Electron Acceleration by Laser Driven Electron Plasma Waves in Plasmas with Different Density Profiles

Thesis Submitted for the Award of the Degree of

DOCTOR OF PHILOSOPHY

In

PHYSICS

By

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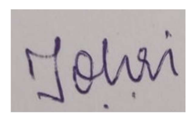
2026

DECLARATION

I hereby declared that the presented work in the thesis entitled **"Electron Acceleration by Laser Driven Electron Plasma Waves in Plasmas with Different Density Profiles"** in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy (Ph.D.) in Physics** is the outcome of original research work carried out by me under the supervision of **Dr. Naveen Gupta**, working as Assistant Professor in the Department of Physics, Lovely Professional University, Phagwara, Punjab, India. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.



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CERTIFICATE

This is to certify that the work reported in the Ph.D. thesis entitled **"Electron Acceleration by Laser Driven Electron Plasma Waves in Plasmas with Different Density Profiles"** submitted in fulfillment of the requirement for the award of the degree of **Doctor of Philosophy (Ph.D.)** in **Physics** is a research work carried out by **Mr. Rohit Johari, Registration No. 42000586**, is bonafide record of his original work carried out under my supervision and that no part of the thesis has been submitted for any other degree, diploma or equivalent course.

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Abstract

The main focus of this thesis is on nonlinear propagation of q-Gaussian and Cosh-Gaussian (ChG) laser beams in nonlinear medium such as plasma. Further, the impacts of self action effects of laser beams in plasmas with temperature and density ramps, have been studied theoretically. In the midst of different existing semi-analytical approaches, variational theory approach has been used to investigate the propagation of q-Gaussian and Cosh-Gaussian (ChG) laser beams in nonlinear medium such as underdense plasmas under nonlinear regime.

Theoretical investigation has been conducted on the interaction between a q-Gaussian laser beam profile and underdense plasmas with temperature ramp and axial density ramp. The study incorporates the influence of self-focusing. The nonlinear propagation of q-Gaussian laser beam in plasma is influenced by two factors: DP 'q' and the ellipticity of the beam. It has been seen that the self-focusing is most pronounced for q=3, while it decreases in the transverse direction where the beam is more elliptical. Increasing the value of q, which moves the q-Gaussian beam closer to an ideal Gaussian distribution, leads to reduced cross-focusing. Consequently, the effect of off-axial rays diminishes, resulting in a decrease in the intensity of the pump. This is due to the self-focusing of the laser beam intensifying with higher slope values, combined with a longer propagation distance.

Theoretical investigations have been conducted to explore the phenomenon of self-focused ChG laser beams interacting nonlinearly with plasmas featuring an axial density ramp. The study focuses on the impact of the laser beam's irradiance profile on its nonlinear interaction with the plasma. It has been observed that the flatness of the laser beam's irradiance profile plays a crucial role in enhancing the coupling of laser energy with the plasma by promoting self-focusing.

One key factor influencing the results is the parameter b associated with the cosh function, which is commonly known as the cosh factor or decentered parameter of the laser beam. The investigation reveals that as the nonlinear refraction of the laser beam reaches a state of equilibrium for self-focusing, an increase in the value of b within the range of $0 \leq b \leq 1$ enhances the extent of self-focusing. At b=1, the beam width is minimized as the contributions from off-axial rays become balanced. However, for b>1, the presence of a central dark region prevents the ChG laser beam from receiving contributions from the axial part of the wave fronts, resulting in reduced nonlinear refraction due to the growing size of the central dark region.

Dedication

"Success in any challenge is a blend of personal determination and the invaluable support of those who believe in you."

I dedicate my humble efforts to special persons of my life

Who were very close to my heart during this research work journey

***My Daughters** (**Diya** and **Tvishi**) and **My Wife** (Mrs. Shikha Johari)*

And

My Father (Mr. J.C. Johari)

And

Along With Hard Working** and **Respected Teachers

Who had made my base strong and ignite the spark for doctorate degree,

Due to which I am able to survive in education field.

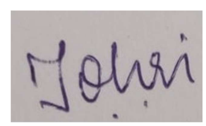
Acknowledgment

It is a best opportunity to acknowledge some special persons of my life for their unconditional kind support, guidance and motivation throughout this research degree awarded in physics.

First and foremost, I am delighted to express my gratitude and thanks to my research supervisor ***Dr. Naveen Gupta*** *for their irreplaceable guidance, consistent encouragement, immeasurable care and critical review throughout this research work. Their mastery of scientific questioning, interpretation, imaginations and suggestions enhanced my cognitive awareness and helped me remarkably in the fulfillment of my proposed research objectives.*

From the bottom of my heart, I deeply express my gratitude to my family for all their unlimited love, care and remarkable support for pursuit of my scientific career. They had encouraged me to follow my passion, especially, my daughters and my wife to support me throughout this journey.

I am glad to express my gratitude to Dr. Kailash Chandra Juglan (HOD), ***Department of Physics*** *and* ***Department of Research and Development of Lovely Professional University*** *for their general support. Especially, I am grateful to Dr. Rajeev Sobti for his kind encouragement during these years.*



Rohit Johari

Preface

The main focus of this thesis is especially on electron acceleration by laser driven electron plasma waves in plasma. The present research work has been divided into total four chapters. The nonlinear Schrodinger wave equation for the evolution of beam envelope is solved numerically by using variational theory. In particular,

Chapter - 1 incorporates the introduction part which involves the particle accelerator and its brief history, types of accelerator. Principle of plasma particle accelerator and effect of wave breaking, methods to generate plasma waves and application of accelerators, research objectives, scope of this study and review of Literature.

C**hapter - 2** includes the electron acceleration by laser driven plasma waves in a plasma with a density gradient of cosh Gaussian laser beams. This analysis has been done by variational theory approach for electron acceleration by cosh – Gaussian laser beams.

C**hapter - 3** emphasizes on inducing electron plasma waves through optically guided q – Gaussian laser beam in plasma channel created by ignitor heater technique and its effect on electron acceleration. This investigation focuses on excitation of electron plasma waves and change in width of cosh Gaussian laser beam.

Chapter - 4 focuses on beat wave excitation of electron plasma wave by cross focused q – Gaussian laser beams in thermal quantum plasma and its effect on electron acceleration. The investigation leads to how q – Gaussian laser beam cross focusing effects electron plasma wave excitation in thermal quantum plasma.

Chapter - 5 focuses on the future and scope of this thesis work.

List of Research Paper Publications (These are the main five papers which contribute to the thesis)

1. Naveen Gupta, Rohit Johari, “Laser-Driven Electron Acceleration by q-Gaussian Laser Pulse in Plasma: Effect of Self-Focusing”, Journal of Applied Spectroscopy 90, 1133-1141. (Published Online : 25 November 2023)
2. Naveen Gupta, Rohit Johari, A. K. Alex, “Electron acceleration by laser driven electron plasma wave in plasma with axial density ramp: Cosh Gaussian laser beam”, Journal of Optics 53. (Published Online : 21 March 2024)
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Additional Research Paper Publications

6. Naveen Gupta, A.K. Alex, Rohit Johari, “Nonlinear interaction of quadruple-Gaussian laser beams with periodic lattice of metallic nanoparticles: self-action effects”, Journal of Optics 53. (Published Online : 27 February 2024)
7. Naveen Gupta, Rohit Johari, A. K. Alex, “Propagation dynamics of q-Gaussian laser beams in preformed collisionless plasma channel: self-focusing, self-channeling, and self-phase modulation”, Journal of Optics 53. (Published Online : 11 March 2024)
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9. Naveen Gupta, A. K. Alex, Rudra Pratap, Rohit Johari,"Self action effects of q-Gaussian laser beam in preformed parabolic plasma channels: effect of nonlinear absorption", Journal of Optics 53. (Published Online : 3 April 2024)
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15. Naveen Gupta, Rohit Johari, Sanjeev Kumar, S. B. Bhardwaj, A. K. Alex, "Electron acceleration by self focused quadruple Gaussian laser beams in thermal quantum plasma: effect of relativistic optical nonlinearity of plasma", Journal of Optics 53. (Published Online : 5 March 2024)
16. Naveen Gupta, Rohit Johari, Sanjeev Kumar, Alex A K, S. B. Bhardwaj, "Dynamics of q-Gaussian laser beam in thermal quantum plasma: self-focusing self-trapping and self-phase modulation", Journal of Optics 53. (Published Online : 14 May 2024)
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plasma with exponential density ramp: combined effect of self-focusing and self-compression", Journal of Optics 53. (Published Online : 29 February 2024)

19. Naveen Gupta, Sanjeev Kumar, Suman Choudhry, A. K. Alex, Rohit Johari, S. B. Bhardwaj, "Quadruple Gaussian laser beam in cubic-quintic nonlinear media: effect of nonlinear absorption", Journal of Optics 53. (Published Online : 2 January 2024)
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21. Naveen Gupta, A. K. Alex, Rohit Johari, Suman Choudhry, Sanjeev Kumarand, S. B. Bhardwaj, "Excitation of electron plasma wave by self-focused Laguerre–Gaussian laser beams in axially inhomogeneous plasma: effect of orbital angular momentum of photons", Journal of Optics 52. (Published Online : 23 December 2023)
22. Naveen Gupta, Rohit Johari, Sanjeev Kumar, Suman Choudhry, A. K. Alex, S. B. Bhardwaj, "Excitation of upper hybrid wave by cross focused q-Gaussian laser beams in graded index plasma channel", Journal of Optics 52. (Published Online : 23 September 2023)
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diffraction managed nonlinear optical media: effect of cubic quintic nonlinearity", Journal of Optics 52. (Published Online : 18 August 2023)

28. Naveen Gupta, Rohit Johari, Dinesh Bhardwaj, A. K. Alex, Siddhanth Shishodia, Nakul Kohli, S. B. Bhardwaj, "Self-focusing, self-trapping and self-phase modulation of elliptical q-Gaussian laser beams in collisionless plasma", Journal of Optics 52. (Published Online : 10 February 2023)
29. Naveen Gupta, Sanjeev Kumar, Suman Choudhry, Rishabh Khatri, Siddhanth Shishodia, Rohit Johari, S. B. Bhardwaj, "Second-harmonic generation of two cross-focused q-Gaussian laser beams by nonlinear frequency mixing in plasmas", Journal of Optics 51. (Published Online : 5 November 2022)
30. Naveen Gupta, Suman Choudhry, Rohit Johari, A. K. Alex, Sanjeev Kumar,S. B. Bhardwaj, "Quadruple Gaussian laser beams in thermal quantum plasma: self-focusing, self-trapping and self-phase modulation", Journal of Optics 51. (Published Online : 21 November 2023)

Conferences/Seminars

1. National conference on recent advancement in physics held on 17th September 2022 at Hansraj Mahila MahaVidyalaya at Jalandhar (Punjab) presenting a poster under emerging scientist category on electron acceleration by elliptical q-Gaussian laser driven electron plasma wave in collisionless plasma.
2. ELI Summer school 2022 held a conference in Hungary from 30th August to 2nd September 2022 in which an online paper presentation was done on Generation of Super Thermal Electrons by Self Focused Cosh Gaussian Laser Beams in Inertial Confinement Fusion Plasma.
3. International conference of students and young researchers in theoretical and experimental physics "Eureka 2023" held in Ukraine from 16th to 18th May 2023 in which online presentation was given "Stimulated Raman scattering of self focused elliptical q-gaussian laser beam in plasma; effect of density ramp "

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Chapter 1

Laser Plasma Based Electron Acceleration

1.1 Introduction

Large particle accelerators, such as the Large Hadron Collider, are commonly linked to state-of-the-art physics research, but their uses go beyond basic science. These intricate devices, which took decades to construct, provide important new light on some of the universe's most profound mysteries, such the structure of protons and neutrons, two of the fundamental particles. Particle accelerators, however, are not limited to these enormous machines. There is more to particle accelerators than big physics! They are essential in many different fields, including medical sterilization, cancer therapy, security screening, and the study of novel materials and biological processes. With ambitious initiatives like the Superconducting Super Collider, we are getting closer to two key restrictions as we push the limits of accelerator technology:

Magnetic Breakdown: As particle acceleration magnetic fields become stronger and stronger, they will eventually overwhelm the structural stability of the materials they are contained in, essentially rupturing the magnets. Electron Stripping: The tremendous energy produced by accelerator electric fields is getting close to the energy needed to separate electrons from their atoms in the materials around them, jeopardizing the system's structural integrity.

Thankfully, a novel technique known as "plasma particle acceleration" presents a viable way to get around the drawbacks of traditional accelerators and reach even greater energy. The problem of electron stripping is not a barrier for plasma accelerators, in contrast to conventional techniques. This is so that there is no chance of the electrons becoming dislodged from the surrounding materials since they work with plasma, a state of matter in which the electrons have already been split from their atoms. As a result, plasma accelerators are able to overcome the structural constraints that prevent further breakthroughs by having the potential to maintain electric forces thousands of times stronger than those of current technologies. This creates fascinating opportunities to reach particle energies that were unthinkable before.

The acceleration power of modern particle accelerators is expressed in millions of electron volts (MeV) per meter. This indicates that for every meter that electrons move inside the machine, they accumulate millions of volts of energy. But conventional radiofrequency (rf) accelerators are almost at the end of their useful life; their maximum tens of MeV per meter is usually reached. The only way to attain larger collision energies with RF technology is to construct longer and more costly accelerators. Acceleration of plasma particles, however, has enormous potential to overcome these restrictions. Its potential acceleration rates are estimated to be in the billions of electron volts (GeV) per meter range. Comparable energy levels might be achieved using a plasma accelerator spanning only a few hundred meters, as opposed to the 87-kilometer Superconducting Super Collider, if it could maintain such high-intensity fields across greater distances. If successful, this innovative technology has the potential to completely change the industry. By allowing far smaller and more efficient machines to produce comparable energy, it has the potential to make gigantic rf accelerators like the Large Hadron Collider (LHC) and the projected International Linear Collider (ILC) obsolete.

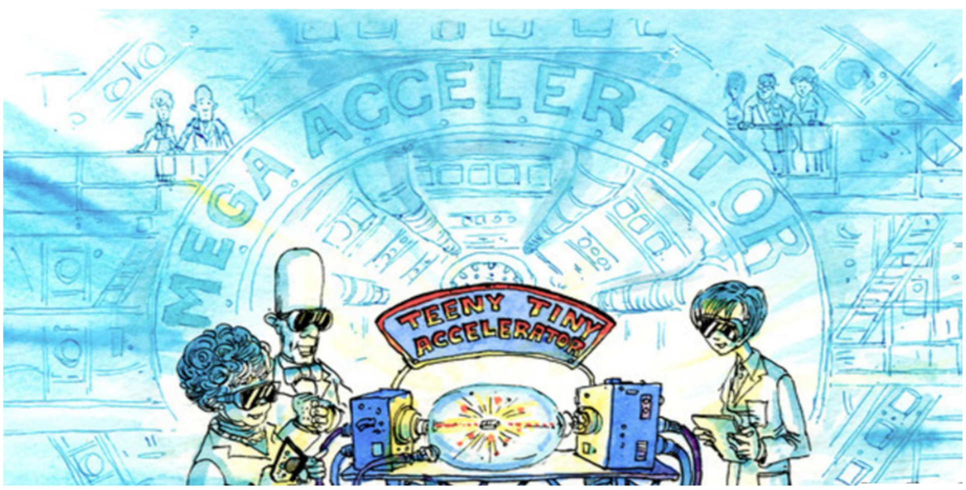

Fig.1.1: Shrinking the size of particle accelerators.

(Courtesy - The Economist Newspaper Article)

1.2 Notion of Acceleration

Our typical notion of acceleration is complicated when approaching near light speed. Particles such as electrons already move very near to the speed of light in high-energy accelerators. Consider an electron that has been accelerated to 50 billion electron volts (GeV); it is only a small bit slower than light. It would have finished just 0.1 millimeters behind if it had been a light pulse racing around Earth. How can we speed up particles that are traveling at this speed already? According to Einstein's theory of relativity, it's not about greatly increasing their velocity, but rather increasing their mass. Plasma accelerators need a unique "wave" in the plasma that moves at almost the speed of light in order to do this. This guarantees that the particles may efficiently acquire energy and do not escape the accelerating electric field.

1.3 Brief History of Particle Accelerators

Particle accelerator history started in 1897 at Cambridge University's Cavendish Laboratory. It was there that J. J. Thomson made a revolutionary discovery using a straightforward table top setup akin to the cathode-ray tubes seen in vintage televisions (fig. 1.2). He discovered a negatively charged particle that was previously unidentified and is now known as the electron. This was the first particle accelerator, opening the door to further discoveries about the underlying components of the cosmos.

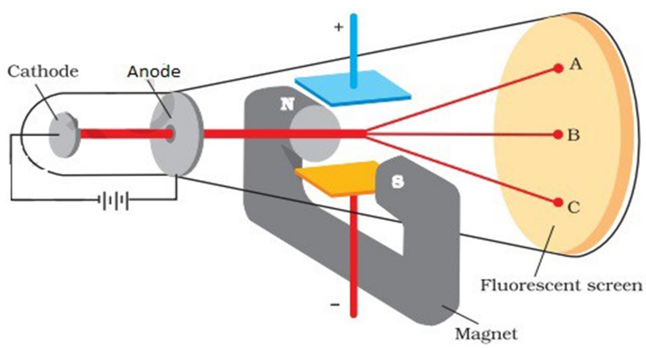

Fig.1.2: Cathode ray tube. (Courtesy – NCERT Book)

By subjecting atoms to radioactive particle bombardment, scientists were able to discover the last two fundamental components of the atom: protons and neutrons. This important finding opened the door to more developments in our understanding of the structure of atoms. Ernest Lawrence transformed the discipline in the 1930s with the invention of the cyclotron, a revolutionary palm-sized particle accelerator. Protons could be accelerated by this novel technology to energies of about 80,000 electron volts (keV).

Fig.1.3: Ernest Lawrence’s accelerator. (Courtesy – Lawrence Livermore National Laboratory)

Particle accelerator technology advanced quickly after these early discoveries, enabling scientists to explore the secrets of the atom in more detail. These formidable devices made it possible to explore the atomic nucleus and, in a remarkable development, to investigate its basic building blocks, the individual nucleons (protons and neutrons) (fig.1.4).

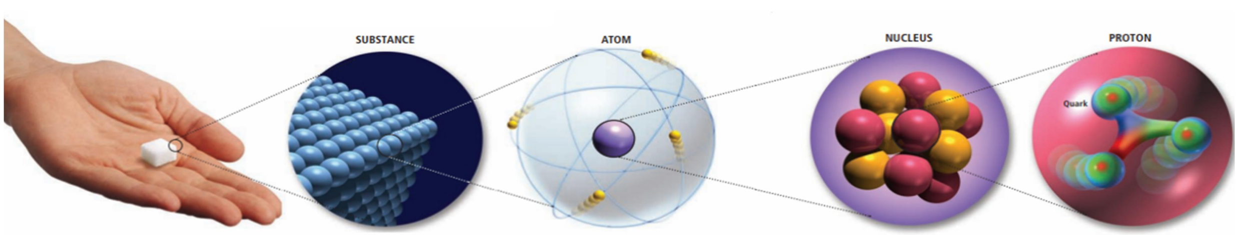

Fig.1.4: Structure of proton obtained by scattering of electron beam.
(Courtesy – Astronomy, Astrophysics article in department of physics in Lehman College)

Driven by breakthroughs in accelerator technology, researchers set out on an amazing exploration that yielded hundreds of fundamental particles. But as these accelerators' energy skyrocketed around the close of the 20th century, an unexpected fact became apparent. In the end, these many particles were made up of just seventeen basic components, exactly as the Standard Model of particle physics predicted. Through a variety of accelerator experiments, all but one of these anticipated particles had been correctly detected by the late 1990s. At the Large Hadron Collider (LHC), the missing Higgs boson was ultimately found, completing the mystery and establishing the Standard Model as the pinnacle of contemporary particle physics.

1.4 Types of Particle Accelerators

There are two forms for accelerators:

1. Synchrotrons, a type of circular accelerator (fig. 1.5)
2. Linac, a type of linear accelerator (fig.1.6)

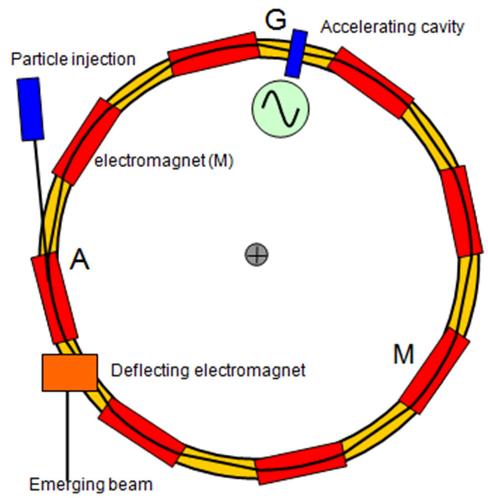

Fig.1.5: Synchrotron.(Courtesy – Article in School Physics)

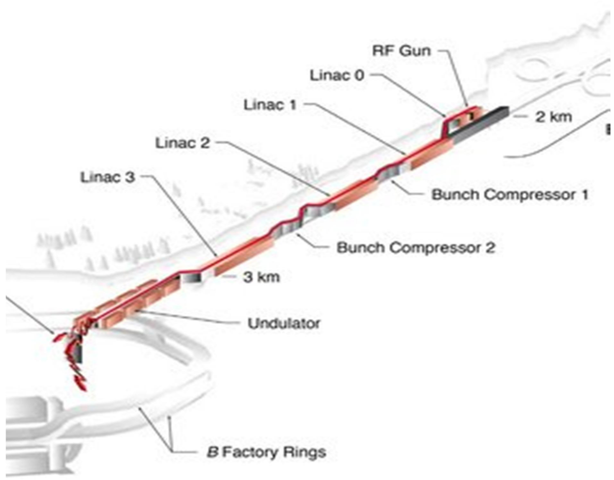

Fig.1.6: Linac .(Courtesy – SLAC National Accelerator Laboratory Article)

These large-scale particle accelerators propel particles to nearly the photon velocity by exerting strong forces. Consider the LHC, the Large Hadron Collider. Two pronuclear beams move in opposite directions within its ring, repeatedly passing through regions designated as radio-frequency cavities. Radio waves in these cavities produce electric fields that alternate between positive and negative charges all the time. The secret is in this deft manipulation. When the field is deliberately changed, the positively charged protons undergo a constant "forward pull," akin to that of a cyclotron. The protons receive an energy injection from this constant push, which causes them to accelerate to amazing speeds. The particles are not left on their own after they attain the appropriate energy levels. Strong magnetic lenses function as focusers, carefully directing them to pre-determined locations of impact inside the ring. These exact collisions are essential for a number of studies that aid in our quest to understand the workings of the universe. High-speed collisions between these particles release incredibly concentrated energy that can build completely new, heavier particles. There is, however, a disadvantage to this procedure. The energy that charged particles lose during their forced circular motion within the accelerator is known as "synchrotron radiation."

Charged particles in particle accelerators lose energy as they bend in a circular pattern due to "synchrotron radiation." In the case of lighter particles like electrons and positrons, this energy loss is very noteworthy. To reduce this energy loss, heavier protons are used in the Large Hadron Collider, which is built for high-energy collisions. Future accelerators, nevertheless, will have to overcome this obstacle if they want to investigate the interactions of lighter particles, such as electrons and positrons. There are two primary methods under consideration:

Linear accelerators: Energy loss due to curvature is eliminated because the particles move along a straight path instead of a circular trajectory.

Large-radius circular accelerators: The synchrotron radiation is reduced by minimizing the curvature by a large increase in the circular path's radius.

Future colliders will need to employ these techniques in order to properly investigate the behavior of lighter particles and uncover other mysteries about the universe.

It comes down to a power struggle in the end regarding accelerator size. The accelerator can be smaller for a given beam energy the more radio-frequency power we can feed into its core construction without running the risk of electrical breakdown. These structures were traditionally made of copper, which has breakdown limits that raise the attainable energy gain per meter to between 20 and 50 million electron volts (MeV).

Nonetheless, researchers are expanding the horizons by investigating:

Elevated frequencies: investigating the possibility of raising the breakdown threshold by experimenting with novel materials and designs that can withstand higher frequencies.

Superconducting cavities: Increasing the strength of the accelerating fields inside accelerators by using these strong cavities, which are currently widely used in accelerators.

These developments are significant first steps that will be carried out before completely novel ideas are realized and upend the well-established field of traditional accelerator technology.

Although current accelerator technology is excellent, creative methods are needed to reach even greater energies. The U.S. Department of Energy initiated a groundbreaking program in 1982 that investigated novel and radical techniques for accelerating charged particles, leading to the development of several exciting concepts. Among these ideas, three have the most promise:

1. A method known as "two-beam acceleration" shows promise. Using a strong yet affordable electron pulse, this technique produces high-frequency radiation inside a specially designed cavity. Another cavity's secondary electron pulse is energized by this radiation. This novel idea is presently being tested on a device known as the Compact Linear Collider (CLIC) at CERN, the European Organization for Nuclear Research.
2. Using muons, heavier cousins of electrons, is another possible approach. Unlike electrons, their larger mass enables them to move in circular accelerators with less energy loss from synchrotron radiation. The problem, though, is that muons are unstable particles that have a very short half-life of two millionths of a second. Although there hasn't been a muon accelerator constructed yet, some scientists are nevertheless excited about its possibilities.

J. M. Dawson's 1970 proposal for plasma-based acceleration is the last fascinating idea. This technique creates a "wake field" that may accelerate particles thousands or even ten thousand times faster than conventional accelerators by using a unique state of matter called plasma in conjunction with strong laser pulses or electron beams. The plasma wakefield accelerator is a novel concept that could lead to a dramatic reduction in the size of these massive machines, much like integrated circuits did for electronics in the 1960s.

1.5 Principle of Plasma Particle Accelerators

Strong electric fields are essential for speeding particles, and they can only be produced in plasmas, a unique state of matter where some electrons have been removed from atoms (fig.1.7). Plasmas have segregated positive and negative charges (ions and electrons), despite being electrically neutral overall. When this barrier is broken, areas with substantial concentrations of positive or negative charges might result. Think of these areas as the magnet's opposite ends. In the same way that an unseen force surrounds a magnet, they inherently produce an electric field that stretches between them. Both negatively charged electrons and positively charged ions are drawn toward one another by this electric field, which functions like a tug-of-war. But unlike larger ions, which essentially stay motionless, electrons react to the attraction considerably more quickly because of their much smaller mass. In plasma-based technologies, the motion of electrons in the electric field produces a special wave-like disturbance that serves as the basis for particle acceleration.

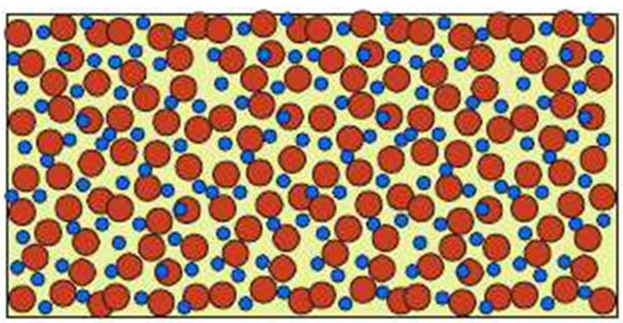

Fig.1.7: Plasma.

The negatively charged electrons gain momentum and speed as they are drawn by the plasma's positive areas. They continue past the positive region after reaching it thanks to this impetus. The electrons are first slowed down by the electric field, which then deftly reverses course to draw them back towards the positive region. Similar to a bouncing ball, this back-and-forth action produces an electron "oscillator." Think of these electron oscillators as springs that wobble. A zone of compressed and sparse electron density is created when a large number of electrons engage in this dance; these regions move across the plasma like a wave (fig.1.8). In plasma-based accelerators, this wave—also referred to as the electron plasma wave, or EPW—is essential for particle acceleration.

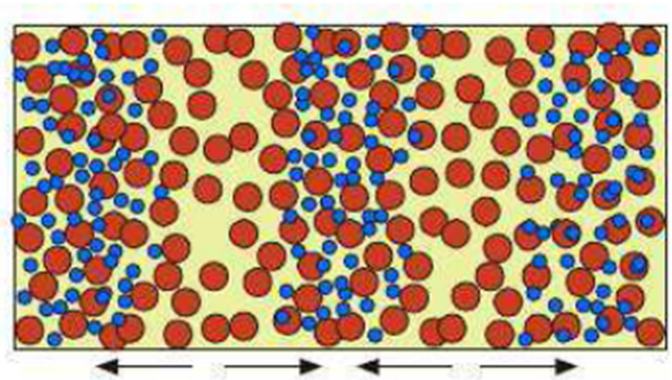

Fig.1.8: Plasma wave.

When a charged particle is introduced into a plasma at a speed equivalent to a plasma wave, an intriguing phenomenon takes place. Particle "surfing" on wave energy, it stays in sync with the electric field of the wave. The particle can gradually collect energy from the wave thanks to this ongoing interaction, which accelerates it significantly.

1.6 Effect of Wave Breaking

Even if they are strong, plasma waves can only accelerate electrons so far. Wave breaking puts a limit on the amount of energy that can be transferred between them. Imagine a wave breaking in the water; the crest hits the trough, causing the wave's smooth flow to be disrupted. A plasma wave likewise "breaks" when electron clusters come into contact, preventing additional acceleration. The maximum amplitude of a plasma wave's oscillations prior to breaking determines the strength of the electric field within it and, in turn, the energy transfer. The density of electrons in the plasma is closely correlated with this amplitude. Scientists can regularly produce plasmas in the range of 10^{16} to 10^{21}electrons per cubic centimetre in lab settings. This corresponds, in principle, to maximal accelerating fields for such plasmas of 100 million to 30 billion volts per centimetre. By contrast, fields produced by current conventional accelerators average about 200,000 volts per centimetre; future models are expected to produce fields as high as one or two million volts per centimetre. Although plasma waves have great promise, their restrictions must be taken into account when assessing how they contribute to particle acceleration.

1.7 Methods to Generate Plasma Waves

The search for strong particle accelerators has prompted research into a number of plasma wave creation techniques. These two encouraging methods are as follows:

1. Wake field method
2. Beat wave method

1.7.1 Wake field method

This process makes use of a big assembly of electrons that are moving through plasma like a boat. It moves and causes some plasma electrons to be displaced, leaving a wake in its wake. The plasma wave is an oscillation that resembles a wave and is caused by this movement. The wake field, a strong electric field, is carried by this wave (fig.1.9). Scientists can use the energy of the wave to accelerate these "surfer electrons" to even greater energies than the original group that formed the wave by carefully injecting a smaller group of electrons into this wake field. Future developments of much more potent and compact particle accelerators may be possible using this method.

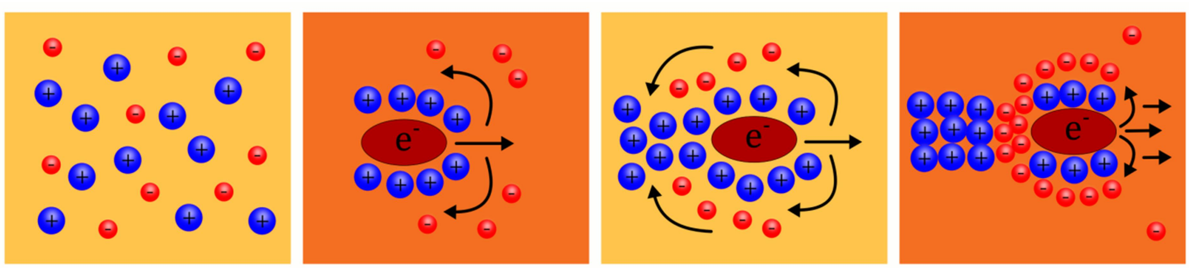

Fig.1.9: Wakefield acceleration.

When pushing aside plasma electrons with the electron bunch, things don't always happen as planned. This resistance restricts the amount of energy that may be used to produce strong waves. Scientists have come up with a really creative way to solve this problem: they shape the electron bunch so that its density increases gradually and ends abruptly. This makes it possible for the incoming bunch to be smoothly accommodated by the plasma electrons at first, and subsequently to feel a stronger push, which results in bigger oscillations and more powerful plasma waves.

1.7.1.1 Limitations of Wake Field Method

Picture a nice ride getting choppy! In plasma, turbulence can break up waves and lose a lot of energy. This occurs because of instabilities that lead the accelerated group and the initial electron bunch (driving bunch) to become "jittery". Due to its higher quantity of lower-energy electrons, the driving bunch is particularly vulnerable to these instabilities. The driving group may produce one of two primary sorts of troublemakers:

1. **The instability of two-streams:**Imagine a group of runners, some of whom move more quickly than others. This erratic tempo may lead to issues! When certain electrons in a group (bunch) move faster than others, this speed differential gets wider as the electrons pass through the plasma. Thus, it becomes more difficult to produce strong and effective plasma waves.

2. **The Weibel instability:** Visualize a group of electrons moving in unison. Instead of flowing smoothly as a whole, they clump together into denser, smaller groupings, resembling tight clusters, in the Weibel instability. This occurs as a result of each electron drawing in surrounding electrons like a little magnet. The bunch as a whole shrinks unevenly when these electrons clump together; instead, it becomes patchy with these concentrated patches. Researchers have found a cunning method to stop these electron clusters from developing. They impart a small, haphazard wiggle to all the electrons in the group, comparable to shrugging shoulders as they advance. Their inclination to clump together is disrupted by this wiggle, which keeps the bunch steady. But this wriggling action, which is equivalent to shaking the electrons excessively, can also spread them out too much.Similar to how guide rails keep electrons on course, scientists can add a guiding magnetic field to stop this from happening.

1.7.2 Beat Wave Method

The beat-wave method uses two strong, different-colored laser beams to create plasma waves instead of a large number of electrons. Think of these beams as distinct-pitch sound waves. Like how sound waves can conflict and cancel each other out in some places, when they join, they generate a new wave pattern with alternating peaks and valleys. The objective is to align the laser beams' frequency difference with the plasma's inherent oscillation frequency. The tremendous vibration of the plasma electrons caused by this "resonance" is akin to pushing a swing at just the right time. Particles could potentially accelerate to very high speeds due to these powerful vibrations, also known as plasma waves.

Comprehending the interaction between plasma waves and laser beams in the beat-wave approach is a challenging task. The wave itself is an extremely complex process, as is the way the lasers pass through the plasma. To address this intricacy, researchers have constructed advanced computer simulations. Millions of charged particles are tracked by these models as they travel through the strong electromagnetic fields produced by the particles and the lasers themselves. Even though every "particle" in the model is actually a large number of actual electrons, these models accurately depict behaviour in the real world, including the consequences of special relativity. This method has worked very well for anticipating the complex interactions that occur in plasma between ions, electrons, and beat waves.

1.8 Applications of Accelerators

The 20th century produced some amazing inventions, such as particle accelerators, which have many uses outside of pure research. They are essential to our health and well-being since they help with food sterilization, radiation therapy for cancer, and CT scan diagnostics. They are also essential for producing radioactive isotopes that are utilized in a variety of medical procedures and examinations. Moreover, X-ray free-electron lasers, which are essential to cutting-edge research in a variety of disciplines like physics, biology, and the life sciences and enable many scientists and engineers to discover new information and breakthroughs, are powered by particle accelerators (fig.1.10).

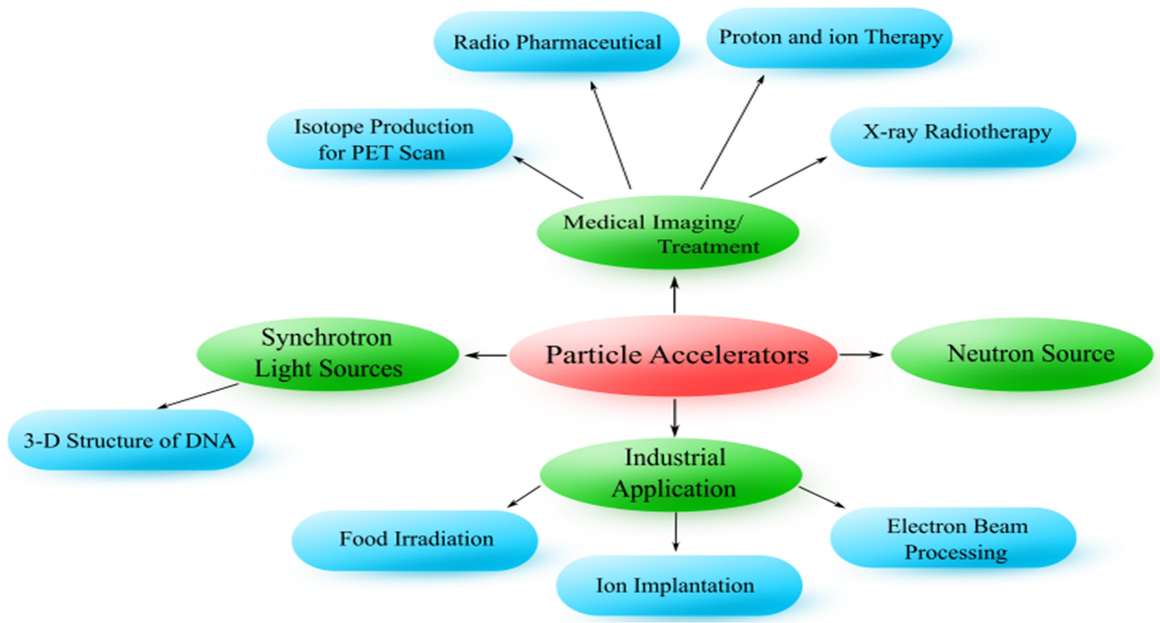

Fig.1.10: Applications of electron accelerators.

1.9 Objectives of the Study

The primary aim of this research is to conduct a theoretical exploration of electron plasma wave excitation caused by intense laser pulses in both uniform and non-uniform plasmas, including those with varying density profiles. Furthermore, the research will explore how the excited plasma waves influence the acceleration of electrons injected from an external source.

- Acceleration of electrons by an electron plasma wave driven by a cosh-Gaussian laser beam in an underdense plasma with an axial density gradient.
- Excitation of electron plasma waves through the interaction of two cross-focused q-Gaussian laser beams in thermal quantum plasmas.
- Electron acceleration induced by a beat wave generated by two cross-focused q-Gaussian laser beams in thermal quantum plasma.
- Optical guidance of a q-Gaussian laser beam within a plasma channel formed by the two pre-pulse technique and its influence on the excitation of electron plasma waves.
- Electron acceleration due to a laser-driven electron plasma wave in a graded index plasma channel created using the two pre-pulse technique.

1.10 Scope of the Study

Particle accelerators are the backbone of high energy physics as well as cancer treatment and diagnose. Currently due to huge infrastructure involved in particle accelerators, cancer treatment is very expensive. Only a very few hospitals and institutes are able to install these huge accelerator facilities. However, the laser driven plasma waves can shrink the size of these accelerators only to a few meters. Thus, by bringing particle accelerators off the shelf laser driven particle acceleration will greatly reduce the cost of cancer treatment. Also, in near future these laser driven particle accelerators may find applications in defense as weapons.

1.11 Research Methodology

We first write the Nonlinear Schrodinger Wave Equation in the form:

$$\hat{F}[\Psi] = 0 \qquad \dots\dots\dots\dots\dots (1.1)$$

We then define a Lagrangian density £ $[\Psi, \Psi^*]$, such that:

$$\frac{\partial £}{\partial \Psi^*} = \hat{F}[\Psi] \qquad \dots\dots\dots\dots\dots (1.2)$$

According to Variational Method, we need to solve the following set of extended Euler-Lagrange equation for the variational parameter $g_i(z)$ with i= 1, 2, 3…... N:

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial q_i}{\partial z}\right)}\right) - \frac{dL}{dz} = 0 \qquad \dots\dots\dots\dots\dots (1.3)$$

Where, L is the average Lagrangian obtained by integration of £ over the transverse coordinates x and y.

$$L = \iint £ \; dxdy \qquad \dots\dots\dots\dots\dots (1.4)$$

For our particular problem, the Lagrangian density corresponding to equation (11) is

$$£ = ik_0\left(\Psi\frac{\partial \Psi^*}{\partial z} - \Psi^*\frac{\partial \Psi}{\partial z}\right) + \left|\frac{\partial \Psi}{\partial x}\right|^2 + \left|\frac{\partial \Psi}{\partial y}\right|^2 - \frac{k_0^2 n_2 |\Psi|^4}{2} + \frac{k_0^2 n_4 |\Psi|^6}{3} \qquad \dots\dots\dots\dots\dots (1.5)$$

And investigate the propagation of the laser beam by using the Variational method with the trial function;

$$\Psi(x, y, x) = \frac{E_{00}}{\sqrt{f_x f_y}}\left\{1 + \frac{1}{q}\left(\frac{x^2}{r_x^2 f_x^2(z)} + \frac{y^2}{r_y^2 f_y^2(z)}\right)\right\}^{-\frac{q}{2}} e^{i\left(x^2 b_x + y^2 b_y + \Phi(z)\right)} \qquad \dots\dots\dots\dots\dots (1.6)$$

Where, $b_x(z), b_y(z)$ are the inverse wave fronts of the beam and $\Phi(z)$ is the longitudinal phase; all being real function of z.

After using the Euler-Lagrangian equation, we get a set of coupled equations for f_x, f_y, a_x, a_y and Φ.

1.12 Review of Literature

T. Tajima and J. M. Dawson (1979)[1]; The idea of laser driven particle acceleration was originally given by Tajima and Dawson. They proposed that an intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. This acceleration mechanism was demonstrated through computer simulations. Applications to accelerators and pulsers were also examined.

J. M. Dawson (1989)[2]; In this paper Dawson explained the physics of laser driven plasma based accelerators. This included wake field method and beat wave method. Relative advantages and disadvantages of both the methods were discussed in detail.

P. Sprangle et al (1996) [3]; This paper provides a comprehensive examination of key topics related to laser acceleration in vacuum, neutral gases, and plasmas. It addresses the limitations of laser vacuum acceleration, focusing on issues such as material damage, electron aperture effects, laser diffraction, and electron slippage. The concept of a self-guided laser beam in a partially ionized atmosphere forms the foundation for an inverse Cherenkov laser acceleration setup. The study explores the nonlinear self-focusing properties of neutral gases and the diffraction effects of ionization, which together enable optical self-guiding. Additionally, the stability of self-guided beams is analyzed and discussed. The paper also includes a review of laser wakefield accelerators and provides a brief overview of laser-driven accelerator experiments.

K. P. Singh (2004) [4]; This paper presents the results from a relativistic three-dimensional single-particle simulation on the direct laser acceleration of electrons in an axial static field. The study reports that when an intense circularly polarized laser pulse interacts with an electron, the electron undergoes rotation along the propagation path of the laser pulse. For two specific magnetic field strengths, the electron and electric field of the laser pulse experience betatron resonance, leading to a significant increase in the electron's energy.

K. P. Singh (2004) [5]; In this study, the author investigated the acceleration of electrons in a vacuum using plane-polarized laser light. The relativistic equations of motion were precisely solved to determine the electron trajectory and energy as a function of laser intensity, laser wave phase θ, and initial electron energy. The results indicated that the peak electron energy increases with both the laser intensity and the initial electron energy.

K. P. Singh (2004) [6]; This research presents a theoretical analysis of electron acceleration by a laser pulse in the presence of a magnetic wiggler. It was found that the inverse free-electron laser resonance condition is satisfied, leading to an increase in the energy gained by the electron at a specific value of the wiggler wave number. The resonance state is influenced by the electron density of the medium and the electron's initial energy. With a properly tapered wiggler period, the resonance can be maintained for a longer duration, allowing the electron to gain significantly more energy.

K. P. Singh (2006) [7]; In this paper, the author utilized a relativistic three-dimensional single-particle code to study the acceleration of electrons generated when nitrogen and oxygen gas atoms are ionized by a laser pulse. The study found that the maximum energy gain of the electrons occurs at the optimal laser spot size. Additionally, for a long-duration laser pulse, electrons produced near the tail region fail to gain sufficient energy.

C. Joshi (2007)[8]; In this paper, Joshi provides a review of the development of laser- and beam-driven plasma accelerators as an experimental field. The study offers a detailed discussion on the excitation of stable plasma accelerating structures by ultra-short laser pulses. It was noted that these accelerating structures can extend up to the centimeter scale when excited by laser pulses and up to the meter scale when excited by ultra-relativistic particle beams.

E. Esarey et al (2009)[9]; In this paper the physics of laser-driven plasma-based accelerators was thoroughly reviewed. These include plasma waves powered by multiple laser pulses, the self-modulated laser wakefield accelerator, the laser wakefield accelerator, the plasma beat wave accelerator, and extremely nonlinear regimes. Electron acceleration in plasma waves and the characteristics of linear and nonlinear plasma waves are addressed. Additionally covered are techniques for injecting and trapping plasma electrons in plasma waves.

R. Zhang et al (2015) [10]; This study focuses on the investigation of laser-driven electron acceleration in a transversely inhomogeneous plasma channel. According to various reports, adjusting the laser polarization angle and the inhomogeneity of the plasma channel can help control the development of instability, thereby influencing electron acceleration and energy gain in such a channel. In other words, by modifying the laser polarization angle and the plasma channel's inhomogeneity, it is possible to reduce the accelerating length and enhance the energy gain within an inhomogeneous plasma channel.

A. Singh and N. Gupta [11]; In this paper, the authors conducted a theoretical investigation on the excitation of electron plasma waves through the interaction of two cosh-Gaussian laser beams in an underdense plasma. The study thoroughly examines the effect of cross-focusing of the two laser beams, influenced by the relativistic nonlinearity of plasma electrons, on the power of the generated plasma wave.

A. Singh and N. Gupta [12]; In this paper, the authors present a theoretical study on the excitation of electron plasma waves through the interaction of two q-Gaussian laser beams in an underdense plasma. The investigation focuses on the effect of cross-focusing of the two laser beams, driven by the ponderomotive nonlinearity of plasma electrons, on the power of the generated plasma wave.

N. Gupta (2019) [13]; In this paper, the author examined the optical guiding of q-Gaussian laser beams within a plasma channel created by an ignitor heater beam. The use of two laser beams was aimed at extending the plasma channel. The study found that laser beams with broadened wings in their transverse intensity distribution experience enhanced optical guiding within plasma channels.

Chapter 2

Electron Acceleration by Laser-Driven Plasma Waves in a Plasma with a Density Gradient: Cosh Gaussian Laser Beam

Abstract

The chapter presents a nonlinear propagation of a Cosh-Gaussian (ChG) laser beam in plasma with axial inhomogeneity that is investigated theoretically. The relativistic mass nonlinearity of plasma electrons in reaction to the laser beam is taken into account in the study. The study also investigates how self-focusing affects electron plasma wave (EPW) excitation. To get semi-analytical solutions for the nonlinear wave equations controlling the laser beam and EPWs, the analysis makes use of variational theory. Additionally, the acceleration of electrons trapped inside the excited EPWs is investigated.

2.1 Introduction

The search for the fundamental nature of matter has, for the past fifty years or so, mostly depended upon one experimental method. Using this method, a particle is accelerated to extremely high speeds and then collides with another particle. Researchers can learn a great deal about the properties of the particles and the forces governing their interactions by dissecting the debris that is created after a collision. Such investigations require a consistent source of energetic particles. Although cosmic rays are a naturally occurring source, the particle flux they produce is diffuse and outside the control of experimenters. As a result, using a particle accelerator—a gadget meant to increase particle speed and energy—is a more workable approach.

In 1928, Ernest O. Lawrence used small-diameter laboratory glassware to build one of the first particle accelerators. Most functioning accelerators today are direct descendants of Lawrence's device, but with much larger and more complicated designs. The biggest accelerators nowadays cover enormous areas—many square kilometers—which means that land availability plays a big role in their design. The particle accelerator is no longer a stand-alone device restricted to a lab; instead, the accelerator is carefully built into the laboratory. Hundreds of millions of dollars are spent on building such a machine, and using many digital computers and a workforce of about a thousand people is required for it to function.

Larger accelerators are at the forefront of high-energy physics research, but smaller devices are also employed in fusion research, materials science, structural biology, nuclear medicine, food sterilization, transmutation of radioactive waste, and cancer treatment[4,5]. Even though these little devices produce electron or proton beams with lesser energies—typically between 100 million and one billion volts—they still need large lab rooms. There are only a few broad categories into which accelerators fall:

1. To begin with, they might be utilized to accelerate heavier particles like protons and antiprotons or lighter particles like electrons and positrons.
2. Secondly, these accelerators have the ability to accelerate particles in several orbits around a circular ring or in a single pass along a straight route.

Take the Large Hadron Collider (LHC) as an illustration. It is essentially a circular ring where two protons collide. The construction of a linear collider for positrons and electrons, which is meant to replace the LHC, is underway. The collision point in the planned collider will start at about half a TeV (trillion electron volts) of energy. Using a circular ring would result in excessive energy loss from synchrotron radiation, therefore at such high energies, electrons and positrons must be accelerated in a straight line. Strong electromagnetic radiation is produced when electrons are forced to move at high speeds along a curved path. The problem is that this radiation can result in a large energy loss that quickly gets worse as the energy of the machine gets higher. For the linear acceleration of electrons and positrons, plasma-based accelerators present a far more effective substitute than traditional RF acceleration.

Conventional accelerators use radio-frequency (RF) cavities to drive particle beams to greater energies. In order to accelerate the particles, this method involves flipping the electrical polarity of particular zones inside the RF cavity. These positively and negatively charged locations attract and repel the particles. However, the maximum strength of the accelerating field is limited when using a metallic framework. Sparks and current discharge from the cavity walls occur when the field exceeds the electrical breakdown threshold, which is reached at a level of 20 million to 50 million volts per meter. A longer acceleration path is needed to reach a given energy since the electric field has to be maintained below the breakdown threshold. The accelerator could be made more compact if it were able to accelerate particles at a rate that is noticeably quicker than that permitted by the electrical breakdown limit. Here's where plasma becomes useful.

The ionized gas known as plasma acts as the accelerating structure in a plasma accelerator . Interestingly, since the gas is first broken down, electrical failure is purposefully included into the design, rather than being a burden. A laser beam or a charged particle beam is the power source for a plasma accelerator, as opposed to microwave radiation. Laser beams might not seem like a good idea for particle acceleration at first. Even though they have powerful electric fields, the direction of propagation is not the primary orientation of these fields. But in order for acceleration to be successful, the accelerator's electric field—also referred to as a longitudinal field—must line up with the particle's direction of travel. Luckily, this need may be satisfied when a laser or charged particle beam interacts with a plasma to produce a longitudinal electric field.

The process operates as follows: a plasma is electrically neutral when it contains equal amounts of positive ions and negative electrons. However, when a powerful laser injects an optical beam into the plasma, a disruption occurs. The beam pushes the heavier positive ions away from the lighter electrons, leaving the ions behind. This creates regions with an excess of positive charge and negative charge. The resulting disruption travels through the plasma as a wave, propagating at nearly the speed of light. A strong electric field forms between the positive and negative regions of the wave, capable of accelerating any charged particles in its path.

Electrons moving in the same direction and at similar speeds as the excited plasma wave become trapped within it. To these electrons, the electric field of the plasma wave appears as a constant, unchanging field. Due to the intense electric field of the plasma wave, these trapped electrons accelerate and reach extremely high energies. The process is as follows:

Envision yourself in a boat that traverses a lake. In our scenario, the boat represents the "drive beam," or laser beam, and the lake is the plasma. These wake fields are formed when waves are created by the propulsion beam entering the lake. A "trailing beam," which is positioned behind the drive beam, is comparable to a wake surfer following a wake in this metaphor. You now place electrons on top of these wakes, which accelerates them. Wake surfers go faster because gravity pulls them along as they essentially ride down a wet hill. Particles, such as electrons, accelerate due to the force of an electric field.

The acceleration gradient is a measurement of how well an accelerator can accelerate a particle beam over a specific distance; it is commonly represented in volts per meter (V/m). It has been shown that acceleration gradients obtained using plasma wake fields can be up to 1000 times higher than those obtained with radiofrequency cavities. However, the infeasibility of present methods, which necessitate the configuration of several plasma sources to obtain high energies, has limited the employment of plasma wake fields in high-energy and particle physics studies.

Tajima and Dawson first the idea of laser-driven wake field acceleration in 1979.They suggested that the nonlinear ponderomotive force acts to produce weak plasma oscillations in response to a massive electromagnetic pulse. A computer simulation was used to illustrate the acceleration mechanism. Since then, numerous studies on wake field acceleration caused by lasers have been published by various researchers. Electron acceleration was studied experimentally by Everette et al. by trapping electrons in relativistic EPWs. It was said that electrons introduced into the plasma at a starting energy of 2 MeV were accelerated to 28 MeV at a rate of 2.8 GeV m^{-1}.Laser-induced electron acceleration within a non-uniform plasma channel was studied by Zhang et al. .

In plasmas, lasers with different irradiance profiles display varying behaviors. However, a review of existing literature indicates that most studies on laser-driven electron acceleration in either vacuum or plasmas have primarily focused on plane waves or Gaussian laser beams. There are also a few investigations into electron acceleration using q-Gaussian beams, which are similar to Gaussian profiles, in addition to the ideal Gaussian beams. Flat-top beams, or top-hat beams, offer a uniform irradiance across their entire cross-section, ensuring consistent intensity throughout the laser system's target. In contrast, Gaussian beams have a cross-sectional irradiance that symmetrically decreases as the distance from the beam's center increases. This characteristic makes flat-top beams more advantageous in applications like semiconductor wafer processing, material interaction, and nonlinear frequency conversion with high-power beams, as they yield more precise and consistent results. Flat-top beams can also achieve cleaner cuts and sharper edges than Gaussian beams in processing systems. One significant drawback of Gaussian laser profiles is the low-intensity "wings" on either side of the beam's core. These wings, which do not meet the necessary intensity for specific applications—such as laser surgery, materials processing, or laser-driven fusion—often waste energy. Additionally, the wings can cause damage to areas outside the targeted region, increasing the affected heat zone. This is particularly problematic in applications requiring minimal damage to surrounding areas and high precision, such as laser surgery or precise materials processing. Consequently, features produced by a Gaussian beam often lack smooth edges, reducing the accuracy of the system.

Flat-top beam profiles, which lack wings and exhibit sharper edge transitions, offer more efficient energy delivery and smaller heat-affected zones compared to Gaussian beams. When used for tasks such as etching, welding, or machining, flat-top beams produce more precise features and cause less damage to surrounding areas. These beams also reduce statistical variability and measurement uncertainty due to their uniform and well-defined irradiance profile. Flat-top beams are particularly beneficial in applications like holography, interferometry, and fluorescence microscopy. Such beams are often modeled using a super-Gaussian profile. However, the hyper-Gaussian irradiance profile also presents a promising approach. In experimental settings, multiple decentered Gaussian laser beams can be combined to produce beams with a consistent irradiance across their cross-section. One such profile is the Cosh-Gaussian (ChG) irradiance profile [22, 23]. To the author's knowledge, no previous study has applied variational theory to the examination of electron acceleration via ChG laser beams in laser-driven wakefield acceleration within relativistic plasmas with an axial density gradient. Thus, the aim of this work is to provide the first theoretical analysis of electron acceleration by ChG laser beams using variational theory.

2.2 Evolution of the Laser Beam

Examine how a laser beam with circular polarization propagates when an electric field vector is applied.

$$E(r,t) = E_0(r,z)e^{-\iota(k_0 z-\omega_0 t)}(e_x + \iota e_y)$$

through a plasma that is axially inhomogeneous and whose equilibrium electron density is represented by

$$n_0(z) = n_0^0(1 + tan(dz)) \tag{2.1}$$

The density profile described by equation (2.1) is commonly referred to as the density ramp. In this context, the plasma density at z=0 is represented by n_0^0, and the parameter d characterizes the rate at which the plasma density increases with longitudinal distance. This parameter d is known as the slope of the density ramp. We have considered this ramp-shaped density profile because it enhances the laser beam's self-focusing ability, which in turn amplifies the energy gained by the accelerated electrons.

The dielectric function of the plasma, where the electron density is given by Eq. (2.1), can be written as

$$\epsilon = 1 - \frac{4\pi e^2 n_0^0}{m_e}(1 + tan(dz)) \tag{2.2}$$

Where m_e represents the effective mass of the plasma electrons when the laser beam is present. Due to the circular polarization of the laser beam, the plasma electrons move in circular orbits with a frequency ω_0. The quiver speed of the electrons approaches the speed of light in a vacuum because of the strong magnetic field induced by the intense laser beam. Consequently, the modified mass m_e of the electrons in Eq. (2.2) must be replaced by $m_0\gamma m$, where γ is the relativistic Lorentz factor and m_0 is the electron's rest mass. According to Polovin and Akhiezer, at equilibrium:

$$-eE_0 = \frac{m_0 v \omega_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} \tag{2.3}$$

where c represents the speed of light in a vacuum, and v denotes the quiver velocity of the electron in the laser beam's field. This relation leads to

$$\gamma = (1 + \beta E_0 E_0^\star)^{\frac{1}{2}}$$

where the coefficient of relativistic nonlinearity is denoted by $\beta = \frac{e^2}{m_0^2 c^2 \omega_0^2}$. Therefore, the plasma channel's effective dielectric function can be expressed as

$$\epsilon = 1 - \frac{\omega_{p0}^2}{\omega_0^2 \gamma}(1 + tan(dz)) \tag{2.4}$$

The equilibrium plasma frequency in this case is $\omega_{p0} = \sqrt{\frac{4\pi e^2}{m_0} n_0^0}$. The relativistic mass nonlinearity in plasma electrons results in a nonlinear optical plasma response to the incoming laser beam since dielectric function of the plasma depends on laser intensity due to the intensity-dependent nature of the relativistic Lorentz factor γ. Writing eq. (2.4) as the total of its nonlinear and linear components.

$$\epsilon(E_0\, E_0^\star) = \epsilon_0 + \phi(E_0\, E_0^\star)$$

we get

$$\epsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega_0^2}$$

$$\phi = \frac{\omega_{p0}^2}{\omega_0^2}\left\{1 - \left(1 + tan\ (dz)\right)(1 + \beta E_0 E_0^{\star})^{-\frac{1}{2}}\right\} \quad (2.5)$$

The propagation of a laser beam in a nonlinear medium, which is defined by a nonlinear dielectric function $\phi(E_0\ E_0^{\star})$, is governed by the wave equation.

$$2\iota k_0 \frac{\partial E_0}{\partial z} = \nabla_{\perp}^2 E_0 + \frac{\omega_0^2}{c^2}\phi(E_0\ E_0^{\star})E_0 \quad (2.6)$$

Since equation (2.6) is nonlinear, there is not a closed-form solution. To get understanding of the laser beam's propagation dynamics, we have employed a partial analytic method called the variation method in the current work. Equation (2.6) is a method of variation issue for the action principle grounded in Lagrangian density, according to this technique.

$$\mathcal{L} = \iota\left(E_0 \frac{\partial E_0^{\star}}{\partial z} - E_0^{\star}\frac{\partial E_0}{\partial z}\right) + |\nabla_{\perp} E_0|^2 - \frac{\omega_0^2}{c^2}\int \phi(E_0 E_0^{\star}) d(E_0 E_0^{\star}) \quad (2.7)$$

The cosh Gaussian function is used in the current analysis to represent the irradiance profile$E_0\ E_0^{\star}$ of the laser beam as

$$E_0 E_0^{\star}(r,\ z) = \frac{E_{00}^2}{f^2} e^{-\frac{r^2}{r_0^2 f^2}} \cosh^2\left(\frac{b}{r_0 f} r\right) \quad (2.8)$$

wherein the laser beam's axial amplitude is represented byE_{00}, while its spot size at the plane of incidence, or z=0, is represented by r_0. The parameter b, associated with the cosh function, is referred to as the cosh-factor. It serves as a key governing parameter that influences the behavior of the pump beam and, subsequently, the energy gained by the accelerated electrons, making it a critical factor of interest. Currently unknown, the function $f(z)$is known as a dimensionless beam width parameter. It has two significances: (1) The beam's instantaneous spot size at a certain place within the medium is obtained through multiplying it by the steady-state beam width,$r_{0.}$(2). Dividing it by the amplitude provides a measure of the instantaneous intensity of the laser beam.

The simplified Lagrangian expression is derived by inserting the given trial function from equation (2.8) into the Lagrangian density and integrating over the entire cross-sectional area of the laser beam:

$$L = \int \mathcal{L}(E_0, E_0^{\star}, \phi) d^2 r$$

The corresponding Euler Lagrange equation

$$\frac{d}{dz}\left(\frac{\partial L}{\partial\left(\frac{\partial f}{\partial z}\right)}\right)-\frac{\partial L}{\partial f}=0 \tag{2.9}$$

provides the differential equation for the evolution of the laser beam's width as

$$\frac{d^2 f}{d\xi^2}=\left(\frac{1+e^{-b^2}(1-b^2)}{2(1+b^2)}\right)\frac{1}{f^3} - \left(\frac{e^{-b^2}}{1+b^2}\right)\left(\frac{\omega_{p0}^2 c^2}{c^2}\right)\frac{\beta E_{00}^2}{f^3}\left(1+tan(d'\xi)\right)(T_1-bT_2) \tag{2.10}$$

where,

$$T_1=\int_0^\infty x^3 e^{-2x^2}\cosh^4(bx)\left\{1+\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)\right\}^{-\frac{3}{2}}dx$$

$$T_2=\int_0^\infty x^2 e^{-2x^2}\cosh^3(bx)\sinh(bx)\left\{1+\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)\right\}^{-\frac{3}{2}}dx$$

$$x=\frac{r}{r_0 f}$$

$$\xi=\frac{z}{k_0 r_0^2}$$

$$d'=dk_0 r_0^2$$

The change in the beam width of the ChG laser beam throughout its distance of propagation in the plasma is determined by Equation (2.10). Initial conditions apply to eq.(2.10) for a laser beam that is initially collimated (i.e., a laser beam with a flat wavefront).

$$f(0)=1$$

and

$$\frac{df}{d\xi}|_{\xi=0}=0$$

The given set of laser and plasma parameters for the current experiment was used to numerically solve Equation (2.10).

$$\omega_0=1.78\times10^{15}\mathrm{rad/sec};\ \ \mathrm{r_0}=20\mu\mathrm{m};\beta\mathrm{E}_{00}^2=3;\ \left(\frac{\omega_{\mathrm{p0}}\mathrm{r_0}}{c}\right)^2=12;\mathrm{T_0}=10^5\mathrm{K}$$

and for various cosh-factor values and the density ramp parameters slope, that is,

$$b=(0,0.5,1,1.1,1.2,1.3)\text{and}d'=(0.025,0.030,0.035)$$

and the related changes in the laser beam's beam width are displayed in figures 1, 2, and 3, respectively. A laser beam exhibits harmonic fluctuations in spot size across the propagation distance when it is transmitted through plasma. The physical meanings of the various components in equation 10 can be used to understand this behavior. The first term on the right-hand side of the equation represents spatial dispersion, which decreases as the cube of the beam width (represented asf^{-3}). It explains how diffraction spread causes the laser pulse to spread out in transverse directions.

The secondary component on the right-hand side of the equation arises from the relativistic mass nonlinearity of plasma electrons under the influence of the high-intensity laser field, which has a complex effect on the beam width f. This word refers to the laser beam's nonlinear refraction. The influence of diffraction in the transverse directions is generally offset by the nonlinear refraction because of the laser-induced nonlinearity in the plasma. As the laser pulse travels through the plasma medium, nonlinear optical refraction and diffraction interact with each other. The prevailing phenomenon ultimately dictates the behavior of the laser beam, determining whether it will converge or diverge.

Upon balancing the two terms on the right-hand side of Equation 2.10, the laser beam reaches a critical intensity, beyond which it begins to converge along the transverse axes. This study addresses the initial convergence of the laser beam's width by selecting an initial intensity that exceeds the critical threshold. As the beam's cross-section reduces, its intensity increases. However, when the intensity becomes excessively high and the plasma electron mass reaches saturation, the nonlinear effects diminish. Consequently, the beam propagates through the plasma as if it were in a vacuum. Once the beam width reaches its minimum value, it begins to expand again as it continues through the plasma, similar to how an optical beam diffracts in a vacuum.

The conflict between nonlinear refraction and diffraction broadening recommences as the laser beam's width increases. This rivalry continues until the beam width is as wide as it can get. The width of the laser beam exhibits oscillating behavior due to these repeated actions.

Additionally, it has been noted that the beam width's minimum and maximum shift downward after each focal location. This can be explained by the fact that the equilibrium electron density increases as longitudinal distance increases. As a result, the laser beam propagates into the plasma, leading to a continued decrease in the plasma's refractive index. As a result, following each focal point, the beam width's peak and trough continue to migrate down, enhancing the self-focusing phenomenon. Additionally, it is seen that the beam's oscillation frequency rises with distance. The physics underlying this finding states that the laser beam's phase velocity through a denser plasma will be higher.

Figures 2.1 and 2.2 illustrate the effect of the cosh factor b on the degree of self-focusing of the laser beam. Figure 2.1 illustrates how the amount of self-focusing grows for $0 \leq b \leq 1$,whereas Figure 2.2 shows how the situation reverses for$b > 1$ (i.e., for $b > 1$ the degree of laser beam self-focusing decreases with an increase in b value). This occurs because, as the value of b increases within the range 0 ≤ b ≤ 1, the intensity uniformity across the laser beam's cross-sectional area improves. As a result, rays deviating from the axis contribute equally to the nonlinear optical effects in the laser beam. Since the laser beam's nonlinear refraction maintains homeostasis for its self-focusing, the extent of the laser beam's self-focusing grows as b increases for $0 \leq b \leq 1$. However, in the case of $b > 1$, the ChG laser beam receives no input from the axial section from the wavefronts to nonlinear refraction due to the presence of a central dark region. Since the extent of the central dark zone grows as the value of b increases for $b > 1$, the degree of self-focusing begins to decrease as the cosh factor value increases for $b > 1$.

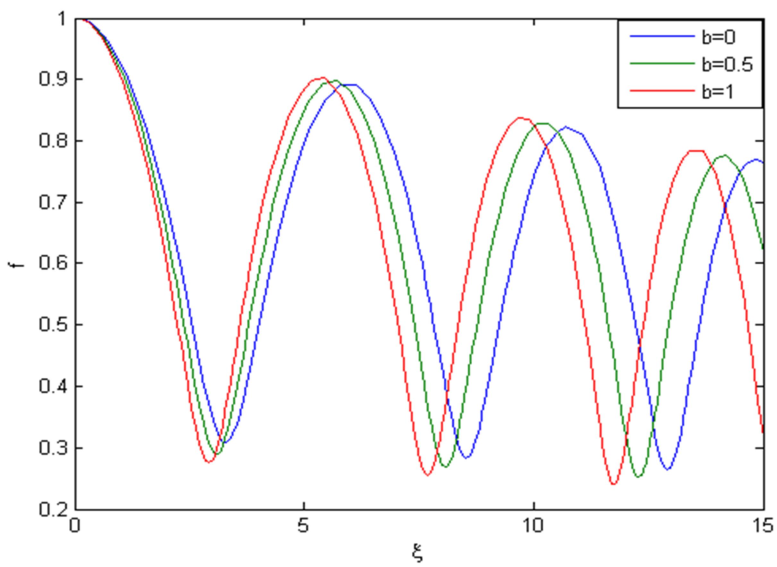

Fig.2.1: Evolution of beam with parameter f for various values of the off-center parameter$b$ for $0 \leq b \leq 1$ at a constant density ramp slope i.e., $d' = 0.030$.

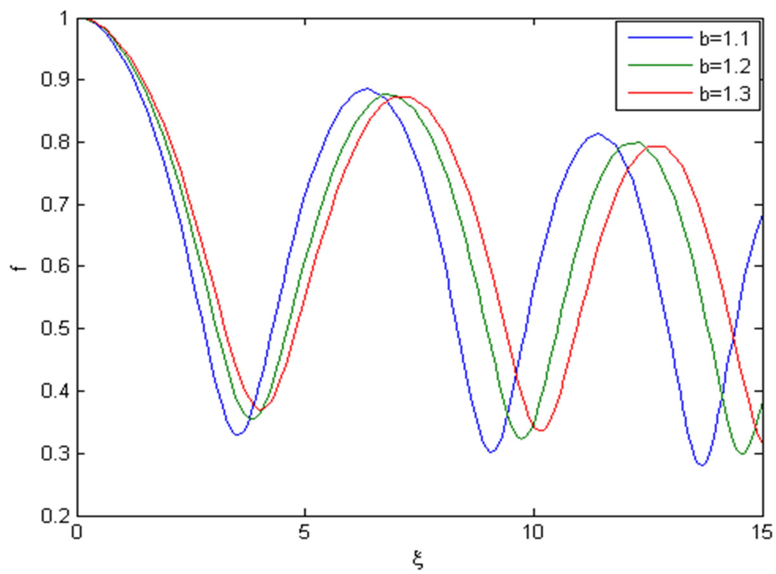

Fig.2.2: Evolution of beam with parameter f for various values of the off-center parameter b where $b > 1$, while keeping the density ramp slope constant i.e., $d' = 0.030$.

Figure 2.3 illustrates how the slope of the density ramp influences the self-focusing ability of the laser beam. It is evident that an increase in the density ramp's slope enhances the self-focusing of the laser beam. As the slope of the density ramp increases, the laser beam encounters a lower refractive index in the deeper regions of the plasma, leading to this effect. As a result, increasing the density ramp's slope increases the laser beam's degree of focus.

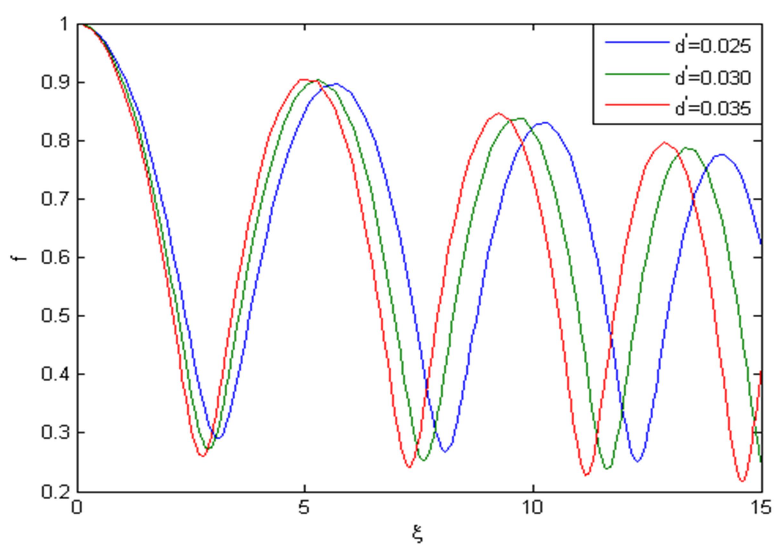

Fig. 2.3: Evolution of beam with parameter f for different values of slope of density ramp at fixed value of decentered parameter i.e., $b = 0.50$.

2.3 Generation of Electron Plasma Waves

The propagation of the stimulated electron plasma wave is governed by the wave equation:

$$2\iota k_{ep}\frac{\partial n_{ep}}{\partial z} = \nabla_{\perp}^{2} n_{ep} + \frac{\omega_{ep}^{2}}{v_{th}^{2}}\left\{1-\left(1+tan(dz)\right)\left(1+\beta E_0 E_0^{\star}\right)^{-\frac{1}{2}}\right\} n_{ep} \tag{2.11}$$

where the thermal velocity of plasma electrons is expressed as$v_{th} = \sqrt{\frac{2KT_0}{m}}$. Thinking of the EPW's Gaussian ansatz as

$$n_{ep} = \frac{n_{00}}{f_{ep}} e^{-\frac{r}{2r_0^2 f_{ep}^2}} \tag{2.12}$$

By following the steps outlined in Section 2, we derive the equations that describe the evolution of the EPW's beam width as follows:

$$\frac{d^2 f_{ep}}{d\xi^2} = \frac{1}{f_{ep}^3} - \left(\frac{\omega_{ep}^2 r_0^2}{v_{th}^2}\right)\frac{n_{00}}{n_0^0}\frac{1}{f_{ep}^3}(1+\tan(d'\xi))(T_3 - bT_4) \tag{2.13}$$

$$T_3 = \int_0^\infty x^3 e^{-x^2} e^{-\frac{f^2}{f_{ep}^2}x}\left\{1+\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)\right\}^{-\frac{3}{2}}\cosh^4(bx)dx$$

$$T_4 = \int_0^\infty x^2 e^{-x^2} e^{-\frac{f^2}{f_{ep}^2}x}\left\{1+\frac{\beta E_{00}^2}{f^2}e^{-x^2}\cosh^2(bx)\right\}^{-\frac{3}{2}}\cosh^3(bx)\sinh(bx)\,dx$$

Poisson's equation may be used to determine the electric field linked to the excited electron plasma wave. Equation (2.13), which illustrates the interaction between the EPW and the pump beam, or ChG laser beam. It illustrates that the density disturbance associated with the electron plasma wave exhibits significant sensitivity to the laser beam's self-focusing, expressed as $E_{ep} = E_{ep} e^{\iota(k_{ep}z - \omega_{ep}t)}\hat{z}$

$$E_{ep} = \frac{\iota m\omega_{ep}^2}{ck_{ep}f_{ep}} e^{-\frac{r}{2r_0^2 f_{ep}^2}} \tag{2.14}$$

Equation (2.14) quantifies the field strength associated with the excited electron plasma wave. The average field strength was calculated by integrating over the cross-sectional area of the laser beam. This calculation utilized Equations (2.10) and (13) under the condition $\frac{n_{00}}{n_0^0} = 0.0001$. Figures (2.4, 2.5, and 2.6) depict the variations in the field strength of the EPW as a function of the propagation distance.

As the excited Electron Plasma Wave (EPW) travels, its electric field strength fluctuates and reaches its peak at the laser beam's focal points. The self-focusing of the laser beam, which dramatically changes the amplitude of the EPW, explains this behavior. When the pump beam self-focuses, its strength increases, increasing the amplitude of the plasma's electron oscillations. As a result, the amplitude of EPW likewise rises. Moreover, the intensity of the excited EPW exhibits an oscillating pattern as the pump beam's widths vary in both transverse directions, with the highest field happening at the point of the pump beam's shortest width.

Figures (2.4) and (2.5) illustrate that the power of the excited electron plasma wave increases with a rising cosh factor within the range $0 \leq b \leq 1$. However, for b > 1, the power of the excited EPW decreases as b increases. This behavior arises from the direct correlation between the power of the stimulated EPW and the degree of self-focusing exhibited by the laser beam.

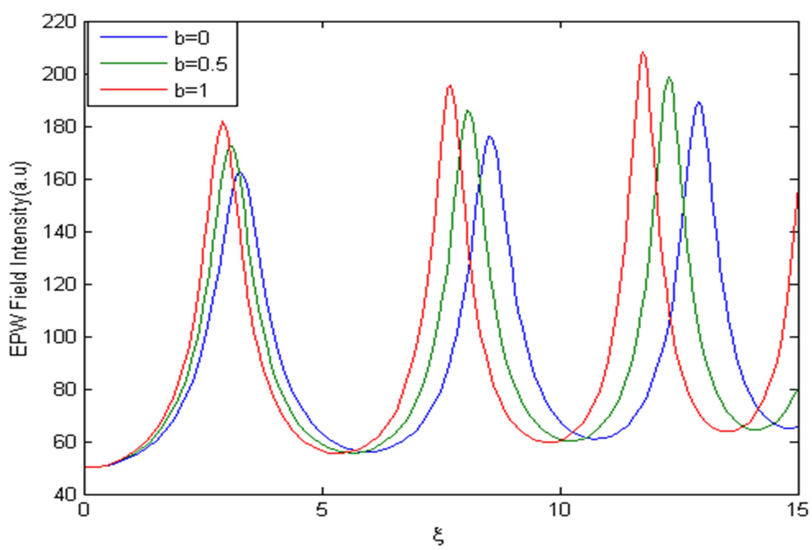

Fig.2.4: Evolution of intensity of EPW for various values of the off-center parameter$b$ for $0 \leq b \leq 1$ at constant value of the density ramp slope i.e., $d' = 0.030$.

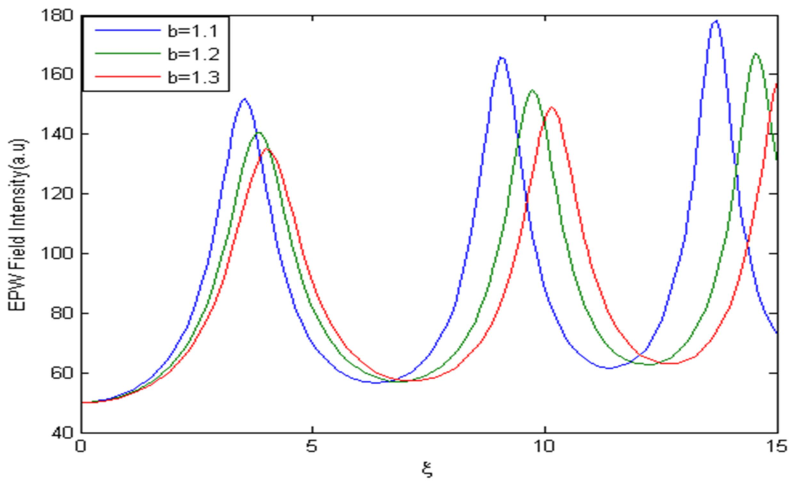

Fig.2.5: Evolution of intensity of EPW for various values of the off-center parameter b for $b > 1$ at constant value of the density ramp slope i.e., $d' = 0.030$.

Figure 2.6 demonstrates the influence of the plasma density ramp's slope on the stimulated EPW power. As the slope of the density ramp increases, the stimulated EPW power rises significantly. This occurs because a steeper density ramp enhances the focusing of the laser beam.

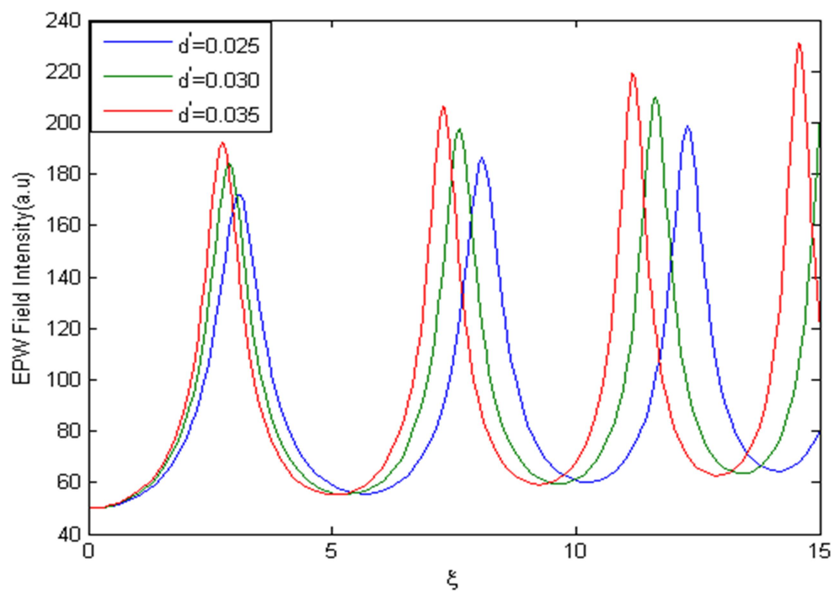

Fig.2.6: Evolution of intensity of EPW for various values of density slope ramp at fixed value of decentered parameter i.e., $b = 0.50$.

2.4 Energy Gained by Electrons

Particle acceleration in accelerator physics does not mean increasing the particle's speed with time. Conversely, energetic electrons created by contemporary particle accelerators travel in a vacuum at nearly the speed of light. An electron beam with a mass of 50 GeV, for example, would finish a laser pulse race around the Earth only a few millimeters behind the light pulse. Particles moving at such high speeds suffer acceleration—a phenomenon that causes their mass to rise in accordance with special relativity—when they absorb energy from electromagnetic fields. Still, their velocity has not changed. An laser-driven accelerator must generate a plasma wave that travels at almost the speed of light in a vacuum in order to accelerate a beam of charged particles that are already moving at extremely high speeds. This guarantees that the charged particles stay within the wave's electric field.

Equation governs the time evolution of the energy obtained by the accelerated electron.

$$\frac{d\gamma_e}{dt} = \frac{e}{mc^2} E_{ep} v_z \tag{2.15}$$

Where the energy is given by $\gamma_e = \sqrt{1 + \frac{p_e^2}{m^2c^2}}$ andv_z represents the velocity of the plasma wave driven by the laser. By using the transformation $\frac{1}{v_z}\frac{d}{dt} = \frac{d}{dz}$, theelectron's obtained energy's magnitude,γ_e may be found using

$$\frac{d\gamma_e}{dz} = \frac{e}{mc^2} E_{ep} \tag{2.16}$$

The energy obtained by the electrons trapped in the excited EPW is displayed after we solved Eq. (2.16) by averaging throughout the laser beam's cross-sectional area in respect to Eqs. (2.10) and (2.14). The resulting changes in the energy produced by the accelerated electrons in respect to the propagation distance for different laser-plasma characteristics are depicted in Figs. 2.7, 2.8, and 2.9.

These graphs clearly show two behaviors in the energy of accelerated electrons:

1. The energy of an electron grows monotonically with propagation distance.
2. The electron energy behaves in a step-like manner, with each step taking place at the focal point of the laser beam. Notably, the electron energy surges rapidly at the point where the laser beam reaches its minimal width.

The electrostatic field generated by the EPW, triggered by the incident laser beam, accelerates electrons. As illustrated in Figures 2.7, 2.8, and 2.9, the field strength of the stimulated EPW initially increases along the propagation distance, reaching a local maximum near the laser beam's focal point. The accelerated electron's energy increases with the propagation length as the strength of the accelerating field intensifies. At the position of the pump beam's minimum beam width, where the accelerating force is strongest, the electron experiences a sharp energy jump. Following this, the electron's energy rises linearly again until the next focal point. Thus, the energy of the accelerated electron increases progressively with distance, punctuated by abrupt steps at the pump's focal points.

As shown in (Fig. 2.7), both the maximum energy achieved by the electron and the rate of energy increase rise as the laser beam's decentered parameter$b$ (where for $0 \leq b \leq 1$) increases. The reason for this is that the electron's energy is susceptible to the laser beam's self-focusing when the excited electric plasma wave (EPW) interacts with the pump beam. The electron accelerates in proportion to the rise in the laser's self-focusing as the decentered parameter$b$where$0 \leq b \leq 1$. In a similar vein, when $b > 1$, the electron′s acceleration decreases as b increases in magnitude (Fig.2.8) owing to a decrease in the laser beam's degree of self-focusing.

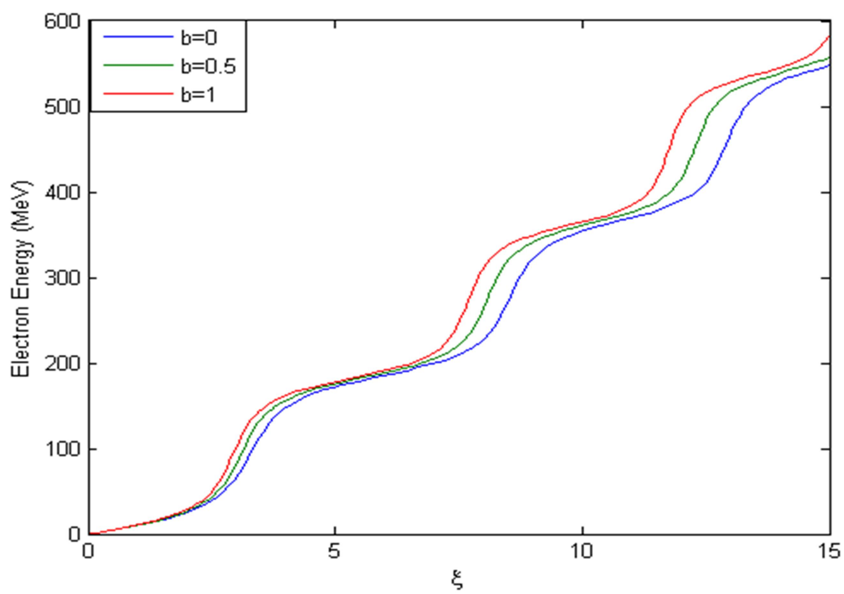

Fig.2.7: Evolution of energy gained by electron for various values of off-center parameter b for $0 \leq b \leq 1$ at constant density ramp slope i.e., $d' = 0.030$.

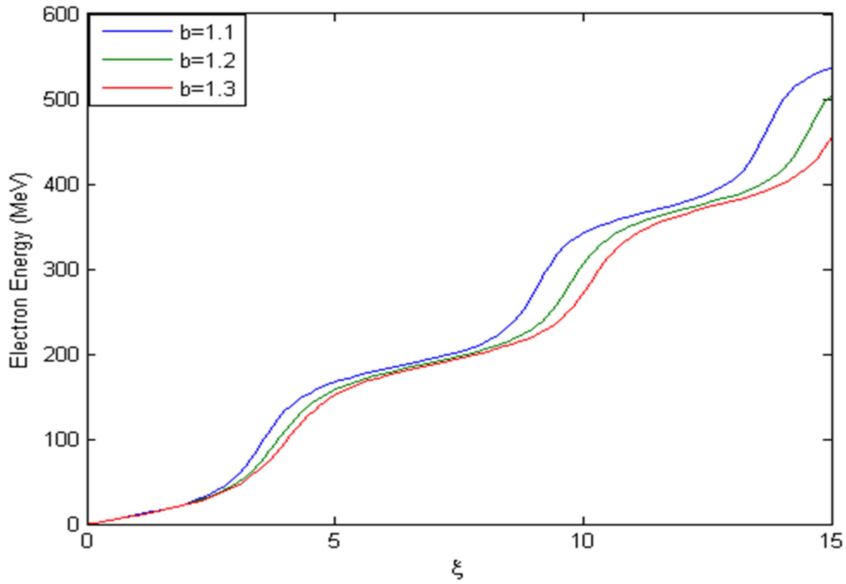

Fig.2.8: Evolution of energy gained by electron for various values of off-center parameter b for $b > 1$ at constant density ramp slope i.e., $d' = 0.030$.

Fig. 2.9 illustrates how the slope of the density ramp influences electron acceleration. As the slope of the density ramp increases, the electrons accelerate more rapidly. This occurs because a steeper density gradient facilitates the laser's self-focusing.

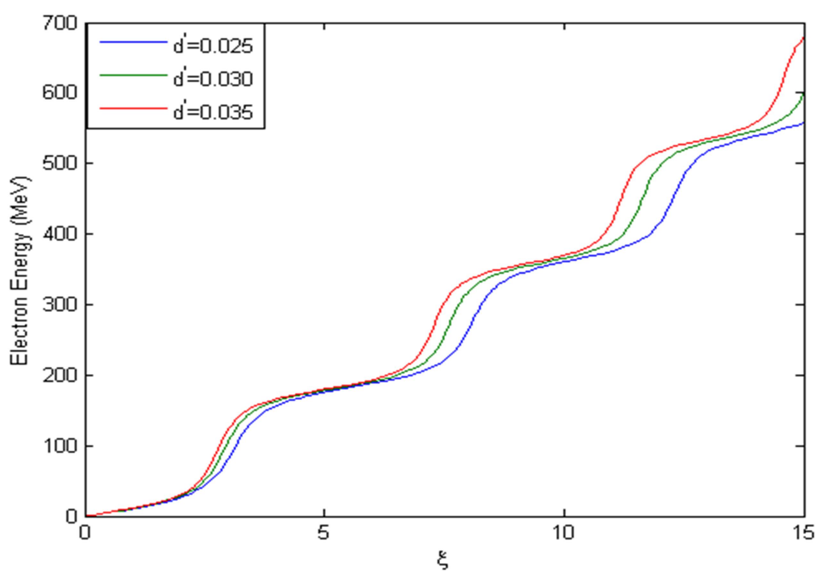

Fig.2.9: Evolution of energy gained by electron for various values of slope of density ramp at fixed value of decentered parameter i.e., $b = 0.50$.

2.5 Results and Conclusion

When the ideal values of the decentered parameter are used, the numerical analysis results show a more noticeable self-focusing impact of the Cosh-Gaussian laser beam. This discovery is useful for producing high-amplitude EPWs because it increases the contact length, which permits electron acceleration.

Chapter 3

Inducing Electron Plasma Waves through Optically Guided q-Gaussian Laser Beam in Plasma Channel Created by Ignitor Heater Technique and its Effect on Electron Acceleration

Abstract

In this chapter theoretical research has been done on the self-action effects (self focussing) and self phase modulation of a q-Gaussian laser beam in a plasma channel produced by the ignitor heater approach. The relativistic mass nonlinearity of the plasma electrons and the density non-uniformity of the parabolic channel work together to direct the third beam (q-Gaussian beam) in the plasma channel. The formulation is based on using the moment theory approach to get the numerical solution of the nonlinear Schrodinger wave equation (NSWE) for the fields of incident laser beams. Dynamical changes in the laser beams' spot sizes and the guided beam's longitudinal phase-shift with propagation distance are given special attention. The irradiance across the laser beams' cross-section has been represented using a q-Gaussian beam profile in order to investigate the effects of the guided laser beams' irradiance profile on the guided beam's propagation dynamics and the strength of the excited EPW. An EPW at the pump frequency is stimulated by the laser beam as it passes through the plasma channel. The optical nonlinearity of the plasma causes this EPW to become nonlinearly coupled to the laser beam. Semi-analytical solutions for the coupled nonlinear wave equations governing the ignitor, heater, guided beam, and EPW have been obtained by using moment theory and the W.K.B. approximation

3.1 OVERVIEW

A resurgence of interest in studying the relationship between matter and light was led to by the invention of the laser in the 1960s. The field of laser-plasma interactions is a whole new field of study that sprang from this groundbreaking discovery. Due to its critical importance for many possible applications, such as nuclear fusion [1-3], plasma wakefield accelerators [4,5], and coherent radiation sources [6,8], this field has attracted a great deal of attention from researchers in recent years. The effectiveness of these applications depends on how well lasers and plasmas couple, which is a function of several nonlinear phenomena [9–11]. The phenomena that these phenomena cover are as diverse as the self-focusing of laser beams and the activation of various wave propagation modes in plasmas. When external magnetic fields are absent, ion sound waves (IAWs) and electron plasma waves (EPWs) are primarily associated with these wave propagation modes.

When it comes to laser-driven nuclear fusion, electron plasma wave (EPW) activation is critical. Nonetheless, the pump beam-excited EPWs are dual in nature. They reflect a substantial portion of the laser energy from the fusion pellet on one side through a process known as stimulated Raman scattering. On the other hand, they produce super-thermal electrons, which lead to the pellet heating up too soon. To be more precise, electrons that align their motion with the stimulated plasma wave and approximately match its speed become constrained inside the structure of the plasma wave. By utilizing the strong electric field produced by the EPW, these trapped electrons are accelerated to attain super-thermal velocities. These super-thermal electrons, with their exceptional penetration powers, play a role in prematurely heating the pellet before it reaches the necessary density needed for fusion ignition. We call this process "preheating." Moreover, energetic ions may be ejected from the ablation layer due to the strong electrostatic attraction between these high-energy electrons and the ions in the surrounding plasma sheath. Afterward, the laser energy that was initially meant to compress the imploding pellet is exhausted by these outwardly propelled energetic ions.

Electron plasma waves (EPWs) have been a subject of intense study by plasma physicists in recent years. Rosenbluth & Liu[12] and Tajima & Dawson[13] presented the first theoretical investigation of stimulating plasma waves by interfering with two laser pulses, and their subsequent use in accelerating charged particles. A model for triggering EPWs in plasma with rippling density was presented by Darrow et al [14, 15]. Gupta et al. [16] explored the effect of intersecting two coaxial laser beams on the excitation of EPWs in an underdense plasma, considering both ponderomotive and relativistic nonlinearities. Tiwari and Tripathi [17] examined the generation of plasma waves through beat-wave interactions in a dense gas cluster medium. In experimental work, Amini and Chen [18] were the first to report on the identification of beat plasma waves in plasma with long-scale lengths. The plasma wave frequency can be precisely determined through aggregate Thomson scattering, which occurs as a result of the optical interference between two counter-propagating CO2 laser beams in a plasma, as demonstrated.

The literature review reveals that the interaction between an optical beam and plasmas is significantly influenced by the amplitude distribution of the laser beam across its cross-sectional profile. However, the majority of previous studies on the generation of EPWs in plasmas through powerful laser beams exhibit only documented ideal or q-Gaussian laser beams. Studies on the excitation of EPWs in plasmas using q-Gaussian beams are concentrated within either prepared plasma conduits or radially homogenous plasmas [19–22]. To the best of the author's knowledge, there has been no previous research on the excitation of EPWs in laser-produced plasma channels using q-Gaussian laser beams. This study aims to, for the first time, investigate the excitation of EPWs by q-Gaussian laser beams within plasma channels generated by the ignitor heater technology, considering the relativistic mass effects of plasma electrons.

3.2 Mechanism related to Channel Formation and Optical Guiding

Plasma channel guiding operates on the premise that a plasma column, characterized by a radial density distribution with a minimum at the center, can function as a lens for a laser beam by leveraging the refractive index's dependence on plasma density. The ability to guide the laser over extended distances is facilitated by maintaining equilibrium between the convergence of light rays via the index of refraction gradient and also the outward dispersion due to geometric diffraction.

To guide very powerful laser beams efficiently, it is necessary to create plasma channels in highly ionized gases so that the guided beam does not introduce extra ionization into the density profile. Any axial density increase may cause refraction caused by ionization., which would avoid providing a guidance effect. To address these needs, Volfbeyn et al. [23] developed a novel method that makes use of two laser beams. The underlying principles of physics are based on the ignitor beam tunnel ionizing the surrounding gas to initiate plasma (see Fig. 3.1).

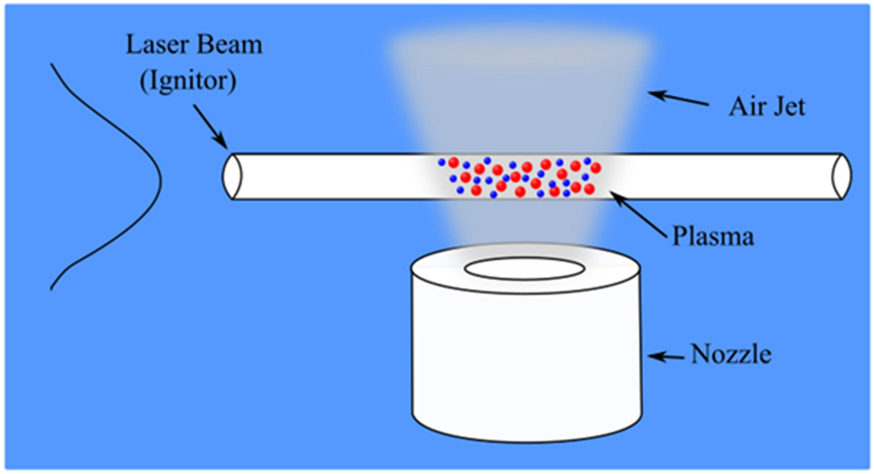

Fig.3.1: Plasma formation by ionization of air.

By heating the preexisting plasma, the heater beam produces a parabolic electron density profile with a concave shape, which causes the density of the plasma to decrease along the central line relative toward the channel's borders, where it increases (Fig. 3.2).

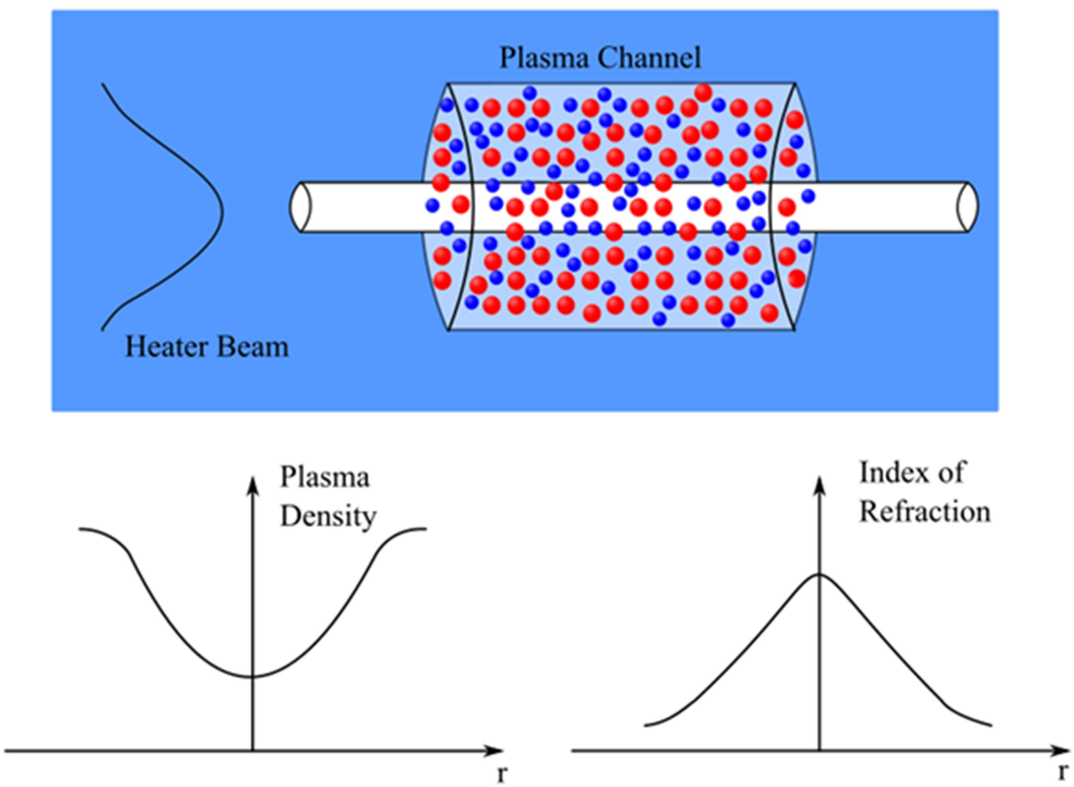

Fig.3.2: Formation of plasma channel by heater beam.

The use of an ignitor beam, in contrast to a single-beam configuration, provides better control over plasma parameters and shot-to-shot reproducibility, while also reducing the energy required for the heating beam. Along the axis, the refractive index increases, whereas it decreases towards the edges.

The third laser beam tends to diffract due to diffraction and converge due to nonlinear refraction when it passes through the plasma channel [24], taking into account the combined effects of the relativistic mass nonlinearity of plasma electrons and the non-uniformity of the plasma channel density. The laser beam travels without changing its beam diameter when its convergence and divergence completely balance each other. This criterion, however, cannot be met across an axially non-uniform plasma channel; as a result, the size of the beam's spot varies as it moves along the plasma channel.

3.3 Plasma Creation by Ignitor Beam

Think about how an ignitor beam travels down the z-axis via air. The ignitor beam's field is provided by

$$E_1(r,z,t) = E_1(r,z)e^{-i(\omega_0 t - k_0 z)}e_x$$

$$E_1E_1^{\star} = \frac{E_{10}^2}{f_1^2}e^{-\frac{r^2}{r_0^2 f_1^2}} \tag{3.1}$$

where the instantaneous beam spot size of the ignitor is denoted by $r_0 f_1$.Therefore, the dimensionless beam width parameter f_1 which gauges the ignitor beam's spot size and axial intensity was created. The beam ionizes the air through tunnel ionization, which primarily consists of nitrogen, the main component of the air we breathe. According to Keldysh [25], an atom's rate of tunneling ionization is determined using

$$\Gamma = \frac{4\pi e^4}{\hbar^3}\left(\frac{E_i}{E_a}\right)^{\frac{5}{2}}\frac{E_a}{|E_1|}e^{-\frac{2}{3}\frac{E_a}{|E_1|}\left(\frac{E_i}{E_a}\right)^{\frac{5}{2}}} \tag{3.2}$$

Where, $E_a = \frac{m^2 e^5}{\hbar^4} \simeq 1.7 \times 10^7 esu$is the atomic unit of the electric field, $E_i$is the atom's ionization energy, and $E_h$is the ionization energy of hydrogen. Liu as well as Tripathi model this so-formed plasma density as

$$\omega_p^2 = \omega_{p0}^2 e^{-\frac{E_a'}{|E_1|}} \tag{3.3}$$

where, $E_a' = \frac{2}{3} E_a \left(\frac{E_i}{E_h}\right)^{\frac{1}{2}}$, $\omega_p^2 = \frac{4\pi e^2}{m_e} n_{e0}$ and $\omega_{p0}^2 = \frac{4\pi e^2}{m_e} n_0$.

The plasma's dielectric function may be found using

$$\epsilon_1 = 1 - \frac{\omega_p^2}{\omega_0^2} \tag{3.4}$$

Using eq.(3.3) in (3.4), we get

$$\epsilon_1 = 1 - \frac{\omega_{p0}^2}{\omega_0^2} e^{-\frac{E_a'}{|E_1|}} \tag{3.5}$$

Taking

$$\epsilon_1 = \epsilon_{01} + \phi_1(E_1 E_1^\star) \tag{3.6}$$

where the dielectric function component off-axis is denoted byϕ_1, and the on-axis component is represented by ϵ_{01}= $\epsilon_1|_{r=0}$

$$\epsilon_{01} = 1 - \frac{\omega_{p0}^2}{\omega_0^2} e^{-f_1 \frac{E_a'}{E_{10}}} \tag{3.7}$$

and

$$\phi_1(E_1 E_1^\star) = \frac{\omega_{p0}^2}{\omega_0^2} \left\{ e^{-f_1 \frac{E_a'}{E_{10}}} - e^{-\left(f_1 \frac{E_a'}{E_{10}} e^{\frac{r^2}{2 r_0^2 f_1^2}} \right)} \right\} \tag{3.8}$$

3.4 Formation of Plasma Channel by Heater Beam

One introduces the second beam (warmer beam) with an electric field vector to extend the plasma and delay the electron-ion recombination process.

$$E_2 = E_2(r,z) e^{-i(\omega_0 t - k_0 z)} e_x$$

$$E_2 E_2^\star = \frac{E_{20}^2}{f_2^2} e^{-\frac{r^2}{r_0^2 f_2^2}} \tag{3.9}$$

via the plasma that the ignitor beam produces. It gives the electrons in the plasma an oscillatory velocity.

$$v_2 = \frac{-eE_2(\nu_{ei}+i\omega_0)}{m_e\omega_0^2} \tag{3.10}$$

Wherein the frequency of electron-ion collision is ν_{ei}. They get heated by rising electron temperatures at a steady pace $Re\left[-e\ E_2^{\star}.\frac{v}{2}\right]$. The electrons' energy balance equation is

$$\frac{3}{2}\frac{dT_e}{dt} = \frac{e^2|E_2|^2\nu_{ei}}{2m_e\omega_0^2} - \frac{3}{2}\delta\nu_{ei}[T_e - T_{e0}] \tag{3.11}$$

where, $T_e$is the electron temperature dependent on the field, T_{e0} is the plasma temperature at equilibrium, and δ represents the average fraction of additional energy lost during each collision. Assuming that$\nu_{ei} = \nu_0\left(\frac{T_e}{T_{e0}}\right)^{-\frac{3}{2}}$represents the fluctuation of electron-ion collision frequency in relation to electron temperature,$\nu_0$denotes the collision frequency between electrons and ions at the equilibrium plasma temperature. The electron temperature can be calculated by integrating Equation (3.11) as

$$T_e = T_{e0}\left[1 + \frac{\beta_2 E_{20}^2}{f_2^2} e^{-\frac{r^2}{r_0^2 f_2^2}}\right] \tag{3.12}$$

where, $\beta_2 = \frac{e^2 M}{6K_0 T_{e0} m_e^2 \omega_0^2}$ is the coefficient of collisional nonlinearityBecause of the gradient in electron partial pressure caused by temperature nonuniformity, electrons migrate from a region of high temperature to one of low temperature. The ions are propelled forward by the space charge field created by this. When the total partial pressures of the ions and electrons are equal, $n_eT_e + n_iT_i = 2n_{e0}T_{e0}$, a stable state is reached. By taking $n_e \simeq n_i$, one can get

$$n_e = \frac{2n_0 e^{-f_1\frac{E_a'}{E_1}}}{\left(1+\frac{T_e}{T_{e0}}\right)} \tag{3.13}$$

Expanding n_e in the powers of r can be done as

$$n_e = n_1 + n_2\frac{r^2}{r_0^2}$$

where,

$$n_1 = \frac{n_0 e^{-f_1\frac{E_a'}{E_{10}}}}{1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}} \tag{3.14}$$

and

$$n_2 = \frac{2n_0 e^{-f_1 \frac{E'_a}{E_{10}}}\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{-\frac{3}{5}}}{5\left(1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right)^2} \frac{\beta_2 E_{20}^2}{f_2^4} \tag{3.15}$$

Thus, the heater beam's linear and nonlinear components of the dielectric permittivity can be expressed as

$$\epsilon_{02} = 1 - \frac{\omega_{p0}^2}{\omega_0^2} \frac{e^{-f_1 \frac{E'_a}{E_{10}}}}{1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}} \tag{3.16}$$

$$\phi_2 = -\frac{\omega_{p0}^2}{\omega_0^2} \frac{2}{5} \frac{\beta_2 E_{20}^2}{f_2^4} e^{-f_1 \frac{E'_a}{E_{10}}} \frac{\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{-\frac{3}{5}}}{1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}} \frac{r^2}{r_0^2} \tag{3.17}$$

3.5 Relativistic Dielectric Function of the Plasma Channel

Analysis of the motion of a circularly polarized q-Gaussian beam:

$$E_3 = E_3(r,z) e^{-i(\omega_0 t - k_0 z)} (e_x + i e_y)$$

$$E_3 E_3^\star = \frac{E_{30}^2}{f_3^2}\left(1 + \frac{r^2}{q r_0^2 f_3^2}\right)^{-q} \tag{3.18}$$

via the channel of plasma. The parameter q indicates that the intensity profile of the laser beam is not Gaussian. Laser beams with lower q values are defined by the wider edges of the intensity distribution. As q increases, the intensity profile of the laser beam goes toward a Gaussian distribution and, for q=∞, becomes exactly Gaussian.

$$\lim_{q \to \infty} E_3 E_3^\star = \frac{E_{30}^2}{f_3^2} e^{-\frac{r^2}{r_0^2 f_3^2}}$$

The plasma channel's dielectric function can be expressed as

$$\epsilon_3 = 1 - \frac{\omega_{p0}^2}{\omega_0^2} \frac{e^{-f_1 \frac{E_a'}{E_{10}}}}{\left(1+\left(1+\frac{\beta_2 E_{20}^2}{f_2}\right)^{\frac{2}{5}}\right)} - \frac{2}{5} \frac{\omega_{p0}^2}{\omega_0^2} \frac{\beta_2 E_{20}^2}{f_2^4} e^{-f_1 \frac{E_a'}{E_{10}}} \frac{\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{-\frac{3}{5}}}{\left\{1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right\}^2} \frac{r^2}{r_0^2} \tag{3.19}$$

With frequency ω_0, the plasma electrons generated by the circularly polarized laser beam travel in circular orbits. The electrons' quiver speed is close to that of light in a vacuum due to the high magnetic field produced by the intense laser beam. Thus, $m_0 \gamma$, where $m_0$is the electron's rest mass and γ is the relativistic Lorentz factor, which replaces the effective massm_e of electrons in equation (3.19). Keeping up with Akhiezer and Polovin at a balance

$$-eE_3 = \frac{m_0 v \omega_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}} \tag{3.20}$$

Where c is speed of light in a vacuum and V is the quiver velocity of the electron in the laser beam's field. Equations (3.18) and (3.20) yield

$$\gamma = \left\{1 + \frac{\beta_3 E_{30}^2}{f_3^2}\left(1 + \frac{r^2}{q r_0^2 f_3^2}\right)^{-q}\right\}^{\frac{1}{2}} \tag{3.21}$$

Thus, given a q-Gaussian laser beam, the plasma channel's effective dielectric function can be expressed as

$$\epsilon_{03} = 1 - \frac{\omega_{p0}'^2}{\omega_0^2} \frac{e^{-f_1 \frac{E_a'}{E_{10}}}}{\left\{1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right\}} \left(1 + \frac{\beta_3 E_{30}^2}{f_3^2}\right)^{-\frac{1}{2}} \tag{3.22}$$

$$\phi_3 = \frac{\omega_{p0}'^2}{\omega_0^2} \frac{e^{-f_1 \frac{E_a'}{E_{10}}}}{\left\{1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right\}} \left(1 + \frac{\beta_3 E_{30}^2}{f_3^2}\right)^{-\frac{1}{2}} - \left\{\frac{\omega_{p0}'^2}{\omega_0^2} \frac{e^{-f_1 \frac{E_a'}{E_{10}}}}{\left\{1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right\}^2} \frac{r^2}{r_0^2}\right\} \left(1 + \frac{\beta_3 E_{30}^2}{f_3^2}\left(1 + \frac{r^2}{q r_0^2 f_3^2}\right)^{-q}\right)^{-\frac{1}{2}} \tag{3.23}$$

where, $\omega_{p0}'^2 = \frac{4\pi e^2}{m_0} n_0$.

3.6 Variations in the Laser Beam Spot Size

Starting with Ampère's and Faraday's laws, for an isotropic, non-conductive, and non-absorptive material ($J = 0, \rho = 0, \mu = 1$), we can derive the following equations:

$$\nabla \times B_j = \frac{1}{c}\epsilon_j \frac{\partial E_j}{\partial t} \tag{3.24}$$

$$\nabla \times E_j = -\frac{1}{c}\frac{\partial B_j}{\partial t} \tag{3.25}$$

When equations (3.24) and (3.25) are combined, it may be demonstrated that the vectors of the electric field E(r,z,t) of the laser intersect with the wave equation, where *j*=1-3.

$$\nabla^2 E_j - \nabla(\nabla . E_j) + \frac{\omega_0^2}{c^2}\epsilon_j E_j = 0 \tag{3.26}$$

The polarization term $\nabla(\nabla . E_j)$ of eqn (3.26) can be disregarded even if E_j contains longitudinal components, given that $\frac{c^2}{\omega_0^2}\left|\frac{1}{\epsilon_j}\nabla^2 ln\, \epsilon_j\right| << 1$ that, in other words, in comparison to the laser wavelength, the transverse derivative of the dielectric function is negligible (under W.K.B approximation). This suggests that, under this approximation, eqn (3.26) simplifies to either the plasma is greatly under dense or the transverse dielectric fluctuations are weak.

$$\nabla^2 E_j + \frac{\omega_0^2}{c^2}\epsilon_j E_j = 0 \tag{3.27}$$

using eqns (3.1), (3.9) and (3.18) in (3.27) we get

$$i\frac{\partial E_j}{\partial z} = \frac{1}{2k_0}\nabla_\perp^2 E_j + \frac{k_0}{2\epsilon_{0j}}\phi_j(E_j E_j^\star)E_j \tag{3.28}$$

The assumption that $\left|\frac{d^2 E_j}{dz^2}\right| << k_o^2 E_j$ implies that the electric field amplitude's characteristic length along the longitudinal direction is much larger than the wavelength. This justifies the omission of the term $\frac{d^2 E_j}{dz^2}$ in the derivation of equation (3.28).

The average squared radius of a laser beam can now be found using the definition of the second-order spatial moment of the intensity distribution which is

$$< a_j^2 >= \frac{1}{I_{0j}} \int_0^{2\pi} \int_0^{\infty} r^2 E_j E_j^{\star} \, r dr d\theta \tag{3.29}$$

where,

$$I_{0j} = \int_0^{2\pi} \int_0^{\infty} E_j E_j^{\star} \, r dr d\theta \tag{3.30}$$

After twice differentiating equation (29) with respect to z, we obtain by substituting the values of $\frac{dE_j}{dz}$ and $\frac{dE_j^{\star}}{dz}$ from equation (28),

$$I_{0j} \frac{d^2}{dz^2} < a_j^2 >= \frac{2}{k_0^2} \left[\frac{1}{k_0} \int_0^{2\pi} \int_0^{\infty} \left| \nabla_{\perp} E_j \right|^2 r dr d\theta + \frac{k_0}{2\epsilon_{0j}} \int_0^{2\pi} \int_0^{\infty} r^2 E_j E_j^{\star} \frac{\partial \phi_j}{\partial r} \, dr d\theta \right] \tag{3.31}$$

Equations (3.1), (3.7)–(3.9), (3.16)–(3.18), (3.22), (3.23), (3.29), and (3.30), when substituted into equation (3.31), collectively form the coupled differential equations that describe the evolution of laser beam spot sizes as a function of propagation distance.

$$\frac{d^2 f_1}{d\xi^2} = \frac{1}{f_1^3} + 2 \frac{E_a'}{E_{10}} \left(\frac{\omega_{p0}^2 r_0^2}{c^2} \right) I_1 \tag{3.32}$$

$$\frac{d^2 f_2}{d\xi^2} = \frac{1}{f_2^3} - \frac{2}{5} \left(\frac{\omega_{p0}^2 r_0^2}{c^2} \right) \frac{\beta_2 E_{20}^2}{f_2^2} e^{-f_1 \frac{E_a'}{E_{10}}} \frac{\left(1 + \frac{\beta_2 E_{20}^2}{f_2^2}\right)^{-\frac{3}{5}}}{\left(1 + \left(1 + \frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right)^2} \tag{3.33}$$

$$\frac{d^2 f_3}{d\xi^2} = \left(1 - \frac{1}{q}\right)\left(1 - \frac{2}{q}\right) \left[\frac{1}{\left(1 + \frac{1}{q}\right) f_3^3} - \frac{1}{2} \frac{\beta_2 E_{20}^2}{f_3^3} \left(\frac{\omega_{p0}^2 r_0^2}{c^2} \right) \right] \frac{e^{-f_1 \frac{E_a'}{E_{10}}}}{\left(1 + \left(1 + \frac{\beta_2 E_{20}^2}{f_2}\right)^{\frac{2}{5}}\right)} K_1$$
$$- \frac{1}{5} \left(\frac{\omega_{p0}^2 r_0^2}{c^2} \right) \frac{\beta_2 E_{20}^2}{f_2^4} e^{-f_1 \frac{E_a'}{E_{10}}} \frac{\left(1 + \frac{\beta_2 E_{20}^2}{f_2^2}\right)^{-\frac{3}{5}}}{\left(1 + \left(1 + \frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right)^2} \left(2 f_3 K_2 + \frac{\beta_3 E_{30}^2}{f_3} K_3 \right) \tag{3.34}$$

where,

$$I_1 = \int_0^\infty x e^{-x} e^{-\left(f_1 \frac{E'_a}{E_{10}} e^x\right)} dx$$

$$K_1 = \int_0^\infty y\left(1+\frac{y}{q}\right)^{-2q-1}\left\{1+\frac{\beta_3 E_{30}^2}{f_3^2}\left(1+\frac{y}{q}\right)^{-q}\right\}^{-\frac{3}{2}} dy$$

$$K_2 = \int_0^\infty y\left(1+\frac{y}{q}\right)^{-q}\left\{1+\frac{\beta_3 E_{30}^2}{f_3^2}\left(1+\frac{y}{q}\right)^{-q}\right\}^{-\frac{1}{2}} dy$$

$$K_3 = \int_0^\infty y\left(1+\frac{y}{q}\right)^{-2q-1}\left\{1+\frac{\beta_3 E_{30}^2}{f_3^2}\left(1+\frac{y}{q}\right)^{-q}\right\}^{-\frac{1}{2}} dy$$

$$x = \frac{r^2}{r_0^2 f_1^2}$$

$$y = \frac{r^2}{r_0^2 f_3^2}$$

$$\xi = \frac{z}{k_0 r_0^2}$$

boundary conditions f_j =1,$\frac{df_j}{d\xi} = 0$ at $\xi = 0$ apply to planar wavefronts at the beginning.

3.7 Evolution of EPW

Due to their unique properties, such as approximate neutrality and collective behavior, plasmas provide a medium for the generation of electron plasma waves (EPWs). In a plasma state, sufficient thermal energy causes atoms to lose their distinct identities as their bonds break. Positively charged nuclei and negatively charged electrons coexist, but their mutual attraction no longer binds them together. This distinctive characteristic imparts plasmas with properties that set them apart from conventional states of matter—solids, liquids, and gases.

In a plasma, free electrons and ions respond strongly to electric and magnetic forces. While maintaining an overall quasi-neutrality, the separation of electrons and positively charged ions allows for perturbations that create localized regions of net positive and net negative charge. This phenomenon resembles the behavior of a parallel-plate capacitor, where non-uniform charge distribution generates an electric field that directs itself from areas of positive to negative charge. This field exerts equal but opposite forces, pulling ions and electrons towards one another.

However, due to their significantly larger mass, ions remain largely stationary, while electrons move towards the ions.

As the electrons accelerate towards equilibrium, they gain momentum, similar to a pendulum returning to its resting position after being displaced. The acquired momentum causes the electrons to overshoot their equilibrium positions, reversing the electric field's direction. This inverted field slows the electrons, eventually forcing them to reverse direction once again, initiating a cyclical process. These oscillations create a phenomenon akin to an electron oscillator.

Intense laser beams can stimulate these electron oscillators. When a laser beam enters the plasma, its radiation pressure displaces the plasma electrons, pushing them away from their equilibrium positions. Upon removal of the laser beam, the displaced electrons rapidly return to equilibrium, creating regions with reduced electron density. This displacement triggers oscillations among the plasma electrons, ultimately leading to the formation of a plasma wave. The wave equation controls how excited EPW propagates.

$$2\imath k_{ep}\frac{\partial n_{ep}}{\partial z} = \nabla_{\perp}^{2}\mathrm{n_{ep}} + \frac{\omega_{\mathrm{ep}}^{2}}{v_{th}^{2}}\phi_3 n_{ep} \quad (3.44)$$

where the thermal speed of plasma electrons is expressed as$v_{th} = \sqrt{\frac{2KT_0}{m}}$. Poisson's equation, which describes the motion of an electron fluid, and the equation of continuity can all be linearized to get this equation. Thinking of the EPW's Gaussian ansatz as

$$n_{ep} = \frac{n_{00}}{f_{ep}} e^{-\frac{r}{2r_0^2 f_{ep}^2}} \quad (3.45)$$

Now we derive the equations governing the evolution of the EPW's beam widths by following the steps in section 6.

$$\frac{d^2 f_{ep}}{d\xi^2} = \left[\frac{1}{f_{ep}^3} - \frac{1}{2}\frac{\beta_2 E_{20}^2}{f_{ep}^3}\left(\frac{\omega_{ep}^2 r_0^2}{v_{th}^2}\right)\right]\frac{e^{-f_1\frac{E_a'}{E_{10}}}}{\left(1+\left(1+\frac{\beta_2 E_{20}^2}{f_2}\right)^{\frac{2}{5}}\right)} K_4$$

$$-\frac{1}{5}\left(\frac{\omega_{ep}^2 r_0^2}{v_{th}^2}\right)\frac{\beta_2 E_{20}^2}{f_{ep}^4} e^{-f_1\frac{E_a'}{E_{10}}}\frac{\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{-\frac{3}{5}}}{\left(1+\left(1+\frac{\beta_2 E_{20}^2}{f_2^2}\right)^{\frac{2}{5}}\right)^2}$$

$$\left(2 f_{ep} K_5 + \frac{\beta_3 E_{30}^2}{f_{ep}} K_6\right) \tag{3.46}$$

where,

$$K_4 = \int_0^\infty y\, e^{-\frac{f_3^2}{f_{ep}^2}y}\left(1+\frac{y}{q}\right)^{-2q-1}\left\{1+\frac{\beta_3 E_{30}^2}{f_3^2}\left(1+\frac{y}{q}\right)^{-q}\right\}^{-\frac{3}{2}} dy$$

$$K_5 = \int_0^\infty y e^{-\frac{f_3^2}{f_{ep}^2}y}\left(1+\frac{y}{q}\right)^{-q}\left\{1+\frac{\beta_3 E_{30}^2}{f_3^2}\left(1+\frac{y}{q}\right)^{-q}\right\}^{-\frac{1}{2}} dy$$

$$K_6 = \int_0^\infty y e^{-\frac{f_3^2}{f_{ep}^2}y}\left(1+\frac{y}{q}\right)^{-2q-1}\left\{1+\frac{\beta_3 E_{30}^2}{f_3^2}\left(1+\frac{y}{q}\right)^{-q}\right\}^{-\frac{1}{2}} dy$$

Equation (3.46) demonstrates the coupling of the electron plasma wave (EPW) with the heater, ignitor, and guided beams. The density fluctuations associated with the EPW are highly sensitive to the self-focusing behavior of the laser beams. The electric field of the stimulated EPW can be calculated using Poisson's equation as

$$\boldsymbol{E_{ep}} = E_{ep} e^{\iota(k_{ep} z - \omega_{ep} t)}$$

$$E_{ep} = \frac{\iota m \omega_{ep}^2}{c k_{ep} f_{ep}} e^{-\frac{r}{2 r_0^2 f_{ep}^2}} \tag{3.47}$$

$$P = \frac{\int E_{ep} E_{ep}^{\star} dr}{\int E_3 E_3^{\star} dr} \tag{3.48}$$

3.8 Electron Acceleration by the Excited EPW

The acceleration of the particle does not follow the intuitive notion of acceleration as being an increase in velocity over time. Charged particles in present-day high energy accelerators travel at nearly the speed of light. An electron from a 50-GeV accelerator falls short of the speed of light by only five parts in 10^{11}, that is, if an electron raced a light pulse around the earth, the electron would cross the finish line only 2.1 millimeter behind the light. When particles travelling at these speeds absorb energy from a field, they are accelerated in the sense that their mass increases in accordance with Einstein's theory of relativity. The particles' velocity, however, increases very little. In order to accelerate (to add mass to) a beam of charged particles already moving at very high speed, a plasma accelerator must create a plasma wave travelling at nearly the speed of light so that charged particles do not outrun the electric field wave.

The energy gained by the accelerated electron is given by

$$\frac{d\gamma}{dt} = \frac{e}{mc^2} E_{ep} v_z \tag{3.49}$$

Here, v_z is the velocity of accelerated electron and $\gamma = \sqrt{1 + \frac{p_e}{m^2c^2}}$. Using transformation $\frac{1}{v_z}\frac{d}{dt} = \frac{d}{dz}$ we get

$$\frac{d\gamma_e}{dz} = \frac{e}{m^2c^2} E_{ep} \tag{3.50}$$

Eq.(3.50) gives the energy of accelerated electron as a function of distance of propagation.

3.9 Discussion

The beam width parameters are expressed in dimensionless terms $f_j$of the laser beams are described by equations (3.32) through (3.34) and the power of the excited EPW is described by equation (3.48) as a function of propagation distance. Equations (3.32) to (3.34) contain two terms on the right-hand side (RHS), with each term corresponding to a distinct physical process that governs the evolution of the beam envelope during propagation. The phrase "diffractive term" refers to the first term in wave equation (3.28), which originates from the Laplacian ($\nabla_\perp^2$) and causes the laser beam to diffraction widen. The subsequent term arises from the nonlinear optical refraction of the laser beam, which is induced by the medium's dielectric properties exhibiting nonlinear behavior under the influence of the laser. One can see the laser beam focus or defocus based on how these two phrases compete with one another. The subsequent set of parameters has been the numerical solution for Equations (3.32)-(3.34) and (3.45):

$$\omega_0 = 1.78 \times 10^{15}\ rad/sec$$

$$r_0 = 15\mu m$$

$$n_0 = 10^{17} cm^{-3}$$

$$T_{e0} = 10^6 K$$

Figures 3.3, 3.4, and 3.5 illustrate how the intensity of the initial ignitor beam influences the evolution of laser beam focal spot sizes with propagation distance. The data presented in Figure 3 reveal that the ignitor beam's spot size expands continuously as it propagates through the atmosphere. This behavior is attributed to ionization-induced defocusing, which enhances natural diffraction through the nonlinear refraction of the ignitor beam. Furthermore, it is observed that higher intensities of the ignitor beam result in a slower defocusing rate. This effect arises because increasing the ignitor beam's intensity reduces the extent of its nonlinear refraction.

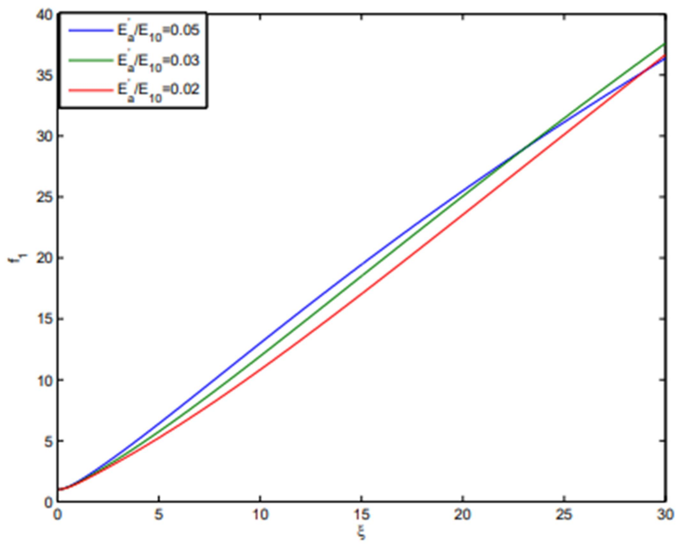

Fig.3.3: Effect of the ignitor beam intensity on the progression of its beam width.

According to the charts in Figure 4, the second beam—the heater beam—first exhibits oscillatory focusing as it moves through the plasma before beginning to increase monotonically in spot size. This is because the ignitor beam is the one producing the plasma. At first, it is strong enough to ionize the air sufficiently to produce a plasma density high enough for the heater beam to focus on itself. However, the ignitor beam's intensity decreases with increasing propagation distance, and as a result, the plasma density falls below what is needed for the heater beam to self-focus..

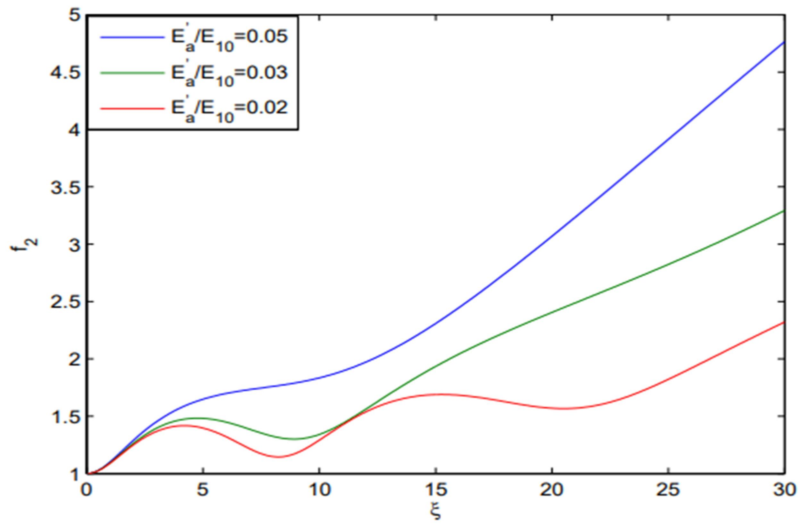

Fig.3.4: Impact of the ignitor beam's intensity on the evolution of the heater beam's width.

Moreover, as the intensity of the ignitor beam increases, the defocusing rate of the heater beam decreases. This reduction is attributed to the diminished defocusing rate of the heater beam with higher ignitor beam intensity, coupled with a decrease in the refractive term that counterbalances the diffractive component as described in equation (3.33).

The data presented in Figure 3.5 demonstrate that the third laser beam undergoes repeated oscillations as it propagates through the plasma channel. However, after each focal spot, the peaks and troughs of the spot size progressively increase. This behavior arises from the significant electric field of the laser beam, which accelerates plasma electrons to velocities approaching that of light in a vacuum. As the q-Gaussian laser beam enters the plasma channel, the increased electron mass leads to a reduction in plasma frequency. At the center of the beam, where the intensity is highest, the electron mass is greatest, resulting in the lowest plasma frequency. Consequently, the refractive index in the central region of the laser beam increases, causing the wavefronts to bend inward and the laser to self-focus. The laser beam's spot size,$r_0 f_3$ diminishes as it becomes self-focused over propagation distance, increasing diffraction in opposition to nonlinear refraction. This reduces the focus rate till$f_3$reached the minimum. Subsequently, diffraction effects become more prominent, though they remain insufficient to counteract self-focusing. This process continues until the diffraction effects eventually dominate over the laser beam's nonlinear optical behavior. The repeated interactions between these effects result in an oscillatory focusing and defocusing of the laser beam, and as the beam propagates along the channel, its spot size undergoes periodic oscillations, manifesting as a scalloped or sausage-like pattern. Furthermore, the scattering of the ignitor beam leads to a reduction in plasma density, which causes the minimum of f_3 to shift upward as the beam propagates.

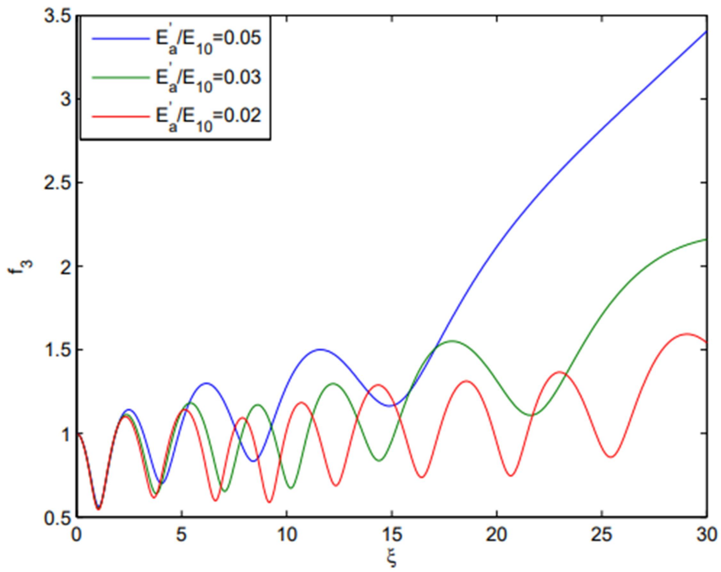

Fig.3.5: Effect of intensity of ignitor beam on the evolution of beam width of the guided beam.

It is also noted that as the intensity of the ignitor beam increases, the effective propagation distance of the third laser beam through the plasma channel is significantly extended. The fundamental physics of this phenomenon lies in the fact that as the power of the ignitor beam increases, the diffraction of the heater beam, which forms the channel, is reduced. This results in a more radially inhomogeneous refractive profile of the plasma channel, thereby enhancing the ability of the third beam to propagate through it.

Figure 3.6 illustrates the influence of the ignitor beam's intensity on the evolution of the excited electron plasma wave (EPW) with propagation distance. It is observed that the peak power of the stimulated EPW occurs at the guided beam's focal points, which coincide with the regions of highest laser intensity. Consequently, these focal points exhibit the maximum amplitude of the density perturbation and the highest EPW power. Furthermore, an increase in the ignitor beam's intensity is associated with a corresponding rise in EPW power. This relationship arises from the direct correlation between the EPW intensity and the degree of self-focusing of the guided beam. As the intensity of the ignitor beam increases, so too does the self-focusing of the guided beam, thereby amplifying the EPW power.

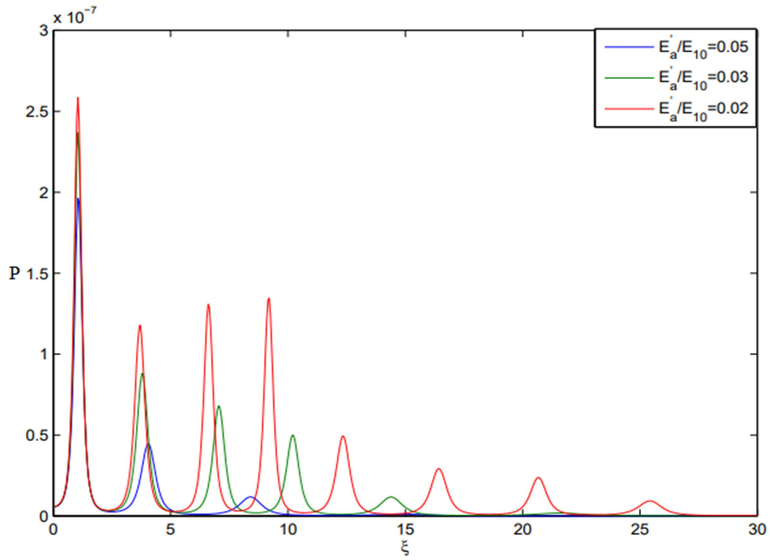

Fig.3.6: Impact of the ignitor beam's intensity on the evolution of the excited EPW's power.

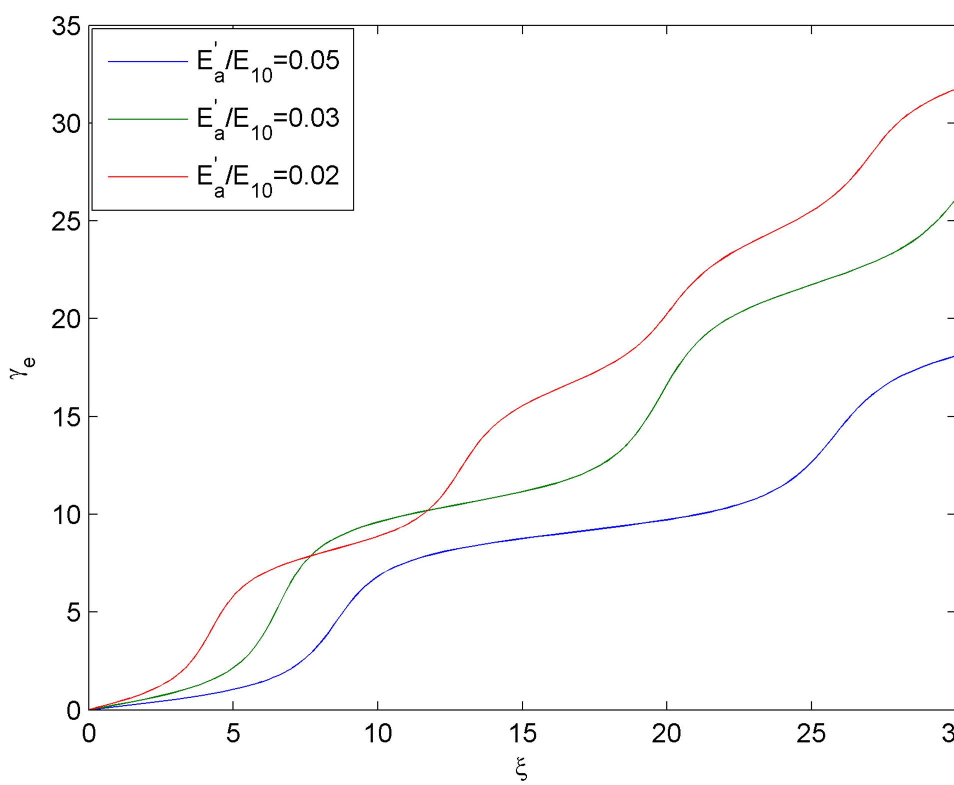

Fig.3.7: Impact of the ignitor beam's intensity on the evolution of the energy of accelerated electron.

Fig.3.7 illustrates the effect of intensity of the ignitor beam on the energy of accelerated electron. It can be seen that with increase in the intensity of ignitor beam there is increase in the energy of accelerated electron. This is because with increase in ignitor intensity there is increase in the power of excited EPW.

Figures 3.8 and 3.9 illustrate the influence of the heater beam's intensity on the variation in spot sizes of the second and third laser beams with respect to propagation distance. As shown in Figure 3.8, the rate of defocusing of the heater beam diminishes as its intensity increases. This behavior can be attributed to the increased intensity of the heater beam, which leads to a faster plasma heating process. Consequently, the enhanced plasma heating results in a reduced rate of defocusing of the heater beam.

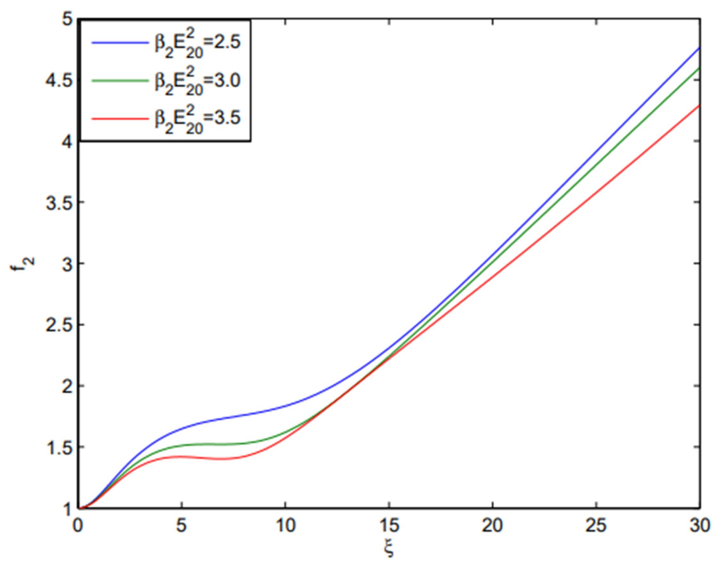

Fig.3.8: Impact of the heater beam's intensity on the evolution of its beam width.

Figure 3.9 demonstrates that an increase in the heater beam's intensity leads to an extended effective propagation distance of the third beam through the plasma channel. This effect arises from the enhanced intensity of the heater beam, which amplifies the radial inhomogeneity in the refractive properties of the plasma channel.

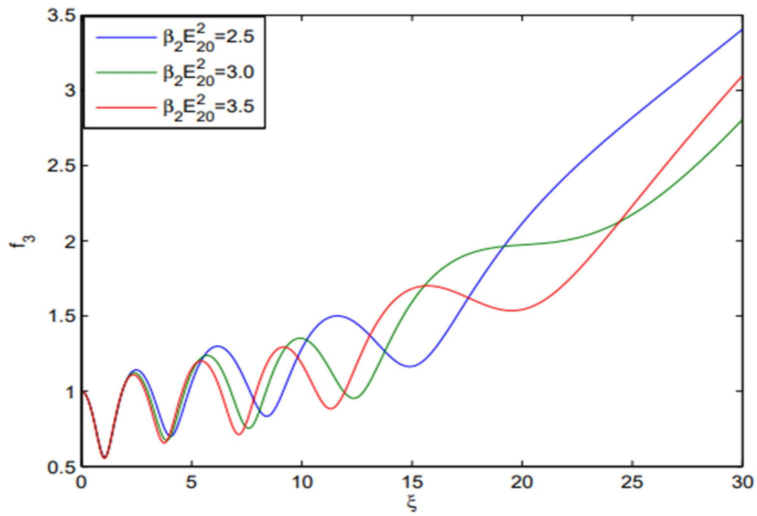

Fig.3.9: Impact of the heater beam's intensity on the evolution of beam width of guided beam.

Figure 3.10 illustrates the influence of the heater beam's intensity on the evolution of the excited EPW's power. It is observed that as the intensity of the heater beam increases, the power of the excited EPW also increases. This is attributed to the enhanced self-focusing of the guided beam and the reduction in the degree of defocusing of the heater beam, both of which occur as the heater beam's intensity increases.

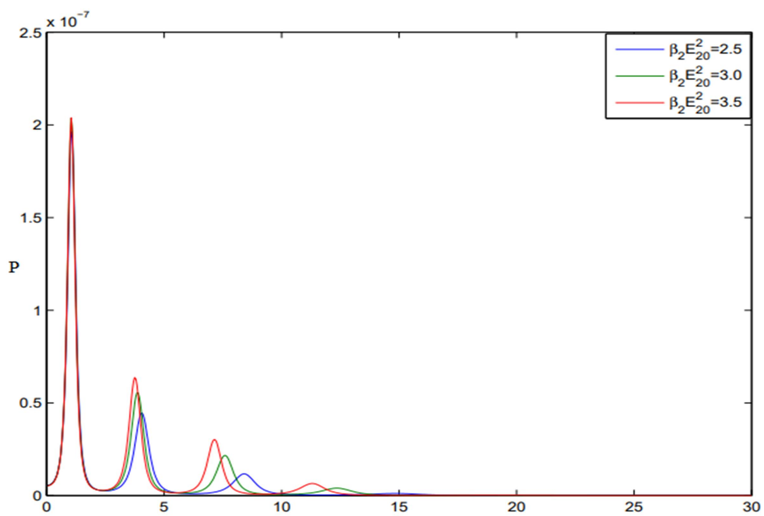

Fig.3.10: Impact of the heater beam's intensity on the evolution of the power of excited EPW.

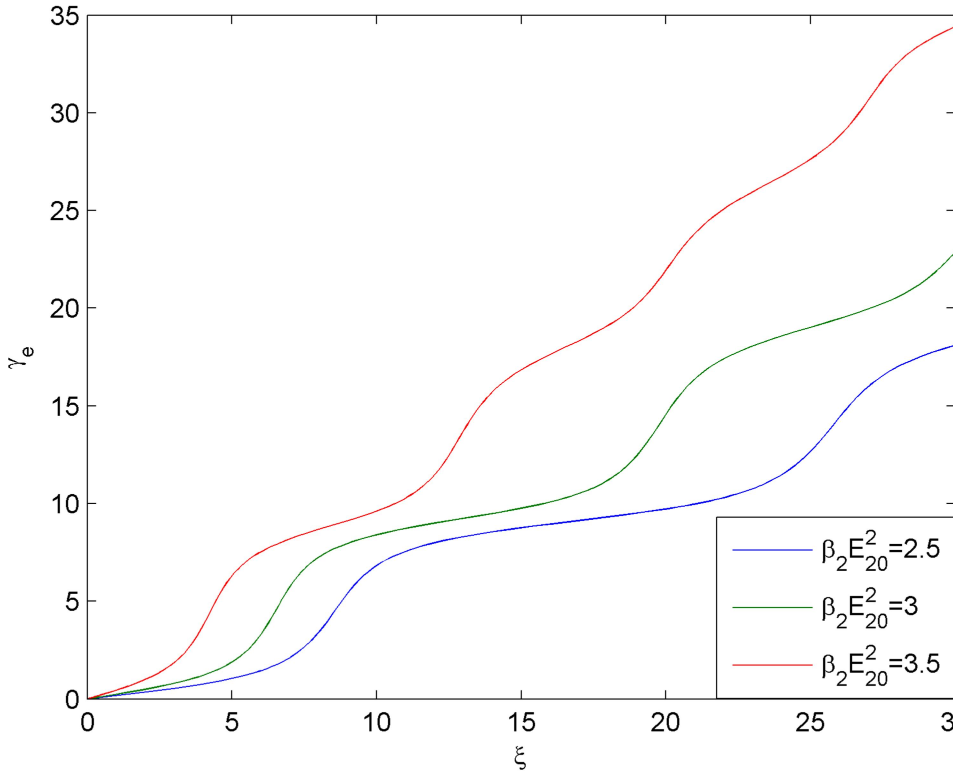

Fig.3.11: Impact of the heater beam's intensity on the energy of accelerated EPW.

Fig.3.11 depicts the effect of intensity of heater beam on the energy of accelerated electron. Again it has been observed that with increase the intensity of heater beam there is substantial increase in the energy gained by the accelerated electron. This is due to increase in the power of excited EPW with increase in the intensity of heater beam.

Fig. 3.12 shows how the intensity of the third beam affects how the spot size changes with propagation distance. It has been noted that a third beam's effective propagation length through the plasma channel increases with its intensity. It results from a rise in the electron mass's relativistic nonlinearity as the third beam's intensity increases.

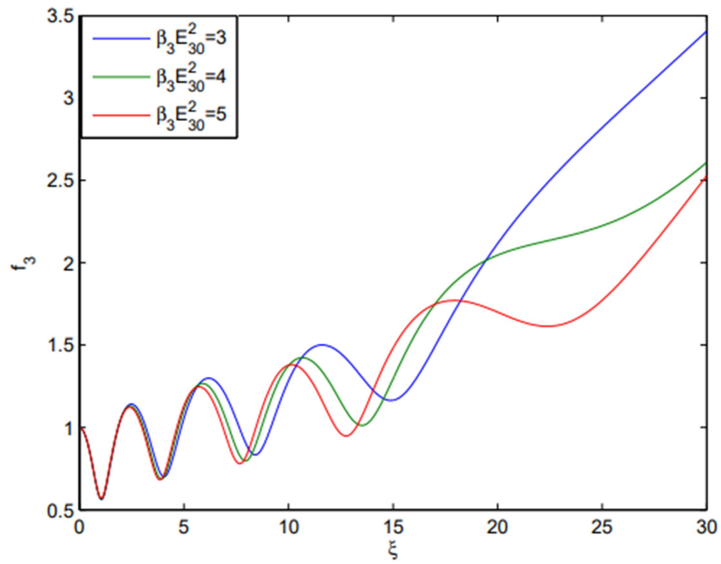

Fig.3.12: Impact of the guided beam's intensity on the evolution of its beam width.

The impact of the third beam's intensity on the excited EPW's power is shown in Fig. 3.11. It has been noted that a rise in the third beam's intensity causes an increase in the excited EPW's power. It results from an increase in the guided beam's effective focusing.

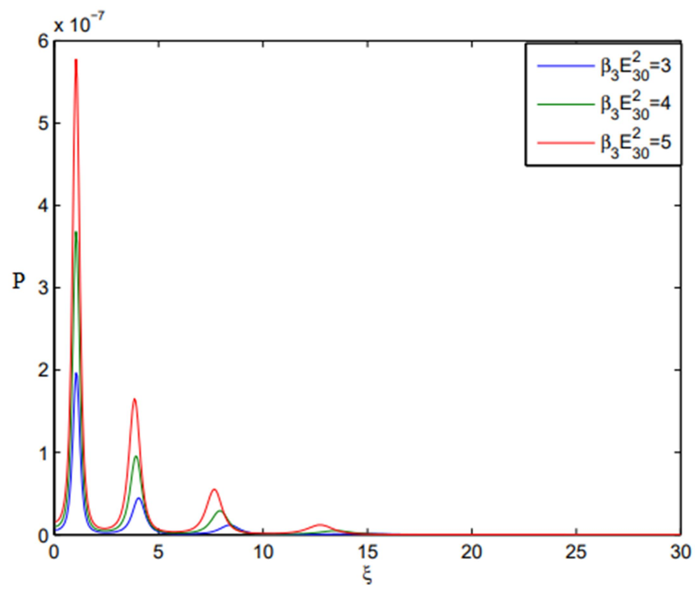

Fig.3.13: Impact of the guided beam's intensity on the evolution of the excited EPW's power.

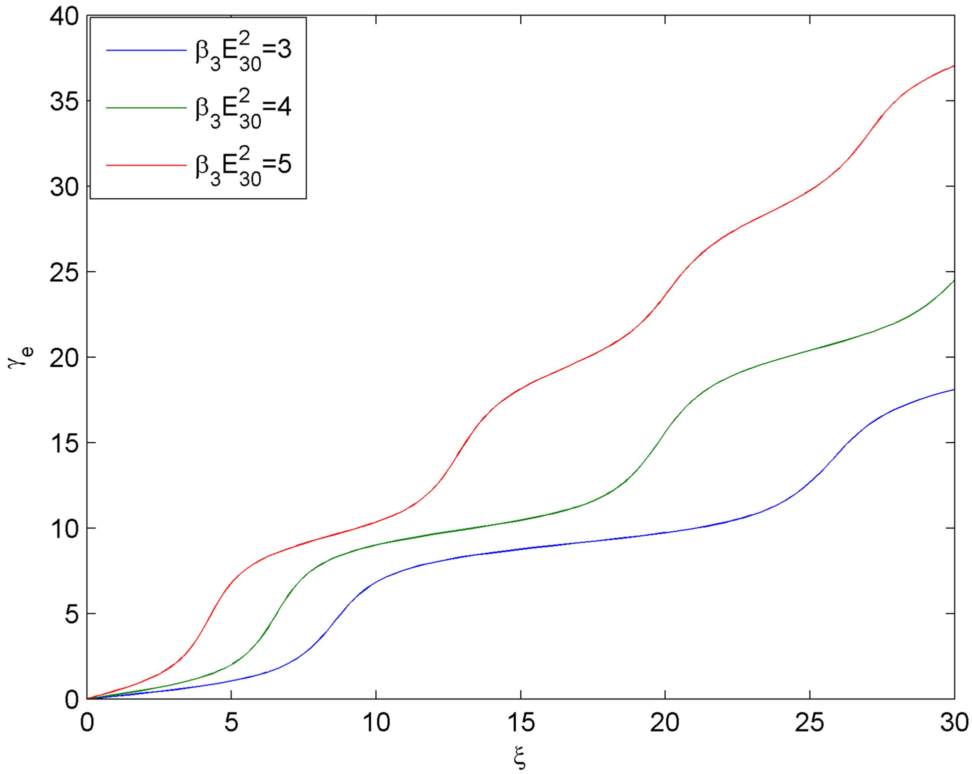

Fig.3.14: Impact of the guided beam's intensity on the evolution of energy of the accelerated electron.

Influence of the intensity of guided beam on energy gained by the accelerated electron has been shown in the fig.14. It can be seen that increase in the intensity of guided beam results in the energy gained by the accelerated electron. This is again due to the increase in the power of excited EPW with increase in the intensity of guided beam.

The influence of the third beam's intensity profile deviation (i.e., the deviation parameter q) from the Gaussian profile on how the spot size evolves with propagation distance is shown in Fig. 3.15. It has been noted that the third beam's effective propagation distance via the plasma channel decreases as q increases. As the value of q increases, the intensity of the laser beam becomes more concentrated toward the axial region of the wavefront. Consequently, laser beams with higher q values exhibit a smaller root mean square (r.m.s.) radius. It is well-established that the diffraction divergence of a laser beam is inversely proportional to its beam width, meaning that as q increases, the magnitude of the diffractive term in equation (3.34) also increases. Moreover, as q rises, the central portion of the laser beam's wavefront becomes less pronounced, leading to a reduction in the amplitude of the nonlinear refractive term. As a result, an increase in q corresponds to a decrease in the effective propagation distance of the third laser beam through the plasma channel.

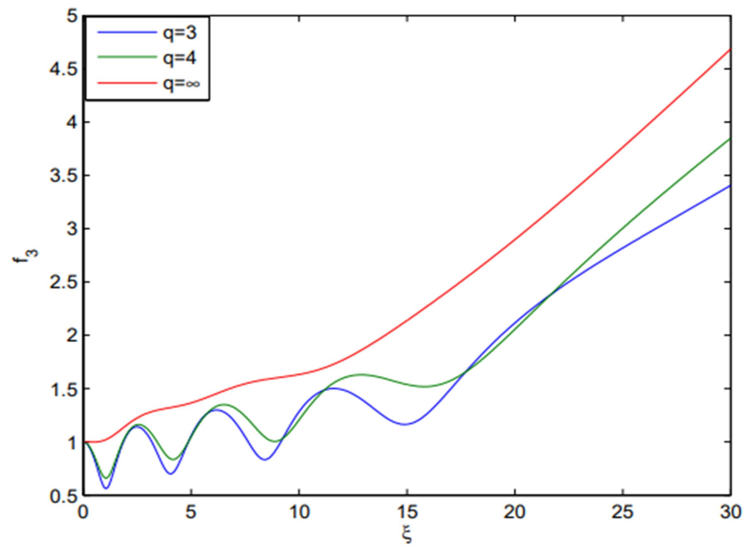

Fig.3.15: Effect of the guided beam's deviation parameter q on the evolution of its beam width.

Figure 3.16 illustrates the effect of the guided beam's deviation parameter (q) on the power of the stimulated EPW. As the value of (q) increases, the power of the excited EPW decreases, which is attributed to the reduction in the guided beam's focus with increasing (q).

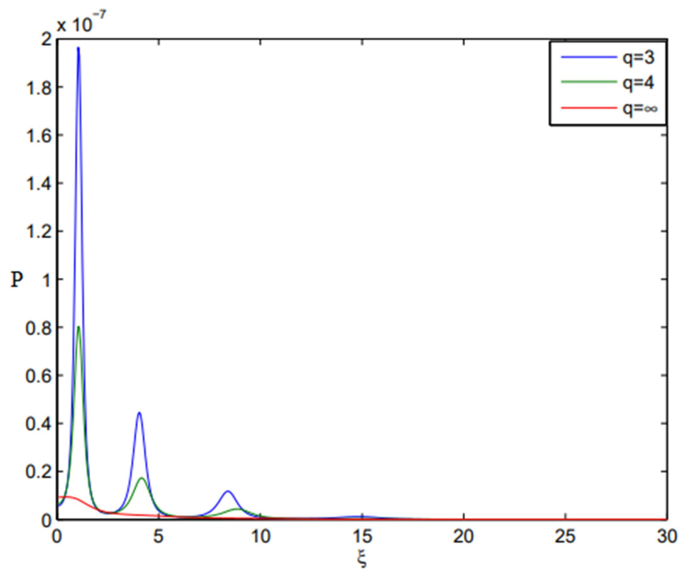

Fig.3.16: Effect of the guided beam's deviation parameter q on the evolution of the excited EPW power.

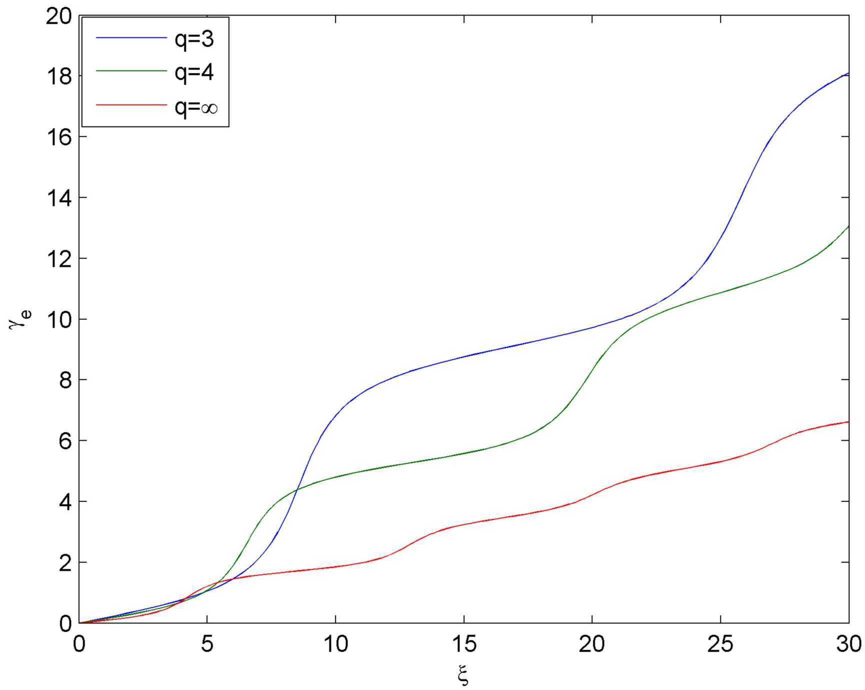

Fig.3.17: Effect of the guided beam's deviation parameter q on the evolution of the energy of accelerated electron.

Fig. 3.17 depicts the effect of deviation parameter q of the guided beam on the energy gained by the accelerated electron. It can be seen that with increase in the deviation parameter q of the guided beam the energy gained by the accelerated electron decreases. This is due to reduction in the power of excited EPW with increase in the deviation parameter q of the guided beam.

3.10 Conclusion

Notably, it has been observed that the pump beam's self-focusing effect has a major impact on the EPW's output. It is evident that the energy acquired by the accelerated electron reduces as the guided beam's deviation parameter q increases. This is because the excited EPW's power decreases as the guided beam's deviation parameter q increases. It has also been shown that the energy obtained by the accelerated electron increases significantly as the heater beam's intensity increases. This is because as the intensity of the heater beam increases, so does the power of excited EPW.

Chapter 4

Beat Wave Excitation of Electron Plasma Waves via Cross-Focused q-Gaussian Laser Beams in Thermal Quantum Plasmas and Their Impact on Electron Acceleration

Abstract

A theoretical study of the excitation of an electron plasma wave in thermal quantum plasmas by beating two q-Gaussian laser beams is presented in this research. The nonlinear Schrodinger problem for the Drude model of the dielectric function of plasma has been numerically solved using moment theory in the W.K.B. approximation. Relativistic mass nonlinearity of plasma electrons has been used to represent optical nonlinearity of plasma. The intensity of both the first and second beams affects the laser-induced nonlinearity in the dielectric characteristics of plasma. Cross-focusing of the two laser beams occurs as a result of the propagation properties of one beam influencing those of the second. The background electron concentration changes as a result of the laser beams' uneven intensity distribution along their wavefronts. As a result, there is a nonlinear coupling between the laser beams and the amplitude of EPW, which is dependent on the background electron concentration. The impacts of laser-plasma characteristics on laser beam cross-focusing and their impact on EPW power have been studied through numerical simulations.

4.1 Introduction

When Maiman invented the laser in 1960, plasma physics underwent a sea change. His invention gave rise to the intricate area of laser-plasma interactions, which will continue to enthrall scientists for years to come. These days, high-power laser beams play a major role in plasma physics, especially when trying to start fusion reactions for the creation of useful energy. Laser-plasma interactions have expanded beyond fusion to encompass a wide range of possible uses. These have practical applications in the real world and include lightning protection, microwave guiding, and precipitation pattern manipulation. Even though they are more sophisticated, other uses include producing terahertz radiation, speeding electrons, and creating X-ray lasers are equally important for advancing science and technology.

Effective laser-plasma interactions are crucial for a wide range of applications. However, diffraction—a fundamental wave phenomenon—causes a light beam to spread as it propagates through a medium or vacuum. This diffraction-induced spreading significantly reduces the effectiveness of laser-plasma coupling, negatively impacting the performance of various applications. Consequently, methods that extend the interaction length between laser beams and plasmas to enhance coupling efficiency are gaining increasing attention. One such approach is self-focusing, a nonlinear phenomenon driven by the plasma's nonlinear response to the incident laser. Under the influence of a high-intensity laser beam, the plasma medium behaves like a convex lens, altering its refractive index. This modification in the refractive properties causes the beam to focus, as the change in light speed across the beam's wavefront results in curvature and spherical convergence.

Lately, low-temperature, high-density plasmas called quantum plasmas have become more common in a variety of settings and modern technology [18]. Applications in laser-produced plasmas [23], fast ignition [24–29], astrophysical systems [20–21], biophotonics [21–22], metallic nanoparticles [19–23], and quantum diodes and wells [30] are among them. The thermal de Broglie wavelength, $\lambda_B = \frac{\hbar}{(m_e K_B T)^{\frac{1}{2}}}$, separates classical and quantum versions of plasmas. Here, ħ, m_e, K_B, and T stand for the reduced Planck constant, electron mass, Boltzmann constant, and plasma temperature, respectively. The spatial extent of a particle's wave function, accounting for quantum uncertainty, is denoted as λ_B. When the electrons' de Broglie wavelength is equal to or greater than the average interaction distance, or$n_e \lambda_B^3 \geq 1$, quantum effects become relevant. Here, $n_e$represents the electron plasma density. Particles can be viewed as point-like in the classical domain because the de Broglie wavelength is modest enough to prevent quantum interference and wave function overlap. Put more simply, when the plasma temperature approaches or drops below the electron Fermi temperature, quantum effects become apparent [31]. The statistical distribution changes from Maxwell-Boltzmann to Fermi-Dirac when quantum effects are dominant. The definition of the Fermi temperature is [31].

$$K_B T_F = E_F \tag{4.1}$$

Where,

$$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n_e)^{\frac{2}{3}} \tag{4.2}$$

In this case, the terms T_F and E_F stand for the temperature and energy of Fermi, respectively. It is clear from equations (1) and (2) that in plasmas with lower temperatures and larger densities, quantum effects are amplified.

Several nonlinear processes, including EPW excitation, stimulated Raman scattering, and stimulated Brillouin scattering, are involved in laser-plasma interactions. Some of these phenomena cause electrons and ions to heat abnormally, while others reduce or change the direction of the incident laser energy, which reduces fuel pellet implosion efficiency in inertial confinement fusion (ICF). Therefore, to fully comprehend the physics of laser-plasma interaction, these phenomena must be thoroughly investigated through extensive research that combines theoretical analysis and experimental study.

Since the early days of plasma physics, it has been known that plasmas can support high-amplitude, nonlinear waves. The groundbreaking work of Akhiezer, Polovin, and Dawson prepared the way for a great deal of study into how these waves behave inside plasmas. Ultra-high-energy particle acceleration is one intriguing feature that attracted attention to this very specialized field. The Electron Plasma Wave (EPW), which is a coherent and dynamic structure created by powerful laser beams interacting with plasmas and taking use of the special features of the medium, is an example of one of these waves.

The overall electrical neutrality of plasma is maintained by the equal distribution of positive and negative charges, or ions and electrons, respectively. This balance is upset when a strong laser beam interacts with an object. Localized excesses of positive and negative charges result from the beam's basic displacement of lighter electrons away from heavier ions. Between these areas with varying charges, there is an electric field created by this charge differential. Both electrons and ions are subject to equal forces from the electric field, which causes them to move together.

Since electrons are somewhat lighter than ions, they go toward areas with positive charge whereas ions stay put most of the time. Electrons constantly accelerate and acquire momentum as they go from negative to positive areas. Electrons are propelled beyond positive regions as well as towards positive regions by this momentum. The outcome is a change in direction of the electric field, which first opposes and slows down the passage of electrons before drawing them back. A longitudinal wave with alternating positive and negative regions moves over the plasma as a result of this cycle being repeated.

Potential uses for the plasma wave produced by this technique include electron accelerators, plasma lasers, ionosphere manipulation, plasma heating, and current drive-in tokamaks (instruments for controlled fusion research). In particular, like a surfer riding a wave, electrons that just so happen to travel in the same direction and at a similar speed as the plasma wave get stuck inside of it. These electrons experience a non-oscillating electric field since they are moving in time with the wave. They are accelerated beyond thermal speeds by this electric field, which makes it easier for them to leave the plasma layer. The following explanation helps to clarify the process:

Suppose you have a boat crossing a lake; the lake is the plasma, and the boat is the 'drive beam' (the laser beam in this example). Wakefields are the waves created by the driving beam entering the lake. A "trailing beam," which is similar to a wake surfer riding the wake, follows behind the drive beam. After that, electrons are deposited onto these wakefields and accelerate there. Wake surfers use gravity to propel them forward while they ride downhill on the sea. In a similar vein, the pull of an electric field causes electrons and other particles to accelerate. By generating high-energy particles on a benchtop size, laser-driven plasma wave accelerators have the potential to herald in a new era and eventually replace large-scale accelerators like the Large Hadron Collider.

Large conventional particle accelerators may become unnecessary with the advent of laser-driven accelerators, which could revolutionize cancer therapy. Better clinical results in cancer radiation therapy are attributed to electrons accelerated by Electron Plasma Waves (EPWs). This benefit results from electrons' propensity to use up most of their energy immediately before stopping their motion when passing through materials. By varying the energy of the beam, ionization can be optimized exactly at the desired target, minimizing damage to healthy tissue.

Numerous elements of Electron Plasma Wave (EPW) excitation through the interaction of two strong laser beams in plasmas have been confirmed by vast literature since it was proposed by Rosenbluth and Liu [49]. Groundbreaking experimental research on the detection of beat plasma waves in plasmas with extended scale lengths was carried out by Amini and Chen [50]. Their research showed that accurate measurements of plasma wave frequencies could be obtained by collective Thomson scattering, which is accomplished by optically combining two counter-propagating CO2 laser beams in plasma. Resonant laser beat waves—a term used to describe the activation of relativistic plasma waves through collinear optical mixing—were experimentally validated by Clayton et al. [51] and later research by Darrow et al. [52, 53]. A model explaining the excitation of electron plasma waves through beat waves in plasma with rippling density was created by Darrow et al. [52, 53]. Leemans et al. [54] examined the nonlinear dynamics of laser-driven plasma beat waves in the presence of strong short-wavelength density ripples using the relativistic Lagrangian oscillator model. Sharma and Chauhan [55] investigated the impact of cross-focusing two coaxial laser beams on beat wave excitation in relativistic plasmas.

Different intensity profile laser beams behave differently in plasmas. The main goal of earlier studies was to describe the propagation of laser beams with uniform or Gaussian profiles. At the Rutherford Appleton Laboratory, experimental investigations by Patel et al. [56] and Nakatsutsumi et al. [57] point to variations from a fully Gaussian intensity profile for the Vulcan Petawatt laser. A q-Gaussian intensity profile appears to be a better fit for the reported laser beam characteristics based on the experimental results. The expression for this profile is $f(r) = f(0)(1 + \frac{r^2}{qr_0^2})^{-q}$. Fitting to experimental data allows for the determination of the parameters q and r_0.The literature that is currently available points to a theoretical research gap concerning the excitation of EPW in quantum thermal plasmas by q-Gaussian irradiance laser beams along their wavefronts. This work intends to investigate, for the first time, how q-Gaussian laser beam cross-focusing affects EPW excitation in thermal quantum plasmas.

4.2 Evolution of Beam Envelopes

Let us study the propagation of an underdense thermal quantum plasma of two coaxial linearly polarized laser beams represented by electric field vectors, E_j (r,z,t) where j = 1,2.By utilizing Maxwell's formulas for an isotropic, nonconducting, non absorbing medium (J = 0, ρ=0,μ=0), the following results are obtained:

$$\nabla \times B_j = \frac{1}{c}\frac{\partial D_j}{\partial t} \tag{4.3}$$

$$\nabla \times E_j = -\frac{1}{c}\frac{\partial B_j}{\partial t} \tag{4.4}$$

The laser beams' magnetic fields are denoted by B_j whereas the electric displacement vectors are represented by $D_j = \epsilon_j E_j$. Equations (4.3) and (4.4) when combined show that the laser beams' electric fields E_j (r,z,t) meet the wave equation:

$$\nabla^2 E_j - \nabla(\nabla . E_j) + \frac{\omega_j^2}{c^2}\epsilon_j E_j = 0 \quad (4.5)$$

The angular frequencies of the laser beams are represented by ω_j in this case. The polarization term $\nabla(\nabla . E_j)$in equation (4.5) can be disregarded even if $E_j$contains longitudinal components since the laser beams' root mean square (r.m.s) beam radii are significantly larger than their vacuum wavelengths[58]. This approximation reduces equation (4.5) to:

$$\nabla^2 E_j + \frac{\omega_j^2}{c^2}\epsilon_j E_j = 0 \quad (4.6)$$

In order to investigate the overall behavior of the beam, it makes sense to use the slowly shifting envelope approximation to distinguish between the rapid oscillations in the laser field phases, denoted by $\psi = \omega_j t - k_j z$, and their amplitudes $A_j(r, z)$[59].

$$E_j = A_j(r, z) e^{\iota\psi j} e_x \quad (4.7)$$

The effective dielectric function of thermal quantum plasma can be described as follows when taking into account quantum mechanical factors such as the Bohm potential, the Drude model, and Fermi gas pressure[18,60,61]:

$$\epsilon_j = 1 - \frac{\omega_{p0}^2}{\gamma(\omega_j^2 - \gamma^{-1}\alpha k_j^4 - k_j^2 v_F^2)} \quad (4.8)$$

Here, the electron quantum diffraction resulting from the quantum correction of density fluctuation is represented by $\alpha = \frac{\hbar}{4m_0^2}$.

The electron plasma frequency is represented as $\omega_{p0} = (\frac{4\pi e^2}{m_0} n_0)^{\frac{1}{2}}$, where e is the electronic charge. The relativistic Lorentz factor is $\gamma = (1 + \Sigma_j A_j A_j^*)^{\frac{1}{2}}$, and the Fermi speed is $v_{F0} = \sqrt{\frac{2K_B T_F}{m_0}}$. Relativistic nonlinearity coefficients are represented by the formula $\beta_j = \frac{e^2}{m_0 c^2 \omega_j^2}$.

Expressing equation (4.8) as

$$\epsilon_j = \epsilon_{0j} + \emptyset_j(A_1 A_1^*, A_2 A_2^*) \quad (4.9)$$

We get,

$$\epsilon_{0j} = 1 - \frac{\omega_{p0}^2}{\omega_j^2} \quad (4.10)$$

$$\emptyset_j(A_1A_1^*, A_2A_2^*) = 1 - \frac{\omega_{p0}^2}{\omega_j^2}\{1 - \frac{\gamma^{-1}}{(1 - \frac{\gamma^{-2}\alpha k_j^4}{\omega_j^2} - \frac{k_j^2 v_F^2}{\omega_j^2}\gamma^{-1}}) \quad (4.11)$$

Here, the nonlinear components of the dielectric function are represented by $\emptyset_j$, and the linear portions are indicated by ϵ_{0j}. Equations (4.7) and (4.9) in equation (4.6) are used to derive

$$\iota \frac{dA_j}{dz} = \frac{1}{2k_j}\nabla_\perp^2 A_j + \frac{k_j}{c^2}\emptyset_j(A_1A_1^*, A_2A_2^*)A_j \qquad (4.12)$$

Where, along the transverse direction, $\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$represents the Laplacian. One of the fundamental equation models for nonlinear waves in many fields of physics is the nonlinear Schrodinger wave equation (NSWE), or Eq. (4.12). It is a general equation that describes a wave train's slowly changing envelope in nonlinear mediums. Since the wave amplitude scale length along the z-axis is significantly bigger than the characteristic scale in the traverse direction, the quantity $\frac{\partial^2 A_j}{\partial z^2}$ has been ignored in the derivation of equation (4.12).

In this analysis, the Nonlinear Schrödinger Wave Equation (NSWE) will be solved using the use of the method of moments [58, 62]. The dynamics of electromagnetic beam propagation across nonlinear media is studied using the method of moments. This method was originally devised by Valasov et al. [63] for the cubic Schrödinger equation (CSE) in 2-D cylindrical symmetry. It was initially used to study the phenomenon of self-focusing. This approach was then used by Zakharov [64] in his seminal work on Langmuir wave collapse in plasma. Subsequently, the technique was expanded to the vectorial cubic Schrödinger equation (CSE) in arbitrary dimensions by Goldman and Nicholson [65] and Goldman et al. [66], eliminating the constraints on radial symmetry.

The spatial moments of approximate test functions that are intended to closely resemble the real solution of the Nonlinear Schrödinger Wave Equation (NSWE) are computed using the moments method. Given a suitable number of parameters, the method of moments yields a qualitative comprehension and reasonably accurate quantitative findings when the genuine solution preserves the initial shape and shows self-similarity with the test functions. In light of our emphasis on the induced focusing of laser beams, we take into account the trial functions listed below [67–70]:

$$A_j A_j^* = \frac{E_{j0}^2}{f_j^2}\left(1 + \frac{r^2}{q_j r_j^2 f_j^2}\right)^{-q_j} \tag{4.13}$$

From the foregoing, the function $f_j(z)$ appears to determine two characteristics of the beam envelope. When multiplied by r_j for any arbitrary value of z, it represents the beam width. Furthermore, during propagation and focussing, f_j calculates the amplitude of the laser beams in the manner $\frac{E_{j0}}{f_j}$ at any arbitrary distance z. The laser beams' intensity distribution deviates from a Gaussian intensity distribution, as indicated by the parameters q_j. The laser beams' intensity distribution converges towards the Gaussian distribution as q_j approaches infinity, exactly becoming Gaussian for $q_j = \infty$, that is to say,

$$\lim_{q_j \to \infty} = \frac{E_{j0}^2}{f_j^2} e^{-\frac{r^2}{r_j^2 f_j^2}}$$

Imperfections in the laser cavity systems are the main source of the amplitude structure of laser beams deviating from the ideal Gaussian profile. The amplitude structure's wings expand as a result of these flaws, which cause a sizable number of photons to be released away from the beam axis. The oscillations of an electromagnetic field, which resemble playground swing movements, can be used to explain this phenomenon.

Like a light wave, the oscillation of the swing has two components: an amplitude that indicates the height achieved in each cycle and a certain frequency that indicates the number of cycles performed in a given amount of time. Similar to two light waves, the relative motion of two kids on swings is determined by their phase relationship: if both of them hit the highest point at the same time every cycle, they swing in phase. On the other hand, they are deemed somewhat out of phase if they reach their peaks at different times.

Imagine a situation when a kid on a playground makes unusual use of a swing. Throughout the swing's cycle, this child alternates between standing on the seat and squatting, instead of the more common back-and-forth action. The child increases the amplitude of the swing's motion by actively opposing centrifugal force by kneeling at the top of the swing and standing up when it is closest to the ground. As a result, the swing gets higher and higher with every new cycle. On the other hand, if the child stands when the swing rises and squats when it falls, they lessen the swing's motion and get shorter in each cycle.

The child's pumping motion in our analogy demonstrates two different traits. First, during each round trip of the swing, the youngster switches positions twice between standing and squatting. Technically speaking, the pumping movement occurs twice as frequently as the swing motion. Second, the relative phase between the child's standing and squatting positions, especially at the highest point of the swing, can increase or decrease the amplitude of the swing's motion.

Likewise, a light wave contained in an extended cavity with mirrors on both ends can be subjected to a similar pumping action. When a light wave's wavelength is exactly matched by the cavity's length, interference occurs when the confined wave reflects off of the cavity's ends. The wave reverberates as a standing wave as a result of this interference, much like sound waves do inside organ pipes. Imagine a situation in which the length of the cavity changes at a frequency that is precisely twice that of the light wave due to the oscillation of one of the mirrors at the end of the cavity. This repeated adjustment of the cavity's length would result in an addition or subtraction of energy from the light wave upon reflection from the mirror. The mirror's vibration would, like the child's pumping motion, either increase or decrease the light wave's intensity based on its phase relationship with the oscillation of the light wave. A stronger oscillating electromagnetic field is produced when the motion of the mirror coincides with the proper phase of the oscillation of the light wave. On the other hand, the wave is de-amplified and the oscillating electromagnetic field is weakened if the motion of the mirror is in phase with the opposite phase.

Using our previous example again, picture a playground full of kids swinging at the same rate but not in unison. At any given moment, some children may be closer to the top of their swing cycle, while others may be closer to the bottom. Now picture a teacher coming into the playground and shouting, "Stand... squat... stand... squat..." over a megaphone at exactly twice the rate at which the children are swinging. Some children will be near or at the top of their swing cycle when the teacher cries, "Stand," while others will be near or at the bottom. When the teacher gives the order, kids who stand close to the bottom will intensify their swing action, while kids who stand close to the top will de-amplify it.

Children who are closer to the bottom of their swing cycle will eventually swing with a much larger amplitude, while those who are closer to the top will swing less. In this situation, kids start swinging at considerably higher altitudes, almost in unison with the teacher's directions and each other. In the meantime, a different set of kids keeps standing and squatting but pretty much stops swinging. These kids, in our analogy, are photons that are emitted out from the central axis and contribute to the wings of the intensity profile being broader. Establishing the overall beam power as stated by

$$N_j = \int_0^{2\pi} \int_0^{\infty} \left|A_j\right|^2 r dr d\theta \tag{4.14}$$

Equations (4.11) and (4.12) correspond to the global Hamiltonian.

$$H_j = \int_0^{2\pi} \int_0^{\infty} \frac{1}{2k_j^2} \left(\left|\nabla_{\perp} A_j\right|^2 - F_j \right) r dr d\theta \tag{4.15}$$

Where,

$$F_j = \frac{1}{2\epsilon_{0j}} \int_0^{A_j A_j^*} \emptyset_j (A_1 A_1^*, A_2 A_2^*) d\left(A_j A_j^*\right) \tag{4.16}$$

$\frac{\partial N_j}{\partial z} = \frac{\partial H_j}{\partial z} = 0$, may be shown, proving that N_j and H_j are invariants of wave equation (4.12). The first invariant, N_j indicates that photon number conservation occurs in laser beams; the second invariant, H_j, connects the laser beams' wavefront curvatures to plasma nonlinearity. These invariants, which are based on the symmetry of the Schrödinger wave equation under gauge transformation and transformation along the z-direction, can be found in the Lagrangian density for equation (4.12). It is noteworthy that when the nonlinear refraction term predominates over the dispersion term, H_j need is not positive definite; rather, it is negative.

The average value of a physical quantity $M_{j(z)}$ is defined in the moment theory method as

$$\langle M_j^2 \rangle = \frac{1}{N_j}\int_0^{2\pi}\int_0^{\infty} A_j M_j(z) A_j^* r dr d\theta (4.17)$$

The mean square radius of laser beams is a number that is particularly significant from the perspective of self-focusing.

$$\langle r_j^2 \rangle = \frac{1}{N_j}\int_0^{2\pi}\int_0^{\infty} A_j r^2 A_j^* r dr d\theta (4.18)$$

By using Goldman et al.'s method (66), the following results are obtained:

$$\frac{d^2}{dz^2}\langle \sigma_j^2(z) \rangle = 4\frac{H_j}{P_j} + \frac{R_j}{P_j} \qquad (4.19)$$

Where

$$R_j = 4\int_0^{2\pi}\int_0^{\infty} (2F_j - \frac{1}{2\epsilon_{0j}} A_j A_j^* \emptyset_j) r dr d\theta (4.20)$$

By calculating the spatial moments [58–62] of the envelope equation (4.12), this can be found directly.

Equations (4.13)–(4.16), (4.18), and (4.20) in (4.19) can be used to create the following set of coupled differential equations that control how laser beam spot sizes change with propagation distance.

$$\frac{d^2 f_1}{d\xi^2} + \frac{1}{f_1}(\frac{df_1}{d\xi})^2 = \frac{\left(1-\frac{1}{q_1}\right)\left(1-\frac{1}{q_1}\right)}{\left(1+\frac{1}{q_1}\right)}\frac{1}{f_1^3} - \frac{1}{2}(1-\frac{1}{q_1})(1-\frac{2}{q_1})(\frac{\omega_{p0}^2 r_1^2}{c^2})J_1 \qquad (4.21)$$

$$\frac{d^2 f_2}{d\xi^2} + \frac{1}{f_2}(\frac{df_2}{d\xi})^2 = (\frac{r_1}{r_2})^4(\frac{\omega_1}{\omega_2})^2(\frac{\epsilon_{01}}{\epsilon_{02}})\{\frac{\left(1-\frac{1}{q_1}\right)\left(1-\frac{1}{q_1}\right)}{\left(1+\frac{1}{q_1}\right)}\frac{1}{f_2^3} - \frac{1}{2}\left(1-\frac{1}{q_1}\right)\left(1-\frac{2}{q_1}\right)\left(\frac{\omega_{p0}^2 r_1^2}{c^2}\right)J_2$$

(4.22)

where

$$J_1 = \frac{\beta_1 E_{10}^2}{f_1^3} T_1 + \frac{\beta_2 E_{20}^2}{f_1^3}((\frac{r_1}{r_2})^2(\frac{f_1}{f_2})^4 T_2$$

$$J_2 = \frac{\beta_1 E_{10}^2}{f_2^3} T_3 + \frac{\beta_2 E_{20}^2}{f_2^3}((\frac{r_1}{r_2})^2(\frac{f_1}{f_2})^4 T_4$$

$$T_1 = \int_0^\infty G_1 x(1 + \frac{x}{q_1})^{-2q_1-1} dx$$

$$T_2 = \int_0^\infty G_1 x(1 + \frac{x}{q_1})^{-q_1} (1 + \frac{1}{q_2} (\frac{r_1}{r_2}\frac{f_1}{f_2})^2 x)^{-2q_1-1} dx$$

$$T_3 = \int_0^\infty G_2 x(1 + \frac{x}{q_1})^{-q_1-1} (1 + \frac{x}{q_2} (\frac{r_1}{r_2}\frac{f_1}{f_2})^2)^{-q_2} dx$$

$$T_4 = \int_0^\infty G_2 x(1 + \frac{x}{q_2} (\frac{r_1}{r_2}\frac{f_1}{f_2})^2)^{-2q_2-1} dx$$

$$G_j = \frac{\left(1 - F^{-1}\frac{\alpha k_j^4}{\omega_j^2} - \frac{k_j^2 v_F^2}{\omega_j^2}\right) F^{-3} + F^{-4}\frac{\alpha k_j^4}{\omega_j^2}}{\left(1 - F^{-1}\frac{\alpha k_j^4}{\omega_j^2} - \frac{k_j^2 v_F^2}{\omega_j^2}\right)^2}$$

$$F = \{1 + \frac{\beta_1 E_{10}^2}{f_1^2}(1 + \frac{x}{q_1})^{-q_1} + \frac{\beta_2 E_{20}^2}{f_2^2}(\frac{1}{q_2}(\frac{r_1}{r_2}\frac{f_1}{f_2})^2 x)^{-q_2}\}^{\frac{1}{2}}$$

$$x = \frac{r^2}{r_1^2 f_1^2}$$

$$\xi = \frac{z}{k_1 r_1^2}$$

Equations (21) and (22) for initially plane wavefronts are subject to the boundary conditions, $f_j = 1, \frac{df_j}{d\xi} = 0$ at $\xi = 0$.

4.3 Various Mechanisms for Excitation of EPW:

Laser beams can excite EPWs using two main mechanisms:

Resonance Absorption:

Resonant absorption is the term for the first phenomenon. A large amount of the laser beam's fluctuating electric field energy is transferred to the nearby electrons when it enters the plasma. However, the laser frequency and the inherent frequency of Electron Plasma Waves (EPWs) coincide close to the critical-density surface. This energy can resonantly drive EPWs to large amplitudes at this depth in the plasma, similar to how a toddler on a swing can grow taller by synchronizing pumping with the swing's natural motion.

Three Wave Mixing:

Three-wave mixing is the second technique used to excite Electron Plasma Waves (EPWs). An incoming laser beam divides into two separate daughter waves during this procedure. In general, when the interacting wave amplitudes are large, this mixing process is most noticeable. The frequency of the primary wave is obtained by adding the frequencies of the two ensuing daughter waves. The creation of EPWs is stimulated by three-wave mixing via two different methods. The first process takes place when the main beam splits into two EPWs which is known as two-plasmon decay. The creation of daughter waves made up of an EPW and a reflected light wave is the outcome of the second mechanism, which is called stimulated Raman scattering. We consider resonance absorption to be the principal mechanism responsible for generating Electron Plasma Waves (EPWs) in our current study.

4.4 Dynamics of Excited EPW:

It is important to understand that ions are only marginally involved in Electron Plasma Waves (EPWs), mostly functioning as a positive background in the plasma. The electron population of the plasma bears the principal responsibility for excitation of epicentres. The nonuniform intensity distribution of the laser beam along its wavefront alters the density of the background plasma. As such, the laser beam properties and the amplitude of the electron plasma wave, which is dependent upon the background electron density, are intimately entwined.

Three basic equations control the evolution of the created plasma wave: Poisson's equation, the equation of motion, and the equation of continuity. These formulas are essential for explaining how the plasma wave behaves and develops:

$$\frac{\partial N}{\partial t} + \nabla\, .\, (Nv) = 0 \tag{4.23}$$

$$m_0 \frac{dv}{dt} = -eE - 3\frac{K_0 T_0}{N_0} \nabla N + \frac{\hbar^2}{2m_0^2} \nabla \left(\frac{1}{\sqrt{N}} \nabla^2 \sqrt{N} \right) \tag{4.24}$$

$$\nabla . E = -4\pi e N \tag{4.25}$$

Here, N is the total electron density and E is the sum of the electron plasma wave electric fields and of the laser beams' electric field.

$$N = n_0 + n'$$
$$E = \sum_j E_j + E'$$

Electron oscillation velocity is given by v = v_e. These equations are analyzed using linear perturbation theory.

$$\frac{\partial^2 n'}{\partial t^2} - v_{th}^2 \nabla^2 n' + \omega_p^2 n' - \frac{\hbar^2}{4m_0^2} \nabla^4 n' = \frac{e}{m_0} n_0 \nabla \sum_j E_j \quad (4.26)$$

Taking,

$$n' = n_1 e^{\iota(\omega t- \quad)}$$

The density perturbation linked to the plasma wave is obtained where k=k_2–k_1.

$$n_1 = \frac{e n_0}{m_0} \frac{1}{(\omega^2 - k^2 v_{th}^2 - \omega_p^2)} \left[\frac{E_{10}}{r_1^2 f_1^3} (1 + \frac{r^2}{q_1 r_1^2 f_1^2})^{-\frac{q_1}{2}-1} + \frac{E_{20}}{r_2^2 f_2^3} (1 + \frac{r^2}{q_2 r_1^2 f_1^2})^{-\frac{q_2}{2}-1} \right] r \tag{4.27}$$

Applying Poisson's equation

$$\nabla . E' = -4\pi e n'$$

$$E' = E_{e.p} e^{\iota(\omega t - kz)}$$

We get

$$E_{e.p} = \frac{\frac{\iota}{k} \omega_{p0}^2}{(\omega^2 - k^2\, v_{th}^2 - \omega_p^2)} \sum_j \frac{E_{j0}}{r_j^2 f_j^2} (1 + \frac{r^2}{q_2 r_1^2 f_1^2})^{-\frac{q_2}{2}-1} r \tag{4.28}$$

The electron wave's normalized power is described as

$$P = \frac{P_{e.p}}{P_1}$$

Where,

$$P_{e.p} = \frac{v_g}{8\pi}\int_0^\infty E_{e.p}E_{e.p}^* 2\pi r dr$$

$$P_1 = \frac{c}{8\pi}\int_0^\infty E_1 E_1^* 2\pi r dr$$

$$v_g = v_{th}(1 - \frac{\omega_{p0}^2}{\omega^2})^{\frac{1}{2}}$$

We get

$$P = 2\left(\frac{v_g}{c}\right)\left(1 - \frac{1}{q_1}\right)\frac{\omega_{p0}^2}{r_1^2 k^2 \omega^4}\int_0^\infty \frac{1}{D(\omega_1,\omega_2)} X(E_{10}, E_{20}) dx \tag{4.29}$$

Where,

$$D(\omega_1,\omega_2) = \{1 - \frac{k^2 v_{th}}{\omega^2} - \frac{\omega_{p0}^2}{(\omega_2-\omega_1)^2}(1 + \sum_j \frac{\beta_j E_{j0}^2}{f_j^2}(1 + \frac{x}{q_j}(\frac{r_1 f_1}{r_j f_j})^{-2})^{-q_j})^{-\frac{1}{2}}\}^2$$

$$X(E_{10}, E_{20}) = [\frac{E_{10}}{f_1}(1 + \frac{x}{q_j})^{-\frac{q_1}{2}-1} + \frac{E_{20}}{f_2}((\frac{r_1 f_1}{r_j f_j})^2)(1 + \frac{x}{q_j}(\frac{r_1 f_1}{r_j f_j})^2)^{-\frac{q_2}{2}-1}]^2$$

4.5 Electron Acceleration by the Excited EPW

The acceleration of the particle does not follow the intuitive notion of acceleration as being an increase in velocity over time. Charged particles in present-day high energy accelerators travel at nearly the speed of light. An electron from a 50-GeV accelerator falls short of the speed of light by only five parts in 10^{11}, that is, if an electron raced a light pulse around the earth, the electron would cross the finish line only 2.1 millimeter behind the light. When particles travelling at these speeds absorb energy from a field, they are accelerated in the sense that their mass increases in accordance with Einstein's theory of relativity. The particles' velocity, however, increases very little. In order to accelerate (to add mass to) a beam of charged particles already moving at very high speed, a plasma accelerator must create a plasma wave travelling at nearly the speed of light so that charged particles do not outrun the electric field wave.

The energy gained by the accelerated electron is given by

$$\frac{d\gamma}{dt} = \frac{e}{mc^2} E_{ep} v_z \tag{4.30}$$

Here, v_z is the velocity of accelerated electron and $\gamma = \sqrt{1 + \frac{p_e}{m^2c^2}}$. Using transformation $\frac{1}{v_z}\frac{d}{dt} = \frac{d}{dz}$ we get

$$\frac{d\gamma_e}{dz} = \frac{e}{m^2c^2}E_{ep} \tag{4.31}$$

Eq. (4.31) gives the energy of accelerated electron as a function of distance of propagation.

4.6 Results and Discussion:

Equations (4.21) and (4.22) include a coupled system of nonlinear differential equations that control the cross-focusing of two coaxial q-Gaussian laser beams in an underdense plasma. The normalized power of the electron plasma wave produced by the interference of these two laser beams is given by equation (4.29). Since there are no analytical solutions for these equations, the dynamics of the beams are studied using numerical computing techniques. Understanding the physical mechanisms underlying the various terms on the right-hand side of equations (4.21) and (4.22) is crucial before moving forward with numerical simulations. The main terms on the right-hand sides of equations (4.21) and (4.22) come from the Laplacian $\nabla^2\perp$ found in the nonlinear Schrödinger wave equation (4.12) and explain the diffraction divergence of the laser beams. The combined effects of nonlinear coupling between the two laser beams and relativistic mass nonlinearity give birth to the secondary terms on the right-hand sides of these equations. The nonlinear refraction of the laser beams is caused by these terms. Whether the laser beams focus or defocus in the plasma environment depends on how the diffractive and refractive factors interact.

Numerical solutions were obtained for equations (4.21)–(4.22) and (4.29) using the following set of parameters:

$$\omega_1 = 1.78 \times 10^{15}\text{rad/sec}$$

$$\omega_2 = 1.98 \times 10^{15}\text{rad/sec}$$

$$r_1 = r_2 = 15\mu\text{m}$$

$$T_0 = 10^7\text{K}$$

$$T_F = 10^9\text{K}$$

The objective of the numerical solution was to examine the effects of laser and plasma parameters on the creation of beat waves in the electron plasma wave and the cross-focusing of laser beams.

The effects of the first laser beam's deviation in intensity distribution (q_1) from a Gaussian profile on the two laser beams' focusing and defocusing behaviors are shown in Figures 4.1 and 4.2. The rhythmic fluctuations in laser intensity with respect to the dimensionless propagation distance are shown in Figures 4.1 and 4.2. Periodic changes in the laser beams' focusing and defocusing are the source of these oscillations. The self-focusing of the laser beams during propagation results in a decrease in their spot sizes ($r_i f_i$), which raises diffraction and cancels out the nonlinear refractive terms in equations (4.21) and (4.22). This slows down the self-focusing process until it reaches$f_j$minimum. Diffraction effects become more noticeable, but they are still insufficient to compensate for self-focusing. This process continues until the laser beams' nonlinear refractive effects outweigh the effects of diffraction. Laser intensity fluctuates periodically as a result of these cyclical processes.

The plots in Figure 4.1 show that a decrease in the degree of self-focusing coincides with an increase in the value of q_1. This pattern emerges because the initial laser beam's intensity focuses more in the axial region of the wavefront as q_1 rises toward greater values.This causes axial rays' diffraction divergence to be more noticeable than off-axial rays'. The laser beams focus more quickly at higher q values. This phenomenon is mostly explained by the off-axial rays' slower focusing characteristic.

As seen in Figure 4.2, a higher value of q_1 indicates that the second laser beam is self-focusing to a greater degree. This effect arises from the fact that the refractive term in equation 4.22 has a greater amplitude when q_1 increases.

Phase-space charts (Figs. 4.3, 4.4) provide useful information to understand the mechanics of laser beam propagation. The phase-space spiral trajectories show that both lasers' beam widths oscillate, with the oscillations being either quasi-periodic or multi-frequency in nature. These plots also show that the laser beam concentrating initially happens gradually. But the pace of focusing increases rapidly with increasing intensity of self-focusing. This acceleration happens because the laser beams' intensities rise as they self-focus, intensifying the nonlinear refractive properties of the plasma. Additionally, it is noted that raising q_1 results in a higher rate of focusing for both laser beams. The science behind the finding is based on the fact that when q_1 increases, the intensity of the original laser beam converges further towards the axial area of the wavefront. Axial rays focus faster than off-axial rays as a result. The explanation for the higher rate of focusing for the second laser beam with greater q_1 values is because relativistic nonlinearity in the refractive properties of the plasma depends on the intensity of both laser beams. The two laser beams are coupled as a result of their mutual influence, such that an increase in one beam's focusing rate also increases the other beam's focusing rate.

The Fermi temperature of the plasma affects the two laser beams' focusing and defocusing behaviours, as seen in Figures 4.5 and 4.6. A larger degree of self-focusing for both laser beams has been reported when the Fermi temperature increases. This happens as a result of the plasma's dielectric constant being raised above its Fermi temperature, which increases the laser beams' ability to focus on themselves.

The effects of variations in plasma density on laser beam focusing and defocusing are shown in Figures 4.7 and 4.8. More self-focusing for both laser beams is correlated with increasing plasma density. Higher plasma densities lead the nonlinear refractive components in equations (4.21) and (4.22) to become more significant, which improves the laser beams' ability to focus on themselves.

The effect of the first laser beam's deviation in intensity distribution from a Gaussian profile on the resultant plasma wave's amplitude is depicted in Figure 4.9. It is noted that the two laser beams' focus locations are where the created plasma wave's amplitude peaks. This is because the regions of unusually high intensity that these focal spots represent serve as sources for the creation of plasma waves. Increasing the amount of q_1 causes the plasma wave's amplitude at the first laser beam's focal places to decrease, whilst the second laser beam's focal locations experience a rise in amplitude (Figure 4.9). Raising q_1 causes this phenomena by increasing self-focusing in the second laser beam and decreasing it in the first. Figures 4.10 and 4.11 illustrate how Fermi temperature and plasma density affect the power of the generated plasma wave. As either the Fermi temperature or the plasma density rises, the resultant plasma wave's power increases noticeably. This enhancement happens as a result of stronger laser beam focusing caused by higher Fermi temperature or plasma density, which raises the power of the ensuing plasma wave.

Figs4.12-4.14 illustrates the effect of deviation parameter q_1 of the beam 1, Fermi temperature of plasma electrons and plasma density, respectively on the energy gained by the accelerated electrons. It can be seen that with increase in the value of either of these parameters there is increase in the energy gained by the accelerated electrons. This is due to the enhancement of the power of excited EPW with increase in the value of either of these parameters.

4.7 CONCLUSIONS:

The author has examined the cross-focusing of two intense coaxial q-Gaussian laser beams in thermal quantum plasma and its subsequent effects on beat wave excitation of electron plasma waves. The current analysis has led to the following significant conclusions:

- The impact of self-focusing of one laser beam increases whereas that of other beam's self-focusing effect decreases when the intensity distribution of the first one deviates from Gaussian distribution.
- Greater amplitude of the ensuing plasma wave and self-focusing of laser beams are both correlated with higher Fermi temperature or density of plasma.
- The focal places where the two laser beams converge are where the created plasma wave's amplitude peaks.
- When one laser beam's q value is raised, the plasma wave's amplitude at its focal points decreases, but the plasma wave's amplitude at the other beam's focal points increases concurrently.

This work is relevant to material processing and other industrial applications that require focused energy from coupled laser beams, like laser-driven fusion for fuel pellet compression.

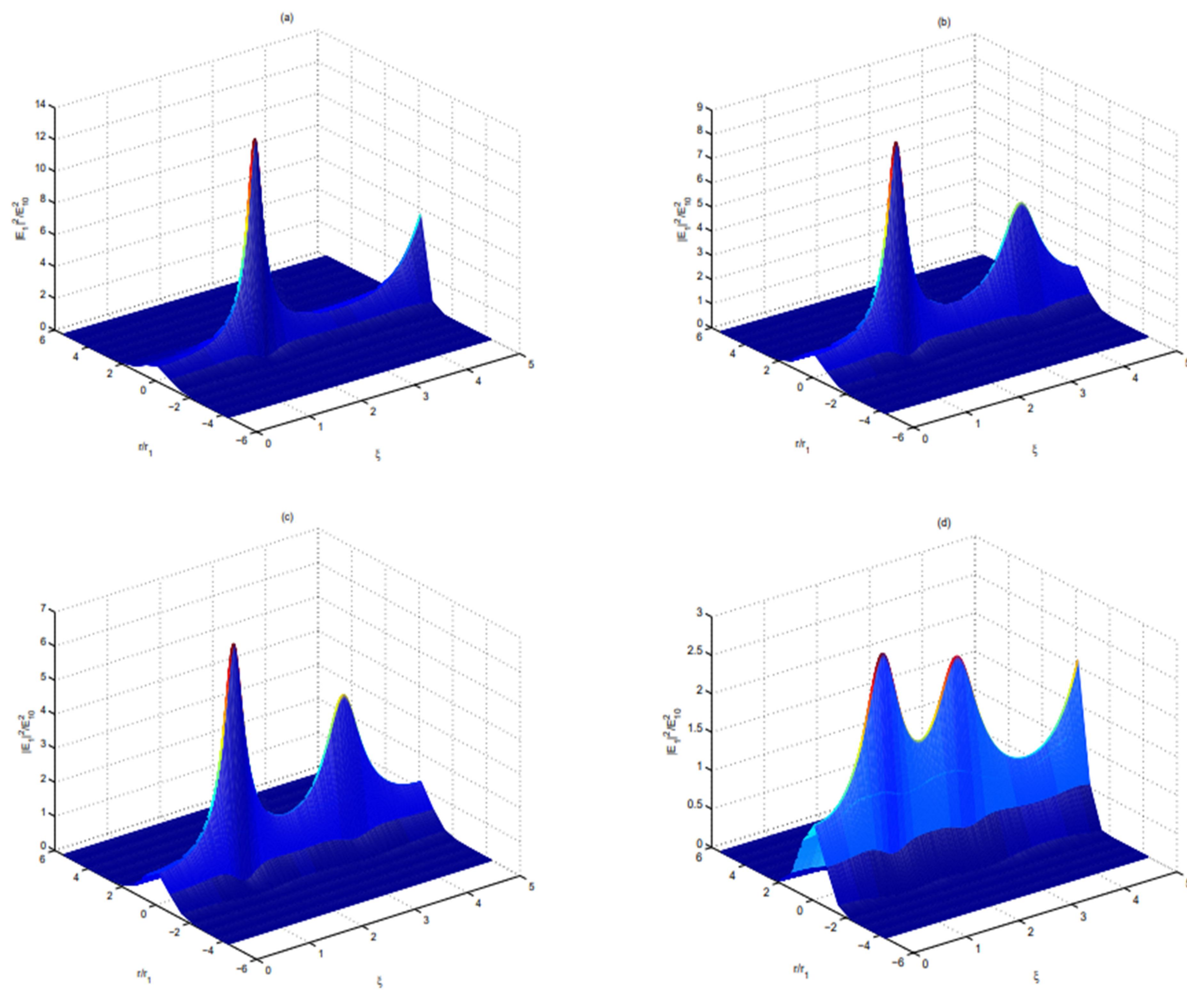

Figure 4.1: Effect of radial distance $\frac{r}{r_1}$ and normalized distance of propagation ξ on the normalized intensity of first beam, putting different values of q_1, (a) q_1=3, (b) q_1=4, (c) q_1=5, (d) q_1=∞ and keeping $(\frac{\omega_{p0} r_1}{c})^2$= 9, βE_{10}^2=3.0, βE_{20}^2=3.50, q_2=3, $T_F = 10^9$K fixed respectively.

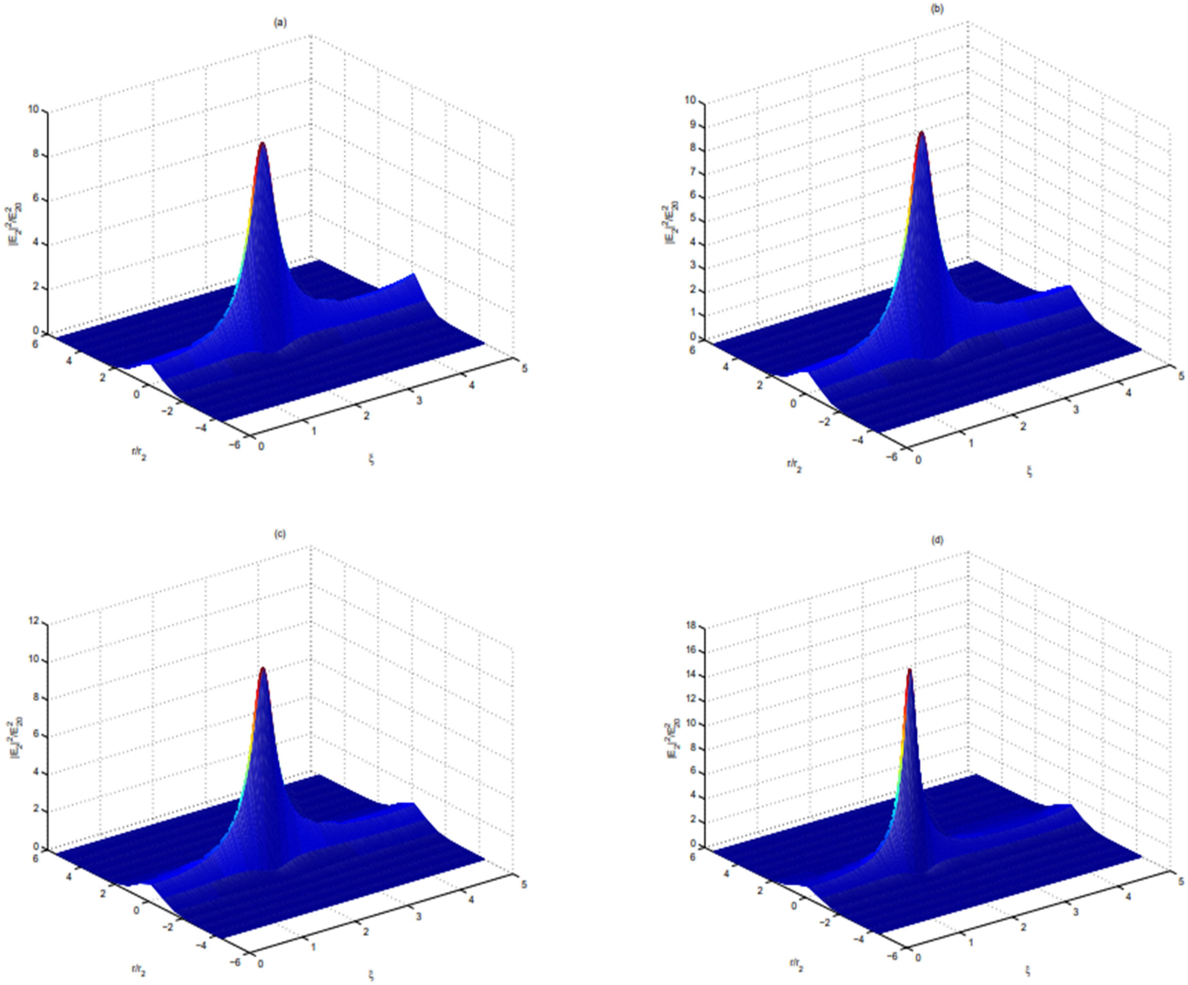

Figure 4.2: Effect of radial distance $\frac{r}{r_1}$ and normalized distance of propagation ξ on the normalized intensity of second beam, putting different values of q_1, (a) q_1=3, (b) q_1=4, (c) q_1=5, (d) q_1=∞ and keeping $(\frac{\omega_{p0} r_1}{c})^2$= 9, βE_{10}^2=3.0, βE_{20}^2=3.50, q_2=3, T_F = 10^9K fixed respectively.

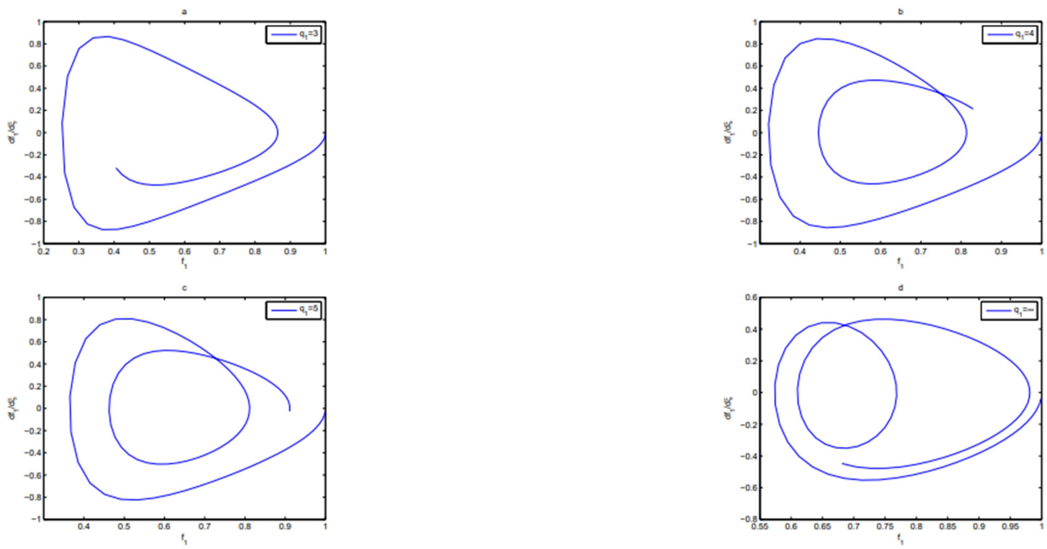

Figure 4.3: Phase space plots for first beam at different values of q_1, (a) q_1=3, (b) q_1=4, (c) q_1=5, (d) q_1=∞ and keeping $(\frac{\omega_{p0} r_1}{c})^2$= 9, βE_{10}^2=3.0, βE_{20}^2=3.50, q_2=3, T_F = 10^9K fixed respectively.

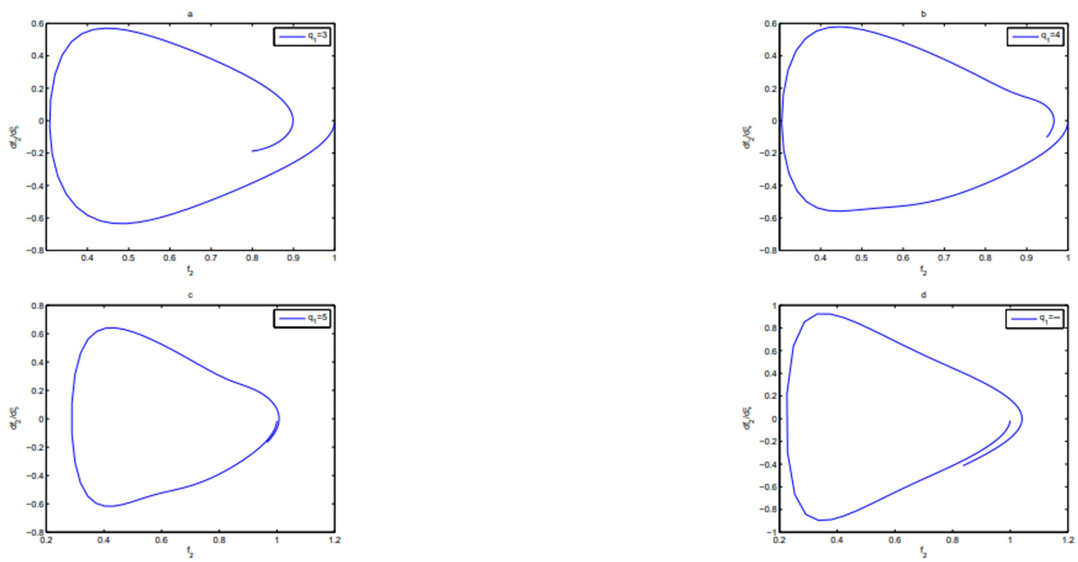

Figure 4.4: Phase space plots for second beam at different values of q_1, (a) q_1=3, (b) q_1=4, (c) q_1=5, (d) q_1=∞ and keeping $(\frac{\omega_{p0} r_1}{c})^2$ = 9, βE_{10}^2=3.0, βE_{20}^2=3.50, q_2=3, T_F = 10^9K fixed respectively.

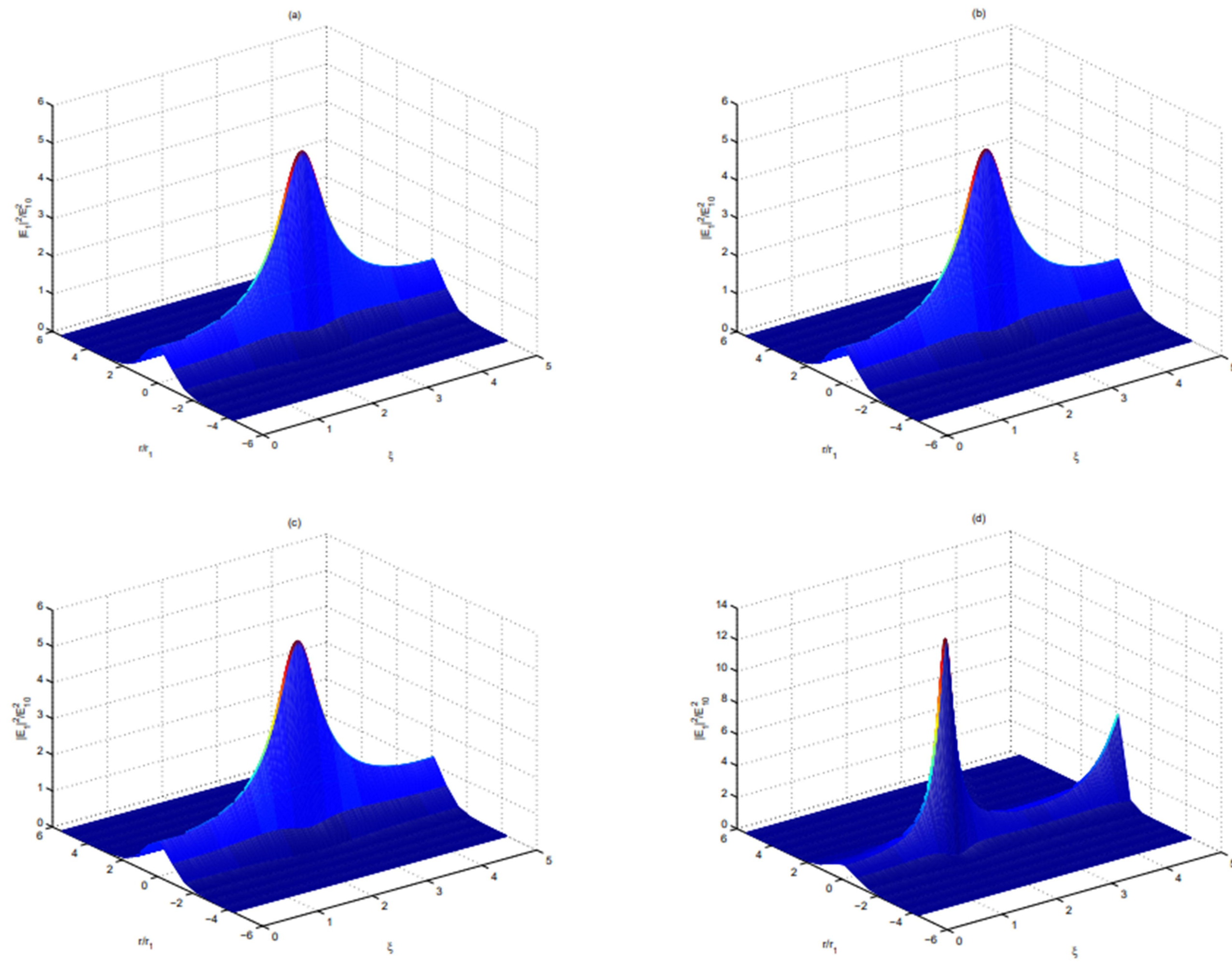

Figure 4.5: Effect of radial distance $\frac{r}{r_1}$ and normalized distance of propagation ξ on the normalized intensity of first beam, putting different values of Fermi temperature, (a) T_F = 10^6K, (b) T_F = 10^7K, (c) T_F = 10^8K, (d) T_F = 10^9K and keeping $(\frac{\omega_{p0}r_1}{c})^2$= 9, βE_{10}^2=3.0, βE_{20}^2=3.50,q_1=3, q_2=3 fixed respectively.

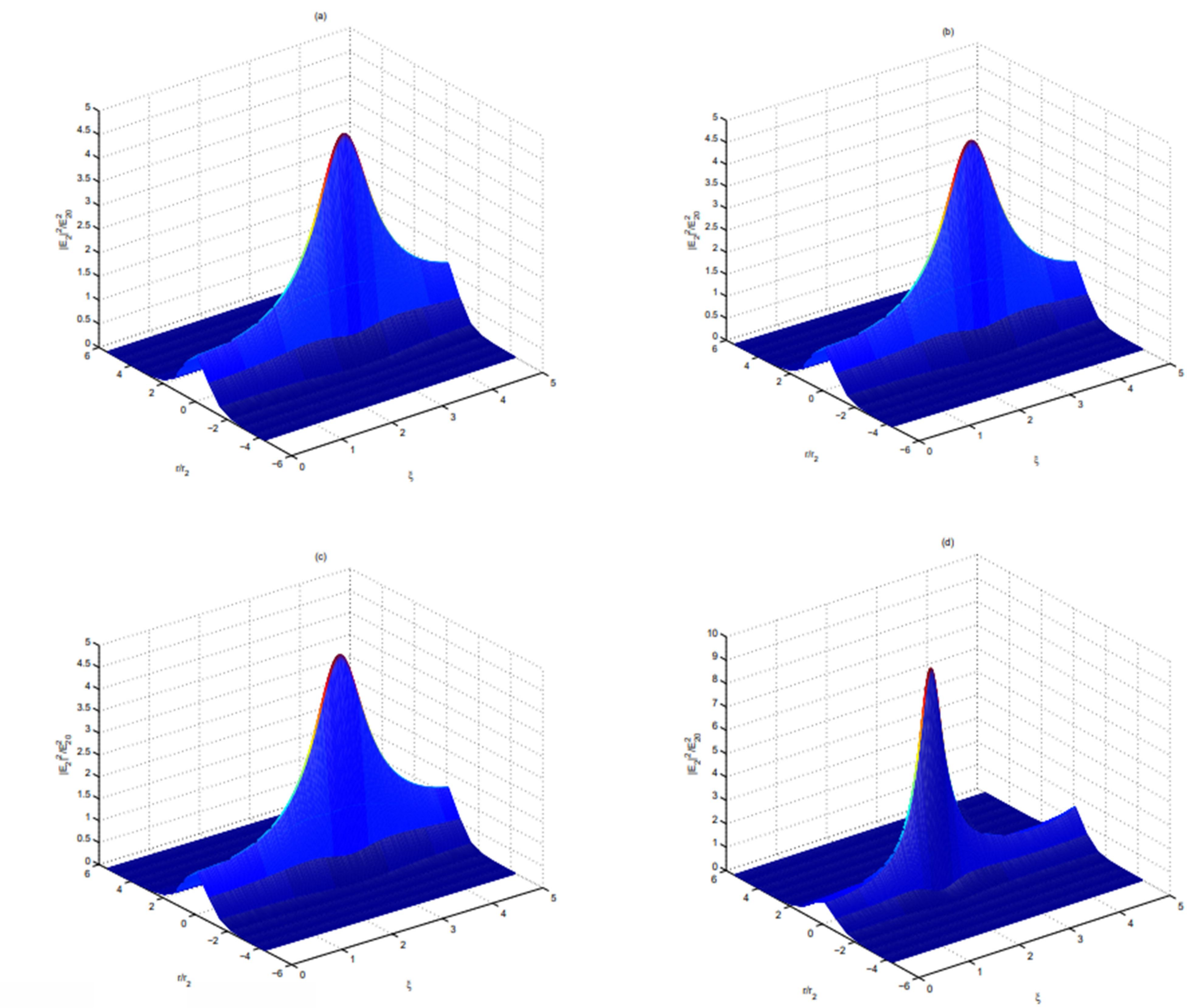

Figure 4.6: Effect of radial distance $\frac{r}{r_1}$ and normalized distance of propagation ξ on the normalized intensity of second beam, putting different values of Fermi temperature, (a) T_F = 10^6 K, (b) T_F = 10^7K, (c) T_F = 10^8K, (d) T_F = 10^9K and keeping $(\frac{\omega_{p0} r_1}{c})^2$= 9, βE_{10}^2=3.0, βE_{20}^2=3.50, q_1=3, q_2=3 fixed respectively

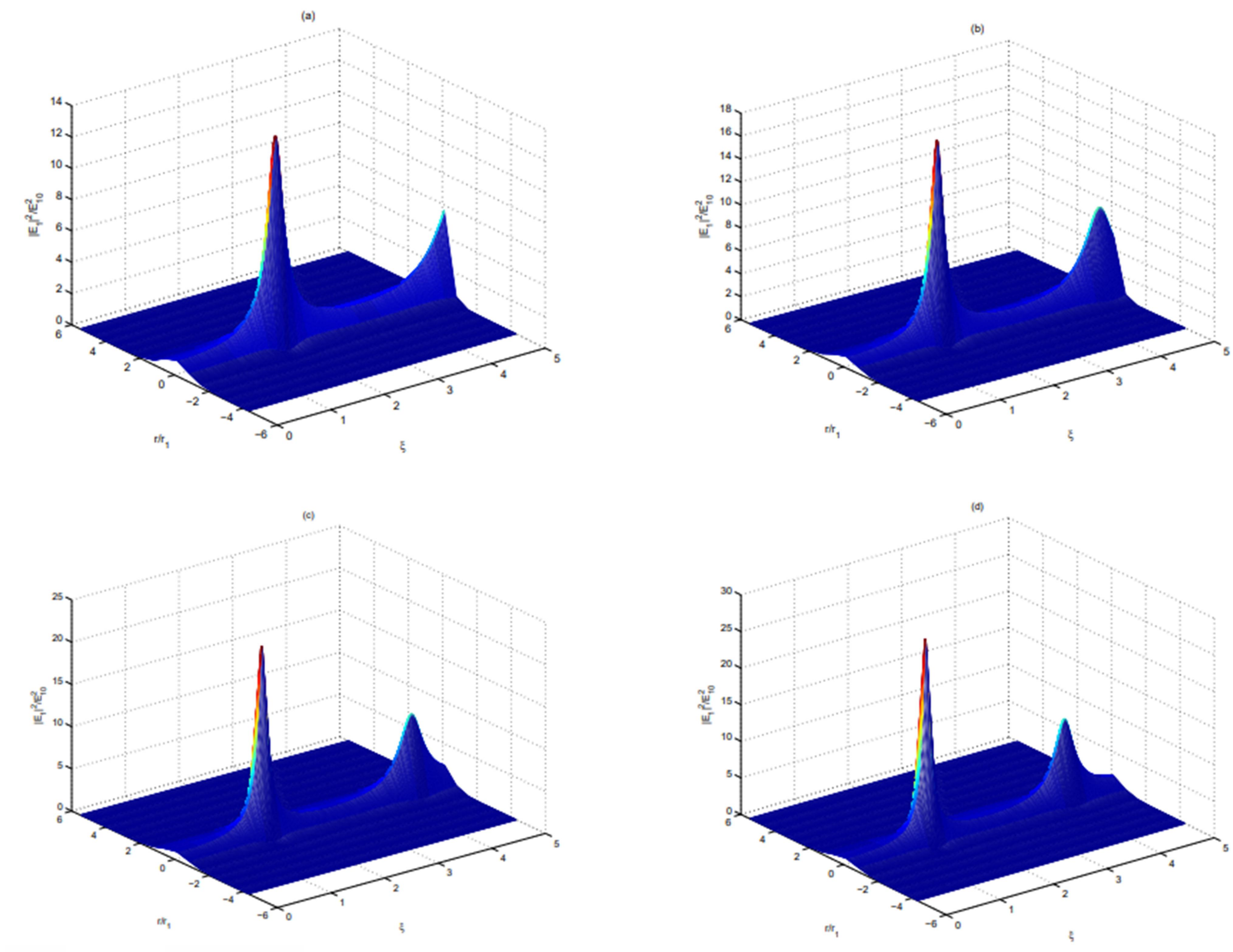

Figure 4.7: Effect of radial distance $\frac{r}{r_1}$ and normalized distance of propagation ξ on the normalized intensity of first beam, putting different values of normalized density, (a) $(\frac{\omega_{p0} r_1}{c})^2 =$ 9, (b) $(\frac{\omega_{p0} r_1}{c})^2$= 10, (c) $(\frac{\omega_{p0} r_1}{c})^2$= 11, (d) $(\frac{\omega_{p0} r_1}{c})^2$= 12 and keeping βE_{10}^2=3.0, βE_{20}^2=3.50, q_1=3, q_2=3, $T_F = 10^9$K fixed respectively.

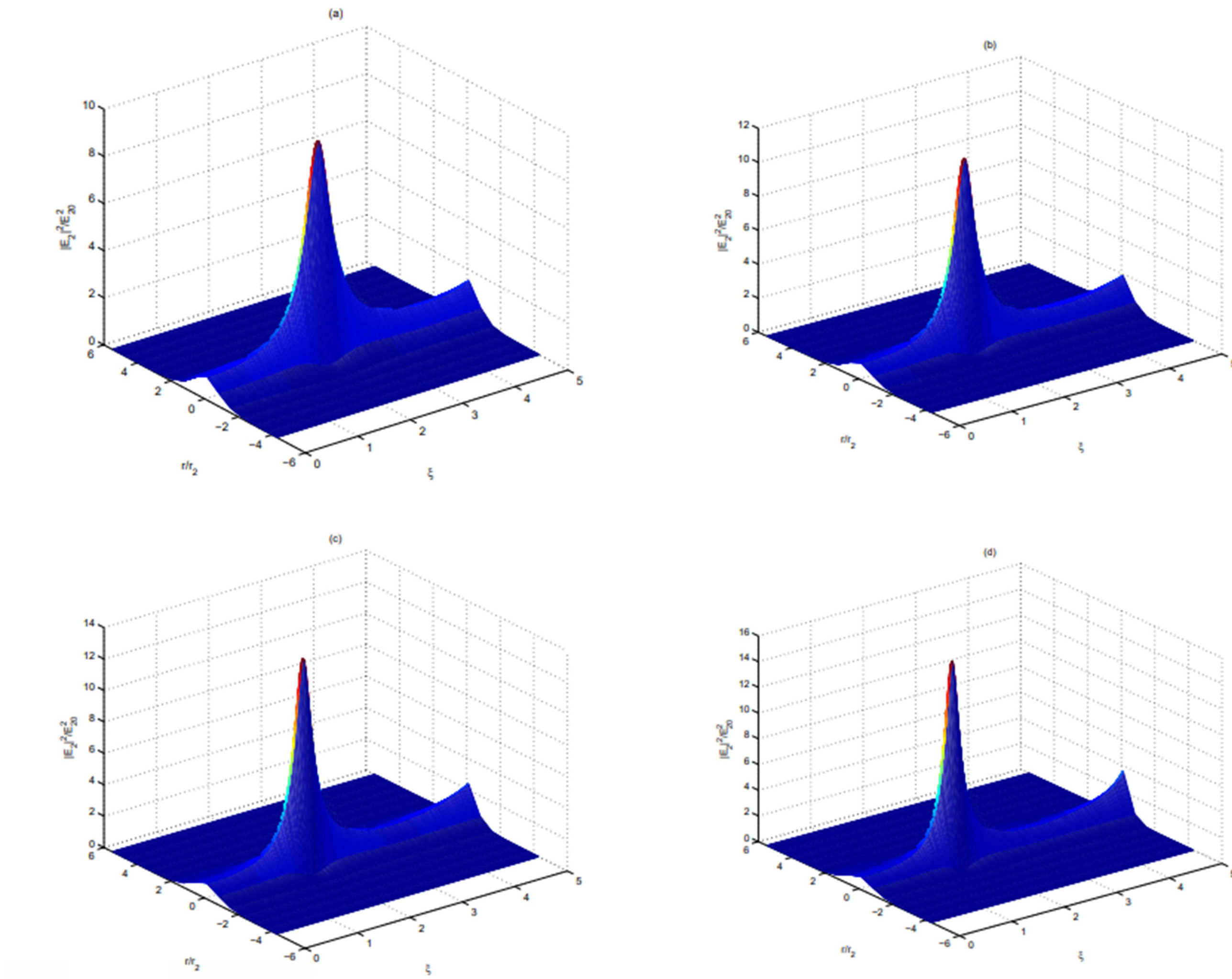

Figure 4.8: Effect of radial distance $\frac{r}{r_1}$ and normalized distance of propagation ξ on the normalized intensity of second beam, putting different values of normalized density, (a) $(\frac{\omega_{p0}r_1}{c})^2$ = 9, (b) $(\frac{\omega_{p0}r_1}{c})^2$ = 10, (c) $(\frac{\omega_{p0}r_1}{c})^2$= 11, (d) $(\frac{\omega_{p0}r_1}{c})^2$= 12 and keeping βE_{10}^2=3.0, βE_{20}^2=3.50, q_1=3, q_2=3, T_F = 10^9K fixed respectively.

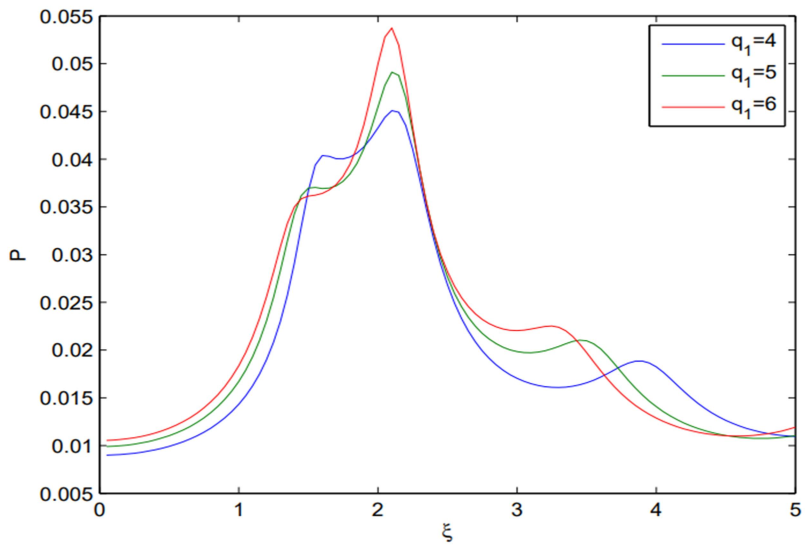

Figure 4.9: Effect of normalized distance of propagation ξ on normalized power P of beat wave, putting different values of q_1=4,5,6 and keeping $(\frac{\omega_{p0} r_1}{c})^2$= 9, β$E_{10}^2$=3.0, β$E_{20}^2$=3.50, q_2=3, T_F = 10^9K fixed.

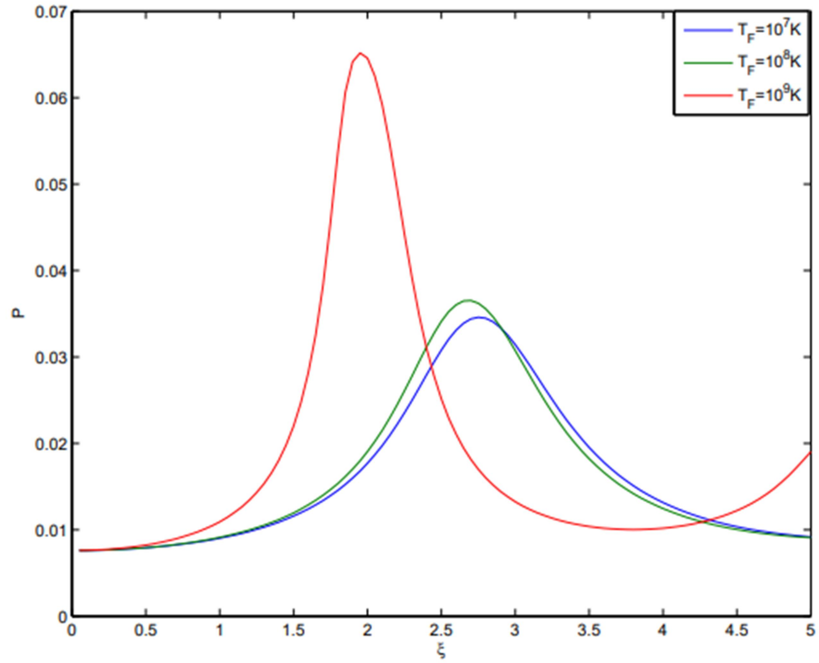

Figure 4.10: Effect of normalized distance of propagation ξ on normalized power P of beat wave, putting different values of Fermi temperature T_F=10^7K, 10^8K, 10^9K and keeping $(\frac{\omega_{p0} r_1}{c})^2 = 9$, βE_{10}^2=3.0, βE_{20}^2=3.50, q_1=3, q_2=3 fixed.

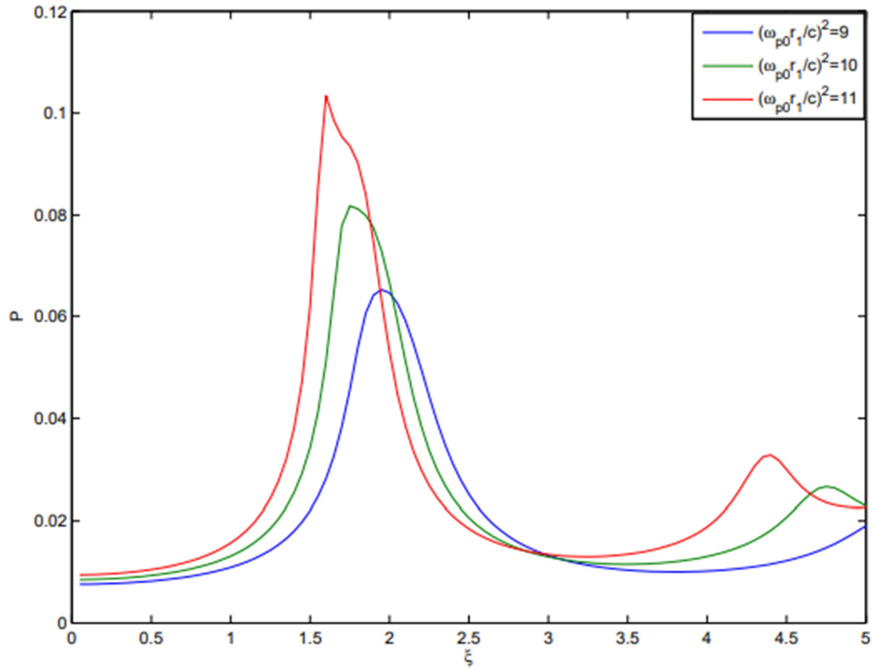

Figure 4.11: Effect of normalized distance of propagation ξ on the normalized power η of beat wave, putting at different values of normalized plasma density $(\frac{\omega_{p0}r_1}{c})^2$ = 9, 10, 11 and keeping βE_{10}^2= 3.0, βE_{20}^2= 3.50, q_1=3, q_2=3, T_F=10^9K fixed.

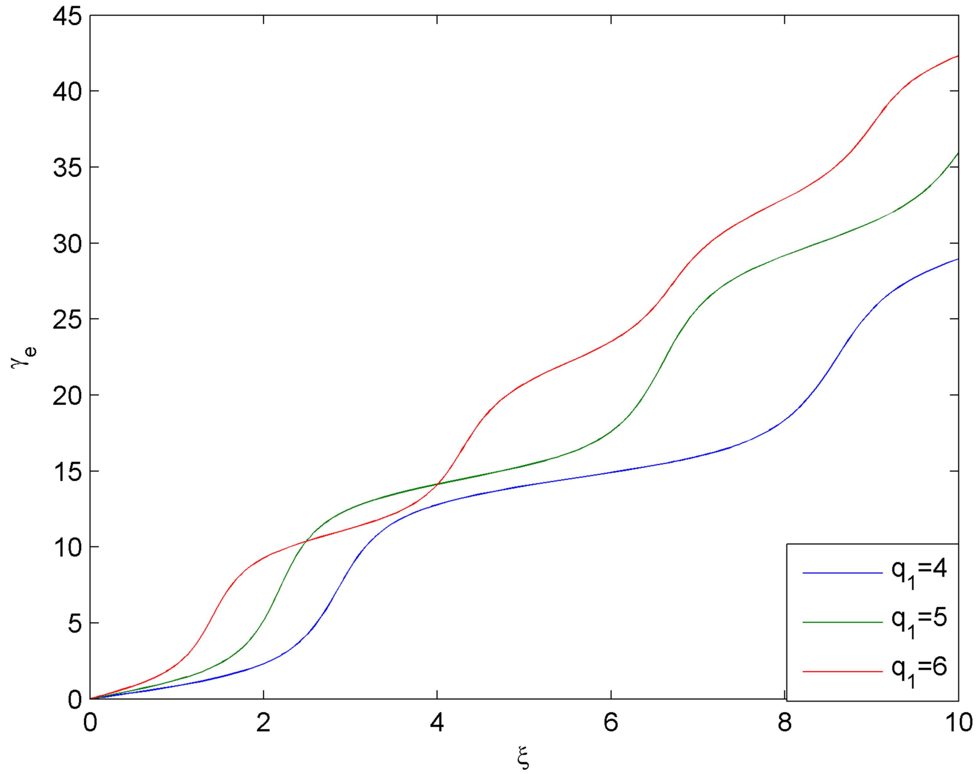

Figure 4.12: Effect of normalized distance of propagation ξ on energy of accelerated electron, putting different values of q_1=4,5,6 and keeping $(\frac{\omega_{p0} r_1}{c})^2$= 9, βE_{10}^2=3.0, βE_{20}^2=3.50, q_2=3, T_F = 10^9K fixed.

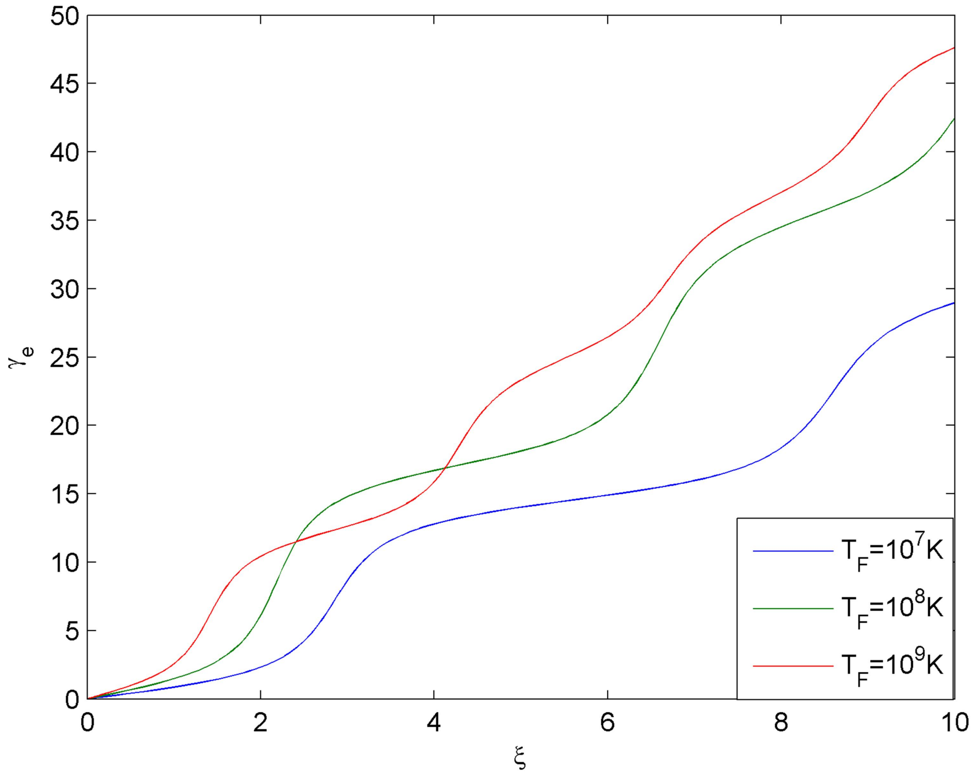

Figure 4.13: Effect of normalized distance of propagation ξ on energy gained by accelerated electron, putting different values of Fermi temperature T_F=10^7K, 10^8K, 10^9K and keeping $(\frac{\omega_{p0} r_1}{c})^2 = 9$, βE_{10}^2=3.0, βE_{20}^2=3.50, q_1=3, q_2=3 fixed.

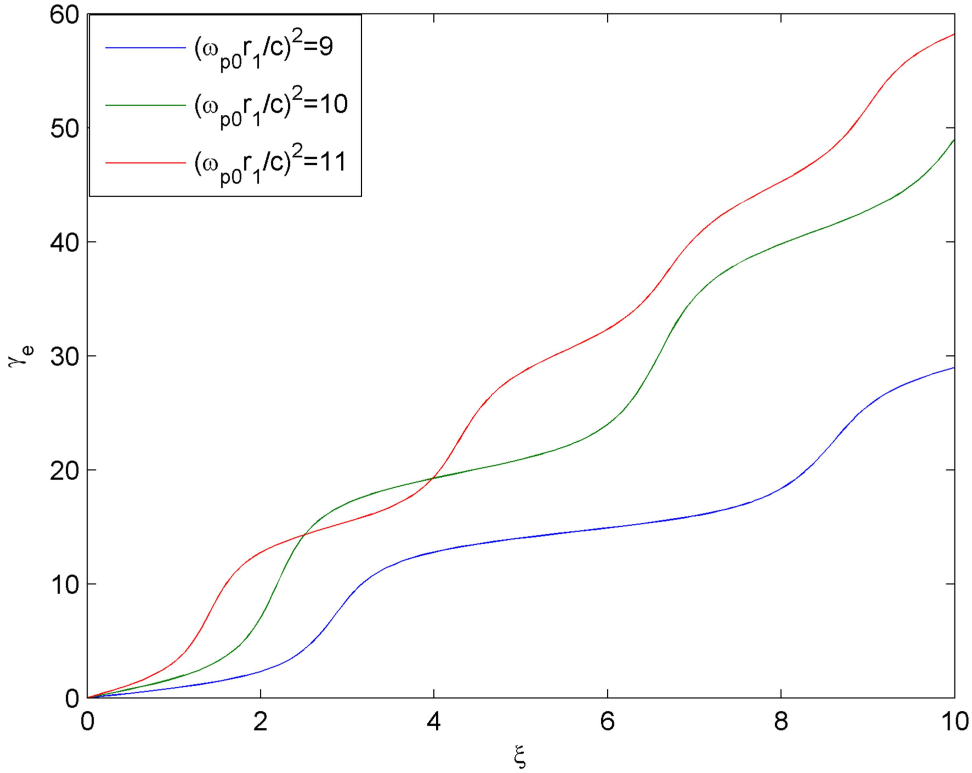

Figure 4.14: Effect of normalized distance of propagation ξ on the energy gained by accelerated electron, putting at different values of normalized plasma density $(\frac{\omega_{p0}r_1}{c})^2$ = 9, 10, 11 and keeping βE_{10}^2= 3.0, βE_{20}^2= 3.50, q_1=3, q_2=3, T_F=10^9K fixed.

Chapter 5

Conclusions and Future Scope

The major objective of this research work is to accelerate electron by laser driven electron plasma waves in plasma with different density profiles as well as to present the theoretical and analytical study of short pulse laser interaction with plasma relevant to the particle acceleration. It is seen in all objectives that electron has been accelerated and the objectives have been achieved. This filed has a versatile application in not only physics but also in chemistry, biology and many other branches of science.

The above work is relevant to material processing and other industrial applications which require focused energy from coupled laser beams like laser driven fusion or fuel pellet compression. It can also be used in cancer treatment so that bad cells can be damaged thereby saving the good cells.

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LASER DRIVEN ELECTRON ACCELERATION BY *q*-GAUSSIAN LASER PULSE IN PLASMA: EFFECT OF SELF-FOCUSING**

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A scheme for electron acceleration by self-focused q-Gaussian laser pulses in under-dense plasma has been presented. The relativistic increase in the mass of plasma electrons gives nonlinear response of plasma to the incident laser pulse resulting in self-focusing. Under the combined effects of the saturation nature of relativistic nonlinearity of plasma, self-focusing and diffraction broadening of the laser pulse, the beam width of the laser pulse evolves in an oscillatory manner. An electron initially on the pulse axis and at the front of the self-focused pulse, gains energy from it until the peak of the pulse is reached. When the electron reaches the tail of the pulse, the pulse begins to diverge. Thus, the deacceleration of the electron from the trailing part of the pulse is less, compared to the acceleration provided by the ascending part of the pulse. Hence, the electron leaves the pulse with net energy gain. The differential equations for the motion of electrons have been solved numerically by incorporating the effect of self-focusing of the laser pulse.

Keywords: *q-Gaussian, self-focusing, variational theory, electron acceleration.*

ЛАЗЕРНОЕ УСКОРЕНИЕ ЭЛЕКТРОНОВ *q*-ГАУССОВЫМ ПУЧКОМ В ПЛАЗМЕ С УЧЕТОМ ЕГО САМОФОКУСИРОВКИ

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Представлена схема ускорения электронов самофокусированными q-гауссовыми лазерными импульсами в неплотной плазме. Релятивистское увеличение массы электронов плазмы приводит к нелинейному отклику плазмы на падающий лазерный импульс, что вызывает самофокусировку. При совместном воздействии насыщающего характера релятивистской нелинейности плазмы, самофокусировки и дифракционного уширения лазерного импульса ширина луча лазерного импульса изменяется колебательно. Электрон, первоначально находящийся на оси импульса и во фронте самосфокусированного импульса, получает от него энергию до достижения пика импульса. Когда электрон достигает хвоста импульса, импульс расходится. Замедление электрона от замыкающей части импульса меньше по сравнению с ускорением, которое обеспечено восходящей частью импульса. Следовательно, электрон покидает импульс с чистым выигрышем в энергии. Дифференциальные уравнения движения электронов решены численно с учетом эффекта самофокусировки лазерного импульса.

Ключевые слова: *q-гауссиан, самофокусировка, вариационная теория, ускорение электронов.*

Introduction. Whenever we think of particle accelerators, we consider them to be meant only for research at the very edge of known physics as these enormous facilities take decades to build [1–3]. However, along with this lofty goal, particle accelerators are being used for decidedly more down-to-earth projects such as medical treatments, diagnostics [4] and sterilization [5], security screening [6], material science [7], biological processes [8], and many more [9]. The use of energetic electrons in these applications depends on the modes of the interaction of electrons with matter and electromagnetic fields. These modes are ionization,

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RESEARCH ARTICLE

Self action effects of *q*-Gaussian laser beam in preformed parabolic plasma channels: effect of nonlinear absorption

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Abstract This paper offers a theoretical examination of the self-action effects observed in intense laser beams as they propagate through preformed parabolic plasma channels. Specifically, detailed investigations have been conducted on the phenomena of self-focusing, self-trapping, and self-phase modulation of the laser beam. To visualize the impact of the beam profile deviating from the ideal Gaussian profile, the field distribution in the medium has been characterized by a *q*-Gaussian distribution. Applying the moment theory approach, the nonlinear partial differential equation governing the slowly varying envelope of the laser beam has been transformed into a set of interconnected ordinary differential equations describing the evolution of beam width and longitudinal phase. The obtained equations were numerically solved to visualize how both laser and medium parameters affect the propagation characteristics of the laser beam.

Keywords Plasma channel · Self focusing · q-Gaussian · Clean energy · Moment theory

Introduction

The discovery of the laser [1] has been instrumental in unveiling the true power and beauty of the interaction between light and matter, leading to the emergence of various new phenomena. This encompasses a range of phenomena, spanning from stimulated scattering to various self-action effects such as self-focusing, self-trapping, self-phase modulation, and more. Due to their intricate and physics-rich nature, these nonlinear phenomena have the potential to captivate researchers for many years. Hence, to enhance the comprehension of the physics of laser-matter interaction, numerous researchers are exploring various facets of these nonlinear phenomena. Significant momentum was gained with the suggestion to construct tabletop accelerators relying on laser-plasma interactions [2–4]. Beyond particle accelerators, numerous applications have emerged from the physics of laser-plasma interactions. Principal among these are X-ray lasers [5, 6], inertial confinement fusion (ICF) [7–9], supercontinuum generation [10], and others. The successful implementation of these applications hinges crucially on the quality and efficiency of laser-plasma coupling. However, the wave property of light, diffraction, poses a challenge to the features of the aforementioned applications by negating them. Hence, endeavors are underway to discover methods that can mitigate the diffraction of laser beams. The diffraction broadening of laser beams can be avoided by employing waveguides or by leveraging the phenomenon of self-focusing [11]. Self-focusing is a strongly nonlinear optical phenomenon that occurs because of the intensity-dependent refractive index of the medium. In this way, even in the absence of any lens-like structure, the medium exhibits behavior akin to a convex lens.

Intense laser beams can induce nonlinear responses in plasmas primarily through three mechanisms. These mechanisms are (1) Relativistic mass nonlinearity [12, 13] (2) nonlinearity due to Ponderomotive force [14, 15] (3) nonlinear Ohmic heating [16–18]. In the relativistic mechanism, alterations in the optical properties of plasma occur without any changes in electron density. Conversely, in the other two mechanisms, this modification takes place through the redistribution of plasma electrons. Relativistic nonlinearity becomes prominent when the power of the incident laser surpasses the threshold power needed for self-focusing, and

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RESEARCH ARTICLE

Propagation dynamics of *q*-Gaussian laser beams in preformed collisionless plasma channel: self-focusing, self-channeling, and self-phase modulation

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Abstract A study was conducted to examine the dynamics of a *q*-Gaussian laser beam as it travels through a preformed collisionless plasma channel. The analysis focused on observing the changes in beam width, phase shift, and self-trapping of the laser beam. The refractive properties of the plasma were assumed to exhibit laser-induced nonlinearity, specifically ponderomotive nonlinearity. By employing a moment theory approach in the W.K.B approximation, a set of coupled differential equations governing the evolution of the laser beam's spot size with propagation distance was derived. These differential equations were then solved numerically to explore the influence of both laser and channel parameters on the propagation dynamics of the laser beam.

Keywords Self-focusing · *q*-Gaussian · Moment theory · Ponderomotive force · Clean energy

Introduction

Since the pioneering demonstration of the laser by Maiman [1], the field of laser technology has witnessed remarkable advancements, particularly with the introduction of chirped pulse amplification (CPA) technology [2]. These developments have opened up new possibilities and avenues for laser-plasma interactions [3]. Various applications have emerged, ranging from practical real-world impacts like guiding microwaves [4], lightning protection [5], triggering rain [6] and snow [7], and inertial confinement fusion (ICF) [8], to more subtle yet significant contributions such as terahertz radiation generation [9], electron acceleration [10–12], and X-ray lasers [13]. To realize the full potential of these applications, efficient coupling of laser energy with plasmas is crucial. However, without an optical guiding mechanism, the interaction length is inherently limited due to diffraction, restricting it to approximately the Rayleigh length. This diffraction broadening phenomenon significantly hinders the effective utilization of laser energy, thus jeopardizing the aforementioned applications [14, 15]. Consequently, the guidance of laser beams through plasmas becomes indispensable. Relativistic self-focusing and ponderomotive self-channeling are the fundamental nonlinear mechanisms employed to guide intense laser beams in plasmas. In a uniform plasma, if the laser power surpasses the critical power $P_c = 17\frac{\omega_0^2}{\omega_p^2}\,[GW]$ (where ω_p represents the plasma frequency and ω_0 denotes the laser frequency), the laser beam can self-guide through the effect of relativistic self-focusing. However, when the laser power is below the critical power, beam diffraction dominates over relativistic self-focusing and ponderomotive self-channeling. Consequently, preformed plasma channels are commonly employed to guide laser beams, overcoming the limitations imposed by diffraction [16, 17].

Laser-plasma interactions are characterized by a multitude of nonlinear effects, encompassing parametric instabilities [18–22] and various self-action phenomena such as self-focusing [23], self-phase modulation [24], and mode conversion. Among these, self-focusing [23, 24] is a highly nonlinear phenomenon that emerges due to the material medium's nonlinear response to the incident laser beam, causing the medium to behave like a positive lens. In laser fusion applications, self-focusing represents one of the most concerning nonlinear phenomena as it can disrupt the required irradiation symmetry for the proper implosion of the target. The intricate

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RESEARCH ARTICLE

Formation of elliptical *q*-Gaussian breather solitons in diffraction managed nonlinear optical media: effect of cubic quintic nonlinearity

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Abstract This paper presents theoretical investigation on the formation of elliptical *q*-Gaussian breather solitons in diffraction managed optical media. The optical nonlinearity of the medium has been modeled by cubic–quintic nonlinearity. To obtain the physical insight into the propagation dynamics of the laser beam, semi-analytical solution of the wave equation for the laser beam has been obtained by using variational theory approach in W.K.B approximation. Emphasis is put on investigating evolutions of transverse dimensions and axial phase of the optical beam.

Keywords Soliton · Clean Energy · Self Focusing · Variational Theory · Breather

Introduction

Since the debut of quantum mechanics in the 1920s, the two different aspects of physical quantities, i.e., waves and particles, have been intimately related in physical theories. Although both the aspects appear to be physically different, there are a number of experimental evidences that show correlation among both. In the past few years, solutions of certain wave equations have revealed another correlation between waves and particles. The surprising fact is that these wave equations are not the part of quantum mechanics but instead have been derived from classical physics [1]. Solutions to these equations describe waves those neither spread in space (i.e., those do not diffract) nor disperse in time. Diffraction and dispersion (Fig. 1) are the inherent properties of all kind of waves whether it is electromagnetic wave, mechanical wave (sound wave) or even matter wave.

However, these new kinds of waves retain their size and shape indefinitely (Fig. 2). These waves can be regarded as a quantity of energy localized permanently to a definite region of space. It can be set in motion but it cannot dissipate by spreading out. When two such waves collide, each comes away from the encounter with its identity intact (Fig. 3). If a wave meets an "anti-wave," both can be annihilated. This kind of behavior is extraordinary in waves, but it is familiar in another context, i.e., with particles. Thus, such waves can be considered as particles and are termed as "solitons."

The first recorded observation of a soliton was made almost 200 years ago by Russell [2], an engineer and naval architect. He reported to the British Association for the Advancement of Science: "I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses. When the boat suddenly stopped-not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation then suddenly leaving it behind rolled forward with great velocity assuming the form of a large solitary elevation a rounded smooth and well-defined heap of water which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback and overtook it still rolling on at a rate of some eight or nine miles per hour preserving its original figure some 30 feet long and a foot to a foot and a half in height. Its height gradually diminished and after a chase of one or two miles I lost it in the windings of the channel."

Our topic of investigation, i.e., spatial optical solitons, arises due to dynamical balance of diffraction with induced focusing of the optical beam in a nonlinear medium. By nonlinear medium, it is meant by a medium whose index

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RESEARCH ARTICLE

Effect of self-focusing of bessel gauss laser beam on excitation of electron plasma wave in collisionless plasma with axial density ramp

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Abstract A theoretical investigation has been conducted to analyze the dynamics of electron plasma waves (EPWs) induced by Bessel–Gauss laser beams in plasmas featuring an axial density ramp. The study involves evaluating how the self-focusing of the laser beam influences the power of the laser-excited EPWs. As the laser beam traverses the plasma, it triggers the generation of an EPW at the frequency ω_{ep}. This EPW, affected by the optical nonlinearity of the plasma, becomes nonlinearly coupled to the laser beam due to the ponderomotive nonlinearity of plasma electrons. Employing variational theory, semi-analytical solutions for the coupled nonlinear wave equations governing the pump wave and EPW have been obtained using the W.K.B approximation technique. The findings highlight a significant impact of the self-focusing effect of the pump beam on the power of the EPW.

Keywords Bessel–Gauss · Plasma waves · Variational theory · Self-focusing · Clean energy

Introduction

Plasma is a collection of charged particles, comprising both positive and negative ions, characterized by their high energy levels that resist easy combination. This state of matter is pervasive throughout the universe [1]. It appears as extremely hot, high-pressure gas in celestial bodies like the sun and stars, as well as in the rarefied gas present in interstellar space and the ionospheric envelope surrounding the Earth. On a more local scale, plasmas are evident in the intense reactions of burning fuel and in gas-discharge devices like neon signs. Physicists have extensively studied the various physical effects exhibited by plasmas over the course of approximately two centuries. Earlier investigations, especially those focusing on gas discharges, played a crucial role in uncovering the nature of electrons and providing insights into atomic structure [2].

The current emphasis on plasmas can be ascribed to two primary factors. Firstly, there is a technological motivation, placing importance on comprehending plasma behavior for the controlled release of thermonuclear energy [3–6]. This involves endeavors to reproduce, within artificially created plasmas, the nuclear reactions observed in the sun. Another technological goal is the advancement of magneto-hydrodynamic generators [7], wherein electric power is generated through the interaction of gas plasma jets with magnetic fields. The second overarching rationale for investigating plasmas is their significance in space and astrophysics. When a plasma is influenced by electromagnetic fields, the motion of its particles becomes non-random. A noteworthy outcome of this imposed order is that plasmas can propagate specific types of waves that share connections with electromagnetic waves but possess distinct and intriguing properties. These waves include high-frequency electron plasma waves [8, 9] (EPWs) and low-frequency ones referred to as ion acoustic waves [10, 11] (IAWs).

The electron plasma wave (EPW) excited through stimulated Raman scattering (SRS) displays an exceptionally high phase velocity, potentially leading to the generation of superthermal electrons in inertial confinement fusion (ICF). These penetrating electrons have the capacity to induce preheating of the fuel, thereby impeding the efficient compression required for achieving high gain [12]. As

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RESEARCH ARTICLE

Beat wave excitation of electron plasma wave by cross-focused q-Gaussian laser beams in thermal quantum plasmas

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Abstract This paper presents a theoretical investigation on excitation of an electron plasma wave by beating two q-Gaussian laser beams in thermal quantum plasmas. Moment theory in W.K.B. approximation has been invoked to find numerical solution of nonlinear Schrodinger equation for Drude model of dielectric function of plasma. Optical nonlinearity of plasma has been modeled by relativistic mass nonlinearity of plasma electrons. The laser-induced nonlinearity in the dielectric properties of plasma depends not only on the intensity of the first beam but also on that of second beam. Therefore, propagation characteristics of one laser beam affects that of second beam, and hence, cross-focusing of the two laser beams takes place. Due to nonuniform intensity distribution along the wavefronts of the laser beams, the background electron concentration gets modified. The amplitude of EPW, that depends on the background electron concentration, thus gets nonlinearly coupled with the laser beams. Numerical simulations have been carried out to investigate the effects of laser-plasma parameters on cross-focusing of the laser beams and further its effect on power of EPW.

Keywords q-Gaussian · Beat wave · Quantum plasma · Clean energy · Self-focusing

Introduction

The introduction of the laser by Maiman [1] in 1960 revolutionized the field of plasma physics, giving rise to a new area of study known as laser-plasma interactions. This field is highly complex yet filled with physics that will continue to engage researchers for years to come. Nowadays, lasers play a pivotal role in plasma physics, driven in part by the proposal of using high-power laser beams to initiate fusion reactions [2–4] for practical energy production. However, laser-plasma interactions have branched out into numerous potential applications. Some of these applications, such as microwave guiding [5], lightning protection [6], and the manipulation of rain and snowfall [7], have direct real-world impacts. Other applications, like the generation of terahertz radiations [8–10], electron acceleration [11–14], and X-ray lasers [15, 16], are more subtle but equally significant for the scientific community and technological advancements.

Most of these applications rely on efficient coupling between lasers and plasmas. Unfortunately, when a light beam travels through a medium or vacuum, it naturally broadens due to the wave property of diffraction. This diffraction broadening fundamentally undermines the efficiency of laser-plasma coupling, thereby jeopardizing the effectiveness of the aforementioned applications. Consequently, there is a growing interest in exploring methods or processes that can enhance the efficiency of laser-plasma coupling by increasing the effective interaction length between laser beams and plasmas. The such nonlinear phenomenon that addresses this challenge is self-focusing, which occurs due to the material medium's nonlinear response to incident laser beams. Under certain conditions, the medium behaves like a convex lens [17], altering its refractive properties when exposed to an intense laser beam in a plasma. This variation in the light's velocity across the beam's wavefront causes the beam to bend and converge spherically, leading to self-focusing of the laser beam.

Recently, the high-density and low-temperature plasmas known as quantum plasmas are encountered in modern

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RESEARCH ARTICLE

Electron acceleration by self focused quadruple Gaussian laser beams in thermal quantum plasma: effect of relativistic optical nonlinearity of plasma

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Abstract A method for achieving relativistic electron acceleration using intense laser beams has been introduced. As intense laser beams propagate through plasmas, they induce an electron plasma wave (EPW) at the pump frequency. Electrons externally introduced into this EPW and subsequently trapped undergo acceleration, reaching exceedingly high energies. To optimize the coupling of laser energy with the plasma, the irradiance across the beam cross-section is modeled using a Quadruple Gaussian (Q.G) profile. The impact of self-focusing of the laser beam on electron energy gain has been thoroughly examined. Applying variational theory based on Lagrangian formulation and a hydrodynamic fluid model of plasma, coupled differential equations governing the evolution of the laser beam's width and the energy of accelerated electrons have been derived. It is observed that the uniformity of irradiance across the beam cross-section significantly enhances the energy acquired by electrons.

Keywords Quadruple Gaussian · Quantum plasma · Clean energy · Self focusing · Variational theory

Introduction

Accelerators serve as the primary tools in high-energy physics, a scientific discipline focused on the study of matter in its elementary forms [1–3]. These instruments not only function as "microscopes" for examining matter but also serve as the means by which various elementary forms of matter are generated for laboratory study. While the conventional perception of an accelerator is that of a particle gun producing high-speed particles that bombard atomic nuclei, it is equally valid to consider accelerators as devices that shine "light" on nuclei, allowing us to effectively "see" and explore their characteristics.

Now the resolving power of a microscope, i.e., its ability to distinguish small objects, depends on the wavelength of the light it employs. The shortest wavelength of visible light is about 4×10^{-7} m; with these waves one can perceive a microbe of about the same length. Biologists employ electron microscopes to scrutinize minute structures. The wavelength of a particle is contingent on both its mass and energy. At the energy levels typical for electron microscopes, which operate in the range of a few thousand electron volts, electrons exhibit wavelengths approximately 10,000 times shorter than those of visible light. This shortened wavelength allows researchers to delve into the intricate details of molecules.

The nucleus of an atom is about 10^{-15} m in diameter. This represents the wavelength of a proton when it possesses an energy of 1 MeV. Consequently, to visually perceive the nucleus, a "microscope" utilizing 1 MeV protons is necessary, and to discern certain internal features, an energy range of approximately 10 to 20 times higher is required. Consequently, a laboratory focused on classical nuclear physics typically incorporates a Van de Graaff accelerator or a cyclotron functioning within the 1 to 20 MeV range.

But physics has pushed beyond this point. At present many of us are interested not in the nucleus as a whole but in the structure of the protons and neutrons (nucleons) of which it is composed. It is the old problem of worlds within worlds, for the proton itself turns out to have a rich structure. It is perhaps 10^{-15} m in diameter, and to resolve it requires an energy of several hundred MeV. To see it in as fine detail

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RESEARCH ARTICLE

Electron acceleration by elliptical q-Gaussian laser pulse in collisionless plasma with exponential density ramp: combined effect of self-focusing and self-compression

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Received: 4 August 2023 / Accepted: 13 November 2023

Abstract A scheme for electron acceleration by self-focused and self-compressed q-Gaussian laser pulses in under dense plasma has been presented. The ponderomotive force acting on the plasma electrons gives nonlinear response of plasma to the incident laser pulse resulting in its self-focusing and self-compression. Under the combined effects of saturation nature of ponderomotive nonlinearity of plasma, self-focusing, self-compression, and diffraction broadening of the laser pulse, the beam width of the laser pulse evolves in an oscillatory manner. An electron initially on pulse axis and at the front of the self-focused pulse, gains energy from it until the peak of the pulse reaches. When the electron reaches at the tail of the pulse, the pulse begins to diverge. Thus, the deacceleration of the electron from the trailing part of pulse is less compared to the acceleration provided by the ascending part of the pulse. Hence, the electron leaves the pulse with net energy gain. The differential equations for the motion of electron have been solved numerically by incorporating the effects of self-focusing and self-compression of the laser pulse.

Keywords Electron acceleration · q-Gaussian · Self compression · Self focusing · Plasma

Introduction

When contemplating particle accelerators, we often associate them exclusively with cutting-edge research in the realm of known physics, given the substantial time investment required to construct these massive facilities [1–3]. However, in addition to this ambitious objective, particle accelerators are finding practical applications in various down-to-earth projects, including medical treatments, diagnostics [4], sterilization [5], security screening [6], material science [7], biological processes [8], and numerous others [9]. These applications utilize energetic electrons and depend on their interactions with matter and electromagnetic fields. Such interactions include ionization, chemical changes, heating, bremsstrahlung, and synchrotron radiation. In industrial settings, electron beam irradiation is commonly employed for specific purposes, with a primary focus on ionization-induced alterations in the properties of irradiated materials. This process is driven by secondary reactions caused by free radicals—molecular fragments with unpaired electrons—that are generated by the electron beam when it bombards the material. These processes can be categorized into two main types: (1) radiation processing and (2) radiation treatment. Radiation processing serves industrial applications such as polymer grafting, cross-linking, and the curing of monomers, oligomers, and epoxy-based composites. On the other hand, radiation treatment is utilized for sterilizing medical products, wastewater treatment, food preservation, decontamination of chimney and flue gases, and the degradation of plastics for use in coatings and inks.

As the world contemplates the construction of the largest accelerator, such as the superconducting supercollider, accelerator technology finds itself approaching its practical limits. There are two primary reasons for this development:

1. The magnetic forces involved have grown so immense that the magnets generating them would be at risk of being torn apart [10].

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RESEARCH ARTICLE

Electron acceleratajon by laser driven electron plasma wave in plasma with axial density ramp: Cosh Gaussian laser beam

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Received: 16 January 2024 / Accepted: 19 February 2024

Abstract This paper presents a theoretical investigation into the nonlinear propagation of a Cosh-Gaussian (ChG) laser beam through plasma with axial inhomogeneity. The study takes into consideration the relativistic mass nonlinearity of plasma electrons in response to the laser beam. Additionally, the research explores the influence of self-focusing on the excitation of electron plasma waves (EPWs). The analysis employs variational theory to derive semi-analytical solutions for the nonlinear wave equations governing the laser beam and EPWs. Furthermore, the study examines the acceleration of electrons trapped within the excited EPWs. The results obtained from numerical analysis indicate a more pronounced self-focusing effect of the Cosh-Gaussian laser beam when optimal values of the decentered parameter are utilized. This finding is beneficial for generating high-amplitude EPWs, enabling electron acceleration by extending the interaction length.

Keywords Self focusing · Cosh-Gaussian laser · Electron plasma wave · Electron accelerator · Density ramp

Introduction

For about 5 decades, the exploration of the fundamental nature of matter has primarily depended on a singular experimental approach. This method involves propelling a particle to high velocities and inducing collisions with another particle [1, 2]. Through the analysis of the resulting debris generated during these collisions, researchers obtain valuable insights into the properties of particles and the forces governing their interactions. To carry out such experiments, a dependable source of energetic particles is essential. Although cosmic rays act as a natural source, their particle flux is widely dispersed and beyond the control of experimenters. As a result, a more practical solution involves the use of a particle accelerator, a device crafted to augment the speed and energy of particles.

In 1928, Ernest O. Lawrence developed one of the earliest particle accelerators, employing laboratory glassware with a small diameter [3]. Today, the majority of operational accelerators can be traced back as direct successors to Lawrence's device, although they have grown considerably in size and complexity. The largest accelerators now cover extensive areas, measuring several square kilometers, prompting the consideration of available land as a crucial factor in their design. The particle accelerator has evolved beyond being a standalone instrument confined to a laboratory; instead, the laboratory is intricately constructed around the accelerator itself. The construction of such a machine entails costs reaching hundreds of millions of dollars, and its operation requires a workforce of approximately 1000 individuals along with the use of numerous digital computers.

Giant accelerators are not only at the forefront of high-energy physics research but smaller machines are also employed across diverse fields such as materials science, structural biology, nuclear medicine, fusion research, food sterilization, nuclear waste transmutation, and cancer treatment [4, 5]. Despite their lower energy levels, typically ranging from 100 million to one billion volts, these compact machines, which generate electron or proton beams, still require substantial laboratory spaces. Accelerators fall into a few general categories. First, they can propel either lighter particles (such as electrons and positrons) or heavier particles (like protons and antiprotons). Second, they can

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RESEARCH ARTICLE

Cross focusing of q-Gaussian laser beams in cubic–quintic nonlinear media and conversion of circular laser beam into elliptic laser beam

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Abstract Variational theory has been used to find the solution of nonlinear Schrodinger wave equation (NSWE) in a semi analytical way with the goal to model the dynamics of two coaxial asymmetric q-Gaussian laser beams in nonlinear optical media. The nonlinearity in the refractive index of the medium has been modeled by cubic-quintic model. Due to the intensity dependence of refractive index, the two laser beams get coupled with each other and thus influence the propagation characteristics of each other. Emphases are put on the dynamical variations of beam widths and longitudinal phases of the laser beams with distance of propagation.

Keywords Self focusing · q-Gaussian · Elliptical beam · Variational theory · Schrodinger wave equation

1. Introduction

The Invention of the laser [1] has played a fundamental role in the development of new fields like nonlinear optics, quantum optics and atom optics, and thus to reveal true power and beauty of light matter interaction through the appearance of several new phenomena [2, 3]. This span a gamut from parametric instabilities to several self-action effects like self-focusing [4], self-phase modulation [5], self-trapping [6], etc. These nonlinear phenomena are extremely complex but are rich in enough exotic physics to keep researchers busy for years to come. Over past few years researchers are attempting to improve upon the understanding of these self-action effects in order to have deep insight into light matter interaction physics.

Optical self-action effects occur via electromagnetic field induced changes in the optical properties of a medium through which they pass. The change in the index of refraction then reacts back on the fields so as to affect the propagation characteristics of the electromagnetic beam [7]. The consequences of self-action effects can be significant, e.g., the phenomenon of self-focusing can lead to the formation of spatial solitons [8], i.e., the optical beams that do not disperse or dissipate but instead maintain their shape and size indefinitely. When collimated optical beams propagate through linear media, i.e., the media whose index of refraction is independent of beam intensity, they broaden with distance of propagation due to light's natural wave property of diffraction. It would seem that this kind of spreading is inevitable and therefore irreducible, since it originates at a fundamental level from the uncertainty principle of quantum mechanics. However, in 1964 Chiao et al. [9] showed that the spreading of an optical beam could in principle be avoided in a nonlinear optical medium, and therefore that the expansion of an optical beam due to diffraction was neither inevitable nor irreducible in such a medium. In nonlinear materials, the presence of laser beam modifies their optical properties like dielectric constant, absorption, or conversion to higher frequencies. The resulting change in the dielectric constant resembles the intensity distribution of the laser beam, resembling an optical lens that increases the refractive index in the beam's center while leaving it unaffected in the beam's tail. This induced lens accumulates the beam energy towards its center. This phenomenon is known as self-focusing. If self-focusing exactly balances diffraction broadening, then the beam propagates in self-trapped mode and is known as spatial soliton.

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RESEARCH ARTICLE

Excitation of electron plasma wave by self-focused Laguerre–Gaussian laser beams in axially inhomogeneous plasma: effect of orbital angular momentum of photons

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Abstract Theoretical investigation has been conducted into the dynamics of electron plasma waves (EPWs) driven by lasers in plasmas featuring axial density ramps. To explore the impact of the laser beams' orbital angular momentum on both the propagation dynamics and the strength of the excited EPWs, the irradiance across the laser beam's cross section has been represented using a Laguerre–Gaussian beam profile. As the laser beam travels through the plasma, it stimulates an EPW at the pump frequency. This EPW becomes nonlinearly coupled to the laser beam due to the optical nonlinearity of the plasma. By employing variational theory and the W.K.B. approximation, semi-analytical solutions for the coupled nonlinear wave equations governing the pump wave and EPW have been derived. Notably, it has been observed that the power of the EPW is significantly influenced by the self-focusing effect of the pump beam.

Keywords Self-focusing · Electron plasma wave · Clean energy · Laguerre–Gaussian · Ponderomotive force

Introduction

The introduction of the laser in 1960 initiated a sequence of events that triggered a resurgence in the study of interactions between light and matter. This revolutionary development gave rise to an entirely novel realm of research termed as laser–plasma interactions. Over the recent years, this field has garnered remarkable attention from researchers, owing to its paramount significance for numerous potential applications, including nuclear fusion [1–3], plasma wakefield accelerators [4, 5], and coherent radiation sources [6–8]. The efficacy of these applications hinges upon the efficiency of the coupling between lasers and plasmas, a factor influenced by various nonlinear processes [9–11]. These processes encompass an array of phenomena, ranging from the self-focusing of laser beams to the excitation of diverse modes of wave propagation within plasmas. In scenarios where external magnetic fields are absent, these modes of wave propagation primarily involve electron plasma waves (EPWs) and ion acoustic waves (IAWs).

In the context of laser-driven nuclear fusion, the activation of electron plasma waves (EPWs) holds paramount importance. However, the EPWs excited by the pump beam have a dual nature. On one side, they reflect a substantial portion of the laser energy out of the fusion pellet through a phenomenon known as stimulated Raman scattering. Conversely, they generate superthermal electrons, which in turn cause premature heating of the pellet. Specifically, electrons that synchronize their motion with the excited plasma wave and roughly match its speed become entrapped within the plasma wave structure. Leveraging the potent electric field of the EPW, these trapped electrons undergo acceleration to attain superthermal velocities (as illustrated in Fig. 1). Possessing remarkable penetration capabilities, these superthermal electrons contribute to the untimely heating of the pellet before it reaches the critical density required for fusion ignition. This phenomenon is termed as 'preheating.' Furthermore, the appealing electrostatic attraction between these superthermal electrons and the ions in the surrounding corona can lead to the ejection of energetic ions from the ablation layer. These outward-propelled energetic ions

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RESEARCH ARTICLE

Stimulated Raman scattering of self-focused Laguerre–Gaussian laser beams in axially inhomogeneous plasma

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Abstract This paper presents a theoretical investigation on stimulated Raman scattering (SRS) of intense Laguerre–Gaussian (LG) laser beams propagating through plasma with axial density ramp. The optical nonlinearity of the plasma has been considered to be originating due the ponderomotive force acting on the plasma electrons due to intensity gradient over the cross section of laser beam. An intense laser beam with frequency ω_0 propagating through plasma gets coupled with a preexisting electron plasma wave (EPW) at frequency ω_{ep} and produces a back scattered wave at frequency $\omega_s = \omega_0 - \omega_{ep}$. Using variational theory semi-analytical solution of the set of coupled wave equations for the pump, EPW and scattered wave has been obtained under W.K.B approximation. It has been observed that power of the scattered wave is significantly affected by the self-focusing effect of pump beam.

Keywords Self-focusing · Stimulated Raman scattering · Laguerre–Gaussian · Clean energy · Ponderomotive force

Introduction

The invention of the laser led to a renaissance in the field of light matter interactions by giving birth to an entirely new area of research known as laser plasma interactions. Since the past few decades, this new field is at vanguard of research due to its importance in many potent applications [1–8]. The impetus was built by the proposal of initiating thermonuclear fusion [1, 3] for viable energy production by using intense laser beams.

Both the allure and the challenges of fusion arise from the nature of the fusion process itself. Fusion fuel is abundant and cheap. The major advantages are: (1). The abundance of fuel—the most easily exploitable fuels are deuterium and tritium. Deuterium occurs naturally in all sources of water specially sea water. Tritium, however, is not readily available naturally, it can easily be manufactured inside the fusion reactor by the bombardment of neutrons with lithium, which also abundant in nature. (2). Cleanest source of energy—Fusion does not produce nuclear waste directly. In laser driven fusion, the goal is to deposit laser energy at a particular density in the plasma in order to derive the compression and subsequent heating of the fuel pellet. If the pellet is compressed sufficiently, it may undergo fusion, leaving to the release of a large amount of energy. It's as if there is a tiny hunk of the sun on Earth.

The successful implosion of the fuel pellet depends on the efficiency of laser plasma coupling which is decided by several nonlinear processes [9–11] ranging from collisional absorption to excitation of several laser driven instabilities [12–15]. In laser plasma instabilities, the pump beam (incident laser beam) splits into two daughter waves. If there is no external magnetic field, these daughter waves will be a scattered electromagnetic wave along with an electron plasma wave (EPW) or ion acoustic waves (IAW).

For laser driven fusion, these instabilities are of serious concern because if they are operative to a significant extent, they make it difficult to achieve high gain. In this context, SRS [16, 17], in which the incident laser beam decays into an EPW and a scattered electromagnetic wave, is of more

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RESEARCH ARTICLE

Optical guiding of *q*-Gaussian laser beams in radial density plasma channel created by two prepulses: ignitor and heater

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Abstract Self-action effects (self focusing) and self phase modulation of *q*-Gaussian laser beam in plasma channel created by ignitor heater technique have been investigated theoretically. The ignitor beam causes tunnel ionization of air. The heater beam heats the plasma electrons and establishes a parabolic channel and also prolongs the channel life by delaying the electron ion recombination. The third beam (*q*-Gaussian beam) is guided in the plasma channel under the combined effects of density nonuniformity of the parabolic channel and relativistic mass nonlinearity of the plasma electrons. Formulation is based on finding the numerical solution of nonlinear Schrodinger wave equation (NSWE) for the fields of incident laser beams with the help of moment theory approach. Particular emphasis is put on dynamical variations of the spot size of the laser beams and longitudinal phase-shift of the guided beam with distance of propagation.

Keywords Self-Focusing · Self-Trapping · Phase Modulation · Bessel Gauss Lasers · Ponderomotive Force

Introduction

After the transistor, lasers [1] are considered to be one of the most successful inventions of 20th century science. When laser made its debut in 1960, some people called it solution in search of a problem. Today lasers have reserved their place in almost every aspect of life: consumer technologies like CD players, super market checkout scanners to higher end technologies. With the advent of chirped pulse amplification (CPA) technique [2], the turn of last century has witnessed a giant leap in laser technology leading to a renaissance in the field of light-matter interactions. This amelioration in laser technology has given birth to an entirely new field of science known as laser–plasma interactions. An agglomeration of nonlinear phenomena viz., parametric instabilities, [3, 4] higher harmonic generation, [5, 6] Self-focusing, [7] self-phase modulation [8], etc is ubiquitous in these laser plasma interactions.

A major impetus behind the investigations on laser plasma interactions was provided by the proposal of initiating fusion reactions [9, 10] for viable energy production by using intense laser beams. Fusion is considered to be the cleanest source of energy as there will be no emission of radioactive end products and green house gases. Thus, it bears the promise to quench humanity's endless thirst for energy without making any harm to global climate. Along the way the field of laser–plasma interactions has branched into a number of potential applications like laser-driven accelerators, [11, 12] X-ray lasers, [13, 14] higher harmonic generation [5, 6], etc. The ultimate breath of most of these applications depends on stable guiding of intense laser beams over longer distances, without significant energy loss. However, due to lights natural wave property of diffraction, a light beam traveling in vacuum or in a medium always broadens in the absence of an optical guiding mechanism. Diffraction broadening of the laser beam is thus the fundamental phenomenon that jeopardizes the feature of aforesaid applications by negating the efficiency of laser–plasma coupling. Hence, there is surging interest to explore the methods that may aid to increase the

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RESEARCH ARTICLE

Spatial frequency chirping of *q*-Gaussian laser beams in graded index plasma channel with ponderomotive nonlinearity

Naveen Gupta[1] · Rohit Johari[1] · A. K. Alex[1] · Suman Choudhry[1] · Sanjeev Kumar[1] · S. B. Bhardwaj[2]

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Abstract Chirping of spatial frequency of *q*-Gaussian laser beams interacting nonlinearly with plasmas with radially inhomogeneous electron density has been investigated theoretically. Due to the radial nonuniformity of the electron density, the index of refraction of the plasma channel resembles to that of a graded index fiber. Chirping or modulation of spatial frequency also known as phase anomaly occurs due to position momentum uncertainty of photons. Due to intensity gradient over the laser cross section, the transverse component of ponderomotive force becomes finite. This results in redistribution of carriers in the irradiated potion of the channel. This results in the enhancement of the radial gradient of the density profile that stimulates the laser beam to get self-focused. The reduction in transverse dimensions of the laser beam in turn leads to spread in transverse momentum of its photons. This transverse momentum spread then modifies the axial phase of the laser beam. Following Virial theory, equations of motion for radius and spatial frequency of the laser beam have been obtained. The equations so obtained have been solved numerically to envision the effect of various laser and plasma parameters on the evolution of beam envelope. Manifestation of axial phase to Berry phase has also been explained.

Keywords *q*-Gaussian · Virial theory · Self-focusing · Gouy phase

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Introduction

The additional accumulation of axial phase of a converging beam in comparison with an infinite plane wave is known as modulation of spatial frequency or Gouy phase shift [1]. In his experiment, Gouy reflected a beam of light emerging from a pin hole, from both a curved and flat mirror. The focusing beam over lapped with the nonfocusing beam in a region near the focus and created a circular diffraction pattern. Gouy then looked at the circular diffraction pattern at several different locations, both before and after the focus. He observed that the central region of the diffraction pattern changed from bright to dark, indicating an axial phase jump of the focusing beam—the Gouy phase shift.

Observing Gouy phase shift is relatively easy, but explaining it is not. Since its discovery, the Gouy phase shift has remained a matter of debate. Curiosity about its origin and physical meaning is still at the vanguard of investigations. Various theories [2, 3] (ranging from classical to quantum) have been used to explain its origin. Classically the phase shift of an optical beam arises due to the contribution of an additional phase in the neighborhood of the beam focal spot arising from the second-order derivative of field amplitude with respect to transverse coordinates. However, in quantum mechanical terms the Gouy phase shift is considered to be originating as a consequence of modification of its transverse dimensions. Converging beams going through focus have finite spatial extent in the transverse plane. The uncertainty relation then induces some distribution over the transverse and consequently longitudinal wave vectors. The net effect of this distribution over wave vectors is an overall phase shift.

Since the discovery of lasers [4], the anomalous behavior of the spatial frequency also known as axial phase or wave number of the optical beams has been drawing attention of

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RESEARCH ARTICLE

Electron plasma wave excitation by self-focused cosh gaussian laser beams in axially inhomogeneous plasma: effect of density ramp

Naveen Gupta[1] · **Sanjeev Kumar**[1,2] · **S. B. Bhardwaj**[3] · **Rohit Johari**[1] · **Suman Choudhry**[1]

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Abstract Dynamics of the laser driven electron plasma waves (EPWs) in plasmas with axial density ramp has been investigated theoretically. The effect of self focusing of the laser beam on the power of laser excited EPW has been incorporated. During its propagation through the plasma, the laser beam excites an EPW at frequency ω_{ep} that due to the optical nonlinearity of plasma gets nonlinearly coupled to the laser beam due to the ponderomotive nonlinearity of plasma electrons. Using variational theory semi analytical solutions of the coupled nonlinear wave equations for the pump wave and EPW have been obtained under W.K.B approximation technique. It has been observed that power of the EPW is significantly affected by the self-focusing effect of pump beam.

Keywords Self-Focusing · Electron plasma wave · Cosh-gaussian · Ponderomotive force · Clean energy

Introduction

Investigations on coupling of intense laser beams with plasmas are at the vanguard of research since past few decades due to its importance in many potential applications including laser fusion [1–3], plasma wake field accelerators [4, 5], X-ray lasers [6, 7], terahertz generation [8], etc. The ultimate breath of these applications depends on the efficiency of laser plasma coupling which is further decided by many different nonlinear processes [9–11]. These processes range from collisional absorption to excitation of copious laser driven instabilities [12–15]. These instabilities can be represented as the resonant coupling of the incident laser beam into two daughter waves. In the absence of external magnetic field these daughter waves can be electron plasma waves, ion acoustic waves along with a scattered electromagnetic wave.

EPWs can be excited in plasmas due to their remarkable properties of quasi neutrality and collective behaviour. Plasma is a state of matter that contains enough heat that atoms lose their individuality. The negatively charged electrons are still attracted by positively charged nuclei, but they are not bound together. This gives a plasma some unusual properties unlike most kind of ordinary matter-solids, liquids and gases-the free-floating electrons and ions of a plasma are strongly affected by electric and magnetic fields. Plasma as a whole is quasi neutral, but as the electrons and positively charged ions are separated, a disturbance can create regions of net negative and net positive charges acting like the plates of a charged parallel plate capacitor. Such an uneven distribution of charges results in an electric field running from positive to negative regions. This electric field pulls the electrons and ions towards each other with equal forces. Due to their large mass ions are lazy and thus remain at rest and the electrons move towards the ions. As the electrons move towards the ions, they steadily gain velocity and momentum like a pendulum moving towards its mean position from an extreme position. Due to this gain in momentum the electrons overshoot their equilibrium positions resulting in reversing the direction of electric field. Now the reversed electric field opposes the electron motion slow them down and then pulling them back again. The process repeats itself, establishing an electron oscillator. In the presence of thermal velocity these electron oscillations lead to a longitudinal wave compression and rarefaction regions of electrons

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RESEARCH ARTICLE

Self-focusing, self-trapping and self-phase modulation of elliptical *q*-Gaussian laser beams in collisionless plasma

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Abstract Theoretical investigation on optical self-action effects of intense *q*-Gaussian laser beams interaction with collisionless plasmas has been investigated in detail. Emphasis on put on investigating the dynamics of beam width and axial phase of the laser beam. Effect of the ellipticity of the cross section of the laser beam also has been incorporated. Using variational theory based on Lagrangian formulation, nonlinear partial differential equation (P.D.E) governing the evolution of beam amplitude has been reduced to a set of coupled ordinary differential equations for the beam widths of the laser beam along the transverse directions. The evolution equation for the axial phase of the laser beam has been obtained by the Fourier transform of the amplitude structure of the laser beam from coordinate space to (k_x, k_y) space. The differential equations so obtained have been solved numerically to envision the effect of laser-plasma parameters on the propagation dynamics of the laser beam.

Introduction

Laser [1] is the one of the most important scientific inventions of the twentieth century. When laser made its debut, it was referred to as solution in search of a problem. Today laser has become ubiquitous in consumer technology, from CD players to supermarket checkout scanners. Higher-end applications of lasers are also abound. This includes medical diagnostics and treatment [2], nuclear fusion [3], particle accelerators [4], decommissioning of explosives [5], etc. The diversity in the applications of lasers can be felt from the fact that currently this instrument is being used for heating as well as for ultra-intense cooling. The same instrument can produce extremely hot state of matter [6] (plasma) as well as extremely cold state of matter [7] (Bose Einstein Condensate). The impact of laser on society has changed over time, and is still changing. Already, lasers have provided the preferred solution to an impressive variety of real-world situations, and it is expected that in coming years it will keep on enhancing quality of life and will contribute wealth to the world economy.

In most of the applications, the laser intensity is the key parameter that decides their ultimate breath. Currently, due to the light's inherent wave property to diffract, the laser power has gotten into bottleneck at the order of few petawatts. Initially, it was believed that diffraction of the laser beam cannot be avoided during its propagation neither through vacuum nor through material media. However, in 1964 Chio et al. [8] showed that in media whose index of refraction depends on the intensity of light, the spreading of an optical beam in principle can be obviated. Hence, the expansion of optical beam due to diffraction is neither inevitable nor irreducible.

Self-focusing and self-trapping are two examples of nonlinear optical effects which may arise from one of many physical mechanisms. Self-focusing describes the formation of a light-induced channel in an illuminated material which confines the optical beam. This channel serves as a lens. Self-trapping occurs when self-focusing substantially exactly counteracts beam spreading due to diffraction. When this happens, the cross section of the light-induced channel remains substantially constant with propagation distance over the distance of the self-trapping.

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RESEARCH ARTICLE

Self-focusing of dark hollow Gaussian laser beams in cubic–quintic nonlinear media

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Abstract The investigation focused on the self-focusing behavior of a specific type of laser beams called dark hollow Gaussian beam has been presented. The approach involved formulating a numerical solution for the nonlinear Schrodinger wave equation using the variational theory approach in the W.K.B approximation. The optical nonlinearity of the medium was described using a cubic–quintic model. The study delved into the detailed examination of the laser beam's spot size dynamics, aiming to gain a thorough understanding of its evolution. Additionally, the investigation explored the dynamics of the laser beam in phase space to gain deeper insights.

Keywords Self-focusing · Cubic–quintic media · Variational theory · Potential well · Dark hollow laser beams

Introduction

The spreading of a propagating beam's transverse dimensions, caused by diffraction, is a universal phenomenon observed in various types of waves, such as sound, electromagnetic waves, and matter waves. This spreading is typically considered inevitable and irreducible due to the fundamental wave property of diffraction inherent in light. However, in 1964, Chio et al. [1] demonstrated that the expansion of an optical beam could potentially be avoided in a nonlinear medium, challenging the notion that diffraction-induced beam spreading is always unavoidable. In nonlinear materials, the presence of an optical beam leads to changes in their optical properties, such as dielectric constants, absorption, or conversion to higher frequencies. This alteration in the dielectric constant resembles the intensity distribution of the laser beam, acting like an optical lens that increases the refractive index at the center of the beam while leaving the tail unaffected. This induced lens effect accumulates the beam's energy toward its center, giving rise to a phenomenon known as self-focusing.

Ever since Askaryan [2] first discovered it, the phenomenon of self-focusing has remained at the forefront of both experimental and theoretical investigations due to its significant relevance in numerous applications. Self-focusing plays a crucial role in the design of ultra-intense laser systems, particularly those employed in laser-driven nuclear fusion. It can impose limitations on the cavity intensity by causing intra-cavity losses or by impeding the release of optical energy. Additionally, self-focusing has the ability to modify the transverse profile of the laser beam through the generation of wavefront distortions.

It is widely recognized that a converging or diverging electromagnetic beam exhibits modulation in its longitudinal phase when compared to a reference on-axis plane wave [3]. This phenomenon, known as self-phase modulation or Gouy phase shift, has been a subject of debate in recent years. Since its discovery [3], various theories (ranging from classical [4] to quantum [5, 6]) have been proposed to explain its origin. Classically, self-phase modulation of a laser beam arises due to the contribution of an additional phase per unit length near the beam's focal spot, resulting from the second-order derivative of the field amplitude with respect to the transverse coordinates. However, from a quantum mechanical perspective, the longitudinal phase shift is considered

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RESEARCH ARTICLE

Excitation of upper hybrid wave by cross focused q-Gaussian laser beams in graded index plasma channel

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Abstract In this paper, a method is presented for exciting an upper hybrid wave (UHW) in a preformed parabolic plasma channel. The plasma channel is magnetized perpendicular to the propagation direction of laser beams. The UHW is generated through the interaction of two q-Gaussian laser beams with frequencies ω_1 and ω_2, employing the ponderomotive nonlinearity. The evolution of the laser beam spot sizes along the propagation distance is described by a set of coupled differential equations derived using the moment theory approach in the W.K.B approximation. The ponderomotive nonlinearity depends on the intensities of both laser beams, resulting in a mutual influence between the two beams, leading to cross-focusing. Numerical simulations are conducted to examine the impact of laser and channel parameters on the cross-focusing of laser beams and its effect on the power of the generated UHW. The results indicate that the intensity profiles of the laser beams, channel depth, and strength of the static magnetic field significantly affect the power of the generated UHW.

Keywords q-Gaussian · Plasma Waves · Moment Theory · Clean Energy · Self Focusing

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Introduction

At the turn of the last century, the introduction of lasers [1] sparked a significant surge in research within the field of plasma physics. The study of plasmas began in the nineteenth century, when Michael Faraday investigated electrical discharges through gases. Modern plasma research dates from 1957 and 1958. During those years, Soviet Sputnik and American Explorer spacecrafts discovered that space near the earth is filled with plasma. At the same time, till then secret research on controlled thermonuclear fusion conducted by the USA, Soviet Union and Europe was revealed at the Atoms for Peace Conference in Geneva, greatly increasing the freely available information on plasmas. Fusion research focuses on producing extremely hot plasmas and confining them in magnetic "bottles," to create the conditions necessary for energy-producing nuclear reactions to occur.

Extensive studies, incorporating both theoretical and experimental approaches, have been conducted to enhance our understanding of this subject. These collective efforts have given rise to various potential applications, such as laser-driven particle accelerators [2–5], inertial confinement fusion [6, 7], X-ray lasers [8–10], laser plasma channeling [11, 12], and supercontinuum generation [13]. The successful realization of these applications relies heavily on the efficient coupling of laser energy with plasmas. Unfortunately, the interaction length between lasers and plasmas is inherently limited by diffraction divergence, restricting it to approximately a Rayleigh length in the absence of an optical guiding mechanism. Diffraction broadening, therefore, represents a fundamental phenomenon that hampers the efficiency of laser–plasma coupling. Consequently, there has been a renewed interest in extending the propagation distance of laser beams through

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RESEARCH ARTICLE

Dynamics of *q*-Gaussian laser beam in thermal quantum plasma: self-focusing self-trapping and self-phase modulation

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Abstract This paper presents theoretical investigation of propagation characteristics of *q*-Gaussian laser beam in thermal quantum plasmas. The field distribution in the medium is expressed in terms of beam width parameter *f* and the parameter *q* describing the deviation of intensity distribution of the laser beam from Gaussian distribution. Due to nonuniform irradiance along the wavefront of the laser beam and high field amplitude associated with it the laser beam gets self-focused. An appropriate nonlinear Schrodinger wave equation for the field of laser beam has been solved with the help of moment theory approach in W.K.B approximation. The behavior of beam width parameter *f* with distance of propagation for different laser-plasma parameters has been investigated. Self-phase modulation and self-trapping of the laser beam also have been investigated for variety of parameters.

Keywords Self-focusing · Clean energy · Plasma · *q*-Gaussian · Lasers

Introduction

Invention of laser by Maiman [1] in 1960, led to a renaissance in the field of plasma physics by giving birth to a completely new area of laser-plasma interactions which is extremely complex but rich in enough exotic physics to keep researchers busy for years to come. Today, plasma physics is one of the major scientific fields to use lasers. A major impetus was provided by the proposal of initiating fusion reactions for viable energy production using high power laser beams [2, 3]. This would quench the world's thirst for energy without worsening global climate change. Along the way, however, the field has branched into a number of potential applications. Some of the applications, such as guiding microwaves [4], lightening protection[5], triggering rain and snow [6] have obvious real world impact. Others, such as generation of terahertz radiations [7], electron acceleration [8], X-ray lasers [9] etc. are subtler but no less important to the scientific community and development of new technologies. Most of these applications require efficient laser-plasma coupling. In the absence of an optical guiding mechanism, a light beam traveling either in vacuum or in a medium always broadens because of light's natural wave property of diffraction. Hence, diffraction broadening is the fundamental phenomenon that negate the efficiency of laser-plasma coupling and thus jeopardize the feature of above mentioned applications. Hence, there is surging interest to explore the methods or processes that may increase the efficiency of laser-plasma coupling by increasing the effective interaction length of laser beams with plasmas. Self-focusing is such a nonlinear phenomenon that arises due to nonlinear response of the material medium to the field of incident laser beams, in such a way that medium starts behaving like a convex lens [10, 11]. When an intense laser beam is shone into a plasma, it changes its dielectric properties. The consequent variations in the light's velocity across the beam's wavefronts causes them to bend into spherically converging one, causing self-focusing of the laser beam.

Laser-plasma interactions are perennially fraught with copious nonlinear effects. This span a gamut from parametric instabilities to several self-action effects such as self-phase modulation, mode conversion etc. [12, 13]. Self-phase modulation is the principal phenomenon responsible for the generation of

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RESEARCH ARTICLE

Third harmonic generation of self-focused *q*-Gaussian laser beams in nonlinear media: effect of cubic quintic nonlinearity

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Received: 9 August 2023 / Accepted: 13 November 2023

Abstract This paper presents theoretical investigation on third harmonic generation (THG) in nonlinear media. Variational method has been adopted to find the semi analytical solution of the wave equation governing the evolution of slowly varying beam envelop in nonlinear medium. Emphasis is put on investigating the evolution of beam width of the laser beam with distance through the medium. When laser beam with frequency ω_0 propagates through a nonlinear medium dominated by $\chi^{(3)}$ nonlinearity, oscillations of the electrons of the medium contain a frequency component $\omega_3 = 3\omega_0$ and thus produce third harmonic of the pump beam. An equation governing the conversion efficiency of third harmonic has been derived. Deviation of intensity profile of the incident laser beam from ideal Gaussian profile has also been incorporated through *q*-Gaussian profile.

Keywords Self focusing · Cubic quintic · *q*-Gaussian · Harmonic generation

Introduction

The quest for short wavelength coherent radiation sources for medical diagnostics and treatment, homeland security, plasma diagnostics, etc. has a long history. For past few decades only two main approaches, namely free electron lasers and synchrotron had been considered for this purpose. However, involvement of large infrastructure, accelerators, beam lines and massive gantries of more than 100 tons, makes these techniques are quite expensive. As a result, access to these facilities is quite limited, specifically for less affluent institutes like universities and hospitals these facilities are not affordable and therefore the research related to them is not growing at a faster pace.

By bringing coherent short wavelength sources off the shelf, the process of laser HHG can reduce the cost of coherent radiation sources. The reduction in cost is not only due to the replacement of accelerator but also due to the fact that there will be no requirement of large building footprints and massive gantries. To understand the generation of higher harmonics in nonlinear media let us see what happens when an intense laser beam passes through a transparent optical medium. The focused light from certain lasers has an electric field as strong as 10^7 V cm^{-1}. Such optical fields are as intense as the cohesive local electric fields in the crystal. Consequently when intense laser beams enter a transparent crystal, they cause a massive redistribution of the electrons and the resulting polarization is no longer proportional to the optical electric field. In fact, at optical fields of 10^7 V cm^{-1} and higher many materials break down completely.

Figure 1 illustrates the characteristic response when an intense optical electric field travels through a nonlinear, or ionic, material. It shows that an intense field in the "right" direction is more effective in polarizing the material than a field in the "left" direction. Such a situation can occur only in a crystal that has a "one-wayness" in its structure, or, to be more precise, one that has no center of symmetry. Such crystals are called as noncentro symmetric crystals. Of the crystals found in nature only about 10% fall in this class, and they usually exhibit the phenomenon called piezoelectricity. When a piezoelectric crystal is subjected to mechanical pressure, its asymmetry leads

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RESEARCH ARTICLE

Generation of superthermal electrons by self-focused Cosh Gaussian laser beams in inertial confinement fusion plasma

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Abstract Theoretical study on the generation of superthermal electrons by intense laser beams in hot dense plasmas during inertial confinement fusion has been presented. In order to obtain uniform heating of the fuel pellet the irradiance over the beam cross section has been modeled by Cosh Gaussian profile. The effect of self-focusing of the laser beam on energy gained by the electrons has been investigated in detail. Following moment theory and hydrodynamic fluid model of plasma coupled differential equations governing the evolution of beam width of the laser beam and energy of the superthermal electrons have been obtained. It has been observed that the uniformity of the irradiance over the beam cross section enhances the energy of superthermal electrons considerably.

Keywords Self focusing · Super thermal electrons · Plasma · Clean energy · Cosh Gaussian

Introduction

The fundamental currency of our universe is energy. It lights up our homes, grows our food and powers our computers. There is no end to world's energy appetite. The need of energy is so great and growing so rapidly around the world that alternate sources of energy to quench humanity's endless thirst of energy without doing any harm to global climate are required. In this regard the quest to tap the energy of nuclear fusion [1, 2] by employing intense laser beams to confine an ultra-hot plasma and generate electric power has been in progress since past few decades. The fusion power plants will be fueled by a form of heavy hydrogen found in ordinary sea water and will produce no harmful emissions-no sooty pollutants, no nuclear waste and no greenhouse gases.

The basic idea of laser fusion is to split an intense laser beam into several smaller beams of equal intensity [3]. The split beams are amplified in energy and subsequently brought back together by a system of mirrors and lenses; the beams are thereby focused on a small region from different directions. A charge of deuterium and tritium fuel is encased in a spherical shell a few millimeters in diameter, made of plastic, glass or some other material, and the resulting fuel pellet is placed at the intersection of the beams; the pellet is thus uniformly illuminated.

The laser beam almost instantly ionizes the atoms in the outermost layer of the pellet, but the material inside a certain critical radius is opaque to the laser energy. Incident energy is consequently absorbed in a dense layer of plasma that surrounds the deuterium tritium fuel. The heated layer of plasma expands and ablates [4], or becomes explosively torn free, from the rest of the pellet; the velocity of the ablated plasma is typically 1000 kilometers per second. An equal and opposite force accelerates the material inside the ablation layer inward, in accordance with Newton's third law, as if it was a rocket propelled by the plasma escaping all around it. In effect the system is a laser-powered spherical rocket whose payload is the rapidly contracting fuel pellet. The energy efficiency of the rocket implosion is low-less than 10 percent because the exhaust velocity (of the ablated material) is much higher than the vehicle velocity (the imploding pellet). The concentric implosive force is sufficient to accelerate the remaining shell to a velocity of several hundred kilometers per second in a billionth of a

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RESEARCH ARTICLE

Spatial frequency modulation of *q*-gaussian laser beams in a collisional plasma exhibiting an axial density ramp

Naveen Gupta[1] · Rohit Johari[1] · Alex A K[1] · Suman Choudhry[1]

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Abstract This study investigates the phenomenon of spatial frequency chirping in q-Gaussian laser beams interacting nonlinearly with plasmas exhibiting an axial density ramp. The chirping, also known as phase anomaly, arises from the inherent uncertainty between a photon's position and momentum.The non-uniform intensity distribution of the laser beam leads to uneven heating of plasma electrons, causing a redistribution of charged particles due to which self focusing of laser beam takes place. The reduction in transverse dimensions subsequently spreads the transverse momentum of photons, modifying the axial phase of the laser beam. Employing variational theory, equations governing the evolution of the laser beam's radius and spatial frequency (axial phase) were derived and solved numerically. This allowed for analysis of how various laser and plasma parameters influence the changes in the laser beam's spatial frequency (axial phase).

Keywords *q*-Gaussian · Spatial frequency · Variational theory · Self focusing · Collisional plasma

Introduction

When a light beam converges, it accumulates an additional axial phase shift compared to a perfectly flat, infinite plane wave. This phenomenon is also known as modulation of spatial frequency (SFM) or the Gouy phase shift. In an experiment, Gouy[1] demonstrated this effect by reflecting light from a pinhole off both a curved and a flat mirror. The focused beam overlapped with the unfocused beam near its focal point, creating a circular diffraction pattern. By observing this pattern at different locations before and after the focus, Gouy noticed a change in the central region – it switched from bright to dark. This observation indicated a sudden jump in the axial phase of the focused beam, confirming the presence of the Gouy phase shift.

While observing the Gouy phase shift in experiments is relatively straightforward, explaining its origin has proven to be a complex and ongoing discussion. Since its discovery, researchers have proposed various theories (both classical and quantum) to understand its physical meaning [2, 3].From a classical perspective, the phase shift is attributed to the additional phase contribution near the beam's focal point, arising from the second derivative of the field amplitude with respect to the transverse coordinates. However, quantum mechanics offers a different explanation. It suggests that the Gouy phase shift arises due to the modification of a converging beam's transverse dimensions as it passes through the focus. This finite size in the transverse plane introduces uncertainty in the beam's wave vectors, affecting both the transverse and longitudinal components. The resulting distribution of wave vectors leads to an overall phase shift.

The anomalous behavior of a light beam's spatial frequency, also known as axial phase or wave number, has been a topic of interest for researchers since the invention of lasers [4]. This phenomenon holds significance in various applications and physical problems. In wave optics, it explains the phase shift experienced by secondary wavelets originating from a primary wavefront. Within lasers, it determines the resonant frequencies of different transverse modes within the cavity. The axial phase jump of optical beams also plays a crucial role in applied physics. One example is optical trapping, where it contributes to the lateral trapping force acting on particles and even provides a mechanism for their

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RESEARCH ARTICLE

Second-harmonic generation of two cross-focused q-Gaussian laser beams by nonlinear frequency mixing in plasmas

Naveen Gupta[1] · S. B. Bhardwaj[2] · Sanjeev Kumar[1] · Suman Choudhry[1] · Rishabh Khatri[1] · Siddhanth Shishodia[1] · Rohit Johari[1]

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Abstract A scheme for second-harmonic generation (SHG) of a pair of q-Gaussian laser beams interacting nonlinearly with underdense plasma has been proposed. Due to the relativistic increase in electron mass under the intense fields of the laser beam, the resulting optical nonlinearity of plasma leads cross-focusing of the laser beams. The resulting nonlinear coupling between the two laser beams makes the oscillations of plasma electrons to contain a frequency component equal to the sum of frequencies of the pump beams. This results in a nonlinear current density at frequency equal to the sum of frequencies of the pump beams. If the frequencies of the pump beams are equal, then the resulting nonlinear current generates a new radiation at frequency twice the frequencies of the pump beams— a phenomenon known as SHG. Starting from nonlinear Schrodinger wave equation a set of coupled differential equations governing the evolution of beam widths of the laser beams and power of generated second-harmonic radiation with longitudinal distance has been obtained with the help of variational theory. The equations so obtained have been solved numerically to envision the effect of laser as well as plasma parameters on the power of generated second-harmonic radiation.

Keywords Cosh Gaussian · Self-focusing · Nonlinear optics · Second-harmonic generation · Clean energy

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Introduction

The invention of the laser [1] is the most towering achievement in the long history of light. It brought an extraordinary technological leap, which has since paved the way for a startling new era in optical science and technology. For the first time, man got a remarkable tool for direct generation and manipulation of coherent light. Laser brought same revolution to optics that transistor brought to electronics and cyclotron brought to nuclear physics. The distinctive qualities of laser derive from its coherence properties, which result in a beam of light with a well-defined optical phase both in space and time. This prescribed phase generally confines the wavelength and frequency of the laser light to a restricted range, so that the beam exhibits a narrow frequency spectrum. Another unique property of laser light is its directionality, which means that the beam can propagate over great distances without significant spreading and can be readily manipulated using conventional optical elements. The phase coherence and directionality of the laser make it possible to create extremely large optical powers and focused intensities that cannot be obtained from incoherent light emitters. These characteristics also allow accurate transfer of information [2], precise calibration of time [3, 4], and measurements of many physical constants [5, 6], among numerous other applications, using laser light. Lasers are now standard components of such commonplace objects as compact-disk players and printers. The everyday presence of lasers does not mean, however, that they have been reduced to performing only pedestrian tasks. Higher-end applications like laser surgery [7], laser-driven particle accelerators [8, 9], inertial confinement fusion [10], etc., are also abound.

Success is never without limitations, and laser is also not an exception. By the virtue of its unique coherence properties, laser light contains only a confined band of frequencies

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RESEARCH ARTICLE

Self focusing of *q*-Gaussian laser beams in collisional plasma with axial density ramp: effect of Ohmic heating

Naveen Gupta[1] · Alex A K[1] · Rudra Partap[1] · Abhay Limbu[1] · Rohit Johari[1]

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Abstract This paper investigates the phenomenon of self focusing in collisional plasmas with a axial density ramp, specifically utilizing *q*-Gaussian laser beam. The interplay between the unique characteristics of *q*-Gaussian laser beam and the varying plasma density along the beam propagation axis introduces intriguing dynamics. Variational theory has been employed to explore the evolution of beam width of the laser beam along the propagation axis. Impact of various parameters like '*q*' parameter of the laser beam, plasma density and initial laser intensity on extent of self focusing of the laser beam has been investigated in detail.

Keywords Self-focusing · *q*-Gaussian · Density Ramp · Variational Theory · Collisional Plasma

Introduction

The phenomenon of self-focusing [1–3] of laser beams in plasmas has garnered significant attention due to its multifaceted applications spanning diverse scientific and technological domains. Self focusing, a nonlinear optical effect, occurs when the intensity of the laser beam causes a local increase in refractive index, leading to the concentration of beam energy over a confined region of space without requiring any lens (Fig. 1).

Understanding self-focusing paves the way for manipulating light in unprecedented ways, opening doors to new technologies in laser surgery [4], micromachining [5], and beyond. This phenomenon holds immense relevance across various applications, ranging from fundamental research in plasma physics [6–8] to cutting-edge technologies [9, 10]. One noteworthy application lies in the field of laser induced plasma channels [11, 12], where self focusing serves as a mechanism to create controlled and guided plasma channels for applications in directed energy, atmospheric studies, and laser induced breakdown spectroscopy.

Moreover, the ability of a laser beam to self focus becomes crucial in the context of laser induced fusion research [13]. The controlled confinement of intense laser pulses through self focusing allows for precise delivery of energy to the fusion target, enhancing the efficiency and success of inertial confinement fusion experiments. This has far reaching implications for the development of sustainable and clean energy sources, positioning self focusing a s a crucial element in the pursuit of controlled nuclear fusion.

In atmospheric studies [14], self focusing plays a pivotal role in the creation of laser filaments. Laser filaments have become a cornerstone in various fields due to their unique prospects. In atmospheric studies the laser filaments help in extending the reach of laser beams over long distances. The self guiding nature of filaments allows lasers to propagate through atmosphere, overcoming challenges posed by atmospheric turbulence. This capability holds promise for applications such as remote sensing, where precise laser beams can be directed over extended ranges for atmospheric probing, pollutant detection, and even communication in challenging environmental conditions.

In the realm of high intensity laser science, filamentation [15] offers a pathway to generate ultrashort pulses and even attosecond pulses, enabling researchers to explore ultrafast processes in matter. The ability to control and tailor laser filaments is instrumental in advancing technologies related to ultrafast optics and high precision spectroscopy.

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RESEARCH ARTICLE

Self-compression of elliptical *q*-Gaussian laser pulse in plasmas with axial density ramp

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Abstract Theoretical investigation on self-compression of a laser pulse with *q*-Gaussian spatial irradiance profile propagating through collisionless plasmas with axial density ramp has been presented. Particularly the dynamics of pulse width, beam widths and axial phase of the laser pulse have been investigated in detail. Effect of the ellipticity of the cross section of the laser pulse also has been incorporated. Using variational theory based on Lagrangian formulation nonlinear partial differential equation (P.D.E) governing the evolution of the pulse envelope has been reduced to a set of coupled ordinary differential equations for the pulse width and beam widths of the laser pulse. The evolution equation for the axial phase of the laser beam has been obtained by the Fourier transform of the amplitude structure of the laser pulse from coordinate space to (k_x, k_y) space. The differential equations so obtained have been solved numerically to envision the effect of laser-plasma parameters on the propagation dynamics of the laser pulse.

Introduction

The dream of intensifying light is as old as human civilization. Ancient people used lenses made up of glass to focus light to burn pieces of papers. However, with the debut of laser [1] in 1960, the twentieth century witnessed a dramatic shift in our perception and understanding of light. The extraordinary properties of (coherence, high intensity and monochromaticity) of laser light have revealed true beauty of light matter interactions.

When laser made its debut, little did its inventors and aficionados realize that it would not only sweep that era of scientists off its feet, but would continue to challenge and mesmerize generations to come. With that high expectation as a benchmark, the laser has proved to be nothing short of a miracle. The laser has become ubiquitous in the almost every field of modern age science and technology [2–6]. Still too many applications are in pipeline and are waiting for their turn.

New opportunities for investigating the interaction of light with matter have been opened by the invention of chirped pulse amplification (CPA) technique [7–9] in which a laser pulse is stretched, amplified and recompressed. This amplification method allows to generate very short laser pulses with peak powers upto petawatt range. Many applications of ultrashort-pulse lasers make use of the very high power that each pulse momentarily provides. Although the average power from the laser may be quite moderate and the total energy within a pulse may also be small, the extremely short duration of each pulse guarantees that the peak instantaneous power is large. In a typical system the interval between pulses is 100,000 times longer than the pulses themselves, and so the peak power is about 100,000 times the average power. For example, a 100-femtosecond pulse with a moderate energy of three microjoules (not enough energy to heat a drop of water by a millionth of a degree Celsius) delivers a peak power of 30 megawatts.

When focused on a tiny spot, such high powers ablate many materials, making ultrashort pulses a tool for

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RESEARCH ARTICLE

Self-focusing of rippled *q*-Gaussian laser beams in plasmas: effect of relativistic nonlinearity

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Abstract Theoretical investigation on self-focusing of *q*-Gaussian laser beam propagating through underdense plasma has been presented. The optical nonlinearity of plasma has been modeled by the relativistic mass nonlinearity of plasma electrons in the field of laser beam. Using variational theory approach, semi-analytical solutions of the wave equations for the fields of main beam and that of ripple have been obtained. Emphasis has been put on the evolutions of the intensities of main beam and that of ripple.

Keywords *q*-Gaussian · Laser ripple · Variational theory · Clean energy · Self-focusing

Introduction

Since the 1930s, when scientists began to realize that the sun and other stars are powered by nuclear fusion, their thoughts turned toward recreating this process in the laboratory for the viable energy production. Because fusion can use atoms present in ordinary water as a fuel, harnessing the process could assure future generations of adequate electric power [1]. The ultimate stakes are so high, fusion will produce no harmful emissions—no sooty pollutants, no nuclear waste and no greenhouse gases. All the stars and the sun use their strong gravitational pull to compress nuclei to high densities. In addition, temperatures in the sun are extremely high, so that the positively charged nuclei have enough kinetic energy to overcome their mutual electrostatic repulsion and draw near enough to fuse. However, such resources are not readily available on the earth. The particles that fuse most easily are the nuclei of deuterium and tritium. To fuse even deuterium and tritium, hydrogen gas has to be heated intensely and also has to be confined long enough that the particle density multiplied by the confinement time exceeds 10^{14} seconds per cubic centimeter. Since the 1950s, fusion research has focused on two ways of achieving this number: inertial confinement and magnetic confinement.

The strategy of inertial confinement fusion (ICF) is to shine a symmetrical array of powerful laser beams onto a spherical capsule containing a D-T mixture. The laser beams vaporize the surface of the pellet that explodes outward. To conserve momentum, the inner sphere of fuel simultaneously shoots inward just like the recoil of a gun when the bullet is fired. Although the fuel is compressed for only a brief moment (about 10^{-10} second), extremely high densities of almost 10^{25} particles per cubic centimeter can be obtained [2, 3].

For the successful realization of ICF, it is highly necessary that the fuel pellet should be heated uniformly. However, due to the nonuniform irradiance (intensity ripples) over the cross sections of the laser beams, the pellet is not heated uniformly that derives an instability known as Rayleigh–Taylor (RT) instability [4–6]: Whenever a not-very-dense fluid (like air) pushes on a denser fluid (like water), the situation is inherently unstable. If the interface between the two fluids is having any imperfection like bumps or divots, then these imperfections will immediately grow with time.

This fundamental instability can even be observed in everyday kitchen scenarios. It may be difficult to imagine an instability in the kitchen, but consider the following question: Why does not the water stay in a glass when you invert it? At first glance, the answer may seem obvious:

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RESEARCH ARTICLE

Quadruple Gaussian laser beams in thermal quantum plasma: self-focusing, self-trapping and self-phase modulation

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Abstract This paper presents a theoretical investigation into the self-action effects of intense laser beams interacting with fusion plasmas. The focus of the study is on the phenomena related to the nonlinear refraction of the laser beam, which are examined in detail. To analyze the impact of illumination uniformity across the beam's phase fronts on its propagation characteristics, the irradiance profile of the beam is modeled using a quadruple Gaussian (Q.G) profile. By employing the variational theory approach, the nonlinear partial differential equation (PDE) governing the beam envelope is transformed into a system of coupled ordinary differential equations that describe the evolution of the beam width and axial phase. These equations are then solved numerically to investigate the influence of both laser and medium parameters on the propagation characteristics of the laser beam.

Keywords Self focusing · Self trapping · Self phase modulation · Moment theory · Quadruple Gaussian laser · Clean energy

Introduction

The pursuit of harnessing intense laser beams [1–3] to achieve nuclear fusion and meet humanity's ever-growing energy needs while mitigating the impact on the global climate has been a prominent area of research in recent years. This endeavor can be likened to creating a miniature sun on Earth to power industrial machinery. Inertial confinement fusion (ICF) serves as the underlying principle for this approach, where an intense laser beam is divided into multiple smaller beams of equal intensity. These divided beams are then amplified in energy and subsequently recombined using a system of mirrors and lenses. Through this process, the beams converge on a small region from different directions. To facilitate the fusion reaction, a fuel pellet containing deuterium and tritium is encapsulated within a spherical shell measuring a few millimeters in diameter. The shell itself can be composed of materials such as plastic, glass, or other suitable substances. The fuel pellet is precisely positioned at the intersection point of the converging laser beams, ensuring uniform illumination of the pellet [4–6].

Upon interaction with the fuel pellet, the intense laser beam swiftly ionizes the atoms situated in the outermost layer. However, the material within a critical radius acts as an obstacle to the laser energy, rendering it opaque. Consequently, the incident energy becomes absorbed by a dense plasma layer enveloping the deuterium tritium fuel. As a result, this heated plasma layer expands and ablates, essentially tearing away explosively from the rest of the pellet. The ablated plasma attains velocities of approximately 1000 kms per second. In accordance with Newton's third law, an equal and opposite force propels the material inside the ablation layer inward, akin to a rocket being propelled by the plasma escaping from all sides. In essence, this system can be likened to a spherical rocket powered by lasers, with its payload being the rapidly contracting fuel pellet. However, the energy efficiency of this implosion process is relatively low, measuring less than 10 percent, mainly due to the exhaust velocity of the ablated material being significantly higher than the velocity of the imploding pellet. Nonetheless, the concentric implosive force exerted is still sufficient to accelerate the remaining shell to velocities of several hundred

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RESEARCH ARTICLE

Quadruple Gaussian laser beam in cubic-quintic nonlinear media: effect of nonlinear absorption

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Abstract An investigation on nonlinear propagation of Quadruple Gaussian (Q.G) laser beams propagating through dissipative media possessing cubic-quintic nonlinearity has been presented. The formulation is based on finding the numerical solution of Ginzburg–Landau equation for the field of incident laser beam followed by moment theory approach in W.K.B approximation. In particular, dynamical variations of beam spot size and longitudinal phase of the laser beam have been investigated in detail. Self-trapping of the laser beam resulting from the balance between diffraction broadening and nonlinear refraction has been also investigated.

Keywords Self focusing · Moment theory · Axial phase · Quadruple Gaussian · Clean energy

Introduction

Lasers [1] are one of the most successful pieces of apparatus born from twentieth century science. Rapid progress in laser technology fueled by the advent of chirped pulse amplification [2] (CPA) technique has enabled the investigation of highly nonlinear interactions of light with matter. Exotic nonlinear phenomena such as higher harmonic generation [3–5] (HHG), self-focusing [6], self-phase modulation [7, 8], and formation of spatial solitons [9, 10] are ubiquitous in laser-matter interactions. These nonlinear phenomena are extremely complex but are bursting with enough physics to keep researchers busy for years to come. Over last five decades, physicists are attempting to improve upon the understanding of these phenomena in order to have deep insight into laser-matter interaction physics. Major impetus was provided by the relevance of laser-matter interactions to potential applications like optical communication, optical device fabrication, etc. Spatial solitons, i.e., optical beams that do not disperse or dissipate, rather maintain their size and shape indefinitely, have attracted considerable interest recently, due to their applications in optical communication, optical device fabrication, ultrafast signal routing systems, etc. Spatial solitons are also familiar with mathematics, mechanical engineering, fluid dynamics, thermodynamics, biology, etc. Even spatial solitons are ubiquitous in nature and can be found in a variety of systems: from matter waves and EM waves [11]. Solitary waves were first observed by Scottish civil engineer John Scott Russel in 1834 when he noticed a curious occurrence during his ride along side a canal. When a horse-drawn barge suddenly stopped, it generated a single wave that continued to wave along the canal for kilometers without any change in the form or speed [12]. Our topic of interest, i.e., optical spatial solitons, arises due to counter balance of self-diffraction with nonlinearity induced self-focusing in conservative media. When very narrow optical beams propagate without affecting the properties of a medium, they undergo natural diffraction and broaden with distance. The narrower the initial beam is, faster it diverges (diffracts [13]). In nonlinear materials, the presence of light modifies their properties (refractive index, absorption, or conversion to other frequencies). The refractive index change resembles the intensity profile of the laser beam, forming an optical lens that increases the index in the beam's center while leaving it unchanged in the beam's tail. This induced lens focuses the beam, a phenomenon called self-focusing [6] that is a precursor of spatial solitons. When

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RESEARCH ARTICLE

Nonlinear interaction of quadruple-Gaussian laser beams with periodic lattice of metallic nanoparticles: self-action effects

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Abstract In this study theoretical exploration of the optical self-action effects involving quadruple-Gaussian (Q.G) laser beams interacting with a periodic lattice of metallic nanoparticles is presented. The numerical solution of the nonlinear Schrödinger wave equation (NSWE) for the incident laser beam's field is achieved through the moment theory approach. The investigation delves into the detailed examination of self-focusing and self-phase modulation of the laser beam. Additionally, the study places emphasis on the self-trapping phenomenon of the laser beam, which arises from the delicate equilibrium between diffraction broadening and the nonlinear refraction of the laser beam.

Introduction

Since the inception of lasers [1], the rapid advancements in high-power laser technology, driven by the introduction of the chirped pulse amplification (CPA) technique [2], have empowered researchers to unveil the true intricacies of light-matter interactions. The propagation of intense laser beams through matter consistently involves extraordinary nonlinear phenomena, including higher harmonic generation (HHG) [3, 4], self-focusing [5], self-phase modulation [6], and the formation of spatial solitons, among others [7, 8]. Despite their inherent complexity, these nonlinear phenomena provide a wealth of physics, captivating researchers and promising years of exploration ahead. To gain profound insights into the physics of light-matter interactions, seasoned physicists have been striving to enhance our understanding of these phenomena over the past few years.

Spatial solitons, optical beams that persist in maintaining their size and shape without dispersion or dissipation, have recently captured considerable interest due to their applications in optical communication, optical device fabrication, ultrafast signal routing systems, and more [9, 10]. These solitons are prevalent in various systems, including liquids, optical fibers, plasmas, and condensed matter. The observation of solitary waves traces back to 1834 when Scottish civil engineer John Scott Russell noted a peculiar incident during his ride alongside a canal. When a horse-drawn barge abruptly stopped, it generated a single wave that continued to travel along the canal for kilometers without any change in form or speed [11]. Optical solitons emerge from a dynamic balance of diffraction broadening with nonlinearity-induced self-focusing in conservative media. In linear media, narrow laser beams naturally broaden with distance due to the diffraction property of light waves—the narrower the beam, the faster it diffracts. However, in bulk nonlinear media, the intense laser beam alters optical properties like the index of refraction, absorptivity, and conversion to higher frequencies. The refractive index for a Gaussian beam in its paraxial portion is greater than at its periphery, causing the radiation phase velocity to be lower near the axis. As optical rays propagate normally to the wave front, they converge to the axis, resulting in a focusing effect and a consequential avalanche-like increase in axial intensity. This phenomenon, known as self-focusing, is a precursor to the formation of spatial solitons. When self-focusing precisely balances diffraction broadening, the beam becomes self-guided at a very narrow width, referred to as an optical spatial soliton. This effect was discovered by Chiao et al [12] in 1964.

A well-established phenomenon is that a converging/diverging electromagnetic beam undergoes an additional

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RESEARCH ARTICLE

Nonlinear interaction of quadruple Gaussian laser beams with collisional plasmas with nonlinear absorption: self focusing and self phase modulation

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Abstract This study explores how an intense laser beam with a quadruple Gaussian profile behaves as it travels through a collisional plasma that exhibits nonlinear absorption. The Drude model is used to describe the plasma's dielectric properties under this nonlinearity. The analysis relies on numerically solving the nonlinear Schrödinger wave equation (NSWE) for the laser beam's field, employing the moment theory approach within the W.K.B approximation. Through simulations, the researchers investigate how various laser and plasma parameters influence the beam's propagation characteristics. They specifically focus on how the beam's width and longitudinal phase change as it travels through the plasma.

Keywords Self focusing · Quadruple Gaussian · Clean energy · Plasma · Moment theory · Lasers

Introduction

Lasers have become one of the most successful scientific instruments born from the 20th century [1]. Advancements in technology, particularly the advent of Chirped Pulse Amplification (CPA) technique [2], have ushered in a new era of highly intense laser beams. The interaction of these beams with plasmas, known as "nonlinear optics of plasmas," is a fundamental and captivating research area brimming with exotic physics that keeps researchers engaged for years to come [1]. Today, plasma physics stands as one of the major scientific fields heavily reliant on lasers [1]. A significant impetus came from the proposal of initiating fusion reactions using high-power laser beams [3–5]. Fusion holds immense promise as a long-term energy source with no greenhouse gas emissions and minimal waste, potentially fulfilling the world's ever-growing energy demands [3–5]. However, beyond inertial confinement fusion (ICF), laser–plasma interaction physics has spurred numerous other applications, including laser-driven electron accelerators [6–8], X-ray lasers [9, 10], and supercontinuum generation [11]. The quality and efficiency of laser energy coupling with plasmas are crucial for the success of all these applications. In the absence of an optical guiding mechanism, light beams propagating through vacuum or any medium naturally broaden due to their inherent wave property of diffraction. This "diffraction broadening" presents a fundamental challenge that hinders efficient laser–plasma coupling and jeopardizes the feasibility of the aforementioned applications. Therefore, researchers are constantly exploring methods to mitigate diffraction and maintain focused laser beams. Conventional optics offers solutions like optical fibers or the phenomenon of self-focusing [12]. Self-focusing is a highly nonlinear phenomenon that arises due to the material medium's nonlinear response to the incident optical beam's field. This response causes the medium to behave like a convex lens, effectively concentrating the beam [12].

In laser–plasma interactions there are mainly three mechanisms leading to self-focusing of the laser beams. These mechanisms are (1) Relativistic [13, 14] (2) Ponderomotive [15, 16] (3) Collisional [17, 18] nonlinearity. The relativistic nonlinearity does not show any transient behaviour. It arises instantaneously when the incident laser power is grater than the threshold power required for self-focusing. In this mechanism change in optical properties of plasma occurs due to change in electron mass when its

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