ENHANCING SUPPLY CHAIN PERFORMANCE BY IMPROVING SYSTEM RELIABILITYAND SUSTAINABILITY

A Thesis

Submitted in partial fulfillment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

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Mathematics

By

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Declaration

I the undersigned solemnly declare that dissertation work is based on my own work carried out during the course of our study under the supervision of Dr. Arunava Majumder. I assert the statements made and conclusions drawn are an outcome of my research work.

I further certify that

I. The work contained in the research is original and has been done by me under the general supervision of my supervisor.

II. The work has not been submitted to any other Institution for any other degree/diploma/certificate in this university or any other University of India or abroad.

III. We have followed the guidelines provided by the university in writing the report.

IV. Whenever we have used materials (data, theoretical analysis, and text) from other sources, we have given due credit to them in the text of the report and giving their details in the references.

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CERTIFICATE OF PUBLICATION OF PAPERS FOR PH.D.

This is to certify that Ms. Richa Nandra pursuing Ph.D. (**Part Time**) programme in Department of Mathematics with Registration Number 41800185 under the Guidance of Dr. Arunava Majumder has the following Publications / Letter of Acceptance in the Referred Journals / Conferences mentioned thereby fulfilling the minimum programme requirements as per the UGC.

Sno.	Title of paper with author names	Name of journal / conference	Published date	Issn no/ vol no, issue no	Indexing in Scopus/ Web of Science/UGC-CARE list
1.	Establishing relation between production rate and product quality in a single-vendor multi-buyer supply chain model, Richa Nandra and Kristina Rangsha Marak,Ramandeep Kaur, Bikash Koli dey, Arunava Majumder	Int. J. Services Operations and Informatics	2/3/2021	Vol no. 11, 315-331	Scopus

2.	A multi-retailer sustainable supply chain model with information sharing and quality deterioration, Richa Nandra, Arunava Majumder, Mowmita Mishra	RAIRO,Operations Research	6/10/2020	55 (2021) S2773-S2794	SCI
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Signature of Guide with Date & UID **Signature of Co-Guide with Date & UID**

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q_i order quantity delivered to i-th buyer by vendor in a single lot (units)

k_i safety factor for i-th buyer

r_i reorder point for i-th buyer (units)

s_i safety factor for i-th buyer (units)

L_i length of lead time for i-th buyer (units)

d_i demand per unit time for buyer i (units per unit time)

h_{bi} holding cost for buyer i per unit per unit time ($ per unit per unit time)

n number of buyers (integer)

o_{bi} ordering cost per order ($ per order)

π_i unit backorder cost ($ per unit backordered)

σ_i standard deviation for the demand

P per unit time production rate (unit per unit time)

$C(P)$ production cost ($ per unit)

Q lot size delivered by vendor

m number of lots delivered to each buyer in one production cycle (positive integer)

S_v setup cost for vendor ($ per setup)

h_v holding cost for vendor ($ per unit per unit time)

R rework cost per unit ($ per unit)

E_v environmental cost parameter for vendors

E_b environmental cost parameters for buyers

E_{bi} environmental cost for buyer i

E_b environmental cost parameters all buyers such that $E_b = \sum_{i=1}^{n} E_{bi}$

S_v social cost parameter for vendor

E_{ri} environmental cost for retailer i

S_{bi} social cost for buyer i

S_{cv} Social Cost for vendor

t production run time (time unit)

N number of defective goods in a production cycle (units)

α percentage of defective items when process shifts "out-of-control"

η(P) elapsed time that the process goes "out-of-control" (exponential random variable)

S_0 initial setup cost of vendor per setup ($/setup)

δ percentage decrease in setup cost, per dollar increase in the investment

O_b Ordering cost of single buyer

O_{ri} Variable ordering cost of single buyer

d demand per unit time for single buyer

q_s lot size delivered by vendor to single buyer

R_p Reorder point of a single buyer

L Length of lead time

θ_{mi} Initial "out − of − control" probability

θ_m $Variable$ "out − of − control" probability

b_r "Percentage decrease in setup cost per dollar increase in investment"

b_m "Percentage decrease in out − of − control probability per dollar increase in investment"

P_{max} Maximum threshold of Production rate

X_i normally distributed demand during lead time period for buyer i with mean $d_i L_i$ and standard deviation $\sigma_i \sqrt{L_i}$

$E(.)$ mathematical expectation

Contents

Abstract

This research study focuses on two the most prominent aspects of supply chain management namely, reliability and sustainability. As we know, supply chain management is a concept that emerged with shifting business focus from manufacturing to customer value. As good customer service is vital to sustain the demand hike, reducing lead time is a vital issue for huge supply chain profitability. This study considers an investment to crash lead time for improved customer service. Change in rate of production may affect product quality. In this dissertation, we studied system reliability as well as product quality in a single-vendor multi-buyer supply chain system with stochastic demand and measured the reliability on the scale of MTTF and "Out-of-control" probability. As the word suggests, reliability is one of the most central characteristics for measuring supply chain management performance. A reliable supply chain system is not only dependent on the proper functioning of its fundamental components, but in deep, it covers environmental, social and legal issues, i.e. sustainability. The inherent relationship of SCM with the environment we live in gives researchers an exciting opportunity to make a profound difference in society with their work. As management theories and principles continue to develop, this provides us with a reason to examine where we have been and consider where we should be going as we move forward. Motivated from this all, this study consists of 7 chapters except chapter 1 and 2, dedicated to our introduction and literature review, all other consists of modeling to pull off profit by upgrading manufactured good quality and sustainability. Along with this new procedure is introduced to abate the chance of system shifting to "out-of-control" state from "in-control" state. The gist of each chapter is given below.

Chapter 1
Introduction and motivation

1. Introduction and motivation

Supply chain management involves series of activities required to plan, control and execute a product's flow. It can be characterized as a most streamlined and cost effective way to increase competitiveness and customer satisfaction. It encompasses incorporated planning and execution of processes used to optimize the flow of materials, information and financial capital. It covers demand planning, sourcing, production, inventory management and storage, transportation, and return for excess or defective products. The success of SCM is determined by the potential and dedication of each partner from supplier to manufacturers and beyond. This requires effective management, collaboration and risk management to create apt association and communication between all the entities.

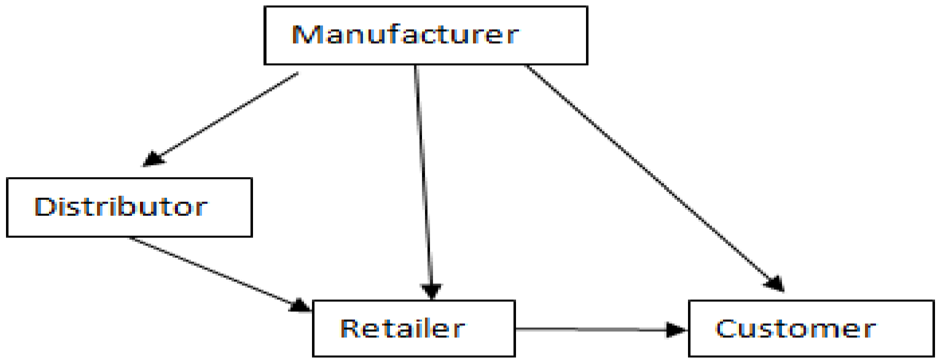


Figure 1.1: Usual production flow of supply chain

Many researchers have identified the critical importance of SCM in business. A firm's managerial ability to integrate and coordinate the complex network of business relationships among supply chain members decides their ultimate success (Habib 2010, Lambert and Cooper 2000). Chopra and Meindl (2013) had identified supply chain as companies of all kind that imperatively look for customer requirement only. **Earlier research on supply chain management has Inventory management, behavior of information flow, planning and operations management as major concerns. Now, these are extended with reliability and sustainability**

development.

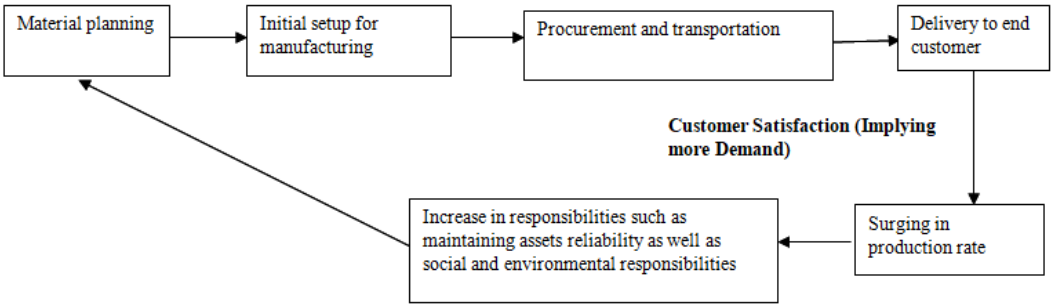


Figure 1.2: Flow of Reliable and sustainable supply chain management

1.1.Centralized and decentralized supply chain

Here, in this work, author has studied two different category of supply chain namely centralized and decentralized separately and has compared their combined and separate cost values of players namely manufacturer/vendor and retailer/buyer. A centralized supply chain had a system in which all up and downstream decision is taken in headquarter regarding optimization of cost and profit. On the other hand, in decentralized individual took responsibility for their optimization on their own. There are certain characteristics on which comparison basis of both supply chain are made and highlighted below:

Table 1.1: Various comparison basis of centralized and decentralized supply chain

Comparison Basis	Centralized supply chain	Decentralized supply chain
Interaction flow	Vertical	Open tree
Administration	Slow	Comparatively faster
Pro	Proper coordination and leadership	Sharing of burden and responsibility
Controlling authority	Lies with the top management	Multiple persons share the power of decision execution.

1.2.Supply chain management with multiple retailers

A single buyer or single vendor problem consists of only a single inventory model of either vendor or buyer. To thrive, every industry must select new technologies and strategies in modern global markets. Vendor-buyer integrated models optimize vendor and buyer inventory decisions overall. Basically, both sellers and buyers are trying to optimize their joint costs and profits. Managing manufacturing, inventory, and the supply chain, therefore, is a concern for any business. In initial modeling, single vendor and single retailer models were optimized, then an analysis of multi-retailer supply chains is presented, where a single vendor manufactures goods and supplies them to multiple buyers at multiple times. The manufacturer received an order quantity and produced its constant multiple quantities in a single setup using the SSMD policy.

1.3.Delivery policies in supply chain

Choice of a suitable delivery policy is an inevitable prerequisite of any successful supply chain working to obtain maximum profit with desirable customer satisfaction level. Depending on different marketing surroundings, policies have their own advantages.

1.3.1. Single set-up single delivery policy

Single set-up single delivery policy (**SSSD**) or `lot-for-lot' policy was offered by Banerjee (1986). A lot is produced in just one setup after receiving an order from retailer with no extra production. Cost incurred to manufacturer includes setup and holding cost while to retailer; these are ordering and holding cost.

1.3.2. Single-setup multi-delivery

Single-setup multi-delivery (**SSMD**) policy or ‘lot-splitting' policy was introduced by Goyal (1986). However, it supports delivery over multiple times to retailer producing integer multiple of received order (Ouyang et al., 2004). The manufacturer makes the same amount as ordered while dividing it into equal parts. Its cost components do not differ from SSSD policy for both parties.

1.3.3. Consignment policy

This is a kind of supply chain policy where the supplier retains ownership of a product until it has been sold by a retail outlet. In simple words, a consignment is the act of sending goods from one party to another party for the purpose of selling them on the latter's behalf. The owner of the goods does not give up ownership of the goods; only the possession is transferred. Its prime intent is to deliver or transport goods.

When customer demand is uncertain, this model is especially advantageous for retailers; it gives them the ability to offer customers more products and to focus more on sales. It allows a retailer to take a smaller financial risk because they do not pay until the product sells. While these policies carry risk for suppliers, one of its benefits is the ability to identify new customers for the product by placing it in front of more prospective customers. The retailer is referred to as consignee in the contract, while the supplier is called the consignor. Consumption is the transfer of ownership from supplier to retailer.

In this day and age, **CP** has emerged as a new policy in SCM which is becoming popular in heath care field. On the basis of its utilities, many other industries such as Wall-Mart, amazon.com etc., are accepting this policy. Despite of some assumptions, the prominent pros of this policy is it lessens the supplier's inventory level as vendor uses retailer's warehouse to stock items.

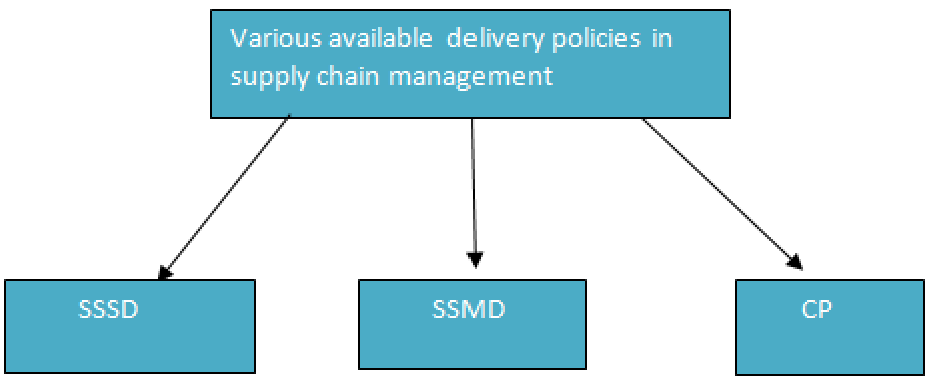


Figure 1.3: Delivery Policies in supply chain

1.4. Various cost components of the players of SCM

Supply chain whether centralized or decentralized has player such as manufacturer/vendor, retailer/buyer those finally satisfy customer by delivering them their demanded product on minimum possible time while maintaining their combined/separate profit values. To execute this, they incurred with various costs which has been listed below one after another.

1.4.1. Ordering cost

Ordering cost is price value of created and processed order and does not depended on the size of product to be delivered. This cost is obtained after multiplying per ordering cost to the time the order is placed.

1.4.2. Holding cost

Holding cost or carrying cost includes the expenditure of a company owner (a manufacturer/supplier/vender or retailer) to hold and store inventory. It constitutes investment, storeroom cost, maintaining cost, taxes, service costs, deterioration and/or inventory risk cost.

It helps calculating the expected profit or in decision making whether to increase or decrease production of goods in hand. This cost change with time therefore is calculated using average inventory level for every listed or included party.

Above case can particularly describe holding cost of retailer when lead time is zero or considered to be constant, in other words order is received at the same time it is placed.

1.4.3. Shortage cost

Shortage cost constitutes of a cost of goods that customer bought from elsewhere, the part of sales that could not be completed, and the extra cost spend in order bring good in demand in shortest possible time. This cost incur to company when inventory goes out of stock.

1.4.4. Backorder Cost

An organization's backorder cost is the expense it incurs when it is unable to respond quickly to an order and promises the client that the order will be delivered at a later date. Normally, It is attributed to the delay in placing order, discrepancies in logistics occurred because of human errors. This cost can be direct, indirect, or uncertain. It generally depends on the product and companies, how long order completion would take. In the meantime, the customer pays for the item, and then the company keeps the purchaser informed about when the item will arrive.

1.4.5. .Lead time crashing cost

Lead time crashing cost is the cost invested by retailer to complete customer demand by finding faster alternatives and ultimately reducing replenishment time. The ultimate objective of this cost is to achieve excellent customer service level and improving organizations goodwill. The achievement of the same helps logistic and network optimization.

1.4.6. Setup cost

Setup cost is the cost associated with the setup of equipment to manufacture products. This cost comprises the cost of raw material, machinery, arrangements, documentation, labor, and crumbing cost of test units. The overall cost is segregated among the number of goods produced of a received order. The real setup cost is also time waste till machine is operational to generate order and the items that are produced defective. Hence to reduce setup time and “out-of-control” situation in production run, many authors have incorporated additional logarithmic investment.

1.4.7. Rework cost

Rework cost is fund spent either to repair or adjust a defective or unacceptable item to sell them as an ideal or acceptable finished good. The actual cost depend on the number of defective items that can be any linear/constant/ exponential percentage of the whole production and are separated after applying inspections on various or last stage of production.

1.4.8. Manufacturing cost

Manufacturing cost, cost of preparing of product from raw to finish, can be considered as fixed and variable cost. It is attributed as variable when production is dependent on its demand. It is important to calculate the unit cost of produced product in order to know if any necessary action is needed to be taken with respect to machinery, procedure or employees.

1.4.9. Transportation cost

When a company transfers its inventory or other assets to another location, it incurs costs for moving. A key element of logistics that ties together separate activities is transportation. Additional or unnecessary transport costs are primarily the result of inefficient supply chain routing, network planning, and resource deployment for finished goods and raw materials alike. Hence, to minimize costs and to provide optimal services to customers, it is essential to integrate these functions and sub-functions into a system of goods movement.

1.5.Reliability and Sustainability

Except manufacturing of products, nowadays it is important for a company to be able to run for a long period of time or emerge as a consistent provider of a product while maintaining its social and environmental responsibilities. Hence, reliability and sustainability has become an important characteristics or components of a supply chain. In the context of products, systems, and services, reliability means the likelihood that they will perform in a required manner for stipulated time, or will operate in a given atmosphere without failure. And sustainability is a way to create long-term value by recognizing the impact an organization has on the environmental, social and economic contexts in which it operates. Since, it is also believed that developing such strategies will foster the longevity of a firm.

1.5.1. *Deterioration and imperfect production*

As the rate of production has a direct impact on system performance. In a long production run, with increased production rate in lieu of augmented demand rate, shifting of an "in-control" system to "Out-of-control" appears to be the most probable cause to happen after an elapsed time. The "In-control" system generates ideal items to sell while the "Out-of-control" system starts producing defective items that are not appropriate for sale. This originates a need of discussing various possibilities of finding the number of defective produced like in a constant percentage, linear or exponential excreta.

1.5.2. Manufacturing system reliability

MTTF (mean time to failure) is one of the important components to measure the system reliability. MTTF refers to the time taken for a non-repairable asset to fail. The MTTF is considered as a reciprocal of linear and quadratic quality function. Since, quality function represents the number of defective items increase with an amplified production rate. Also, this deterioration is taken into account as linearly and exponentially. The experimental results showed the reduction of MTTF with increasing production rate which has a significant impact on obtaining the managerial decisions. Then, the "out-of-control" probability is considered as is an indicator of system reliability. The high value of this entity implies low system reliability. A logarithmic investment function is imposed to reduce the "out-of-control" probability. The "out-of-control" probability is considered as an increasing function of production rate.

1.5.3. The cost components of sustainability

In sustainability aspect, to get technological advancement in order to reduce carbon emission as well as to enhance health, education and safety benefits of workers, necessary investment is proposed to add with respect to environmental and social components for both retailer and manufacturer total cost distinctively.

1.6. Elements affecting performance of SCM

1.6.1. Variable lead time

In a process, the lead time is the period that elapses between the initiation and completion. In SCM it is refers to the period between placing a purchase order for products and receiving them in the warehouse. It is regarded as a vital measure to calculate the efficiency of SCM because it directly influences customer satisfaction. It aids in demand forecasting. It varies based on various specific. The order lead time depends on the number of suppliers involve, the more the supplier the tardier it will be. If it takes longer than usual, to maintain business performance, companies have to raise their inventory levels.

1.6.2. Reorder point

Reorder point is reaching a level where every buyer or retailer gets ready to place an order for replenishment to avoid an out-of-stock situation. Reorder point is calculated by adding safety stock with lead time demand. Safety stock, an indemnity against variation in demand, is quantity stored extra in a storehouse. Lead time demand is the demand of orders received between the times the order is placed to replenish inventory and the anticipated time it will receive. Therefore, the net inventory level of retailer/buyer before and after an order is received will be 'reorder point- lead time demand' and 'order+ reorder point- lead time demand', respectively. Or, it will be average inventory level +safety stock. As it lessens holding cost and evade situations like stock-out, over-stocking and lost sales, so is crucial for the effective functioning of SCM. It enables a company to stay up-to-date with your next batch of inventory. It allows the identification of procurement-related issues and helps in their resolution, resulting in an improved process.

1.6.3. *Backordering*

Backordering does not consider as an ideal situation in SCM as no one wants to wait after an order is placed and money is spent. It clearly indicates a situation when product demand is greater than its supply. It is an important factor of inventory management analysis as it can varies based upon the backorder nature and the number of items on backorder. The effects of stock-outs can be detrimental to a retailer's business practices, customer base, and to credibility of the brand while availability of the same at the promised time can results in garnering positive endorsements.

1.6.4. Uncertainty

To deal with uncertainty following questions may appear

i. What will be the demand/preference of my customer?
ii. How many products should we order or have in stock to fulfill customer demand on time?
iii. What will be his reorder time, or which process will he follow to decide to reorder?
iv. Will the supplier deliver the requested goods on time and demand specifications to avoid a longer lead time?

Decision-makers have to retain safety buffers to prevent loss due to uncertainty to improve chain performance. Those companies which answer these questions well on time gives internally competitive top-line performances.

Uncertainty occurs in logistic supplies; a business may face uncertainty in economics, social, political, technical, environmental issues. These issues can positively or adversely affect core business. The factors that affects/effects an organization includes

- **Natural disasters**
- **Terrorist attacks**
- **Political changes**
- **Strikes**
- **Unreliable system**
- **Logistics, supply chain failures**
- **Unexpected lack of essential production inputs**

When uncertainty-related issues are not resolved on time, it creates high variability in demand, process, or supply, resulting in planning, scheduling, and control problems.

Figure 1.4: Elements affecting performance of SCM

1.6.5. Reliability

A well-functioning supply chain ensures that good quality parts arrive on time so the final product can be manufactured and shipped to a client on time. When quality assurance is provided during procurement, it can reduce the risk of sourcing substandard and hinder further degradation, which further reduces the incidence of complaints and recalls and losses in terms of finances. A good supply chain service is therefore essential for companies because when a supply chain or overall logistics operation is not reliable, it will fail. Hence, under the circumstances in which a system is implemented, reliability assures that it will perform as planned.

1.6.6. *Sustainability*

Sustainable supply chains are beneficial for improving both productivity and reducing costs. Using sustainable practices, all supply chains can be optimized or in other words, efficiencies in buildings, vehicles, and machinery increase significantly when sustainable technologies and resources are used. Overall nowadays, it has become a major goal of a company to acquire sustainability with respect to environment, society and economics. Environmentally sustainable practices help companies in not only decreasing their aggregate carbon footprint but also aids them to optimize their end-to-end operations to achieve more significant cost savings and profitability. Hence, having the capacity to continue existing and developing without exhausting the planet's natural resources is considered sustainability. The plan assumes that resources are finite, and ought to be used carefully and conservatively to ensure that future generations will have enough without negatively impacting current quality of life. On the other hand, social sustainable business involves evaluating both positive and negative business impacts on people. The notion of social sustainability encompasses more than groups of rights holders; it also includes topics that affect them, like health and education.

Chapter 2

Literature Review

2. Literature review

Supply chain management (SCM) is perfect coordination among manufacturers, retailers, and buyers. It synchronizes the production planning, operation scheduling, purchasing, and distribution activities—this synchronization benefits in reducing inventories and in improving customer service levels (Fliedner, 2003). Instead of focusing on individually minimizing their cost, centralized SCM players work in teams for minimizing the total supply cost of a chain as a whole. Their coordination not only reduces overall cost but also reduces lead time (Jha & Shanker, 2013). In an integrated inventory model, lead time plays a crucial role. It covers order preparation, order transit, supplier lead time, delivery time, and setup time (Ouyang & Wu, 1997). Shortening Lead time lowers the safety stock and reduces the loss incurred by stock-out, which gives various advantages in the competitive business environment (Jha & Shanker, 2009).

2.1. Centralized and decentralized supply chain system

Zhu, Gavirneni, and Kapuscinski (2009) measured a two-stage serial supply chain of a retailer facing demand uncertainties and its supplier. With the computation of policy called ‘periodic flexibility’, they manage to bridge the 43% control gap between the understudy supply chains and improve its performance by 11%. Duan and Liao (2013) probed the refill policies of a capacitated supply chain involving a distributor and multiple retailers for centralized and decentralized control policies. They made a cost comparison of these supply chains, suggested centralized better than decentralized. They presented a mechanism for decentralized control for maintaining better coordinating in its members. Furthermore, Rached et al. (2016) studied divergent decentralized and centralized supply chains considering a supplier, a warehouse, retailers and clients. They made their comparison while combining diverse states of affairs of simultaneous upstream and downstream information sharing in a decentralized supply chain.

2.2. Supply chain management with multi-retailer

For the first time, Goyal (1976) developed an integrated inventory model based on a single vendor, a single retailer. The joint economic lot size model presented by Banerjee (1986) adds to Goyal's (1976) model. Further, Goyal (1988) extended

Banerjee's (1986) model to include vendor total production as an integer multiple of buyer's quantity ordered. In a multi-vendor supply chain model with a single seller, Burton and Banerjee (1994) examined the coordination and independence of replenishment policies. The integrated vendor-buyer model proposed by Huang (2002) has imperfect quality products. Then, a coordinated single supplier multi buyer supply chain model with trade credit policy was considered by Sarmah et al. (2008). By synchronizing the production method for the first two models with equal and unequal sized batch transfer for the last model, Hoque (2008) created three different single-vendor multi-buyer models. Guan and Zhao (2011) created a model taking into account multiple retailers and continuous review policies. The system optimizes pricing and inventory management, so that profit can be maximized.

2.3. A tactic to reduce overall total cost

2.3.1. Setup costs reduction

Ouyang et al. (2002) put forward the idea of an inventory model for product's quality improvement and vendor's set up reduction. Brito and Almeida's work single–vendor multi-buyer integrated production inventory was further extended by Jha and Shanker (2013) with controllable lead time. They studied the effect of initial investment on reducing setup cost and on diminishing the production of imperfect quality items. This investment facilitates a cut in each independent setup cost and in reducing the number of imperfect items produced by updating pieces of machinery and brings other moderation in the system. Sarkar and Majumder (2013) incorporated a logarithmic investment function to reduce the high setup cost, which has a remarkable impact on minimizing the overall expected supply chain cost. Sarkar and Moon (2014) integrated the logarithm function used by Ouyang et al. (2002) for reducing setup cost and improving the quality of products. They extend their work by considering variable backorder rates. Recently, using a coordinated supply chain model, Dey et al. (2019) decreased setup costs through discrete investment, and improvement of process quality was achieved through logarithmic investment functions, and expected total profit was minimized.

2.4. Factors affecting performance of SCM

2.4.1. Uncertainties in SCM

The first necessity of an efficient and effective supply chain is integration among supply chain stages (Towill, 1996). Besides integration, uncertainty also needs to be taken care of to run an effective supply chain inventory policy. In the market supply chain, participants face the uncertainty of product demands, raw material supplies, commodity prices, and costs (Liu and Sahinidis, 1997). To incorporate uncertainty into supply chain modeling and optimization, the determination of suitable representation of uncertain parameters is important (Gupta and Maranas, 2003). Three different approaches are being used to represent uncertainty (Gupta and Maranas, 2003; Hameri and Paatela, 2005). First, normal distribution with specified mean and standard deviation is used to model uncertain demand and other parameters. Such an approach is termed a distribution-free approach. In the Second approach or in the fuzzy-based approach, the forecast parameters are considered as a fuzzy numbers. The scenario-based approach is the third approach; several discrete scenarios with associated probability levels are used to illustrate the expected occurrence of particular outcomes (Chen and Lee, 2004).

2.4.2. *Variable lead time*

Lead time can be viewed as the single biggest factor that influences the performance of the supply chain. Controlling lead time will resolve two main issues namely Inventory level and shortage risk. Ouyang, Wu & Ho (2004) used stochastic lead time in an integrated inventory model at the place of deterministic lead time demand and permitted shortage during the lead time and also introduced crash cost to reduce lead time. Huang et al. (2011) considered lead time demand as a compound Poisson process. This dissertation used setup cost as a variable with an added investment. Sarkar et al. (2014) contemplated lead time and ordered quality as decision variables with imperfect product quality and used inspection policies to improve product quality. Huang et al. (2011), Sarkar et al. (2014) developed an integrated inventory model with customer's delays in payment and focused on the improvement of service level.s

Tersine 1994, in his research article, breaks lead time down into five parts: "supplier's lead time, order preparation, order transit, delivery time, and setup time." Liao and

Shyu (1991) presented a probabilistic inventory model with lead time as a unique decision variable to minimize the sum of expected holding cost and the additional cost. Ben-Daya and Raouf (1994) explained both the ordering quantity as well as the lead time as decision variables and lift the shortages constraint. Later Ouyang et al., (1996) extended Ben-Daya and Raouf's (1994) model with shortage constraint, but they commit a mistake. Their mistake was corrected by Moon and Choi (1998), in his extended work. Hariga and Ben-Daya (1999) developed some stochastic inventory models to determine the optimal reduction in the procurement lead time duration with optimal ordering decisions with a mixture of backorders and lost sales and the base stock model. Pan and Yang (2002) have realized the importance of the length of lead time in customer service level, competitive abilities of a firm, and inventory investment in safety stocks. So they considered the lead time as a controllable factor to achieve the lower total expected cost and shorter lead time.

2.4.3. *Deterioration and quality improvement*

Widyadana and Wee (2011) developed two EPQ models: One for uniform distribution and the other for the exponential distribution. These were prepared for deteriorating items. In this paper, they assumed preventive maintenance and corrective times as probabilistic. Ghare and Schrader (1963) were the first authors in history who took deterioration of items in production in exponential form. Later Covert and Philip (1993) after a decade expressed failure, the maximum life of a product and time of deterioration of an item using Weibull distribution. Misra (1995) considered deterioration both in constant and variable form in its production lot size model. Goyal (1987) had scrutinized deterioration and demand that vary with time and ultimately suggested an approach for economic ordering policy. While considering the maximum lifetime of a product, Sett et al. (2012) considered deterioration that changes with time and scrutinized demand that increases quadratically. Sarkar and Saren (2015) had used the supplier-retailers partial trade-credit model to minimize retailers' annual cost and had taken deterioration exponentially. Chang and Dye (1999), Skouri and Papachristos (2003), Skouri et al. (2009), Sarkar (2011), Sarkar et al.(2013), Sarkar and Sarkar(2012), Sarkar and Sarkar(2013), Shaw et al. (2019)etc. had considered different types of deterioration in their papers.

Otten et al. (2016), introduced two echelon production-inventory system, considered a central supplier which is connected to several local suppliers for replenishing their local inventories. In local production systems, demand follows a Poisson process. Otten et al. (2016) examined a cost analysis to maintain an optimal base stock level in local production systems. Kim and Sarkar (2017) extended the idea of Porteus (1986) from a single-stage imperfect manufacturing process to a complex multi-stage imperfect manufacturing process for quality improvement by eliminating all defective items during the production process and with necessary investment in setup cost. They had also considered budget constraints and optimized replenishment intervals, number of shipments, backorder discounts, quality factor, safety factor, and lead time.

2.4.4. System Reliability

From the traditional approach of EMQ to the updated model, many researchers have been considering the production model with unreliable machines. Porteus (1986) has established a relationship between product quality and production rate and has proposed that with reduced setup cost not only the size of the lot gets smaller but also the quality of product improved. Therefore, he proposed to manufacturer to produce larger quantity with acceptable quality and shortened lead time delivery in order to meet the customer's demand and to maintain the goodwill. Rosenblatt and Lee (1986) considered "out-of-control" state timing, τ, as a negative exponentially distributed random variable with a mean of $1/\mu$. The out-of-control state is usually a result of increasing production rates to meet growing customer demand. While a hike in production rate gradually increases the number of defective products. Earlier, it was assumed that in an out-of-control state, the defective items are produced with a constant percentage 'α' while in a real situation, this assumption is not valid. The number of defective items may not remain the same throughout the production process. They have considered linear, exponential, and multi-state deterioration rate in the production process. Khouja and Mehraj (1994) had proved unit production cost and product quality both depend on production rate. The result showed that where quality of product is dependent on production rate (the quality of products deteriorate significantly with an increase in production rate), the optimal production rate occurred to be smaller from the rate that minimizes unit production cost. Alternately, if quality is not affected by production rate, the optimal rate arises as being higher than the lowest unit production cost rate.

Product reliability becomes important nowadays in lieu of felicity, well-being, safety, goodwill, and economic welfare, etc. for critically analyzing the reliability of the product is difficult (Sarkar et al., 2016, Kim and Ha, 2003) presented a coordinated model of single buyer and single seller with a constant deterioration rate and optimized the number of shipment, lot size and quantity of product to be shipped. Kim and Sarkar (2017) extended the idea of Porteus (1986) from a single-stage imperfect manufacturing process to a complex multi-stage imperfect manufacturing process for quality improvement by eliminating all defective items during the production process and with necessary investment in setup cost. They have used a stochastic inventory model. They had also considered budget constraints and optimized replenishment intervals, number of shipments, backorder discounts, quality factor, safety factor, and lead time. Sarkar et al. (2018) have incorporated integrated inventory model for single vendor and multi buyer, they had considered the variable production rate with an imperfect production process.

2.4.5. Sustainable system

Sustainability, another viable characteristic measuring the performance of chain after including social and environmental friendly initiatives in complete process ranging from material selection to disposal of product. Sustainable supply chain practices like conservation of resources, reduction of carbon footprints, bearing social responsibilities etc. had brought numerous benefits to companies including being declared as good global company. Sustainability is balancing between cost reduction and social responsibilities in a supply chain. This change over time thus becomes more complicated (Epstein & Buhovac, 2010). With depletion of resources like air, water and soil, destruction of ecosystems, habitat destruction, pollution, global warming and obligation from government to keep environment save and secure for living, organizations are integrating sustainability practices in their supply chain and has started evaluating them with effective strategies (Laurin &Fantazy, 2017).

For long run survivals, government and people are paying ample attention on an organization's management tactics and practices and with a close eye they are also monitoring their environmental initiatives and outcomes. Since, supply chain management is a basis of an organization, studying sustainable practices in a supply chain with a systematic way has become an emerging area of interest of many researchers. Xu et al. (2016) scrutinized a centralized and decentralized two-echelon sustainable supply chain's decision behaviour and synchronization methods under a cap-and-trade regulation. The article concluded that sustainability level and selling price affect the customer's demand. Tang et al. (2016) put forward a sustainable supply chain network model that reflected on the tendency of enhancing customer's demand which showed that customer's incur a high payment for environmentally sustainable products. In a while, Zhu et al. (2017) supported an emission reduction investment by developing an emission dependant supply chain model for manufacturer and supplier. Tiwari et al. (2018) presented a coordinated sustainable model for minimizing both inventory level and carbon emission with defective items. Considering the emissions trading schemes Modak et al. (2018) implemented a model for two-echelon supply chain management which suggests managers to control shortages and adjustment policies to reduce greenhouse gas emission during manufacturing process.

Chapter 3

Establishing relation between production rate and product quality in a single-vendor multi-buyer supply chain model

3.1. Introduction

In classical approach of supply chain model of single supplier and multiple buyers, the rate of production was inflexible or fixed. While in manufacturing based on machines, production rate changes easily which influences the production to be considered as decision variable instead of a constant (Sarkar et al., 2018). As per the observations, the increased production rate tends to increase tool or die cost of machines. More likely the chances of failures of process increase gradually. These component failures accelerate number of substandard products produced by the machine. For example, the production process where robot is been used repeatedly to increase production rate, it gets deteriorated rapidly (Mehrez et al., 1995). Moreover, high manufacturing rate influences the emission of vulnerable gasses in the environment. Industries also have an intension to improve the overall sustainable development both for the environmental point of view.

The relationship between product quality and production rate was first considered by Porteus (1986). He explained that with the increase in production rate, the product quality deteriorates, and the process shift from 'in control' state to 'out of control' state with a given probability. He also assumed that sub-standard quality products once began to produce continue to be produced till the end of the process. Rosenblatt and Lee (1986) considered that 'in control' (product produced is of perfect quality) process shift to 'out of control' state after a period η. The 'out of control' state was supposed as a negative exponentially distributed random variable with mean 1/μ. This work was extended by Khouja and Mehrez (1994) with an additional assumption regarded mean of random variable. They treated mean is related to production rate and created a quality function in linear and quadratic polynomial.

3.2. Model formulation

To develop the mathematical model, the following assumptions are considered.

3.2.1. Assumptions

1. A single-vendor and multi-buyer supply chain model developed for single type of products.

2. To fulfil the each buyer's demand, the vendor supplies a total of $Q = \sum_{i=1}^{n} q_i$ items. (Dey et al., 2019)

3. Vendor produces mQ quantity in one setup, after receiving the orders from all buyers to reduce setup cost and delivers products over multiple times. It is considered $q_i = Q\frac{d_i}{D}$. The production rate is a variable quantity which varies within the range $P_{\min}\left(P_{\min} > D = \sum_{i=1}^{n} d_i\right)$ and $P_{\max}$. The unit production cost is dependent on the production rate P. The quality of the product deteriorates with increasing production rate.

4. The lead time L_i (for buyer i) has n_i mutually independent components. For the j th component, $a_{i,j}$ = minimum duration, $b_{i,j}$ = normal duration, and $c_{i,j}$ = crashing cost per unit time. For the sake of convenience, it is assumed $c_{i,1} \le c_{i,2} \le ... \le c_{i,n}$ (Ouyang et al., 2004).

5. For the i-th buyer, it is assumed $L_{i,0} = \sum_{j=1}^{n_i} b_{i,j}$. $L_{i,r}$ is the length of lead time with components 1, 2, ...,r crashed to their minimum duration. Thus, $L_{i,r}$ can be expressed as $L_{i,r} = L_{i,0} - \sum_{j=1}^{r}\left(b_{i,j} - a_{i,j}\right),\ r = 1,2,3...,n;$ and the lead time crashing cost per cycle $C_i\left(L_i\right)$ is expressed as $C_i(L_i) = c_{i,r}\left(L_{i,r-1} - L_i\right) + \sum_{j=1}^{r-1} c_{i,j}\left(b_{i,j} - a_{i,j}\right), L \in \left[L_{i,r}, L_{i,r-1}\right]$

6. The lead time crashing cost belongs entirely to the buyers' cost component.

7. The elapsed time after the production system goes "out-of-control" is an exponentially distributed random variable and the mean of the exponential distribution is a decreasing function of the production rate (Dey et al., 2020).

8. Shortages are allowed and are fully backordered.

9. An inspection cost is incurred by the vendor per unit item.

10. The significance of the environmental cost for supply chain's profit.

There are two types of players as buyers and vendor in the proposed model. The buyer's model is developed first, and then vendor's model is discussed, respectively.

3.2.2. Buyer's mathematical model

The expected total cost for buyer $i$is ETC_{bi}= ordering cost + holding cost + shortage cost + lead time crashing cost.

Thus, ETC_{bi}can be written as (See for reference Sarkar et al. (2018))

$$ETC_{bi}(q_i,k_i,L_i)=\left[\frac{O_{bi}d_i}{q_i}+h_{bi}\left\{\frac{q_i}{2}+k_i\sigma_i\sqrt{L_i}\right\}+\frac{\pi_i d_i}{q_i}E(X_i-r_i)^+ +R(L_i)\frac{d_i}{q_i}\right] \tag{3.1}$$

Putting $E(X_i-r_i)^+=\sigma_i\sqrt{L_i}\psi(k_i)$ one can obtain

$$ETC_{bi}(q_i,k_i,L_i)=\left[\frac{O_{bi}d_i}{Q}+h_{bi}\left\{\frac{Q}{2D}d_i+k_i\sigma_i\sqrt{L_i}\right\}+\pi_i\sigma_i\sqrt{L_i}\psi(k_i)\frac{D}{Q}+R(L_i)\frac{D}{Q}\right] \tag{3.2}$$

3.2.3. Vendor's mathematical model

The expression of the expected total cost for the vendor is given by

Expected total cost of vendor i.e. $ETC_v(m,Q,P)=setup\ cost+holding\ cost+rework\ cost+manufacturing\ cost+environment\ cost$

$$ETC_v(m,Q,P)=\frac{S_vD}{mQ}+\frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+RD\alpha f(P)\frac{Q}{2P}+DC(P)+mE_vQ \tag{3.3}$$

The objective of this study is to obtain centralized decisions for both the vendor and the buyers to minimize the joint total supply chain cost. Therefore, the joint total expected cost for both the vendor and the buyers (*EJT C*) can be expressed as (See for reference Sarkar et al., 2020)

$$\begin{aligned}EJTC(Q,k_i,L_i,P,m)=&\sum_{i=1}^{n}\left[\frac{O_{bi}d_i}{Q}+h_{bi}\left\{\frac{Q}{2D}d_i+k_i\sigma_i\sqrt{L_i}\right\}+\pi_i\sigma_i\sqrt{L_i}\psi(k_i)\frac{D}{Q}+R(L_i)\frac{D}{Q}\right]\\&+\frac{S_vD}{mQ}+\frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+RD\alpha f(P)\frac{Q}{2P}+DC(P)+mE_vQ\end{aligned} \tag{3.4}$$

Now, to obtain the global optimum value with respect to decision variable first take first order partial derivative of *EJTC* with respect to the decision variable Q,k_i,L_i,P and m and equal to zero separately, and the global minimum of the objective cost function exists if the second order partial derivatives are all positive. According to our assumption, the number of shipment, m is a positive integer. Thus, it is no need to take the derivative of *EJTC* with respect to m. Besides this, the second order partial derivative of EJTC with respect to L_i is negative:

$$\frac{\partial^2 EJTC(Q,k_i,L_i,P,m)}{\partial L^2} = -\frac{D}{4Q}\pi_i\sigma_i\psi(k_i)L_i^{-3/2} - \frac{1}{4}h_{bi}k_i\sigma_i L_i^{-3/2} < 0$$

Thus $EJTC(Q,k_i,L_i,P,m)$ is concave with respect to L_i for the fixed value of Q,k_i,P and *m*. Therefore in the interval $\left[L_{i,j}, L_{i,j-1}\right]$ $EJTC(Q,k_i,L_i,P,m)$ is attained minimum value for the fixed value of Q,k_i,P and *m*. The following inequality always holds for fixed value Q,k_i,P and for a fixed positive integer *m*.

$$EJTC(Q,k_i,L_i,P,m-1) \geq EJTC(Q,k_i,L_i,P,m)$$
$$EJTC(Q,k_i,L_i,P,m) \leq EJTC(Q,k_i,L_i,P,m+1)$$

Equating to zero of the first order partial derivatives one can obtain the optimum value of the decision variable as follows

$$\frac{\partial EJTC}{\partial Q} = \sum_{i=1}^{n}\left[\frac{h_{bi}}{2D}d_i - \frac{O_{bi}D}{Q^2} - \frac{\pi_i\sigma_i\sqrt{L_i}\psi(k_i)D}{Q^2} - R(L_i)\frac{D}{Q^2}\right] - \frac{S_v D}{mQ^2} + \frac{h_v}{2}\left[m(1-\frac{D}{P}) - 1 + \frac{2D}{P}\right] + \frac{RD\alpha f(P)}{2P} + mE_v = 0$$

$$Q = \sqrt{\frac{2D\left\{\frac{S_v}{m} + \sum_{i=1}^{n}(O_{bi} + \pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i + h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \frac{RD\alpha f(P)}{P} + mE_v}} \qquad (3.5)$$

$$\frac{\partial EJTC}{\partial k_i} = \frac{D}{Q}\pi_i\sigma_i\sqrt{L_i}\left[\Phi(k_i) - 1\right] + h_{bi}\sigma_i\sqrt{L_i} = 0$$
$$\Phi(k_i) = 1 - \frac{h_{bi}Q}{D\pi_i} \qquad (3.6)$$

$$\frac{\partial EJTC}{\partial P} = (m-2)h_v\frac{QD}{2P^2} + DC'(P) + \frac{R\alpha DQ}{2P^2}\left(Pf'(P) - f(P)\right) = 0$$
$$\frac{1}{P^2} = \frac{2h_v DC(P)}{2QD(2-m) + h_v R\alpha DQ\left(f(P) - Pf'(P)\right)} \qquad (3.7)$$

Two different cases are considered with two different functions to define the mean time to failure

Case I: $\frac{1}{f(P)} = \frac{1}{b_1 P}$, (The quality function f(P) is linear in P)

Case II: $\frac{1}{f(P)} = \frac{1}{b_2 P + c_2 P^2}$, (The quality function f(P) is quadratic in P)

Where b_1, b_2, c_2 are non-negative scaling parameters.

Now, one can get the minimized cost, along with the decision variable based on two cases as follows:

Case I: f(P) is linear in P

$$Q_1=\sqrt{\frac{2D\left\{\frac{S_v}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\psi(k_i)+R(L_i))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P_1}\right]+\frac{RD\alpha b_1P_1}{P_1}+mE_v}} \tag{3.8}$$

$$\Phi(k_i^1)=1-\frac{h_{bi}Q_1}{D\pi_i} \tag{3.9}$$

$$P_1=\sqrt{\frac{2a_1D-Q_1h_vD(m-2)}{2Da_2}} \tag{3.10}$$

Then the total cost become

$$\begin{aligned}EJTC(Q_1,k_i^1,L_i,P_1,m)&=\sum_{i=1}^{n}\left[\frac{O_{bi}D}{Q_1}+h_{bi}\left\{\frac{Q_1}{2D}d_i+k_i^1\sigma_i\sqrt{L_i}\right\}+\pi_i\sigma_i\sqrt{L_i}\psi(k_i^1)\frac{D}{Q_1}+R(L_i)\frac{D}{Q_1}\right]\\&+\frac{S_vD}{mQ_1}+\frac{Q_1}{2}h_v\left[m\left(1-\frac{D}{P_1}\right)-1+\frac{2D}{P_1}\right]+RD\alpha b_1P_1\frac{Q_1}{2P_1}+D\left(\frac{a_1}{P_1}+a_2P_1\right)+mE_vQ_1\end{aligned} \tag{3.11}$$

Case II: *f(P)* is quadratic in *P*

$$Q_2=\sqrt{\frac{2D\left\{\frac{S_v}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\psi(k_i)+R(L_i))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P_2}\right)-1+\frac{2D}{P_2}\right]+\frac{RD\alpha(b_2P_2+c_2P_2^2)}{P_2}+mE_v}} \tag{3.12}$$

$$\Phi(k_i^2)=1-\frac{h_{bi}Q_2}{D\pi_i} \tag{3.13}$$

$$P_2=\sqrt{\frac{2a_1D-Q_2h_vD(m-2)}{2Da_2+R\alpha DQ_2b}} \tag{3.14}$$

Then the total cost become

$$\begin{aligned}EJTC(Q_2,k_i^2,L_i,P_2,m)&=\sum_{i=1}^{n}\left[\frac{O_{bi}D}{Q_2}+h_{bi}\left\{\frac{Q_2}{2D}d_i+k_i^2\sigma_i\sqrt{L_i}\right\}+\pi_i\sigma_i\sqrt{L_i}\psi(k_i^2)\frac{D}{Q_2}+R(L_i)\frac{D}{Q_2}\right]\\&+\frac{S_vD}{mQ_2}+\frac{Q_2}{2}h_v\left[m\left(1-\frac{D}{P_2}\right)-1+\frac{2D}{P_2}\right]+RD\alpha(b_2P_2+c_2P_2^2)\frac{Q_2}{2P_2}+D\left(\frac{a_1}{P_2}+a_2P_2\right)+mE_vQ_2\end{aligned} \tag{3.15}$$

3.3. Solution procedure

One can use similar solution procedure as used in Sarkar et al. (2018) to derive the extremum values of the decision variables. The iterative procedure is also applicable here as the closed form solution is unavailable. The following steps are given to develop the solution algorithm

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of Q from (3.5) and (3.12).

Step 1b Obtain $\Phi(k_i)$ from (3.9) and (3.13) and find the values of k_i by inverse normal distribution.

Step 1c Obtain P from (3.10) and (3.14).

Step 1d Perform 1a to 1c by updating the values until no changes occurs (upto a specified accuracy level) in Q, k_i, and P.

Step 2 Obtain the total cost from (3.11) and (3.15).

Step 3 Set$m = 2, 3, \ldots, p$ and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

3.4. Numerical experiments

The following two examples are considered to check the optimality of the model.

Example 1

The parametric values are taken as follows for the illustration of the model numerically

$A_{b1} = \$100/setup, A_{b2} = \$150/setup, A_{b3} = \$100/setup$ $d_1 = 200 units/year,$ $d_2 = 100$ $units/year, d_3 = 100 units/year$, $S_v = \$4000/setup$, $h_v = \$10/unit/week$, $h_{b1} = \$11$ $/unit/week, h_{b2} = \$11/unit/week, h_{b3} = \$12/unit/week$, $\sigma_1 = 9,$ $\sigma_2 = 10, \sigma_3 = 15$, $n = 3, \pi_1 = \$50/unit$, $\pi_2 = \$50/unit, \pi_3 = \$51/unit$, $a_1 = 35\times10^3, a_2 = 0.1$ $E_v = \$12.5/unit$.

The two quality functions are given by

Case I: $\dfrac{1}{f(P)} = \dfrac{1}{10^{-4}P}$

Case II: $\dfrac{1}{f(P)} = \dfrac{1}{10^{-4}P + 10^{-6}P^2}$

Now the lead time is as follows

Table 3.1: Lead time data

Buyer *i*	Lead time component	Normal duration $(b_{i,r})$ (week)	Minimum duration $(a_{i,r})$ (week)	Unit crashing cost $(c_{i,r})$ ($ per unit)
1	1	20	6	0.1
	2	20	6	1.2
	3	16	9	5.0
2	1	20	6	0.5
	2	16	9	1.3
	3	13	6	5.1
3	1	25	11	0.4
	2	20	6	2.5
	3	18	11	5.0

Using this parametric value one can easily obtain the optimised value of decision variable along with optimized total joint minimum cost. The optimal values of the decision variable along with cost are described in the Table 3.2.

Table 3.2: Optimal result table

	M	L_1 (week)	L_2 (week)	L_3 (week)	k_1	k_2	k_3	$C(P)$ ($)	Q (unit/year)	P (unit/year)	$EJTC$ ($/year)
Case 1	6	4	4	4	1.28	1.28	1.26	121.56	169.14	556.37	67265.18
Case 2	7	4	4	4	1.35	1.35	1.32	121.62	149.08	549.32	67940.97
Case 3	8	4	4	4	1.39	1.39	1.36	121.72	137.12	542.92	68764.57

The optimal result for reorder point and MTTF are described in Table 3.3

Table3.3: Optimal values for reorder points and MTTF

	r_1	r_2	r_3	$\frac{1}{f(P)}$ (weeks)
Case 1	41	36	47	19
Case 2	42	37	49	4
Case 3	43	38	50	3

Example 2

When the mean time to failure is independent of P, then all parametric values are same as Example 1, except $\frac{1}{f(P)}$. The expression $\frac{1}{f(P)}$ can be replaced by $\frac{1}{\beta}$, where β is a constant, when the production process shifted to "out-of-control" state, then process independent of production rate. Conversely, one can stated that the quality of the product deteriorate at a constant rate β, which is independent of production rate P. The optimal results for this case are elaborate in the Table 3.4.

Table 3.4: Optimal values of the decision variables for independent MTTF

β	M	L_1 (week)	L_2 (week)	L_3 (week)	k_1	k_2	k_3	$C(P)$ ($)	Q (unit/ year)	P (unit/ year)	$EJTC$ ($/year)
0.25	8	4	4	4	1.35	1.35	1.32	126.56	149.84	558.25	69772.14
0.50	8	4	4	4	1.41	1.41	1.35	126.62	143.48	561.47	69377.71
0.75	9	4	4	4	1.45	1.45	1.39	126.72	128.32	562.05	69687.83
1.00	9	4	4	4	1.49	1.49	1.41	126.74	125.47	564.34	69575.83

3.4.1. Sensitivity analysis

The sensitivity analysis for all parameters for Case 1 and 2 are depicted in Tables 5 and 6, respectively. The values of the parameters are varied between –10% to +10% and the effect of changes is shown by the analysis. The high value of sensitivity (both positive and negative) defines small change in parameter values results significant change in total cost while low value indicates less effect on total cost.

Table 3.5: Sensitivity analysis

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
A_{b1}	-10	-0.036249
	-5	-0.018113
	+5	0.018091
	+10	0.036160
A_{b2}	-10	-0.054407
	-5	-0.027178
	+5	0.027128
	+10	0.054207
A_{b3}	-10	-0.036249
	-5	-0.018113
	+5	0.018091
	+10	0.036160

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
h_{b1}	-10	-0.027821
	-5	-0.013862
	+5	0.013771
	+10	0.027454
h_{b2}	-10	-0.029940
	-5	-0.014916
	+5	0.014814
	+10	0.029529
h_{b3}	-10	-0.043739
	-5	-0.021780
	+5	0.021607
	+10	0.043049
h_v	-10	-0.023807
	-5	-0.011926
	+5	0.011970
	+10	0.023981
π_1	-10	-0.003724
	-5	-0.001807
	+5	0.001708
	+10	0.003328
π_2	-10	-0.004138
	-5	-0.002008
	+5	0.001898
	+10	0.003698
π_3	-10	-0.006827
	-5	-0.003313
	+5	0.003132
	+10	0.006100

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
R	-10	-0.006446
	-5	-0.003224
	+5	0.003228
	+10	0.006458
E_v	-10	-0.265070
	-5	-0.131839
	+5	0.130485
	+10	0.259656
A_v	-10	-0.318763
	-5	-0.158372
	+5	0.156423
	+10	0.310964

Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
A_{b1}	-10	-0.036469
	-5	-0.018223
	+5	0.018201
	+10	0.036379
A_{b2}	-10	-0.054737
	-5	-0.027343
	+5	0.027293
	+10	0.054536
A_{b3}	-10	-0.036469
	-5	-0.018223
	+5	0.018201
	+10	0.036379

Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
h_{b1}	-10	-0.027919
	-5	-0.013911
	+5	0.013819
	+10	0.027550
h_{b2}	-10	-0.030039
	-5	-0.014965
	+5	0.014862
	+10	0.029626
h_{b3}	-10	-0.043849
	-5	-0.021834
	+5	0.021661
	+10	0.043157
h_v	-10	-0.024070
	-5	-0.012056
	+5	0.012095
	+10	0.024229
π_1	-10	-0.003722
	-5	-0.001806
	+5	0.001708
	+10	0.003326
π_2	-10	-0.004136
	-5	-0.002007
	+5	0.001897
	+10	0.003696
π_3	-10	-0.006824
	-5	-0.003311
	+5	0.003130
	+10	0.006097

Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
R	-10	-0.010371
	-5	-0.005189
	+5	0.005197
	+10	0.010401
E_v	-10	-0.263537
	-5	-0.131085
	+5	0.129757
	+10	0.258224
A_v	-10	-0.316909
	-5	-0.157465
	+5	0.155553
	+10	0.309257

3.5. Managerial implications

Significant managerial insights can be driven out from this study which helps managers to build new strategies in achieving a successful and reliable supply chain. The important implications are discussed below.

- Both manufacturer and retailers make their decisions for exact amount of lot-size and order quantity under uncertain demand environment.
- The retailers can reduce the length of the lead time by investing an amount (lead time crashing cost) which results enhanced supply chain performance and customer satisfaction.
- According to the sensitivity of production rate on manufacturing reliability the MTTF of the production system changes. Managers are free to control the rate of production to enhance the manufacturing performance and to reduce MTTF.
- To achieve the environmental sustainability an investment for sustainable development is required to be incurred by the manufacturer.

3.6. Conclusion

This paper is an extension of Khouja and Mehrez's (1994) and Sarkar et al.'s (2018) model. Sarkar et al. (2018) considered a special type of quality function in their model whereas in this model environmental cost parameter for vendor is considered. A single-vendor-multi-buyer echelon supply chain model is considered here along with sustainable environmental cost parameter. A multi-buyer centralised supply chain model was developed here where the environmental cost for vendor is considered. Besides this, a general case is studied when the production system moves to 'out-of-control' state. A relationship between production rate and mean time to machine failure is established in this model in the presence of environmental cost. Finally, the total cost is minimised based on the optimised value of the decision variable.

This model can be extended by considering multi-echelon (more than two) supply chain for multi-item assembled product. Inspection is negligible for this model, thus inspection for vendor and buyer to detect the defective product is one another interesting research in this direction. Inspection, rework and inspection error along with this model will give a new direction of research. This model can also be developed by considering quality of the product as a decision variable as now days (Dey et al., 2020) customer wants more perfect quality product. Sometime demand of a product is depending on the selling price, thus considering selling price dependent demand; this model can give more realistic research direction.

Chapter 4

A multi-buyer sustainable supply chain model with information sharing and quality deterioration

4.1. Introduction

A smooth conduct of supply chain requires appropriate contribution from each of its contributing sector. In other words, information flow between several parties must be continued properly. Thus, an integrated supply chain model retains a valuable contribution in forming a successful supply chain. The pioneer approach on integrated supply chain management was introduced by Goyal (1976) which was later extended with a joint economic lot-size model (Goyal, 1988). A supply chain with various intermediate parties faces difficulties to maintain the sharing of information due to demand uncertainty. Therefore, considering random demand is a matter of concern for supply chain modelling. A very well-known approach to deal with uncertain demand is to handle with a normal distribution. A significant number of articles used this distribution to solve and obtain the managerial decisions in an integrated supply chain management (Liao & Shyu, 1991, Majumder, Guchhait & Sarkar, 2017, Majumder, Jaggi and Sarkar, 2018, Ouyang & Chen, 2002, Ouyang, Wu, & Ho, 2004). An important aspect was left out of the discussion of the literatures, which was existence of multiple retailers. Inclusion of multi-retailer in an integrated channel was introduced by Banerjee and Banerjee (1992). Later on many researchers studied and extended the basic idea of the existence of multi-retailer in their studies (Banerjee & Burton, 1994, Jha & Shankar, 2013, Lu, 1995, Majumder, Jaggi & Sarkar, 2018). Moreover, a smooth conduction of delivery of items is another vital parameter to cope up with customer satisfaction. One of the most useful ways to achieve this is to reduce the lead time. A lead time is composed of many components such as supplier's lead time, order preparation, order transit, delivery time, and the setup time (Liao & Shyu, 1991, Sarkar & Majumder, 2013). Reduction of each component of lead time leads to an achievement of successful supply chain. Therefore, researchers has been creating and implementing efficient methods to reduce lead time from decades. The investment (lead time crashing cost) to shorten lead time was studied by many literatures (Liao & Shyu, 1991, Majumder, Guchhait & Sarkar, 2017, Ouyang, Yeh, & Wu, 1996, Pan &Yang, 2002).

The role of manufacturer in the supply chain has significant importance in maintaining the system reliability. After a certain period of time the system may shift from “in-control” to “out-of-control” state and begin producing defective items. The probability of shifting one state to another state can be reduced by an investment (Porteus, 1986, Rosenblatt & Lee, 1986). Increased production rate is one of the most crucial reasons behind this situation. As an example, in a robotic assembly manufacturing system, increasing rate of production may result the deterioration of the repeatability of robotic arm (Khouja & Mehrez, 1994). As the arm speed is increased to raise the production rate, robot repeatability deteriorates. “Repeatability is defined as the ability of the robot to return to the same point, and is critical for product quality. The deterioration of repeatability results in a decrease in the percentage of conforming units produced by the robot (Mehrez, Offodile & Ahn, 1995)”. Offodile and Ugwu (1991) also supported the idea of deterioration of robotic arm with repeatability. They induced that process variables, especially speeds and weight highly affects robot performance. Conrad and McClamrock (1987) studied a drilling operation which concluded that 10% change in processing rate of the drilling machine results 50% change in tool cost. Therefore, production rate plays an extremely vital role in controlling system reliability as well as production or machine tool cost. Wang et al. (2020) enlightened on the issue of quality deterioration during production process especially when the process reaches to “out-of-control” state. They considered the adaptation of predictive maintenance policy to prevent defective production. Cheng and Li (2020) emphasized that deterioration of machine during production process influenced quality of product. Hence, a rapid quality check and machine maintenance are required to meet the product conformance.

Excess production rate also has an impact of environmental sustainability due to which industries release additional carbon in the environment. Environmental degradation, global warming, and strict governmental rules force industries to adopt green initiatives and incorporate sustainability practices into their supply chain. An additional charge termed as environmental sustainability cost has to be incurred by the companies, which is added for accounting social welfare. This environmental cost is one of the components of total cost of entire supply chain. Though environmental impact is one of the most important concerns in sustainable development, many researchers considered economic and social impact also along with environmental sustainability (Hacking & Guthrie, 2008, Herva & Roca, 2013). As the sustainability in supply chain was limited to optimization of environmental factors only, researchers gradually considered joint decision-making with manufacturing, disposal, and customer service also along with sustainable development (Linton, Klassen & Jayaraman, 2007). Again, sustainable order quantity (SOQ) model and economic order quantity (EOQ) model with sustainability were developed and became a matter of concern (Battini, Persona & Sgarbossa, 2014, Bouchery et al., 2012). Later on a significant number of definitions on several aspects regarding green and sustainable supply chain were stated (Ahi & Searcy, 2013, Khan, Hussain & Saber, 2016).

On the above context of the study, we set the objectives of this article. The objective of this research is to develop a two echelon supply chain system with single-vendor multi-buyer integrated supply chain system under demand uncertainty. Lead time plays a crucial role for customer satisfaction and uncertainty in lead time demand makes the system vulnerable towards reduced profitability. Therefore, reducing lead time is one of the most important tasks for the managers to enhance the customer's demand satisfaction. In addition to that another parameter also plays important role for customer satisfaction such as quality of product. The product quality depends on system reliability as a reliable manufacturing system produces an insignificant number of defectives. Therefore, a study on the effects of reliability under increasing rate of production along with lead time reduction strategy is another important goal of this study. Due to strict government regulations and increasing emission of greenhouse gases, a sustainable supply chain system has become a matter of concern. One of the vital objectives of this study is to analyze the effect of environmental issues and restrictions for the sustainable development on the entire system cost of the chain. Therefore, the aim of this study is to minimize the supply chain system cost under the factors discussed above on the centralized supply chain.

4.2. Model formation

This chapter is a continuation of the preceding one. The model's assumptions are disregarded because they are similar to those in the preceding chapter except, an environmental sustainability cost is incurred by all buyers.

The integrated single-vendor multi-buyer model is developed in this article as described in Porteus (1986). The similar expressions for the expected total cost of each buyer and vendor are used in this article except the environmental sustainability cost for buyers and vendor.

4.2.1. Mathematical model for buyers

Since, E_{bi} is the environmental cost for each buyer i, thus, the environmental cost component of each inventory cycle for each buyer i should be $q_i E_{bi}$. The total cost for buyer i for every inventory cycle is given by (1).

$$ETC_{bi}(q_i, k_i, L_i) = \begin{bmatrix} \frac{O_{bi} d_i}{q_i} + h_{bi}\left\{\frac{q_i}{2} + k_i \sigma_i \sqrt{L_i}\right\} \\ + \frac{\pi_i d_i}{q_i} E(X_i - r_i)^+ + R(L_i)\frac{d_i}{q_i} + q_i E_{bi} \end{bmatrix} \quad (4.1)$$

The expression of expected shortage at the end of the cycle $E(X_i - r_i)^+$ is obtained from (Sarkar and Majumder (2013)) as $E(X_i - r_i)^+ = \sigma_i \sqrt{L_i}\ \Psi(k_i)$.

Where, $\Psi(k_i) = \phi(k_i) - k_i(1 - \Phi(k_i))$,

$\phi(k_i)$ and $\Phi(k_i)$ are standard normal probability density function and distribution function of normal variate, respectively.

Thus, (1) can be written as follows

$$ETC_{bi}(q_i, k_i, L_i) = \begin{bmatrix} \frac{O_{bi} d_i}{Q} + h_{bi}\left\{\frac{Q}{2D} d_i + k_i \sigma_i \sqrt{L_i}\right\} \\ + \pi_i \sigma_i \sqrt{L_i}\ \psi(\mathrm{k_i})\ \frac{D}{Q} + R(L_i)\frac{D}{Q} + Q\frac{d_i}{D}\ E_{bi} \end{bmatrix} \quad (4.2)$$

4.2.2. Mathematical model for vendor

The expression of the expected total cost for the vendor used which is similar as (Sarkar et al., 2018)but environmental cost of vendor. As, in a single production cycle, vendor produces, mQ number of lots, thus, the environmental cost of the vendor should becomemQE_v. Therefore, total cost of the vendor possesses the expression elaborated by (3).

$$ETC_v(m, Q, P) = \begin{Bmatrix} \frac{S_v D}{mQ} + \frac{Q}{2}\ h_v\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \\ + RD\alpha f(p)\frac{Q}{2P} + DC(P) + mQE_v \end{Bmatrix} \quad (4.3)$$

4.2.3. *Significance of quality function* $\boldsymbol{f}(\boldsymbol{P})$ *and production cost* $\boldsymbol{C}(\boldsymbol{P})$

The time in which the manufacturing process shifts "in-control" to "out-of-control" follows an exponential distribution with mean μ (Offodile & Ugwu, 1991). To establish the relation between the production rate and product quality, the mean μ is considered as an increasing function of production rate $f(P)$, which is denoted as the "quality function" (Herva & Roca, 2013). This function relates the manufacturing sustainability to the rate of production. Conventionally, $f(P)$ is increasing in P such that $1/f(P)$ becomes a decreasing function in P. This $1/f(P)$ implies MTTF of the production system. Therefore, higher production rate leads to low MTTF which results degradation of system reliability as reduced MTTF is vulnerable to production of low quality products. Moreover, the manufacturing cost $C(P)$ is considered as a convex function of P as used in many literatures (Herva & Roca, 2013, Porteus, 1986).

4.3. Centralized decision for information sharing

In case of information sharing a centralized model should be implemented. Thus, the total expected cost jointly for all buyers and the vendor is established. In this case, we should consider the sum of costs of all buyers. Therefore, the expected joint total cost can be expressed by (4).

$$EJTC(Q, k_i, L_i, P, m) =$$

$$\sum_{i=1}^{n} \begin{bmatrix} \frac{O_{bi} d_i}{Q} + h_{bi} \left\{ \frac{Q}{2D} d_i + k_i \sigma_i \sqrt{L_i} \right\} + \pi_i \sigma_i \sqrt{L_i}\ \psi(k_i)\ \frac{D}{Q} + R(L_i) \frac{D}{Q}] \\ + \frac{S_v D}{mQ} + \frac{Q}{2} h_v \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] + RD\alpha f(P) \frac{Q}{2P} \\ + DC(P) + Q \left[mE_v + \frac{d_i}{D} E_b \right] \end{bmatrix} \quad (4.4)$$

Where, $E_b = \sum_{i=1}^{n} E_{bi}$.

Now, the objective is to minimize $EJTC$ with five decision variables Q, k_i, L_i, P, and m. To obtain the global optimum value with respect to the decision variables, first we take first order partial derivatives of EJTC with respect to the decision variables $Q, k_i, L_i, P, and\ m$ and put equal to zero. The sufficient condition of the global minimum of the objective cost function $EJTC$, the Hessian matrix of $EJTC$ should be positive definite. But, due to some conditions, obtaining the global minimum through Hessian matrix is restricted for the decision variables Q, k_i, and P only. The reasons are stated below.

1. The number of shipment m must be a positive integer.
2. The second order partial derivative of EJTC with respect to Li is negative ($i = 1,2, \dots, n$).

$$\frac{\partial^2 JTC(Q,k_i,L_i,P,m)}{\partial L_i^2} = -\frac{D}{4Q}\ \pi_i\sigma_i\Psi(k_i)L_i^{-\frac{3}{2}} - \frac{1}{4}h_{bi}k_i\sigma_i L_i^{-3/2} < 0.$$

Thus, $EJTC(Q,k_i,L_i,P,m)$ is concave for L_i for the fixed values of Q, k_i, m, and P. Therefore, in the interval $[L_{i,j}, L_{i,j-1}]$ $EJTC(Q,k_i,L_i,P,m)$ is attained minimum value for the fixed value of Q, k_i, P and m.

As, m is a positive integer, discrete optimization technique is used to obtain the optimal value of$m$. The method follows the following inequalities to find the value of$m$. For fixed values of Q, k_i, P and L_i, the below mentioned inequality holds true.

$$EJTC\big(Q,k_i,L_{i,},P,m-1\big) \geq EJTC\big(Q,k_i,L_{i,},P,m\big) \leq EJTC\big(Q,k_i,L_{i,},P,m+1\big)$$

For optimal m the process requires to find such a value of m so that the above inequality holds.Now, to obtain the decision variables Q, k_i, and P, equate the first order partial derivatives with respect to the variables to zero. The results obtained are stated by (5), (6), and (7).

$$Q = \sqrt{\frac{2D\left\{\frac{S_v}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\ \Psi(k_i)+R(L_i))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+\frac{RD\alpha f(P)}{P}+\left(mE_v+\frac{d_i}{D}E_b\right)}} \tag{4.5}$$

$$\Phi(k_i) = 1 - \frac{h_{bi}Q}{D\pi_i} \tag{4.6}$$

$$\frac{1}{P^2} = \frac{2h_v DC(P)}{2QD(2-m)+\ h_v R\alpha DQ(f(P)-Pf'(P))} \tag{4.7}$$

Two separate cases are considered with two individual functions to explain the MTTF.

Case I: $\frac{1}{f(P)} = \frac{1}{b_1 P}$ (The quality function $f(P)$ is linear in P)

Case II: $\frac{1}{f(P)} = \frac{1}{b_2 P + c_2 P^2}$ (The quality function $f(P)$ is quadratic in P)

Where, b_1, b_2, c_2 are non-negative scaling parameters.

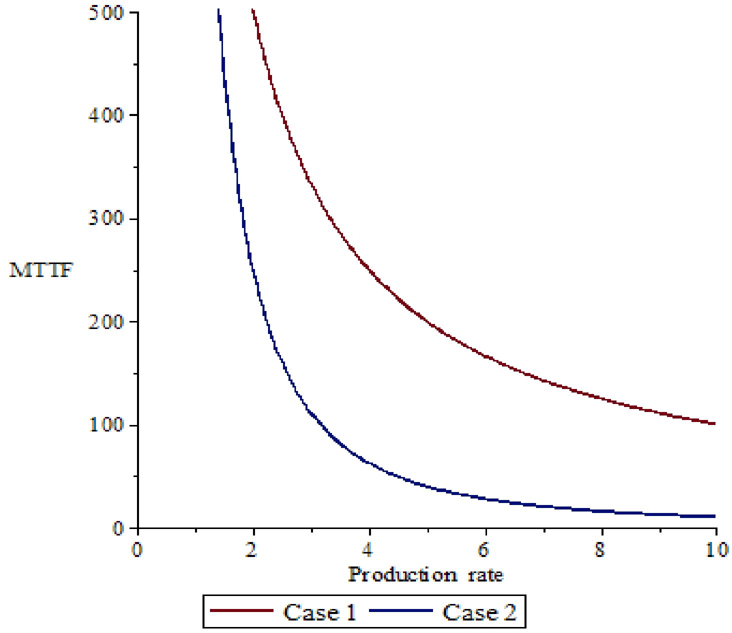


Figure 4.1: Graphical representations of MTTF

From Figure 4.1 the effect of production rate on MTTF is clearly observed. As production increases, the MTTF of the system reduces simultaneously. This is also shown that quadratic quality function affects the system reliability more than the linear case.

We use a special U shaped cost function for production cost $C(P)$ as

$$C(P) = \left(\frac{a_1}{P} + a_2 P\right) \tag{4.8}$$

Where, a_1 and a_2 are constants which give the best fit of the function.

Now, the optimal decisions and expected joint total cost based on two cases are as follows:

Case I: $f(P)$ is linear in P

$$Q_1 = \sqrt{\frac{2D\left\{\frac{S_v}{m}+\sum_{i=1}^{n}\left(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\,\Psi(k_i)+R(L_i)\right)\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P_1}\right)-1+\frac{2D}{P_1}\right]+\frac{RD\alpha b_1 P_1}{P_1}+\left(mE_v+\frac{d_i}{D}E_b\right)}} \tag{4.9}$$

$$\Phi(k^1{}_i) = 1 - \frac{h_{bi}Q_1}{D\pi_i} \tag{4.10}$$

$$P_1 = \sqrt{\frac{2a_1D - Q_1h_vD(m-2)}{2Da_2}} \tag{4.11}$$

Then the total cost becomes

$$EJTC(Q_1, k^1{}_i, L_{i,}, P_1, m) =$$

$$\sum_{i=1}^{n}\begin{bmatrix} \frac{O_{bi}d_i}{Q_1} + h_{bi}\left\{\frac{Q_1}{2D}d_i + k^1{}_i\sigma_i\sqrt{L_i}\right\} + \pi_i\sigma_i\sqrt{L_i}\ \psi(k^1{}_i)\ \frac{D}{Q_1} + R(L_i)\frac{D}{Q_1}] \\ + \ \frac{S_vD}{mQ_1} + \frac{Q_1}{2}\ h_v\left[m\left(1-\frac{D}{P_1}\right) - 1 + \frac{2D}{P_1}\right] + RD\alpha f(p)\frac{Q_1}{2P_1} \\ +D\left(\frac{a_1}{P_1} + a_2P_1\right) + Q_1\left\{mE_v + \frac{d_i}{D}\ E_b\right\} \end{bmatrix} \tag{4.12}$$

Case II: $f(P)$ is quadratic in P

$$Q_2 = \sqrt{\frac{2D\left\{\frac{Sv}{m} + \sum_{i=1}^{n}(O_{bi} + \pi_i\sigma_i\sqrt{L_i}\ \Psi(k_i) + R(L_i))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i + h_v\left[m\left(1-\frac{D}{P_2}\right) - 1 + \frac{2D}{P_2}\right] + \frac{RD\alpha(b_2P_2 + c_2P_2^2)}{P_2} + \left(mE_v + \frac{d_i}{D}E_b\right)}} \tag{4.13}$$

$$\Phi(k^2{}_i) = 1 - \frac{h_{bi}Q_2}{D\pi_i} \tag{4.14}$$

$$P_2 = \sqrt{\frac{2a_1D - Q_2h_vD(m-2)}{2Da_2 + R\alpha D\ Q_2b}} \tag{4.15}$$

Then the total cost becomes

$$EJTC(Q_2, k^2{}_i, L_{i,}, P_2, m) =$$

$$\sum_{i=1}^{n}\begin{bmatrix} \frac{O_{bi}d_i}{Q_2} + h_{bi}\left\{\frac{Q_2}{2D}d_i + k^2{}_i\sigma_i\sqrt{L_i}\right\} + \pi_i\sigma_i\sqrt{L_i}\ \psi(k^2{}_i)\ \frac{D}{Q_2} \\ +R(L_i)\frac{D}{Q_2}] \ + \ \frac{S_vD}{mQ_2} + \frac{Q_2}{2}\ h_v\left[m\left(1-\frac{D}{P_2}\right) - 1 + \frac{2D}{P_2}\right] \\ +RD\alpha(b_2P_2 + c_2P_2^2)\frac{Q_2}{2P_2} + D\left(\frac{a_1}{P_2} + a_2P_2\right) + Q_2\left\{mE_v + \frac{d_i}{D}\ E_b\right\} \end{bmatrix} \tag{4.16}$$

4.3.1. *Proposition 1*

The joint expected total cost *EJTC* in Case 1 is positive definite in Q_1, k_i^1, and P_1 if the following condition.

$$\frac{1}{Q_1}((2a_1 - Q_1h_v(m-2))\sum\left(2\left(\ o_{bi} + \pi_i\sigma_i\sqrt{L_i}\,\psi(k_i^1) + R(L_i) + \frac{S_v}{m}\right)\ .\left(\sum\left(\ Q_1\pi_i\sigma_i\sqrt{L_i}\ \ k_i^1\varphi\ (k_i^1) - \frac{D}{Q_1}\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1 - \Phi(k_i^1)\right)\right)^2\ \right)\right) > \frac{\left(\frac{m}{2}-1\right)^2 h_v^2}{P_1}.\sum\left(\pi_i\sigma_i\sqrt{L_i}\ \ \varphi\ (k_i^1)\ \right)$$

is satisfied.

Proof: See Appendix A.

4.3.3. Proposition 2

The joint expected total cost *EJTC* in Case 2 is positive definite in Q_2, k_i^2, and P_2 if the following condition

$$\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).2\sum\pi_i\sigma_i\sqrt{L_i}\,\varphi\,(k_i^1)\left(o_{bi} + R(L_i) + \frac{S_v}{m} + \pi_i\sigma_i\sqrt{L_i}\,\psi(k_i^1)k_i^1\right) - \left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^1)\right)\right)^2 > \left(m\frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2.\sum\left(\frac{D}{Q_2^2}\,\pi_i\sigma_i\sqrt{L_i}\;k_i^2\varphi\,(k_i^2)\right)$$

is satisfied.

Proof: See Appendix B.

4.4. Solution procedure

This article uses similar iterative solution procedure as used in Sarkar et al. (2018) to derive the extremum values of the decision variables. The iterative procedure is also applicable here as the closed form solution is unavailable. The following steps are given to develop the solution algorithm.

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of Q from (4.9) and (4.13).

Step 1b Obtain $\Phi(k_i)$ from (4.10) and (4.14) and find the values of k_i by inverse normal distribution.

Step 1c Obtain P from (4.11) and (4.15).

Step 1d Perform 1a to 1c by updating the values until no changes occurs (upto a specified accuracy level) in Q, k_i, and P.

Step 2 Obtain the total cost from (4.12) and (4.16).

Step 3 Set$m = 2,3, ... p$ and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

4.5. Numerical experiments

The following two examples are given to check the applicability of the model.

Example

The values of parameter are taken as follows for the illustration of the model numerically

$A_{b1} = \$100$/setup, $A_{b2} = \$150/$setup, $A_{b3} = \$100$/setup, $d_1 = 200$ units/year, $d_2 = 100$ units/year, $d_3 = 100$ units/year, $S_v = \$4000$/setup, $h_v =$ \$10/unit/week, $h_{b1} = \$11$/unit/week, $h_{b2} = \$11$/unit/week, $h_{b3} =$ \$12/unit/week, $\sigma_1 = 9, \sigma_2 = 10, \sigma_3 = 15$, $\pi_1 = \$50/$unit, $\pi_2 = \$50/$unit, $\pi_3 = \$51/$unit, $a_1 = 35 \times 10^3$, $a_2 = 0.1$, $E_v = \$12.5/$unit, $b_1 = 10^{-4}$, $b_2 = 10^{-4}$, $c_2 = 10^{-6}$, $R = \$60/$unit. E_{b1}=\$15.0/unit; E_{b2}= \$16.0/unit;E_{b3} = \$17.0/unit;

Therefore, according to the parameter values, the MTTF functions of Case 1 and 2 transforms as follows.

Case I: $\frac{1}{f(P)} = \frac{1}{10^{-4}\,P}$

Case II: $\frac{1}{f(P)} = \frac{1}{10^{-4}\,P + 10^{-6}\,P^2}$

The lead time data is given by Table 1.

Table 4.1: Lead time data

Buyer i	Lead time component	Normal duration ($b_{i,r}$) (week)	Maximum duration ($a_{i,r}$) (week)	Unit crashing cost ($c_{i,r}$) ($ per unit)
1	1 2 3	20 20 16	6 6 6	0.1 1.2 5.0
2	1 2 3	20 16 13	6 9 6	0.5 1.3 5.1
3	1 2 3	25 20 18	11 6 11	0.4 2.5 5.0

Using these parametric values optimal decision values of the decision variable along with optimized total joint minimum cost are obtained which are illustrated in Table 2.

Table 4.2: Optimal result table

	m	L_1	L_2	L_3	k_1	k_2	k_3	C(P) ($)	Q	P	EJTC ($)
Case 1	6	4	4	4	1.28	1.28	1.26	121.56	169.14	556.37	67265.18
Case 2	7	4	4	4	1.35	1.35	1.32	121.62	149.08	549.32	67940.97

The optimal result for reorder point and MTTF are described in Table 3.

Table 4.3: Optimal values for reorder points and MTTF

	r_1	r_2	r_3	$\frac{1}{f(P)}$ (weeks)
Case 1	41	36	47	19
Case 2	42	37	49	4

4.5.1. Sensitivity analysis

The sensitivity analysis of all cost parameters for Case 1 and 2 are performed in Table 4 and 5, respectively. The cost parameters are varied from -10% to +10% and the changes in expected total cost is observed.

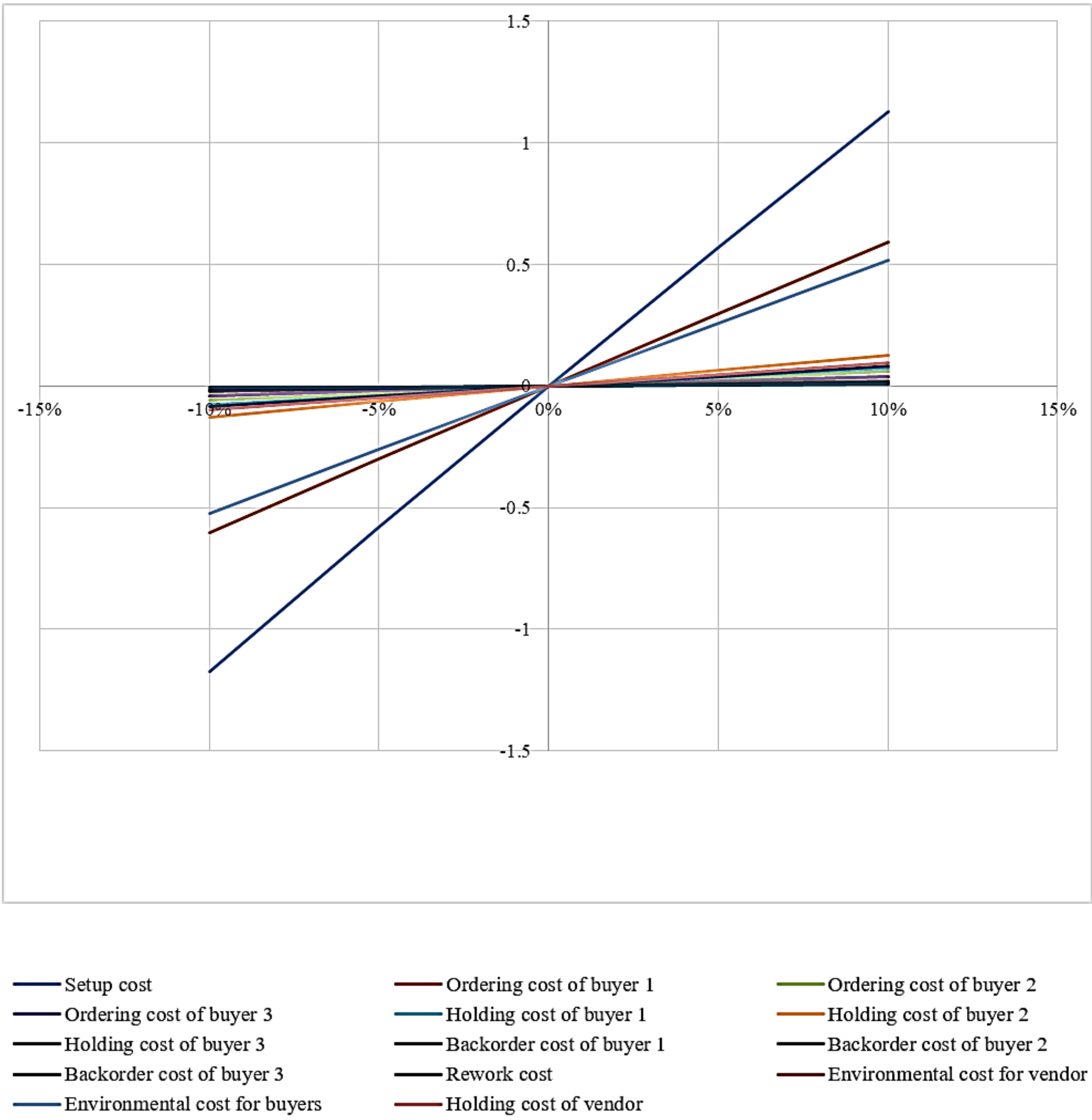
Table 4.4: Sensitivity analysis

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
S_v	-10	-1.175314
	-5	-0.581071
	+5	0.568742
	+10	1.125912
A_{b1}	-10	-0.039403
	-5	-0.019695
	+5	0.019681
	+10	0.039347
A_{b2}	-10	-0.059126
	-5	-0.029547
	+5	0.029515
	+10	0.058999
A_{b3}	-10	-0.039403
	-5	-0.019695
	+5	0.019681
	+10	0.039347

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
h_{b1}	-10	-0.077129
	-5	-0.038351
	+5	0.037940
	+10	0.075485
h_{b2}	-10	-0.129083
	-5	-0.129083
	+5	0.063616
	+10	0.126645
h_{b3}	-10	-0.082873
	-5	-0.041200
	+5	0.040745
	+10	0.081052
h_v	-10	-0.097448
	-5	-0.048733
	+5	0.048748
	+10	0.097508
π_1	-10	-0.015044
	-5	-0.007290
	+5	0.006875
	+10	0.013379

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
π_2	-10	-0.022565
	-5	-0.010934
	+5	0.010313
	+10	0.020068
π_3	-10	-0.016614
	-5	-0.008050
	+5	0.007591
	+10	0.014770
R	-10	-0.007018
	-5	-0.003509
	+5	0.003509
	+10	0.007019
E_v	-10	-0.602509
	-5	-0.299646
	+5	0.296526
	+10	0.590024
E_b	-10	-0.525259
	-5	-0.261380
	+5	0.258950
	+10	0.515533

Figure 4.2: Graphical representation of sensitivity analysis for linear quality function

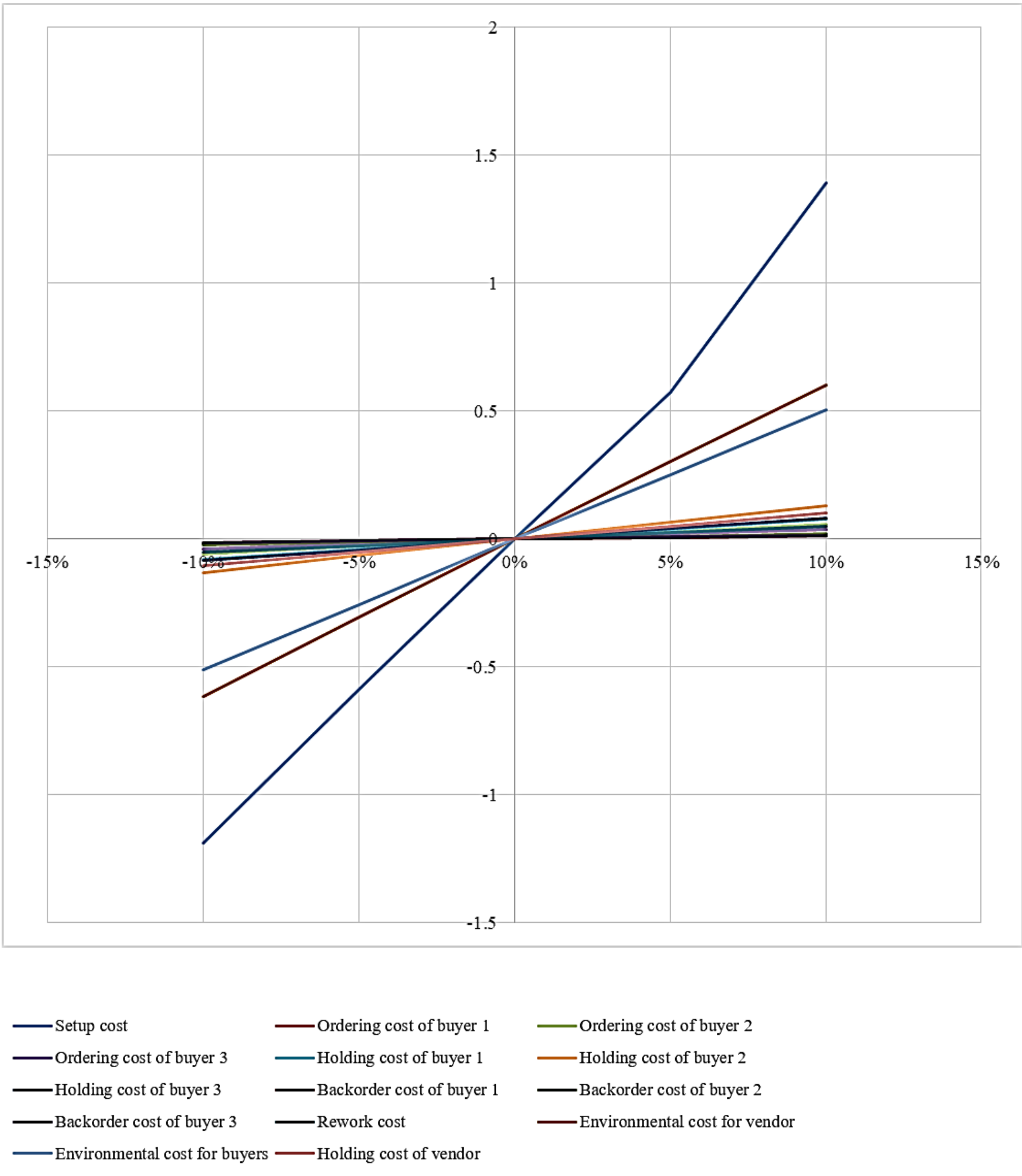


Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
S_v	-10	-1.189376
	-5	-0.588021
	+5	0.575540
	+10	1.139365
A_{b1}	-10	-0.038804
	-5	-0.019395
	+5	0.019381
	+10	0.038749
A_{b2}	-10	-0.058227
	-5	-0.029098
	+5	0.029067
	+10	0.058102
A_{b3}	-10	-0.038804
	-5	-0.019395
	+5	0.019381
	+10	0.038749

Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
h_{b1}	-10	-0.078281
	-5	-0.038923
	+5	0.038506
	+10	0.076609
h_{b2}	-10	-0.131507
	-5	-0.065430
	+5	0.064808
	+10	0.129017
h_{b3}	-10	-0.084111
	-5	-0.041815
	+5	0.041352
	+10	0.082259
h_v	-10	-0.102522
	-5	-0.051257
	+5	0.051247
	+10	0.102481
π_1	-10	-0.015227
	-5	-0.007378
	+5	0.006958
	+10	0.013538
π_2	-10	-0.022841
	-5	-0.011067
	+5	0.010437
	+10	0.020308
π_3	-10	-0.016820
	-5	-0.008149
	+5	0.007683
	+10	0.014949

Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
R	-10	-0.050851
	-5	-0.025421
	+5	0.025413
	+10	0.050818
E_v	-10	-0.616383
	-5	-0.306505
	+5	0.303236
	+10	0.603303
E_b	-10	-0.512709
	-5	-0.255190
	+5	0.252920
	+10	0.503627

Figure 4.3: Graphical representation of sensitivity analysis for quadratic quality function



4.5.2. Numerical discussion

The results of numerical experimentation are illustrated in Table 4.2 and 4.3. The sensitivity analysis of all cost parameters are shown in Table 4.4 and 4.5. According to the results, the optimal lot sizes and total costs for Case 1 and 2 are 169.14 units; $67265.18 and 149.08 units; $67940.97, respectively. As the order quantity of each buyer is defined by $q_i = Q\frac{d_i}{D}$, therefore, the order quantity for buyer 1, 2 and 3 are calculated as 84.57, 42.28, and 42.28, respectively. Accordingly, for Case 2, these are 74.54, 37.27, and 37.27, respectively. MTTF for Case 1 and 2 are 19 weeks and 4 weeks, respectively. Clearly, it is observed that quadratic quality function results lower MTTF than linear case which proves that the system following linear nature of the quality function more reliable than the system having quadratic nature.

4.6. Managerial implications

The managerial opinions drawn by analysing the numerical experimentation are given as follows:

- The system containing the quadratic quality function incurs higher cost and lower supply chain profitability than the system having linear quality function.
- The MTTF is higher if the quality deterioration is a linear function of production rate than the quadratic case. Therefore, the system is more reliable in linear case.
- Setup cost of vendor and environmental costs for vendor and buyers are two of the most sensitive costs which influences the expected total cost of the chain significantly.
- Rework cost, holding costs (vendor and buyers), and ordering costs are three of the less sensitive costs in the chain.

4.7. Conclusion

The study analyzed the reliability of a manufacturing system under two echelon supply chain management with a number of retailers. They considered a special type of quality function in their single echelon model whereas in this model similar function was analyzed under a centralized supply chain with environmental cost parameter for both vendor and buyers. In this paper, uncertainty in demand was assumed with a normal distribution.

The study concluded that for higher degree of quality function the system reliability is diminished which results early MTTF. As production rate varies and reliability is directly considered as a function of production which deteriorates with increased production rate, quadratic nature of quality function is more sensitive to quality deterioration than linear one. The investments were considered to achieve the environmental sustainability for each buyer and vendor. Due to centralization of the model, each buyer's environmental investment was added and considered to be acted like single investment which is beneficial to achieve reduced system cost. Moreover, analysis of sensitivity disclosed that the impacts of changing cost parameters occur for setup and environmental investments.

The model can further be extended a 3PL supply chain model. As inspection is negligible, the model can also be revised with inspection which can help reducing the rework of defective goods. A smart autonomation technology can be used for inspection. Moreover, instead of single item, a multi-item and multi-stage production model can be a great deal of attention.

4.8. Appendices of Chapter 4

Appendix A

The second order derivatives with respect to Q_1, k_i^1, and P_1 are as follows.

$$\frac{\partial^2(EJTC)}{\partial Q_1\,\partial k_i^1} = \sum\left(\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\ (1-\Phi(k_i^1))\right)$$

$$\frac{\partial^2(EJTC)}{\partial Q_1\,\partial P_1} = m\frac{h_v D}{2P_1^2} - \frac{h_v D}{P_1^2}$$

$$\frac{\partial(EJTC)}{\partial k_i^1} = \sum\left(\frac{D}{Q_1}\ \pi_i\sigma_i\sqrt{L_i}\ \{(\Phi(k_i^1)-1)\} + h_{bi}\sigma_i\sqrt{L_i}\right)$$

$$\frac{\partial^2(EJTC)}{\partial k_i^1\,\partial Q_1} = \sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \left\{\left(1-\Phi(k_i^1)\right)\right\}\right)$$

$$\frac{\partial^2(EJTC)}{\partial {k_i^1}^2} = \sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \varphi\ (k_i^1)\right)$$

$$\frac{\partial^2(EJTC)}{\partial k_i^1\ \partial P_1} = 0$$

$$\frac{\partial^2(EJTC)}{\partial P_1 \partial k_i^1} = 0$$

$$\frac{\partial^2(EJTC)}{\partial P_1\ \partial Q_1} = \ m\frac{h_v D}{2P_1^2} - \frac{h_v D}{P_1^2}$$

$$\frac{\partial^2(EJTC)}{\partial {P_1}^2} = \ -\frac{Q_1 h_v D}{P_1^3}(m-2) + \ 2\frac{D}{P_1^3}a_1$$

The Hessian matrix is defined as

$$\mathrm{H} = \begin{bmatrix} \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{Q}_1^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{Q}_1\ \partial \mathrm{k}_\mathrm{i}^1} & \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{Q}_1\ \partial \mathrm{P}_1} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{k}_\mathrm{i}^1\ \partial \mathrm{Q}_1} & \frac{\partial^2 \mathrm{EJTC}}{\partial (\mathrm{k}_\mathrm{i}^1)^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{k}_\mathrm{i}^1\ \partial \mathrm{P}_1} \\ \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{P}_1\ \partial \mathrm{Q}_1} & \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{P}_1\ \partial \mathrm{k}_\mathrm{i}^2} & \frac{\partial^2 \mathrm{EJTC}}{\partial \mathrm{P}_1^2} \end{bmatrix}$$

The first principle minor is $\frac{\partial^2 EJTC}{\partial Q_1^2} > 0$ in all cases.

The second principle minor is

$$\begin{vmatrix} \frac{\partial^2 \boldsymbol{EJTC}}{\partial \boldsymbol{Q}_1^2} & \frac{\partial^2 \boldsymbol{EJTC}}{\partial \boldsymbol{Q}_1 \partial \boldsymbol{k}_i^1} \\ \frac{\partial^2 \boldsymbol{EJTC}}{\partial \boldsymbol{k}_i^1 \partial \boldsymbol{Q}_1} & \frac{\partial^2 \boldsymbol{EJTC}}{\partial (\boldsymbol{k}_i^1)^2} \end{vmatrix} =$$

$$\begin{vmatrix} \sum\left(\frac{2o_{bi}D}{Q_1^3} + \ \pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \ \frac{2\ R(L_i)D}{Q_i^3}\right) + \frac{2\ S_v D}{mQ_i^3} & \sum\left(\ \pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\ (1-\Phi(k_i^1))\right) \\ \sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \left\{\left(1-\Phi(k_i^1)\right)\right\}\right) & \sum(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \varphi\ (k_i^1)\ ) \end{vmatrix}$$

$= (\sum\left(\frac{2o_{bi}D}{Q_1^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_i^3}\right) + \frac{2\,S_vD}{mQ_i^3})\sum(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;\varphi\,(k_i^1)\,) - \left(\sum \pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi(k_i^1)\right)\right)^2 > 0$ holds true.

Since,

$2\sum\pi_i\sigma_i\sqrt{L_i}\varphi\,(k_i^1)(o_{bi}+R(L_i)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\,\psi(k_i^1)k_i^1) > \left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^1)\right)\right)^2$. Note that $0 < 1-\Phi(k_i^1) < 1$, thus, $\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^1)\right) < 1$ implies, $\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^1)\right)\right)^2 \ll 1$.

Third principle minor is

$$\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial (k_i^1)^2} & \frac{\partial^2 EJTC}{\partial k_i^1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial P_1\partial Q_1} & \frac{\partial^2 EJTC}{\partial P_1\partial k_i^2} & \frac{\partial^2 EJTC}{\partial P_1^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial (k_i^1)^2} & 0 \\ \frac{\partial^2 EJTC}{\partial P_1\partial Q_1} & 0 & \frac{\partial^2 EJTC}{\partial P_1^2} \end{vmatrix} = \frac{\partial^2 EJTC}{\partial P_1\partial Q_1}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial (k_i^1)^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial P_1^2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial (k_i^1)^2} \end{vmatrix}$$

$=-\left(m\frac{h_vD}{2P_1^2}-\frac{h_vD}{P_1^2}\right)^2.\sum\left(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;k_i^1\varphi\,(k_i^1)\right) + (\,2\frac{D}{P_1^3}a_1 - \frac{Q_1h_vD}{P_1^3}(m-2)).(\sum\left(\frac{2o_{bi}D}{Q_1^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_i^3}\right) + \frac{2\,S_vD}{mQ_i^3})\sum(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;\varphi\,(k_i^1)\,) - \left(\sum\left[\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi(k_i^1)\right)\right]\right)^2 > 0$ if the following condition holds.

$$(2\frac{D}{P_1^3}a_1 - \frac{Q_1 h_v D}{P_1^3}(m-2)).(\sum\left(\frac{2o_{bi}D}{Q_1^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_i^3} + \frac{2\,S_v D}{mQ_1^3}\right).\sum(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;k_i^1\varphi\,(k_i^1)\,) - \left(\sum\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi(k_i^1)\right)\right)^2\;)$$

$$> \left(m\frac{h_v D}{2P_1^2} - \frac{h_v D}{P_1^2}\right)^2.\sum\left(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;\varphi\,(k_i^1)\right)$$

$$\text{Or,}\frac{1}{Q_1}(\left(2a_1 - Q_1 h_v(m-2)\right)(\sum\left(2\left(o_{bi} + \pi_i\sigma_i\sqrt{L_i}\psi(k_i^1) + R(L_i) + \frac{S_v}{m}\right)\right)\;.\left(\sum(\,Q_1\pi_i\sigma_i\sqrt{L_i}\;k_i^1\varphi\,(k_i^1) - \frac{D}{Q_1}\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^1)\right)\right)^2\;\right)\right)$$

$$> \frac{\left(\frac{m}{2}-1\right)^2 h_v^2}{P_1}.\sum\left(\pi_i\sigma_i\sqrt{L_i}\;\varphi\,(k_i^1)\,\right).$$

Which proves the proposition.

Appendix B

Like Appendix A, the Hessian matrix is

$$H = \begin{bmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} & \frac{\partial^2 EJTC}{\partial k_i^2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial P_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial P_2^2} \end{bmatrix}$$

The first principle minor

$$\frac{\partial^2(EJTC)}{\partial Q_2^2} = \sum\left(\frac{2o_{bi}D}{Q_2^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_2^1)D}{Q_2^3} + \frac{2\,R(L_i)D}{Q_2^3}\right) + \frac{2\,S_v D}{mQ_2^3} > 0$$

The second principle minor

$$\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} \\ \frac{\partial^2 EJTC}{\partial k_i^3 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} \end{vmatrix} > 0. \text{ As,}$$

$$2 \sum \pi_i \sigma_i \sqrt{L_i} \quad \varphi\ (k_i^1) \quad (o_{bi} + \quad R(L_i) + \frac{S_v}{m} + \pi_i \sigma_i \sqrt{L_i}\, \psi(k_i^1) k_i^1) > \left(\sum \pi_i \sigma_i \sqrt{L_i} \left(1 - \Phi(k_i^1)\right)\right)^2$$

The third principle minor

$$\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} & \frac{\partial^2 EJTC}{\partial k_i^2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial P_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial P_2^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} & 0 \\ \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} & 0 & \frac{\partial^2 EJTC}{\partial P_2^2} \end{vmatrix} = \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial (k_i^2)^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial P_2^2} \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} \end{vmatrix}$$

$$= \ -\left(m \frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2 . \sum \left(\frac{D}{Q_2^2}\ \pi_i \sigma_i \sqrt{L_i}\ \ k_i^2 \varphi\ (k_i^2)\right) + \left(2 \frac{D}{P_2^3} a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right). (2 \sum \pi_i \sigma_i \sqrt{L_i} \varphi\ (k_i^1)(o_{bi} + R(L_i) + \frac{S_v}{m} + \pi_i \sigma_i \sqrt{L_i}\, \psi(k_i^1) k_i^1) - \left(\sum \pi_i \sigma_i \sqrt{L_i} \left(1 - \Phi(k_i^1)\right)\right)^2) > 0$$

Only if,

$$\left(2 \frac{D}{P_2^3} a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right). 2 \sum \pi_i \sigma_i \sqrt{L_i}\, \varphi\ (k_i^1) \left(o_{bi} + \ R(L_i) + \frac{S_v}{m} + \pi_i \sigma_i \sqrt{L_i}\, \psi(k_i^1) k_i^1\right) - \left(\sum \pi_i \sigma_i \sqrt{L_i} \left(1 - \Phi(k_i^1)\right)\right)^2 > \left(m \frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2 . \sum \left(\frac{D}{Q_2^2}\ \pi_i \sigma_i \sqrt{L_i}\ \ k_i^2 \varphi\ (k_i^2)\right)$$

holds true which proves the proposition.

Chapter 5

A sustainable centralized supply chain management with the effect of deterioration on system reliability under reduced setup cost

5.1. Introduction

There is a massive impact of the rate of production on the quality of a product. The manufacturing quality deteriorates as the production rate increases. In this context, this research develops a single-vendor multi-buyer supply chain model with variable production cost, poor quality of products, environmental and social factors. A unit production cost is considered as a function of the production rate. Moreover, an increase in production rate deteriorates the product quality, resulting in inferior products. Thus, a relation between the production rate and system reliability is measured through the meantime to system failure (MTTF). High production rate also produces significant carbon emissions in the environment. In order to reduce environmental pollution, industries have to invest an amount either for technological development or as a carbon tax. Recently, the initiatives are aware of investing in social welfare as insurances and other benefits to its labors and the environmental cost as it is important for long-term sustainable development. Besides, investment in technology to reduce greenhouse gas emissions may result in an increase in setup costs. Therefore, an attempt to reduce setup cost is also employed by assuming a variable investment. Considering all these facts, a cost minimization model is developed which is solved to obtain a global optimal strategy.

In integrated supply chain model where quality of production does not remain same throughout the process, the increased production rate draws our attention on the safety of environment as well. Hence, Sarkar et al. (2020) used a realistic approach for single supplier and multiple buyer by viewing production rate as decision variable instead of a parameter in machine manufacturing based system. With increased production, the machine components start dying and results in production of sub-standard goods. This condition more likely appeared in robot-based production where the robot is used repeatedly to raise production rate (Mehraz et al., 1995). The increased production influences the release of vulnerable gases. With rapid climate change, sustainability is becoming a corporate social responsibility. They need to incorporate green initiates and seek effective strategies to attain sustainable development in SCM (Singh et al., 2018). To celebrate century of the EOQ model, an honour to Ford Whitman Harris was presented by Cárdenas-Barrón et al. (2014).

5.2. Model formation

5.2.1. Assumptions

As this research is an addition to the previous research hence most of the assumptions are followed here as well except, Social sustainability cost is incurred by all retailers and the vendor.

5.2.2. Buyer's mathematical model

The various cost components for expected total cost for buyer i's will be ordering cost , holding cost, shortage cost, lead time crashing cost, environment cost and social cost. Hence,

ETC_{bi}= ordering cost + holding cost + shortage cost + lead time crashing cost + Environment Cost + Social Cost.

Or

$$ETC_{bi}(q_i, k_i, L_i) = \begin{bmatrix} \frac{O_{bi}d_i}{q_i} + h_{bi}\left\{\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i}\right\} \\ + \frac{\pi_i d_i}{q_i} E(X_i - r_i)^+ + R(L_i)\frac{d_i}{q_i} + q_i(E_{bi} + S_{bi}) \end{bmatrix} \quad (5.1)$$

The expression of expected shortage at the end of the cycle can be written as

$$E(X_i - r_i)^+ = \sigma_i \sqrt{L_i}\ \Psi(k_i). \quad (5.2)$$

Where, $\Psi(k_i) = \phi(k_i) - k_i(1 - \Phi(k_i))$,

$\phi(k_i)$ and $\Phi(k_i)$ are standard normal probability density function and distribution function of normal variate, respectively.

Thus, (5.1) should be written as follows

$$ETC_{bi}(Q, k_i, L_i) = \begin{bmatrix} \frac{O_{bi}d_i}{Q} + h_{bi}\left\{\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i}\right\} \\ +\pi_i\sigma_i\sqrt{L_i}\ \psi(\mathrm{k_i})\ \frac{D}{Q} + R(L_i)\frac{D}{Q} + Q\,\frac{d_i}{D}\ (E_{bi} + S_{bi}) \end{bmatrix} \quad (5.3)$$

5.2.3. Vendor's mathematical model

The expected total cost for vendor is ETC_v= Setup cost + holding cost + Rework cost + Production cost +Environment Cost + Social cost. (Sarkar et al. (2018), Khan et al. (2018)).

Where, Setup cost = $\frac{S_v D}{mQ}$, Holding cost = $\frac{Q}{2}\ h_v\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right]$

Rework cost $=\frac{RD}{Q}\,E(N)$, Where $E(N)$ is expected (mean) number of defective items in production process. Where, the number of defective units in a production cycle with linear deterioration will be given by

$$N=\begin{cases} 0 & t\geq\tau \\ \alpha P\,(t-\tau)+\alpha\,P\,\int_0^{t-\tau}\beta x\,dx & t>\tau \end{cases}$$ (Rosenblatt and Lee, 1986)

The number of defective units in a production cycle with exponential deterioration will be given by

$$N=\begin{cases} 0 & t\geq\tau \\ \alpha P\,(t-\tau)+\alpha\,P\,\int_0^{t-\tau}(1-e^{-\beta x})dx & t>\tau \end{cases}$$ (Rosenblatt and Lee, 1986)

5.2.4 Proposition 3

The expected number of defective items are same for linear and exponential deterioration which is equal to E (N) = $\frac{\alpha f(P)Q^2}{2P}$.

Proof: See Appendix C

Hence, Rework cost becomes $\frac{\boldsymbol{R\,D\alpha f(P)\,Q}}{\boldsymbol{2P}}$.

Manufacturing cost $=D\;C(P)$,
$C(P) is\ unit\ production\ cost\ and\ considered\ as\ C(P)=a_1P+\frac{a_2}{P}$.

$a_1=$ labor and energy costs and $a_2=$ unit tool cost

In a single production cycle, vendor produces, m*Q number of lots, thus, the social cost of the vendor should become$mQ(E_v+S_{cv})$.

Thus, average cost of vendor (ETC_v) can be written as (See for reference Sarkar et al., 2018, Khan et al., 2016).

Therefore, total cost of the vendor possesses the expression elaborated by (5.4). In this paper, we are investigating the effect of a variable set up cost on vendors expected total cost. In equation (5.4), a fixed setup cost has been taken which is inappropriate for more realistic situation.

$$ETC_v(m,Q,P) = \begin{Bmatrix} \frac{S_vD}{mQ} + \frac{Q}{2}\, h_v \left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \\ +RD\alpha f(p)\frac{Q}{2P} + DC(P) + mQ(E_v + S_{cv}) \end{Bmatrix} \tag{5.4}$$

That's why in this part, setup cost is taken as a variable instead of taking it as a constant and a capital investment cost C_s = B $\ln S_0/S_v$ for $0 < S_v \le S_0$ is used to reduced vendor's total expected cost after a fixed interval of time. Here S_0 is vendor's original set up cost. And B=$\frac{1}{\delta}$with δ is percentage decrease in S or in setup cost with per dollar increase in C_s (Majumder, Jaggi & Sarkar, 2017).

Hence, the expression of expected total cost for the vendor is given by

$$ETC_v(m,Q,P\ ,S_v) = \begin{Bmatrix} \beta\, \mathrm{B}\ (\ln(S_0) - \ln(S_v)) + \frac{S_vD}{mQ} + \frac{Q}{2}\, h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \\ +RD\alpha f(p)\frac{Q}{2P} + DC(P) + mQ(E_v + S_{cv}) \end{Bmatrix} \tag{5.5}$$

Therefore, the joint total expected cost for both the vendor and the buyers is equals to the sum of buyer's expected total cost and vendor's expected total cost.

$$EJTC(Q,k_i,L_{i,},P,m,S_v) = \sum_{i=1}^{n}\left[\begin{matrix} \frac{O_{bi}d_i}{Q} + h_{bi}\left\{\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i}\right\} + \pi_i\sigma_i\sqrt{L_i}\ \psi(\mathrm{k_i})\ \frac{D}{Q} + R(L_i)\frac{D}{Q}] \\ +\beta\, \mathrm{B}\ (\ln(S_{v_0}) - \ln(S_v)) + \frac{S_vD}{mQ} + \frac{Q}{2}\, h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + RD\alpha f(P)\frac{Q}{2P} \\ +DC(P) + Q\left[m(E_v + S_{cv}) + \frac{d_i}{D}\ (E_{bi} + S_{bi})\right] \end{matrix}\right] \tag{5.6}$$

5.2.5 *Proposition 4*

The joint expected total cost *EJTC* in Case 1 is positive definite in Q_1, k_i^1, S_{v1} and P_1 if the following c\\ondition

$(\frac{D^2}{m^2Q_2^4})\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2h_vD}{P_2^3}(m-2)\right).\sum\left(\frac{D}{Q_2^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ k_i^2\varphi\ (k_i^2)\right) + \frac{\beta B}{S^2}\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2h_vD}{P_2^3}(m-2)\right).(2\sum\pi_i\sigma_i\sqrt{L_i}\ \varphi\ (k_i^1)\ (o_{bi}+R(L_i) + \frac{S_v}{m} + \pi_i\sigma_i\sqrt{L_i}\ \psi(k_i^1)k_i^1) \ge \frac{\beta B}{S^2}\{\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2h_vD}{P_2^3}(m-2)\right)*\left(m\frac{h_vD}{2P_2^2} - \frac{h_vD}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2 + \left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1 - \Phi(k_i^1)\right)\right)^2\}$

is satisfied.

Proof: See Appendix D.

5.2.6 *Proposition 6*

The joint expected total cost *EJTC* in Case 2 is positive definite in Q_2, k_i^2, S_{v2} and P_2 if the following condition

$$\left(\frac{D^2}{m^2 Q_2^4}\right)\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).\sum\left(\frac{D}{Q_2^2}\pi_i\sigma_i\sqrt{L_i}\;k_i^2\varphi\left(k_i^2\right)\right) + \frac{\beta B}{S^2}\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).\left(2\sum\pi_i\sigma_i\sqrt{L_i}\quad\varphi\left(k_i^2\right)\quad\left(o_{bi} + R(L_i) + \frac{S_v}{m} + \pi_i\sigma_i\sqrt{L_i}\,\psi(k_i^2)k_i^2\right) > \frac{\beta B}{S^2}\left\{\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right) * \left(m\frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2 + \left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1 - \Phi(k_i^2)\right)\right)^2\right\}$$

is satisfied.

Proof: See Appendix E.

The expected total cost has five decision variables as Q, L_i, k_i, m, S_v, P. To obtain the global optimum solution the classical optimization technique is used which is supposed to take derivatives of the objective function with respect to all decision variables and equate them to zero. The classical optimization technique is used to obtain the values of the decision variables

The values of the decision variables can be obtained are as follow.

$$Q = \sqrt{\frac{2D\left\{\frac{S_v}{m} + \sum_{i=1}^{n}\left(O_{bi} + \pi_i\sigma_i\sqrt{L_i}\,\Psi(k_i) + R(L_i)\right)\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i + h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + \frac{RD\alpha f(P)}{P} + \left(m(E_v + S_{cv}) + \frac{d_i}{D}(E_{bi} + S_{bi})\right)}} \tag{5.7}$$

$$\Phi(k_i) = 1 - \frac{h_{bi}Q}{D\pi_i} \tag{5.8}$$

$$S_v = \beta B\frac{mQ}{D} \tag{5.9}$$

$$\frac{1}{P^2} = \frac{2h_m DC(P)}{2QD(2-m) + h_v R\alpha DQ(f(P) - Pf'(P))} \tag{5.10}$$

Since, *m* (number of lot) is an integer; a discrete optimization technique is used to obtain the optimal number of lot.

$$EJTC\big(Q, k_i, L_{i,}, P, m-1, S_v\big) \geq EJTC\big(Q, k_i, L_{i,}, P, m, S_v\big) \leq EJTC\big(Q, k_i, L_{i,}, P, m+1, S_v\big)$$

The second order partial derivative of EJTC with respect to L_i is negative ($i = 1,2, \dots, n$).

$$\frac{\partial^2 EJTC(Q,k_i,L_i,P,m,S_v)}{\partial L_i^2} = -\frac{D}{4Q}\ \pi_i \sigma_i \Psi(k_i) L_i^{-\frac{3}{2}} - \frac{1}{4} h_{bi} k_i \sigma_i L_i^{-3/2} < 0.$$

Thus, $EJTC(Q, k_i, L_i, P, m, S_v)$ is concave for L_i for the fixed values of Q, k_i, m, P, and S_v. Therefore, in the interval $[L_{i,j}, L_{i,j-1}]$ $EJTC(Q, k_i, L_i, P, m, S_v)$ is attained minimum value for the fixed value of Q, k_i, P, S_v *and* m.

Two separate cases are considered with two individual functions to explain the MTTF with the help of $f(P)$ *as quality fuction*. The quality function is considered as linearly and quadratically increasing function in P $i.e\ f(P) = b_1 P$ *and* $f(P) = b_2 P + c_2 P^2$ which connect production rate with number of defective items. Ideally, as the production rate increases the number of defective items increases in order terms system shifts from perfect state to imperfect state. Therefore, $\frac{1}{f(P)}$ represents MTTF (mean time to failure), a measure of reliability in manufacturing units where non-repairable units of a machine operate before failure (Khauja and Mehraj, 1994).

Two cases are considered for same.

Case I: $\frac{1}{f(P)} = \frac{1}{b_1 P}$ (The quality function $f(P)$ is linear in P)

Case II: $\frac{1}{f(P)} = \frac{1}{b_2 P + c_2 P^2}$ (The quality function $f(P)$ is quadratic in P)

Decision variables for linear quality function (Case 1)

$$Q_1 = \sqrt{\frac{2D\left\{\frac{S_{v_1}}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\,\Psi(k_i)+R(L_i))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_m\left[m\left(1-\frac{D}{P_1}\right)-1+\frac{2D}{P_1}\right]+\frac{RD\alpha b_1 P_1}{P_1}+\left(m(E_v+S_{cv})+\frac{d_i}{D}(E_{bi}+S_{bi}\right)}} \quad (5.11)$$

$$\Phi(k^1{}_i) = 1 - \frac{h_{bi}Q_1}{D\pi_i} \quad (5.12)$$

$$P_1 = \sqrt{\frac{2a_1D - Q_1h_vD(m-2)}{2Da_2}} \quad (5.13)$$

$$S_{v_1} = \beta\, B\, \frac{mQ_1}{D} \quad (5.14)$$

After incorporating all, the combined total expected cost of buyer and supplier (manufacturer) will be as follow.

$$EJTC\left(Q_1, k_i^1, L_{i,}, P_1, m, S_{v_1}\right) =$$

$$\sum_{i=1}^{n}\left[\begin{array}{c} \frac{O_{bi}d_i}{Q_1} + h_{bi}\left\{\frac{Q_1}{2D}d_i + k_i^1\sigma_i\sqrt{L_i}\right\} + \pi_i\sigma_i\sqrt{L_i}\ \psi(\mathrm{k_i})\ \frac{D}{Q_1} + R(L_i)\frac{D}{Q_1}] \\ +\beta\ \mathrm{B}\ (\ln(S_{v_0}) - \ln(S_{v_1})) + \frac{S_{m_1}D}{mQ_1} + \frac{Q_1}{2}\ h_v\left[m\left(1-\frac{D}{P_1}\right) - 1 + \frac{2D}{P_1}\right] + RD\alpha f(P)\frac{Q_1}{P_1} \\ +D\left(\frac{a_1}{P_1} + a_2P_1\right) + Q_1\left[m(E_v + S_{cv}) + \frac{d_i}{D}\ (E_{bi} + S_{bi})\right] \end{array}\right] (5.15)$$

Decision variables for quadratic quality function (Case 2)

$$Q_2 = \sqrt{\frac{2D\left\{\frac{S_{v_2}}{m}+\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\,\Psi(k_i)+R(L_i))\right\}}{\sum_{i=1}^{n}\frac{h_{bi}}{D}d_i+h_v\left[m\left(1-\frac{D}{P_2}\right)-1+\frac{2D}{P_2}\right]+\frac{RD\alpha(b_2P_2+c_2P_2^2)}{P_2}+\left(m(E_v+S_{cv})+\frac{d_i}{D}(E_{bi}+S_{bi}\right)}} \quad (5.16)$$

$$\Phi(k^2{}_i) = 1 - \frac{h_{bi}Q_2}{D\pi_i} \quad (5.17)$$

$$P_2 = \sqrt{\frac{2a_1D - Q_2h_vD(m-2)}{2Da_2 + R\alpha D\ Q_2 b}} \quad (5.18)$$

$$S_{v_2} = \beta\, B\, \frac{mQ_2}{D} \quad (5.19)$$

Hence, combined total cost is given below.

$$EJTC\left(Q_2, k^2{}_i, L_{i,}, P_2, m, S_{v_2}\right) =$$

$$\sum_{i=1}^{n}\left[\begin{array}{c} \frac{O_{bi}d_i}{Q_2} + h_{bi}\left\{\frac{Q_2}{2D}d_i + k^2{}_i\sigma_i\sqrt{L_i}\right\} + \pi_i\sigma_i\sqrt{L_i}\ \psi(k^2{}_i)\ \frac{D}{Q_2} \\ +R(L_i)\frac{D}{Q_2}] + \frac{S_{v_2}D}{mQ_2} + \frac{Q_2}{2}\ h_v\left[m\left(1-\frac{D}{P_2}\right) - 1 + \frac{2D}{P_2}\right] + +\beta\ \mathrm{B}\ (\ln(S_{v_0}) - \ln(S_{v_2})) \\ +RD\alpha(b_2P_2 + c_2P_2^2)\frac{Q_2}{2P_2} + D\left(\frac{a_1}{P_2} + a_2P_2\right) + Q_2\left[m(E_v + S_{cv}) + \frac{d_i}{D}\ (E_{bi} + S_{bi})\right] \end{array}\right] \quad (5.20)$$

5.3. Solution procedure

One can use similar solution procedure as used in Sarkar et al. (2018) to derive the extremum values of the decision variables. The iterative procedure is also applicable here as the closed form solution is unavailable. The following steps are given to develop the solution algorithm

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of Q from (5.11) and (5.16).

Step 1b Obtain $\Phi(k_i)$ from (5.12) and (5.17) and find the values of k_i by inverse normal distribution.

Step 1c Obtain P from (5.13) and (5.18).

Step1d Obtain S_v from (5.14) and (5.19).

Step 1e Perform 1a to 1d by updating the values until no changes occurs (upto a specified accuracy level) in $Q, k_i, P\ ans\ S_v$.

Step 2 Obtain the total cost from (5.15) and (5.20).

Step 3 Set$m = 2$, 3,…, p and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

5.4. Numerical experiment

With the help of examples, the applicability of model has been checked.

Example

The values of parameter are taken as follows for the illustration of the model numerically

$A_{b1} = \$100$/setup, $A_{b2} = \$150/$setup, $A_{b3} = \$100$/setup, $d_1 = 200$ units/year, $d_2 = 250$ units/year, $d_3 = 150$ units/year, $S_m = \$4000$/setup, $h_m = \$10$/unit/week, $h_{b1} = \$11$/unit/week, $h_{b2} = \$11$/unit/week, $h_{b3} = \$12$/unit/week, $\sigma_1 = 7, \sigma_2 = 6, \sigma_3 = 6$, $\pi_1 = \$50/\text{unit}$, $\pi_2 = \$50/\text{unit}$, $\pi_3 = \$51/\text{unit}$, $a_1 = 35 \times 10^3$, $a_2 = 0.1$, $E_m = 12.5/\text{unit}$, $b_1 = 10^{-4}$, $b_2 = 10^{-4}$, $c_2 = 10^{-6}$, $R = \$60/\text{unit}$, $S_{v_0} = 5000$, B= 30000, $\beta = 0.4.$, S_m=$0.01/unit; S_{b1}=$0.02/unit, S_{b2}=$0.01/unit,$S_{b3}$=$0.01/unit , E_v=$6.5/unit, E_{b1}=$11/unit, E_{b2}=$11/unit, E_{b3} =$11/unit, Therefore, according to the parameter values, the MTTF functions of Case 1 and 2 transforms as follows.

Case I: $\frac{1}{f(P)} = \frac{1}{10^{-4}\ P}$

Case II: $\frac{1}{f(P)} = \frac{1}{10^{-4}\ P\ +10^{-6}\ P^2}$

Using the parametric values optimal decision values of the decision variable along with optimized total joint minimum cost are obtained which are illustrated below.

Table 5.1: Optimal results for first case (Linear quality function)

M	L_1	L_2	L_3	k_1	k_2	k_3	C(P)	Q	P	S_v	EJTC
1	4	4	4	1.58	1.58	1.54	118.33	155.18	598.13	310.36	82556.75
2	**4**	**4**	**4**	**1.63**	**1.63**	**1.60**	**118.32**	**140.86**	**591.60**	**563.47**	**82532.32**
3	4	4	4	1.67	1.67	1.64	118.33	129.98	586.01	779.92	82765.35

Table 5.2: Optimal results for second case (Quadratic quality function)

M	L_1	L_2	L_3	k_1	k_2	k_3	C(P)	Q	P	S_v	EJTC
1	4	4	4	1.59	1.59	1.55	118.33	153.56	597.64	307.11	82592.36
2	**4**	**4**	**4**	**1.63**	**1.63**	**1.60**	**118.32**	**139.64**	**591.23**	**558.58**	**82562.43**
3	4	4	4	1.67	1.67	1.64	118.33	129.02	585.79	774.13	82791.58

Here, It has been observed that minimum value for both cases is attained at m=2.

The value of MTTF with minimum of total expected cost is also obtained for both of the cases as below. It has been observed that the linear quality function would be more reliable in comparison to quadratic quality function.

Table 5.3: Comparison of EJTC with MTTF

Cases	EJTC	MTTF
I (Linear quality function)	**82532.32**	16.90
II(Quadratic quality function)	**82562.43**	10.63

The finding includes reduction of expected joint total cost after applying logarithmic investment function to lessen setup cost.

Table 5.4: Comparison of expected joint total cost (EJTC) with variable to fixed set up cost for case 1 and case 2

EJTC fixed setup cost	EJTC variable setup cost
86802.17(m=6)	**82532.32** (m=2)
86832.050(m=6)	**82562.43** (m=2)

5.4.1. Sensitivity analysis

The sensitivity analysis of all cost parameters for Case 1 and 2 are performed in Table 5.5 and 5.6, respectively. The cost parameters are varied from -10% to +10% and the changes in expected total cost is observed.

Table 5.5: Sensitivity analysis

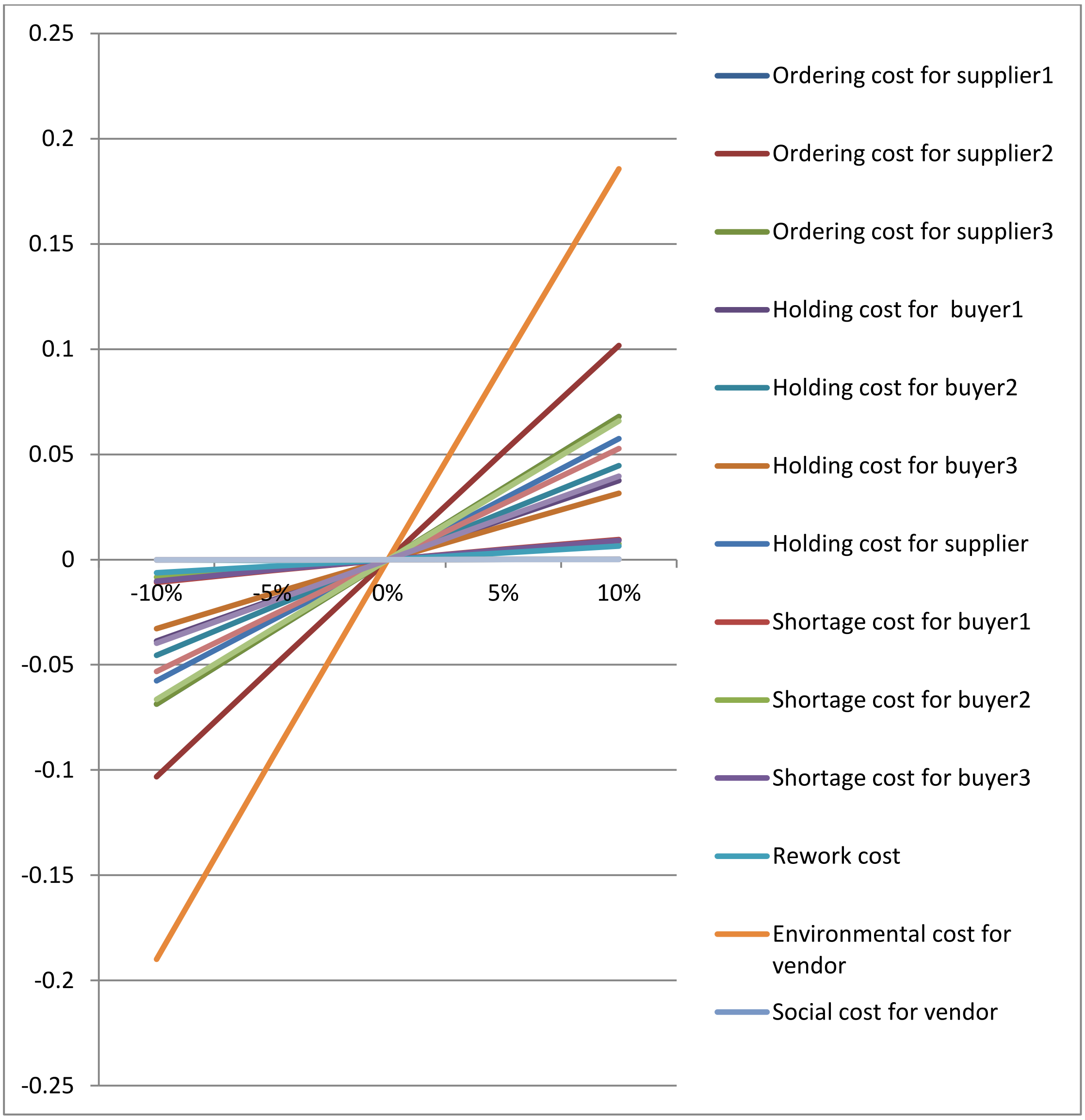
Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
A_{b1}	-10	-0.068594
	-5	-0.034221
	+5	0.034070
	+10	0.067992
A_{b2}	-10	-0.103123
	-5	-0.051388
	+5	0.051050
	+10	0.101768
A_{b3}	-10	-0.068594
	-5	-0.034221
	+5	0.034070
	+10	0.067992
h_{b1}	-10	-0.038762
	-5	-0.019219
	+5	0.018907
	+10	0.037512

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
h_{b2}	-10	-0.045563
	-5	-0.022659
	+5	0.022423
	+10	0.044619
h_{b3}	-10	-0.032758
	-5	-0.016223
	+5	0.015922
	+10	0.031552
h_v	-10	-0.057606
	-5	-0.028792
	+5	0.028768
	+10	0.057510
π_1	-10	-0.010769
	-5	-0.005207
	+5	0.004892
	+10	0.009502
π_2	-10	-0.008395
	-5	-0.004063
	+5	0.003824
	+10	0.007434
π_3	-10	-0.010298
	-5	-0.004978
	+5	0.004675
	+10	0.009078
R	-10	-0.006217
	-5	-0.003108
	+5	0.003108
	+10	0.006216

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
E_v	-10	-0.189939
	-5	-0.094417
	+5	0.093346
	+10	0.185652
Eb_1	-10	-0.053127
	-5	-0.026521
	+5	0.026435
	+10	0.052786
Eb_2	-10	-0.066463
	-5	-0.033164
	+5	0.033031
	+10	0.065930
Eb_3	-10	-0.039813
	-5	-0.019882
	+5	0.019835
	+10	0.039621
S_v	-10	-0.000289
	-5	-0.000144
	+5	0.000144
	+10	0.000289
S_{b1}	-10	-0.000096
	-5	-0.000048
	+5	0.000000
	+10	0.000048
S_{b2}	-10	-0.000060
	-5	-0.000030
	+5	0.000030
	+10	0.000060

Case 1 (Linear quality function)		
Cost parameter	% change	Sensitivity
S_{b3}	-10	-0.000036
	-5	-0.000018
	+5	0.000000
	+10	0.000018

Figure 5.1: Graphical representation of sensitivity analysis for linear quality function

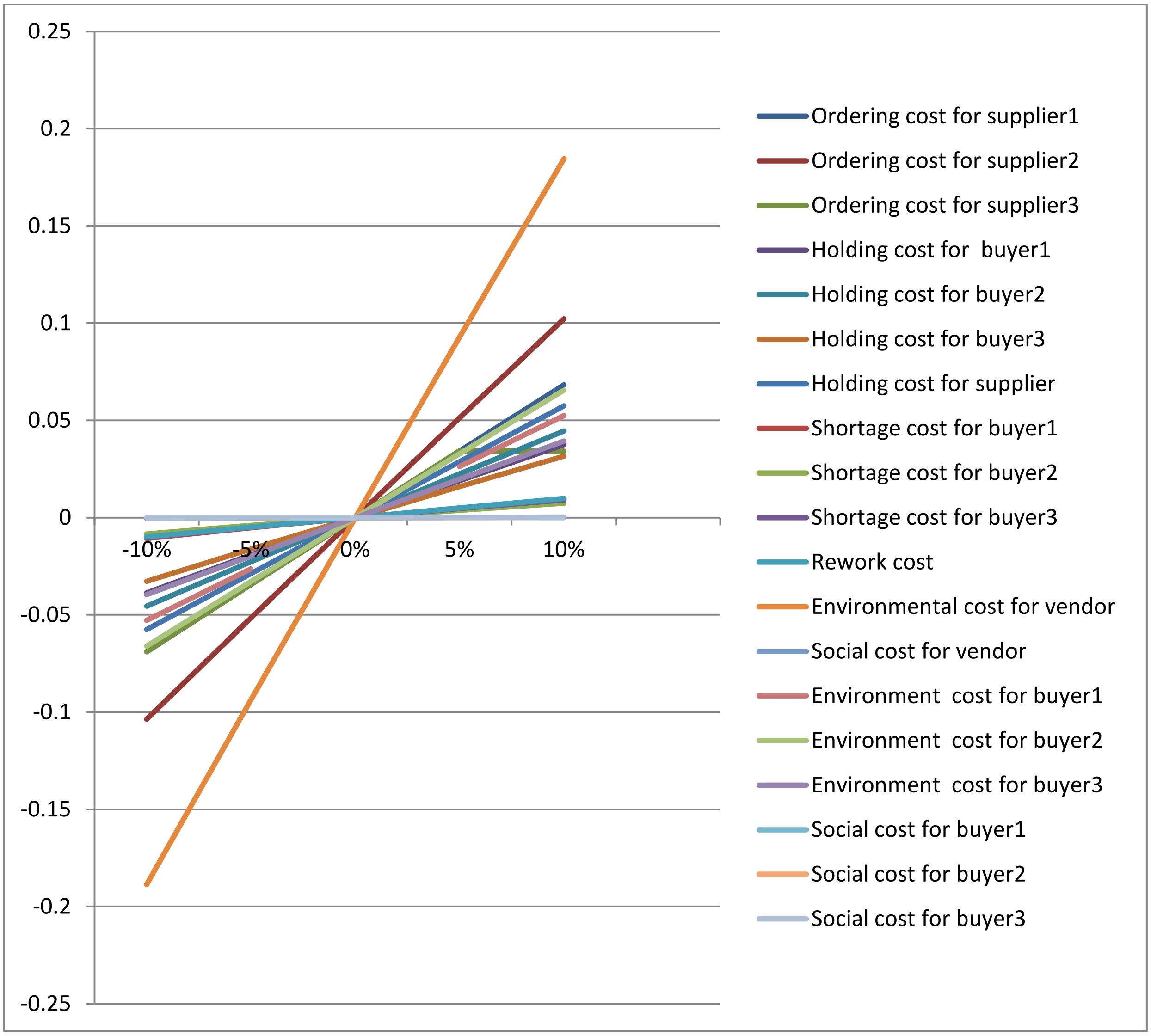


Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
A_{b1}	-10	-0.068916
	-5	-0.034381
	+5	0.034230
	+10	0.068310
A_{b2}	-10	-0.103606
	-5	-0.051629
	+5	0.051288
	+10	0.102243
A_{b3}	-10	-0.068916
	-5	-0.034381
	+5	0.034230
	+10	0.068310
h_{b1}	-10	-0.038758
	-5	-0.019218
	+5	0.018905
	+10	0.037508
h_{b2}	-10	-0.045546
	-5	-0.022650
	+5	0.022414
	+10	0.044600
h_{b3}	-10	-0.032758
	-5	-0.016223
	+5	0.015922
	+10	0.031552
h_v	-10	-0.057526
	-5	-0.028750
	+5	0.028723
	+10	0.057416

Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
π_1	-10	-0.010762
	-5	-0.005203
	+5	0.004888
	+10	0.009495
π_2	-10	-0.008388
	-5	-0.004060
	+5	0.003821
	+10	0.007428
π_3	-10	-0.010292
	-5	-0.004975
	+5	0.004671
	+10	0.009072
R	-10	-0.009880
	-5	-0.004940
	+5	0.004939
	+10	0.009877
E_v	-10	-0.188744
	-5	-0.093830
	+5	0.092778
	+10	0.184535
Eb_1	-10	-0.052798
	-5	-0.026357
	+5	0.026273
	+10	0.052464
Eb_2	-10	-0.066051
	-5	-0.032959
	+5	0.032829
	+10	0.065528

Case 2 (Quadratic quality function)		
Cost parameter	% change	Sensitivity
Eb_3	-10	-0.039567
	-5	-0.019760
	+5	0.019713
	+10	0.039379
S_v	-10	-0.000287
	-5	-0.000144
	+5	0.000144
	+10	0.000287
Sb_1	-10	-0.000096
	-5	-0.000048
	+5	0.000048
	+10	0.000096
Sb_2	-10	-0.000060
	-5	-0.000030
	+5	0.000030
	+10	0.000060
Sb_3	-10	-0.000036
	-5	-0.000018
	+5	0.000018
	+10	0.000036

Figure 5.2: Graphical representation of sensitivity analysis for quadratuc quality function



5.5. Conclusion

The model jointly reduced the expected total cost and set up cost for single vendor and multi-buyers by simultaneously optimizing the order quantity, lead time, number of lots, safety factor, and production rate. The experimental results represented the reduction of MTTF with increasing production rate which has a significant impact on obtaining the managerial decisions. Adding suitable investment in order to obtain social and environment sustainability increase overall total cost hence after applying the logarithmic investment function, setup cost was reduced which results reduction of expected joint total cost. The given sensitivity analysis highlight the fact that the expected total cost is highly effected by environment cost parameter of vendor.

This research can be further extended with the involvement of more players of supply chain. Since deterioration was leading the system to shift from "in-control" state to "out-of-control" state and overall process of production was considered continuous, so step-by–step inspection of production can be taken into account with production of multi items instead of single item.

5.6. Appendices of Chapter 5

Appendix C

The number of defective units in a production cycle with linear deterioration is given by

$$N = \begin{cases} 0 & t \geq \tau \\ \alpha P\,(t-\tau) + \alpha\, P \int_0^{t-\tau} \beta x\, dx & t > \tau \end{cases}$$ (Rosenblatt and Lee, 1986)

E (N) $=\int_0^t (\alpha P\,(t-\tau) + \alpha\, P\, \int_0^{t-\tau} \beta x\, dx)\, \mu\, e^{-\mu\tau}\, d\tau$

$=\int_0^t \alpha\, P(t-\tau)\mu\, e^{-\mu\tau}\, d\tau \; + \; \alpha P\, \int_0^t (\int_0^{t-\tau} \beta\, x\, dx\,)\, \mu\, e^{-\mu\tau}\, d\tau$

$= \alpha\, P\, \mu\, [\int_0^t t\;\, e^{-\mu\tau}\, d\tau - \int_0^t \tau\;\, e^{-\mu\tau}\, d\tau + P\alpha\, \int_0^t \beta\, \frac{x^2}{2}\Big)_0^{t-\tau}\, \mu\, e^{-\mu\tau}\, d\tau$

$= \alpha\, P\, (t + \frac{1}{\mu}\, e^{-\mu\tau} - \frac{1}{\mu}) + \beta\alpha\, P\, [\frac{t^2}{2} + \left(\frac{1}{\mu}\right)^2\, (1 - t\mu - \; e^{-\mu\tau}\,)]$

Now μ = $f(P)$ hence

$$e^{-f(P)t} = 1 - f(P)t + \frac{(f(P)t)^2}{2}$$

E(N) = $\alpha P\ \left(t + \frac{1}{f(P)}\left(1 - f(P)t + \frac{(f(P)t)^2}{2}\right) - \frac{1}{f(P)}\right) + \alpha\,\beta\,P\,[\frac{t^2}{2} + \left(\frac{1}{f(P)}\right)^2 \left[\ \{1 - t.f(P) - \left(1 - f(P)t + \frac{(f(P)t)^2}{2}\right)\right\}\ ]]$

E(N)= $\alpha\frac{f(\,P)}{2}P\,t^2 \quad Since\ t = \frac{Q}{P}$

Hence **E(N)** = $\boldsymbol{\frac{\alpha f(P)Q^2}{2P}}$

The number of defective units in a production cycle with exponential deterioration is given by

$N = \begin{cases} 0 & t \geq \tau \\ \alpha P\ (t-\tau) + \alpha\ P\ \int_0^{t-\tau}(1 - e^{-\beta x})dx & t > \tau \end{cases}$ (Rosenblatt and Lee, 1986)

E(N)=$\int_0^t(\alpha P\ (t-\tau) + \alpha\ P\ \int_0^{t-\tau}(1 - e^{-\beta x})\ dx)\ \mu\ e^{-\mu\tau}\ d\tau$

= $\alpha\ P\ (t + \frac{1}{\mu}\ e^{-\mu\tau} - \frac{1}{\mu})$ + P $\alpha\mu\ \int_0^t\int_0^{t-\tau}\left(1 - e^{-\beta x}\right)dx\ \ e^{-\mu\tau}\ d\tau$

= $\alpha\ P\ \left(t + \frac{1}{\mu}\ e^{-\mu\tau} - \frac{1}{\mu}\right) +\ P\ \alpha\mu\ \int_0^t\ \{(t-\tau) + \frac{e^{-\beta(t-\tau)}}{\beta}\ - \frac{1}{\beta}\}\ e^{-\mu\tau}d\tau$

= $\alpha\ P\ \left(t + \frac{1}{\mu}\ e^{-\mu\tau} - \frac{1}{\mu}\right) +\ \alpha\ P\ t - \alpha P\ (1 - e^{-\mu\tau}\ )\left(\frac{1}{\mu} + \frac{1}{\beta}\right) - \frac{\alpha\mu P}{\beta(\beta-\mu)}(\ e^{-\beta t} -\ e^{-\mu t})$

Now μ = $f(P)$ $hence \quad e^{-f(P)t}\ = 1 - f(P)t + \frac{(f(P)t)^2}{2} \quad and \quad e^{-\beta t}\ = 1 - \beta t + \frac{(\beta t)^2}{2}$

E(N) = P $\alpha\ (t + \frac{1}{f(P)}\left(1 - f(P)t + \frac{(f(P)t)^2}{2}\right) +\ \alpha P\ t - \alpha P t - \alpha P\ (1 - \left(1 - f(P)t + \frac{(f(P)t)^2}{2}\right)\left(\frac{1}{f(P)} + \frac{1}{\beta}\right) - \frac{\alpha f(P)P}{\beta\left(\beta - f(P)\right)}\ (e^{-\beta t}\ -\ 1 - f(P)t + \frac{(f(P)t)^2}{2})$

E(N)= $\frac{\alpha f(P)t^2}{2}P$ Since t = $\frac{Q}{P}$

Hence **E(N) =** $\boldsymbol{\frac{\alpha f(P)Q^2}{2P}}$**.**

Therefore, the expected number of defective items are same for linear and exponential deterioration which is equal to E(N) = $\frac{\alpha f(P)Q^2}{2P}$.

Appendix D

The second order derivatives with respect to Q_1, k_i^1, $S_{v1}$and P_1 are as follows.

The hessian matrix for decision variable will be

$$\begin{matrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\,\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1 \partial P_1} & \frac{\partial^2 EJTC}{\partial Q_1 \partial S} \\ \frac{\partial^2 EJTC}{\partial k_i^1 \partial Q_1} & \frac{\partial^2 EJTC}{\partial {k_i^1}^2} & \frac{\partial^2 EJTC}{\partial k_i^1 \partial\, P_1} & \frac{\partial^2 EJTC}{\partial k_i^1 \partial S} \\ \frac{\partial^2 EJTC}{\partial P_1 \partial Q_1} & \frac{\partial^2 EJTC}{\partial P_1 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial P_1^2} & \frac{\partial^2 EJTC}{\partial P_1 \partial S} \\ \frac{\partial^2 EJTC}{\partial S\,\partial Q_1} & \frac{\partial^2 EJTC}{\partial S\,\partial k_i^1} & \frac{\partial^2 EJTC}{\partial S\,\partial P_1} & \frac{\partial^2 EJTC}{\partial S^2} \end{matrix}$$

Conditions to obtain this as a positive definite are as follow:

$$\boldsymbol{\frac{\partial^2(EJTC)}{\partial Q_1^2} = \sum\left(\frac{2o_{bi}D}{Q_1^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_1^3}\right) + \frac{2\,S_vD}{mQ_1^3} > 0}$$

$$\frac{\partial^2(EJTC)}{\partial Q_1\,\partial k_i^1} = \sum\left(\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\,(1-\Phi(k_i^1))\right)$$

$$\frac{\partial^2(EJTC)}{\partial Q_1\,\partial P_1} = m\frac{h_vD}{2P_1^2} - \frac{h_vD}{P_1^2}$$

$$\frac{\partial^2(EJTC)}{\partial Q_1\,\partial S_v} = -\frac{D}{mQ_i^2}$$

$$\frac{\partial(EJTC)}{\partial k_i^1} = \sum\left(\frac{D}{Q_1}\,\pi_i\sigma_i\sqrt{L_i}\,\left\{\left(\Phi\left(k_i^1\right)-1\right)\right\} + h_{bi}\sigma_i\sqrt{L_i}\right)$$

$$\frac{\partial^2(EJTC)}{\partial k_i^1\,\partial Q_1} = \sum\left(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\,\left\{\left(1-\Phi\left(k_i^1\right)\right)\right\}\right)$$

$$\frac{\partial^2(EJTC)}{\partial\ ({k_i^1}^2)} = \sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \varphi\ (k_i^1)\right)$$

$$\frac{\partial^2(EJTC)}{\partial k_i^1\ \partial P_1} = 0$$

$$\frac{\partial^2(EJTC)}{\partial k_i^1\ \partial S_v} = 0$$

$$\frac{\partial^2(EJTC)}{\partial P_1 \partial k_i^1} = 0$$

$$\frac{\partial^2(EJTC)}{\partial P_1\ \partial Q_1} = m\frac{h_v D}{2P_1^2} - \frac{h_v D}{P_1^2}$$

$$\frac{\partial^2(EJTC)}{\partial {P_1}^2} = -\frac{Q_1 h_v D}{P_1^3}(m-2) + 2\frac{D}{P_1^3}a_1$$

$$\frac{\partial^2(EJTC)}{\partial P_1\ \partial S_v} = 0$$

$$\frac{\partial^2(EJTC)}{\partial S_v \partial Q_1} = -\frac{D}{mQ_1^2}$$

$$\frac{\partial^2(EJTC)}{\partial S_v\ \partial k_i^1} = 0$$

$$\frac{\partial^2(EJTC)}{\partial S_v\ \partial P_1} = 0$$

$$\frac{\partial^2(EJTC)}{\partial S_v^2} = \frac{\beta B}{S^2}$$

$$\begin{vmatrix} \frac{\boldsymbol{\partial^2 EJTC}}{\boldsymbol{\partial Q_1^2}} & \frac{\boldsymbol{\partial^2 EJTC}}{\boldsymbol{\partial Q_1 \partial k_i^1}} \\ \frac{\boldsymbol{\partial^2 EJTC}}{\boldsymbol{\partial k_i^1 \partial Q_1}} & \frac{\boldsymbol{\partial^2 EJTC}}{\boldsymbol{\partial (k_i^1)^2}} \end{vmatrix} =$$

$$\begin{vmatrix} \sum\left(\frac{2o_{bi}D}{Q_1^3} + \pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\ R(L_i)D}{Q_i^3}\right) + \frac{2\ S_v D}{mQ_i^3} & \sum\left(\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\ (1-\Phi(k_i^1))\right) \\ \sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \left\{\left(1-\Phi(k_i^1)\right)\right\}\right) & \sum(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\varphi\ (k_i^1)\ ) \end{vmatrix}$$

$$= (\sum\left(\frac{2o_{bi}D}{Q_1^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_i^3}\right) + \frac{2\,S_vD}{mQ_i^3})\sum(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;\varphi\,(k_i^1)\,) - \left(\sum\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi(k_i^1)\right)\right)^2 > 0$$

$$= \mathbf{2}\sum\boldsymbol{\pi_i\sigma_i\sqrt{L_i}\,\varphi\,(k_i^1)\,(o_{bi}+R(L_i)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\,\psi(k_i^1)k_i^1) > \left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^1)\right)\right)^2}$$

$$\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial(k_i^1)^2} & \frac{\partial^2 EJTC}{\partial k_i^1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial P_1\partial Q_1} & \frac{\partial^2 EJTC}{\partial P_1\partial k_i^2} & \frac{\partial^2 EJTC}{\partial P_1^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial(k_i^1)^2} & 0 \\ \frac{\partial^2 EJTC}{\partial P_1\partial Q_1} & 0 & \frac{\partial^2 EJTC}{\partial P_1^2} \end{vmatrix} = \frac{\partial^2 EJTC}{\partial P_1\partial Q_1}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial(k_i^1)^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial P_1^2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial(k_i^1)^2} \end{vmatrix}$$

$$=-\left(m\frac{h_vD}{2P_1^2}-\frac{h_vD}{P_1^2}\right)^2.\sum\left(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;k_i^1\varphi\,(k_i^1)\right) + (\,2\frac{D}{P_1^3}a_1 - \frac{Q_1h_vD}{P_1^3}(m-2)).(\sum\left(\frac{2o_{bi}D}{Q_1^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_i^3}\right) + \frac{2\,S_vD}{mQ_i^3})\sum(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;\varphi\,(k_i^1)\,) - \left(\sum[\,\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi(k_i^1)\right)]\right)^2$$

$$= (2\frac{D}{P_1^3}a_1 - \frac{Q_1h_vD}{P_1^3}(m-2)).(\sum\left(\frac{2o_{bi}D}{Q_1^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_i^3} + \frac{2\,S_vD}{mQ_1^3}\right).\sum(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;k_i^1\varphi\,(k_i^1)\,) - \left(\sum\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi(k_i^1)\right)\right)^2\;) > \left(m\frac{h_vD}{2P_1^2}-\frac{h_vD}{P_1^2}\right)^2.\sum\left(\frac{D}{Q_1^2}\,\pi_i\sigma_i\sqrt{L_i}\;\varphi\,(k_i^1)\right)$$

$$= \frac{1}{Q_1}((2a_1 - Q_1 h_v(m-2))\,(\sum\left(2\left(o_{bi} + \pi_i\sigma_i\sqrt{L_i}\,\psi\left(k_i^1\right) + R(L_i) + \frac{S_v}{m}\right)\right)\ .\left(\sum\left(Q_1\pi_i\sigma_i\sqrt{L_i}\ k_i^1\varphi\,(k_i^1) - \frac{D}{Q_1}\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)\right)^2\right)\right)$$

$$> \frac{\left(\frac{m}{2}-1\right)^2 h_v^2}{P_1}.\sum\left(\pi_i\sigma_i\sqrt{L_i}\ \varphi\,(k_i^1)\right)$$

$$\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\,\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} & \frac{\partial^2 EJTC}{\partial Q_1\partial S} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial {k_i^1}^2} & \frac{\partial^2 EJTC}{\partial k_i^1\partial\,P_1} & \frac{\partial^2 EJTC}{\partial k_i^1\partial S} \\ \frac{\partial^2 EJTC}{\partial P_1\partial Q_1} & \frac{\partial^2 EJTC}{\partial P_1\partial k_i^2} & \frac{\partial^2 EJTC}{\partial P_1^2} & \frac{\partial^2 EJTC}{\partial P_1\partial S} \\ \frac{\partial^2 EJTC}{\partial S\,\partial Q_1} & \frac{\partial^2 EJTC}{\partial S\,\partial k_i^1} & \frac{\partial^2 EJTC}{\partial S\,\partial P_1} & \frac{\partial^2 EJTC}{\partial S^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\,\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} & \frac{\partial^2 EJTC}{\partial Q_1\partial S} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial {k_i^1}^2} & 0 & 0 \\ \frac{\partial^2 EJTC}{\partial P_1\partial Q_1} & 0 & \frac{\partial^2 EJTC}{\partial P_1^2} & 0 \\ \frac{\partial^2 EJTC}{\partial S\,\partial Q_1} & 0 & 0 & \frac{\partial^2 EJTC}{\partial S^2} \end{vmatrix}$$

(On expanding 4th row)

$$= -\frac{\partial^2 EJTC}{\partial S\,\partial Q_1}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1\,\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} & \frac{\partial^2 EJTC}{\partial Q_1\partial S} \\ \frac{\partial^2 EJTC}{\partial {k_i^1}^2} & 0 & 0 \\ 0 & \frac{\partial^2 EJTC}{\partial P_1^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial S^2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial\left(k_i^1\right)^2} & 0 \\ \frac{\partial^2 EJTC}{\partial P_1\partial Q_1} & 0 & \frac{\partial^2 EJTC}{\partial P_1^2} \end{vmatrix}$$

$$= \frac{\partial^2 EJTC}{\partial S\,\partial Q_1}\frac{\partial^2 EJTC}{\partial P_1^2}\left(-\frac{\partial^2 EJTC}{\partial {k_i^1}^2}\frac{\partial^2 EJTC}{\partial Q_1\partial S}\right) + \frac{\partial^2 EJTC}{\partial S^2}\left(\frac{\partial^2 EJTC}{\partial P_1\partial Q_1}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} & \frac{\partial^2 EJTC}{\partial Q_1\partial P_1} \\ \frac{\partial^2 EJTC}{\partial\left(k_i^1\right)^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial P_1^2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_1^2} & \frac{\partial^2 EJTC}{\partial Q_1\partial k_i^1} \\ \frac{\partial^2 EJTC}{\partial k_i^1\partial Q_1} & \frac{\partial^2 EJTC}{\partial\left(k_i^1\right)^2} \end{vmatrix}\right)$$

$$= \left(\frac{D^2}{m^2\,Q_1^4}\right) * \left(2\frac{D}{P_1^3}a_1 - \frac{Q_1 h_v D}{P_1^3}(m-2)\right) * \left(\sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ \varphi\,(k_i^1)\right) + \frac{\beta B}{S^2} * \left[\left(-\left(m\frac{h_v D}{2P_1^2} - \frac{h_v D}{P_1^2}\right)^2 * \sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ k_i^1\varphi\,(k_i^1)\right) + \left(2\frac{D}{P_1^3}a_1 - \frac{Q_1 h_v D}{P_1^3}(m-2)\right) * \left\{\left(\sum\left(\frac{2o_{bi}D}{Q_1^3} + \right.\right.\right.\right.\right.$$

$2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3} + \frac{2\,R(L_i)D}{Q_i^3}\Big) + \frac{2\,S_v D}{mQ_i^3}\Big) * \sum(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \varphi\ (k_i^1)\ ) - \Big(\sum\Big[\ \pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\ \Big(1 - \Phi(k_i^1)\Big)\Big]\Big)^2\ \}]>0$

Appendix E

The second order derivatives with respect to Q_2, k_i^2, S_{v1} and P_2 are as follows.

$$\frac{\partial^2(EJTC)}{\partial Q_2^2} = \sum\left(\frac{2o_{bi}D}{Q_2^3} + 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_2^1)D}{Q_2^3} + \frac{2\,R(L_i)D}{Q_2^3}\right) + \frac{2\,S_v D}{mQ_2^3} > 0$$

$$\frac{\partial^2(EJTC)}{\partial Q_2\,\partial S_v} = -\frac{D}{mQ_i^2}$$

$$\frac{\partial^2(EJTC)}{\partial Q_2\,\partial k_i^2} = \sum\left(\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_2^2}\ (1 - \Phi(k_2^1))\right)$$

$$\frac{\partial^2(EJTC)}{\partial Q_2\,\partial P_2} = m\frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}$$

$$\frac{\partial(EJTC)}{\partial k_i^1} = \sum\left(\frac{D}{Q_1}\ \pi_i\sigma_i\sqrt{L_i}\ \left\{\left(\Phi\left(k_i^1\right) - 1\right)\right\} + h_{bi}\sigma_i\sqrt{L_i}\right)$$

$$\frac{\partial^2(EJTC)}{\partial k_i^2\,\partial Q_2} = \sum\left(\frac{D}{Q_2^2}\ \pi_i\sigma_i\sqrt{L_i}\ \left\{\left(1 - \Phi\left(k_i^2\right)\right)\right\}\right)$$

$$\frac{\partial^2(EJTC)}{\partial\ ({k_i^2}^2)} = \sum\left(\frac{D}{Q_2^2}\ \pi_i\sigma_i\sqrt{L_i}\ k_i^2\varphi\ (k_i^2)\right)$$

$$\frac{\partial^2(EJTC)}{\partial k_i^2\,\partial P_2} = 0$$

$$\frac{\partial^2(EJTC)}{\partial k_i^2\,\partial S_v} = 0$$

$$\frac{\partial^2(EJTC)}{\partial P_2\partial k_i^2} = 0$$

$$\frac{\partial^2(EJTC)}{\partial P_2\,\partial S_v} = 0$$

$$\frac{\partial^2(EJTC)}{\partial P_2\,\partial Q_2} = m\frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}$$

$$\frac{\partial^2(EJTC)}{\partial {P_2}^2} = -\frac{Q_2 h_v D}{P_2^3}(m-2) + 2\frac{D}{P_2^3}a_1$$

$$\frac{\partial^2(EJTC)}{\partial S_v \partial Q_1} = -\frac{D}{mQ_1^2}$$

$$\frac{\partial^2(EJTC)}{\partial S_v\, \partial k_i^2} = 0$$

$$\frac{\partial^2(EJTC)}{\partial S_v\, \partial P_2} = 0$$

$$\frac{\partial^2(EJTC)}{\partial S_v^2} = \frac{\beta B}{S^2}$$

$$\begin{vmatrix} \boldsymbol{\frac{\partial^2 EJTC}{\partial Q_2^2}} & \boldsymbol{\frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2}} \\ \boldsymbol{\frac{\partial^2 EJTC}{\partial k_i^3 \partial Q_2}} & \boldsymbol{\frac{\partial^2 EJTC}{\partial (k_i^2)^2}} \end{vmatrix} > 0$$

$$\boldsymbol{2\sum \pi_i \sigma_i \sqrt{L_i}\, \varphi\,(k_i^1)\,(o_{bi} + R(L_i) + \frac{S_v}{m} + \pi_i \sigma_i \sqrt{L_i}\, \psi(k_i^1) k_i^1) > \left(\sum \pi_i \sigma_i \sqrt{L_i}\left(1 - \Phi(k_i^1)\right)\right)^2}$$

$$\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} & \frac{\partial^2 EJTC}{\partial k_i^2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial P_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial P_2^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} & 0 \\ \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} & 0 & \frac{\partial^2 EJTC}{\partial P_2^2} \end{vmatrix} = \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial (k_i^2)^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial P_2^2} \begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial (k_i^2)^2} \end{vmatrix}$$

$$= -\left(m\frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2 . \sum\left(\frac{D}{Q_2^2}\,\pi_i \sigma_i \sqrt{L_i}\; k_i^2 \varphi\,(k_i^2)\right) + \left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).(2\sum \pi_i \sigma_i \sqrt{L_i}\, \varphi\,(k_i^1)\,(o_{bi} + R(L_i) + \frac{S_v}{m} + \pi_i \sigma_i \sqrt{L_i}\, \psi(k_i^1) k_i^1) - \left(\sum \pi_i \sigma_i \sqrt{L_i}\left(1 - \Phi(k_i^1)\right)\right)^2) > 0$$

$$\left(2\frac{D}{P_2^3}a_1-\frac{Q_2h_vD}{P_2^3}(m-2)\right).2\sum\pi_i\sigma_i\sqrt{L_i}\,\varphi\,(k_i^1)\left(o_{bi}+\ R(L_i)+\frac{S_v}{m}+\pi_i\sigma_i\sqrt{L_i}\,\psi\left(k_i^1\right)k_i^1\right)$$

$$-\left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi\left(k_i^1\right)\right)\right)^2$$

$$>\left(m\frac{h_vD}{2P_2^2}-\frac{h_vD}{P_2^2}+\frac{RD\alpha c_2}{2}\right)^2.\sum\left(\frac{D}{Q_2^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ k_i^2\varphi\,(k_i^2)\right)$$

$$(\sum\left(\frac{2o_{bi}D}{Q_1^3}+\ 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3}+\ \frac{2\ R(L_i)D}{Q_i^3}\right)+\frac{2\ S_vD}{mQ_i^3})\sum(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ \varphi\,(k_i^1)\,)$$

$$-\left(\sum\pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\left(1-\Phi(k_i^1)\right)\right)^2$$

$$+\left(m\frac{h_vD}{2P_1^2}-\frac{h_vD}{P_1^2}\right)^2.\sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ k_i^1\varphi\,(k_i^1)\right)-\ (\,2\frac{D}{P_1^3}a_1$$

$$-\frac{Q_1h_vD}{P_1^3}(m-2)).\,(\sum\left(\frac{2o_{bi}D}{Q_1^3}+\ 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi(k_i^1)D}{Q_1^3}+\ \frac{2\ R(L_i)D}{Q_i^3}\right)$$

$$+\frac{2\ S_vD}{mQ_i^3})\sum(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ \varphi\,(k_i^1)\,)+\left(\sum\left[\ \pi_i\sigma_i\sqrt{L_i}\frac{D}{Q_1^2}\ \left(1-\Phi(k_i^1)\right)\right]\right)^2$$

$= \sum\left(\frac{D}{Q_1^2}\ \pi_i\sigma_i\sqrt{L_i}\ \ \varphi\,(k_i^1\right)$ [{(1- $2\frac{D}{P_1^3}a_1+\frac{Q_1h_vD}{P_1^3}(m-2)$)*

($\sum\left(\frac{2o_{bi}D}{Q_1^3}+\ 2\pi_i\sigma_i\sqrt{L_i}\frac{\psi\left(k_i^1\right)D}{Q_1^3}+\ \frac{2\ R(L_i)D}{Q_i^3}\right)+\frac{2\ S_vD}{mQ_i^3}$)} +$\left(m\frac{h_vD}{2P_1^2}-\frac{h_vD}{P_1^2}\right)^2$]

This whole value must generate a result greater than zero.

$$\begin{vmatrix}\frac{\partial^2EJTC}{\partial Q_2^2} & \frac{\partial^2EJTC}{\partial Q_2\,\partial k_i^2} & \frac{\partial^2EJTC}{\partial Q_2\partial P_2} & \frac{\partial^2EJTC}{\partial Q_2\partial S}\\ \frac{\partial^2EJTC}{\partial k_i^2\partial Q_2} & \frac{\partial^2EJTC}{\partial {k_i^2}^2} & \frac{\partial^2EJTC}{\partial k_i^2\partial\,P_2} & \frac{\partial^2EJTC}{\partial k_i^2\partial S}\\ \frac{\partial^2EJTC}{\partial P_2\partial Q_2} & \frac{\partial^2EJTC}{\partial P_2\partial k_i^2} & \frac{\partial^2EJTC}{\partial P_2^2} & \frac{\partial^2EJTC}{\partial P_2\partial S}\\ \frac{\partial^2EJTC}{\partial S\,\partial Q_2} & \frac{\partial^2EJTC}{\partial S\,\partial k_i^2} & \frac{\partial^2EJTC}{\partial S\,\partial P_2} & \frac{\partial^2EJTC}{\partial S^2}\end{vmatrix}=\begin{vmatrix}\frac{\partial^2EJTC}{\partial Q_2^2} & \frac{\partial^2EJTC}{\partial Q_2\,\partial k_i^2} & \frac{\partial^2EJTC}{\partial Q_2\partial P_2} & \frac{\partial^2EJTC}{\partial Q_2\partial S}\\ \frac{\partial^2EJTC}{\partial k_i^2\partial Q_2} & \frac{\partial^2EJTC}{\partial {k_i^2}^2} & 0 & 0\\ \frac{\partial^2EJTC}{\partial P_2\partial Q_2} & 0 & \frac{\partial^2EJTC}{\partial P_2^2} & 0\\ \frac{\partial^2EJTC}{\partial S\,\partial Q_2} & 0 & 0 & \frac{\partial^2EJTC}{\partial S^2}\end{vmatrix}$$

(On expanding 4th row)

$$=-\frac{\partial^2 EJTC}{\partial S\,\partial Q_2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2\,\partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial S} \\ \frac{\partial^2 EJTC}{\partial {k_i^1}^2} & 0 & 0 \\ 0 & \frac{\partial^2 EJTC}{\partial P_2^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial S^2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2\,\partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial {k_i^2}^2} & 0 \\ \frac{\partial^2 EJTC}{\partial P_2 \partial Q_2} & 0 & \frac{\partial^2 EJTC}{\partial P_2^2} \end{vmatrix}$$

$$= \frac{\partial^2 EJTC}{\partial S\,\partial Q_2}\frac{\partial^2 EJTC}{\partial P_2^2}\left(-\frac{\partial^2 EJTC}{\partial {k_i^2}^2}\frac{\partial^2 EJTC}{\partial Q_2 \partial S}\right) + \frac{\partial^2 EJTC}{\partial S^2}\left(\frac{\partial^2 EJTC}{\partial P_2 \partial Q_2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial P_2} \\ \frac{\partial^2 EJTC}{\partial \left(k_i^2\right)^2} & 0 \end{vmatrix} + \frac{\partial^2 EJTC}{\partial P_2^2}\begin{vmatrix} \frac{\partial^2 EJTC}{\partial Q_2^2} & \frac{\partial^2 EJTC}{\partial Q_2 \partial k_i^1} \\ \frac{\partial^2 EJTC}{\partial k_i^2 \partial Q_2} & \frac{\partial^2 EJTC}{\partial \left(k_i^2\right)^2} \end{vmatrix}\right)$$

$$=\left(\frac{D^2}{m^2 Q_2^4}\right)\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).\sum\left(\frac{D}{Q_2^2}\,\pi_i\sigma_i\sqrt{L_i}\;k_i^2\varphi\,(k_i^2)\right) + \frac{\beta B}{S^2}\{-\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right)*\left(m\frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2 + \left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).(2\sum\pi_i\sigma_i\sqrt{L_i}\,\varphi\,(k_i^2)\,(o_{bi} + R(L_i) + \frac{S_v}{m} + \pi_i\sigma_i\sqrt{L_i}\,\psi(k_i^2)k_i^2) - \left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^2)\right)\right)^2\} > 0$$

Or $$\left(\frac{D^2}{m^2 Q_2^4}\right)\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).\sum\left(\frac{D}{Q_2^2}\,\pi_i\sigma_i\sqrt{L_i}\;k_i^2\varphi\,(k_i^2)\right) + \frac{\beta B}{S^2}\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right).(2\sum\pi_i\sigma_i\sqrt{L_i}\,\varphi\,(k_i^2)\,(o_{bi} + R(L_i) + \frac{S_v}{m} + \pi_i\sigma_i\sqrt{L_i}\,\psi(k_i^2)k_i^2) > \frac{\beta B}{S^2}\{\left(2\frac{D}{P_2^3}a_1 - \frac{Q_2 h_v D}{P_2^3}(m-2)\right)*\left(m\frac{h_v D}{2P_2^2} - \frac{h_v D}{P_2^2} + \frac{RD\alpha c_2}{2}\right)^2 + \left(\sum\pi_i\sigma_i\sqrt{L_i}\left(1-\Phi(k_i^2)\right)\right)^2\}$$

The above stated matrices will be positive definite if above stated conditions would be satisfied.

Chapter 6

The effect of variable production rate on “out-of-control” probability in an integrated supply chain system

6.1. Introduction

Improving product quality is one of the most salient features in modern business to reach numerous numbers of customers. Thus, every company is working to improve customer satisfaction level by increasing product quality. But, to satisfy customer's demand, companies may, sometimes increase the rate of production which affects the production system. In this research, the author had developed a supply chain system with one vendor and buyer. In vendor's end, an investment is considered to improve the system performance which is measured through a probabilistic value ("out-of-control" probability). Two models are discussed in this paper regarding the independency and dependency of production with the "out-of-control" probability. An objective of centralized system cost is obtained. Besides that, the buyer also incurs an investment to reduce its setup cost.

Porteus (1986) initially developed the logarithmic concept of reducing setup cost and quality improvement. Rosenblatt and Lee (1986) introduced the in-control and out of control state of the production system. In this paper authors discussed about the shifting of manufacturing process from reliable to unreliable state. Khouja and Mehrez (1994) developed an EMQ model with production rate as decision and considered reliability of manufacturing process. Ouyang and Chen (2002) worked on a manufacturing process in an inventory system with setup cost reduction and quality improvement. Majumder et al. (2017) and Sarkar et al. (2017) developed a supply chain system with simultaneous reduction of vendor's setup cost and system imperfection.

6.2. Model formation

To develop the mathematical model, the following assumptions have been considered.

6.2.1. Assumptions

1. The paper considers an integrated centralized chain which minimizes the added cost of vendor and buyer.
2. An SSMD ("Single-setup multi-delivery") policy is adopted by the vendor. Through this policy, buyer's order is delivered over multiple times with equal lots.

3. Shortages are negligible in the models described in this study.
4. No lead time is considered in the models.
5. An investment is assumed to decrease the cost of setup for the buyer logarithmically.
6. The vendor also uses investment to increase system reliability by reducing the "out-of-control" probability.

6.2.2. Buyer's cost equation

Buyer's cost components consist of ordering, holding, and amount of reducing ordering cost. As per classical inventory model, the ordering and holding cost of the buyer are:

$\frac{O_b d}{q}$ and $h_b\left[\frac{q_s}{2}+R_p-DL\right]$ respectively. The concept of decrement in setup cost is taken from Majumder et al. (2017). As, O_{ri} is the initial fixed cost and O_b is the variable cost, the reduction strategy is assumed by a capital investment, $I_{sr}=b_r(\ln O_{ri}-\ln O_b)$.

The expected shortage at the end of a cycle is

$$E(X-R)=\int_R^{\infty}(X-R)dFx=\sigma\sqrt{L}\ \varphi(k)$$

where $\psi(k)=\phi(k)-k[1-\Phi(k)]$, ϕ = standard normal probability density function and Φ = cumulative distribution function of the normal distribution. The safety factor *k* is assumed as a decision variable instead of *R*.

Thus,buyer's total cost is

$$TC_b(q_s,O_b)=\frac{O_b d}{q_s}+h_b\left[\frac{q_s}{2}+R_p-dL\right]+\alpha b_r(\ln O_{ri}-\ln O_b)+\frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k) \qquad (6.1)$$

α is the "annual fractional cost of capital investment.

6.2.3. Vendor's cost equation

Similarly, vendor's cost components consist of setup, holding, amount of reducing "out-of-control" probability, and rework cost. Vendor uses SSMD policy to transfer goods to retail outlets. As per Sarkar and Majumder (2013), the average level of inventory is calculated as

$$\frac{d}{nq_s}\left[\left\{mq_s\left(\frac{q_s}{P}+\frac{(m-1)q_s}{d}\right)-\frac{m^2{q_s}^2}{2P}\right\}-\left\{\frac{{q_s}^2}{d}\left(1+2+3+\cdots(m-1)\right)\right\}\right]$$

$$=\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]$$

So that the vendor's holding cost will be

$$h_v\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right].$$

The investment for reducing "out-of-control" probability can be calculated as $b_m(\ln\theta_{mi}-\ln\theta_m)$.

And, the rework cost for defective goods per unit is $\frac{Rdmq_s\,\theta_m}{2}$ (Majumder et al., 2017).

Therefore, the vendor's cost equation turns to

$$TC_v(P,\theta_m,m)=\frac{S_v d}{mq_s}+h_v\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]$$

$$+\alpha b_m(\ln\theta_{mi}-\ln\theta_m)+\frac{Rdmq_s\,\theta_m}{2} \qquad (6.2)$$

6.2.4. Dependency on production rate

The paper considers the "out-of-control" probability θ_m is an increasing function of production rate P_m such that $\theta_m=f(P)$. Also, consider that θ_{mi}, the initial probability is a constant quantity which is attained when the maximum production rate is obtained by the production system i.e. $\theta_{mi}=f(P_{max})$= constant.

Thus, the cost of vendor is

$$TC_v(q_s,P,m)=\frac{S_v d}{mq_s}+h_v\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]$$

$$+\alpha b_m(\ln(f(P_{max}))-\ln(f(P)))+\frac{Rdmq_s f(P)}{2} \qquad (6.3)$$

6.2.5. Centralized cost equation

To establish the coordination between vendor and buyer a centralized system must be developed. Centralized chain is effective and information sharing is smooth. Also, in this case more reduced cost can be achieved that decentralization. Therefore, the joint cost of both parties is achieved by the following equation.

$$TCRM(q_s, O_b, P, m) = TC_b\left(q_s, O_{b,}k\right) + TC_v(q_s, P, m)$$

$$= \frac{O_b d}{q_s} + h_b\left[\frac{q_s}{2} + R_p + k\sigma\sqrt{L}\right] + \alpha b_r(\ln O_{ri} - \ln O_b) + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k) + \frac{S_v d}{m q_s} + h_v\frac{q_s}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \alpha b_m(\ln(f(P_{max})) - \ln(f(P))) + \frac{Rdmq_s f(P)}{2}$$

$$0 \le O_{ri} \le O_b$$

$$0 \le \theta_m \le 1$$

6.3. Solution methodology

To optimize the centralized cost equation, classical optimization method can be used which is taking derivatives of joint cost $TCRM(q_s, O_b, P, m)$ with the decision variables and equation to zero. Hence, we have

$$\frac{\partial TCRM}{\partial q_s} = -\frac{O_b d}{{q_s}^2} + \frac{h_b}{2} - \frac{S_v d}{m{q_s}^2} + \frac{h_v}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \frac{Rdmf(P)}{2} + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k)$$

$$= -\frac{1}{{q_s}^2}\left[O_b d + \frac{S_v d}{m} + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k)\right] + A$$

Where, $A = \frac{h_v}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \frac{Rdmf(P)}{2} + \frac{h_b}{2}$

Thus, $\frac{\partial TCRM}{\partial q_s} = 0$ implies

$$q_s = \sqrt{\frac{O_b d + d\pi\sigma\sqrt{L}\psi(k) + \frac{S_v d}{m}}{A}} \qquad (6.5)$$

Similarly,

$\frac{\partial TCRM}{\partial O_b} = 0$ implies

$$\frac{\partial TCRM}{\partial O_b} = -\frac{\alpha b_r}{\theta_m} + \frac{d}{q_s} = 0$$

$$or, O_b = \frac{\alpha b_r q_s}{d} \tag{6.6}$$

Now, $\frac{\partial TCRM}{\partial P} = 0$ implies

$$\frac{\partial TCRM}{\partial P} = \left(\frac{Rdmq_s}{2} - \frac{\alpha b_m}{f(P)}\right) f^{'}(P) = 0$$

$$Or, f(P) = \frac{2\alpha b_m}{Rdmq_s} \tag{6.7}$$

Now, consider some specific function which illustrates $f(P)$ more precisely to validate the model. This study considers two cases for dependency and independency of the production with system reliability.

Similarly, $\frac{\partial TCRM}{\partial k} = 0\ implies,$

$$\frac{\partial TCRM}{\partial k} = h_b + \sigma\sqrt{L} + \frac{d\pi\sigma\sqrt{L}}{q_s}[\Phi(k) - 1] = 0$$

$$\Phi(k) = 1 - \frac{q_s h_b}{\pi d} \tag{6.8}$$

Case 1 $\boldsymbol{\theta_m}$ is exponentially dependent on the production rate

In this case, $\theta_m = f(P) = 1 - e^{-aP}$, where a is suitable fitness parameter, which is so chosen that $0 \le \theta_m \le 1$. Based on this consideration, industry's decisions and graphical representation of stated relation are listed as follows.

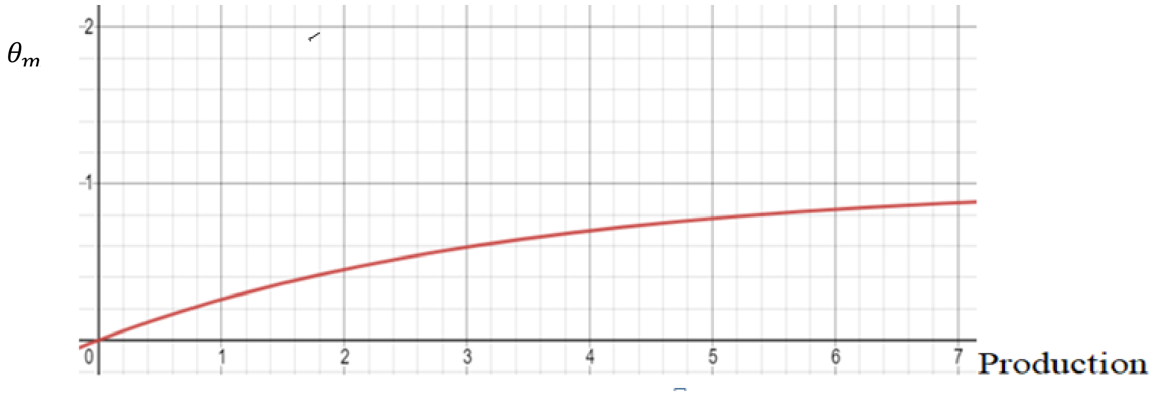


Figure 6.1: Relation between “Out-of-control” probability and production rate

$$q_1 = \sqrt{\frac{O_{b1}d + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[n\left(1 - \frac{d}{P_1}\right) - 1 + \frac{2d}{P_1}\right] + \frac{Rdm\theta_m}{2} + \frac{h_b}{2}}} \tag{6.9}$$

$$O_{b1} = \frac{\alpha b_r q_{s_1}}{d} \tag{6.10}$$

$$P_1 = \frac{2\alpha b_m}{Rdmq_{s1}} \tag{6.11}$$

$$\Phi(k_1) = 1 - \frac{q_{s1}h_b}{\pi d} \tag{6.12}$$

Case II ($\boldsymbol{\theta_m}$ is independent on the production rate)

In this case, θ_m is entirely independent on production rate. Here, θ_m is itself becomes a variable quantity. Thus, the decision variables are as follows.

$$q_{s_2} = \sqrt{\frac{O_{b2}d + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \frac{Rdm\theta_m}{2} + \frac{h_b}{2}}} \tag{6.13}$$

$$O_{b2} = \frac{ab_r q_{s_2}}{d} \tag{6.14}$$

$$\theta_m = \frac{2\alpha b_m}{Rdmq_{s2}} \tag{6.15}$$

$$\Phi(k_2) = 1 - \frac{q_{s2}h_b}{\pi d} \tag{6.16}$$

To obtain a numerical value of each variables, (6.10) and (6.14) should be written in the form of q_2 only which becomes

$$q_{s_1} = \sqrt{\frac{\alpha b_r q_{s_1} + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]+\frac{\alpha b m}{q_{S1}}+\frac{h_b}{2}}} \tag{6.16}$$

$$q_{s_2} = \sqrt{\frac{\alpha b_r q_{s_2} + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]+\frac{\alpha b m}{q_{S_2}}+\frac{h_b}{2}}} \tag{6.17}$$

Both (6.16) and (6.17) are in the form of the equation $x = f(x)$ the solution of which can be obtained by suitable numerical methods like iteration or Newton Raphson. As m is a discrete integer variable, the minimum value can be obtained when the following relation holds.

$$TCRM(m-1) \geq TCRM(m) \leq TCRM(m+1)$$

Therefore, the expected total cost value of case 1 and case 2 will be obtained as below.

$$\text{TCRM}(q_s, O_{b,}k, P, m) = \frac{O_{b1}d}{q_{s1}} + h_b\left[\frac{q_{s_1}}{2} + R_p + k\sigma\sqrt{L}\right] + \alpha b_r(\ln O_{ri} - \ln O_{b1}) + \frac{d}{q_{s_1}}\pi\sigma\sqrt{L}\psi(k_1) + \frac{S_v d}{m\, q_{s1}} + h_v\frac{q_{s1}}{2}\left[m\left(1-\frac{d}{P_1}\right) - 1 + \frac{2d}{P_1}\right] + \alpha b_m(\ln(f(P_{max})) - \ln(f(P_1))) + \frac{Rdmq_{s1}f(P_1)}{2} \text{ (case 1)} \tag{6.18}$$

Using $f(P) = 1 - e^{-aP}$ and $f(P_{max})$= $1 - e^{-aP_{max}}$.

$$\text{TCRM}(q_s, O_{b,}k, \theta_m, m)=\frac{O_{b2}d}{q_{s2}} + h_b\left[\frac{q_{s2}}{2} + R_p - dL\right] + \alpha b_r(\ln O_{ri} - \ln O_{b2}) + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k_2) + \frac{S_v d}{mq_s} + h_v\frac{q_{s2}}{2}\left[m\left(1-\frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \alpha b_m(\ln\theta_{mi} - \ln\theta_m) + \frac{Rdmq_s\,\theta_m}{2} \text{ (case2)} \tag{6.19}$$

6.4. Solution procedure

In this chapter, we will employee two different solution procedures. One for finding decision variables for total cost equation in which we will use "out-of-control" probability as an increasing function of production rate while in second we will obtain total cost and decision variable considering "out-of-control: probability as a independent function respectively. This similar solution procedure has been used by Sarkar et al. (2018) to derive the extremum values of the decision variables and total cost. The iterative procedure is also applicable here as the closed form solution is unavailable. The following steps are given to develop the solution algorithm1 and Solution algorithm2.

6.4.1. Solution algorithm1

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of q_s from (6.9).

Step 1b Obtain the values of O_b from (6.10).

Step 1c Obtain $\Phi(k)$ from (6.12) and find the values of k by inverse normal distribution.

Step 1d Obtain P from (6.11).

Step 1e Perform 1a to 1d by updating the values until no changes occurs (upto a specified accuracy level) in $q_s, k, P\ ans\ O_b$.

Step 2 Obtain the total cost from (6.18).

Step 3 Set$m = 2$, 3,…, p and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

6.4.2. Solution algorithm2

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of q_s from (6.9).

Step 1b Obtain the values of O_b from (6.10).

Step 1c Obtain $\Phi(k)$ from (6.12) and find the values of k by inverse normal distribution.

Step 1d Obtain the values of θ_m from (6.15).

Step 1e Perform 1a to 1d by updating the values until no changes occurs (up to a specified accuracy level) in $q_s, ,k, \theta_m\ and\ O_b$.

Step 2 Obtain the total cost from (6.18).

Step 3 Set$m = 2$, 3,…, p and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

6.5. Numerical experimentation

To achieve numerical results, all constant values of the parameters are taken numerically. The following Table 6.1 represents the values. The results are calculated by using the Method of Iteration.

Table 6.1: values of parameters

Parameters	Values	Parameters	Values
O_{bi}	500	R	60
d	100	θ_{mi}	0.1
h_b	10	b_r	500
S_v	10000	b_m	900
h_v	15	P_{max}	1000

Table 6.2: Results of Case 1

Variables	Values
q_1	73.68
O_{b1}	92.11
P_1	773.52
θ_m	0.000155
m	3
JETC	**6068.961**

Table 6.3: Results of Case 2

Variables	Values
q_1	90.57
O_{b1}	113.22
P	1000
θ_m	0.000414
m	3
JETC	**12746.795**

It is observed that in both cases “out-of-control” probability is reduced significantly from 0.1 to 0.000155 and 0.000414, for Case 6.1and 6.2, respectively. All numerical values of variables are shown in Table 6.2 and Table 6.3.

6.5.1. Sensitivity analysis

The sensitivity analysis and the graphical representation of all cost parameters for Case 1 and 2 are performed in Table 6.4 and 6.5, respectively. The cost parameters are varied from -10% to +10% and the changes in expected total cost is observed.

Table 6.4: Sensitivity analysis of cost parameters for case1

Case 1 (Out-of-control probability as a dependant function)		
Cost parameter	% change	Sensitivity
π	-10	-0.303484
	-5	-0.146440
	+5	0.137104
	+10	0.265929
h_b	-10	-0.701321
	-5	-0.345877
	+5	0.336576
	+10	0.664088
R	-10	-0.173089
	-5	-0.086873
	+5	0.087607
	+10	0.176038
h_v	-10	-1.019809
	-5	-0.508584
	+5	0.505965
	+10	1.009334
S_v	-10	-4.479983

	-5	-2.211853
	+5	2.159660
	+10	4.270740

Figure 6.2: Graphical representation of sensitivity analysis obtained for case1

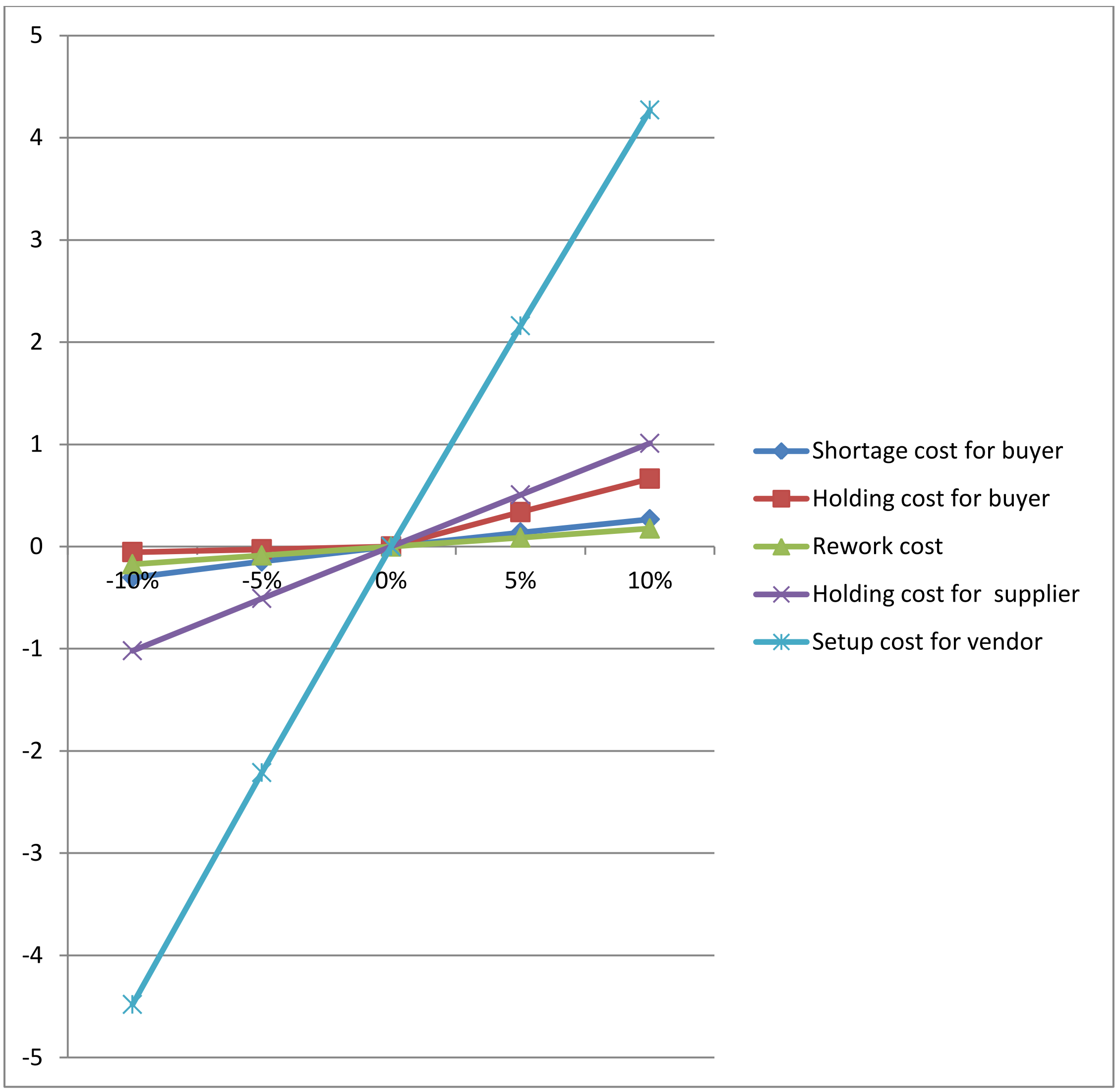
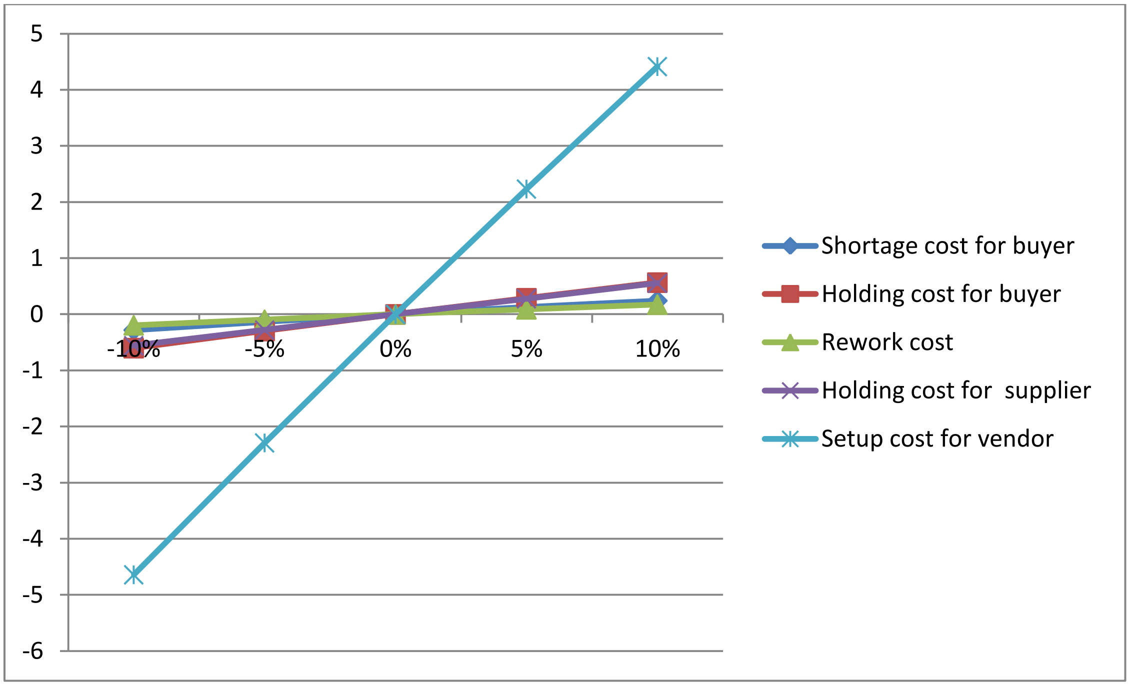


Table 6.5: Sensitivity analysis of cost parameters for case 2

Case 1 (Out-of-control probability as an independent function)		
Cost parameter	% change	Sensitivity
π	-10	-0.279961
	-5	-0.134896
	+5	0.125988
	+10	0.244112
h_b	-10	-0.601517
	-5	-0.296217
	+5	0.287348
	+10	0.566015
R	-10	-0.194188
	-5	-0.094538
	+5	0.089924
	+10	0.175665
h_v	-10	-0.558174
	-5	-0.278659
	+5	0.277811
	+10	0.554781
S_v	-10	-4.639787
	-5	-2.289644
	+5	2.233600
	+10	4.415091

Figure 6.3: Graphical representation of sensitivity analysis obtained for case2



6.6. Conclusion

The study is concerned about the effect of variable rate of production on manufacturing system reliability. The system reliability, in this paper, is measured through "out-of-control" probability which defines the tendency for a manufacturing process to shift an unreliable state. The study concludes that a suitable investment can reduce the propensity of the system to go to "out-of-control" state. There are two different models showing the dependency and independency of "out-of-control" probability on production process. It is also observed that this probability value as well as joint total cost is lower for Case 1 than Case 2 along with the determination of an optimal production level for case 1. Four decision variables were optimized to get the centralized minimum cost for vendor and buyer. Sensitivity analysis for various cost parameters of supply chain has been done for both the cases and observed that setup cost of vendor in controlling "out-of-control' probability emerges as a most sensitive cost.

The above discussed research can be further extended by considering “out-of-control” probability as a different possible function of increasing production rate with variable lead time.

Chapter 6

The effect of variable production rate on “out-of-control” probability in an integrated supply chain system

6.1. Introduction

Improving product quality is one of the most salient features in modern business to reach numerous numbers of customers. Thus, every company is working to improve customer satisfaction level by increasing product quality. But, to satisfy customer's demand, companies may, sometimes increase the rate of production which affects the production system. In this research, the author had developed a supply chain system with one vendor and buyer. In vendor's end, an investment is considered to improve the system performance which is measured through a probabilistic value ("out-of-control" probability). Two models are discussed in this paper regarding the independency and dependency of production with the "out-of-control" probability. An objective of centralized system cost is obtained. Besides that, the buyer also incurs an investment to reduce its setup cost.

Porteus (1986) initially developed the logarithmic concept of reducing setup cost and quality improvement. Rosenblatt and Lee (1986) introduced the in-control and out of control state of the production system. In this paper authors discussed about the shifting of manufacturing process from reliable to unreliable state. Khouja and Mehrez (1994) developed an EMQ model with production rate as decision and considered reliability of manufacturing process. Ouyang and Chen (2002) worked on a manufacturing process in an inventory system with setup cost reduction and quality improvement. Majumder et al. (2017) and Sarkar et al. (2017) developed a supply chain system with simultaneous reduction of vendor's setup cost and system imperfection.

6.2. Model formation

To develop the mathematical model, the following assumptions have been considered.

6.2.1. Assumptions

1. The paper considers an integrated centralized chain which minimizes the added cost of vendor and buyer.
2. An SSMD ("Single-setup multi-delivery") policy is adopted by the vendor. Through this policy, buyer's order is delivered over multiple times with equal lots.

3. Shortages are negligible in the models described in this study.
4. No lead time is considered in the models.
5. An investment is assumed to decrease the cost of setup for the buyer logarithmically.
6. The vendor also uses investment to increase system reliability by reducing the "out-of-control" probability.

6.2.2. *Buyer's cost equation*

Buyer's cost components consist of ordering, holding, and amount of reducing ordering cost. As per classical inventory model, the ordering and holding cost of the buyer are:

$\frac{O_b d}{q}$ and $h_b\left[\frac{q_s}{2} + R_p - DL\right]$ respectively. The concept of decrement in setup cost is taken from Majumder et al. (2017). As, O_{ri} is the initial fixed cost and O_b is the variable cost, the reduction strategy is assumed by a capital investment, $I_{sr} = b_r(\ln O_{ri} - \ln O_b)$.

The expected shortage at the end of a cycle is

$$E(X-R) = \int_R^{\infty}(X-R)dFx = \sigma\sqrt{L}\ \varphi(k)$$

where $\psi(k) = \phi(k) - k[1-\Phi(k)]$, ϕ = standard normal probability density function and Φ = cumulative distribution function of the normal distribution. The safety factor k is assumed as a decision variable instead of R.

Thus,buyer's total cost is

$$TC_b(q_s, O_b) = \frac{O_b d}{q_s} + h_b\left[\frac{q_s}{2} + R_p - dL\right] + \alpha b_r(\ln O_{ri} - \ln O_b) + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k) \qquad (6.1)$$

α is the "annual fractional cost of capital investment.

6.2.3. *Vendor's cost equation*

Similarly, vendor's cost components consist of setup, holding, amount of reducing "out-of-control" probability, and rework cost. Vendor uses SSMD policy to transfer goods to retail outlets. As per Sarkar and Majumder (2013), the average level of inventory is calculated as

$$\frac{d}{nq_s}\left[\left\{mq_s\left(\frac{q_s}{P}+\frac{(m-1)q_s}{d}\right)-\frac{m^2{q_s}^2}{2P}\right\}-\left\{\frac{{q_s}^2}{d}\left(1+2+3+\cdots(m-1)\right)\right\}\right]$$

$$=\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]$$

So that the vendor's holding cost will be

$$h_v\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right].$$

The investment for reducing "out-of-control" probability can be calculated as $b_m(\ln\theta_{mi}-\ln\theta_m)$.

And, the rework cost for defective goods per unit is $\frac{Rdmq_s\,\theta_m}{2}$ (Majumder et al., 2017).

Therefore, the vendor's cost equation turns to

$$TC_v(P,\theta_m,m)=\frac{S_v d}{mq_s}+h_v\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]$$

$$+\alpha b_m(\ln\theta_{mi}-\ln\theta_m)+\frac{Rdmq_s\,\theta_m}{2} \qquad (6.2)$$

6.2.4. *Dependency on production rate*

The paper considers the "out-of-control" probability θ_m is an increasing function of production rate P_m such that $\theta_m=f(P)$. Also, consider that θ_{mi}, the initial probability is a constant quantity which is attained when the maximum production rate is obtained by the production system i.e. $\theta_{mi}=f(P_{max})$= constant.

Thus, the cost of vendor is

$$TC_v(q_s,P,m)=\frac{S_v d}{mq_s}+h_v\frac{q_s}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]$$

$$+\alpha b_m(\ln(f(P_{max}))-\ln(f(P)))+\frac{Rdmq_s f(P)}{2} \qquad (6.3)$$

6.2.5. *Centralized cost equation*

To establish the coordination between vendor and buyer a centralized system must be developed. Centralized chain is effective and information sharing is smooth. Also, in this case more reduced cost can be achieved that decentralization. Therefore, the joint cost of both parties is achieved by the following equation.

$$TCRM(q_s, O_b, P, m) = TC_b\left(q_s, O_{b,}k\right) + TC_v(q_s, P, m)$$

$$= \frac{O_b d}{q_s} + h_b\left[\frac{q_s}{2} + R_p + k\sigma\sqrt{L}\right] + \alpha b_r(\ln O_{ri} - \ln O_b) + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k) + \frac{S_v d}{m q_s} + h_v\frac{q_s}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \alpha b_m(\ln(f(P_{max})) - \ln(f(P))) + \frac{Rdmq_s f(P)}{2}$$

$$0 \le O_{ri} \le O_b$$

$$0 \le \theta_m \le 1$$

6.3. Solution methodology

To optimize the centralized cost equation, classical optimization method can be used which is taking derivatives of joint cost $TCRM(q_s, O_b, P, m)$ with the decision variables and equation to zero. Hence, we have

$$\frac{\partial TCRM}{\partial q_s} = -\frac{O_b d}{{q_s}^2} + \frac{h_b}{2} - \frac{S_v d}{m{q_s}^2} + \frac{h_v}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \frac{Rdmf(P)}{2} + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k)$$

$$= -\frac{1}{{q_s}^2}\left[O_b d + \frac{S_v d}{m} + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k)\right] + A$$

Where, $A = \frac{h_v}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \frac{Rdmf(P)}{2} + \frac{h_b}{2}$

Thus, $\frac{\partial TCRM}{\partial q_s} = 0$ implies

$$q_s = \sqrt{\frac{O_b d + d\pi\sigma\sqrt{L}\psi(k) + \frac{S_v d}{m}}{A}} \qquad (6.5)$$

Similarly,

$\frac{\partial TCRM}{\partial O_b} = 0$ implies

$$\frac{\partial TCRM}{\partial O_b} = -\frac{\alpha b_r}{\theta_m} + \frac{d}{q_s} = 0$$

$$or, O_b = \frac{\alpha b_r q_s}{d} \tag{6.6}$$

Now, $\frac{\partial TCRM}{\partial P} = 0$ implies

$$\frac{\partial TCRM}{\partial P} = \left(\frac{Rdmq_s}{2} - \frac{\alpha b_m}{f(P)}\right) f'(P) = 0$$

$$Or, f(P) = \frac{2\alpha b_m}{Rdmq_s} \tag{6.7}$$

Now, consider some specific function which illustrates $f(P)$ more precisely to validate the model. This study considers two cases for dependency and independency of the production with system reliability.

Similarly, $\frac{\partial TCRM}{\partial k} = 0\ implies,$

$$\frac{\partial TCRM}{\partial k} = h_b + \sigma\sqrt{L} + \frac{d\pi\sigma\sqrt{L}}{q_s}[\Phi(k) - 1] = 0$$

$$\Phi(k) = 1 - \frac{q_s h_b}{\pi d} \tag{6.8}$$

Case 1 $\boldsymbol{\theta_m}$ is exponentially dependent on the production rate

In this case, $\theta_m = f(P) = 1 - e^{-aP}$, where a is suitable fitness parameter, which is so chosen that $0 \leq \theta_m \leq 1$. Based on this consideration, industry's decisions and graphical representation of stated relation are listed as follows.

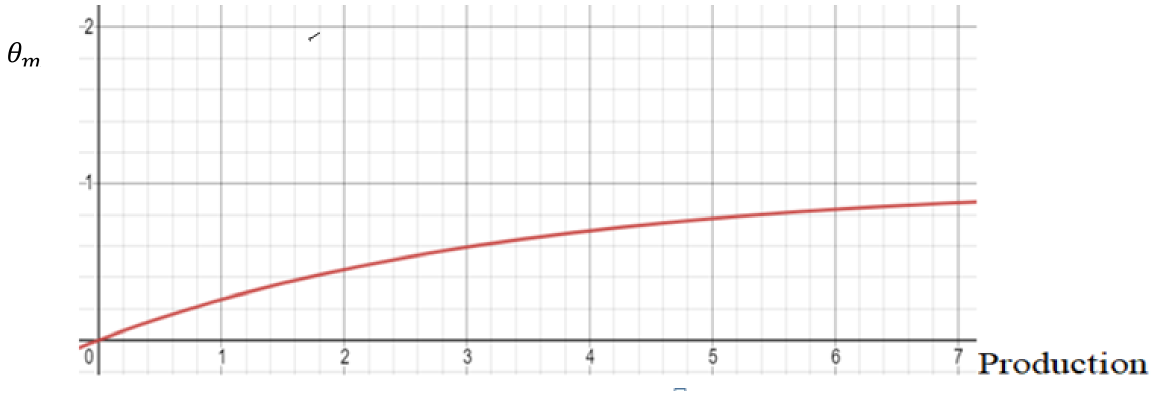


Figure 6.1: Relation between "Out-of-control" probability and production rate

$$q_1 = \sqrt{\frac{O_{b1}d + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[n\left(1 - \frac{d}{P_1}\right) - 1 + \frac{2d}{P_1}\right] + \frac{Rdm\theta_m}{2} + \frac{h_b}{2}}} \tag{6.9}$$

$$O_{b1} = \frac{\alpha b_r q_{s_1}}{d} \tag{6.10}$$

$$P_1 = \frac{2\alpha b_m}{Rdmq_{s1}} \tag{6.11}$$

$$\Phi(k_1) = 1 - \frac{q_{s1}h_b}{\pi d} \tag{6.12}$$

Case II (θ_m is independent on the production rate)

In this case, θ_m is entirely independent on production rate. Here, θ_m is itself becomes a variable quantity. Thus, the decision variables are as follows.

$$q_{s_2} = \sqrt{\frac{O_{b2}d + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \frac{Rdm\theta_m}{2} + \frac{h_b}{2}}} \tag{6.13}$$

$$O_{b2} = \frac{ab_r q_{s_2}}{d} \tag{6.14}$$

$$\theta_m = \frac{2\alpha b_m}{Rdmq_{s2}} \tag{6.15}$$

$$\Phi(k_2) = 1 - \frac{q_{s2}h_b}{\pi d} \tag{6.16}$$

To obtain a numerical value of each variables, (6.10) and (6.14) should be written in the form of q_2 only which becomes

$$q_{s_1} = \sqrt{\frac{\alpha b_r q_{s_1} + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]+\frac{\alpha b m}{q_{s1}}+\frac{h_b}{2}}} \tag{6.16}$$

$$q_{s_2} = \sqrt{\frac{\alpha b_r q_{s_2} + \frac{S_v d}{m} + d\pi\sigma\sqrt{L}\psi(k)}{\frac{h_v}{2}\left[m\left(1-\frac{d}{P}\right)-1+\frac{2d}{P}\right]+\frac{\alpha b m}{q_{s_2}}+\frac{h_b}{2}}} \tag{6.17}$$

Both (6.16) and (6.17) are in the form of the equation $x = f(x)$ the solution of which can be obtained by suitable numerical methods like iteration or Newton Raphson. As m is a discrete integer variable, the minimum value can be obtained when the following relation holds.

$$TCRM(m-1) \geq TCRM(m) \leq TCRM(m+1)$$

Therefore, the expected total cost value of case 1 and case 2 will be obtained as below.

$$\text{TCRM}(q_s, O_{b,}k, P, m) = \frac{O_{b1}d}{q_{s1}} + h_b\left[\frac{q_{s_1}}{2} + R_p + k\sigma\sqrt{L}\right] + \alpha b_r(\ln O_{ri} - \ln O_{b1}) + \frac{d}{q_{s_1}}\pi\sigma\sqrt{L}\psi(k_1) + \frac{S_v d}{m\, q_{s1}} + h_v\frac{q_{s1}}{2}\left[m\left(1-\frac{d}{P_1}\right) - 1 + \frac{2d}{P_1}\right] + \alpha b_m(\ln(f(P_{max})) - \ln(f(P_1))) + \frac{Rdmq_{s1}f(P_1)}{2} \text{ (case 1)} \tag{6.18}$$

Using $f(P) = 1 - e^{-aP}$ and $f(P_{max})$= $1 - e^{-aP_{max}}$.

$$\text{TCRM}(q_s, O_{b,}k, \theta_m, m) = \frac{O_{b2}d}{q_{s2}} + h_b\left[\frac{q_{s2}}{2} + R_p - dL\right] + \alpha b_r(\ln O_{ri} - \ln O_{b2}) + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k_2) + \frac{S_v d}{mq_s} + h_v\frac{q_{s2}}{2}\left[m\left(1-\frac{d}{P}\right) - 1 + \frac{2d}{P}\right] + \alpha b_m(\ln\theta_{mi} - \ln\theta_m) + \frac{Rdmq_s\,\theta_m}{2}$$

(case2) (6.19)

6.4. Solution procedure

In this chapter, we will employee two different solution procedures. One for finding decision variables for total cost equation in which we will use “out-of-control” probability as an increasing function of production rate while in second we will obtain total cost and decision variable considering “out-of-control: probability as a independent function respectively. This similar solution procedure has been used by Sarkar et al. (2018) to derive the extremum values of the decision variables and total cost. The iterative procedure is also applicable here as the closed form solution is unavailable. The following steps are given to develop the solution algorithm1 and Solution algorithm2.

6.4.1. Solution algorithm1

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of q_s from (6.9).

Step 1b Obtain the values of O_b from (6.10).

Step 1c Obtain $\Phi(k)$ from (6.12) and find the values of k by inverse normal distribution.

Step 1d Obtain P from (6.11).

Step 1e Perform 1a to 1d by updating the values until no changes occurs (upto a specified accuracy level) in $q_s, k, P\ ans\ O_b$.

Step 2 Obtain the total cost from (6.18).

Step 3 Set$m = 2$, 3,…, p and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

6.4.2. Solution algorithm2

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of q_s from (6.9).

Step 1b Obtain the values of O_b from (6.10).

Step 1c Obtain $\Phi(k)$ from (6.12) and find the values of k by inverse normal distribution.

Step 1d Obtain the values of θ_m from (6.15).

Step 1e Perform 1a to 1d by updating the values until no changes occurs (up to a specified accuracy level) in q_s, ,k, $\theta_m\ and\ O_b$.

Step 2 Obtain the total cost from (6.18).

Step 3 Set$m = 2$, 3,…, p and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

6.5. Numerical experimentation

To achieve numerical results, all constant values of the parameters are taken numerically. The following Table 6.1 represents the values. The results are calculated by using the Method of Iteration.

Table 6.1: values of parameters

Parameters	Values	Parameters	Values
O_{bi}	500	R	60
d	100	θ_{mi}	0.1
h_b	10	b_r	500
S_v	10000	b_m	900
h_v	15	P_{max}	1000

Table 6.2: Results of Case 1

Variables	Values
q_1	73.68
O_{b1}	92.11
P_1	773.52
θ_m	0.000155
m	3
JETC	**6068.961**

Table 6.3: Results of Case 2

Variables	Values
q_1	90.57
O_{b1}	113.22
P	1000
θ_m	0.000414
m	3
JETC	**12746.795**

It is observed that in both cases "out-of-control" probability is reduced significantly from 0.1 to 0.000155 and 0.000414, for Case 6.1and 6.2, respectively. All numerical values of variables are shown in Table 6.2 and Table 6.3.

6.5.1. Sensitivity analysis

The sensitivity analysis and the graphical representation of all cost parameters for Case 1 and 2 are performed in Table 6.4 and 6.5, respectively. The cost parameters are varied from -10% to +10% and the changes in expected total cost is observed.

Table 6.4: Sensitivity analysis of cost parameters for case1

Case 1 (Out-of-control probability as a dependant function)		
Cost parameter	% change	Sensitivity
π	-10	-0.303484
	-5	-0.146440
	+5	0.137104
	+10	0.265929
h_b	-10	-0.701321
	-5	-0.345877
	+5	0.336576
	+10	0.664088
R	-10	-0.173089
	-5	-0.086873
	+5	0.087607
	+10	0.176038
h_v	-10	-1.019809
	-5	-0.508584
	+5	0.505965
	+10	1.009334
S_v	-10	-4.479983

	-5	-2.211853
	+5	2.159660
	+10	4.270740

Figure 6.2: Graphical representation of sensitivity analysis obtained for case1

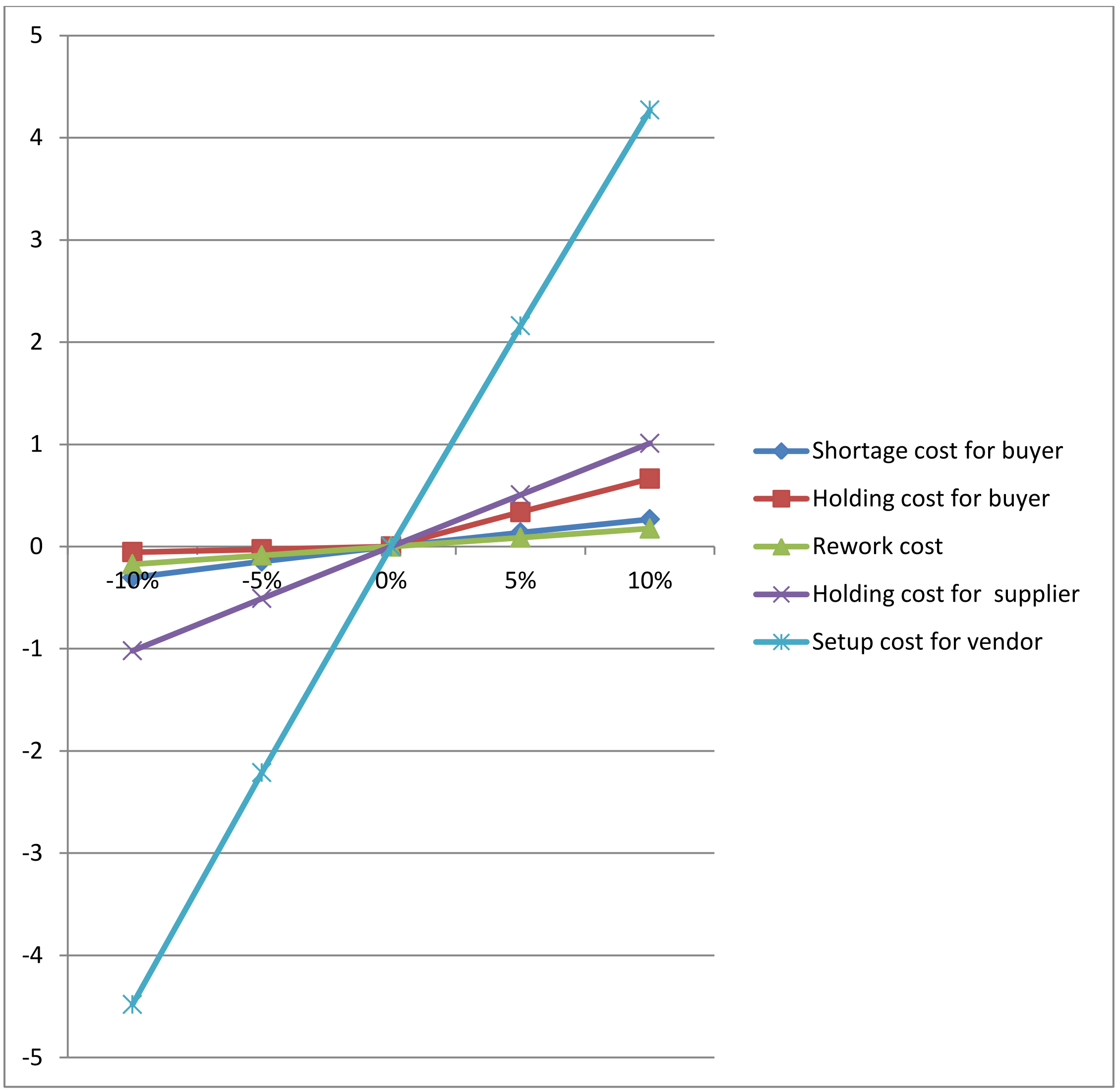
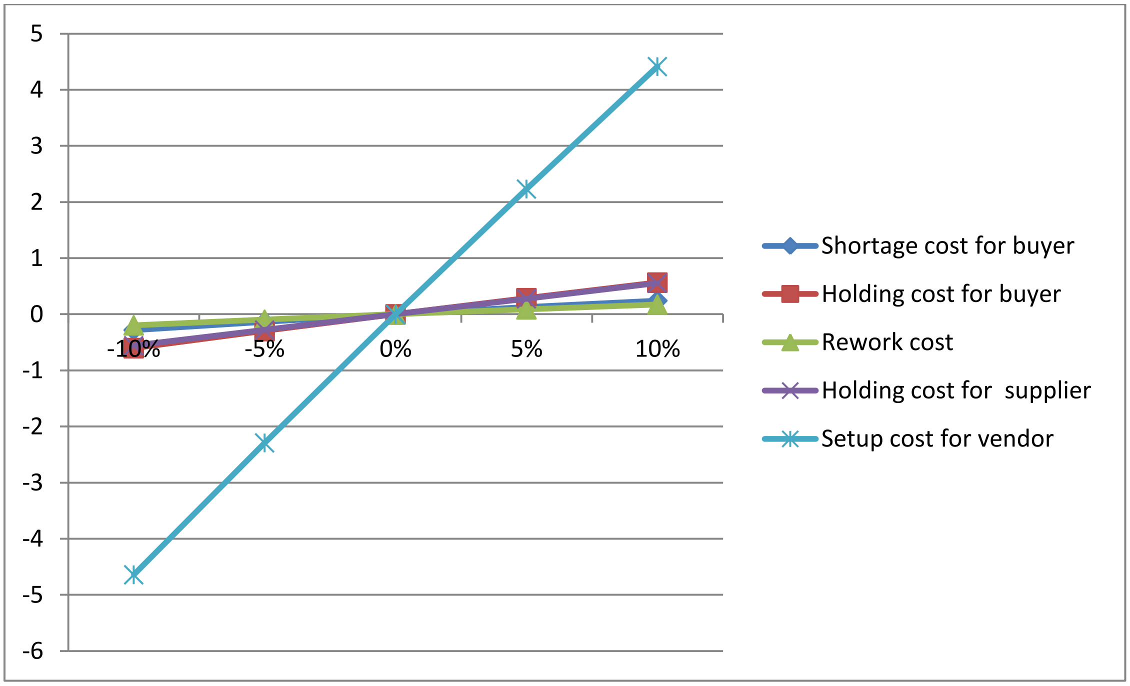


Table 6.5: Sensitivity analysis of cost parameters for case 2

Case 1 (Out-of-control probability as an independent function)		
Cost parameter	% change	Sensitivity
π	-10	-0.279961
	-5	-0.134896
	+5	0.125988
	+10	0.244112
h_b	-10	-0.601517
	-5	-0.296217
	+5	0.287348
	+10	0.566015
R	-10	-0.194188
	-5	-0.094538
	+5	0.089924
	+10	0.175665
h_v	-10	-0.558174
	-5	-0.278659
	+5	0.277811
	+10	0.554781
S_v	-10	-4.639787
	-5	-2.289644
	+5	2.233600
	+10	4.415091

Figure 6.3: Graphical representation of sensitivity analysis obtained for case2



6.6. Conclusion

The study is concerned about the effect of variable rate of production on manufacturing system reliability. The system reliability, in this paper, is measured through “out-of-control” probability which defines the tendency for a manufacturing process to shift an unreliable state. The study concludes that a suitable investment can reduce the propensity of the system to go to “out-of-control” state. There are two different models showing the dependency and independency of “out-of-control” probability on production process. It is also observed that this probability value as well as joint total cost is lower for Case 1 than Case 2 along with the determination of an optimal production level for case 1. Four decision variables were optimized to get the centralized minimum cost for vendor and buyer. Sensitivity analysis for various cost parameters of supply chain has been done for both the cases and observed that setup cost of vendor in controlling “out-of-control’ probability emerges as a most sensitive cost.

The above discussed research can be further extended by considering "out-of-control" probability as a different possible function of increasing production rate with variable lead time.

Chapter 7

Decentralized supply chain system under reliability and sustainability using "Stackelberg game" approach

7.1. Introduction

In this research, the researcher has shifted its interest from centralized supply chain management to decentralized supply chain management considering it will considerably improve its performance. As an optimal supply chain is facilitated by such a system, which encourages the actors to cooperate. The system comes with numerous benefits such as:

1. It has the primary benefit of cutting logistics costs at the local level.
2. This leads more rational wealth distribution more rationally, which results into better macro-economic growth driven by higher consumer purchasing power and lower taxes.
3. This approach brings more flexibility with respect to the system expansion in new markets and experimentation of new products.
4. With respect to the shipping times and trust, a decentralized organization expect to offer better customer service ending in a stronger long-term relationship.

Here, the author is scrutinizing the decentralized supply chain model utilizing the "Stackelberg game" approach for decision making. Decentralization of supply chain is such a system in which vendor and buyer do not need to share any information. The costs of both parties are optimized separately. Since in this research, "Stackelberg model" is used for decentralization. In which, at least one player is deemed to be the leader who will make strategic decisions and commit to a plan before others who are termed as followers. In this study, vendor is playing as the leader while buyer as the follower. While optimizing the cost, the optimal decision values of the retailer are substituted with the vendor's decision variables. Hence, the total costs are calculated separately for both parties. The solution methods with "Stackelberg game approach" are illustrated below.

7.2. Model formulation

As this study is analyzing decentralized supply chain management instead of centralized discussed in chapter 4. Therefore, the assumptions are considered as same as chapter 4.

7.2.1. Decentralized cost equation

To optimize the decentralized cost equation, classical optimization method can be used which is taking derivatives of first buyer cost $ETC_{bi}(q_i, L_i, k_{i,})$ with the decision variables and equate them to zero. Hence, we have

$$ETC_{bi}(q_i, k_i, L_i) = \begin{bmatrix} \frac{O_{bi}d_i}{q_i} + h_{bi}\left\{\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i}\right\} \\ + \frac{\pi_i d_i}{q_i} E(X_i - r_i)^+ + R(L_i)\frac{d_i}{q_i} + q_i E_{bi} \end{bmatrix} \quad (7.1)$$

$$Q = D\sqrt{\frac{\{\sum_{i=1}^{n}(O_{bi}+\pi_i\sigma_i\sqrt{L_i}\,\Psi(k_i)+R(L_i))\}}{\sum_{i=1}^{n} h_{bi}d_i+(d_iE_b)}} \quad (7.2)$$

$$\Phi(k_i) = 1 - \frac{h_{bi}Q}{D\pi_i} \quad (7.3)$$

As, $ETC_{bi}(Q, k_i, L_i)$ is concave for L_i for the fixed values of Q and k_i. Therefore, in the interval $[L_{i,j}, L_{i,j-1}]$ $ETC_{bi}(Q, k_i, L_i)$is attained minimum value for the fixed value of $Q\ and\ k_i$.

The values of the decision variables of buyer, obtained by using the solution algorithm described above are illustrated in Table 7.1.

Table 7.1: Buyer's decisions for decentralization

Parameters	Values
Q	182.01
$k1$	1.93
$k2$	1.93
$k3$	1.90
$r1$	73.23
$r2$	77.09
$r3$	95.54
$L1$	4
$L2$	4
$L3$	4
TERC	9379.68

The decisions of buyers are used to obtain the optimal production rate, production cost, number of shipment, and expected total cost of the vendor.

$$ETC_v(m,Q,P) = \begin{Bmatrix} \frac{S_v D}{mQ} + \frac{Q}{2}\ h_v \left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right] \\ + RD\alpha f(p)\frac{Q}{2P} + DC(P) + mQE_v \end{Bmatrix} \tag{7.4}$$

Hence, the vendor's optimal values are illustrated in Table 7.2.

Table 7.2: Vendor's decisions for decentralization

Parameters	Case 1	Case 2
m	3	3
P	583.86	583.38
$C(P)$	118.33	118.33
$MTTF$	17.12	10.82
$TECV$	210072.30	210215.71

7.3. Comparison of centralized and decentralized

Table 7.1 and 7.2 describes the non-coordination model. It is observed that expected joint total costs for coordinated chain are 206077.21 and 206129.04 for Case 1 and 2, respectively obtained in chapter 4. On the other hand, for non-coordinated chain independent decisions are taken by the buyers which are then followed by the vendor. A comparison table (Table 7.3) is created to show the difference of total costs.

Table 7.3: Comparison between coordinated and non-coordinated chain

	Case 1		Case 2	
Centralized	206077.21		206129.04	
Decentralized	Buyer's cost	9379.68	Buyer's cost	9379.68
	Vendor's cost	210072.30	Vendor's cost	210215.71
	Total	219451.98	Total	219595.39

It is observed that for same parametric values, the entire supply chain cost is lower for coordinated chain than that of non-coordinated chain.

7.4. An another example of decentralized model of supply chain

7.4.1. Model Formulation

The study is analyzing decentralized supply chain management of model discussed in chapter 6 for single buyer and single vendor. Therefore the assumptions are same.

7.4.2. Buyer's mathematical model

Using classical optimization method, the values of decision variables of buyer cost $TC_b(q_s, O_b, k)$ is obtained. Where, $TC_b(q_s, O_b, k)$, have cost components as ordering cost, holding cost, an logarithmic investment to find optimal ordering cost and shortage cost. Hence,

$$TC_b(q_s, O_b, k) = \frac{O_b d}{q_s} + h_b\left[\frac{q_s}{2} + R_p - dL\right] + \alpha b_r(\ln O_{ri} - \ln O_b) + \frac{d}{q_s}\pi\sigma\sqrt{L}\psi(k) \quad (7.5)$$

$$\frac{\partial TCRM}{\partial q_s} = -\frac{O_b d}{{q_s}^2} + \frac{h_b}{2} - \frac{d\pi\sigma\sqrt{L}\psi(k)}{{q_s}^2}$$

Thus, $\frac{\partial TC_b}{\partial q_s} = 0$ implies

$$q_s = \sqrt{\frac{2(O_b d + d\pi\sigma\sqrt{L}\psi(k))}{h_b}} \quad (7.6)$$

Similarly,

$\frac{\partial TC_b}{\partial O_b} = 0$ implies

$$\frac{\partial TC_b}{\partial O_b} = -\frac{\alpha b_r}{O_b} + \frac{d}{q_s} = 0$$

Or, $O_b = \frac{\alpha b_r q_s}{d}$ (7.7)

Similarly,

$\frac{\partial TC_b}{\partial k} = 0$ Implies

$$\frac{\partial TCRM}{\partial k} = h_b\sigma\sqrt{L} + \frac{d\pi\sigma\sqrt{L}}{q_s}[\Phi(k) - 1] = 0$$

$$\Phi(k) = 1 - \frac{q_s h_b}{\pi d} \tag{7.8}$$

After employing the above obtained values, the optimal production rum for vendor is obtained from equation of total expected cost of vendor given below.

7.4.3. Vendor's mathematical model

$$TC_v(q_s, P, m) = \frac{S_v d}{q_s} + h_v \frac{q_s}{2}\left[m\left(1 - \frac{d}{P}\right) - 1 + \frac{2d}{P}\right]$$

$$+\alpha b_m(\ln(f(P_{max})) - \ln(f(P))) + \frac{Rdmq_s f(P)}{2} \tag{7.9}$$

$\frac{\partial TC_v}{\partial P} = 0$ implies

$$\frac{\partial TCM}{\partial P} = \left(\frac{Rdmq_s}{2} - \frac{\alpha b_m}{f(P)}\right) f'(P) + \left[\frac{h_v dmq_s}{2} - h_v dq_s\right]\frac{1}{P^2} = 0$$

$$\text{Or, } P^2 = \frac{h_v dq_s\left[\frac{m}{2} - 1\right]}{\left[\frac{\alpha b_m}{f(P)} - \frac{Rdmq_s}{2}\right] f'(P)} \tag{7.10}$$

Now, consider some specific function which illustrates $f(P)$ more precisely to validate the model. This study considers case for dependency of the production with system reliability.

θ_m is exponentially dependent on the production rate

In this case, $\theta_m = f(P) = 1 - e^{-aP}$, where a is suitable fitness parameter, which is so chosen that $0 \leq \theta_m \leq 1$. Based on this consideration, industry's decisions are summarized as follows.

$$q_{s_1} = \sqrt{\frac{2(O_{b1}d + d\pi\sigma\sqrt{L}\psi(k_1))}{h_b}} \tag{7.10}$$

$$O_{r1} = \frac{\alpha b_r q_{s_1}}{d} \tag{7.11}$$

$$P_1^2 = \frac{h_v dq_{s_1}\left(\frac{m}{2} - 1\right)}{\left[\frac{\alpha b_m}{1 - e^{-aP}} - \frac{Rdmq_{s_1}}{2}\right]\left(\frac{a}{e^{aP}}\right)} \tag{7.12}$$

$$\Phi(k_1) = 1 - \frac{q_{s_1} h_b}{\pi d} \quad (7.13)$$

To obtain a numerical value of each variables, (7.11) should be written in the form of q_1 only which becomes

$$q_{s_1} = \sqrt{\frac{\alpha b_r q_{s1} + d\pi\sigma\sqrt{L}\psi(k_1)}{\frac{h_b}{2}}} \quad (7.14)$$

Now, (7.14) is in the form of the equation $x = f(x)$ the solution of which can be obtained by suitable numerical methods like iteration or Newton Raphson. As m is a discrete integer variable, the minimum value can be obtained when the following relation holds.

$$TC_v(m-1) \geq TC_v(m) \leq TC_v(m+1)$$

Hence, utilizing above values, the total cost of buyer and vendor can be obtained separately as stated below.

$$TC_b(q_{s1}, O_{b1}, k_1) = \frac{O_{b1} d}{q_{s1}} + h_b\left[\frac{q_{s1}}{2} + R_p - dL\right] + \alpha b_r(\ln O_{ri} - \ln O_{b1}) + \frac{d}{q_{s1}}\pi\sigma\sqrt{L}\psi(k_1)$$

(7.15)

$$TC_v(q_{s1}, P, m) = \frac{S_v d}{m q_{s1}} + h_v \frac{q_{s1}}{2}\left[m\left(1 - \frac{d}{P_1}\right) - 1 + \frac{2d}{P_1}\right]$$

$$+\alpha b_m(\ln(f(P_{max})) - \ln(f(P_1))) + \frac{Rdmq_s f(P_1)}{2} \quad (7.16)$$

Using $f(P) = 1 - e^{-aP}$ and $f(P_{max}) = 1 - e^{-aP_{max}}$

7.5. Solution procedure

This similar solution procedure has been used by Sarkar et al. (2018) to derive the extremum values of the decision variables and total cost. The iterative procedure is also applicable here as the closed form solution is unavailable. The following steps are given to develop the solution algorithm1.

Step1 Input values of all cost parameters and set$m = 1$. For each value L_i perform the following steps.

Step 1a Obtain the values of q_s from (7.10).

Step 1b Obtain the values of O_b from (7.11).

Step 1c Obtain $\Phi(k)$ from (6.12) and find the values of k by inverse normal distribution.

Step 1d Obtain P using the solution of q_s obtained in (7.10) from (7.13).

Step 1e Perform 1a to 1d by updating the values until no changes occurs (upto a specified accuracy level) in $q_s, k, P\ ans\ O_b$.

Step 2 Obtain the total cost of buyer and vendor separately from (7.17) and (7.18) respectively.

Step 3 Set$m = 2$, 3,…, p and perform the steps again.

Step 4 Obtain the minimum total cost for $m = j; 1 < j < p$

7.6. Numerical experimentation

To achieve numerical results, all constant values of the parameters are taken numerically. The following Table 7.4 represents the values. The results are calculated by using the Method of Iteration.

Table 7.4: Parameter values

Parameters	Values	Parameters	Values
O_{ri}	500	R	60
d	100	α	0.25
h_b	10	b_r	800
S_v	400	b_m	900
h_v	15	P_{max}	1000
σ	15	π	30
E_v	500	S_{cv}	0.426
a	.0000002	M	3

Table 7.5: Optimal values of decision variables with separate total cost of buyer and vendor

Optimal result values	
Lot size(q_s)=75.94	Production rate(P)=803.054
Safety stock(k1)=0.664	Total cost of buyer=36957.282
Ordering cost(O)=151.87	Total cost of vendor=115387.302

Table 7.6: Cost comparison of centralized and decentralized system

Centralized	6068.961	
Decentralized	Buyer's cost	36957.282
	Vendor's cost	115387.302
	Total	152344.583

The following value is explicitly showing relation amongP, θ and TC_v.

Table 7.7: Explicit relation among P,θ, TC_v

P	θ	TC_v
100	0.000020	115782.354
200	0.000040	115640.067
300	0.000060	115562.508
400	0.000080	115511.450
500	0.000100	115474.913
600	0.000120	115447.560
700	0.000140	115426.546
800	0.000160	115410.171
900	0.000180	115397.3393
1000	0.000200	115387.302

It is observed that for growing values of$P$, the "out-of-control" probability θ value is increasing.

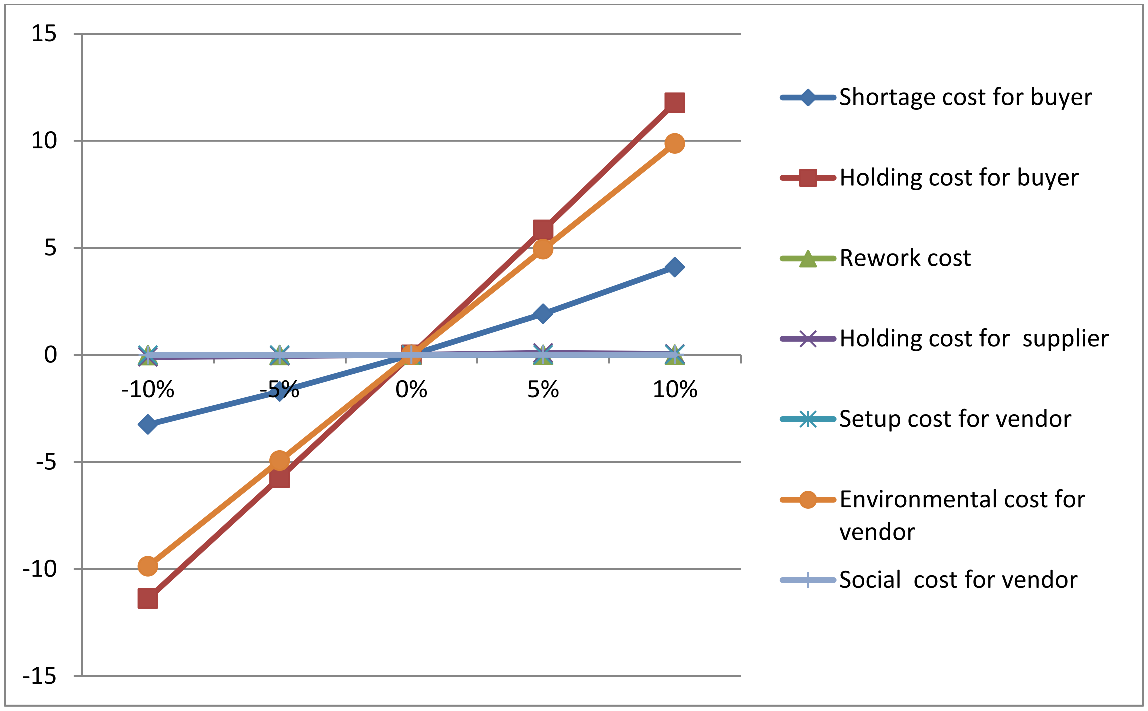
7.6.1. *Sensitivity analysis*

The sensitivity analysis and the graphical representation of all cost parameters of decentralized supply chain are performed in Table 7.7. The cost parameters are varied from -10% to +10% and the changes in expected total cost is observed.

Table7.7: Sensitivity analysis of cost parameters

Case 1 (Out-of-control probability as a dependant function)		
Cost parameter	% change	Sensitivity
π	-10	-3.247088
	-5	-1.710068
	+5	1.919022
	+10	4.094476
h_b	-10	-11.373294
	-5	-5.734551
	+5	5.833358
	+10	11.768724
R	-10	-0.011845
	-5	-0.005923
	+5	0.005923
	+10	0.011845
h_v	-10	-0.092573
	-5	-0.046287
	+5	0.046287
	+10	0.092573
S_v	-10	-0.015216
	-5	-0.007608
	+5	0.007608
	+10	0.015216
S_{cv}	-10	-0.008411
	-5	-0.004205
	+5	0.004205
	+10	0.008411
E_v	-10	-9.871955
	-5	-4.935977
	+5	4.935977
	+10	9.871955

Figure 7.1: Graphical representation of above calculated sensitivity analysis



7.7. Conclusions

The study is concerned about the effect of variable rate of production on manufacturing system reliability. The system reliability is measured through "out-of-control" probability which defines the tendency for a manufacturing process to shift an unreliable state. Also the effect of variable production rate on product reliability was discussed in this study. As a novel approach, the "out-of-control" probability was considered as an increasing function of production rate. The study concludes that a suitable investment can reduce the propensity of the system to go to "out-of-control" state. The research developed a decentralized supply chain system with one vendor and one buyer. The optimal solution for an uncoordinated system was obtained through "Stackelberg game approach". It is also observed that the system reliability diminishes with increasing rate of production. For various cost parameters sensitivity analysis result has observed with an inference of finding influential costs like shortage cost for buyer, holding cost for buyer and environmental cost for vendor in cost parameters of buyers and vendors in decentralized model.

Another vital conclusion was obtained by comparison the expected total cost of supply chain of centralized and decentralized system, which infers that better profitability is obtain by adopting a centralized chain. The supply chain cost is less for centralization than a decentralized chain.

Concluding remarks

The dissertation has explored a numerous practical problems associated with the management of supply chains in the modern world. The study mainly focused on multi-retailer supply chain management as it is more realistic in modern market scenario. Also, manufacturing quality improvement and reliability are two of the important factors to be studied in this research. The study focuses on optimizing the production rate along with minimizing the rate of deterioration and maximizing the quality. As increased production leads to high carbon emission, maintaining the environmental and social sustainability are very important aspects. Moreover, handling the uncertain demand and lead time minimization strategy to increase customer satisfaction are two main motto of this research. The entire thesis is divided into seven chapters. First and second chapters discuss about the motivational background and literature survey along with the research gap of the study. In chapter 3, while establishing a relation between production rate and "mean time to failure", it has been observed that linear quality function is superior to quadratic quality function when production rate is viewed as a variable instead of a constant. In chapter 4, where environmental and social investment is incorporated for system sustainability factor, Sensitivity analysis shows that environmental cost parameter is the most sensitive cost parameter, thus the company needs to think more before investing on technologies or carbon tax to enhance the environmental factor. Chapter 5 concluded total, the applied logarithmic investment to reduce setup cost aid to reduce joint cost to optimize the investment expenditure initiated by vendor and buyer for three bottom line of sustainability in supply chain management. In Chapter 6, "out-of-control" probability is studied as an increasing function of production rate as well as an independent function. It concluded that system reliability can be increased if a fixed optimized production level is produced. Chapter 7, deals with decentralized supply chain model, in this chapter, first half is dedicated to obtain a more reliable system considering an "out-of-control" probability function. And in second half, it concluded that a company should adopt the coordinated strategy than non-coordinated to reduce the supply chain cost.

Comparison of this study with existing literature

One of the unique contributions of this study is to establish a relation between the production rate and “out-of-control” probability. The existing literature such as Sarkar et al. (2017) and Majumder et al. (2017) used “out-of-control” probability as a measure of system reliability. This research proposed two measures of reliability such as “MTTF” and “out-of-control” probability which provides additional choice of measuring the system reliability. Moreover, as a pioneer approach this research considers the “out-of-control” probability as an increasing function of production rate under based on a negative exponential distribution. This assumption brings out an improved realistic result compared to the existing studies. The incorporation of this concept leads to almost 50% reduction of total cost as compared to independent “out-of-control” probability.

This research compares centralized and decentralized supply chain model. Many existing literature such as Thomas et al.(2016) and Sarkar et al. (2016) used decentralized model solved by using the Stackelberg game approach. Independent decisions were obtained for parties involved in the chain. In real scenario, centralized decision provides improved decisions as compared to the decentralized one. Thus, this research compared the centralized and decentralized supply chain to observe the exact scenario. To obtain the decentralized decisions, same Stackelberg game is used as shown in the existing literature. We observe a 6% reduction of total cost for centralized system as compared to the decentralized one

Another approach used in this research deals with observing a change obtained in combined total cost value of supply chain models used by Nandra et al. (2021a, 2021b). These models have considered setup cost as fixed. This study also incorporates an approach to reduce the vendor’s setup cost with the help of a logarithmic investment function. The setup cost becomes variable and obtained setup cost is reduced than fixed one. This reduction of setup cost helps reducing the expected total cost by approximately 5%.

Future extensions

This study is focused on two players of supply chain namely vendor and buyer while this two echelon study of supply chain model can be further extended to three echelon supply chain in which vendor, buyer and supplier mathematical model can be considered and their total cost optimization goal can be reached. Furthermore, various forms of backorder costs can be added in the measurement in addition to variable lead time, safety stock, rework cost, and shortage cost in order to improve the customer satisfaction level as well as to increase the lifespan of a customer in a company.

As we have found that when an out-of-control probability function is expressed and scrutinized in terms of increasing production rate to measure the system reliability not only the system reliability enhanced but also the cost optimization is obtained. We have expressed that function as a negative exponential function in such a way that value of probability function lies between its range and this incorporation has opened the door to use various other expressional relating out of control probability function to increasing production level so that comparative study can be made and useful results can be drawn for firms to increase their profit levels.

Instead of considering static or dynamic deterioration in continuous production process, deterioration with an inspection cost in multi-step production process with various rework cost can be considered. This study is done for production that took place in manufacturing units and has a longer survival time. On the other hand, these models can be used for perishable goods with shorter survival time.

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