STRONG SELF-FOCUSING OF q-GAUSSIAN LASER BEAM IN UNDERDENSE PLASMA

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By

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LOVELY PROFESSIONAL UNIVERSITY, PUNJAB

2024

DECLARATION

I declare that the thesis entitled, Strong self-focusing of q-Gaussian laser beam in underdense plasma, has been prepared by me under the supervision of Prof. Vishal Thakur, department of Physics, Lovely Professional University, Punjab, India. No part of the thesis has formed the basis for the award of any degree or fellowship previously.

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THESIS CERTIFICATE

This is to certify that the thesis titled “Strong self-focusing of q-Gaussian laser beam in underdense plasma”, submitted by Anees Akber Butt (11919628), to the Lovely Professional University, Punjab, for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work done by him under my suppression. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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Abstract

Introduction: Self-focusing of laser beams in plasmas has gained significant importance in recent years due to its relevance in various applications, including fast ignitor laser-driven fusion, laser-driven electron acceleration, and X-ray lasers. In the relativistic regime, where laser powers approach the speed of light and plasma permittivity shows nonlinear behavior, self-focusing becomes a critical phenomenon to understand. The historical survey of self-focusing traces back to Soviet scientists in the 1950s and 1960s, who analyzed the motion of plasma electrons under intense electromagnetic radiation and predicted the possibility of self-focusing due to changes in plasma dielectric constant produced by the beam itself. This discovery sparked worldwide interest and triggered extensive research in this field. Various theoretical techniques were developed to study self-focusing in nonlinear media, and paraxial ray approximation emerged as the simplest and most popular method.

Chapter 2: Literature Review This chapter provides a comprehensive review of previous studies and contributions related to self-focusing of laser beams in plasma. It discusses the pioneering work of Akhiezer, Polovin, Askar'yan, and Akhmanov, which laid the foundation for understanding the phenomenon of self-focusing. The various theoretical techniques developed, including moments theory, variational method, and method of inverse scattering transforms, are also explored. The review emphasizes the significance of self-focusing in laser-driven accelerators, X-ray sources, plasma lenses, and laser-induced fusion. The chapter concludes with a discussion of previous research conducted by Sodha et al., where self-focusing of laser beams in plasma was studied under different conditions.

Chapter 3: Self-Focusing of q-Gaussian Laser Beams with Exponential Plasma Density Ramp This chapter investigates the self-focusing behavior of q-Gaussian laser beams (q-GLB) in plasma with the influence of an exponential plasma density ramp. The q-GLB exhibits oscillatory self-focusing and defocusing behavior during propagation. The introduction of an exponential plasma density ramp effectively counters the defocusing tendency, enabling the q-GLB to achieve a minimal spot size and retain it up to several Rayleigh lengths. The chapter explores how self-focusing increases with increasing q-values and laser intensity, with lower q-values leading to stronger self-focusing. Numerical simulations provide valuable insights into the dynamics of self-focusing and harmonic generation in the presence of the exponential plasma density ramp.

Chapter 4: Self-Focusing of q-Gaussian Laser Beams with Exponential Plasma Density Ramp and Wiggler Magnetic Fields This chapter delves into the fascinating behavior of q-Gaussian laser beams in underdense plasma, considering the combined influence of two key parameters:

the exponential plasma density ramp and the wiggler magnetic field. The results reveal a remarkable synergy between these factors, leading to improved self-focusing and enhanced beam quality. The wiggler magnetic field generates a periodic variation in the magnetic field, leading to high-frequency radiation and influencing the self-focusing behavior of the laser pulse. The interplay between the plasma density ramp and the wiggler magnetic field produces fascinating effects, providing a guiding channel for the laser beam and preventing its spreading due to diffraction.

Chapter 5: Self-Focusing of q-Gaussian Laser Beams with Exponential Plasma Density Ramp and Axial Magnetic Field This chapter explores the mutual effect of an exponential plasma density ramp and an axial magnetic field on the self-focusing behavior of q-Gaussian laser beams in underdense plasma. The axial magnetization direction is found to be crucial in controlling and manipulating self-focusing dynamics. A forward axial magnetic field enhances self-focusing due to the relativistic self-focusing effect, while a reverse magnetic field weakens it due to the magnetic self-channelling instability. The research highlights the importance of the axial magnetization direction in achieving precise and efficient self-focusing, with significant implications for laser-plasma interactions.

Conclusion: The comprehensive study of self-focusing of q-Gaussian laser beams in underdense plasma with the influence of exponential plasma density ramps and axial magnetic fields opens up exciting new avenues for laser technology and high-energy physics research. The findings demonstrate the potential benefits of using exponential plasma density ramps to enhance self-focusing and optimize laser performance, leading to groundbreaking advancements in laser applications across diverse fields. The interplay between plasma density ramps, axial magnetic fields, and q-parameters holds promise for precise beam control and efficient energy transfer, making this research highly valuable for various laser-based applications.

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With utmost honour, I acknowledge the profound impact of the teachings of Prophet Muhammad (peace be upon him) on our journey, guiding us unwaveringly towards the noble pursuit of education. As proud member of his ummah, I draw strength and inspiration from his wisdom to excel in the realm of learning.

And finally, praise be to Almighty Allah, I bow in profound thankfulness for the countless blessings. Alhamdulillah-i-Rabil Alamein! my heart is brimming with gratitude.

Preface

In this study, we delve into the phenomenon of strong self-focusing exhibited by a q-Gaussian laser beam within an underdense plasma. Our research investigates the impact of multiple factors such as exponential plasma density ramp, wiggler magnetic field and axial magnetic field on the self-focusing behavior of q-Gaussian laser beam within the plasma medium.

The self-focusing of a laser beam within a nonlinear medium such as plasma has garnered significant attention from scientists and researchers. This intrigue arises from the remarkable concentration of energy that occurs at the focal point of the converged beam. Our investigation focuses on a q-Gaussian laser beam, probing the depths of the plasma medium to unlock the self-focusing phenomena. This process holds exceptional promise in diverse applications including laser-driven accelerators, laser-driven fusion and x-ray lasers, all of which demand copious amounts of high-energy output. Thus, the principle of self-focusing emerges as a pivotal asset in catering to these energy-intensive applications.

Central to our study is the aspiration to augment the self-focusing effect through the strategic integration of external influencers. By incorporating an exponential plasma density ramp, wiggler magnetic field, axial magnetic field and meticulously selecting laser and plasma parameters, we strategically amplify the self-focusing phenomenon. The outcome is a noteworthy enhancement in the self-focusing behavior of the laser beam, a result that we meticulously document and present in this study.

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CONTENTS

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ABBREVIATIONS AND SYMBOLS

- EM wave: Electromagnetic wave
- THz: Terahertz
- S.F: Self-Focusing
- fs: Femtosecond
- L.B: Laser Beam
- μm: Micrometer
- WKB: Wentzel-Kramers-Brillouin (approximation)
- E: Electric field
- B: Magnetic field
- c: Speed of light in vacuum
- k: Wave-vector
- m: Mass of electrons
- ω: Angular frequency
- ω_p: Angular plasma frequency
- ε: Permittivity
- GLB: Gaussian Laser Beam
- λ: Wavelength

Chapter-1

INTRODUCTION

1.1 Historical Context

Self-Focusing (S.F) is an intriguing phenomenon that occurs when electromagnetic waves travel through plasma. This fascinating discovery emerged from the extensive research conducted by Soviet scientists. In 1956, Akhiezer and Polovin[1,2], delved into the movement of plasma electrons when exposed to intense electromagnetic radiation, which led to the remarkable finding of a nonlinear relativistic contribution to the effective dielectric constant of plasma. Building on this foundation, Askar'yan predicted in 1962 [3], the potential of electromagnetic beam focusing, which could result from changes in the plasma's dielectric constant caused by the beam itself. Akhmanov et al.[4,5] then analysed the characteristics of electromagnetic wave propagation through a medium with nonlinear dielectric constant, publishing their findings in 1966 and 1968. Their discoveries provide fundamental insights into S.F and have significant implications for the field of plasma physics. The authors behind this remarkable discovery utilized the WKB and paraxial approximations to solve the wave equation in a nonlinear medium. Their study showed that under specific conditions, the radiation beam could be focused, with the brightness of the light beam on the direction along axis increasing exponentially. Thus, significance of this process was immediately recognized in the field of laser-induced fusion reactions, which sparked widespread interest in this research area worldwide. Additionally, the availability of high-intensity laser beam (L.B) played a crucial role in triggering further scientific exploration in this field. Subsequently, several theoretical techniques were developed to study the focusing of beams in nonlinear media. These included the moments theory of beam S.F formulated by Vlasov et al. [6], which was later advanced by Lam et al. [7], Furth's variational method was another approach [8], as was the method of inverse scattering transforms, developed by Zakharov and Shabat [9,10]. These sophisticated techniques involve complex mathematical analyses but have contributed immensely to our understanding of S.F and its potential applications. The paraxial ray approximation, developed by Akhamanov et al. [11] , is a popular and straightforward technique used to investigate S.F. Extensive research has been conducted on electromagnetic wave propagation through various media, such as dielectrics, gaseous plasmas and semiconductors, using this technique. Sodha and his team employed this approximation to conduct in-depth studies and published their findings in a book and a review in 1974 and 1976, respectively [12]. The phenomenon of S.F is critical to various applications, such as harmonic generation, laser-driven fusion[13,14], X-ray sources[15], and plasma lenses[16].Additionally, it has significant implications for studying related phenomena such as parametric excitation and instabilities[17], the generation of strong magnetic fields[18,19]and laser-induced fusion[20]. These discoveries have provided new avenues for further research, contributing significantly to the advancement of plasma physics. S.F of L.Bs in plasma has been extensively studied and key contributions to the field have been reviewed by Esarey et al. [21] and Umstadter [22]. Sodha and colleagues[23–28]have conducted research on S.F under varying conditions and found that a specific relation among the starting beam strength and its width is necessary for this laser to propagate without convergence or divergence in a uniform wave guide mode. The relationship can be represented as a curved line on a graph that shows how the beam power and beam width are related, with a minimum for the beam width. Sodha's work has revealed that the beam can undergo oscillatory convergence (S.F), oscillatory divergence, or steady divergence at a certain beam width larger than the minimum for different beam powers. However, the propagation of beam with beam width less than the minimum value was not extensively explored. These studies have contributed significantly to our understanding of S.F phenomena, with implications for various applications, including laser-driven accelerators, X-ray sources, plasma lenses, and the generation of strong magnetic fields.

1.2 Laser Characteristics

A laser is a device that produces coherent radiation with a high directivity. Currently one can obtain power densities ~ $10^{20} W/cm^2$ using Nd: glass and Ti: Sapphire lasers with pulse durations as short as a few laser periods. The transverse modes of a laser are the spatial patterns of light that the laser produces. They are denoted by TEMmn, where m and n are whole numbers. The TEM stands for "transverse electromagnetic," which means that the modes are characterized by their electric and magnetic fields in the plane perpendicular to the direction of propagation of the light. The fundamental mode or TEM_{00} mode is the simplest transverse mode. It has a Gaussian field distribution, which means that the intensity of the light decreases as you move away from the center of the beam. Higher-order transverse modes have more complex field distributions. They can have multiple lobes of intensity and their shapes can be described by Hermite-Gaussian polynomials. The type of transverse mode that a laser produces depends on the shape of the laser cavity and the wavelength of the light. In general, lasers with larger cavities can support more transverse modes.

The beam intensity profile of this primary mode, which is commonly used, is typically represented as follows:

$$I(r) = I(0)exp\left(-\frac{r^2}{r_0^2}\right) \tag{1.1}$$

where $I(r)$is the intensity at r while the parameter $r_0$determines the width of this beam and thus intensity falls to $1/e^{th}$ of the same on the axis at $r = r_0$.The parameter r_0 represents the length with respect to the center up to the limit where the amplitude of the field reduces by a certain factor i.e., $\sqrt{1/\mathrm{e}}$. Unlike other modes, the Gaussian TEM_{00} mode doesn't have any points where the electric field goes to zero. In contrast, the TEM_{10} mode has a zero point along the x-axis, while the TEM_{11} mode has zero points along both the axis. Due to its straightforward spatial profile and absence of zero points in the beam intensity distribution, the Gaussian mode is usually preferred in most research studies.

I. **Gaussian L.B (GLB):** A GLB is a L.B that exhibits a Gaussian-shaped intensity profile. It is the most commonly used type of L.B and is characterized by a bell-shaped intensity distribution in the transverse direction. The intensity of the beam decreases rapidly as one move away from the center of the beam.

II. **Super gaussian L.B:** A super GLB is a light beam that has a flatter intensity profile than the simple Gaussian L.B. It is distinguished by steeper fall in intensity as one move away from the center of the beam. The order of a super Gaussian L.B may be changed to influence its profile.

III. **q-Gaussian L.B:** A q-GLB is a special light beam which possesses a non-Gaussian intensity profile. It is characterized by a power-law behavior of the intensity distribution, which is controlled by a parameter$q$. The value of q determines the degree of non-Gaussianity of the beam, with $q = 1$ corresponding to a Gaussian beam. q-Gaussian beams find applications in fields such as Optical communications, Material processing, optical trapping and nonlinear optics. The q-Gaussian beam profile has a flatter top and steeper sides than a Gaussian beam leading to different propagation and S.F properties in the plasma medium. The self-trapping of a q-GLB in plasma can occur when the laser pulse intensity is high enough to create a plasma channel and the plasma density profile is suitable for S.F to occur. As q-Gaussian beams are special with respect to other beam profiles like other gaussian profiles due to their non-Gaussian nature, which gives them some unique properties and applications. Here are a few reasons in support that q-Gaussian beams are special:

Flexibility: Non-Gaussian (q-Gaussian) beams offer greater flexibility in controlling the shape of the beam profile, which can be adjusted by changing the value of q. This makes them suitable for a diverse variety of uses, including photonic communication, optical manipulation, and quantum optics.

Nonlinearity: Since these q-Gaussian beams exhibit strong nonlinearity in certain regimes, which can result in interesting nonlinear optical features including S.F and self-trapping. This makes them useful for applications such as nonlinear optics and laser-plasma interactions.

Resilience: Compared to Gaussian beams, q-G.L.B are more resistant to beam aberrations such turbulence or air scattering. This makes them ideal for applications that require long-distance propagation or beam shaping in turbulent environments.

Information Theory: It has been demonstrated that q-Gaussian distributions are connected to information theory and statistical mechanics, which has led to their use in industries including image processing, signal processing and encryption. Overall, q-Gaussian beams offer unique properties and applications that make them special compared to other beams.

1.3 Plasma Characteristics

It is frequently stated that plasma is the fourth state, or supplementary one, after solids, liquids, and gases. It is a highly ionized gas consisting of charged particles including electrons and ions that exhibit collective behavior because of their interactions with each other and with external electromagnetic fields. One of the most important characteristics of plasmas is their ability to conduct electricity. This is because free electrons are present, which can move freely through the plasma and carry an electric current. Plasmas are also highly reactive and may be utilised to promote chemical reactions that would be difficult or impossible otherwise, to achieve in other states of matter. Another characteristic of plasmas is their response to electromagnetic fields. Due to the presence of charged particles, plasmas can be influenced by electric and magnetic fields in complex ways. For example, plasmas can be confined and controlled using magnetic fields, which is the principle behind magnetic confinement fusion, a promising approach to sustainable energy production. Plasmas are ubiquitous in the universe and are found in many natural and man-made environments. Examples include lightning, auroras, stars and plasma televisions. The study of plasmas is interdisciplinary; drawing upon physics, chemistry, engineering, and many other fields. Mathematically, plasma can be described using the equations of plasma physics which include the Maxwell equations, the equations of fluid dynamics and the equations of motion for charged particles. These equations can be employed to predict and clarify the behaviour of plasma in a variety of physical systems by taking into consideration the intricate interactions between charged particles, neutrals and electric and magnetic fields. One important equation in plasma physics is the continuity equation, which describes the conservation of mass in plasma. This equation takes the form:

$$\partial\rho/\partial t + \nabla \cdot (\rho v) = 0 \tag{1.2}$$

where, ρ denotes the plasma density,v denotes the velocity and ∇ is the gradient operator. This relation states that differentiation of the density of plasma with time is equal to the divergence of the plasma velocity, which represents the flow of plasma into or out of a given region.

Another key equation is the equation of motion for charged particles, which takes into account the effects of electric and magnetic fields on particle trajectories. For a particle with charge q and mass m, this equation can be written as:

$$m(dv/dt) = q(E + v \times B) \quad (1.3)$$

where, v is the velocity, E is used to denote electric field and B shows the magnetic field. This equation describes the forces acting on a charged particle due to electric and magnetic fields and can be used to understand phenomena like plasma confinement in magnetic fields and the generation of electric fields in plasma waves. Overall, the mathematical equations of plasma physics give a strong weapon for understanding and predicting the behavior of plasma in a wide range of physical systems, from laboratory experiments to astrophysical phenomena.

Plasma is a state of matter that consists of charged and neutral particles, exhibiting collective behavior. In plasma, the ions are generally much heavier than electrons, which mean that they have a relatively small effect on the transmission of higher frequencies of electromagnetic waves in the plasma. However, they can screen space charge fields that arise due to the displacement of electrons. The Debye length is a characteristic screening length of the charged particle potential in plasma.

$$\lambda_D = [\varepsilon_0 T_e / e^2 n_0]^{1/2} \quad (1.4)$$

and a frequency of natural oscillation, called Plasma frequency.

$$\omega_p = (n_0 e^2 / \varepsilon_0 m_0)^{1/2} \quad (1.5)$$

where, m_0 is the electron rest mass. In brief we can say that an ionized gas must satisfy the following three conditions in order to qualify for plasma. To realize the quasi-neutral characteristic of plasma, the dimension of plasma should be greater than the Debye length (λ_D). To accomplish the collective behavior, the charge particle number in the Debye sphere should be very large.

The frequency associated with the characteristic oscillation of plasma should be greater than the collision frequency among the constituent particles. In general, there are two modes of propagation of waves in plasma. First mode is longitudinal wave, known as Langmuir wave. Here the electric field is parallel to the direction of propagation of the wave such that $\vec{k} \times \vec{E} = 0$. Second mode is transverse wave, such as electromagnetic wave, in which the electric field of the wave is perpendicular to the direction of propagation ($\vec{k}.\vec{E} = 0$). The longitudinal oscillations of the electron fluid (n as electron number density) have a single unique frequency called plasma frequency$\omega_p = \sqrt{ne^2/m_e\epsilon_0}$. Magnetic field plays vital role in plasma dynamics as constituent charged particles are prone to the magnetic field due to the Lorentz force. The magnetic field alters the motion of the charged particles in the plasma which leads to the modification of dispersion relation of the Langmuir wave and electromagnetic wave. The presence of the external magnetic field leads to different eigen modes: circularly polarized and extraordinary wave. The general approach to electromagnetic wave in magnetized plasma is started from Maxwell equations

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + 1/c^2\, \partial\vec{E}/\partial t \tag{1.5a}$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \tag{1.5b}$$

The response of the medium is explicit in the current density $\vec{J}$.

1.4 The effective dielectric constant

In materials that exhibit isotropic properties, there is a correlation among the electric displacement vector D and the field intensity vector, E, which can be expressed by a specific equation as given below

$$\vec{D} = \epsilon \vec{E} \tag{1.5c}$$

where, $\in$ is the dielectric constant. If plasma is considered, the Maxwell Equations can be simplified to be the same as those in vacuum if the dielectric constant is replaced with an effective dielectric constant that depends on the electric field intensity. The effective dielectric constant is mathematically represented as:

$$\epsilon_{eff} = \epsilon_0 + \phi(EE^*) \tag{1.6}$$

$$\epsilon_0 = 1 - \left(\omega_p^2/\omega^2\right) \tag{1.7}$$

ω_p represents the plasma frequency and ω shows the frequency of laser. Several different physical processes contribute to the function $\phi(EE^*)$ and have been analyzed extensively by Sodha et al.[24,25] . These mechanisms are briefly described and the final expressions quoted below:

(i) Collisionless Nonlinearity (Due to Ponderomotive forces)

As an electromagnetic wave moves through plasma, the electrons in the plasma undergo a perpendicular variation in the Lorentz force. This force is referred to as the Ponderomotive force and it attempts to push the electrons out of the region of higher field strength. This results in a redistribution of electron density and creates a gradient in the density. The dielectric constant of the plasma is affected by this phenomenon, with the contribution being dependent on the electric field intensity and denoted asϕ,

$$\phi(EE^*) = \frac{\omega_p^2}{\omega^2}\{1 - exp\left(-\frac{3}{4}\alpha\frac{m}{M}EE^*\right)\} \tag{1.8}$$

where, $\alpha = \frac{e^2 M}{6K_B T_0 m_0^2 \omega}$, M is the mass of the heavier particle (neutral or ions) and T_0 the temperature of the plasma.

(ii) Collisional (Ohmic heating) nonlinearity

The nonlinearity which arises in the effective dielectric constant of nonlinear medium such as plasma is a result of the decay of laser energy, which causes heating of the plasma through collisions with electrons, ions, and neutrals. The nature and frequency of these collisions depend on factors such as the plasma density, neutral species density, degree of ionization and thermal speed of the electrons. As and when the time duration of the pulse of the electromagnetic beam is significantly greater than the energy relaxation time of the electrons, thermal nonlinearity becomes dominant, leading to a change in the effective dielectric constant.

$$\epsilon = \epsilon_0 + \phi(EE^*) - i\epsilon_i \tag{1.9}$$

$$\phi(EE^*) = \frac{\omega_p^2}{\omega^2} - \frac{1}{2}(2 + \alpha EE^*)^{\frac{s}{2}-1} \tag{1.10}$$

$$\epsilon_i = \frac{4}{3\sqrt{(\pi)}}\Gamma\left(\frac{5+s}{2}\frac{\omega_p^2}{\omega^2}\right)\frac{v_0}{\omega}(1 + \alpha EE^*) \tag{1.11}$$

M shows particle mass in plasma, k_B is obviously the Boltzmann's constant, plasma temperature(T), ν = $\nu_0(mv^2/2kBT)\frac{s}{2}$ is the collision frequency of electron and v is the random electron speed. The parameter s characterizes the nature of collisions. It has values 0, 1, 2 and -3 corresponding respectively to the cases (i) the collision frequency is velocity independent, (ii) collisions are between electrons and atoms, (iii) the collisions between diatomic molecules and negatively charged particles (electrons) and (iv) the collisions are between electrons and ions.

(iii) Relativistic Nonlinearity

The propagation of high intensity Gaussian L.B through plasma results in the quiver speed of the subatomic particles like electrons of the order of the speed of electromagnetic wave in vacuum; as a consequence, mass of electrons of rest mass m_0 increases to m. This redefines the dielectric constant of the medium as:

$$\epsilon = 1 - \left(\frac{\omega_p^2}{\omega^2}\right) \tag{1.12}$$

$$\omega_p = \omega_{p0}\gamma \tag{1.13}$$

and $$\gamma = (m_0/m) = \sqrt{\left(1 - (v^2/c^2)\right)} \tag{1.14}$$

where v, is the root mean square of time varying quiver speed of the electrons. The parameter γ can be derived from the equations of motion in terms of modified linear energy and momentum of the electron; the final expression for the resulting dielectric constant corresponding to a circularly polarized beam turns out to be

$$\epsilon = \epsilon_0 + \phi(EE^*) \tag{1.15}$$

$$\phi(EE^{*}) = \frac{\omega_p^2}{\omega^2}\{1 - \left(1 + \frac{1}{2}\alpha EE^{*}\right)^{-\frac{1}{2}}\} \tag{1.16}$$

$$\alpha = (e/m_0\omega c)^2 \tag{1.17}$$

These nonlinearities are in general not effective at the same time. This can easily be explained in terms of the various time scales involved. Let τ be the time length of laser pulse, $\tau_s = r_0/c_s$ the time required for the sound to traverse the beam width (c_s the ion sound speed) and τ_H the time for thermal effects to get manifested. The relativistic nonlinearity sets instantaneously but needs intense laser fields so as to produce significant change in the electron mass. Hence for intense light if $\tau < \tau s, \tau H$ there will be only relativistic contribution to the nonlinear dielectric constant. For $\tau > \tau s, \tau_H$ the other nonlinearities will be effective and if the laser is not intense enough the relativistic component will be absent.

1.5 Basic Physics of S.F

When a GLB passes through gaseous plasma, its intensity distribution in the transverse plane can be expressed using a specific equation (1.1). To study the beam's propagation through the plasma, a cylindrical polar coordinate system with the z-axis aligned with the beam's direction is used. The plasma consists of electrons, ions and neutral particles that interact with each other as the electromagnetic wave propagates through them. Momentum is nearly totally randomized in an idealized collision and the temperature of the electrons is precisely the same as that of the massive particles in stability. However, when an electric field is present, electrons gain energy and reach a greater temperature in the steady state, resulting in a power balance between the energy obtained and the energy lost in collisions. [29]. If the applied field has a non-uniform intensity distribution, the electron temperature becomes non-uniform, leading to a corresponding variation in their concentration. The resulting space charge field causes ions to follow the electrons, and a transverse gradient of effective dielectric constant and index of refraction of plasma is established as a result of the redistribution of carriers. Hence the refractive index away from the axis becomes lower than that along the axis. Therefore, at the axis ($r = 0$) the phase velocity of the beam is minimum and increases with increasing r. The wave front of the L.B, which is planar at $z = 0$, becomes curved and tends to converge as a result of varying phase velocities at various r values. S.F is a phrase used to describe this convergence or focusing since it results from a change in the dielectric constant brought on by the monochromatic light beam itself. A process known as beam filamentation instability causes an electromagnetic (EM) beam with random fluctuations in intensity throughout its journey to additionally concentrate into lens-like regions[9],[30,31]. To focus the beam, the nonlinear effects have to overcome natural diffraction. There is thus a competition between nonlinear convergence and diffraction divergence; as a result, the process of S.F is highly reliant on beam power. The possibility of diffraction-free beam propagation arises as the power of the beam is just critical, because nonlinear S.F and natural diffraction cancel one another out. Propagation of the beam under this condition is called 'Uniform Waveguide Propagation'. When nonlinear changes in the dielectric constant become the dominant factor, a L.B can undergo S.F, causing it to converge and oscillate in width between its initial value and its narrowest point. If the influence of diffraction divergence surpasses the S.F effect, the beam will experience either oscillatory or steady divergence, depending on its initial power. In the case of oscillatory defocusing, the beam width fluctuates between its initial value and its maximum width as it propagates through the plasma. On the other hand, in steady divergence, the beam width continuously increases as it travels through the plasma.

1.6 Objectives of the Proposed Work

Based on the interest and need of S.F, the following are the objectives for the work done in this thesis.

1. Self-Focusing of q-Gaussian Laser Beam in plasma under the influence of exponential plasma density ramp.

2. Self-Focusing of q-Gaussian Laser Beam in plasma under the influence of Wiggler magnetic field.

3. Self-Focusing of q-Gaussian Laser Beam in plasma under the influence of axial magnetic field.

1.7 Thesis Outline

The study on the strong S.F of q-GLB in underdense plasma is the subject of this thesis. This research was split into analytical, numerical and simulations-based investigations. The work is organized as follows: Chapter 2 is the review of literature that introduces the underlying theory required to investigate the events presented in this thesis. This provides a brief overview of the relevant plasma physics, such as laser-plasma interaction, plasma density ramp, wiggler magnetic field and the work done till date in this field. Chapter 3 presents the results of strong S.F of q-GLB in underdense plasma under the influence of exponential plasma density ramp. This includes analytically working on wave equation with WKB approximation and then using paraxial ray approximation for obtaining beam width parameter equation. Then final simulations are done. Work related to this study has been published in SCIE journal namely Phycica Scripta, in March2023. Chapter 4 shows the combined effect on S.F of q-Gaussian laser due to the presence of density ramp and wiggler magnetic in underdense plasma. This includes solving nonlinear equation analytically and doing the mathematical analysis to achieve the equation required for beam width parameter. The author has given an oral presentation related to this work in International Conference in 2022. This chapter forms the basis of a manuscript which has got acceptance from the journal of optics (springer). Chapter 5 shows the effect on S.F of q-GLB due to the presence of density ramp in axially magnetized plasma. The author has given an oral presentation related to this work in International Conference Recent advances in fundamental and applied sciences (RAFAS-2023) hosted by school of chemical engineering and physical sciences, lovely professional university. Finally, Chapter 6 summarises the work provided in this thesis, as well as the implications that may be taken from it and future research intentions on the subject.

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CHAPTER-2

2.1 LITERATURE REVIEW

The advent of exuberant, high-intensity laser systems has ignited a profound curiosity among scholars, prompting them to delve into the captivating realm of their interplay with plasmas and semiconductors. This enthralling convergence has yielded a plethora of applications, most notably in the domains of laser-driven fusion and harmonic generation. In the contemporary annals of scientific exploration, the examination of the dynamic interplay between laser beams (referred to as L.B) and plasmas has emerged as an increasingly beguiling and captivating realm of investigation. It was Askar 'yan [1] who initially illuminated the profound insight that an intense beam, when traversing through a medium, can exhibit a mesmerizing dichotomy, oscillating between the realms of convergence and divergence. This captivating phenomenon hinges upon the intricate nonlinear processes that orchestrate their intricate ballet within the plasma medium. In recent decades, the study of L.B interaction with plasmas has become an increasingly intriguing and attractive area of research. He was the first to note that a intense beam going through a medium can either converge or diverge depending on the nonlinear processes taking place in the plasma medium. He realised that when the intense L.B was self-focusing (S.F) the plasma was pushed out towards the edges, creating a pressure that acted in the opposite direction to the central axis.

G. Askar 'yan [2] was the first person to publish a paper on the S.F of L.Bs in plasma. He investigated the energy momentum flux density of the L.B during S.F, which causes plasma to be evacuated towards the edges and pressure from the plasma profile to balance out of the center of L.B. Askar 'yan also measured the necessary optical intensities to modify the dynamic pressure of the gas.

Hora [3] in his research, found that the intricate self-converging effect elicited by an extraordinarily high-energy Laser Beam (L.B) within a plasma medium, driven by the ponderomotive force. It also scrutinizes the phenomenon of Self-Focusing (S.F) within a plasma environment that has been previously established. This investigation delves into the precise mechanisms by which the ponderomotive force, engendered by the intense laser beam, orchestrates the self-convergence of the beam within the plasma. Furthermore, it meticulously examines scenarios in which the plasma exists prior to the laser beam's interaction, shedding light on how the pre-existing plasma conditions impact the laser beam's self-focusing dynamics. In essence, this research contributes profound insights into the intricate realm of laser-plasma interactions and the pivotal role played by ponderomotive forces in shaping these complex phenomena. The ponderomotive force, in this context, exerts its influence by propelling the plasma in the radial direction of the laser pulse. As a consequence of this action, the plasma's density in the central regions diminishes. By applying the principle of equilibrium, Hora was able to ascertain the minimum laser power required. This determination was based on the well-established phenomena of total reflection and diffraction. In essence, Hora's work leveraged the interplay of ponderomotive forces and plasma dynamics to calculate the laser power threshold needed for achieving specific optical effects, such as total reflection and diffraction, within the system.

Litvak [4] conducted research on S.F in magnetoactive plasma when the beam propagates longitudinally. The study investigated the nonlinearity mechanisms in magnetoactive plasma and derived expressions for nonlinear corrections to the refractive index due to heating and nonlinear motion of a single electron. The necessary conditions for S.F were identified and characteristic parameters for S.F of the beam were determined.

Sodha et al. [5] conducted a comprehensive study focused on the propagation and focusing behavior of electromagnetic waves within inhomogeneous dielectric media. Their research findings revealed a noteworthy improvement in the focusing length when the dielectric constant decreased with the propagation distance. Additionally, the study delved into the intriguing phenomena associated with L.B S.F, filamentation, and self-trapping. Specifically, the investigation centered on the behavior of a Quadruple gaussian L.B (GLB) within inhomogeneous magnetized plasma, considering both linear absorption and ponderomotive nonlinearity effects. The study's outcomes shed light on the distinct characteristics of diverging and converging L.Bs, demonstrating oscillatory divergence for the former and oscillatory convergence for the latter.

Sodha et al. [6] glanced the S.F effect of a Gaussian pump pulse in plasmas, taking into account nonlinearity caused by ponderomotive force and non-uniform heating. They assumed the pulse duration is longer than the specific period of nonlinearity and noticed the L.B focusing in a traveling filament. In the case of collisional plasmas, relaxation effects caused the pulse peak to shift to higher values and become severely distorted due to S.F, while in collisionless plasmas, the peak shift was not significant.

Sodha et al. [7] discovered that as the temperature increases and the refractive index decreases in the TEM_{01} mode the electromagnetic energy converges in the x-direction and vice versa. They also observed that in the cylindrical modes of TEM_{00} and TEM_{10} thermal S.F or defocusing of the beam occurs in the case of geometric optics.

Siegrist et al.[8] investigated the S.F phenomenon in plasma under the influence of ponderomotive forces and relativistic effects. He observed that in the case of stationary S.F the entire ponderomotive force effectively counteracts the occurrence of relativistic effects through their coupled mechanisms.

Sodha et al. [9] undertook a comprehensive theoretical investigation that delved into the intricate mechanisms governing the transmission and concentration of electromagnetic (EM) waves within materials characterized by diverse dielectric properties. It was observed like when the dielectric constant decreases with axial distance of propagation, the focusing length increases in the medium. They also examined the S.F/defocusing of L.Bs operating in the TEM_{01} mode and found that the cylindrical symmetry and power of irradiance increase in the medium, becoming concentrated in various directions around the points of maximum irradiance.

Nayyar et al. [10] and colleagues investigated the S.F behavior of a highly energetic, non-GLB functioning in TEM_{01} mode in a highly ionized plasma. They observed that above a critical power, the beam experiences focusing in the Y-direction but diverges in the direction parallel to the X-axis. Furthermore, the normalized beam width parameter f_2 first increases in the Y-direction and reaches a maximum after some distance then decreases as the beam propagates within the medium. Additionally, they found that absorption decreases within the extent of S.F.

Nayyar et al. [11] conducted a comprehensive investigation into the impact of S.F and defocusing of an elliptical L.B in plasmas. They made the intriguing observation that the pump pulse's focal point could vary, leading to scenarios where the L.B exhibited convergence or divergence in both dimensions within specific power ranges. Conversely, in other instances, one dimension displayed convergence while the other dimension demonstrated divergence. These findings significantly contribute to our understanding of laser-beam behavior in plasma environments and hold relevance for a wide range of applications where laser-plasma interactions play a pivotal role.

Askar'yan et al. [12] conducted research on the defocusing of a concentrated beam and detected a thin beam emanating from the zone of focus. They employed a neodymium laser that was single-mode and unmodulated with $1J$ energy and a millisecond pulse as well as a Nd-YAG laser operating in both single and high frequency millisecond-pulse repetition modes and provided experimental evidence to support the defocusing nature of the beam in a nonlinear medium with weak absorption.

Mori et al. [13] used computer simulations to study what happens when a really powerful burst of laser light travels through a very dense plasma. What they saw was quite interesting. First, the L.B starts to concentrate or self-focus because of a phenomenon related to how fast particles are moving, known as relativistic effects. After that, another force called ponderomotive forces kicks in, which continues to make the L.B focus even more. However, something else starts to happen near the edges of the L.B's path – it begins to break up into smaller pieces, kind of like when a stream of water splits into droplets. This splitting of the L.B into smaller bits is called filamentation, and it occurs especially at the edges of where the laser is traveling.

Now, here's the really interesting part. The researchers discovered that this S.F effect is even stronger when they used a specific type of laser light called resonant double frequency illumination compared to using just a single frequency of laser light. So, in simple terms, their study showed that when a super powerful laser goes through dense plasma, it initially focuses because of fast-moving particles and then gets even more concentrated due to some special forces, but it also breaks into smaller pieces near the edges. And when they used a certain kind of laser light, this effect became even more intense.

Aggarwal et al. [14] In the course of their research endeavor, the investigators embarked upon a meticulous exploration of the dynamic behaviors exhibited by a circularly polarized quadruple Gaussian beam as it traverses the intricate milieu of a magnetized plasma. This rigorous inquiry took into profound consideration the emergent nonlinear phenomena arising from the relativistic augmentation of electron mass, thus precipitating consequential alterations in the refractive index of the medium. Central to their investigation was the deliberate imposition of an external magnetic field, a pivotal parameter that delineated three distinct regions of interest within the propagation scenario, namely, the Self-Focusing (S.F) regime, oscillatory behaviors, and smooth divergence patterns. The results of their discerning study unveiled a nuanced interplay between the applied magnetic field and the beam's propagation characteristics, particularly in relation to two distinctive modes of wave polarization: the extraordinary and ordinary modes. Remarkably, their findings disclosed a discernible trend in which the magnetic field exhibited a salutary influence on the Self-Focusing (S.F) behavior when considering the extraordinary mode. In contrast, within the purview of the ordinary mode, the magnetic field exerted a counteractive effect, introducing perturbations that manifested as a diminution in the S.F tendency. This intricate interplay of magnetic fields, relativistic mass increases, and wave polarization modes underscored the profound complexities at play in the transmission of quadruple Gaussian beams through magnetized plasmas, thereby enriching our understanding of these intricate phenomena.

Kurki et al. [15] investigated the S.F of a pulsed laser in collisional plasma and studied the effects of relativistic and ponderomotive forces. They obtained a steady-state solution for the beam transmission in the localized waveform which showed oscillatory behavior similar to that of multiple-beamlet shapes.

Cicchitelli et al. [16] demonstrated that electromagnetic beams in vacuum have a longitudinal component that can be experimentally verified through the polarization independence of electron energy at the focus of a laser. They introduced an improvement to the Lax, Louisell and Knight (L.L.K) paraxial approximation to a Maxwellian exact solution for a Gaussian beam, which accounts for the exact longitudinal field components of the L.B.

Cohen et al. [17] examined the behavior of ponderomotive S.F in plasmas and calculated the space-time evolution of a non-linear, coherent beam taking into account the dominant non-linearity because of the ponderomotive force, and the hydrostatic nature of the plasma response. This research has applications in high-power laser technology such as inertial-confinement fusion and in the heating of magnetically confined plasmas with intense lasers.

Brandi et al. studied the transmission of a L.B in plasma at high irradiance levels by using theoretical and numerical analysis. They considered the non-linear relationship between the optical index of the plasma and the intensity of light and examined the effects of electron expulsion and the decrease of the plasma frequency. The researchers assumed that the defocusing and focusing effects of the beam remain cylindrical and that the plasma is a homogeneous medium along the direction of propagation.

Chen et al. [18] developed three-dimensional equations to describe the movement of a strong laser pulse with arbitrary strength ($a = eA/mc^2$) in a cold, underdense plasma. These equations can be reduced to certain previous one-dimensional models in different limits. Chen et al. found that when $|a| < 1$, an approximate set of equations can be obtained from the averaged Lagrangian. They numerically solved the axisymmetric two-dimensional model equations to investigate the effect of dispersion in the S.F process.

Parashar et al. [19] investigated the generation of harmonics in semiconductors induced by the interaction of the pump pulse. They astutely discerned that, during this interaction, a notable force known as the $V \times B$ force came into play, exerting its influence upon the electrons within the semiconductor medium. This dynamic interplay unfolded in the presence of a magnetic wiggler, a pivotal element in this experimental setup. The electrons in the presence of a magnetic wiggler produces a transverse harmonic current which results to the production of HG. The magnetic field of the wiggler plays an important role in fulfilling the phase matching condition.

Kim et al. [20] investigated the influence of the density ramp structure on the energy of electrons during laser wake field acceleration. They discovered that when the density ramp was lowered, electron energy decreased due to acceleration area latency and reduced acceleration field strength compared to a uniform density. An upward density ramp, on the other hand, resulted in an increase in electron energy due to the stronger acceleration field and the location of the acceleration area. These effects were investigated using both simulations with a 2-dimensional particle-in-cell algorithm and experiments with a $20TW$ laser.

Bulanov et al. [21] proposed that a high power laser with short pulse duration can be guided in a bullet-shaped channel to achieve efficient amplification of its intensity. This can be achieved by creating a longitudinal electric field with the help of a long pulse.

Hafizi et al. [22] investigated the possibility of stably guiding an energetic L.B over long distances using a combination of relativistic focusing and ponderomotive channelling in plasma. They derived an envelope equation, based on the beam's initial spot size and divergence. They found that for power of lasers greater than the critical power for relativistic S.F, significant spot size contraction can be achieved, leading to an enhancement in L.B intensity. Ponderomotive channelling plays a significant contribution in this development of the focusing effect. Hafizi et al. used simulations to demonstrate the stable guiding of a powerful L.B over prolonged distances by both relativistic focusing and ponderomotive channelling. They also found that the envelope equation is valid for arbitrary laser intensity in the long pulse, quasi-static approximation, and neglects instabilities.

Aggarwal et al. [23] investigated the possibility of generating enhanced higher harmonic photons of a millimetre wave by using the plasma-filled waveguide and a magnetic wiggler. Their investigation yielded a noteworthy discovery: the presence of a plasma waveguide, imparting additional momentum to the system, played a pivotal role in fostering the development of second harmonic photons. This consequential revelation led to a discernible augmentation in the intensity of these photons, illuminating the influential role played by the plasma waveguide in enhancing the output of second harmonic radiation. Verma and Sharma [24] developed a theoretical model to investigate the S.F effect. They observed that the propagation of a L.B results in the expansion of plasma, which leads to the creation of a channel with minimum electron density on the axis. The second pulse of lesser duration and intensity is capable of heating the electrons, which increases the rate of ionization and decreases the recombination ratio. This results in a significant enhancement of the plasma channel's lifetime. The formation of the channel by the first pulse requires a delay of approximately 5-10 nanoseconds between the pulses to allow radial ambipolar plasma diffusion. The second laser pulse then undergoes periodic focusing in the channel, leading to high electron heating rates.

S.F of the second pulse occurs, which further enhances the heating rate and increases the lifetime of plasma channel.

Jha et al. [25] conducted research on the mutual influence of relativistic process and ponderomotive type of forces on the movement characteristics and conjugation of a GLB through moderately ionized non-linear medium (plasma). It has been discovered that the nonlinearity due to the above mentioned force opposes the relativistic S.F effect and causes the L.B to defocus. Additionally, the higher current density plays a crucial role in increasing the conjugation uncertainty of the GLB.

Sharma et al. [26] discovered that by utilizing a highly intense laser pulse with a suitable magnetic wiggler, second harmonic radiation can be generated in plasma. To fulfill the critical phase matching condition necessary for the resonant process, a crucial step is taken: the impartation of momentum to the second-order harmonic. This strategic adjustment ensures that the waves align harmoniously, facilitating the desired outcome. Furthermore, the self-focusing (S.F) attribute inherent to the pump pulse plays an instrumental role in this process. It functions as an amplifying mechanism, boosting the amplitude of the second harmonic radiation. In this intricate interplay of momentum and self-focusing, the resonant process unfolds with greater intensity and efficacy.

Upadhyay et al. [27] have developed a new model to know the mechanism of next harmonic generation in a plasma channel. They observed that a low-density plasma channel generated by a pump L.B emits second harmonic radiation, which propagates outward in a hollow cone shape along the central axis of the channel. Through meticulous analysis, the researchers delved into the intricate oscillatory behavior exhibited by second harmonic photons in relation to varying plasma density. In the course of their investigation, they made a compelling revelation: the power of second harmonic radiation attains its zenith at specific, well-defined values of plasma density. This discovery underscores the critical influence of plasma density on the generation and amplification of second harmonic radiation, shedding light on the nuanced dynamics governing this captivating phenomenon.

Sharma et al. [28] conducted an experiment where they visualised about the L.B that when a L.B is directed perpendicularly to the axis of a fiber it undergoes a focusing effect to reduce its dimensions. This effect is due to the nonlinear S.F phenomenon in the dielectric material of the fiber. Their findings elucidated a noteworthy relationship between particular laser intensity levels and the radius of the fiber. In this context, they observed that under specific conditions, the laser beam achieved a precise and accurate focal alignment with the central axis of the fiber. This intriguing outcome highlighted the intricate interplay between laser parameters and fiber dimensions, shedding light on the conditions that enable precise beam focusing within the fiber core.

Parakash et al. [29] investigated S.F and defocusing effects of a fundamental laser beam as it propagates through a non-homogeneous medium. They developed mathematical equations to describe the behavior of the beam width parameter, taking into account the nonlinear index of refraction within the medium. Additionally, they derived expressions for absorption and nonlinearity parameters, which help quantify how the pump pulse interacts with the medium over distance. This research contributes to a better understanding of how laser beams behave in non-uniform media, with potential applications in laser-induced breakdown spectroscopy, laser material processing, and atmospheric laser propagation, among others.

Kant et al. [30] In their comprehensive study delved into the intriguing dynamics of a laser pulse as it traverses through a plasma density ramp. Their investigation yielded a compelling insight: the mere existence of the density ramp within the plasma exerts a profound influence on the behavior of the laser pulse. In contrast to scenarios where the density ramp is absent, where diffraction typically causes the laser pulse to disperse and lose focus, the researchers observed a starkly different phenomenon when the density ramp was present. In this case, the pulse underwent a notable transformation, becoming increasingly focused. It was the distinctive feature of the upward density ramp that emerged as a pivotal contributor to this remarkable phenomenon, playing a significant role in facilitating the enhanced focusing of the laser pulse. This discovery illuminates the pivotal role of plasma density ramps in shaping the behavior of laser pulses as they propagate through complex plasma environments, marking a noteworthy advancement in our understanding of these intricate processes.

Theberge et al. [31] did the study which focused on the generation of the third harmonic in the atmosphere using femtosecond laser pulses. Through both theoretical and experimental approaches, they investigated the THG during filamentation of a powerful femtosecond pump pulse and observed a significant increase in the third harmonic beam generation.

Patil and Takale [32] observed that within the regime of weakly relativistic and ponderomotive plasma dynamics, the presence of an upward-moving plasma density ramp had a significant impact. It led to the acceleration of electrons to higher energy levels over a more extensive distance compared to what occurs in a uniform-density relativistic plasma. Additionally, the study highlighted the crucial roles played by electronic temperature, plasma density profile, and laser intensity in influencing the process of laser Self-Focusing (S.F). This investigation unveiled the intricate interplay of these factors, shedding light on the captivating and complex dynamics within the plasma environment.

Varsheney et al. [33] investigated the S.F behavior of a pump pulse as it penetrates through an inhomogeneous plasma under relativistic conditions. They derived an equation for the beam width parameter that depends on the normalized propagation distance. It was found that the GLB tends to reach an optimal value that depends on both the plasma inhomogeneity and the initial L.B intensity. The authors also provided numerical calculations for different interaction scenarios between the pump pulse and plasma for both underdense and overdense plasmas.

Liu et al. [34] proposed a novel model to explain the acceleration of electrons to energies of around GeV in plasma. The acceleration occurs through laser-magnetic resonance with the highest acceleration being achieved under the influence of the magnetic field (B). This study introduced a novel model to elucidate the acceleration of electrons to energies around GeV within a plasma. This acceleration primarily relies on laser-magnetic resonance, where laser radiation matches the gyration frequency of electrons in existence of a field (B). The model highlights like most significant acceleration of negatively charged particle occurs when an azimuthal (circular or helical) magnetic field is applied, amplifying energy transfer from the laser to the electrons. This research has implications for understanding and predicting electron acceleration processes in plasma, with potential applications in particle acceleration and high-energy physics experiments.

Gupta et al. [35] investigated self-focusing (S.F) in a pump pulse as it passed through plasma with a rippled density profile and was influenced by an external magnetic field. The researchers observed that the beam waist of the pump pulse decreased as it propagated through the density ramp in the plasma. This reduction in beam waist was attributed to the interaction between the density ramp and the magnetic field, which altered the plasma's optical properties and allowed for the beam to maintain its tight focus over multiple Rayleigh lengths. These findings have implications for controlling and optimizing laser-beam behavior in applications like laser-driven fusion and particle acceleration, where precise focusing is critical. The L.B is self-focused within underdense plasma because of Wakefield generation and relativistic mass nonlinearity, resulting in a minimum spot size. The S.F capability of L.Bs becomes stronger with increasing plasma density.

Kaur et al. [36] investigated the increase in third harmonic generation in plasma created by a laser. They created a model to systematically study the enhancement of third harmonic by treating a thin metallic film with a laser. The laser passing through the plasma causes reflections, creating a plasma density ramp which helps in creating phase matching property needed in enhancing the harmonic photon.

Lieu et al. [37] conducted a rigorous investigation into the fascinating realm of higher harmonic generation, specifically focusing on the third harmonic, in the context of a pulse traversing through a density ripple within a medium. Their pioneering work proposed a novel framework for elucidating the intricate process of harmonic generation when subjected to the influence of an intense and high-power laser pulse within the backdrop of a density ripple. The culmination of their research led to an intriguing and consequential conclusion: density ripples within the medium can serve as a valuable tool for achieving the crucial phase matching condition required for efficient harmonic generation. In essence, they revealed that these density ripples, acting as a sort of guiding structure, can significantly enhance the intricate process of harmonic generation when subjected to the potent influence of a high-power laser pulse. This discovery not only advances our understanding of harmonic generation mechanisms but also offers a promising avenue for optimizing and harnessing this phenomenon in practical applications.

Shahi et al. [38] investigated the Self-Focusing (S.F) behavior of a high-intensity ultra-short laser pulse as it propagated through a plasma that was both relativistic and magnetized. Their study aimed to understand how such unique conditions influenced the laser pulse's behavior within the plasma medium. This research provided valuable insights into the complex dynamics of laser-plasma interactions in high-intensity, ultra-short pulse scenarios, shedding light on the effects of relativity and magnetism on the self-focusing phenomena. They considered the third order nonlinearity and external magnetic field and analysed the relativistic effects using the paraxial approach. Their findings indicate that the S.F ability is enhanced by magnetic field. However, the S.F is reduced when the angle is increased subtended by the laser field and the magnetic field. They made use of the density ramp profile for the study of the process of S.F in cold quantum plasma (CQP). They obtained the differential equation and analytical solution is also obtained for the same. Their outcomes reveal that the quantum effect significantly adds to the S.F effect in comparison to classical relativistic effect. But, the ramped density profile gives rise to the higher oscillations and enhanced focusing of L.B in cold quantum plasma.

Habibi et al. [39] investigated the S.F phenomenon in cold quantum plasma by utilizing the density ramp profile. They derived a differential equation and obtained an analytical solution for it. Their findings indicate that the S.F effect is significantly influenced by quantum effects rather than classical relativistic effects. Moreover, the density ramp profile causes higher oscillations and stronger focusing of the L.B in cold quantum plasma.

Singh et al. [40] embarked on an investigation into the intriguing relativistic effects associated with self-convergence and self-channelling phenomena induced by high-power pump pulses as they traversed through dense plasmas. To delve into and comprehend these phenomena, the researchers adopted a rigorous analytical approach. They utilized the moment approach to formulate a differential equation that effectively described the evolution of the laser beam's width parameter as it propagated through the plasma medium. Subsequently, in order to gain deeper insights into the mathematical behavior of this complex system, they employed the Runge-Kutta numerical method. This method facilitated the systematic exploration of the intricate interplay between high-power laser pulses and dense plasmas, shedding light on the relativistic phenomena of self-convergence and self-channelling that occur under these demanding conditions. The focus of the study was on understanding strong self-focusing (S.F) in the pump pulse, where the laser beam intensifies and narrows significantly as it interacts with the dense plasma. This research contributes to our knowledge of laser-plasma interactions and their potential applications.

Sharma et al. [41] conducted a study on the behavior of q-GLB in relativistic plasma. They used a non-paraxial theory to consider nonlinearity and the relativistic decrease in plasma frequency. They analysed the dynamics of the guided beam with respect to the q-parameter through both analytical and numerical simulations. The outcomes derived from the simulations unveiled a significant relationship between the focusing length and the q-Gaussian profile. This discovery underscored how the specific characteristics and parameters of the q-Gaussian profile influenced the focal point's position or distance within the simulated system.

Xeiong et al. [42] undertook a comprehensive exploration of how an external magnetic field, oriented within the yz-plane, influenced the self-focusing (S.F) behavior of high-power laser beams as they propagated through underdense and magnetized cold plasma. Their investigation yielded noteworthy insights. One key finding was that the self-focusing of the laser beam was influenced by the angle between the y-axis and the magnetic field. Specifically, a larger angle between these two axes resulted in a weaker self-focusing effect on the laser beam. However, it's important to note that the mere presence of the magnetic field itself had an overall enhancing effect on the self-focusing behavior of the laser beam. This complex interplay between the magnetic field's orientation and its intrinsic influence on the self-focusing phenomena illuminated the intricate dynamics at play when high-power laser beams interact with magnetized plasma under these conditions. This behavior is attributed to the interaction between the magnetic field, charged particles in the plasma, and the laser beam, which affects the plasma's refractive index and influences the self-focusing process.

Gill et al. [43] investigated into the intricate dynamics surrounding the propagation of a super-Gaussian beam. They conducted their investigation by deriving a differential equation that described the beam's evolution and subsequently obtained an analytical solution to gain insights into its behavior. What set this study apart was the meticulous consideration of the influence of a magnetic field on the beam's propagation. The researchers went on to analyze the conditions under which dark and bright rings could form within the beam. This analysis took into account higher-order terms of the dielectric function, considering their impact on the optical beam's characteristics, specifically focusing on the profiles of dark and bright rings. One key revelation of their study was the significant strengthening of the self-focusing effect when a dark ring pattern was formed by the optical beam. This finding underscores the intricate interplay between beam characteristics, magnetic fields, and self-focusing phenomena, offering valuable insights into the behavior of super-Gaussian beams in complex optical environments. Conversely, when the beam produces a bright ring pattern, the self-focusing effect is diminished or reversed. The reasons for these effects are likely due to complex interactions between the beam, the dielectric function, and the medium it passes through, requiring further detailed analysis for a complete understanding.

Kant et al. [44] conducted an exploration of the ponderomotive self-focusing (S.F) phenomenon. Their focus was centered on a short pulse laser beam (L.B) and its interaction within an environment characterized by a density ripple. Ponderomotive self-focusing refers to the self-focusing of a laser beam caused by the ponderomotive force resulting from the beam's intensity variations. The phenomena of ponderomotive S.F initially supports the beam to acquire a minimum spot size, but eventually leads to defocusing. To maintain the least possible spot size over larger lengths, the authors used an upward density ramp and studied the resulting enhanced S.F effects.

Kant et al. [45] investigated the resonant enhancement of THG of an ultra-short pulse within an environment characterized by wiggler magnetic field. Their investigation revolved around the intricate interplay between these elements that when a specific wiggler wave number is present, it satisfies the phase matching conditions, leading to an increase in the efficiency of energy conversion.

Kant et al. [46] conducted a study on resonant second harmonic generation and observed that applying a magnetic wiggler can enhance this phenomenon. Their investigation revealed a fascinating mechanism: the laser pulses induce a distinctive response in electrons, driving them to undergo oscillatory motion and experience a longitudinal ponderomotive force. As a consequence of this intricate interplay, a transverse second harmonic current is generated within the system. This current, in turn, acts as a driving force behind the production of second harmonic electromagnetic radiation. Importantly, as this process unfolds, the amplitude of the second harmonic radiation gradually saturates, causing it to deviate from the primary beam. This complex cascade of events underscores the multifaceted dynamics at play in the generation and propagation of second harmonic radiation in the presence of intense laser fields. Furthermore, their research unveiled an intriguing phenomenon: a Gaussian beam, when coupled with a magnetic wiggler possessing an appropriate period, can indeed generate second harmonic radiation. Crucially, when the phase matching conditions are precisely met for this specific wiggler period, an amplification of the intensity of the second harmonic pulse ensues. This enhancement is facilitated by the Self-Focusing (S.F) effect induced by the primary pulse, a phenomenon that contributes to the intensification of the second harmonic radiation. This synergy between the Gaussian beam, the magnetic wiggler, and the phase matching conditions sheds light on the intricacies of optimizing second harmonic generation in this unique and precisely controlled experimental setup. The primary wave then creates a channel of plasma, in which it experiences regular focusing.

Litvak et al. [47] investigated S.F during longitudinal propagation in magnetoactive plasma. They identified the nonlinearity mechanisms in magnetoactive plasma and derived expressions for the nonlinear corrections caused by heating, refractive index, and nonlinear motion of a single electron. They also obtained the necessary conditions for S.F and determined the characteristic parameters of the beam required for S.F.

Sharma et al. [48] delved into the realm of third-order harmonic generation (THG) induced by a high-power laser pulse within a plasma environment, employing a magnetic wiggler denoted as W_B. Their study revealed a pivotal factor influencing the efficiency of this nonlinear optical process: the critical role played by phase matching conditions. Remarkably, the researchers discerned that shorter laser pulses, characterized by higher amplitudes, were more likely to satisfy the phase matching conditions. This phenomenon resulted in the efficient generation of third harmonics. The rationale behind this behavior lies in the broader spectral bandwidth and increased energy inherent to shorter laser pulses. These attributes render them particularly adept at aligning with phase matching criteria, thereby enhancing the efficiency of third harmonic generation. This discovery sheds light on the intricate interplay between laser pulse characteristics and the optimization of THG processes in plasma, offering valuable insights into the dynamics of nonlinear optics in such environments. In some cases, a magnetic wiggler, a device that manipulates electron trajectories, can be used to balance for potential mismatches within these first and third harmonic photons, further enhancing phase matching and THG efficiency.

Kauret al. [49] conducted research on the interaction between parallel Gaussian electromagnetic beams in relativistic magneto plasma. They discovered that S.F occurs at shorter propagation distances in relativistic magneto plasma. Additionally, they observed that at higher magnetic field values, S.F occurs earlier and the beam displays oscillatory behavior as the intensity increases. [49]

Cohen et al. [18] Cohen et al. conducted research on the dynamics of ponderomotive S.F in plasma. They analyzed the space-time evolution of a non-linear coherent beam and identified parameters with dominant non-linearity in terms of the ponderomotive force and plasma response. These findings are significant for heating magnetically confined plasmas with intense pulsed free electron lasers. Additionally, they developed a set of three-dimensional equations for intense laser pulse propagation in cold, underdense plasma, which can be reduced to previous one-dimensional models in different limits. Chen et al. also derived an approximate set of equations from the averaged Langrangian for this process. The researchers numerically solved the axisymmetric two-dimensional model equations to demonstrate the effect of dispersion on the S.F process.

Bulanov et al. [21] conducted a study on the S.F of an ultra-short, high-intensity laser pulse in plasma, which can result in amplification and channelling in a narrow, bullet-shaped channel. Plasma turbulence is generated in the region occupied by the pulse and behind it, leading to electron heating. They observed the formation of horseshoe structures due to the transverse nonuniformity of the pulse, which can be used to focus and accelerate electrons and protons. The modulation of the pulse is quickly and strongly influenced by the induced focusing of the electromagnetic radiation. They also found that a regular longitudinal electric field is produced in the wake of a wide pulse, located behind the sharp edge of a initial pulse.

Gupta and Suk [50] examined the S.F and spot size variation in semiconductor plasma. They demonstrated that the coupling of two co-propagating L.Bs can lead to S.F by creating a high amplitude plasma wave resonantly in the narrow gap semiconductor. Due to the ponderomotive force of electrons resulted by the plasma beat wave, the media becomes nonlinear. As a result, the incident L.B self-focuses in the semiconductor.

Takale et al. [51] utilized the parabolic wave equation method for the theoretical formulation of Hermite-GLBs and carried out numerical computations using the Runge-Kutta method. They derived differential equations for beam width parameters and observed synchronized periodic oscillations of beam width parameters in transverse directions under small scale spatial manipulations in optical traps.

Milani et al. [52] investigated the behavior of a GLB undergoing ponderomotive S.F in warm, collisional plasma. They found that at first, the collision frequency causes S.F, but as the frequency increases, the S.F length decreases and the beam experiences defocusing. Collisional nonlinearity is less affected by higher order axial electron temperatures. To improve focusing, higher order paraxial theory was applied with a ramping density profile. Patil et al. further discovered that a plasma density ramp can significantly enhance S.F, leading to a more focused beam compared to uniform density relativistic plasma, particularly when traversing multiple Rayleigh lengths.

Nanda et al. [53] investigated the S.F behavior of a HchG beam and found that proper choice of beam parameters and a density transition can lead to powerful S.F. In their investigation, the researchers made another noteworthy observation: the coexistence of a density transition and a magnetic field had a synergistic effect on intensifying the Self-Focusing (S.F) phenomenon exhibited by the laser beam within a magnetized plasma featuring a density profile characterized by a ramping gradient. The combined influence of the density transition and the magnetic field acted as a cooperative force, enhancing the laser beam's ability to self-focus effectively within this dynamic plasma environment. This observation underscores the intricate interplay of various factors, such as plasma density profiles and magnetic fields, in modulating the behavior of laser beams in complex plasma settings. The selection of appropriate decentered parameters is crucial for achieving S.F. On the other hand, Kant et al. demonstrated that the density transition and initial intensity of the L.B have significant effects in maintaining the laser plasma interaction as a compelling phenomenon.

Jha et al. [54] discovered that when a L.B propagates obliquely through dense plasma, it generates second harmonic photons. They observed regarding the second harmonic generation that the efficiency increases as the angle of incidence increases for a specific value of plasma electron density. However, they found that the detuning length decreases as the angle of incidence increases. Meanwhile, Kant et al. showed that the initial intensity of the L.B and the presence of a density transition holds a profound influence on the continuity and sustenance of the laser-plasma interaction as a captivating and enduring phenomenon. This transitional variation in plasma density serves as a pivotal factor in shaping and modulating the intricate dynamics of the interaction, making it a central element in the study of laser-plasma phenomena. The density transition introduces a nuanced dimension to the interaction, contributing to its resilience and perpetuation as a fascinating and scientifically compelling process.

Kant et al. [55] observed that the S.F phenomenon in plasma is stronger when a density transition is present. To reduce the amplitude of oscillations, the plasma density was slowly increased. As the L.B enters the ramped density region, it experiences a slower narrowing of the channel, resulting in a contraction of the oscillation amplitude of the spot size. This causes the L.B to become more focused. The S.F/defocusing nature of the beam is affected by the decentred parameter and linear absorption.

Aggarwal et al. [56] reported an increase in the efficiency of second harmonic generation in clusters due to the influence of a wiggler magnetic field. They observed that the group velocity of the second harmonic wave is faster than that of the pump pulse, resulting in pulse slippage. To achieve a high efficiency of third harmonic generation, a wiggler strength of at least 10 kG is necessary.

Kant et al. [57] conducted a study on the resonant harmonic generation of a short pulse laser in a plasma channel and found that when the L.B's intensity is high enough, the plasma electrons move at relativistic speeds. The interaction between the laser and plasma takes into account the relativistic effects. Within the region characterized by a gradual density ramp, the laser beam (L.B) undergoes a remarkable transformation. It perceives a progressively narrowing channel, resulting in a reduction in the amplitude of oscillations in the spot size. This effect indicates that the presence of the density ramp is instrumental in shaping the behavior of the laser beam, causing it to undergo a distinctive focusing phenomenon. Conversely, in scenarios where the density ramp is absent, the dominant influence is the diffraction effect, which tends to cause the laser beam to disperse and lose focus as it propagates through the plasma. This effect is particularly pronounced when the plasma density is lower. As the plasma density increases, the phenomenon of Self-Focusing (S.F) manifests itself at an earlier stage and with greater intensity. This means that the laser beam begins to self-focus more quickly and efficiently in denser plasma environments. These observations highlight the intricate interplay between plasma density, density ramps, and diffraction effects in shaping the behavior of laser beams as they traverse through plasma, shedding light on the dynamic nature of laser-plasma interactions. In the absence of density transition, f doesn't show much increment after passing through many Rayleigh lengths, it reaches a very low value and maintains it for a considerable distance. Therefore, the enhancement in the L.B's S.F is observed.

Gill et al. [58] utilized a variational approach to analyze the propagation characteristics of a cosh-GLB in magneto plasma. They derived a nonlinear Schrodinger equation and discussed the required phase modulation. The researchers concluded that the S.F/defocusing enhancement of the beam is significantly influenced by the decentered parameter b and magnetic field.[58]

Kant et al. [59] conducted research on the propagation of the L.B(Hermite-Gaussian beams) in plasma when a density ramp is applied. By utilizing the WKB approximation and paraxial approximation, they derived a differential equation and observed a reduction in the spot size of the L.B near the axis of propagation. They discovered that S.F can be improved by selecting appropriate and optimized parameters.

Aggarwal et al. [60] studied the behavior of q-GLB as it passes an exponential density ramp. They utilized paraxial and Wentzel-Kramers-Brillouin (WKB) approximation to derive an equation for the beam width parameter(f). Based on their analysis, they drew the following conclusions.

Amin et al. [61] conducted research on the behavior of CO2 laser propagation in plasma with a transverse magnetic field. They observed that the magnetic field causes the L.B to split into filaments, which can be controlled to manipulate the propagation of the beam. The q-Gaussian distribution, which has been used in statistical mechanics, has been found to have various applications in physics, including plasma and laser physics. The q-GLB has a non-Gaussian spatial intensity distribution that exhibits unique S.F and self-trapping behavior, leading to the creation of high-energy electrons, ions, and gamma-rays through various nonlinear mechanisms due to the nonlinearity associated with the q-Gaussian distribution[62].

Sandeep et al. [63] researchers undertook an investigation focused on the intriguing phenomenon of second harmonic generation (SHG) within a collisional plasma environment featuring a density ramp. This unique setup gave rise to a remarkable effect known as frequency up-conversion, wherein a laser beam (L.B) underwent a transformation, resulting in the generation of second harmonic radiation with an increased frequency. This study delved into the intricacies of this process, shedding light on the complex dynamics of laser-plasma interactions within the context of collisional plasma and density gradients. They used the variational method to find a semi-analytical solution to the wave equation for the L.B's slowly varying envelope. The study revealed that when a L.B with frequency ω_0 propagates through plasma, it induces oscillations of the plasma electrons at the same frequency which generates an electron plasma wave (EPW) that beats with the pump beam to produce its second harmonic. The research focused on investigating the impact of various parameters, such as laser and plasma properties, on the propagation dynamics of the pump beam and the efficiency of second harmonic conversion.

Salih et al. [64] conducted a study on the relativistic S.F of a q-GLB propagating in an unmagnetized plasma using a heuristic approach. The analysis focused on the full width at half maximum (FWHM), intensity distribution, and critical power of the q-Gaussian profile. The study found that the FWHM of the q-Gaussian beam is strongly dependent on the q-factor, with a small part of the beam energy contained within the FWHM for high q-values. The diffraction term is more affected by the q-parameter as compared with the nonlinear term. The critical power(P_c) required for S.F becomes extremely high as q approaches 1, making it difficult to wave-guide such a beam. The critical power(P_c) of a q-GLB for S.F is altered by the value of q, with lower q-values leading to significant deviations in FWHM from a normal Gaussian beam.

Gunjan et al. investigated the phenomenon of S.F exhibited by an intense cosh-GLB in a magnetized plasma under relativistic effects [65]. They established a model that describes the beam's field distribution using parameters like the beam width (f_0) and decentered parameter (b), while considering the field (B) aligned with the moving direction of laser. The author derived a nonlinear equation governing f by utilizing the WKB approximation.

The findings of this study indicate that the S.F behavior of the cosh-GLB is notably influenced by parameters such as the decentered parameter (b), relative plasma density, magnetic field parameter and laser intensity parameter (E_{00}^2). Notably, higher values of these parameters lead to a stronger and earlier S.F effect. The study underscores the significance of the higher-order paraxial approximation in capturing off-axial components, which results in a substantial improvement in the S.F behavior. In terms of applications, the intensified S.F observed in the magnetized plasma has the potential to create energetic laser radiation over larger distances beyond the Rayleigh length. This outcome holds promise for various practical applications that require focused L.B.

Gupta et al. explored the excitation of electron plasma waves (EPW) through the utilization of self-focused cosh-GLBs within collisionless plasmas, focusing particularly on the influence of a density ramp [66]. Their work, delved into the dynamic behavior of EPW under the context of an axial density ramp in plasma. The study uniquely considered the S.F phenomenon inherent in the L.B, assessing its direct impact on the power of EPW engendered by the laser. This interplay emerges as the laser propagates through the plasma, exciting an EPW with a characteristic frequency (ω_{ep}). Importantly, due to the plasma's optical nonlinearity, a nonlinear coupling between this EPW and the L.B arises. Employing the framework of variational theory and the WKB approximation technique, the authors have obtained solutions (semi-analytical) for the mutually coupled wave equations leading both the pump wave and the EPW. In terms of key findings, the study underscores the critical role played by the L.B S.F tendencies, which have a pronounced impact on the power and behavior of the EPW. The interaction between the L.B and the plasma, a driving force in the excitation of EPW, results in a nonlinear coupling that significantly influences the dynamics. Additionally, the presence of a density ramp introduces a crucial parameter that shapes the intricate relationship between the self-focused L.Bs and the behavior of EPW. This insight contributes to a deeper understanding of the complex interdependence of these phenomena and provides fresh insights into the mechanisms governing their interactions.

Qiangbo et al. [67], delve into the realm of Laguerre–Gaussian beams propagation characteristics within nonlinear plasma in their study published in Optical and Quantum Electronics. The authors establish a second-order differential equation governing the beam width using Maxwell's equations. This equation is derived through a thoughtful application of the W–K–B approximation and a paraxial-like approximation. Serving as a fundamental equation, it provides a basis for comprehending how Laguerre–Gaussian beams undergo S.F within nonlinear plasma. The study then proceeds to dissect the influence of crucial parameters on the S.F tendencies of Laguerre–Gaussian beams within this intricate nonlinear plasma setting. By establishing a foundational equation and meticulously exploring the nuanced effects of vital parameters, the study enriches our comprehension of the intricate interplay between these specialized beams and the complex medium of nonlinear plasma.

Butt et al. [68] investigated the influence of an exponential plasma density ramp on the Self-Focusing (S.F) behavior of q-Gaussian Laser Beams (q-GLBs) within a plasma medium. Through numerical analysis and appropriate boundary conditions, they found that the presence of the exponential density ramp significantly enhanced the efficiency of S.F. This enhancement resulted in earlier and more pronounced S.F of the q-GLB as it propagated through the plasma. This study contributes to a deeper understanding of laser-plasma interactions and their potential applications. Additionally, the study highlighted the role of parameters such as the q value and intensity parameter (αA_{00}^2) in shaping the S.F behavior of the L.B. This work has great applications in the areas such as engineering, applied optics, laser-induced fusion, and the study of L.B propagation in atmospheric conditions.

Gunjan Purohit's study, investigated the impact of the magnetic field on the S.F behavior of a cosh-GLB within a magnetized plasma in presence of the ponderomotive effect [69]. Through analytical and numerical approaches, the research explores the interaction of relativistic and ponderomotive nonlinearities in a collisionless magnetized plasma setting. The results reveal that the mutual effect of these nonlinearities enhances the S.F of the L.B. Moreover, the study compares these findings with those obtained under only relativistic nonlinearity. Notably, the influence of an external magnetic field on the beam's S.F behavior is also examined. The outcomes highlight the significance of the magnetic field and provide insights into the interplay between various parameters, contributing to our understanding of laser plasma coupling in the existence of a high magnetic field.

Butt et al. [70] conducted a study to investigate the combined effects of a wiggler magnetic field and an exponential plasma density ramp on a q-GLB within an underdense plasma environment. Their research revealed that the introduction of a wiggler magnetic field, in conjunction with an exponential plasma density ramp, resulted in a reduction in the beam width parameter of the q-GLB, leading to enhanced S.F behavior. The wiggler type of magnetic field played a important role in pushing plasma electrons away from the beam axis, thereby countering the defocusing tendency induced by the plasma density ramp. This combination allowed the q-GLB to achieve a minimal spot size and maintain it over several Rayleigh lengths. Their findings also indicated that the S.F behavior in magnetized plasma was influenced by the q parameter, with higher q values leading to stronger S.F effects. These results have significant implications for a wide range of laser-based applications, including advanced particle accelerators, inertial confinement fusion (ICF), and sustainable energy systems. The study employed numerical methods and graphical representation to analyze the beam width parameter (f) as a function of the normalized propagation distance (ξ), considering specific boundary conditions. Computational simulations were carried out with defined parameters, such as $\omega r_0/c = 18, \alpha {A_{00}}^2 = 0.9, d = 10$, and $\omega_{p0}/\omega = 0.8$. The research highlighted the critical interplay between the exponential plasma density ramp, wiggler magnetic field, and the q parameter in influencing the S.F behavior of q-GLB in plasma, offering insights with potential applications in optimizing laser energy concentration and acceleration rates.

In recent study, Purohit et al. [71] investigated THz field generation using a S.F cosh-GLB in magnetized plasma with density ripples. They explore how the laser's S.F interacts with pre-existing ripples, leading to nonlinear current and THz field generation. Using advanced theory, they derive expressions for the L.B's properties and the THz field's electric vector. The study reveals intensified S.F in the magnetized plasma's higher-order paraxial region. The findings have implications for high-power THz radiation generation and deepen our understanding of plasma-laser interactions.

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Chapter-3

Self-focusing of a q-Gaussian laser beam under the influence of an exponential plasma density ramp in plasma

3.1. Introduction:

Due to a wide range of potential applications, including laser-driven acceleration, x-ray lasers, harmonic generation, etc., the self-focusing of a powerful laser beam has drawn tremendous interest in the last few decades [1–6]. The propagation of powerful laser pulses $(I > 10^{17} W/cm^2)$ in extremely dense plasmas has seen a sudden spike in demand as a result of the rapid igniter concept for inertial confinement fusion [7]. For quite some time, this topic has been researched in regards to particle acceleration driven by lasers [8]. The primary distinction between the two applications is in their points of interest. Strong pulse propagation itself is the key area of interest for applications involving quick igniters whereas plasma waves created in the long-term consequences of the main pulse are significant phenomena for laser wake field acceleration. At these incredibly high intensities, the generation of wake fields and the transmission of the pulse are both highly nonlinear events. As a result, the traditional analytical methods used to examine linear propagation are ineffective, necessitating the use of numerical simulation. Particle simulations have been widely used to study the overall nonlinear propagation problems [9]. The laser wavelength is smaller than the plasma wave length and the diffraction length is larger in comparison, which is the fundamental characteristic for propagation in under dense plasma [10]. For these applications to operate, a high intensity laser beam must cover a large distance with high precession. When a laser beam interacts with plasma, the plasma dielectric constant is adjusted, allowing the electron an oscillatory velocity [11]. Due to the significance of these applications, the non-linear phenomenon (self-focusing) has been explored by various researchers [3,7,10–19]. In addition to that Studying the self-focusing of q-Gaussian beams is important for optimizing the design and performance of high-power laser systems, as well as for developing more efficient and effective laser-plasma interaction systems. q-Gaussian beams have been shown to have advantages over traditional Gaussian beams in some laser-plasma applications, including improved beam quality and reduced energy loss. Therefore, understanding the self-focusing behavior of q-Gaussian beams is crucial for advancing these fields. The self-focusing process was started when the plasma was represented as a nonlinear medium, and it was supposed that the effective dielectric constant was a measure of the amplitude of the electric vector [13]. The nonlinear dielectric constant was later developed and used in laser plasma interactions. As a result, three distinct zones emerged: stable divergence, self-focusing divergence and oscillatory divergence. Then it was revealed that magneto plasma, ramping density plasma and rippling density plasma of the electric vector amplitude all display self-focusing in plasma [14]. The notion of something like the nonlinear dielectric constant was then developed and applied to laser and plasma. When ponderomotive nonlinearity is addressed, Wang and Zhou [15] discovered that plasma density is enhanced radially from the central axis, leading in plasma cavitation at lower laser temperatures [16]. To explain the self-focusing of electromagnetic beams, Liu and Tripathi devised a non-paraxial hypothesis and gradually extended the eikonal in powers of r^2. They noticed that intense minimal ray self-convergence affects the formation of ring-shaped laser intensity profiles in a very brief Rayleigh length [17]. Sharma et al. [18] later used a similar method to explore the thermal self-focusing of the laser in collisional plasma beyond the near-axis approximation. They claim that the self-focusing range improves with laser power. The temperature dependency of plasma ionisation and recombination processes has an additional impact on the dynamics of laser beam propagation. Rising recombination and ionisation cause reduced electron heating and the focusing effect [19]. The self-focusing benefits for right-handed polarisation and the benefits of defocusing for left-handed polarisation are also increased by a decrease in plasma temperature and an increase in the oblique magnetic field[20]. Wani et al. [21] reported that semiconductor quantum plasma (SCQP) had enhanced the accuracy of self-focusing. The Hermite cosh gaussian (HchG) beam grants freedom to the decentred parameter, significantly altering the laser beam's ability to focus on its own quip. The density ramp is absolutely essential in lowering the beam width parameter because it is designed to impart strong laser pulses to a defined set of Rayleigh lengths in order to generate stronger laser plasma interactions. To generate a q-Gaussian laser beam, one can use various methods such as using a spatial light modulator or a laser gain medium. A q-plate, which is a birefringent plate, can also convert a Gaussian beam into a q-Gaussian beam. A diffractive optical element can manipulate the phase of a laser beam to create a specific output profile. One can also design the laser resonator with an appropriate gain medium and cavity geometry to obtain a q-Gaussian beam directly. The most commonly used methods are the q-plate and SLM techniques. These methods require careful control over the laser system parameters and the use of advanced optical components and techniques. When a q-GLB goes through plasma, the pulsing motion of electrons induced by the laser electric field causes relativistic self-focusing. The ponderomotive force causes extreme charge carrier rearrangement [16]. This article is mostly about the ponderomotive process used to focus q-GLB for propagation in underdense plasma under a plasma density ramp of type $n(\xi) = \mathrm{n}_0 \exp(\xi/\mathrm{d})$. The findings are examined after the development of analytical formulas for q-Gaussian beams.

3.2. Theoretical Considerations:

2.1. The q-GLB and its field distribution

The electric field distribution of q-Gaussian laser beam propagating along the z-axis in plasma is as follows:

$$\vec{\mathrm{E}}(r,\mathrm{z},\mathrm{t}) = \hat{\mathrm{x}}\mathrm{A}(r,z,t)exp[-i(\omega t - kz)] \tag{3.1}$$

Where A is a function of space (r,z) and time (t), the unit vector along the x-axis is denoted by $\hat{\mathrm{x}}$, the laser frequency is denoted by ω, and the laser wave vector is denoted by k. $\vec{\mathrm{E}}(r,\mathrm{z},\mathrm{t})$ is the electric field linked with the q-Gaussian laser beam (q-GLB).

The q-GLB's initial intensity profile at $z = 0$ is given by M. Yadav et al. [7] in 2020 as

$$A^2 = A_0^2\left(1 + \frac{r^2}{qr_0^2}\right)^{-q} g(t) \tag{3.2}$$

where, $g(\mathrm{t}) = \exp(-t^2/t_0^2)$, The amplitude of a q-Gaussian laser beam is given by A_0, laser spot size is indicated by r_0 and the laser pulse duration by t_0, $where\ t_0 = 40(fs)$. In this situation, the parameter q indicates the deviation of the laser intensity distribution from the standard Gaussian profile. Strongly q-Gaussian characteristics are shown by the smaller q values. The typical Gaussian distribution results from the q parameter tendency to infinity(∞).

3.3. Exponential plasma density ramp and non-linear dielectric constant:

3.1. Nonlinear Dielectric Constant

We investigate the propagation of a q-GLB in a nonlinear medium defined by the dielectric constant given by Sodha et al. [17].

$$\varepsilon = \varepsilon_0 + \Phi(EE^*)\ , \tag{3.3}$$

where, $\varepsilon_0 = 1 - \omega_p^2/\omega^2$ is the part of the dielectric function which is linear, having ω_p as the plasma frequency and the intensity dependent non-linear part represented by Φ. $\omega_p^2 = 4\pi n(\xi)e^2/m$, $n(\xi) = n_0 exp(\xi/d)\ , where\ \xi = (z/R_d)$, $\omega_{p0}^2 = 4\pi n_0 e^2/m$, $\varepsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega^2}exp(\xi/d)$. Now e denotes the electronic charge, m represents the rest mass of the electron, ω represents the frequency of incident laser beam, The equilibrium electron density is denoted by n_0, and the equilibrium plasma frequency is denoted by ω_{p0} . The diffraction length is denoted by R_d, the normalised propagation distance is ξ, and the freely adjustable dimensionless parameter is denoted by d. Now, in the case of collisionless plasma, the ponderomotive force is the primary cause for the nonlinearity of the dielectric constant and the nonlinear component of the dielectric constant is defined by $\Phi(\mathrm{AA}^*) \approx (1/2)\varepsilon_2 \mathrm{A}_0^2$, $\varepsilon_2 = 2\left(\omega_p^{2/}/\omega^2\right)\alpha$, $\alpha = e^2M/6m^2\omega^2K_bT_0$ where M represents the mass of the scatterer in the plasma, ω is the laser frequency employed, K_b represents the Boltzmann constant, and T_0 represents the equilibrium plasma temperature.

3.4. Self-focusing:

The self-focusing phenomenon in laser beam propagation is governed by a fascinating wave equation derived from Maxwell's equations. Let's walk through the steps to derive this equation and explain the physical significance at each stage. We begin with Maxwell's equations, which describe the behavior of electromagnetic fields. Two of these equations are particularly relevant.

Maxwell's Equations: We start with Maxwell's equations, which are fundamental equations describing the behavior of electromagnetic fields:

Faraday's Law:

$$\vec{\nabla} \times H = \frac{1}{c}\frac{\partial D}{\partial t} \tag{3.4}$$

Ampere's Law:

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial H}{\partial t} \tag{3.5}$$

Gauss's Law for Magnetic Fields:

$$\vec{\nabla}.\vec{\mathrm{B}} = 0 \tag{3.6}$$

Gauss's Law for Electric Fields:

$$\vec{\nabla} \cdot \vec{D} = 0 \tag{3.7}$$

Applying Curl to Ampere's Law We take the curl of Ampere's law (equation 3.5) to get:

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E} \right) = -\frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} \tag{3.8}$$

Simplifying the Curl Expression Using vector calculus identity Using vector calculus identity $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{A} \right) = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A} \right) - \nabla^2 \vec{A}$, we can simplify equation (9) as follows:

$$\vec{\nabla}\left(\vec{\nabla}.\vec{E} \right) - \nabla^2 \vec{E} = -\frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} \tag{3.9}$$

Using Gauss's Law for Electric Fields, which states $\nabla \cdot D = 0$, we can substitute this into the equation.

$$-\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} + \vec{\nabla}\left(\vec{\nabla}.\vec{E} \right) = 0 \tag{3.10}$$

Using value of $\nabla.E$ in Eq. (10), we get;

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} + \vec{\nabla}\left(\frac{\vec{E}\nabla.\varepsilon}{\varepsilon} \right) = 0 \tag{3.11}$$

Using $D = \varepsilon\mathrm{E}$ in (11), we obtain final Eq.

$$\nabla^2 \vec{E} - \frac{\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} + \vec{\nabla}\left(\frac{\vec{E}\nabla.\varepsilon}{\varepsilon} \right) = 0 \tag{3.12}$$

Introducing ω and k^2 which leads to the resultant equation for laser beam propagation, equation (3.12), is given by:

$$\nabla^2\vec{E} + k^2\vec{E} + \nabla\left(\frac{\vec{E}\nabla.\varepsilon}{\varepsilon}\right) = 0 \quad (3.13)$$

$$\nabla^2\vec{E} + k^2\vec{E} + \left[-\vec{E}\left(\frac{\nabla\varepsilon}{\varepsilon}\right)^2 + \vec{E}\left(\frac{\nabla^2\varepsilon}{\varepsilon}\right)\right] = 0 \quad (3.14)$$

$$\nabla^2\vec{E} + \left[k^2 + \left(\frac{\nabla\varepsilon}{\varepsilon}\right)^2 + \left(\frac{\nabla^2\varepsilon}{\varepsilon}\right)\right]\vec{E} = 0 \quad (3.15)$$

$$\nabla^2\vec{E} + \left[k^2 + \nabla\left(\frac{\nabla\varepsilon}{\varepsilon}\right)\right]\vec{E} = 0 \quad (3.16)$$

$$\nabla^2\vec{E} + [k^2 + \nabla^2(\ln\varepsilon)]\vec{E} = 0 \quad (3.17)$$

$$\nabla^2\vec{E} + k^2\left[1 + \frac{1}{K^2}\nabla^2(\ln\varepsilon)\right]\vec{E} = 0 \qquad \because k^2 = \varepsilon\frac{\omega^2}{c^2} \quad (3.18)$$

The last part of eq. (18) on left-hand side can be ignored provided that $K^{-2}\nabla^2(ln\varepsilon) \ll 1$ where k represents the wave number. Thus,

$$\nabla^2\vec{E} + k^2\vec{E} = 0 \quad (3.19)$$

Hence, we can write as the wave equation that regulates laser beam propagation is expressed as follows:

$$\nabla^2\vec{E} + \left(\frac{\omega^2}{c^2}\right)\varepsilon\vec{E} + \nabla\left(\frac{\vec{E}\nabla.\varepsilon}{\varepsilon}\right) = 0 \quad (3.20)$$

$$\nabla^2\vec{E} + \left(\frac{\omega^2}{c^2}\right)\varepsilon\vec{E} = 0 \quad (3.21)$$

In cylindrical co-ordinate system, we can write this equation as

$$\frac{\partial^2\vec{E}}{\partial z^2} + \frac{\partial^2\vec{E}}{\partial r^2} + \frac{1}{r}\frac{\partial\vec{E}}{\partial \mathrm{r}} + \frac{\omega^2}{c^2}\varepsilon\vec{E} = 0 \quad (3.22)$$

For slowly converging or diverging cylindrically symmetric beam, the solution of equation (16) is of the following form;

$$\vec{\mathrm{E}}(r, \mathrm{z}) = \mathrm{A}(r, z, t)\exp[-i(\omega \mathrm{t} - kz)] \quad (3.23)$$

$$where,\ k^2 = \frac{\varepsilon_0}{c^2}\omega^2 = \varepsilon\frac{\omega^2}{c^2}\left(1 - \omega_p^2 exp[i(\omega t - kz)]\right).$$

Differentiating equation (22) twice w. r. t, we get “z" and "r”

$$\frac{\partial\vec{E}}{\partial \mathrm{r}} = \exp[-i(\omega \mathrm{t} - kz)]\frac{\partial \mathrm{A}}{\partial \mathrm{r}} \quad (3.24)$$

$$\frac{\partial^2\vec{E}}{\partial r^2} = \exp[-i(\omega \mathrm{t} - kz)]\frac{\partial^2 A}{\partial r^2} \quad (3.25)$$

$$\frac{\partial\vec{E}}{\partial \mathrm{z}} = \exp[-i(\omega \mathrm{t} - kz)]\left[-\frac{i\omega A}{c}\sqrt{1 - \frac{\omega_{p0}^2}{\omega^2}\exp(\xi/\mathrm{d})} + \left(\frac{i\omega}{2cdR_d}\right)\frac{\omega_{p0}^2}{\omega^2}\frac{zAexp(z/dR_d)}{\sqrt{1 - \frac{\omega_{p0}^2}{\omega^2}\exp(\xi/\mathrm{d})}} + \frac{\partial A}{\partial \mathrm{z}}\right] \quad (3.26)$$

$$\frac{\partial^2\vec{E}}{\partial z^2} = \exp[-i(\omega t - kz)]\left[\frac{\partial^2 A(r,z)}{\partial Z^2} + \left(\frac{i\omega}{c}\right)\frac{\partial A(r,z)}{\partial Z}\left(\frac{\omega_{p0}^2 z exp\left(\frac{z}{dR_d}\right)}{\omega^2 dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}} - \right.\right.$$

$$\left.\left.2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}\right)\right] + \frac{\omega A}{c}\exp[i(\omega t - kz)]\left[\frac{i\omega_{p0}^2 z exp\left(\frac{z}{dR_d}\right)}{\omega^2 dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}} + \frac{\omega_{p0}^2 Z exp\left(\frac{z}{dR_d}\right)}{c\omega^2 dR_d}\right] +$$

$$\frac{\omega}{c}\exp[i(\omega t kz)]\frac{iA\omega_{p0}^2 z exp\left(\frac{z}{dR_d}\right)}{\omega^2 d^2 R_d^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}}\left[exp\left(\frac{z}{dR_d}\right) + \frac{\omega_{p0}^2 exp\left(\frac{z}{dR_d}\right)}{4\omega^2\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)}\right] - \frac{\omega^2 A}{c^2}\exp[i(\omega t -$$

$$kz)]\left[\frac{\omega_{p0}^4 z^2 exp\left(\frac{z}{dR_d}\right)}{4\omega^2 d^2 R_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)} + \left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)\right] \tag{3.27}$$

Substituting the values derived from equations (3.24), (3.25), (3.26), and (3.27) into equation (3.21), while neglecting the terms involving $\frac{\partial^2 A}{\partial z^2}$, the application of the WKB approximation leads to a simplified form of a parabolic wave equation as follows:

$$\frac{i\omega}{c}\left(2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)} - \frac{\frac{\omega_{p0}^2}{\omega^2}z exp\left(\frac{z}{dR_d}\right)}{dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}}\right)\left(\frac{\partial A}{\partial z}\right) - \frac{\frac{\omega_{p0}^2}{\omega^2}A exp\left(\frac{z}{dR_d}\right)}{dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}} -$$

$$\frac{\frac{\omega_{p0}^2}{\omega^2}Az exp\left(\frac{z}{dR_d}\right)}{d^2{R_d}^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}}\left(exp\left(\frac{z}{dR_d}\right) + \frac{\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)}{4\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)}\right) = \frac{\omega^2}{c^2}\left(\frac{\frac{\omega_{p0}^2}{\omega^2}AZ exp\left(\frac{z}{dR_d}\right)}{dR_d} - \right.$$

$$\left.\frac{\left(\frac{\omega_{p0}^2}{\omega^2}\right)^2 AZ^2\left(exp\left(\frac{z}{dR_d}\right)\right)^2}{4d^2{R_d}^2\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)}\right)\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial z} + \frac{\omega^2}{c^2}\Phi(AA^*)A \tag{3.28}$$

The behaviour of a complex amplitude illustrated by the above equation can be expressed as follows,

$$A = A_0(r,z)\exp[-ikS(r,z)] \quad \text{where,} \quad k = \frac{\omega}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)} \tag{3.29}$$

where, A_0 and S are real functions of r and z and $S(r,z)$ is the eikonal of the beam that governs its convergence and divergence. Equating R_E and Imag. parts on both sides of the equation after substituting the A, one obtains:

$$\frac{c^2}{\omega^2 A_0}\left(\frac{\partial^2 A_0}{\partial z^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}\right)+\Phi(A_0^2)=\left(2\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)\right)-\frac{\omega_{p0}^2}{\omega^2}\frac{zexp\left(\frac{z}{d\mathrm{R}_d}\right)}{d\mathrm{R}_d}\right)\frac{\partial S}{\partial z}+\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)\right)\left(\frac{\partial S}{\partial r}\right)^2-\frac{\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)}{d\mathrm{R}_d}\left(S+z-\frac{\omega_{p0}^2}{\omega^2}\frac{zexp\left(\frac{z}{d\mathrm{R}_d}\right)\left(S-\frac{z}{2}\right)}{2d\mathrm{R}_d\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)\right)}\right) \quad (3.30)$$

For a cylindrically symmetric q-Gaussian beam, the solutions of Eq. (3.29) as followed by (Sodha et al., 1976) [16] can be written in the form as follows:

$$A_0^2=\frac{A_{00}^2}{f^2}\left(1+\frac{r^2}{qr_o^2f^2}\right)^{-q} \quad (3.31)$$

$$\mathrm{S}=\frac{r^2}{2}\ \beta\,(z)+\Phi(z), \quad (3.32)$$

where, A_{00} represents the amplitude and the inverse radius of curvature of the wave front is $\beta(z)=1/f(z)\,df/\,dz$.

Differentiating (S) w.r.t ‘z’ and ‘r’ respectively

$$\frac{\partial s}{\partial z}=\frac{r^2}{2}\frac{\partial\beta(z)}{\partial z}+\frac{\partial\Phi(z)}{\partial z} \quad (3.33)$$

$$\frac{\partial s}{\partial z}=\frac{r^2}{2f(z)}\frac{\partial^2\beta(z)}{\partial z^2}-\frac{r^2}{2f^2(z)}\left(\frac{\partial f(z)}{\partial z}\right)+\frac{\partial\Phi(z)}{\partial z}^2 \quad (3.34)$$

$$\frac{\partial s}{\partial z}=\frac{r^2}{2f(z)}\frac{\partial^2 f(z)}{\partial z^2}-\frac{r^2}{2f^2(z)}\left(\frac{\partial f(z)}{\partial z}\right)+\frac{\partial\Phi(z)}{\partial z}^2 \quad (3.35)$$

$$\frac{\partial S(r,z)}{\partial r}=2r\frac{\beta(z)}{2}=r\beta(z)=\frac{r}{f(z)}\frac{\partial f(z)}{\partial z} \quad (3.36)$$

Now differentiating A_0 w.r.t ‘r’ twice

$$\frac{\partial A_0}{\partial r}=-\frac{A_{00}}{f^3}\frac{1}{r_0{}^2}\left(1+\frac{r^2}{qr_o^2f^2}\right)^{-\frac{q}{2}-1} \quad (3.37)$$

$$\frac{\partial^2 A_0}{\partial r^2}=\frac{A_{00}}{f^5}\frac{(q+2)}{r_0{}^4}\frac{r}{q}\left(1+\frac{r^2}{qr_o^2f^2}\right)^{-\frac{(q+4)}{2}} \quad (3.38)$$

Substituting the value of $\partial S(r,z)/\,\partial Z$, $\partial S(r,z)/\,\partial r$, $\partial A_0/\,\partial r$, $\partial A_0^2\,/\,\partial r^2$ and A_0^2 in real part equation, we get

$$\frac{c^2\left(1+\frac{r^2}{qr_0^2f^2}\right)^{\frac{q}{2}}}{\omega^2\frac{A_{00}}{f^2}}\left[\frac{2A_{00}}{\{f^6}\frac{q+2}{qr_0^4}r^2+2\frac{A_{00}}{f^2}\frac{1}{r_0^2f^2}\left(1+\frac{r^2}{qr_o^2f^2}\right)^{-\left(\frac{q+2}{2}\right)}\right]+\frac{A_{00}^2r^2}{2r_0^4f^4}\left(\frac{\alpha\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)=\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)r^2\beta^2+\phi(z)+z+\left[2-2\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)-\frac{z\omega_{p0}^2exp\left(\frac{z}{dR_d}\right)}{\omega^2 dR_d}\right]\left(\frac{r^2}{2}\frac{\partial\beta}{\partial z}+\frac{\partial\phi}{\partial z}\right)+$$

$$\frac{r^2\beta z\omega_{p0}^2\left(exp\left(\frac{z}{dR_d}\right)\right)^2}{4\omega^4 d^2R_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)}-\frac{r^2\beta\omega_{p0}^2 exp\left(\frac{z}{dR_d}\right)}{2d\omega^2R_d}-\frac{\omega_{p0}^2 z exp\left(\frac{z}{dR_d}\right)}{2d\omega^2R_d\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)}\phi(z)+$$

$$\frac{z^2\omega_{p0}^2 exp\left(\frac{z}{dR_d}\right)\phi(z)}{4\omega^2 dR_d\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)} \tag{3.39}$$

By comparing the coefficients of r^2, we obtain:

$$\frac{2(q+2)}{qf^4}\left(\frac{qr_0^2}{f^2+r^2}\right)^{-\frac{q}{2}}+\frac{\alpha A_{00}^2}{2f^4}\left(\frac{\omega_{p0}^2}{\omega^2}\right)\exp\left(\frac{z}{dR_d}\right)=\left(\frac{z}{2dR_d}\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{z}{dR_d}\right)+\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{z}{dR_d}\right)-1\right)R_d{}^2\frac{\partial^2 f}{\partial z^2}-\frac{z}{2d}\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{z}{dR_d}\right)\frac{1}{f}\left(\frac{\partial f}{\partial z}\right)^2+\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{z}{dR_d}\right)\left[1-\frac{\omega_{p0}^2}{\omega^2}\frac{z\exp\left(\frac{z}{dR_d}\right)}{2d\left(1-\frac{\omega_{p0}^2}{\omega^2\exp\left(\frac{z}{dR_d}\right)}\right)}\right]\frac{\partial f}{\partial z} \tag{3.40}$$

Since, $\xi = z/R_d, z=\xi R_d\ and\ z/dR_d=\xi/d.$ Therefore, we can write as:

$$\frac{\partial f}{\partial z}=\frac{\partial f}{\partial \xi}\times\frac{\partial \xi}{\partial z}=\frac{1}{R_d}\frac{\partial f}{\partial \xi}\ and\ \frac{\partial^2 f}{\partial \xi^2}=\frac{1}{R_d^2}\frac{\partial^2 f}{\partial \xi^2} \tag{3.41}$$

Substituting the above values in the equation (3.40), we have established the differential equation of the beam-width parameter under the paraxial approximation as follows:

$$\frac{2(q+2)}{qf^4}\left(\frac{qr_0^2}{f^2+r^2}\right)^{-\frac{q}{2}}+\frac{\alpha A_{00}^2}{2f^4}\left(\frac{\omega_{p0}^2}{\omega^2}\right)\exp\left(\frac{\xi}{d}\right)=\left(\frac{\xi}{2d}\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{\xi}{d}\right)+\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{\xi}{d}\right)-1\right)\frac{\partial^2 f}{\partial \xi^2}-\frac{\xi}{2d}\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{\xi}{d}\right)\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2+\frac{\omega_{p0}^2}{\omega^2}\exp\left(\frac{\xi}{d}\right)\left[1-\frac{\omega_{p0}^2}{\omega^2}\frac{\xi\exp\left(\frac{\xi}{d}\right)}{2d\left(1-\frac{\omega_{p0}^2}{\omega^2\exp\left(\frac{\xi}{d}\right)}\right)}\right]\frac{\partial f}{\partial \xi} \tag{3.42}$$

This (Eq. 3.42) is the required equation of beam width parameter.

3.5. Result and discussion:

Equation (3.42) is numerically solved for different plasma parameters and a q-Gaussian laser beam using Runge-Kutta method and propagation dynamics is determined, using boundary conditions. The results are presented in the graphical form. Consider a plasma exposed to a q-GLB, Figures 3.1, 3.2, 3.3 and 3.4 demonstrate the fluctuation of the beam-width parameter (f) as a function of the normalised propagation distance in an under dense plasma with an exponential plasma density ramp. We use the boundary conditions $f=1$ and $\frac{\partial f}{\partial \xi}=0$ at $\xi=0$, For the initial plane wave front of the beam. To validate the preceding approach, we use computational simulations to solve the beam-width parameter equation. We used these parameters for numerical calculations: $\omega r_0/c=18,\ \alpha A_{00}^2=0.9, d=10$ and $\omega_{p0}/\omega=0.8.$

In Fig. 3.1, The self-focusing of q- Gaussian laser beam(q-GLB) in plasma with and without exponential plasma density ramp has been compared for different parameters like $\omega r_0/c = 18$, $\alpha A_{00}^2 = 0.9, d = 10$, $\omega_{p0}/\omega = 0.8$. It is obviously noticeable from Fig. 3.1 that the process of self-focusing (f = 0.03) becomes strong at ξ = 1, when observed with exponential plasma density transition in comparison to without plasma density ramp. Hence, it can be mentioned that efficient self-focusing is observed by taking exponential plasma density transition into consideration.

The variation of beam width parameter (f) of the q-Gaussian laser beam possessing normalized propagation distance (ξ) for the various values of q is shown in fig.3.2. This figure displays the effect of q parameter on self-focusing of q-GLB. A visible change in self-focusing is seen for smaller q-values like $q = 3, 4\ and\ 5$ as we increase the value of q-parameter, the self-focusing becomes stronger. The focusing of the beam dominates over diffraction because of the nonlinear effect. For q = 5 (the green line), diffraction effects are relatively weaker but reduced focussing is observed for lower values of q. In comparison to the bigger value of q in the intensity distribution, the smaller value of q has extended wings, which may demand more power for self-focusing. This occurs as a result of the fact that the intensity of q-Gaussian laser beam decreases as it moves from the off-axial zone toward the axial region. Thus, our findings concur with those of Aggarwal et al. [19]. It is possible to observe that for $q = 5$, the laser spot size tends to zero as the propagation distance increases. However, this does not mean that the laser beam can actually achieve a zero-spot size. In practice, the smallest achievable spot size of a laser beam is limited by various factors, such as diffraction and the quality of the focusing optics. Therefore, the spot size cannot actually reach zero, and there will always be a minimum spot size that can be achieved. If the laser spot size becomes extremely small, the laser intensity at the focus point can become extremely high. This can lead to various nonlinear optical effects, such as self-focusing, self-phase modulation, and even ionization of the medium. In some cases, these effects can be beneficial for certain applications, such as laser plasma interactions, while in other cases, they can be detrimental, leading to material damage or reduced beam quality. Therefore, controlling the laser spot size is crucial for optimizing the performance of laser systems and ensuring their safe and efficient operation. The laser intensity is proportional to the inverse square of the spot size, meaning that as the spot size decreases, the intensity increases significantly. This can be advantageous for applications that require high-intensity laser beams, such as material processing and laser-based fusion. However, it is important to carefully consider the consequences of high-intensity laser beams and take appropriate safety measures to prevent potential hazards.

The laser undergoes self-focusing only after the appreciable digging of the channel. The significance of the channel formation is related to the development of a waveguide like structure that reinforces the propagation of pulse with minimum losses. The channel shell localises the interaction of the pulse with plasma and its nonlinear effect. The induced plasma channel can perform as a light pipe to guide the laser pulse. Vranic et al. [22] discovered that a low-density channel limits ponderomotive force radial expelling, resulting in the radiative trapping of ions near the channel wall. Through waveguide structure, electron beams can be accelerated by using a lower power laser than in gas jet or cell [23].

The laser spot size decreases as the power of the beam improves until medium damage disrupts the action or defocusing commences. Only after a considerable increase in channel digging does the laser begin to self-focus. The importance of channel development is connected to the production of a "waveguide"-like structure that accelerates pulse propagation with minimal losses. The channel shell localises the pulse's interaction with plasma and its nonlinear consequence. The produced plasma channel can act as a light pipe, channelling the laser pulse.

In an underdense plasma possessing exponential density ramp, Fig. 3.3(a) and Fig. 3.3(b) shows the variability of the beam-width parameters f as a function of the normalised propagation distance ξ for various values of ω_{p0}/ω. The electrons are expelled from the increased intensity zone into the low plasma density region because of the ponderomotive force. The electron mass variation that occurs from the intense laser and the variation in electron density as a result of wake field development, is what causes the plasma to be nonlinear. In the absence of an exponential plasma density ramp, the beam-width parameter monotonically decreases up to a Rayleigh length due to nonlinear factors. After reaching a low value, the beam-width parameter grows as the effect of diffraction takes over and the laser beam begins to diverge as a result of nonlinearity saturation. The laser oscillates in such a way that focusing and defocusing occur alternately as a result of this phenomenon. The findings are consistent with Thakur et al. [23].

For various intensity parameters, Fig. 3.4 illustrates the fluctuation of the beam width parameter f with the normalised propagation distance represented by ξ . The graph clearly demonstrates that the self-focusing length reduces when the value of the intensity parameter αA_{00}^2 is increased, supporting the findings of Nanda et al. [24].

Since the introduction of an exponential plasma density ramp profile with the form $n(\xi) = n_0 \exp(\xi/d)$ prevents laser defocusing by causing the beam width parameters to gradually grow up to a Rayleigh length. The intense self-focusing of the laser in plasma with a density ramp is demonstrated by the saturation behaviour of the beam width parameter. This is due to the relativistic mass effect being significantly stronger in the region of rising exponential plasma density after the laser has been focused. As a result, during the propagation of an exponential plasma density ramp, the laser concentrates more. The plasma dielectric constant, on the other hand, rapidly lowers as the laser beam penetrates further into the plasma because the equilibrium electron density is a decreasing function of the laser propagation distance [25]. As a result, the laser is sharper focused and the self-focusing effect is strengthened. The supposition of an underdense plasma, however, led to the selection of the minimal exponential plasma density. The length of the exponential plasma density ramp is thought to prevent the laser from defocusing to its highest capability, and greater focusing is shown by extending the exponential density ramp [26]. However, the exponential plasma density shouldn't be any higher, else, the extremely dense plasma effect could cause the laser to get reflected.

In collision-less plasma with no ramp, Takale et al. investigated the self-focusing and defocusing of Hermite-Gaussian laser beams. They demonstrated that the laser can only focus up to 0.5 R_d. However, as observed in fig. 3.3, the laser in the current work focuses up to the greater lengths while under an exponential plasma density ramp. It follows that by using the exponential plasma density ramp, the laser self-focusing action is improved and made more precise. Consequently, the exponential plasma density ramp is crucial in improving laser focusing. In the region of rising plasma density, the relativistic effect is more severe. As a result, the laser beam concentrates more while moving up the exponential plasma density ramp. The equality between diffraction and that of nonlinear refraction is observed to cause periodic self-focusing. Under the assumption that solitary waves aid in charge particle acceleration, Gaussian pulse laser propagation in a prepared plasma channel is crucial [27]. In addition to that the exponential plasma density ramp can enhance the self-trapping effect of a q-Gaussian laser beam by providing a guiding structure and a confinement mechanism for the laser beam. This can lead to the generation of high-intensity laser pulses and the acceleration of charged particles in the plasma. This study demonstrates the benefits of propagating the q-Gaussian laser beam over the gaussian laser beam in plasma under ponderomotive nonlinearities and suggest using the q-Gaussian laser beam over the gaussian profile for improved electron acceleration. As the q-Gaussian laser pulse travels in the nonlinear medium (plasma) up to several Rayleigh lengths in such a way that neither being absorbed nor being beam broken-up owing to the filamentation in the current work and the self-channelling of the laser pulse might become significant. Therefore, such pulse propagation is essential in a quick igniting strategy. In a pulse with propagation characteristics like a soliton, nonlinear and dispersion effects are significant. Solitons develop and spread, based on how the laser is focused and defocused on its own.

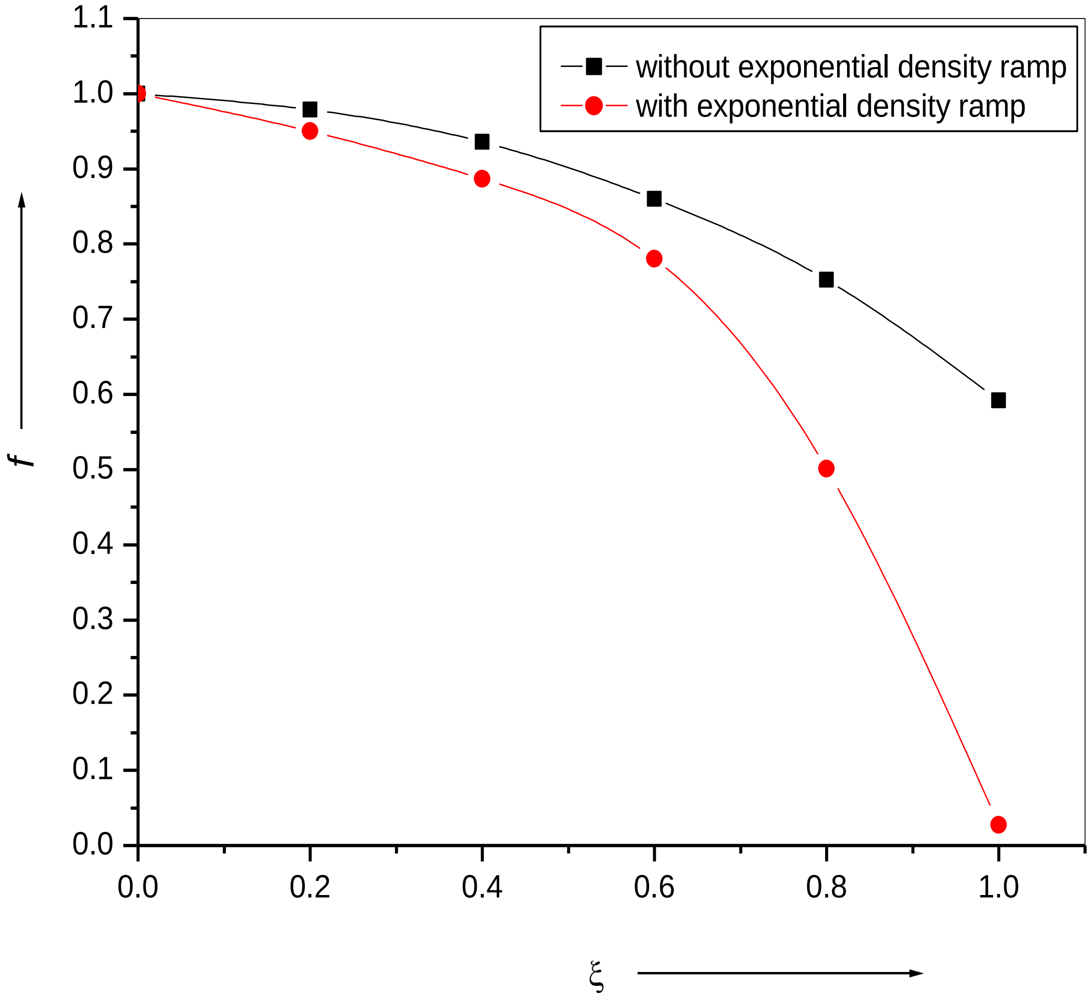

Fig.3.1 Variation of beam width parameter (f) *of the q-Gaussian laser beam with normalized propagation distance*(ξ)*. The other parameters are* $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10\ q = 5$ *and* $\omega_{p0}/\omega = 0.8$.

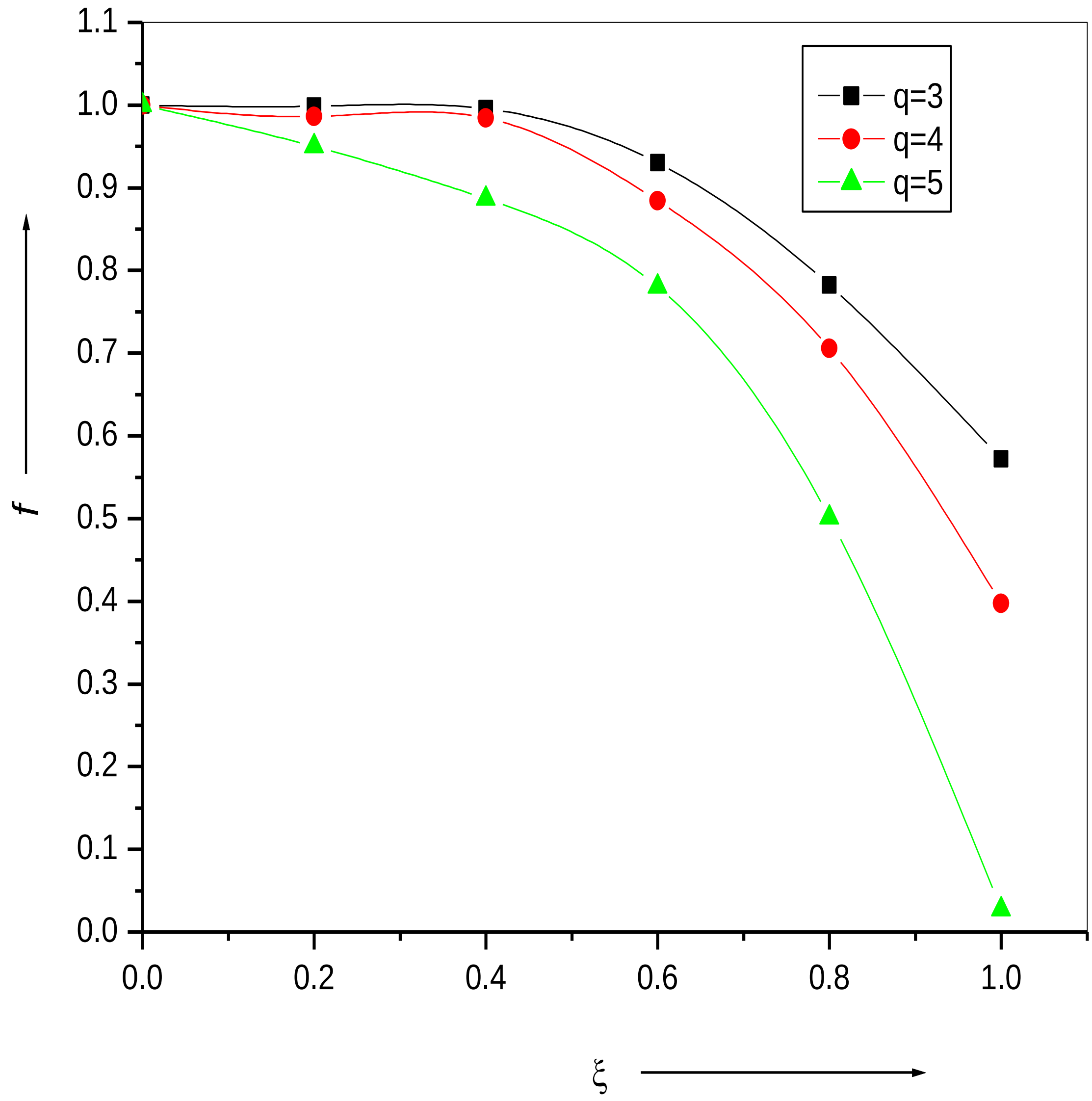

Fig.3.2 Variation of beam width parameter (f) *of the q-Gaussian laser beam having normalized propagation distance*(ξ) *for various values of* q. *The other parameters are* $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10\ and\ \omega_{p0}/\omega = 0.8.$

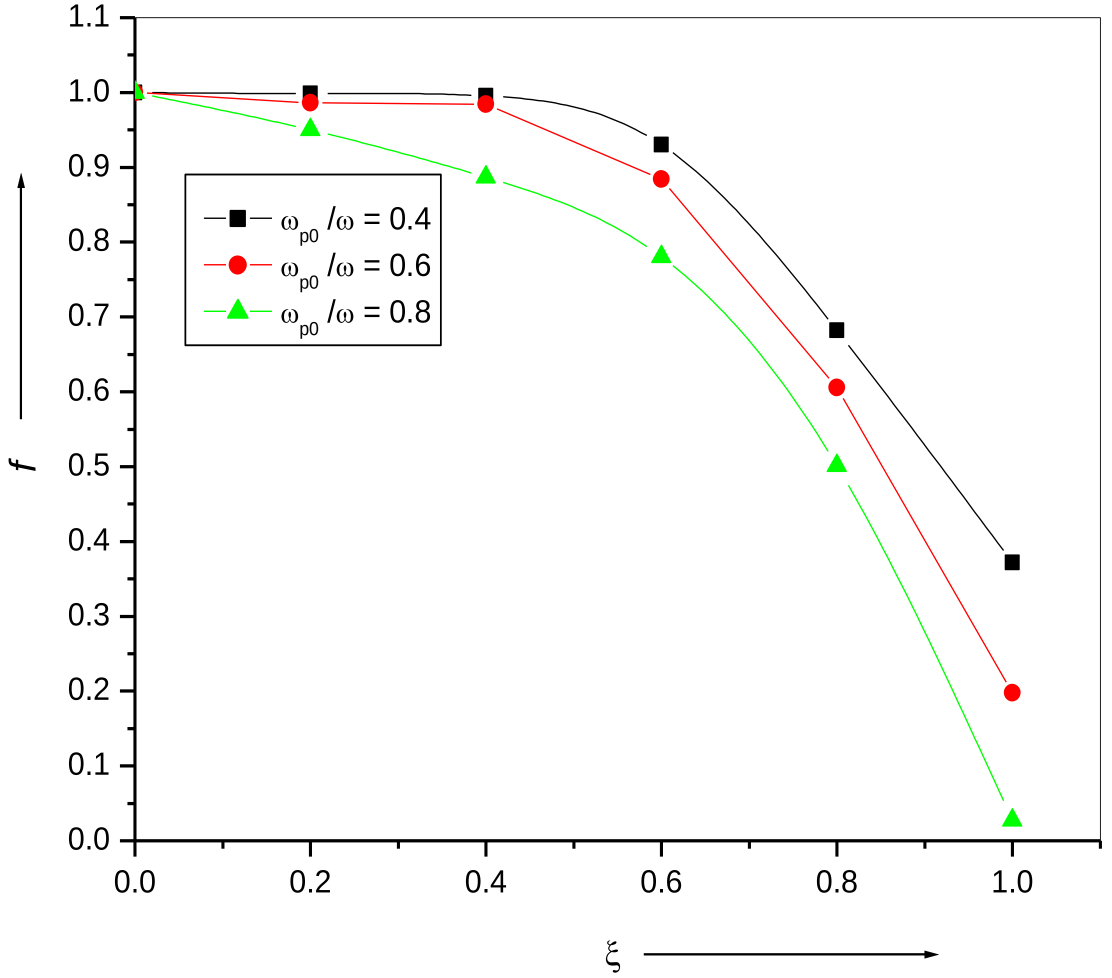

Fig.3.3(a) Variation of beam width parameter (f)*of the q-Gaussian laser beam with normalized propagation distance*(ξ) *for various values of* ω_{p0}/ω*. The other parameters are* $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10$ *and* $q = 5$.

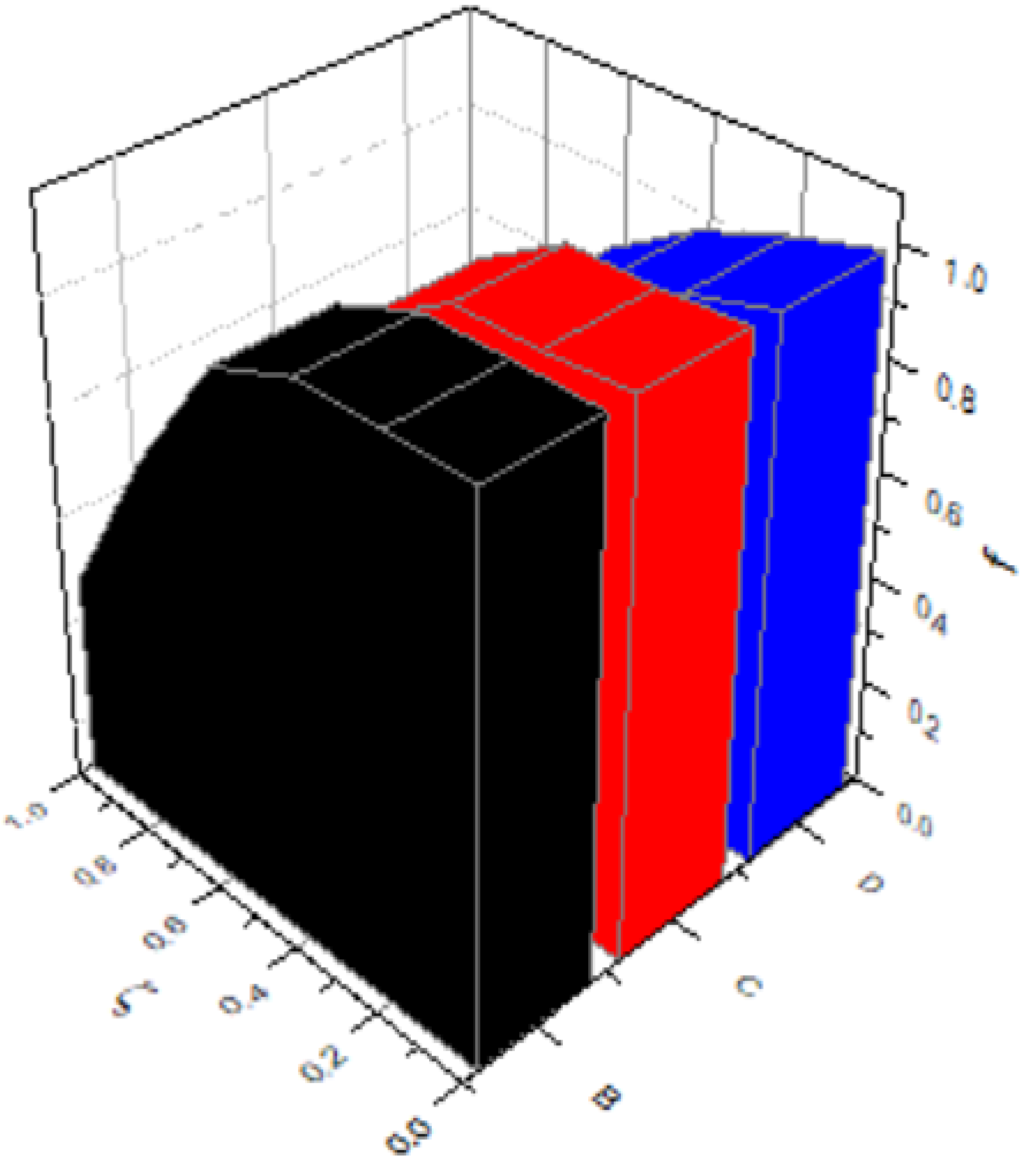

Fig.3.3(b) is the 3-D representation for the variation of beam width parameter (f)*of the q-Gaussian laser beam with normalized propagation distance*(ξ) *for various values of* ω_{p0}/ω*, where B, C, D represents the values of* ω_{p0}/ω *such that B=0.4, C=0.6 and D=0.8. The other parameters are* $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10$ *and* $q = 5$.

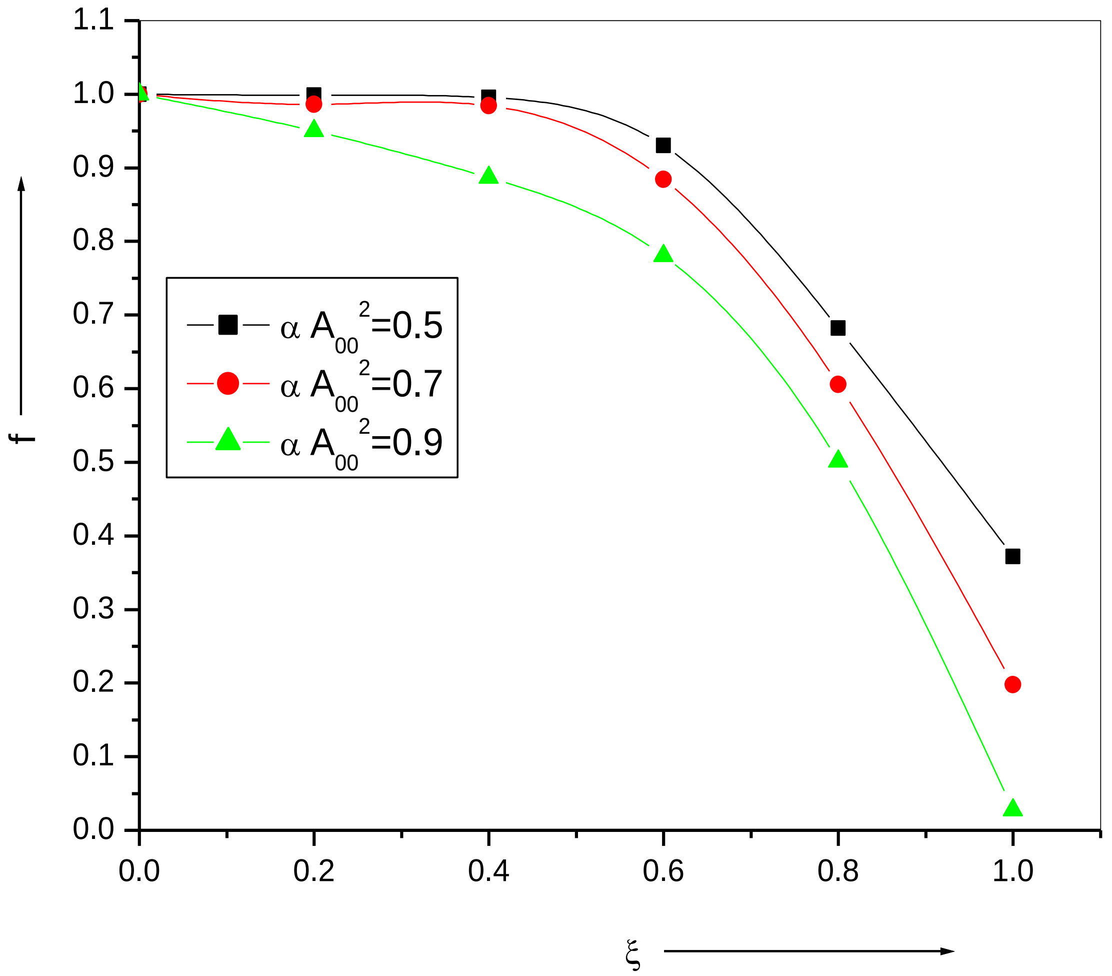

Fig.3.4 Change of beam width parameter (f)*of the q-Gaussian laser beam with normalized propagation distance*(ξ) *for various values of* αA_{00}^2. *The other parameters are* $\omega_{p0}/\omega = 0.8, \omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10$ *and* $q = 5$.

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Chapter 4

Combined Effect of Plasma Density Ramp and Wiggler Magnetic Field on Self-Focusing of q-Gaussian Laser Beam in Underdense Plasma

4.1 Introduction:

Laser-plasma interaction have been extensively studied over the last few decades. Recently, dielectric laser accelerators and terahertz-driven accelerators have been emerging. In addition to that, lasers find extensive use in numerous high-end applications, including medical treatments and nuclear fusion. [1–8]. q-Gaussian laser beams are a type of laser beam that has a non-Gaussian intensity profile and are described by a q-exponential distribution. Unlike Gaussian beams, which have a bell-shaped intensity distribution, q-Gaussian beams have a more complex, power-law-like distribution. These beams are characterized by a parameter called q, which controls the shape of the beam profile and exhibit a non-homogeneous spatial intensity distribution, with the highest energy concentration located at the beam's center. This non-uniformity in intensity creates self-focusing and self-trapping of the beam in plasma [9]. For numerous years, researchers have been investigating the theoretical and practical aspects of the dynamic relationship between q-Gaussian laser beams and plasma [10]. The investigation of q-GLB in plasma was initiated in the 1990s after researchers discovered that the non-Gaussian intensity profile of q-Gaussian beams had the potential to improve self-focusing in plasma. Researchers have also investigated the potential use of q-Gaussian laser beams in laser-plasma particle accelerators, where high-intensity laser beams can generate a wave of charged particles that can be used to accelerate other particles to high energies. In 1998, Esarey et al. [11] conducted a study to investigate the behavior of super-Gaussian and simple Gaussian laser beams in plasma. The aim was to gain insights into the underlying physical mechanisms that govern the behavior of these beams in a plasma medium. Their study revealed that Gaussian beams undergo self-focusing in a plasma medium, resulting in the generation of high-intensity laser pulses with significant applications in various fields, including fusion energy and medical treatments. Conversely, super-Gaussian beams experience less self-focusing and more diffraction than Gaussian beams due to their wider spatial profiles. In 2010, Kourakis et al. [12] pioneered the use of q-GLB in the scientific study of laser-plasma interactions. They showed that the self-focusing and self-trapping of q-GLB is more pronounced than that of Gaussian beams due to their non-uniform distribution of intensity. These studies have shown that the self-focusing and self-trapping of the beam is significantly affected by the existence of these external fields. Singh et al., 2015 [13] studied the flow of q-GLB in a non-linear medium with a linear density ramp. They demonstrated that the self-focusing and self-trapping of the beam is significantly enhanced under the inclusion of the density ramp. Kant and Wani[14] looked into the effects of a plasma density ramp and a linear absorbance coefficient on laser beam self-focusing and defocusing. Their investigation unveiled that absorption serves to weaken the self-focusing effect, whereas density transformation leads to the earlier but more potent self-focusing of cosh-Gaussian laser beams. The exceptional mode benefits from enhanced self-focusing, while the ordinary mode experiences a weakened effect in the presence of a magnetic field. In 2021, Amin et al.[15] studied the propagation of CO_2 laser in a plasma with a transverse magnetic field. It was seen that the transverse magnetic field causes the beam to break up into filaments which can be used to control the propagation of the beam. Since the introduction of q-Gaussian distribution in statistical mechanics, it has been found to have significant applications in various areas of physics, including plasma physics and laser physics. In the context of light-plasma interplay, q-GLB have been demonstrated to display unique self-focusing and self-trapping behavior due to their non-Gaussian spatial intensity distribution. The nonlinearity associated with the q-Gaussian distribution can lead to enhanced self-focusing of the laser beam, which can in turn result in the creation of high-energy electrons, ions, or even gamma-rays through various nonlinear mechanisms[16]. These laser beams have been widely studied in recent years due to their ability to maintain their shape over longer distances compared to traditional one [17]. Recent studies have focused[18,19] on observing the movement of the beam width parameter .The use of density ramp in plasma, where the density of plasma changes exponentially along the direction of motion, has been shown to enhance the self-action of the laser beam. This is because the ramped density can create a gradient in the refractive index, resulting in a stronger self-focusing effect [20]. In quantum plasma, self-focusing is significantly increased compared to classical relativistic cases due to beam weakness at high intensity in the latter case. In a thermal, collisionless quantum plasma, diffraction prevails, leading to beam width increase and oscillatory behavior [21]. Overall, the study of q-GLB interaction with external magnetic fields and density gradients is an active and rapidly evolving area of research which has shown that these beams have unique properties which make them well-suited for a variety of applications [22]. As the comprehension of the interaction between q-Gaussian beams and plasma advances we are likely to witness novel applications of these beams in coming years.

This manuscript proposes a new theoretical model to address a critical gap in current research related to the dynamics of q-GLB in self-generated channels within underdense plasmas, while taking into account the impact of wiggler magnetic fields in presence of exponential plasma density ramp. This is achieved through the ponderomotive mechanism, which induces pulsing motion of electrons in plasma and causes self-focusing. The organization of the paper is as below: In section 2, a mathematical formulation in underdense plasma is derived for the beam width parameter. In section 3, wiggler magnetic field and nonlinear electrical susceptibility is discussed followed by discussion of self-focusing in section 4. Section 5 is devoted to computation results, supported by graphical results while considering plasma density, wiggler magnetic field, and laser intensity. In the end, brief conclusions of the current study are presented.

4.2 Theoretical considerations

4.2.1. The q-GLB and its field distribution

In the context of plasma propagation, q-Gaussian laser beams (q-GLB) propagate along the z-direction with an x-polarized configuration. The electric field distribution associated with q-Gaussian beams traveling along the x-axis in plasma can be expressed as:

$$\vec{E}(r,z,t) = \hat{x}A(r,z,t)exp[-i(\omega t - kz)] \tag{4.1}$$

Here, A represents the amplitude of the laser field as a function of space $(r,z)$and time (t) and ω and k are the laser frequency and wave vector, respectively. The initial intensity profile of the q-GLB in plasma is given by Yadav et al. [12] in 2020 as:

$$A^2 = A_0^2\left(1 + \frac{r^2}{qr_0^2}\right)^{-q} g(t) \tag{4.2}$$

Where r_0 and t_0 are responsible for determining the size of the spot and the duration of the pulse, respectively. The term q in the equation represents the divergence of the laser intensity distribution from the standard Gaussian profile. Smaller q values indicate strongly q-Gaussian characteristics, while a Gaussian distribution is obtained when q tends towards infinity (∞) [22]. The function $g(t)$ is defined as $g(t) = exp(-t^2/t_0^2)$

For non-relativistic motion of electron, we have $m_0 = m, \gamma = 1$, The oscillatory velocity related to the electron is given by $v = eE/m\omega$.

$$A = A_0(r,z)\, exp[-ikS(r,z)] \tag{4.3}$$

where, $k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_{p0}^2 exp\left(\frac{z}{dR_d}\right)}{\omega^2\left(1 - \frac{\omega_c}{\omega}\right)}}$, The functions A_0 and S depend on the radial coordinate r and the distance z, and the eikonal of the beam, denoted by $S(r,z)$, determines whether the beam converges or diverges.

4.3. Wiggler magnetic field and Nonlinear Electrical Susceptibility

4.3.1. Nonlinear Permittivity

The focus of our research is to analyse how a q-GLB travels through a nonlinear medium, which is characterized by a dielectric constant outlined in the work of Sodha et al.[23]

$$\varepsilon = \varepsilon_0 + \Phi(EE^*) \tag{4.4}$$

where, $\varepsilon_0 = 1 - \omega_p^2/\omega^2$, is the linear part of the dielectric function having ω_p as the frequency of plasma and the intensity dependent non-linear part represented by Φ. $\omega_p^2 = \frac{4\pi n(\xi)e^2}{m}$, $n(\xi) = n_0 exp(\xi/d)$ $\xi = z/R_d$. In the current manuscript where ξ stands for normalized propagation distance, R_d represents the diffraction length $\omega_{p0}^2 = 4\pi n_0 e^2/m$, the symbol n_0 refers to the electron density at equilibrium, while the symbol ω_{p0} represents the initial plasma frequency. Additionally, the symbol d is a dimensionless parameter that is freely adjustable.

$$\varepsilon_0 = 1 - \omega_{p0}^2/\omega^2 \, exp(\xi/d) \tag{4.5}$$

To derive the expression for the part of the dielectric function that varies with intensity in a nonlinear manner, let's begin with the dielectric function $\epsilon(\omega)$ in the presence B:

$$\epsilon(\omega) = 1 - \frac{\omega_{p0}^2}{\omega^2 - \omega_c^2} \tag{4.6}$$

Here, ω_{p_0} is the unperturbed plasma frequency, ω is the angular frequency of the electromagnetic wave and ω_c is the cyclotron frequency. Introduce the concept of a wiggler magnetic field B_w in the y-z plane, given by $\boldsymbol{B}_w = B_w e_y Sin(K_w z), where$ k_w is the wiggler wave number. Consider the interaction between the electrons in the plasma and the wiggler magnetic field. This interaction leads to a modification in the dielectric function and d is a characteristic length associated with this interaction. Incorporate the effect of the modified dielectric function into the expression for the intensity-dependent behavior, which is achieved through the exponential term exp $(\xi/$d). Now, the expression for the dielectric function in the presence of the wiggler magnetic field is:

$$\epsilon(\omega) = 1 - \frac{\omega_{p0}^2}{\omega^2 - \omega_c^2 \frac{\omega_{p0}^2}{\omega^2} Sin^2(K_w z)} \tag{4.7}$$

Express the complex dielectric function as:

$$\epsilon(\omega) = 1 - \frac{c^2 k^2}{\omega^2} \tag{4.8}$$

where, k is the wave number of the electromagnetic wave. Use the relationship $k = \omega/c$ to replace k in the dielectric function expression. Express $Sin^2(k_w z)$ in terms of trigonometric identities: $2Sin^2\theta = 1 - cos2\theta$.

Substitute the trigonometric identity into the dielectric function expression and simplify. Recognize that the modified dielectric function corresponds to the real part of ϵ i.e.,

$$\Phi(EE^*) = Re(\epsilon) \frac{\omega_{p0}^2}{\omega^2 - \omega_c^2 \frac{\omega_{p0}^2}{\omega^2} Sin^2(k_w z)} \tag{4.9}$$

Rearrange the expression to match the given form:

$$\Phi(EE^*) = \frac{\omega_{p0}^2 exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega c}{\omega}\right)} \tag{4.10}$$

Where, $\omega_c = eB_W/mc$, is the cyclotron frequency , e denotes the electronic charge, $\mathbf{B}_W = B_w\left(\hat{e}_y \sin \boldsymbol{k}_w z\right)$ is the wiggler magnetic field, m denotes the electron mass and c denotes the speed of light.

4.4 Self-focusing

The equation that regulates laser beam transmission can potentially be derived to a wave equation. The self-focusing phenomenon in laser beam propagation is governed by a fascinating wave equation derived from Maxwell's equations. Let's walk through the steps to derive this equation and understand the physical significance at each stage.

We begin with Maxwell's equations, which describe the behavior of electromagnetic fields. Two of these equations are particularly relevant:

Faraday's Law:

$$\nabla \times H = \frac{1}{c}\frac{\partial D}{\partial t} \tag{4.11}$$

Ampere's Law:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial H}{\partial t} \tag{4.12}$$

$$\nabla . \mathrm{B} = 0 \tag{4.13}$$

$$\nabla \cdot D = 0 \tag{4.14}$$

Applying Curl to Ampere's Law We take the curl of Ampere's law (equation 3.5) to get:

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} \tag{4.15}$$

Simplifying the Curl Expression using vector calculus identity $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$, we can simplify equation (9) as follows:

$$\nabla\left(\nabla . \vec{E}\right) - \nabla^2 \vec{E} = -\frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} \tag{4.16}$$

Using Gauss's Law for Electric Fields, which states $\nabla \cdot D = 0$, we can substitute this into the equation

$$-\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} + \nabla\left(\nabla . \vec{E}\right) = 0 \tag{4.17}$$

Using value of $\nabla . E$ in Eq. (10), we get;

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} + \nabla\left(\frac{E \nabla . \varepsilon}{\varepsilon}\right) = 0 \tag{4.18}$$

Using $D = \varepsilon \mathrm{E}$ in (11), we obtain final Eq.

$$\nabla^2 \vec{E} - \frac{\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} + \nabla\left(\frac{\vec{E} \nabla . \varepsilon}{\varepsilon}\right) = 0 \tag{4.19}$$

Introducing ω and k^2 which leads to the resultant equation for laser beam propagation, equation (3.12), is given by:

$$\nabla^2\vec{E} + k^2\vec{E} + \nabla\left(\frac{\vec{E}\nabla.\varepsilon}{\varepsilon}\right) = 0 \tag{4.20}$$

$$\nabla^2\vec{E} + k^2\vec{E} + \left[-\vec{E}\left(\frac{\nabla\varepsilon}{\varepsilon}\right)^2 + \vec{E}\left(\frac{\nabla^2\varepsilon}{\varepsilon}\right)\right] = 0 \tag{4.21}$$

$$\nabla^2\vec{E} + \left[k^2 + \left(\frac{\nabla\varepsilon}{\varepsilon}\right)^2 + \left(\frac{\nabla^2\varepsilon}{\varepsilon}\right)\right]\vec{E} = 0 \tag{4.22}$$

$$\nabla^2\vec{E} + \left[k^2 + \nabla\left(\frac{\nabla\varepsilon}{\varepsilon}\right)\right]\vec{E} = 0 \tag{4.23}$$

$$\nabla^2\vec{E} + [k^2 + \nabla^2(\ln\varepsilon)]\vec{E} = 0 \tag{4.24}$$

$$\nabla^2\vec{E} + k^2\left[1 + \frac{1}{K^2}\nabla^2(\ln\varepsilon)\right]\vec{E} = 0 \qquad \because k^2 = \varepsilon\frac{\omega^2}{c^2} \tag{4.25}$$

Equation (4.25) has a term on the left-hand side that can be ignored under certain conditions. Specifically, we can neglect this term if $k^{-2}\nabla^2(ln\varepsilon) \ll 1$, where $k^2 = (\omega^2/c^2)\varepsilon$ represents the wave number. This condition provides a criterion to simplify the equation. Understanding this condition is important for analysing the behavior of waves and fields in different media and for designing optical systems. Thus,

$$\nabla^2\vec{E} + k^2\vec{E} = 0 \tag{4.26}$$

Hence, we can write as the wave equation that regulates laser beam propagation is expressed as follows:

$$\nabla^2\vec{E} + \left(\frac{\omega^2}{c^2}\right)\varepsilon\vec{E} + \nabla\left(\frac{\vec{E}\nabla.\varepsilon}{\varepsilon}\right) = 0 \tag{4.27}$$

$$\nabla^2\vec{E} + \left(\frac{\omega^2}{c^2}\right)\varepsilon\vec{E} = 0 \tag{4.28}$$

In cylindrical co-ordinate system, we can write this equation as

$$\frac{\partial^2\vec{E}}{\partial z^2} + \frac{\partial^2\vec{E}}{\partial r^2} + \frac{1}{r}\frac{\partial\vec{E}}{\partial \mathrm{r}} + \frac{\omega^2}{c^2}\varepsilon\vec{E} = 0 \tag{4.29}$$

For slowly converging or diverging cylindrically symmetric beam, the solution of equation (28) is of the following form,

$$\vec{\mathrm{E}}(r,\mathrm{z}) = \mathrm{A}(r,z,t)\exp[-i(\omega\mathrm{t} - kz)] \tag{4.30}$$

$where,\ k^2 = \frac{\varepsilon_0}{c^2}\omega^2 = \varepsilon\frac{\omega^2}{c^2}\left(1 - \omega_p^2 exp[i(\omega t - kz)]\right)$. By introducing the eikonal of the beam, the behaviour of a complex amplitude illustrated by the above equation needs to be expressed. To find the expression for A, we substitute the values of A_0 and S obtained from equation (3) into equation (29) and equate the real and imaginary parts on both sides of the resulting equation. This yields the following equations which are important for understanding the behavior and properties of the beam.

Real part:

$$-2\frac{\partial S}{\partial z}+\frac{S\omega^2 exp\left(\frac{z}{dR_d}\right)}{c^2k^2dR_d}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}+\frac{z\omega^2 exp\left(\frac{z}{dR_d}\right)}{c^2k^2dR_d}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\frac{\partial S}{\partial z}-\frac{zS\omega^4 exp\left(\frac{z}{dR_d}\right)}{2c^4k^4d^2R_d^2}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}+$$

$$\frac{z\omega^2 exp\left(\frac{z}{dR_d}\right)}{c^2k^2dR_d}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}-\frac{z^2\omega^4 exp\left(\frac{z}{dR_d}\right)}{4c^4k^4d^2R_d^2}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}-\left(\frac{\partial S}{\partial r}\right)^2+\frac{1}{2A_0^2k^2}\frac{\partial^2A_0^2}{\partial r^2}-\frac{1}{4A_0^4k^2}\left(\frac{\partial A_0^2}{\partial r}\right)^2+$$

$$\frac{1}{2rA_0^2k^2}\left(\frac{\partial A_0^2}{\partial r}\right)+\Phi\left(\frac{A_0^2}{\varepsilon_0}\right)=0 \qquad (4.31)$$

Imaginary part:

$$\frac{\omega^2 exp\left(\frac{z}{dR_d}\right)}{c^2k^2dR_d}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}-\frac{1}{A_0^2}\left(\frac{\partial A_0^2}{\partial z}\right)+\frac{z\omega^2 exp\left(2\frac{z}{dR_d}\right)}{c^2k^2d^2R_d^2}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}+$$

$$\frac{z\omega^4 exp\left(\frac{z}{dR_d}\right)}{4c^4k^4d^2R_d^2}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\frac{z\omega^2 exp\left(\frac{z}{dR_d}\right)}{2c^2k^2dR_dA_0^2}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\frac{\partial A_0^2}{\partial z}-\frac{\partial^2 S}{\partial r^2}-\frac{1}{A_0^2}\frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r}-\frac{1}{r}\frac{\partial S}{\partial r}=0 \qquad (4.32)$$

The solutions for a cylindrical q-Gaussian beam can be expressed in the form of equations, which describe the beam properties.

$$A_0^2=\frac{A_{00}^2}{f^2}\left(1+\frac{r^2}{qr_o^2f^2}\right)^{-q} \qquad (4.33)$$

$$S=\frac{r^2}{2}\beta(z)+\Phi(z) \qquad (4.34)$$

The quantity $\beta(z)=\frac{1}{f(z)}\frac{df}{dz}$ represents the inverse radius of curvature of the wave front, and it is defined as the ratio of the derivative of the beam's focal length with respect to z to the focal length itself. In the paraxial approximation, we derived a differential equation for the beam-width parameter by equating the coefficients of r^2 in the relevant equations and up on solving we got the required balanced equation as below.

Comparing Coefficients of r^2 from all the terms are as:

$$2\frac{\partial S}{\partial z}=\frac{1}{R_d^2}\frac{\partial^2 f}{\partial \xi^2}r^2 \qquad (4.35)$$

$$\frac{S\omega^2 exp\left(\frac{z}{dR_d}\right)}{c^2k^2dR_d}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}=\frac{1}{2f}\frac{\partial f}{\partial \xi}\frac{\omega_{p0}^2 exp\left(\frac{z}{dR_d}\right)}{c^2dR_d\left(1-\frac{\omega_c}{\omega}\right)k^2}r^2 \qquad (4.36)$$

$$\frac{z\omega^2 exp\left(\frac{z}{dR_d}\right)}{c^2k^2dR_d}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\frac{\partial s}{\partial z}=r^2\frac{S\omega^2 exp\left(\frac{z}{dR_d}\right)}{c^2k^2dR_d}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\frac{1}{2f}\frac{\partial^2 f}{\partial \xi^2}-\frac{1}{2f}\frac{\partial f}{\partial \xi} \qquad (4.37)$$

$$\frac{zs\omega^4 exp\left(\frac{z}{dR_d}\right)}{2c^4k^4d^2{R_d}^2}\left(\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)^2=r^2\frac{z}{4f}\frac{\partial f}{\partial \xi}\frac{\omega^4}{c^4k^4d^2{R_d}^2}exp\left(\frac{z}{dR_d}\right)\left(\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)^2 \qquad (4.38)$$

$$\left(\frac{\partial S}{\partial r}\right)^2=-\frac{1}{f^2}\left(\frac{\partial f}{\partial \xi}\right)^2r^2 \qquad (4.39)$$

$$\frac{1}{2A_0^2k^2}\frac{\partial^2 A_0^2}{\partial r^2}=\left(\frac{r_0^2f^2q}{1+r^2}\right)^q\left(\frac{2A_{00}^2(q+1)}{r_0^4f^6q}-\frac{A_{00}^2(q+1)}{r_0^4f^6q}\right)r^2=\left(\frac{r_0^2f^2q}{1+r^2}\right)^q\frac{A_{00}^2(q+1)}{r_0^4f^6q}r^2 \qquad (4.40)$$

$$-\frac{1}{4A_0^4k^2}\left(\frac{\partial A_0^2}{\partial r}\right)^2=\frac{1}{A_0^4k^2}\left\{\frac{A_{00}^2}{r_0^4f^8}\left(1-\frac{2(q+1)r^2}{r_0^2f^2q}\right)\right\}r^2 \tag{4.41}$$

$$\frac{1}{2A_0^2k^2}\frac{\partial A_0^2}{\partial r^2}=-\frac{1}{qr_0^4k^2f^4}r^2 \tag{4.42}$$

$$\phi(A_0^2)=\frac{\omega^2}{\omega^2-\omega_p^2}\frac{A_{00}^2}{2r_0^4f^4}\left(\alpha\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{dR_d}\right)\right)r^2 \tag{4.43}$$

Since, $\xi=z/R_d, z=\xi R_d\ and\ z/dR_d=\xi/d.$ Therefore, we can write as:

$$\frac{\partial f}{\partial z}=\frac{\partial f}{\partial \xi}\times\frac{\partial \xi}{\partial z}=\frac{1}{R_d}\frac{\partial f}{\partial \xi}\ and\ \frac{\partial^2 f}{\partial \xi^2}=\frac{1}{R_d^2}\frac{\partial^2 f}{\partial \xi^2} \tag{4.44}$$

Substituting the above values in the equation (3.31), we have established the differential equation of the beam-width parameter under the paraxial approximation as follows:

$$\left(\frac{1}{\left(R_0\frac{\omega}{c}\right)^2\left(1-\frac{\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)f}\right)\frac{\partial^2 f}{\partial \xi^2}+\frac{exp\left(\frac{\xi}{d}\right)}{2df\left(R_0\frac{\omega}{c}\right)^2\left(1-\frac{\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)^2}\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\left(\frac{\partial f}{\partial \xi}+\xi\frac{\partial^2 f}{\partial \xi^2}\right)+$$

$$\frac{\omega^4}{c^4\left\{\frac{\omega^2}{c^2}\left(1-\frac{\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)\right\}^2}\frac{exp\left(\frac{\xi}{d}\right)}{d^2\left(R_0\frac{\omega}{c}\right)^2\left(1-\frac{\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)}\left(\frac{\omega_{p0}^2}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)^2\frac{\xi}{4f}\frac{\partial f}{\partial \xi}+\frac{2A_{00}^2(q+1)(q+2)}{\left(R_0\frac{\omega}{c}\right)^2\left(1-\frac{\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)qf^8}+$$

$$\left(\frac{A_{00}^2}{2f^3}\right)\left(\frac{\alpha\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2}\right)=\frac{1}{\left(R_0\frac{\omega}{c}\right)^2\left(1-\frac{\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)f^4}\left(\frac{q+1}{q}\right)+\frac{1}{\left(R_0\frac{\omega}{c}\right)^2\left(1-\frac{\omega_{p0}^2exp\left(\frac{\xi}{d}\right)}{\omega^2\left(1-\frac{\omega_c}{\omega}\right)}\right)f^2}\left(\frac{\partial f}{\partial \xi}\right)^2$$

(4.45)

This Eq. (45) is the desired equation of beam width parameter.

4.5. Analysis and outcome

This research explores the self-focusing (S.F) behavior of a q-GLB in an exponential plasma density ramp under the influence of a wiggler magnetic field (B). To investigate this behavior, we numerically solve the equation for the beam-width parameter (Eq.45) using the Runge-Kutta method for various plasma parameters and the q-Gaussian laser beam. We apply appropriate boundary conditions and present the outcome in graphical form. The figures (Fig. 4.1-4.4) gives the change of f as a function of the ξ in an underdense plasma under the existence of a wiggler magnetic field and an exponential plasma density ramp, when the initial plane wave front of the beam is subject to the boundary conditions f=1 and $\partial f/\partial$ξ= 0 at ξ = 0 We validate our approach using computational simulations with specific parameters for numerical calculations such as $\omega r_0/c=18,\ \alpha A_{00}^2=0.9, d=10$ and $\omega_{p0}/\omega=0.8$.

Figure 1 presents the fluctuation of beam width parameter (*f*) on normalized distance of propagation (ξ) for different measures of ω_c/ω. One may obviously observe that with the presence of wiggler magnetic field, self-focusing of the laser beam enhances. This occurs when the magnetic field is oriented in such a way that it causes the plasma density to increase in regions where the q-GLB is already experiencing self-focusing. The phase shift induced by the wiggler magnetic field reinforces the self-focusing effect, leading to a stronger and earlier self-focusing behavior as compared to the result observed by Butt et al.[5].

It has been observed that the result-oriented S.F of the beam occurs at $\xi = 0.1$. But, in the case we saw stronger self-focusing for q-Gaussian laser beam occurs at $\xi = 0.8$ for $\omega_c/\omega = 0.5$. The wiggler magnetic field can also induce a phase shift that suppresses the self-focusing effect of the q-GLB. The phase shift induced by the field cancels out the self-focusing effect, leading to a weaker or delayed self-focusing behavior. Understanding the interplay between the plasma density ramp and wiggler magnetic field is important for optimizing the self-focusing behavior of q-GLB in plasma-based applications. This can help to maximize the concentration of laser energy in a small region of the plasma, leading to more efficient energy transfer and higher acceleration rates.

Fig. 4.2 shows the relationship between the focal length (f) and the dimensionless parameter (ξ) for a constant q value, while the intensity parameter is varied. The other parameters are consistent with Fig. 1. The results suggest that as the laser intensity parameter increases, there is stronger and earlier self-focusing. The most intense focusing occurs at $\alpha A_{00}^2 = 0.8.$ It is because at this value, the balance between the diffraction and nonlinear terms is optimal, leading to the most intense self-focusing effect.

Thus, the results of Fig. 4.2 demonstrate the significant impact of the laser intensity parameter on the self-focusing process of a laser beam propagating through a plasma and shows that for lower values of the intensity parameter, the self-focusing effect is weaker. However, as the intensity parameter increases, the nonlinear term becomes more significant, and the laser beam self-focuses earlier in the propagation process. The results are in line with the results reported by Nanda et al. [24].

Figure 3 presents the modification of beam width parameter (f) on normalized distance of propagation (ξ) for different measures of ω_{P_0}/ω. Rest of the parameters are same as; $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10$. One may obviously observe that, self-focusing of q-Gaussian laser beam laser beam improves and happens sooner with the rise in the relative density parameter ω_{P_0}/ω. Here, in the current analysis, strong self-focusing of the q-Gaussian laser beam occurs at $\xi = 0.8.$ The ponderomotive force expels electrons from the high intensity region into the low plasma density region, while the electron mass variation from the intense laser and the variation in electron density due to wake field development cause the plasma to be nonlinear. Without an exponential plasma density ramp, the beam-width parameter monotonically decreases up to a Rayleigh length due to nonlinear factors. However, when a wiggler magnetic field is introduced, it enhances the transverse focusing effect on the motion of electrons, leading to an increase in plasma density and a stronger nonlinearity in the plasma. This, in turn, strengthen the self-focusing effect of the laser beam. Additionally, the magnetic field also modifies the ponderomotive force acting on electrons, further affecting the dynamics of self-focusing. Strong self-focusing of the cosh Gaussian laser beam under exponential density was studied by Thakur et al.[2] which supports our results.

Figure 4 depicts the self-focusing of the q-Gaussian laser beam for different values of q, where the other parameters are held constant at $\omega r_0/c = 18$, $\alpha A_{00}^2 = 0.9 \; and \;\; d = 10$ .A visible change in self-focusing is seen for smaller q-values like $q = 1, 3, \; and \; 5$. Our numerical simulations showed that increasing the value of q resulted in stronger self-focusing of the laser beam. As seen in the figure, the self-focusing effect is more pronounced for higher values of q, indicating that the laser beam exhibits stronger nonlinearity and is more susceptible to self-focusing. Overall, our study highlights the complex interplay between exponential plasma density ramp, wiggler magnetic field, and the value of q in determining the self-focusing of q-Gaussian laser beams in underdense plasma. These findings are of great significance for the development of laser-based particle accelerators and high-energy physics research supporting the results of Valkunde et al.[25].

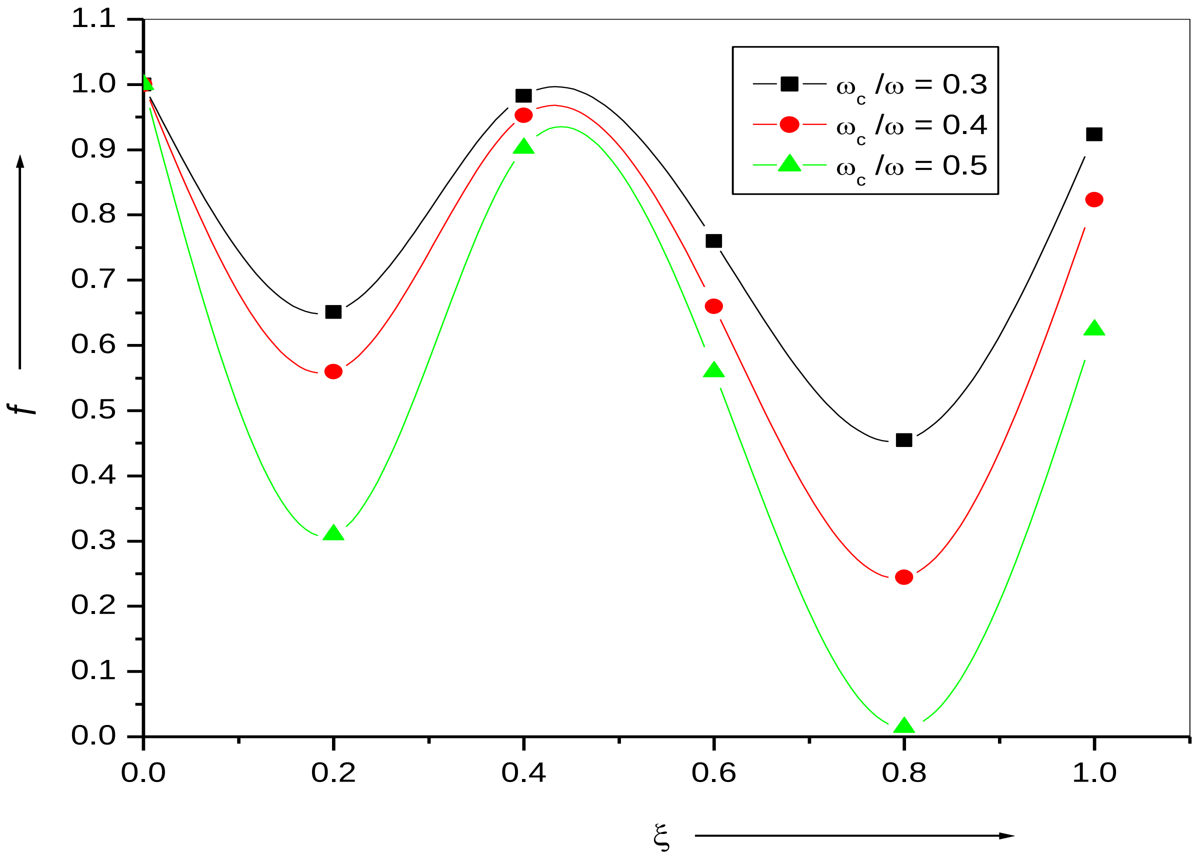

Fig.4.1 Fluctuation of beam width parameter of the pump laser beam with normalized propagation distance at different values of ω_c/ω *. The other parameters are* $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9,\ d = 10$ *and* $\omega_{p0}/\omega = 0.8$.

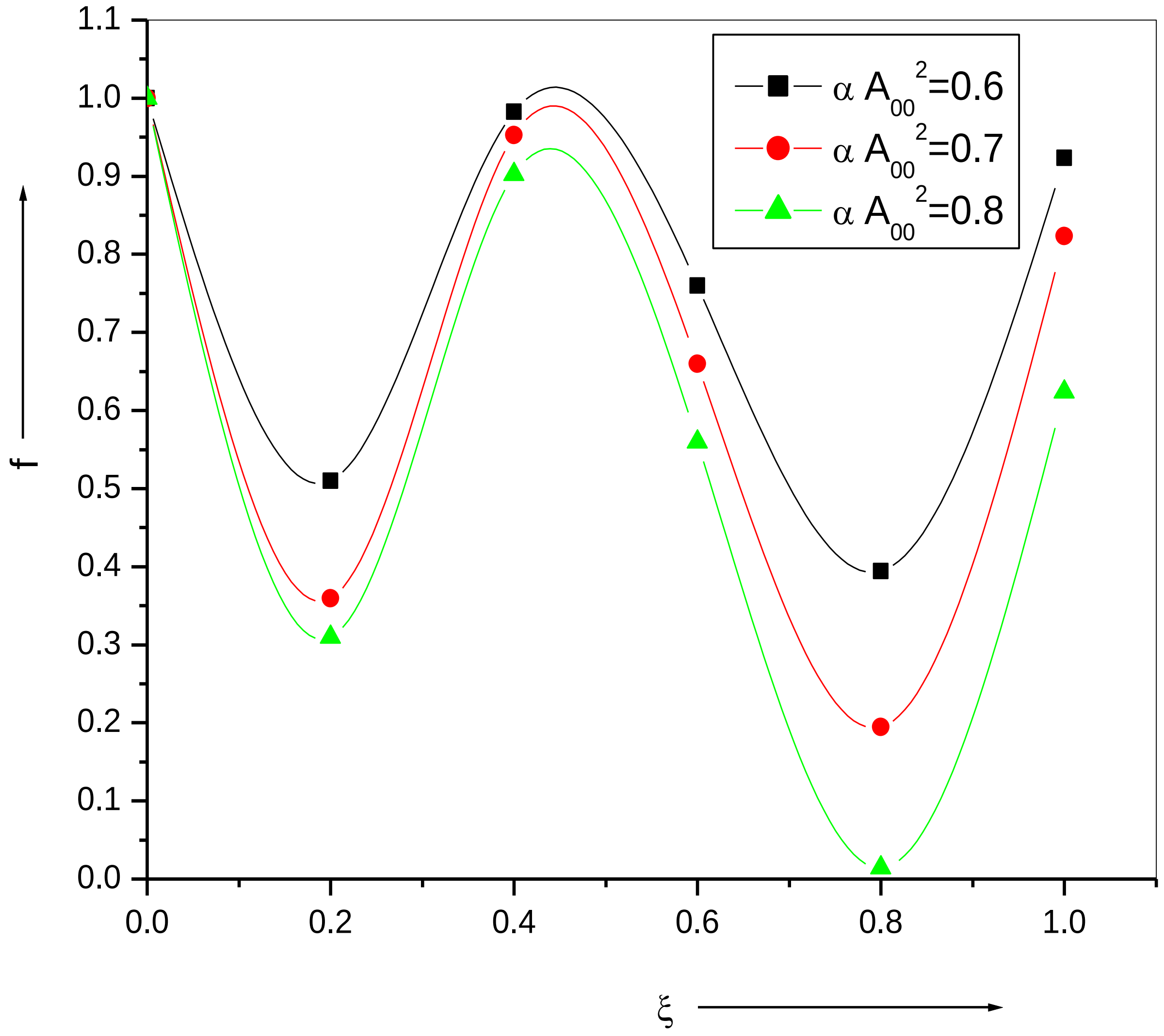

Fig.4.2 Fluctuation of beam width parameter of the pump laser beam with normalized propagation distance at different values of αA_{00}^2. *The other parameters are* $\omega r_0/c = 18$, $\alpha A_{00}^2 = 0.9, d = 10$ *and* $\omega_{p0}/\omega = 0.8$.

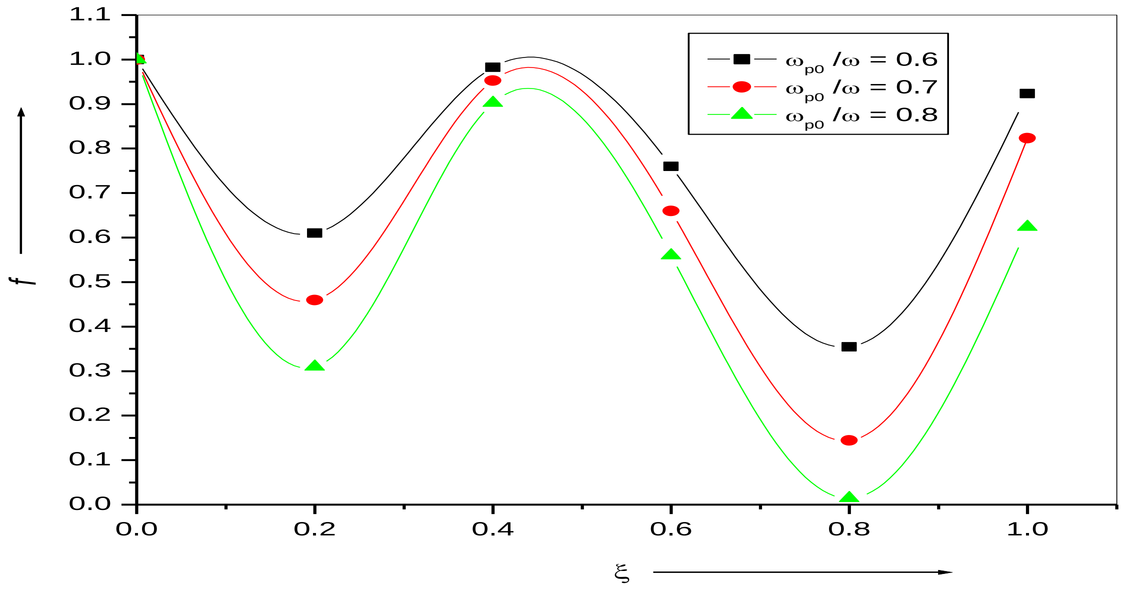

Fig.4.3 Variation of beam width parameter of the pump laser beam with normalized propagation distance at different values of ω_{p0}/ω *. The other parameters are* $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10$.

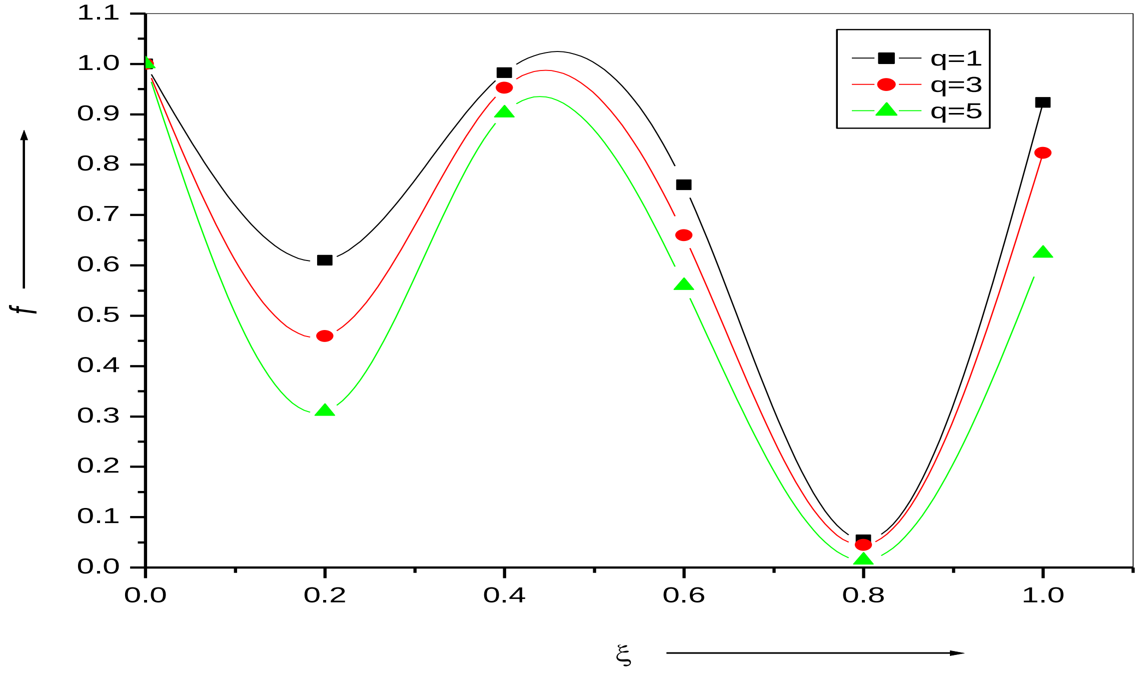

Fig.4.4 Variation of beam width parameter of the pump laser beam with normalized propagation distance at different values of q. The other parameters are $\omega r_0/c = 18,\ \alpha A_{00}^2 = 0.9, d = 10$.

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Chapter 5

Enhanced self-focusing of q-Gaussian laser beam in underdense plasma under the combined effect of density ramp and axial magnetic field

5.1 Introduction:

Laser-plasma interaction stands as a prominent approach to generate energetic particles [1–5] and THz radiations [6–9]. To enhance this interaction, it becomes imperative to augment the laser's penetration within the plasma medium while minimising divergence. An essential technique employed for achieving an increased propagation length up to several Rayleigh lengths in plasma is through the utilization of self-focusing of laser beam. In the year 1975,Hora introduced, the concept of the relativistic self-focusing effect [10]. As a high-intensity laser beam passes through plasma, it causes the electron to oscillate. This motion alters the plasma's refractive index by acting like a positive lens (converging lens) and reducing the beam width [11,12]. As the beam continues its path through the plasma, the reduction in beam width leads to an increase in spatial diffraction. Eventually, at a certain point where the beam width reaches its minimum, the defocusing effect caused by diffraction becomes stronger which causes the beam width to increase again. This interplay between focusing and defocusing creates oscillations in the beam width. Numerous researchers have dedicated their efforts to investigating the behavior of laser beams in plasma under various conditions [13–16]. However, it is worth to acknowledge that quantum effects become noticeable in plasmas characterized by low temperatures and high densities. The pioneering theoretical contributions on quantum plasmas were made by Klimontovich et al. [17]and Bohm [18]. In the realm of quantum plasma, nonlinear phenomena exhibit greater intensity compared to their classical counterparts. As a result, when a laser beam travels through quantum plasma, it undergoes oscillations with higher frequency and lower amplitude. This leads to a more pronounced laser self-focusing effect when compared to the classical regime [19]. As the T_f increases, the effects of quantum nature become increasingly significant. Thus, in the thermal quantum plasma, where the T_f is considerably higher, nonlinear phenomena and S.F of laser become even stronger [20]. The investigation of quantum plasma necessitates the development of new or refined models that differ from classical approaches [21,22]. Various mathematical models, including Wigner-Poisson, Schrödinger-Poisson, and quantum hydrodynamics, have been utilized to understand the characteristics of quantum plasma. The Wigner-Poisson model focuses on the statistical behavior of plasma particles, while the Schrödinger-Poisson model examines the hydrodynamic aspects. On the other hand, the quantum hydrodynamic model incorporates quantum mechanics through the inclusion of the Bohm potential, which modifies the wave properties [23].

Extensive investigations have been conducted on laser self-focusing in plasma, considering various laser pulse shapes [24–26]. These shapes include cosh-Gaussian, q-Gaussian, Hermite-cosh-Gaussian, Airy-Gaussian, and chirping Gaussian beams [27-32]. Additionally, the effects of plasma temperature, density, electron-ion collision frequency, and magnetic field on self-focusing have been explored by numerous researchers [33–35]. In this study, our focus is on analysing the propagation q-Gaussian laser beam in underdense plasma, utilizing the simple approximation method. This study aims to explore the impact of an axial magnetic field and a plasma density ramp on the self-focusing of a laser beam and the distribution of beam intensity in underdense plasma. The main objective is to enhance the laser intensity in the focal region and improve the interaction between lasers and underdense plasma. By employing the governing equation for laser beam propagation in plasma and utilizing the paraxial and Wentzel-Kramers-Brillouin (WKB) approximations, we derive an equation that describes the laser beam width along the direction of propagation. Initially, we investigate how the presence of an axial magnetic field and a plasma density ramp affect the divergence or convergence of the laser beam. Subsequently, we examine the relationship between the beam width parameter and the normalized propagation distance. Additionally, we analyse the fluctuations of the beam width parameter in both the forward and reverse magnetic field along the propagation direction for different beam intensities.

5.2 Conceptual mathematics

In plasma, an x-polarized q-GLB makes its way in z-direction. The electric field distribution of q-Gaussian beams moving towards the x-axis in a nonlinear medium is as under:

$$\vec{\mathrm{E}}(r,\mathrm{z}) = \hat{\mathrm{x}}\mathrm{A}(r,z,t)\exp[-i(\omega\mathrm{t} - kz)] \quad (5.1)$$

where A denotes the laser field amplitude as a relation of space $(r,z)$and time (t), $\hat{\mathrm{x}}$ represents the unit vector in the x-axis direction, ω represents the frequency of laser, and the wave vector is signified by k. $\vec{\mathrm{E}}(r,\mathrm{z})$ is the electric field associated with the q-GLB.

The q-GLB initial intensity profile with at $\mathrm{z} = 0$ is supported by M. Yadav et al. [12] in 2020 as

$$A^2 = A_0^2\left(1 + \frac{r^2}{qr_0^2}\right)^{-q} g(t) \quad (5.2)$$

where, $g(t) = exp(-t^2/t_0^2)$, A_0 indicates the amplitude of q-Gaussian laser beam, the constants r_0 and t_0 control the size of the laser spot and the duration of the laser pulse where, $t_0 = 40fs$. The parameter q in this case reflects how far the distribution of laser intensity deviates from a typical Gaussian profile [37]. When q is less than one, the laser intensity displays significant q-Gaussian features. As q approaches infinity, the laser intensity distribution resembles a conventional Gaussian distribution. We have $m_0 = m = 1$ for non-relativistic electron motion. $v = eE/m$ denotes the oscillation velocity connected to the electron.

5.3 Non-linear dielectric constant and Axial Magnetic field

To determine the dielectric constant for the transmission of a q-GLB through a nonlinear medium caused by ponderomotive nonlinearity, the method used by Sodha and colleagues in their research paper [38] was applied.

$$\varepsilon = \varepsilon_0 + \Phi(EE^*) \tag{5.3}$$

Where ε_0 and $\Phi(EE^*)$ are the linear and intensity dependent measure of dielectric function. The non-linear measure of dielectric function is varying according to physical circumstances but its dependence on intensity of input beam is a common feature. For collisionless magnetized plasma ε_0 and $\Phi(EE^*)$ can be written as[39]

$$\varepsilon_0 = 1 - \frac{\omega_{p0}^2 exp(\xi/d)}{\omega^2(1-\sigma\omega_c/\omega)} \tag{5.3a}$$

$$\Phi(EE^*) = \frac{\omega_{p0}^2 exp(\xi/d)}{2\omega^2(1-\sigma\omega_c/\omega)} 1 - exp[-\alpha\, A_{00}^2] \tag{5.3b}$$

where, the parameter A_{00} represents the peak amplitude or intensity of the laser beam. It describes the maximum electric field strength of the laser beam. α is a coefficient that determines the strength of the nonlinear response. The term $exp[-\alpha\, A_{00}^2]$ represents the exponential decay factor in the equation and accounts for the nonlinear effects and saturation mechanism involved in the self-focusing of a q-Gaussian laser beam in plasma. It helps regulate the self-focusing process and prevents it from becoming unbounded as the laser intensity increases, thereby maintaining a stable and controlled self-focused beam. When the relative motion of plasma ions and electrons produces a restoring electric field ($E = 4\pi n_0 ed$), each electron moves in a simple harmonic motion with the plasma frequency ($\omega_p = 4\pi e^2 n_0/m$). In this formula d, n_0, m and e stand for charge layer displacement from its original location, electron density, electron rest mass, and electronic charge respectively. The presence of stationary ions results in a positive surface charge on one end of the plasma slab and a negative surface charge on the next, under two conditions i.e. (i) electron density (constant) and (ii) fluctuating electron density. Each electron is subjected to an electric force ($F = -4\pi n(\xi)e^2 d$) that is directed towards its equilibrium location. The dynamics of an electron can be explained by the equation $md = (-4\pi n_0 e^2)d$, while the refractive index is given by $1 - \omega_p^2/\omega^2$. If the value of ω is lesser than the ω_p , attenuation occurs because the index of refraction is totally fictitious. Conversely, if ω is larger than the plasma frequency (ω_p), the index of refraction is real. We utilized an exponential plasma density ramp plasma density ramp profile, denoted as $n(\xi) = n_0 \exp(\xi/d)$ in presence of axial magnetic field in under dense plasma. The letter $\xi =$

z/R_d is the normalised propagation distance, d is a dimensionless parameter that may be adjusted and R_d is the diffraction length. In this study, we have introduced a ramped density in plasma and a magnetization that changes the refractive index to $\epsilon_0 = 1 - \omega_p^2/\omega^2(1 - \sigma\omega_c/\omega)$, where ω_c is the cyclotron frequency and is given by $\omega_c = eB_0/mc$. $\boldsymbol{B_0}$ is the magnetic field which is assumed to be applied along the forward direction as well as in reverse (opposite) direction of the propagation of laser beam. σ is the parameter associated with magnetization like $\sigma = +1$ for forward, $\sigma = -1$ for reverse magnetization and 0 for unmagnetized case. The plasma frequency $\omega_p^2 = 4\pi n(\xi)e^2/m$, varies along the z-axis and can be expressed as $\omega_p^2 = 4\pi e^2 n_0 \exp(\xi/d)/m = \omega_{p0}^2 \exp(\xi/d)$. The parameters in the components of the dielectric constant are represented as $\alpha = 3m\alpha_0/4M$, $\alpha_0 = e^2/6m^2\omega^2 k_B T_0$. M indicates a scatterer mass present in a plasma. The symbols, ω, k_B, T_0 denote the frequency of the incident laser, the Boltzmann constant, and the equilibrium temperature of the plasma, respectively.

5.4. Self-focusing

Using an approach like that employed by Sodha et al. [8], the wave equation for a changing field was obtained from Maxwell's equations. In simplest terms, it is an equation that defines how a wave behaves in a medium with changing characteristics such as electric and magnetic fields across time and space. The equation has a generic form, which is represented as below:

$$\nabla^2 E + \left(\frac{\omega^2}{c^2}\right)\varepsilon E + \nabla\left(\frac{E\nabla.\varepsilon}{\varepsilon}\right) = 0 \tag{5.4}$$

Equation (5) has a term on the left-hand side (L.H.S) that can be ignored due to certain conditions. Specifically, we can neglect this term if $k^{-2}\nabla^2(ln\varepsilon) \ll 1$ where $k^2 = (\omega^2/c^2)$ represents the wave number. This condition provides a criterion to simplify the equation. Understanding this condition is important for analysing the behavior of waves and fields in different media and for designing optical systems. Thus,

$$\nabla^2\vec{E} + \left(\frac{\omega^2}{c^2}\right)\varepsilon\vec{E} = 0 \tag{5.5}$$

In cylindrical co-ordinate system, we can write this equation as

$$\frac{\partial^2\vec{E}}{\partial z^2} + \frac{\partial^2\vec{E}}{\partial r^2} + \frac{1}{r}\frac{\partial\vec{E}}{\partial r} + \frac{\omega^2}{c^2}\varepsilon\vec{E} = 0 \tag{5.6}$$

For slowly converging or diverging cylindrically symmetric beam, the solution of equation (16) is of the following form,

$$\vec{E}(r, z) = A(r, z, t)\exp[-i(\omega t - kz)] \tag{5.7}$$

where, $$k = \omega/c\sqrt{1 - \omega_p^2(\xi)/\omega^2(1 - \sigma\omega_c/\omega)}$$

$$= \omega/c\sqrt{1 - \omega_{p0}^2 exp(\xi/d)/\omega^2(1 - \sigma\omega_c/\omega)}$$

Differentiating equation (22) twice w. r. t, we get “z" and "r”

$$\frac{\partial \vec{E}}{\partial \mathrm{r}} = \exp[-i(\omega \mathrm{t} - kz)]\frac{\partial \mathrm{A}}{\partial \mathrm{r}} \tag{5.8}$$

$$\frac{\partial^2 \vec{E}}{\partial r^2} = \exp[-i(\omega \mathrm{t} - kz)]\frac{\partial^2 A}{\partial r^2} \tag{5.9}$$

$$\frac{\partial \vec{E}}{\partial \mathrm{z}} = \exp[-i(\omega \mathrm{t} - kz)]\left[-\frac{i\omega A}{c}\sqrt{1-\omega_{p0}^2 exp(\xi/d)/\omega^2(1-\sigma\omega_c/\omega)} + \left(\frac{i\omega}{2cdR_d}\right)\frac{\omega_{p0}^2}{\omega^2}\ \frac{zAexp(z/dR_d)}{\sqrt{1-\omega_{p0}^2 exp(\xi/d)/\omega^2(1-\sigma\omega_c/\omega)}} + \frac{\partial A}{\partial \mathrm{z}}\right] \tag{5.10}$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \exp[-i(\omega \mathrm{t} - kz)]\left[\frac{\partial^2 A(r,z)}{\partial Z^2} + \left(\frac{i\omega}{c}\right)\frac{\partial A(r,z)}{\partial \mathrm{Z}}\left(\frac{\omega_{p0}^2 zexp\left(\frac{z}{d\mathrm{R}_d}\right)}{\omega^2 dR_d\sqrt{1-\omega_{p0}^2 exp(\xi/d)/\omega^2(1-\sigma\omega_c/\omega)}} - 2\sqrt{1-\omega_{p0}^2 exp(\xi/d)/\omega^2(1-\sigma\omega_c/\omega)}\right)\right] + \frac{\omega \mathrm{A}}{c}\exp[i(\omega \mathrm{t} - kz)]\left[\frac{i\omega_{p0}^2 zexp\left(\frac{z}{d\mathrm{R}_d}\right)}{\omega^2 dR_d\sqrt{1-\omega_{p0}^2 exp(\xi/d)/\omega^2(1-\sigma\omega_c/\omega)}} + \frac{\omega_{\mathrm{p0}}^2 \mathrm{Z} exp\left(\frac{z}{d\mathrm{R}_d}\right)}{c\omega^2 dR_d}\right] + \frac{\omega}{c}\exp[i(\omega \mathrm{t} kz)]\frac{iA\omega_{p0}^2 zexp\left(\frac{z}{d\mathrm{R}_d}\right)}{\omega^2 d^2 R_d^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)}}\left[exp\left(\frac{z}{d\mathrm{R}_d}\right) + \frac{\omega_{p0}^2 exp\left(\frac{z}{d\mathrm{R}_d}\right)}{4\omega^2\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)\right)}\right] - \frac{\omega^2 \mathrm{A}}{c^2}\exp[i(\omega \mathrm{t} - kz)]\left[\frac{\omega_{p0}^4 z^2 exp\left(\frac{z}{d\mathrm{R}_d}\right)}{4\omega^2 d^2 R_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)\right)} + \left(1-\frac{\omega_{p0}^2}{\omega^2}exp\left(\frac{z}{d\mathrm{R}_d}\right)\right)\right] \tag{5.11}$$

Substituting the values derived from equations (3.24), (3.25), (3.26), and (3.27) into equation (3.21), while neglecting the terms involving $\frac{\partial^2 A}{\partial z^2}$, the application of the WKB approximation leads to a simplified form of a parabolic wave equation as follows:

By introducing the eikonal of the beam, the preceding equation illustrates the behaviour of a complex amplitude, which may be stated as follows:

$$\mathrm{A} = A_0(r,z)\exp[-ikS(r,z)] \tag{5.12}$$

A_0 and S depend on the radial coordinate r and the distance z, and the eikonal of the beam, denoted by $\mathrm{S}(r,z)$, determines whether the beam converges or diverges. To find the expression for A, we substitute the values of A_0 and S obtained from equation (8) into equation (7) and equate the real part on both sides of the equation. This method is similar to that used by Sodha et al.[38] we get

Real part:

$$2\frac{\partial s}{\partial z}+\left(\frac{\partial s}{\partial r}\right)^2=\frac{1}{k^2A_0}\left(\frac{\partial^2 A_0}{\partial r^2}+\frac{\partial A_0}{\partial r}\right)+\frac{1}{k^2A_0}\left[1-\left(4-5\frac{\omega_p^2}{\omega^2}\right)\frac{\omega^2}{k^2c^2}-\frac{4}{\epsilon_0}\frac{\omega^2}{k^2c^2}\phi(EE^*)\right] \tag{5.13}$$

The solutions for a cylindrical q-Gaussian beam can be described in the shape of equations (10) and (11), which explain the beam's properties.

$$A_0^2=\frac{A_{00}^2}{f^2}\left(1+\frac{r^2}{qr_o^2f^2}\right)^{-q} \tag{5.14}$$

$$S=\frac{r^2}{2}\beta(z)+\Phi(z) \tag{5.15}$$

The variable $\beta(z)=\frac{1}{f(z)}\frac{df}{dz}$ indicates the inverse radius of curvature of the wave front and it is defined as the ratio of the derivative of the beam's focal length with respect to z to the focal length itself. We generated a differential equation for the f (beam width parameter) in the paraxial approximation by equating the coefficients of r^2 in the pertinent equations as follows:

$$\frac{\partial^2 f}{\partial \xi^2}+\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2=\frac{1}{qf^3}(2q+5)+\frac{f}{2A_{00}}\left(1-\frac{\omega^2(4-5\omega_p^2/\omega^2)}{K^2C^2}\right)-\frac{2\omega^2}{c^2K^2A_{00}}\frac{\omega_p^2(1-exp[-\alpha A_{00}^2])}{2\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)\left(1-\frac{\omega_p^2}{\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)}\right)} \tag{5.16}$$

Substituting the value of k^2, we get

$$\frac{\partial^2 f}{\partial \xi^2}+\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2=\frac{1}{qf^3}(2q+5)+\frac{f}{2A_{00}}\left(1-\frac{(4-5\omega_p^2/\omega^2)}{\left(1-\frac{\omega_p^2}{\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)}\right)}\right)-\frac{\omega_p^2(1-exp[-\alpha A_{00}^2])}{\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)\left(1-\frac{\omega_p^2}{\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)}\right)^2} \tag{5.17}$$

Normalizing the parameters, we obtain the required equation of beam width parameter.

$$\frac{\partial^2 f}{\partial \xi^2}+\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2=\frac{1}{qf^3}(2q+5)+\frac{f}{2A_{00}}\left(1-\frac{(4-5\omega_p^2/\omega^2)}{\left(1-\frac{\omega_p^2}{\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)}\right)}\right)-\frac{\omega_p^2(1-exp[-\alpha A_{00}^2])}{\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)\left(1-\frac{\omega_p^2}{\omega^2\left(1-\sigma\frac{\omega_c}{\omega}\right)}\right)^2} \tag{5.18}$$

Eq. (3.18) is the required equation of beam width parameter.

5.5 Results and discussion:

To analyse the development of the q-Gaussian beam profile, we need to solve a differential equation numerically, considering appropriate boundary conditions. Using this technique, we can determine how the beam width parameter (f) changes in relation to normalized propagation distance (ξ). The Runge-Kutta method has been used to solve the equation (Eq. 3.18), and we considered several parameters in an underdense plasma to visually display our findings in Figure 1-6. The beam starts out with a flat wavefront and is bounded by the criteria $f=1$ and

$\partial f / \partial \xi = 0$ at $\xi = 0$. We carried out simulations with certain settings for the numerical computations to confirm the accuracy of our methodology. The values of these parameters were $\omega_c/\omega = 0.8$, ω_{p0}/ω = 0.9 and $\sigma = 0, \pm 1$.

In plasma physics, the behavior of laser beams in a plasma is influenced by a variety of factors, including the interaction of the laser with the plasma's magnetic field. When considering the promotion or defocusing of a laser beam, the direction of the axial magnetic field plays a significant role. When a laser beam propagates through a plasma with a forward axial magnetic field (in the same direction as the laser propagation), it tends to undergo self-focusing as shown in fig.1. This is due to a phenomenon called the relativistic self-focusing effect. The interaction between the laser beam and the forward magnetic field causes the laser to experience a focusing force which causes self-focusing. This force arises from the interaction of the laser's electric field with the plasma electrons, which are displaced by the Lorentz force caused by the axial magnetic field. As a result, the laser beam focuses into a smaller region. Exploring different values of αA_{00} (parameter representing the degree of non-paraxiality) in the plot of beam width parameter vs normalized distance illustrates that self-focusing of q-Gaussian beam is affected by αA_{00}. Higher values of αA_{00} promote stronger self-focusing effects, leading to a reduction in the beam width and enhanced confinement of the laser energy in the underdense plasma. On the other hand, when a laser beam propagates through a plasma with a reverse axial magnetic field (opposite to the direction of propagation), it tends to undergo defocusing as shown in fig.2. This is due to a phenomenon called the magnetic self-channelling instability. The interaction between the laser beam and the reverse magnetic field results in the generation of plasma waves, known as the Raman forward-scattering instability. These plasma waves act as a scattering mechanism for the laser beam, causing its energy to disperse over a larger area and leading to defocusing. The physics behind the defocusing and self-focusing of a q-gaussian laser beam in plasma with reverse and forward axial magnetic fields, respectively, is influenced by the generation of plasma waves and the relativistic self-focusing effect. The direction of the magnetic field determines the nature of the interaction between the laser beam and the plasma, leading to either defocusing or self-focusing behavior. Comparing the self-focusing behavior of q-Gaussian laser beams with different axial magnetization directions ($\sigma = +1, 0 \; and \; -1$) in the plot of beam width parameter with respect to normalized distance demonstrates that the direction of axial magnetization influences the self-focusing process. The self-focusing effect is most pronounced for $\sigma = +1$ and least pronounced for $\sigma = -1$, while $\sigma = 0$ exhibits intermediate self-focusing behavior as observed in fig. 3. This suggests that the axial magnetization direction can be utilized to control and manipulate the self-focusing dynamics of q-Gaussian laser beams in underdense plasmas. Plotting the beam width parameter vs normalized propagation distance for different values of ω_{p0}/ω in an underdense plasma reveals that self-focusing of q-Gaussian laser beams occurs more prominently for higher values of ω_{p0}/ω. This can be attributed to the stronger interaction between the laser beam and plasma, leading to enhanced self-focusing effects.

Examining the relationship between the beam width parameter and normalized distance for various values of ω_c/ω demonstrates that self-focusing of q-Gaussian laser beams is influenced by ω_c/ω. Higher values of ω_c/ω lead to increased self-focusing due to the stronger coupling between the laser beam and the plasma electrons via the electron cyclotron resonance. From fig. 6, Varying the value of q (parameter describing the degree of non-Gaussianity of the laser beam) in the graph reveals that self-focusing of q-Gaussian beams is influenced by q. For certain values of q, the laser beam exhibits enhanced self-focusing behavior compared to Gaussian beams, indicating the potential for tighter focusing and better confinement.

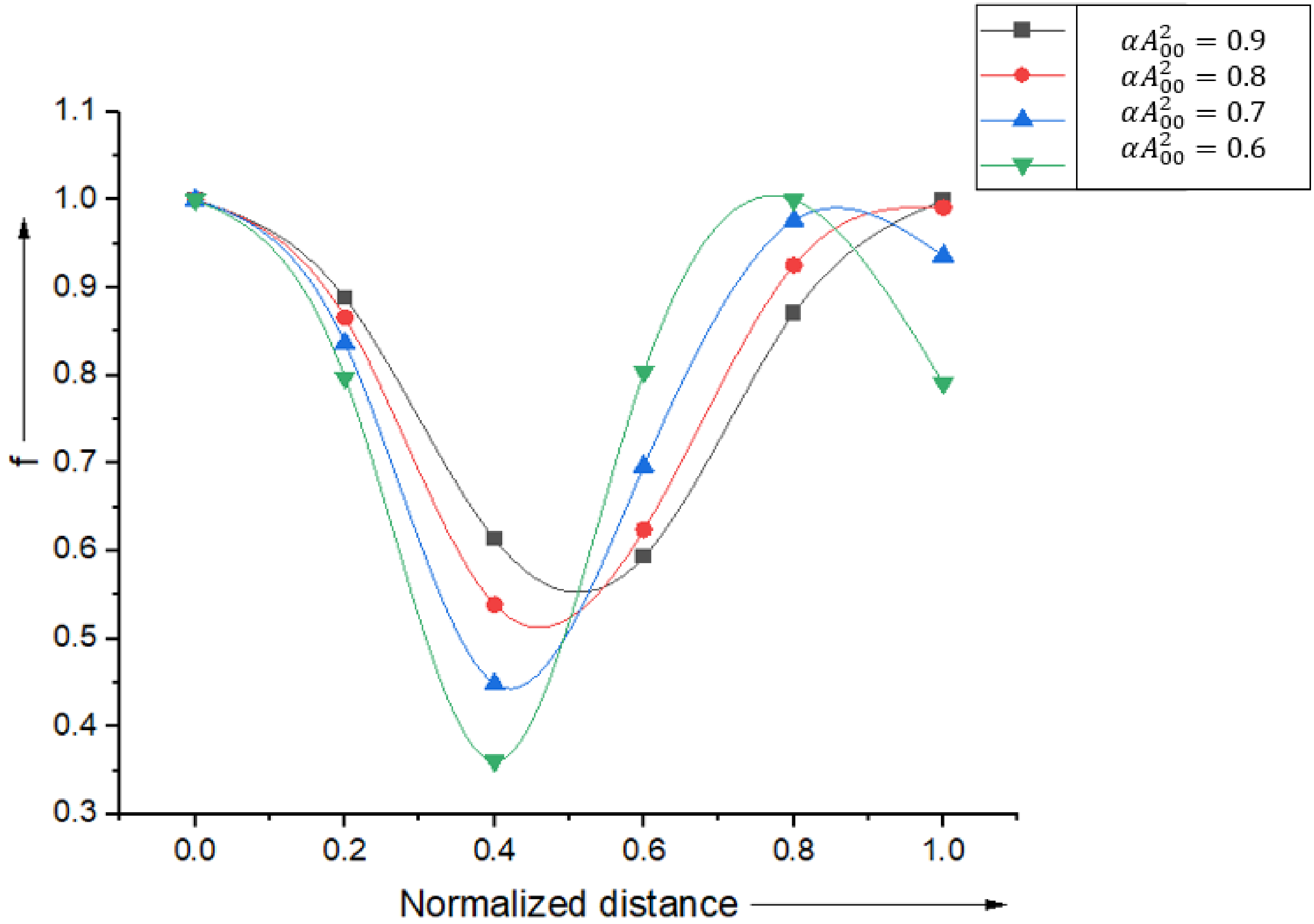

Fig.5.1 Variation of beam width parameter (f) of the pump laser beam with normalized propagation distance at different values of αA_{00}^2 *normalized intensity of incident laser. The other parameters are and* $\omega_{p0}/\omega = 0.8$, $\sigma = +1$

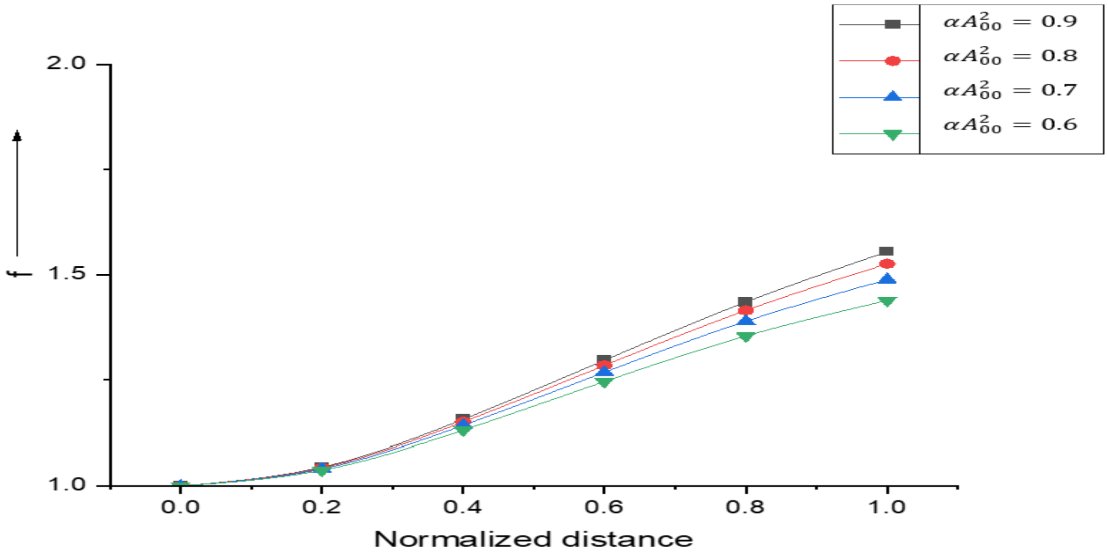

Fig.52 Variation of beam width parameter (f) of the pump laser beam with normalized propagation distance at different values of αA_{00}^2 *normalized intensity of incident laser. The other parameters are and* $\omega_{p0}/\omega = 0.9$, $\omega_c/\omega = 0.8$ *and* $\sigma = -1$.

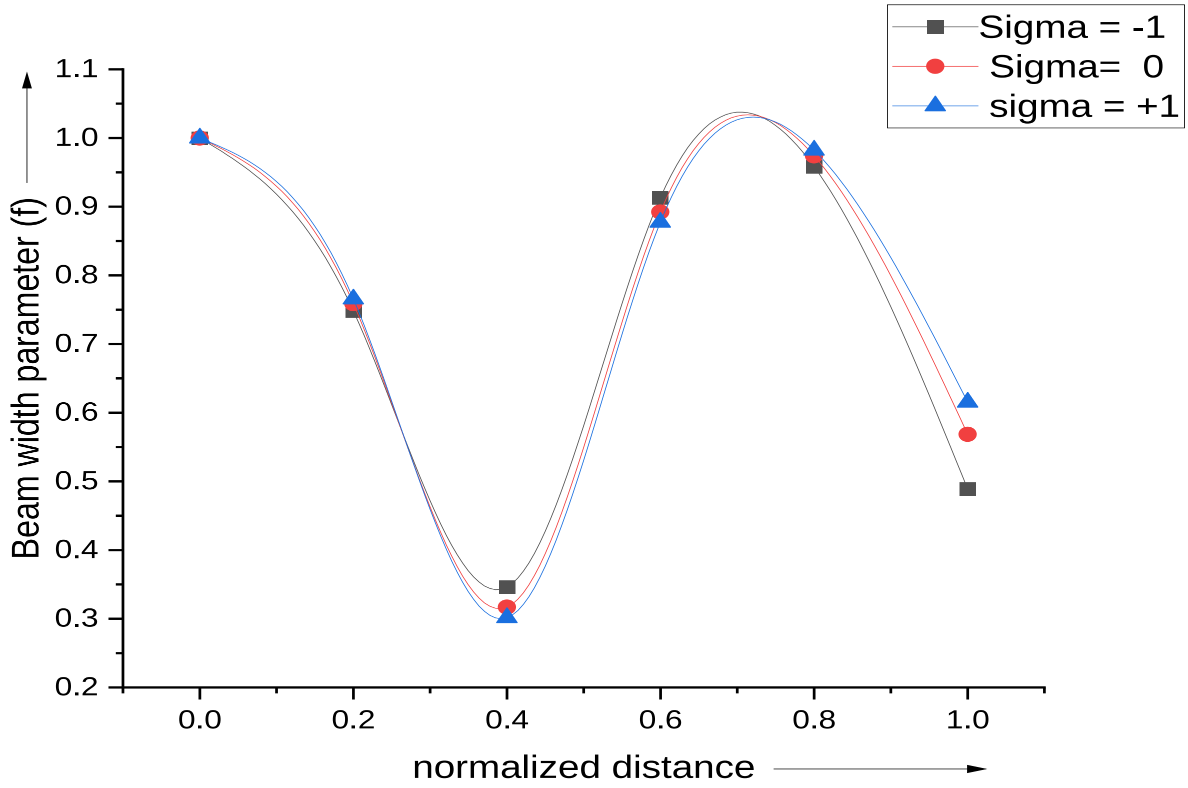

Fig.53 Variation of beam width parameter (f) of the pump laser beam with normalized propagation distance at different values of σ.The other parameters are normalized intensity of incident laser $\alpha A_{00}^2 = 2.9$ $\omega_{p0}/\omega = 0.9$, $\omega_c/\omega = 0.8$ *and* $\sigma = 0, \pm 1$.

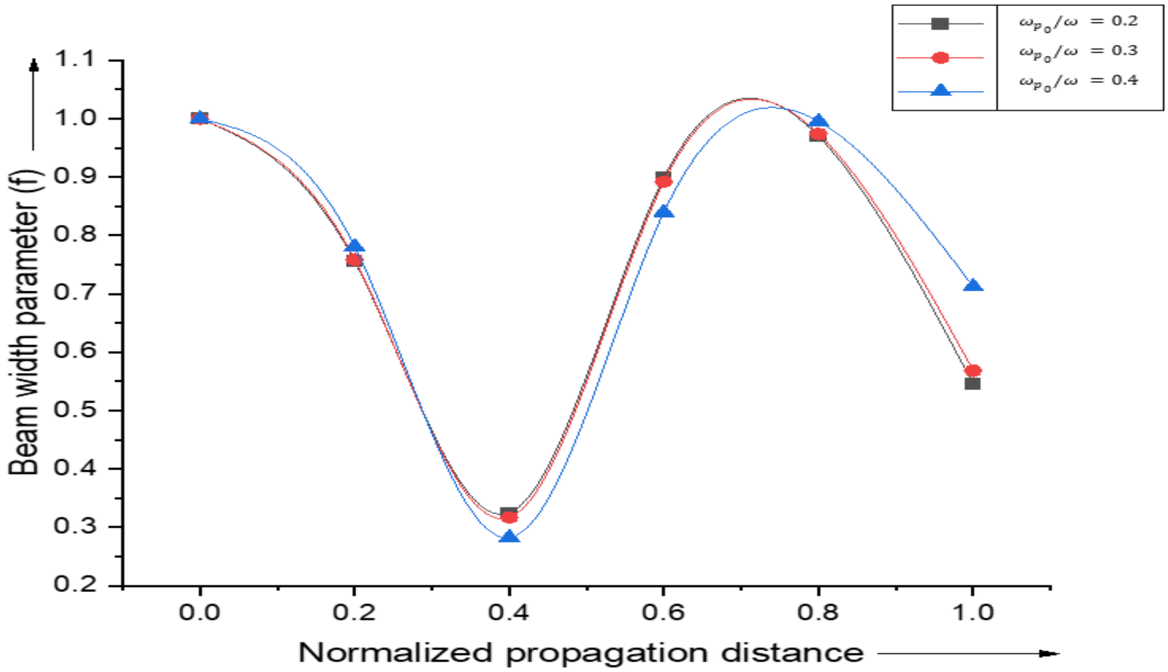

Fig.5.4 Variation of beam width parameter (f) of the pump laser beam with normalized propagation distance at different values of ω_{p0}/ω. The other parameters are at optimum values $\alpha A_{00}^2 = 2.9,\ \omega_c/\omega = 0.8$ *and* $\sigma = +1$.

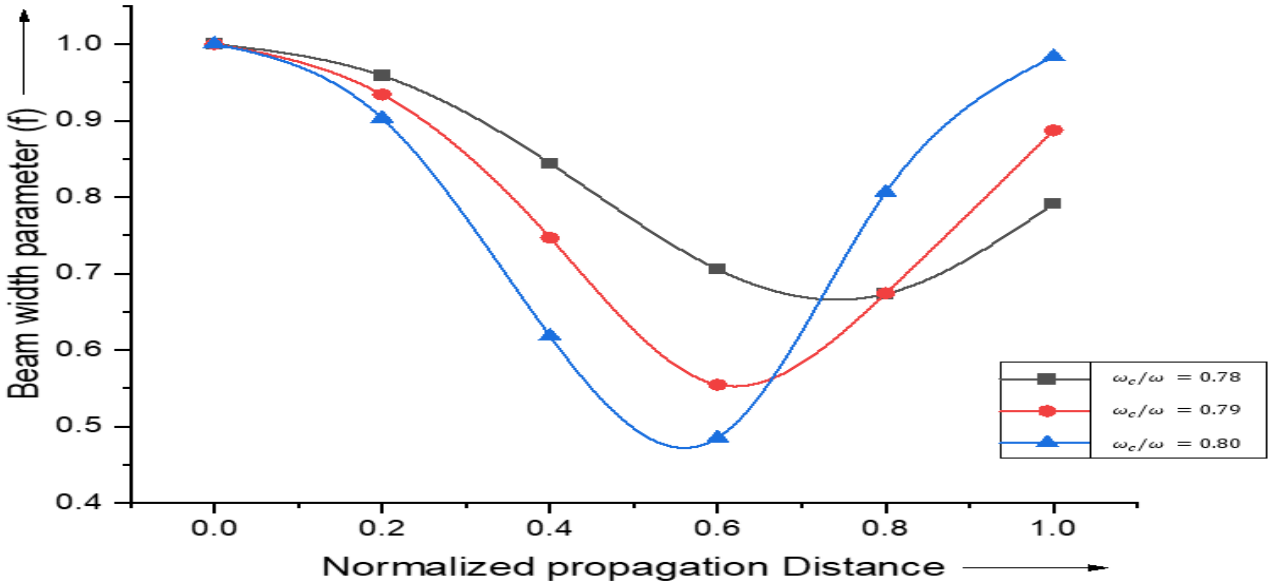

Fig.5.5 Variation of beam width parameter (f) of the pump laser beam with normalized propagation distance at different values of ω_c/ω. The other parameters are $\alpha A_{00}^2 = 2.9$, $\omega_{p0}/\omega = 0.9$ *and* $\sigma = +1$.

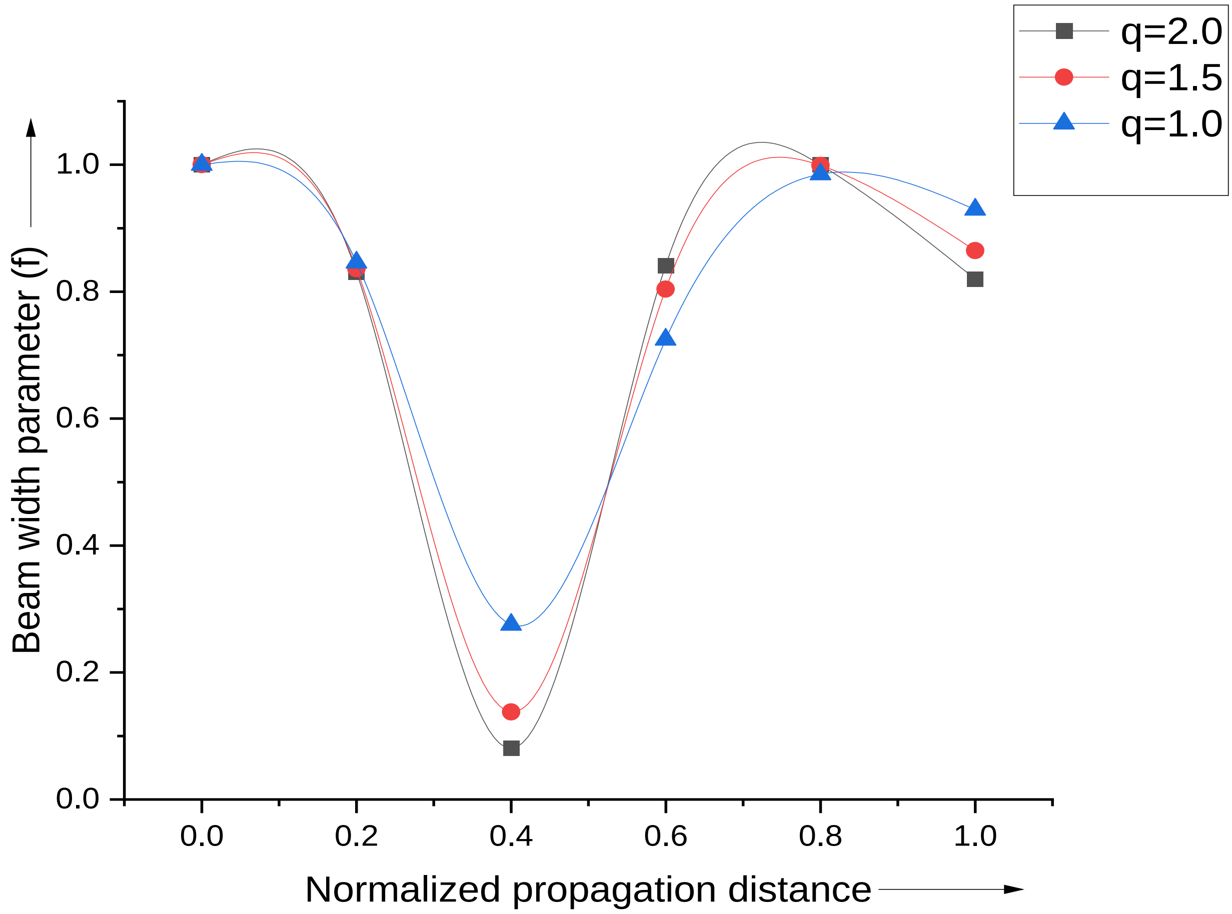

Fig.5.6 Variation of beam width parameter (f) of the pump laser beam with normalized propagation distance at different values of q. The other parameters are $\alpha A_{00}^2 = 2.9$, $\omega_{p0}/\omega = 0.9$, $\omega_c/\omega = 0.8$ *and* $\sigma = +1$.

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CHAPTER 6

6.1 Conclusions

In chapter-3, the study investigated self-focusing behavior of a q-Gaussian laser beam (q-GLB) in plasma with the influence of an exponential plasma density ramp. The q-GLB exhibited oscillatory self-focusing and defocusing behavior during propagation. The introduction of an exponential plasma density ramp effectively countered the defocusing tendency, enabling the q-GLB to achieve a minimal spot size and retain it up to several Rayleigh lengths. It was observed that self-focusing increased with increasing q-values and laser intensity, with lower q-values leading to stronger self-focusing. Numerical simulations provided valuable insights into the dynamics of self-focusing and harmonic generation in the presence of the exponential plasma density ramp. The results highlighted the importance of the plasma density ramp in enhancing self-focusing and beam confinement, which is crucial for applications such as laser-induced fusion, charged particle acceleration, and atmospheric propagation of laser beams. The exponential plasma density ramp effectively localized the interaction of the laser pulse with plasma and minimized losses during propagation, resulting in improved self-focusing action. Additionally, the exponential density ramp played a key role in optimizing laser focusing, preventing the beam from defocusing to its maximum capability. Comparisons between q-GLB with and without the exponential plasma density ramp revealed significant improvements in self-focusing with the latter, indicating the importance of this ramp for precise and efficient self-focusing of the laser beam. The findings also suggested that the q-Gaussian laser beam holds promise for various exciting applications, particularly for electron acceleration and fast igniting strategies.

Overall, this work provides valuable insights into the behavior of q-Gaussian laser beams in plasma, shedding light on the potential benefits of using exponential plasma density ramps to enhance self-focusing and optimize laser performance. The results of this study have implications for engineering, applied optics, laser-induced fusion and other laser-based technologies. With the increasing interest in high-intensity laser-plasma interactions, these findings contribute to advancing our understanding of nonlinear phenomena and their potential applications in cutting-edge research and practical applications.

In the comprehensive study of chapter-4, we delved into the fascinating behavior of q-Gaussian laser beams in underdense plasma. Our investigation focused on the combined influence of two key parameters: the exponential plasma density ramp and the wiggler magnetic field, on the self-focusing ability of the laser beam. The results revealed a remarkable synergy between these factors, leading to improved self-focusing and enhanced beam quality.

The introduction of an exponential plasma density ramp offered a profound advantage by creating a gradient in plasma density. This gradient modified the refractive index of the plasma, enabling better control over the laser beam propagation. Concurrently, the wiggler magnetic field added another dimension to the phenomenon. It generated a periodic variation in the magnetic field, leading to high-frequency radiation and influencing the self-focusing behavior of the laser pulse. The interplay between the plasma density ramp and the wiggler magnetic field produced fascinating effects. The magnetic field provided a guiding channel for the laser beam, preventing its spreading due to diffraction. Furthermore, the transverse ponderomotive force induced by the wiggler magnetic field pushed plasma electrons away from the laser beam axis, creating an ion channel that trapped the laser beam and strengthened self-focusing. Through rigorous numerical simulations, we explored the evolution of the beam width parameter in this complex scenario. We observed that increasing the q parameter of the q-Gaussian laser beam led to enhanced self-focusing in magnetized plasma. Interestingly, lower q-values exhibited strong self-focusing, indicating the nonlinear nature of the phenomenon.

Our findings hold significant implications for various laser-based applications. Improved self-focusing is of great importance in advanced particle accelerators, laser-induced fusion energy research, and sustainable energy systems. Moreover, this research can contribute to the development of efficient X-ray generation for medical imaging and treatment.

In conclusion, the study of q-Gaussian laser beams in underdense plasma, considering the influence of exponential plasma density ramps and wiggler magnetic field provides exciting new avenues for laser technology and high-energy physics research. The unique self-focusing behavior observed in this interplay opens up new possibilities for precise beam control and efficient energy transfer, leading to groundbreaking advancements in laser applications across diverse fields.

In chapter-5, we explored the mutual effect of an exponential plasma density ramp and an axial magnetic field on the self-focusing behavior of q-Gaussian laser beams (q-GLB) in underdense plasma. To understand this complex phenomenon, we derived the equation governing the beam width parameter under the WKB and paraxial ray approximation. The results revealed a remarkable interplay between these optical parameters, significantly enhancing self-focusing and leading to a narrower beam with higher intensity at the focal spot. This improvement in self-focusing allows for more efficient energy transfer to the plasma, which is of great importance in various applications, including inertial confinement fusion and advanced laser-plasma interactions. One key finding was the influence of the q-parameter, which characterizes the degree of non-Gaussianity in the laser beam. It plays a crucial role in shaping the behavior of the q-Gaussian laser beams and the formation of filaments in the beam.

Additionally, the q-parameter helps mitigate the negative effects of diffraction and dispersion, resulting in greater precision and efficiency in laser-plasma interactions.

Further investigation focused on the direction of the axial magnetic field and its impact on self-focusing. A forward magnetic field (in the same direction as the laser propagation) was found to enhance self-focusing due to the relativistic self-focusing effect. This interaction caused the laser beam to experience a focusing force, leading to its confinement into a smaller region. In contrast, a reverse axial magnetic field (opposite to the direction of propagation) resulted in defocusing due to the magnetic self-channelling instability. Here, plasma waves generated by the interaction between the laser beam and the reverse magnetic field acted as scattering mechanisms, causing the laser energy to disperse over a larger area.

We explored various scenarios by varying parameters such as the intensity of the incident laser, the direction and strength of the magnetic field and the degree of non-Gaussianity. The findings demonstrated how these factors influenced the self-focusing behavior of q-Gaussian laser beams. Higher values of certain parameters, such as the axial magnetic field strength and the q-parameter, led to stronger self-focusing effects. The research highlighted the importance of the axial magnetization direction in controlling and manipulating self-focusing dynamics. For example, a forward axial magnetic field promoted more pronounced self-focusing, while a reverse magnetic field weakened the effect. This observation opens up new possibilities for tailored laser-plasma interactions based on magnetic field configurations.

In conclusion, this study sheds light on the intricate behavior of q-Gaussian laser beam in underdense plasma, considering the combined influence of an exponential plasma density ramp and an axial magnetic field. The enhanced self-focusing observed in this scenario holds great promise for improving energy transfer efficiency and precision in laser-based applications. This research paves the way for further innovation and exploration in the field of laser-plasma interactions, offering new avenues for advancing laser technology and high-energy physics.

CHAPTER 7

Future Scope

This comprehensive study delves deep into the fascinating self-focusing behavior of q-Gaussian laser beams as they interact with underdense plasma. It goes to great lengths to consider the intricate impact of exponential plasma density ramps, wiggler magnetic fields, and axial magnetic fields, shedding new light on this complex phenomenon. By meticulously accounting for these multifaceted factors, this research not only advances our current understanding but also paves the way for a multitude of promising future endeavours. It beckons researchers to embark on a captivating journey towards uncharted discoveries and profound insights, awaiting those who explore the captivating world of this scientific inquiry. Prospective avenues include the development of dynamic control strategies for the plasma density ramp in response to changing conditions, the exploration of nonlinear phenomena and higher-order effects, and the extension of the study to multi-dimensional scenarios to understand self-focusing across diverse plasma geometries. Additionally, the optimization of laser and plasma parameters, experimental validation, and application of self-focusing principles to advanced fields like compact accelerators and fusion offer exciting prospects. Investigating synergistic effects between external factors and integrated simulations can refine predictive models, while advancements in plasma manipulation techniques and collaborations with high-power laser facilities could lead to practical implementations and groundbreaking discoveries. Embracing these opportunities, the future scope of this research holds the potential to shape laser-based technologies, energy research, and high-energy physics, yielding transformative insights into self-focusing phenomena and their applications.

APPENDIX / LIST OF PUBLICATIONS

1) A.A. Butt, N. Kant, V. Thakur, Self-focusing of a q-Gaussian laser beam under the influence of an exponential plasma density ramp in plasma, Phys. Scr. (2023). Doi: 10.1088/1402-4896/acc617. (Published)
2) A.A. Butt, V. Sharma, N. Kant, V. Thakur, Combined effect of plasma density ramp and wiggler magnetic field on self-focusing of q-Gaussian laser beam in underdense plasma, Journal of Optics. (2023). Doi: 10.1007/s12596-023-01359-8 (Published)
3) A.A. Butt, V. Sharma, N. Kant, V. Thakur, enhanced self-focusing of q-Gaussian laser beam in underdense plasma under the combined effect of density ramp and axial magnetic field, Journal of optics (2023) (Published)
4) Vishal Thakur, Danish Nazir, Niti Kant, Vinay Sharma, Anees Akber Butt*, The Effect of Density Ramp and Magnetic Field on Self-Focusing of q-Gaussian Laser Beam in Underdense Plasma, Journal of optics. (Published)
5) Relativistic Self-focussing of a q-Gaussian Laser Beam in Magnetized Plasma, Vishal Thakur, Vinay Sharma, Anees Akber But, Danish Nazir* (Accepted)

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PAPER

Self-focusing of a q-gaussian laser beam under the influence of an exponential plasma density ramp in plasma

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Keywords: optical energy, self-focusing, self-trapping, q-gaussian laser beam, beam width parameter, exponential plasma density ramp

Abstract

The goal of the current manuscript is to investigate how a plasma density ramp affects the ability of a q-Gaussian laser beam (q-GLB) to self-focus in plasma. The laser beam exhibits oscillatory self-focusing and defocusing behaviour with increasing distance of propagation. We used an exponential plasma density ramp to combat this defocusing tendency, allowing the particular laser beam to achieve a minimal spot size and nearly retain it up to several Rayleigh lengths. In addition to that, it has been observed that self-focusing increases with increasing q-values and the laser intensity. However, the lower values of q suggest strong self-focusing. By warily selecting the plasma and laser parameters, the density ramp could play a key role in the self-focusing of q-Gaussian laser. Self-focusing is seen to become stronger as propagation distance increases and the q-parameter affects the behaviour of the beam-width parameter in the plasma as shown.

1. Introduction

Due to a wide range of potential applications, including laser-driven acceleration, x-ray lasers, harmonic generation, etc, the self-focusing of a powerful laser beam has drawn tremendous interest in the last few decades [1–6]. The propagation of powerful laser pulses ($I > 10^{17} W/cm^2$) in extremely dense plasmas has seen a sudden spike in demand as a result of the rapid igniter concept for inertial confinement fusion [7]. For quite some time, this topic has been researched in regards to particle acceleration driven by lasers [8]. The primary distinction between the two applications is in their points of interest. Strong pulse propagation itself is the key area of

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RESEARCH ARTICLE

Combined effect of plasma density ramp and wiggler magnetic field on self-focusing of q-Gaussian laser beam in underdense plasma

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Abstract This study has been conducted to examine the combined impact of a wiggler magnetic field and an exponential plasma density ramp on a q-Gaussian laser beam (q-GLB) in underdense plasma. Our observations revealed that the implementation of the wiggler magnetic field in the presence of exponential plasma density ramp led to a reduction in the beam width parameter of q-GLB and enhances self-focusing by pushing plasma electrons away from the beam axis. The wiggler magnetic field also combats the defocusing tendency of plasma density ramp, hence enabling q-GLB to attain a minimal spot size and retain it up to several Rayleigh lengths. It has been revealed that the self-focusing in magnetized plasma is affected by the q parameter, with higher values leading to stronger self-focusing. The implications of these findings have potential to revolutionize a diverse range of laser-based applications including advanced particle accelerators, ICF, and sustainable energy systems.

Keywords Self-focusing · q-Gaussian laser beam · Wiggler magnetic field · Energy efficiency

Introduction

The captivating realm of laser-plasma interaction, particularly the intriguing phenomenon of self-focusing, has been an engrossing subject of exploration and research over the past few decades. Its allure stems from its pervasive presence in a myriad of high-end applications, ranging from cutting-edge medical treatments to the awe-inspiring realms of nuclear fusion. Within the realms of this captivating interplay, a plethora of possibilities unfolds [1–9]. q-Gaussian laser beam is a type of laser beam that has a non-Gaussian intensity profile and is described by a q-exponential distribution. Unlike Gaussian beam which has a bell-shaped intensity distribution, q-Gaussian beams have a more complex and power-law-like distribution. These beams are characterized by a parameter called q, which controls the shape of the beam profile and exhibit a non-homogeneous spatial intensity distribution, with the highest energy concentration located at the beam's centre. This non-uniformity in intensity creates self-focusing and self-trapping of the beam in plasma [10]. For numerous years, researchers have been investigating the theoretical and practical aspects of the dynamic relationship between q-Gaussian laser beams and plasma [11–13]. The investigation of q-GLB in plasma was initiated in the 1990s after researchers discovered that the non-Gaussian intensity profile of q-Gaussian beam had the potential to improve self-focusing in plasma. Researchers have also investigated the potential use of q-Gaussian laser beam in laser-plasma particle accelerators, where high-intensity laser beam can generate a wave of charged particles that can be used to accelerate other particles to high energies. Devi et al. [14, 15] conducted a study in which they investigated the role of magnetic field on self-focusing of super-Gaussian laser beam under relativistic effect when it propagates in plasma. The aim was to gain insights into the underlying physical mechanisms that govern the behaviour of this beam in a plasma medium. Their study revealed that self-focusing is better when cyclotron effects are taken into account. At higher intensity and higher spot size of the laser

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RESEARCH ARTICLE

Enhanced self-focusing of q-Gaussian laser beam in underdense plasma under the combined effect of density ramp and axial magnetic field

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Abstract In this manuscript, we observed the synergistic potential of an exponential plasma density ramp and an axial magnetic field to enhance the self-focusing of q-Gaussian laser beams (q-GLB). Our study includes the derivation of the equation governing the beam width parameter, using the Wentzel-Kramers-Brillouin (WKB) and paraxial ray approximations for a comprehensive analysis. It is found that the forward magnetic field enhances self-focusing, while the reverse magnetic field weakens it in underdense plasma when compared to unmagnetized case. The combination of these optical parameters significantly improves self-focusing, resulting in a narrower beam with higher intensity at the focal spot. This leads to more efficient energy transfer to the plasma. The q-parameter determines the non-Gaussian degree and influences the formation of filaments in the beam. This approach can also mitigate the negative effects of diffraction and dispersion, leading to greater precision and efficiency in laser-plasma interaction. In the realm of laser-plasma interactions, the study's findings, have direct applications in enhancing the precision and efficiency of high-power q-Gaussian laser systems, offering advancements in laser-driven fusion, plasma heating, and particle acceleration. By shedding light on the non-Gaussian behaviour of the system and the combined influence of the external factors, this research opens up new avenues for innovation and exploration in the field.

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Keywords Self-focusing · Axial magnetic field · Energy efficiency · q-Gaussian laser beam · Underdense plasma · Inertial confinement fusion

Introduction

Laser-plasma interaction stands as a prominent approach to generate energetic particles [1–5] and THz radiations [6–9]. To enhance this interaction, it becomes imperative to augment the laser's penetration within the plasma medium while minimizing divergence. An essential technique employed for achieving an increased propagation distance up to several Rayleigh lengths in plasma is through the utilization of self-focusing of laser beam. In 1975, Hora, introduced the concept of the relativistic self-focusing effect [10]. As a high-intensity laser beam passes through plasma, it causes the electron to oscillate. This motion alters the plasma's refractive index by acting like a positive lens (converging lens) and reducing the beam width [11, 12]. As the beam continues its path through the plasma, the reduction in beam width leads to an increase in spatial diffraction. Eventually, at a certain point where the beam width reaches its minimum, the defocusing effect caused by diffraction becomes stronger which causes the beam width to increase again. This interplay between focusing and defocusing creates oscillations in the beam width. Numerous researchers have dedicated their efforts to investigating the behaviour of laser beams in plasma under various conditions [13–16]. However, it is worth to acknowledge that quantum effects become noticeable in plasmas characterized by low temperatures and high densities. Specifically, when the plasma temperature is at or below the Fermi electron temperature, denoted as $T_f = \hbar^2 (3\pi^2 n_e)^{2/3} / 2m_e K_B$, where n_e represents the electron plasma density, m_e represents the electron's rest mass, K_B

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RESEARCH ARTICLE

The effect of density ramp on self-focusing of q-Gaussian laser beam in magnetized plasma

Anees Akber Butt[1] · Danish Nazir[1] · Niti Kant[2] · Vinay Sharma[1] · Vishal Thakur[1]

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Abstract This study investigates the impact of a density ramp on the self-focusing of q-Gaussian laser beam (q-GLB) in magnetized plasma. Utilizing the q-Gaussian distribution to capture non-Gaussian features, we derive the nonlinear differential equation governing beam width parameter evolution under Wentzel–Kramers–Brillouin (WKB) and paraxial approximations. Our findings reveal that the density ramp significantly influences laser beam propagation which results in self-trapping. However, the action is more pronounced with higher ramp parameters, showing the importance of the ramp in dynamics of the system. Furthermore, non-Gaussian properties result in the formation of multiple filaments, with the quantity of these filaments determined by the q-parameter. Density ramp combined with magnetic fields enhances self-focusing and self-trapping, yielding more efficient laser-to-plasma energy transfer. These findings have potential applications in laser-plasma-based technologies, such as particle acceleration and inertial confinement fusion.

Keywords Self-focusing · Magnetized plasma · Density ramp · Optical energy · Energy efficiency · WKB approximation

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Introduction

The evolution of laser technology, in conjunction with the chirped pulse amplification method, has made possible the fabrication of high power lasers that can produce extremely intense (about 10^{21}–$10^{22}\,\mathrm{W/cm^2}$) focused beams [1–3]. These beams generate electric fields that are much stronger than those produced by traditional accelerators, and when subatomic particles are subjected to these fields (of the order of $10^{12}\,\mathrm{V/cm}$), their dynamics become relativistic during the laser period. As a result, researchers have gained a better understanding of the behavior of particles in relativistic conditions, through both experimental [4] and theoretical studies [5–8]. This has led to the design and development of laser-driven particle accelerators, which are at the forefront of contemporary research in this field. Overall, these advances in laser technology and particle acceleration have opened up exciting new possibilities for exploring the behavior of particles under extreme conditions. Over the past several decades, substantial research has concentrated on laser-plasma interactions because of their prospective uses in various domains, including particle acceleration, high harmonic generation [9], X-ray [10] and gamma-ray generation, and fusion energy research [11, 12]. One of the fundamental phenomena associated with laser-plasma interactions is the nonlinear phenomena of laser beams in plasma. Because of the nonlinear behavior of the nonlinear medium in the presence of a strong beam of light, self-focusing along with self-trapping occur, leading in a balance between self-focusing and diffraction, culminating to stable movement of the high intense laser beam. In recent years, the use of q-Gaussian laser beams in plasma has gained considerable attention due to their unique properties, such as non-Gaussian beam profile, self-similar propagation, and robustness against beam breakup. Their use in combination

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CONFERENCES ATTENDED AND PAPERS PRESENTED

1. International Conference on Recent Advances in Fundamental and Applied Sciences (RAFAS-2021) held on June 25-26, 2021, Organized by school of chemical engineering and physical sciences, Lovely professional university, Punjab.
2. National level webinar on career opportunities in physics organized by department of physics mLAC, Bangalore on 3rd July 2021.
3. Two Days National Conference on Chandra's Contribution in Plasma Astrophysics (October 19-20, 2021) School of Physical Sciences, JNU, New Delhi.
4. International conference on commerce, Management & Interdisciplinary subjects (ICCMIS) 28-29 October 2021.
5. International conference on mathematical sciences and its recent Advancements (ICMSRA-2022) held from May 5 to 7,2022 organised by department of mathematics, Rathinam college of Arts and science, Coimbatore-21.
6. International Conclave on Materials, Energy & Climate, 12 December to 14 December 2022 Organized by Indira Gandhi Delhi Technical University for Women, Delhi, India

   (Delivered **Oral On "**Self-focusing of q-Gaussian laser beam under the influence of axial magnetic field in a nonlinear medium")
7. International Conference on Recent Advances in Fundamental and Applied Sciences RAFAS-2021) held on March 24-25, 2023, Organized by school of chemical engineering and physical sciences, Lovely professional university, Punjab.