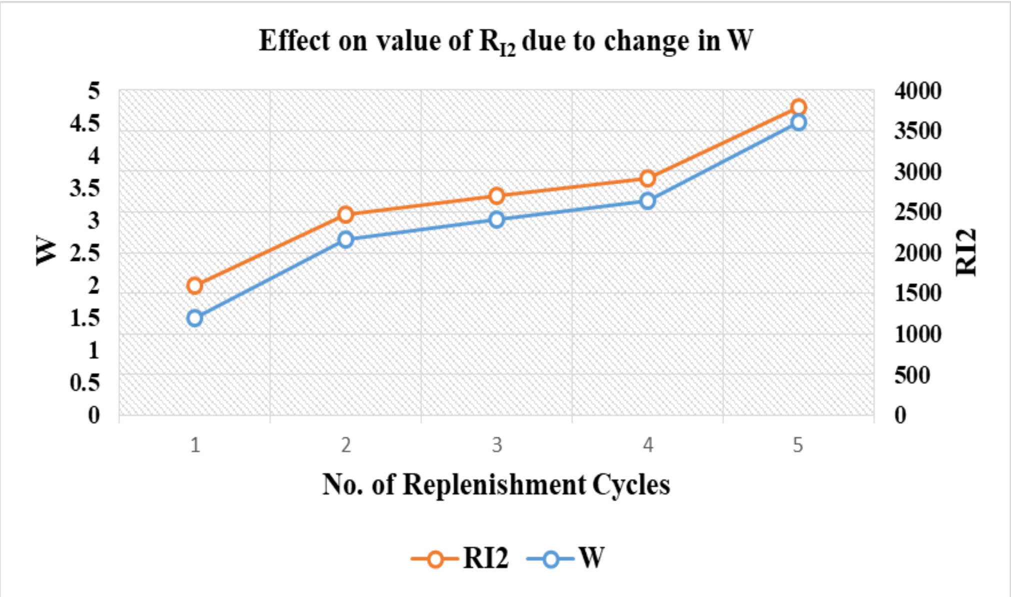


Figure 5.9. Effect on value of R_{I2} due to change in W

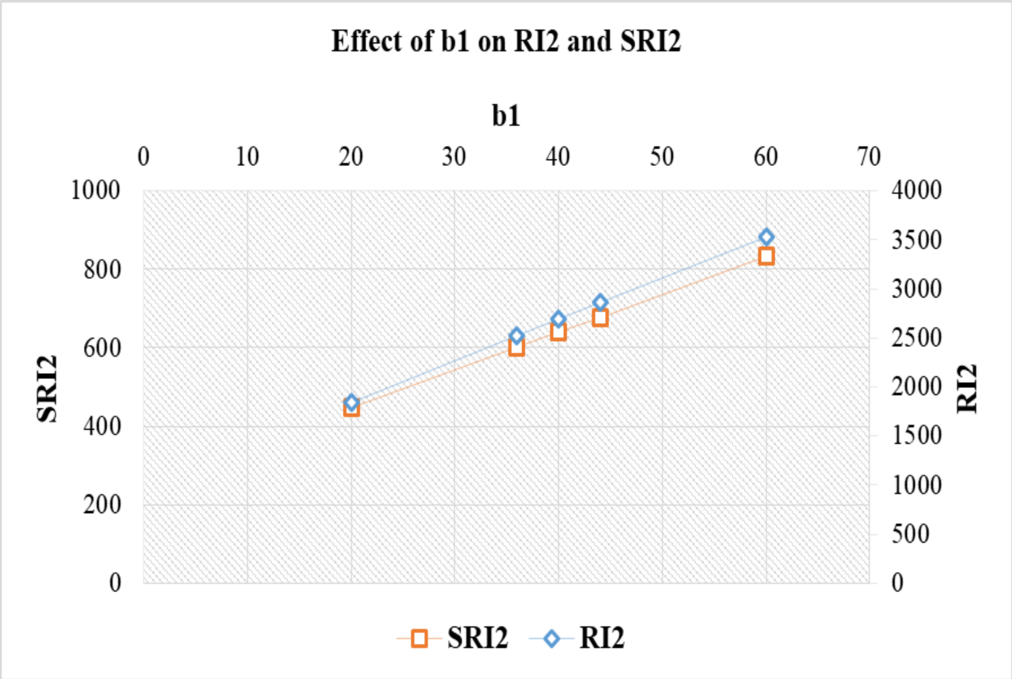


Figure 5.10. Effect on value of R_{I2} and SR_{I2} due to change in b_1

5.10.1. *Sensitivity Analysis Take Aways*

From table 5.5, following interpretations are made:

1. When there is an increase in the value of Y, there is increase in the value of total cost of retailer.
2. An increase in the value of W, has a positive relation with the value of total cost of retailer as well.
3. As the value of b_1 increases, so is the value of total cost of retailer and with a fall in the value of $b_{1,}$ retailer's cost experiences a fall.
4. It can be manifested that there is direct relation between b_1 and retailer's total cost.
5. The increase in the parameters of LC, MC and c increases the cost of the supplier.
6. The decrease in the parameters of LC, MC and c have direct relation with the total cost of the supplier.

Explanations of Figures contained in Sensitivity analysis section

1. In Figure 5.4, the total cost of 4 retailers as (depicted in table 4), is plotted to see how the values act. It demonstrates well that the values first decrease and then gradually increases. Giving convexity of the total cost of four different retailers.
2. The value of total cost of supplier SR_{I2} how it is affected by the change in values of c (purchasing cost of the supplier) is displayed by Figure 5.5. Both have direct relation.
3. The effect on total cost of supplier SR_{I2} by variations in the value of Labour Cost (LC) and Machinery Cost (MC) is demonstrated in Figure 5.6, as both these costs increases so is the cost of SR_{I2} also rises and vice versa.
4. More the ordering cost (Y) of the retailer more will be the total cost, this positive relation between Y and R_{I2} can be established from Figure 5.7.
5. Figure 5.8, shows the relation among R_{I2} and SR_{I2} with the scaling constant of inflation function (b_2). There is direct relation among the variables.
6. As the value of wholesale price for the retailer rises so is the total inventory of the retailer, this directly proportional relation between the two variables (R_{I2} and W) is seen in Figure 5.9.
7. The relation among R_{I2} and SR_{I2} with the increasing demand rate (b_1) is shown in Figure 5.10. The relation is directly proportional.

5.11. Comparative study with the existing model

Geetha and Udayakumar [5] (2020)	k 100	W -	c 0.75	h 0.155	R 0.2	a 500	b 0.5	TC 1423.30
Present paper When inflation is constant	Y 100	W 3	c 0.75	h 0.155	α 0.2	b_1 500	b_2 0.5	TC 1387.85
Present paper When inflation is variable	Y 100	W 3	c 0.75	h 0.155	Variable Linear rise in inflation	b_1 500	b_2 0.5	TC 1874.22

Table 5.6: A Comparative study with the existing model

Under this specific segment, we have made a comparison study with the pre – existing literature [5], and the findings are that if we take the inflation as constant (same as in [5]), in our model, we get the reduced total cost of the retailer. And we took variable inflation rate with linear rise in inflation in each replenishment cycle, the cost of the retailer increases. It thus implies that the because of variable rise in inflation the total inventory cost rises and so retailer needs more funds to bear the increased cost and thus they increase the cost of the product as well. This increased cost is then bought to Equipoise by following the Case2 of the present model.

5.12. Short note on how to handle inflation variability in FMCG sector considering the given model.

Inflation variability introduces challenges in cost forecasting, supplier coordination, and pricing strategies. Our model addresses these by:

1. **Dynamic Cost Adjustments**: The model dynamically adjusts retailer costs in each replenishment cycle based on real-time inflation trends.
2. **Strategic Credit Period Allocation**: By linking credit periods to equipoise cost, we ensure that inflation-induced cost burdens are evenly distributed.
3. **Predictive Sensitivity Analysis**: The model incorporates sensitivity analysis with variations of -10%, +10%, -50%, and +50% in inflation rates, enabling proactive decision-making for suppliers and retailers.
4. **Mathematical Proofs for Stability**: Theorems and lemmas are provided to establish the feasibility and optimality of our coordination model, ensuring its applicability in competitive market conditions.

5.13. Conclusions

Our research carefully studies the market changes caused by inflation. It’s observed that inflation changes randomly so we can't keep inflation same in the whole-time frame of the model. Keeping this vibrant nature of inflation as kernel of our research we created this near to market reality model. We considered a linear rise in inflation in each replenishment cycle, resulting in harnessing every minute swing in inflation.

Thereafter calculating the values of the total cost of four different retailers (as shown in table 2, by applying Algorithm 1, we used algorithm 2, to calculate the next part of the article that is Equipoise. Equipoise of the inventory cost of the retailers is calculated in coordination with the supplier. After attaining the Equipoise in the cost of a product, the profits will automatically flow towards the retailers in the form of new Equipoise cost (if Equipoise cost is greater than the earlier cost), if the case is reversed, the supplier will offer credit period to the remaining retailers. In this way, supplier too gains as he/she will now allow credit term to the limited retailers' only.

Theorem to prove that the layout under Case 2 is tenable is mentioned. Even the layout is free from any kind of stalemate point is also demonstrated in theorem 2 with the help of Lemma 1. Theorem 3 is proved to express the optimality of the total cost of the retailer. Sensitivity analysis has been done with -10%, +10%, -50% and +50%. The effect of this is explained with the help of figures as well and for complete comprehension, the sensitivity analysis take aways has been enumerated additionally.

The managerial stance, the new model will allow managers greater leeway in making decisions. The supplier should take necessary precautions especially in Case 2, as he / she will be solely responsible in managing the Equipoise. Trade credit period has been reduced considerably and the number of retailers to whom it will be given are also reduced, a profitable affair for supplier furthermore.

The inventory model of Equipoise is overriding all the factors which were affecting the inventory cost of the different retailers in the nabe environment to be towering high than each other. The equipoise cost will be a beneficial game for the consumer as well. Concluding we can affirm that the equipoise concept is profitable for all the contenders in the market.

The prospects for future research, we propose the following pointers.

(i). Shortages can be incorporated and partially backlogged.

(ii). Extension can be done taking into consideration fuzzy demand.

(iii). The case 2, can be enlarged for multiple supplier – multiple retailers.

Chapter 6

Fuzzification under Finite planning horizon for deteriorating items and seasonal demand

Abstract: Seasonal products play a crucial role in the Fast-Moving Consumer Goods (FMCG) industry, which is recognized as the fastest-growing sector encompassing a wide range of products, including electronics, electrical goods, computer accessories, fashion items, and perishable food items—essentially, any goods that are prone to deterioration or quick obsolescence. This article examines the ramp-type demand that characterizes these products, highlighting their inherent obsolescence and limited usability tied to specific seasons. Within the context of the FMCG sector, strategic planning accounted for a finite horizon, as seasonal products are not only time-sensitive but also subject to deterioration, necessitating agile inventory and demand management practices. In the inventory model, both the deterioration rate and the percentage of defective products within the total quantity are treated as fuzzy variables. Trapezoidal fuzzy numbers are employed for the fuzzification process, and the subsequent defuzzification of the numerical model is carried out using the sign distance method. The article concludes with a summary of findings, an outline of future research possibilities, and managerial insights.

6.1 Introduction

Various researchers have extended the traditional EOQ and EPQ models to incorporate new parameters to better reflects real – world scenarios. These enhancements address factors such as product deterioration, varying demand types, fuzzy set theory, and seasonal demand along with finite planning horizon. In today's interconnected business environment, collaboration is essential for survival; no entity—be it a supplier, or retailer can thrive in isolation. By fostering coordination and understanding local market dynamics prior to inventory accumulation, can significantly boost profits, minimize losses, address customer complaints, effectively manage seasonal demand, and even explore alternative markets. Effective coordination can stabilize the market and can also alleviate financial constraints for all parties involved.

This impact is first evident in the types of inventories held by suppliers and retailers, as consumer demand shifts in response to seasonal changes. Consequently, the study of seasonal demand becomes crucial. This demand is not solely dictated by the four seasons but can also fluctuate on a monthly or weekly basis. For instance, the beginning of a school term can spike the demand for stationery, while certain products may be in higher demand on specific days of the week due to cultural practices. Conversely, some products may see negligible demand on certain days, such as meat consumption in India on days. The consumer-packaged goods market is highly diverse, encompassing various factors that coexist in the market, including lost sales, shortages, finite/infinite replenishment cycles, seasonal demand, inventory management, inflation, and fuzzy or crisp environments. These goods, characterized by quick turnover and popularity on e-commerce platforms, also face issues like obsolescence and cater to a broad spectrum of consumers, from urban shoppers to rural populations. Effective coordination between suppliers and retailers in the CPG sector is crucial for minimizing holding costs, anticipating seasonal demand shifts, and finding new markets for surplus inventory as seasons change.

6.2 Literature Review

The impact of inventory deterioration was pioneered by Ghare and Schrader [41], whose model was basic, and subsequent researchers expanded on their work. In 1973, Covert and Philip [42] developed an EOQ model that accounted for variable deterioration rates, using the Weibull distribution and other conditions such as no shortages, constant demand, and instantaneous supply. Raafat [31] provided a comprehensive review in 1991, defining deterioration as decay, spoilage, or physical depletion. Building on this, Goyal and Giri [32] conducted a review in 2001 of the progress made in deteriorating inventory research since Raafat's survey, classifying inventory items into three main groups: those that are obsolete, those that deteriorate, and those that do neither. In 2012, Bakker et al. [33] expanded on by incorporating price discounts, lost sales, single and multiple items, payment delays, and one or two warehouses.

In 2009, Skouri et. Al. [65] highlighted the fluctuating nature of demand for seasonal products, which can be characterised by three distant time periods: an initial increase, a period of steady demand, and a subsequent decrease, reflecting a ramp-type demand pattern. This pattern is particularly relevant for categories such as Fast-Moving Consumer Goods (FMCG), Consumer Packaged Goods, seasonal items, and fashionable products. The concept of ramp-type demand was initially introduced by Hill in 1995 [66], and since then, various researchers have expanded upon his model by incorporating additional parameters. In 2011, Skouri et. Al. [67] incorporated deterioration to the ramp-type demand model. Lin et al. [68] explored ramp-type demand in 2012, considering a stock-dependent consumption rate, simplifying the work of Deng et al. [69]. The latter's research in 2012 addressed the inconclusive findings of Mandal and Pal (1998) [70] and Wu and Ouyang (2000) [71]. Panda et al. [72] advanced the field in 2013 by examining perishable items with ramp-type demand that is both price and time-dependent. Saha [73] introduced a quadratic ramp-type demand model in 2014. In 2017, Chandra [77] presented a model featuring ramp-type demand with time-dependent holding costs and discounts offered for backorders, where demand was both ramp-type and a linear function of time. Finally, in 2018, Singh et al. [78] developed a ramp-type demand model considering time-dependent deteriorating items with completely backlogged shortages.

The products available seasonally, along with fashionable items, FMCG, newly launched products, and electronic items exhibit a ramp-type demand pattern. Singh et al. [48] proposed an EOQ model in 2017 for a finite planning horizon, considering both scenarios with and without payment delays. In 2018, Singh et al. [49] introduced an inventory model within a finite planning horizon that incorporates trade credit. The following year, Singh et al. [50] expanded their model to include trade credit policies in both centralized and decentralized planning horizons. Shaikh et al. [79] focused in 2020 on ramp-type demand for deteriorating inventory, emphasizing preservation and trade credit facilities. Jaggi et al. [30] discussed in 2018 the necessity of a centralized policy for consumer goods with displayed stocks to enhance the joint profits of suppliers and retailers, although the benefits tend to skew towards the retailer. Li et al. [31] explored in 2020 the optimization of inventory for fast-moving consumer products by replenishing and salvaging stocks at the start of each cycle, with a particular emphasis on leveraging e-commerce platforms to market these consumer-packaged goods. In their 2021, Barman et al. [97] introduced variable and cloud fuzzy demand rates along with considerations for inflation.

6.3 Defining the Problem

In this article, we tackle the challenge for inventory management for deteriorating items along with defective products. The fuzzy logic is applied to these two variables – deterioration rate (θ) and defective percentage (k) and then defuzzified to analyse their impact on the total cost. The results show that the defuzzified cost is lower than the original total cost, which demonstrates the effectiveness and worthiness of the proposed model which suggests better profitability and cost savings in FMCG industry.

6.4 Assumptions and notations

The article is based on the below-mentioned assumptions. The notations used in the paper are also listed after the assumptions in the form of separate subsection.

6.4.1 Assumptions

- The demand rate is ramp type.
- Lead time is zero. Incorporating a zero lead time assumption in the proposed model enhances its applicability to the FMCG industry by ensuring a highly responsive and cost-efficient inventory system.
- No shortages are taken as in case of FMCG, customers have lot of choices and mostly goes for substitutes and those who wait for the product are negligible.
- Deterioration rate is taken.
- The planning horizon – finite and replenishment cycles are of unequal length.
- Seasonal products of FMCG are taken into frame.
- One supplier and one retailer are taken for the framing of the model.

- Replenishment of stocks/inventory is instant or instantaneous.
- Defective products result from imperfect manufacturing and worker handling, with the defective percentage (k) considered as an interval trapezoidal fuzzy number.
- The deterioration rate is considered a trapezoidal fuzzy number.

6.4.2 Notations

Y = Ordering cost per unit.

W = wholesale cost per unit (for the retailer and W > c).

c = purchasing cost per unit (for the supplier).

h = holding cost per unit.

s = selling price per unit.

K = Defective percentage

θ = Deterioration rate

θ_i = Fuzzy deterioration rate where i=1,2,3,4

n = number of replenishment cycles.

R(t) = Ramp type demand which is a function of time t and

R(t) = D_0{t-(t-μ)H(t-μ)} > 0, moreover $D_0 > 0$

where H(t-μ) is the Heaviside's function which further follows the below identity:

$$H(t-\mu) = \begin{cases} 1, t \geq \mu \\ 0. t < \mu \end{cases}$$

Thus,

$$R(t) = \begin{cases} D_0 t(1-k), & t_i \leq t \leq \mu \\ D_0 \mu(1-k), & \mu \leq t \leq t_{i+1} \end{cases}$$

S_{i+1} = inventory level during (i + 1)th cycle at the time t_i without coordination.

R_I = Retailer's total cost.

S_I = Supplier's total cost.

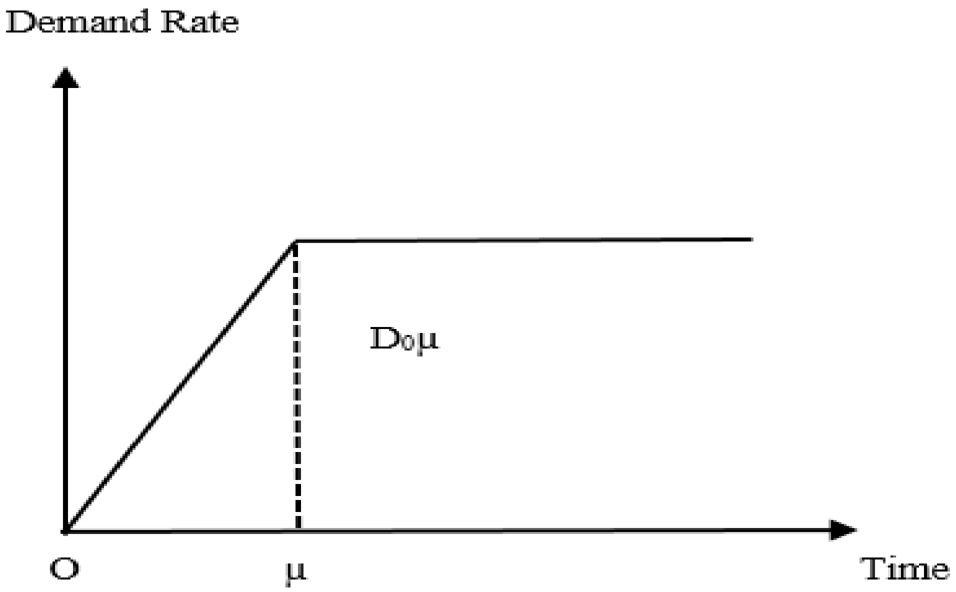


Figure 6.1: Ramp Type Demand, where demand increases from O to μ and then becomes constant.

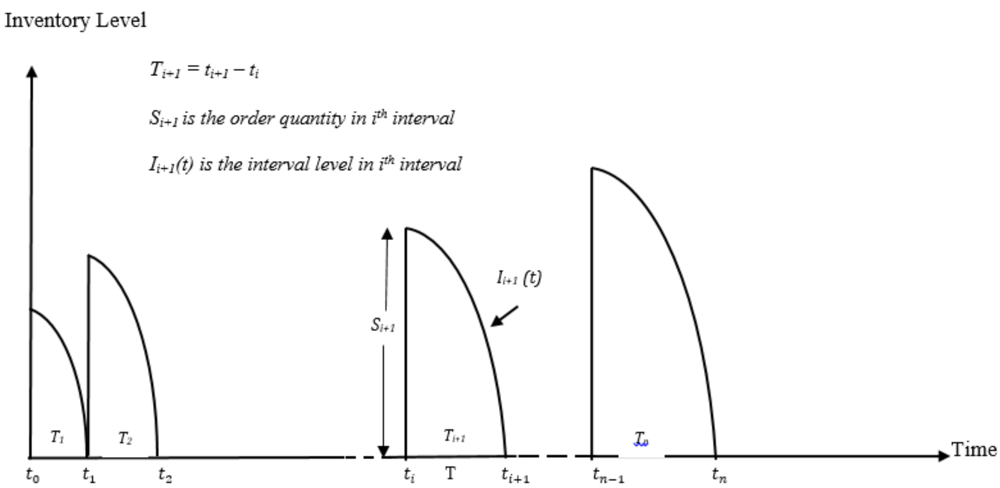


Figure 6.2: Graphical representation of the system discussed in the article.

6.5 The Mathematical Formulation of the model

Here

$$R(t) = \begin{cases} D_0 t(1-k), & t_i \le t \le \mu \\ D_0 \mu(1-k), & \mu \le t \le t_{i+1} \end{cases}$$

The differential equation of the inventory level in (i+1) th cycle will be given as follows:

$$\frac{dI_{i+1}(t)}{dt} + (\theta)I_{i+1}(t) = = -R(t),\ t_i \le t \le t_{i+1} \quad (6.1)$$

Boundary conditions are $I_i\ (t_{i+1}) = 0$ and $I_i\ (t_i) = S_i$

The above equation (6.1) from demand function R(t) can be further reduced to

$$\frac{dI_{i+1}(t)}{dt} + (\theta)I_{i+1}(t) = = -(1-\mathrm{k})D_0 t, \qquad t_i \le t \le \mu \quad (6.2)$$

$$\frac{dI_{i+1}(t)}{dt} + (\theta)I_{i+1}(t) = = -(1-\mathrm{k})D_o \mu, \qquad \mu \le t \le t_{i+1} \quad (6.3)$$

Solving the equation (6.2) we will get:

$$I_{i+1}(t) = S_{i+1} - (1-\mathrm{k})\frac{D_0}{\theta^2}(e^{\theta t}(-1+t\theta) + e^{\theta t_i}(1-\theta t_i) \quad (6.4)$$

For solution Refer Appendix A.6.1

Solving the equation (6.3) we will get:

$$I_{i+1}(t) = \frac{(1-\mathrm{k})D_0\mu}{\theta}\left[e^{\theta(t_{i+1}-t)} - 1\right] \quad (6.5)$$

From (6.4) and (6.5)

$$S_{i+1} = (1-\mathrm{k})\frac{D_0\mu}{\theta}\left[e^{\theta(t_{i+1}-t)} - 1\right] + (1-\mathrm{k})\frac{D_0}{\theta^2}((-1+t\theta) + e^{\theta(t_i-t)}(1-\theta t_i)$$

and when $t = \mu$,

$$S_{i+1} = \frac{(1-\mathrm{k})D_0\mu}{\theta}\left[e^{\theta(t_{i+1}-\mu)} - 1\right] + (1-\mathrm{k})\frac{D_0}{\theta^2}((-1+\mu\theta) + e^{\theta(t_i-\mu)}(1-\theta t_i)$$

Now,

(a) The ordering cost/ cycle = Y

(b) The purchase cost/ cycle = cS_{i+1} (for Supplier) and $W*S_{i+1}$ (for Retailer)

(c) the holding cost/cycle is $h\int_{t_i}^{t_{i+1}} I_{i+1}(t)dt$

$=h\int_{t_i}^{\mu} I_{i+1}dt + \int_{\mu}^{t_{i+1}} I_{i+1}dt$

$$= h\left\{\left(\frac{D_0\mu(1-k)}{\theta^2}\left(1-e^{\theta(t_{i+1}-\mu)}\right)+\theta(t_{i+1}-\mu)\right)+\left(S_{i+1}(\mu-t_i)-\frac{D_0(1-k)}{\theta^2}\left(\frac{\theta(\mu^2-t_i^2)}{2}-\frac{1-\theta t_i}{\theta}\left(e^{\theta(t_i-\mu)}-1\right)\right)\right\} \quad (6.6)$$

Now, the retailer's cost during the finite planning horizon = ordering cost + purchasing cost + inventory holding cost

$$R_I = n*Y + W*\sum_{i=0}^{n-1} S_{i+1} + h\left\{\left(\frac{D_0\mu(1-k)}{\theta^2}\left(1-e^{\theta(t_{i+1}-\mu)}\right)+\theta(t_{i+1}-\mu)\right)+\left(S_{i+1}(\mu-t_i)-\frac{D_0(1-k)}{\theta^2}\left(\frac{\theta(\mu^2-t_i^2)}{2}-\frac{1-\theta t_i}{\theta}\left(e^{\theta(t_i-\mu)}-1\right)\right)\right\}$$

$$R_I = n*Y + W*\sum_{i=0}^{n-1}\frac{(1-k)D_0\mu}{\theta}\left[e^{\theta(t_{i+1}-\mu)}-1\right]+(1-k)\frac{D_0}{\theta^2}((-1+\mu\theta)+e^{\theta(t_i-\mu)}(1-\theta t_i) + h\left\{\left(\frac{D_0\mu(1-k)}{\theta^2}\left(1-e^{\theta(t_{i+1}-\mu)}\right)+\theta(t_{i+1}-\mu)\right)+\left(S_{i+1}(\mu-t_i)-\frac{D_0(1-k)}{\theta^2}\left(\frac{\theta(\mu^2-t_i^2)}{2}-\frac{1-\theta t_i}{\theta}\left(e^{\theta(t_i-\mu)}-1\right)\right)\right\}$$

$$R_I = n*Y + W*\sum_{i=0}^{n-1}\frac{(1-k)D_0\mu}{\theta}\left[e^{\theta(t_{i+1}-\mu)}-1\right]+(1-k)\frac{D_0}{\theta^2}((-1+\mu\theta)+e^{\theta(t_i-\mu)}(1-\theta t_i)+h\left\{\left(\frac{D_0\mu(1-k)}{\theta^2}\left(1-e^{\theta(t_{i+1}-\mu)}\right)+\theta(t_{i+1}-\mu)\right)+\left(\frac{(1-k)D_0\mu}{\theta}\left[e^{\theta(t_{i+1}-\mu)}-1\right]+(1-k)\frac{D_0}{\theta^2}((-1+\mu\theta)+e^{\theta(t_i-\mu)}(1-\theta t_i)*(\mu-t_i)\right)-\frac{D_0(1-k)}{\theta^2}\left(\frac{\theta(\mu^2-t_i^2)}{2}-\frac{1-\theta t_i}{\theta}\left(e^{\theta(t_i-\mu)}-1\right)\right)\right\} \quad (6.7)$$

$$\frac{dR_I}{dt_i} = W\left(\frac{(1-k)D_0}{\theta^2}\left[-\theta^2 t_i e^{\theta(t_i-\mu)} + \left(-1+\mu\theta+e^{\theta(t_i-\mu)}(1-\theta t_i)\right) - (\mu - t_i)\theta^2 t_i e^{\theta(t_i-\mu)} + \theta t_i + \left(e^{\theta(t_i-\mu)} - 1\right) - (1 - \theta t_i)e^{\theta(t_i-\mu)}\right]\right)$$

Under this situation, the supplier's cost will be directly related to the behaviour of replenishment policy of the retailer and thus, the supplier's cost will be sum of manufacturing cost and set up cost. The set-up is further sum of labour cost (LC) and machinery set up cost (MC).

$$S_I = n * (LC + MC) + c * \sum_{i=0}^{n-1} S_{i+1} \quad (6.8)$$

Also,

$S_{i+1} = \sum_{i=0}^{n-1} S_{i+1}$ is the order quantity during the finite planning horizon. (6.9)

6.6 Fuzzification of the Model

Precision in determining values for established parameters in inventory management poses a challenge for decision-makers, introducing uncertainty into crucial factors. Henceforth, the degradation rate (θ) and the percentage of defects in quantity (k) are considered as trapezoidal fuzzy interval types. The operations of fuzzy arithmetic for trapezoidal fuzzy numbers are briefly introduced, drawing from the works of Chen (1985), Mahata & Goswami (2013), and (Ranu Singh and Vinod Kumar Mishra, 2023). Expanding upon these fundamental definitions and findings, the model under consideration is then subjected to fuzzification. Let $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3, \tilde{\theta}_4)$ and $(\tilde{k}_1, \tilde{k}_2, \tilde{k}_3, \tilde{k}_4)$ represent trapezoidal fuzzy numbers, as illustrated in Figure 6.3. Subsequently, the crisp total average cost function is transformed into a fuzzy cost function. Since both the degradation rate (θ) and the defective percentage in quantity (k) are represented as trapezoidal fuzzy numbers, the total cost R_I is also considered a trapezoidal fuzzy number.

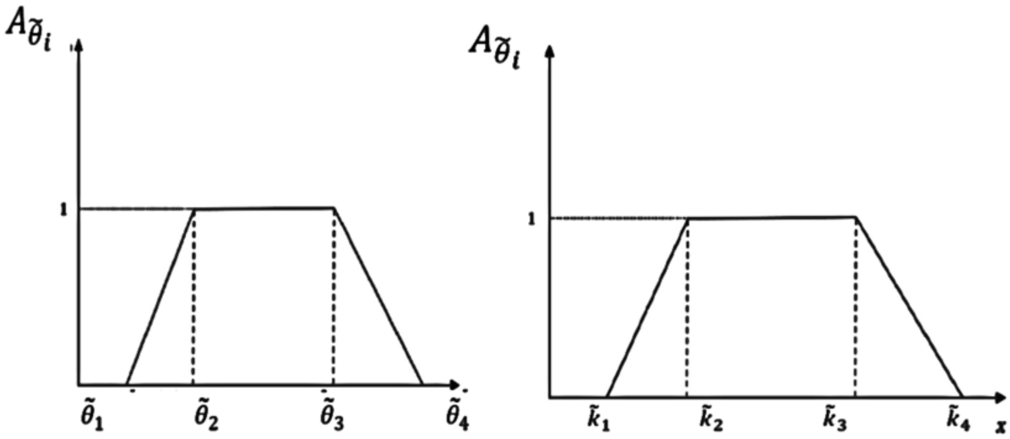


Figure 6.3: Trapezoidal fuzzy numbers for θ and k where θ is deterioration rate and k are defective percentage in quantity.

$$R_I = (\tilde{R}_{I1}, \tilde{R}_{I2}, \tilde{R}_{I3}, \tilde{R}_{I4}) \quad (6.10)$$

where

$$\tilde{R}_{Ii} = n*Y + W*\sum_{i=0}^{n-1}\frac{(1-\tilde{k}_{5-i})D_0\mu}{\tilde{\theta}_{5-i}}\left[e^{\tilde{\theta}_i(t_{i+1}-\mu)}-1\right] + (1-\tilde{k}_i)\frac{D_0}{\widetilde{\theta^2_{5-i}}}\left(\left(-1+\mu\tilde{\theta}_i\right)+e^{\tilde{\theta}_i(t_i-\mu)}\left(1-\tilde{\theta}_i t_i\right)+h\left\{\left(\frac{D_0\mu(1-\tilde{k}_i)}{\widetilde{\theta^2_{5-i}}}\left(1-e^{\tilde{\theta}_i(t_{i+1}-\mu)}\right)+\tilde{\theta}_i(t_{i+1}-\mu)\right)+\left(\frac{(1-\tilde{k}_i)D_0\mu}{\tilde{\theta}_i}\left[e^{\tilde{\theta}_i(t_{i+1}-\mu)}-1\right]+(1-\tilde{k}_i)\frac{D_0}{\widetilde{\theta^2_{5-i}}}\left(\left(-1+\mu\tilde{\theta}_i\right)+e^{\tilde{\theta}_i(t_i-\mu)}\left(1-\tilde{\theta}_i t_i\right)*(\mu-t_i)\right)-\frac{D_0(1-\tilde{k}_i)}{\widetilde{\theta^2_{5-i}}}\left(\frac{\theta(\mu^2-t_i^2)}{2}-\frac{1-\tilde{\theta}_i t_i}{\tilde{\theta}_i}\left(e^{\tilde{\theta}_i(t_i-\mu)}-1\right)\right)\right\} \quad (6.11)$$

Further, the sign distance method is used to defuzzify the fuzzy cost function ($\tilde{R}_{Ii}$). Thus,

$$R_{Id} = \frac{1}{4}\left(\tilde{R}_{I1}+\tilde{R}_{I2}+\tilde{R}_{I3}+\tilde{R}_{I4}\right) \quad (6.12)$$

For the optimality of the equation (6.12), we differentiate Equation (6.12) w.r.t. t_i and then putting it to zero. That is:

$$\frac{\partial \tilde{R}_{Id}}{\partial t_i} = W\left(\frac{(1-\tilde{k}_i)D_0\mu}{\tilde{\theta}_i}\left[-\widetilde{\theta_{5-\iota}^2}t_i e^{\tilde{\theta}_i(t_i-\mu)} + \left(-1+\mu\tilde{\theta}_i + e^{\tilde{\theta}_i(t_i-\mu)}(1-\tilde{\theta}_i t_i)\right) - (\mu - t_i)\widetilde{\theta_{5-\iota}^2}t_i e^{\tilde{\theta}_i(t_i-\mu)} + \tilde{\theta}_i t_i + \left(e^{\tilde{\theta}_i(t_i-\mu)} - 1\right) - (1-\tilde{\theta}_i t_i)e^{\tilde{\theta}_i(t_i-\mu)}\right]\right) \quad (6.13)$$

6.7 Numerical Example

Given W= 3, Y= 500, θ= 0.1, k= 0.35, D_0= 600, $\tilde{k}_1 = 0.30, \tilde{k}_2 = 0.32, \tilde{k}_3 = 0.34, \tilde{k}_4 = 0.36$, $\tilde{\theta}_1$ =0.01, $\tilde{\theta}_2 = 0.02, \tilde{\theta}_3 = 0.03, \tilde{\theta}_4 = 0.04$, c = 2, h = 0.0375, LC = 30, MC =30, the value of $t_i's, R_I, S_I and\ R_{Id}$ are calculated by using equations (6.7) and (6.8) and then values of R_{id} is calculated by using equation (6.12) with the help of Mathematica 12.0

t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
0	0.89	3.64	9.98	12.40	24.85	45.23	120

Table 6.1: Value of t_i ‘s

Costs	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7
$\boldsymbol{R_I}$	5154.64	5147.78	5144.36	5172.25	5189.65	5198.93
$\boldsymbol{S_I}$	3757.48	3754.93	3627.67	3723.42	3738.81	3776.90
R_{Id}	4145.21	4144.44	4142.01	4179.99	4187.62	4190.09
$\boldsymbol{R_I - R_{Id}}$	1009.43	1003.34	1002.35	992.26	1002.03	1008.84

Table 6.2: Values of R_I, S_I and R_{Id}

The last row of table 6.2 shows the improvement in cost of retailer after defuzzification of the model.

6.8 Graphical Representation

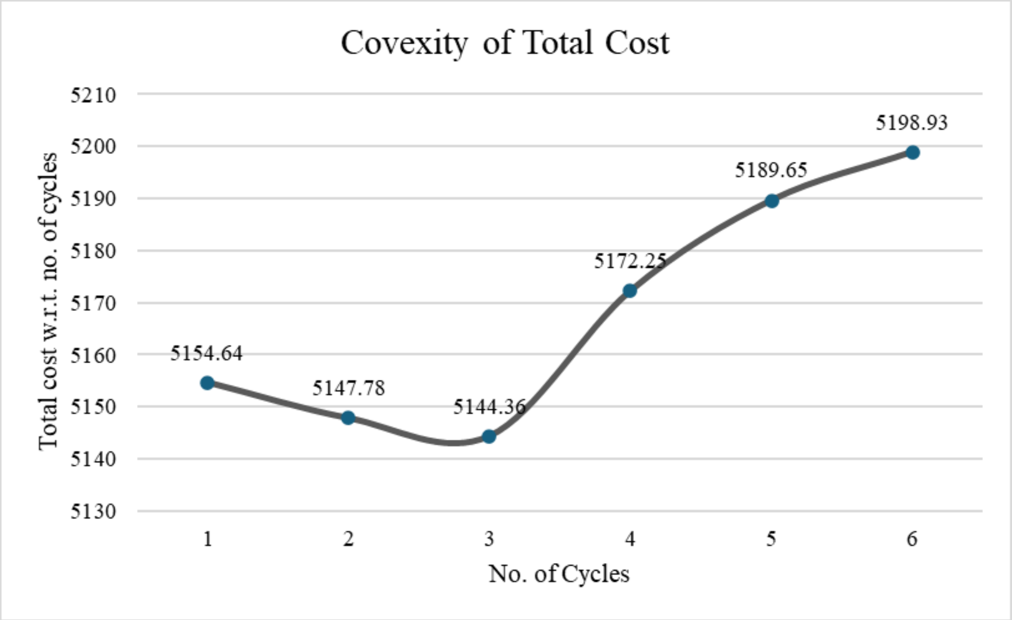


Figure 6.4: convexity of total cost of retailer.

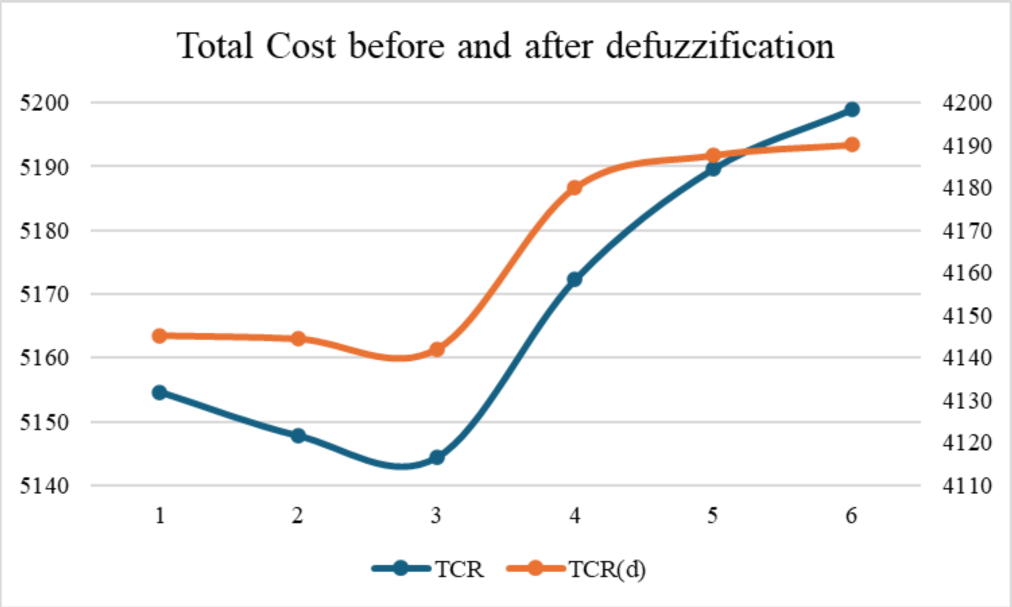


Figure 6.5: total cost of retailer before and after defuzzification.

6.9 Conclusion

The article provides valuable insights into products that experience deterioration over their shelf life, which can manifest as direct decay or physical spoilage. It highlights that all types of inventories are subject to deterioration for various reasons. Additionally, obsolescence is considered a form of deterioration, particularly relevant to consumer-packaged goods and fast-moving consumer goods (FMCG).

The article focuses on the ramp-type demand observed in seasonal products within the FMCG industry. It addresses the separation of imperfect or defective goods from inventory and proceeds to fuzzify the deterioration rate and the percentage of defective products. The findings indicate that incorporating fuzzy variables leads to a significant reduction in costs, thereby increasing the overall profitability of the inventory chain management system in FMCG.

For future research, the article suggests expanding the model by fuzzifying other variables and analysing their impact on the total cost of the system. This approach aims to provide masnagers in FMCG firms and the industry with additional strategies to enhance their profits.

Chapter 7

Conclusion and Future Scope

7.1 Concluding Remarks

The development of “Inventory Models with Trade Credit for Fast-Moving Consumer Goods (FMCG) in a Finite Planning Horizon” was driven by the necessity to create models that closely mirror the realities of the FMCG industry. Given the rapid turnover, perishability, and high sensitivity of these goods to inflationary conditions and seasonal fluctuations, the proposed models provide a robust framework for optimizing inventory costs and trade credit policies while maximizing profitability.

The key contributions of this thesis are structured across six chapters:

- **Chapter 1** introduces the fundamental concepts of operations research and inventory management, establishing the theoretical foundation for the study.
- **Chapter 2** provides an extensive literature review, identifies research gaps, and formulates research objectives, ensuring that the study builds upon existing work while addressing unexplored areas.
- **Chapter 3** develops an inventory model that incorporates deteriorating goods and trade credit, emphasizing retailer cost minimization. The analysis demonstrates an inverse relationship between the retailer’s interest earnings and total inventory cost, making this model particularly relevant for highly perishable FMCG categories such as dairy, fresh produce, and bakery items.
- **Chapter 4** extends the supplier-retailer coordination framework by incorporating seasonal ramp demand and trade credit considerations. The results confirm that strategic coordination benefits both suppliers and retailers, particularly in industries where demand surges during specific periods, such as beverages, fashion apparel, and personal care products.
- **Chapter 5** takes up the variable nature of inflation along with coordination of supplier and retailer for inventory and time dependent demand. Rise in

inflation is considered linear along with change in inflation rate in every replenishment cycle. Cycle lengths for inventory replenishment are unequal within a finite horizon of supply chain management of FMCG industry. Mathematical model of theoretical portion is solved by using algorithm taking the help of Mathematica 12.0. The cost of four different retailer is calculated in 1st part of the chapter. Then by applying next algorithm equipoise cost of four retailers were attained. The trade credit and coordination were given a new dimension by using it at both ends – the supplier and the retailer. Since inflation rate is different and according to the market in every replenishment cycle thus the model presented in the chapter is capable in addressing almost every substantiable market problem of FMCG/ CPG industry.

- **Chapter 6** explores inventory models under uncertainty by introducing fuzzified deterioration rates and defective product considerations. The findings suggest that incorporating fuzzy optimization techniques reduces total costs while enhancing profitability, making the approach particularly relevant for quality-sensitive FMCG segments such as pharmaceuticals and cosmetics.

  The comprehensive findings of this thesis affirm that the proposed inventory models offer mathematically rigorous and practically viable solutions for real-world challenges in FMCG supply chain management. By leveraging advanced optimization techniques and integrating trade credit mechanisms, these models provide enhanced decision-making tools for both suppliers and retailers.

7.2 Applications in the FMCG Industry

The results of this research have direct applications across multiple FMCG sectors, including:

1. **Grocery and Packaged Foods:** The variable inflation model helps manage cost fluctuations in staple items like grains, dairy, and processed foods, where prices are highly sensitive to supply chain disruptions.
2. **Pharmaceuticals and Healthcare:** The fuzzy model for defective products can be applied to minimize losses in medical supplies and over-the-counter drugs,

where even minor defects can lead to significant regulatory and financial consequences.

3. **Personal Care and Cosmetics:** The deterioration-based model supports efficient inventory turnover for beauty products, which often have expiration dates and quality degradation concerns.
4. **Beverages and Seasonal Products:** The seasonal demand model is crucial for managing stock levels in industries with peak consumption periods, such as cold beverages in summer and chocolates during festive seasons.
5. **Cleaning and Hygiene Products:** The credit-based coordination model ensures that suppliers and retailers maintain competitive pricing for essential household products, allowing stable inventory replenishment despite economic fluctuations.

By implementing these models, FMCG firms can enhance supply chain efficiency, minimize wastage, stabilize pricing strategies, and improve financial sustainability while maintaining service quality for consumers.

7.3 Future Research Directions

To extend and refine the contributions of this thesis, the following avenues for future research are suggested:

(a) **Integration of Dynamic Discounting Strategies:** Future models can incorporate promotional discounts alongside inflation variability in each replenishment cycle to analyze their impact on inventory costs and demand elasticity.

(b) **Extension to Multi-Supplier and Multi-Product Frameworks:** Expanding the model to include multiple suppliers and heterogeneous product categories would better capture the complexities of real-world FMCG supply chains.

(c) **Utilization of Advanced Computational Techniques:** The application of optimization algorithms using MATLAB, CPLEX, or AI-driven predictive analytics can enhance the computational efficiency of the proposed inventory models.

(d) **Incorporation of Primary Data for Empirical Validation:** While this study primarily relies on secondary data, future research can leverage primary data collected from FMCG firms to validate the theoretical models under real-world business conditions.

By addressing these research directions, inventory management strategies can be further refined to remain adaptive to the evolving dynamics of the FMCG industry. The integration of advanced mathematical modeling techniques with practical supply chain applications ensures that the field of inventory optimization continues to develop, offering valuable contributions to both academic research and industry practice.

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Appendices

Appendix A: Thesis Chapters Appendices

Appendix A.3.1.

Solution of Equation 3.2

$$\frac{dN_{i+1}(t)}{dt} + \theta N_{i+1}(t) = -v$$

Integrating factor = $e^{\int \theta dt} = e^{\theta t}$

Thus, solution will be given as:

$$N_{i+1}(t)e^{\theta t} = -\int ve^{\theta u}du + c$$

$$-N_{i+1}(t)e^{\theta t} = -\int_{t}^{t_{i+1}} ve^{\theta u}du$$

$$N_{i+1}(t) = e^{-\theta t}\int_{t}^{t_{i+1}} ve^{\theta u}du$$

Appendix A.3.2.

Solution of equation 3.5

$$RET_{TC} = n * C + \sum_{i=0}^{n-1} IC \int_{t_i}^{t_{i+1}} N_{i+1}(t)\, dt + \sum_{i=0}^{n-1} A * OQ_{(i+1)}$$

$$RET_{TC} = n * C + \sum_{i=0}^{n-1} IC \int_{t_i}^{t_{i+1}} \left\{ e^{-\theta t}\int_{t}^{t_{i+1}} ve^{\theta u}du \right\} dt + \sum_{i=0}^{n-1} A \left\{ e^{-\theta t_i} \int_{t_i}^{t_{i+1}} v\, e^{\theta t}\, dt \right\}$$

$$RET_{TC} = n*C + \sum_{i=0}^{n-1} IC \int_{t_i}^{t_{i+1}} \left\{\frac{v}{\theta} e^{\theta(t_{i+1}-t)} - \frac{v}{\theta}\right\} dt + \sum_{i=0}^{n-1} A\left\{\frac{v}{\theta} e^{-\theta t_i}\left\{e^{\theta t_{i+1}} - e^{\theta t_i}\right\}\right\}$$

$$RET_{TC} = n*C + \sum_{i=0}^{n-1} IC \left\{\left(\frac{-v}{\theta^2} e^{\theta(t_{i+1}-t_{i+1})} - \frac{v}{\theta} t_{i+1}\right) - \left(\frac{-v}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{v}{\theta} t_i\right)\right\} + \sum_{i=0}^{n-1} A\left\{\frac{v}{\theta}\left\{e^{\theta(t_{i+1}-t_i)} - 1\right\}\right\}$$

$$RET_{TC} = n*C + \sum_{i=0}^{n-1} \frac{IC}{\theta} \left\{-\frac{v}{\theta} - vt_{i+1} + \frac{v}{\theta} e^{\theta(t_{i+1}-t_i)} + vt_i\right\} + \sum_{i=0}^{n-1} A\left\{\frac{v}{\theta} e^{\theta(t_{i+1}-t_i)} - \frac{v}{\theta}\right\}$$

$$RET_{TC} = n*C + \sum_{i=0}^{n-1} \left(\frac{IC}{\theta} + A\right)\left\{\frac{v}{\theta} e^{\theta(t_{i+1}-t_i)} - \frac{v}{\theta}\right\} - \sum_{i=0}^{n-1} \frac{IC}{\theta}\{v(t_{i+1} - t_i)\}$$

$$RET_{TC} = n*C + \sum_{i=0}^{n-1} \left(\frac{IC}{\theta} + A\right)\left\{\frac{v}{\theta} e^{\theta(t_{i+1}-t_i)} - \frac{v}{\theta}\right\} - \sum_{i=0}^{n-1} \frac{IC}{\theta}\{v(t_{i+1} - t_i)\}$$

$$RET_{TC} = n*C + \sum_{i=0}^{n-1} \left(\frac{IC}{\theta} + A\right)\int_{t_i}^{t_{i+1}} ve^{\theta(t-t_i)} dt - \frac{IC}{\theta} vZ$$

Appendix A.3.3.

Theorem 3.1. $RET_{TC}(n, t_0, t_1, t_2, \ldots., t_n)$is decreasing convex function in n.

Proof of theorem 3.1

We assume,

$RET_{TC} = n * C + \sum_{i=0}^{n-1}\left(\frac{IC}{\theta} + A\right) f(n, 0, Z) - U$ where$f(n, 0, Z) =$

$\int_{t_i}^{t_{i+1}} ve^{\theta(t-t_i)}dt.$

Thus, to prove this theorem we need to prove that $f(n, 0, Z)$ is convex in n. for this, the interval (0, Z) will be divided into n sections and the interval (n+1, Z) will be divided into n+1 sections and the division in both the cases will be random.

Now, without any loss of generality, we cut the interval (t_{n-1}, Z) is into 2 segments where $t_{n+1} = Z$

The next step, is to prove $f(n + 1,0, Z) < f(n, 0, Z)$

Consider,

$f(n + 1,0, Z) - f(n, 0, Z)$

$$= \int_{t_{n-1}}^{t_n} ve^{\theta(t-t_{n-1})}dt + \int_{t_n}^{Z} e^{\theta(t-t_n)}dt - \int_{t_{n-1}}^{Z} ve^{\theta(t-t_{n-1})}\,dt$$

$$= \int_{t_{n-1}}^{t_n} ve^{\theta(t-t_{n-1})}dt + \int_{t_n}^{Z} e^{\theta(t-t_n)}dt - \int_{t_{n-1}}^{t_n} ve^{\theta(t-t_{n-1})}\,dt$$

$$- \int_{t_n}^{Z} ve^{\theta(t-t_{n-1})}\,dt$$

$$= \int_{t_n}^{Z} ve^{\theta(t-t_n)} - ve^{\theta(t-t_{n-1})}dt$$

Now, since, e^t is an increasing function, thus we can say that, $f(n + 1,0, Z) < f(n, 0, Z)$

As a result, $f(n, 0, Z)$ is strictly decreasing function in n.

Thus, the next step is to prove the convexity of $f(n, 0, Z)$ in n.

For this, we will consider,

$f(n, 0, Z) - f(n - 1,0, Z) - [f(n + 1,0, Z) - f(n, 0, Z)]$

$$= \int_{t_{n-1}}^{Z} v[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}]dt - \int_{t_n}^{Z} v[e^{\theta(t-t_n)} - e^{\theta(t-t_{n-1})}]dt$$

$$= \int_{t_{n-1}}^{t_n} v[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}]dt + \int_{t_n}^{Z} v[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}]dt$$

$$- \int_{t_n}^{Z} v[e^{\theta(t-t_n)} - e^{\theta(t-t_{n-1})}]dt$$

$$= \int_{t_{n-1}}^{t_n} v[e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}]dt + \int_{t_n}^{Z} v[2e^{\theta(t-t_{n-1})} - e^{\theta(t-t_{n-2})}$$

$$- e^{\theta(t-t_n)}]dt$$

Now since, e^t is a convex function, therefore,

$f(n,0,Z) - f(n-1,0,Z) < [f(n+1,0,Z) - f(n,0,Z)]$

As a result, $f(n,0,Z)$ is convex in n.

Hence, $RET_{TC}(n, t_0, t_1, t_2, \dots, t_n)$ is also decreasing convex function in n.

Appendix A.3.4.

Theorem 3.2. The unique solution to nonlinear system of equations derived by obtaining first derivative of $RET_{TC}(n, t_0, t_1, t_2, \dots, t_n)$ is the optimal ordering interval for given n.

Proof of theorem 3.2

$$\frac{\partial RET_{TC}(n, t_0, t_1, t_2, \dots, t_n)}{\partial t_i}$$

$$= \left(\frac{IC}{\theta} + A\right)\left(\sum_{i=0}^{n-1} -\frac{v}{\theta^2}\left(e^{\theta(t_{i+1}-t_i)}\right)\right) - \frac{Av}{\theta^2}\left(\sum_{i=0}^{n-1}\left(e^{\theta(t_{i+1}-t_i)}\right)\right)$$

$$+ v - Av\left(\sum_{i=0}^{n-1} E_{i+1}\,(t_{i+1} - t_i - \eta)\right)$$

$$\frac{\partial RET_{TC}(n, t_0, t_1, t_2, \dots, t_n)}{\partial t_i}$$

$$= \left(\frac{IC}{\theta} + A\right)\left(\sum_{i=0}^{n-1} -\frac{v}{\theta^2} e^{\theta T_{i+1}}\right) - \frac{Av}{\theta^2}\left(\sum_{i=0}^{n-1} e^{\theta T_{i+1}}\right) + v$$

$$- Av\left(\sum_{i=0}^{n-1} E_{i+1}\,(T_{i+1} - \eta)\right)$$

$$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\dots.,t_n)}{\partial {t_i}^2} = \left(\frac{IC}{\theta}+A\right)\left(\sum_{i=0}^{n-1}\frac{v}{\theta^3}\left(e^{\theta(t_{i+1}-t_i)}\right)\right)+\frac{Av}{\theta^3}\left(\sum_{i=0}^{n-1}\left(e^{\theta(t_{i+1}-t_i)}\right)\right) + Av\left(\sum_{i=0}^{n-1}E_{i+1}\right)$$

$$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\dots.,t_n)}{\partial {t_i}^2} = \left(\frac{IC}{\theta}+A\right)\left(\sum_{i=0}^{n-1}\frac{v}{\theta^3}e^{\theta T_{i+1}}\right)+\frac{Av}{\theta^3}\left(\sum_{i=0}^{n-1}e^{\theta T_{i+1}}\right) + Av\left(\sum_{i=0}^{n-1}E_{i+1}\right)$$

$$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\dots.,t_n)}{\partial t_i \partial t_{i+1}} = -\left[\left(\frac{IC}{\theta}+A\right)\left(\sum_{i=0}^{n-1}\frac{v}{\theta^3}\left(e^{\theta(t_{i+1}-t_i)}\right)\right)+\frac{Av}{\theta^3}\left(\sum_{i=0}^{n-1}\left(e^{\theta(t_{i+1}-t_i)}\right)\right) + Av\left(\sum_{i=0}^{n-1}E_{i+1}\right)\right]$$

$$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\dots.,t_n)}{\partial t_i \partial t_{i+1}} = -\left[\left(\frac{IC}{\theta}+A\right)\left(\sum_{i=0}^{n-1}\frac{v}{\theta^3}e^{\theta T_{i+1}}\right)+\frac{Av}{\theta^3}\left(\sum_{i=0}^{n-1}e^{\theta T_{i+1}}\right) + Av\left(\sum_{i=0}^{n-1}E_{i+1}\right)\right]$$

$$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\ldots.,t_n)}{\partial t_i \partial t_{i-1}} = -\left[\left(\frac{IC}{\theta}+A\right)\left(\sum_{i=0}^{n-1}\frac{v}{\theta^3}\left(e^{\theta(t_i-t_{i-1})}\right)\right)+\frac{Av}{\theta^3}\left(\sum_{i=0}^{n-1}\left(e^{\theta(t_i-t_{i-1})}\right)\right) + Av\left(\sum_{i=0}^{n-1}E_{i+1}\right)\right]$$

$$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\ldots.,t_n)}{\partial t_i \partial t_{i-1}} = -\left[\left(\frac{IC}{\theta}+A\right)\left(\sum_{i=0}^{n-1}\frac{v}{\theta^3}\left(e^{\theta T_i}\right)\right)+\frac{Av}{\theta^3}\left(\sum_{i=0}^{n-1}\left(e^{\theta T_i}\right)\right) + Av\left(\sum_{i=0}^{n-1}E_{i+1}\right)\right]$$

$$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\ldots.,t_n)}{\partial t_i \partial t_k} = 0 \; for\ all\ k \neq i-1, i, i+1$$

Moreover,

$\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\ldots.,t_n)}{\partial {t_i}^2} > \left|\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\ldots.,t_n)}{\partial t_i \partial t_{i-1}}\right| + \left|\frac{\partial^2 RET_{TC}(n,t_0,t_1,t_2,\ldots.,t_n)}{\partial t_i \partial t_{i+1}}\right|$ for all i = 1, 2, …. , n-1.

Thus, the Hessian matrix will be a diagonal matrix and all the diagonal elements will be positive and it will be positive definite.

$$\nabla^2 RET_{TC}(n,t_0,t_1,t_2,\ldots.,t_n) = \begin{bmatrix} \frac{\partial^2 RET_{TC}}{\partial t_1^2} & \frac{\partial^2 RET_{TC}}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 RET_{TC}}{\partial t_2 \partial t_1} & \frac{\partial^2 RET_{TC}}{\partial t_2^2} & \frac{\partial^2 RET_{TC}}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 RET_{TC}}{\partial t_3 \partial t_2} & \frac{\partial^2 RET_{TC}}{\partial t_3^2} & \frac{\partial^2 RET_{TC}}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\ \ldots. & \ldots. & \ldots & \ldots & \ldots & \ldots. & \ldots. & \ldots. & \ldots. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 RET_{TC}}{\partial t_{n-1} \partial t_{n-2}} & \frac{\partial^2 RET_{TC}}{\partial t_{n-1}^2} & \frac{\partial^2 RET_{TC}}{\partial t_{n-1}\, \partial t_n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 RET_{TC}}{\partial t_n \partial t_{n-1}} & \frac{\partial^2 RET_{TC}}{\partial t_n^2} \end{bmatrix}$$

Appendix A.4.1.

Solution of Equation 4.2.

$\frac{dI_{i+1}(t)}{dt} = -D_0 t, \qquad t_i \leq t \leq u$

Thus, solution will be given as:

$$\int_{t_i}^{t} \frac{dI_{i+1}(t)}{dt} = -\int_{t_i}^{t} D_0 t dt$$

$$I_{i+1}(t) - I_{i+1}(0) = -\frac{D_0(t^2 - t_i^2)}{2}$$

Now, S_{i+1} units enters/ ordered at the beginning of each replenishment cycle thus the above identity reduces to,

$$I_{i+1}(t) = S_{i+1} - \frac{D_0(t^2 - t_i^2)}{2}$$

Appendix A.4.2.

Solution of equation 4.3.

$\frac{dI_{i+1}(t)}{dt} = -D_o \mu, \qquad u \leq t \leq t_{i+1}$

Thus, solution will be given as:

$$\int_{t}^{t_{i+1}} \frac{dI_{i+1}(t)}{dt} = -\int_{t}^{t_{i+1}} D_0 \mu dt$$

$I_{i+1}(t) = D_0 u(t_{i+1} - t), \;\; u \leq t \leq t_i$ and $I_{i+1}(t_i) = 0$

Appendix A.5.1

Solution of Equation 5.1.

Solution will be given as

$$\int_{t_i}^{t_{i+1}} I_{i+1}(t) e^{kt} dt = -\int_{t}^{t_{i+1}} e^{ku}(b_1 u + b_2 e^{\alpha_i u})\, du$$

$$\int_{t_i}^{t_{i+1}} I_{i+1}(t) dt = -e^{-kt} \int_{t}^{t_{i+1}} e^{ku}(b_1 u + b_2 e^{\alpha_i u})\, du$$

$$I_{i+1}(t_{i+1}) - I_{i+1}(t) = -e^{-kt}\int_{t}^{t_{i+1}} (b_1 u + b_2 e^{\alpha_i u})\, du$$

Now, $I_{i+1}(t_{i+1}) = 0, thus,\ I_{i+1}(t) = e^{-kt}\int_{t}^{t_{i+1}}(b_1 u + b_2 e^{\alpha_i u})\, du,\ t_i \le t \le t_{i+1}$

Appendix A.5.2

Solution of Equation 5.4

Ordering cost for first cycle = $Ye^{\alpha_1(t_1 - t_0)}$

Ordering cost for second cycle = $Ye^{\alpha_1(t_1 - t_0) + \alpha_2(t_2 - t_1)}$

Ordering cost for first and second cycle = $Y(e^{\alpha_1(t_1-t_0)} + e^{\alpha_1(t_1-t_0)+\alpha_2(t_2-t_1)})$

Similarly total ordering cost = $Y(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)})))$

Appendix A.5.3

Solution of Equation 5.5

Total holding cost = $\sum_{i=0}^{n-1} h * e^{\alpha_{i+1}(t_{i+1}-t_1)} \int_{t_i}^{t_{i+1}} I_{i+1}(t)dt$

$$= \sum_{j=0}^{n-1} h * e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)} \int_{t_i}^{t_{i+1}} e^{-kt}(\int_{t}^{t_{i+1}} e^{ku}(b_1 u + b_2 e^{\alpha_i u})\, du)dt$$

$$= \sum_{j=0}^{n-1} h * e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)} \int_{t_i}^{t_{i+1}} ((\frac{b_1}{k}(t_{i+1}e^{k(t_{i+1}-t)} - \frac{e^{k(t_{i+1}-t)}}{k} - t + \frac{1}{k}) + (\frac{b_2}{\alpha_{i+1}+k}(e^{\alpha_{i+1}t_{i+1}} * e^{k(t_{i+1}-t)} - e^{\alpha_{i+1}t})))dt$$

$$= \sum_{j=0}^{n-1} h * e^{\sum_{i=0}^{j}\alpha_{i+1}(t_{i+1}-t_i)} (\frac{b_1 t_{i+1}}{k}\int_{t_i}^{t_{i+1}} e^{k(t_{i+1}-t)}dt - \frac{b_1}{k^2}\int_{t_i}^{t_{i+1}} e^{k(t_{i+1}-t)}dt + \frac{b_1}{k}\int_{t_i}^{t_{i+1}}(-t + \frac{1}{k})dt + \frac{b_2 e^{\alpha_{i+1}t_{i+1}}}{\alpha_{i+1}+k}\int_{t_i}^{t_{i+1}} e^{k(t_{i+1}-t)}dt - \frac{b_2}{\alpha_{i+1}+k}\int_{t_i}^{t_{i+1}} e^{\alpha_{i+1}t}dt)$$

$$= \sum_{j=0}^{n-1} h * e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}((-\frac{b_1 * t_{i+1}}{k^2}(1 - e^{k(t_{i+1}-t_i)}) + \frac{b_1}{k^3}(1 - e^{k(t_{i+1}-t_i)}) + \frac{b_1}{k^2}(t_{i+1} - t_i) - \frac{b_1}{2k}(t_{i+1}^2 - t_i^2) - \frac{b_2 * e^{\alpha_{i+1}t_{i+1}}}{k(\alpha_{i+1} + k)}(1 - e^{k(t_{i+1}-t_i)}) - \frac{b_2 * e^{\alpha_{i+1}(t_{i+1}-t_i)}}{\alpha_{i+1}(\alpha_{i+1} + k)})$$

$$h(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * \frac{1}{2k^3} b_1(2 - 2e^{k(-t_j+t_{1+j})} - 2kt_j + k^2 t_j^2 + 2e^{k(-t_j+t_{1+j})}kt_{1+j} - k^2 t_{1+j}^2) + \frac{b_2(\frac{e^{-kt_j+t_{1+j}\alpha_{1+j}}(-e^{kt_j} + e^{kt_{1+j}})}{k} + \frac{e^{t_j\alpha_{1+j}} - e^{t_{1+j}\alpha_{1+j}}}{\alpha_{1+j}})}{k + \alpha_{1+j}})$$

Appendix A.5.4

Solution of Equation 5.6

Total purchasing cost = $W(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * Q_{j+1}$

$$where\ Q_{j+1} = e^{-kt_j} \int_{t_j}^{t_{j+1}} e^{kt}(b_1 t + b_2 e^{\alpha_{j+1}t})\, dt$$

$$= W(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * (e^{-kt_j}(\int_{t_j}^{t_{j+1}} e^{kt} b_1 t dt + \int_{t_j}^{t_{j+1}} b_2 e^{(\alpha_{j+1}+k)t}\, dt))$$

$$= W(\sum_{j=0}^{n-1}((e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * ((b_1(\frac{e^{k(t_{j+1}-t_j)} * t_{j+1}}{k} - \frac{e^{k(t_{j+1}-t_j)}}{k^2} - \frac{t_j}{k} + \frac{1}{k^2}) + b_2(\frac{e^{\alpha_{j+1} * t_{j+1}} * e^{k(t_{j+1}-t_j)} - e^{t_j * \alpha_{j+1}}}{k+\alpha_{j+1}}))))$$

Appendix A.5.5

Proof of Theorem 5.1: In – depth analysis of Figure 2, indicates that there will be no circulation proffer offered by any retailer as the proffer will travel from highest to lowest inventory cost.

Let's suppose that, the layout is untenable, thus, there will be set of two nabe retailers, $C_{R_m}(t_i)\ and\ C_{R_n}(t_i), m, n\ \in M_i\ s.t. C_{R_m}$, is the retailer having largest value of inventory cost in the nabe and $C_{R_n}(t_i)$, is the retailer having lowest inventory cost in the nabe. Moreover, $C_m - C_n$ is the highest inventory cost difference in the nabe. Following this, the retailer $C_{R_m}(t_i)$ will make proffer to $C_{R_n}(t_i)$and $C_{R_n}(t_i)$will accept the proffer, resulting in CostPact between $C_{R_m}(t_i)\ and\ C_{R_n}(t_i)$. Since, $C_{R_n}(t_i)$ is increased at time $t_{i+1}, n\ \in M_i$ holds.

Appendix A.5.6

Proof of Theorem 5.2

Since according to Lemma 1,

$D(t_i) > D(t_{i+1})$, this implies, that eventually, the differences among the inventory costs is reducing with the every proffer made.

Moreover, theorem 1, proves that there is no stalemate point in the layoutM_i, thus the layout will reach Equipoise. (As the value of k increases, the equipoise is attained in lesser loops. (Refer Algorithm 2 for Equipoise).

Appendix A.5.7

Lemma 5.1: Let $D(t_i) = high_{i \in M_i} C_i(t_i) - \ low_{i \in M_i} C_i(t_i)$. As long as $C_i(t_i) \neq \phi$, we have $D(t_i) > D(t_{i+1})$

Proof: To start with, let us consider, $i \notin M_i$. Since retailer, R_i does not pass a proffer to any other retailer in the layout, there will be no change in its inventory cost. Thus, $high_{i \in M_i} \left[\frac{C_i(t_i) - C_j(t_i)}{k}\right] = 0$ holds.

Now, suppose that a retailer has a highest inventory cost in M_i. As no nabe retailer will make a proffer to such a retailer, the cost will be down at time t_{i+1}. If a retailer has a lowest inventory cost in M_i, the nabe retailer will make a proffer to such retailer. Thus, the inventory cost will rise at time t_{i+1}. Suppose, $C_i^{high2}(t_i)$ be the second highest inventory cost among all the retailers in the layout. Then, the cost will not exceed $C_i^{high2}(t_i)$ at time t_{i+1} as it will go up only when it accepts a proffer made by only such retailer which has higher inventory cost than $C_i^{high2}(t_i)$. Even if this scenario takes place, the increment in the inventory cost will be at the maximum, half (taking k=2), of the difference between the inventory costs. Thus, we have, $C_i^{high}(t_i) > C_i^{high2}(t_{i+1})\ and\ C_i^{high}(t_i) > C_i^{high}(t_{i+1})$.

On the other side, the retailer $C_i^{low}(t_i)$ will accept the proffer and its inventory cost will raise at time (t_{i+1}). Suppose, $C_i^{low2}(t_i)$ is the second lowest inventory cost among all the retailers in the layout. Then, it will at the maximum, decreases without any proffer only when it is in contact with such a retailer which have $C_i^{low}(t_i)$. Thus, we have, $C_i^{low}(t_i) \leq C_i^{low2}(t_{i+1}), and\ C_i^{low}(t_i) \leq C_i^{low}(t_{i+1})$. As a result, $D(t_i) > D(t_{i+1})\ holds\ true$.

Appendix A.5.8

Proof of Theorem 5.3:

Proof:

$$\frac{\partial R_{I2}}{\partial t_i} = h\{(\alpha_i - \alpha_{1+i}) * (e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * (\frac{1}{2k^3} b_1(2 - 2e^{k(-t_j+t_{1+j})} - 2kt_j + k^2 t_j^2 + 2e^{k(-t_j+t_{1+j})} k t_{1+j} - k^2 t_{1+j}^2) + \frac{b_2(\frac{e^{-kt_j+t_{1+j}\alpha_{1+j}}(-e^{kt_j}+e^{kt_{1+j}})}{k} + \frac{e^{t_j\alpha_{1+j}} - e^{t_{1+j}\alpha_{1+j}}}{\alpha_{1+j}})}{k+\alpha_{1+j}}) + (e^{\sum_{i=0}^{j} \alpha_{i+1}(t_{i+1}-t_i)}) * \frac{(-e^{k(-t_{-1+i}+t_i)}k + e^{k(-t_i+t_{1+i})}k)b_1}{k^3} + \frac{b_1(-1+e^{k(-t_{-1+i}+t_i)} + e^{k(-t_{-1+i}+t_i)}kt_i - e^{k(-t_i+t_{1+i})}kt_{1+i})}{k^2} + \frac{e^{k(-t_{-1+i}+t_i)+t_i\alpha_i} b_2}{k+\alpha_i} + \frac{e^{t_i\alpha_i} b_2(-\frac{1}{k} + \frac{e^{k(-t_{-1+i}+t_i)}}{k} - \frac{1}{\alpha_i})\alpha_i}{k+\alpha_i} + \frac{e^{t_i\alpha_{1+i}} b_2}{k+\alpha_{1+i}} - \frac{e^{k(-t_i+t_{1+i})+t_{1+i}\alpha_{1+i}} b_2}{k+\alpha_{1+i}}\} + Y\{e^{\sum_{i=0}^{i-1} \alpha_{i+1}(t_{i+1}-t_i)} * (\alpha_i - \alpha_{i+1})\} + e^{\sum_{i=0}^{i} \alpha_{i+1}(t_{i+1}-t_i)} * (\alpha_i - \alpha_{i+1}) * (b_1(\frac{e^{k(t_{i+1}-t_i)} * t_{i+1}}{k} - \frac{e^{k(t_{i+1}-t_i)}}{k^2} - \frac{t_i}{k} + \frac{1}{k^2}) + b_2(\frac{e^{\alpha_{i+1}*t_{i+1}} * e^{k(t_{i+1}-t_i)} - e^{t_i*\alpha_{i+1}}}{k+\alpha_{i+1}})) + e^{\sum_{i=0}^{i} \alpha_{i+1}(t_{i+1}-t_i)} * (b_1(-\frac{1}{k} + \frac{e^{k(-t_i+t_{1+i})}}{k} - e^{k(-t_i+t_{1+i})} * t_{1+i}) + \frac{b_2(-e^{k(-t_i+t_{1+i})+t_{1+i}*\alpha_{1+i}}k - e^{t_i*\alpha_{1+i}}\alpha_{1+i})}{k+\alpha_{1+i}}))\}$$

$$\frac{\partial^2 R_{I2}}{\partial t_i^2}$$

$$= \frac{e^{(-t_i+t_{1+i})\alpha_{1+i}} h b_1 (2k^2 - 2e^{k(-t_i+t_{1+i})} k^2 + 2e^{k(-t_i+t_{1+i})} k^3 t_{1+i})}{2k^3}$$

$$- \frac{e^{(-t_i+t_{1+i})\alpha_{1+i}} h b_1 (-2k + 2e^{k(-t_i+t_{1+i})} k + 2k^2 t_i - 2e^{k(-t_i+t_{1+i})} k^2 t_{1+i}) \alpha_{1+i}}{k^3}$$

$$+ e^{(-t_i+t_{1+i})\alpha_{1+i}} Y \alpha_{1+i}^2 + e^{(-t_i+t_{1+i})\alpha_{1+i}} \alpha_{1+i}^2 (b_1 (\frac{1}{k^2} - \frac{e^{k(-t_i+t_{1+i})}}{k^2} - \frac{t_i}{k} + \frac{e^{k(-t_i+t_{1+i})} t_{1+i}}{k})$$

$$+ \frac{(-e^{t_i \alpha_{1+i}} + e^{k(-t_i+t_{1+i}) + t_{1+i}\alpha_{1+i}}) b_2}{k + \alpha_{1+i}})$$

$$+ e^{(-t_i+t_{1+i})\alpha_{1+i}} h \alpha_{1+i}^2 (\frac{b_1 (2 - 2e^{k(-t_i+t_{1+i})} - 2k t_i + k^2 t_i^2 + 2e^{k(-t_i+t_{1+i})} k t_{1+i} - k^2 t_{1+i}^2)}{2k^3}$$

$$+ \frac{e^{\mathrm{kt}_i + t_{1+i}\alpha_{1+i}} (-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2}{k(k + \alpha_{1+i})}) - 2e^{(-t_i+t_{1+i})\alpha_{1+i}} \alpha_{1+i} (b_1 (-\frac{1}{k} + \frac{e^{k(-t_i+t_{1+i})}}{k}$$

$$- e^{k(-t_i+t_{1+i})} t_{1+i}) + \frac{b_2 (-e^{k(-t_i+t_{1+i}) + t_{1+i}\alpha_{1+i}} k - e^{t_i \alpha_{1+i}} \alpha_{1+i})}{k + \alpha_{1+i}})$$

$$+ e^{(-t_i+t_{1+i})\alpha_{1+i}} (b_1 (-e^{k(-t_i+t_{1+i})} + e^{k(-t_i+t_{1+i})} k t_{1+i})$$

$$+ \frac{b_2 (e^{k(-t_i+t_{1+i}) + t_{1+i}\alpha_{1+i}} k^2 - e^{t_i \alpha_{1+i}} \alpha_{1+i}^2)}{k + \alpha_{1+i}})$$

$$\frac{\partial R_{I2}}{\partial t_{i+1}}$$

$$= e^{(-t_i+t_{1+i})\alpha_{1+i}} (e^{k(-t_i+t_{1+i}) + t_{1+i}\alpha_{1+i}} b_2 + e^{k(-t_i+t_{1+i})} b_1 t_{1+i}) + e^{(-t_i+t_{1+i})\alpha_{1+i}} Y \alpha_{1+i}$$

$$+ e^{(-t_i+t_{1+i})\alpha_{1+i}} \alpha_{1+i} (b_1 (\frac{1}{k^2} - \frac{e^{k(-t_i+t_{1+i})}}{k^2} - \frac{t_i}{k} + \frac{e^{k(-t_i+t_{1+i})} t_{1+i}}{k})$$

$$+ \frac{(-e^{t_i \alpha_{1+i}} + e^{k(-t_i+t_{1+i}) + t_{1+i}\alpha_{1+i}}) b_2}{k + \alpha_{1+i}})$$

$$+ e^{(-t_i+t_{1+i})\alpha_{1+i}} h \alpha_{1+i} (\frac{b_1 (2 - 2e^{k(-t_i+t_{1+i})} - 2k t_i + k^2 t_i^2 + 2e^{k(-t_i+t_{1+i})} k t_{1+i} - k^2 t_{1+i}^2)}{2k^3}$$

$$+ \frac{e^{\mathrm{kt}_i + t_{1+i}\alpha_{1+i}} (-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2}{k(k + \alpha_{1+i})}) + e^{(-t_i+t_{1+i})\alpha_{1+i}} h (\frac{b_1 (-2k^2 t_{1+i} + 2e^{k(-t_i+t_{1+i})} k^2 t_{1+i})}{2k^3}$$

$$+ \frac{e^{\mathrm{kt}_i + t_{1+i}\alpha_{1+i}} (-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2 \alpha_{1+i}}{k(k + \alpha_{1+i})})$$

$$\frac{\partial^2 R_{I2}}{\partial t_i \partial t_{i+1}}$$

$$= -e^{k(-t_i+t_{1+i})+(-t_i+t_{1+i})\alpha_{1+i}} h b_1 t_{1+i} + e^{(-t_i+t_{1+i})\alpha_{1+i}}(-e^{k(-t_i+t_{1+i})+t_{1+i}\alpha_{1+i}} k b_2$$

$$- e^{k(-t_i+t_{1+i})} k b_1 t_{1+i})$$

$$+ \frac{e^{(-t_i+t_{1+i})\alpha_{1+i}} h b_1(-2k + 2e^{k(-t_i+t_{1+i})}k + 2k^2 t_i - 2e^{k(-t_i+t_{1+i})}k^2 t_{1+i})\alpha_{1+i}}{2k^3}$$

$$- e^{(-t_i+t_{1+i})\alpha_{1+i}}(e^{k(-t_i+t_{1+i})+t_{1+i}\alpha_{1+i}} b_2 + e^{k(-t_i+t_{1+i})} b_1 t_{1+i})\alpha_{1+i} - e^{(-t_i+t_{1+i})\alpha_{1+i}} Y \alpha_{1+i}^2$$

$$- e^{(-t_i+t_{1+i})\alpha_{1+i}} \alpha_{1+i}^2 (b_1(\frac{1}{k^2} - \frac{e^{k(-t_i+t_{1+i})}}{k^2} - \frac{t_i}{k} + \frac{e^{k(-t_i+t_{1+i})} t_{1+i}}{k})$$

$$+ \frac{(-e^{t_i\alpha_{1+i}} + e^{k(-t_i+t_{1+i})+t_{1+i}\alpha_{1+i}}) b_2}{k + \alpha_{1+i}})$$

$$- e^{(-t_i+t_{1+i})\alpha_{1+i}} h \alpha_{1+i}^2 (\frac{b_1(2 - 2e^{k(-t_i+t_{1+i})} - 2kt_i + k^2 t_i^2 + 2e^{k(-t_i+t_{1+i})} k t_{1+i} - k^2 t_{1+i}^2)}{2k^3}$$

$$+ \frac{e^{\mathrm{kt}_i + t_{1+i}\alpha_{1+i}}(-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2}{k(k + \alpha_{1+i})})$$

$$- e^{(-t_i+t_{1+i})\alpha_{1+i}} h \alpha_{1+i} (\frac{b_1(-2k^2 t_{1+i} + 2e^{k(-t_i+t_{1+i})} k^2 t_{1+i})}{2k^3}$$

$$+ \frac{e^{\mathrm{kt}_i + t_{1+i}\alpha_{1+i}}(-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2 \alpha_{1+i}}{k(k + \alpha_{1+i})}) + e^{(-t_i+t_{1+i})\alpha_{1+i}} \alpha_{1+i} (b_1(-\frac{1}{k} + \frac{e^{k(-t_i+t_{1+i})}}{k}$$

$$- e^{k(-t_i+t_{1+i})} t_{1+i}) + \frac{b_2(-e^{k(-t_i+t_{1+i})+t_{1+i}\alpha_{1+i}} k - e^{t_i\alpha_{1+i}} \alpha_{1+i})}{k + \alpha_{1+i}})$$

$$\frac{\partial R_{I2}}{\partial t_{i-1}}$$

$$= \frac{e^{(-t_{-1+i}+t_i)\alpha_i} h b_1(-2k + 2e^{k(-t_{-1+i}+t_i)}k - 2e^{k(-t_{-1+i}+t_i)}k^2 t_i)}{2k^3}$$

$$- e^{(-t_{-1+i}+t_i)\alpha_i} Y \alpha_i - e^{(-t_{-1+i}+t_i)\alpha_i} \alpha_i (b_1(\frac{1}{k^2} - \frac{e^{k(-t_{-1+i}+t_i)}}{k^2} - \frac{t_{-1+i}}{k}$$

$$+ \frac{e^{k(-t_{-1+i}+t_i)} t_i}{k}) + \frac{(-e^{t_{-1+i}\alpha_i} + e^{k(-t_{-1+i}+t_i)+t_i\alpha_i}) b_2}{k + \alpha_i})$$

$$- e^{(-t_{-1+i}+t_i)\alpha_i} h \alpha_i (\frac{b_1(2 - 2e^{k(-t_{-1+i}+t_i)} - 2kt_{-1+i} + 2e^{k(-t_{-1+i}+t_i)} k t_i)}{2k^3}$$

$$+ \frac{e^{\mathrm{kt}_i + t_i\alpha_i}(-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2}{k(k + \alpha_i)}) + e^{(-t_{-1+i}+t_i)\alpha_i}(b_1(-\frac{1}{k} + \frac{e^{k(-t_{-1+i}+t_i)}}{k}$$

$$- e^{k(-t_{-1+i}+t_i)} t_i) + \frac{b_2(-e^{k(-t_{-1+i}+t_i)+t_i\alpha_i} k - e^{t_{-1+i}\alpha_i} \alpha_i)}{k + \alpha_i})$$

$$\frac{\partial^2 R_{I2}}{\partial t_i \partial t_{i-1}}$$
$$= -e^{k(-t_{-1+i}+t_i)+(-t_{-1+i}+t_i)\alpha_i} h b_1 t_i + e^{(-t_{-1+i}+t_i)\alpha_i}(-e^{k(-t_{-1+i}+t_i)+t_i\alpha_i} k b_2$$
$$- e^{k(-t_{-1+i}+t_i)} k b_1 t_i)$$
$$+ \frac{e^{(-t_{-1+i}+t_i)\alpha_i} h b_1(-2k + 2e^{k(-t_{-1+i}+t_i)} k - 2e^{k(-t_{-1+i}+t_i)} k^2 t_i)\alpha_i}{2k^3}$$
$$- e^{(-t_{-1+i}+t_i)\alpha_i}(e^{k(-t_{-1+i}+t_i)+t_i\alpha_i} b_2 + e^{k(-t_{-1+i}+t_i)} b_1 t_i)\alpha_i$$
$$- e^{(-t_{-1+i}+t_i)\alpha_i} Y \alpha_i^2 - e^{(-t_{-1+i}+t_i)\alpha_i} \alpha_i^2 (b_1(\frac{1}{k^2} - \frac{e^{k(-t_{-1+i}+t_i)}}{k^2} - \frac{t_{-1+i}}{k}$$
$$+ \frac{e^{k(-t_{-1+i}+t_i)} t_i}{k}) + \frac{(-e^{t_{-1+i}\alpha_i} + e^{k(-t_{-1+i}+t_i)+t_i\alpha_i}) b_2}{k + \alpha_i})$$
$$- e^{(-t_{-1+i}+t_i)\alpha_i} h \alpha_i^2 (\frac{b_1(2 - 2e^{k(-t_{-1+i}+t_i)} - 2k t_{-1+i} + 2e^{k(-t_{-1+i}+t_i)} k t_i)}{2k^3}$$
$$+ \frac{e^{\mathrm{kt}_i + t_i\alpha_i}(-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2}{k(k+\alpha_i)}) - e^{(-t_{-1+i}+t_i)\alpha_i} h \alpha_i (\frac{e^{k(-t_{-1+i}+t_i)} b_1 t_i}{k}$$
$$+ \frac{e^{\mathrm{kt}_i + t_i\alpha_i}(-e^{\mathrm{kt}_i} + e^{\mathrm{kt}_{1+i}}) b_2 \alpha_i}{k(k+\alpha_i)}) + e^{(-t_{-1+i}+t_i)\alpha_i} \alpha_i (b_1(-\frac{1}{k} + \frac{e^{k(-t_{-1+i}+t_i)}}{k}$$
$$- e^{k(-t_{-1+i}+t_i)} t_i) + \frac{b_2(-e^{k(-t_{-1+i}+t_i)+t_i\alpha_i} k - e^{t_{-1+i}\alpha_i}\alpha_i)}{k + \alpha_i})$$

$$\frac{\partial^2 R_{I2}}{\partial t_i \partial t_k} = 0 \; for \; all \; k \neq i-1, i, i+1$$

Moreover,

$$\frac{\partial^2 R_{I2}}{\partial t_i^2} > \left|\frac{\partial^2 R_{I2}}{\partial t_i t_{i-1}}\right| + \left|\frac{\partial^2 R_{I2}}{\partial t_i t_{i+1}}\right|, \forall \, i = 1,2,3, \ldots\ldots, n-1$$

Thus, the Hessian matrix will be a diagonal matrix and all the diagonal elements will be positive and it will be positive definite.

$$\nabla^2 R_{I2} = \begin{pmatrix} \frac{\partial^2 R_{I2}}{\partial t_1^2} & \frac{\partial^2 R_{I2}}{\partial t_1 \partial t_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 R_{I2}}{\partial t_2 \partial t_1} & \frac{\partial^2 R_{I2}}{\partial t_2^2} & \frac{\partial^2 R_{I2}}{\partial t_2 \partial t_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 R_{I2}}{\partial t_3 \partial t_2} & \frac{\partial^2 R_{I2}}{\partial t_3^2} & \frac{\partial^2 R_{I2}}{\partial t_3 \partial t_4} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 R_{I2}}{\partial t_{n-1} \partial t_{n-2}} & \frac{\partial^2 R_{I2}}{\partial t_{n-1}^2} & \frac{\partial^2 R_{I2}}{\partial t_{n-1} \partial t_n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 R_{I2}}{\partial t_n \partial t_{n-1}} & \frac{\partial^2 R_{I2}}{\partial t_n^2} \end{pmatrix}$$

Thus, the Equation (5.10), will have unique solution with the global minimum and the unique solution is the optimal replenishment cycle length.

Appendix A.6.1.

Solution of equation 6.2

$$\frac{dI_{i+1}(t)}{dt} = -(1-\text{k})D_0 t - (\theta) I_{i+1}(t)$$

$$I_{i+1}(t) == \int_t^{\mu} -(1-\text{k})D_0\, u\, e^{\theta(u-t)}\, \text{du}$$

$$I_{i+1}(t) == S_{i+1} + \int_t^{\mu} (1-\text{k})D_0\, u\ e^{\theta(u-t)}\, \text{du}\ , \qquad t_i \le t \le \mu$$

$$I_{i+1}(t) = S_{i+1} + (1-\text{k}) \left[\frac{u * e^{\theta(u-t)}}{\theta} - \frac{1}{\theta^2} e^{\theta(u-t)} \right]_t^{\mu}$$

$$I_{i+1}(t) == S_{i+1} + (1 - \text{k} \frac{\mu * e^{\theta(\mu-t)}}{\theta} - (1-\text{k}) \frac{1}{\theta^2} e^{\theta(\mu-t)} - (1-\text{k}) \frac{t}{\theta} + (1 - \text{k}) \frac{1}{\theta^2}$$

List of Publications

1. List of articles that are published and are the part of thesis.

1. Mishra, N.K., and Sharma, S., (2023, September). "Supplier-retailer coordination with trade credit for deteriorating items for FMCG." In *AIP Conference Proceedings* (Vol. 2800, No. 1). AIP Publishing. https://doi.org/10.1063/5.0165088.
2. Mishra, N.K., and Sharma, S., (2023, June). "Supplier–retailer inventory coordination with trade credit for seasonal demand in FMCG sector." In *AIP Conference Proceedings* (Vol. 2768, No. 1). AIP Publishing. https://doi.org/10.1063/5.0148297.
3. Mishra, N.K., and Sharma, S., (2023). "Effect of Variable Inflation on Supply Chain Management in FMCG Sector with Single Supplier-Multiple Retailers and Trade Credit." *Mathematical Modelling of Engineering Problems*, *10*(2), pp. 627. ISSN: 2369-0739; Volume 10, Issue 2. https://doi.org/10.18280/mmep.100233.

2. List of articles published during the research process but that are not part of thesis.

1. Nitin Kumar Mishra, Ajay Pratap Singh, Ranu, and Sakshi Sharma "To analyse Weibull demand in the production inventory model under a finite planning horizon" European Chemical Bulletin, Vol. 12, No. 3, pp. 1571-1583. ISSN 2063-5346. doi: 10.31838/ecb/2023.12.3.117.

3. List of articles communicated or under review and are part of thesis.

1. Sakshi Sharma and Nitin Kumar Mishra, "Fuzzification under Finite planning horizon for deteriorating items and seasonal demand" – under review in Mathematical Finance.

List of Conferences

1. Presented a research article titled, “Supplier-retailer coordination with trade credit for deteriorating items for FMCG.” at the 3rd International IOP Conference on Recent Advances in Fundamental and Applied Sciences (RAFAS 2021) 25-26 June 2021 held at Lovely Professional University, Punjab, India.
2. Presented a research article, “Supplier – retailer inventory coordination with trade credit for seasonal demand in FMCG sector.” in the International AIP Conference on Computational Applied Sciences &Its Applications, organized by the Department of Applied Sciences, University of Engineering & Management Jaipur, on 28- 29 April 2022.

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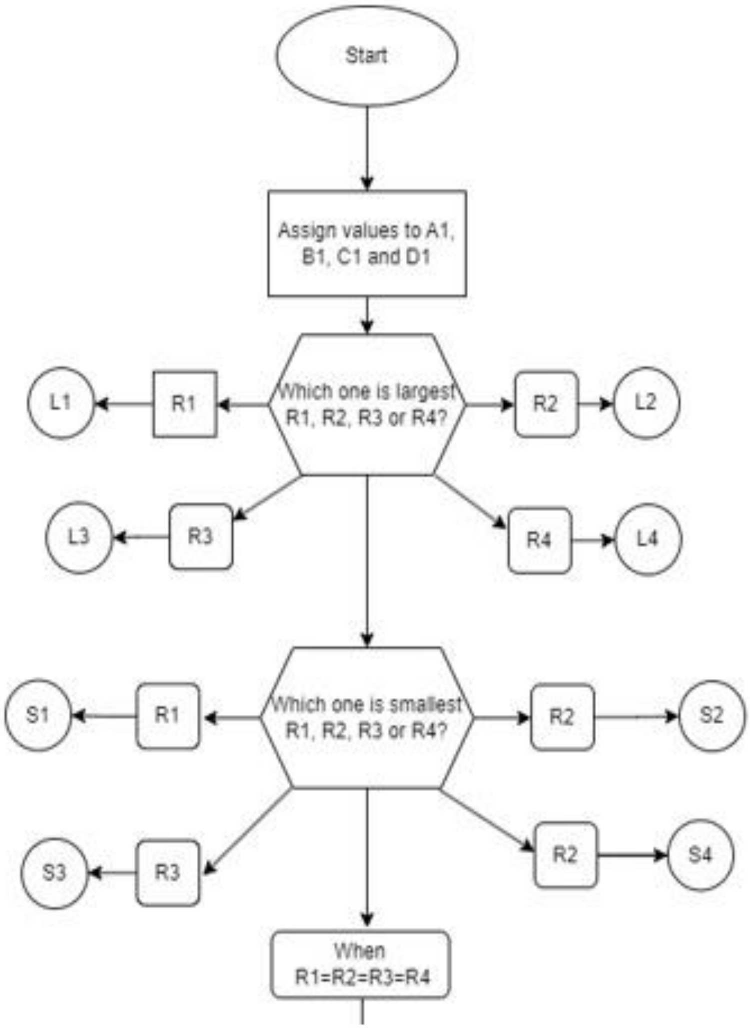
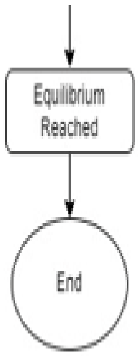
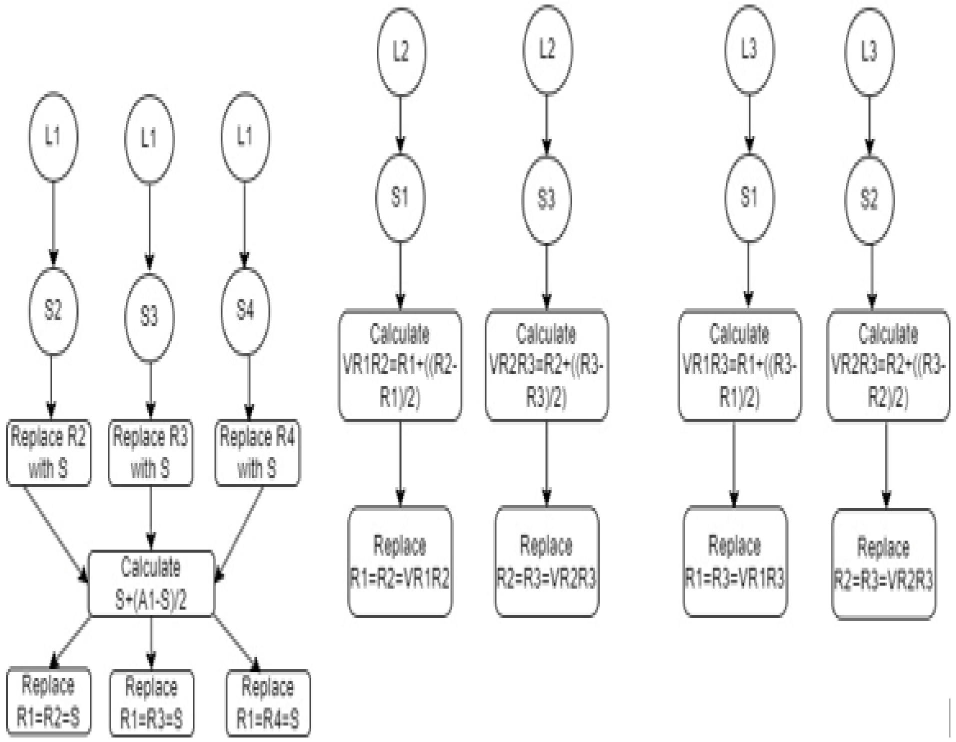



Equilibrium Reached
End


L1
L1
L1
S2
S3
S4
Replace R2 with S
Replace R3 with S
Replace R4 with S
Calculate S+(A1-S)/2
Replace R1=R2=S
Replace R1=R3=S
Replace R1=R4=S
L2
L2
S1
S3
Calculate VR1R2=R1+((R2-R1)/2)
Calculate VR2R3=R2+((R3-R3)/2)
Replace R1=R2=VR1R2
Replace R2=R3=VR2R3
L3
L3
S1
S2
Calculate VR1R3=R1+((R3-R1)/2)
Calculate VR2R3=R2+((R3-R2)/2)
Replace R1=R3=VR1R3
Replace R2=R3=VR2R3

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RESEARCH ARTICLE | SEPTEMBER 08 2023

Supplier-retailer coordination with trade credit for deteriorating items for FMCG

Nitin Kumar Mishra; Sakshi Sharma

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Supplier – Retailer Inventory Coordination with Trade Credit for Seasonal Demand in FMCG Sector

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Abstract. Seasonal products are vital for the Fast Moving Consumer Goods (FMCG) industry. The FMCG sector is the fastest-growing sector and broad areas under this include electronics, electrical, computer accessories, fashion industry, eatables, generally those items which are perishable or become obsolete easily. These products follow the ramp type demand and the same is considered in this article focuses. Taking into account, the FMCG sector, the planning horizon is kept finite. Supplier – retailer coordination is used to decide the trade credit period between the two. Also, its different cases and subcases are discussed. In this article, the total cost of the retailer and the supplier in no coordination and coordination cases are derived. Trade credit policy is discussed in detail, which includes both the interest earned and interest charged parameters. The optimality of the supplier's joint cost is also obtained. Numerical examples are discussed to support the theoretical outcomes. The same is solved by using Mathematica 12.0 software. A new algorithm has been framed. The sensitivity analysis and concluding remarks mark the end of the work for the supplier–retailer coordination under seasonal products with ramp type demand for all consumer packaged goods or FMCG industry.

Keywords. Inventory Management; Trade Credit; Seasonal Demand; Supplier – Retailer Coordination; FMCG (Fast Moving Consumer Goods); Ramp–Type Demand, Consumer Packaged Goods

INTRODUCTION

In today's world, one alone can't survive, the same condition applies to the market also. Alone players (be it suppliers or retailers/sellers or buyers) can't survive. Moreover, if they have coordination before stocking up their goods by understanding their local market very well, their profits can be increased manifolds, the losses can be reduced, customers' grievances can be met successfully, lost sales can be reduced, seasonal demand can be taken care of properly, and even substitutable markets can be searched if there is a two – way coordination between the supplier and the retailer. The coordination between the parties can ease the position of the market and even the financial constraints among the players in the market can also be taken care of. This coordination can lead to a very important aspect of the present-day market and that is trade credit.

The general trend says that whenever we make a purchase, we pay the seller, but in big businesses where there is the involvement of millions and even more, a trust factor plays a vital role and this gave rise to a term called trade credit, in which a seller allows the buyer to pay for their purchased items in a period. This given period is such that no loss is borne by the seller or supplier of goods. The main important point here to be kept in mind is, that this credit period is beneficial for both the parties though

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Supplier – retailer inventory coordination with trade credit for seasonal demand in FMCG sector FREE

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Supplier-Retailer Coordination With Trade Credit for Deteriorating Items for FMCG

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Abstract. In the prevailing market conditions, an integrated approach of the supplier and the retailer is required for successfully managing the supply chains. The present time also focuses on the requirement of trade credit as well, as it's been proved till now that it is for the profit of both the supplier and the retailer. The inventories deteriorate with time, especially chemicals in the laboratory, thin films when exposed to different atmospheric forces, the items having a shorter shelf life, the items like fruits, food products, perishable products, consumer goods, and fashionable items (car accessories, mobile phones, clothes), pieces of machinery are also exposed to wear and tear. The research work focuses on a mathematical model with constant demand, deteriorating inventory, supplier – retailer coordination for permissible delay in payments, and the retailer's interest earned when the replenishment cycle is shorter than the credit period rate, for various types of inventories related to different mechanical and engineering industries and even for fast-moving consumer goods or consumer packaged goods. Thus the paper attempts to calculate the total cost of the retailer with different parameters. A numerical example is also discussed and sensitivity analysis at the last completes the paper.

INTRODUCTION

The classical EOQ and EPQ models are extended by many researchers from time to time by adding new parameters accommodating more real-world scenarios; like deterioration, trade credit, permissible delay in payments, inflation, different types of demand, fuzzy set theory, single and multi-echelon supply chain and so on. The effect of deterioration on the inventory was firstly bought under the scope of study by Ghare and Schrader [1]. The model developed by two of them were simple and later many researchers extended their work, as in 1973 Covert and Philip [2] developed an EOQ model in which they had considered the variable rate of deterioration, and Weibull distribution was used along with other conditions in which no shortages were allowed, also constant demand was taken with instantaneous supply.

Generally, the norms of the market say that the simple way is, we buy a product and we pay for that. But in practicality in maximum businesses, there is an option called Trade Credit. In this the supplier allows the retailer a particular period, when the purchases are made, to pay for that. It is within a permissible limit so that no losses are incurred by the supplier. Many times to counter the risk of default, the supplier offers a discount to encourage earlier payments. In 1973, Haley and Higgins [3], by considering the classical lot size model, introduced the relation

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Effect of Variable Inflation on Supply Chain Management in FMCG Sector with Single Supplier-Multiple Retailers and Trade Credit

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Keywords:
equipoise, trade credit, FMCG, single supplier, multiple retailers, variable inflation rate

ABSTRACT

The main contribution of this article is to inspect the role of variable inflation rate in every replenishment cycle in supply chain management. An inventory, time, and inflation-dependent demand are contemplated. Some completely novel concepts are introduced. Inflation and its effect are studied in every single replenishment cycle, furthermore, equipoise is attained under a single supplier – multiple retailers' coordination in a highly dynamic and competitive free market. Moreover, the equipoise cost of the retailer is used to ultimately reduce the costs of both the supplier and retailers. The algorithms are discussed in detail to understand the working of our new model, which is close to real-world based problems encountered in inventory management. Numerical examples are discussed in support of theoretical aspects mentioned with the help of Mathematica software 12.0.

1. INTRODUCTION

Inflation is the result of a greater supply of money than its demand. Greater demand than supply, leads to inflationary conditions in an economy. Buzacott [1] was the first researcher who wrote something about inflation in inventory models by formulating a minimum cost model for a single item in inventory and studied the effect of inflation. Guria et al. [2] framed an inventory policy for an item with inflation and demand depending on the selling price by adding a new parameter of immediate part payments to the wholesaler. In 1985, Goyal [3] gave an EOQ model with permissible delays in payments. Later in 1995, Aggarwal and Jaggi [4] extended their work and gave a relation between sales revenue and delay period along with the rate of interest. Supplier-Retailer coordination/setup is taken up by immmerable researchers. Esmaeili and Nasrabadi [5] in 2020 talked about the single vendor-multiple retailers set up.

Thusly, the main contribution of this article is to examine the role of variable inflation rates in inventory management. The main reason behind considering the variable inflation in each replenishment cycle is the competitive and fast changing scenario exiting in the present world and as a result, the prices rise faster as compared to earlier. A supplier with multiple retailers' setups is considered. Equipoise of total inventory cost for multiple retailers in a given competitive environment is calculated. The supplier and multiple retailers' trade credits are considered and the process is controlled by the supplier showing the well-coordinated supply chain management. As per our literature review, the variable inflation per cycle is not taken into account by any researcher.

2. LITERATURE REVIEW

Well, there is a lot to discuss in the literature review, here we will consider those research articles only which are related to the scope of our research. Till now, much has been talked about factors that impact the cost and profit of a supplier and a retailer. Demand patterns, deterioration, trade credit, shortages, availability of raw material, transportation cot, set up cost, holding cost, purchase cost, ordering cost, lost sales, opportunity cost, labor cost, machinery cost, inflation, seasonal patterns of a product and many more are all pillars that need to be studied well to make a model in inventory management that reduces cost or maximizes the profit of a supplier or retailer or the whole system. Inventory-led and time-led demand is such, which can't be overlooked to tackle inventory management paraphernalia. Inventory managers can't neglect the impact and after-effects of inflation, and the dynamic nature of inflation is the most cumbersome thing to be taken care of. In a market where there are high inflationary conditions and dynamic forces are most active, coordination among the players of the market can highly affect some important costs related to the inventory system. In the futuristic and promising industry of FMCG, everything is or will be found, which is included in Inventory Management System.

In 2009, Mirzazadeh et al. [6] in their inventory model, under finite production rate talked about an uncertain inflationary setup. In 2011, Tripathi [7] discussed important parameters of pricing and ordering policy under inflation with permissible delay in payments. In 2013, Shastri et al. [8] added manufacturers along with suppliers and retailers for deteriorating items in inflationary situations. Weibull Deterioration is added by Singh and Panda [9] in 2015 along with the above-taken parameters. In 2015 itself, Kiniwa et al. [10] added deflation in the price stabilization process. In 2020, Geetha and Udayakumar [11] wrote about deteriorating items along with inflation-induced demand which was time-dependent as well in infinite horizon. In 2020, Sundararajan et. al. [12] studied the impact of delays in payments along with

Analysis of a Supply Chain Model with Weibull Demand and Deterioration under Finite Planning Horizon: Mathematical Inventory Model, Optimality, and Sensitivity Analysis

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Abstract

A significant financial loss can occur in a supply chain system due to the degradation of items in inventory. In this study, we have analyzed a supply chain model that examines deterioration in a finite planning horizon model with Weibull market demand. We have discussed the formulation of a mathematical inventory model with a finite planning horizon and its solution. To illustrate the optimality of replenishment cycles in this inventory model, we have provided a numerical example. We have determined the total optimal cost, which is a convex nonlinear function by establishing the positive definiteness of the Hessian matrix. We have conducted a sensitivity analysis of all parameters using different tables and graphs. In addition, we have dedicated a separate section to discuss the results obtained and provide managerial insights.

Keywords: -Weibull Demand, Finite Planning Horizon, Supply Chain Management, deterioration

Introduction:

In 2014, Jaggi explained that lead time refers to the duration between placing an order and receiving the items, which indicates the time needed for delivery. Lead time can vary for different inventory problems and may require clarification. In the pre-processing, processing, and post-processing stages, companies analyze lead time for manufacturing, inventory management in the supply chain process, and project management. By comparing outcomes to predetermined benchmarks, they can identify inefficiencies. Shortening lead time can simplify procedures, accelerate production, and improve output and profits. Lead time can be influenced by manufacturing processes and inventory management. For instance, keeping finished goods in local warehouses can increase lead time compared to finishing specific parts elsewhere. Late delivery of components due to transportation issues can disrupt or delay manufacturing, affecting productivity.

The Weibull distribution can be characterized as a generalized gamma distribution with both shape and size parameters represented by k. When dealing with items that degrade according to the Weibull distribution, demand can vary depending on the selling price in different