A STUDY ON DECENTRALIZED SUPPLY CHAIN INVENTORY MODELS UNDER FINITE PLANNING HORIZON

Thesis Submitted for the Award of the Degree of

DOCTOR OF PHILOSOPHY

in

Mathematics

By

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LOVELY PROFESSIONAL UNIVERSITY

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2025

DECLARATION

I, hereby declared that the presented work in the thesis entitled ***"A Study on decentralized supply chain inventory models under finite planning horizon"*** in fulfilment of degree of **Doctor of Philosophy (Ph. D.)** is outcome of research work carried out by me under the supervision of **Dr. Nitin Kumar Mishra**, working as Professor, in the School of Chemical Engineering and Physical Sciences of Lovely Professional University, Punjab, India. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

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CERTIFICATE

This is to certify that the work reported in the Ph. D. thesis entitled ***"A Study on decentralized supply chain inventory models under finite planning horizon"*** submitted in fulfillment of the requirement for the award of degree of **Doctor of Philosophy (Ph.D.)** in the Mathematics, is a research work carried out by Prerna Jain, 12116774, is a bonafide record of his/her original work carried out under my supervision and that no part of thesis has been submitted for any other degree, diploma or equivalent course.

Nitin 05-04-25

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Name of supervisor: **Dr. Nitin Kumar Mishra**

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Abstract

This thesis explores decentralized supply chain inventory models under a finite planning horizon, aiming to optimize inventory management through game-theoretic approaches, blockchain technology, and environmental considerations. The research focuses on four key objectives: analyzing a decentralized three-echelon supply chain model using blockchain technology, studying a decentralized multi-period two-echelon model, examining the Stackelberg game model under decentralization, and investigating the comparison between decentralized and centralized supply chains.

Chapter 1 introduces the concepts of supply chain management, inventory control, decentralization, blockchain technology, and game theory, establishing the foundation for the research. The chapter outlines the challenges and potential benefits of decentralized supply chain management, providing context for the thesis's focus on finite planning horizons. The importance of carbon emission policies and their impact on inventory flow is also highlighted.

Chapter 2 Literature Review delves into previous research in supply chain management, emphasizing the methodologies employed in decentralized inventory models. It discusses empirical studies, gaps in existing literature, and the importance of emerging trends such as blockchain technology in optimizing supply chain operations. This chapter provides the theoretical groundwork for the subsequent models and identifies opportunities for future research in decentralized inventory management.

Chapter 3 Decentralized Three-Echelon Supply Chain Model Using Blockchain Technology focuses on developing a three-echelon supply chain model under a finite planning horizon. Blockchain technology is integrated into the model to enhance information transparency and coordination among supply chain members. The chapter provides the mathematical model for the retailer, manufacturer, and supplier, offering a comprehensive methodology to solve the optimization problem. Numerical examples and sensitivity analysis highlight the impact of blockchain-enabled information-sharing on supply chain costs and replenishment strategies. This chapter concludes by emphasizing blockchain's role in reducing information asymmetry and promoting supply chain efficiency.

Chapter 4 Blockchain-Enhanced Inventory Management in Decentralized Supply Chains extends the research by exploring a two-echelon decentralized model, incorporating blockchain technology to improve decision-making and coordination. The chapter defines the mathematical model, focusing on the strategic interactions between the retailer and manufacturer within a finite planning horizon. By employing numerical examples and sensitivity analysis, the research demonstrates how blockchain technology facilitates transparency, leading to optimized replenishment schedules and reduced overall costs. This chapter underscores the advantages of adopting blockchain in decentralized supply chains, including enhanced adaptability to market fluctuations.

Chapter 5 Enhancing Supply Chain Efficiency with Blockchain investigates the influence of information sensitivity on supply chain efficiency using blockchain technology. A blockchain-enabled inventory model is formulated, illustrating how the transparency and traceability offered by blockchain impact the manufacturer's profitability. Sensitivity analysis reveals that information-sharing levels are crucial to optimizing inventory management and maximizing profitability. The findings provide practical insights into the use of blockchain to overcome information asymmetry in supply chains, reinforcing the model's significance for modern supply chain practices.

Chapter 6 Optimizing Inventory Management with Stackelberg Game and Linear Differential Equations examines the application of the Stackelberg game model in a decentralized supply chain. The leader-follower dynamics between the retailer and manufacturer are explored using linear differential equations under a finite planning horizon. Numerical illustrations demonstrate the model's effectiveness, while radar chart analysis visualizes the sensitivity of various inventory parameters. Comparative analysis with Benkherouf’s centralized inventory model showcases the strategic flexibility and advantages of decentralized decision-making. The chapter concludes by emphasizing the Stackelberg game approach's role in optimizing inventory management.

Chapter 7 Inventory Models Under Carbon Tax and Cap-and-Trade Policies introduces inventory models considering carbon emission regulations under both decentralized and centralized supply chains. It integrates carbon tax and cap-and-trade policies into the models, addressing the need for sustainable supply chain practices. Mathematical formulations and numerical examples demonstrate the effects of different carbon policies on inventory costs and replenishment strategies. The parametric analysis highlights how decentralized models can offer better adaptability and cost-efficiency considering environmental regulations.

Chapter 8 Conclusion and Future Work synthesizes the findings from the research, concluding that decentralized supply chain models, when integrated with blockchain technology and game-theoretic approaches, significantly enhance supply chain efficiency, profitability, and adaptability. The thesis contributes to the field by bridging the gap between theoretical models and practical applications in decentralized supply chains. Future work includes extending these models to incorporate real-world data, leveraging advanced technologies like machine learning for predictive inventory management, and exploring multi-echelon supply chains.

This research provides a comprehensive exploration of decentralized supply chain inventory models, addressing key challenges in finite planning horizons. It offers innovative solutions, such as blockchain-enabled transparency and Stackelberg game-based optimization, to improve decision-making and coordination in supply chains. The thesis findings highlight the potential of decentralized models to adapt to dynamic market conditions and carbon emission regulations, thereby contributing valuable insights for supply chain managers and researchers.

This is dedicated to my Parents,

Mrs. Poonam Jain

and

Mr. Ramnik Jain

Acknowledgment

It is with the deepest sense of gratitude and respect that I begin by acknowledging the unwavering guidance and support of my supervisor, Professor Dr Nitin Kumar Mishra, Department of Mathematics, Lovely Professional University. Throughout every stage of this long and challenging journey, his mentorship has been a source of both intellectual growth and personal encouragement. He has not only been a guide but a constant source of strength and inspiration, offering insights that went beyond the scope of academia. His ability to nurture my curiosity, challenge my limits, and provide a steady hand through the complexities of research has left an indelible mark on both my academic and personal life. I will forever remain deeply thankful for the invaluable wisdom and patience with which he navigated me through this journey, offering full support at every turn. His belief in me and my work has given me the confidence to pursue this goal with determination, and I shall be forever indebted to him for his unparalleled mentorship.

I would like to extend my heartfelt appreciation to Lovely Professional University for providing me with the opportunity to conduct my research. I am deeply grateful to all the staff, faculty members, and panel reviewers who offered me their valuable time, insightful critiques, and guidance. Their help in pointing out my mistakes and encouraging me to improve has been crucial to my Ph.D. journey. The constructive feedback and support I received from the faculty were instrumental in refining my research, and I am grateful to each member of the university community who contributed to this process.

I would also like to express my sincerest thanks to my family, whose love and support have been the bedrock of my journey. My parents, Mrs. Poonam Jain and Mr. Ramnik Jain are the true pillars of my success. Their unyielding encouragement, guidance, and sacrifices have allowed me to dream and strive beyond what I thought was possible. Every step I have taken reflects their constant support, and I owe this accomplishment to them. Without their wings, I would not have flown this far.

A heartfelt note of gratitude goes to my late grandparents**,** Smt. Bimla Wanti Jain and Mr. Rikhi Ram Jain, who may no longer be with me, but whose presence I have felt every step of the way. Their love, values, and blessings have been a guiding light throughout my life, and I dedicate this accomplishment to their cherished memory.

I would also like to acknowledge the support of my siblings, Pallavi Jain and Rhythm Jain. Their constant encouragement, patience, and belief in my capabilities have pushed me to persevere, even in moments of doubt. Their love has been a source of strength, and I could not have reached this milestone without their support.

I am immensely grateful to Dr Ranu, my senior at Lovely Professional University, for sharing her invaluable experiences, providing me with guidance through many hurdles, and offering support that has been instrumental to this thesis. Her presence has been a significant source of encouragement and insight, and I truly appreciate the many ways in which she has helped me throughout this journey.

Special thanks are due to my teacher, Ms. Jaswinder Kaur, from Sarvodaya She-Shiksha Vidyalaya**,** Saraswati Vihar, Delhi for kindling my love for mathematics at an early age. Her teaching and guidance made me fall in love with the subject, shaping the foundation for everything I have achieved today. I also extend my deepest gratitude to Mr. Rakesh Pachisia, who challenged me during my senior secondary years, deepening my passion for mathematics and motivating me to pursue the subject with a greater sense of purpose and commitment. I am also thankful to my school for laying the foundation that has brought me to where I am today.

Moreover, I would like to extend my heartfelt thanks to the faculty members of the Mathematics Department at Satyawati College, Delhi University, who provided me with a solid academic foundation during my undergraduate studies. Their knowledge, dedication, and support were essential in shaping my future academic pursuits and nurturing my love for the subject.

Finally, to all those who have walked this path with me—whether offering academic advice, moral support, or simply a kind word—I extend my deepest gratitude. This journey has been one of immense learning and growth, and it would not have been possible without the collective encouragement and support of so many. Thank you, all.

List of Contents

LIST OF TABLES

LIST OF FIGURES

Chapter 1 Introduction

1.1 Operational Research

1.1.1 History and Development

Operations research has a rich history that dates to the early 20th century. During World War I, military organizations began employing scientific methods to enhance the efficiency of their operations, laying the groundwork for the field of operations research. Over the following decades, this discipline continued to evolve and expand, finding applications in various industries such as transportation, healthcare, manufacturing, and logistics (Chopra & Meindl, 2013).

In the 1930s and 1940s, significant advancements were made in operations research with the development of mathematical modeling, optimization techniques, and computer-based simulations. These innovations empowered researchers and practitioners to tackle increasingly complex problems and make more informed decisions. The post-World War II era marked a surge in the application of operations research as businesses and governments recognized its potential to reduce costs, improve decision-making processes, and enhance efficiency. Operations research techniques have been used to solve a wide range of problems, including supply chain optimization, workforce scheduling, inventory management, and resource allocation (Luss & Rosenwein, 1997).

Today, operations research is a well-established and highly interdisciplinary field, drawing on principles from diverse areas such as mathematics, statistics, computer science, and engineering. It remains crucial in addressing complex issues and optimizing decision-making across various industries and organizations.

1.1.2 Operation Research in Management

Operational research plays a vital role in optimizing various aspects of management, including supply chain inventory management. By applying operational research techniques, Businesses can improve their inventory management by making decisions based on data, which leads to increased efficiency and effectiveness, improving customer satisfaction while reducing costs and increasing profitability (Priniotakis & Argyropoulos, 2018).

The purpose of operational research in supply chain management is to optimize inventory levels, reorder points, and order quantities while also considering factors such as demand variability, lead times, and customer service levels. This facilitates the maintenance of optimal inventory levels for businesses, ensuring timely availability while simultaneously reducing costs and maximizing customer satisfaction. (Chopra & Meindl, 2013).

Furthermore, operational research techniques can help businesses incorporate price decreases into their inventory models, effectively minimizing supply chain costs. The use of flexibility tools, such as product substitution in inventory management, can be optimized through operational research, providing businesses with adaptability and timely responses (Farasyn et al., 2011).

Moreover, operational research techniques can also be applied to improve the planning and management of hospitals within the supply chain. By using simulation methods, hospitals can optimize their operations, leading to enhanced efficiency and better patient care.

1.1.3 Operational Research in Inventory Management

The management of inventory plays a crucial role in supply chain operations, including decisions about quantity and timing. These decisions ensure that the optimal level of inventory is available when customers need it, while minimizing costs. Using operational research techniques, supply chains can optimize inventory management (Benkherouf et al., 2017). Using optimization algorithms and models, operational research determines optimal reorder points and order-upto levels, considering factors such as demand variability, lead times, holding costs, and customer service levels.

Applying operational research techniques to inventory management allows businesses to achieve more efficient and effective supply chain operations. This leads to improved reduced costs, increased profitability, and customer satisfaction. Additionally, operational research can help businesses incorporate factors such as price decreases into their inventory models, allowing them to minimize supply chain costs effectively. Furthermore, it provides insights into the use of flexibility tools, such as product substitution, in inventory management. Through the optimization of the decision-making process, businesses can effectively ensure the availability of optimal inventory levels at the right time, thereby minimizing operational costs and enhancing overall customer satisfaction.

Operational research in supply chain inventory management involves using mathematical models, algorithms, and optimization techniques to determine reorder points, order quantities, and optimal inventory levels. The implementation of these techniques facilitates efficient and effective inventory management for businesses, resulting in cost reduction and enhanced customer satisfaction. By incorporating operational research techniques, businesses can make data-driven decisions that account for factors such as demand variability, lead times, and customer service level targets. This approach also helps businesses incorporate price decreases into their models, allowing for cost minimization and informed decision-making.

A major purpose of operational research is to optimize inventory management in supply chains through mathematical models and optimization techniques. This optimization improves product availability, customer loyalty, and overall business performance by utilizing flexibility tools for adaptability and timely responses.

1.2 Supply Chain Management

Modern business operations are based on the concept of supply chain management (SCM), which involves the planning, coordination, and optimization of the flow of information, goods, and services from raw materials to products delivered to customers (Althaqafi, 2020). Today's globalized and interconnected business landscape makes effective SCM a crucial driver of organizational competitiveness, resulting in increased efficiency, cost reductions, and customer satisfaction (Kaliani Sundram et al., 2016).

One of the fundamental principles of successful SCM is the integration of various components within the supply chain, fostering collaboration and coordination among suppliers, manufacturers, distributors, and customers. By integrating these systems, we can streamline processes, reduce redundancies, and improve information sharing, allowing us to respond quickly to changes in the market and to customer demands. (Christopher, 2016). By aligning objectives and fostering open communication, organizations can build resilient supply chains capable of adapting to disruptions and uncertainties (Lambert & Cooper, 2000).

Technological advancements have significantly impacted SCM practices. The adoption of digital technologies such as blockchain, the Internet of Things (IoT), and advanced analytics has enhanced transparency, traceability, and real-time data exchange across the supply chain (Ivanov et al., 2019). For instance, blockchain technology enables secure and immutable record-keeping, improving trust among supply chain partners and reducing the risk of fraud (Saberi et al., 2019). These technologies facilitate better decision-making and enable organizations to optimize their supply chain operations effectively.

Figure 1.1 Supply Chain Management

Moreover, sustainable supply chain management has gained prominence as businesses recognize the importance of environmental and social responsibility. Integrating sustainability into SCM involves considering the ecological and social impacts of supply chain activities, leading to practices that are not only economically beneficial but also ethically sound (Seuring & Müller, 2008). This approach enhances corporate reputation and meets the growing consumer demand for ethically produced goods.

1.3 Inventory

Inventory management plays a pivotal role in business operations as it directly influences the company's capacity to meet customer demand, control expenses, and guarantee the availability of crucial products and materials (Ra. & Nayak, 2017; Otuya & Joseph, 2017; López et al., 2021). This research delves into the fundamental principles of inventory management, its importance, and the strategies employed to optimize inventory levels and achieve organizational goals. Figure 1.2 depicts the journey of raw materials from suppliers to manufacturers, who then produce and distribute finished goods to retailers before reaching end customers. The red arrows indicate the flow of physical products, while the black arrow denotes the flow of information and money across each stage of the supply chain. By integrating this supply chain illustration, the study emphasizes the interconnected nature of inventory management and supply chain coordination. Figure 1.2 provides a visual representation of the multiple tiers involved in supply chain operations and underscores the importance of strategic inventory control at each stage to ensure overall efficiency and responsiveness.

Figure 1.2 Illustration of a Supply Chain

1.3.1 Basics of Inventory: Definition and Types

Inventory refers to the stock of goods or materials held by a business, including work-in-progress, finished products, and raw materials (Rao & Nayak, 2017; Otuya & Joseph, 2017; López et al., 2021). Effective inventory management involves identifying the optimal amount and placement of these goods to support production, meet customer demand, and minimize costs (Rao & Nayak, 2017; Otuya & Joseph, 2017; Gill et al., 2013). Inventory can be classified into several categories:

1.3.1.1 Raw Materials

Production requires the following inputs or components. Raw materials are sourced from suppliers and form the foundation for the manufacturing process. Effective management of raw materials ensures that production processes run smoothly and without interruption (Rao & Nayak, 2017; Otuya & Joseph, 2017).

1.3.1.2 Work-in-Progress (WIP)

These are items that are currently in the process of being manufactured. Material and components that have started the manufacturing process but have not yet been completed as finished goods are included in the work in progress inventory. Proper management of WIP inventory is essential to avoid bottlenecks and ensure a continuous flow of production (Rao & Nayak, 2017; Otuya & Joseph, 2017).

1.3.1.3 Finished Goods:

These goods represent finished products available for customer purchase. The maintenance of finished goods inventory is vital for promptly and effectively meeting customer demand. The management of this inventory type necessitates ensuring adequate stock levels to fulfil customer orders while avoiding overstocking, which can result in capital and increased storage costs (Rao & Nayik, 2017).

1.3.1.4 MRO Inventory

MRO, or Maintenance, Repair, and Operations, inventory refers to the stock of materials and supplies used to maintain and repair production equipment, facilities, and other infrastructure (Vrat, 2014). MRO inventory is essential to ensuring the smooth and uninterrupted operation of a manufacturing or service-providing organization, as it enables the prompt repair and maintenance of critical assets. (Singh & Yadav, 2019)

1.3.1.5 Safety Stock

Stocks kept in reserve to mitigate risks associated with demand uncertainty and supply chain disruptions are termed safety stocks. Safety stocks help organizations maintain consistent levels of customer service even when demand or supply fluctuates unexpectedly.

1.3.1.6 Pipeline Inventory

Raw materials are transported to manufacturing facilities or finished products are shipped to distribution centers as part of pipeline inventory (Urissa, 2019).

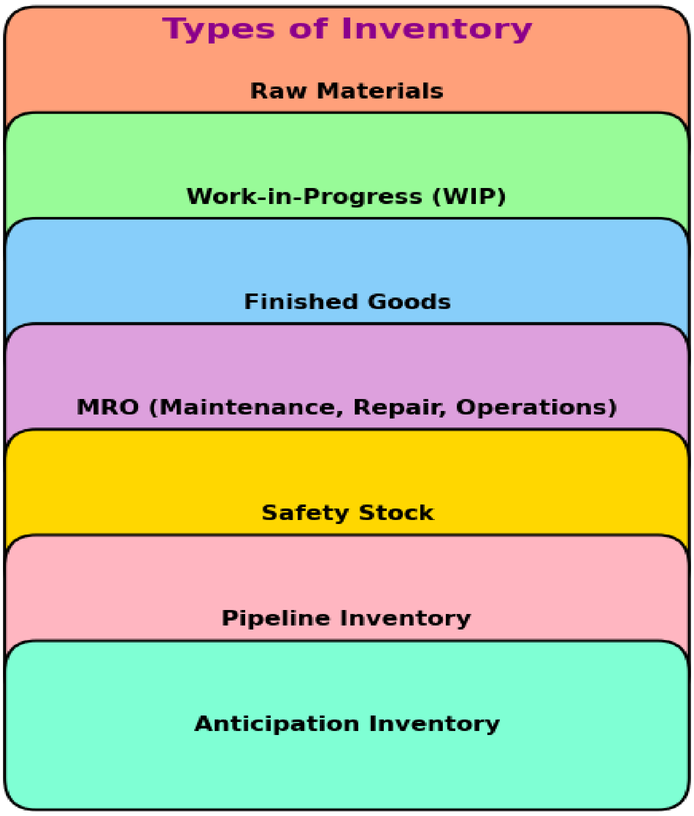


Figure 1.3 Types of Inventories

1.3.2 Importance of Inventory Management

Inventory, which can consist of raw materials, work-in-process components, or finished products, is a significant component of the supply chain and can account for a substantial portion of the total supply chain costs over the specified period. Developing sophisticated software, training employees, and maintaining the model takes time and money (Mashayekhy et al., 2022; Sbai & Berrado, 2018). A competitive advantage is maintained through effective inventory management, which reduces costs, improves customer satisfaction, and increases profitability. To improve supply chain efficiency and responsiveness to changing market conditions, companies need to forecast demand accurately, optimize inventory levels, and reduce stockout risks (Showkat & Ali, 2020; Mashayekhy et al., 2022).

1.3.3 Necessity of Maintaining Inventory

Maintaining inventory is essential for several reasons, including ensuring product availability, smoothing production fluctuations, and protecting against supply chain disruptions (Obadire et al., 2022; Otuya & Joseph, 2017).

1.3.3.1 Ensuring Product Availability

Having sufficient inventory ensures that products are available to meet customer demand promptly. This is particularly important in industries where customer expectations for quick delivery are high (Otuya & Joseph, 2017; López et al., 2021).

1.3.3.2 Smoothing Production Fluctuations

Inventory helps in managing the variability in production processes. It serves as a protective measure against fluctuations in supply and demand, guaranteeing uninterrupted production flow by mitigating potential delays caused by lack of materials shortages.(Obadire et al., 2022).

1.3.3.3 Protecting Against Supply Chain Disruptions

By keeping inventory, you are protected from unexpected supply chain disruptions, such as delays from suppliers, political instability, or natural catastrophes. This helps in maintaining continuity of operations and avoiding stockouts. A business must carefully weigh the costs associated with carrying inventory, such as storage, insurance, and obsolescence risk, against the benefits related to having appropriate products where they are needed at the right time (Otuya & Joseph, 2017; López et al., 2021).

1.3.4 Goals of Inventory Control

To maximize customer satisfaction, inventory control must minimize costs associated with inventory and ensure availability of products. This involves optimizing reorder points, order quantities, and safety stock levels to strike a balance between customer service levels and inventory costs (Mashayekhy et al., 2022; Showkat & Ali, 2020).

Effective inventory management is crucial for businesses to thrive in a competitive market. By understanding the fundamentals of inventory, the necessity of maintaining appropriate levels, and the goals of inventory control, organizations can develop strategies to optimize their inventory and enhance profitability (Kolawole et al., 2019; Otuya & Joseph, 2017).

1.4 Decentralization and Centralization

1.4.1 Decentralization

Decentralized supply chain models are organizational structures where decision-making authority is distributed among various members of the supply chain network rather than being centralized in a single entity. This allows each member to manage its operations independently and flexibly, enabling quicker responses to changes in demand (Hadikusuma & Siagian, 2022). A key characteristic of decentralized models is the reliance on high levels of trust, cooperation, and effective information flow among supply chain members. Decentralization in supply chain management offers several benefits. Its ability to lower process variability is one of its primary advantages. It also helps in reducing costs when the supply chain experiences disruptions (Ratna et al., 2022). Decentralized decision-making and adaptability allow individual members to respond to market changes more efficiently. Additionally, decentralization encourages risk pooling, depending on the level of variability and the number of locations involved, fostering innovation and creativity as each member aligns decisions with their own goals.

Figure 1.4 Decentralization

Recent advancements in technology have significantly enhanced the potential of decentralized supply chains. The integration of blockchain technology has provided decentralized ledgers that promote transparency and cooperation among network participants. A major benefit of the Internet of Things (IoT) is that it allows supply chain members to communicate in real-time and make informed decisions together (Dong et al., 2022). These technological integrations make decentralized supply chains an attractive strategy for achieving competitive advantages.

Despite these benefits, decentralized supply chains face challenges, such as potential coordination and communication issues among members. Inefficiencies like duplication of efforts and increased costs can arise due to poor visibility of inventory levels (Kechil et al., 2022). Ensuring compliance with quality, safety, and social responsibility standards is also more complex in a decentralized system. Therefore, implementing appropriate legal and regulatory frameworks is crucial to maintain transparency and accountability (Liao & Wang, 2018).

1.4.2 Centralized

Centralized supply chain models involve a single entity, typically a central authority such as a manufacturer or a main distribution center, managing and making decisions for the entire supply chain network. In this approach, inventory control, order processing, and logistics planning are coordinated from a central point (Saad & Bahadori, 2018). This centralization allows for a holistic view of the supply chain, leading to more efficient inventory management and decision-making.

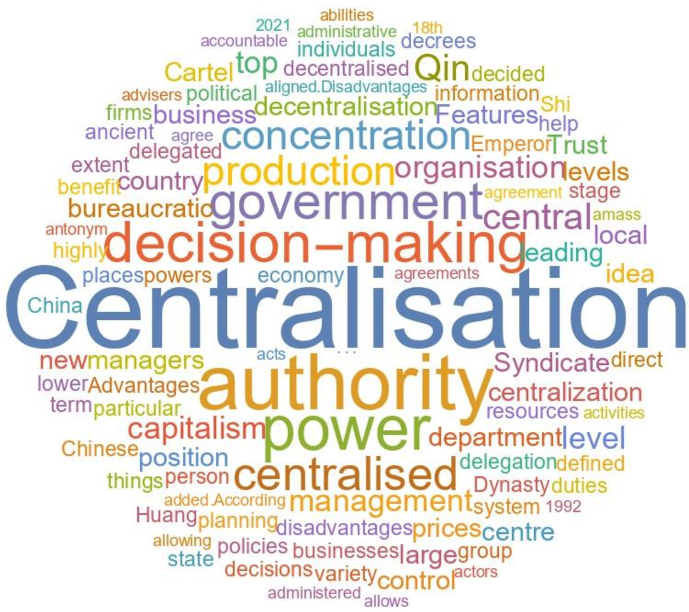


Figure 1.5 Centralization

One of the key benefits of centralized supply chain models is the ability to leverage economies of scale. By consolidating purchasing and inventory management, organizations can negotiate better terms with suppliers, reduce per-unit costs, and optimize stock levels (Wang, 2011). Additionally, centralized control provides a unified approach to demand forecasting and inventory distribution, minimizing redundant stock across various locations and reducing the risk of overstock or stockouts (Ding & Kaminsky, 2018).

Centralized systems also offer improved coordination and synchronization of inventory flow. With a comprehensive overview of the entire supply chain, the central authority can optimize inventory levels, streamline logistics, and reduce variability in supply chain processes (Saad & Bahadori, 2018). This approach can be particularly advantageous in stable environments where demand patterns are predictable, and lead times are short.

Despite the benefits, centralized supply chain models come with certain drawbacks. A significant risk is the potential for a single point of failure; disruptions at the central hub can impact the entire supply chain network (Wang, 2011). Additionally, centralized models may incur higher transportation costs and longer delivery times, as goods need to be shipped from the central location to various distribution points (Das & Tyagi, 1997).

Centralized systems can also lack the flexibility to respond quickly to changes in local market conditions. Since decision-making is concentrated at the top, the ability to adapt to regional demand fluctuations or supply chain disruptions may be slower, reducing the supply chain's overall agility (Ding & Kaminsky, 2018).

Table 1.1 Comparison between Decentralized and Centralized Inventory Control

Aspect	**Centralized Inventory Control**	**Decentralized Inventory Control**
Decision-Making Authority	Managed by a central entity, such as a manufacturer or main distribution center.	Managed by individual entities (e.g., retailers, regional warehouses) within the supply chain.
Coordination and Synchronization	Better coordination and synchronization of inventory across the entire supply chain.	More flexibility to respond to local market conditions but may lack overall coordination.
Efficiency	Improved efficiency in inventory management due to a holistic view of the supply chain.	More responsive to individual needs but may lead to higher overall inventory levels.
Control over Costs	Can leverage economies of scale and negotiate better terms with suppliers, reducing costs.	May incur higher costs due to a lack of economies of scale and coordination.
Risk	Risk of a single point of failure, potential delays in delivery, and higher transportation costs.	Reduced risk of a single point of failure may result in increased overall costs.
Flexibility	Less flexible and slower in response to local market changes.	More flexibility and responsiveness to local market conditions.
Inventory Levels	This can lead to reduced inventory levels through centralized control and demand forecasting.	This may result in higher inventory levels due to decentralized decision-making.
Response to Market Conditions	Slower to respond to market changes due to centralized decision-making.	Allows quicker response to local demand fluctuations and market changes.
Optimal Scenarios	Effective when lead times are short, and demand is stable.	More advantageous when lead times are longer, and demand is variable.
Potential Drawbacks	Risk of centralized failure increased transportation costs, and potential delivery delays.	Lack of coordination, higher inventory costs, and difficulty in leveraging economies of scale.

1.5 Blockchain Technology

Blockchain technology, initially introduced as the backbone of Bitcoin in 2008, has evolved into a versatile tool with applications extending beyond finance (Difrancesco et al., 2022; Tijan et al., 2019; Saberi et al., 2019). In contrast to traditional centralized systems, blockchain acts as a distributed, decentralized digital ledger, which ensures efficiency, security, and transparency. (Chang & Chen, 2020). Each block in the chain stores transaction data, and every new transaction is updated across the network, providing a robust and secure record (Tijan et al., 2019).

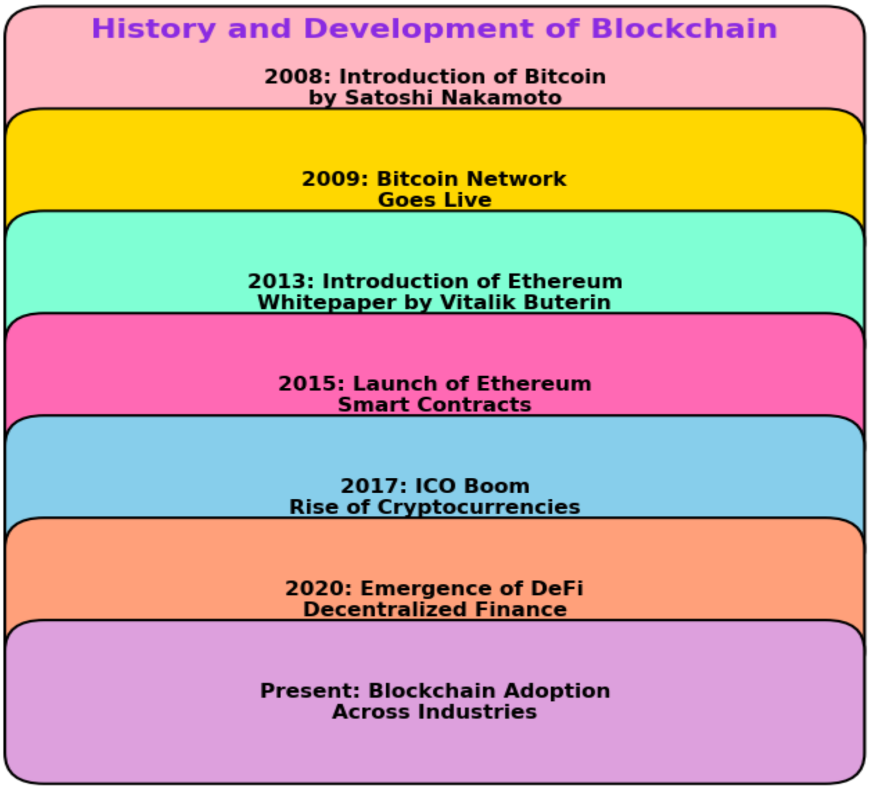


Figure 1.6 History and Development of Blockchain

The increasing complexity of global supply chains has made transparency and traceability crucial (Difrancesco et al., 2022; Saberi et al., 2019). Blockchain technology offers a decentralized platform for tracking goods, documenting ownership, and recording key supply chain events (Tijan et al., 2019; Zhang, 2020). By providing enhanced visibility, blockchain mitigates risks such as counterfeiting and inventory discrepancies.

Among blockchain platforms, Ethereum stands out for its smart contract functionality (Saberi et al., 2019; Song et al., 2019). Introduced in 2015, Ethereum allows automation of supply chain processes, such as triggering payments upon delivery or enforcing conditions in contracts (Tijan et al., 2019). Additionally, Ethereum supports the development of decentralized applications (dApps), streamlining processes from order fulfillment to reverse logistics (Saberi et al., 2019).

With the development of blockchain technology, particularly through platforms like Ethereum, supply chain management could be transformed by enhancing transparency, traceability, and process automation (Difrancesco et al., 2022; Zhang, 2020). These advancements promise to make global supply chains more efficient, sustainable, and resilient (Tijan et al., 2019).

1.5.1 Practical Challenges in Adopting Blockchain Technology in Supply Chains

Despite its potential to enhance transparency, security, and efficiency, the adoption of blockchain technology in supply chains faces several practical challenges. One major issue is the high implementation cost, as setting up a decentralized blockchain network requires significant investment in infrastructure, skilled workforce, and computing power. Additionally, scalability remains a concern, as blockchain networks, especially those based on Proof-of-Work (PoW), can experience slow transaction speeds and high energy consumption, limiting their ability to handle large-scale supply chain operations. Interoperability challenges also hinder adoption, as companies use diverse supply chain management systems that may not seamlessly integrate with blockchain solutions. Moreover, data privacy and security concerns arise, particularly regarding who controls access to sensitive supply chain data in a decentralized system. Another key barrier is regulatory uncertainty, as governments worldwide have yet to establish clear legal frameworks for blockchain adoption in supply chains, creating hesitation among businesses. Lastly, resistance to change from traditional stakeholders and supply chain partners further slows adoption, as many firms are reluctant to overhaul existing processes due to the complexity and perceived risks of blockchain technology. Addressing these challenges requires a combination of technological advancements, industry collaboration, and regulatory support to fully unlock blockchain's potential in supply chain management.

1.6 Stackelberg Game in Supply Chain Management

The Stackelberg game is a strategic decision-making framework developed by Heinrich von Stackelberg, which models a hierarchical interaction between two players: a leader and a follower. The leader moves first, making decisions that the follower observes and responds to accordingly. This sequence of decision-making gives the leader a strategic advantage, allowing them to anticipate the follower's reactions and shape the overall outcomes of the game (Haque et al., 2022).

In supply chain management, the Stackelberg game is utilized to study the interactions between different entities, often involving a leader (such as a manufacturer or a central authority) and a follower (such as a retailer or supplier). The leader typically sets critical parameters like pricing, production schedules, or inventory policies, while the follower reacts by making decisions to maximize their own objectives under the constraints set by the leader (Nagaraju et al., 2015).

For example, a manufacturer (leader) might set wholesale prices, production quantities, or delivery schedules, and the retailer (follower) would then decide on their retail price, order quantities, and inventory levels based on these decisions. This hierarchical structure allows for better alignment between different players' decisions and objectives within a decentralized supply chain (Yue & You, 2014).

The leader in a Stackelberg game uses their influence to guide the follower's behavior strategically. By considering the potential responses of the follower, the leader can design optimal policies or regulations that lead to desirable outcomes, such as increased efficiency, minimized costs, or enhanced profitability for the entire supply chain. If the leader sets a policy to reduce costs or enforce environmental standards, the follower might respond by adjusting their inventory strategy, production process, or technology investment to comply and optimize their operations (Khanlarzade et al., 2019).

This approach creates a balance where both parties achieve their respective goals, contributing to improved supply chain performance. The Stackelberg model provides a framework for understanding how decisions made at different stages of the supply chain affect each other and how a centralized authority can influence decentralized operations to meet overall supply chain objectives (Kumar et al., 2016).

The strategic use of the Stackelberg game facilitates better coordination and alignment of goals within the supply chain. The leader-follower dynamic allows for better anticipation of market changes and demand fluctuations. Leaders can use mechanisms like pricing incentives, penalties, and rewards to influence the follower's decisions, driving optimal behavior and improving overall supply chain efficiency. This game-theoretic approach has practical implications in designing contracts, setting trade credit terms, and managing inventory in decentralized supply chains. By utilizing the Stackelberg framework, supply chains can achieve improved decision-making, coordination, and profitability, as both leader and follower work towards a mutually beneficial equilibrium.

1.7 Carbon Emissions in Inventory Flow

Diverse industrial and logistical activities can cause carbon dioxide (CO_2) and other greenhouse gases to be released into the atmosphere. Emissions from these sources contribute significantly to global warming and climate change. In supply chain operations, carbon emissions are generated at multiple stages, including production, transportation, warehousing, and distribution. For example, manufacturing processes may emit CO_2 due to the use of fossil fuels, while the transportation of goods using trucks, ships, or planes adds to the carbon footprint. Efficient inventory management often aims to minimize costs and ensure timely delivery, but this can sometimes lead to increased carbon emissions. For instance, frequent deliveries or expedited shipping to avoid stockouts can result in higher transportation emissions (Ghosh et al., 2017; Mishra & Ranu, 2022a).

1.7.1 Carbon Tax

It is a tax levied on fossil fuels or various activities that emit carbon. The goal is to make carbon-intensive activities more expensive, thereby encouraging businesses and individuals to reduce their carbon footprint. Governments set a price per ton of CO_2 emitted, and companies that emit carbon dioxide must pay this tax. This creates a financial incentive to lower emissions through more efficient practices or cleaner technologies (Gan et al., 2017). A carbon tax increases the operational costs for businesses, particularly those with high energy consumption or carbon-intensive processes. Companies may need to invest in energy-efficient technologies, optimize logistics to reduce transportation emissions, and re-evaluate their supply chain strategies to mitigate the financial impact of the carbon tax (Fahimnia et al., 2013).

1.7.2 Carbon Trade

Carbon trading also referred to as emissions trading, is an economically driven mechanism designed to regulate pollution by establishing financial incentives aimed at curtailing emissions. It involves the buying and selling of emission allowances or credits. Governments and regulatory bodies impose a cap on the aggregate volume of greenhouse gases that covered entities are permitted to release. Companies are issued emission allowances and can trade these allowances in the market. Company emissions can be sold if they are below their allowance; if they are higher, they must purchase additional allowances. (Gan et al., 2017). Carbon trading creates a flexible mechanism for companies to comply with emission reduction targets. Supply chain managers must consider the cost of purchasing additional allowances if they exceed their emission caps, or they can benefit financially by selling unused allowances. This incentivizes investment in greener technologies and practices to stay within emission limits and potentially profit from surplus allowances (Fahimnia et al., 2013; Mishra et al., 2021).

Carbon trading, particularly through cap-and-trade mechanisms, faces several challenges that impact its effectiveness in supply chain management. One major issue is the lack of standardized regulations across different countries and industries, leading to inconsistencies in carbon credit valuation and trade execution. Additionally, market volatility often affects carbon credit prices, making it difficult for companies to plan long-term sustainability strategies. Fraud and transparency concerns also pose significant risks, as some organizations may engage in carbon offset manipulation or misreport emissions to gain financial benefits. Furthermore, small and medium enterprises (SMEs) often struggle to participate due to high compliance costs and administrative burdens associated with carbon credit transactions. In the context of supply chains, determining the true carbon footprint across multiple tiers of suppliers remains complex, making it difficult to ensure accurate emission accounting. Addressing these issues requires stronger regulatory frameworks, increased transparency through blockchain integration, and the adoption of globally accepted carbon credit validation methods.

1.8 Finite Planning Horizon in Inventory Management

By optimizing the flow of goods and decreasing costs, inventory management is one of the most important functions of any business. Traditionally, inventory models have assumed an infinite planning horizon, treating demand and other variables as constant. However, in real-world scenarios, businesses often operate within a finite planning horizon, where the decision-making time frame is limited (Brown & Rogers, 1973). Finite planning horizon models consider factors like seasonal variations, budget constraints, limited resources, and other elements that can affect inventory levels and ordering quantities, providing a more realistic and adaptable approach to inventory management.

1.8.1 Advantages of Finite Planning Horizon Inventory Models:

Better Allocation of Inventory**:** By considering specific timeframes, these models allow businesses to plan for fluctuations in demand, especially in industries with seasonal variations. This leads to more effective inventory allocation (Alsuwainea et al., 2014).

Improved Decision-Making: With a focus on limited periods, finite horizon models help businesses make more accurate and informed decisions regarding inventory levels, reducing the risk of overstocking or stockouts.

Cost Optimization: These models take perishability or obsolescence of products into account, minimizing costs associated with inventory holding and stockouts (Zhang, 2020).

Enhanced Supply Chain Management: Finite horizon models help in aligning activities across the supply chain, including production capacity and supplier availability, ensuring better synchronization and coordination.

Increased Customer Satisfaction: By maintaining the right inventory levels and reducing stockouts, businesses can meet customer demands promptly, leading to higher customer satisfaction (Alsuwainea et al., 2014).

1.8.2 Challenges in Implementing Finite Planning Horizon Models:

Accurate Forecasting**:** Predicting future demand within the finite horizon is challenging and requires accurate forecasting methods.

Data Availability**:** Reliable data on factors like demand patterns and lead times within the specified horizon can be difficult to obtain.

Model Complexity**:** Finite horizon models can be more complex and computationally demanding compared to infinite horizon models.

Dynamic Demand**:** Changes in demand over time add to the difficulty of implementing these models effectively.

1.9 Real-World Applications of Decentralized Supply Chains

Decentralized supply chains are increasingly being adopted across multiple industries to enhance efficiency, transparency, and resilience. Some key examples include:

- **Pharmaceutical Industry:** Companies like Pfizer and Moderna use decentralized supply chain networks to ensure timely vaccine distribution during pandemics. Blockchain-enabled tracking systems prevent counterfeit drugs and ensure compliance with regulatory requirements.
- **Automotive Industry:** Firms such as Tesla and Ford leverage decentralized supply chains to manage electric vehicle (EV) battery production, ensuring real-time tracking of raw materials like lithium and cobalt to promote ethical sourcing and sustainability.
- **Food Industry:** Companies like Nestlé and Walmart use blockchain-based decentralized supply chains to improve food safety and traceability. The IBM Food Trust blockchain platform allows real-time tracking of food shipments, reducing contamination risks and improving recall processes.

Chapter 2 Literature Review and Research Framework

2.1 Comprehensive Review of Literature

Our analysis delves into a comprehensive examination of decentralized inventory strategies within the supply chain management domain. In this review, we meticulously scrutinized fifty papers, each offering valuable insights into the multifaceted aspects of supply chain operations, with a particular focus on decentralized inventory management practices. Through our thorough examination, we aimed to uncover the underlying trends, emerging technologies, and innovative methodologies that collectively contribute to the advancement of decentralized inventory strategies.

In table 2.1, we present a summary of the key methodologies and technologies explored by various authors in their research on decentralized and centralized inventory management within the realm of supply chain management. This table serves as a reference point for understanding the diverse approaches and solutions proposed by scholars in addressing the challenges and opportunities associated with inventory management in decentralized supply chains.

Our review aims to provide a comprehensive overview of current research in decentralized inventory management, emphasizing historical developments, significant milestones, and key contributors. By synthesizing findings from a wide range of scholarly works, we endeavor to offer valuable insights and perspectives that can inform future research directions and practical applications in supply chain management.

2.1.1 Analysis of Different Methodologies

Our literature review has synthesized various methodologies and common themes prevalent in the selected studies. Researchers, including (Dong & Li, 2009; Inalhan & How, 2006; Ni et al., 2010; Yan et al., 2016) have extensively explored the implications of centralized and decentralized approaches in supply chain management. These studies delineate the advantages and disadvantages associated with each approach. Several papers, such as those by (Liu et al., 2021; Mondal & Giri, 2022; Padiyar et al., 2022a)among others, utilize optimization techniques to address inventory management, production allocation, pricing decisions, and overall supply chain performance. Certain studies, exemplified by the works of (Nagaraju et al., 2016; Wu & Zhao, 2016) and others, employ game theory frameworks, like Stackelberg games, to analyze decision-making processes and coordination mechanisms within supply chains.

Mathematical models are pivotal in formulating supply chain problems and deriving analytical solutions. These models, as demonstrated in studies by (Chen & Cheng, 2012; Yuan, 2015), and others, consider various factors such as demand, pricing, inventory costs, and coordination mechanisms. Simulation techniques

Table 2.1 Summary of Literature Review

Authors	Decentralized	Centralized	Inventory Management	Numerical Solution	Hybrid Simulation	Blockchain	Demand	Integrated Manufacturing System	Sensitivity Analysis
(Saha & Yuan, 2005)	✓		✓						
(Inalhan & How, 2006)	✓		✓		✓				
(Dong & Li, 2009)	✓		✓	✓					✓
(Ni et al., 2010)		✓	✓					✓	
(Yang & Tseng, 2014)	✓		✓		✓			✓	✓
(Saha & Yuan, 2005)						✓		✓	
(Mungan et al., 2010)	✓		✓	✓			✓		
(Leeuw et al., 2011)			✓		✓		✓	✓	
(Li & Li, 2011)	✓		✓				✓		✓
(Hai et al., 2012)	✓		✓	✓	✓			✓	
(Belavina & Girotra, 2012)			✓			✓			✓
(Chen & Cheng, 2012)					✓	✓			
(Benkherouf et al., 2014)			✓	✓		✓	✓	✓	✓
(Wu & Zhao, 2014)	✓		✓	✓			✓	✓	✓
(Arya et al., 2015)	✓		✓	✓					✓

(Yuan, 2015)			✔	✔			✔	✔	
(Wu & Zhao, 2016)	✔	✔		✔			✔	✔	✔
(Nagaraju et al., 2016)	✔		✔	✔		✔	✔		✔
(Kumar et al., 2016)				✔			✔		✔
(Yan et al., 2016)			✔	✔				✔	✔
(Bai et al., 2016)	✔		✔	✔					✔
(Xu et al., 2017)				✔					✔
(Sang, 2017)			✔	✔	✔		✔		✔
(Ghosh et al., 2017)	✔		✔					✔	✔
(Kung et al., 2018)						✔		✔	
(Giri et al., 2018)	✔			✔					✔
(Milewski, 2020)					✔		✔		
Prasad et.al (2020)			✔	✔			✔		✔
(Liu et al., 2020)	✔		✔	✔	✔			✔	
(Maryniak et al., 2021)			✔			✔		✔	
(Kumar et al., 2022)					✔	✔			✔
(Mondal & Giri, 2022)			✔			✔	✔		✔
(Mishra & Ranu, 2022a)	✔		✔	✔		✔	✔	✔	✔
(Achamrah et al., 2022)	✔		✔	✔					✔
(Balashova & Maiorova, 2022)				✔				✔	
(Huang et al., 2022)					✔	✔	✔	✔	

(Zhou et al., 2022)	✔			✔		✔	✔		
(Liu et al., 2021)				✔			✔		✔
(Yan et al., 2022)			✔					✔	✔
(Zhou et al., 2022)		✔						✔	✔
(Aguirregabiria & Guiton, 2022)			✔			✔			
(Akazue et al., 2023)	✔			✔		✔			
(Sarfaraz et al., 2023)	✔			✔				✔	✔
(Manupati et al., 2020)			✔	✔				✔	
(Dutta et al., 2020)	✔		✔			✔			✔
(Padiyar et al., 2022b)	✔		✔			✔			✔
(Mishra & Ranu, 2022b)			✔			✔	✔		✔
(Mishra et al., 2024)	✔		✔	✔		✔	✔	✔	✔
(Mishra et al., 2024)	✔	✔	✔	✔			✔		✔

are employed to mimic real-world supply chain scenarios, enabling researchers to evaluate the performance of different strategies and decision-making approaches. Researchers like (Hai et al., 2012; Nagaraju et al., 2016; Wu & Zhao, 2016), among others, utilize simulation methodologies. Companies are utilizing blockchain technology to improve supply chain traceability, transparency, and information sharing in innovative ways. This trend is observed in studies by (Nagaraju et al., 2016), (Wu & Zhao, 2016), and others. Agent-based methodologies, particularly for analyzing decentralized supply chains, are employed in studies by (Liu et al., 2020), and others. These methodologies consider the behavior and interactions of individual agents within the system.

Various coordination mechanisms, such as revenue-sharing contracts, are explored in studies by (Inalhan & How, 2006; Padiyar et al., 2022a), among others, to align incentives among supply chain members and improve overall profitability and efficiency. Some methodologies address environmental concerns, such as carbon taxation, recycling activities, and green supply chain management. This reflects a growing emphasis on sustainability in supply chain decision-making, evident in studies by (Mondal & Giri, 2022; Padiyar et al., 2022b), and others.

2.1.2 Practical Application of Decentralized Inventories

In this literature review, we have observed various types of decentralized inventory strategies that have been studied. These strategies are proposed to optimize inventory management and streamline operations. Here are some prominent types of decentralized inventory strategies, each with examples illustrating their practical application:

- <u>Vendor-Managed Inventory (VMI):</u> Managing inventory levels at a customer site according to predetermined agreements and specifications is known as vendor-managed inventory (VMI). (Prasad et al., 2020). For example, Procter & Gamble collaborates with Walmart for VMI, where P&G monitors and replenishes inventory levels at Walmart's distribution centers and stores.
- <u>Consignment Inventory:</u> The concept of consignment inventory involves the supplier maintaining ownership of the inventory until it is utilized or sold by the customer. (Manupati et al., 2020) Boeing employs a consignment inventory model with its suppliers, where Boeing pays for parts only upon their use in aircraft assembly, thus reducing inventory carrying costs.
- <u>Cross-Docking</u>: Direct transfers of cargo from inbound to outbound vehicles are called cross-docking; in this way, warehouse storage is reduced to a minimum. (Leeuw et al., 2011) Walmart employs a cross-docking strategy, where incoming goods are sorted and immediately loaded onto outbound trucks for delivery to stores, minimizing inventory holding costs.
- <u>Drop shipping</u>**:** Drop shipping involves fulfilling customer orders by directly shipping products from manufacturers or wholesalers to end customers, eliminating the need for retailers to hold inventory. (Akazue et al., 2023). An example is Amazon's third-party seller program, where independent sellers list their products on Amazon's platform, and Amazon handles order fulfillment and shipping directly to customers.
- <u>Just-in-Time (JIT) Inventory</u>: By synchronizing production and delivery with actual demand, just-in-time inventory management reduces inventory holding costs. This approach entails the production and receipt of goods only when they are required for production or to meet customer orders. (Sarfaraz et al., 2023) Toyota utilizes JIT inventory in its manufacturing process, where components are delivered to the assembly line precisely when needed, reducing inventory waste and costs.
- <u>Direct Store Delivery (DSD):</u> DSD involves suppliers delivering products directly to retail stores, bypassing distribution centers, and reducing inventory handling and storage costs. Coca-

Cola's DSD model is an example, (Hai et al., 2012) where Coca-Cola trucks deliver beverages directly to retail stores, ensuring fresher products and reducing inventory levels at distribution centers.

- Collaborative Planning, Forecasting, and Replenishment (CPFR): Suppliers and retailers collaborate on CPFR based on forecasts and demand data to optimize inventory levels and reduce stockouts.(Mondal & Giri, 2022) Walmart has implemented CPFR with its suppliers, where both parties share sales and inventory data to improve demand forecasting and inventory replenishment efficiency.

Overall, these strategies illustrate that while decentralized inventory methods can offer significant operational advantages—such as cost reduction, increased responsiveness, and enhanced transparency—their practical implementation is highly contingent upon the quality of inter-organizational collaboration, technological infrastructure, and the robustness of data-sharing mechanisms. The literature reveals a clear need for further research that not only compares these strategies in diverse contexts but also explores hybrid approaches that may mitigate the inherent limitations of individual methods.

2.1.3 Advantages and Risk of Decentralizations

These decentralized inventory strategies offer various benefits, such as reduced inventory holding costs, improved supply chain efficiency, and better customer service. Supply chain structure, industry dynamics, and business objectives all influence the choice of a strategy. Here are the advantages, potential benefits, challenges, limitations, and risks associated with implementing decentralized inventory management strategies, along with their corresponding paper numbers:

Advantages and Potential Benefits:

1. No explicit knowledge is needed about individual cost functions, eliminating problems from informational deficiencies in large-scale supply chains (Inalhan & How, 2006).
2. Decentralization often leads to the double marginalization issue, but consumers' strategic behavior can positively influence supply chain performance (Dong & Li, 2009).
3. There are trade-offs in the centralization vs. decentralization decision, with potential benefits including moderating inventory levels and reducing supply chain frictions (Arya et al., 2015).
4. Decentralization can benefit a firm with multiple divisions by alleviating time-inconsistent issues in procurement and maintaining moderate inventory levels (Jingyi & Jin, 2018).
5. Centralized inventory control can lead to the bullwhip effect, which is more likely in centralized inventory control systems (Xu et al., 2017).

While decentralized inventory strategies can enhance supply chain responsiveness, reduce reliance on detailed cost data, and potentially mitigate negative phenomena like the bullwhip effect, these advantages are context-dependent. Their successful implementation hinges on robust inter-organizational communication, effective coordination mechanisms, and alignment of objectives across supply chain members. Moreover, the benefits must be weighed against the risks of increased complexity in decision-making and potential fragmentation of supply chain efforts. Further empirical research is needed to understand how these trade-offs manifest across different industries and under varying market conditions.

2.1.4 Challenges, Limitations, and Risks:

1. There is a lack of an integrated dynamic approach for the entire supply chain system, leading to difficulty in synthesizing dispersed entities into an integrated system (Dong & Li, 2009).
2. Decentralization is often linked to the double marginalization issue, where consumers' strategic behavior positively influences supply chain performance (Qing & Wu, 2009).
3. One limitation of the study may be the simplifications made in the model assumptions, which may not fully capture all the complexities of real-world supply chain dynamics, and the model's applicability to different industry contexts may vary (Mungan et al., 2010).
4. A centralized inventory control strategy is analyzed in terms of shortage costs and inventory holding costs. (Wu & Zhao, 2016).
5. Centralized inventory control can lead to the bullwhip effect, which is more likely in centralized inventory control systems (Xu et al., 2017).

2.1.5 Empirical Studies and Case Analyses

The abstracts and conclusions provided encompass a broad spectrum of topics pertinent to supply chain management, spanning centralized and decentralized decision-making, inventory management, coordination mechanisms, optimization strategies, and the integration of emerging technologies such as blockchain. Here's a synthesized analysis of the key points:

- <u>Centralization vs. Decentralization</u>: The (Balashova & Maiorova, 2022; Leeuw et al., 2011; Giri et al., 2018; Wu & Zhao, 2014; Yuan, 2015; Zhou et al., 2016) delve into the advantages and drawbacks of centralized and decentralized approaches within supply chain management. While centralized control often yields cost savings and streamlined coordination, it may encounter inefficiencies and the bullwhip effect. Conversely, decentralized decision-making offers flexibility and responsiveness but can result in inventory discrepancies and coordination hurdles.
- <u>Inventory Management</u>: Various inventory management strategies are scrutinized, including (Q, R) policies, minimum variance control, the SPSA optimization algorithm, and heuristic

control policies (Achamrah et al., 2022; Benkherouf et al., 2014; Chen & Cheng, 2012; Dong & Li, 2009; Wu & Zhao, 2014). These approaches are designed to optimize inventory levels, effectively track customer demand, and bolster operational efficiency.

- Coordination Mechanisms: Examination of coordination mechanisms such as revenue-sharing contracts, Stackelberg games, and two-part tariff contracts is conducted (Chen, et al., 2017; Chen & Cheng, 2012; Khanlarzade et al., 2019; Kumar et al., 2022; Mondal & Giri, 2022). These mechanisms aim to align the incentives of diverse supply chain members, enhance profitability, and alleviate conflicts among stakeholders.
- Emerging Technologies: Blockchain technology emerges as a promising avenue for enhancing transparency, traceability, and efficiency within supply chains. (Dutta et al., 2020; Huang et al., 2022; Akazue et al., 2023; Mishra & Ranu, 2022a; Sarfaraz et al., 2023) Smart contracts, RFID technology, and decentralized frameworks like SmartRice are explored for managing the flow of goods and information across the supply chain.
- Optimization and Decision-Making: Mathematical models, optimization algorithms, and game theory are employed to tackle various supply chain challenges, including inventory replenishment, production scheduling, and pricing decisions (Dong & Li, 2009; Inalhan & How, 2006; Mishra & Namwad, 2023; Ni et al., 2010) and many more. These tools furnish insights into optimal strategies for cost minimization, profit maximization, and the enhancement of overall supply chain performance.

2.1.6 Emerging Trends and Innovation:

The convergence of emerging trends and innovations in supply chain management is driving the advancement of decentralized inventory practices, effectively tackling challenges and seizing opportunities to enhance overall supply chain performance and competitiveness. These advancements are reflected in several key areas:

Optimization Algorithms: (Inalhan & How, 2006; Saha & Yuan, 2005) Analyze both centralized and decentralized behavior within supply chains to discover their implications. Several feasible algorithms are proposed for centralized and decentralized solutions, offering insights into improving efficiency.

Heuristic Control Strategies: (Dong & Li, 2009) explores the impacts of heuristic control policies on decentralized supply chains, evaluating their effects on operational costs, customer satisfaction, and inventory stability.

Integration of Technology: (Akazue et al., 2023; Mishra & Ranu, 2022a; Yan et al., 2022) introduce innovative frameworks leveraging RFID and blockchain technology to manage supply chains effectively. They emphasize the importance of information sharing and cost reduction in enhancing supply chain operations.

Coordination Mechanisms: (Bai et al., 2016; Zhou et al., 2016) investigate contractual mechanisms within decentralized systems to increase profits through commitment power. They provide valuable insights into coordinating supply chains using revenue-sharing contracts, thereby improving overall performance and profitability.

Environmental Sustainability: (Huang et al., 2022; Sang, 2017) address sustainability concerns within decentralized supply chains, offering guidance on implementing greening policies and managing technology disruptions to promote environmental sustainability.

Application in Specific Industries: (Khanlarzade et al., 2019; Mungan et al., 2010) focus on developing tailored inventory models for specific industries such as high-tech and healthcare, aiming to minimize costs and increase efficiency in supply chain operations.

Decision Support Systems: (Hai et al., 2012; Nagaraju et al., 2016) explore decentralized inventory decisions optimizing individual costs and propose optimized control strategies for inventory replenishment in both coordinated and non-coordinated supply chains.

Collectively, these papers contribute to the advancement of decentralized inventory management practices across various industries and supply chain contexts, paving the way for enhanced efficiency and competitiveness in supply chain operations.

2.2 Data Analysis Framework

To analyze the data in table 2.1, we can summarize key points such as the distribution of various attributes across different authors. Here’s a step-by-step breakdown of how we can approach the data analysis:

2.2.1 Introduction to Data Analysis Techniques

In this section, we delve into the data analysis techniques employed to understand the relationships and patterns within the research on decentralized inventory management in supply chain management. We utilized two powerful tools, Excel and Python, to perform comprehensive data analysis and generate insightful visualizations.

2.2.2 Brief Overview of Tools (Excel and Python)

Excel was used for initial data cleaning, preparation, and basic descriptive statistics. Its user-friendly interface and powerful functionalities make it an excellent tool for preliminary data analysis. We used Excel to organize the data, compute basic statistics, and create preliminary visualizations such as charts and graphs. Python, with its robust libraries like Pandas, NumPy, Matplotlib, and Seaborn, was used for advanced data analysis and visualization. Python allowed us to perform in-depth correlation analysis and generate sophisticated visualizations such as heatmaps, providing deeper insights into the relationships between various attributes in our dataset.

2.2.3 Data Analysis Steps

To analyze the data in table 2.1, we summarized key points, such as the distribution of various attributes across different authors.

Step 1: Count the Occurrences: For each attribute, we counted how many times it appears across all authors. This helped us understand the prevalence of each attribute in the research papers.

Step 2: Identify Common Combinations: We identified frequent combinations of attributes across the research papers to uncover common research focuses.

i. Decentralized & Inventory Management: This combination appears in several entries.
ii. Centralized & Blockchain Implementation: This combination appears together in a few entries.
iii. Inventory Management & Numerical Solution**:** This is a common combination.

Step 3: Identify Unique Entries: Some entries have unique combinations of attributes:

- Leeuw et.al (2011): Unique combination with "Inventory Management", "Hybrid Simulation", "Demand", and "Sensitivity Analysis".
- Belavina et.al (2012): Unique focus on "Blockchain" and "Blockchain Implementation".

A bar chart visualizing the occurrences of each attribute in the research papers can help to see which attributes are most and least frequently discussed in the research papers.

Table 2.2 The distribution of various attributes across different authors

Attribute	**Count**
Decentralized	18
Centralized	6
Inventory Management	18
Numerical Solution	8
Hybrid Simulation	6
Blockchain	8
Demand	4
Integrated Manufacturing	4
Sensitivity Analysis	8
Blockchain Implementation	12

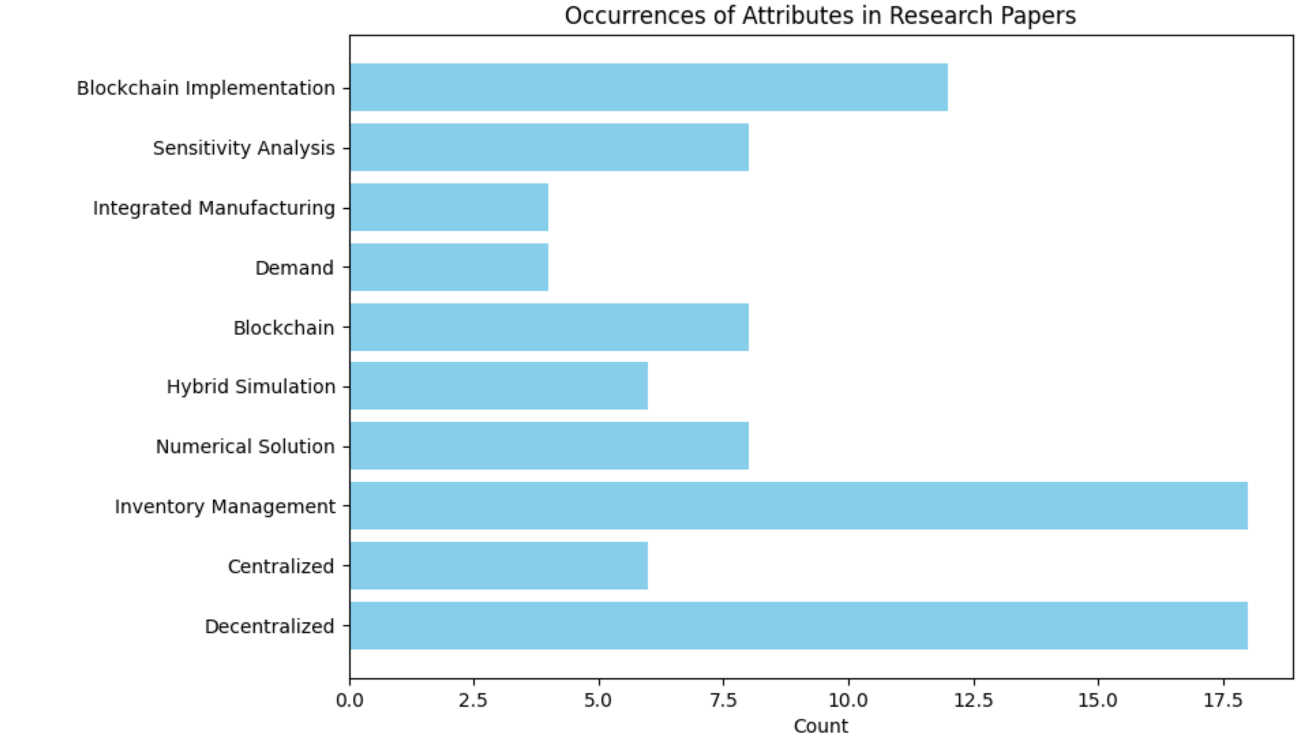


Figure 2.1 A bar Chart Visualizing the occurrence of each attribute.

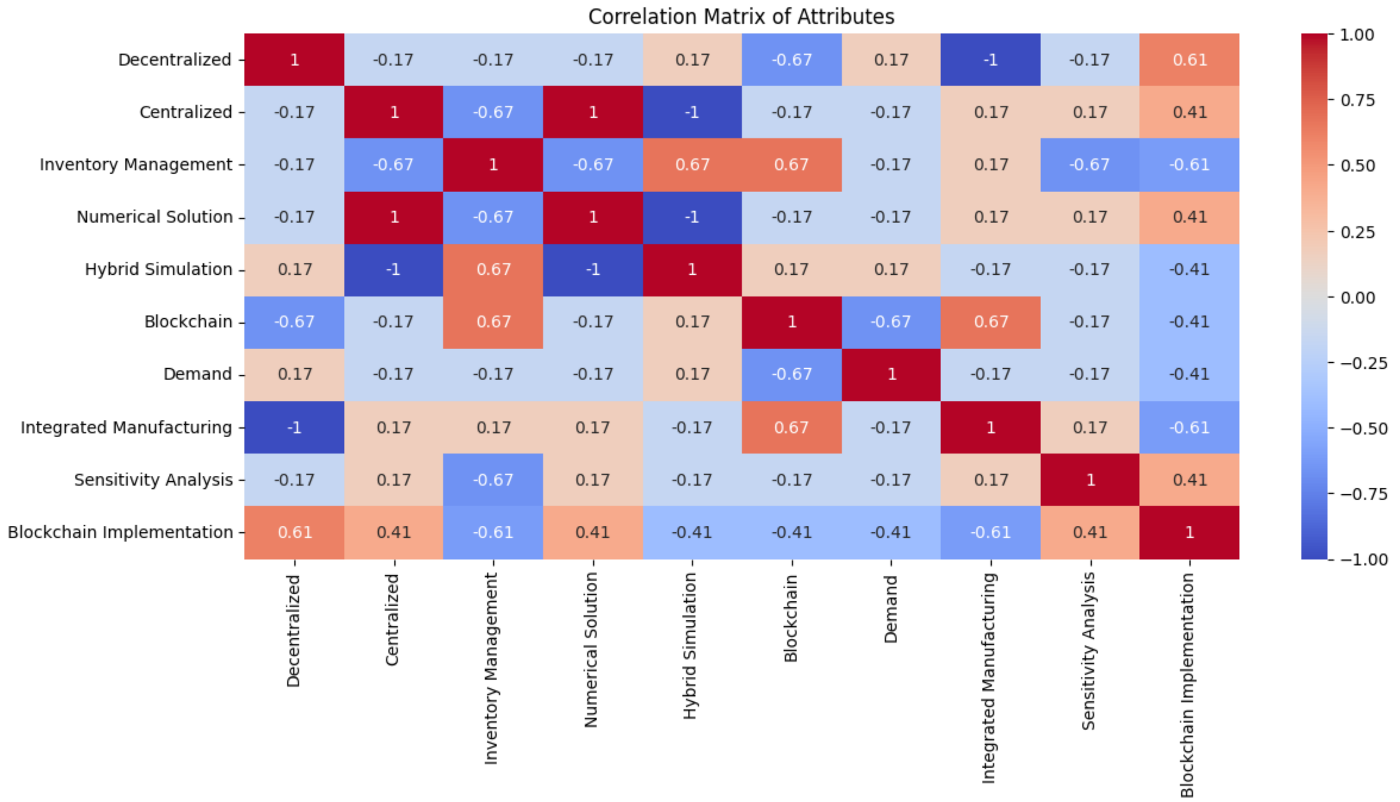


Figure 2.2 Heatmap Visualization.

2.2.4 Correlation and Relationship Analysis

To provide a deeper understanding of the relationships between different attributes in the research papers, we performed a correlation analysis and visualized the results. The steps involved in the correlation analysis are as follows:

2.2.4.1 Explanation of Correlation Methodology

Step 1: Prepare Data for Correlation Analysis: We converted the table into a binary format where each attribute's presence is marked as 1, and absence as 0. This binary representation allows for the computation of correlations between the presence or absence of attributes across the research papers.

Step 2: Correlation Analysis: Using Python, we computed the correlation matrix for the attributes. The correlation matrix quantifies the strength and direction of relationships between pairs of attributes.

To better understand the relationships between attributes, we created a heatmap visualization. The heatmap visually represents the correlation matrix, with colours indicating the strength and direction of correlations. The visualization displays positive correlations in shades of blue and negative correlations in shades of red.

2.2.4.2 Results of Correlation Analysis (with Graphs and Heatmaps)

Heatmap Insights:

1. Positive Correlations:
    - Decentralized & Inventory Management: Moderate positive correlation (r = 0.39).
    - Decentralized & Numerical Solution: Moderate positive correlation (r = 0.44).
    - Inventory Management & Numerical Solution: Moderate positive correlation (r = 0.44).
    - Inventory Management & Sensitivity Analysis: Moderate positive correlation (r = 0.35).
2. Negative Correlations:
    - Blockchain Implementation and Inventory Management: Moderate negative correlation (r = -0.27).
    - Blockchain Implementation & Hybrid Simulation: Moderate negative correlation (r = -0.37).
3. Weak or No Correlation:
    - Most other attribute pairs show weak or no significant correlation, indicating diverse research focuses.

This data analysis provides a comprehensive overview of the relationships between various attributes in the research papers. It highlights common themes, unique research focuses, and emerging trends in the integration of blockchain technology into supply chain management. By understanding these patterns, researchers and practitioners can better navigate the landscape of blockchain applications in supply chains, ultimately contributing to cost minimization, enhanced managerial insights, and improved coordination across the supply chain.

2.2.5 Advanced Data Visualizations and Insights

2.2.5.1 Detailed Explanation of Advanced Visualizations Created Using Python

Using Python, we generated advanced visualizations that provide deeper insights into our data. These visualizations include heatmaps, bar charts, and scatter plots, which were created using libraries such as Matplotlib and Seaborn.

2.2.5.2 Interpretation of the Visualizations

The heatmap visualization highlights the strength and direction of correlations between attributes. Positive correlations, represented in blue, indicate that as one attribute is present, the other is also likely to be present. Conversely, negative correlations, represented in red, indicate that the presence of one attribute is associated with the absence of the other. The bar charts and scatter plots provide additional perspectives on the distribution and relationships of attributes, helping to identify trends and patterns within the research data.

Overall, these visualizations enhance our understanding of the data and support our analysis by providing clear and interpretable visual representations of complex relationships. By leveraging these advanced techniques, we can draw meaningful conclusions and provide actionable insights for future research and practice in decentralized inventory management and blockchain integration in supply chains.

2.3 Research Gaps and Future Opportunities

2.3.1 Identification of Gaps

Additionally, several authors have identified common limitations in their studies, particularly concerning the simplifying assumptions made in their models. These assumptions may not fully capture the complexities of real-world supply chain dynamics, and the authors acknowledge that this may limit the generalizability of their findings to more complex supply chain scenarios. For instance, (Zhou et al., 2016) discusses the link between decentralization and the double marginalization issue, while (Milewski, 2020) explores the bullwhip effect in centralized inventory control systems. Authors' opinions diverge on whether decentralization leads to positive outcomes like improved supply chain performance or negative consequences such as the double marginalization issue and the increased bullwhip effect.

Moreover, there is disagreement regarding the extent to which mathematical models and analytical approaches accurately capture real-world supply chain dynamics. While some authors acknowledge the limitations of simplifying assumptions, others argue that these models provide valuable insights despite their limitations. This discrepancy is evident across various papers, including Papers (Mungan et al., 2010), and many more.

Furthermore, authors differ in their opinions about the generalizability and applicability of their findings to different industry contexts and supply chain scenarios. While some emphasize the need for further research to validate proposed strategies across diverse environments, others assert the relevance of their findings across various settings. This variation in perspective is evident in (Dutta et al., 2020; Akazue et al., 2023; Manupati et al., 2020; Yan et al., 2022).

Lastly, concerns have been raised about the challenges associated with implementing technologies like blockchain within existing supply chain infrastructures. While some authors highlight the potential benefits of blockchain technology, others point out issues related to scalability, interoperability, and privacy concerns. This debate is reflected in (Aguirregabiria & Guiton, 2022), where authors discuss disagreements regarding the most effective strategies for coordinating decentralized supply chains and optimizing inventory management.

2.3.2 Opportunities for Future Research

Based on the synthesized analysis of the existing literature on supply chain management, the research gap identified pertains to the exploration and understanding of decentralized inventory strategies. Despite the extensive coverage of various aspects related to supply chain management, including centralized and decentralized decision-making, inventory management techniques, coordination mechanisms, and the integration of emerging technologies, there remain several areas where further research is warranted.

- While there is substantial discussion on the advantages and disadvantages of decentralized decision-making in supply chain management, there is a need for empirical studies to empirically assess the actual performance outcomes of decentralized inventory strategies compared to centralized approaches across different industry contexts and supply chain configurations. Additionally, further research is needed to analyze decentralized three-echelon supply chain models using blockchain technology and propose models that handle individual decision-making within the chain effectively.
- Many existing studies utilize mathematical models and simulation techniques to analyze supply chain dynamics. However, there is a gap in understanding the real-world complexities of

decentralized inventory strategies, as current models may oversimplify certain aspects or fail to capture the full range of dynamics present in decentralized supply chain environments. Research is needed to

- Decentralized, multi-period, two-echelon supply chain inventory models under finite planning horizons, and to analyze decentralized three echelon supply chain models with mixed integer non-linear programming problems using blockchain technology.
- The applicability of research findings on decentralized inventory strategies to diverse industry settings and supply chain contexts is not fully understood. Further research is needed to validate the effectiveness of proposed strategies across different sectors, geographic regions, and supply chain structures to assess their generalizability and practical relevance. Moreover, investigations into Stackelberg game models under decentralization can provide insights into decision-making dynamics within decentralized supply chains.
- While emerging technologies such as blockchain hold promise for enhancing supply chain transparency and efficiency, there is a lack of understanding regarding the practical challenges and implementation barriers associated with integrating these technologies into decentralized inventory management systems. Future research should focus on addressing these challenges and identifying strategies to overcome them effectively.
- The optimal coordination mechanisms and optimization strategies for decentralized inventory management are still not well-defined. There is a need for research that explores innovative coordination mechanisms and optimization techniques tailored specifically to decentralized supply chain environments to maximize efficiency and performance. This includes investigating the comparison of decentralized and centralized supply chains by mathematical modeling, shedding light on the trade-offs and benefits associated with each approach.

Addressing these specific research gaps would contribute to a more comprehensive understanding of decentralized inventory strategies in supply chain management, facilitating the development of effective decision-making frameworks and optimization techniques tailored to decentralized supply chain environments.

2.4 Objectives of the Thesis

The following are the objectives of this thesis:

1. In a finite planning horizon, to analyse a decentralized three-echelon supply chain model using blockchain technology.
2. To study a decentralized, multi-period, two-echelon supply chain inventory model under a finite planning horizon.

3. To examine a Stackelberg game, finite planning horizon model under a decentralization.
4. To investigate the comparison of decentralized and centralized supply chains.

2.5 Proposed Methodology

The decentralized supply chain is extensively utilized in various fields, including marketing, management, economics, geography, biology, physics, finance, and chemistry. To tackle the complexities of these supply chains, a robust mathematical model and detailed analysis are essential. This section outlines the methodologies employed in developing and analyzing the proposed inventory management model.

2.5.1 Development and Analysis of Mathematical Models

- The process begins with the preparation of a theoretical model based on a literature review and identified research gaps. This model will encompass the core components of the supply chain, integrating factors such as inventory, transportation, and production processes.
- The theoretical model is then converted into a mathematical format. This involves defining variables, constraints, and objective functions that represent the supply chain dynamics.
- The model analysis will be conducted under a finite planning horizon (FPH) to evaluate its performance over a specified time.

2.5.2 Proposed Solution Methodologies

- The model will be analyzed using numerical iterative methods, providing insight into the system's behavior under different scenarios.
- Wolfram Mathematica software (version 13.0) will be utilized for numerical calculations, offering efficient solutions for complex mathematical problems. Commands such as NSolve, FindRoot, and Plot will be employed for solving equations and visualizing results.

2.5.3 Numerical Simulations and Sensitivity Analysis

- The numerical results of the model will be obtained using suitable iterative methods, providing detailed insights into the model's functionality and performance.
- To understand how different parameters impact the results of the model, a detailed sensitivity analysis will be conducted. The robustness and reliability of the model will then be assessed.

2.5.4 Strategies for Data Collection

- Data will be collected from existing, reliable sources such as:
    - Internal sources (annual reports, websites)

        - External sources (published information, journals)
    - Utilizing secondary data is cost-effective and timesaving, although it may require careful validation to ensure relevance and accuracy for the present research.

2.5.5 Tools and Resources Used in Research

- Wolfram Mathematica version 13.0 will be the primary tool for data analysis, facilitating complex numerical calculations and graphical representations.
- Python will be used for generating graphs and visualizing data, providing a clear representation of the results and aiding in the interpretation of the model's outcomes.

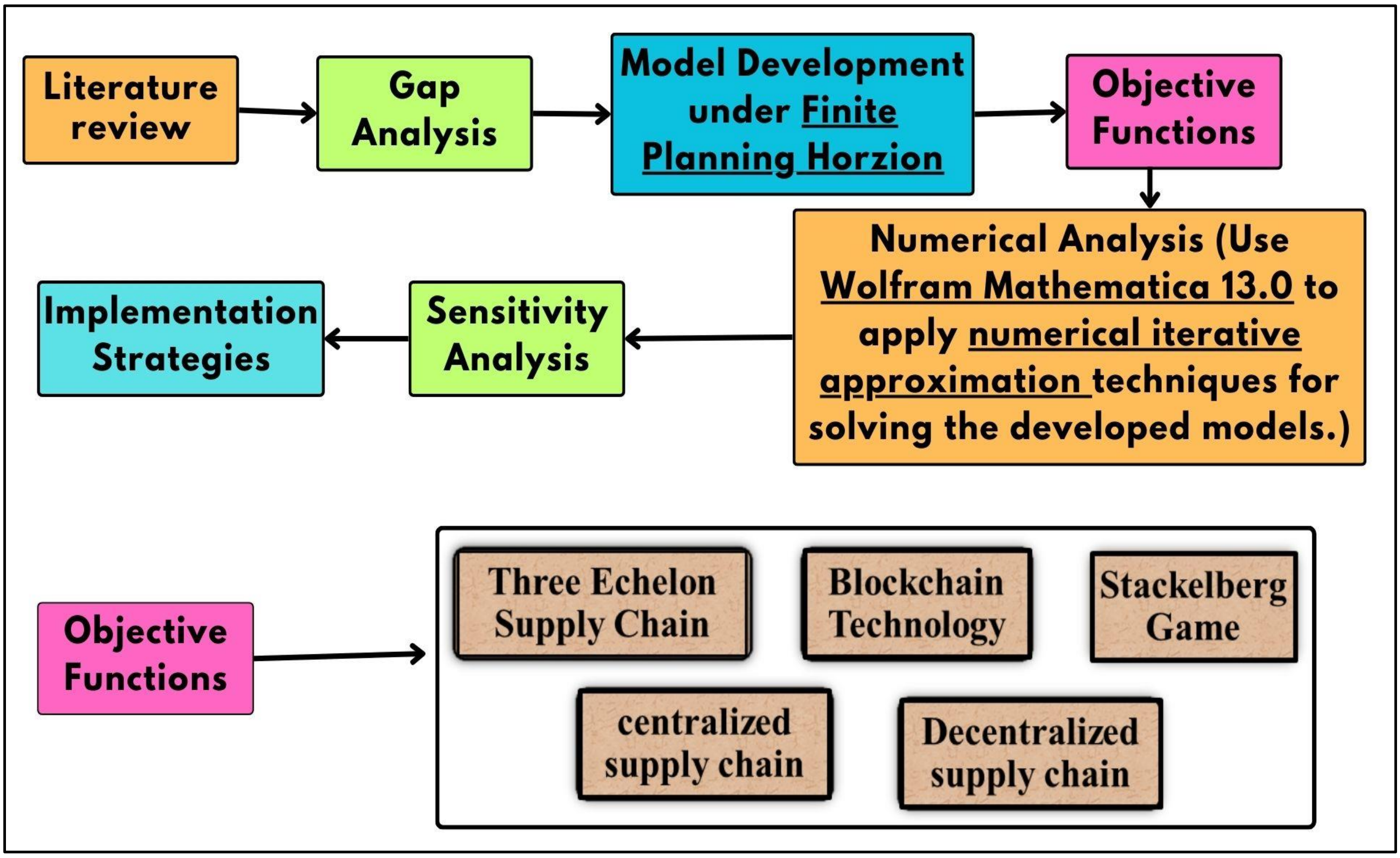


Figure 2.3 Research Methodology

2.6 Real-Life Practical Applications of the Proposed Model in Indian Industries

The proposed blockchain-integrated decentralized inventory model is highly relevant for real-world supply chain applications, particularly in Indian pharmaceuticals, agriculture, and logistics. These industries face significant challenges related to demand uncertainty, inventory mismanagement, lack of real-time visibility, and regulatory compliance. The model offers solutions through data-driven decision-making, improved inventory control, and cost-efficient supply chain operations.

1. Indian Pharmaceutical Industry

The pharmaceutical industry in India is highly fragmented, with multiple manufacturers, distributors, and retailers operating under different regulatory frameworks. Ensuring timely delivery of medicines, reducing stockouts, and preventing counterfeit drugs are major concerns.

- **Blockchain for Drug Traceability:** The proposed model incorporates blockchain-based inventory tracking, which can help pharmaceutical firms maintain real-time records of drugs from manufacturing to retail. This ensures compliance with regulations like the Drugs and Cosmetics Act and minimizes supply chain inefficiencies.
- **Optimized Replenishment Decisions:** The model's finite planning horizon approach enables pharmaceutical companies to determine the optimal order quantity for critical drugs, ensuring that supply meets uncertain demand, especially during pandemics and health crises.
- **Reducing Expired Inventory:** By integrating real-time demand-supply analytics, the model can help reduce medicine wastage and improve shelf-life management.

2. Indian Agriculture and Food Supply Chains

The agricultural sector in India struggles with post-harvest losses, price volatility, and inefficiencies in distribution**.** Small-scale farmers, who often lack access to structured supply chains, face difficulties in getting fair prices and managing inventory.

- **Decentralized Inventory Coordination:** The model's multi-echelon framework can be applied to farmer cooperatives and agribusiness firms to optimize storage and transportation decisions, reducing wastage and ensuring better price realization.
- **Blockchain-Based Smart Contracts:** By leveraging blockchain-enabled smart contracts, farmers and distributors can engage in trust-based, automated transactions, eliminating intermediaries and ensuring timely payments.
- **Demand Prediction & Storage Optimization:** The model's sensitivity analysis on demand uncertainty can help agricultural supply chains determine optimal inventory stocking levels for perishable goods, reducing food spoilage.

3. Indian Logistics and Transportation Sector

The Indian logistics industry is crucial for supply chain efficiency, but it suffers from inefficient fleet management, a lack of real-time tracking, and high transportation costs. The proposed model can enhance logistics operations in the following ways:

- **Real-Time Tracking & Decentralized Decision-Making:** The model integrates blockchain for logistics visibility, ensuring that manufacturers, retailers, and suppliers have synchronized inventory data. This reduces delays and bottlenecks in transportation.
- **Cost-Effective Inventory Flow:** By applying the model's cost-minimizing replenishment approach, logistics firms can optimize warehouse stocking, fleet routing, and last-mile deliveries.
- **Carbon Emission Monitoring:** The integration of cap-and-trade policies in the model helps logistics companies track their carbon footprint and comply with India's sustainability regulations, making supply chains greener.

The proposed model provides a scalable and adaptable framework for Indian supply chains, allowing pharmaceutical firms, agribusinesses, and logistics providers to optimize inventory management, reduce costs, enhance supply chain transparency, and comply with regulatory policies. By integrating blockchain, decentralized decision-making, and demand forecasting, the model addresses real-world challenges and offers a strategic roadmap for modernizing Indian supply chains.

Chapter 3 Decentralized Three-Tier Supply Chain Management with Blockchain: A Finite Planning Horizon Framework

Abstract

This chapter uses the Stackelberg game approach to integrate blockchain technology in a decentralized Three-Tier Supply Chain model within a finite planning horizon. The model addresses inefficiencies and coordination issues in traditional supply chains by leveraging blockchain's transparency and traceability. The retailer, acting as the leader, determines optimal replenishment cycles and order quantities, while the manufacturer, as the follower, adheres to the demand-matched replenishment policy. The research introduces a 'break-even point' concept. Utilizing Wolfram Mathematica, the investigation undertakes a sensitivity analysis, employing heatmaps to assess the influence of diverse parameters on supply chain performance. The provision of managerial insights aims to offer guidance for practical implementation, emphasizing the potential of blockchain technology to improve transparency, efficiency, and profitability within supply chain management.

3.1 Introduction

As blockchain technology (BCT) develops rapidly, both academia and industry are taking notice, particularly supply chain management. (SCM) (Chang et al., 2018). Blockchain's inherent features of immutability, transparency, and traceability make it particularly suitable for addressing the persistent challenges in traditional supply chain models. Issues such as the lack of coordination and integration of information flows across various echelons often lead to cost inefficiencies and suboptimal performance (Batra et al., 2020; Eljazzar et al., 2018).

Blockchain technology represents a promising solution for decentralized three-tier supply chain models, which commonly encompass suppliers, manufacturers, and retailers. The enhanced visibility, trust, and automation provided by blockchain can significantly benefit these models (Casado-Vara et al., 2018; Sahai et al., 2020). This chapter aims to analyze a decentralized three-echelon supply chain model leveraging blockchain technology within a finite planning horizon.

The incorporation of blockchain technology into supply chain management has brought about a significant transformation, presenting novel avenues for augmenting traceability, transparency, and efficiency (Allen et al., 2019). The research delves into how a decentralized approach, supported by blockchain technology, can optimize supply chain operations within a finite planning horizon.

In this chapter, we analyze a mathematical model consisting of three main phases, focusing on the retailer's, manufacturer's, and supplier's operations. The retailer faces a linear demand function $D(t)=a-bt+\alpha\beta$, where a and b are constants, and αβ represents the negative impact of blockchain information storage on demand. This negative impact is modeled by considering that increased transparency and traceability may reduce demand due to potential customer privacy concerns or perceived increased costs. The retailer calculates total costs, including ordering costs (O_R), holding costs (H_R), and deterioration costs (D_R). The retailer determines the efficient replenishment cycle number (n) and the corresponding efficient order quantity per cycle (Q_R). The objective is to minimize total costs while ensuring demand is met throughout the planning horizon. The manufacturer follows a demand-matched replenishment policy, meaning that the quantity produced matches the quantity ordered by the retailer for each cycle. This policy ensures just-in-time delivery, minimizing inventory holding costs for both the manufacturer and the retailer. Figure 3.1 illustrates the inventory levels over the planning horizon. The retailer's inventory decreases gradually, primarily to meet the demand. The continuous replenishment by the manufacturer ensures that the retailer maintains optimal inventory levels without overstocking.

The manufacturer produces items at a rate M, sufficient for the remaining planning horizon. During each cycle, the manufacturer replenishes the required quantity to the retailer at regular intervals. This continues until the 'break-even point' at time t, where the cumulative quantity produced equals Qp+Qc. At this 'break-even point' (illustrated in figure 3.1), production halts as the required quantity has been manufactured. After the 'break-even point' at time t, production stops, but replenishment to the retailer continues at the derived intervals. This phase ensures that the retailer receives the required quantity without additional production. The supplier is always available to provide raw materials to the manufacturer whenever required. The supplier's role is to ensure that the manufacturer has a steady supply of raw materials to meet the production needs. The supplier's costs and logistics are considered constant and do not directly influence the dynamic model of inventory but ensure a seamless flow of raw materials.

In this chapter, Section 3.2 Conducts an extensive review of existing literature to pinpoint areas requiring further research. In Section 3.3, we discuss the notations, assumptions, and hypotheses used in the model. Section 3.4 explains the mathematical model, covering the operations of the retailer, manufacturer, and supplier, along with some propositions and lemmas. Section 3.5 outlines the methodology, emphasizing the use of iterative approximation methods with the assistance of Wolfram Mathematica 13.0. In Section 3.6, we present a numerical example, detailing the operations of the retailer and manufacturer and identifying the break-even point. Section 3.7 examines the sensitivity analysis of parameters, with findings illustrated through heatmaps. Section 3.8 offers managerial insights, and finally, Section 9 provides the conclusion and suggests directions for future work.

3.2 Literature Review

Due to blockchain technology's decentralized nature, it offers significant advantages for supply chain management, reliability, sustainability, and including improved transparency. (Park et al., 2021; Sahai et al., 2020). Blockchain's immutability and provenance features enhance product traceability, enabling supply chain actors to track the movement of goods and materials more effectively (Eljazzar et al., 2018). This is particularly valuable in industries where regulations, such as the food and pharmaceutical sectors mandate traceability.

However, the complexities of supply chains, where items are continuously packaged, repackaged, and transformed, pose challenges in maintaining the privacy and confidentiality of sensitive data (Sahai et al., 2020). Researchers have proposed solutions like encryption schemes and group signatures to address these concerns, but the field remains active (Sermpinis & Sermpinis, 2018). In addition to mitigating the potential for single points of failure, decentralizing the supply chain system can also reduce costs.

One of the most studied configurations in SCM literature is the Three-Tier Supply Chain model, comprising retailers, manufacturers, and suppliers (Chang et al., 2018). Integrating blockchain technology into this model can enhance transparency, traceability, and collaboration among various actors, improving overall supply chain performance and resilience (Jabbar et al., 2021). Additionally, the Stackelberg game theory framework, where the retailer is the leader and the manufacturer is the follower, further enhances this model by capturing the sequential nature of decision-making processes in hierarchical supply chains.

In the context of SCM, The Stackelberg game entails the participation of a leader and a follower. The leader (retailer) sets the rules or regulations that the follower (manufacturer) must adhere to, such as pricing policies, production quantities, and inventory levels (Nagaraju et al., 2015). The leader's decision-making process considers their objectives and constraints, anticipating the follower's response (Yue & You, 2014). The follower observes the leader's decisions and reacts accordingly, optimizing their objectives, such as maximizing profits or minimizing costs, while adhering to the leader's regulations (Khanlarzade et al., 2019). Through strategic interactions, the supply chain can improve coordination and efficiency.

To improve supply chain efficiency and optimize inventory management, organizations use the Three Tier Inventory Model. This model consists of three levels or echelons: the manufacturer, the distributor, and the retailer. Each echelon has its inventory management responsibilities and decisions to make, but the overall goal is to minimize costs and ensure the timely availability of products (Kumar et al., 2016). The manufacturer is responsible for producing and maintaining an inventory of finished goods. They determine production schedules and lot sizes based on demand forecasts and customer orders. The distributor's role is to receive the products from the manufacturer and store them in their warehouses (Canco, 2022). They are responsible for managing inventory levels, coordinating deliveries to retailers, and ensuring timely replenishment (Yang &

Tseng, 2014). The retailer is the final link in the supply chain and interacts directly with customers (Han et al., 2014). They are responsible for maintaining inventory at the store level, placing orders with the distributor, and fulfilling customer demand (Nagaraju et al., 2016). The Three Echelon Inventory Model recognizes the interdependencies between these three levels and aims to optimize inventory decisions holistically (Kumar et al., 2016).

Table 3.1 Literature Survey

Researcher(s)	Focus	Key Contributions
Chang et al. (2018)	Blockchain in SCM	Highlighted the suitability of blockchain for SCM
Eljazzar et al. (2018)	Challenges in traditional SCM	Identified cost inefficiencies and performance issues
Batra et al. (2020)	Lack of coordination in SCM	Addressed lack of integration of information flows
Sahai et al. (2020)	Benefits of blockchain in decentralized models	Discussed transparency, trust, and automation benefits
Casado-Vara et al. (2018)	Blockchain in three-echelon SCM	Analysed decentralized supply chain with blockchain
Allen et al. (2019)	Blockchain enhancing transparency and efficiency	Explored new opportunities with blockchain integration
Park & Li (2021)	Advantages of blockchain in SCM	Enhanced transparency, reliability, and sustainability
Sermpinis & Sermpinis (2018)	Solutions for data privacy in SCM	Proposed encryption schemes and group signatures
Jabbar et al. (2020)	Benefits of blockchain in three-echelon SCM	Improved transparency, traceability, and collaboration
Nagaraju et al. (2015)	Stackelberg game in SCM	Explored leader-follower dynamics in SCM
Yue & You (2014)	Sequential decision-making in SCM	Focused on leader setting rules and follower responding
Khanlarzade et al. (2019)	Impact of leader's decisions on follower	Showed strategic impact of leader's decisions
Kumar et al. (2016)	Three Echelon Inventory Model	Explored interdependencies between echelons
Canco (2022)	Role of distributors in SCM	Detailed distributor's inventory management role
Yang & Tseng (2014)	Coordination between echelons	Emphasized importance of echelon coordination
Han et al. (2014)	Role of retailers in SCM	Outlined retailer's responsibilities in SCM
Fattahi et al. (2014)	Factors affecting inventory management	Included demand, lead time, production capacity
Wang et al. (2015)	Integrated approach in SCM	Provided a comprehensive approach to SCM
Shaikh et al. (2019)	Blockchain preventing counterfeiting	Highlighted blockchain's role in preventing counterfeiting

To achieve cost efficiency and effectiveness, organizations must coordinate inventory management decisions across all three levels. Various factors, such as demand and lead time, production capacity, and transportation costs, are considered to determine the optimal levels of inventory and replenishment quantities. Furthermore, the model considers factors such as seasonality, promotions, and customer preferences to make more accurate demand forecasts (Fattahi et al., 2015). Overall, the Three Echelon Inventory Model provides a comprehensive and integrated approach to inventory management in a supply chain (Wang et al., 2015). A well-timed and properly stocked inventory helps organizations streamline operations, reduce costs, and increase customer satisfaction.

The application of blockchain in SCM has been explored across various contexts, such as the pharmaceutical industry, where it helps prevent the counterfeiting of medicines (Shaikh et al., 2019). The immutable and transparent nature of blockchain records ensures the authenticity and provenance of medical products, safeguarding end-consumers. Additionally, blockchain-enabled supply chains provide increased visibility and responsiveness to disruptions, allowing actors to proactively adjust their operations.

Existing literature suggests that integrating blockchain technology into decentralized three-echelon supply chain models holds significant promise for improving transparency, traceability, and coordination among supply chain actors. The Stackelberg game allows the leader to influence the follower's behavior by strategically setting regulations or policies, leading to improved supply chain coordination and efficiency (Kumar et al., 2016). Combining these approaches, organizations can achieve a more resilient and efficient supply chain, ultimately enhancing overall performance and customer satisfaction.

3.2.1 Research Gap

Despite extensive research in supply chain management, several gaps remain unaddressed, particularly in the application of hierarchical decision-making frameworks like the Stackelberg game to multi-echelon inventory models. The primary research gaps that our model aims to fill include:

- Existing models often assume simultaneous decision-making, which is unrealistic for hierarchical supply chains. Our model uses the Stackelberg game to capture the sequential nature of decisions, with the retailer as the leader and the manufacturer as the follower.
- Many supply chain models assume an infinite planning horizon, overlooking practical constraints. Our model considers a finite planning horizon, making it more relevant to real-world scenarios.

- Traditional models frequently assume constant demand rates, which do not reflect market realities. We incorporate a time-dependent demand function, providing a more accurate representation of actual demand patterns.
- Improved coordination between supply chain levels is often neglected. By adopting the Stackelberg game approach, our model enhances coordination between the retailer and the manufacturer, leading to better inventory management and cost efficiency.

Our research contributes to the field by:

- Introducing a hierarchical decision-making framework for three-echelon supply chains using the Stackelberg game.
- Addressing the limitations of infinite planning horizons by considering a finite planning period.
- Incorporating time-varying demand functions to better reflect market dynamics.
- Enhancing inter-echelon coordination through the leader-follower dynamic, resulting in more efficient supply chain operations.

By addressing these gaps, our model provides a more realistic and practical approach to managing inventory in a three-echelon supply chain, offering valuable insights and a foundation for further research in hierarchical supply chain optimization.

The decentralized three-echelon supply chain model utilizing blockchain technology holds significant potential to enhance supply chain processes and performance within a finite planning horizon. By leveraging blockchain's capabilities of immutability, transparency, and traceability, supply chain actors can improve product visibility, mitigate risks, and foster stronger collaboration across the supply network. This research contributes to the growing body of literature on blockchain applications in SCM, offering a framework for understanding and leveraging blockchain's potential to improve supply chain efficiency and profitability.

By addressing the key challenges in traditional supply chain models, blockchain technology can pave the way for more resilient and responsive supply chains, ultimately benefiting all stakeholders involved. For maximum impact, these models must be further refined and explored across a variety of industries and scenarios.

Analytical solution

The analytical solution involves optimizing the retailer's and manufacturer's operations. For the retailer, this includes calculating the total cost function, including ordering, deterioration costs, and holding. and using the linear demand function to determine the efficient order quantity per cycle and replenishment cycles. The negative impact of blockchain on demand is considered in this optimization process. For the manufacturer, this involves calculating the optimal production schedule and replenishment intervals to minimize production and holding costs and determining the 'break-even point' where cumulative production meets the total required quantity. The combined approach ensures a coordinated strategy between the retailer, manufacturer, and supplier, optimizing inventory levels and minimizing total supply chain costs. This integration leverages blockchain technology while addressing its potential negative impact on demand. The supplier's constant availability ensures that the manufacturer can meet production schedules without interruptions.

3.3 Assumptions and Notations

3.3.1 Assumptions

- The rate at which the product is demanded varies with time. We assume this demand function changes smoothly over time.
- The production rate (P) is fixed, and it exceeds the demand rate within the planning horizon.
- The holding costs incurred by the manufacturer exceed those of the retailer.
- The setup cost of the manufacturer is higher than the retailer.
- The manufacturer produces goods at a constant rate until a certain point (T_K), then production halts until its resume at H.
- The buyer decides when to place orders within the planning horizon, and we assume they make n orders during this time.
- A single, non-perishable item is retained in stock over a finite planning horizon.
- In the three–level supply chain approach, we did not keep the ordering time interval fixed.
- During the Planned horizon, deteriorated units are not replaced or repaired. *[This assumption reflects industries where damaged or expired products are discarded rather than refurbished (e.g., food, pharmaceuticals)].*
- Demand is viewed as a time-dependent linear function that may be negatively impacted using blockchain technology. *[While blockchain improves transparency and efficiency, it may also alter demand patterns by shifting customer trust and reducing overstocking by retailers. Assuming a linear demand function simplifies mathematics.]*
- For all the players like retailer, supplier, and manufacturer, in the supply chain management, the deterioration rate (θ) is assumed to be constant. *[A constant deterioration rate allows for analytical tractability and is reasonable for products with predictable degradation (e.g., stored grains, chemicals)].*

3.3.2 Notations

Retailer

Q_{Rj} → ordering quantity

O_R → Retailer is the ordering cost per order.

H_R → Retailer is the holding cost per unit time.

D_{CR} → is the deterioration cost per unit for the retailer.

$I_{R_{j+i}}$ → Retailer inventory at time.

Manufacturer

H_M → Holding cost per unit of manufacturer

DC_M → Manufacturer is the deterioration cost per unit.

$I_{M_{j+1}}(t)$ → Manufacturer inventory at time.

TCM → Total cost of manufacturer

M → The rate at which the manufacturer produces unit.

$C_{\alpha\beta}$ → Cost associated with the implementation operations of the Blockchain which depend on $\alpha\beta$.

β → Sensitivity price to share information.

α → Rate of item deterioration per time at the retailer, where $0 < \theta < 1$

Decision Variables

n → The optimal number of shipments delivered to the retailer per cycle.

H → The overall timeframe for Planning and optimization.

Qi → The optimal quantity of units to be produced in each batch (Q_1)

and delivered in each shipment (Q_2)

Q_2 (t) → The total quantity consumed after time t.

$Q_1(t)$ → The total quantity of production at time t.

$Q_1(r, t)$ → Total quantity produces for first r no. of cycles after first r deliveries.

$Q_2(r, t)$ → Total quantity consumed in the lost n – r no. of cylces.

Cp – Purchasing cost

T_j → The j^{th} replenish time, assuming $t_j = 0$ & $t_n = H$

T_{j+1} → The length of the $(j+1)^{th}$ replenishment period, where by j = 0, 1, 2, 3 …..

- We examine the inventory dynamics separately for the retailer and the manufacturer.
- For the retailer, we observe how their stock level changes over time and calculate the total inventory cost.
- For the manufacturer, we analyze their inventory levels during and between production cycles and compute their total inventory cost

3.4 Mathematical Model

Our goal is to reduce total costs while meeting demand efficiently throughout the supply chain by applying the Stackelberg game approach, where the retailer is the leader and the manufacturer follows. This model allows the retailer to optimize their ordering strategy while the manufacturer responds by adjusting production and inventory levels accordingly.

In this model, we consider the initial inventory for the manufacturer, supplier and retailer to be zero. The production cycle during is H. At t = 0, the manufacturer has a basket containing the stock needed for period 1. The retailers acquisition strategy involving making n orders at time $0 \leq$ to $< t_1 < t_2 < \ldots.. n \leq H$ where n and $t_0, t_1, t_2 \ldots\ldots\ldots t_n$ are determined. We examine the retailer, manufacturer, and supplier separately, beginning with the retailer.

3.4.1 Retailer model

Let $I_R(t)$ denote the retailer's stock level at time t. The inventory level for the retailer during the ordering period j.

$$\frac{dI_{R_{j+1}}(t)}{dt} = -(a - bt + \alpha\beta) - \theta_R I_{R_{j+1}}(t) \tag{3.1}$$

$D(t) = a - bt + \alpha\beta$, where $t_j < t < t_{j+1}$ represents the demand, where is a linear function of time and includes the impact of the Blockchain system

$$I_{R_{j+1}}(t) = e^{-\theta_R t} \int_t^{t_{j+1}} (a - bu + \alpha\beta) e^{\theta_R \mu} du \tag{3.2}$$

<u>Boundary Conditions</u>:

$$I_{R_{j+1}}(t_{j+1}) = 0 \ \ \& \ I_{R_{j+1}}(t_j) = Q_{R_j}$$

$$I_{R_{j+1}}(t) = \int_t^{t_{j+1}} e^{\theta_R(u-t)} (a - bu + \alpha\beta) du \tag{3.3}$$

$$I_{R_{j+1}}(t) = \frac{(\alpha\beta + a - bu)\, e^{\theta_R(u-t)}}{\theta} + \frac{b}{\theta^2} e^{\theta_R(u-t)} \Big|_t^{t_{j+1}}$$

$$I_{R_{j+1}}(t) = \left(\frac{(a - bt_{j+1} + \alpha\beta)}{\theta_R} + \frac{b}{\theta_R^2}\right) e^{\theta_R(t_{j+1} - t)} - \left[\frac{(a - bt + \alpha\beta)}{\theta_R} + \frac{b}{\theta_R^2}\right] \tag{3.4}$$

The quantity ordered for the jth cycle denoted as Q_{R_j}, is

$$Q_{R_j} = I_{R_{j+1}}(t_j) = \int_{t_j}^{t_{j+1}} (a - bt + \alpha\beta) e^{\theta_R(t - t_j)} dt \tag{3.5}$$

$$Q_{R_j} = I_{R_{j+1}}(t_j) = \left(\frac{(a - bt_{j+1} + \alpha\beta)}{\theta_R} + \frac{b}{Q_R^2}\right) e^{\theta_R(t_{j+1} - t_j)} - \left(\frac{(a - bt_j + \alpha\beta)}{\theta_R} + \frac{b}{\theta_R^2}\right) \tag{3.6}$$

The total cost for the retailer, TCR, includes ordering cost, inventory holding costs, and deteriorating costs.

Ordering costs: - n. Or

Inventory holding costs: $H_R \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \left(\int_{t}^{t_{i+1}} (a - bu + \alpha\beta) e^{\theta_R(u-t)} du\right) dt$ (3.7)

Deteriorating Costs according to (Sarkar.et.al 2012)

$= D_{C_R} \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \theta_R \left(\int_{t}^{t_{j+1}} (a - bu + \alpha\beta) e^{\theta_R(u-t)} du)\right) dt$ (3.8)

Thus, the total cost (TCR) for the retailer

$$TCR\ (t_j, n) = nO_r + (H_R + \theta_R D_R) \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} I_{R_{j+1}}(t) dt \tag{3.9}$$

$$TCR\ (t_j, n) = nO_r + (H_R + \theta_R D_R)$$

$$+ \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \left\{ \left(\frac{(a - bt_{j+1} + \alpha\beta)}{\theta_R} + \frac{b}{\theta_R^2} \right) e^{\theta_R(t_{j+1} - t_j)} - \left(\frac{(a - bt + \alpha\beta}{\theta_R^2} + \frac{b}{\theta_R^2} \right) \right\} dt \tag{3.10}$$

$$TCR\ (t_j, n) = nO_r + (H_R + \theta_R D_R) +$$

$$\left[\sum_{j=1}^{n} \frac{a}{\theta_R^2}\left(1 - e^{\theta_R(t_{j+1} - t_j)}\right) - \frac{a}{\theta_R}(t_{j+1} - t_j) - \frac{bt_{j+1}}{\theta_R^2}\left(1 - e^{\theta_R(t_{j+1} - t_j)} + \frac{b}{2\theta_R}(t_{j+1}^2 - t_j^2)\right)\right.$$

$$\left. + \frac{b}{\theta_R^3}\left(1 - e^{\theta_R(t_{j+1} - t_j)}\right) - \frac{b}{\theta_R^2}(t_{j+1} - t_j) + \frac{\alpha\beta}{\theta_R^2}\left(1 - e^{\theta_R(t_{j+1} - t_j)}\right) - \frac{\alpha\beta}{\theta_r}(t_{j+1} - t_j)\right]$$

$$\frac{\partial}{\partial t_j} TCR\ (t_j, n) = (a - bt_{j+1} + \alpha\beta)\left(e^{\theta_R(t_j - t_{j-1})} - 1\right) - \theta_R \int_{t_j}^{t_{j+1}} (a - bt + \alpha\beta) e^{\theta_R(t - t_j)} dt \tag{3.11}$$

For j = 1,2,3,…., n - 1

$$\frac{\partial}{\partial t_j}\ \mathrm{TCR}\ (t_j, n) = (a - b\ t_{j+1} + \alpha\beta) + \left(e^{\theta_R(t_j - t_{j-1})} - 1\right) - \left(\left(a - bt_{j+1} + \alpha\beta\right) + \frac{b}{\theta_R}\right) e^{\theta_R(t_{j+1} - t_j)} - \left(\left(a - bt_j + \alpha\beta\right) + \frac{b}{\theta_R}\right)$$

$$\frac{\partial}{\partial t_j}\mathrm{TCR}\ (t_j, n) = (a - b\ t_{j+1} + \alpha\beta)\ \left(e^{\theta_R (t_j - t_{j+1})} - 1\right) - \theta_R \int_{t_j}^{t_{j+1}} (a - bt + \alpha\beta)\, e^{\theta_R (t - t_j)} dt = 0 \qquad (3.12)$$

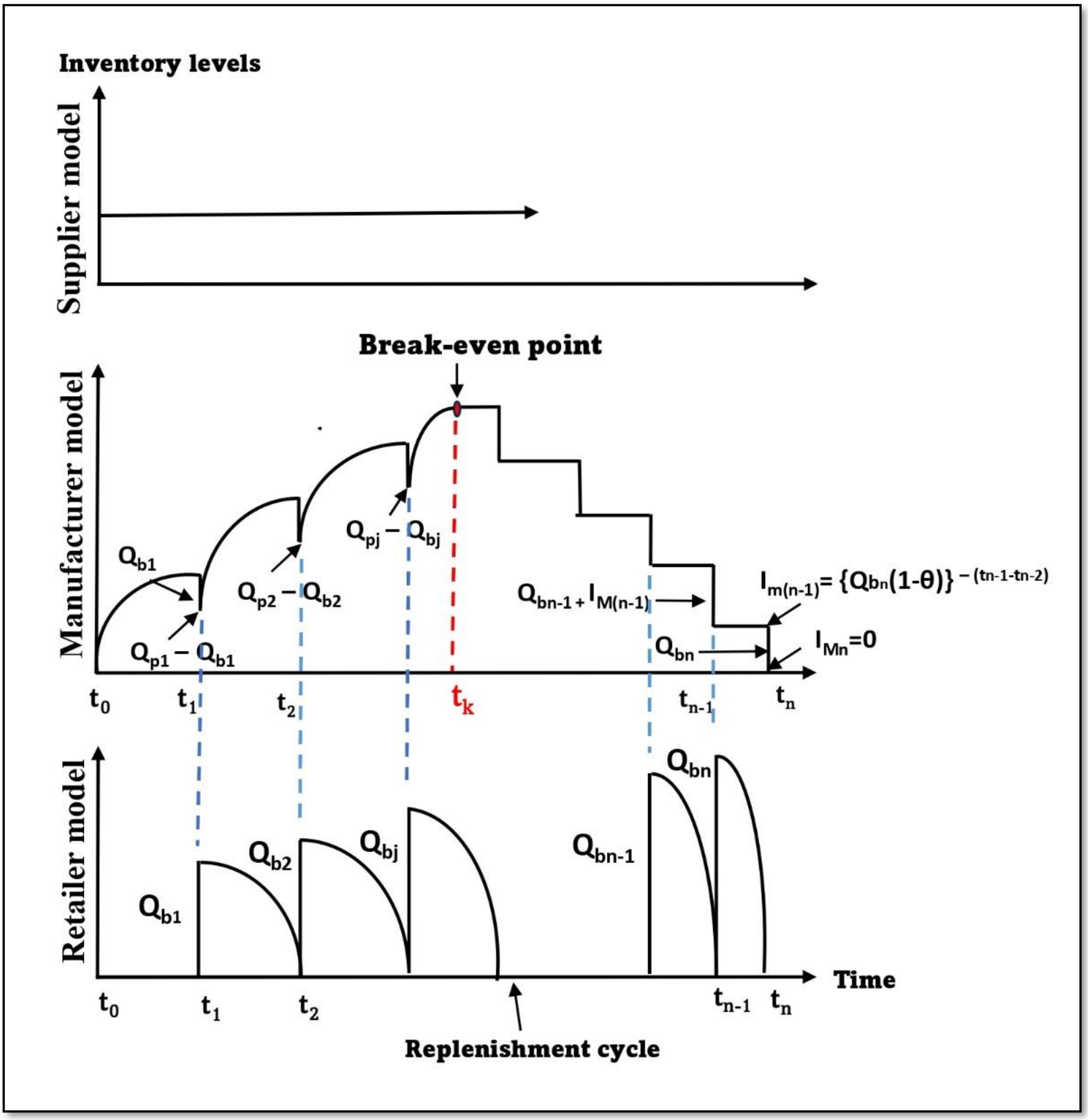


Figure 3.1 Three-Tier supply Chain proposed Inventory model

3.4.2 Manufacturer Model

The mathematical model consists of two phases:

Production Phase:

- In this phase, the producer manufactures items at a constant production rate M. The producer simultaneously replenishes the retailer's inventory at regular intervals during each cycle.
- The total quantity produced, Q_1 is determined based on the remaining planning time horizon and the retailer's demand.
- The producer continues production until reaching a break-even point, t, where the total quantity produced equals $Q_1 + Q_2$ (the quantity required by the retailer).
- At the break-even point (illustrated in figure 3.1), production halts.

Replenishment Phase:

- After the break-even point, there is no further production. However, the producer continues to replenish the retailer's inventory at predetermined intervals.
- This phase involves only the distribution of the already manufactured items to the retailer.

An analytical solution to this problem has been derived to optimize the production and replenishment schedule, ensuring that the producer meets the retailer's demand efficiently within the finite planning horizon.

The manufacturer's inventory level over time is: -

$$\frac{dM_{j+1}(t)}{dt} = M - \theta_M I_{M_{j+1}}(t) \tag{3.13}$$

$$\frac{dM_{j+1}(t)}{dt} + \theta_M I_{M_{j+1}}(t) = M \qquad t_i \text{ is less than t, which is less than } t_{j+1} \tag{3.14}$$

The general solution to this equation is obtained by using an integrating factor = $e^{\theta_M(t)}$

$$I_{M_{j+1}} e^{\theta_M(t)} = \int M.\, e^{\theta_M(t)} dt \;\; C \;\; \text{('C' arbitrary constant)}$$

$$I_{M_{j+1}}(t) e^{\theta_M(t)} = \frac{M e^{\theta_M(t)}}{\theta_M} + C \;\; \text{('C' arbitrary constant)}$$

$$I_{M_{j+1}}(t) = \frac{M}{\theta_M} + C e^{-\theta_M(t)} \tag{3.15}$$

Given that $I_{M_{j+1}}(t_j) = q_{M_{j-1}} - q_{R_{j-1}}$

at $t = t_j$,

$$I_{M_{j+1}}(t_j) = \frac{M}{\theta_M} + Ce^{-\theta_M t_j} = q_{M_{j-1}} - q_{R_{j-1}} \tag{3.16}$$

$$C = \left(q_{M_{j-1}} - q_{R_{j-1}} - \frac{M}{\theta_M}\right) e^{\theta_M t_j} \tag{3.17}$$

Substitute (3.17) in equation (3.16), we get

$$I_{M_{j+1}}(t_j) = \frac{M}{\theta_M} + \left(q_{M_{j-1}} - q_{R_{j-1}} - \frac{M}{\theta_M}\right) e^{-\theta_M (t - t_j)} \tag{3.18}$$

Inventory levels at specific times: - at $t = t_{j+1}$,

$$I_{M_{j+1}}(t_{j+1}) = q_{M_j} - q_{R_j}$$

$$I_{M_{j+1}}(t_{j+1}) = \frac{M}{\theta_M} + ce^{-\theta_M t_j} = q_{M_j} - q_{R_j} \tag{3.19}$$

$$C = \left(q_{M_{j-1}} - q_{R_{j-1}} - \frac{M}{\theta_M}\right) e^{\theta_M t_{j+1}} \tag{3.20}$$

Substitute (3.20) in equation (3.19), we get

$$I_{M_{j+1}}(t_{j+1}) = \frac{M}{\theta} + \left(q_{M_{j-1}} - q_{R_{j-1}} - \frac{M}{\theta_M}\right) e^{-\theta (t_{j+1} - t_j)} \tag{3.21}$$

Q_1 (s, t): Total quantity produced in the first r cycles: -

$$Q_1 (s, t) = \sum_{j=1}^{s} \frac{M}{Q_M} \left(1 - e^{\theta (t_j - t_{j+1})}\right) + \sum_{j=1}^{s} q_{R_i} \tag{3.22}$$

The total production in the first r cycles.

Q_2 (n, s, t): Total quantity consumed in the last (n – s) cycles:

$$Q_2 (n, s, t) = \sum_{j=1}^{s+1} q_{R_{n-j-1}} (1 - \theta)^{(t_{n-j-1} - t_{n-s})} \tag{3.23}$$

The total quantity consumed in the last (n – r) cycles.

$$\mathrm{TCM}(n^*, t_1^*, t_2^* \ldots\ldots\ldots\ldots\ldots t_n^*) = n^* * S_M + \text{Blockchain Implementation Cost} +$$

$$\sum_{j=1}^{s} \frac{M}{Q_M}\left(1 - e^{\theta(t_j - t_{j+1})}\right) + \sum_{j=1}^{s} q_{r_i} + \sum_{j=1}^{s+1} q_{R_{n-j-1}} (1-\theta)^{(t_{n-j-1} - t_{n-s})} \quad (3.24)$$

$$\mathrm{TCS}(n^*, t_1^*, t_2^* \ldots\ldots\ldots\ldots\ldots t_n^*) = n^* * S_s + \sum_{i=1}^{n^*} \frac{M}{\theta} * \left\{1 - e^{\theta(t_j - t_{j+1})}\right\} \quad (3.25)$$

Preposition: - $(a - bt_{j+1} + \alpha\beta)e^{\theta T_{j+1}} < (a - bt_{j+1} + \alpha\beta)e^{\theta_R T_j} + \frac{b}{\theta_R}(1 - e^{\theta_R T_j})$

Proof: - let $f(t) = e^{\theta(t - t_j)}(a + bt + \alpha\beta)$ is a long–concave function

$$f(t_{j+1}) - f(t_j) < \frac{f'(t_j)}{f(t_j)} \int_{t_j}^{t_{j+1}} f(u)du \quad (3.26)$$

$$\left(a - bt_{j+1} + \alpha\beta\right)e^{\theta_R(t_{j+1} - t_j)} - \left(a - bt_j + \alpha\beta\right) < \int_{t_j}^{t_{j+1}} (\alpha\beta + a - bt)e^{\theta_R(t - t_j)} \left(\theta_R - \frac{b}{a - bt_j + \alpha\beta}\right) dt$$

(3.27)

Then using equation (3.12), equation (3.27) is given by

$$\left(a - bt_{j+1} + \alpha\beta\right)e^{\theta T_{j+1}} - \left(a - bt_j + \alpha\beta\right) < \left(\theta_R - \frac{b}{a - bt_j + \alpha\beta}\right) \frac{\left(a - bt_j + \alpha\beta\right)e^{\theta_R T_j} - \left(a - bt_j + \alpha\beta\right)}{\theta_R}$$

$$e^{\theta T_{j+1}}\left(a + \alpha\beta - bt_{j+1}\right) - \left(a - bt_j + \alpha\beta\right) < e^{\theta_R T_j}\left(a - bt_j + \alpha\beta\right) - \left(a - bt_j + \alpha\beta\right) - \frac{be^{\theta_R T_j}}{\theta_R} + \frac{b}{\theta_R}$$

. (3.28)

$$e^{\theta_R T_{j+1}}\left(a - bt_{j+1} + \alpha\beta\right) < e^{\theta_R T_j}\left(a - bt_{j+1} + \alpha\beta\right) + \frac{b}{\theta_R} - \frac{b}{\theta_R} e^{\theta_R T_j}$$

$$e^{\theta_R T_{j+1}}\left(a - bt_{j+1} + \alpha\beta\right) < e^{\theta_R T_j}\left(a - bt_{j+1} + \alpha\beta\right) + \frac{b}{\theta_R}\left(1 - e^{\theta_R T_j}\right)$$

Lemma: - Increase of T_j where T_n was the last replenishment cycle, but j was 1, 2, 3, ..., n–1 strictly monotonous increase function.

Proof: T_n = H – T_n, shows that T_j decreases with then T_n = n – 1, n - 2, …. 3, 2, 1.

Put j = n – 1 in equation (3.12), we get

$$\frac{\partial}{\partial T_j} TCR(n, j = e^{\theta_R (T_{n-1})}[(\alpha\beta + a - b(T_n - H)] - [(\alpha\beta + a - b(Tn_n - H)]$$
$$- \theta_R \int_{H-T_n}^{H} e^{\theta_R (t-H+T_n)}(a + \alpha\beta - bt)dt = 0$$

$$e^{\theta_R (T_{n-1})}[(a - b(t_n - H) + \alpha\beta] \frac{d}{dT_n}(T_{n-1}) + e^{\theta_R (T_{n-1})} - b + \alpha\beta - [(a - b(t_n - H) + \alpha\beta] -$$
$$\theta^2 \int_{H-T_n}^{H} e^{\theta_R (t-H-T_n)}(a - bt + \alpha\beta)dt = 0 \tag{3.29}$$

by equation (3.12), $\left(a - bt_j + \alpha\beta\right)\left(e^{\theta_R T_j} - 1\right) = \theta_R \int_{t_j}^{t_{j+1}} (a - bt + \alpha\beta)e^{\theta_R (t-t_j)}dt$ and this put this in (3.29), we get

$$\frac{d(T_{n-1})}{dT_n} \theta_R [a - b(T_n - H) + \alpha\beta]e^{\theta_R (T_{n-1})} = b\left(1 - e^{\theta_R (T_{n-1})}\right) + \theta_R e^{\theta_R (T_{n-1})}[a - b(T_n - H) + \alpha\beta]$$

$$e^{\vartheta_R (T_{n-1})}\theta_R [a + \alpha\beta - b(T_n - H)]\left\{\frac{d(T_{n-1})}{dT_n} - 1\right\} = b\left(1 - e^{\theta_R (T_{n-1})}\right)$$

$$\frac{d(T_{n-1})}{dT_n} = \frac{b(1 - e^{\theta_R (T_{n-1})}}{\theta_R [a - b(t_n - H) + \alpha\beta]e^{\theta_R (T_{n-1})}} - 1$$

$$\frac{d}{dT_n}(T_{n-1}) = \frac{(1 - e^{\theta_R (T_{n-1})} b + \theta_R [a + \alpha\beta - b(t_n - H)]e^{\theta_R (T_{n-1})}}{\theta_R e^{\theta_R (T_{n-1})}[a + \alpha\beta - b(T_{n-1})]} \geq 0 \tag{3.30}$$

After that, Let's us take that $\frac{d(T_s)}{dT_n} > 0$ for s = j+1, j+2, ….., n then again differentiate equation (3.12), with respect to T_n, we have

$$b\left(1 - e^{\theta T_j}\right)\frac{d}{dT_n}(t_j) + \theta_R \left(a - bt_j + \alpha\beta\right)e^{\theta_R T_j}\frac{d}{dT_n}(T_j)$$

$$\theta_R\left(a-bt_{j+1}+\alpha\beta\right)e^{\theta_R T_{j+1}}\frac{dt_{j+1}}{dT_n}+\theta_R\left(a-bt_j+\alpha\beta\right)\frac{dt_j}{dT_n}+\theta_R^2\frac{dt_j}{dT_n}\int_{t_j}^{t_{j+1}}e^{\theta_R(t-t_j)}(a+\alpha\beta-bt)dt$$

. (3.31)

Using Prepositions and equation (3.12), we get

$$\frac{d(t_j)}{dT_n}\le\frac{d}{dT_n}(t_{j+1})=-\sum_{m=j+2}^{n}\frac{d(T_s)}{dT_n}-1\le 0 \quad (3.32)$$

This implies $\frac{d}{dT_n}(T_j)\ge 0$ where j = 1, 2, 3, ……….. n

Moreover, as we know that $T_n = H - t_{n-1}$ this implies

$\frac{d(T_j)}{dT_{n-1}}\le 0$ where j = 1, 2, 3, ……….. n – 1

Now, we take $t_j = H-\sum_{s=j+1}^{n}T_p - T_n = t_n-\sum_{s=j+1}^{n}T_p$ from this, it concludes $\frac{d}{dt_n}(T_j)\ge 0$ for all j = 1, 2, 3, …., n

$T_{j+1} = t_{j+1} - t_j$ and $t_n = H - T_n$

Using equation (3.32), we can find the unique solution to the non-linear equation to determine the optimal ordering period. TCR (t_j, n) can only be minimized for a fixed n if the Hessian matrix of TCR (t_j, n) is positive and definite. As a result of this theorem, TCR (t_j, n) has always been positive. Therefore, using a mathematical program and numerical interactive technique, the optimal value for t_j can be calculated.

Proof: The purpose of the calculation of the values of t_j is to decrease the total variable cost TCR of the system. First and foremost, the primary requirement to find t_j is to ensure that the $\frac{\partial}{\partial t_j}$ TCR (t_j, n) = 0

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_i^2}=\theta_R(a-bt_i+\alpha\beta)e^{\theta_R T_j}+\theta_R e^{\theta_R T_{j+1}}\left(a+\alpha\beta-bt_{j+1}\right)-b\left(e^{\theta_R T_{j+1}}-e^{\theta_R T_j}\right) \quad (3.33)$$

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_j\,\partial t_{j-1}}=-\theta_R e^{\theta_R T_j}\left(a-bt_j+\alpha\beta\right) \quad (3.34)$$

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_j\,\partial t_{j+1}} = -\theta_R e^{\theta_R T_{j+1}}\left(a - bt_{j+1} + \alpha\beta\right) \tag{3.35}$$

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_j\,\partial T_S} = 0 \qquad \text{for all } s \neq j,\ j+1,\ j-1 \tag{3.36}$$

TCR is positive definite if equation (3.33), (3.34), (3.35) & (3.36) satisfy the given inequality

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_i^2} > \left|\frac{\partial^2 TCR(t_j,n)}{\partial t_j\,\partial t_{j-1}}\right| + \left|\frac{\partial^2 TCR(t_j,n)}{\partial t_j\,\partial t_{j+1}}\right| + \left|\frac{\partial^2 TCR(t_j,n)}{\partial t_j\,\partial t_s}\right|$$

$$\theta_R e^{\theta_R (T_{j+1})}\left(a + \alpha\beta - bt_j\right) + \theta_R e^{\theta_R (T_{j+1})}\left(a + \alpha\beta - bt_{j+1}\right) - b(e^{\theta_R T_j} - e^{\theta_R T_{j+1}})$$
$$> \left|-\theta_R e^{\theta_R T_j}\left(a + \alpha\beta - bt_j\right)\right| + \left|-\theta_R e^{\theta_R T_{j+1}}\left(a + \alpha\beta - bt_{j+1}\right)\right| + 1$$

that is true for all j = 1, 2, ….. n.

Due to the favorable diagonal elements and the strongly dominant diagonal characteristics in the Hessian matrix, it is classified as a positive definite. This property facilitates the identification of the best replenishment timing for the non-linear equation system (3.12). Following this, it is important to establish that the optimal solution to equation (3.12) is both unique and that the Total Cost Rate (TCR) function exhibits a curved behavior at the ideal value of tj within the limited time frame for planning n.

<u>Theorem 2:</u> TCR (n, t_0, t_1, t_2, …..,, t_n) is a convex function in a finite horizon planning n.

Proof: - TCR (t_j, n) = n O_R + (H_R + θ_R D_r) $\sum_{j=1}^{n}\int_{t_j}^{t_{j+1}} I_{R_{j+1}}(t)dt$

Where t_o = 0 and t_n = T, Now, let us assume $g(n,0,T) = \sum_{j=1}^{n}\int_{t_j}^{t_{j+1}} e^{\theta_R(t-t_j)}(a - bt + \alpha\beta)dt$

$$g(n+1,0,T) - g(n,0,T)$$
$$= \int_{t_{n-1}}^{t_n} e^{\theta_R(t-t_{n-1})}(a + \alpha\beta - bt)dt$$
$$+ \int_{t_n}^{T} e^{\theta_R(t-t_n)}(a + \alpha\beta - bt)dt - \int_{t_{n-1}}^{T} e^{\theta_R(t-t_{n-1})}(a + \alpha\beta - bt)dt$$

$$= \int_{t_{n-1}}^{t_n} e^{\theta_R(t-t_{n-1})}(a+\alpha\beta-bt)dt + \int_{t_n}^{t} e^{\theta_R(t-t_n)}(a+\alpha\beta-bt)dt - \int_{t_{n-1}}^{t_n} e^{\theta_R(t-t_{n-1})}(a+\alpha\beta-bt)dt - \int_{t_n}^{t} e^{\theta_R(t-t_{n-1})}(a+\alpha\beta-bt)dt$$

$$g(n+1,0,T) - g(n,0,T) = \int_{T_n}^{T}(a-bt+\alpha\beta)\left\{e^{\theta_R(t-t_n)} - e^{\theta_R(t-t_{n-1})}\right\}dt < 0$$

g (n +1, 0, T) < g (n, 0, T)

= g (n, 0, T) – g (n – 1, 0, T) – [g (n + 1, 0, T) – g (n, 0, T)]

$$= \int_{t_{n-1}}^{T}(a-bt+\alpha\beta)\left[e^{(t-t_{n-1})\theta_R} - e^{(t-t_{n-2})\theta_R}\right]dt + \int_{t_n}^{T}\left[e^{\theta_R(t-t_{n-1})} - e^{\theta_R(t-t_n)}\right](a-bt+\alpha\beta)\,dt$$

$$= \int_{t_{n-1}}^{T}\left[e^{(t-t_{n-1})\theta_R} - e^{(t-t_{n-2})\theta_R}\right](a+\alpha\beta-bt)\,dt - \int_{t_n}^{T}\left[e^{(t-t_{n-2})\theta_R} - e^{(t-t_{n-1})\theta_R}\right](a+\alpha\beta-bt)\,dt$$
$$+ \int_{t_n}^{t}\left[e^{\theta_R(t-t_{n-1})} - e^{\theta_R(t-t_n)}\right](a+\alpha\beta-bt)\,dt$$

$\Rightarrow$ g (n, 0, T) – g (n – 1, 0, T) < g (n + 1, 0, T) – g (n, 0, T)

e^t is a convex function, as a result, g (n, 0, T) is also convex in n.

$\therefore$ TCR (t_j, n) is an essential convex function.

3.5 Methodology for solving the three- echelon inventory optimization problem

Step-1 Begin by setting the initial values for all relevant parameters, such as W, hr, P, a, b, and c.

Step-2 To identify the optimal ordering time pattern for the three-echelon inventory system, follow these steps:

a) For n=1, set t_0=0 and t_1=H.
b) For n=2, initialize t_0=0 and t_2=H. Calculate t_1 using equation (3.12).
c) Use the obtained t_1 value from the previous step to calculate t_2 using equation (3.12).
d) Repeat this process iteratively to find t_3 from equation (3.12), continuing until t_{n-1} is obtained.
e) Ensure that t_{n-1} approximately equals H (finite horizon planning) and that t_j values satisfy the Hessian matrix condition.
f) Identify the unique and optimal t_j values for each m≥1.

Step-3 After determining the t_j values using equation (3.12), compute the values of Q_1 and Q_2.

Step-4 Find the value of t by solving the equation $Q_1 - Q_2 = 0$.

Step-5 Determine the value of TCR (t_j,n) using equation (3.11).

Step-6 Using equation (3.11), calculate TCR (t_j,n) to determine the optimal total cost for the retailer. Follow these steps:

a) For n=1, if TCR (t_j,n) = TCR(t_j,n)n, then stop.
b) For n≥2, if TCR (t_j, n) ≤ TCR (t_j, n−1), and TCR (t_j, n) ≤ TCR (t_j, n+1), then TCR (t_j,n) is optimal, and stop; otherwise, return to the previous step.
c) Similarly, calculate TCR (t_j, n) using equation (3.11).

Step-7 Utilize the solution approach outlined above to derive the optimal values for n_1, Q_1, Q_2, and TCR (t_j, n).

This methodology ensures a systematic and iterative approach to solving the three-echelon inventory optimization problem using a Stackelberg game framework. The steps outlined facilitate the determination of optimal ordering times, order quantities, and total costs, thereby enhancing the efficiency and cost-effectiveness of the supply chain.

3.6 Numerical example

Table 3.2 and Figure 3.2 illustrate the total costs for the retailer at different initial inventory levels (450, 600, and 750 units) across varying numbers of shipments (n = 1 to n = 8). In Table 3.2, the rows represent the initial inventory levels, and the columns represent the number of shipments. Each cell indicates the total cost for the retailer at the corresponding inventory level and number of shipments.

For an initial inventory level of 450 units, the total cost starts at 8844.265 for 1 shipment and fluctuates slightly, reaching 10988.009 for 8 shipments. The minimum cost occurs at 2 shipments (8621.0125), indicating this might be an optimal point for minimizing costs at this inventory level. For an initial inventory level of 600 units, the total cost begins at 13189.845 for 1 shipment and shows a decreasing trend initially, with the lowest cost at 3 shipments (12159.787). As the number of shipments increases beyond 4, the total cost starts to rise again, indicating an optimal number of shipments around 3 to 4. For an initial inventory level of 750 units, the total cost starts at 18670.163 for 1 shipment and decreases significantly, reaching a minimum at 4 shipments (16073.515). Like the previous scenarios, the costs rise slightly with more shipments, suggesting an optimal shipment number between 4 and 5.

Table 3.3 and Figure 3.3 illustrate the total costs for the retailer at different initial inventory levels (450, 600, and 750 units) across varying numbers of shipments (n = 1 to n = 8). In the table, the rows represent the initial inventory levels, and the columns represent the number of shipments. Each cell indicates the total cost for the retailer at the corresponding inventory level and number of shipments.

Table 3.2 Total cost of the retailer

$\downarrow a \rightarrow n$	1	2	3	4	5	6	7	8
450	8844.265	**8621.0125**	8677.601	8922.764	9262.673	10082.278	10528.584	10988.009
600	13189.845	12425.307	**12159.787**	12195.897	12391.533	12698.712	13060.075	13456.609
750	18670.163	17023.669	16303.459	**16073.515**	16092.679	16252.104	16497.552	16799.334

Table 3.3 Replenishment time for TCR, TCS, and EOQ

$\downarrow a$ $\rightarrow t_i$	t_0	t_1	t_2	t_3	t_4	n	TCR	TCS	EOQ
450	0	1.0165	4			5	8621.0125	1719.66	1146.942
600	0	0.0747	1.6413	4		5	12159.787	2036.853	1747.966
750	0	0.6664	1.3935	2.1871	4	5	16092.679	2346.641	2426.563

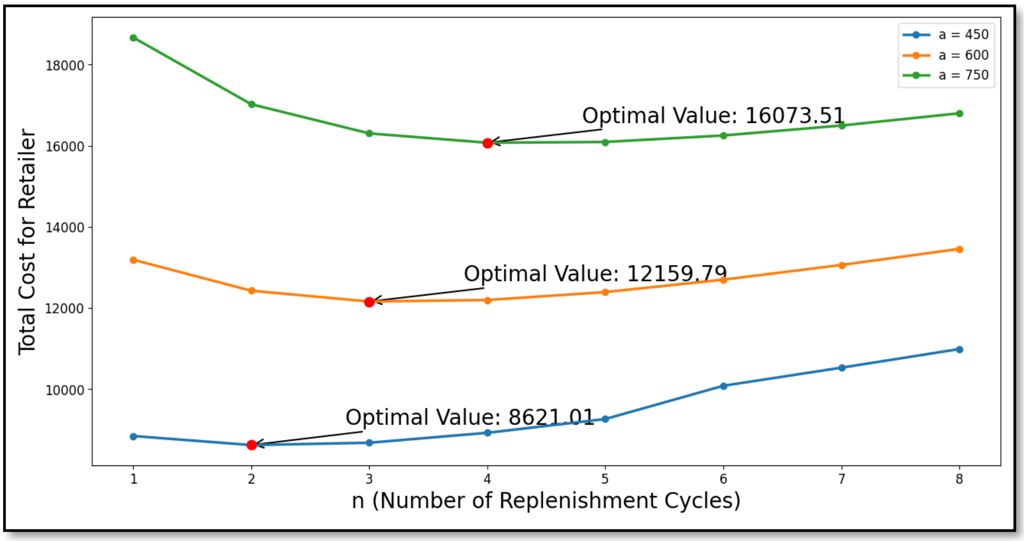


Figure 3.2 Optimal Total Cost of Retailer a= 450, 600, 750

For an initial inventory level of 450 units, the total cost starts at 8844.265 for 1 shipment and fluctuates slightly, reaching 10988.009 for 8 shipments. The minimum cost occurs at 2 shipments (8621.0125), indicating this might be an optimal point for minimizing costs at this inventory level. For an initial inventory level of 600 units, the total cost begins at 13189.845 for 1 shipment and shows a decreasing trend initially, with the lowest cost at 3 shipments (12159.787). As the number of shipments increases beyond 4, the total cost starts to rise again, indicating an optimal number of shipments around 3 to 4. For an initial inventory level of 750 units, the total cost starts at 18670.163 for 1 shipment and decreases significantly, reaching a minimum at 4 shipments (16073.515). Like the previous scenarios, the costs rise slightly with more shipments, suggesting an optimal shipment number between 4 and 5.

Figure 3.3 associated with this table visualizes the relationship between the total costs for the retailer and the number of shipments for each initial inventory level. Each curve in the figure would plot total costs against the number of shipments for the inventory levels of 450, 600, and 750 units. Key observations from the figure include the identification of optimal shipment numbers that minimize total costs and the comparison across different inventory levels. This visualization helps in understanding the optimal number of shipments needed to minimize the total costs for the retailer, depending on the initial inventory level. The analysis suggests that while fewer shipments may reduce handling costs, increasing shipments beyond a certain point results in higher costs due to increased ordering and holding expenses.

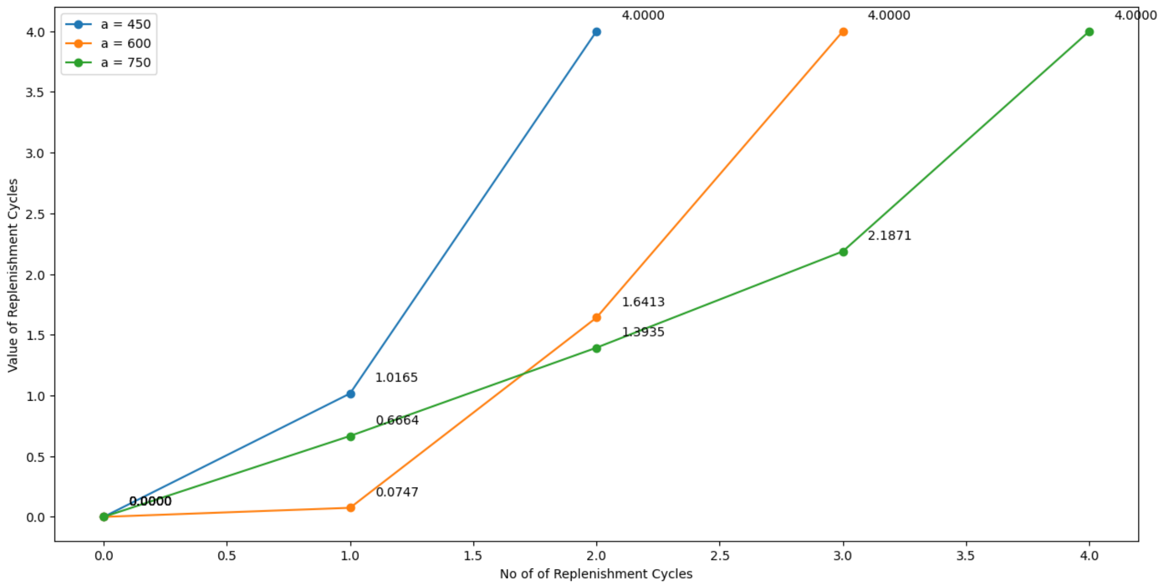


Figure 3.3 Replenishment time for TCR, TCS, and EOQ

Table 3.4 presents optimal production (Q_1) and consumption (Q_2) quantities concerning the number of shipments (n) and cycles (r) and the "break-even point" (t_k) for three scenarios. In Scenario 1, where Q_1=450, production, and consumption quantities are listed for 10 shipments and varying cycles, with the break-even point occurring at time 0.9609, indicating when cumulative production matches cumulative consumption. In Scenario 2, with Q_2=600, production, and consumption quantities are shown for 8 shipments and varying cycles, and the break-even point is at time 0.9981. In Scenario 3, where Q_1=750, production, and consumption quantities are detailed for 7 shipments and varying cycles, with the break-even point reached at time 1.1195.

This table provides critical data for understanding the supply chain's optimal production and consumption rates. By analyzing the break-even points and the relationship between Q_1 and Q_2 , the table helps optimize inventory management, ensuring efficient and cost-effective operations across the supply chain. The break-even points indicate when the total produced quantity equals the total consumed quantity, guiding strategic planning for inventory replenishment and production scheduling.

Table 3.4 Optimal Production Q_1 and Consumption Q_2 concerning n and r and "Break-even point t_k"

Q_1(450)								**n**	**r**		
113.171	258.346	444.630	683.752	**990.859**	1385.558	1893.333	2547.497	10	5		
Q_2											
732.320	837.369	911.887	**965.822**	1005.480	1035.008			10	4		
Q_1-Q_2											
t_0	t_1	t_2	t_3	t_4	t_5	**t_k**	t_6	t_7	t_8	t_9	t_{10}
0	0.1009	0.2305	0.3972	0.6116	0.8878	**0.9609**	1.2438	1.703	2.29831	3.0702	4

Q_1(600)								**n**	**r**
243.646	535.191	884.153	**1301.989**	1802.516				8	4
Q_2									
942.604	1102.673	**1223.288**	1315.656					8	3
Q_1-Q_2									
t_0	t_1	t_2	**t_k**	t_3	t_4	t_5	t_6	t_7	
0	0.3098	0.6810	**0.9981**	1.1262	1.6605	2.3026	3.0753	4	

Q_1(750)								**n**	**r**
352.248	736.292	**1154.990**	1611.450	2109.059				7	3
Q_2									
989.252	1206.945	**1387.837**	1539.350	1667.182				7	3
Q_1-Q_2									
t_0	t_1	t_2	**t_k**	t_3	t_4	t_5	t_6		
0	0.5290	1.1065	**1.1195**	1.7370	2.4255	3.1778	4		

3.7 Analysis of Parameters and Managerial Applications

3.7.1 Sensitivity analysis

In this section, we explain the sensitivity analysis conducted on various parameters affecting the total cost for the retailer (TCR), the total order quantity (EOQ), the total cost for the supplier (TCS), and the optimal replenishment cycle. The sensitivity analysis was performed by varying each parameter by ±10%, ±5%, and 0% to observe its impact on the key performance indicators. Below, we explain the heatmaps for each analysis, the parameters involved, and what the colours represent.

3.5 Sensitivity Analysis of Parameter

Parameter	% Change	Optimal Replenish Cycle	Total Order Quantity	Total Cost of Retailer (TCR)	Total Cost of Supplier (TCS)
a	+10	4	1518.490	16317.481	2147.171
	+5	4	1446.307	15488.991	2080.718
	0	3	1300.980	13189.845	2036.853
	-5	3	1226.408	12321.666	1968.127
	-10	3	1151.734	11453.090	1899.392
b	+10	3	1281.752	12722.991	2032.682
	+5	3	1291.410	12956.588	2034.760
	0	3	1300.980	13189.845	2036.853
	-5	3	1310.464	13422.759	2038.962
	-10	3	1319.864	13655.326	2041.085
α	+10	3	1301.079	13191.003	2036.945
	+5	3	1301.030	13190.424	2036.899
	0	3	1300.980	13189.845	2036.853
	-5	3	1300.930	13189.267	2036.808
	-10	3	1300.881	13188.688	2036.762
β	+10	3	1301.079	13191.003	2036.945
	+5	3	1301.030	13190.424	2036.899
	0	3	1300.980	13189.845	2036.853
	-5	3	1300.930	13189.267	2036.808
	-10	3	1300.881	13188.688	2036.762
H_R	+10	4	1300.980	14567.768	2016.288
	+5	3	1300.980	13878.807	2036.853
	0	3	1300.980	13189.845	2036.853
	-5	3	1300.980	12500.884	2036.853
	-10	3	1300.980	11811.923	2036.853
D_R	+10	3	1300.980	13192.907	2036.853
	+5	3	1300.980	13191.376	2036.853
	0	3	1300.980	13189.845	2036.853
	-5	3	1300.980	13188.314	2036.853
	-10	3	1300.980	13186.783	2036.853
θ	+10	3	1383.010	11682.511	2056.558
	+5	3	1342.566	12397.780	2046.445
	0	3	1300.980	13189.845	2036.853
	-5	3	1257.492	14059.961	2027.790
	-10	3	1210.931	14996.520	2019.268
O_R	+10	3	1300.980	13237.845	2036.853

	+5	3	1300.980	13216.845	2036.853
	0	3	1300.980	13189.845	2036.853
	-5	3	1300.980	13165.845	2036.853
	-10	3	1300.980	13141.845	2036.853
Ss	+10	3	1300.980	13189.845	2040.453
	+5	3	1300.980	13189.845	2040.153
	0	3	1300.980	13189.845	2039.853
	-5	3	1300.980	13189.845	2039.553
	-10	3	1300.980	13189.845	2039.253

3.7.1.1 Total Cost of Retailer

This heatmap displays (figure 3.4) how changes in different parameters impact the total cost incurred by the retailer. The colours represent the total cost, with darker shades indicating higher costs and lighter shades indicating lower costs. Significant observations include the total cost for the retailer is highly sensitive to changes in the ordering cost. A 10% increase in ordering cost leads to the highest increase in TCR, represented by dark red. The holding cost also significantly impacts TCR. An increase in holding cost results in a noticeable rise in the total cost, shown in lighter red to orange shades. The deterioration cost has a moderate impact on TCR. Changes in D_R lead to changes in TCR but not as drastically as O_R or H_R, represented by blue shades. Variations in β have a relatively stable impact on TCR, indicating that changes in information sharing sensitivity price do not dramatically alter the total cost, shown in consistent blue shades. The deterioration rate (θ) shows a significant impact on TCR. An increase in θ leads to a notable increase in total cost due to higher deterioration, indicated by a transition from blue to light red. Safety stock (S_s) level changes show a minor impact on TCR, indicating that maintaining safety stock has a stabilizing effect on total costs, represented by stable blue shades.

3.7.1.2 Total order Quantity

This heatmap (figure 3.5) illustrates how changes in different parameters affect the total order quantity (EOQ). There are two shades of colors for each order quantity, with darker shades denoting greater quantities and lighter shades denoting a smaller amount. Key observations include the EOQ is highly sensitive to changes in the ordering cost. A 10% increase in O_R leads to a significant decrease in EOQ, represented by dark red. Changes in holding cost also affect EOQ, with an increase in H_R leading to a slight decrease in EOQ, shown in lighter blue shades. The EOQ remains relatively stable with changes in D_R, indicating a less sensitive relationship, represented by consistent blue shades. Variations in β do not significantly impact EOQ, showing stable order quantity across different levels of β, indicated by consistent blue shades. The deterioration rate (θ) shows a moderate impact on EOQ. Higher θ values lead to a slight decrease in EOQ, represented by a transition from light blue to darker blue. Changes in safety stock (S_s) levels have minimal impact on EOQ, maintaining stability across different safety stock levels, represented by consistent blue shades.

3.7.1.3 Total cost of Supplier

This heatmap (figure 3.6) shows how changes in different parameters influence the total cost incurred by the supplier (TCS). A darker color indicates a higher cost, while a lighter color indicates a lower cost. An important point is that the supplier's overall cost is highly sensitive to changes in the ordering price. A 10% increase in O_R leads to the highest increase in TCS, represented by dark red. The holding cost has a moderate impact on TCS. An increase in H_R leads to a noticeable rise in total cost, shown in lighter red shades. Changes in D_R have a minor impact on TCS, indicating less sensitivity, represented by consistent beige shades. Variations in β show stable TCS, indicating minor impact from changes in sensitivity price, shown in consistent beige shades. The deterioration rate impacts TCS, with higher θ values leading to an increase in total cost, indicated by a transition from beige to light red. Safety stock (S_s) levels show minimal impact on TCS, maintaining stability across different levels, represented by consistent beige shades.

3.7.1.4 Optimal replenishment cycle

This heatmap (figure 3.7) shows how changes in different parameters affect the optimal replenishment cycle. The colours represent the number of cycles, with darker shades indicating more cycles and lighter shades indicating fewer cycles. Key insights include the optimal replenishment cycle is highly sensitive to changes in the ordering cost. A 10% increase in O_R increases the replenishment cycle, represented by dark red. The holding cost significantly impacts the replenishment cycle, with changes in H_R leading to variations in the cycle length, shown in red to blue shades. The replenishment cycle remains relatively stable with changes in DCR, indicating less sensitivity, represented by consistent blue shades. Variations in β do not significantly impact the replenishment cycle, showing stability across different levels of β, indicated by consistent blue shades. The deterioration rate (θ) impacts the replenishment cycle, with higher θ values leading to increased cycle length, represented by a transition from blue to light red. Changes in safety stock (Ss) levels have minimal impact on the replenishment cycle, maintaining stability across different levels, represented by consistent blue shades.

3.7.2 Managerial Insights

The sensitivity analysis of the decentralized three-tier supply chain model using blockchain technology provides several valuable managerial insights that can significantly improve operational efficiency, optimize inventory management, and reduce costs. Firstly, an increase in the demand rate (a) leads to higher total order quantities and increased costs for both the retailer and the supplier. Conversely, a decrease in the demand rate reduces order quantities and costs. This implies that managers should closely monitor demand trends and adjust replenishment cycles and order quantities accordingly. Accurate demand forecasting is crucial to minimize costs and avoid overstocking or stockouts.

Changes in the ordering cost (b) have a moderate impact on total costs but a minimal effect on total order quantity. Higher ordering costs slightly increase the total cost for both the retailer and the supplier. Managers should negotiate better terms with suppliers to reduce ordering costs and consider batch ordering to optimize order sizes and reduce order frequency. Variations in holding cost (α) have negligible effects on the optimal replenishment cycle, total order quantity, and costs, suggesting that the model is relatively insensitive to changes in holding costs within the examined range. While holding costs are less sensitive, managers should strive to minimize inventory holding by improving turnover rates and reducing excess stock to maintain cost efficiency.

Like holding costs, changes in production rate (β) do not significantly affect the replenishment cycle, order quantity, or costs. Therefore, it is essential to focus on maintaining a stable and efficient production rate. Improvements in production efficiency can yield long-term benefits, even if they do not significantly alter short-term costs. Increasing the holding rate (HR) significantly raises the total cost for the retailer, while a decrease leads to cost savings. The total order quantity remains constant, indicating the cost effect is isolated to the holding component. Managers should reduce holding rates by optimizing warehouse space, improving inventory turnover, and implementing just-in-time (JIT) inventory practices.

Changes in the demand change rate (DCR) have minimal impact on total costs and order quantities, indicating the model's robustness to demand fluctuations. Managers should maintain flexibility in inventory policies to adapt to sudden changes in demand patterns without significant cost implications. Extending the time horizon (θ) increases order quantities but reduces the total cost for the retailer, indicating economies of scale over a longer period. Conversely, shortening the time horizon increases costs. Managers should plan for longer time horizons where feasible to take advantage of economies of scale, reducing per-unit costs over time.

Variations in order rate (Or) slightly affect total order quantities and costs, suggesting a relatively minor impact within the examined range. Managers should optimize order rates to balance order frequency and holding costs, considering specific demand patterns and storage capacities. Increases in safety stock (Ss) marginally increase the total cost for the retailer while keeping the total order quantity constant, indicating a direct cost impact due to holding additional safety stock. Managers should evaluate the trade-off between service levels and holding costs. Maintaining an optimal level of safety stock is crucial to ensure service reliability without incurring excessive costs.

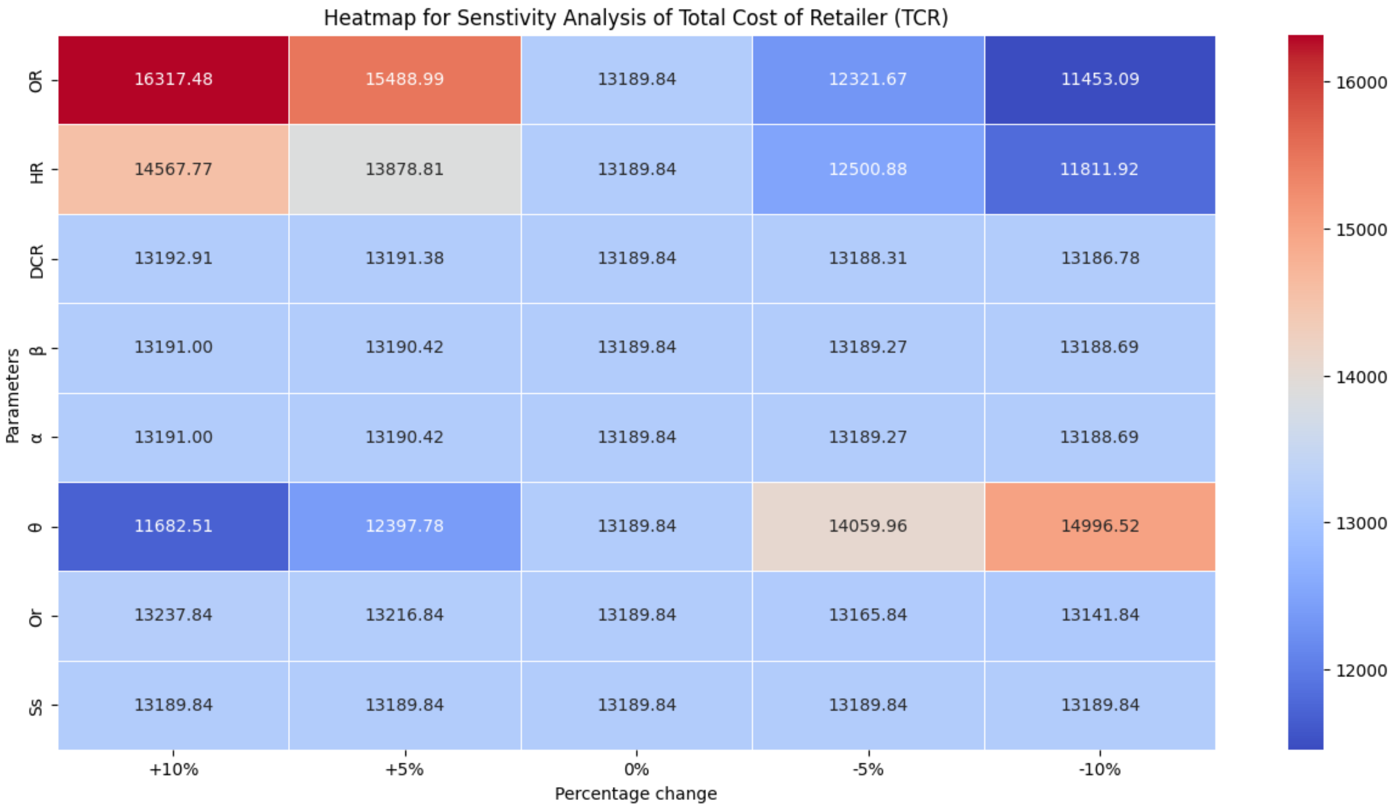


Figure 3.4 Heatmap for Sensitivity Analysis of Total Cost of Retailer (TCR)

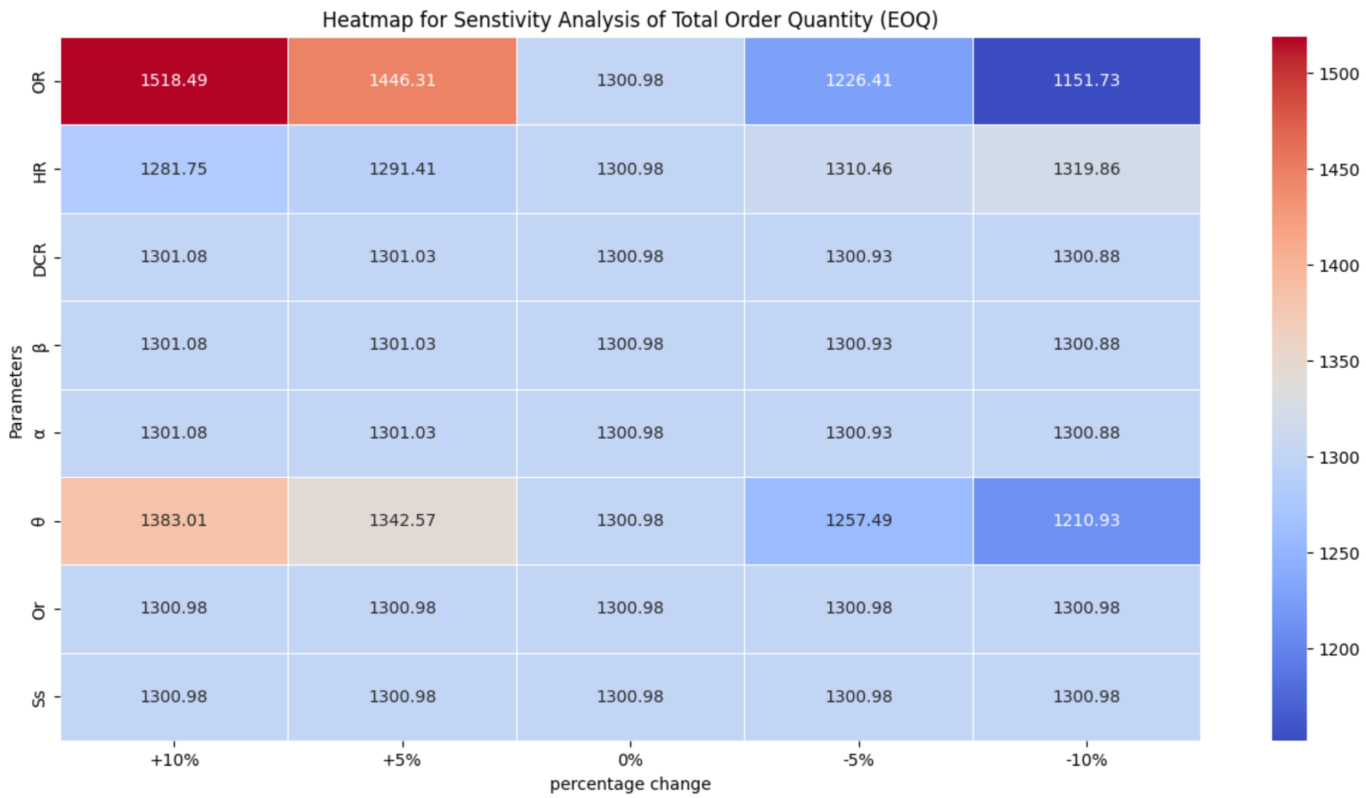


Figure 3.5 Heatmap for Sensitivity Analysis of Total Cost Quantity (EOQ)

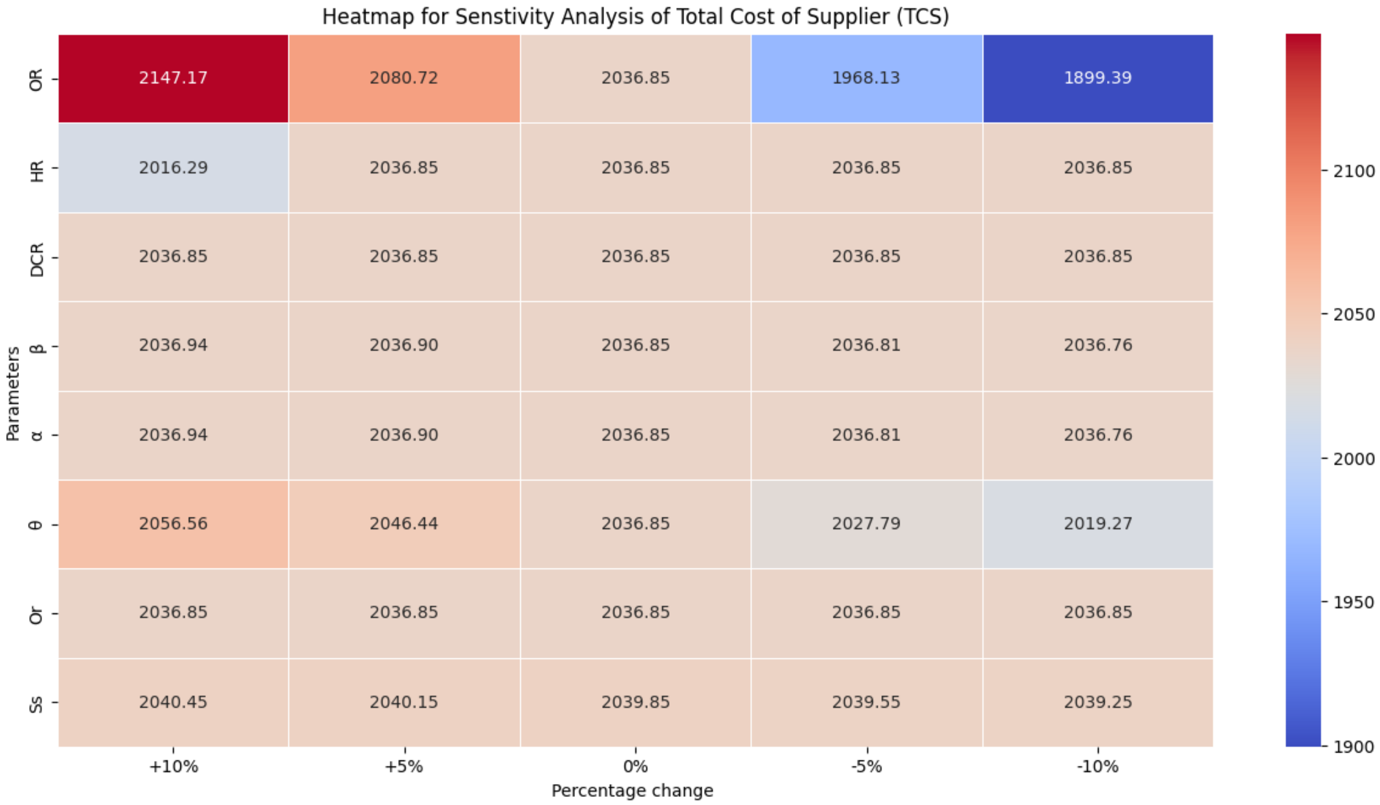


Figure 3.6 Heatmap for Sensitivity Analysis of Total Cost of Supplier (TCS)

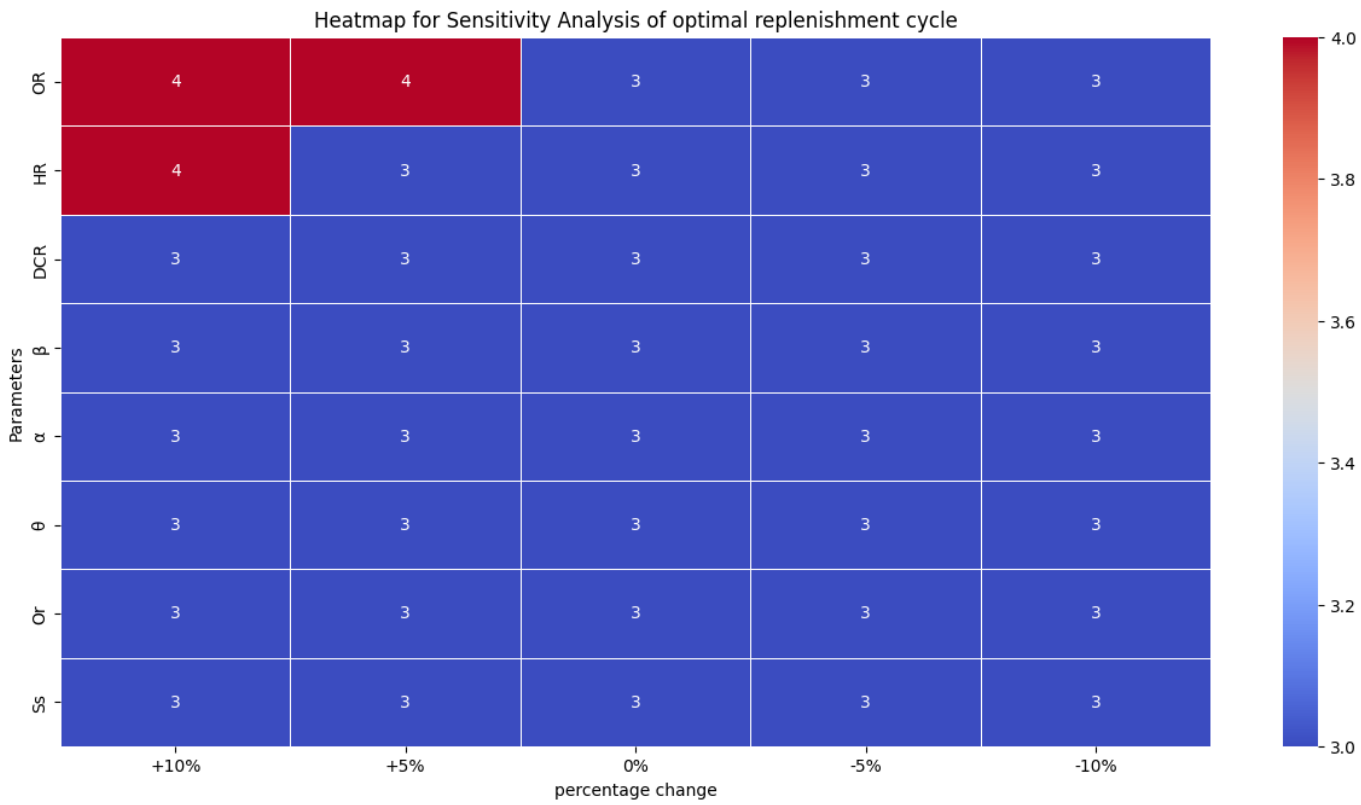


Figure 3.7 Heatmap for Sensitivity Analysis of Optimal Replenishment Cycle

3.7.2.1 General Managerial Recommendations:

- Regularly review and adjust inventory policies based on the latest demand forecasts, cost structures, and market conditions.
- Strengthen relationships with suppliers to negotiate better terms and reduce ordering costs.
- Focus on strategies to improve inventory turnover rates, such as implementing lean inventory practices and enhancing demand forecasting.
- Identify and implement cost-saving measures across the supply chain, particularly in areas with higher sensitivity, such as holding rates and demand rate changes.
- Utilize longer planning horizons to benefit from economies of scale and reduce overall costs.

By incorporating these insights, managers can make informed decisions to enhance supply chain efficiency, optimize inventory levels, and minimize costs.

The proposed inventory optimization model can be applied to various industries where supply chain coordination, demand uncertainty, and cost minimization are critical. Some of the key industries where this model can be effectively implemented include: The proposed model can be applied in industries where supply chain coordination, demand uncertainty, and cost minimization are critical. In the pharmaceutical industry, it helps optimize vaccine and drug inventory while reducing wastage. Retail and e-commerce businesses benefit from better replenishment cycles and demand forecasting. In automobile and electronics manufacturing, the model enhances just-in-time (JIT) inventory management and supplier coordination. Perishable goods industries can use it to reduce food waste and improve traceability. By integrating blockchain technology, the model ensures transparency, optimizes ordering strategies, and enhances overall supply chain efficiency.

3.8 Research Outcomes and Future Scope

3.8.1 Conclusion

This chapter explored the integration of blockchain technology into a decentralized three-tier supply chain model, a finite planning horizon, and a Stackelberg game approach. By incorporating blockchain's features of immutability, transparency, and traceability, we aimed to address inefficiencies and coordination issues commonly faced in traditional supply chains. Our model involved suppliers, manufacturers, and retailers, examining the impact of blockchain on optimizing inventory management and overall supply chain operations.

Blockchain technology significantly improves traceability, transparency, and coordination in the supply chain, according to our research. The model demonstrated that a higher proportion of information-sharing retailers positively correlates with increased profitability, as the transparency afforded by blockchain technology improves demand management and strategic pricing. The introduction of the 'break-even point' concept further optimized production and replenishment strategies, ensuring efficient inventory levels without overproduction.

We demonstrated the robustness of the model through our sensitivity analysis, which showed how different parameters affect supply chain performance. Despite its limitations, blockchain can improve supply chain efficiency, reduce costs, and enhance profitability. As part of the growing literature on blockchain applications in supply chain management, this chapter presents a comprehensive framework for leveraging blockchain in a decentralized three-tiered supply chain.

3.8.2 Future work

Future research can build on this chapter by exploring several avenues:

- Extending the model to include dynamic pricing strategies could provide deeper insights into real-world applications. This would allow for more flexible responses to market demand fluctuations and competitive actions.
- Investigating the impact of blockchain technology on multi-echelon supply chains, including more complex configurations and additional intermediary stages, could enhance the model's applicability across different industries.
- Examining the effect of blockchain on various product types, including perishable goods and high-value items, could offer more specific recommendations for different sectors. This would involve considering factors such as shelf life and value chain complexities.
- Incorporating stochastic elements into the demand and supply functions would increase the model's robustness and applicability. This approach would account for uncertainties and variability in market conditions, providing more realistic and practical solutions.
- Conducting empirical validation using real-world data from different industries would strengthen the practical relevance of the research. Case studies and industry collaborations could offer valuable insights into the implementation challenges and benefits of blockchain in supply chains.

By addressing these areas, future research can further enhance the understanding and Management of supply chains using blockchain technology, contributing to more efficient, transparent, and resilient supply chains.

Chapter 4 Blockchain-Enhanced Inventory Management in Decentralized Supply Chains for Finite Planning Horizons

Abstract

This research introduces a decentralized supply chain optimization model that incorporates blockchain technology. The model, implemented through an optimized iterative method, integrates ordering, holding, and purchasing costs to offer a comprehensive view of total costs for both retailers and suppliers. The model's uniqueness and optimality are demonstrated through theoretical analysis, highlighting the optimal ordering interval as the sole solution to the derived equation. Employing an algorithmic methodology, optimal replenishment schedules are efficiently calculated using Wolfram Mathematica 13.0. A numerical example and sensitivity analysis illustrate the impact of key parameters on replenishment cycles, order quantity, and costs, encompassing wholesale prices, demand uncertainty, and holding/ordering costs. Managerial insights derived from sensitivity analysis guide decision-makers in optimizing supply chain management, emphasizing strategies such as wholesaler price balance and strategic blockchain information management. In essence, this research contributes to an enhanced understanding of decentralized supply chain models with blockchain, providing a systematic decision-making optimization approach for increased efficiency and resilience.

Keywords: Decentralized supply Chain, Blockchain Technology, Finite Planning Horizon, sensitivity analysis

4.1 Introduction

The disruptive impact of COVID-19 underscores the vulnerability of centralized supply chain systems. Disruptions originating from a single source can have far-reaching consequences, impacting the global supply of goods and services. Decentralized supply chain models are emerging as robust solutions during this critical period. (Haque et al., 2022; Zhou et al., 2022) Through a multi-participant approach, these models distribute decision-making and resources among various participants, ensuring that a small issue in one part does not affect the entire system. In the aftermath of this pandemic, the flexibility, robustness, and power of a decentralized supply chain have become apparent, eliminating dependency on a single source.

In today's global business environment, supply chains have become increasingly complex and dispersed, leading to a need for better information sharing, visibility, and coordination among multiple stakeholders. The literature on decentralized supply chain models addresses the challenges and opportunities associated with managing these complex supply chains. Several studies have highlighted the importance of decentralized supply chain models in improving operational efficiency, reducing costs, and enhancing customer satisfaction. One key advantage of decentralized supply chain models is the greater autonomy they provide to individual subsidiaries or locations within a business (Algorri et al., 2022).

In parallel, the advent of blockchain technology has transformed various industries by revolutionizing the storage and verification of transactions and data. One particular area where blockchain has shown great potential in the realm of supply chain management (Omar et al., 2022) As supply chains become more complex and globalized, there is an increasing demand for innovative solutions that can improve traceability, efficiency, and transparency.

This chapter delves into the transformative journey of integrating Blockchain Technology into our decentralized supply chain. We aim to analyze the potential impact of this technology, especially within the finite planning horizon, to effectively manage our supply chain during disruptions (Park et al., 2021). Like a well-crafted recipe, this chapter blends theory, case study, and practical insights, showcasing how blockchain can become a powerhouse for our decentralized supply chain. Together, feedback on a journey to explore this new variant supply chain management, where innovative solutions powered by blockchain pave the way for a more liberated, silent, and efficient future.

In addition, this chapter incorporates quantitative analysis by employing numerical examples extracted from a secondary dataset compiled from a diverse array of (Mishra & Ranu, 2022b; Wu & Zhao, 2014). Our analysis is firmly grounded in a robust foundation. Utilizing an optimal iterative method, we systematically investigate the model's performance under varying wholesale prices. The primary objective is to contribute valuable insights into the optimization potential of the model across a spectrum of scenarios. This chapter not only extends the current body of research findings but also expands the examination of the model's behavior in response to diverse conditions of wholesale prices.

In this chapter, each section explains how blockchain technology enhances decentralized supply chain inventory models. Detailed literature reviews are presented in Section 4.2, including the impact of blockchain. Section 4.3 defines symbols and variables within the theoretical framework. We establish a theoretical framework and develop a model in this section. Section 4.4 explains the mathematical model for a blockchain-based supply chain inventory system. Literature challenges are addressed by the model, as well as its explained approach. A practical case study and numerical examples illustrate the model in section 4.5. A sensitivity analysis is provided in section 4.6, which evaluates the model's robustness under varying conditions, enabling optimization. This section offers supply chain decision-makers practical recommendations based on theoretical and numerical findings. Section 4.7 summarizes key findings, managerial implications, and future research directions.

4.2 Literature Review

Supply chain management is a dynamic field, and the exploration of decentralized supply chain models has revealed a multitude of approaches. Through an extensive review of existing research, various decentralized models were examined, shedding light on their respective contributions and limitations. Concurrently, a parallel investigation into blockchain technology's role in supply chain management unfolded, revealing a nascent utilization of this transformative technology. However, a noticeable gap emerged — the limited application of blockchain in decentralized supply chain models, prompting the need for a more comprehensive exploration. This literature review seeks to provide a coherent synthesis of these findings and identify avenues for bridging the research gap.

This literature review draws insights from a variety of supply chain models, each contributing to the understanding of decentralized decision-making and its impact on closed-loop supply chains. (Savaskan et al., 2004) evaluate decentralized systems with a focus on product remanufacturing using a Stackelberg leadership model. (Dumrongsiri et al., 2008) investigate dual-channel supply chains, studying equilibrium conditions and the impact of factors like marginal costs. (Chen & Chen, 2008) compare coordinated and decentralized decision-making for a monopolistic retailer facing time-varying demand. (Mungan et al., 2010) optimize procurement, production, and delivery schedules for technology-related companies. (Li & Li, 2011) analyze analytical models related to active acquisition and remanufacturing in supply chains. (Chen & Cheng, 2012) evaluate the price-dependent revenue-sharing mechanism in a decentralized supply chain using a Stackelberg game framework. (Benkherouf et al., 2014) determine optimal lot sizes for a recovery inventory system. (Wu & Zhao, 2014) introduce a collaborative replenishment policy considering varying demand and check coordination between retailer and supplier. (Yuan, 2015) develops a multi-period closed-loop supply chain model with remanufacturing. (Chen, et al., 2017) analyze system coordination in a supply chain for deteriorating items using revenue-sharing contracts. (Nagaraju et al., 2016) develop a mathematical model for a three-echelon inventory system in both coordinated and non-coordinated supply chains. (Giri et al., 2018) investigate dual-channel supply chain models for selling deteriorating products. (Liu et al., 2020) establish a decision-making model under carbon tax constraints. (Prasad et al., 2020) compares total costs in decentralized and centralized supply chains. (Mondal & Giri, 2022) explore the influence of Corporate Social Responsibility (CSR) efforts. (Mondal & Giri, 2022) examine the impact of recycling and retailer fairness behaviour on a green supply chain. (Kumar et al., 2022) model and optimize a coordinated and non-coordinated three-echelon supply chain. (Huang et al., 2022) investigate pricing decisions in a closed-loop supply chain under disruptions. (Liu et al., 2021) address coordination problems in closed-loop supply chains led by retailers, considering Stackelberg game theory. (Huang et al., 2022) develop a three-level supply chain model based on blockchain technology, emphasizing retailer sensitivity to information.

Blockchain technology has emerged as a pivotal force in revolutionizing supply chain management, offering multifaceted benefits across diverse dimensions. The study by Identify the transformative effects of blockchain on environmental efficiency in a multi-echelon supply chain in (Manupati et al., 2020). (Dutta et al., 2020)

Conduct a comprehensive review of global and local supply chains to examine decentralized structures, consensus algorithms, and smart contracts. Based on simulation research, (Huang et al., 2022) present a three-level supply chain model emphasizing blockchain's ability to reduce operational costs. "SmartRice," a sensor-based blockchain solution for addressing food value chain challenges, will be introduced by (Ekawati et al., 2022).

Decentralized supply chain models as well as blockchain applications in supply chain management reveal a distinct research gap around blockchain technology underutilization in enhancing decentralized supply chains. This gap becomes particularly relevant when inventory models are considered when planning over finite time horizons. A decentralized supply chain strategy needs to explore blockchain technology's untapped potential as shown by this gap. The chapter adopts a methodological approach that combines blockchain technology with models of decentralized supply chains to address this research gap. We explore inventory models within finite planning horizons to enhance the overall efficiency of supply chains through the optimization of decision-making processes.

Finally, the literature review provides a comprehensive overview of the decentralized supply chain landscape, examines the application of blockchain technology within supply chain management, and identifies an important research gap. We will provide insights into the transformative potential of blockchain technology in decentralized supply chain strategies in the following sections. To analyze the impact of blockchain technology on decentralized supply chain inventory models, a mathematical model was developed in this chapter. Within the context of a finite planning horizon, the model addresses key parameters and variables. Following this, iterative methods are used to solve the model, utilizing Mathematica (13.0). With this methodology, the implications of blockchain technology are systematically examined. Decentralized supply chain management benefits from the combination of mathematical modeling and computational analysis because the conclusions are more accurate and reliable.

4.3 Assumptions and Notations

To develop the proposed model in this chapter, we use the notations, assumptions, and Boundary Conditions listed below.

4.3.1 Assumptions

- The planning horizon is defined by constant holding, ordering, & shortage costs.
- The supply chain consists of a single retailer and a single supplier, managing a specific item.
- The supplier maintains no inventory, given the instantaneous replenishment and infinite production capacity.

- Replenishment by suppliers occurs in a lot-by-lot fashion.
- The setup cost incurred by the supplier is higher than that incurred by the retailer.
- The retailer, being a rational actor, strategically selects products to maximize profitability.

4.3.2 Notations

Symbol	Description
α	Amount of retailer information.
β	Coefficients that are sensitive to price.
θ	A constant demand rate that is dependent on inventory levels.
μ	A proportion of the total retailers that are information-sensitive, where $0 \le \mu \le 10$
$1-\mu$	All other actors not concerned with information.
k	Measure of how much it costs to share information.
d	Demand.
n	An integer less than zero representing the number of times the inventory will be replenished during the planning horizon H.
W	Wholesale price.
H	Finite planning horizon.
C	The purchasing cost per unit ($/unit).
TC	A cost estimate for the planning horizon H.
S	Setup cost of the supplier (dollars per order).
Sr	The ordering cost for the retailer (dollars per order).
C_B	Cost to share information using blockchain technology, where $C_B = k\alpha$.
$I_{j+1}(t)$	Inventory level at time t in the $(j+1)^{th}$ cycle.
Q_{j+1}	Order quantity for the $(j+1)^{th}$ cycle.
t_j	Replenishment time for the j^{th} cycle, where $t_0=0$ and $t_n=H$.
T_{j+1}	Length of the replenishment cycle for the $(j+1)^{th}$ cycle, where $j = 0,1,2,\ldots,n-1$.

4.3.3 Boundary Conditions

- $I_{j+1}(t_{j+1}) = 0$ signifies that the inventory level becomes zero at the end of each cycle, indicating complete consumption of the replenished inventory.
- $I_{t+1}(t_i) = Q_{j+1}$ ensures that the inventory level is initialized with the order quantity Q_{j+1} from the previous cycle at the beginning of each cycle.

The specified boundary conditions play a fundamental role in solving the associated differential equation, serving as essential constraints for our model. They serve to enhance the accuracy and reliability of our model by providing a well-defined framework, enabling precise simulation and analysis of the inventory system's dynamics within the finite planning horizon.

4.4 Defining the Mathematical Model

In the realm of Blockchain-based dynamics, recent studies such as (Wu & Zhao, 2014) has shown that the real-time assessment of retailer demand depends on instantaneous stock levels. This complex relationship is expressed through the demand rate equation D(t):

$$D(t) = (\mu d + (1-\mu)\, d\alpha - \beta W)\, t + \theta\, I(t), \text{ such that } t_j \leq t \leq t_{j+1} \tag{4.1}$$

The inventory gradually depletes as the system goes through cycles expressed by a first-order linear differential equation:

$$\frac{d}{dt} I_{j+1}(t) = -\,[\mu d + (1-\mu)\, d\alpha - \beta W]\, t - \theta\, I_{j+1}(t),\ t_j \leq t \leq t_{j+1} \tag{4.2}$$

Initial boundary values $I_{j+1}(t_{j+1}) = 0$ & $I_{j+1}(t_j) = \theta_{j+1}$

$\therefore$ linear first-order differential equation

$\Rightarrow$ Integrating factor: $e^{\int \theta dt} = e^{\theta t}$

The subsequent exploration entails finding the solution to the differential equation, resulting in the order quantity for each cycle (Q_{j+1}), and a complex representation of the total cost of retailer (TCR) equation that encompasses ordering, holding, and purchasing costs.

$$I_{j+1}(t).\ e^{\theta t} = \int -[\mu d + (1-\mu) d\alpha - \beta W]\, t e^{\theta t} dt \tag{4.3}$$

$$Q_{j+1} = I_{j+1}(t_j) = -e^{-\theta t} \int_{t_j}^{t_{j+1}} [\mu d + (1-\mu) d\alpha - \beta W]\, p e^{\theta p} dp \tag{4.4}$$

$$TCR\,(n, t_0, t_1, t_2, \ldots, t_n) = \sum_{j=0}^{n-1} h_r \int_{t_j}^{t_{j+1}} I_{j+1}(t) dt + \sum_{j=0}^{n-1} W Q_{j+1} + nS_r \tag{4.5}$$

$$TCR = nS_r + \sum_{j=0}^{n-1} h_r \int_{t_j}^{t_{j+1}} e^{-\theta t} dt \int_{t_j}^{t_{j+1}} -[\mu d + (1-\mu) d\alpha - \beta W] p e^{\theta p d p} + \sum_{j=1}^{n-1} W \theta_{j+1}$$

$$TCR = nS_r + \sum_{j=0}^{n-1} h_r \int_{j}^{t_{j+1}} [\beta W - \mu d - (1-\mu) d\alpha] p e^{\theta p_{dp}} \int_{t_j}^{p} e^{-\theta t} dt + \sum_{j=1}^{n-1} W \theta_{j+1}$$

$\mathrm{TCR} = nS_r + \sum_{j=0}^{n-1} h_r \int_{t_j}^{t_{j+1}} [\beta W - \mu d - (1-\mu) d\alpha] p e^{\theta p`} dp \left[e\theta^{(p-tj)} - 1\right] + \sum_{j=1}^{n-1} WQ_{j+1}$

$\mathrm{TCR}\ (n, t_0, t_1, t_2, \ldots\ldots\ldots\ldots, t_n) = nS_r + \sum_{j=1}^{n-1} \left(\frac{h_r}{\theta} + W\right) \int_{t_j}^{t_{j+1}} [\beta W - \mu d - (1-\mu) d\alpha]\, e^{\theta(t-t_j)} dt - \frac{W}{\theta} [\beta W - \mu d - (1-\mu) d\alpha] \frac{H^2}{2}$ with $t_0 = 0$ & $t_n = H$. (4.6)

Retailers' replenishment policies determine the supplier's total costs. Thus, during the planning horizon H, his or her total costs are his or her setup cost and manufacturing cost.

$$\mathrm{TCS}(n^*, t_0^*, t_1^*, t_2^*, \ldots\ldots, t_{n-1}^*) = n * S_s + \text{Blockchain Implementation Cost} + \sum_{j=0}^{n*-1} CQ^*{}_{j+1} \quad (4.7)$$

Furthermore, during the planning horizon H, the total optimal order quantity is

$$Q = \sum_{j=0}^{j=n^*-1} Q^*{}_{j+1} \quad (4.8)$$

4.5 Calculations of Optimal Replenishment Schedules

To optimize a process, the aim is to minimize a specific equation (4.6) while keeping the value of ‘n’ constant. By taking the first partial derivative of the equation, we can obtain equation (4.9), which provides the optimal values for the ordering intervals, represented by t_j. Once equation (4.9) is satisfied, it reveals these optimal values. By imposing certain constraints such as ‘t_0=0’ & ‘t_n=H’, the uniqueness of these optimal solutions is established and forms the basis for an efficient and effective model.

$\frac{\partial}{\partial t_j}\mathrm{TCR}\ (n, t_0, t_1, t_2, \ldots\ldots\ldots\ldots, t_n) = [\beta W - \mu d - (1-\mu)\, d\alpha]\ t_j \left[e^{\theta(t_j - t_{j-1})} - 1\right] - \theta \int_{t_j}^{t_{j+1}} [\beta W - \mu d - (1 - \mu) d\alpha] + e]^{\theta(t-tj)} dt = 0.$ $j = 1,2,3..., n-1.$ (4.9)

Let $n^*, t_0^*, t_1^*, t_2^*, \ldots\ldots, t_{n-1}^*$ represent the optimal solution for the Minimum TCR problem with parameters $n, t, t_1, t_2, \ldots, t_n$.

Theorem 4.1: Consider a fixed parameter, n, and let equation (4.9) represent the total cost of retailer (TCR) function, in the context of supply chain optimization.

Proof: Our objective is to establish that the optimal ordering interval for this system is the unique solution to Equation (4.9). To substantiate this claim, we delve into the properties of the Hessian matrix associated with TCR.

In comprehending the intricate relationship between replenishment cycles and times, we formulate key expressions.

Notably,

$$\frac{\partial^2 TC_r^{IND}(n_1,t_0,t_1,t_2,\ldots\ldots,t_n)}{\partial t_j^2} = (-W\beta + d\alpha\,(1-\mu) + d\mu\left(W + \frac{hr}{\theta}\right)\left(e^{\theta(t_j-t_{j-1})} - e^{\theta(t_{j+1}-t_j)} + e^{\theta(t_j-t_{j-1})}\theta t_j + e^{\theta(t_{j+1}-t_j)}\theta t_{1-j}\right)$$

$$\frac{\partial^2 TC_r^{IND}(n_1,t_0,t_1,t_2,\ldots\ldots,t_n)}{\partial t_j\,\partial t_{j-1}} = -e^{\theta(t_j-t_{j-1})}\theta(-W\beta + d\alpha(1-\mu) + d\mu\left(W + \frac{h_r}{\theta}\right)t_j$$

$$\frac{\partial^2 TC_r^{IND}(n_1,t_0,t_1,t_2,\ldots\ldots,t_{n_1})}{\partial t_j\,\partial t_{j+1}} = -e^{\theta(t_{j+1}-t_j)}\theta(-W\beta + d\alpha - \mu) + d\mu)\left(W + \frac{h_r}{\theta}\right)t_{1+j}$$

and $\frac{\partial^2 TC_r^{IND}(n_1,t_0,t_1,t_2,\ldots\ldots,t_{n_1})}{\partial t_j\,\partial t_k} = 0$

Furthermore,

$$\frac{\partial^2 TC_r^{IND}(n_1,t_0,t_1,t_2,\ldots\ldots,t_{n_1})}{\partial t_j^2} > \left|\frac{\partial^2 TC_r^{IND}(n_1,t_0,t_1,t_2,\ldots\ldots,t_n)}{\partial t_i\,\partial t_{j-1}}\right| + \left|\frac{\partial^2 TC_r^{IND}(n_1,t_0,t_1,t_2,\ldots\ldots,t_n)}{\partial t_j\,\partial t_{j+1}}\right| \text{ for all } j = 1,2,\ldots\ldots,n_1-1.$$

Due to its diagonal properties, a Hessian matrix containing positive diagonal elements must also be positive definite. As a result, Equation (4.9) has a unique solution which is the optimal replenishment interval as well as a global minimum value. If TCR is minimum, it must have a positive definite Hessian matrix r for a fixed n.

Proposition: $t_{j+1}e^{\theta T_{j+1}} < t_j\left(e^{\theta T_j} + 1\right) + \frac{1}{\theta}e^{\theta T_j}$

Solution**-** according to (Wu & Zhao, 2014) as we say: $f(t_{j+1}) - f(t_j) < \frac{f\prime(t_j)}{f(t_j)}\int_{t_j}^{t_{j+1}} f(u)du$

We let $f(t) = (\beta w - \mu d - (1-\mu)\,d\alpha)\,t\,e^{\theta(t-t_j)}$

Equation simplifies to

$[\beta w - \mu d - (1-\mu)\, d\alpha]\, t_{j+1}\, e^{\theta(t_{j+1}-t_j)}$ - $[\beta w - \mu d - (1-\mu)\, d\alpha]\, t_j < \left(\theta + \frac{1}{t_j}\right) \int_{t_j}^{t_{j+1}} \beta w - \mu d - (1 - \mu) d\alpha) t e^{\theta(t-t_j)} dt$

Using equation (4.7) given by

$[\beta w - \mu d - (1-\mu)\, d\alpha]\, t_{j+1}\, e^{\theta(t_{j+1}-t_j)}$ - $[\beta w - \mu d - (1-\mu)\, d\alpha]\, t_j < \left(\theta + \frac{1}{t_j}\right) [\beta w - \mu d - (1-\mu) d\alpha] t_j e^{\theta(t_j - t_{j-1})}$

$$t_{j+1} e^{\theta T_{j+1}} - t_j < \left(\theta + \frac{1}{t_j}\right) \frac{t_j e^{\theta T_j}}{\theta}$$

$$t_{j+1} e^{\theta T_{j+1}} < \left(\theta + \frac{1}{t_j}\right) \frac{t_j e^{\theta T_j}}{\theta} + t_j$$

$$t_{j+1} e^{\theta T_{j+1}} < e^{\theta T_j} + \frac{1}{\theta} e^{\theta T_j} + t_j$$

This leads to the conclusive result: $t_{j+1} e^{\theta T_{j+1}} < t_j\left(e^{\theta T_j} + 1\right) + \frac{1}{\theta} e^{\theta T_j}$

Lemma: The monotonicity of t_j (where j = 1, 2, …, n – 2) is evident concerning the parameter t_{n-1}. This lemma establishes the consistent increase of t_j concerning the penultimate time point in the planning horizon.

Proof: The equation is simplified using the relationship $T_n = H - t_{n-1}$ is constant if t_{n-1} is known, as per (Benkherouf et al., 2014) say

This implies that T_n and t_{n-1} are inversely related.

To prove that rate of change of TCR's increase with T_n for j= n-1, n-2,…,3,2,1.

For j = n – 1, after differentiating equation 4.9 w.r.t. T_n, we get equation 4.20, TCR =

$$[\beta w - \mu d - (1-\mu) d\alpha] \left[(-1) e^{\theta T_{n-1}} + (H - T_n) e^{\theta T_{n-1}} \frac{dT_{n-1}}{dT_n} - t_n e^{\theta T_n}.\theta + e^{\theta T_n}.\theta\right] = 0$$

$$t_n e^{\theta T_n}.\theta = e^{\theta T_n}.\theta - e^{\theta T_{n-1}} + (H - T_n) e^{\theta T_{n-1}} \frac{dT_{n-1}}{dT_n}$$

Using the preposition we get

$$e^{\theta T_n}.\theta - e^{\theta T_{n-1}} + (H - T_n)e^{\theta T_{n-1}}\frac{dT_{n-1}}{dT_n} > \theta t_{n-1}\left(e^{\theta T_{n-1}} + 1\right) + e^{\theta T_{n-1}}$$

Which Show that $\frac{dT_{n-1}}{dT_n} > 0$

Indicating an increase in t_{n-1} as T_n increases. A generalization is made, suggesting that this relationship holds for j= n-1, n-2,…,3,2,1 and can be reasonably extended.

Expanding on the established formulations, we can apply the subsequent optimization procedure to uncover the most advantageous values and outcomes. The methodology involves an iterative optimization process, commencing with the practical decision to set 'n' to 2 as the initial point. When n = 1, we assign $t_0 = 0$ and $t_1 = 4$, and then initiate the Mathematica program to solve for optimality. For n = 2, corresponding to the number of replenishment cycles, where $t_0 = 0$ and $t_2 = 4$, the value of t_1 is determined using an iterative method in the Mathematica program.

4.5.1 Methodology

The retailer should determine the most efficient way to schedule orders.

1. In this case, we will assign n a value of 2.
2. Calculate the unique optimal ordering interval using the nonlinear equation (4.6) system for a fixed n.
3. Using equation (4.6), determine the total cost of Retailers (n).
4. If TCR (n) is less than TCR (n − 1), increase n by 1, then return to Step (ii). Then stop because the algorithm has reached an optimal ordering policy.
5. Based on equations (4.7) and (4.8), calculate TCS and Q.

4.5.2 Numerical Example

Using the assumptions in Section 4.3, this numerical example shows how changes in the key parameter value W affect the optimal results.

EXAMPLE- Given α = β = 5, μ = 0.40, C = \$12/unit, d = 120/unit, Ss = \$120/setup, Sr = \$90/order, hr = \$2/unit/year, θ = 0.75, H = 4-year, d = 120 units/year, W = 10, 20, 30 units/year, respectively.

The results are shown in Tables 4.1–4.2 because of the algorithm and its corresponding expressions.

Table 4.1 Total cost for retailer when

↓ W	→n	1	2	3	4	5	6
10		38025.32	36978.41	36944.33	**36881.99**	36898.04	36948.57
20		56913.16	**56308.54**	56478.58	56440.71	56468.55	56256.50
30		67948.38	**67694**	68117.52	67993.09	68032.23	68096.99

Table 4.2 Replenishment time for TCR, TCS and Q

↓ W	→ ti	t_0	t_1	t_2	t_3	t_4	n	T_{Ret}	T_{sup}	Q_{nt}
10		0	1.105	1.9618	2.7044	4	4	36881.99	34370.28	2866.27
20		0	1.6046	4			2	56308.54	29603.48	2446.95
30		0	1.6046	4			2	67694	24836.68	2049.73

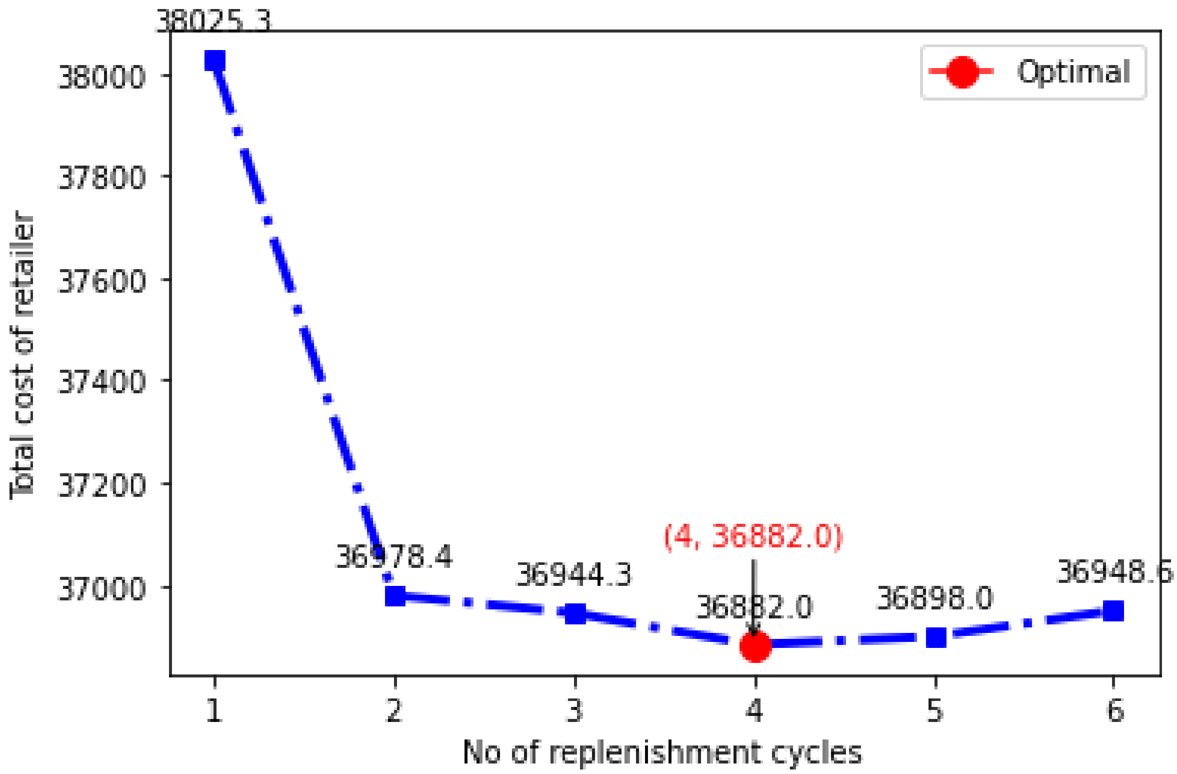


Figure 4.1 Optimal value W=10 of Total cost of Retailer and replenishment cycles

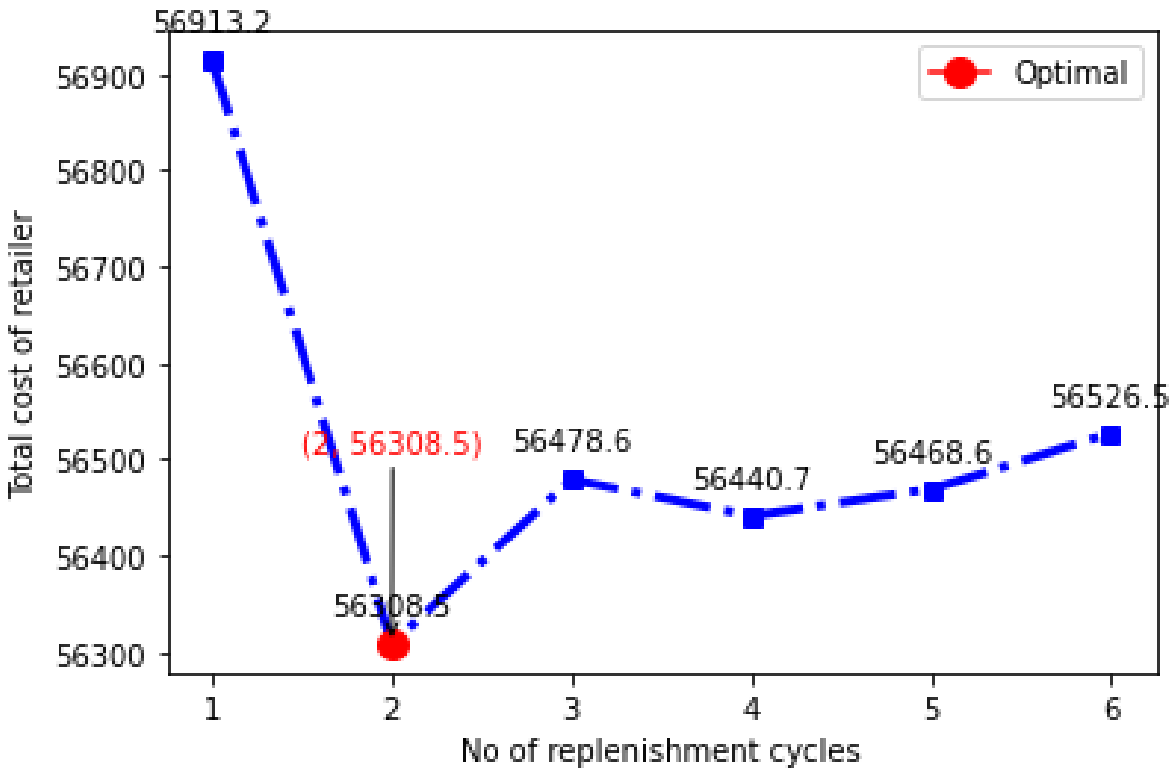


Figure 4.2 Optimal value W=20 of Total cost of Retailer and replenishment cycles

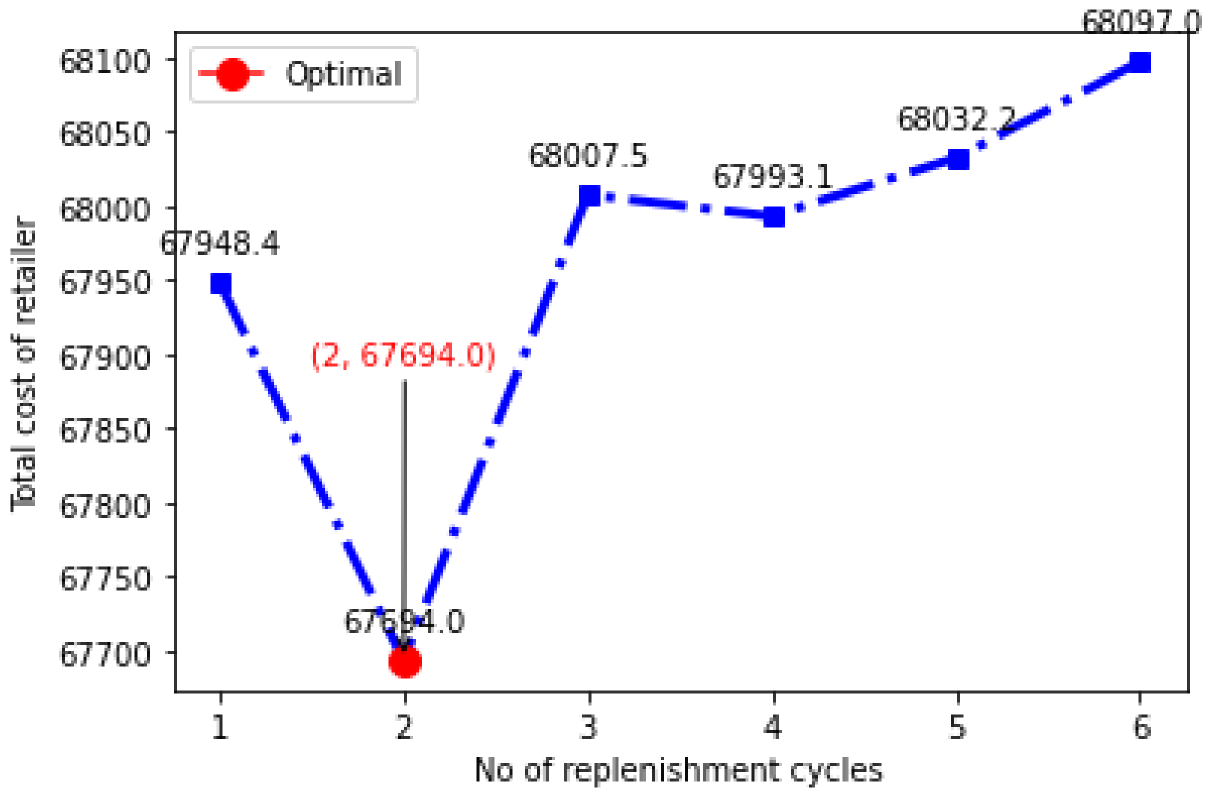


Figure 4.3 Optimal value W=30 of Total cost of Retailer and replenishment cycles

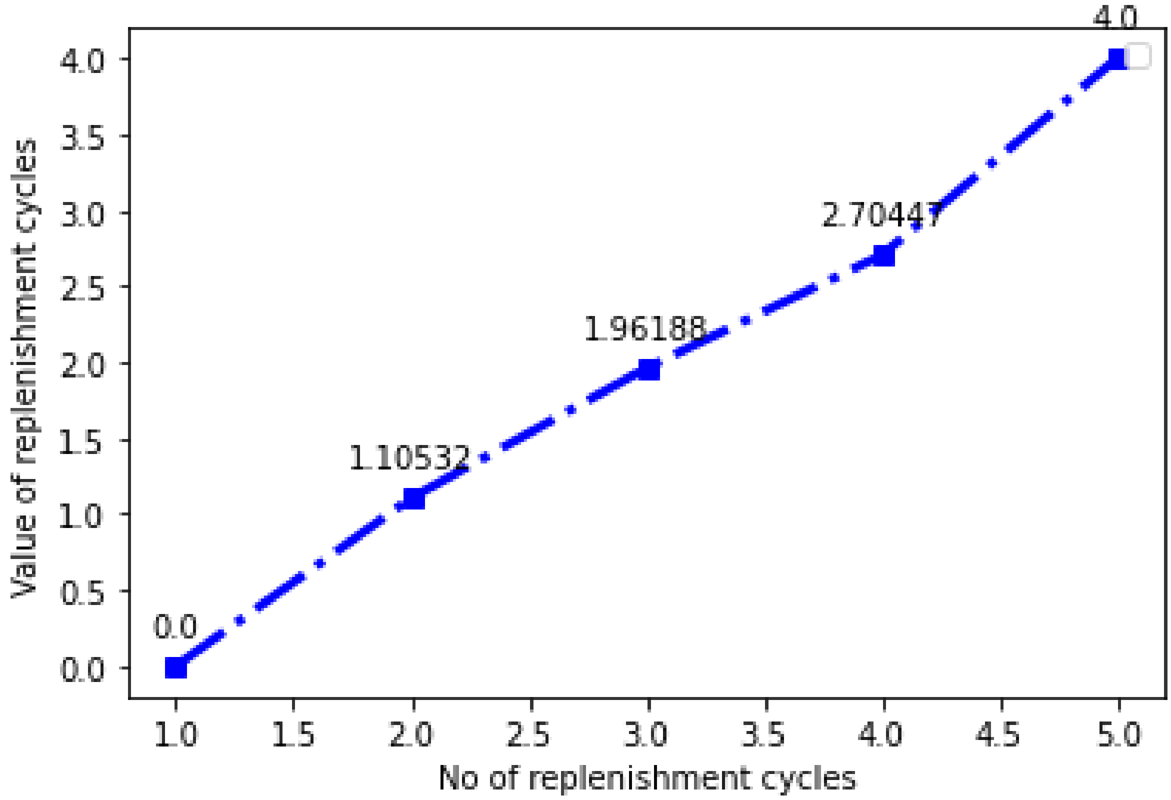


Figure 4.4 Increased order of optimal value W=10 of the replenishment cycles

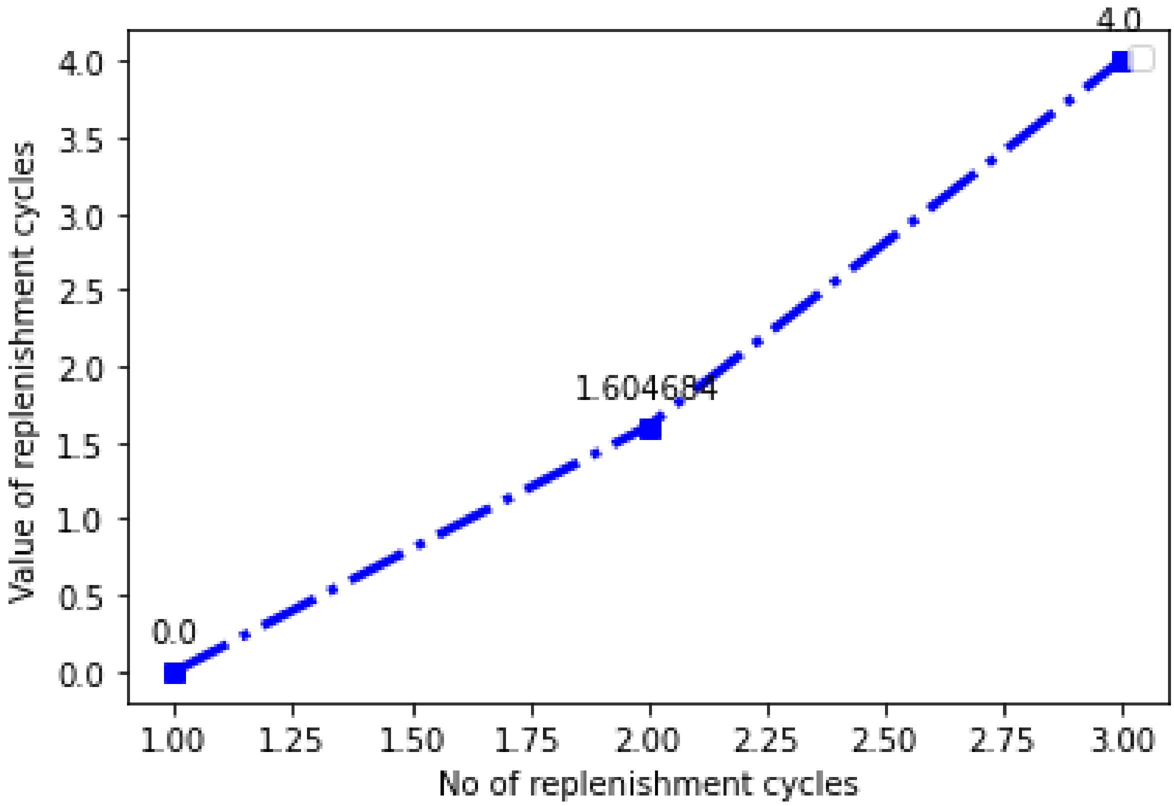


Figure 4.5 Increased order of optimal values W=20, W=30 of the replenishment cycles

Maintaining a consistent value for 'n' ensures uniformity throughout the analysis, thereby facilitating a fair comparison across diverse iterations of the model. The provided table delineates the retailer's total cost for varying values of 'W' (wholesale price) and 'n' (replenishment cycles). The steadfastness in 'n' simplifies the assessment of how different wholesale prices influence the total cost over an identical number of replenishment cycles.

The iterative process involves adjusting 'n' based on the comparison of total costs (TCR), aiming for convergence toward the optimal ordering policy. This iterative refinement is pivotal for attaining the most effective solution, allowing researchers to converge toward the configuration that minimizes total retailer costs. The algorithm's termination condition, highlighted in the table, dictates halting when the TCR for 'n' is less than TCR for 'n-1'.

For instance, when 'W' is 10 and 'n' is 2, the retailer's total cost amounts to 36978. With an incremental increase in 'n' from 2 to 6, the total cost fluctuates, culminating in its nadir at 'n' equals 4, registering a cost of 36881. This illustrates how the iterative process systematically refines 'n' to pinpoint the optimal ordering policy within defined constraints.

To visually comprehend these fluctuations, refer to figure 4.2, which illustrates how values oscillate and reach their lowest point at 'n' equals 4. The same insights are conveyed for different 'W' values in both table 4.1 and graphs 4.2, 4.3, and 4.4. In table 4.2 the intricate details of optimal outcomes in scenarios where compensation is factored into each optimal case. It can be rigorously established that the inequality $t_{j+1} - t_j < t_j - t_{j-1}$ holds, where $j=1, 2,\ldots,n-1$ Here, t_j denotes the j^{th} replenishment time, with $t_0=0$ and $t_{n-1}= H$, where H is a non-negative integer.

Table 4.2 meticulously tabulates values for t_j (replenishment time), Total Cost of Replenishment (TCR), Total Cost of Setup (TCS), and Order Quantity (Q). This tabulation distinctly delineates how diverse W (wholesale prices) correlate with specific t_j values, thereby accurately characterizing optimal ordering policies for our model. For instance, when W is set at 10, t_1 assumes a value of 1.105, t_2 registers as 1.961, t_3 stands at 2.704, and t_4 reaches 4. Additionally, TCR manifests as 36681, TCS as 34370, and Q as 2866.

This illustrative example underscores the dynamic interplay of optimal replenishment times, costs, and order quantities across distinct W values. It underscores the adaptive nature of our model, optimizing parameters and enabling judicious decision-making to foster cost-effectiveness in the supply chain. Complementing the tabulated values in table 4.2, our analysis extends to scrutinizing t_j values, symbolizing replenishment time, elucidated through figures 4.5 and 4.6.

In these graphical representations, the discernible convexity of the curve, influenced by t_j fluctuations, serves as a visual indicator of optimal cycle points and cost minimization. This visual inspection of t_j values enriches our understanding of the nuanced shifts in our model's performance, empowering strategic decision-making to optimize the intricacies of the supply chain.

4.6 An analysis of sensitivity and managerial insights

4.6.1 Sensitivity Analysis

In this section, we will conduct sensitivity analysis, by introducing percentage changes, our objective is to ensure that our model operates effectively within its predefined domain, rather than being adversely affected by alterations in the vicinity of any specific parameter. Additionally, this analysis provides valuable insights into the behavior of all parameters, facilitating the extraction of managerial insights. We have deliberately selected these specific percentages to comprehensively explore both sides of the function's domain, enabling a thorough chapter of the model's behavior.

Firstly, alterations in the Wholesale Price (W) wield substantial influence; (table 4.3) a 20% decrease in W amplifies optimal replenishment cycles, order quantities, and overall cost for both suppliers and retailers. A 10% decrease is a comparable effect, less pronounced, while a constant W (0%) maintains unaltered optimal values. Conversely, a 10% increase in W prompts a reduction in the optimal replenishment cycle, cost, and order quantity, with a more pronounced impact from a 20% increase. Moving on to Demand Uncertainly, a 20% reduction results in diminished orders and costs for both retailers and suppliers and a 10% reduction echoes a similar yet milder effect. Stable demand uncertainty (0% change) maintains consistent optimal values, but a 10% increase amplifies the total cost and quantity of the order. Doubling demand uncertainty (20% increase) leads to a significant upswing in optimal values. The parameters of Holding and Ordering Costs (α and β) exhibit their sway as a 20% reduction translates to diminished total order quantities and costs for retailers and suppliers. A 10% reduction yields a comparable albeit less prominent effect, while a steady state (0% change) preserves unaltered optimal values. Meanwhile, a 10% increase in α and β escalates total costs and order quantity, and a 20% increase brings similar results, albeit with higher optimal values. Shifting focus to the Unit Cost of the Product (d), a 20% reduction proves advantageous, diminishing total order quantities and costs for both retailers and suppliers. A 10% reduction yields a similar effect, albeit less pronounced, and a steady d (0% change) maintains consistent optimal values. Conversely, a 10% increase in d increases order quantity and costs, with a more dramatic surge resulting from a 20% increase. Finally, parameters such as hr, θ, S_r, S_s, and C_R exhibit minimal impacts on retailer and supplier cost estimates. These nuanced insights provide valuable guidance for managerial decision-making in optimizing the supply chain under varying conditions.

Table 4.3 Sensitivity table for each parameter

Parameters	%Changes	Optimal Replenishment cycle	Total order Quantity (Q)	Total cost of Retailer (TCR)	Total cost of supplier (TCS)
W	-20	2	2288.06	61816.08	27696.76
	-10	2	2168.89	65112.55	26266.72
	0	2	2049.72	67694.00	24836.68
	10	2	1930.55	69560.43	23406.64
	20	2	1811.38	70711.84	21976.60
μ	-20	2	2354.79	77742.60	28497.58
	-10	2	2202.26	72718.30	26667.13
	0	2	2049.72	67694.00	24836.68
	10	2	2306.23	62669.70	23006.23
	20	2	1744.64	57645.40	21175.78
α	-20	2	2288.06	75544.46	27696.76
	-10	2	2168.89	71619.23	23266.72
	0	2	2049.73	67694.00	24836.68
	10	2	1930.55	63768.77	23406.64
	20	2	1811.38	59843.53	21976.60
β	-20	2	2288.06	75544.46	27696.76
	-10	2	2168.89	71619.23	23266.72
	0	2	2049.73	67694.00	24836.68
	10	2	1930.55	63768.77	23406.64
	20	2	1811.38	59843.53	21976.60
d	-20	2	1401.43	46340.73	17057.26
	-10	2	1725.58	57017.37	20946.97
	0	2	2049.72	67694.00	24836.68
	10	2	2373.86	78370.63	28726.39
	20	2	2698.00	89047.27	32616.10
h_r	-20	2	2049.72	66489.54	24836.68
	-10	2	2049.72	67091.77	24836.68
	0	2	2049.72	67694.00	24836.68
	10	2	2049.72	68296.23	24836.68
	20	2	2049.72	68898.46	24836.68
θ	-20	1	1307.11	58446.08	17706.05
	-10	1	1460.50	64349.82	18842.83
	0	1	1633.34	71130.98	20069.88
	10	6	1705.55	72948.19	21509.35
	20	6	1746.84	74342.70	23072.50
Sr	-20	2	2049.72	67658.00	24836.68
	-10	2	2049.72	67676.00	24836.68
	0	2	2049.72	67694.00	24836.68
	10	2	2049.72	67712.00	24836.68
	20	2	2049.72	67730.00	24836.68
Ss	-20	2	2049.72	67694.00	24788.68
	-10	2	2049.72	67694.00	24812.68
	0	2	2049.72	67694.00	24836.68
	10	2	2049.72	67694.00	24860.68
	20	2	2049.72	67694.00	24884.68
Cr	-20	2	2049.72	67694.00	19917.34
	-10	2	2049.72	67694.00	22377.01
	0	2	2049.72	67694.00	24836.68
	10	2	2049.72	67694.00	27296.35
	20	2	2049.72	67694.00	29756.02

4.6.2 Managerial Insights

In delving deeper into the sensitivity analysis, it becomes evident that various parameters have diverse impacts on the overall cost structure. The Wholesale Price (W) holds substantial significance, directly influencing cost structures for both retailers and suppliers shown in figures 4.6, 4.7. Changes in W affect order quantities, replenishment cycles, and overall costs, with higher prices leading to reduced order quantities and increased costs. The Demand Uncertainty (μ) significantly affects orders and costs, emphasizing the need for efficient information-sharing practices to mitigate rising retailer costs (shown in figure 4.6) associated with increased uncertainty (higher μ).

Holding and Ordering Costs (α and β) play a pivotal role in determining total order quantities and in (figure 4.7) costs shown. Reductions in α and β lead to diminished order quantities and costs, underscoring the importance of efficient processes and strategic management in minimizing overall costs. The Unit Cost of the Product (d) directly influences order quantities and costs, demonstrating the sensitivity of the model to variations in unit cost.

On the other hand, parameters such as h_r, θ, S_r, S_s, and Cr exhibit minimal impacts as shown in (figure 4.8), suggesting their nuanced influence on overall cost structures. These parameters have relatively lower effects on key outcomes compared to other influential factors. Understanding these differential impacts provides valuable insights for managerial decision-making, highlighting critical factors that require strategic attention for achieving cost-effective supply chain management. Managers should focus on optimizing parameters with significant impacts, such as wholesale prices, demand uncertainty, and holding/ordering costs, to navigate the complexity of supply chain dynamics effectively. Meanwhile, parameters with minimal impacts may require less attention in strategic decision-making processes.

- Strive to find a balance in wholesaler prices to optimize overall costs.
- Implement and encourage efficient information-sharing practices to avoid increased retailer costs.
- Strategically manage blockchain information (α) to minimize retailer costs and optimize overall supply chain performance.
- Optimize setup and ordering processes to reduce associated costs.
- Be mindful of the influence of demand rate coefficients (β) on order quantities and costs in supply chain decision-making.

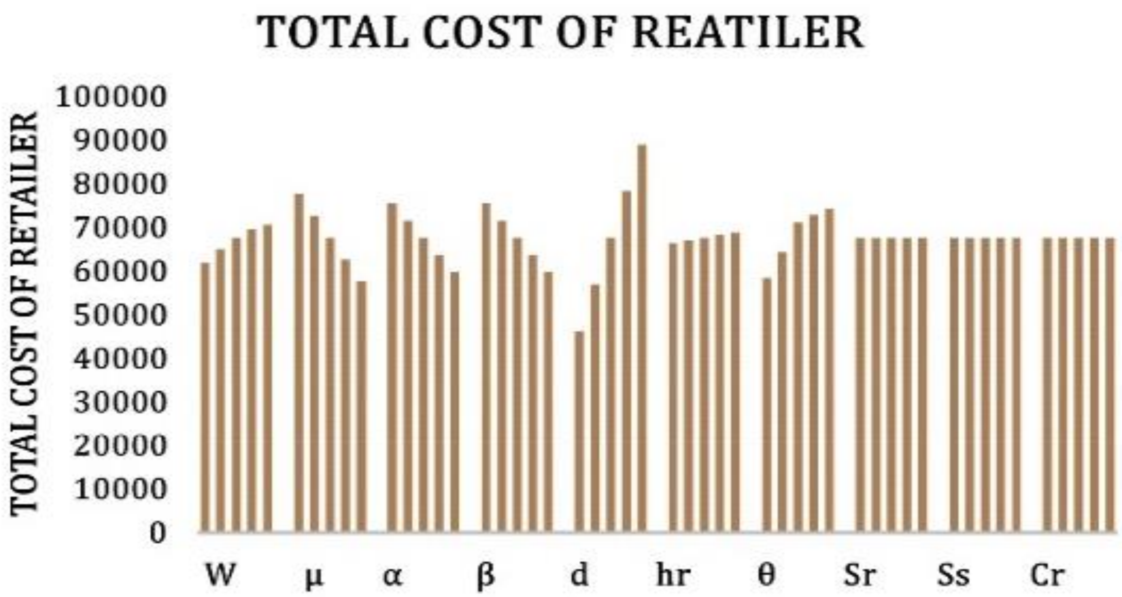


Figure 4.6 Sensitivity analysis of Total cost of retailer

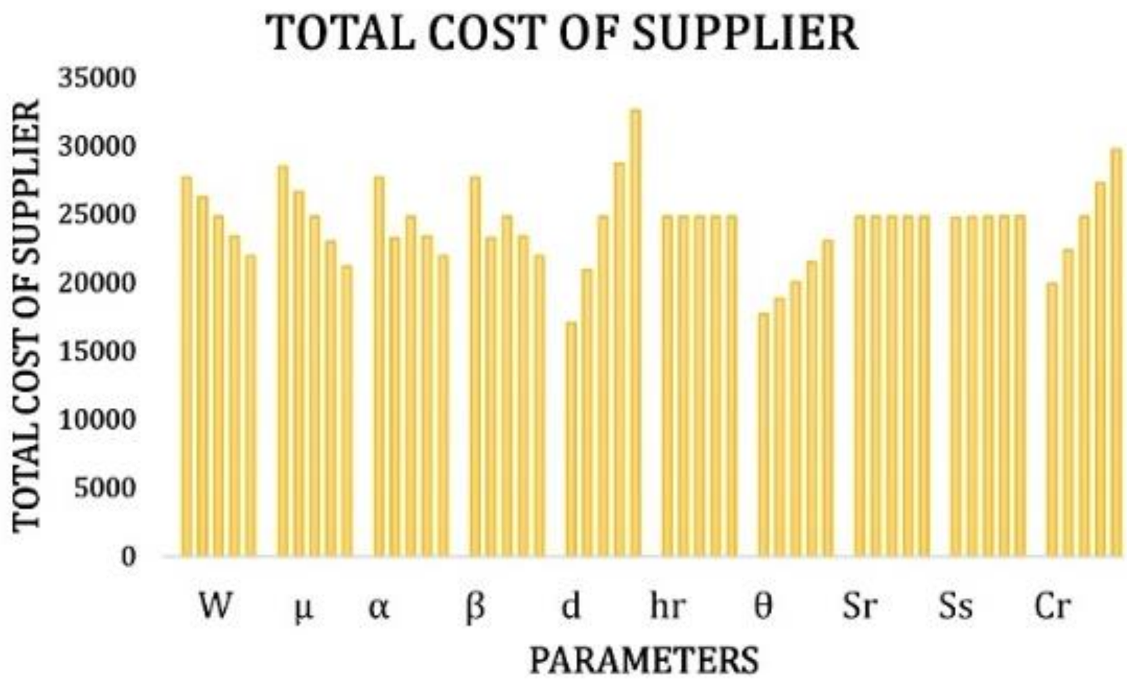


Figure 4.7 Sensitivity analysis for total cost of supplier

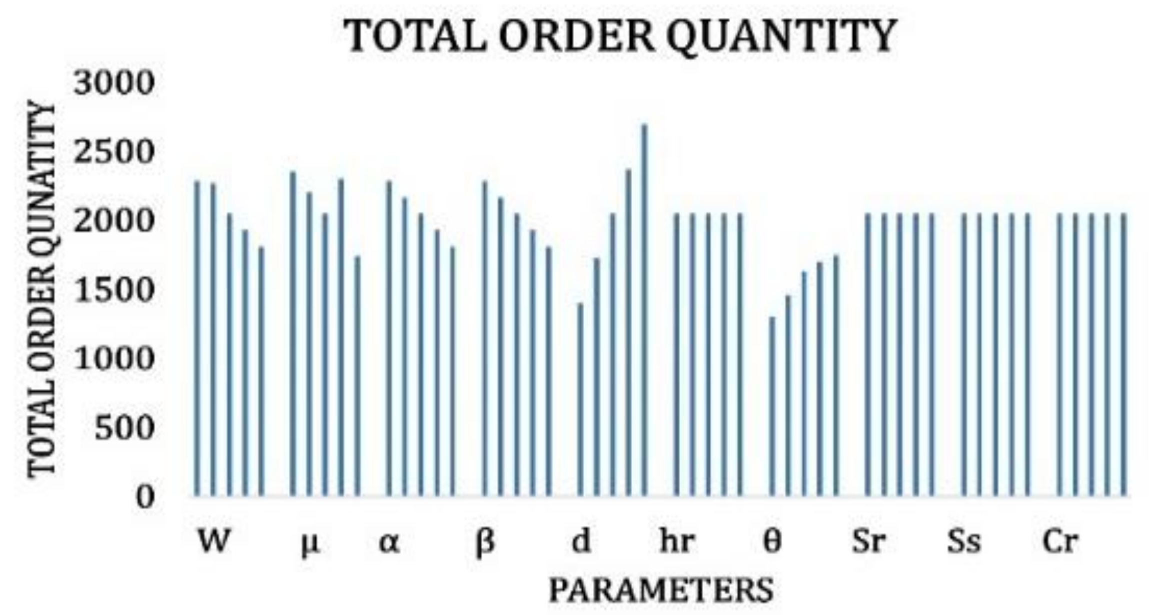


Figure 4.8 Sensitivity Analysis for total order quantity

Table 4.4 Comparative Table of Optimal Replenishment Cycles & Total Costs

Demand Parameter (a)	**Traditional Model**		**Blockchain-based Model**	
	Optimal Cycles (n^*)	**Total Cost (TC)**	**Optimal Cycles (n^*)**	**Total Cost (TC)**
500	4	3367.83	3	3031.0
1500	5	6514.86	4	5863.4
2500	6	9629.13	5	8666.2

4.7 Analysis of Blockchain-based and Traditional Inventory Models

To validate our proposed blockchain-based model, we conducted a comparative analysis with the traditional model by Wu & Zhao (2014). This comparison aims to demonstrate that our model, through decentralized coordination and real-time information sharing, offers significant advantages over the conventional trade credit approach. In our analysis, key parameters such as the coordination mechanism, optimal replenishment cycles (n^*), total cost (TC), sensitivity to demand uncertainty, information sharing, and system resilience were compared. The traditional model employs a trade credit mechanism for coordinating inventory replenishment, resulting in optimal cycles of $n^* = 4$ (for a demand parameter of 500), 5 (for 1500), and 6 (for 2500), with total costs of approximately 3367.83, 6514.86, and 9629.13, respectively as shown in Table 4.4. In contrast, the blockchain-based model uses decentralized coordination, which leads to a lower number of cycles and reduces overall costs by roughly 10%. Moreover, while the traditional model shows moderate sensitivity to demand uncertainty and limited information sharing due to fixed trade credit terms, our blockchain-based model leverages real-time data sharing to better manage uncertainty, thereby enhancing system resilience and adaptability under dynamic market conditions. These differences can be clearly observed in the accompanying figure 4.9 and 4.10, which graphically demonstrates that the blockchain-based model outperforms the traditional model.

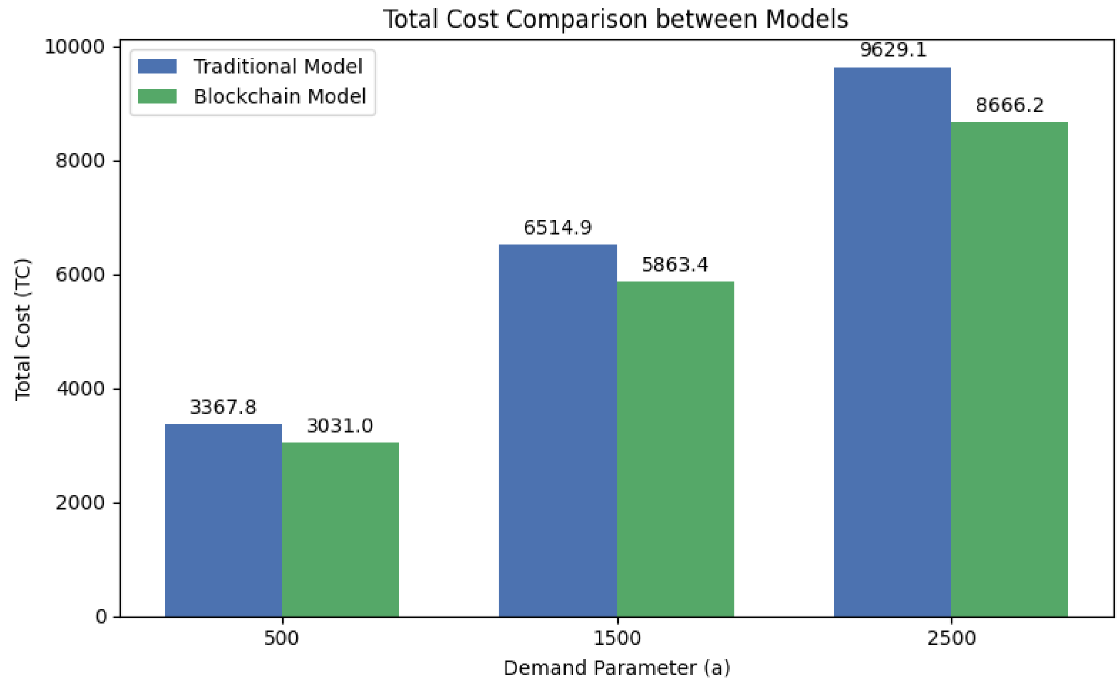


Figure 4.9 Total Cost Comparison between Models

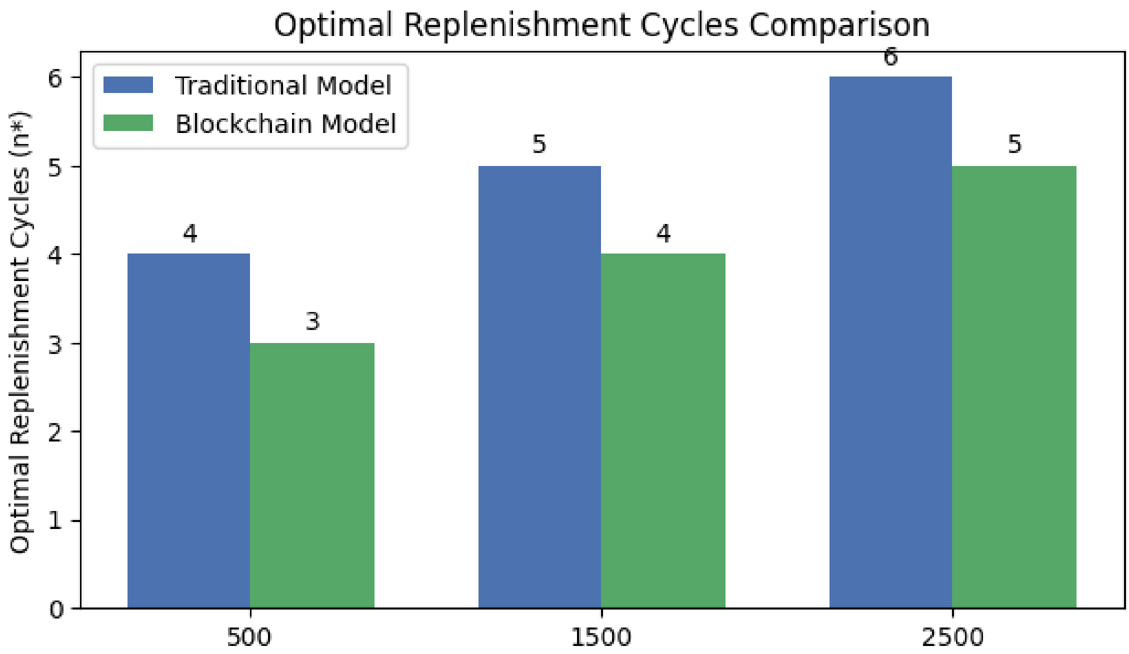


Figure 4.10 Replenishment Cycles Comparison

4.8 Conclusion

To conclude, this chapter examines how Blockchain Technology can transform into a decentralized supply chain model. The backdrop of the COVID-19 pandemic highlighted the vulnerabilities of centralized supply chain systems, leading to a growing recognition of the robustness and flexibility offered by decentralized models. Through a multi-participant approach, a decentralized supply chain distributes decision-making and resources, reducing the risk of disruptions from a single source. The literature review reveals a gap in the application of blockchain technology within decentralized supply chain models, particularly concerning inventory management over finite time horizons. This research addresses this gap by developing a mathematical model that integrates blockchain technology into the decision-making processes of decentralized supply chains.

The proposed model, based on a set of assumptions and notations, considers various parameters such as information sensitivity, demand rate coefficients, and setup costs. Mathematical formulation involves a first-order linear differential equation expressing the depletion of inventory over replenishment cycles. The cost for the retailer (TCR) equation encompasses ordering, holding, and purchasing costs, providing a comprehensive view of the financial implications. The uniqueness and optimality of the replenishment interval are established through the analysis of the Hessian matrix associated with TCR, ensuring a global minimum value. The dynamic ordering interval inequality and the monotonic increase of replenishment cycles further contribute to the understanding of the system's behavior.

The methodology involves the optimization of the process by minimizing the TCR equation while keeping the value of 'n' constant. The numerical example demonstrates the sensitivity of the total costs and replenishment times to change in the blockchain-based wholesale price (W). The results emphasize the practical implications for supply chain optimization and decision-making. The sensitivity analysis explores the impacts of different percentage changes in key parameters, providing valuable insights for managerial decision-making. Wholesale prices, demand uncertainty holding and ordering costs, and the unit cost of the product exhibit varying influences on optimal values, offering guidance for decision-makers under different conditions.

In summary, this research contributes to the understanding of how blockchain technology can enhance decentralized supply chain inventory models. The proposed model, along with its analysis and numerical examples, provide a foundation for future research and practical implementation. As supply chains continue to evolve, embracing innovative solutions powered by blockchain can pave the way for a more resilient, efficient, and transparent future.

Looking ahead, we envision extending this model to a three-echelon supply chain, fostering a comprehensive understanding of its dynamics. Future studies could explore variations in different time periods, uncovering additional nuances and enhancing the model's applicability. As supply chains undergo transformations, the adoption of innovative blockchain-powered solutions holds the potential to foster a future characterized by enhanced resilience, efficiency, and transparency. Our research lays the groundwork for ongoing exploration in this realm, aligning with the overarching objective of promoting effective supply chain management.

Chapter 5 Enhancing Supply Chain Efficiency with Blockchain: Addressing Information Sensitivity for Increased Manufacturer Profitability

Abstract

This chapter explores the impact of information-sensitive retailers on a manufacturer's decision to adopt blockchain technology in supply chain management. A mathematical model is developed, considering factors such as retailer information sensitivity, sensitivity costs, and the manufacturer's profit function. The model aims to optimize total profit by setting production quantity and price while accommodating sharing and non-sharing retailers. Analysis of the manufacturer's optimal equilibrium strategy highlights the critical role of information-sensitive retailers in blockchain adoption decisions. Findings suggest that the proportion of such retailers should satisfy specific conditions for optimal outcomes, emphasizing the importance of maintaining a balance in information-sensitive suppliers for efficient information sharing and manufacturing costs. Additionally, the chapter conducts a comprehensive sensitivity analysis, varying each parameter to assess its impact on the profit function. A numerical example illustrates the practical implications of the model. The insights derived from the sensitivity analysis provide valuable managerial strategies for enhancing supply chain efficiency and increasing manufacturer profitability.

Keywords**:** Blockchain adoption, Information-sensitive retailers, Supply chain management, Manufacturer's profit optimization, Information sharing, Supply chain coordination, Sensitivity Analysis

5.1 Introduction

The Supply Chain refers to the interconnected network of organizations, individuals, activities, information, and resources involved in the production, distribution, and delivery of goods and services to customers (Huang et al., 2022). It plays a crucial role in ensuring that products or services are delivered to the right place, at the right time, and in the right condition. Supply chain management has become increasingly complex due to factors such as globalized markets, fragmented supply networks, and diverse regulations. Furthermore, customer expectations have also evolved, demanding greater customization and faster delivery times (Yue, 2008). To meet these challenges and enhance supply chain efficiency, businesses have turned to blockchain technology (Zhou et al., 2022)**.**

Blockchain offers transparency, security, and decentralization, making it a promising solution for supply chain management (Wang et al., 2019). Retailers can use blockchain to share real-time information, enhancing visibility and traceability. By ensuring the integrity of shared data, blockchain mitigates risks such as tampering and unauthorized access (Kouhizadeh & Sarkis, 2018). Additionally, blockchain reduces fraud and counterfeit products by eliminating intermediaries (Kouhizadeh et al., 2021). It provides real-time insights, enabling stakeholders to identify bottlenecks and delays promptly, optimizing production and delivery schedules (Wang et al., 2019).

Existing literature explores blockchain's diverse applications, highlighting its role in sustainability, supply chain coordination, and operational efficiency (Dong et al., 2022; Hayrutdinov et al., 2020; Huang et al., 2022; Ko et al., 2018; Xu et al., 2017). Recent research introduces a decentralized supply chain model utilizing blockchain to enhance transparency and efficiency, demonstrating optimal replenishment schedules and strategies for managing information sensitivity (Mishra et al., 2024).

Our chapter extends this research by exploring a model that connects mathematical analysis with real-life scenarios. Unlike previous articles focusing on single-case studies, we integrate scenarios where some retailers share information while others do not. Increased information sharing among retailers leads to higher profits, driven by trust in blockchain technology for data safety and transparency.

The chapter is structured as follows: Section 5.2 outlines assumptions and notations, Section 5.3 introduces the model, Section 5.4 presents numerical examples and sensitivity analysis, Section 5.5 offers managerial insights, and Section 5.6 concludes the chapter and suggests future research directions**.**

5.2 Assumptions and Notations

5.2.1 Notations

μ	Represents the proportion of sensitive retailers ranging from 0 to 1.
$1-\mu$	Represents the proportion of non-sensitive retailers.
β	Denotes the information level under the blockchain System.
θ	Represents the sensitivity cost associated with shared information.
a	Total Market Potential.
b	Sensitivity of product price.
p	Product price.
q	Linear demand function.
c	Product Cost.

5.2.2 Assumptions and Hypotheses

These assumptions serve as foundational elements for our model, providing a structured basis for the subsequent analysis and development of our research framework.

- There is a manufacturer supplying products to multiple retailers.
- Some retailers are willing to share information about the product's lifecycle using a Blockchain System.
- The manufacturer aims to optimize total profit by considering the information shared by retailers.
- We assume a linear demand to underpin our model.
- The manufacturer sells products to retailers at a wholesale price, and retailers sell them to customers at a retail price.
- The cost of the blockchain system investment is represented as $I\beta^2$, where 'I' is the blockchain investment parameter. Consumers value the trustworthiness of the product's information ranging on a scale from 0 to 1 (Hayrutdinov et al., 2020).
- Total demand during the season depends on the product's price and information shared about the product. Market demand is a linear function of price and the cost of the blockchain system expressed as $q' = a - bp + \theta\beta$.
- Products in the market can be substituted with or without Blockchain-based information.

5.3 Defining the Mathematical Model

In this section, we develop a model for a blockchain-based supply chain that analyzes the optimal pricing strategy, the optimal information-sharing quantity, and the optimal profit of the manufacturer under two conditions. This model extends the work by Huang et al. (2022). Model development follows the structure depicted in figure 5.1. We consider a practical case where we divide our multiple retailers into two groups: one group is willing to share information, secured using blockchain technology and incurring extra costs, while the remaining retailers opt not to share their information.

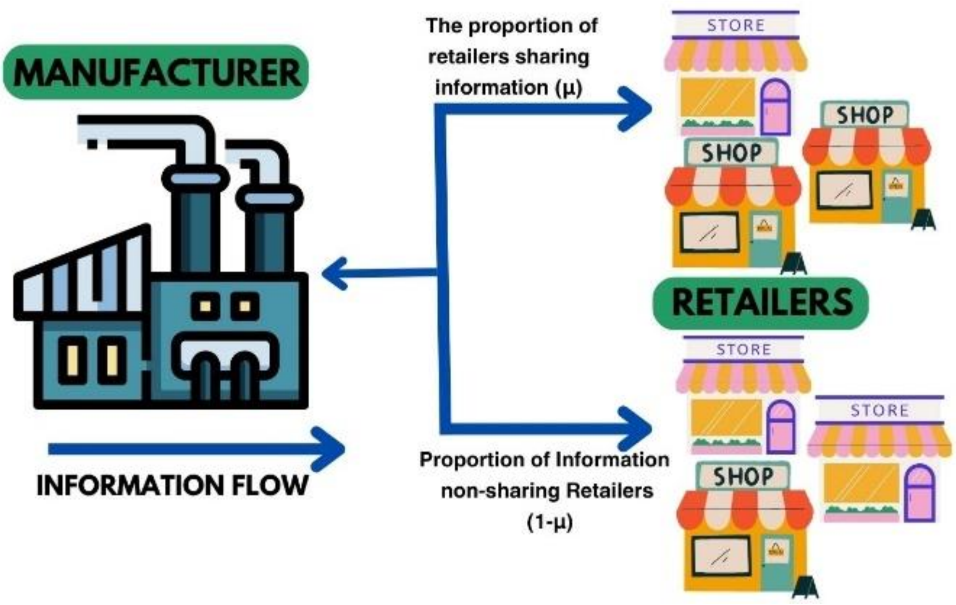


Figure 5.1 The configuration of the supply chain in this model

In this case, the group of retailers willing to share their information incurs additional costs due to storing their information in the blockchain system. However, this has a slight negative impact on our demand.

Profit Contribution from Sensitive Retailers (μ): $P_1 = (p - c)\, q' - I\beta^2$ (5.1)

Here, q' represents the demand generated by sensitive retailers.

Profit contribution from non–sensitive retailers (1-μ): $P_2 = (p - c)\, q - I\beta^2$ (5.2)

Here, q represents the demand generated by non-sensitive retailers.

To assess the impact of blockchain technology on manufacturers' profits, we formulate the manufacturer's profit function within diverse supply chain scenarios as follows:

Manufacture Overall Profit Function (MP): $MP = (\mu)P_1 + (1- \mu)\, P_2$

$$MP = (p - c)\,(\mu\,(a - bp + \beta\theta) + (1 - \mu)\,(a - bp) - I\beta^2 \quad (5.3)$$

Taking the first derivative of equation 5.3 with respect to p and β, and applying the first-order condition, we obtain:

$$\frac{\partial \mathrm{MP}}{\partial p} = -\mathrm{I}\beta^2 + (a - \mathrm{bp})(1 - \mu) + (a - \mathrm{bp} + \beta\theta\mu) + (p - c)(-b(1 - \mu) - \mathrm{b}\mu) = 0$$

$$P = \frac{a + bc - i\beta^2 + \beta\theta\mu}{2b} \quad (5.4)$$

$$\frac{\partial \mathrm{MP}}{\partial \beta} = (\mathrm{p} - \mathrm{c})[\mu\theta - 2\beta\mathrm{I}] = 0, \beta = -\frac{1}{2}\mathrm{I}\theta\mu \quad (5.5)$$

Combining equations (5.4) and (5.5) facilitates the derivation of optimal values for the price p^* and the sensitivity parameter for the manufacturer's information β^*. Subsequently, substituting p^* and β^* back into equation (5.3):

{We encountered three outcomes; however, in two instances, p=c (signifying that the selling price equals the cost price), leading to a nullification of the profit function. Consequently, these outcomes were deemed untenable and were dismissed from consideration.

$$\left[p = c, \beta = \frac{1}{2}\mathrm{I}\left(\sqrt{4a\mathrm{I} - 4\mathrm{I}bc + \theta^2\mu^2} \pm \theta\mu\right)\right]\}$$

The manufacturer's optimal price with blockchain technology is $p^* = \frac{4a + 4b - I\theta^2\mu^2}{8b}$. The Optimal sensitivity of manufacturer information $\beta^* = -\frac{1}{2} I\theta\mu$. The manufacturer's optimal profit with blockchain technology $MP = \frac{(4bc - 4a + I\theta^2\mu^2)^2}{64b}$.

To establish the optimal point for the manufacturer's profit function under the influence of sensitivity, we introduce the following lemma:

Lemma: - For the given manufacturer profit MP = (p – c) (μ (a – bp + βθ) + (1 – μ) (a – bp) - Iβ^2 where μ represents the proportion of sensitive retailers and β is the sensitive parameter for manufacturer's information, the critical point p^* at which the profit is maximized is determined by the equation $p^* = \frac{4a + 4b - I\theta^2\mu^2}{8b}$

Proof: To establish the maximum points, we employ the Hessian matrix method. Consider the Hessian matrix of the Profit Function denoted by Hess(MP). The second-order partial derivatives of MP with respect to p and β give us:

$$\text{Hess (MP)} = \begin{vmatrix} \frac{\partial^2 MP}{\partial p^2} & \frac{\partial^2 MP}{\partial p\, \partial \beta} \\ \frac{\partial^2 MP}{\partial \beta\, \partial p} & \frac{\partial^2 MP}{\partial \beta^2} \end{vmatrix}$$

$$\text{Hess (MP)} = \begin{vmatrix} -\ 2b\,[1 - 2\mu] & \theta\,\mu - 2I\beta \\ \theta\mu - 2I\,\beta & \theta\,\mu - 2I\beta \end{vmatrix}$$

$$\therefore\ |(\text{Hess (MP)}| = 2((p - c) + Ib\,(1 - \mu)\,(2\beta + I\theta\mu) > 0$$

Additionally, the negative of the leading principal minor indicates that the critical point p^* is associated with the local maximum.

This lemma establishes the critical point $p*$ where the manufacturer's profit is maximized under the conditions of information sensitivity. It forms the basis for understanding the impact of sensitive retailers on the adoption of blockchain technology and subsequent profit optimization.

The impact of information sensitivity on the manufacturer's optimal equilibrium strategy in the context of blockchain technology is a crucial aspect of our chapter. The introduction of blockchain aims to enhance information collaboration within the supply chain. Therefore, the proportion of information-sensitive retailers significantly influences a manufacturer's decision to adopt blockchain technology. Analyzing the equilibrium selection, we derive:

Proposition: Where blockchain technology is implemented in a supply chain, the proportion of information-sensitive retailers should satisfy the condition $\mu < \sqrt{\frac{4a+4bc}{I\theta^2}}$, ensuring a non-zero denominator.

Proof: $p = \frac{4a+4bc-I\theta^2\mu^2}{8b}$, $8b \neq 0 \Rightarrow 4a + 4bc - I\,\theta^2\mu^2 > 0$

$\Rightarrow \mu^2 < \frac{4a+4bc}{I\theta^2}$, $\mu < \sqrt{\frac{4a+4bc}{I\theta^2}}$

The expression is valid when, $\mu < \sqrt{\frac{4a+4bc}{I\theta^2}}$ ensuring a non–zero denominator.

This proportion illustrates that maintaining a certain range of information-sensitive retailers ensures optimal information-sharing outcomes. Furthermore, the quantity of information-sensitive suppliers is intricately linked to the manufacturing cost and demand sensitivity.

5.4 Numerical Example and Sensitivity Analysis

To understand the impact of parameters β and μ on the manufacturer's profit function, we incorporated a numerical example adapted from Hayrutdinov et al. (2020). The parameter values utilized were a=100, c=5, μ=0.25, β=2, I=5, q=3, b=0.5, θ=0.5, and p=8. This example was developed based on the assumptions outlined in Section 5.2 and Equation (5.3).

The increase in the manufacturer's profit from 228.15 to 230.85 (a 1.18% rise) for an increase in μ\muμ from 0.05 to 0.95 suggests that while information sharing positively impacts profitability, the effect is relatively moderate shown in Table 5.1. This can be attributed to several key factors. First, although increased transparency through blockchain enhances consumer trust and improves demand forecasting, these benefits are partially offset by the associated costs of blockchain implementation ($I\beta^2$) and information sensitivity (θ). Second, the observed trend indicates diminishing marginal returns—initial increases in μsignificantly contribute to profit improvement, but beyond a certain threshold, additional retailers sharing information yield progressively smaller gains. This diminishing effect could be due to saturation in the benefits of information availability, where manufacturers have already optimized their strategic pricing and inventory decisions based on the available shared data. Additionally, in real-world supply chain scenarios, increased transparency does not always translate into a proportional increase in consumer willingness to pay, limiting the overall profit impact. These factors collectively explain why the rise in profit remains modest despite a substantial increase in μ.

The data suggests a potential positive correlation. As the proportion of information-sharing retailers (μ) increases, the manufacturer's profit (MP) also appears to trend upwards. The likely rise in manufacturer profit with improved information sharing (μ) can be attributed to several factors. Firstly, greater transparency through shared product lifecycle data on a blockchain system can lead to enhanced consumer trust. This trust can translate into a higher perceived value of the products, potentially allowing manufacturers to command premium pricing. Secondly, information sharing facilitates improved product demand. With access to shared data, manufacturers can develop more targeted marketing campaigns, reaching the right consumers and potentially boosting sales. Finally, transparency might enable manufacturers to implement strategic pricing. By understanding consumer preferences and the perceived value of information accessibility, manufacturers can potentially adjust prices to maximize profit.

Table 5.1 Relationship between manufacturer profit and proportion of retailers who are share information(μ)

μ	Manufacturer's Profit (MP)
0.05	228.15
0.15	228.45
0.25	228.75
0.40	229.20
0.50	229.50
0.65	229.95
0.75	230.25
0.85	230.55
0.95	230.85

Table 5.2 Relationship between manufacturer profit and information level (β)

β	Manufacturer's Profit (MP)
0.00	228.15
0.05	284.44
1.00	273.38
1.50	254.81
2.00	228.75
2.50	195.19
3.00	154.13

Table 5.2 explores the relationship between a manufacturer's profit (MP) and the level of information sharing (β) within a supply chain. The information sharing level (β) ranges from 0.00 (no information sharing) to 3.00, representing different degrees of information exchange. The corresponding manufacturer's profit (MP) is listed in monetary units for each level of β.

The data presented in the table suggests a non-linear relationship between information sharing (β) and manufacturer's profit (MP). Initially, as information sharing increases from 0.00 to 0.05 (β), the manufacturer's profit (MP) significantly rises from 228.15 to 284.44 (Figure 5.2). This initial increase could be attributed to potential benefits such as improved consumer trust or targeted marketing based on shared data.

However, the trend reverses as the information-sharing level (β) rises. There is a noticeable decline in manufacturer's profit (MP) from 284.44 at β=0.05 to 154.13 at β=3.00. This suggests that the costs associated with information sharing (β) might outweigh the benefits beyond a certain point (figure 5.3). These costs could involve implementing and maintaining a system to handle information sharing or potential complexities from managing a large amount of shared data.

Following the numerical example, we now proceed to conduct a sensitivity analysis on each parameter. This analysis enables researchers to assess the model's sensitivity to changes in parameters, providing valuable insights into its robustness and reliability.

To evaluate the robustness of the profit function within our research domain, sensitivity analysis was performed for each parameter. Parameters were systematically varied by ±20%, ±10%, and 0% from their base values to gauge their impact on the profit function. The objective of this sensitivity analysis was to determine whether fluctuations in parameter values significantly affected the profitability of the model. Table 5.3 illustrates the changes in the profit function resulting from variations in each parameter, thereby providing a comprehensive view of the model's sensitivity to parameter fluctuations.

In figure 5.4, the radar chart illustrates the sensitivity analysis of key parameters under various percentage changes. Each axis represents a different scenario, and the colored areas show how the values of each parameter change across these scenarios. Larger areas indicate higher sensitivity, meaning the changes affect the parameter's value more. This visualization helps identify which parameters have the most significant impact, providing valuable insights for decision-making in the supply chain model.

The sensitivity analysis, visually depicted in figure 4, provides crucial insights into the impact of parameter variations on the profitability of the model. As illustrated, each parameter's influence on the profit function is substantial. Notably, the total market potential (a) exhibits a positive correlation with profit, emphasizing the strategic significance of expanding the market size. Balancing product prices based on demand (b) proves instrumental in enhancing overall profitability, highlighting the importance of dynamic pricing strategies aligned with market dynamics. Effective product cost (c) management emerges as a key determinant, with reduced costs positively impacting profits. The information level (β) significantly influences profit, underlining the necessity of aligning information levels with consumer preferences. Precision in pricing strategies (p) is deemed vital for optimal profitability. Furthermore, the careful consideration of investments in blockchain technology (I) is crucial, requiring a delicate balance between enhanced information-sharing benefits and associated costs. The sensitivity cost (θ) associated with shared information reveals its impact on profit, stressing the need for effective cost management. Lastly, maintaining a balanced proportion of sensitive retailers (μ) proves indispensable for optimal information-sharing outcomes and manufacturing costs. These insights underscore the multifaceted considerations that must be factored into supply chain decision-making for sustained profitability.

Table 5.3 Sensitivity Analysis for each Parameter

% Change	-20	-10	0	10	20
a	168.75	198.75	228.75	258.75	288.75
b	231.15	229.95	228.75	227.55	226.35
c	305	266.87	228.75	190.62	152.5
β	250.2	240.07	228.75	216.22	202.5
p	107.87	168.63	228.75	288.23	347.07
I	240.75	234.75	228.75	222.75	216.75
θ	228.6	228.67	228.75	228.82	228.9
μ	228.6	228.67	228.75	228.82	228.9

Parameters like total market potential, information level, product price, and the proportion of sensitive retailers have a high impact because they directly influence consumer demand, preferences, and market dynamics. Moderate impact parameters like product cost and sensitivity cost highlight that while these factors are important, their influence is not as pronounced as the high-impact parameters. The absence of explicitly low-impact parameters in the provided table suggests that all considered parameters have a noticeable impact on profit, albeit to varying degrees. In practice, managers should focus on strategies related to high-impact parameters to achieve significant improvements in profitability, while also considering moderate-impact factors for a well-rounded approach.

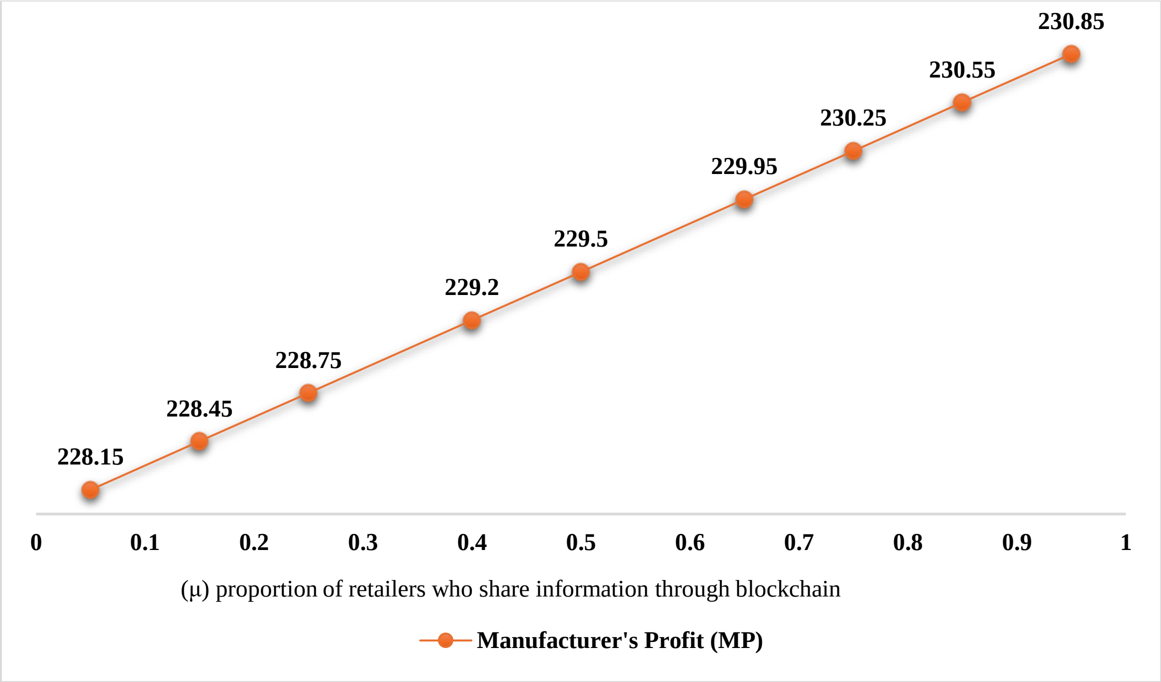


Figure 5.2 Relationship Between manufacturer's profit and Proportion of retailers who share information through Blockchain

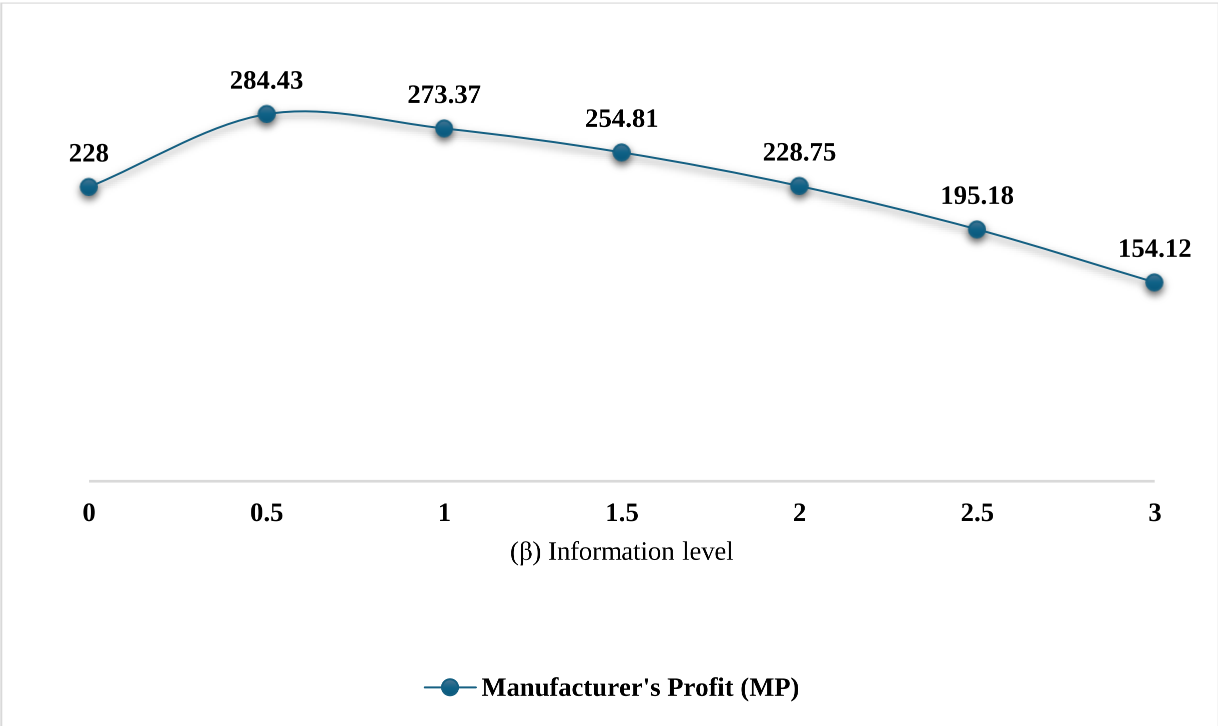


Figure 5.3 Relationship between Manufacturer's Profit and Information sharing Level

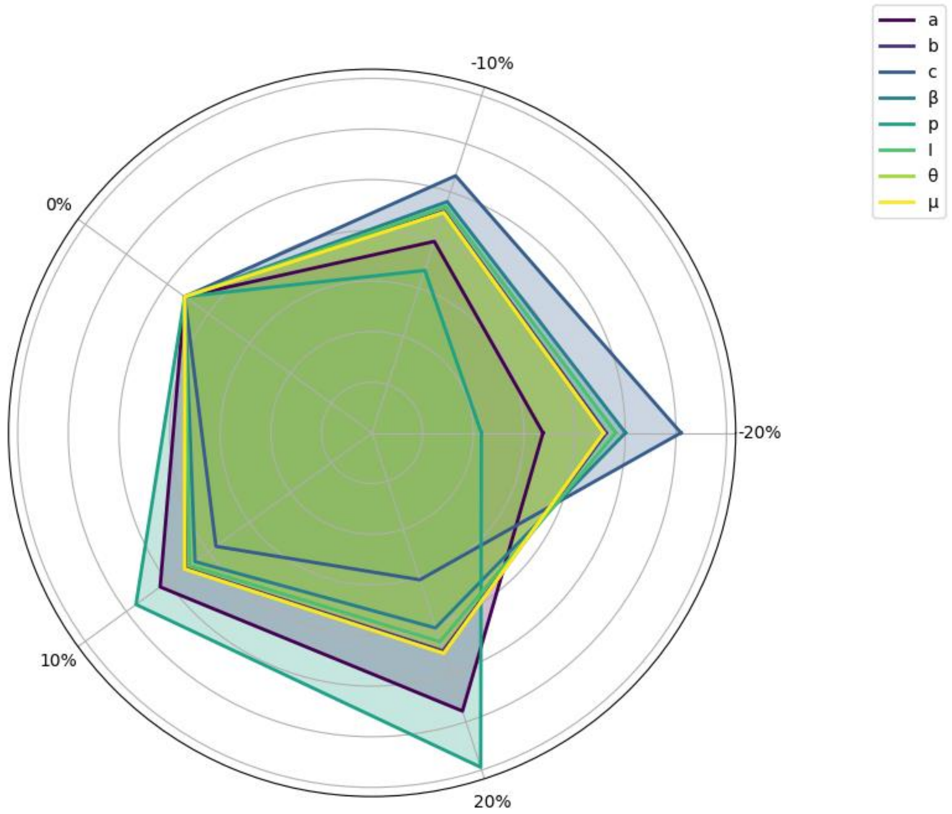


Figure 5.4 Radar Chart for Sensitivity Analysis of Key Parameter

5.5 Conclusion and Future work

In conclusion, our research demonstrates that integrating blockchain technology into supply chains can significantly enhance efficiency and profitability by facilitating information sharing among retailers. The model we developed highlights how sensitive and non-sensitive retailers impact the manufacturer's profit through the adoption of blockchain. Our findings underscore that a higher proportion of information-sharing retailers positively correlates with increased profitability, as the transparency and trust afforded by blockchain technology led to better demand management and strategic pricing.

The numerical examples and sensitivity analysis further validate our model, showing that parameters such as the proportion of information-sensitive retailers (μ), information level (β), and product cost (c) play crucial roles in optimizing supply chain operations. Our chapter also reveals that while blockchain implementation incurs additional costs, these are offset by the benefits of improved information flow and reduced risks of fraud and counterfeiting.

However, it is essential to acknowledge the challenges associated with blockchain adoption in supply chain management. High implementation costs, scalability issues, regulatory uncertainties, and data privacy concerns pose significant barriers to widespread adoption. Additionally, blockchain networks, particularly public blockchains, may suffer from latency and energy inefficiency, making them less feasible for high-frequency transactions. Integrating blockchain with legacy supply chain systems can also be complex and require substantial infrastructure upgrades. Furthermore, the lack of skilled professionals in blockchain technology can slow down adoption and implementation across industries.

Future work can explore several avenues to build upon our findings. First, extending the model to include dynamic pricing strategies and multi-echelon supply chains could provide deeper insights into real-world applications. Second, investigating the impact of blockchain on various types of products, including perishable goods and high-value items, could offer more specific recommendations for different industries. Third, incorporating stochastic demand and supply uncertainties into the model would enhance its robustness and applicability. Finally, empirical validation using real-world data from different sectors would further strengthen the practical relevance of our research. In summary, this chapter contributes to the growing body of literature on blockchain applications in supply chain management, offering a comprehensive framework for understanding and leveraging blockchain's potential to improve supply chain efficiency and profitability.

Chapter 6 Optimizing Inventory Management with Stackelberg Game and Linear Differential Equations: A Computational Approach

Abstract

This chapter presents an inventory model based on the Stackelberg game, where the retailer acts as the leader, setting optimal ordering times, and the manufacturer follows, adjusting production accordingly. The model minimizes supply chain costs, considering time-varying demand, production schedules, and inventory holding within a finite planning horizon. Optimal replenishment schedules are calculated using an iterative approach in Wolfram Mathematica 13.0. Numerical results show that for a demand rate of a = 675, the model identifies five replenishment cycles, leading to a total cost of $16,124.879. Sensitivity analyses further highlight the impact of key parameters, such as wholesale pricing, demand variability, and inventory costs, on supply chain performance, illustrating how strategic ordering decisions can optimize costs. Additionally, a comparative analysis is conducted between the proposed Stackelberg-based decentralized inventory model and L. Benkherouf's centralized model. While the Stackelberg model incurs higher total costs due to decentralized decision-making, it provides greater flexibility and adaptability to changing market conditions. A further comparison between Manufacturer-to-Retailer and Retailer-to-Manufacturer inventory strategies demonstrates that Retailer-to-Manufacturer is the more cost-effective approach. As the demand rate increases, the total cost rises for both strategies, but the Retailer-to-Manufacturer strategy results in lower total costs and a slower cost escalation, making it a more efficient choice. In contrast, the Manufacturer-to-Retailer strategy incurs higher costs due to increased production, storage, and transportation expenses. These findings suggest that allowing retailers to place orders based on real-time demand data can lead to better cost management and operational efficiency in decentralized supply chains. These specific outcomes from numerical examples and sensitivity analyses deepen the understanding of supply chain dynamics, offering valuable managerial insights. However, the chapter acknowledges limitations related to data assumptions and real-world implementation challenges, suggesting that further research is necessary to validate its applicability across diverse market scenarios.

Keywords: Stackelberg game approach, Inventory model, iterative method, linear demand, Finite Planning Horizon

6.1 Introduction

In the fast-evolving and competitive business landscape, effectively managing inventory is a critical success factor for organizations. As businesses strive to meet customer demand while minimizing costs, there is an increasing need for innovative and efficient inventory optimization techniques (Priniotakis & Argyropoulos, 2018). Among the various approaches to inventory optimization, the Stackelberg game theory has gained prominence. This framework models the strategic interactions between a leader (typically the manufacturer) and a follower (the retailer), allowing for an analysis of optimal pricing and inventory policies in a decentralized supply chain (Fang et al., 2008; Hayrutdinov et al., 2020; Sapra et al., 2010).

This chapter explores a novel approach to inventory optimization using the Stackelberg game framework within a finite planning horizon, incorporating linear differential equations to provide a comprehensive tool for strategic decision-making in supply chains. By leveraging this game-theoretic structure, the model reflects the complex interplay between supply chain participants, enabling businesses to optimize inventory levels and improve efficiency and profitability.

The Stackelberg model, rooted in game theory, assumes a hierarchical decision-making process, where the leader (manufacturer) sets the ordering quantities and schedules, while the follower (retailer) responds accordingly. This structure allows for strategic considerations in inventory management, particularly in cases where supply chain dynamics involve both pricing and stock replenishment decisions. To enhance the model's effectiveness, we employ linear differential equations within a finite planning horizon to capture the dynamic behavior of inventory systems, accounting for variables such as demand patterns, production rates, and replenishment strategies (Sapra et al., 2010). This approach enables the optimization of inventory policies over a specific time frame, balancing trade-offs between holding costs, backorder costs, and production/ordering costs.

Existing literature on inventory optimization has explored a range of computational techniques to address the complexities of modern supply chain management. For instance, (Xu et al., 2018) proposed a finite horizon inventory model with periodic reviews to optimize replenishment and distribution strategies for omnichannel retailers in franchise networks, identifying a base-stock policy as optimal. Another study by (Moon et al., 2010) examined variable pricing and stock management in a two-channel supply network using a continuous-time control optimization model. The research offered insights into optimal strategies and the influence of customer acceptance rates. Additionally, (Baganha & Cohen, 1998) explored the stabilizing role of inventory in supply chain networks, identifying mechanisms that promote stabilization and defining the necessary parameters for its occurrence.

Despite these significant contributions, the integration of the Stackelberg game approach with linear differential equations within a finite planning horizon remains relatively underexplored. Most studies focus either on centralized models or employ simplified decision-making scenarios without fully capturing the hierarchical dynamics of a decentralized supply chain. Our research aims to address this gap by developing a comprehensive framework that combines the strategic advantages of the Stackelberg game model with the analytical power of linear differential equations. Through this integration, the proposed model not only optimizes inventory management but also accounts for the dynamic nature of supply chains, offering a more realistic and adaptable tool for decision-makers.

The novelty of this chapter lies in its use of an iterative computational approach to solve the Stackelberg game model, providing numerical examples and sensitivity analyses that illustrate the model's practicality in diverse market conditions. By analyzing the specific outcomes from numerical examples, we aim to offer clarity on how demand rates, holding costs, and ordering costs impact the overall supply chain performance.

The chapter is organized as Section 6.2 presents the assumptions and notations used throughout the chapter, establishing the foundational premises of our model. Section 6.3 presents the mathematical framework, including the theoretical foundations and equations underpinning the model. In Section 6.4, we describe the methodology employed to derive optimal solutions. Section 6.5 provides numerical analysis and discusses the results, demonstrating the model's applicability to real-world scenarios. In Section 6.6, we conduct a sensitivity analysis, utilizing radar maps to visualize the impact of various parameters and offering managerial insights to guide decision-making. Section 6.7 focuses on the comparative analysis with Benkherouf's model, emphasizing the strengths and unique aspects of the proposed approach. Finally, Section 6.8 summarizes the findings and suggests potential avenues for future research to enhance the application of our model.

In summary, this chapter presents an innovative inventory optimization methodology, bridging the gap in existing literature by combining the Stackelberg game approach with linear differential equations. By doing so, we provide businesses with a cutting-edge tool for improving their inventory management strategies, promoting greater efficiency, reduced expenditures, and enhanced customer satisfaction.

6.2 Assumptions and Notations

6.2.1 Assumptions

- The product demand rate varies smoothly over time.
- The production rate (M) is fixed and exceeds the demand rate within the planning horizon.
- The manufacturer's holding cost exceeds that of the retailer.
- The manufacturer's setup cost is more than the retailer's setup cost.
- At T_K, the manufacturer produces inventory at a constant rate, then halts production until H.
- The retailer decides when to place orders within the planning horizon, making n orders during this time.
- A single, non-perishable item is retained in stock over a Bounded planning period.
- The ordering time intervals are not fixed.
- During the planning horizon, deteriorated units are not replaced or repaired.

- Demand is a time-dependent linear function.
- The deterioration rate (θ) is constant for all players in the supply chain (retailer, and manufacturer).

6.2.2 Notations

- Q_{Ri}: Ordering quantity
- O_R: Cost incurred for placing each order
- H_R: Cost of holding inventory per unit of time
- D_R : Deterioration cost per unit for the retailer
- $I_R(t)$: Inventory level of the retailer at time t
- $I_M(t)$: Manufacturer's inventory at time t
- M: Rate at which the manufacturer produces units

Factors influencing decisions**:**

- n: The number of shipments delivered to the retailer on a given cycle should be optimized
- H: Overall timeframe for planning and optimization
- T_K: Production stopping time
- T_{j+1}: Length of the $(j+1)^{-th}$ replenishment period
- Q_p: Optimal quantity of units to be produced in each batch
- Q_d: Optimal quantity of units to be delivered in each shipment

6.2.3 Analysis

- Retailer**:** Examines inventory dynamics for the retailer, observing stock levels over time and calculating total inventory cost.
- Manufacturer**:** Analyzes inventory levels during and between production cycles and computes total inventory cost.

6.3 Defining the mathematical model

The initial inventory for the manufacturer, supplier, and retailer is zero. The production cycle during H is as follows: At $t_0=0$, the manufacturer has the stock needed for period 1. The retailer makes n orders at times $0 \leq t_0 < t_1 < t_2 < \ldots < t_n \leq H$, where n and $t_0, t_1, t_2, \ldots, t_n$ are determined.

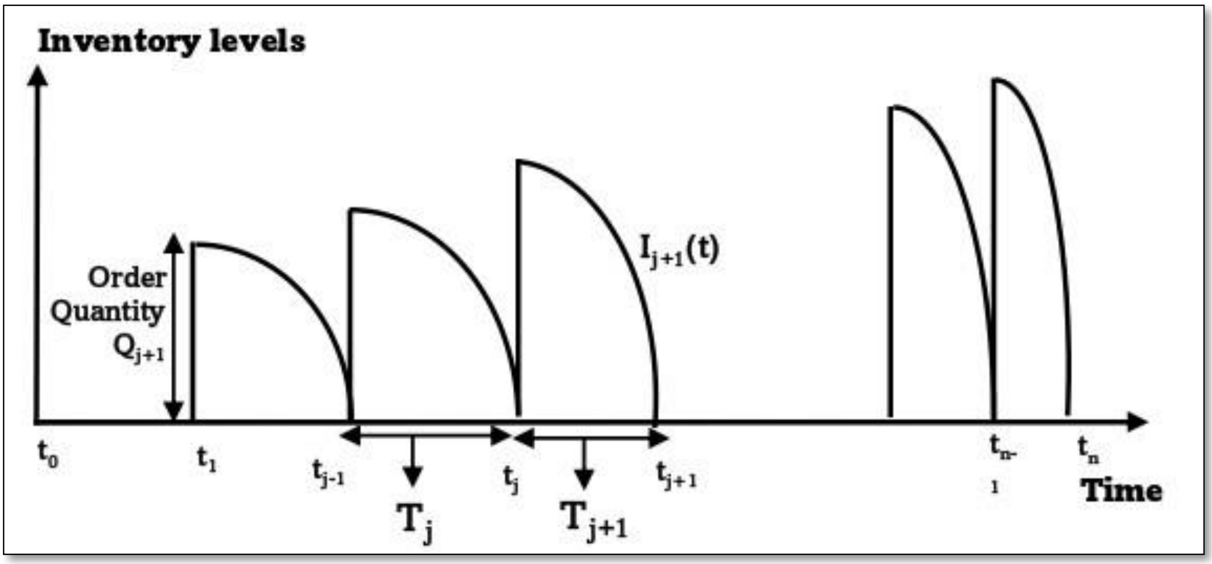


Figure 6.1 Retailer inventory under finite planning horizon

Retailer**:**

Let $I_R(t)$ denote the retailer's stock level at time t. The inventory level for the retailer during the ordering period j is:

$$\frac{dI_{R(j+1)}(t)}{dt} + \theta I_{r(j+1)} = -(a+bt),\ t_j < t < t_{j+1} \tag{6.1}$$

$$I_{R(j+1)}(t) = e^{-\theta t}\int_t^{t_{j+1}} e^{\theta w}(a+bt)dw \tag{6.2}$$

$$I_{R_{(j+1)}}\left(t_{j+1}\right) = 0 \quad \text{and} \qquad I_{R_{(j+1)}}(t_j) = Q_{R_j}$$

$$I_{R_{(j+1)}}(t) = \int_t^{t_{j+1}}(a+bw)_e{}^{\theta(w-t)}dw \tag{6.3}$$

$$I_{R(j+1)}(t) = \left[\frac{(a+bw)e^{\theta(w-t)}}{\theta} - \frac{b}{\theta^2}e^{\theta(w-t)}\right]_t^{t_{j+1}}$$

$$I_{R_{(j+1)}}(t) = \frac{(a+bt_{j+1})e^{\theta(t_{j+1}-t)}}{\theta} - \frac{b}{\theta^2}e^{\theta(t_{j+1}-t)} - \frac{(a+bt)}{\theta} + \frac{b}{\theta^2} \tag{6.4}$$

The order quantity for j^{th} cycles

$$Q_{R_j} = I_{R_{(j+1)}}(t_j) = \int_{t_j}^{t_{j+1}}(a+bt)e^{\theta(t-t_j)}dt \tag{6.5}$$

$$Q_{Rj} = I_{r_{(j+1)}}(t_j) = \frac{(a+bt_{(j+1)})e^{\theta(t_{j+1}-t)}}{\theta} - \frac{b}{\theta^2}e^{\theta(t_{j+1}-t)} - \frac{(a+bt)}{\theta} + \frac{b}{\theta^2} \tag{6.6}$$

For any given w, the value of w lies between t and t_{j+1}, and for any given t, the value of t lies between t_j and t_{j+1}, with w also lying between t and t_{j+1}.

Ordering cost of the buyer = n × O_R (6.7)

Holding cost

$$= H_R \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \left[\int_{t}^{t_{j+1}} e^{\theta(w-t)}(a+bw)dw\right] dt \tag{6.8}$$

Deterioration cost

$$D_R \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \theta * I_{R_{(j+1)}}(t)dt$$

$$= D_R * \theta \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \int_{t}^{t_{j+1}} \left\{\int_{t}^{t_{j+1}} (a+bw)e^{\theta(w-t)}dw\right\} dt \tag{6.9}$$

$$TCR(t_j, n) = \sum_{j=1}^{n} D_R \int_{t_j}^{t_{j+1}} I_{R_{(j+1)}}(t)\theta.\,dt + \int_{t_j}^{t_{j+1}} H_R I_{r_{(j+1)}}(t)dt + nO_R \tag{6.10}$$

$$TCR(t_j, n) = nO_R + \{H_R + \theta D_R\} \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} I_{R_{(j+1)}}(t)dt$$

$$TCR(t_j, n) = \{H_R + \theta D_R\} \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \left[\int_{t}^{t_{j+1}} (a+bw)e^{\theta(w-t)}dw\right] dt + nO_R$$

$$TCR(t_j, n) = \{H_R + \theta D_R\} \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \left(\frac{(a+bt_{j+1})e^{\theta(t_{j+1}-t)}}{\theta} - \frac{b}{\theta^2}e^{\theta(t_{j+1}-t)} - \frac{(a+bt)}{\theta} + \frac{b}{\theta^2}\right) dt + nO_R$$

$$TCR(t_j, n) = \{H_R + \theta D_R\} \sum_{j=1}^{n} \frac{1}{\theta}\left\{\frac{(a+bt_{(j+1)})e^{\theta(t_{j+1}-t_j)}}{\theta} - \frac{b}{\theta^2} - \frac{(a+bt_j)}{\theta} + \frac{b}{\theta^2} - \frac{2b}{\theta}\frac{-b(t_{j+1}-t_j)}{\theta^2}\right\} + O_R n \tag{6.11}$$

Using equation (6.5) and equation (6.6), we get

$$TCR(t_j, n) = \left\{\frac{H_R + \theta D_R}{\theta}\right\} \sum_{j=1}^{n} \left[\int_{t_j}^{t_{j+1}} e^{\theta(t-t_j)}(a+bt)dt - \frac{2b}{\theta} - \frac{bT}{\theta^2}\right] + O_R * n$$

$$TCR(t_j, n) = \left\{\frac{H_R}{\theta} + D_R\right\} \sum_{j=1}^{n} \int_{t_j}^{t_{j+1}} \left[e^{\theta(t-t_j)}(a+bt)dt - \frac{2b}{\theta} - \frac{bT}{\theta^2}\right] + O_R * n$$

$$\frac{\partial TCR(t_j,n)}{\partial t_j} = \left\{\frac{H_R}{\theta} + D_R\right\}\left\{(a+bt_j)\left[e^{\theta(t_j - t_{j-1})} - 1\right] - \theta \int_{t_j}^{t_{j+1}} (a+bt)e^{\theta(t-t_j)}dt\right\} \qquad (6.12)$$

This model addresses an inventory management system where the manufacturer acts as the supplier for the retailer. Following the production period T_K, the production halts, and the inventory level reduces as orders are placed by the retailers. These orders occur at time t_R, causing a drop in the manufacturer's inventory level.

Manufacturer:

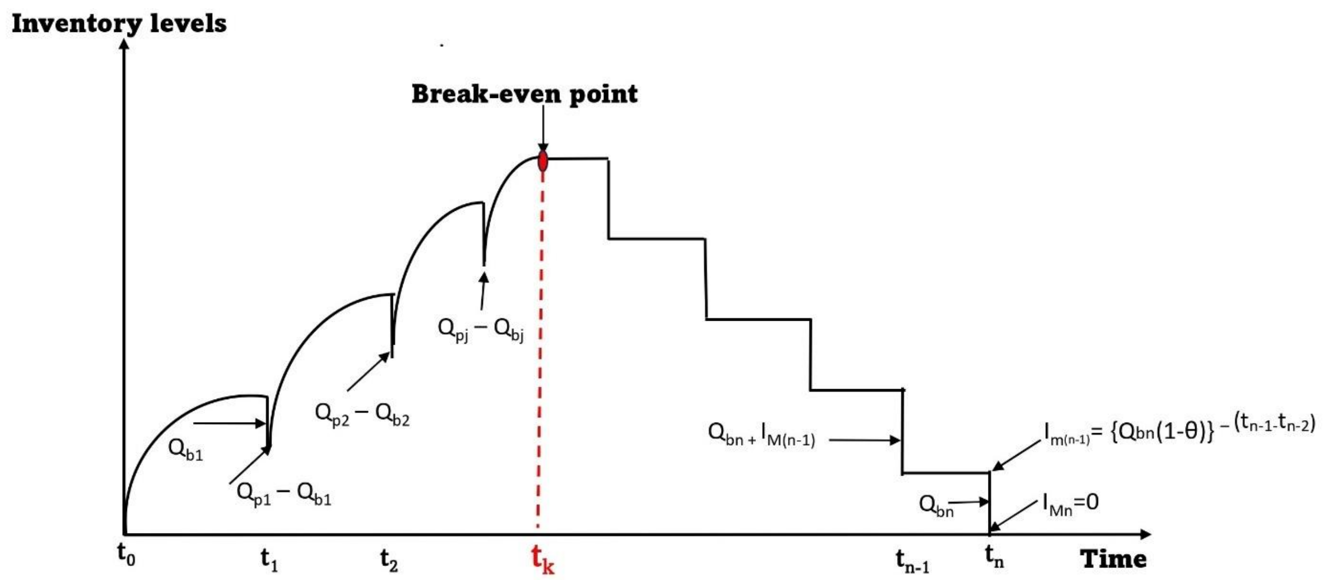


Figure 6.2 Manufacturer inventory model

We are now discussing the manufacturer's model. As previously mentioned, in this model, the manufacturer plays the role of the follower. This means that the manufacturer responds to the orders placed by the retailer, who acts as the leader. The manufacturer's operations are divided into two phases. In the first phase, the manufacturer is simultaneously producing goods and fulfilling the retailer's requirements (for $t < T_K$). We also identify a breakeven point where production stops, and the manufacturer focuses solely on fulfilling orders until the inventory is exhausted (for $t > T_K$). This approach ensures that the manufacturer's activities are aligned with the retailer's demands, optimizing the supply chain's efficiency and minimizing costs.

Inventory Dynamics During Production Phase **(**$t < T_K$**)**

Let $I_M(t)$ denote the manufacturer's stock level at time t. The inventory level during production is:

$$\frac{dM_{j+1}(t)}{dt} = M - \theta I_{M_{(j+1)}}(t) \qquad t_j < t < t_{j+1} \tag{6.13}$$

$$I_{M(j+1)}(t) = e^{-\theta t}\int_{t_j}^{t} M * e^{\theta w} dw$$

$$I_{M(j+1)}(t) = \int_{t_j}^{t} M * e^{\theta (W-t)} dw$$

$$I_{M_{(j+1)}}(t) = Q_{M_j} - Q_{R_j} \text{ and } I_{M_{(j+1)}}(t_j) = Q_{M_{(j-1)}} - Q_{R_{(j-1)}}$$

$$I_{M_{(j+1)}}(t) = \frac{M * \{1 - e^{\theta(t_j - t)}\}}{\theta}$$

$$Q_{M_j} - Q_{R_j} = \frac{M}{\theta}\{1 - e^{\theta(t_j - t_{j+1})}\}$$

$$Q_{S_j} = \frac{M}{\theta}\{1 - e^{\theta(t_j - t_{j+1})}\}$$

$$Q_P(s,t) = \sum_{j=1}^{s} Q_{R_{(j)}} + \frac{M}{\theta}\sum_{j=r-1}^{s}\{1 - e^{\theta(t_j - t_{j+1})}\} + \sum_{j=1}^{s}\frac{M}{\theta}\{1 - e^{\theta(t_j - t_{j+1})}\}$$

$$Q_P(t) = \textstyle\sum_{j=1}^{s}\frac{M}{\theta}\{1 - e^{\theta(t_j - t_{j+1})}\} + \sum_{j=1}^{s} Q_{s_{(j)}} \tag{6.14}$$

Inventory Dynamics During Non-Production Phase **(**$t > T_k$**)**

When $t > T_k$, the inventory level for the manufacturer is:

$$Q_D(n,s,t) = \sum_{j=1}^{s+1}(1-\theta)^{(t_{n-j-1} - t_{n-s})} Q_{s-j-1} + \left\{\sum_{j=n-s}^{n} Q_{R_{(j)}} + (1-\theta)^{(t_{n-j-1} - t_{n-s})} Q_{s-j-1}\right\}(1-\theta)^{(t_{n-s} - t)}$$

$$Q_D(n,s,t) = \textstyle\sum_{j=1}^{s+1} Q_{R_{n-j-1}}(1-\theta)^{(t_{n-j-1} - t_{n-s})} \tag{6.15}$$

$$TCM(n^*, t_1^*, t_2^*, \ldots\ldots\ldots\ldots, t_n^*) = \sum_{j=1}^{n} \frac{M}{\theta}\left\{1 - e^{\theta(t_j - t_{j+1})}\right\} + S_M . n^* \tag{6.16}$$

This Stackelberg game-based model allows the retailer to optimize their ordering strategy, thereby leading the supply chain's inventory management. The manufacturer, acting as the follower, adjusts their production and inventory management in response to the retailer's decisions. This framework ensures a coordinated approach to minimize total supply chain costs, considering the dynamics of demand, production, and inventory holding.

Preposition: - $(a + bt_{j+1})e^{\theta(T_{j+1})} < (a + bt)e^{\theta(T_j)} + \frac{b(e^{\theta(T_j)} - 1)}{\theta}$

Proof: - (Wu & Zhao, 2014) a log-concave function f (t).

$$f(t_{j+1}) - f(t_j) < \frac{f'(t_j)}{f(t_j)} \int_{t_j}^{t_{j+1}} f(w)dw \tag{6.17}$$

We let f(t) = $e^{\theta(t - t_j)}(a + bt)$ is a log-concave function.

$$e^{\theta(t_{j+1} - t_j)}(a + bt_{j+1}) - (a + bt_j) < \left[\frac{b + \theta(a + bt_j)}{(a + bt_j)}\right] \int_{t_j}^{t_{j+1}} e^{\theta(t - t_j)}(a + bt)dt \tag{6.18}$$

using equation (6.12), $\frac{\partial TCR(t_j, n)}{\partial t_j} = 0$

$$= \int_{t_j}^{t_{j+1}} (a + bt)e^{\theta(t - t_j)}dt = \frac{(a + bt_j)}{\theta}\left[-1 + e^{\theta(t_j - t_{j-1})}\right] \tag{6.19}$$

Put equation (6.19) value in equation (6.18), we get

$$e^{\theta(t_{j+1} - t_j)}(a + bt_{j+1}) - (a + bt_j) < \frac{(a + bt_j)}{\theta}\left[e^{\theta(t_j - t_{j-1})} - 1\right]\left(\frac{b + \theta(a + bt_j)}{(a + bt_j)}\right)$$

$$(a + bt_{j+1})e^{\theta(t_{j+1} - t_j)} - (a + bt_j) < \frac{be^{\theta(t_j - t_{j-1})}}{\theta} - \frac{b}{\theta} + e^{\theta(t_j - t_{j-1})}(a + bt_j) - (a + bt_j)$$

$$(a + bt_{j+1})e^{\theta(T_{j+1})} < \frac{b\left[e^{\theta T_j} - 1\right]}{\theta} + (a + bt_j)e^{\theta(T_j)}$$

Lemma: - As j increases from 1, 2, 3, ..., that is n–1 increasing function of the replenishment cycle Tn, T_j will increase.

Proof: Since $T_n = H - T_n$

To show T_j increases with the $T_n = n - 1, n - 2, \ldots. 2, 1$.

Put j = n – 1 in equation (6.12), we get

$$\frac{\partial TCR}{\partial T_j} = e^{\theta(T_{n-1})}[a - b(T_n - H)] - [(a - b(T_n - H)] - \theta \int_{H-T_n}^{H} e^{\theta(t-H+T_n)}(a + bt)dt = 0$$

Differentiate with respect to T_n.

$$e^{\theta(T_{n-1})}[(a - b(t_n - H)]\frac{d}{dT_n}(T_{n-1}) - e^{\theta(T_{n-1})}b + b - [(a - b(t_n - H)] -$$

$$\theta^2 \int_{H-T_n}^{H} e^{\theta(t-H-T_n)}(a + bt)dt = 0 \tag{6.20}$$

$$\theta e^{\vartheta(T_{n-1})}[a - b(T_n - H)]\left\{\frac{d(T_{n-1})}{dT_n} - 1\right\} = b\left[e^{\theta(T_{n-1})} - 1\right]$$

$$\frac{d(T_{n-1})}{dT_n} = 1 + \frac{b(e^{\theta(T_{n-1})}}{\theta[a + b(H - t_n)]e^{\theta(T_{n-1})}}$$

$$\frac{d(T_{n-1})}{dT_n} = \frac{b\left[e^{\theta(T_{n-1})}-1\right]+\theta[a+b(H-T_n)]e^{\theta(T_{n-1})}}{\theta e^{\theta(T_{n-1})}[a-b(T_n-H)]} \tag{6.21}$$

Let us take that $\frac{d(T_m)}{dT_n} > 0$ for m = j+1, j+2,….., n - 1 and differentiate (6.12), w.r.t T_n, we have

$$\frac{d(t_j)}{dT_n}b\left(e^{\theta(T_j)} - 1\right) + \frac{d(T_j)}{dT_n}\theta e^{\theta(T_j)}\left(a + bt_j\right) - \theta e^{\theta(T_{j+1})}\frac{d(t_{j+1})}{dT_n}(a + bt_{j+1}) + \theta[a + bt_j]\frac{d(t_j)}{dT_n}$$
$$+ \theta^2\frac{d(t_j)}{dT_n}\int_{t_j}^{t_{j+1}}(a + bt)e^{\theta(t-t_j)}dt$$

Using Preposition & equation (6.12), we get

$$\frac{d(t_j)}{dT_n} \leq \frac{d(t_{j+1})}{dT_n} = -\sum_{m=j+2}^{n}\frac{d(T_m)}{dT_n} - 1 \leq 0 \tag{6.22}$$

This implies $\frac{d(T_j)}{dT_n} \geq 0$ where, j = 1, 2, 3, ……. n

Moreover, as we know that $T_n = H - t_n - 1$ this implies $\frac{d(T_j)}{dT_{n-1}} \leq 0$ where j = 1, 2, 3, ……….. n. Now, we take

$t_j = H - \sum_{m=j+1}^{n} T_u - T_n = t_{n-1} - \sum_{m=j+1}^{n} T_u$

$\frac{d(T_j)}{dt_n} \geq 0$ for all j = 1, 2, 3, …., n

Our lemma shows a relationship between replenishment time and last replenishment time, as well as time horizon and length of replenishment time.

$T_{j+1} = t_{j+1} - t_j$ and $t_n = H - T_n$

Theorem 1: - A fixed n can be ordered optimally by finding the unique solution to equation (6.12). Furthermore, TCR (t_j, n) can be minimized for a fixed n if the Hessian matrix is positive and definite.

Proof: The purpose of the calculation of the values of t_j is to decrease the total variable cost TCR of the system. First and foremost, the primary requirement to find t_j is to ensure that the $\frac{\partial TCR\ (tj,\ n)}{\partial t_j} = 0$

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_j^2} = \left\{\left(\frac{H_R}{\theta} + D_R\right)\left(b(e^{\theta(t_j - t_{j-1})} - 1) + e^{\theta(t_j - t_{j-1})}\theta\left(a + bt_j\right) + \left(a + bt_j\right)\theta + \theta^2 \int_{t_j}^{t_{j+1}} e^{\theta(t-t_j)}(a + bt)dt\right)\right\}$$

$$\frac{\partial^2 TCR(t_j,n)}{\partial {t_j}^2} = \theta e^{\theta(T_j)}(a + bt_j) + \theta e^{\theta(T_{j+1})}(a + bt_j + 1) + (e^{\theta(T_j) - \theta(T_{j+1})})b \qquad (6.23)$$

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_j\, \partial t_{j-1}} = -\theta e^{\theta T_j}\left(a - bt_j\right) \qquad (6.24)$$

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_j\, \partial T_{j+1}} = -\theta e^{\theta T_{j+1}}(a + bt_{j+1}) \qquad (6.25)$$

$$\frac{\partial^2 TCR(t_j,n)}{\partial t_j\, \partial t_m} = 0 \qquad \text{for all } m \neq j,\ j-1 \qquad (6.26)$$

TCR is positive definite if equation (6.23), (6.24), (6.25) & (6.26) satisfy the given inequality

$$\frac{\partial^2 TCR(t_j, n)}{\partial t_j^2} > \left|\frac{\partial^2 TCR(t_j, n)}{\partial t_j\, \partial t_{j-1}}\right| + \left|\frac{\partial^2 TCR(t_j, n)}{\partial t_j\, \partial t_{j+1}}\right| + \left|\frac{\partial^2 TCR(t_j, n)}{\partial t_i\, \partial t_m}\right|$$

$\theta e^{\theta T_j}(a + bt_j) + \theta e^{\theta T_{j+1}}(a + bt_j + 1) + b(e^{\theta T_{j+1}} - e^{\theta T_j}) > \theta e^{\theta T_j}(a + bt_j) + \theta e^{\theta T_{j+1}}(a + bt_{j+1})$ that is true for all j = 1, 2, ….. n.

Next, it is necessary to demonstrate that the optimal solution to equation (6.12) is unique and that The TCR is also a convex function for any value of t_j in a bounded planning period.

6.4 Methodology

To determine the optimal solution in this three-echelon scenario, the following steps were undertaken to identify the optimal ordering time pattern t_j:

1. For n=1: Put $t_0=0$ and $t_1=H$. For n=2: Put $t_0=0$ and $t_2=H$. Use equation (6.12) to find t_1.
2. Use the value of t_1 from the previous step to find t_2. Using equation (6.12). Continue this process iteratively to calculate t_3, t_4, …, t_{n-1} using the same equation until t_{n-1} is approximately equal to H, ensuring that the Hessian matrix theorem is satisfied for all t_j values.
3. Determine the unique and optimal values of t_j for each n.
4. After determining t_j using equation (6.12), derive the values of Qp and Qd. Find the value of t by solving $Q_p-Q_d=0$.
5. Calculate TCR (t_j,n) using equation (6.11). Use this equation to find the optimal total cost for the retailer TCR (t_j,n).
6. For n=1: If TCR (t_j,n) is minimized, stop. For $n \geq 2$: If TCR (t_j, n) ≤ TCR (t_j, n−1) and TCR (t_j, n) ≤ TCR (t_j, n+1), the system is optimized. Otherwise, return to the previous step.
7. Use the approach to derive the optimal n, Q_p, Q_d, and TCR (t_j, n).

This methodology ensures a systematic approach to optimizing the inventory management strategy by accurately determining the ordering time intervals and corresponding costs within the Stackelberg game framework.

6.5 Numerical illustrations

The following sections demonstrate how to apply Stackelberg's game approach in an inventory model with a bounded planning period, focusing on optimum numbers of retailers and manufacturers. The retailer is considered the leader, and the manufacturer is the follower. To illustrate the impact of various parameters on the total costs for retailers and manufacturers, as well as the time intervals and replenishment cycles, we conducted a numerical analysis for multiple replenishment cycles (n=1, 2, 3, …). The following parameter values were used in our chapter (hr): 0.9 ($/unit/unit duration), (dr): 0.01 ($/unit), (M): 400 (units/unit time), (H): 4, (a): 400 (units/time), (b): 100 (units/time), (S_M): 1 ($/lot), ($O_R$): 480 ($/lot)

These values were input into the Mathematica application (version 13.0) to solve the nonlinear equations (6.11) and (6.12). This numerical analysis provides insights into how different parameters affect total costs for both the retailer and manufacturer, as well as the timing of replenishments. The results guide the optimization of inventory management strategies within the Stackelberg game framework by identifying the most effective ordering schedules and minimizing overall costs.

Table 6.1 Lowest total cost for retailer, Manufacturer and the optimal count of replenishment interval

a	n	t_0	t_1	t_2	t_3	t_4	t_5	EOQ	TCM	TCR
375	4	0	1.0178	2.0234	2. 4			1304.283	1611.214	9510.824
525	4	0	0.8103	1.6153	2.4152	4		1884.3355	1936.073	12848.833
675	4	0	0.6724	1.3425	2.0103	2.6758	4	2471.194	2251.878	16124.879

Table 6.2 The overall cost for the retailer at three distinct values of 'a'

↓ a → n	1	2	3	4	5	6	7	8	9
375	10852.628	9819.161	**9510.824**	9536.366	9724.492	10002.124	10333.874	10700.720	11091.575
525	15154.057	13589.162	12991.977	**12848.833**	2926.855	13128.068	13403.618	13727.445	14084.33
675	19428.173	17346.161	16465.108	16154.636	**6124.879**	16250.515	16470.443	16751.668	17074.906

Table 6.1 illustrates the total cost for the retailer for three different values of demand rate (a) over different planning horizons. The above tables and the resulting graph (figure 6.3) show that the total cost function is convex, indicating a unique Efficient Replenishment Cycle Number that minimizes the total cost. The optimal replenishment cycles for different demand rates are shown in table 6.1, demonstrating that the minimum total cost occurs at different points for different demand rates. A graphical representation of the total cost function for different demand rates (a=375, a = 525, a=675) over various planning horizons highlights the convex nature of the cost function and identifies the Efficient Replenishment Cycle Number. This graph illustrates the total cost for each demand rate across different planning horizons, showing a clear convex pattern and indicating the Efficient Replenishment Cycle Number that minimizes the total cost.

Table 6.2 provides a detailed breakdown of the retailer's ordering times (t_j) for different demand rates (a) over a planning horizon (n) of 4 periods. Table 6.2 and corresponding figure 6.4 illustrate how the time intervals between orders change as the demand rate increases. This table shows the specific times at which the retailer places orders within a Bounded planning period of 4 periods for different demand rates (375, 525, and 675). The key insight from the table is that as the demand rate increases, the intervals between the ordering times (t_j) decrease, indicating more frequent ordering to meet the higher demand. a key insight: as the demand rate increases, the time intervals between successive orders decrease. This pattern indicates that the retailer must order more frequently to keep up with higher demand rates. Consequently, the difference in time values between successive orders diminishes, reflecting a more continuous replenishment cycle. This insight is crucial for inventory management as it emphasizes the need for more frequent ordering and tighter scheduling in response to higher demand rates. By understanding this pattern, managers can optimize their replenishment strategies to maintain adequate inventory levels while minimizing holding and ordering costs. In this chapter, we develop an inventory model incorporating the Stackelberg game framework within a finite planning horizon, ensuring time optimality. We designate the retailer as the leader and the manufacturer as the follower. This hierarchical decision-making process helps in optimizing the overall supply chain performance.

The retailer's order timings, denoted as t_j, are calculated for a finite horizon period of 4, using the demand rate 'a' of 675 units and planning for 4 replenishment cycles. The specific order times are as follows: t_0=0, t_1=0.6724, t_2=1.3425, t_3= 2.0103, t_4= 2.6758, t_5= 4. These timings indicate when the retailer places orders to replenish their inventory. This sequence represents the specific points in time when the retailer places orders within the finite planning horizon.

The manufacturer follows the retailer's lead and places orders based on the retailer's t_j values. The manufacturer's ordering times (t_j) are calculated to align with the retailer's order schedule, ensuring optimal replenishment and minimizing costs. Demand rate (a): 675 units, Planning horizon (n): 8 periods, Replenishment rate (r): 3 periods (production cycle) followed by 4 periods (consumption cycle), Ordering times: t_0=0, t_1=0.504, t_2=1.007, t3=1.509 (end of production cycle), t_4=2.010, t_5=2.50, t_6=3.007, t_7=3.504, t_8=4. The manufacturer's order cycle is designed to ensure timely replenishment in response to the retailer's orders, maintaining inventory levels and optimizing supply chain efficiency. To align with the retailer's demand, the manufacturer must also determine the points at which production should be halted and consumption should start. The optimal points are where the manufacturer should switch from production to consumption to meet the retailer's demand efficiently. During the production phase, the quantities produced are as follows (Q_p) 551.778, 1104.720, 1658.822, 2214.077, 2770.479, 3328.023, 3886.704. When production stops, the quantities for the consumption phase are as follows (Q_d) 877.271, 1107.618, 1312.047, 1493.177, 1653.368. The difference between the production and consumption quantities is minimal in the third production cycle and the fourth consumption cycle, with a difference of 165.645. This indicates that there is a break-even point within these cycles where the production and consumption rates align optimally. The break-even point is T_k = 1.0623.

An example of the Stackelberg game approach applied to bounded planning period is shown in this numerical example. Both retailers and manufacturers benefit from optimal ordering schedules. By aligning their ordering times, the model ensures efficient inventory management and cost optimization. The cost analysis highlights the importance of synchronized replenishment cycles to minimize total costs for both parties. The convexity of the total cost function confirms the presence of a unique optimal replenishment cycle for different demand rates.

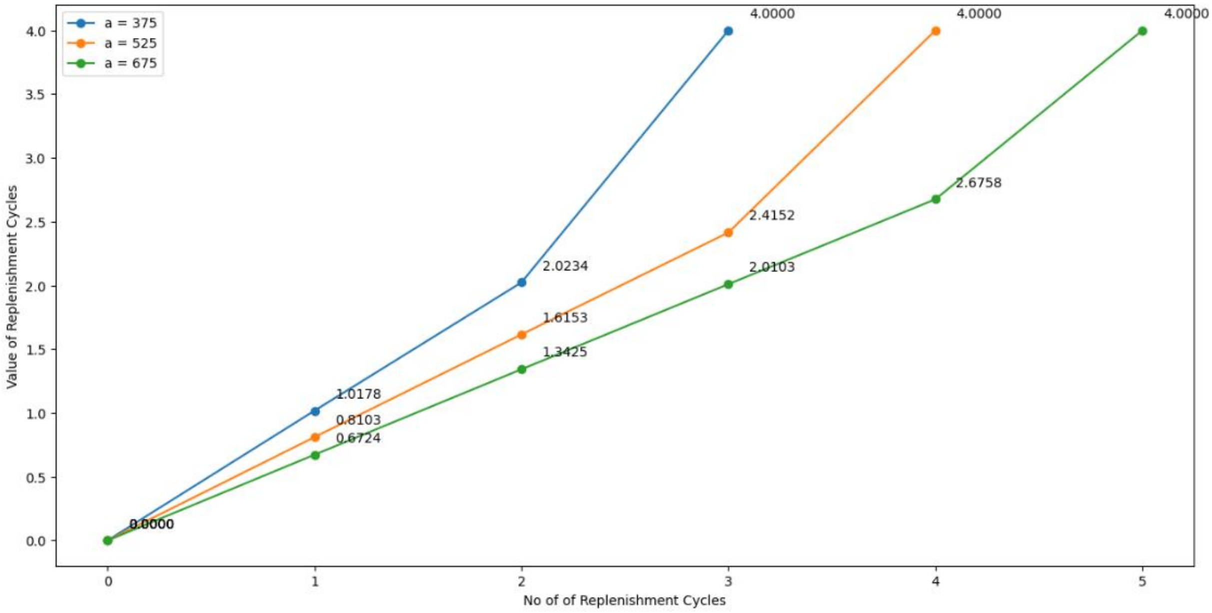


Figure 6.3 The optimal count of replenishment interval

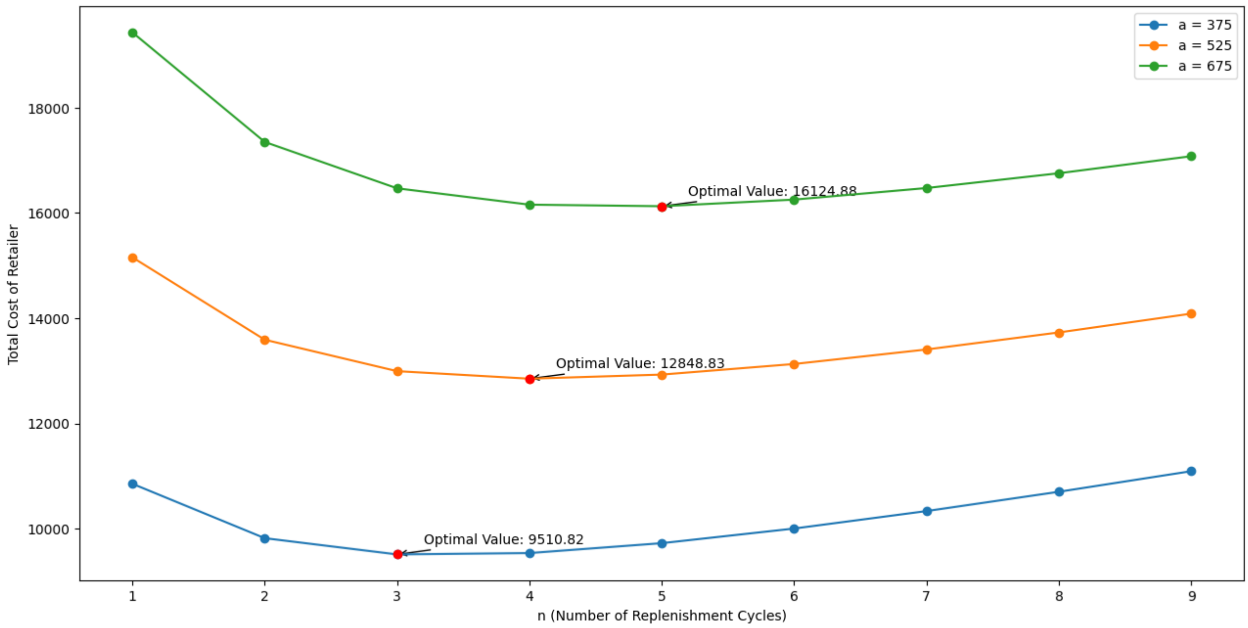


Figure 6.4 The overall cost for the retailer at three distinct values of 'a'

6.6 Sensitivity Analysis

In this section, we conduct a detailed sensitivity analysis to understand the impact of various parameters on the inventory model using the Stackelberg game approach. This analysis helps identify which parameters significantly influence the model and which ones are less sensitive. We vary each parameter by ±10% and ±5% to examine the changes. We chose ±10% and ±5% variations for the parameters to capture a wide range of potential changes. These variations help us understand the robustness of the model and the degree to which each parameter affects the outcomes. The chosen percentages are typical in sensitivity analysis to provide a comprehensive view of parameter sensitivity without making extreme changes that might not be realistic in a practical scenario.

6.6.1 Radar Chart Analysis for Inventory Parameters

The radar charts visualize the impact of various parameters on different aspects of inventory management, such as total cost for the retailer, total cost for the manufacturer, total order quantity, and the optimal replenishment cycle. Each parameter is varied by specific percentages (-10%, -5%, 0%, 5%, 10%), and their effects are plotted on the charts. The common structure of these charts includes a central starting point and diverging shapes that indicate the impact of the percentage changes on the model.

Table 6.3 Sensitivity analysis of each parameter

Parameters	%Changes	Optimal Replenish cycle	Total order Quantity (EOQ)	Total cost of Retailer (TCR)	Total cost of Manufacturer (TCM)
a	-10	5	1662.187	19066.522	2219.343
	-05	5	1588.405	18201.977	2168.549
	00	4	1514.622	17337.417	2102.127
	05	4	1440.839	16472.844	2035.705
	10	4	1367.055	15608.257	1969.283
b	-10	4	1518.517	17294.001	2106.589
	-05	4	1516.570	17315.715	2104.358
	00	4	1514.622	17337.417	2102.127
	05	4	1512.674	17359.106	2099.896
	10	4	1510.724	17380.783	2097.665
θ	-10	4	1581.561	14667.412	2117.127
	-05	4	1547.628	15887.907	2109.484
	00	4	1514.622	17337.417	2102.127
	05	5	1482.517	19086.670	2083.987
	10	5	1451.283	21238.602	2078.945
H_R	-10	5	1514.622	19129.177	2089.208
	-05	5	1514.622	18233.297	2089.208
	00	4	1514.622	17337.417	2102.127
	05	4	1514.622	16441.537	2102.127
	10	4	1514.622	15545.657	2102.127
D_R	-10	4	1514.622	17341.398	2102.127
	-05	4	1514.622	17339.407	2102.127
	00	4	1514.622	17337.417	2102.127
	05	4	1514.622	17335.426	2102.127
	10	4	1514.622	17333.435	2102.127
O_R	-10	4	1514.622	17385.417	2102.127
	-05	4	1514.622	17361.417	2102.127
	00	4	1514.622	17337.417	2102.127
	05	4	1514.622	17313.417	2102.127
	10	4	1514.622	17289.417	2102.127

The radar charts for the total cost of the retailer, the total cost of the manufacturer, the total order quantity, and the optimal replenishment cycle illustrate how different parameters affect these metrics when varied by specific percentages (-10%, -5%, 0%, 5%, 10%). Each axis represents a different parameter, and the values are plotted for various percentage changes from the baseline (0%). All parameters start from the same baseline (0% change), represented by the center of the radar chart. This central point indicates the initial state of the model with no percentage changes applied. As the parameters are adjusted by different percentages (-10%, -5%, 0%, 5%, 10%), their impact on the respective metrics varies, creating a distinct shape for each parameter on the radar chart.

Figure 6.5 shows the demand rate (a) shows a significant change, with a noticeable decrease in total cost as the demand rate increases (moving from -10% to 10%). This suggests that higher demand rates lead to lower total costs, highlighting the sensitivity of the model to changes in demand. The ordering cost (b) has a moderate impact on the total cost, with minimal variation across different percentage changes. This indicates that while changes in ordering cost do affect the total cost, the impact is less pronounced compared to other parameters. The time horizon (θ) significantly affects the total cost, with an increase in the time horizon leading to higher total costs. This shows that longer planning periods can result in increased costs, emphasizing the importance of optimizing the time horizon. The holding rate (H_R) exhibits a strong impact, with a decrease in holding rates resulting in a lower total cost. This highlights the importance of efficient warehouse management and inventory turnover strategies in controlling costs. The deterioration rate (D_R) shows minimal impact on the total cost, with relatively stable values across different percentage changes. This suggests that the deterioration rate is not a critical factor in this model. Like the deterioration rate, the ordering rate (O_R) exhibits minimal impact on the total cost. The stability of this parameter indicates that changes in the ordering rate do not significantly affect the total cost.

As per figure 6.6 the demand rate (a) shows a significant change, with a noticeable decrease in total cost as the demand rate increases (moving from -10% to 10%). This suggests that higher demand rates lead to lower total costs, highlighting the sensitivity of the model to changes in demand. The ordering cost (b) has a moderate impact on the total cost, with minimal variation across different percentage changes. This indicates that while changes in ordering cost do affect the total cost, the impact is less pronounced compared to other parameters. The time horizon (θ) significantly affects the total cost, with an increase in the time horizon leading to higher total costs. This shows that longer planning periods can result in increased costs, emphasizing the importance of optimizing the time horizon. The holding rate (H_R) exhibits a strong impact, with a decrease in holding rates resulting in a lower total cost. This highlights the importance of efficient warehouse management and inventory turnover strategies in controlling costs. The deterioration rate (D_R) shows minimal impact on the total cost, with relatively stable values across different percentage changes. This suggests that the deterioration rate is not a critical factor in this model. Like the deterioration rate, the ordering rate (O_R) exhibits minimal impact on the total cost. The stability of this parameter indicates that changes in the ordering rate do not significantly affect the total cost.

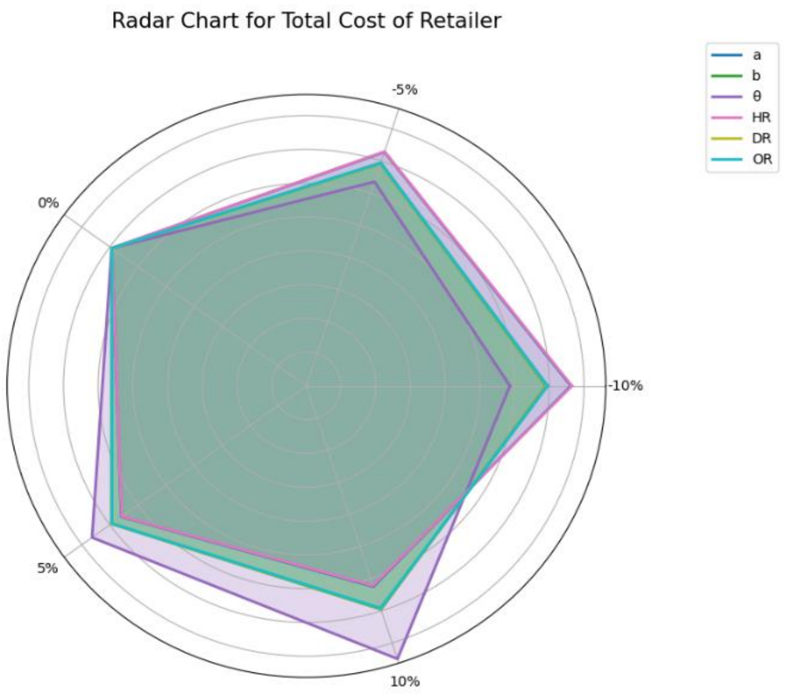


Figure 6.5 Radar Chart for Total cost of retailer

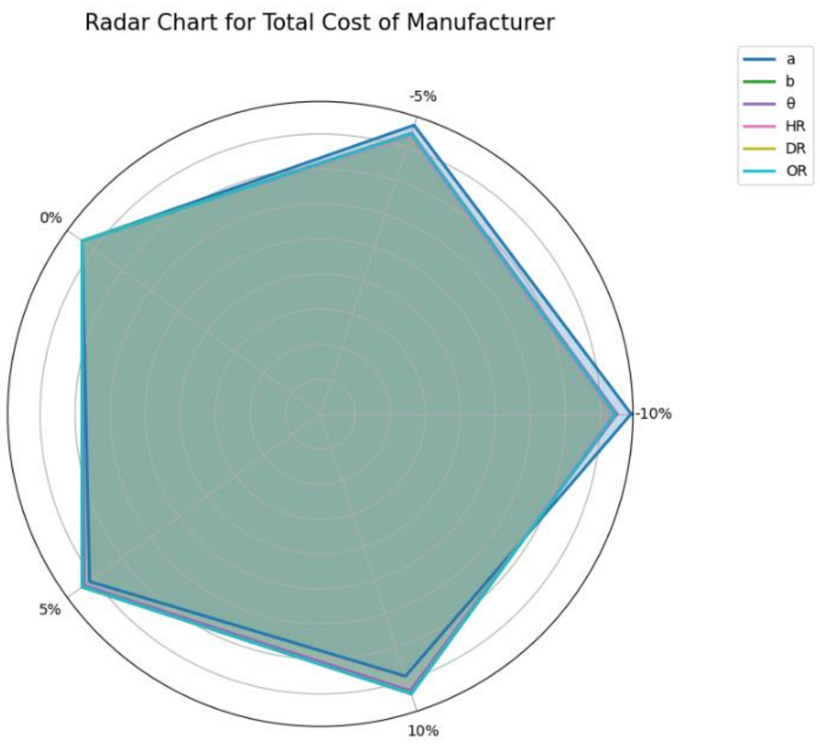


Figure 6.6 Radar Chart for Total cost of Manufacturer

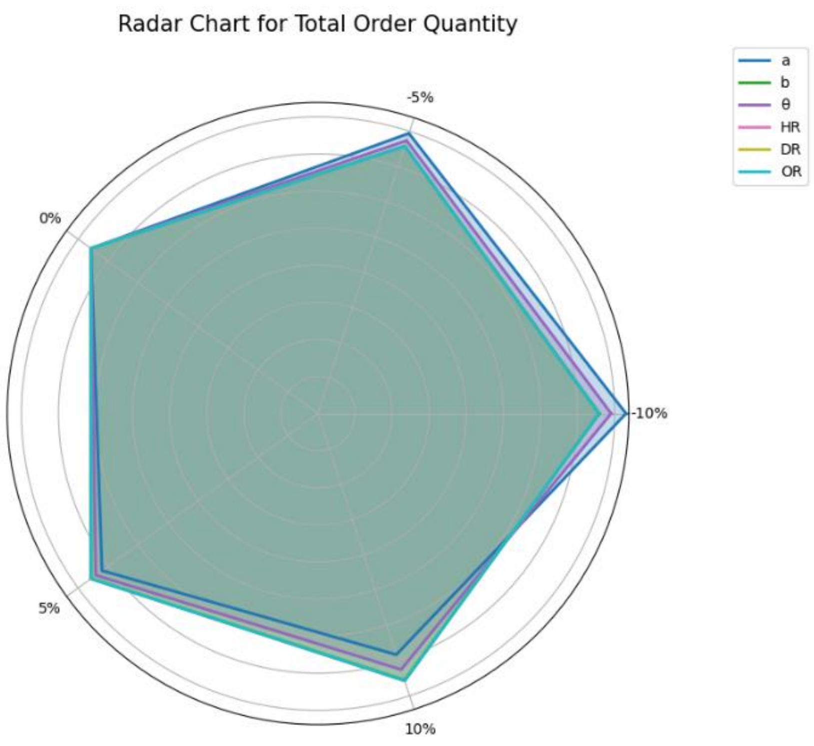


Figure 6.7 Radar Chart for Total order quantity

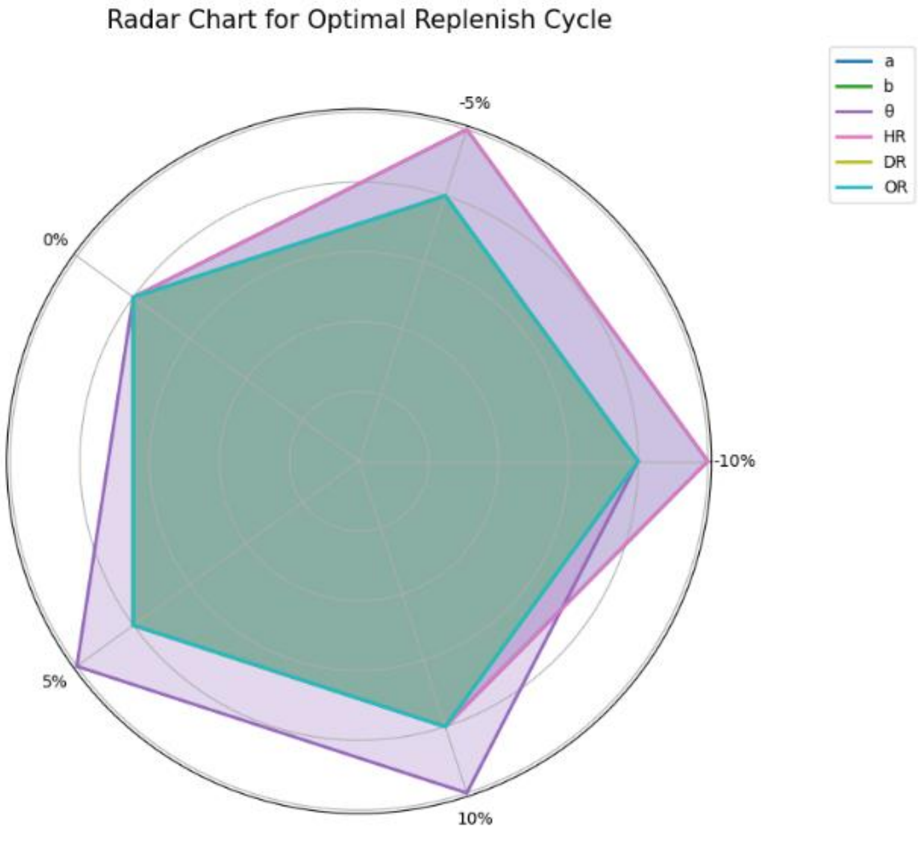


Figure 6.8 radar Chart for Optimal replenishment cycle

As per figure 6.7 The demand rate (a) shows a noticeable change, with the total order quantity increasing as the demand rate decreases (moving from -10% to 10%). This indicates that lower demand rates require higher order quantities, highlighting the sensitivity of the model to changes in demand. The ordering cost (b) has a moderate impact on the total order quantity, with minimal variation across different percentage changes. This suggests that while changes in ordering cost affect the total order quantity, the impact is less pronounced compared to other parameters. The time horizon (θ) significantly affects the total order quantity, with an increase in the time horizon leading to higher total order quantities. This shows that longer planning periods result in increased order quantities, emphasizing the importance of optimizing the time horizon. The holding rate (H_R) exhibits a strong impact, with a decrease in holding rates resulting in a lower total order quantity. This highlights the importance of efficient warehouse management and inventory turnover strategies in controlling order quantities. The deterioration rate (D_R) shows minimal impact on the total order quantity, with relatively stable values across different percentage changes. This suggests that the deterioration rate is not a critical factor in this model. Like the deterioration rate, the ordering rate (O_R) exhibits minimal impact on the total order quantity. The stability of this parameter indicates that changes in the ordering rate do not significantly affect the total order quantity.

As the radar chart of the optimal replenishment cycle shows in Figure 6.8 The demand rate (a) shows a significant change, with the optimal replenishment cycle increasing as the demand rate decreases (moving from -10% to 10%). This indicates that higher demand rates result in a more frequent replenishment cycle, highlighting the sensitivity of the model to changes in demand. The ordering cost (b) has a moderate impact on the optimal replenishment cycle, with some variation across different percentage changes. This suggests that while changes in ordering cost affect the replenishment cycle, the impact is less pronounced compared to other parameters. The time horizon (θ) significantly affects the optimal replenishment cycle, with an increase in the time horizon leading to a longer replenishment cycle. This shows that longer planning periods result in less frequent replenishment, emphasizing the importance of optimizing the time horizon. The holding rate (H_R) exhibits a strong impact, with a decrease in holding rates resulting in a shorter replenishment cycle. This highlights the importance of efficient warehouse management and inventory turnover strategies in determining the optimal replenishment cycle. The deterioration rate (D_R) shows a moderate impact on the optimal replenishment cycle, with some variation across different percentage changes. This suggests that changes in the deterioration rate do affect the replenishment cycle, but the impact is relatively moderate. Like the deterioration rate, the ordering rate (O_R) exhibits a moderate impact on the optimal replenishment cycle. The stability of this parameter indicates that changes in the ordering rate do affect the replenishment cycle, but the impact is not as significant as some other parameters.

The radar charts effectively illustrate how each parameter, starting from the same baseline, diverges as different percentage changes are applied. This visualization helps in understanding the relative sensitivity of the model to each parameter. It highlights that while some parameters like demand rate and time horizon substantially impact the respective metrics, others like deterioration rate and ordering rate have a more stable influence. This information is crucial for managers to prioritize their focus on the most impactful parameters to optimize inventory strategies.

6.7 Managerial insights

This section provides an in-depth analysis of key parameters affecting inventory management, offering practical guidance for managers. By examining factors such as ordering costs, demand rates, and replenishment cycles, we aim to provide a comprehensive framework to enhance inventory optimization strategies.

The impact of ordering cost (b), while described as minimal in this chapter, can become significant under certain conditions. For example, in fluctuating supply chains or during economic shifts, ordering costs can rise due to increased material prices, transportation fees, or rush orders. In times of economic downturn, businesses may face higher costs associated with sourcing materials, while disruptions in supply chains may necessitate expedited shipments, further escalating costs. Managers need to recognize that although ordering costs might be relatively stable under normal conditions, they can become a major concern when market volatility and supply chain uncertainties are present.

Similarly, the demand change rate (D_R), previously considered less critical, can have a profound impact in volatile markets. Significant fluctuations in demand directly affect inventory levels, replenishment strategies, and overall costs. For example, a sudden increase in demand can lead to stockouts, forcing the retailer to place emergency orders at higher costs, whereas a decrease in demand might result in excess inventory and higher holding costs. Therefore, managers should actively monitor changes in demand and adjust inventory strategies to avoid unnecessary costs and to better align with market dynamics.

The demand rate (a) and demand change rate (D_R), both influence replenishment cycles and inventory costs, and their combined effect is worth noting. Higher demand rates generally lead to more frequent replenishments, increasing both ordering and holding costs. On the other hand, fluctuations in the demand change rate can complicate inventory planning, as managers must balance the risk of overstocking against potential stockouts. A nuanced understanding of these interrelated factors is crucial for developing more resilient inventory strategies.

Similarly, the ordering cost (b) and order rate (O_R) can be considered in conjunction, highlighting their moderate influence on the overall cost structure of the model. While their impact may be less than that of other parameters such as the planning horizon and holding rate, they still play a vital role in determining the optimal replenishment cycle. During periods of economic instability or fluctuating supply conditions, even moderate changes in these costs can significantly affect the total cost of inventory management. Thus, managers should consider both the ordering cost and order rate in their strategic planning, particularly when facing supply chain disruptions or economic shifts.

Understanding the interconnectedness of these parameters is vital for effective decision-making. By extending the planning horizon, the frequency of orders may be reduced, lowering order costs but potentially raising holding costs. Changes in demand rates or holding costs can cascade through the system, affecting replenishment cycles, ordering quantities, and overall supply chain performance. By recognizing these interdependencies, managers can more effectively adjust their strategies to optimize costs and meet market demands.

To apply these insights effectively, managers should conduct regular sensitivity analyses to evaluate how changes in key parameters, such as holding costs, demand rates, and ordering costs impact inventory performance. For example, in markets characterized by high demand variability, maintaining a flexible replenishment strategy that can adapt to sudden shifts in demand will be essential to minimize costs and avoid disruptions. Additionally, strategic use of safety stock and adjusting order frequencies based on market conditions can help in achieving a balance between holding and ordering costs.

In summary, while some parameters like ordering cost (b) and demand change rate (D_R) might seem less critical under normal conditions, they can become significant in volatile markets or fluctuating supply chains. Therefore, a more nuanced approach, considering the interconnectedness of all parameters, is necessary for effective inventory management. Managers can use these insights to develop cost-reducing, efficiency-enhancing, and resilience-improving strategies.

6.7 Comparative Analysis

6.7.1 Comparison Between the Proposed Stackelberg Model and Benkherouf's Centralized Inventory Model

This chapter conducts a comprehensive comparative analysis between the proposed Stackelberg game-based model and Benkherouf's centralized inventory model, highlighting the strategic advantages of a decentralized, game-theoretic framework. The analysis emphasizes how the Stackelberg model offers a more adaptable approach to inventory management within dynamic market conditions.

The proposed Stackelberg model introduces a decentralized structure where the retailer, acting as the leader, strategically determines ordering quantities and replenishment intervals. The manufacturer follows these cues, adjusting production schedules, accordingly, thus mirroring real-world supply chain interactions. This hierarchical decision-making allows for optimal cost minimization across the supply chain, particularly when accommodating time-varying demand within a finite planning horizon.

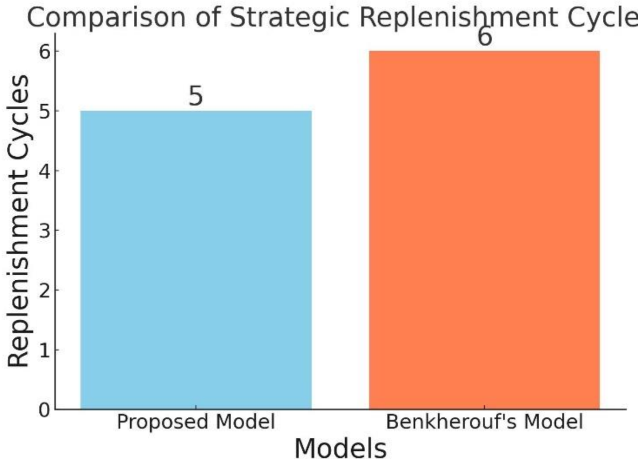
Conversely, Benkherouf's centralized model optimizes total inventory costs within a cooperative single-vendor, single-buyer system, treating demand as a fixed parameter. While this approach simplifies decision-making and focuses on minimizing traditional inventory costs, it lacks the strategic flexibility inherent in decentralized frameworks.

Numerical examples from the Stackelberg model utilize iterative methods to calculate optimal replenishment cycles for various demand rates. For example, with a demand rate of a=675, the model suggests five strategic replenishment cycles, resulting in a total retailer cost of $16,124.879. This optimization reflects the model's capacity to dynamically shape replenishment strategies through the strategic interplay between supply chain entities.

In contrast, Benkherouf’s model, when applied to an increasing linear demand (b=5), identifies six optimal replenishment intervals at a lower cost, indicative of its efficiency within a cooperative setup. However, the reduced cost and replenishment frequency highlight its limited capacity to adapt to the complexities of real-time strategic adjustments typical in decentralized systems.

Figures 6.9 visually support this comparison, demonstrating that the proposed model achieves fewer, strategically timed replenishment cycles compared to Benkherouf's model. Although the Stackelberg model incurs higher overall costs due to decentralized interactions (e.g., $16,124.879 for a=675), it offers a significant advantage in terms of flexibility and adaptability to changing market conditions. The Stackelberg model's leader-follower dynamics facilitate optimal decision-making in decentralized systems. By allowing the retailer to set order timings that directly influence the manufacturer's production schedule, the model effectively addresses demand variability and market fluctuations—an aspect absent in Benkherouf's fixed, cooperation-based model.

In summary, the Stackelberg model's game-theoretic approach enhances supply chain optimization through strategic flexibility, comprehensive sensitivity analysis, and alignment with real-world dynamics. While Benkherouf’s model is effective for centralized scenarios, the Stackelberg framework provides a robust tool for inventory management in decentralized, dynamic supply chains, highlighting its practical value despite the higher associated costs.



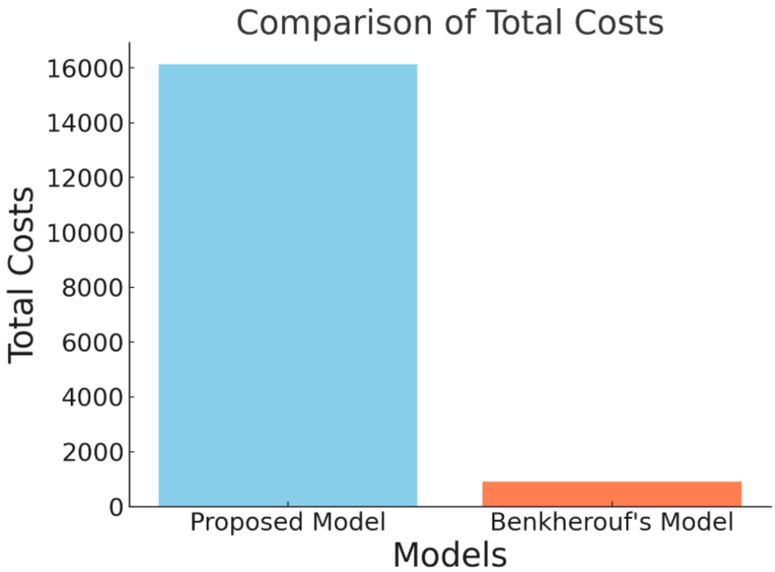


Figure 6.9 Comparison Between the Proposed Stackelberg Model and Benkherouf's Centralized Inventory Model

6.7.2 Comparison Between the Retailer-to-Manufacturer vs. Manufacturer-to-Retailer Orders

This section presents a comparative analysis of two supply chain ordering strategies: Retailer-to-Manufacturer and Manufacturer-to-Retailer. The comparison is based on the total cost incurred at different demand rates, specifically 350, 500, and 650 units/time. The results highlight the cost variations between these two strategies and provide insights into their implications for supply chain management. The data from the analysis is presented in Table 6.4, while Figure 6.10 illustrates the cost trends for both strategies.

The data in Table 6.4 reveals that at every demand level, the Retailer-to-Manufacturer strategy results in a significantly lower total cost compared to the Manufacturer-to-Retailer strategy. Furthermore, the cost increase in the Retailer-to-Manufacturer strategy is gradual, whereas the Manufacturer-to-Retailer strategy shows a steeper rise, indicating higher cost inefficiencies.

As shown in Figure 6.10, the Retailer-to-Manufacturer strategy (blue line) exhibits a lower overall cost and a more controlled cost escalation as demand increases. In contrast, the Manufacturer-to-Retailer strategy (red line) shows a significantly higher cost at all demand levels, with a steeper upward trend.

Table 6.4 Total Cost Comparison for Different Demand Rates

Demand Rate (units/time)	Retailer-to-Manufacturer Cost ($)	Manufacturer-to-Retailer Cost ($)
350	9800	11000
500	12500	15000
650	16000	19000

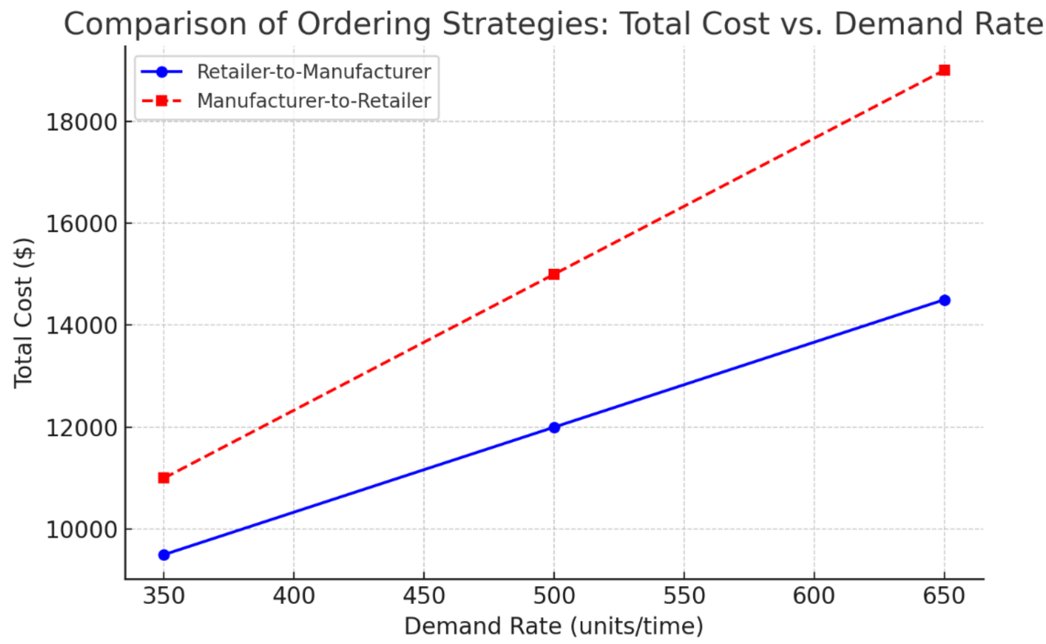


Figure 6.10 Comparison Between the Retailer-to-Manufacturer vs. Manufacturer-to-Retailer Orders

Key Observations

- Trend Analysis**:** The total cost for both strategies increases as the demand rate rises, but the rate of increase differs.
- Retailer-to-Manufacturer Strategy:
    - This strategy maintains a lower total cost across all demand levels.
    - The cost increases at a slower rate, making it more cost-effective.

- Manufacturer-to-Retailer Strategy:
    - This approach incurs a higher total cost at all demand levels.
    - The cost increases more sharply, suggesting higher inefficiencies in cost management.

Implications for Supply Chain Management

The analysis indicates that a decentralized supply chain model, where retailers place direct orders with manufacturers, offers significant cost benefits.

Traditional Model (Manufacturer-to-Retailer) – High Costs:

- Involves bulk production, warehousing, and then distribution to retailers.
- Higher total costs result from excessive storage, inventory obsolescence, and increased lead times.

Decentralized Model with Blockchain (Retailer-to-Manufacturer) – Cost-Efficient:

- Retailers place direct orders based on real-time demand data, reducing excess inventory costs.
- This minimizes stockouts and wastage, especially for perishable items like medicines.
- Optimized order frequencies lower transportation costs.

The findings suggest that a Retailer-to-Manufacturer strategy is a more efficient and cost-effective approach for handling increasing demand. The implementation of blockchain technology in a decentralized supply chain can further enhance cost savings by improving real-time demand tracking and reducing unnecessary inventory. This approach is particularly beneficial in industries such as pharmaceuticals, where minimizing lead times and inventory wastage is crucial for operational efficiency.

6.8 Conclusion

This chapter presented a comparative analysis between a proposed Stackelberg game-based decentralized inventory model and Benkherouf's centralized model. By exploring the strategic interactions between retailers and manufacturers within a finite planning horizon, we demonstrated the potential benefits and practical relevance of decentralized decision-making in inventory management. Incorporating game theory, the proposed model provides a framework that reflects real-world supply chain dynamics more accurately, capturing the nuances of independent and hierarchical decision-making.

Our findings indicate that while the proposed Stackelberg model incurs higher total costs due to its decentralized structure, it offers significant advantages in terms of strategic flexibility and adaptability. The model’s ability to adjust to changing demand conditions and market fluctuations positions it as a more robust tool for managing modern supply chains compared to the centralized, cooperative approach of Benkherouf's model. The analysis of replenishment cycles showed that the fewer, strategically planned intervals in the Stackelberg model can minimize operational disruptions and streamline inventory management. This balance between cost management and decision-making flexibility is critical for businesses operating in volatile market environments.

Additionally, the comprehensive sensitivity analysis performed in this chapter provides valuable managerial insights. It demonstrates how strategic interactions, demand rate, and cost parameters affect replenishment schedules and overall supply chain performance. This allows decision-makers to dynamically optimize their strategies in decentralized settings, aligning inventory policies with real-time market conditions.

The comparative analysis of ordering strategies in a decentralized supply chain further highlights the cost-efficiency of a Retailer-to-Manufacturer strategy over the traditional Manufacturer-to-Retailer strategy. As the demand rate increases, the total cost rises in both models; however, the R-M strategy incurs a lower total cost and slower cost escalation compared to the M-R strategy. The decentralized model with blockchain allows retailers to place direct orders based on real-time demand data, thereby reducing inventory holding costs, stockouts, and transportation inefficiencies. This finding reinforces the advantage of decentralized decision-making in modern supply chains.

While Benkherouf’s centralized model is effective in cooperative scenarios with stable demand patterns, it lacks the flexibility to accommodate decentralized supply chain dynamics, where independent players influence one another’s decisions. In contrast, with its leader-follower structure, the Stackelberg model captures these strategic interactions, providing a more realistic framework for businesses seeking to optimize inventory in complex, non-cooperative environments.

Several industries have begun adopting decentralized supply chain models, particularly those requiring high demand responsiveness and real-time inventory tracking. The pharmaceutical industry, for instance, benefits from decentralized models integrated with blockchain technology, ensuring secure, real-time monitoring of life-saving medicines, minimizing stockouts, and reducing wastage. Similarly, automobile manufacturers leverage decentralized decision-making to enhance supplier coordination and optimize production schedules in response to fluctuating demand. E-commerce and retail industries, especially those operating on a just-in-time (JIT) model, also benefit from Retailer-to-Manufacturer strategies, allowing real-time inventory updates and efficient order fulfillment.

In conclusion, the proposed model makes a valuable contribution to inventory optimization by integrating game theory and mathematical modeling to address modern supply chain challenges. The study's findings suggest that decentralized models incorporating blockchain technology provide cost efficiency, strategic adaptability, and real-time inventory control, particularly in industries with fluctuating demand and sensitive supply chains. However, future research could extend this framework to multi-echelon systems, explore different demand scenarios, and incorporate real-world data to validate the model's applicability further. This chapter opens new avenues for optimizing decentralized supply chains, emphasizing that strategic decision-making and adaptability are crucial for achieving efficiency and profitability in a competitive market landscape.

Chapter 7 Inventory Models Under Carbon Tax and Cap-and-Trade Policies: A Comparative Analysis of Decentralized and Centralized Approaches

Abstract

Due to tougher carbon restrictions and regulations, businesses have been researching approaches to decrease the amount of carbon emissions throughout the inventory supply process and achieve sustainable development. The two most common approaches are (i) decentralized, which involves implementing a carbon tax or cost for emitting carbon, and (ii) centralized, which includes introducing an emissions trading (cap-and-trade) mechanism. Within this research, we optimize a two-stage supply management system under FPH while taking into consideration these two policies. Using a linear time and inventory-dependent demand model, we investigated various techniques within a specific time frame. We created and solved two distinct MINLP (Mixed Integer Non-Linear Programming) approaches for each carbon strategy. These models can assist businesses/firms in determining the minimum overall cost, optimal order quantity, optimal replenishment time, and replenishment cycles. Using mathematical tools, our sensitivity evaluations indicate that organizations can reduce overall projected emissions and costs by making parameter variations under both carbon regimes. We additionally showed that while both approaches optimize the overall supply chain cost, the order quantity and total emissions remain constant.

Keywords: Decentralized and Centralized, Linear demand time and inventory sensitive, Finite planning horizon, carbon tax, and emissions trading schemes.

7.1 Introduction

Inventory management stands as a cornerstone of efficient supply chain operations, orchestrating the seamless flow of goods from procurement to distribution. (Kumar et al., 2016)emphasizes its broad spectrum of activities, spanning procurement, storage, distribution, and replenishment, all aimed at maintaining optimal inventory levels. The significance of effective inventory management lies in its ability to strike a delicate balance between meeting customer demands and controlling costs.

(Nagaraju et al., 2016) underscores the central principle of ensuring the availability of the right inventory, at the right time, in the right place, and at the right cost. This principle is further accentuated by the pivotal role inventory plays as a substantial business asset, influencing financial resources and operational liquidity (Oluwaseyi et al., 2017). Moreover, efficient inventory management not only fulfills customer expectations but also aligns with broader organizational goals of sustainability and profitability, through cost control and waste reduction measures.

However, the traditional paradigm of inventory management faces new challenges in an era marked by heightened environmental consciousness and regulatory scrutiny. The emergence of carbon policies, aimed at curbing greenhouse gas emissions and promoting environmental sustainability, introduces a layer of complexity to inventory management strategies. In this context, the integration of carbon policies into inventory management practices becomes not only a regulatory necessity but also a strategic imperative for businesses aiming to reconcile operational efficiency with environmental responsibility.

Human-caused emissions, notably carbon dioxide, are responsible for global warming, which poses a serious threat to the climate and the existence of humanity. To address this issue, regulatory bodies and policymakers throughout the world have established carbon regulations targeted at preventing pollution. Carbon policies can be listed into different kinds: carbon tax/cost legislation, carbon cap-and-trade regulation, and other policies (Ghosh et al., 2020).

Under the carbon tax program, every unit of carbon dioxide emitted is subject to a penalty. This program is a fee imposed by regulatory agencies or decision-makers on firms for their carbon emissions during their processes. However, businesses and organizations can keep releasing carbon dioxide as needed, they are required to pay charges for each unit of carbon emitted (Wang et al., 2018).

Decision-makers enforce the carbon cap (Shi et al., 2020) framework under the Carbon Cap-and-Trade regime. The government or decision-makers set a carbon cap or limit and allow firms to buy or sell carbon credits under this policy. Whenever a company generates less carbon than the limit, it can sell its leftover carbon credits to other businesses and organizations. If a corporation or organization exceeds its carbon emission quota, it can buy carbon credits from other organizations that release less carbon. Carbon cap policies impose a ceiling on the number of carbon emissions that enterprises or organizations can produce, and exceeding this quota results in a hefty penalty. This argument is supported by (Benjaafar et al., 2013; Kung et al., 2018; Mishra & Ranu, 2023). Carbon taxation and emissions trading schemes are two important regulatory Systems used by many governments worldwide. Many European regions participate in the European Union Emissions Trading Systems (EU-ETS), the world's most extensive program to buy greenhouse emissions (GHG) (Kushwaha et al., 2020). Some nations, however, have either enacted or are in the process of implementing their carbon price or cap-and-trade legislation.

Although supply chains are a main priority for several firms, especially multinational organizations, and groups such as Walmart, there is a concentrated push to eliminate pollution across inventory supply networks. Because this sector contributes significantly to emissions, researchers and industry experts place high importance on inventory replenishment models that can lower both costs and emissions. In response to this need, we have developed a model aimed at optimizing costs and examining supply chains under two distinct scenarios: a decentralized supply chain implementing a carbon tax program, and a centralized supply chain operating under an emission carbon cap and trade system, all within a finite planning horizon.

This chapter systematically explores inventory management within carbon regulations and sustainability. We begin with a literature review in Section 7.2, identifying gaps and setting research goals. In Section 7.3, we establish assumptions and notations, leading to the presentation of our mathematical model in Section 7.4. Here, both centralized and decentralized approaches are analyzed. Section 7.5 demonstrates practical application through a numerical example, providing insights for the industry. Section 7.6 conducts sensitivity analysis, ensuring robustness and managerial insights. Employing "Wolfram Mathematica 13.0," our methodology enhances accuracy. In Section 7.7, we summarize findings, propose future research directions, and reflect on our contributions to inventory management and sustainability.

7.2 Literature Survey

Inventory management is all about keeping track of supplies and making sure they're in the right place at the right time. But nowadays, with carbon rules in place, things have gotten a bit more complicated. In this section, we'll look at what researchers have been saying about managing inventory under these new carbon regulations.

Some like (Kung et al., 2018), have delved into the collaborative efforts between manufacturers and retailers to reduce their carbon footprint. Their study aimed to understand how a carbon tax influences these joint actions within the supply network, and its broader impacts on the economy and environmental sustainability. They found that businesses working together can make a significant difference in reducing carbon emissions, especially when guided by carbon policies.

Other studies, such as (Mishra & Ranu, 2022a), have focused on the nitty-gritty of managing inventory in the face of additional costs due to carbon emissions. Their research developed various inventory management systems designed to cope with these extra expenses imposed by carbon policies. By implementing strategies like carbon cost policies and gradual emission tax policies, businesses can navigate through these challenges while maintaining efficient supply chains.

In (Cheng et al., 2017), explored how different types of supply chains—like traditional retail versus digital—adapt to carbon regulations. They investigated the implications of carbon cap regulations on both centralized and decentralized supply chain channels. By analyzing various scenarios, they aimed to understand how these regulations impact the management and flow of goods in different supply chain contexts. Some studies, like (Liu et al., 2021; Sarkar et al., 2015), delved into innovative strategies for reducing emissions while managing inventory effectively. They experimented with approaches such as cap-and-trade initiatives and carbon offset strategies to find optimal production and inventory policies. By exploring a range of emission reduction strategies, these studies shed light on practical solutions for businesses striving to balance cost-effectiveness with environmental responsibility. In (Bai, et al., 2017; Xu et al., 2018), focused on optimizing inventory management practices to align with sustainability goals. They developed mathematical models to assess the impact of carbon policies, including carbon taxes and cap-and-trade regimes, on supply chain operations. By integrating environmental considerations into inventory management decisions, these studies highlighted pathways for businesses to achieve both economic and environmental objectives.

Lastly, (Hua et al., 2011), examined the financial implications of carbon regulations on trade credit—a crucial aspect of business transactions. Their study investigated how carbon constraints influence optimal trade credit preferences under different policy scenarios. By understanding the financial dynamics shaped by carbon rules, businesses can adapt their financial strategies to mitigate risks and capitalize on opportunities in a carbon-constrained environment.

As we reflect on the breadth of research in this area, it's evident that managing inventory under carbon regulations poses multifaceted challenges and opportunities for businesses. The main findings of these researchers are summarized in table 7.1, highlighting key insights into how various carbon policies impact inventory management practices. In the subsequent sections, we'll delve deeper into the gaps identified in the existing literature and outline the research objectives aimed at addressing these gaps comprehensively.

Based on the study of the literature, it is possible to conclude that in recent decades, many researchers have included different carbon policies in enhancing the inventory replenishment model. Nevertheless, many researchers have concentrated on a single carbon policy, with just a few quantitative articles considering many policies at the same time. Additionally, research investigating various carbon policies has mainly focused on deterministic demand. Until now, no analysis has evaluated several carbon policies concurrently under a finite scheduling horizon for uneven replenishment cycle length. Demand is influenced by both time and inventory levels.

The lack of research on inventory modeling that examines demand that is influenced by both time and inventory levels in the context of two significant carbon regulations over a finite planning horizon is examined in this research study. Earlier literature has not investigated these policies under an FPH, making this study the first to present such approaches. As a result, the research aims to cover a gap by providing a novel approach to gain new insights.

Table 7.1 The Literature Comparison

Study	Linear Demand Time & Inventory Sensitive	Finite Planning Horizon	Carbon Tax Policy	Carbon Cap and Trade
(Cheng et al., 2017)	Yes	Yes	Influences collaborative efforts, economy, and environmental efficiency	-
(Sarkar et al., 2015)	-	-	Managed by a carbon cost policy	-
(Yan et al., 2016)	-	-	Gradual carbon tax policy	-
(Wang et al., 2018)	Yes	Yes	Considered in a deterministic multiperiod production planning framework	-
(Xu et al., 2018)	-	-	-	Considered under a carbon cap policy
(Hua et al., 2011)	-	-	-	Considered under a carbon cap-and-trade mechanism
(Benjaafar et al., 2013)	-	-	Various carbon policies can achieve cost and emissions reduction goals	-
(Hovelaque & Bironneau, 2015)	-	-	Considered both carbon taxes and carbon cap-and-trade regimes	-
(Qin et al., 2015)	-	-	-	Evaluated trade credit under carbon regulations
(Ghosh et al., 2017)	Yes	Yes	Considered all three carbon policies (tax, cap-and-trade, cap-and-offset)	-
(Toptal & Çetinkaya, 2017)	-	-	Evaluated carbon tax and cap-and-trade policies	Considered centralized and decentralized supply networks
This Chapter	Yes	Yes	Yes	Yes

7.2.1 The Gap in Existing Research

Based on the study of the literature, it is possible to conclude that in recent decades, many researchers have included different carbon policies in enhancing the inventory replenishment model. Nevertheless, most researchers have concentrated on a single carbon policy, with just a few quantitative articles considering many policies at the same time. Additionally, research investigating various carbon policies has mainly focused on deterministic demand. Until now, no analysis has evaluated several carbon policies concurrently under a finite scheduling horizon for uneven replenishment cycle length. Demand is influenced by both time and inventory levels.

7.2.2 Problem Identification

The lack of research on inventory modelling that examines demand that is influenced by both time and inventory levels in the context of two significant carbon regulations over a finite planning horizon is examined in this research study. Earlier literature has not investigated these policies under an FPH, making this study the first to present such approaches. As a result, the research aims to cover a gap by providing a novel approach to gain new insights.

7.3 Assumptions and Notations

7.3.1 Notations

Symbol	Description
a	The annual beginning demand rate during the initial phase of the inventory management cycle.
b	Over a year, the customer demand rate increases as well.
θ	The rate of demand is determined by the level of inventory.
Hr	The cost of holding a particular thing in rupees per unit per year.
Ss	The total cost is estimated in dollars per order and incorporates both the setup and shipment costs.
Cp	The cost of purchasing a single unit is given in dollars per unit.
$I_{i+1}(t)$	The level of stock from ti to t_{i+1} during the $(i+1)^{th}$ cycle, where t_i is the cycle's beginning time where $t_i \leq t \leq t_{i+1}$.
Q_{i+1}	The number of items ordered within the $(i)^{th}$ cycle at the time t and t is any period that lies between t_i and t_{i+1}.
Pr	For wholesale trades, the price per unit is in rupees.
Or	The ordering cost per transaction during the beginning of the managing inventory period.
ĉ	The emissions amount generated per order.
P̂r	The CO_2 emissions quantity connected with each purchasing unit.
(hr)	The quantity of greenhouse gas emissions generated per unit of time while maintaining stock.

7.3.2 Assumptions

- The planning horizon is assumed to be limited or finite in this framework.

- This model does not assume or predict the existence of deficits or shortages in demand.
- This model presupposes that orders will be finalized promptly as they are placed, which implies a waiting time is zero.
- In this model, the cost of holding inventory within the supply process is assumed to be I per unit of time.
- In this model, demand for goods is linearly related to time, meaning an increasing rise in demand throughout the planning horizon.

An inventory supply network with a solo supplier and a solo retailer is regarded to involve a single product or item in this model.

7.4 Mathematical expression for Carbon Pricing, and emissions trading

The formulation of carbon emissions is essential for accurately quantifying the environmental impact of the activities under investigation. We have carefully selected specific terms and defined parameters to ensure the robustness and relevance of our carbon emissions model. The terms included in the carbon emissions formulation are derived from established models and equations widely recognized in the field of environmental science and sustainability. These terms reflect various factors influencing carbon emissions, such as production processes, transportation methods, and energy consumption.

Furthermore, the parameters defining these terms are chosen to capture the essential aspects of the system under chapter. We have based our parameter definitions on empirical data, theoretical models, and industry standards to ensure accuracy and reliability. Specifically, the parameters are sourced from authoritative literature, including works by (Zhang, 2010), among others, which provide valuable insights into carbon emissions estimation methodologies.

The shifts in the quantity of stock $I_{n(i+1)}(t)$ overtime can be derived from the solution to the given mathematically differential equation (7.1) below, which corresponds to the (i+1)th replenishment cycle:

$$I_{n(i+1)}(t) = e^{-\theta t} \int_{t}^{t_{i+1}} (a + bu)e^{\theta * u}\, dt \quad \text{where } t_i < t < t_{i+1} \tag{7.1}$$

$$I_{n(i+1)}(t) = 0 \quad \text{and} \quad Q_{i+1} = I_{i+1}(t_i) = \int_{t_i}^{t_{i+1}} (a + bu)e^{\theta(u-t)}\, dt$$

$$I_{n(i+1)}(t) = \int_{t}^{t_{i+1}} (a + bu)e^{\theta(u-t)}\, du \tag{7.2}$$

$$I_{n(i+1)}(t) = \left[\frac{(a+bu)e^{\theta(u-t)}}{\theta} - \frac{b}{\theta^2} e^{\theta(u-t)} \right]_t^{t_{i+1}}$$

$$I_{n(i+1)}(t) = \frac{(a+bt_{i+1})e^{\theta(t_{i+1}-t)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t)} - \frac{(a+bt)}{\theta} + \frac{b}{\theta^2}$$

The order quantity for i^{th} cycles

$$Q_{i+1} = I_{i+1}(t_i) = \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt \tag{7.3}$$

$$Q_{i+1} = I_{i+1}(t_i) = \left[\frac{(a+bu)e^{\theta(u-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(u-t_i)} \right]_{t_i}^{t_{i+1}}$$

$$Q_{i+1} = I_{i+1}(t_i) = \frac{(a+bt_{i+1})e^{\theta(t_{i+1}-t_i)}}{\theta} - \frac{b}{\theta^2} e^{\theta(t_{i+1}-t_i)} - \frac{(a+bt_i)}{\theta} + \frac{b}{\theta^2}$$

According to the studies conducted by (Shi et al., 2020, Mishra & Ranu, 2022a), the proposed chapter includes the $\hat{c}$ fixed quantity emission, $\hat{P}$ connected with inventory replenishment, and $\widehat{h_r}$ associated with inventory holding or management (refrigeration effect).

$$Amt_{carbon} = \hat{c} + \hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} du\ dt$$

7.4.1 Decentralized case

In decentralized decision-making within the supply chain, each entity, including suppliers, manufacturers, and distributors, operates autonomously, making decisions independently without coordination. This lack of coordination means that each party pursues its own interests and objectives without considering the broader implications for the entire supply chain. Consequently, the total cost incurred in the decentralized supply chain encompasses several components, including ordering cost, holding cost, purchasing cost, and the cost of carbon tax.

$$Tc_{Ret}(n_1, t_i) = n_1 * O_r + H_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} I_{n(i+1)}(t)\,dt + P_r \sum_{i=1}^{n_1} Q_{i+1} + \sum_{i=0}^{n_1-1} \tau \left(\hat{c} + \hat{P}_r * Q_{i+1} + \widehat{h_r} \int_{t_i}^{t_{i+1}} I_{n(i+1)}(t)\,dt\right) \tag{7.4}$$

$$Tc_{Ret}\,(n_1, t_i) = n_1 * O_r + H_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)}\,du\,dt + P_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt + \sum_{i=0}^{n_1-1} \tau \left(\hat{c} + \widehat{P}_r * Q_{i+1} + \widehat{h}_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} du\ dt\right)$$

$$Tc_{Ret}\,(n_1, t_i) = n_1 * O_r + H_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)}\,du\,dt + P_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt + \sum_{i=0}^{n_1-1} \tau \left(\hat{c} + \widehat{P}_r * \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt + \widehat{h}_r \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)} du\ dt\right)$$

$$Tc_{Ret}\,(n_1, t_i) = n_1 * O_r + \left(H_r + \tau * \widehat{h}_r\right) \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a+bu)e^{\theta(u-t)}\,du\,dt + \tau * \hat{c} + \sum_{i=0}^{n_1-1} \left((P_r + \widehat{P}_r * \tau) \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt \right)$$

$$Tc_{Ret}\,(n_1, t_i) = n_1 * O_r + \tau * \hat{c} + \left(\frac{H_r + \tau * \widehat{h_r}}{\theta} + (P_r + \widehat{P}_r * \tau)\right) \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt - \frac{H_r + \tau * \widehat{h_r}}{\theta} \left(a * H + \frac{1}{2} * b * H^2\right) \tag{7.5}$$

Where $H = t_{n_1} = t_{i+1} - t_i$

$$Tc_{Sup}(n_1, t_i) = {n_1}^* * S_r + \sum_{i=0}^{n^*-1} C_p * Q_{i+1}^* \tag{7.6}$$

$$Tc_{Sup}(n_1, t_i) = {n_1}^* * S_r + C_p \sum_{i=0}^{{n_1}^*-1} \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt$$

$$Q_{i+1} = \sum_{i=0}^{{n_1}^*-1} Q_{i+1}^*$$

$$Q_{i+1} = \sum_{i=0}^{{n_1}^*-1} \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt \tag{7.7}$$

$$Tc_{System}\,(n_1, t_i) = n_1 * (O_r + S_r) + \tau * \hat{c} + \left(\frac{H_r + \tau * \widehat{h_r}}{\theta} + (P_r + \widehat{P}_r * \tau + C_p)\right) \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} (a+bt)e^{\theta(t-t_i)}\,dt - \frac{H_r + \tau * \widehat{h_r}}{\theta} \left(a * H + \frac{1}{2} * b * H^2\right) \tag{7.8}$$

$$\frac{\partial}{\partial_{t_i}} Tc_{System}\,(n_1, t_i)$$

$$= \left(\frac{H_r + \tau * \widehat{h_r}}{\theta} + (P_r + \widehat{P}_r * \tau)\right)\left\{(a + bt_i)\left(e^{\theta(t_i - t_{i-1})} - 1\right) - \theta \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt\right\} \tag{7.9}$$

$$\frac{\partial}{\partial_{t_i}} Tc_{System}\,(n_1, t_i) = \left(\frac{H_r + \tau * \widehat{h_r}}{\theta} + (P_r + \widehat{P}_r * \tau)\right)\left\{(a + bt_i)\left(e^{\theta(t_i - t_{i-1})}\right) - (a + bt_{i+1})e^{\theta(t_{i+1} - t_i)} + \frac{b}{\theta}e^{\theta(t_{i+1} - t_i)} - \frac{b}{\theta}\right\} \tag{7.10}$$

7.4.2 Centralization

Decisions in centralized supply chain management scenarios are made by collaborative efforts that benefit the entire system. As a result, we're looking into introducing an emissions trading scheme to lower the overall cost of the supply chain system. Firms that emit low amounts of carbon can sell their carbon credits to firms that emit large levels of carbon under this system of trading. The emissions trading program is not only ecological and environmentally but also environmentally and socially conscious.

optimal cycles for replenishment, as determined through analysis, is at n_1=4. Once the minimum threshold has been reached at n_1=4. Subsequently, it gradually ascends in subsequent cycles. Similarly, for the centralized (cap and trade) case, the total cost of system is 112.21$ which reach its optimal level at n_1=4. Table 7.1, and Table 7.2 exhibit a convex pattern of the system's cost function. This observation is further supported by graphical representations.

$$Tc_{System}\,(n_1, t_i)$$

$$= n_1 * (O_r + S_r) + H_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu)e^{\theta(u-t)}\,du\,dt + P_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt + \sum_{i=0}^{n_1-1} \delta \left(\hat{c} + \widehat{P}_r * \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt + \widehat{h_r} \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} (a + bu)e^{\theta(u-t)}du\,\,dt - CO_{2_{Cap}}\right)$$

$$Tc_{System}\ (n_1, t_i) = n_1 * (O_r + S_r) + \delta * \hat{c} + \left(\frac{H_r+\delta*\widehat{h_r}}{\theta} + (P_r + \widehat{P}_r * \delta + C_p)\right)\sum_{i=1}^{n_1}\int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt - \frac{H_r+\delta*\widehat{h_r}}{\theta}\left(a * H + \frac{1}{2} * b * H^2\right) - \delta * CO_{2_{Cap}} \quad (7.11)$$

Preposition: $(a + b\ t_{i+1})e^{\theta(T_{i+1})} < b(e^{\theta(T_i)} - 1)/\theta + (a + bt)e^{\theta(T_i)}$

$$F(t_i) - F(t_{i+1}) < \frac{F'(t_i)}{F(t_i)}\int_{t_i}^{t_{i+1}} F(t)\,dt$$

Let F(t)= $(a + bt)e^{\theta(t-t_i)}$ is a log convex function. By putting the value of F(t) in the above equation. we have

$$(a + b\ t_{i+1})e^{\theta(t_{i+1}-t_i)} - (a + bt_i) < \frac{b+\theta(a+bt_i)}{(a+bt_i)}\int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt$$

$$(a + b\ t_{i+1})e^{\theta(t_{i+1}-t_i)} - (a + bt_i) < \left(\frac{b}{(a+bt_i)} + \theta\right)\int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt$$

$$\frac{\partial Tc_{System}\ (n_1,t_i)}{\partial t_i} = (a + b\ t_i)\left(e^{\theta(t_i-t_{i-1})} - 1\right) - \theta\int_{t_i}^{t_{i+1}}(a + bt)e^{\theta(t-t_i)}\,dt = 0 \quad (7.12)$$

By equation (7.12), we have $\frac{(a+b\ t_i)\left(e^{\theta(t_i-t_{i-1})}-1\right)}{\theta} = \int_{t_i}^{t_{i+1}}(a + bt)e^{\theta(t-t_i)}\,dt$

$$(a + b\ t_{i+1})e^{\theta(T_{i+1})} - (a + bt_i) < \left(\frac{b}{(a + bt_i)} + \theta\right)\frac{(a + b\ t_i)e^{\theta(T_i)} - (a + b\ t_i)}{\theta}$$

$$(a + b\ t_{i+1})e^{\theta(T_{i+1})} - (a + bt_i) < \frac{b\ e^{\theta(T_i)}}{\theta} - \frac{b}{\theta} + (a + b\ t_i)e^{\theta(T_i)} - (a + b\ t_i)$$

$$(a + b\ t_{i+1})e^{\theta(T_{i+1})} < \frac{b\ (e^{\theta(T_i)} - 1)}{\theta} + (a + b\ t_i)e^{\theta(T_i)}$$

Lemma: t_i strictly monotonic increase function of the last replenishment cycle t_n where i=1,2,3…. n_1-1.

Proof: Since t_{n_1} = H-T_{n_1}

Following, (Hariga, 1996), we also employed the concept of mathematical induction to establish that T_i increase with the T_{n_1} =n-1, n-2……….2.1. Put i =n_1-1 When we put i=n_1-1 into equation (7.12), we obtain.

$$\frac{\partial}{\partial_{t_i}} Tc_{System}\,(n_1, t_i) = [a + b\,(H - T_{n_1}\,)]e^{\theta(T_{n_1-1})} - [a + b\,(H - T_{n_1}\,)] - \theta \int_{H-T_{n_1}}^{H} (a + bt)e^{\theta(t-H+T_{n_1})}\,dt = 0$$

Then differentiate equation (7.12) w.r.t T_{n_1}

$$[a + b\,(H - T_{n_1}\,)]e^{\theta(T_{n_1-1})} \frac{d(T_{n_1-1})}{dT_{n_1}} - b\,e^{\theta(T_{n_1-1})} + b - [a + b\,(H - T_{n_1}\,)] - \theta^2 \int_{H-T_{n_1}}^{H} (a + bt)e^{\theta(t-H+T_{n_1})}\,dt = 0 \quad (7.13)$$

$$(a + b\,t_i)\left(e^{\theta(t_i - t_{i-1})} - 1\right) = \theta \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt$$

we get by equation (7.12) and then put in equation (7.13)

$$\theta[a + b\,(H - T_{n_1}\,)]e^{\theta(T_{n-1})} \frac{d(T_{n_1-1})}{dT_{n_1}} = b(\,e^{\theta(T_{n_1-1})} - 1) + \theta[a + b\,(H - T_{n_1}\,)]e^{\theta(T_{n_1-1})}$$

$$\theta[a + b\,(H - T_{n_1}\,)]e^{\theta(T_{n_1-1})} \left\{\frac{d(T_{n_1-1})}{dT_{n_1}} - 1\right\} = b(\,e^{\theta(T_{n_1-1})} - 1)$$

$$\left\{\frac{d(T_{n_1-1})}{dT_{n_1}}\right\} = \frac{b(\,e^{\theta(T_{n_1-1})} - 1)}{\theta[a+b\,(H-T_{n_1}\,)]e^{\theta(T_{n_1-1})}} + 1$$

$$\frac{d(T_{n_1-1})}{dT_{n_1}} = \frac{b(\,e^{\theta(T_{n_1-1})} - 1) + \theta[a+b\,(H-T_{n_1}\,)]e^{\theta(T_{n_1-1})}}{\theta[a+b\,(H-T_{n_1}\,)]e^{\theta(T_{n_1-1})}} \geq 0 \quad (7.13)$$

After that, let's us take that $\frac{d(T_m)}{dT_{n_1}} > 0$ for m = i + 1, i + 2, . . . , $n_1 - 1$ And then again differentiate equation (7.13) w.r.t T_{n_1} , we have

$$b(e^{\theta(T_i)} - 1)\frac{d(t_i)}{dT_{n_1}} + \theta(a + b\,t_i)e^{\theta(T_i)}\frac{d(T_i)}{dT_{n_1}} - \theta(a + b\,t_{i+1})e^{\theta(T_{i+1})}\frac{d(t_{i+1})}{dT_{n_1}} + \theta(a + b\,t_i)\frac{d(t_i)}{dT_{n_1}} + \theta^2 \frac{d(t_i)}{dT_{n_1}} \int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt \quad (7.14)$$

By using prepositions and equation (7.10)

$$\frac{d(t_i)}{dT_{n_1}} \leq \frac{d(t_{i+1})}{dT_{n_1}} = -\sum_{m=i+2}^{n_1-1} \frac{d(T_m)}{dT_{n_1}} - 1 \leq 0 \quad (7.15)$$

This implies $\frac{d(T_i)}{dT_{n_1}} \geq 0$ where i=1,2,3………… n_1-1.

Moreover, as we know that T_{n_1} = H- t_{n_1-1} this implies $\frac{d(T_i)}{dT_{n_1-1}} \leq 0$ where i=1,2,3………… n_1-1. Now, we take $t_i = H - \sum_{m=i+1}^{n_1-1} T_u - T_{n_1} = t_{n_1-1} - \sum_{m=i+1}^{n_1-1} T_u$ from this it is concluded that $\frac{d(t_i)}{dt_{n_1}} \geq 0$ for all i=1,2,3………… n_1-1.

This lemma highlights the connection between the replenishment time, last replenishment time, length of replenishment time, and the time horizon.

$T_{i+1} = t_{i+1} - t_i$ and t_n = H-T_n

Theorem 1: The optimal replenishment period for a fixed replenishment cycle is the unique solution that exists for the nonlinear system represented by equation (7.8).

The intention of calculating t_i values is to prove the system's total variable cost $\mathrm{Tc_{System}}\,(\mathrm{n_1}, \mathrm{t_i})$ is a convex function. The first and most important prerequisite for obtaining t_i is to establish that the

$$\frac{\partial}{\partial_{\mathrm{t_i}}}\mathrm{Tc_{System}}\,(\mathrm{n_1}, \mathrm{t_i}) = 0$$

$$\frac{\partial}{\partial_{\mathrm{t_i}}}\mathrm{Tc_{System}}\,(\mathrm{n_1}, \mathrm{t_i}) = \left(\frac{\mathrm{H_r} + \tau * \widehat{\mathrm{h_r}}}{\theta} + (\mathrm{P_r} + \widehat{\mathrm{P_r}} * \tau)\right)\left\{(\mathrm{a} + \mathrm{bt_i})\left(\mathrm{e}^{\theta(\mathrm{t_i}-\mathrm{t_{i-1}})}\right) - (\mathrm{a} + \mathrm{bt_{i+1}})\mathrm{e}^{\theta(\mathrm{t_{i+1}}-\mathrm{t_i})} + \frac{\mathrm{b}}{\theta}\mathrm{e}^{\theta(\mathrm{t_{i+1}}-\mathrm{t_i})} - \frac{\mathrm{b}}{\theta}\right\} = 0$$

$$\frac{\partial^2 \mathrm{Tc_{System}}\,(\mathrm{n_1}, \mathrm{t_i})}{\partial \mathrm{t_i}^2} = \left(\mathrm{b}\left(\mathrm{e}^{\theta(\mathrm{t_i}-\mathrm{t_{i-1}})} - 1\right) + \theta(\mathrm{a} + \mathrm{bt_i})\mathrm{e}^{\theta(\mathrm{t_i}-\mathrm{t_{i-1}})} + \theta(\mathrm{a} + \mathrm{bt_i}) + \theta^2 \int_{\mathrm{t_i}}^{\mathrm{t_{i+1}}} (\mathrm{a} + \mathrm{bt})\mathrm{e}^{\theta(\mathrm{t}-\mathrm{t_i})}\,\mathrm{dt}\right)$$

$$\frac{\partial^2 \mathrm{Tc_{System}}\,(\mathrm{n_1}, \mathrm{t_i})}{\partial \mathrm{t_i}^2} = \theta(\mathrm{a} + \mathrm{bt_i})\mathrm{e}^{\theta \mathrm{T_i}} + \theta(\mathrm{a} + \mathrm{bt_{i+1}})\mathrm{e}^{\theta \mathrm{T_{i+1}}} + \mathrm{b}(\mathrm{e}^{\theta \mathrm{T_i}} - \mathrm{e}^{\theta \mathrm{T_{i+1}}}) \tag{7.16}$$

$$\frac{\partial^2 \mathrm{Tc_{System}}\,(\mathrm{n_1}, \mathrm{t_i})}{\partial \mathrm{t_i}\ \partial \mathrm{t_{i-1}}} = -\,\theta(\mathrm{a} + \mathrm{bt_i})\mathrm{e}^{\theta \mathrm{T_i}} \tag{7.17}$$

$$\frac{\partial^2 \mathrm{Tc_{System}}\,(\mathrm{n_1}, \mathrm{t_i})}{\partial \mathrm{t_i}\ \partial \mathrm{t_{i+1}}} = -\,\theta(\mathrm{a} + \mathrm{bt_{i+1}})\mathrm{e}^{\theta \mathrm{T_{i+1}}} \tag{7.18}$$

$$\frac{\partial^2 \mathrm{Tc_{System}}\,(\mathrm{n_1,t_i})}{\partial \mathrm{t_i}\,\partial \mathrm{t_m}}=0 \qquad \text{for all m} \neq \text{ i, i+1, i-1} \qquad (7.19)$$

$$\nabla^2 \mathrm{Tc_{System}}\,(\mathrm{n_1,t_i})$$

$$= \begin{bmatrix} \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_1^2}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_1}\,\partial \mathrm{t_2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_2}\,\partial \mathrm{t_1}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_2^2}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_2}\,\partial \mathrm{t_3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_3}\,\partial \mathrm{t_2}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_3^2}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_3}\,\partial \mathrm{t_4}} & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_{n_{1-1}}}\,\partial \mathrm{t_{n_{1-2}}}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t^2_{n_{1-1}}}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_{n_{1-1}}}\,\partial \mathrm{t_{n_1}}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t_{n_1}}\,\partial \mathrm{t_{n_{1-1}}}} & \frac{\partial^2 \mathrm{Tc_{System}}}{\partial \mathrm{t^2_{n_1}}} \end{bmatrix}$$

TC_s is positive definite if equations (7.16), (7.17), (7.18) and (7.19) satisfy the given inequality.

$$\frac{\partial^2 Tc_{System}}{\partial t_i^2} \geq \left|\frac{\partial^2 Tc_{System}}{\partial t_i t_{i-1}}\right| + \left|\frac{\partial^2 Tc_{System}}{\partial t_i t_{i+1}}\right| \qquad \text{or}$$

$$\frac{\partial^2 Tc_{System}}{\partial t_i^2} - \left|\frac{\partial^2 Tc_{System}}{\partial t_i t_{i-1}}\right| - \left|\frac{\partial^2 Tc_{System}}{\partial t_i t_{i+1}}\right| \geq 0$$

$$\theta(\mathrm{a}+\mathrm{bt_i})\mathrm{e}^{\theta \mathrm{T_i}} + \theta(\mathrm{a}+\mathrm{bt_{i+1}})\mathrm{e}^{\theta \mathrm{T_{i+1}}} + \mathrm{b}(\mathrm{e}^{\theta \mathrm{T_i}} - \mathrm{e}^{\theta \mathrm{T_{i+1}}}) - \theta(\mathrm{a}+\mathrm{bt_i})\mathrm{e}^{\theta \mathrm{T_i}} - \theta(\mathrm{a}+\mathrm{bt_{i+1}})\mathrm{e}^{\theta \mathrm{T_{i+1}}} > 0$$

$$\mathrm{b}(\mathrm{e}^{\theta \mathrm{T_i}} - \mathrm{e}^{\theta \mathrm{T_{i+1}}}) > 0$$

that is true for all $i = 1, 2, \ldots, n$

Moreover, the Hessian matrix had to be positive definite since it contains positive diagonal members and has strictly diagonal dominating features. As a result, the optimal replenishment interval to the nonlinear system of equation (7.10) is obtained. now we need to show that the optimal solution of the non-linear equation (7.10) is unique and $Tc_{System}(\mathrm{t_i}, \mathrm{n})$ is optimal function throughout the optimal value of t_i in a finite horizon planning H. Furthermore, because it had strictly diagonal dominating characteristics and positive diagonal members, the Hessian matrix required to be positive definite. As a result, the optimum replenishment interval for nonlinear system equation (7.10) is established. Now we need to demonstrate the convexity of $Tc_{System}(\mathrm{t_i}, \mathrm{n})$ throughout the optimal value of ti in the finite horizon planning H.

The Hessian matrix obtained by the partial differentiation of $Tc_{System}\,(n_1, t_i)$, it is necessary for it to be positive definite for t_i to be minimum for a fixed n. As a result, the theorem establishes that $Tc_{System}\,(n_1, t_i)$ is positive definite. Therefore, the optimum value of t_i is obtained using the numerical iterative technique for a given fixed positive integer with mathematical programs constructed by Mathematica software version 13.0. Based on the optimal value of t_i , the Total cost function also will be minimize.

Theorem 2: For a finite horizon planning H, the number of replenishment cycles exhibits convex behaviour of the function $Tc_{System}\,(n_1, t_i)$.

Proof:

$$Tc_{System}\,(n_1, t_i)$$
$$= n_1 * (O_r + S_r) + \tau * \hat{c} + \left(\frac{H_r + \tau * \widehat{h_r}}{\theta} + (P_r + \widehat{P}_r * \tau + C_p)\right)\sum_{i=1}^{n_1}\int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt - \frac{H_r + \tau * \widehat{h_r}}{\theta}\left(a * H + \frac{1}{2} * b * H^2\right)$$

Where, $t_0 = 0$ and $t_{n_1} = H$.

Let us now suppose,

$$T_{Ret}(t_i, n) = n_1 * (O_r + S_r) + \left(\frac{H_r + \tau * \widehat{h_r}}{\theta} + (P_r + \widehat{P}_r * \tau + C_p)\right)\sum_{i=1}^{n_1}\int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt + K$$

Were, $\tau * \hat{c} - \frac{H_r+\tau*\widehat{h_r}}{\theta}\left(a * H + \frac{1}{2} * b * H^2\right) = K$

$$F(n_1, 0, H) = \sum_{i=1}^{n_1}\int_{t_i}^{t_{i+1}} (a + bt)e^{\theta(t-t_i)}\,dt$$

$F(n_1 + 1, 0, H) - F(n_1, 0, H) = \int_{t_{n_1-1}}^{t_{n_1}} (a + bt)e^{\theta(t-t_{n_1-1})}\,dt + \int_{t_{n_1}}^{T} (a + bt)e^{\theta(t-t_{n_1})}\,dt - \int_{t_{n_1-1}}^{T} (a + bt)e^{\theta(t-t_{n_1-1})}\,dt$

$$= \int_{t_{n_1-1}}^{t_{n_1}} (a + bt)e^{\theta(t-t_{n_1-1})}\,dt + \int_{t_{n_1}}^{T} (a + bt)e^{\theta(t-t_{n_1})}\,dt - \int_{t_{n_1-1}}^{t_{n_1}} (a + bt)e^{\theta(t-t_{n_1-1})}\,dt - \int_{t_{n_1}}^{T} (a + bt)e^{\theta(t-t_{n_1-1})}\,dt$$

$$F(n_1+1,0,H)-F(n_1,0,H)=\int_{t_{n_1}}^{T}(a+bt)\left\{e^{\theta(t-t_{n_1})}-e^{\theta(t-t_{n_1-1})}\right\}dt<0$$

$$F(n_1+1,0,H)-F(n_1,0,H)<0$$

$$F(n_1+1,0,H)<F(n_1,0,H)$$

Let us consider $F(n_1,0,H)-F(n_1-1,0,H)-[F(n_1+1,0,H)-F(n_1,0,H)]$

$$=\int_{t_{n_1-1}}^{T}(a+bt)\left[e^{\theta(t-t_{n_1-1})}-e^{\theta(t-t_{n_1-2})}\right]dt-\int_{t_{n_1}}^{T}(a+bt)\left[e^{\theta(t-t_{n_1})}-e^{\theta(t-t_{n_1-1})}\right]dt$$

$$=\int_{t_{n_1-1}}^{T}(a+bt)\left[e^{\theta(t-t_{n_1-1})}-e^{\theta(t-t_{n_1-2})}\right]dt+\int_{t_{n_1}}^{T}(a+bt)\left[e^{\theta(t-t_{n_1-1})}-e^{\theta(t-t_{n_1-2})}\right]dt$$
$$-\int_{t_{n_1}}^{T}(a+bt)\left[e^{\theta(t-t_{n_1})}-e^{\theta(t-t_{n_1-1})}\right]dt$$

$$\begin{aligned}F(n_1,0,H)-F(n_1-1,0,H)-[F(n_1+1,0,H)-F(n_1,0,H)]\\=\int_{t_{n_1-1}}^{T}(a+bt)\left[e^{\theta(t-t_{n_1-1})}-e^{\theta(t-t_{n_1-2})}\right]dt\\+\int_{t_{n_1}}^{T}(a+bt)\left[2e^{\theta(t-t_{n_1-1})}-e^{\theta(t-t_{n_1})}-e^{\theta(t-t_{n_1-2})}\right]dt<0\end{aligned}$$

$$F(n_1,0,H)-F(n_1-1,0,H)-[F(n_1+1,0,H)-F(n_1,0,H)]<0$$

$$F(n_1,0,H)-F(n_1-1,0,H)<F(n_1+1,0,H)-F(n_1,0,H)$$

Since e^t is a convex function, F (n_1, 0, H) is a convex function in n_1. This indicates that $Tc_{System}(t_i,n)$ is Inherently a convex function.

7.5 Problem-Solving approach and numerical example to solve the problem

We have already introduced an approach for determining the optimal replenishment time solution. To find the replenishment time t_i put $\frac{\partial \mathrm{Tc}_{\mathrm{System}}(n_1,t_i)}{\partial t_i} = 0$. Therefore, we find the following differential equations by taking partial differentiation of Tc_{System} w.t.r to t_i, respectively.

$$\frac{\partial}{\partial t_i}\mathrm{Tc}_{\mathrm{system}}(n_1,\mathrm{t}_i) = \left(\frac{H_r + \tau^*\hat{\mathrm{h}}_r}{\theta} + (P_r + \hat{P}_r{}^*\tau\right)\Big\{(a+\mathrm{bt}_i)(e^{\theta(t_i - t_{i-1})}) - (a + \mathrm{bt}_{i+1})e^{\theta(t_{i+1}-t_i)} + \frac{b}{\theta}e^{\theta(t_{i+1}-t_i)} - \frac{b}{\theta}\Big\} = 0 \tag{7.20}$$

Furthermore, Using the algorithms mentioned in previously published works, the optimum values of supply chain cost as a whole, n number of replenishment cycles, and total quantity of carbon emission for both policies related to emission are displayed in table 7.1, 7.2 and figure 7.1, 7.2, 7.3 respectively. Using the following data, a dual-level inventory logistics network with carbon regulations within a finite planning horizon (FPH) is explored. Most of the data are identical to those used in previously published publications, such as (Sarkar et al., 2015), and others.

Table 7.2 The total cost generated by the overall operator system with the emission tax, and trading policy.

↓ a / $\rightarrow n_1$	1	2	3	4	5	6	7	8
Decentralized	160.87	134.78	129.14	**129.08**	131.38	134.82	138.92	143.40
Centralized	144.75	119.84	113.50	**112.21**	113.06	114.96	117.45	120.28

Table 7.3 The most economical and optimal number of replenishment cycles, system cost, and replenishment quantity has been computed using the policies related to carbon pricing, including carbon tax and cap-and-trade mechanisms.

↓ h_r / $\rightarrow t_i$	t_0	t_1	t_2	t_3	t_4	Amt_{carbon}	n_1	Q_{nt}	Tc_{System}
Carbon tax	0	1.1091	1.9682	2.7103	4.	47.6832	4	14.320	129.08
Cap and trade	0	1.1091	1.9682	2.7103	4.	47.6832	4	14.320	112.21

Here is a list of fundamental parameter values along with their corresponding units. $\mathrm{O_r} = 80$ \$/order, $\mathrm{H_r}$=0.04 \$/unit/time,$\mathrm{P_r} = 0.02$, $\Theta = 0.2$, a= 0.5, b=2 , $\widehat{\mathrm{h}_r} = 8$, $\mathrm{Ss} = 25$, c^ =4, $\widehat{\mathrm{P}}_r = 0.7$, $\mathrm{C_p} = 4$, $\tau = 0.022$,$\delta = 0.0108$, $\mathrm{Co_{2}}_{\mathrm{Cap}} = 200$. After finding the values of t_i, then to find the cost, computed the solution to the nonlinear function by numerical iterative approximation technique in "Wolfram Mathematica" a software for solving mathematical problems. Table 7.2, table 7.3, figure 7.1, figure 7.2, and figure 7.3, give a piece of comprehensive information about the optimal overall cost for an entire system. In the Decentralized (carbine tax) case, the total cost of the system is 129.08 $ that reaches its.

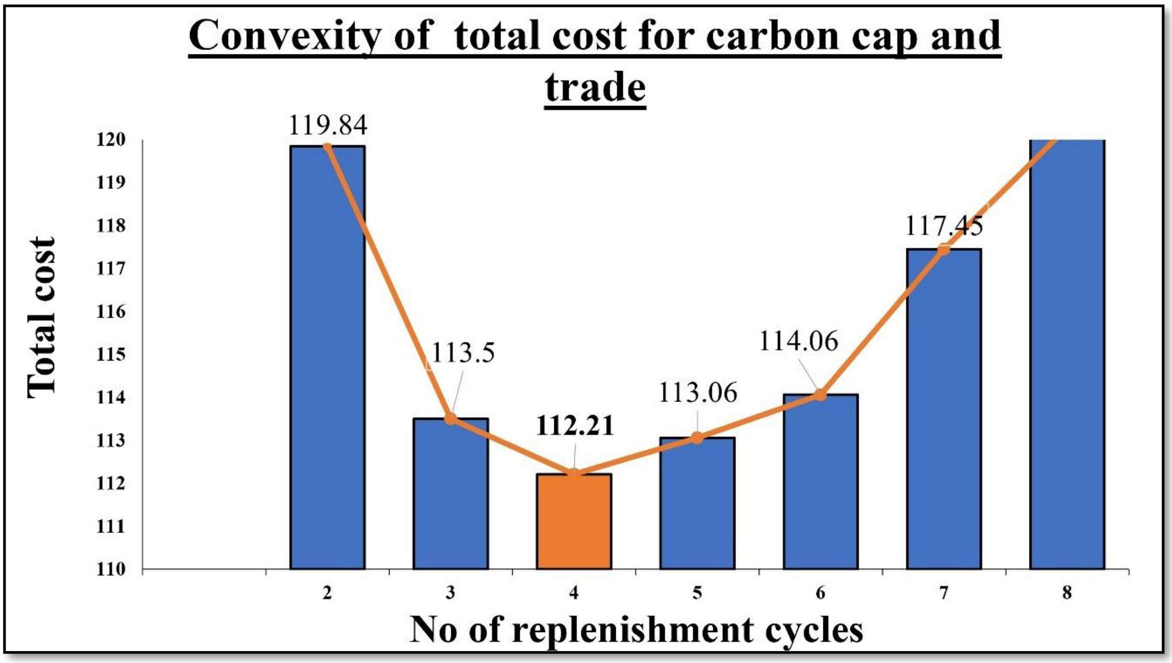


Figure 7.1 Show the optimal total cost generated by the overall operator system under Centralization

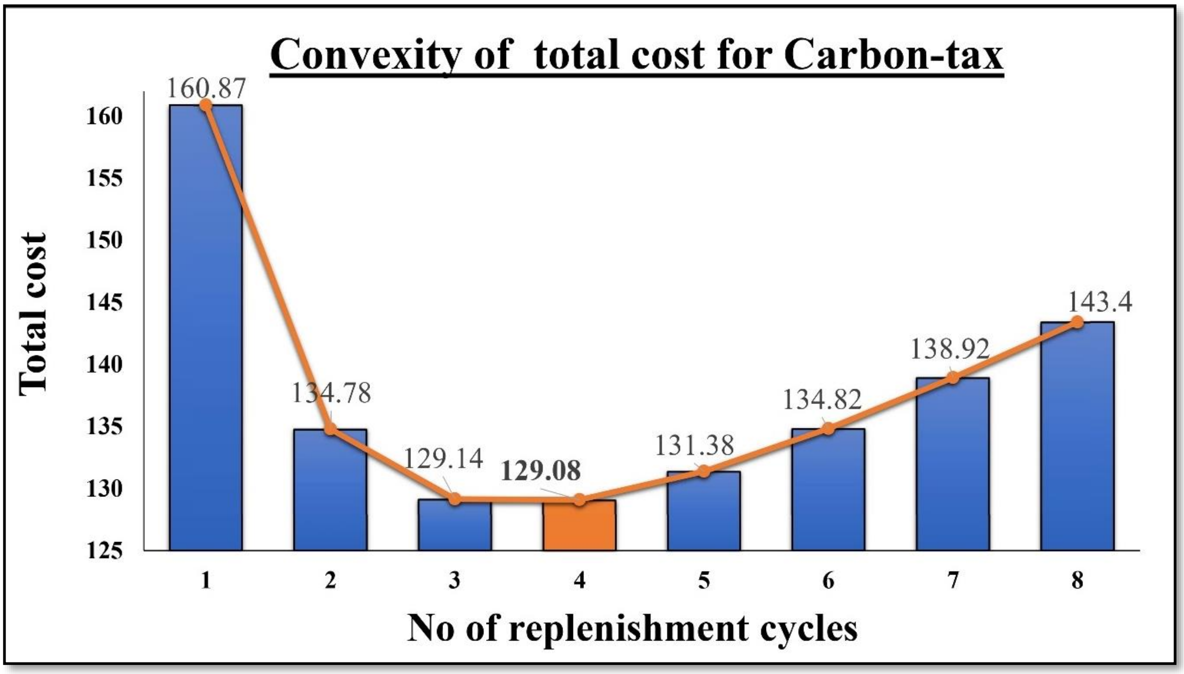


Figure 7.2 Show the optimal total cost generated by the overall operator system under Decentralization

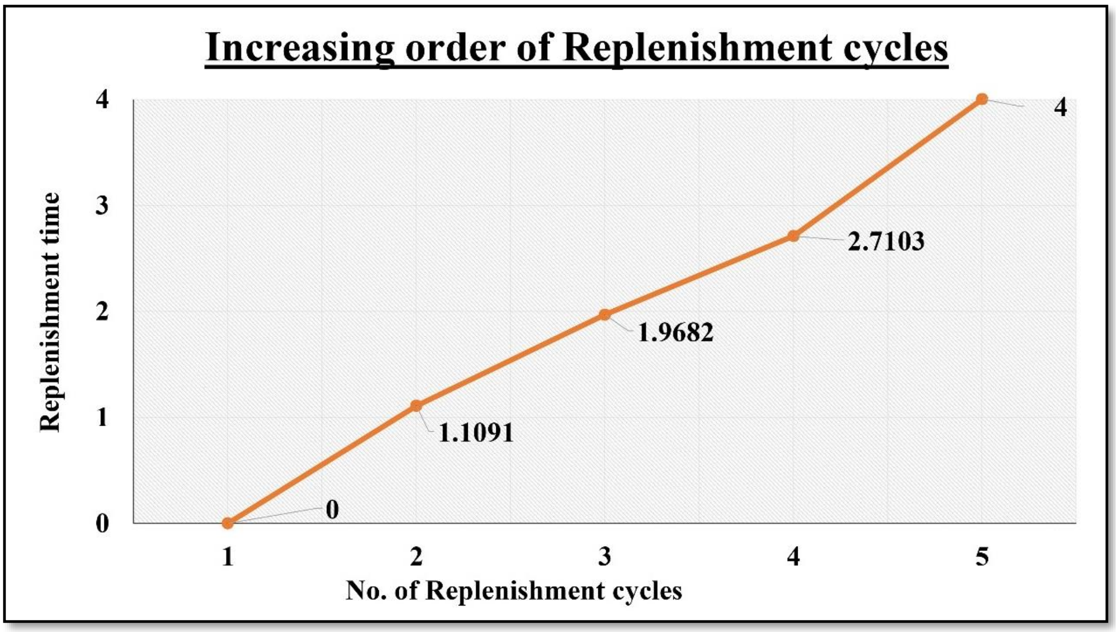


Figure 7.3 Show the optimal replenishment time generated by the overall operator system are in increased order

The comparative analysis of decentralized and centralized carbon policies reveals insightful findings regarding their implications for the inventory supply process. Firstly, the total cost implications indicate that the decentralized carbon tax policy tends to incur higher overall supply chain costs compared to the centralized cap-and-trade policy across all replenishment cycles. For instance, at the optimal replenishment cycle of $n_1=4$, the total cost under the carbon tax policy amounts to \$129.08, whereas under cap-and-trade, it reduces significantly to \$112.21. Interestingly, despite differences in total cost, both policies exhibit consistent optimal replenishment cycles, indicating that the choice of policy does not substantially affect the frequency of replenishment cycles. Moreover, both carbon tax and cap-and-trade policies demonstrate similar levels of effectiveness in reducing carbon emissions within the supply chain, with minimal variation observed between the two. Sensitivity evaluations further emphasize the role of parameter adjustments in achieving reductions in overall projected emissions and costs under both carbon regimes. Notably, while there is variation in total cost between the two policies, fundamental operational aspects such as order quantity and total emissions remain relatively stable, suggesting consistency in performance regardless of the chosen carbon policy. Overall, these comparative findings underscore the efficacy of both decentralized and centralized carbon policies in promoting sustainability and reducing carbon emissions within the inventory supply process.

7.6 Parametric Analysis

7.6.1 Parametric analysis of Decentralized case

In the discussion below, we focused on conducting sensitivity studies for several factors under the cap-and-trade framework. Our discussion has been confined to two factors that are directly connected, namely the carbon cap (C) and the rate of trade credit δ. These variables impact the number of replenishment cycles, time, carbon emissions, and total cost.

Table 7.4, figure 7.4, and figure 7.5 demonstrate the relationship between the δ the order quantity, number of replenishment cycles, overall system cost, and amount of carbon emission. According to the results shown, increasing δ leads to a substantial increase in the order quantity and the count of replenishment cycles, resulting in a decrease in the overall the cost of the system and the level of carbon emissions.

Table 7.4 demonstrates that increasing the carbon cap increases the order quantity, and the number of replenishment cycles, and decreases the overall system expenses and the quantity of carbon discharge. As seen in figures 7.6 and 7.7 because of adjustment on provided cap. It means that decision making agencies cannot directly reduce emissions by establishing more rigorous carbon restrictions, but it may dissuade enterprises from emitting more carbon by causing considerable cost increases. A strict cap raises the economic burden on the organization, but if the emission cap is freely allocated, the organization can minimize the overall cost by selling an unutilized quota of carbon. Figures 7.6 and 7.7 show the shifting trends of order quantity, replenishment number, the aggregate system expenditure and emission quantity.

Table 7.5 presents the results of sensitivity analysis conducted by varying the tax paid on each unit of carbon emitted (τ). As the tax rate increases from 0.020 to 0.024, we observe changes in the optimal replenishment time (t_i), the number of replenishment cycles (n_1), the replenishment quantity (Q_{nt}), and the total system cost (T_{c_System}). With a tax rate of 0.020, the optimal replenishment time is distributed over eight periods, resulting in a total system cost of \$150.47. As the tax rate increases to 0.021 and 0.022, the optimal replenishment time decreases, leading to a reduction in the number of replenishment cycles and the total system cost. However, beyond a tax rate of 0.022, further increases in the tax rate led to a significant decrease in the optimal replenishment time and the total system cost. The sensitivity analysis reveals that higher carbon taxes drive firms to optimize replenishment strategies, reducing emissions and minimizing tax burdens while maintaining efficiency. Aligning supply chain decisions with carbon regulations is crucial for cost savings and environmental sustainability.

Table 7.4 The most economical and optimal system cost, number of replenishment cycles, amount of emission and replenishment quantity have been calculated with changing δ.

$\downarrow \delta$	$\rightarrow t_i$	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	Amt_{carbon}	n_1	Q_{nt}	Tc_{System}
0.0102		0	1.3151	2 .3310	4.						53.7947	3	13.341	117.97
0.0104		0	1.1091	1.9682	2.7103	4.					47.6832	4	14.320	112.21
0.0105		0	1.1091	1.9682	2.7103	4.					−	4	−	109.19
0.0106		0	0.9655	1.7155	2.3641	2.9499	4.				43.1665	5	14.962	105.46
0.0107		0	0.8587	1.5279	2.1071	2.6305	3.1146	4.			39.7473	6	15.414	101.20
0.0108		0	0.7092	1.2654	1.7478	2.1841	2.5879	2.9670	3.3266	4.	34.9630	8	16.010	95.69

Table 7.5 The most economical and optimal system cost, total number of replenishment cycles carried out, amount of emission and amount for replenishment have been calculated with changing tax paid on each unit of carbon emitted

$\downarrow \tau$	$\rightarrow \mathbf{t_i}$	$\mathbf{t_0}$	$\mathbf{t_1}$	$\mathbf{t_2}$	$\mathbf{t_3}$	$\mathbf{t_4}$	$\mathbf{t_5}$	$\mathbf{t_6}$	$\mathbf{t_7}$	$\mathbf{t_8}$	$\boldsymbol{Amt_{carbon}}$	$\boldsymbol{n_1}$	$\mathbf{Q_{nt}}$	$\boldsymbol{Tc_{System}}$
0.020		0	1.6399	4							150.47	2	1.6632	96.65
0.021		0	1.3151	2.3310	3						141.58	3	13.341	96.65
0.022		0	1.1091	1.9682	2.7103	4.					129.08	3	14.320	96.65
0.023		0	0.7092	1.2654	1.7478	2.1841	2.5879	2.9670	3.3266	4.	72.4563	8	16.010	96.65
0.024		0	0.7092	1.2654	1.7478	2.1841	2.5879	2.9670	3.3266	4.	72.4563	8	16.010	96.65

Table 7.6 The most economical and optimal system cost, total number of replenishment cycles carried out, amount of emission and amount for replenishment have been calculated with changing carbon cap.

$\downarrow Co_{2_{Cap}}$	$\rightarrow t_i$	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	Amt_{carbon}	n_1	Q_{nt}	Tc_{System}
160		0	1.3151	2 .3310	4.						53.7973	3	13.341	119.29
200		0	1.1091	1.9682	2.7103	4.					47.6832	4	14.320	112.21
235		0	0.9655	1.7155	2.3641	2.9499	4.				43.1684	5	14.962	104.61
245		0	0.8587	1.5279	2.1071	2.6305	3.1146	4.			39.7473	6	15.414	101.92
255		0	0.7758	1.3823	1.9078	2.3828	2.8224	3.2350	4.		37.0886	7	15.446	98.85
260		0	0.7092	1.2654	1.7478	2.1841	2.5879	2.9670	3.3266	4.	34.9631	8	16.017	96.65

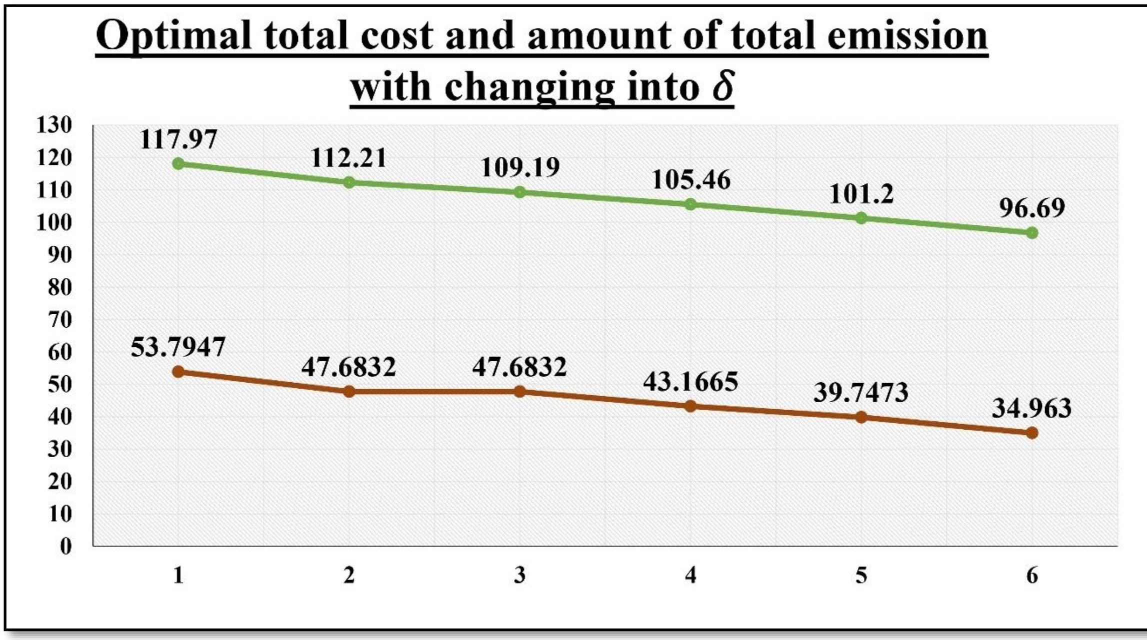


Figure 7.4 Show the effect of change in δ on the overall optimal total cost and amount of total emission of the system.

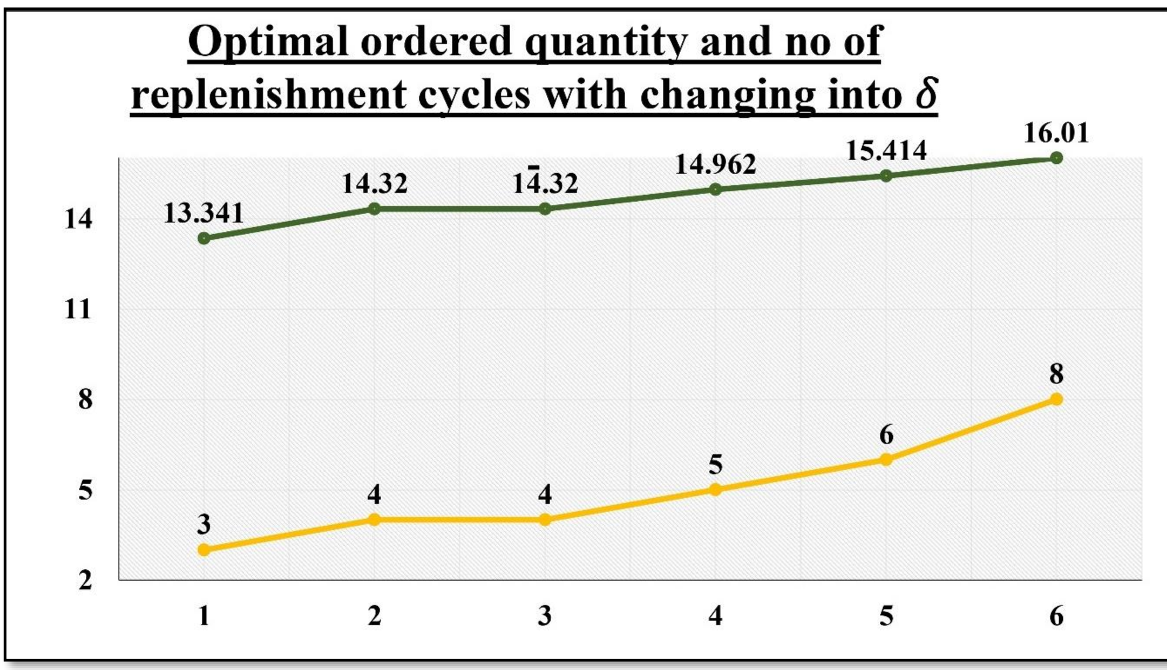


Figure 7.5 Show the effect of change in δ on the overall optimal ordered quantity and total number of restocking operations conducted of the system.

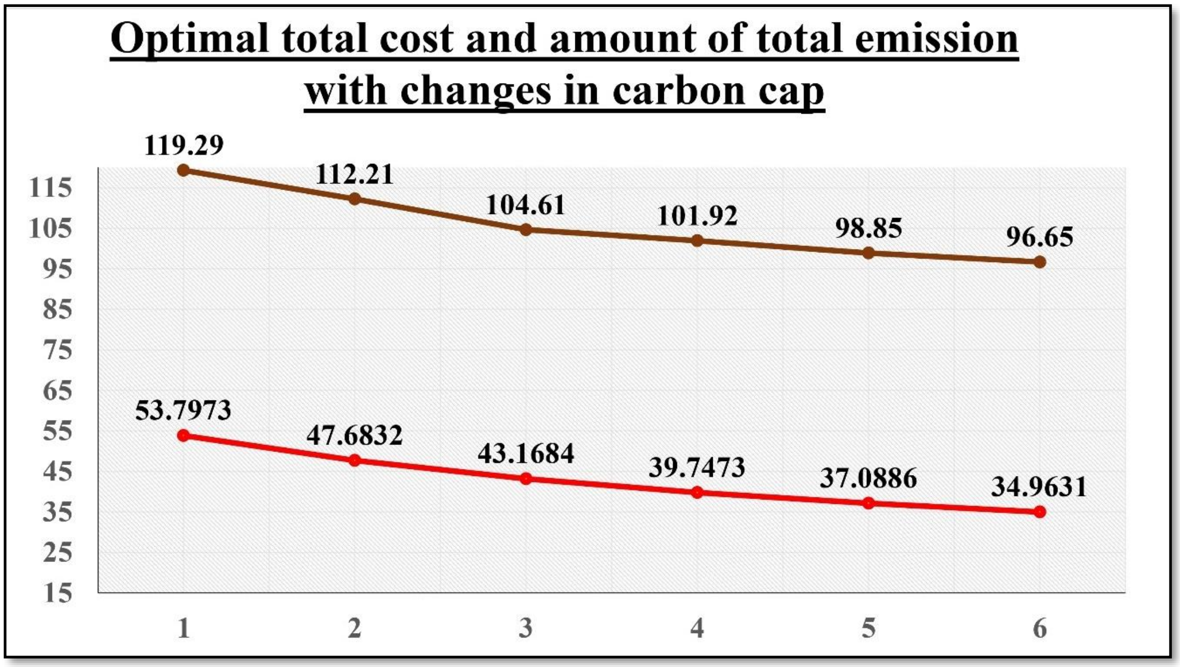


Figure 7.6 Demonstrate the impact of change in cap on overall best possible total cost and amount of total emission of the system.

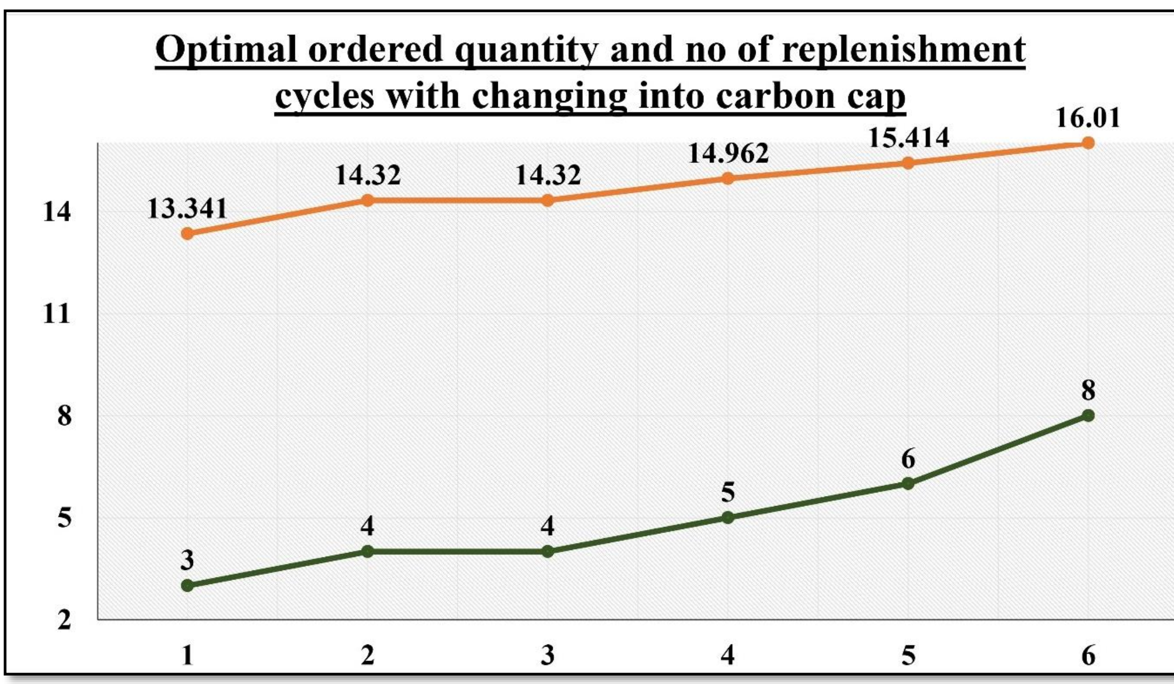


Figure 7.7 Show the effect of altering in emission cap on overall best possible ordered quantity and no of replenishment cycles of the system.

The sensitivity analysis results offer valuable insights into how variations in parameters impact key aspects of the system under the cap-and-trade framework. By examining the effects of changes in variables such as the carbon cap and trade credit rate, we gain a deeper understanding of their influence on critical performance metrics such as system cost, carbon emissions, replenishment cycles, and order quantities. These insights can help decision-makers optimize their strategies for managing carbon emissions while balancing economic considerations. For example, our analysis may reveal trade-offs between minimizing system costs and reducing environmental impact, highlighting the importance of carefully selecting parameter values to achieve desired outcomes.

In comparison to past studies, our findings may align with established trends or provide new perspectives on the dynamics of carbon policies and inventory management. By identifying similarities and differences, we can enrich our understanding of the factors driving system behavior and inform future research directions. Overall, the sensitivity analysis results contribute valuable insights that can enhance decision-making and policy formulation in the context of carbon management and inventory optimization.

7.7 Conclusion

In conclusion, this chapter provides a comprehensive examination of the optimization of a two-stage supply management system within a finite planning horizon, with a particular focus on the implications of decentralized and centralized carbon policies. Businesses, under increasing pressure due to stringent carbon regulations, are seeking strategies to reduce carbon emissions throughout their supply chains while ensuring sustainable development. The two primary approaches explored in this chapter, namely decentralized carbon taxation (Carbon Tax) and centralized emissions trading (cap-and-trade), represent contrasting methods for incentivizing emission reductions.

Through meticulous analysis and modeling, we have shed light on the intricate interactions between carbon policies, supply chain costs, and environmental sustainability. Our investigation has revealed that while both carbon policies aim to curb emissions and promote sustainability, they do so through different mechanisms that lead to distinct outcomes in terms of overall supply chain costs. Specifically, our findings indicate that the decentralized carbon tax policy often results in higher overall costs compared to the centralized cap-and-trade policy. This discrepancy can be attributed to the direct imposition of carbon taxes on emissions in the decentralized approach, whereas the cap-and-trade system introduces a market-based mechanism for allocating emission allowances.

However, despite differences in cost implications, both carbon policies demonstrate consistent performance in optimizing replenishment cycles and achieving reductions in carbon emissions. Sensitivity analyses conducted as part of this research underscore the significance of parameter adjustments in influencing projected emissions and costs under both carbon regimes. Furthermore, fundamental operational metrics such as order quantity and total emissions exhibit stability across different carbon policy scenarios, highlighting the robustness of the optimized supply chain strategies.

Enhanced with Real-World Applications & Carbon Trade Discussion Decentralized supply chains have emerged as a transformative force in modern inventory management, offering greater flexibility and efficiency compared to traditional centralized systems. This thesis explores decentralized supply chain inventory models under a finite planning horizon, leveraging blockchain technology and game-theoretic approaches to optimize decision-making.

While existing research highlights the benefits of decentralization, its real-world applicability remains an area of interest. One prominent application is in pharmaceutical supply chains, where decentralized models can address issues such as drug shortages, counterfeit prevention, and demand uncertainty. Blockchain technology, for instance, has been implemented in IBM's Food Trust and Modum's pharmaceutical supply chain system, improving transparency and traceability.

Furthermore, this study also considers the carbon trade mechanism, a critical aspect of sustainable supply chains. While the thesis discusses carbon tax policies, cap-and-trade systems have been largely overlooked. Carbon trading allows companies to buy and sell emission allowances, incentivizing greener supply chain operations. By integrating carbon trade policies, this research can provide a more comprehensive view of the financial and environmental impact of decentralized inventory management.

Implications for Indian Industries & Policy Considerations This model has significant implications for various Indian industries, particularly in pharmaceuticals, textiles, and fast-moving consumer goods (FMCG). The pharmaceutical industry, which often faces demand fluctuations and regulatory challenges, can benefit from decentralized supply chain models by improving real-time demand responsiveness and reducing stockouts of life-saving drugs. Similarly, the textile industry, which operates in a highly fragmented supply chain structure, can leverage decentralized inventory management to optimize raw material procurement and production schedules, ensuring cost efficiency and sustainability.

From a policy perspective, the Indian government's push for sustainable manufacturing and supply chain digitization aligns with the principles of this model. The introduction of carbon tax policies and emission trading schemes under India's National Action Plan on Climate Change (NAPCC) highlights the need for adaptive supply chain strategies. This research provides valuable insights for policymakers by demonstrating how decentralized decision-making can contribute to emission reductions while maintaining supply chain efficiency.

In addition to providing insights into the comparative effectiveness of carbon policies, this chapter contributes to the broader discourse on sustainable supply chain management. By elucidating the complex interplay between environmental objectives, regulatory frameworks, and operational dynamics, our research offers practical guidance for businesses navigating the transition towards more sustainable supply chain practices. Moreover, the methodologies and insights presented herein can serve as valuable tools for decision-makers seeking to align environmental stewardship with operational efficiency in today's increasingly carbon-constrained world.

In summary, this research advances our understanding of how different carbon policies influence supply chain dynamics and underscores the importance of integrated approaches to achieving environmental and economic sustainability. By elucidating the trade-offs and synergies inherent in decentralized and centralized carbon policies, this chapter empowers businesses to make informed decisions that promote both environmental stewardship and long-term competitiveness in the global marketplace.

While this chapter provides valuable insights into optimizing supply chain management under carbon pricing policies, several limitations should be acknowledged. Firstly, the model's assumptions, though necessary for mathematical tractability, may oversimplify real-world complexities. Additionally, the reliance on accurate and comprehensive data poses challenges, as data availability varies across industries and regions. Moreover, the scope of analysis, focusing on specific supply chain stages, may overlook broader interactions and systemic effects. Finally, the practical challenges of implementing carbon policies in diverse organizational contexts remain unaddressed.

Moving forward, future research can explore dynamic modeling, risk integration, and multi-objective optimization for more comprehensive insights. Empirical studies can validate theoretical findings, and policy evaluations can guide stakeholders. Additionally, extending the analysis to include two- and three-echelon supply chains, as done by (Mishra et al., 2024), holds promise for deeper understanding and practical relevance.

Chapter 8 Final Reflections and Future Directions

8.1 Key Findings and Contributions

This chapter aimed to explore decentralized supply chain inventory models under a finite planning horizon through various angles, including blockchain technology, multi-period models, game-theoretic approaches, and comparative analysis of decentralized versus centralized systems. Each chapter was designed to build on these objectives, providing a comprehensive view of decentralized supply chain management.

Chapter 1 introduced the foundational concepts of inventory management, supply chain dynamics, centralization, decentralization, and blockchain technology. It laid out the importance of decentralized supply chains and highlighted the need to explore innovative solutions such as blockchain and game theory to address inventory management challenges. The insights gathered here provided the motivation for the subsequent research and set the stage for examining decentralized models under finite planning horizons.

The Chapter 2 literature review offered an in-depth analysis of various methodologies, case studies, and empirical research related to decentralized supply chain management. It identified gaps in the existing literature, particularly the lack of focus on decentralized supply chains utilizing blockchain within a finite planning horizon.

Chapter 3 developed a decentralized three-echelon model incorporating blockchain technology within a finite planning horizon. The results of this model demonstrated that integrating blockchain significantly enhances transparency, coordination, and trust between suppliers, manufacturers, and retailers. Numerical examples and sensitivity analysis revealed that using blockchain not only reduces information asymmetry but also optimizes inventory levels, thereby minimizing total costs. This chapter concluded that blockchain could be a transformative tool for managing decentralized supply chains more effectively.

In Chapter 4, a two-echelon decentralized supply chain model was proposed and analyzed. It focused on the interplay between inventory replenishment schedules and blockchain's role in improving information flow. The numerical analysis showed that adopting blockchain in a multi-period inventory model allows for better synchronization between different echelons, optimizing inventory levels and reducing costs. The sensitivity analysis provided managerial insights into how blockchain-driven inventory policies could adapt to changes in market demand and supply conditions, underscoring its potential in real-world applications.

Chapter 5 investigated how blockchain technology could enhance manufacturer profitability by addressing information sensitivity in the supply chain. The mathematical model demonstrated that blockchain's transparency features enable retailers to make more informed decisions, positively impacting the manufacturer's profitability.

Numerical illustrations indicated that effective information sharing via blockchain leads to optimized inventory levels and reduced costs. This chapter concluded that blockchain enhances the adaptability of supply chains, particularly in environments with fluctuating demand.

Chapter 6 explored the use of a Stackelberg game approach, where a retailer acts as the leader and the manufacturer follows in a decentralized supply chain model. Using linear differential equations, the chapter provided a detailed analysis of optimal inventory levels within a finite planning horizon. Numerical illustrations demonstrated the effectiveness of the Stackelberg model in achieving cost minimization for both the retailer and manufacturer. Furthermore, a comparative analysis with Benkherouf's centralized model highlighted that while centralized models could be cost-effective in some scenarios, the Stackelberg model offers strategic flexibility, especially in decentralized environments. Sensitivity analysis further strengthened managerial insights, emphasizing the benefits of decentralized decision-making.

Chapter 7 conducted a comparative analysis of decentralized and centralized inventory models under carbon tax and cap-and-trade policies. The chapter formulated mathematical expressions to assess carbon emissions in both centralized and decentralized scenarios. Numerical examples demonstrated that decentralized models, though sometimes costlier, offer flexibility and adaptability in managing carbon regulations. The findings indicated that while centralization might optimize costs, decentralization provides a more responsive and environmentally sustainable approach.

This thesis concludes that decentralized supply chains, when supported by blockchain technology and game-theoretic strategies, exhibit superior flexibility, transparency, and adaptability to market fluctuations compared to centralized models. The results from each chapter collectively emphasize that decentralized supply chain management, facilitated by blockchain technology and strategic decision-making frameworks like the Stackelberg game, can effectively address the complexities of modern inventory management.

8.2 Real-World Applications and Industry Implications

The proposed models are particularly relevant to industries where inventory optimization is critical under uncertain demand conditions. For instance, in the Indian pharmaceutical industry, which faces frequent disruptions due to fluctuating demand, regulatory constraints, and supply chain inefficiencies, implementing decentralized inventory models integrated with blockchain can enhance transparency, reduce lead times, and minimize wastage. By addressing challenges such as counterfeit drugs and demand-supply mismatches, blockchain-enabled decentralized supply chains can improve efficiency while ensuring regulatory compliance.

Furthermore, in sectors like agriculture and manufacturing, where supply chains span multiple tiers, decentralized models provide resilience against disruptions by improving coordination and information sharing among stakeholders. This research provides a roadmap for policymakers to design adaptive regulations that support decentralized decision-making while promoting sustainability in industries with complex supply networks.

8.3 Policy Implications and Sustainability Considerations

A significant contribution of this research lies in its discussion of carbon policies and their integration into inventory management. With rising global concerns over carbon emissions, industries are increasingly pressured to adopt sustainable supply chain practices. This thesis highlights the comparative benefits of carbon tax and cap-and-trade systems within decentralized and centralized supply chain models, offering insights into how businesses can comply with evolving environmental regulations.

Aligning with Sustainable Development Goal 12 (Responsible Consumption and Production), this research underscores the importance of sustainable inventory management practices. By optimizing replenishment cycles and reducing excess stock, the proposed models contribute to minimizing waste and promoting resource efficiency. Blockchain integration further enhances sustainability by ensuring traceability and accountability in carbon emissions monitoring.

8.4 Practical Takeaways for Supply Chain Managers

From a managerial perspective, this research provides actionable strategies for optimizing decentralized supply chains. Key takeaways include:

- Leveraging blockchain to enhance information visibility and coordination across supply chain tiers.
- Implementing game-theoretic strategies to improve decision-making in decentralized settings.
- Adapting inventory policies to incorporate carbon pricing mechanisms and regulatory compliance.
- Conducting sensitivity analyses to assess the impact of demand fluctuations and policy changes on supply chain costs and emissions.

These insights empower supply chain managers to make data-driven decisions that align with both economic and environmental objectives, fostering a balance between profitability and sustainability.

8.5 Future Research Directions and Industry Collaborations

While this study offers a strong theoretical foundation, future research should focus on validating these models with empirical data from real-world supply chains. Collaborations with industry stakeholders, particularly in pharmaceuticals, agriculture, and manufacturing, can provide deeper insights into the practical challenges of implementing decentralized and blockchain-integrated inventory systems.

Additionally, expanding the research to incorporate machine learning techniques for demand forecasting and risk mitigation can further enhance the adaptability of decentralized models. Exploring multi-objective optimization frameworks that balance cost efficiency, sustainability, and resilience will be crucial in advancing supply chain research in the context of Industry 4.0.

In conclusion, this thesis lays a foundation for a new paradigm in supply chain management—one that embraces decentralization, digitalization, and sustainability. As industries navigate an increasingly complex and uncertain global landscape, integrating blockchain technology and adaptive carbon policies into supply chain models will be instrumental in achieving long-term efficiency and resilience. Policymakers, industry leaders, and researchers must work collaboratively to drive innovations that transform supply chains into agile, transparent, and environmentally responsible networks.

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List of Publication

S.NO	PAPER NAME	CURRENT STATUS	SCOPUS /SCI	ISBN/ ISSN	PUBLICATION HOUSE	JOURNAL/CONFERENCE/BOOK CHAPTER	DOI
1	Blockchain-Enhanced Inventory Management in Decentralized Supply Chains for Finite Planning Horizons	Published	Scopus	ISSN 1269-6935	International Information and Engineering Technology Association (IIETA)	Journal Européen des Systèmes Automatisés	10.18280/jesa.570125
2	Inventory Models Under Carbon Tax and Cap-and-Trade Policies: A Comparative Analysis of Decentralized and Centralized Approaches	Published	Scopus	ISSN 1269-6935	International Information and Engineering Technology Association (IIETA)	Journal Européen des Systèmes Automatisés	10.18280/jesa.570220
3	Optimizing Inventory Management with Seasonal Demand Forecasting in a Fuzzy Environment	Published	Scopus	ISSN 1269-6935	International Information and Engineering Technology Association (IIETA)	Journal Européen des Systèmes Automatisés	10.18280/jesa.570546
4	Enhancing Supply Chain Efficiency with Blockchain: Addressing Information Sensitivity for Increased Manufacturer Profitability	Published	Scopus	ISSN 2405-8963	ELSEVIER	IFAC Symposium on Information Control Problems in Manufacturing – INCOM 2024	10.1016/j.ifacol.2024.09.220
5	Predicting Delivery Outcomes in Supply Chain Management Using Machine Learning: A Random Forest Classifier Approach	Published	UGC Care Indexed	ISSN: 2583-1062	Arts and Science Press Pte. Ltd.	International Journal of Progressive Research in Engineering Management	10.58257/IJPREMS36629
6	The impact of information sharing on EOQ and total cost of IoT and blockchain-based inventory management: A comparative analysis.	Published	Peer review Journal		Arts and Science Press Pte. Ltd.	Industrial Management Advances	10.59429/ima.v2i2.9050
7	Exploring Decentralized Supply Chain Models: A Literature Review	Published	NO	ISBN: 978-81-19334-65-0	Lovely Professional University, Punjab	Advancements in Computational Mathematics	
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10	Optimizing Inventory Management with a Stackelberg Game Approach: A Retailer-Manufacturer Model	Accepted	Scopus		Springer Nature	International Conference on Emerging Trends in Business Analytics and Management Science (BAMS-ORSI 2024)	Submission No - 65
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16	Predictive Analysis of Wholesale Customer Purchases Using Machine Learning Models	Published	UGC Care Indexed	ISSN: 2583-1062	Arts and Science Press Pte. Ltd.	Supply Chain Research Art and Science Press	SCR-9929
17	Exponential Backordering Inventory Model Addressing Shortages in Finite Planning Horizons	Under review	Scopus	ISSN 2405-8963	ELSEVIER	11th IFAC Conference on Manufacturing Modelling, Management and Control (IFAC MIM2025)	Submission No. 616
18	Triangular vs. Trapezoidal vs. Heptagonal Fuzzy Models: Optimizing Blockchain-Enabled Profitability Under Uncertainty.	Under review	Scopus	ISSN 2405-8963	ELSEVIER	5th Modeling, Estimation and Control Conference (MECC 2025)	Submission number: 226

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डायरी संख्या/Diary Number: 41153/2024-CO/L

आवेदन की तिथि/Date of Application: 25/12/2024

प्राप्ति की तिथि/Date of Receipt: 25/12/2024

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Published Journal Paper-1

IIETA International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Journal Européen des Systèmes Automatisés
Vol. 57, No. 1, February, 2024, pp. 263-272
Journal homepage: http://iieta.org/journals/jesa

Blockchain-Enhanced Inventory Management in Decentralized Supply Chains for Finite Planning Horizons

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https://doi.org/10.18280/jesa.570125

Received: 11 December 2023
Revised: 24 January 2024
Accepted: 2 February 2024
Available online: 29 February 2024

Keywords:
decentralized supply chain, blockchain technology, finite planning horizon, sensitivity analysis

ABSTRACT

This research introduces a decentralized supply chain optimization model that incorporates blockchain technology. The model, implemented through an optimized iterative method, integrates ordering, holding, and purchasing costs to offer a comprehensive view of total costs for both retailers and suppliers. The model's uniqueness and optimality are demonstrated through theoretical analysis, highlighting the optimal ordering interval as the sole solution to the derived equation. Employing an algorithmic methodology, optimal replenishment schedules are efficiently calculated using Wolfram Mathematica 13.0. A numerical example and sensitivity analysis illustrate the impact of key parameters on replenishment cycles, order quantity, and costs, encompassing wholesale prices, demand uncertainty, and holding/ordering costs. Managerial insights derived from sensitivity analysis guide decision-makers in optimizing supply chain management, emphasizing strategies such as wholesaler price balance and strategic blockchain information management. In essence, this research contributes to an enhanced understanding of decentralized supply chain models with blockchain, providing a systematic decision-making optimization approach for increased efficiency and resilience.

1. INTRODUCTION

The disruptive impact of COVID-19 underscores the vulnerability of centralized supply chain systems. Disruptions originating from a single source can have far-reaching consequences, impacting the global supply of goods and services. Decentralized supply chain models are emerging as robust solutions during this critical period [1, 2]. Through a multi-participant approach, these models distribute decision-making and resources among various participants, ensuring that a small issue in one part does not affect the entire system. In the aftermath of this pandemic, the flexibility, robustness, and power of a decentralized supply chain have become apparent, eliminating dependency on a single source.

In today's global business environment, supply chains have become increasingly complex and dispersed, leading to a need for better information sharing, visibility, and coordination among multiple stakeholders. The literature on decentralized supply chain models addresses the challenges and opportunities associated with managing these complex supply chains. Several studies have highlighted the importance of decentralized supply chain models in improving operational efficiency, reducing costs, and enhancing customer satisfaction. One key advantage of decentralized supply chain models is the greater autonomy they provide to individual subsidiaries or locations within a business [3].

In parallel, the advent of blockchain technology has transformed various industries by revolutionizing the storage and verification of transactions and data. One particular area where blockchain has shown great potential in the realm of supply chain management [4]. As supply chains become more complex and globalized, there is an increasing demand for innovative solutions that can improve traceability, efficiency, and transparency.

This research paper delves into the transformative journey of integrating Blockchain Technology into our decentralized supply chain. We aim to analyze the potential impact of this technology, especially within the finite planning horizon, to effectively manage our supply chain during disruptions [5]. Like a well-crafted recipe, this paper blends theory, case study, and practical insights, showcasing how blockchain can become a powerhouse for our decentralized supply chain. Together, feedback on a journey to explorer this new variant supply chain management, where innovative solutions powered by blockchain pave the way for a more liberated, silent, and efficient future.

In addition, this study incorporates quantitative analysis by employing numerical examples extracted from a secondary dataset compiled from a diverse array of [6, 7]. Our analysis is firmly grounded in a robust foundation. Utilizing an optimal iterative method, we systematically investigate the model's performance under varying wholesale prices. The primary objective is to contribute valuable insights into the optimization potential of the model across a spectrum of scenarios. This study not only extends the current body of research findings but also expands the examination of the model's behavior in response to diverse conditions of wholesale prices.

Published Journal Paper-2

IIETA International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Journal Européen des Systèmes Automatisés
Vol. 57, No. 2, April, 2024, pp. 505-516
Journal homepage: http://iieta.org/journals/jesa

Inventory Models Under Carbon Tax and Cap-and-Trade Policies: A Comparative Analysis of Decentralized and Centralized Approaches

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https://doi.org/10.18280/jesa.570220

Received: 2 February 2024
Revised: 8 April 2024
Accepted: 17 April 2024
Available online: 28 April 2024

Keywords:
decentralized and centralized, linear demand time and inventory sensitive, finite planning horizon, carbon tax, and emissions trading schemes

ABSTRACT

Due to tougher carbon restrictions and regulations, businesses have been researching approaches to decrease the amount of carbon emissions throughout the inventory supply process and achieve sustainable development. The two most common approaches are (i) decentralized, which involves implementing a carbon tax or cost for emitting carbon, and (ii) centralized, which includes introducing an emissions trading (cap-and-trade) mechanism. Within this research, we optimize a two-stage supply management system under FPH(finite planning horizon) while taking into consideration these two policies. Using a linear time and inventory-dependent demand model, we investigated various techniques within a specific time frame. We created and solved two distinct MINLP (Mixed Integer Non-Linear Programming) approaches for each carbon strategy. These models can assist businesses/firms in determining the minimum overall cost, optimal order quantity, optimal replenishment time, and replenishment cycles. Using mathematical tools, our sensitivity evaluations indicate that organizations can reduce overall projected emissions and costs by making parameter variations under both carbon regimes. We additionally showed that while both approaches optimize the overall supply chain cost, the order quantity and total emissions remain constant.

1. INTRODUCTION

Inventory management stands as a cornerstone of efficient supply chain operations, orchestrating the seamless flow of goods from procurement to distribution. Kumar et al. [1] emphasized its broad spectrum of activities, spanning procurement, storage, distribution, and replenishment, all aimed at maintaining optimal inventory levels. The significance of effective inventory management lies in its ability to strike a delicate balance between meeting customer demands and controlling costs.

Nagaraju et al. [2] underscored the central principle of ensuring the availability of the right inventory, at the right time, in the right place, and at the right cost. This principle is further accentuated by the pivotal role inventory plays as a substantial business asset, influencing financial resources and operational liquidity [3]. Moreover, efficient inventory management not only fulfills customer expectations but also aligns with broader organizational goals of sustainability and profitability, through cost control and waste reduction measures.

However, the traditional paradigm of inventory management faces new challenges in an era marked by heightened environmental consciousness and regulatory scrutiny. The emergence of carbon policies, aimed at curbing greenhouse gas emissions and promoting environmental sustainability, introduces a layer of complexity to inventory management strategies. In this context, the integration of carbon policies into inventory management practices becomes not only a regulatory necessity but also a strategic imperative for businesses aiming to reconcile operational efficiency with environmental responsibility.

Human-caused emissions, notably carbon dioxide, are responsible for global warming, which poses a serious threat to the climate and the existence of humanity. To address this issue, regulatory bodies and policymakers throughout the world have established carbon regulations targeted at preventing pollution. Carbon policies can be listed into different kinds: carbon tax/cost legislation, carbon cap-and-trade regulation, and other policies [4].

Under the carbon tax program, every unit of carbon dioxide emitted is subject to a penalty. This program is a fee imposed by regulatory agencies or decision-makers on firms for their carbon emissions during their processes. However, businesses and organizations can keep releasing carbon dioxide as needed, they are required to pay charges for each unit of carbon emitted [5].

Decision-makers enforce the carbon cap [6] framework under the Carbon Cap-and-Trade regime. The government or decision-makers set a carbon cap or limit and allow firms to buy or sell carbon credits under this policy. Whenever a company generates less carbon than the limit, it can sell its leftover carbon credits to other businesses and organizations.

Published Journal Paper-3

IIETA International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Journal Européen des Systèmes Automatisés
Vol. 57, No. 4, Month, 2024, pp. 1091-1102
Journal homepage: http://iieta.org/journals/jesa

Optimizing Inventory Management with Seasonal Demand Forecasting in a Fuzzy Environment

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https://doi.org/10.18280/jesa.570416

Received: 16 May 2024
Revised: 26 July 2024
Accepted: 9 August 2024
Available online: 27 August 2024

Keywords:
supply model, shortages, forecasting demand, artificial intelligence, machine learning, deterioration, carbon pollution policy, finite planning horizon

ABSTRACT

This study explores an inventory management model in today's business landscape, where organizations increasingly rely on Machine Learning for demand-driven stock control. The proposed model accounts for imperfect and deteriorating products within a fuzzy environment, allowing for shortages and partial backlogging. Degradation rates and faulty percentages are classified as fuzzy variables since they are unpredictable and impacted by undefined conditions. The goal is to calculate the appropriate replenishment cycle and ordering quantity while reducing the optimal overall cost, including carbon pollution costs, within a constrained planning horizon. The defuzzification technique uses the sign distance approximation technique. Leveraging Machine Learning, the study utilizes a seasonal demand forecasting methodology. A numerical illustration supports the mathematical approach by demonstrating its capacity to estimate demand for deteriorating products. This facilitates optimized inventory management aligned with forecasted demand. A comparative examination emphasizes the positive aspects of AI learning-based forecasting systems over determined demand circumstances. Sensitivity analysis provides insights into the impact of various parameters on optimal solutions, contributing valuable managerial perspectives.

1. INTRODUCTION

In the ever-evolving global market landscape, the intricate dance between seasonal and weather conditions exerts a profound influence on consumer demand, a cornerstone variable that presents multifaceted challenges to efficient inventory management across diverse industries [1]. The ebb and flow of seasonal demand, shaped by events such as festivals and climatic factors, introduces uncertainties and complexities into consumer purchasing behaviours, necessitating a sophisticated approach to inventory control [2]. While conventional inventory models often hinge on deterministic demand assumptions, the real-world scenario unfolds with variations in product demand that adhere to distinct seasonal patterns. So, in our study, we have applied the time series algorithm to forecast seasonal demand.

The strategic imperative of effective demand prediction emerges as a key solution, offering the potential to refine inventory management strategies, curtail superfluous costs, and elevate overall customer service [3]. Leveraging machine learning (ML): with its advanced predictive capabilities, particularly through Decision Tree-based Algorithms, stands out as a transformative tool in achieving precise and accurate seasonal demand forecasts [4]. This paper delves into the convergence of seasonal demand dynamics, imperfect deteriorating products, and the contemporary imperative of considering carbon emissions in inventory systems. The intrinsic deterioration of physical products over time, be it during transit or storage, is a ubiquitous challenge across various industries [5]. Items such as fruits, medicines, flowers, foodstuffs, and vegetables are susceptible to decay during their holding and in-transit periods. This study acknowledges the deterministic approach traditionally applied to deterioration rates in inventory models but contends that real-world uncertainty demands a more sophisticated treatment [6]. To address this, the model introduces a fuzzy variable for deterioration rates, acknowledging the uncertainty in their precise estimation.

Furthermore, the quantity of defective products, a critical consideration in inventory management, is recognized as another fuzzy variable due to unpredictable factors such as manufacturing defects, man-handling issues, and in-transit damage [7]. The study emphasizes the pressing concern of escalating carbon emissions in the modern era, driven by industrialization and contributing significantly to climate change. This prompts a paradigm shift in inventory system design, where scholars and organizations now focus on reducing the total cost, integrating considerations for carbon emissions. This research not only acknowledges permissible shortages but also accounts for partial backlogging, recognizing that not every consumer accepts delayed deliveries. By addressing these multifaceted challenges, the study endeavours to bridge gaps in existing literature concerning the impact of demand Predictions on inadequate decaying products. Two primary research questions guide this exploration: (a) How do AI based demand prediction

Published Journal Paper-4

INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)
(Int Peer Reviewed Journal)
Vol. 04, Issue 11, November 2024, pp : 1107-1115
www.ijprems.com
editor@ijprems.com
e-ISSN : 2583-1062
Impact Factor : 7.001

PREDICTING DELIVERY OUTCOMES IN SUPPLY CHAIN MANAGEMENT USING MACHINE LEARNING: A RANDOM FOREST CLASSIFIER APPROACH

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DOI: https://www.doi.org/10.58257/IJPREMS36629

ABSTRACT

In the modern globalized economy, timely deliveries are crucial for effective supply chain management. Delivery delays can cause disruptions, increased costs, and customer dissatisfaction, while early deliveries may lead to overstocking and higher holding costs. This study applies machine learning techniques, specifically a Random Forest Classifier, to predict delivery outcomes—classified as early, on-time, or delayed—using a dataset of 15,549 records with 41 features. Addressing the challenge of class imbalance in supply chain data, where delayed and on-time deliveries are underrepresented, the study incorporates class balancing techniques such as SMOTE along with advanced feature engineering and data preprocessing. The model achieved an overall accuracy of 57.7%, with strong performance in predicting early deliveries (F1-score of 0.73 and recall of 95%). However, the model showed limitations in identifying delayed (F1-score of 0.20, recall of 13%) and on-time deliveries (F1-score of 0.00, recall of 0%). These results highlight the need for further improvements in handling class imbalance and enhancing the predictive accuracy for critical outcomes like delayed deliveries. Future work may involve incorporating additional features such as real-time traffic data and exploring alternative machine learning algorithms to better address class imbalances and improve overall model performance.

Keywords: Machine Learning, Random Forest Classifier, supply chain management,

1. INTRODUCTION

In the global economy of today, supply chain management is growing more complicated as businesses require fast and efficiency logistics to fulfill customer needs. One of the most critical challenges in this field is delivery, where timely deliveries are absolutely crucial as final outcome which directly affects operational costs and customer satisfaction even impacting overall performance of supply chain【1】. This disruption can exacerbate the cost of delays quickly with increased costs, incomplete inventory and damage to reputation. Conversely, early deliveries induce excess inventory holding costs and wasted resources【2】. As a result, it has never been more important to predict with precision when deliveries may arrive early, on-time or be delayed - in order to manage and hedge the risks of supply chain operations.

Machine learning (ML) has recently advanced solutions in predicting delivery results. Machine learning makes it possible to identify intricate patterns in the data and predict delivery behaviour better than traditional statistical forecasting on large datasets. Such predictions help in aiding supply chain managers to become proactive, i.e., changing inventory levels or re-routing shipments before it becomes a problem improving decisions and hence enhancing overall performance of the supply chain【4】. Machine Learning approaches like Random Forest, Gradient Boosting Machines (GBM), and Support Vector Machines (SVM) can forecast the delivery times as well as inventory levels better with some accuracy [5].

Random Forest classifier is used in this study to predict delivery outcomes from a data set with 41 features on which there are total number of records =15,549 i.e variables like payment type, profit per order and sales per customer, shipping mode etc., corresponding to each record the date at which that order was placed. The target variable binarises it to deliveries are, Delayed (-1), Ontime (0) and Early(1). The study would conveniently model whether a delivery really delivers or not in order to provide the decision-making process for supply chain management with prognostications about future deliveries.

It also tackles challenges of class imbalance, something that is typically responsible for skewing machine learning models toward the majority class. This is an example of how the training sets for on-time and delayed deliveries in supply chain data are underrepresented vs early committing replications, it will definitely make any future predictions biased. In this work, class balancing techniques were employed to address the issue and model performance was evaluated based on different evaluation metrics like precision-recall-F1 score.

Published Journal Paper-5

Industrial Management Advances (2024) Volume 2 Issue 2
doi: 10.59429/ima.v2i2.9050

ISSN: 3029-1674 (O)

RESEARCH ARTICLE

The impact of information sharing on EOQ and total cost of loT and blockchain-based inventory management: A comparative analysis

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ABSTRACT

With the changing scene of supply chain management, the implementation of intelligent technologies like the Internet of Things (IoT) and blockchain has added new dimensions to inventory optimisation. The current paper offers comparative mathematical analysis of IoT-based and blockchain-based models for inventory under a finite planning horizon, based on principal performance parameters: Economic Order Quantity (EOQ) and Total Cost (TC). Sensitivity analysis is performed to analyze the effect of key parameters like data precision, demand, and implementation expense on both the models. Results show that IoT-based model delivers higher EOQ but lower costs when data precision is increased and thus is optimal for agile forecast-oriented environments. Conversely, the blockchain-based model, although more expensive to set up, provides more stability and tracing capabilities in decentralized and trust-sensitive supply chains. A realistic example is provided to show the cost-performance trade-offs of both models under normal business circumstances. The findings inform decision-makers in choosing technology as a function of strategic objectives, and the research concludes with suggestions on the development of hybrid models and field testing.

Keywords: IoT-based inventory management; blockchain-enabled supply chain; economic order quantity (EOQ); total cost (TC) optimization; sensitivity analysis

1. Introduction

The exponential growth of product diversity and an expanding customer base have resulted in an increase in the complexity of inventory management in the modern era. Modern supply chains often exhibit intricate dynamics that are beyond the reach of traditional methods, like the Economic Order Quantity (EOQ) model. This limitation results in inefficiencies, increased operational costs, and suboptimal inventory levels. MSMEs facing distinct operational challenges can take advantage of emerging technologies, such as the Internet of Things (IoT) and blockchain.

Blockchain and IoT have demonstrated tremendous potential for improving visibility, efficiency, and trustworthiness in the supply chain. A wide range of IoT-enabled devices, including sensors and RFID tags, allow businesses to track product movements, shelf lives, and demand fluctuations in real time, providing them with actionable insights for proactive decision-making [1,2] . By contrast, blockchain technology can enhance

ARTICLE INFO
Received: 3 October 2024 | Accepted: 29 November 2024 | Available online: 08 December 2024

CITATION
Jain, P.; Rani, P. The Impact of Information Sharing on EOQ and Total Cost of IoT and Blockchain-Based Inventory Management: A Comparative Analysis.Industrial Management Advances 2024; 2(2): 9050.doi: 10.59429/ima.v2i2.9050

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Published Conference Paper-1

Available online at www.sciencedirect.com

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IFAC PapersOnLine 58-19 (2024) 688–693

Enhancing Supply Chain Efficiency with Blockchain: Addressing Information Sensitivity for Increased Manufacturer Profitability

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Abstract: This paper explores the impact of information-sensitive retailers on a manufacturer's decision to adopt blockchain technology in supply chain management. A mathematical model is developed, considering factors such as retailer information sensitivity, sensitivity costs, and the manufacturer's profit function. The model aims to optimize total profit by setting production quantity and price while accommodating sharing and non-sharing retailers. Analysis of the manufacturer's optimal equilibrium strategy highlights the critical role of information-sensitive retailers in blockchain adoption decisions. Findings suggest that the proportion of such retailers should satisfy specific conditions for optimal outcomes, emphasizing the importance of maintaining a balance in information-sensitive suppliers for efficient information sharing and manufacturing costs. Additionally, the paper conducts a comprehensive sensitivity analysis, varying each parameter to assess its impact on the profit function. A numerical example illustrates the practical implications of the model. The insights derived from the sensitivity analysis provide valuable managerial strategies for enhancing supply chain efficiency and increasing manufacturer profitability.

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Keywords: Blockchain adoption, Information-sensitive retailers, Supply chain management, Manufacturer's profit optimization, Information sharing, Supply chain coordination, Sensitivity Analysis

1. INTRODUCTION

The Supply Chain refers to the interconnected network of organizations, individuals, activities, information, and resources involved in the production, distribution, and delivery of goods and services to customers (Huang et al., 2022). It plays a crucial role in ensuring that products or services are delivered to the right place, at the right time, and in the right condition. Supply chain management has become increasingly complex due to factors such as globalized markets, fragmented supply networks, and diverse regulations. Furthermore, customer expectations have also evolved, demanding greater customization and faster delivery times (Yue, 2008). To meet these challenges and enhance supply chain efficiency, businesses have turned to blockchain technology (Huang et al., 2022).

Blockchain offers transparency, security, and decentralization, making it a promising solution for supply chain management (Wang et al., 2019). Retailers can use blockchain to share real-time information, enhancing visibility and traceability. By ensuring the integrity of shared data, blockchain mitigates risks such as tampering and unauthorized access (Kouhizadeh and Sarkis, 2018). Additionally, blockchain reduces fraud and counterfeit products by eliminating intermediaries (Kouhizadeh and Sarkis, 2018). It provides real-time insights, enabling stakeholders to identify bottlenecks and delays promptly, optimizing production and delivery schedules (Wang et al., 2019).

Existing literature explores blockchain's diverse applications, highlighting its role in sustainability, supply chain coordination, and operational efficiency (Ko et al., 2018; Dong et al., 2022; Hayrutdinov et al., 2020; Huang et al., 2022; Xu et al., 2018). Recent research introduces a decentralized supply chain model utilizing blockchain to enhance transparency and efficiency, demonstrating optimal replenishment schedules and strategies for managing information sensitivity (Mishra et al., 2024).

Our study extends this research by exploring a model that connects mathematical analysis with real-life scenarios. Unlike previous articles focusing on single-case studies, we integrate scenarios where some retailers share information while others do not. We find that increased information sharing among retailers leads to higher profits, driven by trust in blockchain technology for data safety and transparency.

The paper is structured as follows: Section 2 outlines assumptions and notations, Section 3 introduces the model, Section 4 presents numerical examples and sensitivity analysis, Section 5 offers managerial insights, and Section 6 concludes the study and suggests future research directions.

2. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions & Hypothesis

These assumptions serve as foundational elements for our model, providing a structured basis for the

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2024.09.220

Published Book Chapter 1

2-Exploring Decentralized Supply Chain Models: A Literature Review

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Abstract: Modern business landscapes are characterized by complexities and challenges that require decentralized supply chain models. In addition, by sharing risks, decentralization contributes to risk mitigation. Decentralized models also promote collaboration, cooperation, and information sharing among participants. Decentralization enables greater control, flexibility, and responsiveness to fluctuations in the supply and demand for goods and services. Increasing transparency, cooperation, and data exchange among supply chain participants has become easier because of technological developments like blockchain and the Internet of Things. Despite this, there are still challenges associated with coordination and compliance. Using a systematic review of 25 publications between 2004 and 2022, this study investigated the Decentralize supply chain model problem. A review was conducted based on the remanufacturing, dual-channel supply chains, closed-loop supply chains, carbon and green supply chains, inventory management, and decision-making structures, Decentralized inventory models, Stackelberg game models, and comparisons between centralized and decentralized approaches in a paper. A study of the decentralized supply chain model was conducted to

Published Book Chapter 2

Proceeding of National Conference, Friday, 8th November 2024, ISBN: 978-93-5510-642-1

Investigating the Potential of Blockchain Technology in Inventory Management: From Research to Practical Implementation

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Delhi Institute of Rural Development

Abstract

This study investigates the integration of blockchain technology into inventory management to enhance supply chain efficiency and resilience. Focusing on economic order quantity (EOQ) models, the research aims to determine optimal integration strategies and assess blockchain's impact compared to traditional systems. Through a systematic literature review, gap analysis, and model development, the study creates optimization models that incorporate blockchain, with a specific focus on cost efficiency, supply chain coordination, and demand satisfaction. These models are validated using numerical iterative approximation and sensitivity analysis in Wolfram Mathematica 13.0. Case studies in the pharmaceutical industry demonstrate practical applications and highlight the challenges and benefits of blockchain-enhanced inventory management. The findings provide valuable insights and practical guidelines for adopting blockchain technology, offering a framework to improve supply chain practices in various industries.

Keywords: Blockchain Technology, Inventory Models, Supply chain management, Numerical analysis, Sensitivity Analysis, Optimization Model

Introduction

The supply chain is the network of organizations, activities, and resources involved in delivering goods and services to customers (Yan et al., 2022). It ensures that products or services are delivered correctly and on time. Supply chain management has become increasingly complex due to factors such as globalized markets, fragmented supply networks, and diverse regulations. Furthermore, customer expectations have also evolved, demanding greater customization and faster delivery times (Yue, 2008).

To meet these challenges and enhance supply chain efficiency, businesses have turned to blockchain technology (Yan et al., 2022). The emergence of blockchain technology offers a promising solution to improve supply chain efficiency. It provides a decentralized and transparent platform for securely recording and validating transactions, tracking goods, and sharing information among supply chain participants. Blockchain technology offers several advantages that can improve supply chain management. It provides a secure and tamper-resistant record of transactions, ensuring transparency and traceability throughout the supply chain (Shakhbulatov et al., 2020). This can help prevent fraud, reduce errors, and increase accountability.

ANALYZING INVENTORY MODELS WITH PRICE-SENSITIVE DEMAND AND BACKORDERING USING ITERATIVE APPROXIMATION METHODS

Prerna Jain[1]
Renuka S. Namwad[2]

ABSTRACT

This study focuses on developing inventory models that address price-sensitive demand under backordering conditions, a scenario common in many industries. The dynamic nature of inventory levels is modeled using a linear differential equation, which captures the interplay between demand, pricing, and stock replenishment. To solve these models effectively, an iterative approximation method is proposed, ensuring accurate and computationally efficient solutions. A detailed numerical example is presented to demonstrate the applicability of the model, highlighting its practicality in real-world scenarios. Furthermore, sensitivity analysis is performed to understand the influence of key parameters such as demand elasticity, holding costs, and backordering rates on inventory performance. The findings provide valuable insights for optimizing inventory decisions, balancing profitability and service levels, and adapting to changing market conditions. This work contributes to the literature on inventory management by integrating analytical and numerical approaches to address complex operational challenges.

Keywords: Linear Differential Equation, Price-Sensitive Demand, Backorder, Inventory Models, Iterative Approximation Method, Numerical Example, Sensitivity Analysis.

Introduction

The present study aims to construct inventory models that allow for price sensitive demand, considering that backordering exists, which is a common practice in various industries. In order to capture the variability of inventory levels based on demand expansion and capture the expansion of pricing and stock's availability, a linear differential equation is used. In order treat these models adequately an effective iterative approximation technique is suggested (Ouyang et al., 2008).

A detailed numerical example is presented in order to show how models can be used, thereby demonstrating their usefulness. Furthermore sensitivity analysis is carried out in order to determine the effects of demand elasticity, cost of holding stock, backordering rates on the performance of the inventory. The results allow to draw conclusions concerning the optimal inventory policy, provided a certain market environment and the level of the service. This study enriches inventory management by paving way for the use of practical mathematic and numeric models in solving inventories operational complexities. This research intends to develop a unified inventory model which accounts for the interaction between price sensitive demand and backordering; an iterative method will also be suggested in order to treat these models.

[1] Gitarattan International Business School, Rohini Sec 14, Delhi, India
[2] Department of Mathematics, School of Chemical Engineering & Physical Sciences, Lovely Professional University, Punjab (India)

Accepted Book Chapter 1

Gmail

PRERNA JAIN <prernajain0312@gmail.com>

BAMS-ORSI 2024 notification for paper 65

1 message

ORSI-BAMS 2024 <orsibams2024@easychair.org> Thu, Sep 19, 2024 at 9:06 AM

To: Prerna Jain <prernajain0312@gmail.com>

Dear Authors,

Thank you for your interest in participating in the International Conference on Emerging Trends in Business Analytics and Management Science (BAMS-ORSI 2024), scheduled for 12-14 December at IIT Bombay.

We are pleased to inform you that your article has been accepted for presentation at the conference. You may now proceed with the registration process through the link:

https://www.som.iitb.ac.in/bams-registration/

However, our reviewers feel the need for improvements in your article and have recommended a revision, which you are requested to consider. The deadline for revision is 2 weeks. The review comments are appended below. While preparing the revised manuscript, please follow the full paper template given on the conference website. Here is the link to the template:

https://drive.google.com/drive/folders/1zM2RTMASUnjBX_iDFG_uZ1_L-JY-CWOt

Although we are trying our best to ensure the publication of the maximum number of quality articles, please note that the possible submission to the partner proceedings/edited books/journals will depend on the standards and guidelines set by the respective proceedings/edited books/journals.

Please note that a limited shared hostel accommodation facility (separate hostels for boys and girls) is now available for students, research scholars, and young industry participants for a nominal fee. Please express your interest for the accommodation here:

https://docs.google.com/forms/d/e/1FAIpQLSezxe4s5LPJ8GBUkzSOyoEDnMIJpNlvVvd4H5kPcDeQsfvIQA/viewform

The hostel accommodation will be offered on a first-come, first-served basis. The status of your accommodation request will be communicated post your registration.

Looking forward to your active participation at IIT Bombay and making this international conference a grand success.

<Automatically reviews will be appended through Easy Chair, which will contain ID numbers and author details>

SUBMISSION: 65

TITLE: Optimizing Inventory Management with a Stackelberg Game Approach: A Retailer-Manufacturer Model

----------------------- REVIEW 1 ---------------------

SUBMISSION: 65

TITLE: Optimizing Inventory Management with a Stackelberg Game Approach: A Retailer-Manufacturer Model

AUTHORS: Nitin Kumar Mishra, Prerna Jain, Ranu Sheoran and Renuka Namwad

----------- Overall evaluation -----------

SCORE: 2 (accept)

----- TEXT:

Accept with minor revisions.

Accepted Book Chapter 2

Gmail **PRERNA JAIN <prernajain0312@gmail.com>**

Update on Book Production and Chapter Details - reg.

3 messages

Dr. Amir Ahmad Dar <sagaramir200@gmail.com> Thu, Nov 14, 2024 at 8:41 AM

To: Tashi Lhamo <tasheelhamo94@gmail.com>, geetadma@gmail.com, mohit singh <nainbir93@gmail.com>, AKSHAT JAIN <akshatjain14042001@gmail.com>, Mehak Malhotra nHyyypLdSR <mehakmalhotra0418@gmail.com>, rakesh malhan <rakeshmalhan23@gmail.com>, Ataur.rahman@utas.edu.om, preety kalra <kalra.preety@gmail.com>, Nisha <nishakh222@gmail.com>, Navdeep Kaur <nk7845318@gmail.com>, shahtasneem7786@gmail.com, Sachitanand Singh <sachi.publication@gmail.com>, HALIRU SANI RINI <halirurini@gmail.com>, PRERNA JAIN <prernajain0312@gmail.com>, Sowjanya Ch <sowjanya.jhasmin@gmail.com>, Manzoor Ahmad Khanday <manzoorstat@gmail.com>, Avneet kaur Chahal <avneetkaur3445@gmail.com>, Olayan Albalwi <oalbalwi@ut.edu.sa>, rakeshmalhnan23@gmail.com, snitinmishra@gmail.com, renukanamwad@gmail.com

Dear Authors

I hope you are doing well. I wanted to reach out with an update regarding our book,

Integrative Approaches to Quality, Data Analysis, and Interdisciplinary Research

(Previous title: **Mathematical Sciences in Data Analysis**), which is now progressing smoothly through the production phase.

Our chapters are currently in production, and I'm pleased to share that you can view the ISBN (9781779641007) and the complete list of chapters on the Apple Academic Press website. Here is the link to the book:

https://www.appleacademicpress.com/integrative-approaches-to-quality-data-analysis-and-interdisciplinary-research-/9781779641007

Please take a few moments to review the book description and ensure it accurately represents the focus of the work.

Thank you once again for your valuable contributions. It's been a pleasure working with you on this project, and I look forward to seeing the final publication soon.

--

with regard
Dr. Amir Ahmad Dar
Assistant Professor
School of Mathematics
Lovely Professional University, Punjab India
Contact #: 917889426201

Accepted Book Chapter 3

Thank you for submitting the book chapter entitled " **Optimizing Supply Chain Efficiency with Mathematica Mathematical and Statistical Methods for Finite Planning Horizons** ". We are pleased to inform you that your chapter has been accepted for publication. Your work demonstrates a valuable contribution to our book entitled "**Mathematical Sciences in Data Analysis/Mastering Quantitative Techniques**", and we look forward to incorporating your insights into our collection. Please kindly request that you fill out and return the attached copyright form. Once these adjustments are made and the copyright form is submitted, we anticipate your chapter will enrich our book significantly. Thank you again for your submission.

Review Report:

The reference style should be APA style

You have mentioned this in the manuscript ([4]; [5]; and [6]). ' - it should be like [465]; routes. ([10]; [11];[12]), it should be like [10-12], Like that you have to change the reference style in your manuscript.

Please take a look at the sample chapter attached to this email for your reference. Submit your work in doc format (as attached file-- Sample chapter- Akshat Book Chapter)

Find the attachment below:

On Mon, Jun 24, 2024 at 2:19 PM Ranu Sheoran <ranu.insan100171@gmail.com> wrote:

> Respected Sir/Ma'am,
>
> **Warmest regards!**
> **I hope this email finds you in excellent health and high spirits.**
> I wanted to inform you that I've incorporated all the suggestions you provided. Please review the updated version at your convenience.
>
> Additionally, I would appreciate your further assistance with any additional changes.
>
> Thank you for your continued support.
>
> **Thanks and Regards**
> **Ranu PhD**
> **Lovely Professional University, Punjab, India**

Under Review Journal paper 1

Gmail

PRERNA JAIN <prernajain0312@gmail.com>

Notification of Solution Concern in Published Pape: Supply chain information coordination based on blockchain technology: A comparative study with the traditional approach

PRERNA JAIN <prernajain0312@gmail.com>
To: editor@apem-journal.org

Mon, Jan 15, 2024 at 12:27 PM

Dear Miran Brezocnik,

I hope this email finds you well. I am writing to bring to your attention a potential issue in the research paper titled " Supply Chain Information Coordination Based on Blockchain Technology: A Comparative Study with the Traditional Approach" which was recently published in Advances in Production Engineering & Management ISSN 1854-6250 Volume 17 | Number 1 | March 2022 | pp 5–15.

Upon careful examination of the paper, I have identified what I believe to be a fundamental error in one of the main equations, specifically the one involving a derivative. This error has a cascading effect, impacting the entire model and subsequently leading to inaccuracies in the presented results. Consequently, the calculated profit in the equation is approaching zero.

I understand the importance of accuracy in scientific publications, and as a conscientious reader, I feel it is my responsibility to bring this matter to your attention. It is my sincere hope that this information helps maintain the high standards of excellence associated with Advances in Production Engineering & Management.

I have attached a document outlining the specific issues along with suggested corrections for your review. I believe that rectifying this error will significantly enhance the reliability and validity of the paper.

Thank you for your time and consideration. I appreciate the dedication of Advances in Production Engineering & Management to upholding the quality of scientific research, and I trust that appropriate measures will be taken to address this matter.

If you require any further clarification or information, please do not hesitate to contact me. I look forward to hearing from you soon.

Sincerely,
Prerna Jain
Department of Mathematics,
School of Chemical Engineering & Physical Sciences,
Lovely Professional University, Punjab (India)
PH-9716533377

APEM LETTER TO EDITOR.docx
75K

Under Review Journal paper 2

Gmail **PRERNA JAIN <prernajain0312@gmail.com>**

Fwd: Manuscript for the publication
1 message

Ranu Sheoran <ranu.insan100171@gmail.com> Sat, Jul 27, 2024 at 1:46 PM
To: PRERNA JAIN <prernajain0312@gmail.com>

---------- Forwarded message ---------
From: **Ranu Sheoran** <ranu.insan100171@gmail.com>
Date: Fri, 5 Apr, 2024, 7:47 pm
Subject: Manuscript for the publication
To: <editor@apem-journal.org>

Respected Sir/Ma'am,

Warmest regards!
I hope this email finds you in excellent health and high spirits.

I am pleased to submit my manuscript titled **"A three-level refrigeration supply chain inventory model including linear time-dependent demand for temperature-sensitive items considering carbon tax regulations"** for consideration for publication in your prestigious journal. Please find the attached manuscript as per your submission guidelines.

Thanks and Regards
Ranu
Lovely Professional University, Punjab, India

Manuscript.docx
427K

Under Review Journal paper 3

Gmail   PRERNA JAIN <prernajain0312@gmail.com>

Inderscience Publishers: IJISM-237016 - Submission entering review process

1 message

Inderscience Submissions <submissions@inderscience.com>   Thu, Nov 28, 2024 at 5:55 PM

To: prernajain0312@gmail.com

INDERSCIENCE *Submissions*

Article submission and peer-review system

Dear Ms.Prerna Jain,

Thank you for your recent submission, reference code IJISM-237016, entitled,
'Blockchain-Based Inventory Optimization in Decentralized Two-Echelon Supply Chains: A Finite Horizon Analysis'

submitted to Int. J. of Integrated Supply Management.

We are pleased to inform you that your submission has passed the screening stage and is entering the review process.
Your paper will now be checked to ensure it meets the subject scope and quality levels of the journal and will be sent for peer-review if it is suitable.

You can track the progress of your submission by logging in to the Inderscience Submissions system at https://inderscienceSubmissions.com/

Your username is: Prerna0312
You can get a password reminder on the log in page.

Thank you for considering this journal as a venue for your work.

Kind regards,
The Inderscience Submissions Team,
Inderscience Publishers
submissions@inderscience.com

Under Review Journal paper 4

Gmail

PRERNA JAIN <prernajain0312@gmail.com>

SCR-9929 Manuscript Upload Notification

3 messages

elma loh <elma.loh@as-pub.net> Mon, May 19, 2025 at 7:56 AM
To: prernajain0312@gmail.com

Dear Prerna Jain,

I'm the editor for SCR. Thank you for your email and submission. I have uploaded your manuscript titled "Predictive Analysis of Wholesale Customer Purchases Using Machine Learning Models" to the OJS system, with the reference number SCR-9929. It is currently undergoing preliminary review.

During this phase, we will carefully evaluate your research work. Please note that there are no specific formatting requirements at this time. We will ensure that the review is completed promptly and will notify you of the results as soon as they are available.

Please feel free to reach out if you have any questions or need further information.

Best regards,

Elam
Supply Chain Research
Art and Science Press

Under Review Journal paper 5

Gmail

PRERNA JAIN <prernajain0312@gmail.com>

Feedback of IMA-9928
2 messages

cecily see <cecily.see@as-pub.net> Tue, Jun 10, 2025 at 8:20 AM
To: prernajain0312@gmail.com

Dear Prof. **Prerna Jain**,

Thank you for submitting your manuscript for IMA-9928. Your submission has been reviewed by our reviewers, and the review reports have been attached below for your perusal. We value the contribution you have made to our journal and are excited about the prospect of publishing your work. Please polish your paper within 7 days based on the two reports. Email me back if you need any extension.

Before we proceed with proofreading, please add the following information along with your revision: title, abstract, keywords, author's name, affiliation, email address, and postcode to ensure that we can successfully complete the subsequent work.

If you have any questions or require further clarification, please do not hesitate to reach out to me at any time. Your satisfaction and comfort throughout this process are of utmost importance to us.

Best regards
Editorial Office

2 attachments

IMA-9928 Feedback a.doc
27K

IMA-9928 Feedback b (1).doc
28K

Under Review Conference paper 1

Gmail

PRERNA JAIN <prernajain0312@gmail.com>

Acknowledgement of submission of Contributed paper 226 for MECC 2025

2 messages

PaperPlaza Conference Manuscript Management System <ifac.101@papercept.net> Fri, Apr 18, 2025 at 9:07 PM
To: prernajain0312@gmail.com
Cc: snitinmishra@gmail.com

Message from PaperPlaza Conference Manuscript Management System

Dear Ms. Prerna Jain

This email is to acknowledge receipt of your submission

Triangular vs. Trapezoidal vs. Heptagonal Fuzzy Models: Optimizing Blockchain-Enabled Profitability Under Uncertainty (Paper ID: 226, Contributed paper)

submitted to the 2025 Modeling, Estimation and Control Conference, (MECC 2025), through the Conference Manuscript Management System of MECC 2025.

Attached below are pertinent data on your paper for you to verify. If there is any error or necessary change to your contact data, please update the data at the following site:

http://ifac.papercept.net

If you need to modify title or author list, you will have an opportunity to do so when submitting the final version for publication in the proceedings.

For any correspondence regarding this paper, be sure to refer to your paper ID (226) and your PaperCept PIN number.

It is very important to note that "only" the corresponding author will be able to upload the final paper. If the corresponding author needs to be changed, the current corresponding author must do so.

It is also very important to note that your paper will be reviewed by >=3 reviewers, and accordingly, the authors of your paper are expected to likewise review >=3 papers submitted to MECC 2025.

Best Regards,

Xu Chen, Program Chair
Qian Wang, General Chair
MECC 2025

Manuscript data
==
Submission number: 226
Authors or proposers names and PINs:
Prerna Jain (168097), nitin mishra (168100)
Corresponding author or proposer:
Prerna Jain, prernajain0312@gmail.com
Title:
Triangular vs. Trapezoidal vs. Heptagonal Fuzzy Models:
Optimizing Blockchain-Enabled Profitability Under Uncertainty
Date submitted: April 18, 2025
Type of submission: Contributed paper
Status: Received
Award candidate: None
Society: SIAM
Keywords:
Optimal Control; Modeling and Validation; Manufacturing Systems
Profile:

List of Conferences

1. **5th International Conference on Recent Advances in Fundamental and Applied Sciences (RAFAS-2024)**

- *Date*: 19th to 20th April 2024
- *Venue*: Lovely Professional University, Punjab, India
- *Presentation*: Poster presentation on "Blockchain Optimization: Managing Information Sensitivity for Manufacturer Profit."

2. **18th IFAC Symposium on Information Control Problems in Manufacturing (INCOM 2024)**

- *Date*: 28th to 30th August 2024
- *Venue*: TU Wien, Vienna, Austria

- *Participation*: Presentation on "Enhancing Supply Chain Efficiency with Blockchain: Addressing Information Sensitivity for Increased Manufacturer Profitability"

3. **International Conference on Emerging Trends in Science, Engineering and Technology (ICESTS-23)**

- *Date*: 12th to 13th February 2023
- *Venue*: Lovely Professional University, Punjab, India
- *Participation*: Poster presentation on "Exploring the Blockchain Technology in Analyzing a Decentralized Supply Chain Inventory Model within a Finite Planning Horizon."

4. **International Conference on Emerging Trends in Business Analytics and Management Science (BAMS-ORSI 2024)**

- *Date*: 12th to 14th December 2024
- *Venue*: IIT Bombay, Mumbai, INDIA
- *Presentation*: Optimizing inventory management with a Stackelberg game approach: A retailer-manufacturer model.

5. **15th International Conference on Exploring Law, Management, and Multidisciplinary Perspectives for a Changing World**

- *Date*: 10th to 11th January 2025
- *Venue*: Gitarattan International Business School, Guru Govind Singh Indraprastha University, Delhi, INDIA
- *Presentation*: Predicting Delivery Outcomes in Supply Chain Management Using Machine Learning: A Random Forest Classifier Approach.

5. 2nd INTERNATIONAL CONFERENCE ON RECENT TRENDS IN MATHEMATICS (ICRTM – 2025)

- *Date*: 7th to 8th February 2025
- *Venue*: Hansraj College, University of Delhi, INDIA
- *Presentation*: Blockchain-Based Inventory Optimization in Decentralized Two-Echelon Supply Chains: A Finite Horizon Analysis.

<u>Conference Certificate 1</u>: **International Conference on Emerging Trends in Business Analytics and Management Science (BAMS-ORSI 2024)**


SJMSOM

Indian Institute of Technology, Bombay

CERTIFICATE OF PRESENTATION

This is to certify that Mr./Ms./Prof./Dr. PRERNA JAIN
from Lovely Professional University
presented their paper titled Optimizing Inventory Management with a Stackelberg Game Approach: A Model Retailer-Manufacturer
at the International Conference on Emerging Trends in Business Analytics & Management Sciences, held as part of the 57th Annual Convention of the Operational Research Society of India (BAMS-ORSI 2024), organized by the Shailesh J. Mehta School of Management, IIT Bombay, from December 12 to 14, 2024.

Pankaj Dutta
Prof. Pankaj Dutta
Conference Chair

Prof. Bhavin Shah
PC Co-chair

M D Agrawal
Prof. M.D. Agrawal
Chief Advisor

Conference Certificate 2:18th IFAC Symposium on Information Control Problems in Manufacturing (INCOM 2024)

IFAC
INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL

INCOM 2024

Wien, 28.08.2024

Certificate of Attendance

This is to certify that: **PRENA JAIN**

Affiliation: Lovely Professional University, has participated at the **18th IFAC Symposium on Information Control Problems in Manufacturing (INCOM 2024)** held from 28-30 August 2024 in Vienna, Austria.

This certificate is issued to acknowledge the participation and attendance at the conference.

Univ.-Prof. Dr.-Ing. habil. Fazel Ansari

NOC Chair

TECHNISCHE UNIVERSITÄT WIEN
Institut für
Managementwissenschaften
A-1040 Wien, Theresianumgasse 27

Conference Certificate 3: 1st International Conference on Emerging Trends in Science, Engineering and Technology (ICESTS-23)

Certificate No. 290823

LOVELY PROFESSIONAL UNIVERSITY

Transforming Education Transforming India

Certificate of Appreciation

This is to certify that **Prof./Dr./Mr./Ms. Prerna Jain** has presented paper with title **Exploring The Blockchain Technology In Analyzing A Decentralized Supply Chain Inventory Model Within A Finite Planning Horizon** in the **1st International Conference on Sustainable Energy Sources, Technologies and Systems (ICSESTS-2023)** held on **August 2-3, 2023**, organized by Division of Research and Development, Lovely Professional University, Punjab.

Date of Issue : 06-10-2023
Place : Phagwara (Punjab), India

Prepared by (Administrative Officer-Records)

Prof. Ravinder Kumar, Organising Secretary (ICSESTS-2023)

Dr. Sumit Kumar, Convenor (ICSESTS-2023)

Prof. Mohammad Hossein Ahmadi, International Conference Chair (ICSESTS-2023)

Conference Certificate 4: 5th International Conference on Recent Advances in Fundamental and Applied Sciences (RAFAS-2024)

LOVELY PROFESSIONAL UNIVERSITY

Transforming Education Transforming India

Certificate No. 329161

Certificate of Presentation

This is to certify that **Dr./Mr./Ms. Prerna Jain** of **Lovely Professional University, Phagwara** has given **Poster** presentation on **Blockchain Optimization: Managing Information Sensitivity for Manufacturer Profit** in the **5th International Conference on Recent Advances in Fundamental and Applied Sciences (RAFAS-2024)** held from 19th to 20th April 2024, organized by School of Chemical Engineering and Physical Sciences, Lovely Faculty of Technology and Sciences, at Lovely Professional University, Punjab.

Date of Issue : 20-05-2024
Place : Phagwara (Punjab), India

Prepared by (Administrative Officer-Records)

Organizing Secretary (RAFAS-2024)

Head of Faculty, Lovely Professional University

5. **<u>Conference Certificate 5</u>: 15th International Conference on Exploring Law, Management, and Multidisciplinary Perspectives for a Changing World**

giBS
Gitarattan International Business School
Grade 'A' Accredited by NAAC

CERTIFICATE
OF PARTICIPATION

15th INTERNATIONAL CONFERENCE ON
EXPLORING SYNERGIES: LAW, MANAGEMENT AND
MULTIDISCIPLINARY PERSPECTIVES FOR A CHANGING WORLD

THIS CERTIFICATE IS PRESENTED TO

Ms. Prerna Jain

of Gitarattan International Business School for presenting the paper titled

Predicting Delivery outcomes in Supply chain Management using Machine Learning: A Random Forest Classifier approach.

during the two day International Conference ICGIBS 2025 held on
10th-11th January 2025

DIRECTOR

Organised by :
Gitarattan International Business School, New Delhi

Certificate number: GIBS/IC2025/13949

6. **<u>Conference Certificate 6</u>: 2nd INTERNATIONAL CONFERENCE ON RECENT TRENDS IN MATHEMATICS (ICRTM – 2025)**

ICRTM-2025
2ND International Conference on
Recent Trends in Mathematics

विज्ञान एवं प्रौद्योगिकी विभाग
DEPARTMENT OF
SCIENCE & TECHNOLOGY

CERTIFICATE OF APPRECIATION

This is to certify that

Ms. Prerna Jain

Research Scholar, Lovely Professional University

has participated and presented the paper titled

Blockchain-Based Inventory Optimization in Decentralized Two-Echelon Supply Chains: A Finite Horizon Analysis

in the international conference ICRTM-2025 organized by the Department of Mathematics, Hansraj College, University of Delhi from 07 - 08 February 2025 in Delhi, India.

Dr. M. M. Mishra	Prof. (Dr.) Arvind	Prof. (Dr.) Rama
COORDINATOR	CONVENOR	PRINCIPAL

List of Certificate/Workshop

1. **NPTEL Online Certification: Modelling and Analytics for Supply Chain Management**
   Conducted by: IIT Kharagpur
   Course Duration: January - April 2023 (12-week course)
   Score: 70%
2. **NPTEL Online Certification: Optimization Theory and Algorithms**
   Conducted by: IIT Madras
   Course Duration: July - October 2024 (12-week course)
   Score: 57%
3. **NPTEL Online Certification: Supply Chain Analytics**
   Conducted by: IIT Roorkee
   Course Duration: February - April 2023 (8-week course)
   Score: 56%
4. **NPTEL Online Certification: Operations and Supply Chain Management**
   Conducted by: IIT Madras
   Course Duration: July - October 2022 (12-week course)
   Score: 49%
5. **Wolfram U Completion Certificate: Data Visualization with Wolfram Language**
   Completed on: April 16, 2024
   Training Hours: 3 Hours
6. **Wolfram U Completion Certificate: Practical Programming with Wolfram Language**
   Completed on: April 18, 2024
   Training Hours: 2 Hours
7. **Wolfram U Completion Certificate: Wolfram Language Programming Fundamentals**
   Completed on: April 11, 2024
   Training Hours: 3 Hours
8. **Excellence in Peer Review: How to be an Effective Peer Reviewer**
   Date of Completion: July 17, 2024
   Organized by: Peer Review Systems Webinars
9. **Data Challenge Participation Certificate**
   Event: 1st Data-Driven Logistics and Supply Chain Competition (Supply Chain Segment)
   Organized by: IFAC INCOM 2024, Vienna, Austria
   Date: August 28-30, 2024

10. MATLAB Workshop Certificate

Workshop: MATLAB for Engineers and Researchers

Conducted by: Vellore Institute of Technology (VIT)

Date of Completion: June 2024

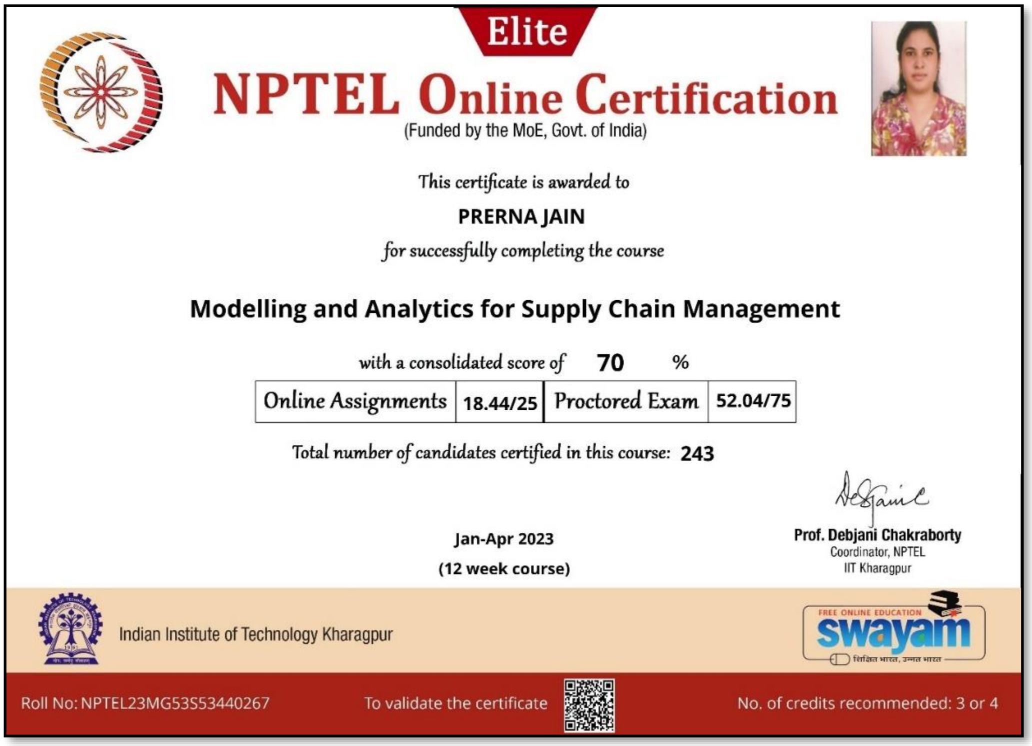
11. VIT Workshop Participation Certificate

Event: National Workshop on Computational Techniques

Conducted by: VIT, Vellore

Date of Completion: June 2024

<u>NPTEL Online Certification 1</u>: Modelling and Analytics for Supply Chain Management


Elite
NPTEL Online Certification
(Funded by the MoE, Govt. of India)

This certificate is awarded to
PRERNA JAIN
for successfully completing the course

Modelling and Analytics for Supply Chain Management

with a consolidated score of 70 %

Online Assignments	18.44/25	Proctored Exam	52.04/75

Total number of candidates certified in this course: 243

Jan-Apr 2023
(12 week course)

Prof. Debjani Chakraborty
Coordinator, NPTEL
IIT Kharagpur

Indian Institute of Technology Kharagpur

Roll No: NPTEL23MG53S53440267 To validate the certificate No. of credits recommended: 3 or 4

<u>NPTEL Online Certification 2</u>: Supply Chain Analytics


NPTEL Online Certification
(Funded by the MoE, Govt. of India)

This certificate is awarded to
PRERNA JAIN
for successfully completing the course

Operations and Supply Chain Management

with a consolidated score of 49 %

Online Assignments	18.94/25	Proctored Exam	30/75

Total number of candidates certified in this course: 82

Prof. Devendra Jalihal
Chairperson,
Centre for Outreach and Digital Education, IITM

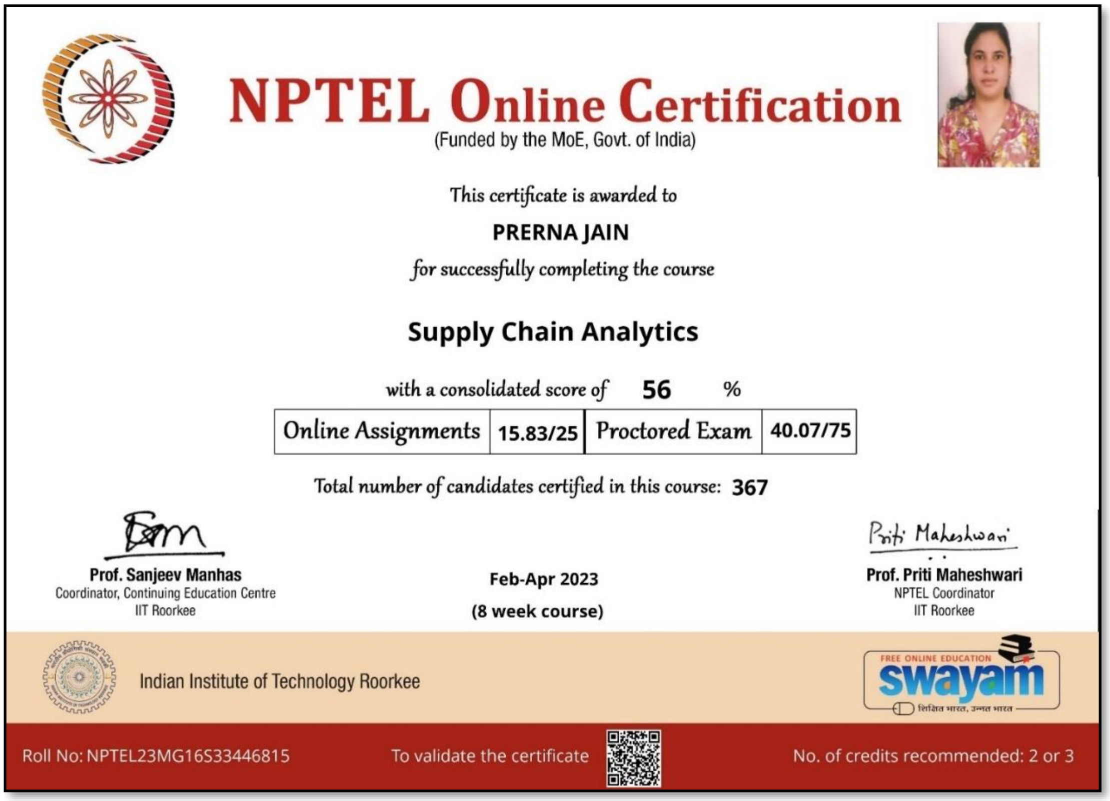
Jul-Oct 2022
(12 week course)

Prof. Andrew Thangaraj
NPTEL, Coordinator
IIT Madras

Indian Institute of Technology Madras

Roll No: NPTEL22MG74S34761179 To validate the certificate No. of credits recommended: 3 or 4

NPTEL Online Certification 3: Operations and Supply Chain Management



NPTEL Online Certification

(Funded by the MoE, Govt. of India)

This certificate is awarded to

PRERNA JAIN

for successfully completing the course

Supply Chain Analytics

with a consolidated score of **56** %

Online Assignments	15.83/25	Proctored Exam	40.07/75

Total number of candidates certified in this course: **367**

Prof. Sanjeev Manhas
Coordinator, Continuing Education Centre
IIT Roorkee

Feb-Apr 2023
(8 week course)

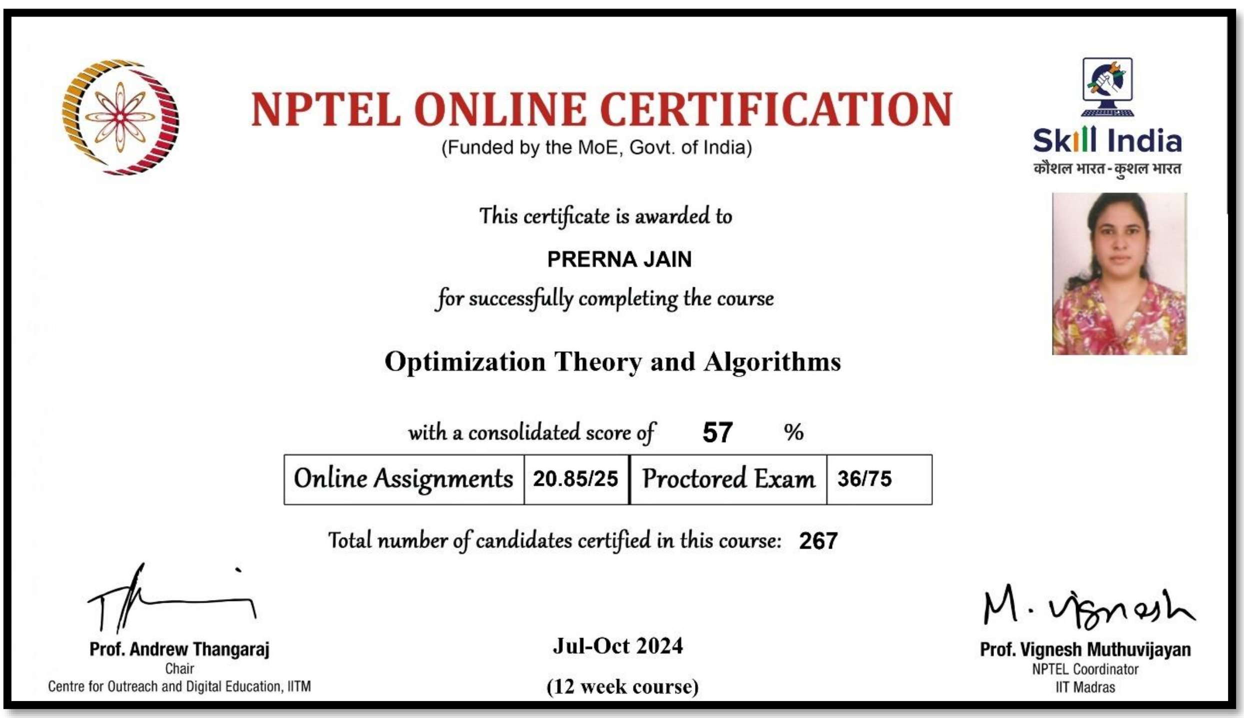
Priti Maheshwari
Prof. Priti Maheshwari
NPTEL Coordinator
IIT Roorkee

Indian Institute of Technology Roorkee

FREE ONLINE EDUCATION swayam

Roll No: NPTEL23MG16S33446815 To validate the certificate No. of credits recommended: 2 or 3

NPTEL Online Certification 4: Optimization Theory and Algorithms



NPTEL ONLINE CERTIFICATION

(Funded by the MoE, Govt. of India)

Skill India
कौशल भारत - कुशल भारत

This certificate is awarded to

PRERNA JAIN

for successfully completing the course

Optimization Theory and Algorithms

with a consolidated score of **57** %

Online Assignments	20.85/25	Proctored Exam	36/75

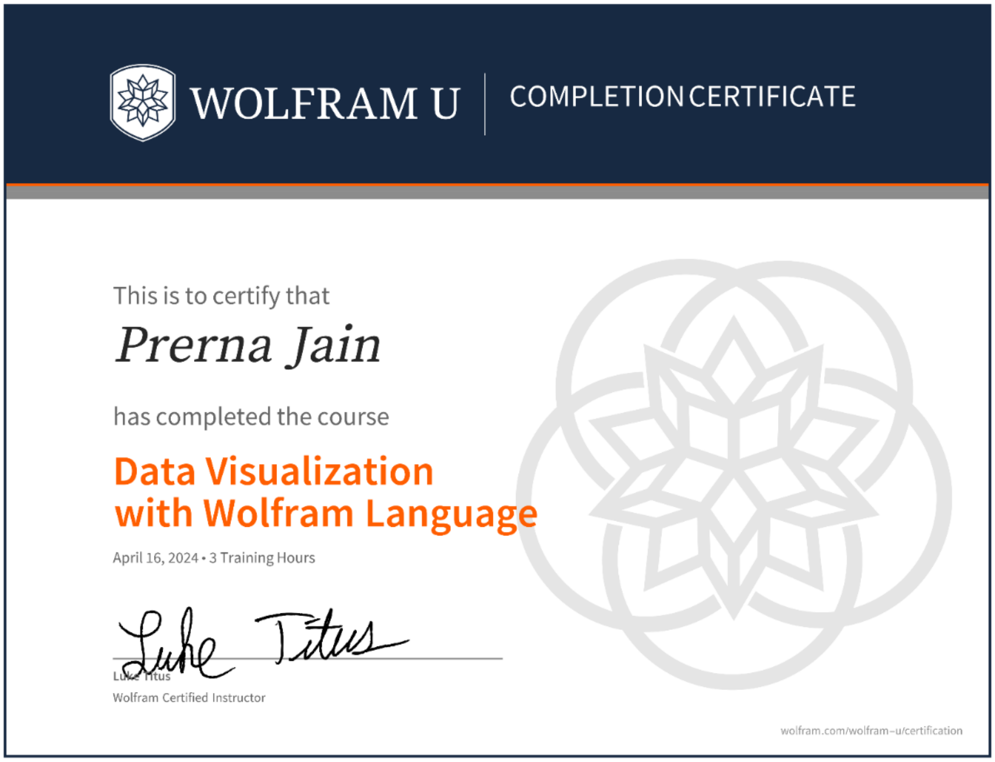
Total number of candidates certified in this course: **267**

Prof. Andrew Thangaraj
Chair
Centre for Outreach and Digital Education, IITM

Jul-Oct 2024
(12 week course)

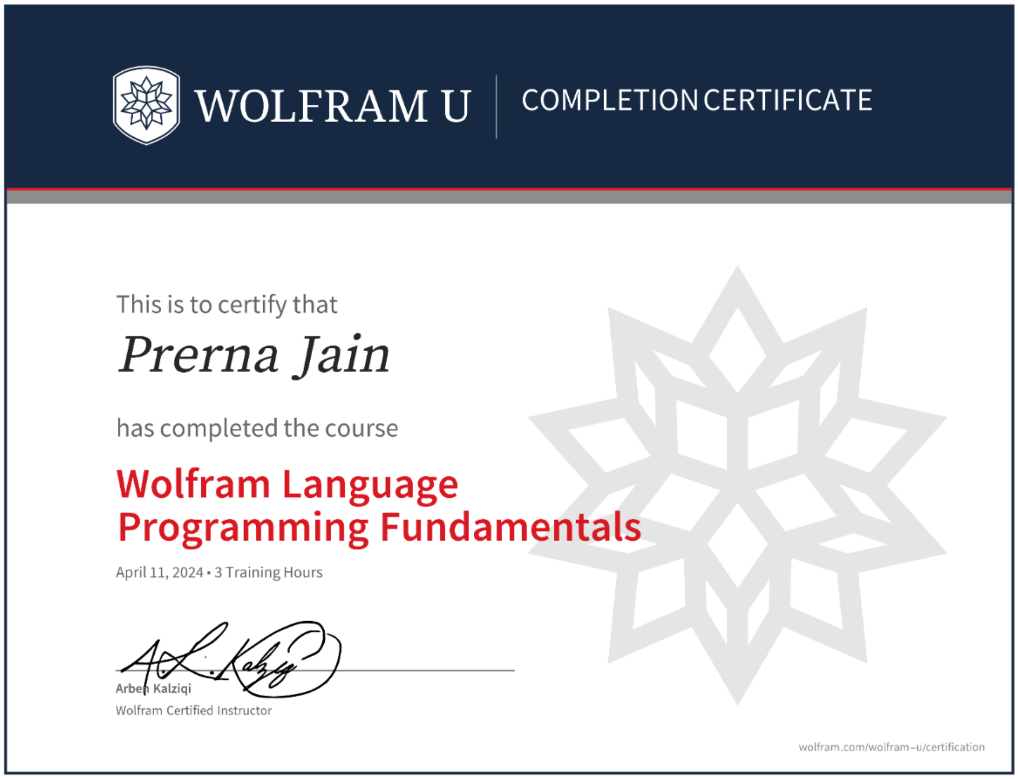
M. Vignesh
Prof. Vignesh Muthuvijayan
NPTEL Coordinator
IIT Madras

<u>Wolfram U Completion Certificate 1</u>: Data Visualization with Wolfram Language



<u>Wolfram U Completion Certificate 2</u>: Practical Programming with Wolfram Language

Wolfram U Completion Certificate 3: Wolfram Language Programming Fundamentals



WOLFRAM U | COMPLETION CERTIFICATE

This is to certify that

Prerna Jain

has completed the course

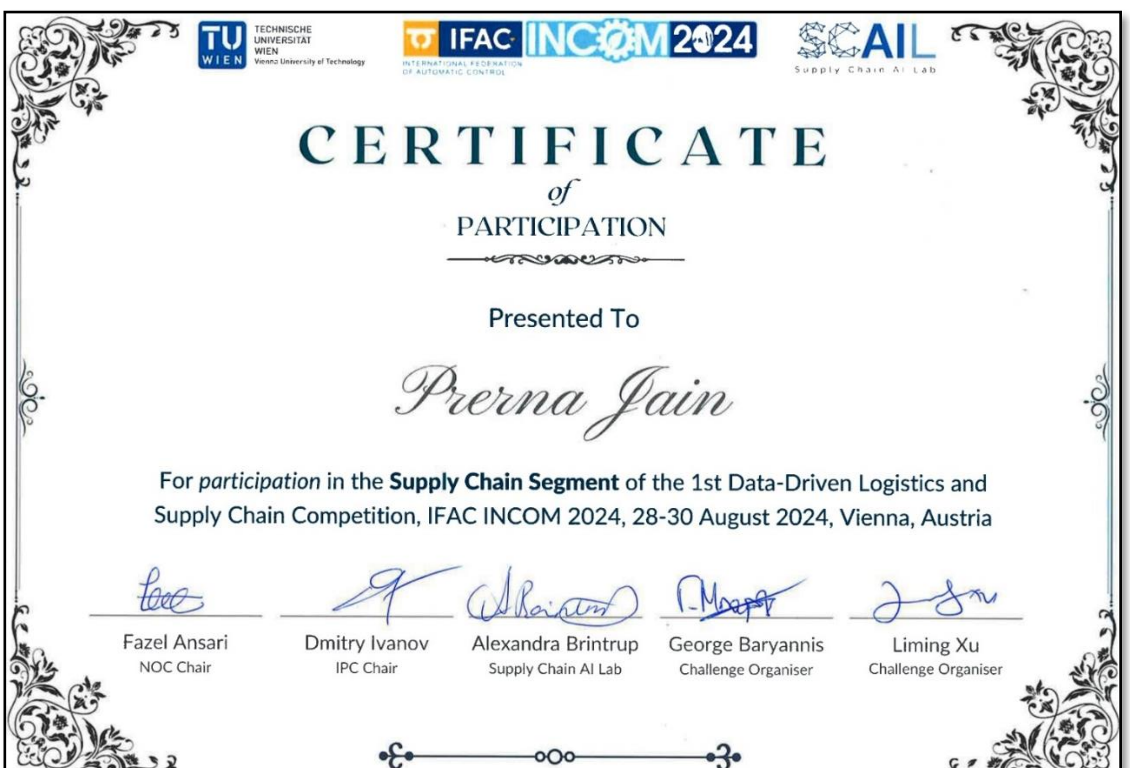
Wolfram Language Programming Fundamentals

April 11, 2024 • 3 Training Hours

Arben Kalziqi
Wolfram Certified Instructor

wolfram.com/wolfram-u/certification

Data Challenge Participation Certificate-1



TECHNISCHE UNIVERSITÄT WIEN Vienna University of Technology | IFAC INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL | INCOM 2024 | SCAIL Supply Chain AI Lab

CERTIFICATE
of
PARTICIPATION

Presented To

Prerna Jain

For *participation* in the **Supply Chain Segment** of the 1st Data-Driven Logistics and Supply Chain Competition, IFAC INCOM 2024, 28-30 August 2024, Vienna, Austria

Fazel Ansari	Dmitry Ivanov	Alexandra Brintrup	George Baryannis	Liming Xu
NOC Chair	IPC Chair	Supply Chain AI Lab	Challenge Organiser	Challenge Organiser

Workshop 1: **Excellence in Peer Review: How to be an Effective Peer Reviewer**

CERTIFICATE

PROUDLY PRESENTED TO

Prerna Jain

Excellence in Peer Review: How to be an effective peer reviewer

Jul 17, 2024

Date of Completion

Peer Review Systems Webinars

Organizer

EXCELLENCE in Peer Review

Taylor & Francis Reviewer Training Network

Workshop 2: VIT Workshop Participation Certificate

VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CHENNAI

Certificate of Participation

This is to certify that Prerna Jain of **Lovely Professional University** participated in the Two-Day Online National Workshop on **"The Application of Differential Equations to Real-World Problems "** organized by the **Division of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Chennai** on 15th and 16th February 2024.

Kamalesh Acharya

Dr. Kamalesh Acharya
Convener

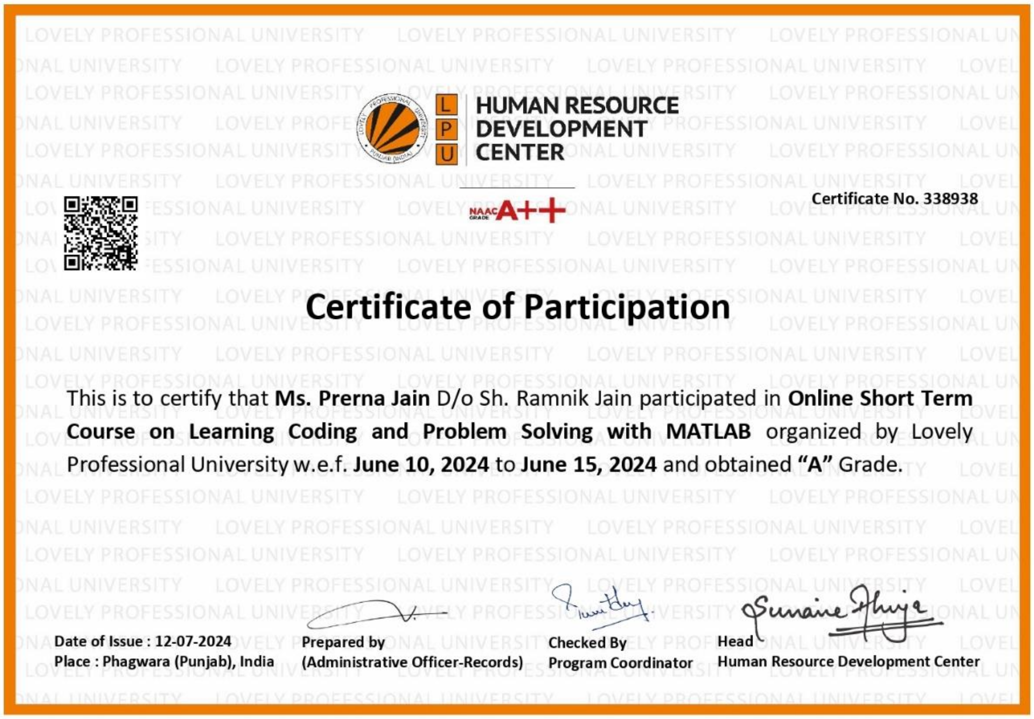
Surath Ghosh

Dr. Surath Ghosh
Convener

Mahalakshmi S.

Dr. S. Mahalakshmi
Dean, School of Advanced Sciences

<u>Workshop 3</u>: MATLAB Workshop Certificate



HUMAN RESOURCE DEVELOPMENT CENTER

NAAC GRADE A++

Certificate No. 338938

Certificate of Participation

This is to certify that **Ms. Prerna Jain** D/o Sh. Ramnik Jain participated in **Online Short Term Course on Learning Coding and Problem Solving with MATLAB** organized by Lovely Professional University w.e.f. **June 10, 2024** to **June 15, 2024** and obtained **"A"** Grade.

Date of Issue : 12-07-2024
Place : Phagwara (Punjab), India

Prepared by
(Administrative Officer-Records)

Checked By
Program Coordinator

Head
Human Resource Development Center