DESIGN AND ANALYSIS OF OPTIMIZED BACKSTEPPING BASED CONTROLLER FOR A CLASS OF FRACTIONAL ORDER CHAOTIC SYSTEMS

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DECLARATION

I, hereby declared that the presented work in the thesis entitled "**DESIGN AND ANALYSIS OF OPTIMIZED BACK STEPPING BASED CONTROLLER FOR A CLASS OF FRACTIONAL ORDER CHAOTIC SYSTEMS**" in fulfilment of degree of **Doctor of Philosophy (Ph.D.)** is outcome of research work carried out by me under the supervision of Dr. Anuj Jain, working as Professor in the School of Electronics and Electrical Engineering, Lovely Professional University, Punjab, India and Dr.Manoj Kumar Shukla, Assistant Professor, Department of Robotics and Automation, Symbiosis Institute of Technology, Symbiosis International (Deemed University), Pune. In keeping with general practice of reporting scientific observations, due acknowledgements have been made whenever work described here has been based on findings of other investigator. This work has not been submitted in part or full to any other University or Institute for the award of any degree.

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CERTIFICATE

This is to certify that the work reported in the Ph.D. thesis entitled "**DESIGN AND ANALYSIS OF OPTIMIZED BACK STEPPING BASED CONTROLLER FOR A CLASS OF FRACTIONAL ORDER CHAOTIC SYSTEMS**" submitted in fulfillment of the requirement for the award of degree of **Doctor of Philosophy (Ph.D.)** in Electronics and Communication Engineering, is a research work carried out by A. Narmada, 41900481, is bonafide record of his/her original work carried out under my supervision and that no part of thesis has been submitted for any other degree, diploma or equivalent course.

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Mrs A. NARMADA

Dedicated

To

My Beloved

Mother & Husband

ABSTRACT

Fractional-order calculus employs orders other than integers to move mathematical modelling beyond conventional differentiation and integration. Researchers from diverse disciplines use FOC because it helps them model real-world processes better than ordinary integer-order calculus does. The unique quality of fractional-order systems is their ability to hold continuous information from every past state along with their most recent condition. Systems described using fractional-order calculus retain memory of past events to provide a better representation of time-varying dynamics.

In this work, various Non-linear controllers such as Sliding-Mode controller, Dynamic Surface Controller, Fuzzy Dynamic Surface Controller and Backstepping controllers are utilized to decrease the error present in the Nonlinear system. Out of these, the backstepping controller exhibits better performance in comparison to others in the case of actuator faults. The performance of Backstepping controller to handle tracking control problems in nonlinear systems under uncertain internal parameters and external interference can be enhanced by integrating a deep learning architecture that is Recurrent neural Network (RNN) is with backstepping controller. An RNN network improves control performance when compared with the backstepping controller and allowing for higher tracking precision.

The research shows the RNN-based backstepping controller provides superior control results over other optimization controllers such as Event-triggered backstepping control with fuzzy-based reinforcement learning, Sliding mode control with neuro-fuzzy adaptive backstepping and Backstepping control with echo state networks. The proposed backstepping controller with RNN was highly effective and outperformed for minimizing system tracking error, it reduces errors better and maintains better stability.

When two non-linear systems are synchronised, they produce same chaotic signal which is unpredictable in nature. The chaotic system generates noise-like waveform, which is difficult to predict and sensitive to changes in initial conditions. Chaotic synchronization has potential applications in the areas of secure communication, image encryption, information processing, chemical processing, biological engineering and other fields.

RNN based backstepping controller is used to synchronise Chaotic Fractional order systems. Chaotic signal is unpredictable in nature and chaotic signal generated by

Master system is used for masking the speech signal and same chaotic signal generated by the receiver when both the systems are in synchronous with each other, is used for unmasking the speech signal. The effectiveness of RNN-backstepping controller in secure communication and image encryption for fractional-order chaotic systems is studied in this research while demonstrating its versatility and practical implementation potential.

Keywords: Fractional order calculus, Nonlinear Controllers, Chaotic systems, Backstepping controller, RNN, Synchronization

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ABBREVIATIONS

AF	Actuator Faults
ANN	Artificial Neural Network
ANN-BSC	Artificial neural network based backstepping controller
ESN	Echo State Network
FIS	Fuzzy Inference System
FLS	Fuzzy Logic System
FO	Fractional Order
FOS	Fractional Order System
MSE	Mean Square Error
NFABSM	Neuro fuzzy adaptive backstepping sliding mode control
NFADBSC	Neuro fuzzy adaptive backstepping dynamic surface control
PSNR	Peak Signal to Noise Ratio
RBFNN	Radial Basis Function based Neural Network
RLEBSC	Reinforcement learning for event triggered backstepping control
RMSE	Root Mean Square Error
RNN	Recurrent neural network
SMC	Sliding Mode Control

Chapter-1

INTRODUCTION

1.1. Introduction

Automation in control systems is extremely crucial to the rapid growth of engineering and science, since, it has been integrated into every system such as a spacecraft, a missile guidance system, an industrial process, and robotic systems [1]. The control action to be executed can be proportional, derivative, integral, or a mixture of any two or all of them. A gain-adjustable amplifier is essentially a proportional controller with a step input, and the proportional controller generates a steady-state inaccuracy known as offset [2]. High sensitivity is produced by the "integrated-controller" when a "proportional-controller" and a "derivative-controller" action are integrated.

Generally practical systems are Nonlinear in nature. The application of fractional order calculus (FOC) has been considered for mathematical modelling of Non-linear systems, as FOC more aptly characterises the behaviour of non-linear systems. Though the FOC was first developed in early 1960s, the Fractional Order Non-Linear (FONL) systems have gained interest recently in the control systems community [3-7]. Some of the areas such as thermodynamics, physics, practices, electrical circuit theory, signal processing, mechatronic systems, chaos theory, chemical mixing, and biological systems, have a significant role played by fractional order calculus [8-11].

1.2 History of "Fractional-Order Calculus"

The chronology of the FOC is given in Table 1.1 [12-13]

Table 1.1: Fractional Calculus History

Author name	year	Introduced
Joseph-Louis Lagrange	2722	Highlighted the role of differential operators in understanding fractional operators, noting limitations in applying the law of indices.
Laplace	1812	Defined the fractional derivative using integral approaches; Lacroix first presented the fractional order derivative.
Joseph B. J. Fourier	1822	Proposed fractional derivative of trigonometric functions.

Niels Henrik Abel	1823	First fractional calculus in solving the Tautochrone problem, serves as an early application example.
Liouvile	1832	Conducted major studies on Fourier fractional integrals and the fractional derivative of exponential functions.
Reimann	1847	Systematically studied fractional derivatives for power functions; Leibniz and Euler made earlier attempts.
N. Sonin	1869	Published on differentiation with arbitrary indices, building on Cauchy's Integral formula.
Reisz	1949	Developed fractional integration theory for multivariable functions, addressing ambiguities in Riemann and Liouville definitions.
Caputo	1967	Offered a simplified definition of fractional operators based on series expansion, which leads to the Mittag-Leffler function.
B Ross	1974	explained brief history and exposition of the fundamental theory of fractional calculus
Mandelbrot	1980's	Introduced fractional geometry, sparking interest in fractional Brownian motion and anomalous diffusion processes.
Glasgow, 2nd Int. Conf. on fractional calculus.	1984	Presented papers on Historical Background and advancements in Fractional Calculus .
Nihon University, 3rd Int. Conf. in FOC Tokyo Japan	1989	Was dedicated to the area of mathematics called fractional calculus
Raspini	2000	Investigated a fractional symmetric wave equation, raising fundamental questions in physics.
Shahi and Khosravi	2021	Developed novel methods for solving fractional differential equations using machine learning techniques.

Gonzalez et al.	2023	Introduced applications of fractional calculus in complex systems and chaos theory, broadening the scope of its practical implications.
International Conference on Mathematical Sciences	2024	Focused sessions on fractional calculus and its applications, highlighting interdisciplinary research that integrates fractional calculus into engineering and physics.

1.3 Nonlinear Controllers

To achieve better results in control, a system needs to be modelled accurately. Active control methods as well as sliding mode control and dynamic surface control serve as examples of controllers. These approaches reduce how far the estimated results differ from true values in systems that behave nonlinearly.

Backstepping controller: In the design of backstepping controller, system is broken into small subsystems while building controllers for every section. This method offers a systematic design process, especially for recognized dynamics systems [13,17, 22,25]. Companies that build robots and aerospace equipment choose backstepping control because it works well with unpredictable conditions. By designing virtual controls and Lyapunov functions, this technique enables stability in multiple applications throughout the robotics and aerospace sectors.

Dynamic Surface Controller: The control design procedure becomes simpler through Dynamic Surface Control (DSC) since this method eliminates complicated and challenging derivative calculations for state variables [5,6,14, 83,84]. Low-pass filters installed in DSC allow the system to reduce fast changes in control signals, so that the overall control results in smoother action. Systems can produce fast reliable responses because of this improvement which makes them resilient to both unpredictable events in the system and external disturbances. The technology proves advantageous specifically in robotic and automotive systems because quick decision processes combined with precise control are crucial to the operation.

Sliding-Mode Controller: Sliding-Mode control (SMC) creates a controller that guides system movement across state space surface to overcome external interruptions. SMC [7,8,15,75,76,78,80] maintains its performance stability despite external influences and plant design changes making it an effective tool for handling system nonlinear characteristics. Chattering issues occur regularly during sliding mode control yet different methods are available to handle them. This controller builds automotive and robotic systems that work well despite unknown parameters present in the system.

Fuzzy controller: Fuzzy logic controllers (FLC) use fuzzy set theory [5,6,8,11,12,15,18,23] to design systems when available information remains unclear and imprecise. These systems work with linguistic rules instead of mathematical equations allowing smooth navigation of unpredictable conditions. A fuzzy controller uses expert knowledge to control systems without needing an exact mathematical description. FLCs work best under these conditions and help to produce smart electronic devices plus automated systems that behave as real.

1.3.1 Optimized Controllers

Backstepping controllers are increasingly used in nonlinear systems due to their enhanced performance and flexibility. Optimization techniques are commonly employed to fine-tune controllers, particularly chaotic systems to achieve the desired control objective.

Particle Swarm Optimization (PSO): This method uses multiple individuals to find optimization solutions from natural routines observed in bird and fish groups. During controller parameter optimization PSO [102,221] finds Backstepping controller settings to reach the desired control performance. The method alters particle placements through an iterative process based on personal learning and neighbour implications to achieve optimal results.

Genetic Algorithms (GA): Genetic Algorithm functions [22,139,221] like natural evolution to help users find optimized solutions. By working with multiple potential solutions GA adds them to the population and uses selection crossover and mutation to find better solutions. A fractional order backstepping controller needs GA optimization tools to search through parameter options and make the design resilient to disturbances and uncertainties.

1.3.2 Deep learning Controllers

Deep neural network architectures like “Convolutional Neural Network (CNN)” [23] and “Recurrent Neural Network (RNN)” [24] help researchers solve complex fractional order nonlinear system problems by handling distinctive dynamic functions with non-integer orders. CNNs shows excellent performance in system identification when dealing with complex input-output relationships from multidimensional datasets. Through training on suitable datasets, CNN models learn to recognize fractional order system behaviours for better system predictions and control process improvement. RNNs handle time-dependent data patterns perfectly so they work well for systems that follow fractional order dynamics. The models maintain past input information and use it to identify repeated patterns that develop and change with time [25]. By integrating reinforcement learning with CNNs and RNNs developers can make adaptive control policies that automatically optimize system performance based on real-time environmental feedback. Combining deep learning for system detection with traditional backstepping control designs helps enhance stability while preserving better tracking results [26]. CNN and RNN integration with fractional order systems demonstrates great potential even though model interpretability and dataset size requirements pose obstacles.

RNNs are artificial neural networks designed specifically to deal with sequential data by remembering the previous inputs in their internal memory. In contrast to feed-forward networks, where each input is processed independently, RNNs introduce a hidden state that allows information to carry over.

The hidden layer of an RNN has feedback connections, which is the main idea behind it. The network processes the current input and the information from previous inputs at the same time. This makes RNNs a great choice for modelling dynamic and nonlinear systems. In the proposed work, the RNN is utilised as an approximator to estimate unknown nonlinear dynamics and uncertainties inherent in the system. The RNN changes its parameters in real time by always learning from the tracking error and system states. This makes control more accurate.

The input layer receives the current input signal. This input can represent system states, error signals, or any time-varying data. The input is then passed to the hidden layer through weighted connections. The RNN weights are updated using an adaptive learning law, which keeps the learning process stable. The hidden layer plays a central role in the RNN. It processes the current input along with information from the previous time step, which is fed back through a recurrent connection. Because of this feedback, the hidden layer is able to store and use past information, giving the RNN its memory property. The output layer generates the network output based on the current hidden state. Depending on the application, this output could be an estimated signal, a system response, or a control-related quantity. The RNN works by using the same structure over and over again at each time step and always updating its hidden state. This makes it especially suitable for dynamic and time-dependent systems, where the present behaviour depends on past states.

The basic structural representation for RNN is shown in figure 1.1.

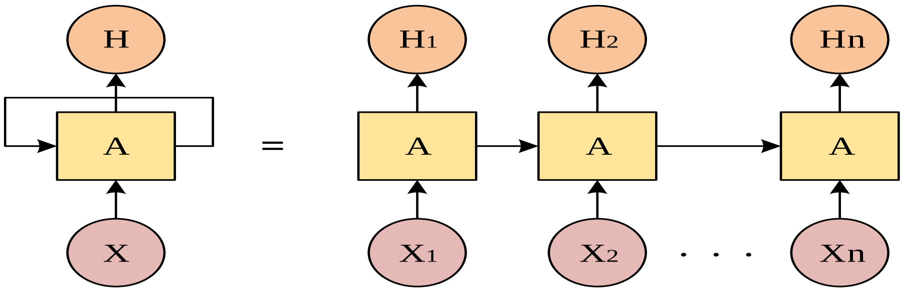


Figure 1.1: Basic architecture of RNN

Where, $X = [X_1, X_2, \ldots . X_n]$ indicates input layer, $H = [H_1, H_2, \ldots . H_n]$ denotes output layer and $A = [A_1, A_2, \ldots . A_n]$ represent hidden layer in RNN [200]. In this work, it assumes that only output of RNN is used to for controller design and nonlinear system. The current state is evaluated as follows;

$$H_n = f(H_{n-1}, X_n) \quad \text{(e.q., 1.1)}$$

Where, X_n denotes input, H_{n-1} signifies previous sate, H_n represent current state, this estimated value error is denoted as

$$E = X - \dot{X} \quad \text{(e.q., 1.2)}$$

$$H_n = \tanh(E_H H_{n-1} + E_{XH} X_n) \quad \text{(e.q., 1.3)}$$

Where, E_H denotes error at input neuron and E_{XH} indicates error at recurrent neuron. This error value is solved using RNN at the output layer. This will enhance the performance of the system. RNN learns the features for the time-series data by the use of memory about the past inputs in the neural network's internal state. Further, it uses historical as well as the current data for the forecasting of the future information.

1.4 Applications of Fractional Calculus

There are several applications that used fractional calculus and improved real-world performance. The key application detailed about following:

Biological system: Today the fractional-order model succeeds at mathematical modelling of biological systems that integer-order modelling cannot handle effectively. Through drug release studies they show pharmaceutical factories can deliver medicine at different rates [27]. Experimental studies validate these methods and help medical device developers better understand how electrodes connect to heart tissue when designing critical pacemakers and defibrillators. By using fractional calculus to model neural activity in the field of neuro-engineering can improve brain-computer interface technologies.

Engineering Application: Modern technology uses fractional-order systems across many work areas. Industrial automation programs demonstrate how fractional-order PID controllers [28] help motor systems work more effectively than conventional controllers. These controllers bring effective control results to complex systems that need robotics and process control power. Researchers use fractional-order models to improve fuel cell and lithium-ion battery performance and efficiency [29].

Secure Communication: Based on research, now applications of fractional calculus includes cryptographic advancement through chaos-based encryption methods [30] and image protection using the fractional-order fuzzy logic system model (FFLSM) approach [31]. These methods use the unique features of fractional-order systems to create reliable security measures that resist multiple threat types. Data protection needs to drive this program's development as a modern requirement.

1.5 Challenges in Fractional-Order System

The complexity of these systems creates unique problems during controller design. Key challenges include:

Computational complexity: Doing mathematical work with fractional derivatives and integrals needs high amounts of processing power. Stronger algorithms are needed to manage the growing amount of computing work.

Instability and Oscillation: Standard control techniques face stability and fluctuation problems as common problems. To avoid instability in fractional-order systems excellent control practices are needed.

Robust Synchronization: Maintaining reliable synchronization amongst chaotic systems poses a hard technical difficulty. Controller performance degrades significantly in fractional-order systems because these systems respond strongly to small changes in initial state and model parameters.

1.6 Motivation

Modern control technologies need development because today's complicated systems demand better ways to deal with uncertain behavior across healthcare robots, aerospace and renewable energy sectors. Even though traditional control engineering depends mainly on integer-order calculus it regularly proves ineffective at modeling complex dynamic systems and processes with memory effects and past influences. Most of the real-world systems, show nonlinear behavior, uncertainties, disturbances, and dynamics that depend on memory. Most traditional control methods use mathematical models based on integer-order mathematical models, which are generally written as

$$\dot{x}(t) = f(x(t), u(t), t) \qquad \text{(e.q., 1.4)}$$

Although these models have been successfully implemented in a variety of applications, they frequently provide an insufficient representation of systems whose present behaviour depends on past inputs and previous states. A general fractional-order nonlinear system can be expressed as

$$D_t^{\alpha} x(t) = f(x(t), u(t), t) \qquad \text{where } 0 < \alpha < 1 \quad \text{(e.q., 1.5)}$$

The use of the fractional-order calculus introduces memory and hereditary properties to the system model, making it a more accurate representation of complex dynamics than traditional integer-order formulations. An important objective in non-linear systems is to ensure accurate tracking of a desired reference signal despite the presence of uncertainties and disturbances. The combined effects of nonlinearities, unc.ertainties, and actuator faults pose serious challenges to conventional control strategies. These challenges motivate the development of control techniques that improves tracking accuracy, maintaining stability because of internal and external disturbances & faults in actuators, by combining the benefits of deep learning architecture.

1.7 Problem Statement

Control engineers have not fully explored using FOC to design systems that control nonlinear dynamics. Modern control techniques prove ineffective at keeping the system stable when durability issues appear alongside inner parameter changes and outside disturbances.

Standard control strategies including backstepping [13,17], sliding mode control [7,8] and dynamic surface control [5,6] struggle to provide optimal performance when dealing with these real-world problems.

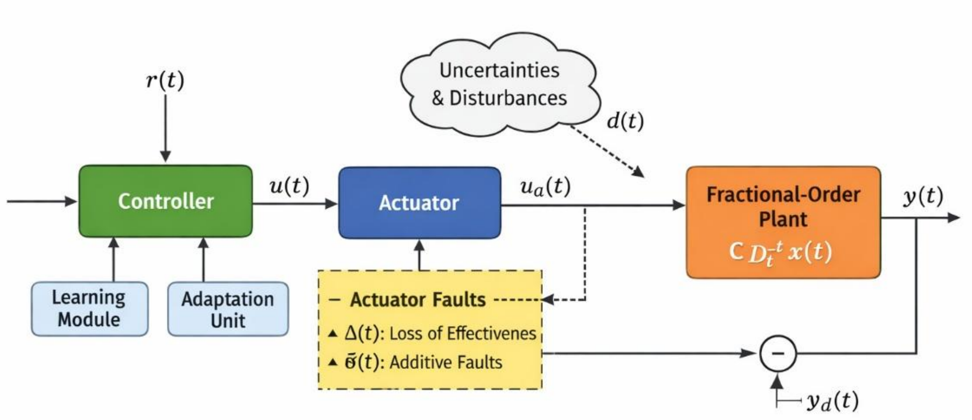


Figure 1.2: Block Diagram of Non-linear fractional order system with Controller

In practical applications, the system may also be affected by external disturbances as shown in the above figure 1.2 and actuator faults, which can be represented as:

$$u_a(t) = \Delta(t)u(t) + \sigma(t) \text{ and}$$

$$e(t) = y(t) - y_d(t) \qquad \text{(e.q., 1.6)}$$

Where $u_a(t)$ is the actuator signal, $\Delta(t)$ models loss of effectiveness. $\sigma(t)$ is additive fault components and $u(t)$ represents control input to be designed such that the system output $y(t)$ accurately tracks a desired reference signal $y_d(t)$. But, standard control strategies including backstepping, sliding mode control and dynamic surface control struggle to provide optimal performance when dealing with these real-world problems. To overcome these problems, controllers are integrated with optimization techniques to boost tracking precision by reducing errors with superior performance.

1.8 Objectives

Research objectives of this work are

i. To evaluate various control methods for fractional order non-linear systems with actuator fault.

ii. To propose an RNN-based backstepping controller for the Genesio-Tesi chaotic system.

iii. To optimize the RNN-based backstepping controller for fractional–order nonlinear systems.

iv. To develop a secure communication and image encryption solution using the RNN based backstepping controller for a fractional-order chaotic system.

1.9 Research Contribution

The major contributions in the control systems field includes

i. To formulate an RNN-based backstepping controller capable of accurately modelling fractional dynamic behaviours while ensuring robust tracking performance.
ii. To introduce the new control framework that synergizes RNNs with the backstepping technique, expanding the existing theoretical model for fractional-order systems.
iii. To conduct extensive and analyse the proposed controller's performance against traditional control methods focusing on error metrics, response time, and overall stability.
iv. To employ Backstepping controller with RNN for real time applications for transmitting information signals and images securely.

1.10 Thesis Organization

Chapter 1: Introduction

Discuss briefly about Fractional Calculus, Nonlinear fractional order control systems, objectives, applications, challenges and existing work are discussed.

Chapter 2: Literature Survey

Discuss about conventional Nonlinear Controllers and Optimized Controllers.

Chapter 3: Objective 1

Deals with the comparison of performance of various controllers when applied to FONL systems with actuator fault.

Chapter 4: Objective 2

Describes the design of adaptively coupled recurrent neural network-based backstepping controller for the Genesio-Tesi chaotic system

Chapter 5: Objective 3

Presents the comparison of optimized controllers with RNN based backstepping controllers.

Chapter 6: Objective 4

Illustrates the synchronization of chaotic FONL systems by using Backstepping controller with RNN for transferring information and images securely.

Chapter 7: Conclusion and Future Scope

Conclusions with future scope of RNN based Backstepping Controller is discussed.

1.11 Summary

This chapter has defined the purpose of the study and offered a background to the research. Thus, it has described the weaknesses of conventional control methods in handling FONL systems and highlighted the opportunity for RNN's cooperation with backstepping control. The rest of the chapters will provide additional elaboration on each aspect of the study, as well as the analysis and discussion of the results.

Chapter-2

LITRATURE SURVEY

2.1 Introduction

Controlling a system means management of its input and output so that it behaves in the way that is intended. In general, the nature of all real-time systems is nonlinear. Fractional-order calculus, provide better mathematical modelling of Nonlinear systems. Fundamentally, FOC is an extension of n-fold integration and integer order differentiation to any Fractional order. This chapter discusses about Fractional Calculus (FC) and various Non-linear controllers

2.2 Conventional Fractional-Order Controllers

The use of FC is very commonly used in control [32-34]. Different approaches for solving “Fractional order differential equations” and using them in control applications have been presented by Pudlubny, [35]. According to Peng et al, [36], improved characteristics are obtained when a FO controller is applied to a FO system that indicates the better representation of real-world systems. Pudlubny proposed first theory of generalisation of Proportional+Integral+Derivative(PID) control, known as Fractional Order PID (FOPID) control. Various methods for creating and fine-tuning FOPID controllers have been presented in [37]. Due to their improved performance over the conventional PID controller, FO-PID controllers are now used in real-time applications.

Other control techniques like H_∞ and state-space approach have also been expanded to use FC. The domain of FC is described in terms of optimal control strategies [38]. Odibat and S Momani proposed the fractional order Riccati equation in [39]. In light of all the advancement, fractional-calculus has been applied to address a variety of practical issues in control systems, including flexible manipulator [40], hydraulic actuator [41] in a mathematical modelling of muscular blood vessels, automatic generation control [42], buck converter[43], flexible transmission [44], active suspension [45], chaos suppression [46], etc.

2.3 Stability of Fractional Order Nonlinear Systems

Stability is one of a system's key characteristics. Stability of dynamical systems that can be studied in either the time or frequency domain. Despite the fact that there has been little research on the stability analysis of fractional order systems, significant conclusions have been reached.

Even though the stability study of FONLS is more complex than that of their linear counterpart, progress has nonetheless been achieved in this area. Recent research has suggested a new definition of power law stability in terms of the Mittag-Leffler function, which is detailed below.

Theorem 2.1: Trajectory $\mathbf{x}(t) = 0$ of the system $D_t^p\mathbf{x}(t) = \mathbf{f}(t, \mathbf{x})$ is t^{-q} asymptotically stable, if there is a positive real q so that:

$$\forall \|\mathbf{x}(\mathbf{t})\| \text{ with } t \leq t_0,\ \exists\ N(\mathbf{x}(\mathbf{t})), \text{ such that } \forall\ t \geq t_0, \|\mathbf{x}(\mathbf{t})\| \leq Nt^{-p}$$

The components of $\mathbf{x}(t)$ slowly decay towards **0** following t^{-p}.

The above definition has been further used by Li et. al. to propose a new stability condition for FONLS, i.e. Mittag-Leffler stability, which is expressed in form of a theorem as given below:

Theorem 2.2: The solution of following fractional order non-autonomous system

$$D_t^p\mathbf{x}(t) = \mathbf{f}(t, \mathbf{x}, u) \quad \text{(e.q.,2.1)}$$

is said to be Mittag-Leffler stable if

$$\|\mathbf{x}(t)\| \leq \{m[\mathbf{x}(t_0)]E_p(-\lambda(t - t_0)^p)\}^b \quad \text{(e.q.,2.2)}$$

where t_0 is the initial time, $p \in (0, 1)$, $\lambda > 0$, $b > 0$, $m(0) = 0$, $m(\mathbf{x}) \geq 0$, E_q is the Mittag-Leffler function as defined previously and $m(\mathbf{x})$ is locally Lipschitz on $\mathbf{x} \in \mathbb{B} \in \mathbb{R}^n$ with Lipschitz constant m_0.

A few lemmas that are crucial to comprehending the stability conditions are reproduced below before moving on to the next criterion for FONLS stability.

Lemma 2.1: Power law for fractional order derivative: Let $x(t) \in \mathbb{R}$ be a real continuously differentiable function. Then for any $p = 2^n$, $n \in N$,

$$D_t^q x^p(t) \leq px^{p-1}(t)D_t^q x(t) \quad \text{(e.q.,2.3)}$$

where $0 < q < 1$ is the fractional order.

From the above lemma following corollary and theorem can be deduced as follows:

Corollary 2.1: Let $x(t) \in \mathbb{R}$ be a continuous and differentiable function. Then, for any time t,

$$\frac{1}{2}D_t^q x^2(t) \leq x(t)D_t^q x(t), \qquad \forall q \in (0,1) \quad \text{(e.q.,2.4)}$$

Theorem 2.3: The fractional order system (1.18), with control law $u = \beta(\mathbf{x})$ is stable if for $p = 2^n$, $n \in N$,

$$\mathbf{x}^{p-1} D_t^{\alpha} \mathbf{x}(t) = \mathbf{x}^{p-1} \mathbf{f}(\mathbf{x}, \beta(\mathbf{x})) \leq 0 \qquad \text{(e.q.,2.5)}$$

and the system with $u = \beta(\mathbf{x})$ is asymptotically Mittag-Leffler stable if $\mathbf{x}^{p-1} \mathbf{f}(\mathbf{x}, \beta(\mathbf{x})) < 0$.

The fractional order extension of the Lyapunov stability requirements can be defined using the aforementioned equations and Mittag-Leffler stability. The Lyapunov stability criterion has been attempted to be extended to fractional order systems by a number of academics. For the same, the following theorem can be stated.

Theorem 2.4: Let $\mathbf{x} = \mathbf{0}$ be an equilibrium point for the non-autonomous fractional order system (1.18). Assume that there exists a Lyapunov function $V(t, \mathbf{x}(t))$ and class $\mathcal{K}$ functions $\gamma_i (i = 1, 2, 3)$ satisfying

$$\gamma_1 \|\mathbf{x}\| \leq V(t, \mathbf{x}(t)) \leq \gamma_2 \|\mathbf{x}\| \qquad \text{(e.q.,2.6)}$$

$$\text{and} \quad D_t^q V(t, \mathbf{x}(t)) \leq -\gamma_3 \|\mathbf{x}\| \qquad \text{(e.q.,2.7)}$$

where $t \geq 0$, $q \in (0, 1)$. Then $\mathbf{x} = \mathbf{0}$ is Mittag-Leffler stable, asymptotically.

The above results can be stated in more usable form as below:

The stability of the system (1.18) with a controller defined as $u = \beta(\mathbf{x})$ is guaranteed

Theorem 2.5: by the Lyapunov function $V(t, \mathbf{x}(t))$ and its fractional order derivative if following condition is met, where γ is class $\mathcal{K}$ function and g is some function of states:

$$D_t^q V(t, \mathbf{x}(t)) \leq -\gamma g(\mathbf{x}, \beta(\mathbf{x})) \qquad \text{(e.q.,2.8)}$$

$$\text{Also if} \qquad D_t^q V(t, \mathbf{x}(t)) < -\gamma g(\mathbf{x}, \beta(\mathbf{x}) \qquad \text{(e.q.,2.9)}$$

then it can be concluded that, the corresponding controller u stabilizes the system globally and asymptotically.

Further, Tavazoei and Haeri proposed stability conditions for FONLS which are based on the linearization of the systems via Jacobian method. These are given below in form of theorems:

Theorem 2.6: (For commensurate order systems): The equilibrium points of the system (1.18) are asymptotically stable for $q_1 = q_2 = \cdots = q_n = q$ if all the eigenvalues $\lambda_i (i = 1, 2, \ldots, n)$ of the Jacobian matrix $\mathbf{J} = \partial \mathbf{f} / \partial \mathbf{x}$, where $\mathbf{f} = [f_1, f_2, \ldots f_n]^T$, evaluated at a particular equilibrium point E^*, satisfy the condition:

$$|\arg(\mathrm{eig}(J))| = |\arg(\lambda_i)| > q\frac{\pi}{2}, \qquad i = 1,2 \ldots, n \qquad \text{(e.q.,2.10)}$$

Theorem 2.7: (For incommensurate order systems): For an incommensurate fractional-order system (1.18) with $q_1 \neq q_2 \neq \cdots \neq q_n$ and if m is the LCM of the denominators u_i's of q_i's, where $q_i = \frac{v_i}{u_i}$, $v_i, u_i \in \mathbb{Z}^+$, for $i = 1,2,\ldots,n$ and $\gamma = 1/m$, then the system is asymptotically stable if,

$$|\arg(\lambda)| > \gamma\frac{\pi}{2} \quad \text{(e.q.,2.11)}$$

for all the roots λ of the following equation

$$det(diag([\lambda^{mq_1}\lambda^{mq_2}\ldots\lambda^{mq_n}]) - J) = 0 \quad \text{(e.q.,2.12)}$$

Stability is one of a system's key characteristics. Matignon [47] provided the initial definition of Bounded input & Bounded Output(BIBO) stability in time domain for FO systems (FOS). “Linear Matrix Inequality” technique has been employed for examining the stability of Fractional Order Linear systems (FOLS) and results demonstrated in [48]. Frequency-domain techniques like Nyquist criterion were proposed in [49]. Bonnet & Partington [49] and Chen and Moore [50] studied FONL delayed systems. Stability of commensurate & non-commensurate Fractional order - Linear Time Invariant (FOLTI) systems is investigated by Matignon depending on pole locations determined from the transfer function of the system. This research was motivated with the evolution of Laplace transform of FODE. With the aim of providing a stability criterion that makes use of the State Space model of FOLTI systems, Tavazoei and Haeri [51-52] extended these findings. Deng et. al. created further versions for various types of FOS in [53]. The stability of FONL Systems is not extensively studied, but in the past ten years, attention has turned to this topic.

The stability and impulsive Caputo differential equations were covered in a survey by Ravi Agarwal[54]. In [55-60], stability of FONL systems is checked by employing Lyapunov stability. According to energy balance method, [60] proposed Lyapunov stability for FONL systems of non-commensurate orders. By constructing quadratic Lyapunov functions, Quan Xu [61] used linear state feedback control and adaptive control to address the stability problem of FO non-autonomous systems. Based on Lyapunov stability, Authors R. S. & Zhang [62] presented single state “Adaptive feedback control law” to achieve stabilisation of 3-dimensional chaotic systems. For FONL systems with unidentified coefficients, the problem of Mittage-Leffler stabilisation is discussed in[63]. In this study, Backstepping with adaptive law is proposed to obtain an asymptotic Mittag-Leffler stabilisation using fractional-order update laws. In [64], authors looked into a number of Mittag-Leffler stability requirements for fractional Lyapunov functions for FONL systems. Wang et al [71]studied the stability of commensurate strict feedback FONL systems using Adaptive Mittag-Leffler stabilisation technique. Based on Mittag-Leffler stability theory, the active control method is employed in [65] to reduce chaos. In the domain of dynamic information storage or retrieval, brain-like associative learning, and other issues, the neurodynamic features are effective instruments for solving numerous issues. Mittag-Leffler stability analysis is used in [66-67] to examine the stability of Fractional Order Neural Networks.

It is possible to regulate FONL systems using a linear State-Feedback Controller. The State-Feedback Controller is suggested in [68-69] for FONL lower triangular systems depending on Lyapunov function. The challenge of linear state feedback controller are discussed in [70] utilising Linear Matrix Inequality (LMI). A technique to design a non-linear system that takes data losses into consideration is suggested in [71]. For nonlinear systems which are controllable and observable [72], presents a unified high gain state feedback controller to address an acceptable tracking problem. Using basic state feedback gains, the chaos control of 3 different FO systems is described in [70].

One of the leading techniques to design a controller is LMI. It is proved to be a good technique in designing controllers for Integer & Fractional Order systems. For FONL systems, Linear matrix inequality(LMI) stability requirements are stated in [64], and a state feedback controller based on the LMI Approach is built in [73]. LMI technique is utilised in [74],[75] to stabilise “Fractional Order Chaotic Systems(FOCS)”.

Sliding-Mode controller (SMC) was proposed in [76], for Integer and Fractional order systems. To synchronize Chaotic Fractional order systems with undetermined factors, a SMC is proposed in [77],[78], and an adaptive SMC is presented for FOCS in [79]. To synchronise two chaotic systems, authors in [80] developed an active sliding mode control technique. The SMC was adopted by Liu et al. [81] as it provides higher convergence for the tracking error, while the interval excitation controller with persistent conditions reduces the estimation error.

Liu et al. [82] employed to reduce tracking error in FONL Systems using Dynamic Surface Control(DSC). The dynamic surface control model uses the persistent excitation constraint to achieve convergence. However, the complexity of using persistent excitation is high; hence, the model makes use of the composite learning law on interval excitation. Liu et al. [83]developed an adaptive NN based backstepping controller for FONL systems with AF. The employed control model reduced the complexity of solving the complex equations of the backstepping controller. Moreover, the system used command filters that reduce tracking errors by using the command signals in a fractional filter.

2.4. Hybrid Controllers

Backstepping with Adaptive fuzzy logic & DSC method is presented in [84], for FONL systems of strict-feedback type influenced by uncertain parameters & unidentified external disturbances. To estimate unknown parameters, Fuzzy Logic can be used. The stabilisation of nonlinear non-autonomous fractional-order systems are proposed in [85] using an adaptive intelligent fuzzy technique. In case of the FO satisfying $0 < \propto < 1$ and $1 < \propto < 2$, the stabilisation problem of uncertain FOCS is examined in [86] using T-S fuzzy model. To compensate harmonic current and stabilise the DC voltage quickly, a Backstepping sliding mode adaptive fuzzy controller has been developed in [87] for a 3-Φ active filter. To estimate unknown nonlinear functions in [88] and to account for external disturbances and input saturation, a Backstepping control with an adaptive fuzzy approach is adopted in [89].

A hybrid controller that takes the features from BS and DSC has been reported by Zhiyao Ma and Hongjun Ma [90] for SISO strict feedback FONL. The unknown nonlinear function concerning external disturbance encountered is obtained by using fuzzy logic systems. The hybrid control model is initially set with the change in coordinates, where, the surface error is described by the fractional order adaptation law.

For the stability analysis, fuzzy controller with command-filtered backstepping was employed by Ha et al. [91].To operate the system at high frequencies, virtual input to the system was obtained from a command filter of fractional order. For reducing the error because of filtering, error compensation mechanism was also adopted. Fuzzy logic along with Gaussian function was implemented to estimate the non-linear parameters of the controller and also to minimize the complexity of backstepping control parameter calculation. A SMC with neuro fuzzy network system was reported by Song et al. [92] in order to solve the issue related to finite time based fractional order nonlinear system. Taking the uncertainties and disturbances into consideration, an adaptive hybrid BS recursive technique was utilized. Thus the approximate unknown nonlinear error in the system is highly reduced. However, the model results in high shattering effect, which is the major problem of sliding mode controller.

Controllers like backstepping, dynamic surface control, sliding mode control, active control, fuzzy controller, are designed to minimize approximation error in FONL system. Even though several controllers are available for the accurate tracking of virtual signal in accordance with reference trajectory, Still Fractional order nonlinear strict feedback system suffers from unknown parameters. This nonlinear system includes internal parameter uncertainty and external disruptions. To deal with such problem of actuator fault, parameter uncertainty and disturbance [93], the backstepping controller is widely adopted. The backstepping controller improves the asymptotic tracking and stability concerning virtual control input.

But still, these controllers have the problem of slow error tracking, computational burden, increasing derivative terms etc. that results in unstable virtual control input. Also, it is difficult to cope with the system with severe uncertainty, which results in nonlinearity in the parameter. Hence optimized controllers are developed.

2.5 Optimized Controllers

Traditionally, the unknown function of the fractional order controllers is optimized by using optimization algorithms like “Genetic Optimization” and “Particle Swarm Optimization”. Later, optimized algorithms are extended to use neural networks so that accuracy in detecting approximation error is obtained [94]. A full state constraint neural quantized control for an FO system is reported. The controller effectively detects the approximation error when the system has subjected with quantization, nonlinearity and faults. Further, the neural network is used with other limitations of input saturation, output constraints, and time varying pseudo state constraints [62][95]. This motivated to study the performance of various artificial intelligence based optimized controllers for FONL system.

A backstepping controller with an adaptive law is proposed for stabilizing using Mittag-Leffler stabilisation in [71]. Backstepping method is discussed in [96] for stabilising a class of FOCS. An adaptive controller is proposed using Backstepping approach for estimating unknown system parameters of FOCS in [97], [98]. For robotic manipulators, authors in [99] suggested a fractional-order calculus-based finite-time adaptive Backstepping fault-tolerant tracking control. A novel adaptive technique was developed in [100] by integrating Backstepping, Sliding-Mode, fuzzy with neuro networks and adaptive control to stabilise the fractionally uncertain chaotic systems. An optimized Backstepping controller is employed for a Nonlinear pendulum system for stabilising the position of the pendulum's ball in the required position, is proposed in [101]. For improving the behaviour of a system and also its robustness to parameter variation, an Adaptive Backstepping controller was developed in [102]. Authors in [103] developed Backstepping controller for an autonomous quad rotor. The suggested ideal Backstepping controller used the PSO method to automatically choose the controller parameters.

Like other scientific disciplines, the study of neural networks has undergone a long journey of evolution by crossing so many difficulties. Because neural networks (NNs) have such broad applications in areas like image processing, signal processing, associative memories, optimization, etc.., they have drawn a lot of attention in present era. The control problem of FONL systems is addressed in [104], by employing a neural network Backstepping technique, which simultaneously addresses the issue of "explosion of terms". For strict-feedback FONL systems, an adaptive neural control algorithm was proposed by Jang-Hyun Park et al. in [105].

The Back stepping control scheme with Adaptive Neural Network for FONS with actuator faults, with unknown parameters, was proposed by authors in [106]. For stability analysis of FONL, authors in [107] developed Fractional order “Complex Valued Hopfield Neural Networks(CVHNNs)” with time delay. To address the tracking issue because of unknown regularly time-varying disturbances, Renwei Zuo et al.[108] suggested an adaptive neural control. Authors developed a multilayer neural networks (MNNs) to control FONL systems in [109]. The proposed strategy in [110] is designed using "dynamic surface control" with "minimum learning parameter" technique. It compensates nonlinearities in the system using Radial Basis Function Neural Networks(RBFNN). Adaptive neural network architecture was used for solving the tracking issues of nonlinear systems suffering from unidentified regularly time-varying disturbances in [111]. Backstepping controllers with Neural Networks are proposed to resolve the issues of FONL systems with unmeasured states [112], chaotic systems with unknown backlash-like hysteresis[113] because of input quantization & external disturbance and tracking issues of uncertain FONL systems [114].

Further, Reinforcement Learning (RL) has been integrated with BS control to minimize the problem of “explosion of parameters” of controller. To reduce the zero behavior in the FONL, the event triggering mechanism is used with BS [115]. Thus the model ensures better stability with ease in computation. The optimality of the controller was achieved by using two fuzzy logic systems that uses weight learning laws to converge the solution to a global optimal value. This global optimal weight of fuzzy was achieved by using gradient decent optimizer.

An extended form of DSC was used by Song et al. [116] for controlling strict feedback FONL with input constraints. The unknown nonlinear system function was realized by using neuro-fuzzy system that uses FO filters to reduce number of parameters in DSC. To deal with uncertainty and disturbances, the compensation mechanism was brought in the controller. The recursive steps of the controller were treated with fractional order update laws so that the error in the system could be reduced.

An RNN is a statistical learning model that is used to approximate functions based on many inputs. Networks of linked "neurons" that communicate with one another are a frequent way to visualize NNs [92]. Because their connections may be changed to corresponding numerical weights, neural networks are able to adapt to new inputs and learn from them. The hidden, input, and output layers make up 3 layers of an RNN. The input layer distributes inputs to the synapses of the hidden layer after receiving them. The precise RNN result is generated by the output layer. From input to output, intermediary calculations are carried out via a hidden layer.

To improve tracking precision and reduce approximation error, an adaptive backstepping controller can be designed using a deep learning process. So, an adaptively coupled RNN based backstepping controller is proposed in this work. This controller is utilized to synchronize chaotic FONL systems. The synchronisation of chaotic FONL systems can be used for transmitting speech signal and encrypted image safely.

2.6 Synchronization of FONL systems

When two or more chaotic systems are identical, synchronised, the trajectories of states of both the systems follow the same path. In [117], Weihua Deng and Changpin discussed the synchronisation of chaos in the fractional Chen system. The Pecora-Carroll approach was used to replicate the authors' impulsive synchronisation method for discrete-time chaotic systems in [118]. M.S. Tavazoei et al. [79] introduced a controller using active sliding mode theory to synchronise chaotic FO systems. Sliding mode control was suggested by Hong Zhang et al. [77] to synchronise two connected

Newton-Leipnik systems. Using a Backstepping strategy, Manoj Kumar Shukla et al.[98] suggested a method to synchronise and stabilize Fractional order chaotic systems(FOCS) with unspecified parameters. Adaptive neural network synchronisation method for the synchronisation of non-identical FOCS, which is able to guarantee prescribed performance is proposed in [119].

Systems with Nonlinear nature and chaotic behavior have been extensively utilized in various fields over the last few years [120]. In reality, majority of practical systems are nonlinear and chaotic in their behaviour [121]. Chaos plays an essential role in various domains, including the study of biological systems such as the brain, human heart, secure communication systems, smart systems, robotics engineering, data encryption, and the design of nonlinear oscillators [122]. Fractional-order calculus is a widely utilized mathematical tool across various scientific fields, providing more accurate descriptions of complex real systems and nonlinear phenomena [123]. Fractional calculus has obtained significant attention recently as it provides précised models when compared to Integer Order calculus [124].

Fractional calculus finds applications in control systems, bioengineering, image encryption systems, analog filters, and oscillator's circuit theory [125]. Consequently, fractional-order chaotic systems offer greater research potential and application value in the study and implementation of chaotic systems [126]. A proficient image encryption scheme is developed based on Memristor Hopfield neural network [127]. Optimization algorithms applied to adjust encryption parameters and cipher text image is generated by executing confusion and diffusion operations [128]. In [129], systems are synchronized using impulsive and event-triggered control methods. In [130], multiple layer encryption network is used for image encryption. An S-box and encryption key are generated for each layer, and employed.

Some of the existing works related to communication and image encryption are discussed in this section. To improve the performance of observer, artificial Harris hawks optimisation (HHO) method with Sliding-Mode Observer approach was presented in [131] to synchronize systems. Xu et al. [120] discussed a Nonlinear FOCS based on image encryption. A Hopfield Neural network (HNN) was utilized for the encryption of image. The FOCS is solved by a domain decomposition approach. Results illustrates that the proposed technique achieved better performance compared to other existing methodologies. The Multisim circuit simulation was utilized to implement the 4-neurons-based HNNFO system. Taheri et al. [132] designed chattering-free method based on fractional-integral SMC to synchronise different FONL systems which are chaotic in nature. Utilizing the fractional order integral operator and boundedness property in FO system states, a robust, model-free approach was designed to withstand uncertainties and external disturbances. The results show the efficacy of the technique and the suggested encryption algorithm.

Bai et al. [133] used backstepping method with event-triggering to synchronize fractional order non-linear chaotic system. To reduce computational effort in recurrent differentiation of virtual control signals, an event-triggered control technique is used. The results illustrate the efficacy of proposed technique to addresses chaos synchronization. Chen et al. [134] employed Backstepping Sliding Mode with adaptive Neural Network Control on FOCS. The Pade delay approximation scheme was utilized to address input delay, reducing the complexity of analyzing FO chaotic systems with inputs delay. The proposed controller guarantees system stability based on Lyapunov stability. The results denote the efficacy of proposed technique.

Chen et al. [135] discussed about Fractional-Order Discrete HNN (FODHNN) to encrypt the image. The Control strategies were developed using the stability principles of FODHNN to achieve synchronisation in discrete fractional neural networks. In the image encryption algorithm, FODHNN was utilized to generate pseudo-random number generator. Results showed that, proposed algorithm exhibits strong encryption characteristics. In the following table 2.1, the advantages and disadvantages of various controllers have been discussed.

Table 2.1 Comparison of Various Controllers

Author	Method	Advantages	Drawbacks
Shouli Gao et al. [136]	FASFM	Better tracking performance and concentration	Less robustness.
Mengmeng Gao et al. [137]	Finite-time SFC	Better stabilization and improved the efficiency of control strategy	Need to resolve the control problems better.

Author	Method	Advantages	Drawbacks
Mengru Kong & Liang Liu [138]	SFC with Lyapunov-Krasovski functional	Guaranteed the stability	Need to consider fault-tolerant control and finite-time control problem of the system.
Rene Osorio Sanchez et al. [139]	SFC-based GE	Ensured better performance of tracking controller with minimum error, minimum settling time and small overshoot	Required to minimize the error between indictor current and reference current.
Xue-Jun Xie &Mengmeng Jiang [140]	Dynamic SFC	Eliminate the obstructive conditions on time-delay non-linearities	Analyze the design of output feedback controller.
Tognetti et al. [141]	Robust PID	Efficient for unstable and uncertain systems	Relay work issues
Veerasamy et al. [142]	PID- HNN	Better in transient and consistent state performance	Need an advanced controller and improve the speed of the system.
Amuthambigaiyin Sundari, K., and P. Maruthupandi et al. [143]	PID	Reduced the constraints of the two tank system and accomplished an optimal control.	To reduce the constraints in the system, a better control model could be required.

Author	Method	Advantages	Drawbacks
Xia et al. [1444]	Optimal fractional-order PID control design	Enhanced the control system's performance	Need advanced techniques to enhance the stability.
Zamani et al. [145]	Optimal fractional-order PID controller	More proficient than ordinary PID	Minimize time delay and required to improve seismic execution.
Rasheed,LUAY THAMIR et al. [146]	PID- CCM	peak overshoot, Reduced peak time, rise time, and settling time.	Required to improve control strategies
Mobayen et al. [147]	NTSMC	Robust performance with singularity-free dynamics and finite time convergence	Improve control gain and chattering effect.
Saeedi et al. [148]	SMC	Managed the issue of arrangement and tracing tasks.	Required to improve the tracking task.
Azar et al. [149]	SMHOESO	Huge decrease in regulator energy, expanded control signal perfection, and exact following of the reference signal	Need to improve the execution time.
Mobayen et al. [150]	Adaptive second-order sliding mode control technique	Offered robustness, and efficiency.	Relay work issues could be studied.

Author	Method	Advantages	Drawbacks
Sun et al. [151]	SMSC-ADSMO	Better speed control with improved performance	Need to improve the speed, ability and torque.
Hou et al. [152]	Hierarchical SMC	Better tracking execution with unidentified disturbance	Need better hardware system.
Faisal Jamsheed and Sheikh Javed Iqbal [153]	ANN-based online adaptive control model	Quick response, computation implicity and adaptability	Need to improve optimal convergence rate and stability.
Wenshan Bi [154]	NN-based adaptive control	Ensured the stability and lessened the tracking error	Required to focus on fractional order multi-agent system.
Junchang Zhai et al. [155]	FTANTC	Resolved the tracking issue of non-strict-feedback systems and finite time stability performance.	Need to minimize the complexity.
Tien-Loc Le at al. [156]	Hybrid NN and CMAC model	Effectively controlled the dynamic time-varying plants	More time complexity and less stability.
Xingqiang Zhao et al. [157]	ISC with NN-based sliding mode scheme	Accurately track the pre-set trajectory, effective attenuate chattering and high-speed plant switching.	Less stability.

Author	Method	Advantages	Drawbacks
Prashant J. Gaidhane, Shirish Adam [158]	ABC-GWO	Offered feasible and reliable solution	Need to resolve the convergence issues.
Gauri Sahoo et al. [159]	MHH based FOF-PID controller	Established better frequency regulation	Need not deal with system uncertainty.
Jian Zhong Shi [160]	T2F-PID	Better control effects when system had disturbance, structure uncertainty or parameters uncertaininty.	Not yet proved the stability, need proper tuning algorithm for parameter optimization and requires to verify its robustness and practicability in real time.
Lakshmanan Shanmugam & Young Hoon Joo [161]	Lyapunov functional with interval T2F-based sampled data controller	Minimized the computational complexity	Need to improve the stabilization criteria.
Amin Taghieh et al. [162]	IT3NF-RC	More accurate approximation of complex nonlinearities and unknown models.	Required to improve the resistance to uncertainties and parameter fluctuations.
Xinghu Yu et al. [163]	ABC with non-parametric uncertainties model	Easily track the target signals by the outputs of reactor system with smaller errors	More disturbance.
Rafet Can Umutlu et al. [164]	ABC with non-linear disturbance and uncertainties model	Guaranteed the control goals and better efficiency	Need to enhance the stability analysis.

Author	Method	Advantages	Drawbacks
Maryam Shahriari-kahkeshi et al. [165]	HG-OFBC model	Compensated the effect of input non-linearity and filtering error and eliminated the explosion complexity	Required to tune the adjustable parameters and minimize the online computation burden.
Xinyao Li et al. [166]	Enhanced backstepping controller	Achieved asymptotic output tracking for a provided reference signal and ensured stability	Need to resolve the parameterization problem that occurs in determining parametric uncertainties of the system.
Xinyao Li &Changyun Wen [167]	Smooth ABC	Asymptotically stable even with the existence of bounded disturbances and uncertainties	Not offered the effectiveness of different fraction order in stabilizing unstable system.

2.7 Summary

In this Chapter, various controllers such as PID control, the state feedback control, sliding mode, backstepping, fuzzy networks-based controller, adaptive backstepping, neural networks-based control, and, hybrid controllers, optimized controllers for non-linear systems have been discussed. Finally, the limitations of various controllers are also summarized. Additionally, when compared to existing controllers it is realized that the Adaptive Backstepping Controller model has achieved better outcome and attracted researchers because of its better stabilization ability. Thus, from this survey, it is observed that the Adaptive backstepping control(ABC) with optimization can advantage robotic, control engineering practitioners in multiple ways.

Chapter-3

EVALUATION OF VARIOUS CONTROLLERS FOR FRACTIONAL ORDER NON-LINEAR SYSTEMS

3.1. Introduction

The FOC describes the fractional order system (FOS) under non-linearity. Fractional order calculus can solve technical problems that cannot be solved by integer order calculus [168]. Researchers widely use fractional order calculus systems due to their dynamic nature, infinite memory capacity, and compact expression than integer calculus [169-171]. In addition, the FOS offers a larger response range due to the analogue circuitry of non-linear metrics [172].

Consequently, the control of non-linear systems is the main research topic because a control model is required to reduce error factors in certain non-linear systems. Various controllers such as active control, dynamic area control, backstepping control, sliding mode control and fuzzy controllers [173-179] are used to reduce the approximation error in non-linear systems of fractional order. Even when multiple controllers are available to track recorded and actual data accurately, failure of the controller model can occur due to an error in the non-linear system. Therefore, the system is unsuitable for real systems with instability problems.

In addition, actuator faults (AF) and unknown external disturbances may be issued for the non-linear system. To deal with this, several researchers employ fault-tolerant control schemes. A fault-tolerant system with a modified backstepping controller [180] has been implemented to cope with infinite faults (AF) in the non-linear system. The control model used was improved by using fuzzy logic for control decision-making. Furthermore, a robust and adaptive control model with Riemann-Liouville integral [181] was adopted, taking into account the non-linearity and uncertainty of both internal and external factors of the fractional calculation. The performances for AF with non-linear characteristics have resulted in high errors. The decentralized fault-tolerant system for handling different error states has been adopted with the neural network architecture [182]. Here, the backstepping DSC technique was employed to stabilize interconnected non-linear systems. Meanwhile, the control model has a high computational explosion problem. Therefore, a non-fragile fault-tolerant control model with distributed delay parameters is introduced [183]. Razumikhin's theorem has treated the distributed delay to make it asymptotically stable. In addition, feedback-based tracking control is implemented with neural network-based tracking controls [184] that consider time delay and data quantization to approximate errors.

The tracking error is reduced based on Lyapunov direct technique and frequency distribution model. However, in FONL systems with fault, the number of parameters rises with complexity despite each controller having an advantage over the others. Hence, the virtual control and actual control signals generated by the controllers are highly pretended to be approximation errors. Hence, this paper presents the performance study of various controllers used in controlling non-linear systems. The objective of the work is as follows: various controllers, such as the BS controller, SMC, DSC and fuzzy DSC, are designed to reduce approximation error in case of stuck type and loss of effectiveness AF conditions. Finally, the tracking error of each controller is analyzed with the respective virtual and actual control signals. Thereby, the highly stable control model in fault conditions is validated.

3.2. Preliminaries

The non-linear system has constant parameters of N, $\Re$ and $\Re^{+}$ denotes the natural, real and positive numbers with m vectors of $\Re^{m}$ and $n \times m$ matrix of $\Re^{n\times m}$. The open set of the vectors is denoted by Ω_c with radius c. The fractional order integral of a non-linear system and Caputo fractional derivative is defined as,

$$I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} \qquad \text{(e.q.3.1)}$$

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(t)}{(t-\tau)^{\alpha+1-n}} d\tau \qquad \text{(e.q.3.2)}$$

Here, α denotes the fractional order term and ranges within $\alpha \in [n-1, n)$ with $\Gamma(\alpha) = \int_0^{+\infty} \tau^{\alpha-1} e^{-\tau} d\tau$. While taking the Laplace transform (LT) for the above equation,

$$\int_0^{\infty} e^{-st} D_t^{\alpha} f(t) dt = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0) = s^{\alpha} F(s) - s^{\alpha-1} f(0) \qquad \text{(e.q.3.3)}$$

where, s is the LT coefficient that converts the time domain to the s-domain. $F(s)$ is the LT of $f(t)$. When $\alpha \in (0,1)$, then Mittag Leffler function is written as,

$$E_{\alpha,\gamma}(G) = \sum_{j=0}^{\infty} \frac{G^j}{\Gamma(\alpha j+\gamma)} \qquad \text{(e.q.3.4)}$$

Here, $\alpha, \gamma \in \Re^{+}$ and $G \in C$ is the function named as mittag-leffler. By taking the LT of e.q.3.4,

$$L\{t^{\gamma-1}E_{\alpha,\gamma}(-at^{\alpha})\} = \frac{s^{\alpha-\gamma}}{s^{\alpha}+a}. \qquad \text{(e.q.3.5)}$$

Lemma 1 [18]:If $\alpha \in (0,2)$, $\beta \in \Re$ and $(\pi\alpha/2) < \eta \leq min\{\pi, \pi\alpha\}$having $\eta \in \Re^{+}$, it holds

$$|E_{\alpha,\gamma}(G)| \leq \frac{C}{1+|G|} \qquad \text{(e.q.3.6)}$$

where, $C \in \Re^{+}$,$\eta \leq |arg(\upsilon)| \leq \pi$and $|\upsilon| \geq 0$. To maintain the stability of FO system, an asymptotic stability criterion is used based on the class-K function.

Lemma 2 **[8]**: An equilibrium point in the fractional order non-linear system is assumed as $x = 0$, then the function is represented as, $D_t^{\alpha}x(t) = f(t, x(t))$, in which $f(.)$ is lipsichiz continuous function. The lipsichiz continuous function involves Lyapunov function (V) and class-K function $g_k; k = 1,2,3..$ to obtain stability. the inequality in this case is,

$$g_1(\|x(t)\|) \leq V \leq g_2(\|x(t)\|),\ D_t^{\alpha}V \leq -g_3(\|x(t)\|) \qquad \text{(e.q.3.7)}$$

Thus $D_t^{\alpha}x(t) = f(t, x(t))$ is asymptotically stable.

Lemma **3** [22]: Suppose$x(t)$ be a smooth function, and$x(t) \in \aleph^{1}$ as $\aleph^{1}[I, \Omega_c]$is the set of continuous mappings then,

$$D_t^{\alpha_1}D_t^{\alpha_2}x(t) = D_t^{\alpha_1+\alpha_2}x(t) \qquad \text{(e.q.3.8)}$$

$\alpha_1 + \alpha_2 \leq 1$and $\alpha_1, \alpha_2 \in \Re^{+}$. In addition, the following lemma is used to check the derivative stability the FO system.

Lemma 4 [8]: If $x(t) \in \aleph^{1}$ then,

$$\frac{1}{2}D_t^{\alpha}(x^T(t)x(t)) \leq x^T(t)D_t^{\alpha}x(t) \qquad \text{(e.q.3.9)}$$

3.2.1 Problem formulation

The fractional order non-linear system with output variable $g_i(t)$and $f_j(t)$ is given as:

$$\begin{cases} D_t^{\alpha}\upsilon_1(t) = g_1(\upsilon_1(t)) + h_1(\bar{\upsilon}_1(t)\upsilon_2(t)) \\ D_t^{\alpha}\upsilon_i(t) = g_i(\bar{\upsilon}_i(t)) + h_i(\bar{\upsilon}_i(t)\upsilon_{i+1}(t)) \\ D_t^{\alpha}\upsilon_n(t) = g_n(\upsilon(t)) + \sum_{j=1}^{m} f_j(t) \end{cases} \qquad \text{(e.q.3.10)}$$

where, D_t^{α} denotes the Caputo Fractional order derivative of order 0< α <1, $i = 1, 2,3, \dots n-1$. The state vectors in the system are represented as $\bar{\upsilon}_i(t) = [\upsilon_1(t), \dots \upsilon_i(t)]^T \in \Re^i$, $h_i(\bar{\upsilon}_i(t)): \Re^i \mapsto \Re$and $g_i(\bar{\upsilon}_i(t)): \Re^i \mapsto \Re$ indicates unknown and known continuous functions, $f_j(t)$ represents actuator fault signals and 'm' denotes the number of faulty actuators respectively.

There are two sorts of AF($f_j(t)$): loss of efficacy and stuck type, which are modelled as follows:

$$f_j(t) = \rho_j \vartheta_j(t) + \beta_j\left(\bar{f}_j(t) - \rho_j \vartheta_j(t)\right) \quad \text{(e.q.3.11)}$$

$$\beta_j = \begin{cases} 1, f_j(t) = \bar{f}_j(t) \\ 0, f_j(t) = \rho_j \vartheta_j(t) \end{cases} \quad \text{(e.q.3.12)}$$

where, $j = 1,2, \ldots m$. In loss of effectiveness fault, $\beta_j = 0$, however, for stuck type fault $f_j(t) = \bar{f}_j(t)$ by keeping $\beta_j = 1$.

In loss of effectiveness fault, $\beta_j = 0$, however, for stuck type fault $f_j(t) = \bar{f}_j(t)$ by keeping $\beta_j = 1$ and

$$f_j(t) = 0 \quad \text{for} \quad t < t_{fj}$$

$$f_j(t) = \rho_j \vartheta_j(t) \quad \text{for } t \geq t_{fj} \quad \text{(e.q.3.12a)}$$

Where t_{fj} represents fault occurrence time.

$\rho_j \epsilon [0,1]$ represents fault magnitude (loss of effectiveness).

Effect of Fault Magnitude & Fault Timing:

- An increase in fault magnitude leads to a corresponding increase in the ultimate bound of the tracking error.
- Early fault occurrence results in longer transient response.
- Late fault occurrence has minimal impact on steady-state tracking.

Let us consider the reference signal as $\upsilon_d(t) \in \Re$; thus, the tracking error is noted by the difference in the output variable and the reference signal $\Phi_1(t) = \upsilon_1(t) - \upsilon_d(t)$. Several controllers, such as sliding mode, backstepping, dynamic surface controllers and fuzzy dynamic surface controller, are analyzed to evaluate the accuracy error in the non-linear system. Those controllers are designed by considering certain assumptions, which are described below.

Assumption 1: Consider L_∞ is space of bounded signal with $\upsilon_d(t)$ and $D_t^\alpha \upsilon_d(t)$ are in $\aleph^1 \cap L_\infty$.

Assumption 2: $\bar{h}_i < h_i(\bar{\upsilon}_i(t)) < \dot{h}_i$, where $\bar{h}_i$ and $\dot{h}_i$, are real positive numbers.

Assumption 3: The parameter bounded on Ω_c are $g_i^{(k)}(\bar{\upsilon}_i(t))$ and $h_i^{(k)}(\bar{\upsilon}_i(t))$, which is a large open set when state variables are equivalent to Ω_c.

Assumption 4: Up to $m - 1$ input suffers by stuck type fault in some places, while in other places, loss of effectiveness is encountered. These are controlled using the control objective of the non-linear equation (3.10).

The unknown continuous function $h_i(\upsilon_i(t))$ is defined for all the controllers as,

$$h_i(v_i(t)) = \xi_i(v_i(t)) + \varepsilon_i(t); i = 1,2,3, \ldots n-1 \quad \text{(e.q.3.13)}$$

where, $\varepsilon_i(t)$is the optimal approximation error that occurred in each state and $\xi_i(t)$is the regression.

3.3 Analysis of controllers on FOS

In this section, four diffenernt controll techniques are adopted to check the stability of FONL systems under above mentioned lemmas.

3.3.1 Design of backstepping controller

This section describes the step-by-step recursive design of the back step controller.

Step 1: The FONL system for the error in the 1st state $(e_1(t))$are represented as follows,

$$D_t^\alpha e_1(t) = g_1(v_1(t)) + h_1(v_1(t)v_2(t)) - D_t^\alpha v_d(t) \quad \text{(e.q.3.14)}$$

$$D_t^\alpha e_1(t) = g_1(v_1(t)) - D_t^\alpha v_d(t) + h_1(v_1)\varpi_1(t) + h_1(v_1)\bar{\varpi}_i^c(t) + h_1(v_1)e_2(t) \quad \text{(e.q.3.15)}$$

The virtual control signal is denoted as $\varpi_1(t)$, which is nothing but the control signal obtained through the virtual control approach [185, 186]. The tracking error in the following state is denoted as $e_2(t) = v_2(t) - \varpi_1^c(t)$where, $\varpi_1^c(t)$is the approximate of $\varpi_1(t)$at α^{th} fractional filter.

$$D_t^\alpha \varpi_1^c(t) = -\lambda_1(\varpi_1^c(t) - \varpi_1(t)) \quad \text{(e.q.3.16)}$$

where, $\lambda_1$is the large positive term,$\varpi_1^c(t)$ is an approximation error, and the value is very small. If the approximation error is high, then the control model violates the Lyapunov stability function. The compensation for that error is designed by,

$$D_t^\alpha q_1(t) = -k_1 q_1(t) + h_1(v_1)\bar{\varpi}_1^c(t) + h_1(v_1)q_2(t) \quad \text{(e.q.3.17)}$$

At initial condition,$q_2(0) = e_1(0)$,$q_2(t)$is the compensated signal given at the second state. $k_1 \in \mathfrak{R}^+$is a real and constant value. Based on the compensated signal, the tracking error$(\bar{e}_1(t))$is evaluated with respect to previous tracking errorsand given as,

$$\bar{e}_1(t) = e_1(t) - q_1(t) \quad \text{(e.q.3.18)}$$

Lyapunov Function:

$$V_1 = \frac{1}{2}\bar{e}_1^2(t) \quad \text{(e.q.3.18a)}$$

Taking the Caputo fractional derivative:

$$D_\alpha^t V_1 \le \bar{e}_1(t) D_\alpha^t \bar{e}_1(t) \quad \text{(e.q.3.18b)}$$

Then, the fractional order with backstepping control error is defined from equation (3.18), and equation (3.15) is rewritten as,

$$D_t^{\alpha} e_1(t) = g_1(v_1(t)) - D_t^{\alpha} v_d(t) + h_1(v_1)\varpi_1(t) + h_1(v_1)e_2(t) + k_1 q_1(t) - h_1(v_1)q_2(t) \quad \text{(e.q.3.19)}$$

The virtual signal for the first state with the unknown continuous function of equation (3.13) on (3.19) is given as,

$$\varpi_1(t) = \frac{1}{h_1(v_1)}\left[-k_1 e_1(t) - \frac{\bar{e}_1(t)\varphi_1(t)\xi_1^T(v_1)\xi_1(v_1)}{2\sigma_1^2} + D_t^{\alpha} v_d(t) - {}^1\!/_2\, \bar{e}_1(t)\right] \quad \text{(e.q.3.20)}$$

Here, $\varphi_1(t)$is the adjustable vector. The virtual signal at this step is evaluated based on the reference signal and $\sigma_i.\sigma_i \in \Re^+$is a small value to achieve low tracking error.

$Step i:$ The fractional order non-linear system and error at i^{th}state having $i = 2,3,..n-1$is represented here.

The analysis for compensated signal and fractional filter for previous stage approximation errors is given as,

$$D_t^{\alpha}\varpi_i^c(t) = -\lambda_i(\varpi_i^c(t) - \varpi_i(t)) \quad \text{(e.q.3.21)}$$

$$D_t^{\alpha} q_i(t) = -k_i q_i(t) + h_i(\bar{v}_i)\bar{\varpi}_i^c(t) + h_i(\bar{v}_i)q_{i+1}(t) - h_{i-1}(\bar{v}_{i-1})q_{i-1}(t) \quad \text{(e.q.3.22)}$$

Before and after updating the compensated signal, the tracking error at i^{th}state is stated as$e_i(t) = v_i(t) - \varpi_{i-1}^c(t)and\bar{e}_i(t) = e_i(t) - q_i(t)$. The FOS in (3.9) with the compensated signal at (3.22) is represented as,

$$D_t^{\alpha}\bar{e}_i(t) = g_i(\bar{v}_i) + h_i(\bar{v}_i)v_{i+1}(t) - D_t^{\alpha}\varpi_{i-1}^c(t) - h_i(\bar{v}_i)\bar{\varpi}_i^c(t) - h_i(\bar{v}_i)q_{i+1}(t) + h_{i-1}(\bar{v}_{i-1})q_{i-1}(t) + k_i z_i(t) \quad \text{(e.q.3.23)}$$

$$D_t^{\alpha}\bar{e}_i(t) = g_i(\bar{v}_i) + h_i(\bar{v}_i)\varpi_i(t) + h_i(\bar{v}_i)e_{i+1}(t) - D_t^{\alpha}\varpi_{i-1}^c(t) - h_i(\bar{v}_i)q_{i+1}(t) + k_i z_i(t) + h_{i-1}(\bar{v}_{i-1})q_{i-1}(t)$$

$$\text{and} \qquad \bar{\varpi}_i^c(t) = \varpi_i^c(t) - \varpi_i(t). \quad \text{(e.q.3.24)}$$

The compensated signal for n^{th}state $q_n(t)$is given by,

$$D_t^{\alpha} q_n(t) = -k_n q_n(t) - h_{n-1}(\bar{v}_{n-1})q_{n-1}(t) \quad \text{(e.q.3.25)}$$

At this condition, the virtual control signal is represented by,

$$\varpi_i(t) = \frac{1}{h_i(\bar{v}_i)}\left[-k_i e_i(t) - \frac{\bar{e}_i(t)\varphi_i(t)\xi_i^T(\bar{v}_i)\xi_i(\bar{v}_i)}{2\sigma_i^2} + D_t^{\alpha}\varpi_{i-1}^c(t) - {}^1\!/_2\, \bar{e}_i(t) - h_{i-1}(\bar{v}_{i-1})e_{i-1}(t)\right] \quad \text{(e.q.3.26)}$$

$$V_i = V_{i-1} + \frac{1}{2}\bar{e}_i^2(t) \quad \text{(e.q.3.26a)}$$

Fractional Derivative of V_i is given by

$$D_{\alpha}^{t}V_i \leq \bar{e}_i(t)D_{\alpha}^{t}\bar{e}_i(t) \quad \text{(e.q.3.26b)}$$

Substituting eq.3.24,

$$D_t^{\alpha}V_i \leq -k_i\bar{e}_i^2(t) + \bar{e}_i(t)h_i(v_i)e_{i+1}(t) + \bar{e}_i(t)\Delta_i(t) \quad \text{(e.q.3.26c)}$$

Where $\Delta_i(t)$ represents bounded approximation and filtering errors

The above equation justifies the virtual control law.

By constructing the Lyapunov function recursively at each backstepping step and applying fractional-order stability theory, all closed-loop signals are guaranteed to be uniformly ultimately bounded despite actuator faults.

Step n: To simplify on nth state, the compensated signal on the previous state is considered with the FOS equation. Then, the model obtained is based on the Actuator Fault on the non-linear system and represented as,

$$D_t^{\alpha}\bar{e}_n(t) = g_n\big(v_n(t)\big) + \sum_{j=1}^{m} f_j(t) - D_t^{\alpha}\varpi_{n-1}^{c}(t) + k_nq_n(t) + h_{n-1}(\bar{v}_{n-1})q_{n-1}(t) \quad \text{(e.q.3.27)}$$

The fault signal $\vartheta_j(t)$having $\vartheta^*(t)$as the actual actuator output must be designed by the controller.

$$\vartheta_j(t) = a_{j1}\vartheta^*(t) + a_{j2}; j = 1,2,\dots m \quad \text{(e.q.3.28)}$$

$$\vartheta^*(t) = -k_ne_n(t) - \frac{\bar{e}_n(t)\varphi_n(t)\xi_n^T(\bar{v}_n)\xi_n(\bar{v}_n)}{2\sigma_n^2} - \frac{1}{2}\bar{e}_n(t) - h_{n-1}(\bar{v}_{n-1})e_{n-1}(t) + D_t^{\alpha}\varpi_{n-1}^{c}(t) \quad \text{(e.q.3.29)}$$

Here,$\sigma_n$is a positive integer number. By analysing the equations (3.28), (3.29), (3.27), (3.25) and (3.17) in (3.10), the system stability is analysed for the backstepping controller.

3.3.2 Design of Dynamic Surface Controller

To synthesize the model, the controller is considered with $v_d(t) = \varpi_1(t) = \varpi_i^c(t)and\bar{\varpi}_i^c(t) = \varpi_i^c(t) - \varpi_1(t) = 0$.

Step 1: The FOS under non-linearity and error at the first stage of the dynamic surface controller is represented as,

$$D_t^{\alpha}e_1(t) = g_1(v_1(t)) - D_t^{\alpha}\varpi_1^c(t) + h_1(v_1)\varpi_2(t) + h_1(v_1)\bar{\varpi}_2^c(t) + h_1(v_1)e_2(t) \quad \text{(e.q.3.30)}$$

The virtual control in this case is$\varpi_2(t)$, however, the error is represented as$e_2(t) = v_2(t) - \varpi_2^c(t)$. Here $\varpi_2^c(t)$ is the auxiliary signal that approximates the virtual control$\varpi_2(t)$. The use of an auxiliary signal will improve the stability of FOS since it is framed based on approximation error and reference signal at the second stage. Then, the fractional dynamic surface is given as,

$$D_t^{\alpha}\varpi_2^c(t) = -\lambda_2(\varpi_2^c(t) - \varpi_2(t)) \quad \text{(e.q.3.31)}$$

$\lambda_2 \in \Re^+$and it is a large constant value. The virtual control is given as,

$$\varpi_2(t) = \frac{1}{h_1(v_1)}[-k_1 e_1(t) - g_1(v_1) - \xi_1^T(v_1)\hat{\varphi}_1(t) + D_t^{\alpha}\varpi_1^c(t)] \quad \text{(e.q.3.32)}$$

Here, $\hat{\varphi}_1(t)$is the approximation of φ and the estimated error is noted as$\bar{\varphi}(t) = \hat{\varphi}(t) - \varphi$. The above-mentioned estimation error is framed due to the calculation of poor constant values in the control model. When t is high in the range of 20, the dynamic surface control violating the law of stability and increases complexity. Hence, it is essential to evaluate the design parameters carefully for the particular controller. Substituting the above equation into the fractional-order non-linear system of equation (3.10), the fractional-order approximation error obtained is represented as follows,

$$D_t^{\alpha}e_1(t) = -k_1 e_1(t) - \xi_1^T(v_1)\bar{\varphi}(t) + h_1(v_1)[\bar{\varpi}_2^c(t) + e_2(t)] \quad \text{(e.q.3.33)}$$

Step i: The fractional order non-linear system and error at i^{th}state having $i = 2,3,..n - 1$is represented.

$$D_t^{\alpha}\bar{e}_i(t) = g_i(\bar{v}_i) + h_i(\bar{v}_i)\varpi_{i+1}(t) + \xi_i^T(\bar{v}_i)\varphi - D_t^{\alpha}\varpi_i^c(t) + h_i(\bar{v}_i)\varpi_{i+1}^c(t) + h_i(\bar{v}_i)e_{i+1}(t) \quad \text{(e.q.3.34)}$$

In which $e_{i+1}(t) = v_{i+1}(t) - \varpi_{i+1}^c(t)and\bar{\varpi}_{i+1}^c(t) = \varpi_{i+1}^c(t) - \varpi_{i+1}(t)$. Then the virtual control signal at i^{th} state is designed as,

$$\varpi_{i+1}(t) = \frac{1}{h_i(\bar{v}_i)}[-k_i e_i(t) - g_i(\bar{v}_i) - \xi_i^T(\bar{v}_i)\hat{\varphi}(t) + D_t^{\alpha}\varpi_i^c(t) - h_{i-1}(\bar{v}_{i-1})e_{i-1}(t)] \quad \text{(e.q.3.35)}$$

where, $k_i \in \Re^+$. Substituting equation (35) in (34), then,

$$D_t^{\alpha}\bar{e}_i(t) = -k_i e_i(t) - \xi_i^T(\bar{v}_i)\bar{\varphi}(t) - h_{i-1}(\bar{v}_{i-1})e_{i-1}(t) + h_i(\bar{v}_i)\bar{\varpi}_{i+1}^c(t) + h_i(\bar{v}_i)e_{i+1}(t) \quad \text{(e.q.3.36)}$$

The auxiliary component at the fractional dynamic surface at i^{th}state is presented by the equation below.

$$D_t^{\alpha}\varpi_{i+1}^c(t) = -\lambda_{i+1}(\varpi_{i+1}^c(t) - \varpi_{i+1}(t)) \quad \text{(e.q.3.37)}$$

Step n: To simplify on nth state with AF, equation (3.27) is used. The actual control input signal by dynamic surface control is provided below:

$$\vartheta^*(t) = \frac{1}{h_n(v_n)}[-k_n e_n(t) - \xi_n^T(\bar{v}_n)\hat{\varphi}_n(t) + D_t^{\alpha}\varpi_1^c(t) - g_n(v_n) - h_{n-1}(\bar{v}_{n-1})e_{n-1}(t)] \quad \text{(e.q.3.38)}$$

Based on the above-mentioned control equation, the approximation error in the model is found.

3.3.3 Design of Fuzzy Dynamic Surface Controller

The DSC model for fractional order non-linear system is presented in sub-section 3.2. In that case, the unknown function of $h_i(v_i(t))$ is defined by equation (3.13). That is stationary, while for the fuzzy dynamic surface controller model, the unknown function is formulated using a fuzzy system.

Dynamic surface control has the problem of direct calculation of design parameters, which is solved by fuzzy logic. Based on the rules, the design parameter guidelines are set so that Lyapunov stability is maintained. The unknown continuous function of a non-linear system is defined by using fuzzy rules, fuzzification and de-fuzzification processes. The rule base for formulating the unknown variable in the dynamic surface is represented as follows:

Rule: If v_1 is P_1^l, v_i is P_i^l and v_n is P_n^l; then y is Q^l.

Here $l = 1,2,..N$, the fuzzy sets are represented as P_i^l and Q^l $i = 1,2,...n$. The fuzzy basis membership function for $h_i(v_i(t))$ is represented as,

$$h_l(t) = \frac{\prod_{i=1}^{n} \mu_{P_i^l}(v_i(t))}{\sum_{l=1}^{N}\left(\prod_{i=1}^{n} \mu_{P_i^l}(v_i(t))\right)} \quad \text{(e.q.3.39)}$$

At the de-fuzzification process, the output y is expressed as,

$$y(v_i(t)) = \frac{\sum_{l=1}^{N} \bar{y} \prod_{i=1}^{n} \mu_{F_i^l}(v_i)}{\sum_{l=1}^{N} \prod_{i=1}^{n} \mu_{F_i^l}(v_i)} \quad \text{(e.q.3.40)}$$

The membership function of Q^l is maximum at $\bar{y}$. The equations (3.38) and (3.39) are updated in fractional order dynamic surface system to generate control output.

3.3.4. Design of Sliding Mode Controller

The fractional order sliding surface of the system is represented as,

$$S(t) = \sum_{i=1}^{n} k_i I_t^{1-\alpha} e_i(t) \quad \text{(e.q.3.41)}$$

Based on the convergence factor of the above equation, the constant $k_i \in \Re^+$ is selected.

Step 1: The FOS of sliding surface in i^{th} state having $i = 1,2,3,..n-1$ is stated as follows,

$$D_t^\alpha e_i(t) = \sum_{i=1}^{n} -k_i \left[h_i(v_{i+1}(t)) - g_i(v_i(t)) + D_t^\alpha h_i(v_d(t))\right] + k_n \sum_{j=1}^{m} f_j(t) \quad \text{(e.q.3.42)}$$

The fault signal is represented in equation (28). Here, $a_{j1}$and $a_{j2}$are positive parameters that are equivalent to$\Re^{+}$. Then, by knowing those parameters, the fractional derivatives in the sliding surface are designed as,

$$D_t^{\alpha} e_i(t) = \sum_{i=1}^{n} k_i h_i(v_{i+1}(t)) + k_n g_i(v_i(t)) - \sum_{i=1}^{n} k_i D_t^{\alpha} h_i(v_d(t)) + k_n \vartheta^*(t) + k_n \sum_{j=1}^{m} \sigma_j f_j(t) + k_n \sum_{i=1}^{n} \sigma_i \xi_i^T(v_i(t))\, \varphi - k_n \sum_{j=1}^{m} \sigma_j f_j(t) \quad \text{(e.q.3.43)}$$

$$D_t^{\alpha} e_i(t) = \sum_{i=1}^{n-1} k_i h_i(v_{i+1}(t)) - \sum_{i=1}^{n} k_i D_t^{\alpha} h(v_d(t)) + k_n\big[g_i\big(v_i(t)\big) + \vartheta^*(t) + \sum_{i=1}^{n} \sigma_i \xi_i^T(v_i(t))\, \varphi\big] \quad \text{(e.q.3.44)}$$

The actual virtual control signal generated by the controller is stated by the equation below.

$$\vartheta^*(t) = -g_i(v_i(t)) - \sum_{i=1}^{n} \sigma_i \xi_i^T\,(v_i)\hat{\varphi}_i(t) + \frac{1}{k_n}\left[\sum_{i=1}^{n} k_i[D_t^{\alpha} v_d(t) - h_i(v_{i+1}(t))]\right] \quad \text{(e.q.3.45)}$$

Based on this control input signal, the sliding mode controller eliminates errors in fractional-order non-linear systems. The obtained control input is fed to the tracking error control signal equation (3.28) to obtain overall control of tracking error in FOS.

3.3.5 Stability analysis

Theorem 1: Consider the system (3.9) with fractional filters (3.16) and (3.21) for a backstepping controller, (3.32) for dynamic surface control and fuzzy dynamic surface control and (3.42) for a sliding mode controller, which is developed based on assumptions 1 and 3. For any $k_i \in \Re^{+}$has $|\bar{\varpi}_i^c(t)| < k_i \;\forall\; t = 0$ and $i = 1$ to $n - 1$. This holds when λ_i is large.

Proof: $\varpi_i(t) \in L_\infty$.Then, based on assumption 3, $|\dot{\varpi}_i(t)|$ is bounded. $D_t^{\alpha}\varpi_i(t)$is continuous with respect to α, which is bounded between $[D_t^{0}\varpi_i(t) = \varpi_i(t) \; and \; D_t^{1}\varpi_i(t) = \dot{\varpi}_i(t)]$, there is $d_i \in \Re^{+}$ such that $|D_t^{\alpha}\varpi_i(t)| \le d_i$. Thus equation (3.20) of backstepping, (3.31) of dynamic surface control and (3.41) of sliding mode control could be analysed.

$$D_t^{\alpha}\bar{\varpi}_i^c(t) = -\lambda_i\bar{\varpi}_i^c(t) - D_t^{\alpha}\varpi_i(t) \quad \text{(e.q.3.46)}$$

The function $y_i(t)$satisfies$-2d_i \le y_i(t) \le 2d_i$, such that

$$D_t^{\alpha}\bar{\varpi}_i^c(t) + y_i(t) = -\lambda_i\bar{\varpi}_i^c(t) + d_i \quad \text{(e.q.3.47)}$$

From the above equation,

$$\bar{\varpi}_i^c(s) = \frac{Y_i(s)}{s^{\alpha}+\lambda_2} \quad \text{(e.q.3.48)}$$

where, $\varpi_i^c(s)$ and $Y_i(s)$ are obtained by taking the Laplace transform of $\varpi_i^c(t)$ and $y_i(t)$. Thus, the above equation could be solved using equation (3) as,

$$\varpi_i^c(t) = d_i t^\alpha E_{\alpha,1+\alpha}(-\lambda_i t^\alpha) - y_i(t) * E_{\alpha,0}(-\lambda_i t^\alpha) \quad \text{(e.q.3.49)}$$

Here, $-\lambda_i t^\alpha = \pi$. Thus, the condition, $\pi\alpha \le |\arg(-\lambda_i t^\alpha)| \le \pi$ in Lemma 1 is satisfied. That is expressed as

$$d_i t^\alpha E_{\alpha,1+\alpha}(-\lambda_i t^\alpha) \le \frac{d_i C_i}{\lambda_i} \quad \forall \ t > t_1 \quad \text{(e.q.3.50)}$$

where, $t_1 \in \Re^+$ and $C_i \in \Re^+$ are constants which are determined by the fractional order α. Thus one has

$$\left|y_i(t) * t^{-1} E_{\alpha,0}(-\lambda_i t^\alpha)\right| = \left|\int_0^t y(t-\tau) \frac{E_{\alpha,0}(-\lambda_i \tau^\alpha)}{\tau} d\tau\right| \le 2d_i \int_0^t y(t-\tau) \frac{E_{\alpha,0}(-\lambda_i \tau^\alpha)}{\tau} d\tau \quad \text{(e.q.3.51)}$$

$$= 2d_i \int_0^t \sum_{k=0}^{\infty} \frac{(-\lambda_i)^k \tau^{\alpha k-1}}{\Gamma(\alpha k)} d\tau = 2d_i \sum_{k=0}^{\infty} \frac{(-\lambda_i \tau^\alpha)^k}{\Gamma(\alpha k+1)} = 2d_i E_{\alpha,1}(-\lambda_i \tau^\alpha) \quad \text{(e.q.3.52)}$$

By Lemma 1, one knows that

$$E_{\alpha,1}(-\lambda_i \tau^\alpha) \le \frac{C_{2i}}{1+\lambda_i \tau^\alpha} \quad \text{(e.q.3.53)}$$

With $C_{2i} \in \Re^+$ is a constant. According to the above equation, when t tends to infinity, the term $E_{\alpha,1}(-\lambda_i \tau^\alpha)$ converges at zero. As a result, for any values of ε_i, which is greater than zero and $t > t_2$, with $t_2 \in \Re^+$, one has

$$2d_i E_{\alpha,1}(-\lambda_i \tau^\alpha) \le \frac{\varepsilon_i}{2} \quad \text{(e.q.3.54)}$$

From this, the value of λ_i is chosen such that $\lambda_i \ge 2d_i \, {}^{C_i}/_{\varepsilon_i}$.

Remark 1: From theorem-1, the upper bound $\left(2d_i \, {}^{C_i}/_{\lambda_i}\right)$ has been derived for error variable $|\varpi_i^c(t)|$. From equation (3.50), the positive constant C_i is evaluated when fractional order and λ_i are available. When t is high, the sliding mode controller, dynamic surface control and fuzzy dynamic surface control are vulnerable since they are extremely slow convergence. In this case, the value is calculated as,

$$E_{\alpha,1}(-\lambda_i t^\alpha) = \frac{1}{\lambda_i t^\alpha} - \frac{1}{\pi\alpha} \int_0^\infty \frac{\sin(\pi\alpha) e^{-f^{1/\alpha}} df}{f^2 + 2\lambda_i \tau^\alpha cos\pi\alpha + \lambda_i^2 t^{2\alpha}} \quad \forall t > 0 \quad \text{(e.q.3.55)}$$

The other parameter d_i is estimated based on $D_t^\alpha \varpi_i^c(t)$, but it requires more mathematical evaluation.

Remark 2: Stability analysis, which is estimated based on evaluated approximation error based on adopted control techniques, is important. To analyse the convergence of the parameter λ_i is used that is a high value. When the value of λ_i is very high, the computation burden increases. The values of the coefficient vary for each control model utilized. To calculate $[a_{j1}, a_{j2}]$ matrix with known ρ_j and σ_j, the real parameters are lies in $\Re^+$. Then, it is stated that

$$\sum_{j=1}^{p}(1-\sigma_j)\,\rho_j a_{j1} = 1 \qquad \text{(e.q.3.56)}$$

and also,

$$\sum_{j=1}^{p}(1-\sigma_j)\,\rho_j a_{j1} = -\sum_{j=1}^{p}\sigma_j \bar{f}_j \qquad \text{(e.q.3.57)}$$

When the obtained values are small, the values of $\varphi_i(t)$ converges. When it is high, the fault signal has control of the FOS; thus, the updated reference signal $v_d(t)$ is wrongly updated. This results in unstable non-linear system output.

3.4 Results and Discussions

The section provides the performance of various controllers used for FOS under non-linearity. The system is considered with a fractional order of 0.9. Although, any value of fractional order alpha can be chosen but system which is used here for simulation purpose shows chaotic behaviour when alpha is near to 0.9 as per the literature. So the value is chosen as 0.9.

The controller parameters are selected based on stability requirements, robustness against actuator faults, and control effort considerations. The gains k_i>0 are chosen to dominate the nonlinear uncertainties and approximation errors in each backstepping step. Larger values of k_i improve convergence speed but may increase control effort. Hence, moderate values are selected to balance fast convergence and smooth control action. The Lyapunov decay rate, λ directly influences the convergence rate in the fractional Lyapunov inequality:

$$D_t^{\alpha} V \leq -\lambda V + \delta \qquad \text{(e.q., 3.58)}$$

A larger λ, improves transient performance but increases sensitivity to noise. Therefore, λ is selected such that Mittag–Leffler stability is ensured while maintaining robustness.

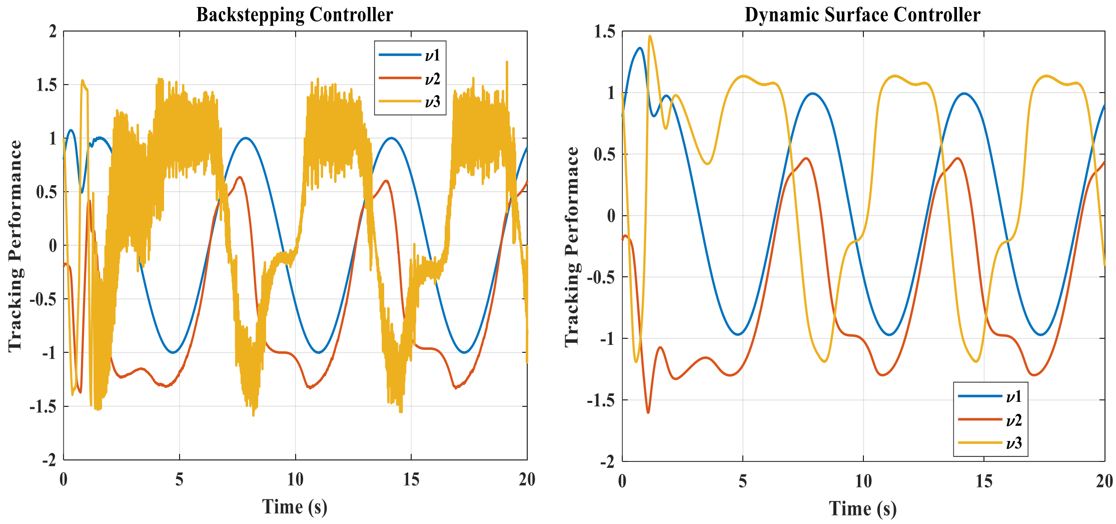
In dynamic surface control, fractional-order filters are employed to avoid the "explosion of complexity" problem. The filter order α is chosen close to the system fractional order α to preserve system dynamics and ensure smooth virtual control signals. In dynamic surface control, fractional-order filters are employed to avoid the "explosion of complexity" problem. The filter order is chosen close to the system fractional order α to preserve system dynamics and ensure smooth virtual control signals. The value of the Fractional order filter is 0.92 and System fractional order α is 0.9

At initial condition, $\varphi_1(0) = \varphi_2(0) = \varphi_3(0) = 0$ and $a_{11} = {}^{1}/_{0.6}$, $a_{21} = 0.5$, $a_{31} = 0$, $a_{12} = {}^{2}/_{0.6}$, $a_{22} = 0.5$ and $a_{32} = 0$. Let $\upsilon(0) = [1.3, 0.3, ..0.2]^T$ and the external disturbance in the system is zero. The input trajectory for the recursive states are initialized as $x_1 = 0.8$, $x_2 = -2$ and $x_3 = 1$.

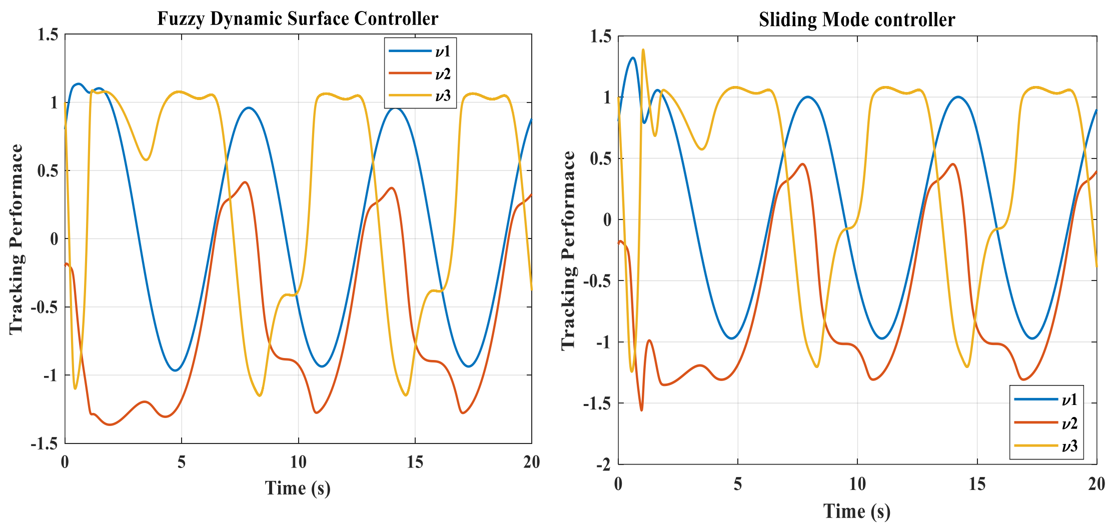
The parameters for backstepping controllers are as follows: The other design parameters are the real positive constant value of 3, that is $k_1 = k_2 = k_3 = 3$, $\lambda_1 = \lambda_2 = 20, t \in [0,20)$. Let the value of $\beta_1=1$, $\beta_2= \beta_3=0$ and $\rho_1=0.6$, $\rho_2= \rho_3=0$ in equation (10) and (11). The input variables are uniformly distributed with Gaussian membership at an interval of [-3, 3] for each recursive set.

The parameters of dynamic surface controller and fuzzy dynamic surface controller, $\phi_1(t) = [2, -1]^T$, $\phi(t) = 0_{2x1}$, $\lambda_1 = \lambda_2 = \lambda_3 = 2$, $\varpi = 1$, $t = [0,10]$ and $k_1 = k_2 = k_3 = 3$. The value of $\beta_1=1$, $\beta_2= \beta_3=0$ and $\rho_1=0.6$, $\rho_2= \rho_3=0$ in equation (3.10) and (3.11). The fuzzy system's membership function is obtained from Gaussian function.

The parameters of sliding mode controllers are given as, $\phi_1(t) = [2, -1]^T$, $\lambda_1 = \lambda_2 = \lambda_3 = 2$, $\varpi = 1$, $t = [0,10]$ and $k_1 = k_2 = k_3 = 3$. At initial condition $\phi(t) = 0_{2x1}$ and $\xi = 5$. Other all parameters of each controller design are similar.



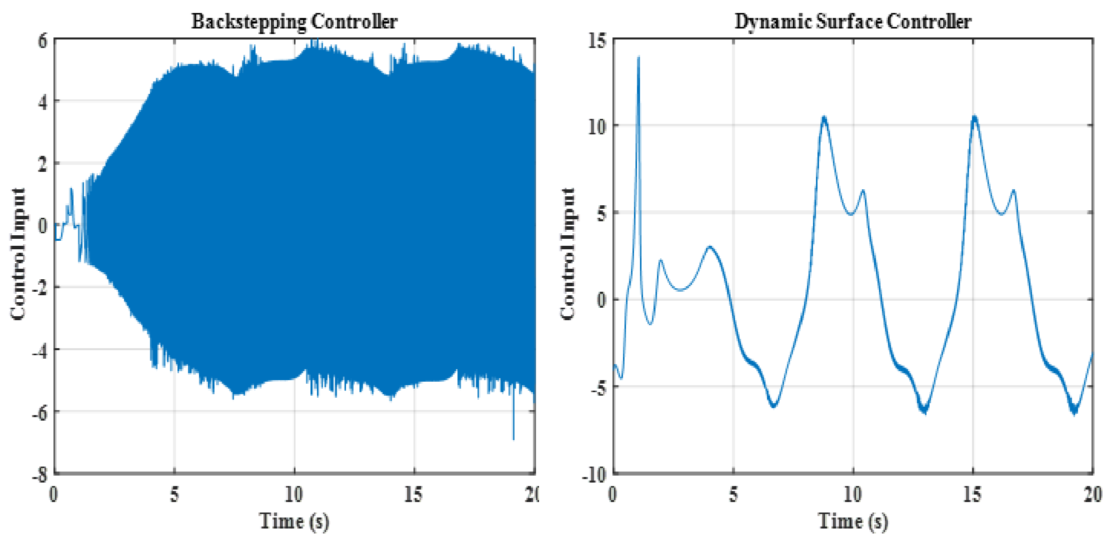
a) b)



c) d)

Figure 3.1: Tracking performance of various controllers: a) backstepping controller, b) dynamic surface controller, c) fuzzy dynamic surface and d) Sliding mode controller

The control performance of three-state controller model used is shown in Figure 3.1. The three states in each controller allow better tracking of non-linear systems with different error factors. The error in the first state is updated and reduced in the second state and vice versa. However, the AF reduction caused by backstepping controllers and dynamic surface controllers is worse and can be seen in Figures3.1 a) and b). Better tracking in each state is achieved for a fuzzy-based dynamic surface control as in Figure 3.1 c). Meanwhile, the recursive tracking update for the sliding-mode controller (Figure 3.1 d) is obtained as each state output is noticed.



a) b)

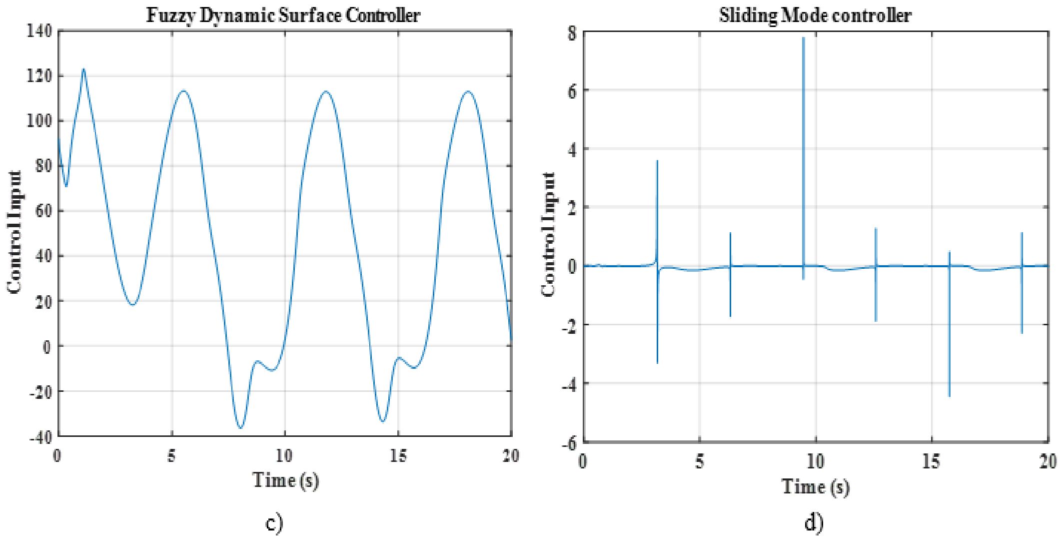


Figure 3.2: Control input of a) backstepping controller, b) dynamic surface, c) fuzzy dynamic surface and d) sliding mode

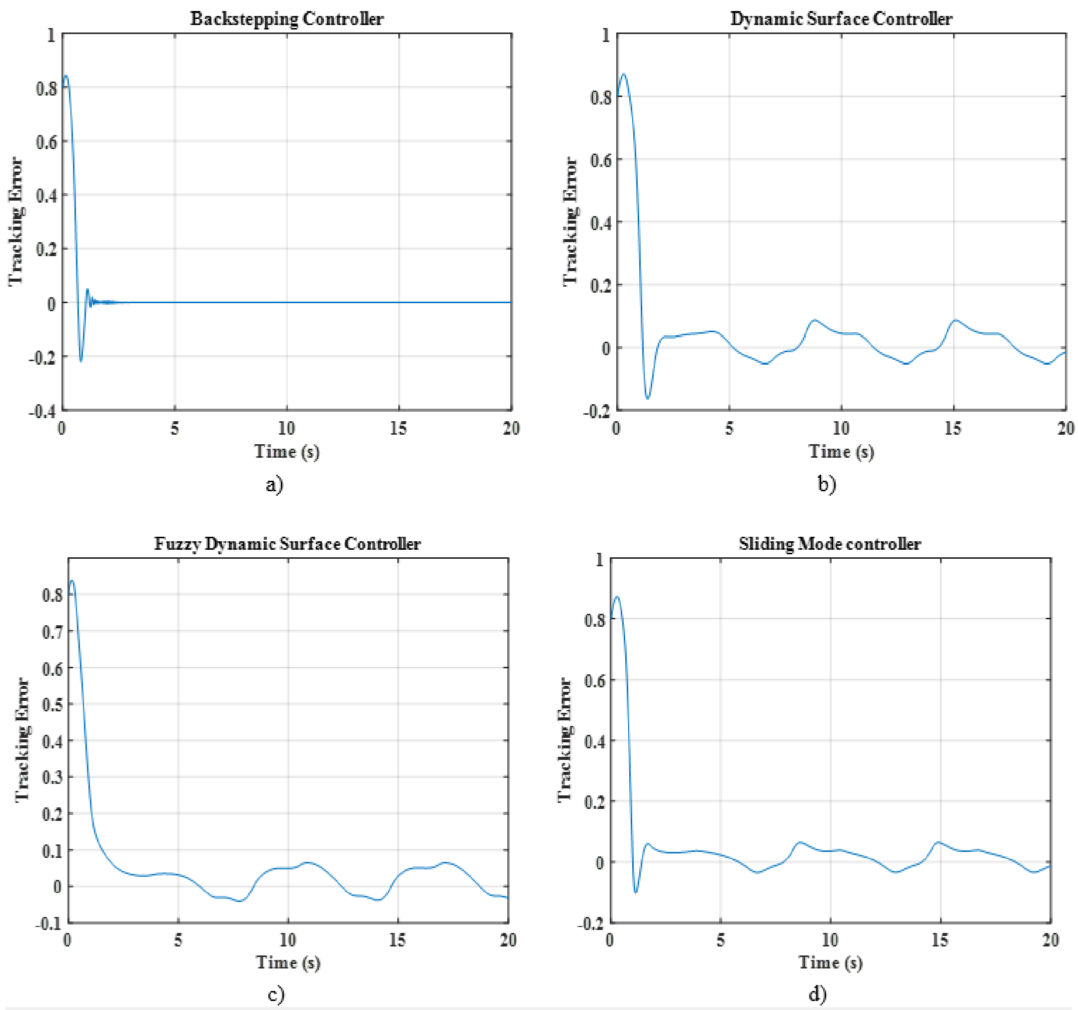


Figure 3.3: Tracking approximation error of controller a) backstepping controller, b) dynamic surface controller, c) fuzzy dynamic surface controller and d) sliding mode controller

The control signal generated by the backstepping, sliding mode, dynamic surface and fuzzy dynamic surface controller to compensate for non-linearity in FOS is shown in Figure 3.2. This generated control signal depends on the system's known and unknown functions.

The approximation error introduced into the system when subjected to control signals from different controllers is shown in Figure 3.3, which has the average tracking error for a non-linear system of order 0.9.

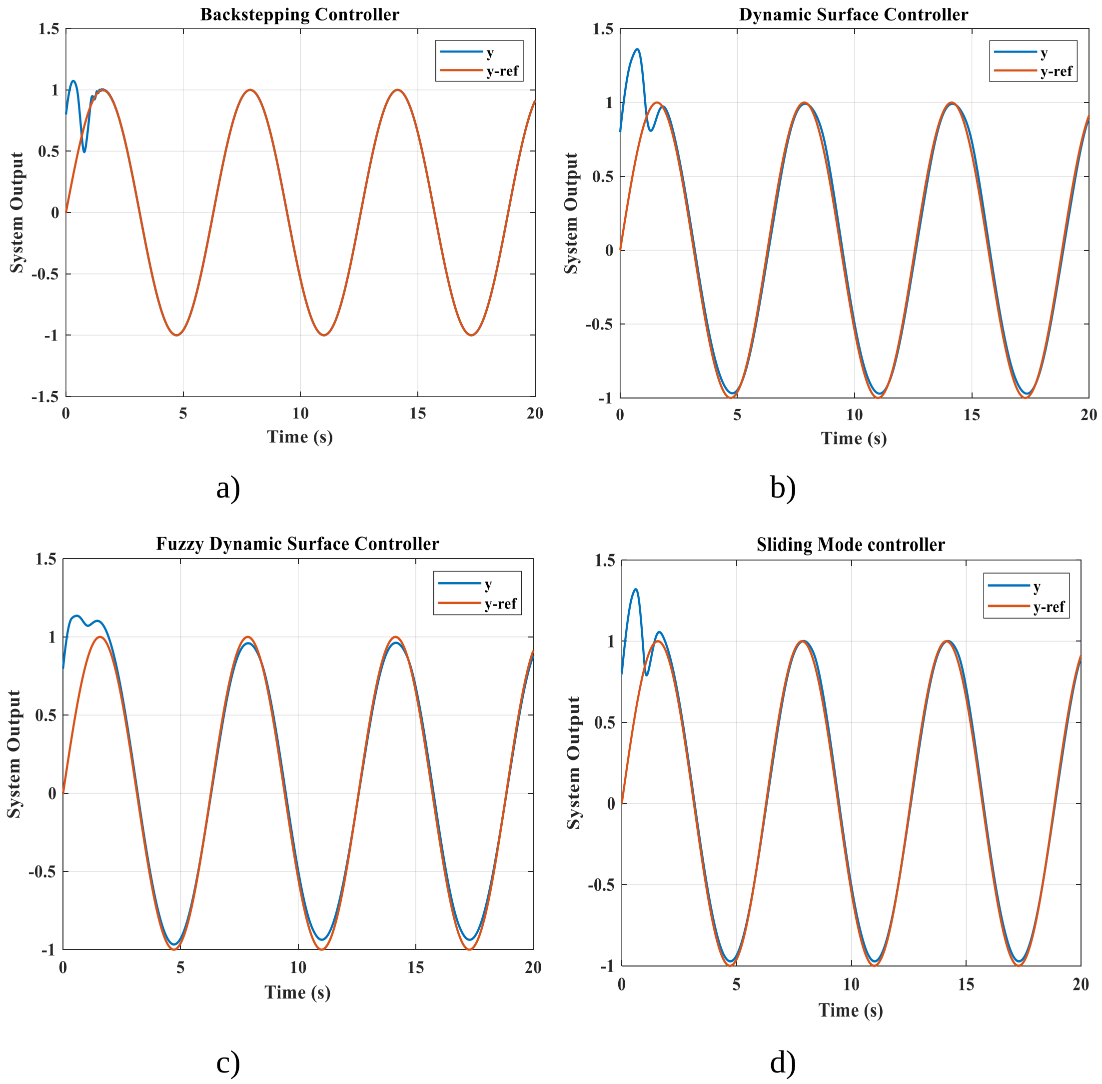


Figure 3.4: Control performance with respect to reference signal a) backstepping controller, b) dynamic surface controller, c) fuzzy dynamic surface controller and d) sliding mode controller

The controlled system output of various controllers on the utilized system is depicted in Figure 3.4 a)-d). The graph's initial condition was found to have an anomaly in the controlled output. This deviation is nothing more than the control system's error concerning the reference signal.

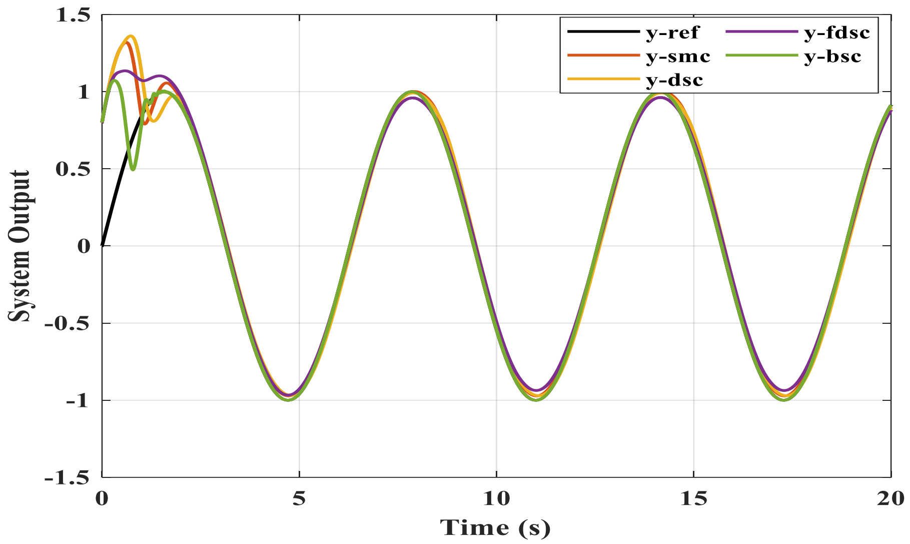


Figure 3.5: Comparison of tracking performance of different controllers

The track and error signals are compared in Figures 3.5 and 3.6 in relation to the reference signal. Both figures show that the dynamic surface controller and the sliding mode controller experienced large deviations from the reference. However, a lesser error is observed with backstepping controllers, with the error clearing after 2.5 to 20 seconds. Better performance regarding the controller of sliding mode is also achieved due to the optimal continuous function selected by the fuzzy-based dynamic surface controller. The average error from Figure 3.6 for each controller is available numerically in Table 3.1.

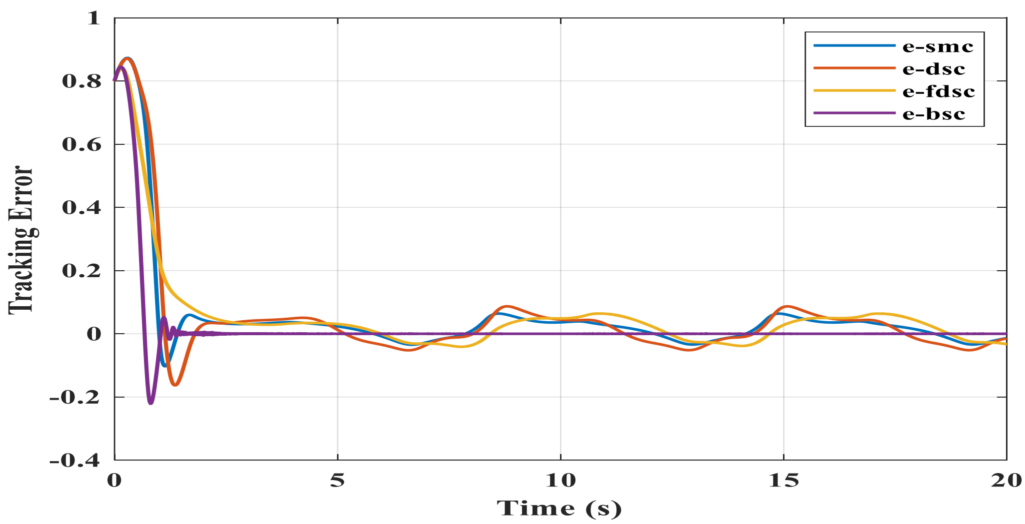


Figure 3.6: Comparison of tracking error of different controllers

Table 3.1: Performance evaluation of various controllers

Method	Error	Response time	Oscillation
Sliding mode controller	0.051	2.36s	High overshoot and moderate undershoot
Dynamic surface controller	0.0597	2.35s	High overshoot and low undershoot
Fuzzy dynamic surface controller	0.061	3.01s	Low overshoot and undershoot
Backstepping controller	0.0358	1.58s	Low overshoot and high undershoot

3.5. Summary

In this chapter the performance of four different controllers is analysed to maintain tracking control of a non-linear system. The tracking error of fuzzy-based dynamic surface and dynamic surface controller is 0.061 with 3.01s response time and 0.0597 with 2.35s response time and for the sliding mode, backstepping controller reduce the tracking error is 0.051 and 0.0358, with a minimum response time of 2.36s and 1.58s, respectively. Although many derived terms exist, the tracking error for backstepping was very small.

Chapter-4

RNN BASED BACKSTEPPING CONTROLLER FOR FRACTIONAL ORDER CHAOTIC SYSTEM

4.1. Introduction

The real world problems of chaotic system could be solved by fractional order calculus [187], which has attention grabbing features such as hereditary, memory etc. FOC is more versatile than the Calculus of integer order [188]. The FOC makes use of non-integer derivative [189] hence it has the capability to model non-integer-derivative. In addition, the FOS offers larger response range due to analog circuitry of non-linear metrics.

The control of nonlinear system is easier with fractional order however; approximation error [190] issue arises in the system. Hence it is essential to model control system to achieve reduced error factors. The controllers such as backstepping, dynamic surface control, sliding mode control, active control, fuzzy controller, optimized controller, and neural network based controller [191] are designed to reduce approximation error in fractional order nonlinear system. Even though several controllers are available for the accurate tracking of virtual signal in accordance with reference trajectory, faults make controllers vulnerable [192]. Thus the system is unsuitable to use in real-world system with instability issues.

To deal with such problem of actuator fault, parameter uncertainty and disturbance [193], the backstepping controller is widely adopted. The backstepping controller improves the asymptotic tracking and stability concerning virtual control input. But the problem of such controller is increased derivative terms and complexity, which makes unstable operation of fractional order chaotic system. Hence Universal Approximator Based Backstepping Control with Command Filter [194] is adopted, which reconstruct the unknown non-linearity in the system. Also, it is utilized with negative feedback to analyse uncertain parameters. The weight of neural network is optimized by convex optimization, which makes increased system metrics. Followed by that event triggered consensus control [195] was established for non-strict feedback system. When subjecting the control on strict feedback system, the boundedness and zero behaviour makes system highly sensitive. To saturate the unknown input, time delay based backstepping control [196] with neural network is reported. Based on the obtained delay the auxiliary dynamic system is analysed under lyapunov theorem.

4.2. Proposed methodology

The non-linear system has been encountered with various problems of tracking its input signal. The problem which would get worsens with fault cases. However, to solve the real-world problems fractional order calculus is used for mathematical modelling of non-linear system which further improves the attributes of non-linear system. For that system, the proposal intends to maintain the stability of tracking using well-known backstepping controller. Meanwhile, the backstepping controller has number of derivative terms and thereby increases the complexity of the controller. However, the accuracy in case of actuator fault in the system is easily maintained by the backstepping controller. Hence here to improve tracking precision and reduce approximation error, an adaptive backstepping controller is designed using a deep learning process. To do so here, an adaptively coupled recurrent neural network based backstepping controller is adopted. Then the control signal performance is examined on Genesio-Tesi chaotic system.

4.3. Preliminaries and Problem Formulation

This section explains fractional calculus ideas and the system description with non-linear strict feedback method. It then builds a model of actuator failures, including loss of control effectiveness faults. Finally, it addresses control process issues and suggests an approximation technique based on proposed recurrent neural network based back-stepping controller.

4.3.1 Preliminaries of Fractional calculus

Equations for FOC is described in this section.

Definition 1***:*** Fractional-order integral for β in a $G(t)$ function is given below;

$$I^{\beta}G(t) = \frac{1}{\Gamma(\beta)}\int_{t_o}^{t}\frac{G(\tau)}{(t-\tau)^{1-\beta}}d\tau, t_o \le t, \beta > 0 \quad \text{(e.q.,4.1)}$$

The Gamma function, $\Gamma(\beta)$ is evaluated using below equation

$$\Gamma(\beta) = \int_{0}^{\infty} e^{-t} \times t^{\beta-1}\, dt \quad \text{(e.q.,4.2)}$$

Definition **2*:*** The β^{th}Caputo derivative [24] is as follows;

$$D^{\beta}G(t) = \frac{1}{\Gamma(1-\beta)}\int_{t_o}^{t}\frac{G^{'}(\tau)}{(t-\tau)^{\beta}}d\tau, 1 > \beta > 0 \quad \text{(e.q., 4.3)}$$

Definition 3: The following equation is describes Mittag-Leffler function;

$$E_{\beta_1,\beta_2}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\cdot\beta_1+\beta_2)} \qquad \text{(e.q., 4.4)}$$

Applying Laplace transform to above equation

$$L\{t^{\beta_2-1}E_{\beta_1,\beta_2}(-\alpha t^{\beta_1})\} = \frac{A^{\beta_1-\beta_2}}{A^{\beta_1}+\alpha} \qquad \text{(e.q., 4.5)}$$

Where, $\beta_1 > 0$and $\beta_2 > 0$, z represent complex number,

Fractional calculus's geometric and physical relevance has been thoroughly discussed by Podlubny in [35].

4.3.2 System description

This work considered a fractional order non-linear strict feedback system as described below.

$$D^{\beta}X_i = f_i(\overline{X_i}) + X_{i+1} + d_i, \qquad where\ i = 1, \dots\dots\dots\dots n-1$$

$$D^{\beta}X_n = U + f_n(\overline{X_n}) + d_n$$

$$Y = X_1 \qquad \text{(e.q., 4.6)}$$

Where, $\overline{X_n} = [X_1, X_2, \dots, X_n]^T \in R^n$ represent state vector, $Y \in R$ indicates output of the system, U indicates actuator output, $f_i(\cdot) \in R,\ i = 1\ to\ n$ represent unknown function and $d_i(\cdot) \in R, i = 1, \dots, n$ signifies unknown external disturbance [197]. In this work actuator fault is applied and modelled as below.

$$U_j^F(t) = v_{j,h}(t) + \rho_{j,h}U_j(t), t \in [t_{jh,k}, t_{jh,e})h = 1,2,3,\dots \qquad \text{(e.q., 4.7)}$$

Where, $\rho_{j,h} \epsilon [0,1]$ represent actuator efficiency factor, $v_{j,h}(t)$ indicates unknown disturbance, the j^{th} actuator's h^{th} fault model starts at time $t_{jh,k}$ and finishes at $t_{jh,e}$. $\rho_{j,h} = 1,\ v_{j,h=0}$ denote the normal operating condition, $\rho_{j,h} = 1,\ v_{j,h=\neq}0$ represent biased fault, $0 < \rho_{j,h} < \bar{\rho}_{j,h} < 1,\ v_{j,h=0}$ signifies actuator will lose some of its effectiveness and U_j^F indicates total loss of effectiveness. Based on fractional order nonlinear strict feedback system, actuator fault is framed as described below.

$$D^{\beta}X_i = f_i(\overline{X_i}) + X_{i+1} \qquad where\ i = 1, \dots\dots\dots\dots n-1$$

$$D^{\beta}X_n = U + f_n(\overline{X_n})$$

$$Y = X_1 \quad \text{(e. q., 4.8)}$$

Assumption 1: There must exist an unknown positive constant $\bar{v}_{j,h}$ for the unknown disturbance term $v_{j,h}$ such that $v_{j,h} \le \bar{v}_{j,h}$.

Assumption 2: The system can nevertheless be pushed to do the designated control duties even in the event that $l - 1$ actuators fail totally at the same time.

Remark 1***:*** In this work, actuator faults of the system are considered. Even a minute fault can lead to poor tracking or the system becomes unstable, therefore, actuator faults need to be compensated. The purpose of the stabilization problem is to select an appropriate controller. To maintain its stability, a novel backstepping controller with RNN is proposed in this work. RNN is included in this work to reduce the error and for increasing the tracking efficiency.

4.4. Proposed backstepping controller with RNN

To get minimum tracking error, a deep learning architecture with backstepping controller is proposed.

4.4.1 Controller design

Backstepping control is most often used nonlinear controller design techniques. It is utilized for the control and synchronization of IONL systems since its development by Kristic et al [198]. It uses a methodical methodology based on Lyapunov theory and ensures that systems are globally stable. The n-order system with fractional order non-linear strict feedback form from equation (8) is considered as follows in the backstepping controller.

$$D^{\beta}\dot{X}_1 = f_1(X_1) + G_1(X_1)X_2$$

$$D^{\beta}\dot{X}_2 = f_2(X_1, X_2) + G_1(X_1, X_2)X_3$$

$$D^{\beta}\dot{X}_3 = f_3(X_1, X_2, X_3) + G_1(X_1, X_2, X_3)X_4$$

$$\cdot$$

$$D^{\beta}\dot{X}_{n-1} = f_{n-1}(X_1, X_2, X_3, \dots, X_{n-1}) + G_{n-1}(X_1, X_2, X_3, \dots, X_{n-1})X_n$$

$$D^{\beta}\dot{X}_n = f_n(X_1, X_2, X_3, \dots, X_n) + G_n(X_1, X_2, X_3, \dots, X_n)U \quad \text{(e.q., 4.9)}$$

Where f_i denotes smooth function, G_j indicates differential with $X \in R^n, U \in R$, $f_i(0) = 0, G_i(0) \neq 0, i = 1,2, \dots, n$.

Step 1**:** Taking into account the initial subsystem of equation (9), X_2 is considered as virtual control input, and select

$$D^{\beta}\dot{X}_2 = \frac{1}{G_1(X_1)}\big(U_1 - f_1(X_1)\big) \qquad \text{(e.q., 4.10)}$$

The initial subsystem has been modified to $\dot{X}_1 = U_1$. The origin of the initial subsystem $X_1 = 0$ is asymptotically stable, and the associated Lyapunov function is as follows.

$$V_1(X_1) = \frac{X_1^2}{2}. \qquad \text{(e.q., 4.11)}$$

Choosing $U_1 = -X_1 k_1$ with $k_1 > 0$, it changes equation (4.10) as follows:

$$D^{\beta}\dot{X}_2 = \varphi_1(X_1) = \frac{1}{G_1(X_1)}\big(-X_1 k_1 - f_1(X_1)\big) \qquad \text{(e.q., 4.12)}$$

Step 2: The subsystem(X_1, X_2), is modified as in equation (4.14) after taking X_3 as a virtual control input.

$$D^{\beta}\dot{X}_3 = \frac{1}{G_2(X_1,X_2)}\big(U_2 - f_2(X_1, X_2)\big) \qquad \text{(e.q., 4.13)}$$

$$\dot{X}_1 = G_1(X_1)X_2 + f_1(X_1)$$
$$\dot{X}_2 = U_2 \qquad \text{(e.q., 4.14)}$$

The above equation (4.14) is in the form of backstepping method. The control law of U_2 is as follows;

$$U_2 = -\frac{\partial V_1}{\partial X_1}G_1(X_1) - k_2\big(X_2 - \varphi_1(X_1)\big) + \frac{\partial \varphi_1}{\partial X_1}[f_1(X_1) + G_1(X_1)X_2] \qquad \text{(e.q., 4.15)}$$

Where, $K_2 > 0$, the above control rule asymptotically stabilises $(X_1, X_2) = (0,0,)$. The Lyapunov function is as equation (16)

$$V_2(X_1, X_2) = V_1(X_1) + \frac{1}{2}\big(X_2 - \varphi_2(X_1)\big)^2 \qquad \text{(e.q., 4.16)}$$

Solving equation (14) in (13) provides equation (17)

$$D^{\beta}\dot{X}_3 = \varphi_1(X_1,X_2) = \frac{1}{G_2(X_1,X_2)}\Big[-\frac{\partial V_1}{\partial X_1}G_1(X_1) - k_2\big(X_2 - \varphi_1(X_1)\big)$$

$$+\frac{\partial \varphi_1}{\partial X_1}[f_1(X_1) + G_1(X_1)X_2] - f_2(X_1,X_2)\Big] \quad \text{(e.q., 4.17)}$$

Step 3: The subsystem (X_1, X_2, X_3) is reformed into (4.19), and it utilized X_4 as a virtual control input.

$$D^{\beta}\dot{X}_4 = \frac{1}{G_3(X_1,X_2,X_3)}\big(U_3 - f_3(X_1,X_2,X_3)\big) \quad \text{(e.q., 4.18)}$$

$$\dot{X}_1 = G_1(X_1)X_2 + f_1(X_1)$$
$$\dot{X}_2 = G_2(X_1,X_2)X_3 + f_2(X_1,X_2)$$
$$\dot{X}_3 = U_3 \quad \text{(e.q., 4.19)}$$

The backstepping approach is used in equation (4.19). The control law of U_3 is as follows;

$$U_3 = -\frac{\partial V_2}{\partial X_2}G_2(X_1,X_2) - k_3\big(X_3 - \varphi_2(X_1,X_2)\big)$$
$$+\frac{\partial \varphi_2}{\partial X_2}[f_2(X_1,X_2) + G_2(X_1,X_2)X_3]$$
$$+\frac{\partial \varphi_2}{\partial X_1}[f_1(X_1) + G_1(X_1)X_2] \quad \text{(e.q., 4.20)}$$

Where, $K_3 > 0$, the above control rule asymptotically stabilises $(X_1, X_2, X_3) = (0,0,0)$. The Lyapunov function is as equation (4.21)

$$V_2(X_1,X_2,X_3) = V_2(X_1,X_2) + \frac{1}{2}\big(X_3 - \varphi_2(X_1,X_2)\big)^2 \quad \text{(e.q., 4.21)}$$

Solving equation (20) in (18) provides equation (22)

$$D^{\beta}\dot{X}_4 = \varphi_1(X_1,X_2,X_3) = \frac{1}{G_3(X_1,X_2,X_3)}$$
$$\Big[-\frac{\partial V_2}{\partial X_2}\times G_2(X_1,X_2) - k_3\big(X_3 - \varphi_2(X_1,X_2)\big)$$
$$+\frac{\partial \varphi_2}{\partial X_2}[f_2(X_1,X_2) + G_2(X_1,X_2)X_3]$$
$$+\frac{\partial \varphi_2}{\partial X_1}[f_1(X_1) + G_1(X_1)X_2] - f_3(X_1,X_2,X_3)\Big] \quad \text{(e.q., 4.22)}$$

Step n: The actual control rule U where it can asymptotically stabilize (4.9) as follows:

$$U = \frac{1}{G_n(X_1,\ldots\ldots,X_n)}$$

$$\left[-\frac{\partial V_{n-1}}{\partial X_{n-1}} \times G_{n-1}(X_1,\ldots\ldots,X_{n-1}) - k_n\big(X_n - \varphi_{n-1}(X_1,\ldots\ldots,X_{n-1})\big)\right.$$

$$+\frac{\partial \varphi_{n-1}}{\partial X_{n-1}}[f_{n-1}(X_1,\ldots\ldots,X_{n-1}) + G_{n-1}(X_1,\ldots\ldots,X_{n-1})X_n]$$

$$\left.+\frac{\partial \varphi_{n-1}}{\partial X_1}[f_1(X_1) + G_1(X_1)X_2] - f_n(X_1,\ldots\ldots,X_n)\right] \quad \text{(e q., 4.23)}$$

Where, $K_3 > 0$, the above control rule asymptotically stabilises $(X_{1,}X_2,X_3) = (0,0,0)$ [199]. The Lyapunov function is as equation (4.24)

$$V_2(X_1,\ldots\ldots,X_n) = V_{n-1}(X_1,\ldots\ldots,X_n)$$

$$+\frac{1}{2}\big(X_n - \varphi_{n-1}(X_1,\ldots\ldots,X_{n-1})\big)^2 \quad \text{(e.q., 4.24)}$$

The Backstepping controller can be applied to any fractional order non-linear systems to stabilize the states of the system. The designed backstepping controller is applied to another benchmark system i.e., Coullet system and the results are presented in Results section. The Coullet system is a three-dimensional continuous-time nonlinear chaotic dynamical system and its mathematical representation is given by

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = X_3$$
$$\dot{X}_3 = -aX_3 - bX_2 + cX_3 - dX_1^3 \quad \text{(e.q., 4.25)}$$

4.4.2 Recurrent neural network

Conventional backstepping has a serious issue with explosion of terms, which results from repeatedly differentiating virtual controllers. Consequently, the controller's complexity rises sharply, restricting the application of the backstepping approach to higher order systems. To overcome these issues RNN is included in this method. This will enhance the performance of the backstepping controlling in tracking the error. This RNN is adaptively coupled with backstepping controller.

RNN learns the features for the time-series data by the use of memory about the past inputs in the neural network's internal state. Further, it uses historical as well as the current data for the forecasting of the future information. The basic structural representation for RNN is shown in figure 4.1.

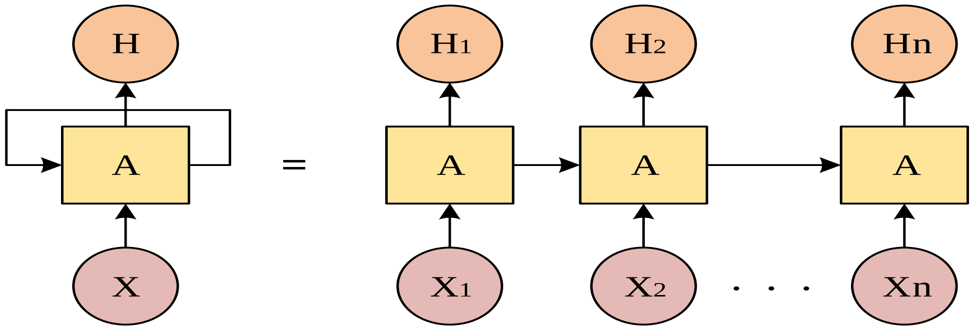


Figure 4.1: Basic architecture of RNN

Where, $X = [X_1, X_2, \ldots.X_n]$ indicates input layer, $H = [H_1, H_2, \ldots.H_n]$ denotes output layer and $A = [A_1, A_2, \ldots.A_n]$ represent hidden layer in RNN [200]. In this work, it assumes that only output of RNN is used to for controller design and nonlinear system. The current state is evaluated as follows;

$$H_n = f(H_{n-1}, X_n) \qquad \text{(e.q., 4.26)}$$

Where, X_n denotes input, H_{n-1} signifies previous sate, H_n represent current state $\rho_{j,h}$ indicates the approximation error and U_j value is estimated value from backstepping controller. This estimated value error is denoted as

$$E = X - \dot{X} \qquad \text{(e.q., 4.27)}$$

$$H_n = \tanh(E_H H_{n-1} + E_{XH} X_n) \qquad \text{(e.q., 4.28)}$$

Where, E_H denotes error at input neuron and E_{XH} indicates error at recurrent neuron. This error value is solved using RNN at the output layer. This will enhance the performance of the system.

Training Process:

The Recurrent Neural Network is trained using an online adaptive learning scheme**,** where the weights are updated using the equation

$$W(k+1) = W(k) + \eta\dot{W} \qquad \text{(e.q., 4.29)}$$

Where $W(k)$ is the current weight

$W(k+1)$ is the updated weights

$\dot{W}$ is the weight rate of change and η is the learning rate and its value is 0.001

Gaussian radial basis functions (RBFs) are used as activation function in the hidden layer and linear activation function is used in the output layer.

Gaussian radial basis function: $\phi_j(x) = \exp\left(-\frac{\|x-\mu_j\|^2}{\sigma^2}\right)$ (e.q., 4.30)

Variance: $\sigma^2 = 25,\ \mu_j$ is the learning rate=0.001

A small learning rate (0.001) is widely used to guarantee smooth adaptation.

Computational Complexity: The computational complexity of Backstepping Controller is approximately $O(n)$. Whereas the computational complexity of RNN based model is $O(N^2 + Nn)$. Where n = 3, represents the Number of States and N=7, represents the number of Neurons.

Loss function: The estimation error is defined as

$$e(t) = x_1(t) - \hat{x}_1(t) \quad \text{(e.q., 4.31)}$$

Loss function, $L(t) = \frac{1}{2}e^2$(t) (e.q., 4.32)

4. 5. Results & Discussions

4.5.1. Genesio-Tesi Chaotic System

The proposed RNN based backstepping controller designed for general “strict feedback nonlinear system" can be used to stabilize Genesio-Tesi system. Numerical simulations of the proposed method model in strict feedback system are implemented on MATLAB software. Then the resulting performance of control signal, error, and stability are evaluated.

This system is one of the chaos paradigms, this system was proposed by Genesio and Tesi [201] in 1992. Here, the aforementioned stabilization strategy is demonstrated through the consideration of a chaotic Genesio-Tesi system. It combines several characteristics of chaotic systems. It is mathematically modelled using 3 differential equations [202]. The state model of the Genesio-Tesi system is described as

$$\dot{X}1 = X2$$

$$\dot{X}2 = X3$$

$$X3 = -PX3 - QX2 - RX1 + X1^2 + U \quad \text{(e.q., 4.33)}$$

Where, $X_1, X_2,$ and X_3 and indicates the system states, P, Q and R represent unknown positive real constants. Suppose, the Genesio-Tesi system is chaotic for the factors P=1.2, Q=2.92 and R=6. Figure 4.2 shows the state variables$X_1, X_2,$ and X_3. Trajectories of chaotic system are shown in figure 4.3. These figures are obtained before applying the RNN based backstepping controller in the system.

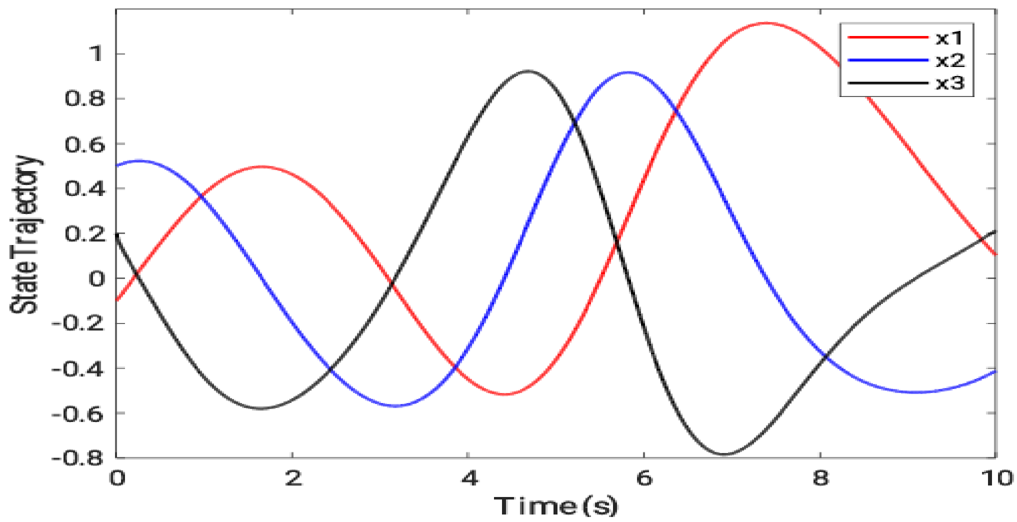


Figure 4.2 State variables of Genesio-Tesi System

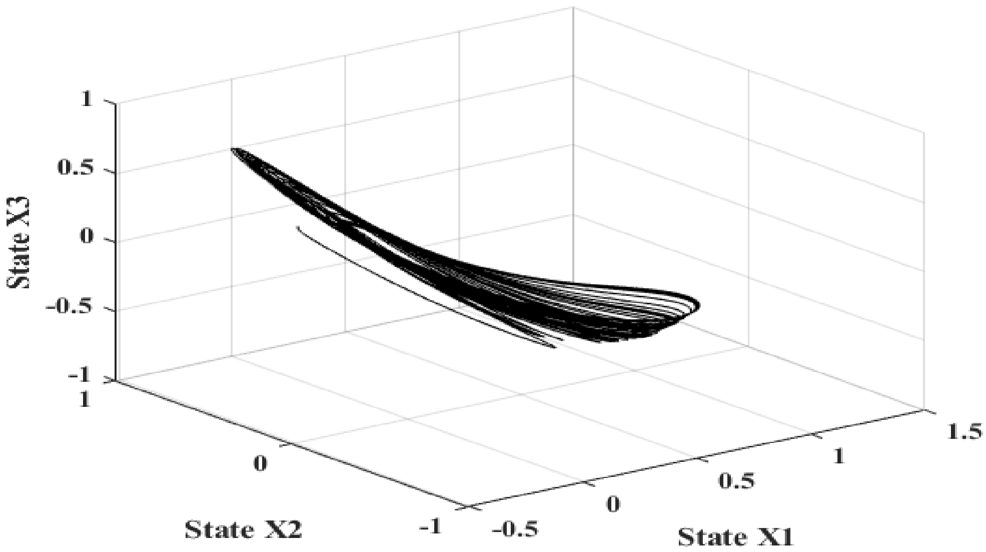


Figure 4.3: Chaotic trajectory of the FO Genesio -Tesi system

The Control signal of the RNN based backstepping controller is obtained optimally within less time. Control signal for the proposed technique is shown in below figure 4.4. The efficacy of the suggested backstepping control technique for stabilizing Genesio-Tesi system is confirmed with these results.

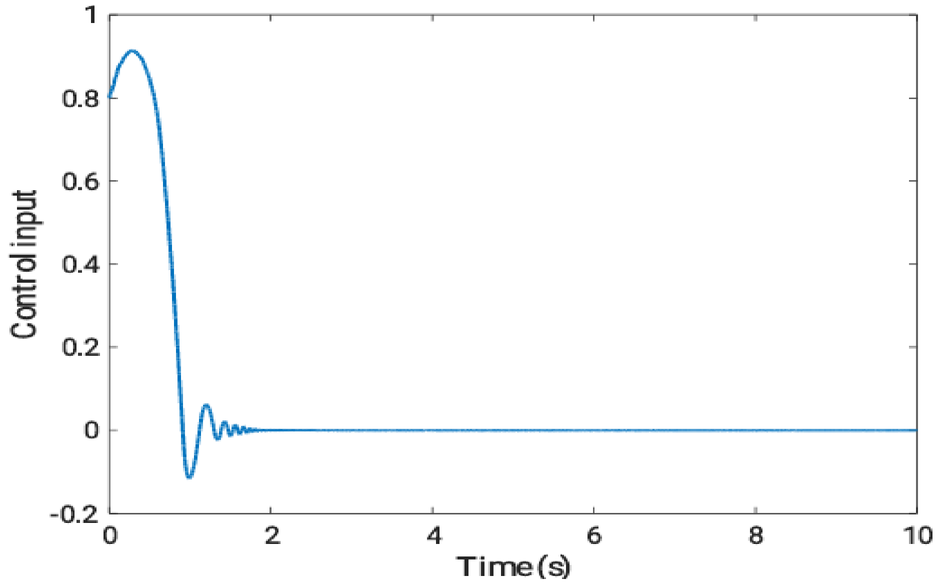
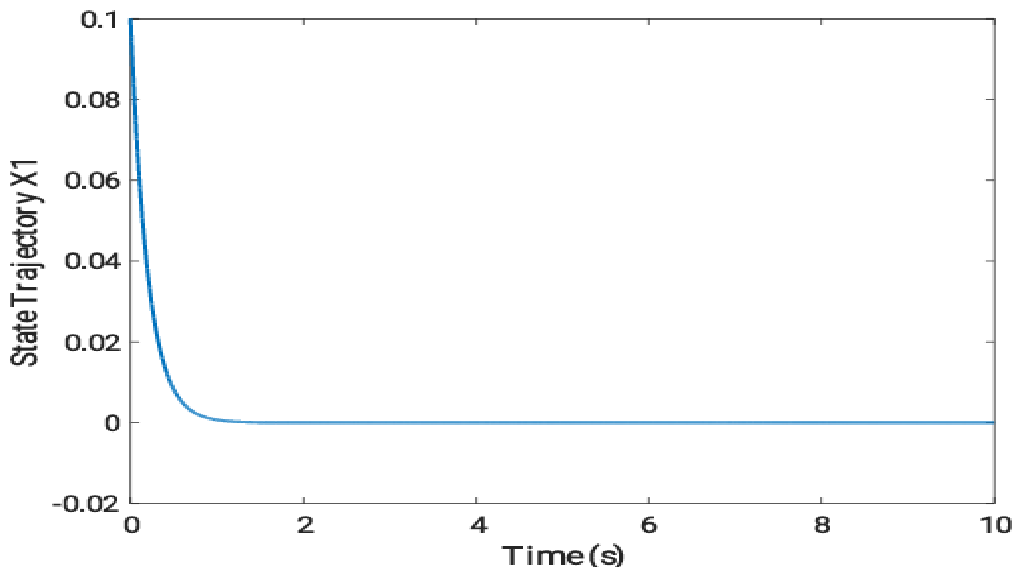
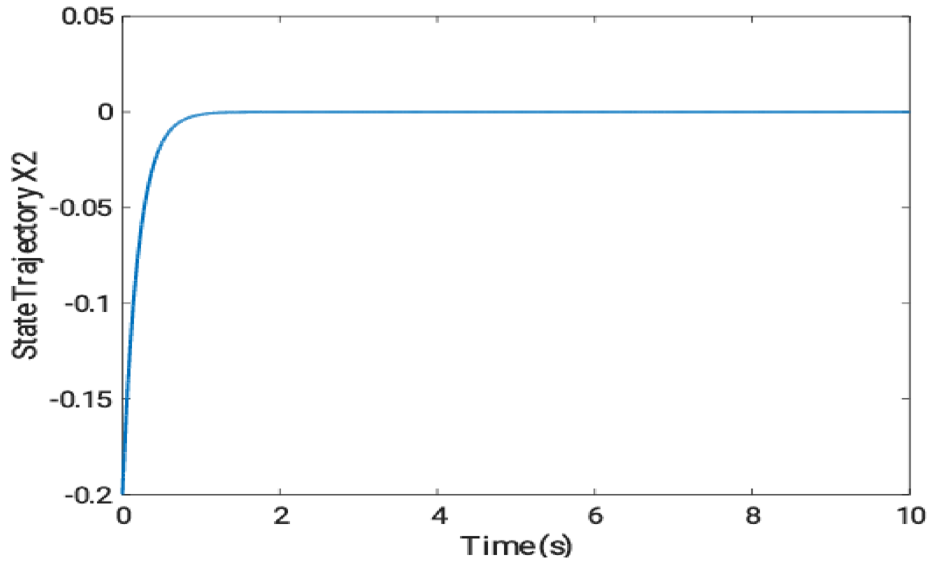


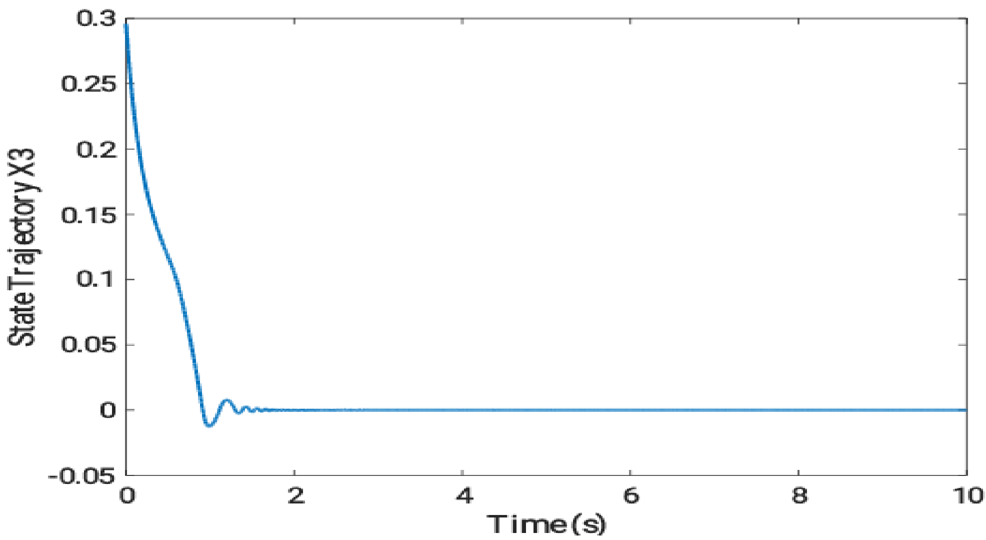
Figure 4.4: Control signal for proposed method



(a)



(b)



(c)

Figure 4.5: Controller time response of states (a) X1, (b) X2 and (c) X3

The above figure 4.5 shows the stabilization of states of Genesio system with the controller. This will optimally obtained the stable zero value in less time. This shows the time convergence of the proposed controller. By varying the control gain parameters in backstepping controller, error response is vaulted. Control input and tracking error response by varying control gains are shown in below figure 4.6.

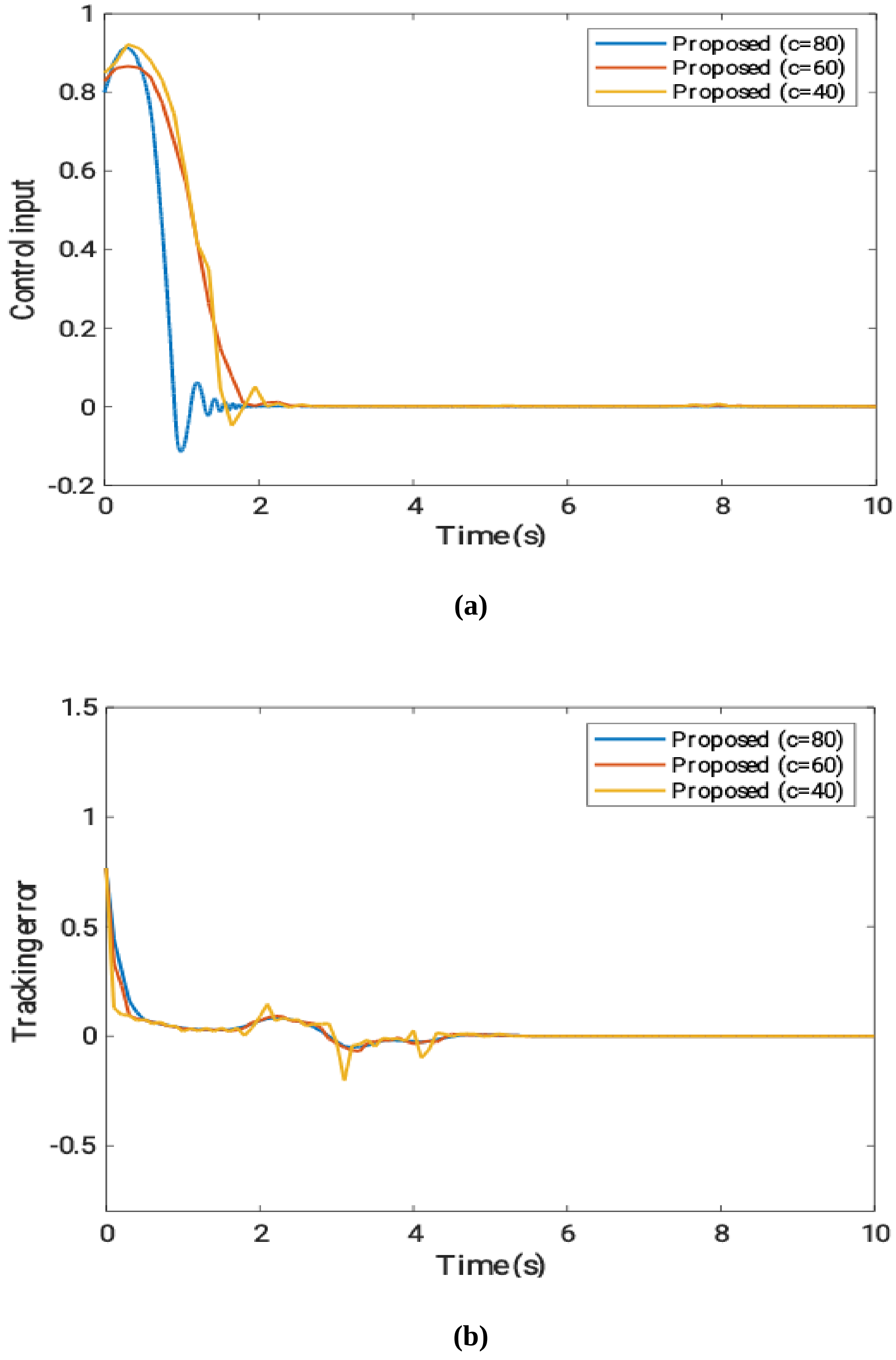


Figure 4.6: Response of the system under varying gain values (a) control input (b) tracking error

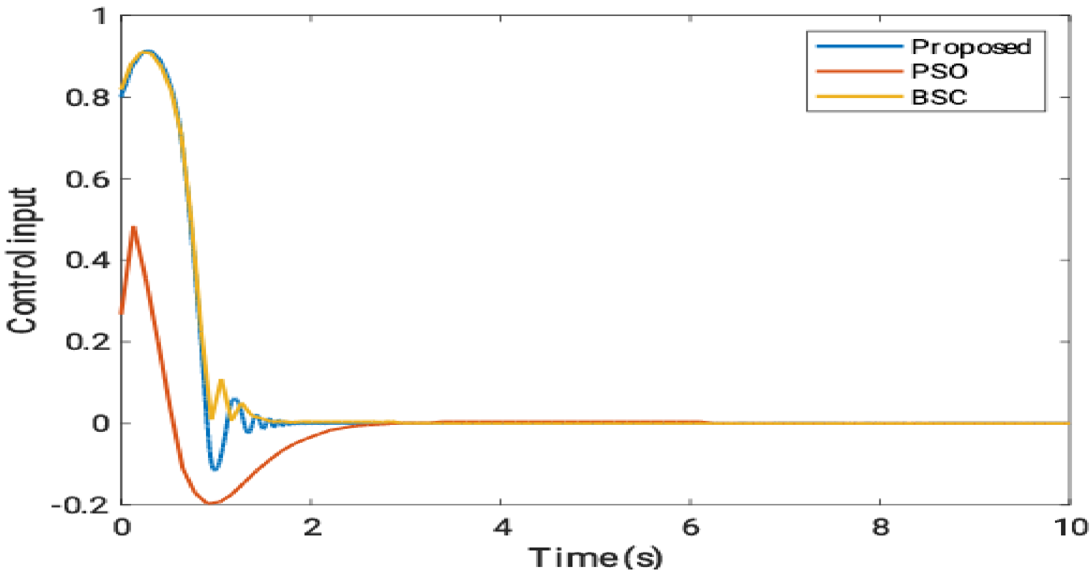
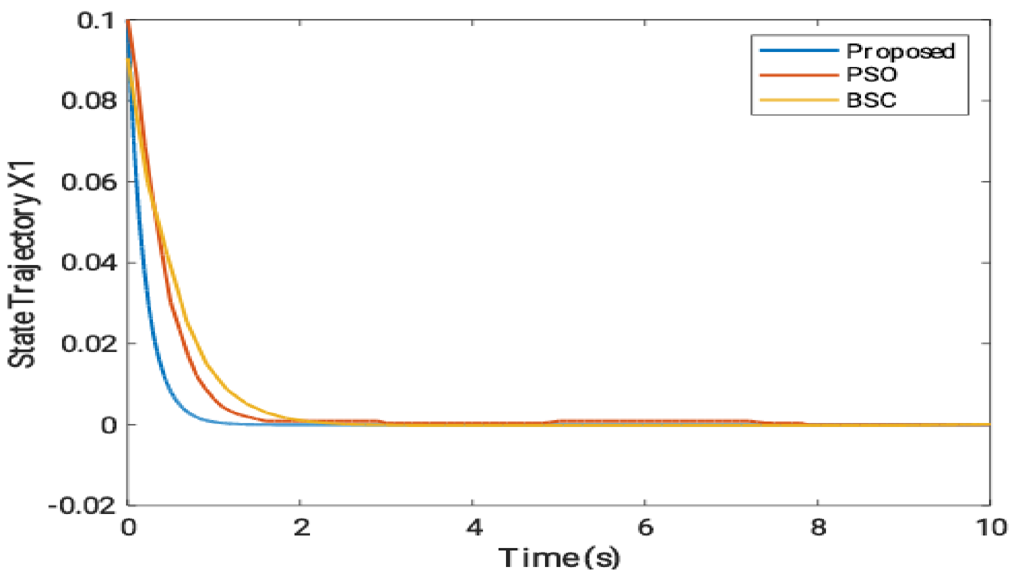
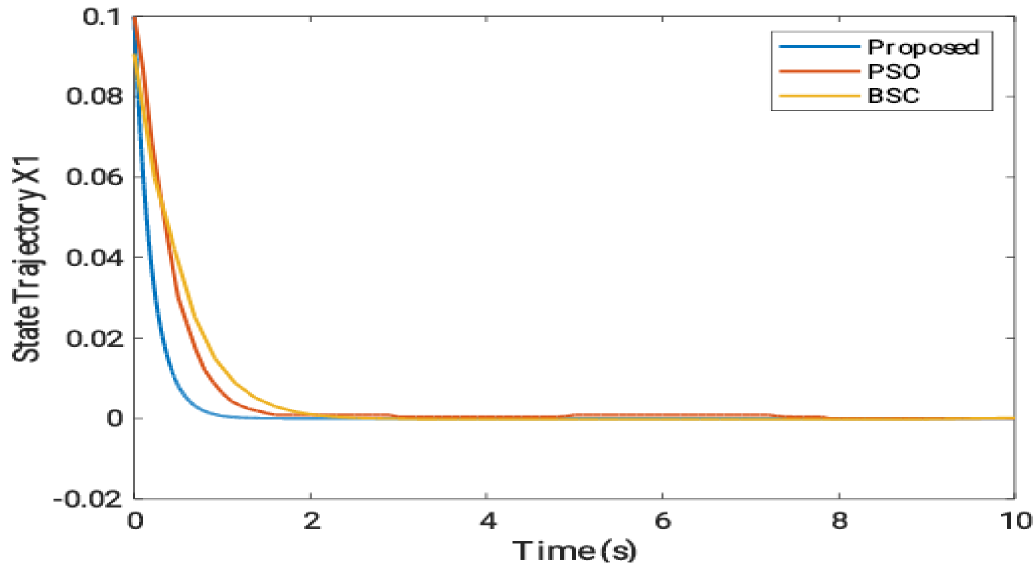


Figure 4.7: Comparison of control input

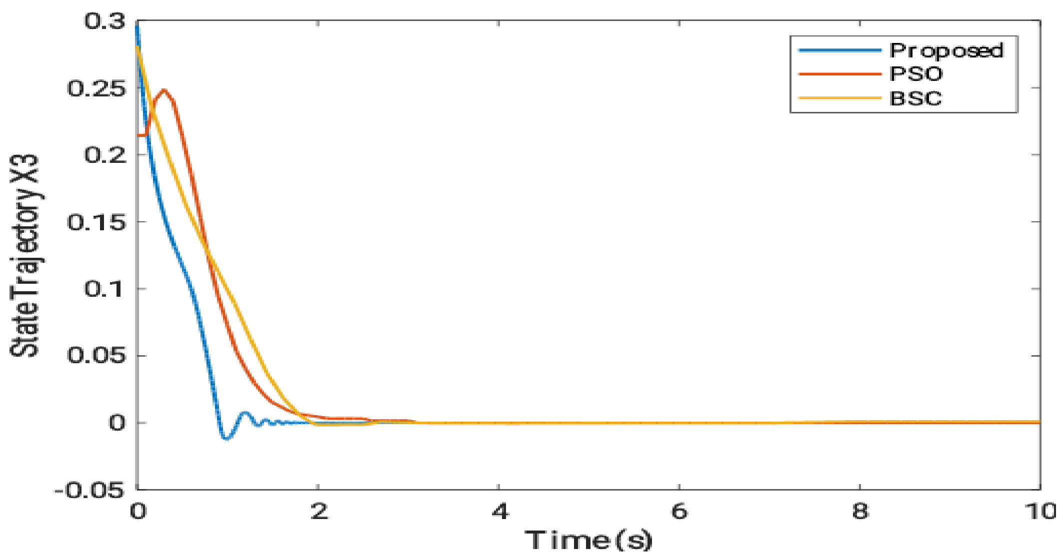
Simulation results in above figure 4.7 represent comparative analysis of control signal. Proposed controller with RNN is compared with particle swarm optimization (PSO) algorithm in [203] and backstepping controller (BSC) in the Genesio system's chaotic control. PSO is a population-based optimization, this selects the parameters using this algorithm. In the proposed method backstepping controller with RNN is used, while for the comparison without RNN is used. Due to the use of control effort in the proposed controller this saturation of the control signal is obtained.



(a)



(b)



(c)

Figure 4.8: Comparative analysis of state variables (a) X1, (b) X2, (c) X3.

The above figure 4.8 demonstrates the comparison of settling time of state variables. The proposed method leads to faster convergence of state's of the system to zero. This method is a much faster control of the system's chaos. Given that all three of the states are either confined or getting closer to zero, it is evident that the chaotic behaviour is suppressed and the system is stable. Also, this shows that the proposed backstepping controller can be used to stabilise fractional-order Genesio-Tesi systems. Moreover, all unknown parameters of the system are estimated to the point where they approach a constant value.

4.5.2 Coullet system

The Coullet system is a third-order autonomous nonlinear dynamical system introduced by Pierre Coullet while studying nonlinear instabilities and chaotic behavior in physical systems. It is widely used as a benchmark chaotic system in nonlinear control, synchronization, secure communication, and chaos suppression studies. The backstepping controller designed in the previous chapter is applied to the Coullet system. MATLAB simulations are carried out, and the results are presented here. The values of parameters of Coullet system are a=0.8, b=-1.1, c=-0.45, d=-1.0 and the Fractional order of the Coullet system is considered as 0.9 for simulation.

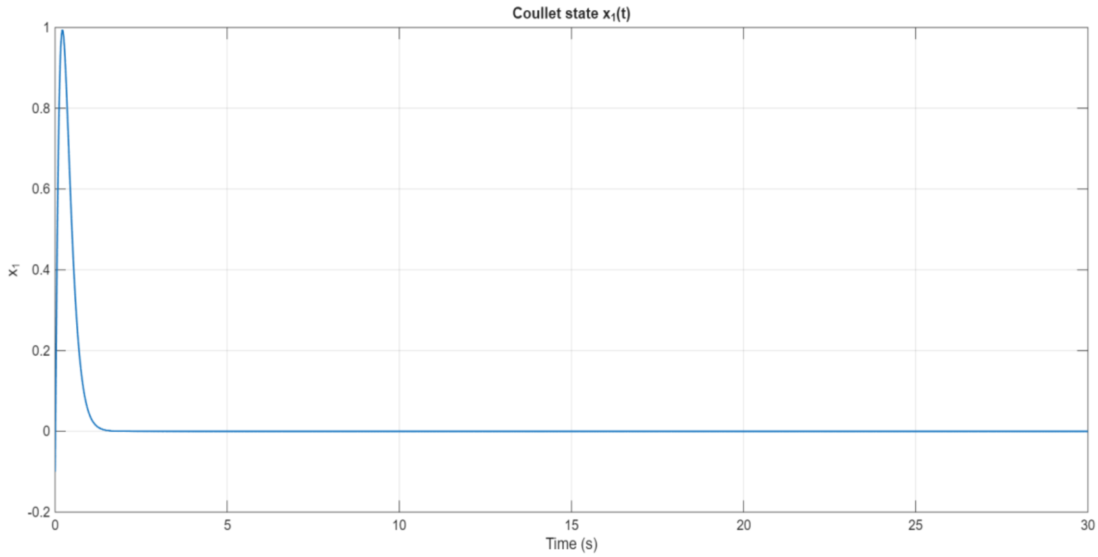


Figure 4.9 (a): Regulation of State X_1 of Coullet system

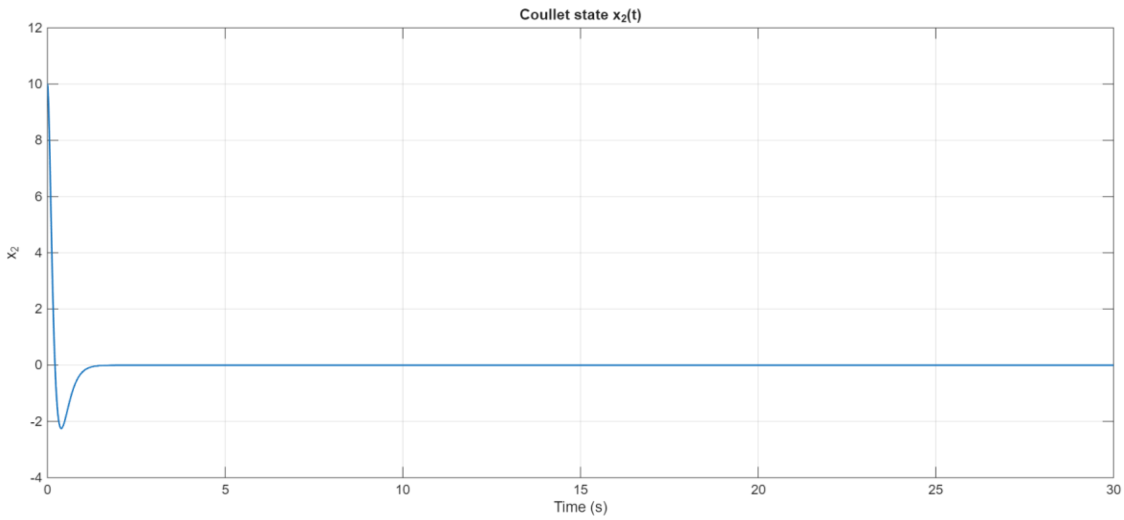


Figure 4.9 (b): Regulation of State X_2 of Coullet system

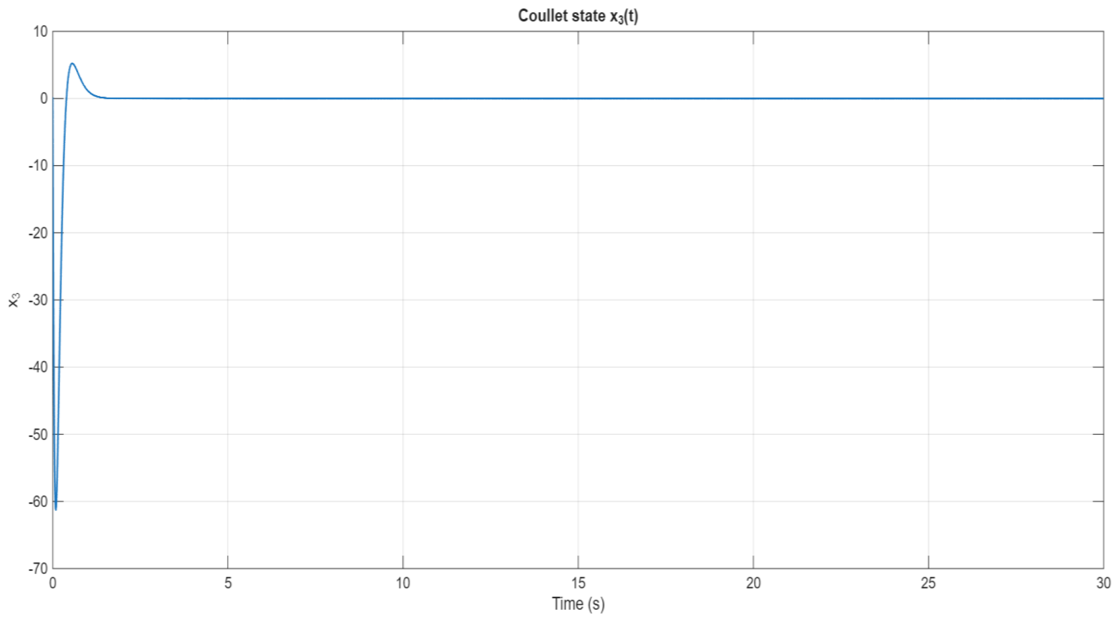


Figure 4.9 (c): Regulation of State X_3 of Coullet system

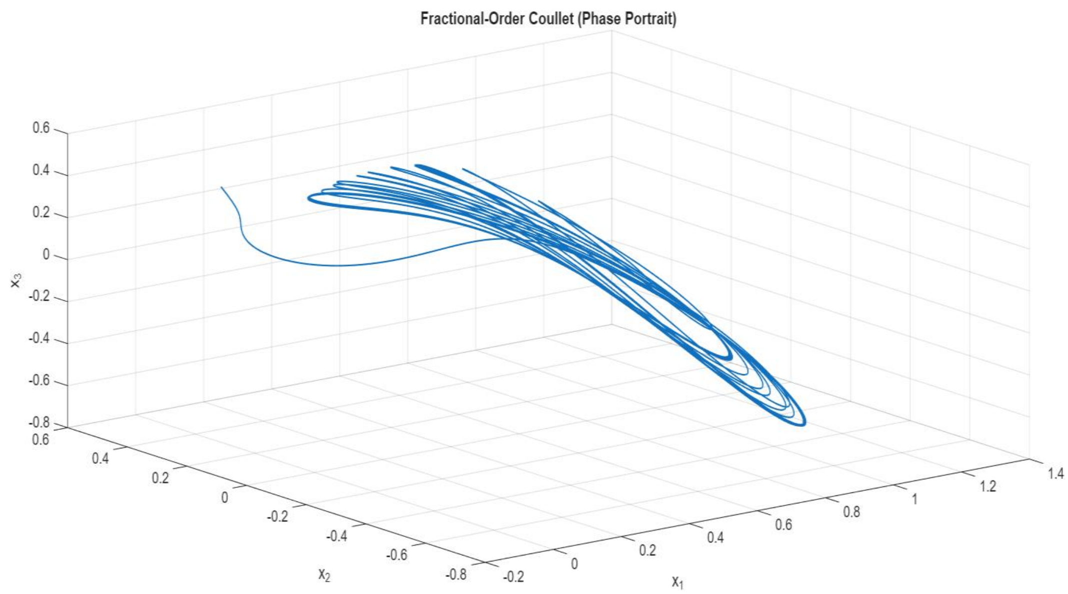


Figure 4.9 (d): Phase portrait of Coullet system

4.6. Summary

In this Chapter, adaptively coupled RNN with backstepping controller is proposed for a FONL strict feedback system. The results shows that the Tracking error is minimum with Backstepping controller with RNN. So, Backstepping controller with RNN is efficient when compared to PSO and Backstepping controller without optimization technique in minimizing the Tracking error.

Chapter-5

ANALYSIS OF OPTIMIZED CONTROLLERS ON FRACTIONAL ORDER NON-LINEAR SYSTEM

5.1. Introduction

The nonlinear behaviour of dynamic models is described effectively by fractional order calculus, which has the ability to solve complex engineering problems more effectively than integer order system [204]. The fractional order calculus has features such as high memory, dynamic nature and hereditary than integer order calculus [205-207]. Due to the analogical circuits of nonlinear system, the fractional order calculus has wider response [208]. Along with these advantages the fractional order calculus has the ability to solve synchronization problem of nonlinear chaotic system [209]. Thus the control and stability analysis of nonlinear system is the major research topic for fractional order calculus [210-211].

The control models bring a virtual control signal to system for the reduction of approximation error on fractional order nonlinear system [212]. To tackle this issue, several controllers are developed such as, backstepping controller, sliding mode controller, dynamic surface controller, active control, fuzzy control etc. [213-217]. A finite time backstepping control has been adopted to deal with actuator fault on nonlinear system [218]. In order to reduce the complex derivative a command filtering technique is incorporated. However, the computational burden is inherently solved by SMC. The unknown terms of the FO system is brought by Szász–Mirakyan operator [219], which is a universal approximate used to solve uncertain restriction of the input time delay fractional order system.

Backstepping control is implemented to reduce the problem of Dynamic Surface Control in [220] and it is used in such a manner that the instability and oscillation problem of the system can be minimized. These controllers have the problem of slow error tracking, computational burden, increasing derivative terms etc. that results in unstable virtual control input. Also, it is difficult to cope with the system with severe uncertainty, which results in nonlinearity in the parameter. Hence optimized controllers are developed.

Traditionally, the unknown nonlinear function of the fractional order controllers is optimized by using optimization algorithms such as particle swarm optimization, genetic optimization [221]. Later, the optimized algorithms are extended to use neural networks so that accuracy in detecting approximation error is obtained [222]. A full state constraint neural quantized control for FOS is reported in [223]. The controller effectively detects the approximation error when the system has subjected with quantization, nonlinearity and faults. Further, the neural network is used with other limitations of input saturation, output constraints, and time varying pseudo state constraints [224-226].

5.2. Proposed Methodology

To reduce the error of tracking in nonlinear system, many controllers are used however, the unknown function of the system reduce the system stability. Hence an virtual control signal is generated by using a novel RNN based backstepping controller. This will enhance the performance of the backstepping controlling in tracking the error. This RNN is adaptively coupled with backstepping controller. The traditional controllers in literature have the problem of higher derivatives, increasing number of parameters etc. Hence to cope with these issues optimized controllers are adopted with neural network technology. In this work, analyze the performance of various intelligence based controllers on Genesio-Tesi system with proposed controller. Thereby, the stability and reference signal tracking capability of controller is analyzed on fractional order system.

5.3. Preliminaries

The derivative and definition of the Caputo integral is expressed as follows [228];

Definition 1. The FO derivative for the function $F(t)$is expressed as;

$$I^r F(t) = \frac{1}{T(r)} \int_{t_0}^{t} (t-\mu)^{r-1} F(\delta) ds \qquad \text{(e. q., 5.1)}$$

From the above equation, integral order is represented by $t \geq t_0$ and$r > 0$. The gamma function is indicated by$T(r)$.

$$T(t) = \int_0^{\infty} t^{\delta-1} E^{-t} dt \qquad \text{(e. q., 5.2)}$$

Definition 2. The Caputo fractional-order derivative of order $r$for a function h belonging to $D^n(0,T]$can be expressed as:

$$D^r h(t) = \begin{cases} \frac{1}{T(n-r)} \int_0^t (t-\delta)^{n-r-1} h^{(n)}(\delta) d\delta, r \in (n-1,n), r = n \\ h^{(n)}(t), \end{cases} \qquad \text{(e. q., 5.3)}$$

From the above equation, $r \in [n-1, n], n \in \square and T > 0$

Lemma 1. Fractional derivatives, like integer-order derivatives, exhibit linearity. For any constantsϖandζ, the linear property of Caputo fractional derivatives is expressed as follows [229];

$$_{t_0}D_t^{\beta}\,[\varpi F(t) + \zeta h(t)] = \varpi t_0 D_1^{\beta} F(t) + \zeta_{t_0} D_t^{\beta} h(t) \quad \text{(e. q., 5.4)}$$

5.4. Recurrent neural network (RNN)

The drawback in back stepping controller is explosion of terms, which increases complexity of the controller and limits the applicability of the back stepping approach for higher-order systems. To overcome these challenges, proposed method incorporates an RNN. This will enhance the performance of the back stepping controller in tracking errors effectively. The RNN is adaptively linked with the back stepping controller. Figure 5.1 represents the structure of RNN.

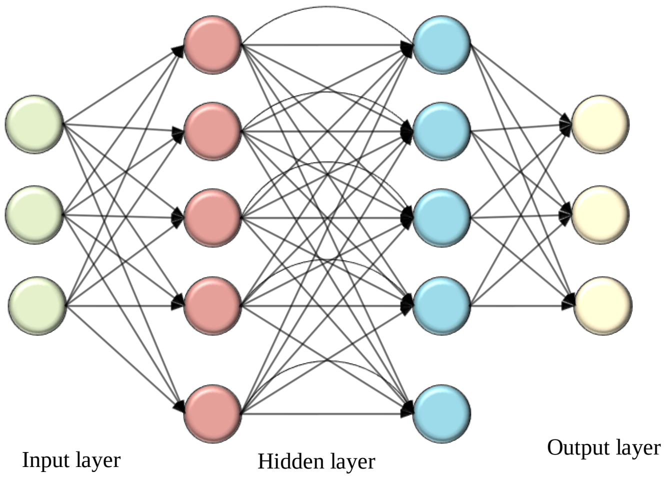


Figure 5.1: Strucure of RNN

An RNN is deep neural network which emulates the interconnected structure found in human brain neurons [227]. The connections of neural networks are utilized for transmitting the signals to neurons/ nodes which is similar to synapses. Neurons and their connections typically involve weights that adjust the learning process. The weight may vary to modify the signal strength as it goes from the input to output layers. An ANN includes intermediate/hidden layers placed among input & output layers.

An RNN generally needs a minimum of three hidden layers. The fundamental architecture of RNNs comprises input unit, hidden unit and output units. The hidden layer performs mathematical operations for generating the outputs by adjusting weights. In the RNN model, information moves from input to hidden units. It includes a directional loop that compares the error between the hidden layer and the previous one, as well as adjusting the weights among these hidden layers.

In this study, it is assumed that only the output of the RNN is employed for designing the controller in the nonlinear system. The current state is computed and expressed as follows:

$$H_n = f(H_{n-1}, X_n) \qquad \text{(e. q., 5.5)}$$

From the above equation, input is denoted by$X_n$previous state byH_{n-1}, current state is denoted byH_n, the approximation error is indicated by$\rho_{j,h}$, and the estimated value from the back stepping controller is represented byU_j. This estimated value error is denoted as

$$E = X - \dot{X} \qquad \text{(e. q., 5.6)}$$

$$H_n = \tanh(E_H H_{n-1} + E_{XH} X_n) \qquad \text{(e. q., 5.7)}$$

The error in the input neuron is denoted byE_H, error in the recurrent neuron is represented byE_{XH}. The error value is processed by the RNN at the output layer, leading to enhanced system performance.

5.4.1 Objective function

In the proposed RNN-based backstepping architecture, the primary objective is to accurately approximate the unknown nonlinear dynamics of the Genesio–Tesi chaotic system. To achieve this, the objective function is defined as the Mean Squared Error (MSE) between the true system states and the RNN-predicted states over the training dataset. The MSE is selected due to its smooth, convex nature and its suitability for nonlinear regression problems. The objective function is mathematically expressed as:

$$J = \frac{1}{N}\sum_{k=1}^{N}\left\|X(K) - \hat{X}(K)\right\|^2 \qquad \text{(e.q., 5.8)}$$

Where $X(K)$ is the state vector of Genesio-Tesi system and $\hat{X}(K)$ is the State vector estimated by RNN Network and ‘N’ is the Number of training samples. Minimization of this objective function ensures accurate state estimation, which is critical for effective compensation of nonlinear uncertainties in the backstepping control law.

5.4.2 Optimization Technique and Training Strategy

The RNN training is performed using the Levenberg–Marquardt (LM) optimization algorithm, which is well suited for nonlinear least-squares optimization problems encountered in neural network training. The LM algorithm adaptively combines the robustness of gradient descent with the fast local convergence of the Gauss–Newton method, enabling efficient and stable convergence even in highly nonlinear error landscapes.

Training data are generated directly from the Genesio–Tesi chaotic system using the MATLAB ode45 solver, which employs an adaptive Runge–Kutta scheme to accurately capture the system's chaotic dynamics. This ensures that the RNN is exposed to sufficiently rich and representative state trajectories during learning. To further improve training efficiency and eliminate subjective selection of the training goal, a Lizard Optimization Algorithm (LOA) is integrated with the LM training process. In this hybrid optimization framework, the LM algorithm updates the network weights and biases, while the LOA optimally selects the target MSE within the predefined range of $10^{-8} \leq MSE \leq 10^{-4}$. The training process terminates when either the optimized MSE goal is achieved or the maximum epoch limit is reached, ensuring both accuracy and computational efficiency.

5.4.3 Hyperparameter Selection and Network Configuration

Several hyper parameters are carefully selected to balance learning accuracy and computational complexity. The RNN architecture consists of a single hidden layer with 10 neurons, which is sufficient to provide approximation capability without overfitting. The hyperbolic tangent sigmoid (tansig) activation function is employed in the hidden layer to capture nonlinear relationships, while a linear (purelin) activation function is used in the output layer to facilitate accurate state estimation.

The training process is carried out for a maximum of 100 epochs using the LM algorithm. To ensure generalization capability and prevent overfitting, the dataset is randomly divided into training, validation, and testing subsets in the ratio of 80%, 10%, and 10%, respectively.

To strengthen the performance evaluation and demonstrate consistency of the proposed approach, statistical analysis is conducted over multiple independent simulation runs with different initial conditions. The tracking error statistics are quantified in terms of mean and variance. The mean tracking error is observed to be 3.1×10^{-4}, while the variance of the tracking error is 1.6×10^{-5}. These low values indicate not only high tracking accuracy but also low dispersion across runs, thereby confirming the robustness and repeatability of the proposed RNN-based controller.

5.5. Backstepping controller with RNN

Backstepping control is a frequently utilized technique in nonlinear controller design [230]. It has been extensively used for controlling and synchronizing IONL systems. This method uses a structured approach based on Lyapunov theory to ensure global stability.

The design of Backstepping controller with RNN is explained in the previous chapter for FONL chaotic system. Similarly, ANN, ESN and neuro-fuzzy methods are implemented on Genesio-Tesi system and obtained the respective results for analysis of RNN performance. Similar method used for analysis is explained below.

5.6. Artificial neural network

An ANN is a kind of statistical learning models that are often unknown and are used to estimate or approximate functions based on many inputs. Networks of linked "neurons" that communicate with one another are a frequent way to visualize ANNs [227]. Because their connections may be changed to correspond to numerical weights, neural networks are able to adapt to new inputs and learn from them. The hidden, input, and output layer make up three layers of an ANN. The input layer passes inputs to hidden layer after receiving them [228]. The precise ANN result is generated by the output layer. From input to output, intermediary calculations are carried out via a hidden layer. The aim of this ANN based technique is to identify the optimal parameter for backstepping controller. Architecture of ANN is provided in the below figure 5.2.

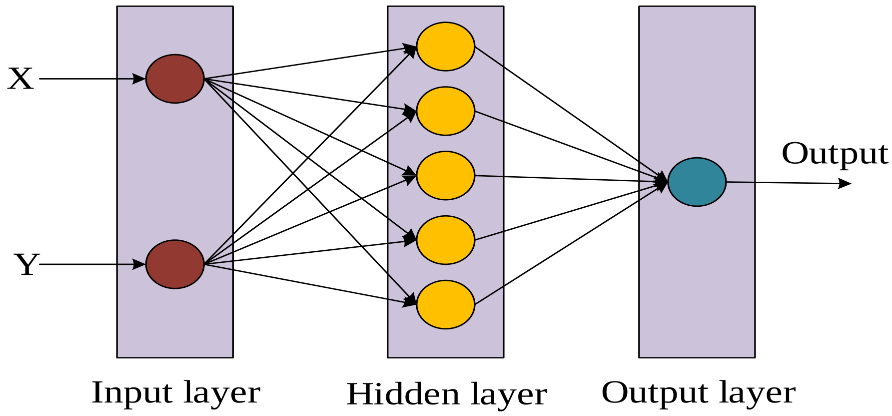


Figure 5.2: Layers of ANN

Neural network in real world application is represented as follows;

$$y_k(s, w_k) = \sum_{i=1}^{h} w_{kj}\varphi_{kj}\left(\sum_{i=1}^{n} v_{kj}s_i + \theta_j\right) = w_k^T \phi_k(.) \quad \text{(e. q., 5.9)}$$

$$w_k = \begin{bmatrix} w_{k1} \\ \vdots \\ w_{kh} \end{bmatrix} \quad \text{(e. q., 5.10)}$$

$$\phi_k = \begin{bmatrix} \varphi_{k1}\left(\sum_{i=1}^{n} v_{1j} s_i + \theta_1\right) \\ \vdots \\ \varphi_{kh}\left(\sum_{i=1}^{n} v_{hj} s_i + \theta_h\right) \end{bmatrix} \quad \text{(e. q., 5.11)}$$

$\phi_k(.)$was evaluated using the below equation.

$$\phi_k(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{(e. q., 5.12)}$$

Where, y_k indicates output, v_{kj}represents randomly selected value between the interval of -1 to 1. h, n and m denotes number of neurons available in hidden, input and output layer respectively.

5.7. Echo state network (ESN) backstepping control

The ESN uses a high-dimensional projection mechanism, which is comparable to the kernel's role in kernel-based learning techniques, to capture the input dynamics. linear regression techniques are employed to calculate the linear readout layer weights, and the output signals are linear combination of the reservoir's echo states. RBFNN is widely utilized to describe system uncertainty to control FONL system. It makes sense to build an extended dynamic approximate by combining an RBFNN with a dynamic system in order to improve estimation capacity.

RBFNN and a leaky ESN are combined using certain threshold functions to create a fractional order ESN [231].The classic RBFNN requires the construction of several neurons in order to achieve its approximation capability. This raises the computational load and complexity of the model. Because of this, scientists frequently lessen the computing load by reducing the ideal weight vector to norm of scalar parameters. FOESN method will improve universal approximator. Figure 5.3 represent the structure of ESN.

$$\dot{P}(z) = -\mu P(z) + tanh\left(W^{in}\gamma + W_d P(z) + W_{fby}\right) \quad \text{(e. q., 5.13)}$$

Where, $P(z)$ indicates activation function, $tanh(.)$denotes hyperbolic tangent function, each reservoir neuron's leakage rate is represented by $\mu > 0$. The connection weight matrices for internal, and feedback and input connections are shown by the symbols W^{in}, $W_{fby,}$ W_d respectively. Output of this network is evaluated as follows;

$$C = W^T P(z) \quad \text{(e. q., 5.14)}$$

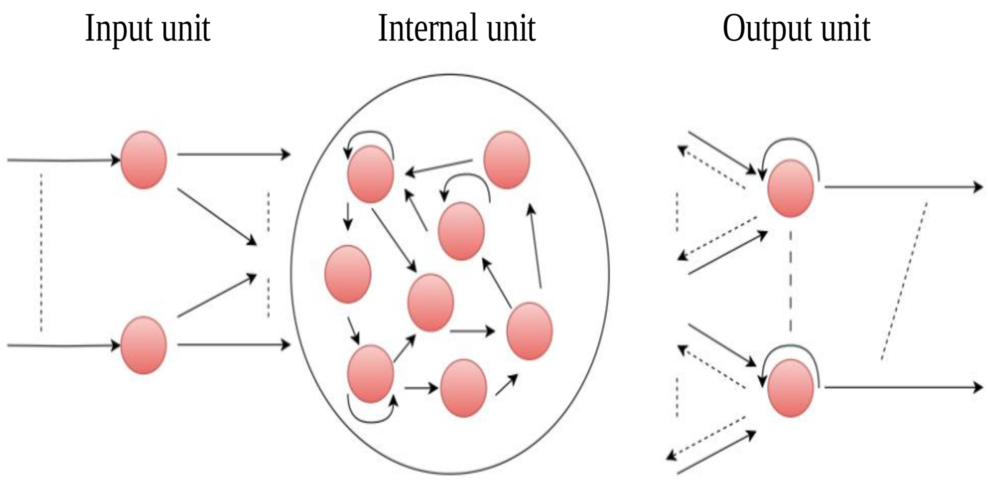


Figure 5.3: Structure of ESN

Where, W^T denotes output weight matrix. The operations in an ESN's hidden layer, or reservoir, are largely responsible for determining its computational complexity. Because of recurrent connections and nonlinear activations, these operations have a high complexity. Increasing the number of layer will increase the complexity of this method.

5.8. Neuro-fuzzy systems

ANN and fuzzy logic are integrated in neuro-fuzzy systems. While fuzzy logic and neural networks both communicate information, reasoning, and learning, they do it in distinct ways and with various benefits and drawbacks. Artificial neural networks provide some amazing capabilities, such the capacity to learn, adapt, and generalise, whereas fuzzy logic offers a reasoning process using imperfect and inadequately correct information. Although this kind of knowledge is nearly hard to express, neural networks may learn from the example. Conversely, fuzzy logic lacks auto-adjustment but offers approximate inference. The fuzzy modelling and learning techniques based on set databases provide the foundation of the neuro-fuzzy methodology. In order for the associated Fuzzy Inference System (FIS) with the minimum error to match the provided pairs of input-output data, the parameters of the membership functions are calculated. This learning process is comparable to the neural network learning process. Figure 5.4 illustrates the operation of neuro fuzzy method.

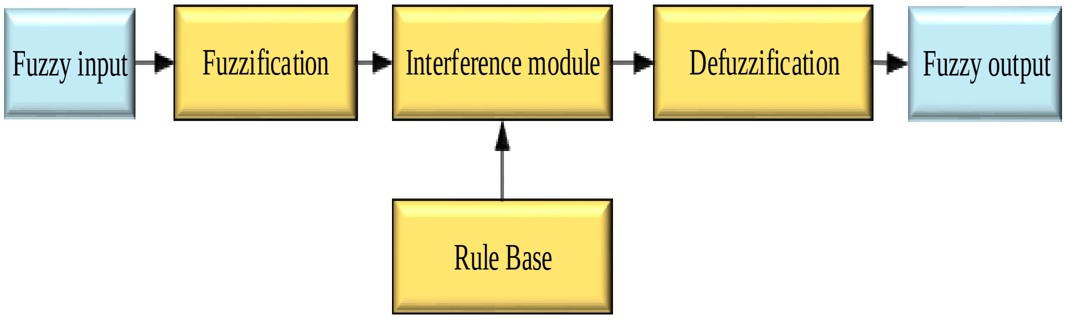


Figure 5.4 Fuzzy logic operations

Neuro with fuzzy system modeling is used as a tool in decision making. Fuzzification, defuzzification, and fuzzy rule base are the three basic parts of a FLS [232]. The following inference rules comprise the fuzzy rule base:

R_l: If $X_1$is F_1^l and $X_2$is F_2^l and $X_n$is F_n^l; then y is G^l . Where$l = 1,2,\ldots\ldots, N$, F_i^l and G^l are set of fuzzy. A FIS is created using a specified input/output data set, and certain algorithms are used to define the membership function parameters.

5.9. Results and Discussion

RNN with backstepping controller is implemented and analyzed the performance in MATLAB tool. In this work, various intelligence-based controllers on Genesio-Tesi system are used to analyze the performance. The following controllers are used to analyze the Genesio-Tesi system.

1. Neuro fuzzy adaptive backstepping DSC (NFADBSC)
2. Reinforcement learning for event triggered backstepping control (RLEBSC)
3. Neuro fuzzy adaptive backstepping sliding mode control (NFABSM)
4. Echo state network backstepping control (ESN)
5. Artificial neural network based backstepping controller (ANN-BSC)

These methods are implemented and obtained the results for analysis. Thereby, the stability and reference signal tracking capability of controller is analyzed on fractional order system. Figure 5.5 shows the response of state trajectories of the system. This the output for RNN with backstepping controller, it obtains stable value with less distortion.

This research demonstrates a significant improvement in finding the optimal control signal for the RNN-based backstepping controller. The above figure illustrates the control signal for the RNN-BSC, it achieved in a much shorter time frame. These results strongly support the effectiveness of the backstepping control strategy with an RNN component in stabilizing the fractional-order Genesio-Tesi system.

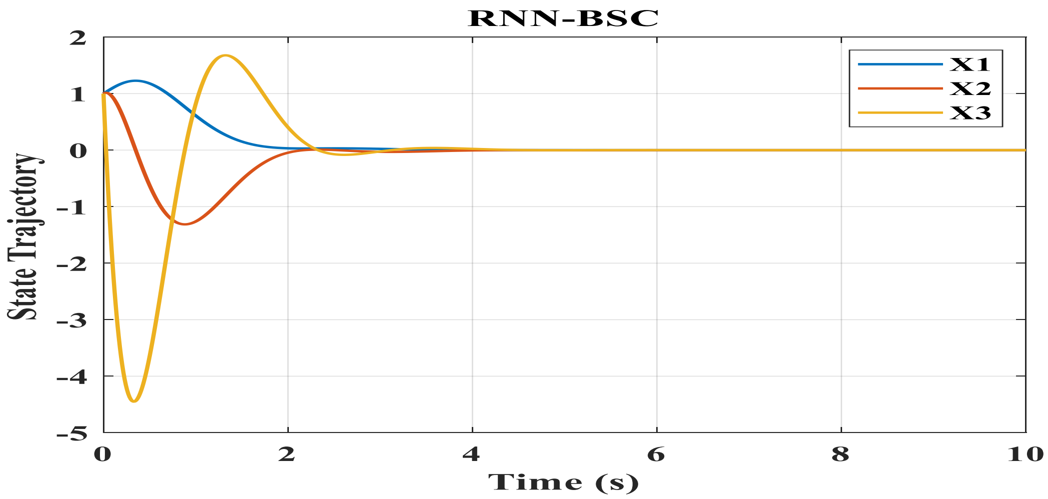


Figure 5.5 System state trajectories

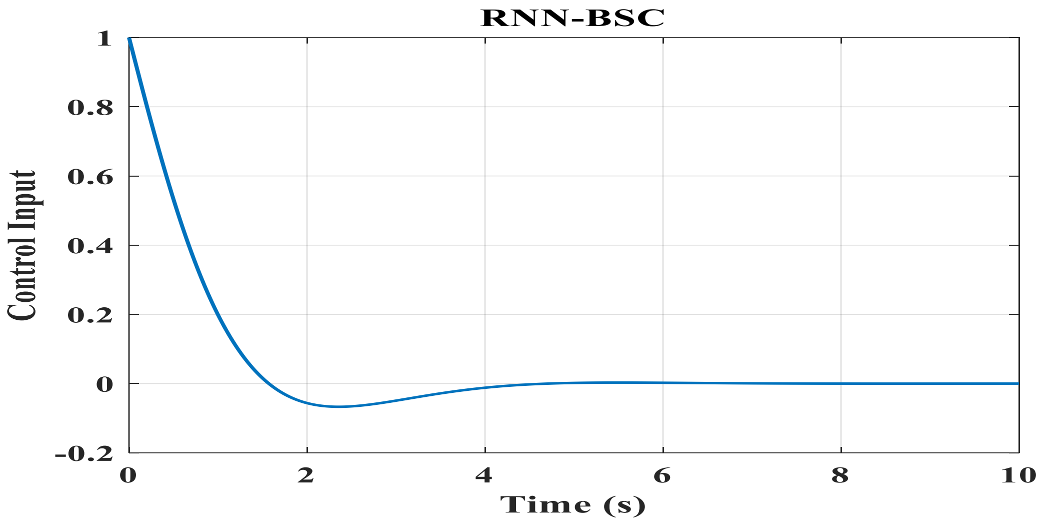
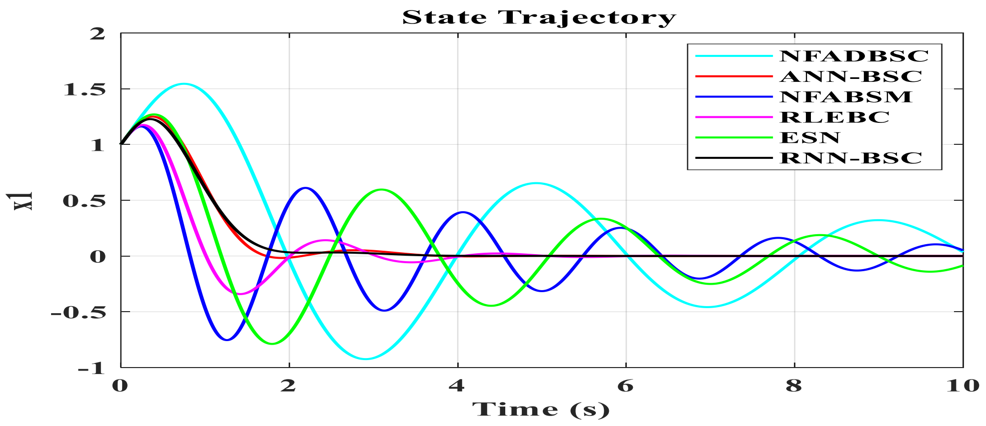


Figure 5.6 Control singal for RNN with BSC

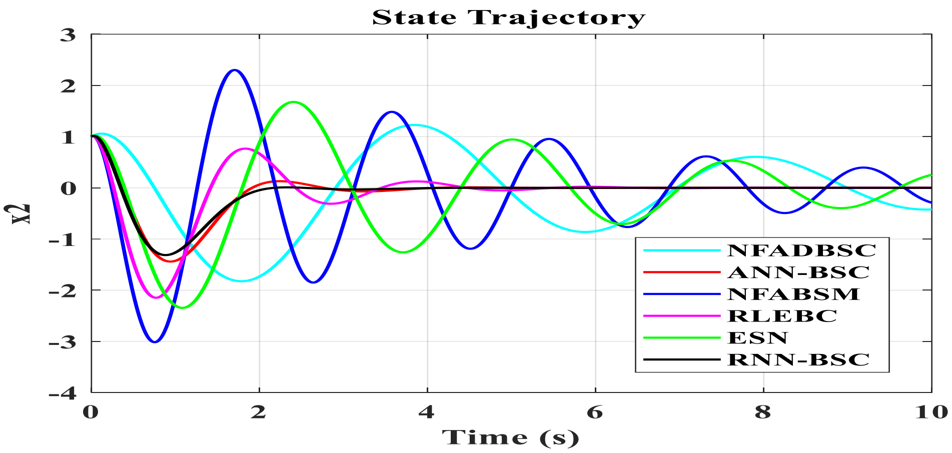
5.10 Analysis of proposed work with existing methods

Controlling a FO Genesio-Tesi system is challenging due to its complex dynamics. Traditional methods often struggle to find the optimal control signal in a timely manner.

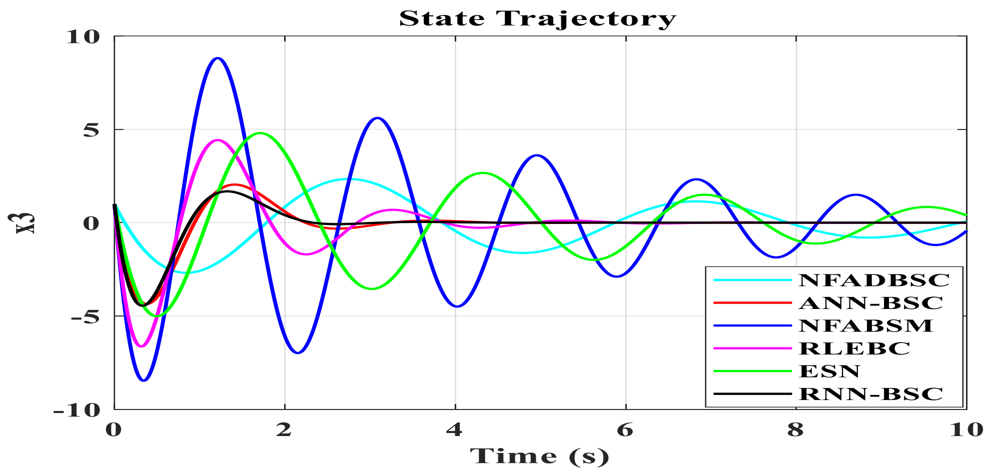
Controlling a FO Genesio-Tesi system is challenging due to its complex dynamics. Traditional methods often struggle to find the optimal control signal in a timely manner.



(a)



(b)



(c)

Figure 5.7 Comparative analysis of controllers for the stabilization of states

This research introduces a RNN-based backstepping controller designed to overcome these limitations. Figure 5.7 a-c shows the performance comparison of various control methods. For this comparison, existing literature controllers like NFADBSC, RLEBSC, NFABSM, ESN and ANN-BSC are compared with RNN with BSC. The results are taken for control signal responses for each approach over time. Notably, the RNN-based backstepping controller's signal reaches the desired value significantly faster than those of other methods. This analysis revealed that the RNN controller achieved its target faster than existing controllers. By finding the optimal control signal quicker, the RNN-based backstepping controller ensures the system reaches its desired state with improved accuracy and reduced settling time.

This figure 5.8 compares the tracking error of various control methods for the FO Genesio-Tesi system. The RNN-based BSC exhibits lower tracking error compared to other controllers. While as NFADBSC, ANN-BSC, NFABSM, and ESN controller has more error till 8 seconds. This signifies the effectiveness of the RNN with BSC in driving the system's output closer to the desired trajectory.

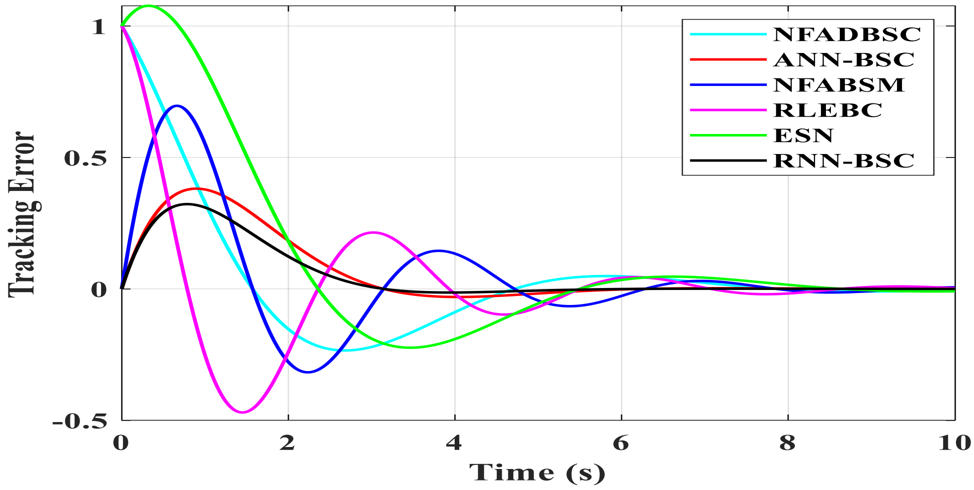


Figure 5.8 Analysis of tracking error with existing methods

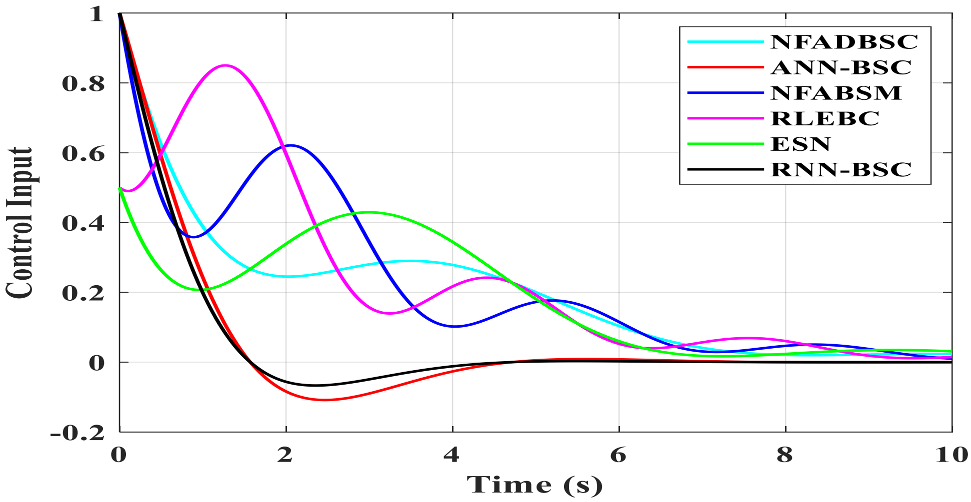


Figure 5.9 Comparative analysis of control Input Signal for different Controller

This figure 5.9 shows the control input signal analysis for different controllers. The signal reaches its target value within a short time frame. The control input signal for the RNN-based backstepping controller, rapidly reaches its target value. While the NFADBSC and RLEBC takes high range of time to reach its stable output. This quick convergence demonstrates the efficacy of the proposed technique in finding the optimal control signal.

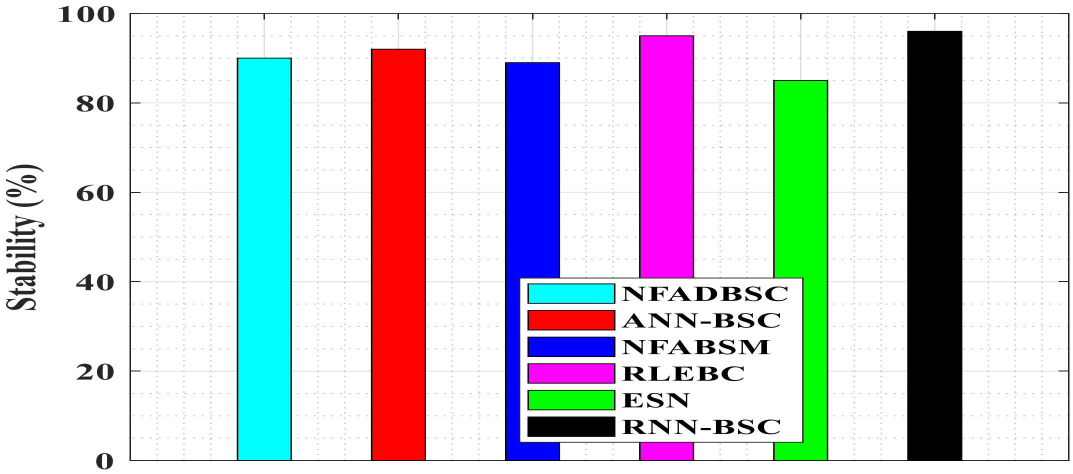


Figure 5.10 Stability analysis for Fractional-Order Genesio -Tesi System

Figure 5.10 compares the stability analysis of various control signals for the FO Genesio-Tesi system. The RNN-based backstepping controller achieves the desired stability value than other controllers. The Stability of the RNN based BSC controller is significantly higher than those of NFADBSC, ANN-BSC, NFABSM, and ESN controllers.

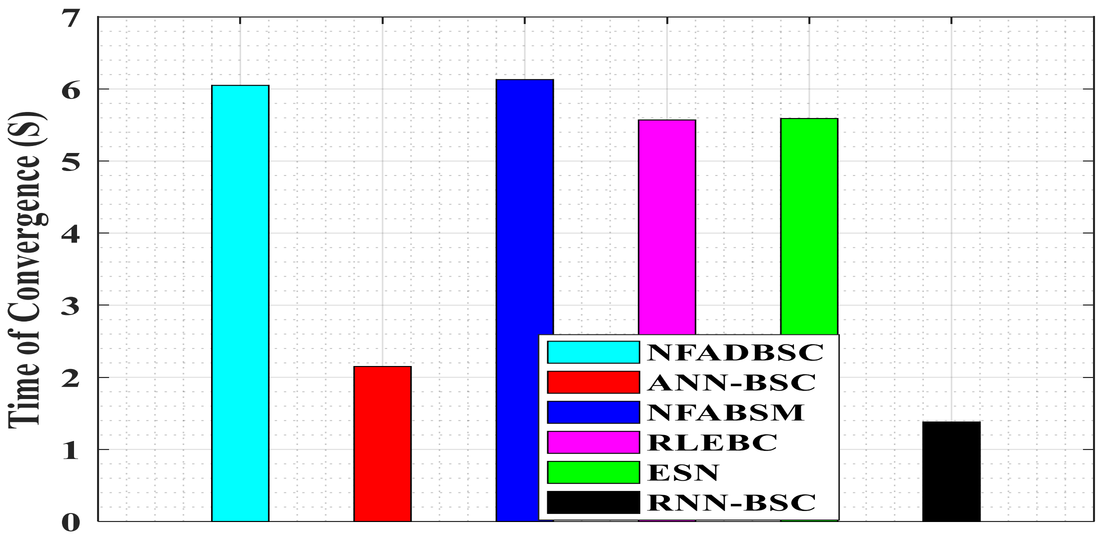


Figure 5.11.: Comparative analysis for time of convergence

Figure 5.11 illustrates a comparison of convergence times for different methods. The y-axis represents the time taken to converge in seconds, while the x-axis shows the various algorithms. Among the algorithms evaluated, proposed RNN-BSC demonstrated the fastest convergence, followed by RLEBC and ESN. Conversely, NFADBSC exhibited the slowest convergence. ANN-BSC and NFABSM displayed moderate convergence times. These findings suggest that RNN-BSC is well-suited for applications requiring rapid convergence, while NFADBSC might be less suitable for time-sensitive tasks.

Table 5.1: Comparative analysis

Methods	**Stability**	**Convergence time (S)**	**Error Tracking (S)**
NFAD-BSC	70	6.05	7.58
ANN-BSC	82	2.15	5.42
NFABSM	69	6.13	8.68
RLEBC	73	5.57	8.94
ESN	75	5.59	8.73
RNN-BSC	**86**	**1.38**	**4.84**

5.11. Summary

In this work, the performance of the fractional-order Genesio–Tesi nonlinear system using a backstepping controller with RNN is compared with conventional optimized controllers. The RNN-based backstepping controller exhibits superior performance compared to other control schemes due to its ability to capture temporal system dynamics through internal feedback. RNN-BSC achieves the highest stability value (86%), indicating superior robustness and better handling of system uncertainties compared to other approaches such as NFAD-BSC (70%) and NFABSM (69%). It provides the fastest convergence (1.38 s), significantly improving response speed over methods like ESN (5.59 s) and RLEBC (5.57 s). RNN-BSC also yields the lowest tracking error (4.84), demonstrating high precision in following the desired trajectory.

Chapter-6

APPLICATIONS OF RNN BASED BACKSTEPPING CONTROLLER

6.1. Introduction

Nonlinear systems with chaotic behaviour have been extensively utilized in various fields over the last few years [233]. In reality, majority of real-world systems are nonlinear, and chaotic in nature [234]. Chaos plays an essential role in various domains, including the study of biological systems such as the brain, human heart, secure communication systems, smart systems, robotics engineering, data encryption, and the design of nonlinear oscillators [235]. Fractional-order calculus is a widely utilized mathematical tool across various scientific fields, providing more accurate descriptions of complex real systems and nonlinear phenomena [236]. Fractional calculus has obtained significant attention recently due to its ability to provide more précised models compared to integer-order calculus [237].

Fractional calculus finds applications in control systems, bioengineering, image encryption systems, analog filters, and oscillator's circuit theory [238]. Consequently, fractional-order chaotic systems offer greater research potential and application value in the study and implementation of chaotic systems [239]. A proficient image encryption scheme is developed based on Memristor Hopfield neural network [240]. Optimization algorithms applied to adjust encryption parameters and cipher text image is generated by executing confusion and diffusion operations [241]. In reference [242], systems are synchronized using impulsive and event-triggered control methods. In reference [243] multiple layer encryption network is used for image encryption. An S-box and encryption key are generated for each layer, and employed.

In reference [242], to address the stability issue of FOCS, an adaptive neuro-fuzzy SMC technique is proposed with back stepping control technique. To synchronize similar systems and systems of varying dimensions, Back stepping technique has been shown to be more successful [245]. Back stepping controller is selected for synchronisation due to its extraordinary performance with nonlinear dynamical systems. Various synchronisation methods which include combination synchronisation, combination-combination synchronisation, difference synchronisation, difference-difference synchronisation and have been explored [246].

An image encryption technique using fractional integral sliding mode control with fuzzy logic has been published in [247]. An audio encryption strategy has been employed to synchronize mismatched fractional-order systems. In reference [248], design includes a sliding surface that is comparable to one of the states of the system. A Lyapunov function is used to show that surface is stable. The stability of the proposed controller was analyzed, along with the continual convergence of error in synchronisation problem in [249]. In this study, a nonlinear fractional-order PID controller with “sliding surface” was developed to synchronize systems which are affected by uncertainty, time-varying time delays and disturbances [239]. Consequently, investigating finite-time chaos synchronisation problems in fractional-order dynamical systems with uncertainties becomes imperative [250]. Therefore, the design of a controller capable of effectively minimizing synchronisation errors within finite time, while maximizing information transmission security, emerges as a significant and powerful challenge [251].

6.2. Proposed methodology

In this work, a novel approach for secure communication and Image encryption within FOCS is proposed.

The concept of chaotic synchronisation is utilized to address the issues of secure transmission of speech signal as well as image encryption. For Secure communication, two similar Non-linear FOCS in “Master-Slave” configuration are considered and a backstepping controller with RNN is designed to synchronize these two systems. Even though several controllers such as backstepping, dynamic surface control, sliding mode control, active control, fuzzy controller are designed to reduce approximation error in fractional order nonlinear systems, the backstepping controller is widely adopted as it improves the tracking and stability of the FONL systems. But still these controllers have the problem of slow error tracking, computational burden, increasing derivative terms etc. that results in unstable virtual control input. Also, it is difficult to cope with the system with severe uncertainty, which results in nonlinearity in the parameter. Hence improved controllers are developed. Traditionally, the unknown nonlinear function of the fractional order controllers is optimized by using optimization algorithms such as PSO, genetic optimization. Later, the optimized algorithms are extended to use neural networks so that accuracy in detecting approximation error is obtained. To reduce approximation error, an adaptive backstepping controller is designed using a deep learning process. To do so here, an adaptively coupled recurrent neural network based backstepping controller is adopted. The backstepping controller leverages the inherent learning capabilities of RNNs to obtain precise synchronisation between sender's encryption system and the receiver's decryption system. This synchronisation phase is crucial for successful decryption. The system's security relies on a large key space encompassing controller structure, system parameters, initial conditions and fractional orders. Furthermore, to optimize the performance of the RNN-based controller, a frilled lizard optimization strategy was employed. This strategy focuses on minimizing the Root Mean Square Error (RMSE) of a designated variable, ultimately leading to faster and more accurate decryption. The efficacy of this approach is validated through extensive simulations. In this work, a novel Communication strategy is proposed to transfer information securely using synchronisation of two distinct fractional-order systems, accounting for uncertainties and time delays. A backstepping controller with RNN developed in chapter 4 is used to synchronize two identical chaotic FONL systems. And the control rule *U*is ,

$$U = \Big[\frac{1}{G_n(X_1,\dots\dots,X_n)} - \frac{\partial V_{n-1}}{\partial X_{n-1}} \times G_{n-1}(X_1,\dots\dots,X_{n-1}) - k_n\big(X_n - \varphi_{n-1}(X_1,\dots\dots,X_{n-1})\big) + \frac{\partial \varphi_{n-1}}{\partial X_{n-1}}[f_{n-1}(X_1,\dots\dots,X_{n-1}) + G_{n-1}(X_1,\dots\dots,X_{n-1})X_n] + \frac{\partial \varphi_{n-1}}{\partial X_1}[f_1(X_1) + G_1(X_1)X_2] - f_n(X_1,\dots\dots,X_n)\Big] \qquad \text{(e. q., 6.1)}$$

$$V_2(X_1,\dots\dots,X_n) = V_{n-1}(X_1,\dots\dots,X_n) + \frac{1}{2}\big(X_n - \varphi_{n-1}(X_1,\dots\dots,X_{n-1})\big)^2 \qquad \text{(e.q., 6.2)}$$

6.2.1. Secure Communication using synchronisation

In transmitting a speech signal securely, a Chaotic signal $(C(t))$ is obtained by integrating the states of the Master system and then the speech signal $\mathrm{m(t)}$is mixed with chaotic signal $\mathrm{C(t)}$.

$$C(t) = Ax_1(t) + Bx_2(t) \qquad \text{(e.q., 6.3)}$$

The following formula is used for masking speech signal m(t) using C(t).

$$s(t) = m(t) + C(t) \qquad \text{(e.q.,6.4)}$$

Signal $\mathrm{s(t)}$ is a mixture of speech signal and chaotic signal that is sent through the communication channel.

The control signal generated at $\mathrm{h_1}$is a mixture of states of master system. The composite signal ($\mathrm{s(t)}$ and $\mathrm{h_1}$) is transferred from Master system which is used for synchronisation at the slave system. The fractional order derivatives, initial conditions and system parameters form the complex key set. This complex key set should be known to Slave system. Since key is very complex, it is not easy to synchronize systems for intruder to encode the information. The Master and Slave slave systems are synchronized by using controller h. The controller h is a combination of h_2 generated at slave system and controller h_1 that is received from Master System. To retrieve the information signal at receiver, a sequence y(t), which is chaotic in nature is utilised.

The block diagram for secure communication is shown in below figure 6.1.

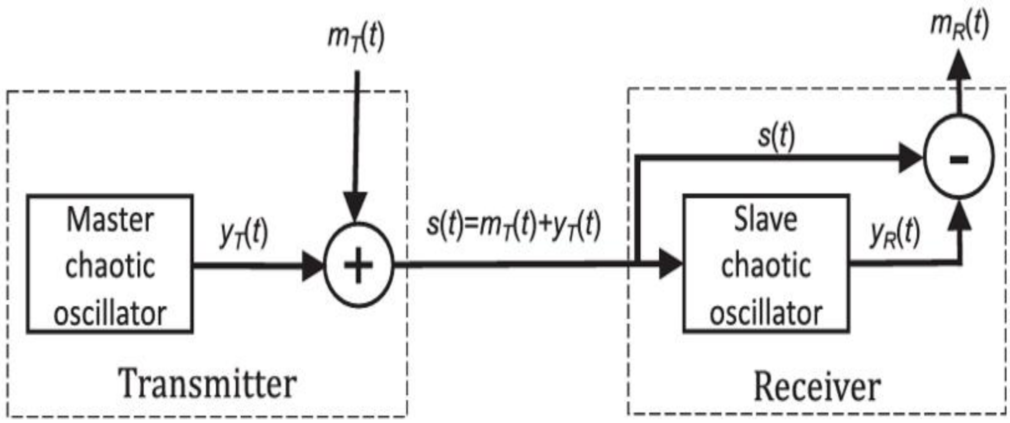


Figure 6.1: Block diagram for secure communication

Signals $s'(t)$ & $y(t)$ are used for unmasking the information signal $m'(t)$, according to the following operation.

$$m'(t) = s'(t) - y(t) \quad \text{(e.q., 6.5)}$$

6.2.2. Image Encryption and Decryption using synchronisation

The scheme for synchronisation of fractional order Genesio Tesi systems used for image encryption and decryption is presented in this section. Initially, a chaotic time series is generated which consists of three states x_1, x_2 andx_3. A specific combination of these three chaotic states (which will be included in key) is used for encrypting image at sender. Instead of sending key, a combination of states of master system and different parameters of system are transmitted to receiver.

Controller received at slave system is used to get synchronisation with Master system. The order of fractional derivatives and parameters of Master must be known to Slave system. The chaotic sequence created at the sender's and receiver's end should be same for synchronizing Master & Slave systems. At sender and receiver ends, the states will be combined in the same manner to create key and this used for decryption.

a. Encryption

Step 1: Consider an image I of size m × n (where 'm' represents number of rows and 'n' represents number of columns in image I. An image J is produced by shuffling the pixels of the original image randomly. In natural images, neighbouring pixels are strongly correlated and shuffling breaks this correlation. Therefore, it will be hard for attackers to hack the information but the histogram of the image remains the same.

Step 2: Choose the order of fractional derivative and appropriate parameters of the system then simulate the Master system. Design a controller by using proposed method explained in Section 3. Now, split it into 2 parts, one which consists of States of the Master and the other part consists of States of the Slave system. So, the section of controller consisting of master states should be sent as a part of key. Select proper values for A, B, C and mix them in a specific manner.

$$x(t) = Ax_1(t) + Bx_2(t) + Cx_3(t) \qquad \text{(e.q., 6.6)}$$

Step 3: The following technique is used to create a pseudo-random vector y of 8-bit integers using the chaotic sequence vector x.

$$\text{Let } abs(x(j)) = h$$

$$y(j) = round(mod(h) - floor(h \times 105\,, 256)) \qquad \text{(e.q., 6.7)}$$

where j = 1, 2, ..., m

From the expression (e. q., 6.7), a set of unsigned integers (range: 0-255) is generated.

Step 4: K1 is the key image obtained from vector y (size: m×n).

Step 5: Image J is encrypted with the help of key image by performing bitwise XOR operation and it has to be sent to the Receiver.

$$E_{image} = J \oplus \text{K1} \qquad \text{(e.q., 6.8)}$$

b. Decryption

Step 1:

The expression for Control signal is given as

$$U = \frac{1}{G_n(X_1 ......X_n)} \begin{bmatrix} \frac{\partial V_{n-1}}{\partial X_{n-1}} G_{n-1}(X_1, X_2, ...X_{n-1}) \text{-} \, k_n(X_n - \emptyset_{n-1} \\ \left((X_1, X_2..X_{n-1})\right) \\ + \frac{\partial \emptyset_{n-1}}{\partial X_{n-1}} \, [f_{n-1}(X_1, X_2, ...X_{n-1}) + G_{n-1} \\ (X_1, X_2, ...X_{n-1})X_n\} \\ + \frac{\partial \emptyset_{n-1}}{\partial X_1} [f_1(X_1) + G_1(X_1) - f_n(X_1, X_2, ...X_n) \end{bmatrix}$$

$$+ \frac{1}{G_n(Y_1 ......Y_n)} \begin{bmatrix} \frac{\partial V_{n-1}}{\partial Y_{n-1}} G_{n-1}(Y_1, Y_2, ...Y_{n-1}) \text{-} \\ k_n(Y_n - \emptyset_{n-1}\left((Y_1, Y_2..Y_{n-1})\right) \\ + \frac{\partial \emptyset_{n-1}}{\partial Y_{n-1}} \, [f_{n-1}(Y_1, Y_2, ...Y_{n-1}) + G_{n-1} \\ (Y_1, Y_2, ...Y_{n-1})Y_n\} \\ + \frac{\partial \emptyset_{n-1}}{\partial X_1} [f_1(Y_1) + G_1(Y_1) - f_n(Y_1, Y_2, ...Y_n) \end{bmatrix} \quad \text{(e.q., 6.9)}$$

Where $X_1, X_2, \& \ X_3$ and $Y_1, Y_2, \& \ Y_3$ are the states of master and slave systems.

Controller '*U*' is formed by combining the controller received from the master (*U1*) which consists of states of master system and part *U2* which is designed at Slave system consisting of states of Slave system. Then controller 'u' is implemented at slave system to achieve synchronisation between Master-Slave systems. Fractional Order and Parameters of the system must be transmitted to the receiver. The chaotic states obtained after synchronisation y1, y2 and y3 are then mixed in the similar manner at Master's end. So, the vector y, similar to vector x at the master's end, is obtained as,

$$y = Ay_1 + By_2 + Cy_3 \quad \text{(e.q., 6. 10)}$$

key image K2 is generated at the Slave system by following the similar procedure adapted at master's end.

Step 2: By using key image K2, the equivalent of original image J^ can be recovered by performing bitwise XOR operation as shown below.

$$J\hat{} = E' \oplus K2 \quad \text{(e.q., 6.11)}$$

Original image I^' can be obtained by reshuffling J^ in exactly reverse manner as in case of encryption. As explained in sections *E* and *F*, information is transferred securely in the form of either speech signal or image.

6.3 RESULTS AND DISCUSSIONS

For the simulation of both schemes, MATLAB software is used.

A. Results for Secure Communication

The fractional order value considered for the simulation is 0.9. Initial conditions for Master and Slave systems are specified in below table.

The states of Master system are: $x_1 = -0.3, x_2 = 0.1$ & $x_3 = 1.8$ and the slave system are: $y_1 = -0.3, y_2 = 0.1$ & $y_3 = 1.8$ respectively.

The simulation results show that synchronisation errors decays with time and state trajectories of slave system approach state trajectories of master system as shown in figure 6.2. The convergence of synchronisation errors is illustrated in Figure 6.2(a) and Figure 6.2(b)-6.2(d) illustrates synchronisation of corresponding states of Master & Slave system.

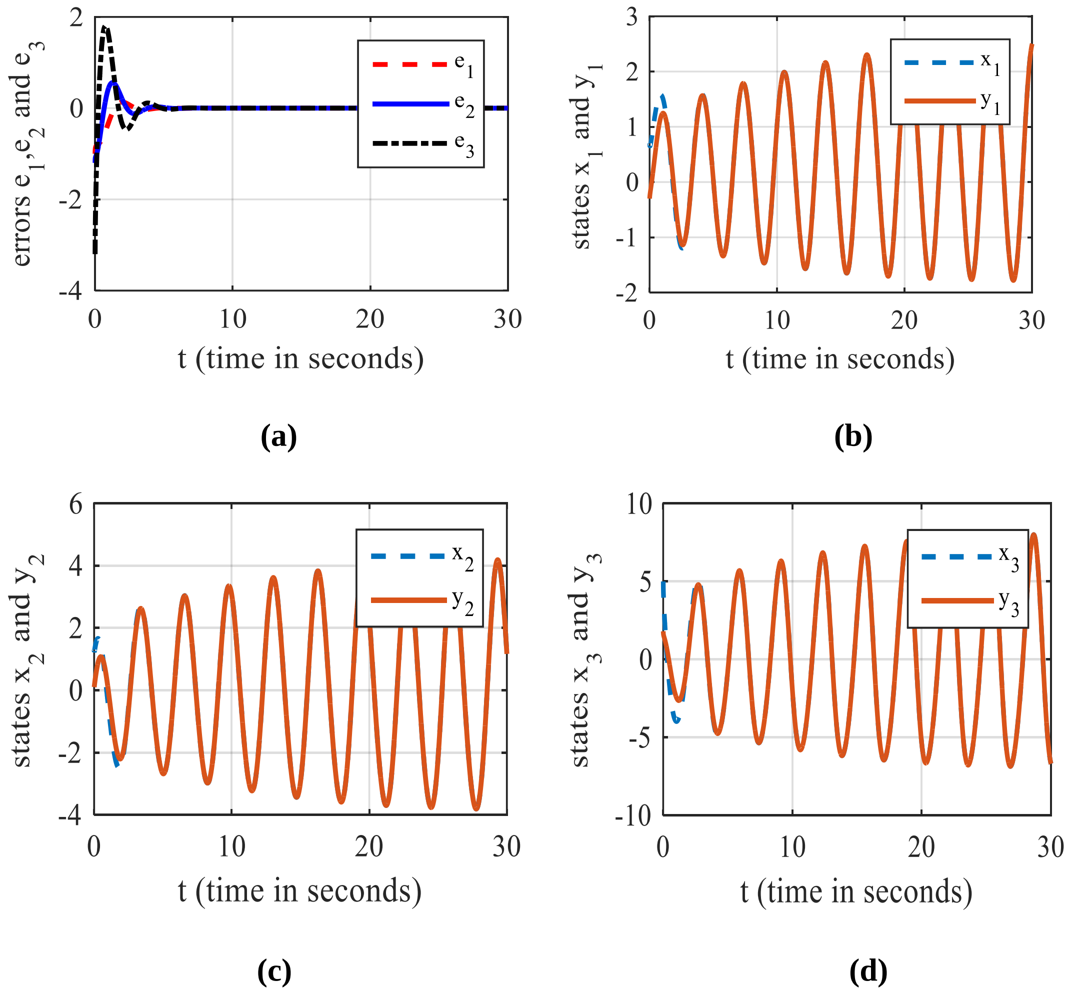
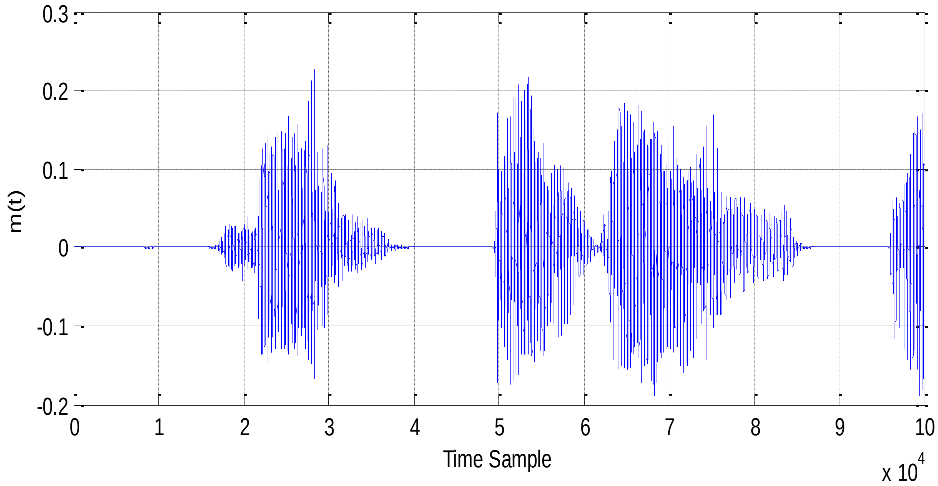
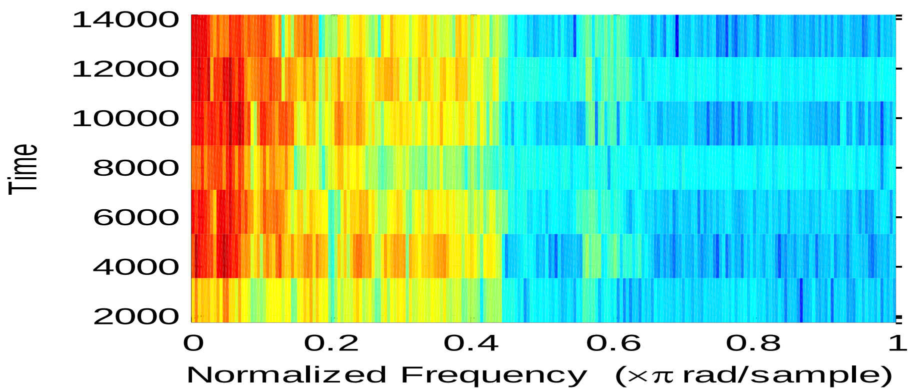


Figure 6. 2: (a) Error Convergence (b)-(d) Synchronisation of states x_1- y_1, x_2- $y_2$and x_3- y_3.

Figure 6.3(a) & 6.3(b) shows the speech signal and its spectrogram. Masked signal s(t) and its spectrogram are shown in figure 6.4. Figure 6.5 presents, retrieved message signal and spectrogram of retrieved signals. Spectrograms of signals illustrate that, it is difficult to decrypt the speech signal from communication channel**.**

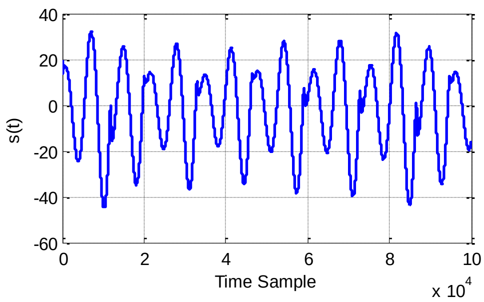


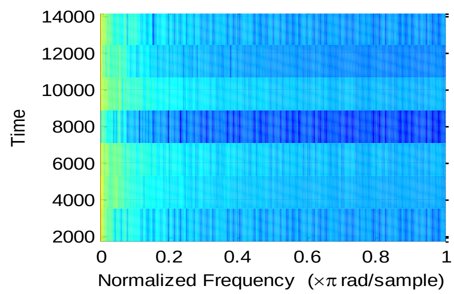
(a)



(b)

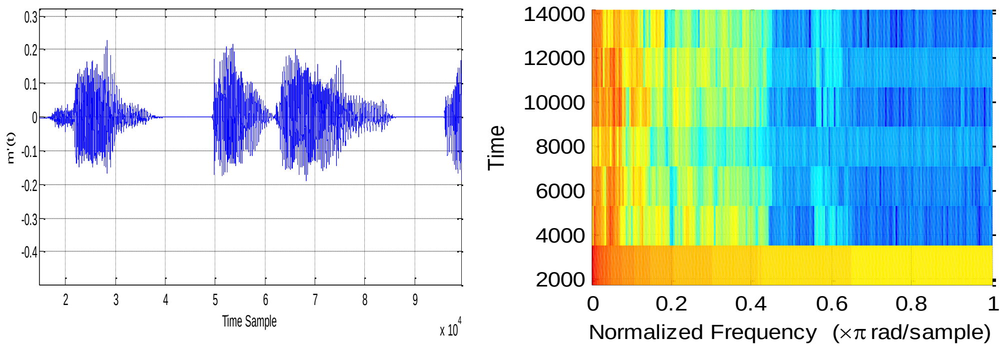
Figure 6.3: (a) Speech signal m(t) (b) it's Spectrogram





(a) **(b)**

Figure 6.4: (a) Encrypted version of $s(t)$ and (b) it's Spectrogram



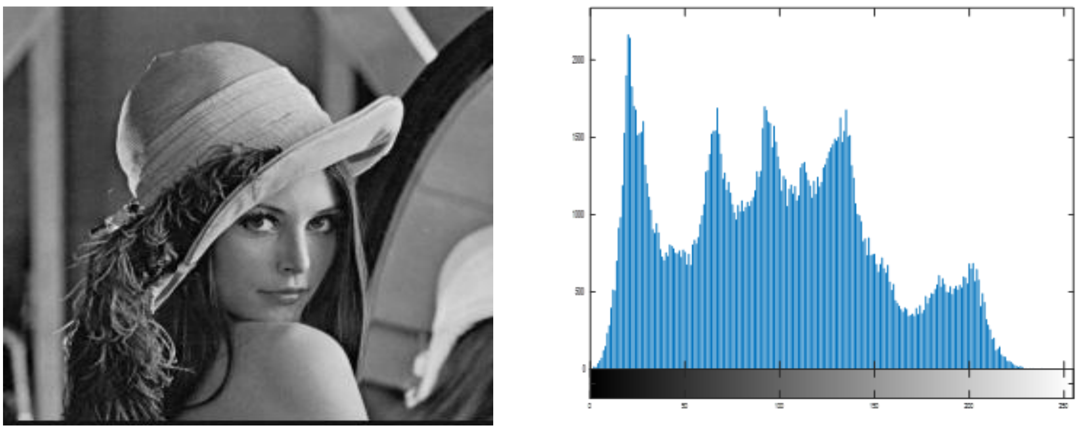
(a) **(b)**

Figure 6.5: (a) Retrieved speech signal $m'(t)$ (b) Spectrogram of $m'(t)$.

B. Resuls for Image Encryption

Encryption scheme is applied on the image shown in figure 6.6. With a step size of 0.001 simulations is carried for 100 seconds to synchronize systems. Synchronisation has been achieved after 4 seconds as shown in figure 6.2. The shuffled version of original image and it's histogram are presented in Figure 6.7(a) and 6.7(b). Encrypted image and its histogram are shown in figure 6.8. The image and histogram following encryption are displayed in figure 6.9.

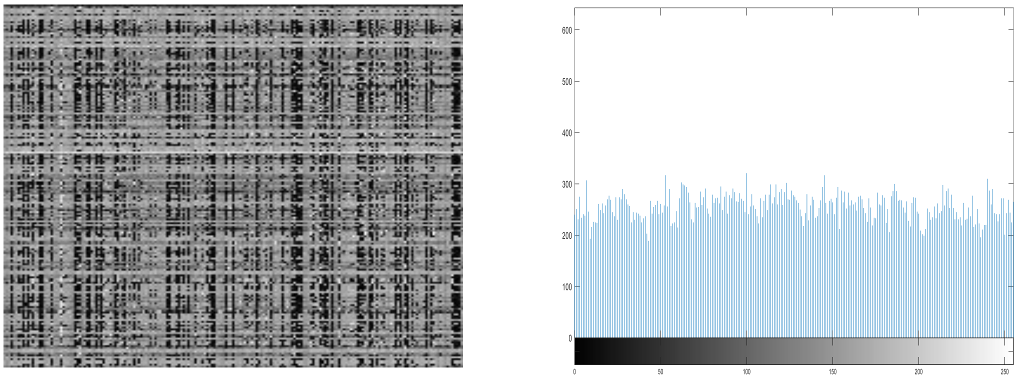
The retrieved image and its histogram are displayed in figure 6.8 after employing suggested decryption scheme.



(a) **(b)**

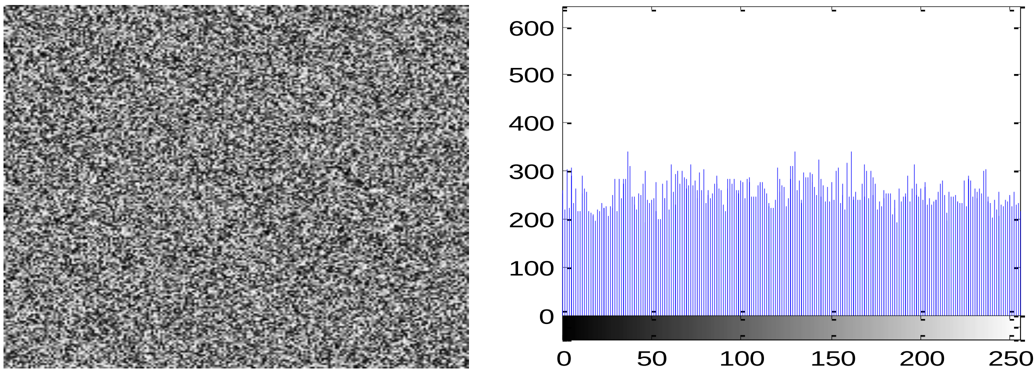
Figure 6.6: (a) Original image (b) it's Histogram

The proposed method improves the parameters like the mean square error(MSE), peak signal to noise ratio(PSNR), correlation coefficient(CC) and entropy (E)values and are calculated and compared with the applications using backstepping controller



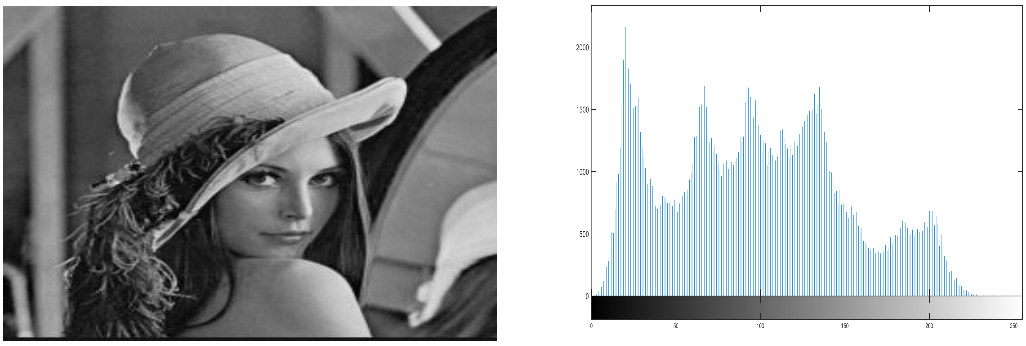
(a) **(b)**

Figure 6.7: (a) Schuffled image and (b) it's Histogram.



(a) **(b)**

Figure 6.8: (a) Encrypted image (b) it's Histogram



(a) **(b)**

Figure 6.9: (a) Retrieved image (b) it's Histogram.

The following expression is used to calculate the entropy of information:

$$E(s) = \sum_{i=0}^{2^N-1} P(s_i) log_2(\frac{1}{p(S_i)}) \qquad \text{(e. q., 6.12)}$$

where, $P(s_i)$ represents the probability of appearance of the information s_i. For a gray-scale image having 256 intensity levels, each level is described by 8 bits and hence, for a fully random encrypted image the entropy should be 8.

The CC between the Gray-scale values of two adjacent pixels x, y in an image is given as:

$$r_{xy} = \frac{cov(x,y)}{\sqrt{D(x)}\sqrt{D(y)}} \quad \text{(e.q.,6. 13)}$$

With, $E(x) = \frac{1}{N}\sum_{i=1}^{N} x_i$, $D(x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - E(x))^2$ and

$$cov(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - E(x))(y_i - E(y)) \quad \text{(e. q., 6. 14)}$$

MSE, calculates the difference of retrieved and actual image and is expressed as:

$$MSE = \frac{1}{mn}\sum_{p=0}^{m-1}\sum_{q=0}^{n-1}|X(p,q) - Y(p,q)|^2 \quad \text{(e. q., 6.15)}$$

Where, $Y(p,q)$ is the retrieved image pixel value after decryption at the (p,q) position and $X(p,q)$ is the actual image pixel and.

PSNR is calculated between actual and encrypted image. For good encryption, PSNR value should be low.

$$PSNR = 10\log\left(\frac{V^2}{MSE}\right) \quad \text{(e. q., 6. 16)}$$

Where, V is the maximum intensity level.

All these parameters are compared when Secure communication and Image encryption applications used Backstepping Controller [252] for synchronization in table 6.3.

Table 6.1: Comparison of parameters

Name of the Controller	PSNR	MSE	Entropy	Correlation Coefficient
Backstepping Controller	4.764632	0.31822	7.8658	0.00163
Backstepping with RNN	4.365842	0.22721	7.9126	0.00142

6.4. Summary

In this Chapter, RNN with Backstepping controller is employed to synchronize two non-linear fractional order chaotic systems. This synchronisation scheme is used to transmit information securely and for the encryption of the image. The RNN-based controller shows a lower MSE (0.22721) compared to the conventional method (0.31822), indicating better accuracy and reduced reconstruction error. A slightly higher entropy (7.9126) in the RNN-based approach suggests better randomness and stronger encryption/security characteristics. The RNN-based controller achieves a lower correlation (0.00142), which implies better decorrelation of data, an important factor in secure communication and image encryption. Although PSNR is slightly lower in the RNN-based method, this is acceptable in encryption applications where higher distortion (lower PSNR) enhances security.

Chapter-7

CONCULSION AND FUTURE SCOPE

7.1 Conclusion

This chapter presents the conclusion of the research work which includes three different methodologies for fractional order control systems.

In the first contribution, the performance of four different controllers is analysed to maintain tracking control of a non-linear system. The approximate error for the fuzzy dynamic surface controller, dynamic surface controller, sliding mode and backstepping controllers are compared. The backstepping controller works best, with the lowest error (0.0358) and the fastest response time (1.58 s). The fuzzy dynamic surface controller has a higher error (0.061) and a slower response (3.01 s), but it tracks more smoothly with fewer oscillations. The sliding mode and dynamic surface controllers work okay, with errors of about 0.05–0.06 and response times of about 2.35 s, but they have more overshoot. However, when comparing the error factor based on the virtual control signal, the backstepping controller reduced the error to very small.

In the second contribution, adaptively coupled RNN with Backstepping controller is proposed for a non-linear strict feedback system. The performance of the Backstepping controller is improved by integrating Backstepping controller with RNN. This proposed controlling method is applied in genesio-tesi system and implemented in the MATLAB tool to validate the proposed method. Results from the simulation prove that the proposed control technique works effectively.

In the third contribution, the performance of back stepping controller with RNN \was compared with various optimized controllers. The analysis indicated that the backstepping controller with RNN was acquired better performance than the other optimized controllers. When compared to other optimized controllers, the back stepping controller with RNN achieves a low error value, thereby enhancing the system's performance. The results indicate that RNN-BSC achieves the most effective performance, with the highest stability (86), fastest convergence time (1.38 s), and minimum tracking error (4.84 s). The ANN-BSC method also demonstrates satisfactory performance, showing good stability (82) and relatively quick convergence (2.15 s). On the other hand, NFAD-BSC, NFABSM, RLEBC, and ESN take longer to converge (more than 5.5 seconds) and have larger tracking errors (more than 7.5 seconds), which means they are less efficient at tracking.

In the final contribution, Backstepping with RNN controller is utilized for synchronizing two chaotic FONL systems. This synchronization scheme is used to transmit information securely and for the encryption of the image. The method applied here is different from existing techniques, to synchronize transmitter and receiver systems. It is very difficult for the intruder for decoding the signal as the control signal is very complex in nature. In this application, when compared with the conventional backstepping controller, the RNN-based backstepping method shows noticeable improvement. It produces a lower reconstruction error (MSE reduced from 0.31822 to 0.22721) and a smaller correlation coefficient (0.00142), which means it does a better job of decorrelating. The small rise in entropy (7.9126) suggests better randomness and security, even though the PSNR value is a little lower.

Limitation of the work: Practical issues such as channel noise, packet loss, synchronization mismatch, and delay are not explicitly modeled, which may affect decryption accuracy in real communication systems.

7.2. Future Scope

This research work could be extended to Discrete FONL systems. Chaos synchronization of incommensurate systems could also be used for secure communication which will offer better security. In the future, this work will be utilized in Image Encryption and Secure Communication applications using hybrid controllers and also the proposed work can be tested under noisy channels.

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List of Publications:

1. Narmada, A., et al. "Evaluation of various controllers in fractional order non-linear systems with actuator fault." Multimedia Tools and Applications 83.24 (2024): 64945-64962.

List of Conferences:

1. Narmada, A., Anuj Jain, and Manoj Kumar Shukla. "Recurrent Neural Network Based Backstepping Controller for Genesio-Tesi Chaotic System." 2024 Sixth International Conference on Computational Intelligence and Communication Technologies (CCICT). IEEE, 2024.
2. Narmada, A., Anuj Jain, and Manoj Kumar Shukla. "Recurrent Neural Network with Backstepping controller based Fractional-Order Chaotic System based Solution for Secure Communication and Image Encryption." 2024 International Conference on IoT Based Control Networks and Intelligent Systems (ICICNIS). IEEE, 2024.
3. Narmada, A., Anuj Jain, and Manoj Kumar Shukla. "RNN Based Optimized Backstepping Controller of Fractional Order Nonlinear System." 2025 International Conference on Computational, Communication and Information Technology (ICCCIT). IEEE, 2025.